AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

ICE-EM MATHEMATICS 9

THIRD EDITION

Peter Brown
Michael Evans
Garth Gaudry
David Hunt
Robert McLaren
Bill Pender
Brian Woolacott

INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

First published 2017
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5

Cover designed by Loupe Studio
Typeset by diacriTech
Printed in China by C & C Offset Printing Co. Ltd.

A Catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-108-40432-7 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

Cambridge University Press acknowledges the Australian Aboriginal and Torres Strait Islander peoples of this nation. We acknowledge the traditional custodians of the lands on which our company is located and where we conduct our business. We pay our respects to ancestors and Elders, past and present. Cambridge University Press is committed to honouring Australian Aboriginal and Torres Strait Islander peoples' unique cultural and spiritual relationships to the land, waters and seas and their rich contribution to society.

Contents

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Preface

ICE-EM Mathematics Third Edition is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Curriculum and its state variations.

The program and textbooks were developed in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. In 2012, AMSI and Cambridge University Press collaborated to publish the Second Edition of the series to coincide with the introduction of the Australian Curriculum, and we now bring you the Third Edition.

The series

ICE-EM Mathematics Third Edition provides a progressive development from upper primary to middle secondary school. The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

For the Third Edition, the series has been carefully edited to present the content in a more streamlined way without compromising quality. There is now one book per year level and the flow of topics from chapter to chapter and from one year level to the next has been improved.

The year 10 textbook incorporates all material for the 10A course, and selected topics in earlier books carefully prepare students for this. *ICE-EM Mathematics Third Edition* provides excellent preparation for all of the Australian Curriculum's year 11 and 12 mathematics courses.

For the Third Edition, *ICE-EM Mathematics* now comes with an Interactive Textbook: a cutting-edge digital resource where all textbook material can be answered online (with students' working-out), additional quizzes and features are included at no extra cost. See 'The Interactive Textbook and Online Teaching Suite' on page xiii for more information.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Author biographies

Peter Brown

Peter Brown studied Pure Mathematics and Ancient Greek at Newcastle University, and completed postgraduate degrees in each subject at the University of Sydney. He worked for nine years as a mathematics teacher in NSW state schools. Since 1990, he has taught Pure Mathematics at the School of Mathematics and Statistics at the University of New South Wales (UNSW). He was appointed Director of First Year Studies at UNSW from 2011 to 2015. He specialises in Number Theory and History of Mathematics and has published in both areas. Peter regularly speaks at teacher inservices, Talented Student days and Mathematics Olympiad Camps. In 2008 he received a UNSW Vice Chancellor's Teaching Award for educational leadership.

Michael Evans

Michael Evans has a PhD in Mathematics from Monash University and a Diploma of Education from La Trobe University. He currently holds the honorary position of Senior Consultant at the Australian Mathematical Sciences Institute at the University of Melbourne. He was Head of Mathematics at Scotch College, Melbourne and has also taught in public schools, returning to classroom teaching in recent years. He has been very involved with curriculum development at both state and national levels. In 1999, Michael was awarded an honorary Doctor of Laws by Monash University for his contribution to mathematics education, and in 2001 he received the Bernhard Neumann Award for contributions to mathematics enrichment in Australia.

Garth Gaudry

Garth Gaudry was Head of Mathematics at Flinders University before moving to UNSW, where he became Head of School. He was the inaugural Director of AMSI before he became the Director of AMSI's International Centre of Excellence for Education in Mathematics. Previous positions include membership of the South Australian Mathematics Subject Committee and the Eltis Committee appointed by the NSW Government to enquire into Outcomes and Profiles. He was a life member of the Australian Mathematical Society and Emeritus Professor of Mathematics, UNSW.

David Hunt

David Hunt graduated from the University of Sydney in 1967 with an Honours degree in Mathematics and Physics. He then obtained a master's degree and a doctorate from the University of Warwick. He was appointed to a lectureship in Pure Mathematics at UNSW in early 1971, where he is currently an honorary Associate Professor. David has taught courses in Pure Mathematics from first year to Master's level and was Director of First Year Studies in Mathematics for five years. Many of David's activities outside UNSW have centred on the Australian Mathematics Trust. In 2016 David was awarded the Paul Erdos medal, in recognition of his contributions to education, as well as his work with the International Mathematical Olympiad movement.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Robert McLaren

Robert McLaren graduated from the University of Melbourne in 1978 with a Bachelor of Science (Hons) and a Diploma of Education. He commenced his teaching career in 1979 at the Geelong College and has taught at a number of Victorian Independent Schools throughout his career. He has been involved in textbook writing, curriculum development and VCE examination setting and marking during his teaching life. He has taught mathematics at all secondary levels and has a particular interest in problem-solving. Robert is currently Vice Principal at Scotch College in Melbourne.

Bill Pender

Bill Pender has a PhD in Pure Mathematics from Sydney University and a BA (Hons) in Early English from Macquarie University. After a year at Bonn University, he taught at Sydney Grammar School from 1975 to 2008, where he was Subject Master for many years. He has been involved in the development of NSW Mathematics syllabuses since the early 1990s, and was a foundation member of the Education Advisory Committee of AMSI. He has also lectured and tutored at Sydney University and at UNSW, and given various inservice courses. Bill is the lead author of the NSW calculus series *Cambridge Mathematics*.

Brian Woolacott

Brian Woolacott graduated from the University of Melbourne in 1978 with a Bachelor of Science and a Diploma of Education. In 1979 he started his teaching career at Scotch College, Melbourne, and during his career has taught at all secondary levels. For 13 years Brian was the Co-ordinator of Mathematics for Years 9 and 10, co-authoring a number of textbooks for the Year 9 and 10 levels during this time. Brian is currently the Dean of Studies at Scotch College.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Acknowledgements

We are grateful to Professor Peter Taylor OA, former Director of the Australian Mathematical Trust, for his support and guidance as chairman of the Australian Mathematical Sciences Institute Education Advisory Committee, 2003-2011.

We gratefully acknowledge the major contributions made by those schools that participated in the Pilot Program during the development of the *ICE-EM Mathematics* program.

We also gratefully acknowledge the assistance of:

Sue Avery
Robin Bailey
Claire Ho
Nikolas Sakellaropolous
James Wan

We also gratefully acknowledge the hard work and professionalism of Mark Jelinek in the development and editing of this Third Edition.

The author and publisher wish to thank the following sources for permission to reproduce material:

Images: © Getty images / miniature, **p.250** / Wavebreakmedia Ltd, **p.254**.

Every effort has been made to trace and acknowledge copyright. The publisher apologises for any accidental infringement and welcomes information that would redress this situation.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

How to use this resource

The textbook

Each chapter in the textbook addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply Understanding, Fluency, Problem-solving and Reasoning skills in every exercise.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

Problem-solving lies at the heart of mathematics, so *ICE-EM Mathematics* gives students a variety of different types of problems to work on, which help them develop their reasoning skills. Challenge exercises at the end of each chapter contain problems and investigations of varying difficulty that should catch the imagination and interest of students. Further, two 'Review and Problem-solving' chapters in each 7–10 textbook contain additional problems that cover new concepts for students who wish to explore the subject even further.

The Interactive Textbook and Online Teaching Suite

Included with the purchase of the textbook is the Interactive Textbook. This is the online version of the textbook and is accessed using the 16-character code on the inside cover of this book.

The Online Teaching Suite is the teacher version of the Interactive Textbook and contains all the support material for the series, including tests, worksheets, skillsheets, curriculum documentation and more.

For more information on the Interactive Textbook and Online Teaching Suite, see page xiii.

The Interactive Textbook and Online Teaching Suite are delivered on the *Cambridge HOTmaths* platform, providing access to a world-class Learning Management System for testing, task management and reporting. They do not provide access to the *Cambridge HOTmaths* stand-alone resource that you or your school may have used previously. For more information on this resource, contact Cambridge University Press.

AMSI's TIMES and SAM modules

The TIMES and SAM web resources were developed by the *ICE-EM Mathematics* author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your *ICE-EM Mathematics* book and are available to teachers and students via the AMSI icon on the dashboard of the Interactive Textbook and Online Teaching Suite.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

The Interactive Textbook and Online Teaching Suite

Interactive Textbook

The Interactive Textbook is the online version of the print textbook and comes included with purchase of the print textbook. It is accessed by first activating the code on the inside cover. It is easy to navigate and is a valuable accompaniment to the print textbook.

Students can show their working

All textbook questions can be answered online within the Interactive Textbook. Students can show their working for each question using either the Draw tool for handwriting (if they are using a device with a touch-screen), the Type tool for using their keyboard in conjunction with the pop-up symbol palette, or by importing a file using the Import tool.

Once a student has completed an exercise they can save their work and submit it to the teacher, who can then view the student's working and give feedback to the student, as they see appropriate.

Auto-marked quizzes

The Interactive Textbook also contains material not included in the textbook, such as a short auto-marked quiz for each section. The quiz contains 10 questions which increase in difficulty from question 1 to 10 and cover all proficiency strands. There is also space for the student to do their working underneath each quiz question. The auto-marked quizzes are a great way for students to track their progress through the course.

Additional material for Year 5 and 6

For Years 5 and 6, the end-of-chapter Challenge activities as well as a set of Blackline Masters are now located in the Interactive Textbook. These can be found in the 'More resources' section, accessed via the Dashboard, and can then easily be downloaded and printed.

Online Teaching Suite

The Online Teaching Suite is the teacher's version of the Interactive Textbook. Much more than a 'Teacher Edition', the Online Teaching Suite features the following:

- The ability to view students' working and give feedback - When a student has submitted their work online for an exercise, the teacher can view the student's work and can give feedback on each question.
- For Years 5 and 6, access to Chapter tests, Blackline Masters, Challenge exercises, curriculum support material, and more.
- For Years 7 to 10, access to Pre-tests, Chapter tests, Skillsheets, Homework sheets, curriculum support material, and more.
- A Learning Management System that combines task-management tools, a powerful test generator, and comprehensive student and whole-class reporting tools.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

1

Number and Algebra

Algebra

Algebra is the language of mathematics. The development of algebra revolutionised mathematics. It is used by engineers, architects, applied scientists and economists to solve practical problems. Every time you see a formula, algebra is being used.

We are now going to review and consolidate our knowledge of algebra so that we will be able to use it in more advanced work.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

1A Substitution

Substitution occurs when a pronumeral is replaced by a numerical value. This allows the term or expression to be evaluated.

When evaluating an expression, it is important to remember the order in which operations are to be applied.

Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - powers
  - multiplication and division from left to right
  - addition and subtraction from left to right.

Example 1

a Evaluate $2x$ when $x = 3$.

b Evaluate $5a + 2b$ when $a = 2$ and $b = -3$.

c Evaluate $2p(3p - q)$ when $p = 1$ and $q = -2$.

Solution

a $$\begin{aligned} 2x &= 2 \times 3 \\ &= 6 \end{aligned}$$

b $$\begin{aligned} 5a + 2b &= 5 \times 2 + 2 \times (-3) \\ &= 10 - 6 \\ &= 4 \end{aligned}$$

c $$\begin{aligned} 2p(3p - q) &= 2 \times 1 \times [3 \times 1 - (-2)] \\ &= 2 \times 5 \\ &= 10 \end{aligned}$$

Exercise 1A

Example 1a

1 Evaluate $3a$ when $a = 7$.

2 Evaluate $12x$ when $x = -5$.

Example 1b

3 Evaluate $7m - 4n$ when:

a $m = 3$ and $n = 5$

b $m = 2$ and $n = -5$

c $m = -1$ and $n = 2$

d $m = -3$ and $n = -2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Evaluate $a + 2b - 3c$ when:

a $a = 3, b = 5$ and $c = 2$

b $a = 3, b = -5$ and $c = 2$

c $a = -3, b = 5$ and $c = 2$

d $a = 3, b = 5$ and $c = -2$

e $a = -3, b = -5$ and $c = 2$

f $a = 3, b = -5$ and $c = -2$

5 Evaluate $2x - 3y$ when:

a $x = \frac{1}{2}$ and $y = \frac{3}{4}$

b $x = -\frac{1}{3}$ and $y = \frac{1}{6}$

c $x = \frac{2}{5}$ and $y = -\frac{1}{4}$

d $x = -\frac{2}{3}$ and $y = -\frac{3}{4}$

6 Evaluate $a + 2b - 3c$ when:

a $a = 0.3, b = 0.5$ and $c = 0.2$

b $a = 1.3, b = -0.5$ and $c = 1.2$

c $a = -2.3, b = 1.5$ and $c = 0.2$

d $a = 2.3, b = 2.5$ and $c = -2.3$

7 Evaluate $p^2 - 2q$ when:

a $p = 3$ and $q = 2$

b $p = 7$ and $q = -2$

c $p = -7$ and $q = 2$

d $p = -7$ and $q = -2$

e $p = -\frac{1}{3}$ and $q = \frac{5}{6}$

f $p = -\frac{2}{5}$ and $q = \frac{1}{4}$

Example 1c

8 Evaluate $2m(m - 3n)$ when:

a $m = 3$ and $n = 5$

b $m = 2$ and $n = -5$

c $m = -1$ and $n = 2$

d $m = -3$ and $n = -2$

e $m = \frac{1}{2}$ and $n = \frac{1}{3}$

f $m = \frac{1}{3}$ and $n = \frac{1}{2}$

9 Evaluate $\frac{x+y}{3}$ when:

a $x = 6$ and $y = 5$

b $x = -6$ and $y = 5$

c $x = 6$ and $y = -5$

d $x = -6$ and $y = -5$

10 Evaluate $\frac{p+2q}{3r}$ when:

a $p = 7, q = 4$ and $r = 2$

b $p = 7, q = -2$ and $r = 2$

c $p = -7, q = 4$ and $r = 4$

d $p = -7, q = -2$ and $r = 2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

1B Like terms

Demitri has three pencil cases, each containing the same number, x, of pencils. So he has a total of $3x$ pencils. If he gets two more pencil cases with x pencils in each, then he has $3x + 2x = 5x$ pencils in total.

$3x$ and $2x$ are said to be **like terms**.

If Lee has x packets of chocolates each containing y chocolates, then she has $x \times y = xy$ chocolates. If David has twice as many chocolates as Lee, he has $2 \times xy = 2xy$ chocolates.

Together they have $2xy + xy = 3xy$ chocolates.

$2xy$ and xy are **like terms**. The pronumerals are the same and have the same indices. The **coefficients** of these terms are 2 and 1.

The terms $2x$ and $3y$ are not like terms.

Similarly, the terms $2x^2y$ and $3xy^2$ are not like terms since the indices of x and y differ.

Example 2

Which of the following are pairs of like terms?

a $3x$ and $2x$

b $3m$ and $2n$

c $3x^2$ and $3x$

d $2x^2y$ and $3yx^2$

e $2mn$ and $3nm$

f 5^2y and $6y^2x$

Solution

a $3x$ and $2x$ are like terms.

b $3m$ and $2n$ are not like terms. (The pronumerals are different.)

c $3x^2$ and $3x$ are not like terms. (The indices of x are different.)

d $2x^2y$ and $3yx^2$ are like terms. (The multiplication order is not important.)

e $2mn$ and $3nm$ are like terms. (The multiplication order is not important.)

f $5x^2y$ and $6y^2x$ are not like terms. (x^2y is not the same as xy^2.)

Like terms

- **Like terms** contain the same pronumerals, with each pronumeral having the same index.
- We can add and subtract like terms.
- Unlike terms cannot be added or subtracted to form a single term.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

Consider the following examples.

Example 3

Simplify each expression, if possible.

a $4a + 7a$ **b** $3x^2y + 4x^2y - 2x^2y$ **c** $5m + 6n$

d $9b + 2c - 3b + 6c$ **e** $3z + 5yx - z - 6xy$ **f** $6x^3 - 4x^2 + 5x^3$

Solution

a $4a + 7a = 11a$

b $3x^2y + 4x^2y - 2x^2y = 5x^2y$

c $5m + 6n$. No simplification is possible because $5m$ and $6n$ are unlike terms.

d $$\begin{aligned} 9b + 2c - 3b + 6c &= 9b - 3b + 2c + 6c \\ &= 6b + 8c \end{aligned}$$

e $$\begin{aligned} 3z + 5yx - z - 6xy &= 3z - z + 5xy - 6xy \\ &= 2z - xy \end{aligned}$$

f $$\begin{aligned} 6x^3 - 4x^2 + 5x^3 &= 6x^3 + 5x^3 - 4x^2 \\ &= 11x^3 - 4x^2 \end{aligned}$$

Example 4

Simplify each expression.

a $\dfrac{x}{2} + \dfrac{x}{3}$ **b** $\dfrac{3x}{4} - \dfrac{2x}{5}$

Solution

a $$\begin{aligned} \frac{x}{2} + \frac{x}{3} &= \frac{3x}{6} + \frac{2x}{6} \\ &= \frac{5x}{6} \end{aligned}$$

b $$\begin{aligned} \frac{3x}{4} - \frac{2x}{5} &= \frac{15x}{20} - \frac{8x}{20} \\ &= \frac{7x}{20} \end{aligned}$$

Exercise 1B

Example 2

1 Which of the following are pairs of like terms?

a $11a$ and $4a$ **b** $6b$ and $-2b$ **c** $12m$ and $5m$

d $14p$ and $5p$ **e** $7p$ and $-3q$ **f** $-6a$ and $7b$

g $4mn$ and $7mn$ **h** $3pq$ and $-2pq$ **i** $6ab$ and $-7b$

j $4ab$ and $5a$ **k** $11a^2$ and $5a^2$ **l** $6x^2$ and $-7x^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

m $4b^2$ and $-3b$ **n** $2y^2$ and $2y$ **o** $-5a^2b$ and $9a^2b$

p $6mn^2$ and $11mn^2$ **q** $6\ell^2s$ and $17\ell s^2$ **r** $12d^2e$ and $14de^2$

s $-4x^2y^2$ and $12x^2y^2$ **t** $-5a^2bc^2$ and $8a^2bc^2$ **u** $6a^2bc$ and $6a^2bc^2$

Example 3a, b

2 Simplify each expression by collecting like terms.

a $7a + 2a$ **b** $8b + 3b$ **c** $11c - 6c$

d $15d - 8d$ **e** $6x^2 + 4x^2$ **f** $9a^2 - 6a^2$

g $4a + 5a - 6a$ **h** $7f - 3f + 9f$ **i** $6m - 4m - 9$

j $8p + 10p - 17p$ **k** $9a^2 + 5a^2 - 12a^2$ **l** $17m^2 - 14m^2 + 8m^2$

m $6ab + 9ab - 12ab$ **n** $14a^2d - 10a^2d - 6a^2d$

3 Copy and complete:

a $2a + \ldots = 7a$ **b** $5b - \ldots = 2b$ **c** $8mn + \ldots = 12mn$

d $11pq - \ldots = 6pq$ **e** $4x^2 + \ldots = 7x^2$ **f** $6m^2 - \ldots = m^2$

g $8ab - \ldots = -2ab$ **h** $5m^2n - \ldots = -6m^2n$ **i** $-7a^2b + \ldots = a^2b$

j $-6\ell m + \ldots = \ell m$

Example 3d, e

4 Simplify each expression by collecting like terms.

a $5c - 2d + 8c + 6d$ **b** $7m + 2n + 5m - n$ **c** $8p + 6 + 3p - 2$

d $9 - 2m + 5 + 7m$ **e** $8m - 6n - 2n - 7m$ **f** $10ab + 11b - 12b + 3ab$

g $6a^2 - 2 + 7ab - 9b$ **h** $-3x^2 + 5x - 2x^2 + 7x$ **i** $4p^2 - 3p - 8p - 3p^2$

Example 3f

5 Simplify each expression by collecting like terms.

a $19xy + 6yx - 4xy$ **b** $8xy^2 + 9xy^2 - y^2x$ **c** $-4x^2 + 3x^2 - 3y - 7y$

d $6v^2z - 11z + 7v^2z - 14z$ **e** $6yz - 11x + 10zy + 15x$ **f** $7x^3 + 6x^2 - 4y^3 - x^2$

g $8x^2 - 12x^2 + y^2 + 12y^2$ **h** $2x^2 + x^2 - 5xy + 7yx$ **i** $-3ab^2 + 4a^2b - 5ab^2 + a^2b$

Example 4a

6 Simplify each expression.

a $\frac{x}{4} + \frac{x}{3}$ **b** $\frac{a}{2} + \frac{a}{5}$ **c** $\frac{c}{6} + \frac{c}{7}$

d $\frac{z}{3} + z$ **e** $\frac{c}{5} + \frac{c}{10}$ **f** $\frac{x}{4} + x$

7 Simplify each expression.

a $\frac{z}{2} - \frac{z}{3}$ **b** $\frac{z}{3} - \frac{z}{5}$ **c** $\frac{x}{7} - \frac{x}{8}$

d $x - \frac{x}{3}$ **e** $\frac{x}{4} - \frac{x}{8}$ **f** $c - \frac{c}{7}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 4b

8 Simplify each expression.

a $\frac{2x}{3}+\frac{x}{4}$ **b** $\frac{5x}{7}+\frac{x}{3}$ **c** $\frac{3x}{4}+\frac{x}{2}$

d $\frac{5x}{3}+\frac{x}{2}$ **e** $\frac{7x}{11}-\frac{x}{2}$ **f** $\frac{2x}{3}-\frac{x}{2}$

g $\frac{5x}{11}-\frac{2x}{3}$ **h** $\frac{7x}{3}+\frac{5x}{4}$ **i** $2x-\frac{7x}{4}$

9 In each part, the expression in each box is obtained by adding the expressions in the two boxes directly below it. Fill in the contents of each box. (The first one has been done for you.)

a

<table>
<tr><td></td><td></td><td colspan="2">$9a+14b$</td><td></td><td></td></tr>
<tr><td></td><td colspan="2">$5a+5b$</td><td colspan="2">$4a+9b$</td><td></td></tr>
<tr><td colspan="2">$2a+b$</td><td colspan="2">$3a+4b$</td><td colspan="2">$a+5b$</td></tr>
</table>

b

<table>
<tr><td></td><td></td><td colspan="2"></td><td></td><td></td></tr>
<tr><td></td><td colspan="2"></td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">$m+3n$</td><td colspan="2">$2m+n$</td><td colspan="2">$4m+3n$</td></tr>
</table>

c

<table>
<tr><td></td><td></td><td colspan="2"></td><td></td><td></td></tr>
<tr><td></td><td colspan="2"></td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">$2p-3q$</td><td colspan="2">$4p+q$</td><td colspan="2">$5p-2q$</td></tr>
</table>

d

<table>
<tr><td></td><td></td><td colspan="2"></td><td></td><td></td></tr>
<tr><td></td><td colspan="2"></td><td colspan="2">$6a+2b$</td><td></td></tr>
<tr><td colspan="2">$3a+2b$</td><td colspan="2"></td><td colspan="2">$4a+b$</td></tr>
</table>

e

<table>
<tr><td></td><td></td><td colspan="2">$11x+2y$</td><td></td><td></td></tr>
<tr><td></td><td colspan="2"></td><td colspan="2">$6x-3y$</td><td></td></tr>
<tr><td colspan="2">$4x-y$</td><td colspan="2"></td><td colspan="2"></td></tr>
</table>

f

<table>
<tr><td></td><td></td><td colspan="2">$7d+e$</td><td></td><td></td></tr>
<tr><td></td><td colspan="2"></td><td colspan="2">$4d-5e$</td><td></td></tr>
<tr><td colspan="2"></td><td colspan="2">$5d+3e$</td><td colspan="2"></td></tr>
</table>

1C Multiplication and division

Any two terms, like or unlike, can be multiplied together to produce a single term. This is different from addition and subtraction, where only like terms can be combined into a single term.

For a single term, it is conventional to use alphabetical order when you simplify your answer.

For example, write $3abc$ instead of $3bca$, and write $4x^2yz$ instead of $4yx^2z$.

Example 5

Simplify each expression.

a $4\times 3a$ **b** $2d\times 5e$ **c** $4m\times 5m$

d $3p\times 2pq$ **e** $3x\times(-6)$ **f** $-5ab\times(-3bc)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

a $4 \times 3a = 12a$ **b** $2d \times 5e = 10de$ **c** $4m \times 5m = 20m^2$

d $3p \times 2pq = 6p^2q$ **e** $3x \times (-6) = -18x$ **f** $-5ab \times (-3bc) = 15ab^2c$

Example 6

Simplify each expression.

a $24x \div 6$ **b** $3 \times 12a \div 4$ **c** $-18x^2 \div (-3)$

Solution

a $24x \div 6 = 4x$

b $3 \times 12a \div 4 = 36a \div 4$
$= 9a$

c $-18x^2 \div (-3) = 6x^2$

Example 7

Simplify each expression.

a $\frac{15a}{3}$ **b** $\frac{12x}{21}$ **c** $\frac{-24xy}{6y}$

Solution

a $\frac{15a}{3} = 5a$ **b** $\frac{12x}{21} = \frac{4x}{7}$ **c** $-\frac{24xy}{6y} = -4x$

Multiplication and division

- To multiply fractions, multiply the numerators and multiply the denominators.
- When the numerator and denominator have a common factor, the factor should be cancelled; that is, the numerator and the denominator should be divided by the common factor.

Example 8

Rewrite each expression as a single fraction.

a $\frac{2a}{5} \times \frac{a}{4}$ **b** $\frac{3x}{7} \times \frac{5y}{12}$ **c** $\frac{4p}{9} \times \frac{3}{2p}$ **d** $\frac{15}{x} \times \frac{2}{3x}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $\dfrac{2a}{5} \times \dfrac{a}{4} = \dfrac{\cancel{2} \times a \times a}{5 \times \cancel{4}_{2}}$ (Multiply the numerators and the denominators.) (Divide 2 into the numerator and the denominator.)

$= \dfrac{a \times a}{5 \times 2}$

$= \dfrac{a^2}{10}$

b $\dfrac{3x}{7} \times \dfrac{5y}{12} = \dfrac{\cancel{3} \times x \times 5 \times y}{7 \times \cancel{12}_{4}}$ (Multiply the numerators and the denominators.)

$= \dfrac{5 \times x \times y}{7 \times 4}$

$= \dfrac{5xy}{28}$

c $\dfrac{4p}{9} \times \dfrac{3}{2p} = \dfrac{{}^{2}\cancel{4} \times \cancel{p} \times \cancel{3}}{{}_{3}\cancel{9} \times \cancel{2} \times \cancel{p}}$

$= \dfrac{2}{3}$

d $\dfrac{15}{x} \times \dfrac{2}{3x} = \dfrac{{}^{5}\cancel{15} \times 2}{x \times \cancel{3} \times x}$

$= \dfrac{5 \times 2}{x \times x}$

$= \dfrac{10}{x^2}$

Recall that to divide by a fraction, we multiply by its reciprocal.

Example 9

Rewrite each expression as a single fraction.

a $\dfrac{2x}{3} \div \dfrac{3x}{5}$

b $\dfrac{6a}{7b} \div \dfrac{2ab}{3}$

Solution

a $\dfrac{2x}{3} \div \dfrac{3x}{5} = \dfrac{2\cancel{x}}{3} \times \dfrac{5}{3\cancel{x}}$

$= \dfrac{2 \times 5}{3 \times 3}$

$= \dfrac{10}{9}$

b $\dfrac{6a}{7b} \div \dfrac{2ab}{3} = \dfrac{{}^{3}\cancel{6}\cancel{a}}{7b} \times \dfrac{3}{\cancel{2}\cancel{a}b}$

$= \dfrac{3 \times 3}{7 \times b \times b}$

$= \dfrac{9}{7b^2}$

Your answer should always be expressed in simplest form – that is, the highest common factor of the numerator and denominator is one.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Exercise 1C

Example 5

1 Simplify:

a $2 \times 3a$ **b** $4b \times 5$ **c** $4a \times 3b$ **d** $5c \times 2d$

e $4f \times (-5g)$ **f** $-3m \times 4n$ **g** $-2p \times (-3q)$ **h** $-6\ell \times (-5n)$

i $a \times a$ **j** $m \times m$ **k** $2a \times 4a$ **l** $-2m \times (-4m)$

m $7a \times 8ab$ **n** $2p \times 3pq$ **o** $-2mn \times 5n$ **p** $-6cd \times (-2de)$

2 Copy and complete:

a $8a \times \ldots = 16a$ **b** $9b \times \ldots = 18b$ **c** $8a \times \ldots = 16ab$

d $5m \times \ldots = 15mn$ **e** $3a \times \ldots = 12a^2$ **f** $6p \times \ldots = 30p^2$

g $-5b \times \ldots = 10b^2$ **h** $-3\ell \times \ldots = 24\ell^2$ **i** $4m \times \ldots = 12m^2n$

j $4d \times \ldots = 28d^2e$ **k** $-3ab \times \ldots = 15ab^2c$ **l** $-2de \times \ldots = 10d^2ef$

Example 6

3 Simplify:

a $15x \div 5$ **b** $27y \div 3$ **c** $24a^2 \div 8$

d $32m \div 16$ **e** $3 \times 12t \div 9$ **f** $7 \times 15p \div 21$

g $21t \div 12 \times 4$ **h** $24x \div 8 \times 3$ **i** $18y \div 6 \times 2$

j $-18x^2 \div 9$ **k** $-16a^2 \div (-4)$ **l** $-24x^2 \div (-8)$

Example 7

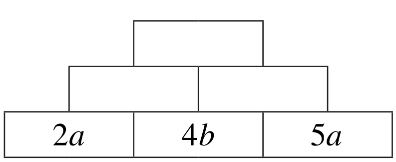
4 Simplify each expression by cancelling common factors.

a $\dfrac{4x}{6}$ **b** $\dfrac{3a}{9}$ **c** $-\dfrac{12m}{18}$ **d** $\dfrac{14p}{21}$ **e** $\dfrac{22x^2}{33}$

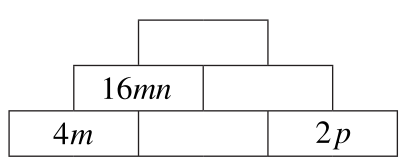
f $\dfrac{15xy}{20}$ **g** $-\dfrac{3ab}{b}$ **h** $\dfrac{12ab}{4a}$ **i** $\dfrac{2xy}{6xy}$ **j** $-\dfrac{4xy}{8x}$

5 In each part, the expression in each box is obtained by multiplying together the terms in the two boxes directly below it. Fill in the empty boxes.

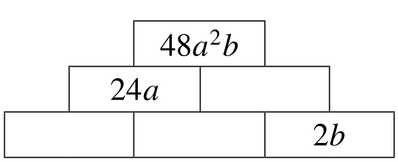
a



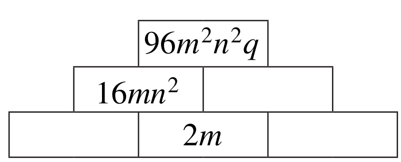
b



c



d



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 8

6 Rewrite each expression as a single fraction. Cancel common factors first.

a $\frac{2b}{3} \times \frac{9}{4}$ **b** $\frac{3x}{5} \times \frac{2}{3}$ **c** $\frac{2y}{5} \times \frac{y}{4}$

d $\frac{3b}{4a} \times \frac{2ab}{9}$ **e** $\frac{2}{5a} \times \frac{1}{4a}$ **f** $\frac{p}{6q} \times \frac{9p}{4q}$

g $\frac{20x}{3y} \times \frac{6}{5xy}$ **h** $\frac{14mp}{9p} \times \frac{3np}{7p}$ **i** $\frac{2yz}{5xz} \times \frac{3xy}{4yz}$

Example 9

7 Rewrite each expression as a single fraction. Simplify your answer by cancelling common factors.

a $\frac{2b}{3} \div \frac{4}{9}$ **b** $\frac{3x}{5} \div \frac{3}{4}$ **c** $\frac{2y}{5} \div \frac{y}{4}$

d $\frac{p}{6} \div \frac{9p}{4}$ **e** $\frac{5}{6a} \div \frac{1}{4a}$ **f** $\frac{p}{6q} \div \frac{4p}{9q}$

g $\frac{8x}{5} \div 4$ **h** $\frac{9y}{2} \div 18$ **i** $\frac{3a}{4} \div \frac{5a}{2}$

j $\frac{4m}{5n} \div \frac{12mn}{7}$ **k** $\frac{5p}{6} \div \left(-\frac{10p}{3}\right)$ **l** $-\frac{9y}{4xy} \div \frac{3x}{16y}$

1D Simple expansion of brackets

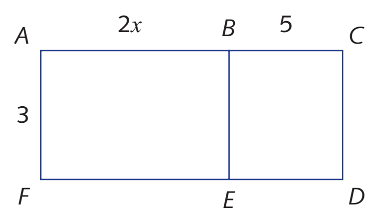
This section looks at how algebraic products that contain brackets can be rewritten in different forms. Removing brackets is called **expanding**.

We are familiar with the distributive law for numbers from Year 7 and Year 8.

The distributive law can be illustrated using a diagram, as shown below.

Area of rectangle $ACDF$ = area of rectangle $ABEF$ + area of rectangle $BCDE$

$$\begin{aligned}\text{That is, } 3(2x+5) &= 3 \times 2x + 3 \times 5 \\ &= 6x + 15\end{aligned}$$

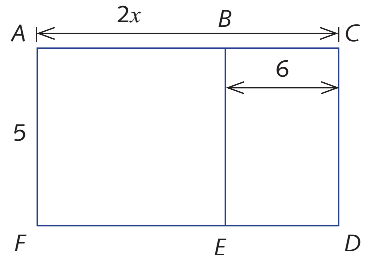


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Area of rectangle $ABEF$ = area of rectangle $ACDF$ − area of rectangle $BCDE$

That is, $5(2x - 6) = 5 \times 2x - 5 \times 6$

$= 10x - 30$



Simple expansion of brackets

- In general: $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
- Each term in the brackets is multiplied by the term outside the brackets.

Example 10

Expand the brackets.

a $2(a + 3)$ **b** $3(x - 2)$ **c** $4(2m - 7)$

d $p(p + n)$ **e** $-2(x - 4)$ **f** $-5(3x + y)$

Solution

a $2(a + 3) = 2a + 6$ **b** $3(x - 2) = 3x - 6$ **c** $4(2m - 7) = 8m - 28$

d $p(p + n) = p^2 + np$ **e** $-2(x - 4) = -2x + 8$ **f** $-5(3x + y) = -15x - 5y$

Example 11

Expand and simplify each expression.

a $5(a + 1) + 6$ **b** $4(2b - 1) + 7$ **c** $6(d + 5) - 3d$

Solution

a $5(a + 1) + 6 = 5a + 5 + 6$
$= 5a + 11$

b $4(2b - 1) + 7 = 8b - 4 + 7$
$= 8b + 3$

c $6(d + 5) - 3d = 6d + 30 - 3d$
$= 3d + 30$

The distributive law can be used to simplify expressions that initially appear quite complicated.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 12

Expand and collect like terms for each expression.

a $2(b+5)+3(b+2)$

b $3(x-2)-2(x+1)$

c $5(a+1)-2(a-4)$

d $3a(2a+3b)+2a(3a-2b)$

Solution

a $2(b+5)+3(b+2) = 2b+10+3b+6$
$= 5b+16$

b $3(x-2)-2(x+1) = 3x-6-2x-2$
$= x-8$

c $5(a+1)-2(a-4) = 5a+5-2a+8$
$= 3a+13$

Note: When expanding the second bracket, be **very careful**. The -2 is multiplied by each term in the second bracket, and $-2\times(-4)=+8$.

d $3a(2a+3b)+2a(3a-2b) = 6a^2+9ab+6a^2-4ab$
$= 12a^2+5ab$

Example 13

Expand and simplify each expression.

a $\frac{3}{5}\left(6x+\frac{7}{3}\right)$

b $\frac{4}{3}(6x+11)+\frac{2}{3}$

Solution

a $\frac{3}{5}\left(6x+\frac{7}{3}\right) = \frac{3}{5}\times 6x+\frac{\cancel{3}^{1}}{5}\times\frac{7}{\cancel{3}_{1}}$
$= \frac{18x}{5}+\frac{7}{5}$

b $\frac{4}{3}(6x+11)+\frac{2}{3} = \frac{4}{3}\times 6x+\frac{4}{3}\times 11+\frac{2}{3}$
$= 8x+\frac{44}{3}+\frac{2}{3}$
$= 8x+\frac{46}{3}$

Note: It is neither necessary nor desirable to write $\frac{46}{3}$ as $15\frac{1}{3}$ in an algebraic expression.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 1D

Example 10a, b, c

1 Expand each expression using the distributive law. For parts **a**, **e**, and **i** check your answers by substituting values for the pronumeral.

a $5(x + 3)$ **b** $2(b + 7)$ **c** $3(2a + 1)$
d $6(2d + 5)$ **e** $4(a - 7)$ **f** $3(b - 5)$
g $6(d - 2)$ **h** $8(f - 4)$ **i** $2(4f - 5)$
j $3(2g - 6)$ **k** $5(3p - 2)$ **l** $6(5q - 1)$

Example 10e, f

2 Expand each expression using the distributive law. For parts **a**, **e**, **i** and **m** check your answers by substituting values for the pronumeral.

a $-2(a + 4)$ **b** $-3(b + 6)$ **c** $-6(2a + 7)$
d $-3(4b + 9)$ **e** $-2(3a - 1)$ **f** $-6(4b - 7)$
g $-5(2b - 7)$ **h** $-7(3b - 2)$ **i** $-4(3b - 5)$
j $-5(4b - 7)$ **k** $-9(3x + 2)$ **l** $-12(4y - 5)$

3 Expand each expression using the distributive law.

a $\frac{1}{2}(2x + 6)$ **b** $\frac{1}{3}(9x + 18)$ **c** $\frac{2}{3}(12p + 6)$
d $\frac{3}{4}(20q + 40)$ **e** $\frac{1}{2}(6x - 24)$ **f** $-\frac{1}{2}(10\ell - 6)$
g $-\frac{1}{4}(16p - 20)$ **h** $-\frac{2}{3}(9m - 15)$ **i** $-\frac{4}{5}(25n - 100)$

4 Expand:

a $\frac{2}{3}\left(6x + \frac{3}{4}\right)$ **b** $\frac{1}{2}\left(5y + \frac{2}{5}\right)$ **c** $\frac{3}{5}\left(\frac{x}{6} + \frac{1}{3}\right)$
d $\frac{2}{7}\left(\frac{3x}{4} + \frac{5}{6}\right)$ **e** $\frac{1}{3}\left(\frac{y}{2} - \frac{3}{4}\right)$ **f** $-\frac{3}{5}\left(\frac{a}{3} - \frac{2}{3}\right)$

Example 10d

5 Expand:

a $a(a + 4)$ **b** $b(b + 7)$ **c** $c(c - 5)$
d $2g(3g - 5)$ **e** $4h(5h - 7)$ **f** $2i(5i + 7)$
g $3j(4j + 7)$ **h** $-k(5k - 4)$ **i** $-\ell(3\ell - 1)$
j $-2m(5m - 4)$ **k** $-3n(5n + 7)$ **l** $-4x(3x - 5)$

6 Expand:

a $3a(2a + b)$ **b** $4c(2c - d)$ **c** $5d(2d - 4e)$
d $2p(3q - 5r)$ **e** $-3x(2x + 5y)$ **f** $-2z(3z - 4y)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

g $2a(3 + 4ab)$ **h** $5m(2m - 4n)$ **i** $5x(2xy + 3)$

j $3p(2 - 5pq)$ **k** $-6y(2x - 3y)$ **l** $-10b(3a - 7b)$

7 A student expanded brackets and obtained the following answers. In each part, identify and correct the student's incorrect answer and write the correct expansion.

a $4(a + 6) = 4a + 6$ **b** $5(a + 1) = 5a + 6$ **c** $-3(p - 5) = -3p - 15$

d $a(a + b) = 2a + ab$ **e** $2m(3m + 5) = 6m + 10m$ **f** $4a(3a + 5) = 7a^2 + 20a$

g $3a(4a - 7) = 12a^2 - 7$ **h** $-6(x - 5) = 6x + 30$ **i** $3x(2x - 7y) = 6x^2 - 21y$

Example 11

8 Expand and collect like terms for each expression.

a $8(a + 2) + 7$ **b** $5(b + 3) + 10$ **c** $2(c + 7) - 9$

d $5(g + 2) + 8g$ **e** $4(h + 1) + 3h$ **f** $6(i - 5) - 3i$

g $2(j - 3) - j$ **h** $2a(4a + 3) + 7a$ **i** $5b(2b - 3) + 6b$

j $2a(4a + 3) + 7a^2$ **k** $3b(3b - 5) - 7b^2$ **l** $5(2 - 4p) + 20p$

m $4(1 - 3q) + 15q$ **n** $2a(3a + 2b) - 6a^2$ **o** $7m(4m - 3n) + 30mn$

9 Expand and collect like terms for each expression. Express the answer as a single fraction.

a $\frac{2}{3}(x + 3) + \frac{x}{6}$ **b** $\frac{1}{4}(x + 2) + \frac{x}{3}$ **c** $\frac{3}{5}(x - 1) + \frac{2x}{3}$

d $\frac{5}{6}(x - 4) + \frac{3x}{4}$ **e** $\frac{3}{7}(3x + 5) + \frac{x}{3}$ **f** $\frac{1}{2}(4x - 1) + \frac{2x}{5}$

g $\frac{3}{4}(2x + 1) - \frac{x}{3}$ **h** $-\frac{1}{2}(3x + 2) - \frac{2x}{5}$ **i** $-\frac{2}{3}(4x - 3) - \frac{x}{4}$

Example 12

10 Expand and collect like terms for each expression.

a $2(y + 1) + 3(y + 4)$ **b** $3(x - 1) + 4(x + 3)$ **c** $2(3b - 2) + 5(2b - 1)$

d $3(a + 5) - 2(a + 7)$ **e** $5(b - 2) - 4(b + 3)$ **f** $2(3y - 4) - 3(2y - 1)$

g $x(x - 2) + 3(x - 2)$ **h** $2p(p + 1) - 5(p + 1)$ **i** $4y(3y - 5) - 3(3y - 5)$

j $2x(2x + 1) - 7(x + 1)$ **k** $2p(3p + 1) - 4(2p + 1)$ **l** $3z(z + 4) - z(3z + 2)$

m $3y(y - 4) + y(y - 4)$ **n** $4z(4z - 2) - z(z + 2)$

11 **a** Copy and complete:

i $12 \times 99 = 12 \times (100 - 1) = 12 \times \ldots - 12 \times \ldots = \ldots - \ldots = \ldots$

ii $14 \times 53 = 14 \times (50 + \ldots) = 14 \times \ldots + 14 \times \ldots = \ldots + \ldots = \ldots$

b Use a similar technique to the one used in part **a** to calculate the value of each product.

i 14×21 **ii** 17×101 **iii** 70×29

iv 8×121 **v** 13×72 **vi** 17×201

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

12 Expand:

a $x(x^2 + 3)$ **b** $x(x^2 + 2x + 1)$ **c** $2x(x^2 - 3x)$

d $2x^2(3 - x)$ **e** $5a(3a + 1)$ **f** $6a^2(1 + 2a - a^2)$

1E Binomial products

In the previous section, we learnt how to expand a product containing one pair of brackets, such as $3(2x - 4)$. Now we are going to learn how to expand a product with two pairs of brackets, such as $(x + 2)(x + 5)$. Such expressions are called **binomial products**.

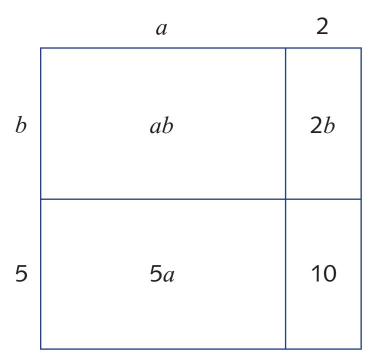
We first look at expanding the expression $(a + 2)(b + 5)$ using the distributive law.

The expression in the second pair of brackets, $(b + 5)$, is multiplied by both a and 2, giving:

$$\begin{aligned}(a + 2)(b + 5) &= a(b + 5) + 2(b + 5)\\ &= ab + 5a + 2b + 10\end{aligned}$$

Each term in the second pair of brackets is multiplied by each term in the first, and the sum of these is taken.

The procedure can be illustrated with the following diagram.



Area of the rectangle = sum of the areas of the smaller rectangles.

That is, $(a + 2)(b + 5) = ab + 5a + 2b + 10$.

Binomial products

- In general:

$$\begin{aligned}(a + b)(c + d) &= a(c + d) + b(c + d)\\ &= ac + ad + bc + bd\end{aligned}$$

- Each term in the second pair of brackets is multiplied by each term in the first, making a total of four applications of multiplication.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 14

Expand $(x + 4)(x + 5)$.

Solution

The expansion can be completed using a table.

	x	4
x	x^2	$4x$
5	$5x$	20

$$\begin{aligned}(x + 4)(x + 5) &= x(x + 5) + 4(x + 5)\\ &= x^2 + 5x + 4x + 20\\ &= x^2 + 9x + 20\end{aligned}$$

The x^2 and $9x$ are unlike terms and cannot be added.

To perform a quick check, substitute $x = 1$:

$$\begin{aligned}(x + 4)(x + 5) &= (1 + 4)(1 + 5)\\ &= 5 \times 6\\ &= 30\end{aligned}$$

and $$\begin{aligned}x^2 + 9x + 20 &= 1^2 + 9 \times 1 + 20a\\ &= 1 + 9 + 20\\ &= 30\end{aligned}$$

Remember that this is not a proof that the expansion is correct. It means that it is probably correct.

With practice you should be able to complete the expansion in one step.

Example 15

Expand and collect like terms for each expression.

a $(x + 3)(x - 2)$

b $(x - 2)(x + 5)$

c $(x - 4)(x - 3)$

d $(2y + 1)(3y - 4)$

Solution

a $$\begin{aligned}(x + 3)(x - 2) &= x^2 - 2x + 3x - 6\\ &= x^2 + x - 6\end{aligned}$$

b $$\begin{aligned}(x - 2)(x + 5) &= x^2 + 5x - 2x - 10\\ &= x^2 + 3x - 10\end{aligned}$$

c $$\begin{aligned}(x - 4)(x - 3) &= x^2 - 3x - 4x + 12\\ &= x^2 - 7x + 12\end{aligned}$$

d $$\begin{aligned}(2y + 1)(3y - 4) &= 6y^2 - 8y + 3y - 4\\ &= 6y^2 - 5y - 4\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 1E

Example 14

1 Expand and collect like terms. For parts **a** and **d**, substitute a value for the pronumeral as a check.

a $(x+3)(x+4)$ **b** $(a+5)(a+8)$ **c** $(a+3)(a+9)$

d $(3+x)(5+x)$ **e** $(2+x)(x+1)$ **f** $(a+8)(9+a)$

Example 15a, b, c

2 Expand and collect like terms. For parts **a** and **d**, substitute a value for the pronumeral as a check.

a $(x-5)(x-1)$ **b** $(x-3)(x-2)$ **c** $(p-6)(p+4)$

d $(x-7)(x+6)$ **e** $(x+11)(x-3)$ **f** $(x+3)(x-8)$

g $(x+6)(x-9)$ **h** $(x-11)(x-7)$ **i** $(x+7)(x-4)$

Example 15d

3 Expand and collect like terms.

a $(2x+3)(3x+4)$ **b** $(3x+2)(5x+4)$ **c** $(5x+1)(x+2)$

d $(2a-5)(a-3)$ **e** $(3a-1)(2b+5)$ **f** $(4m+3)(2m-1)$

g $(2p+5)(3p-2)$ **h** $(5x-2)(3x-8)$ **i** $(2x-7)(3x-1)$

j $(6x-5)(x+2)$ **k** $(7x+9)(x-5)$ **l** $(2b+3)(4b-2)$

4 Expand and collect like terms.

a $(2a+b)(a+3b)$ **b** $(m+3n)(2m+n)$ **c** $(4c+d)(2c-3d)$

d $(4x+5y)(2x-y)$ **e** $(3x-2a)(x+5a)$ **f** $(3x-y)(2x+5y)$

g $(2a+b)(3b-a)$ **h** $(2p+5q)(3q-2p)$ **i** $(2p-5q)(3q-2p)$

5 Write down each expansion, if possible, in one step.

a $(x+2)(x+5)$ **b** $(x+3)(x-7)$ **c** $(x-6)(x-4)$

d $(2x+1)(x+3)$ **e** $(3x+2)(x+5)$ **f** $(4x+1)(3x-1)$

g $(2x-3)(3x-5)$ **h** $(5x-2)(3x+7)$ **i** $(4x+3)(2x-1)$

j $(4x-7)(3x+5)$ **k** $(2x-1)(2x+1)$ **l** $(x-4)(2x+5)$

6 Zak expanded a product using the distributive law but, unfortunately, he erased the terms in the second pair of brackets in each case. Fill in the missing terms in the second pair of brackets.

a $(x+6)(\ldots\ldots) = x^2+7x+6$ **b** $(x+5)(\ldots\ldots) = x^2+8+15$

c $(x+7)(\ldots\ldots) = x^2+5x-14$ **d** $(x+3)(\ldots\ldots) = x^2-2x-15$

e $(2x+1)(\ldots\ldots) = 6x^2-x-2$ **f** $(3x+4)(\ldots\ldots) = 3x^2+x-4$

g $(4x+3)(\ldots\ldots) = 8x^2-22x-21$ **h** $(3x+1)(\ldots\ldots) = 15x^2-13x-6$

7 Fill in the blanks.

a $(x+5)(x+7) = x^2+\ldots x+\ldots$ **b** $(x+\ldots)(x+6) = x^2+9x+\ldots$

c $(x+4)(x-\ldots) = x^2-2x-\ldots$ **d** $(2x+3)(x+\ldots) = 2x^2+7x+\ldots$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

e $(4x - 1)(x - \ldots) = 4x^2 - \ldots x + 3$

f $(\ldots x + 1)(3x - 5) = 6x^2 - \ldots x - \ldots$

g $(\ldots x + \ldots)(2x + 5) = 4x^2 + 12x + \ldots$

h $(\ldots x - 3)(\ldots x + \ldots) = 12x^2 - x - 6$

8 Expand and collect like terms.

a $(x + 3)(x - 3)$ **b** $(x + 3)(x + 3)$ **c** $(x - 5)(x - 5)$

d $(7x - 1)(7x + 1)$ **e** $(2x - 5)(x + 3)$ **f** $(7x + 1)(7x + 1)$

g $(2 - x)(x + 2)$ **h** $(2x + 3)(2x + 3)$ **i** $(5a - 1)(5a + 1)$

9 Expand and collect like terms.

a $\left(\frac{a}{2} + 2\right)\left(\frac{a}{3} + 1\right)$ **b** $\left(\frac{2b}{3} + 2\right)\left(\frac{b}{5} - 2\right)$ **c** $\left(\frac{2x}{5} + \frac{1}{2}\right)\left(\frac{x}{5} - 2\right)$

d $\left(\frac{y}{4} + 3\right)\left(\frac{y}{3} - \frac{3}{4}\right)$ **e** $\left(\frac{3m}{4} + 1\right)\left(\frac{2m}{3} - 3\right)$ **f** $\left(\frac{5b}{4} + \frac{1}{5}\right)\left(\frac{b}{5} - \frac{1}{2}\right)$

1F Perfect squares

A **perfect square** is an expression such as $(x + 3)^2, (x - 5)^2$ or $(2x + 7)^2$. The expansions of these have a special form.

$$\begin{aligned}(x + 3)^2 &= (x + 3)(x + 3)\\ &= x(x + 3) + 3(x + 3)\\ &= x^2 + 3x + 3x + 9\\ &= x^2 + 6x + 9\end{aligned}$$

Note that the form of the answer is $x^2 + \text{twice } (3 \times x) + 3^2$.

$$\begin{aligned}(x - 5)^2 &= (x - 5)(x - 5)\\ &= x^2 - 5x - 5x + 25\\ &= x^2 - 10x + 25\\ &= x^2 + \text{twice}(-5 \times x) + (-5)^2\end{aligned}$$

Another example:

$$\begin{aligned}(3x + 7)^2 &= 3x(3x + 7) + 7(3x + 7)\\ &= 9x^2 + 21x + 21x + 7^2\\ &= 9x^2 + 42x + 49\\ &= (3x)^2 + \text{twice}(7 \times 3x) + 7^2\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Perfect squares

- In general:

$$(a + b)^2 = a^2 + 2ab + b^2$$

- Similarly:

$$(a - b)^2 = a^2 - 2ab + b^2$$

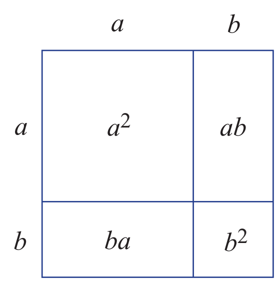
- To expand $(a + b)^2$, take the sum of the squares of the terms and add twice the product of the terms.

The result can be illustrated geometrically.

$$\text{Area of square} = (a + b)^2$$

This is the same as the sum of the

$$\begin{aligned}\text{areas of rectangles} &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$



Example 16

Expand:

a $(x + 3)^2$

b $(x - 6)^2$

Solution

a $\begin{aligned}(x + 3)^2 &= x^2 + 2 \times 3x + 9\\ &= x^2 + 6x + 9\end{aligned}$

b $\begin{aligned}(x - 6)^2 &= x^2 - 2 \times 6x + 36\\ &= x^2 - 12x + 36\end{aligned}$

Example 17

Expand each expression.

a $(2x + 3)^2$

b $(ax - b)^2$

Solution

a $\begin{aligned}(2x + 3)^2 &= (2x)^2 + 2 \times 2x \times 3 + 3^2\\ &= 4x^2 + 12x + 9\end{aligned}$

b $\begin{aligned}(ax - b)^2 &= (ax)^2 - 2 \times ax \times b + b^2\\ &= a^2x^2 - 2abx + b^2\end{aligned}$

The expansion of a perfect square can be performed mentally. With practice you will be able to write down the answer without a middle step.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 1F

Example 16a

1 Expand each expression. Check your answers to parts **a**, **c** and **e** by substitution.

a $(x+1)^2$ **b** $(x+5)^2$ **c** $(x+6)^2$ **d** $(x+20)^2$

e $(a+8)^2$ **f** $(2+x)^2$ **g** $(9+x)^2$ **h** $(x+a)^2$

Example 16b

2 Expand each expression. Check your answers to parts **a**, **c** and **e** by substitution.

a $(x-4)^2$ **b** $(x-7)^2$ **c** $(x-6)^2$ **d** $(x-1)^2$

e $(x-5)^2$ **f** $(x-20)^2$ **g** $(x-11)^2$ **h** $(x-a)^2$

Example 17a

3 Expand each expression.

a $(3x+2)^2$ **b** $(2a+b)^2$ **c** $(2a+3b)^2$ **d** $(3a+4b)^2$

e $(2x+3y)^2$ **f** $(2a+3)^2$ **g** $(3x+a)^2$ **h** $(5x+4y)^2$

Example 17b

4 Expand each expression.

a $(3x-2)^2$ **b** $(4x-3)^2$ **c** $(2a-b)^2$ **d** $(2a-3b)^2$

e $(3a-4b)^2$ **f** $(2x-3y)^2$ **g** $(3c-b)^2$ **h** $(4x-5)^2$

5 Expand:

a $\left(\frac{x}{2}+3\right)^2$ **b** $\left(\frac{x}{3}-2\right)^2$ **c** $\left(\frac{2x}{5}-1\right)^2$ **d** $\left(\frac{3x}{4}+\frac{2}{3}\right)^2$

6 Maria has two pieces of paper. One is $10 \text{ cm} \times 10 \text{ cm}$ and the other is $6 \text{ cm} \times 6 \text{ cm}$.

a Is it possible for these two square pieces of paper to cover a square of size $16 \text{ cm} \times 16 \text{ cm}$ completely?

b How does the answer to part **a** show that $(10+6)^2 \neq 10^2 + 6^2$?

c How much more paper is needed to cover the $16 \text{ cm} \times 16 \text{ cm}$ square?

d How does your answer to part **c** reinforce the result that

$$(10+6)^2 = 10^2 + 2 \times 10 \times 6 + 6^2\,?$$

7 Tom sees the following written on the board in a mathematics classroom:

'Now, $6 + 4 = 10$

Squaring both sides gives

$$(6+4)^2 = 10^2$$

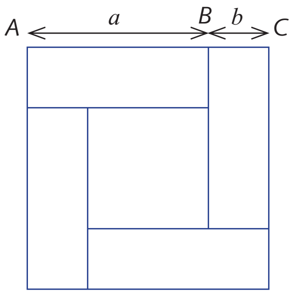
so, $6^2 + 4^2 = 10^2$

That is, $36 + 16 = 100$'

Tom, realising that $36 + 16 \neq 100$, tells the teacher there is a mistake. Can you explain what mistake had been made?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

8 The figure opposite is made from rectangles and a square, with $AB = a$ and $BC = b$.



a Explain how the diagram geometrically proves that $(a - b)^2 + 4ab = (a + b)^2$.

b By expanding, show algebraically that $(a - b)^2 + 4ab = (a + b)^2$.

9 Evaluate the following squares, using the method shown in the example below. (You may be able to do these mentally.)

$$\begin{aligned} 21^2 &= (20 + 1)^2 \\ &= 20^2 + 2 \times 20 \times 1 + 1^2 \\ &= 400 + 40 + 1 \\ &= 441 \end{aligned}$$

a 31^2 **b** 19^2 **c** 42^2 **d** 18^2 **e** 51^2

f 101^2 **g** 99^2 **h** 201^2 **i** 301^2 **j** 199^2

10 Evaluate the following using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$.

a $(1.01)^2$ **b** $(0.99)^2$ **c** $(4.01)^2$ **d** $(4.02)^2$

e $(0.999)^2$ **f** $(1.02)^2$ **g** $(3.01)^2$ **h** $(0.98)^2$

11 Expand and collect like terms.

a $(x - 2)^2 + (x - 4)^2$ **b** $(x - 2)^2 + (x + 2)^2$ **c** $(2x - 3)^2 + (x - 1)^2$

d $(2x + 5)^2 + (2x - 5)^2$ **e** $(x - 4)^2 + (x + 5)^2$ **f** $(2x - 4)^2 + (x - 3)^2$

12 Expand and collect like terms.

a $x^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2$

b $x^2 + (x - 1)^2 + (x - 2)^2 + (x - 3)^2$

c $x^2 - (x - 1)^2 + (x - 2)^2 - (x - 3)^2$

13 Expand:

a $\left(\frac{x}{2} + 1\right)^2 + \left(\frac{x}{2} - 1\right)^2$ **b** $\left(\frac{x}{3} + 3\right)^2 + \left(\frac{2x}{3} + 1\right)^2$

c $\left(\frac{3x}{4} + 2\right)^2 + \left(\frac{x}{2} + 3\right)^2$ **d** $\left(\frac{2x}{5} - \frac{1}{4}\right)^2 + \left(\frac{x}{5} + \frac{1}{2}\right)^2$

14 Identify which of the following is not a perfect square expansion.

a $16x^2 - 8x + 1$ **b** $\frac{9x^2}{4} + 15x + 25$ **c** $4x^2 - 3x + \frac{9}{4}$ **d** $\frac{x^2}{4} + \frac{x}{3} - \frac{1}{9}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

1G Difference of two squares

In this section, we look at a special type of expansion: one that produces the **difference of two squares**.

As an example, consider $(x + 2)(x - 2)$.

$$\begin{aligned}(x + 2)(x - 2) &= x^2 - 2x + 2x - 2^2 \\ &= x^2 - 2^2\end{aligned}$$

Similarly,

$$\begin{aligned}(2x + 3)(2x - 3) &= 4x^2 - 6x + 6x - 3^2 \\ &= (2x)^2 - 3^2\end{aligned}$$

Note that:

- the product $(x + 2)(x - 2)$ is of the form:

  (sum of two terms) × (difference of those terms)

- the answer is of the form:

  (first term) squared – (second term) squared

In general:

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

Example 18

Expand:

a $(x - 5)(x + 5)$

b $(3x - 4)(3x + 4)$

Solution

a Using the result $(a + b)(a - b) = a^2 - b^2$, we get

$(x - 5)(x + 5) = x^2 - 25$

b $$\begin{aligned}(3x - 4)(3x + 4) &= (3x)^2 - 4^2 \\ &= 9x^2 - 16\end{aligned}$$

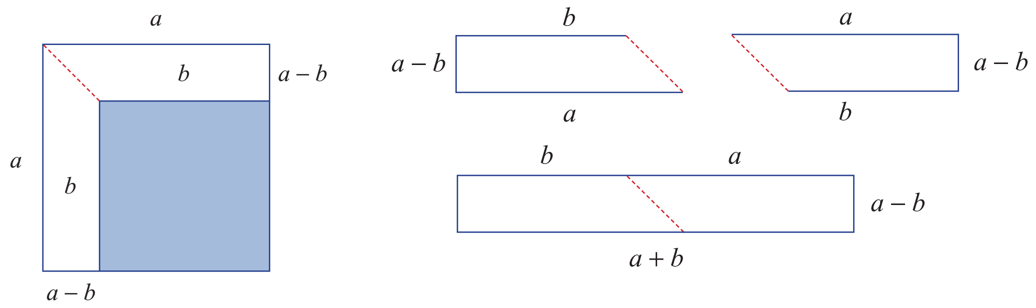
Difference of two squares

In general:

$(a + b)(a - b) = a^2 - b^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

The result is illustrated by the following diagrams.



The diagram on the left shows that the unshaded region has area $a^2 - b^2$.

The diagram on the right, above, shows that the unshaded region has area $(a + b)(a - b)$.

Exercise 1G

Example 18a

1 Expand by using the difference of two squares.

a $(x - 4)(x + 4)$ **b** $(x - 7)(x + 7)$ **c** $(a - 1)(a + 1)$

d $(a - 9)(a + 9)$ **e** $(c - 3)(c + 3)$ **f** $(d - 2)(d + 2)$

g $(z - 7)(7 + z)$ **h** $(10 + x)(10 - x)$ **i** $(x - 5)(x + 5)$

Example 18b

2 Expand:

a $(2x - 1)(2x + 1)$ **b** $(3x - 2)(3x + 2)$ **c** $(4a - 5)(4a + 5)$

d $(3x - 5)(3x + 5)$ **e** $(2x + 7)(2x - 7)$ **f** $(5a + 2)(5a - 2)$

g $(2r - 3s)(2r + 3s)$ **h** $(2x + 3y)(2x - 3y)$ **i** $(5a + 2b)(5a - 2b)$

3 Expand by using the difference of two squares.

a $\left(\frac{2x}{3} + 1\right)\left(\frac{2x}{3} - 1\right)$ **b** $\left(\frac{x}{2} + 3\right)\left(\frac{x}{2} - 3\right)$

c $\left(\frac{x}{3} + \frac{1}{2}\right)\left(\frac{x}{3} - \frac{1}{2}\right)$ **d** $\left(\frac{2x}{5} + \frac{3}{4}\right)\left(\frac{2x}{5} - \frac{3}{4}\right)$

4 The following are either difference of squares expansions or perfect square expansions. Which are which?

a $a^2 - 1$ **b** $a^2 - 2a + 1$ **c** $x^2 - 9$

d $4x^2 - 25$ **e** $4a^2 + 12a + 9$ **f** $9a^2 - 6a + 1$

5 Evaluate the following products, using the method shown in the example below. (You may be able to do these mentally.)

$$\begin{aligned} 51 \times 49 &= (50 + 1)(50 - 1) \\ &= 50^2 - 1^2 \\ &= 2499 \end{aligned}$$

a 21×19 **b** 31×29 **c** 18×22 **d** 32×28

e 17×23 **f** 59×61 **g** 101×99 **h** 102×98

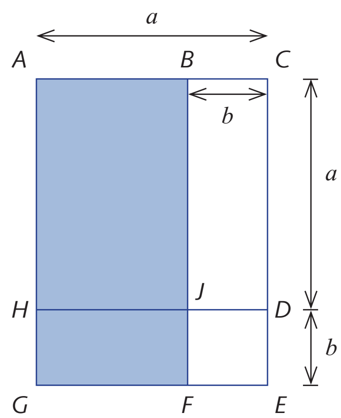
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Evaluate using the difference of two squares identity.

a 1.01×0.99 b 5.01×4.99 c 8.02×7.98

d 1.05×0.95 e 10.01×9.99 f 20.1×19.9

7 Look at the figure opposite.



a What is the length of AB?

b What is the length of AG?

c What is the area of the shaded region?

d Explain why:

$$\text{area } ABFG = \text{area } ACDH + \text{area } HDEG - \text{area } BCDJ - \text{area } JDEF$$

e What has this got to do with the difference of squares expansions?

1H Miscellaneous questions

In this section we present some harder practice questions.

Example 19

Expand and collect like terms.

a $(x+2)^2 - (x+1)^2$ b $(x+6)^2 - (x-6)^2$

Solution

a
$$\begin{aligned}(x+2)^2 - (x-1)^2 &= x^2 + 4x + 4 - (x^2 + 2x + 1)\\ &= 2x + 3\end{aligned}$$

b
$$\begin{aligned}(x+6)^2 - (x-6)^2 &= x^2 + 12x + 36 - (x^2 - 12x + 36)\\ &= 24x\end{aligned}$$

Exercise 1H

Example 19

1 Expand and collect like terms in each case.

a $(x+2)^2 - (x+4)^2$ b $(x+6)^2 - (x+5)^2$

c $(a-4)^2 - (a-4)(a+4)$ d $(a+6)(a-5) - (a+3)(a-2)$

e $(3x+2)(x+1) - (x+2)(x+3)$ f $(2b-5)(3b+2) + (b+5)(b+6)$

g $(a+2b)(2a-b) - (a+b)^2$ h $(2x+y)(y-2x) + (2x-y)^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

2 Evaluate $3m^3 - 2n$ when:

a $m = 2$ and $n = 3$

b $m = -4$ and $n = -5$

c $m = -\frac{1}{2}$ and $n = \frac{3}{4}$

d $m = \frac{1}{3}$ and $n = \sqrt{2}$

3 Evaluate $\dfrac{2x^2 + y}{y^2}$ when:

a $x = 3$ and $y = 4$

b $x = -\frac{1}{2}$ and $y = \frac{3}{2}$

c $x = \sqrt{3}$ and $y = -2$

d $x = \sqrt{6}$ and $y = -4$

4 Evaluate $a + 2b - c$ when:

a $a = \frac{1}{4}$, $b = \frac{1}{2}$ and $c = \frac{3}{8}$

b $a = \frac{b}{3}$, $b = -\frac{1}{4}$ and $c = \frac{5b}{6}$

c $a = \frac{1}{c}$, $b = \frac{3c}{2}$ and $c = \frac{3}{8}$

d $a = c^2$, $b = \frac{1}{c^2}$ and $c = \frac{2}{3}$

5 Evaluate $(x^2 - 2)^2$ when:

a $x = 2$ **b** $x = -2$ **c** $x = \frac{2}{3}$ **d** $x = -\sqrt{2}$ **e** $x = \sqrt{3}$

6 Simplify each expression by collecting like terms.

a $\frac{3}{7}a^2 + \frac{2}{3}a^2$ **b** $\frac{3a}{b} + \frac{5a}{2b}$ **c** $\frac{2x}{y^3} + \frac{7x}{3y^3}$ **d** $\frac{2a^2b}{5} - \frac{2a^2b}{5} + \frac{6m}{n^2}$

7 Copy and complete:

a $\frac{2a^2b}{5} - \ldots = \frac{3a^2b}{10}$

b $\frac{2x^2}{y} - \ldots = \frac{5x^2}{3y}$

c $\frac{3m}{2n^2} + \ldots = \frac{7m}{2n^2}$

d $\frac{p^3}{2n^2} - \ldots = -\frac{p^3}{n^2}$

8 For each value of x, find x^2 and simplify.

a $x = \sqrt{5}$ **b** $x = 3a$ **c** $x = -2b$ **d** $x = a + b$ **e** $x = 3m - 2n$

9 For each value of x, find $x^2 + 3x$ and simplify.

a $x = 3a$ **b** $x = -2b$ **c** $x = a - b$ **d** $x = 2m + n$

10 For each value of x, find $(x - 1)^2$ and simplify.

a $x = 3a$ **b** $x = -2b$ **c** $x = 2a + 1$ **d** $x = -4b + 2$

11 Expand and collect like terms in each case.

a $x(2x^2 + 3x + 4)$

b $3a(a^2 - 4a + 1)$

c $(m + 2)(m^2 + 3m + 2)$

d $(p - 3)(2p^2 + 5p + 3)$

e $(x - 1)(x^2 + x + 1)$

f $(x + 1)(x^2 - x + 1)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

12 If $x = 1, y = 3, z = 5$ and $w = 0$, find the exact value of $\sqrt{3xy} + \sqrt{3xz} + \sqrt{3wy}$.

13 Expand and collect like terms.

a $(x + y - z)(x - y + z)$ **b** $(a - b + c)(a + b - c)$ **c** $(2x - y + z)(2x + y - z)$

d $(x + z + 1)^2$ **e** $(x + y + z)^2$ **f** $(x + y - z)^2$

14 Find $\dfrac{2ab}{a+b}$ if:

a $a = 2$ and $b = 3$ **b** $a = 10$ and $b = 50$ **c** $a = \dfrac{1}{2}$ and $b = \dfrac{1}{3}$

d $a = \dfrac{1}{3}$ and $b = \dfrac{1}{4}$ **e** $a = \dfrac{2}{3}$ and $b = \dfrac{3}{4}$ **f** $a = \dfrac{1}{x}$ and $b = \dfrac{1}{y}$

15 Find $\dfrac{a}{c}$ and $\dfrac{a-b}{b-c}$ if:

a **i** $a = 4, b = 2$ and $c = \dfrac{4}{3}$ **ii** $a = \dfrac{3}{4}$, $b = \dfrac{3}{5}$ and $c = \dfrac{1}{2}$

iii $a = 3$, $b = \dfrac{3}{2}$ and $c = 1$

b In each of the above, show that $\dfrac{1}{c} - \dfrac{1}{b} = \dfrac{1}{b} - \dfrac{1}{a}$.

16 Simplify by first removing brackets.

a $a - (b - c) + a + (b - c) + b - (c + a)$

b $3a - 2b - c - (4a - 3b + 5c)$

c $a - (b + a - (b + a))$

d $-(-(a - b - c))$

e $-(-2x - (3y - (2x - 3y) + (3x - 2y)))$

17 Expand and collect like terms.

a $2[(a + b)(a - b) + (a + c)(a - c) + a(b + c)] + (a - b)^2 + (a - c)^2 + (b - c)^2$

b $(a - b)(a^2 + a + b) + (a + b)(a^2 - a + b)$

c $\left(\dfrac{1}{a} - \dfrac{1}{b}\right)^2 \left(\dfrac{1}{a} + \dfrac{1}{b}\right)^2$

d $\left(a + \dfrac{1}{a}\right)^2 \left(a - \dfrac{1}{a}\right)^2$

e $\left(2a + \dfrac{1}{a}\right)^2 - \left(a - \dfrac{1}{a}\right)^2$

f $\left(a + \dfrac{1}{b}\right)\left(b - \dfrac{1}{a}\right)$

g $(a + b)^2 - [2b(a + b) - (a + b)(b - a)]$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Review exercise

1 Evaluate $8m - 3n$ when:

a $m = 3$ and $n = 5$ **b** $m = 2$ and $n = -5$

c $m = -1$ and $n = 2$ **d** $m = -3$ and $n = -2$

2 Evaluate $2a + 3b - 3c$ when:

a $a = 3, b = 5$ and $c = 2$ **b** $a = 3, b = -5$ and $c = 2$

c $a = -3, b = 5$ and $c = 2$ **d** $a = 3, b = 5$ and $c = -2$

e $a = -3, b = -5$ and $c = 2$ **f** $a = 3, b = -5$ and $c = -2$

3 Evaluate $x^2 - 2x$ when:

a $x = 3$ **b** $x = -2$ **c** $x = -7$

d $x = \frac{2}{3}$ **e** $x = -\frac{2}{3}$ **f** $x = -1.1$

4 Simplify:

a $7a + 2a$ **b** $8b + 3b$ **c** $9a - 6a$

d $7a + 5a - 6a$ **e** $7x - 3x + 9x$ **f** $7a^2 + 3a^2 - 12a^2$

5 Copy and complete:

a $2a + \ldots = 11a$ **b** $5b - \ldots = 3b$ **c** $3mn + \ldots = 16mn$

d $11pq - \ldots = 5pq$ **e** $8ab - \ldots = -5ab$ **f** $3m^2n - \ldots = -8m^2n$

g $-11a^2b + \ldots = 2a^2b$ **h** $-7\ell m + \ldots = 4\ell m$ **i** $14xy^2 - \ldots = -7xy^2$

6 Expand each expression and simplify if possible.

a $3(x + 2)$ **b** $4(b + 6)$ **c** $5(3b - 2)$

d $-6(2d + 5)$ **e** $2(4x - 15)$ **f** $3(4g - 6)$

g $-3(2p + 2)$ **h** $-5(3b - 5)$ **i** $-5(4b - 7)$

j $15(g + 2) - 8g$ **k** $5(h + 1) + 6h$ **l** $16(x - 1) - 13x$

m $2(y - 1) - y$ **n** $5a(2a + 3) - 7a$ **o** $-5b(2b - 3) + 16b$

p $2(3z - 1) - 4(2y - 1)$ **q** $z(z - 1) + 3(z - 2)$ **r** $6y(2y - 5) - 5(2y - 5)$

7 Expand each expression and collect like terms.

a $(x + 2)(x + 4)$ **b** $(a - 11)(a - 4)$ **c** $(a - 5)(a - 5)$

d $(p - 7)(p + 4)$ **e** $(5x + 2)(2x + 2)$ **f** $(2x - 1)(4x - 5)$

g $(4x - 6)(11x + 1)$ **h** $(4x + 3)(2x - 1)$ **i** $(7a - 1)(7a + 3)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 Expand:

a $(x+11)^2$ **b** $(x+6)^2$ **c** $(x-15)^2$ **d** $(x-10)^2$

e $(x-2y)^2$ **f** $(2a+5b)^2$ **g** $(5x+2)^2$ **h** $(5x-6)^2$

9 Expand:

a $(x-6)(x+6)$ **b** $(z-7)(z+7)$ **c** $(p-1)(p+1)$

d $(5x-1)(5x+1)$ **e** $(7x-5)(7x+5)$ **f** $(10-3a)(10+3a)$

g $(5a+2b)(5a-2b)$ **h** $(12x+y)(12x-y)$ **i** $(8x+3a)(8x-3a)$

10 Expand and collect like terms.

a $(3x+y)(2x-y)$ **b** $(3x-4a)(3x+2a)$ **c** $(3c+4b)(5c+4b)$

d $(3x+5y)^2$ **e** $(a-2b)^2$ **f** $(5\ell+2m)^2$

g $(3x-y)(3x+y)$ **h** $(5m+2n)(5\mathrm{m}-2\mathrm{n})$ **i** $(3x+5a)^2$

11 Expand and collect like terms.

a $\left(\frac{a}{2}+1\right)\left(\frac{a}{3}-2\right)$ **b** $\left(\frac{2x}{3}+6\right)(x-4)$ **c** $\left(\frac{a}{2}-1\right)\left(\frac{a}{2}+1\right)$

d $\left(\frac{3x}{5}-1\right)\left(\frac{2x}{5}+3\right)$ **e** $\left(\frac{5b}{6}-3\right)(b+2)$ **f** $(a-6)\left(\frac{a}{3}+4\right)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Challenge exercise

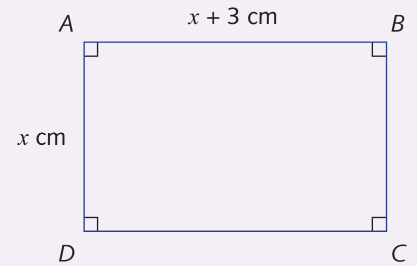
1 **a** Show that the perimeter of the rectangle opposite is $(4x + 6)$ cm.

b Find the perimeter of the rectangle if $AD = 2$ cm.

c Find the value of x if the perimeter is 36 cm.

d Find the area of the rectangle $ABCD$ in terms of x.

e Find the area of the rectangle if $AB = 6$ cm.

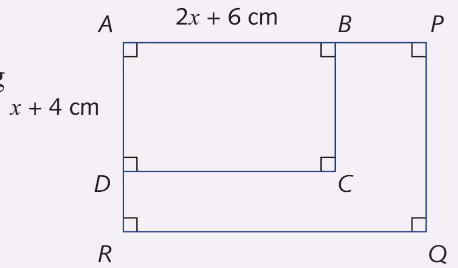


2 Rectangle $ABCD$ is $(x + 4)$ cm wide and $(2x + 6)$ cm long. A new rectangle, $APQR$, is formed by increasing the length of each side of $ABCD$ by 50%.

a Find AP in terms of x.

b Find AR in terms of x.

c Find the area of $APQR$ in terms of x.



d Show that the difference between the areas of the two rectangles is $\frac{5}{2}x^2 + \frac{35}{2}x + 30$.

e What fraction of the area $ABCD$ is the answer in part **d**?

3 We know that $(a + b)(c + d) = ac + ad + bc + bd$.

Expand $(a + b + c)(d + e + f)$. Draw a diagram to illustrate your answer.

4 8, 9, 10 and 11 are four consecutive whole numbers.

a **i** What is the product of the outer pair (8×11)?

ii What is the product of the inner pair (9×10)?

b Find the difference between the answers to parts **i** and **ii**.

c If the smallest of the four consecutive whole numbers is denoted by n, express each of the other numbers in terms of n.

d Express the following in terms of n:

i the product of the outer pair

ii the product of the inner pair

iii the difference between the answers to part **ii** and part **i**.

5 6, 8, 10 and 12 are four consecutive even whole numbers.

a **i** What is the product of the outer pair (6×12)?

ii What is the product of the inner pair (8×10)?

b If the smallest of the four consecutive even numbers is denoted by n, express each of the other numbers in terms of n.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Express the following in terms of n:

i the product of the outer pair

ii the product of the inner pair

iii the difference between the answers to part **ii** and part **i**.

d What happens if you take four consecutive odd whole numbers and complete parts **a, b** and **c** with these numbers?

6 **a** 7, 8 and 9 are consecutive whole numbers. $8^2 - 7 \times 9 = 1$.
Prove that this result holds for any three consecutive numbers $n - 1$, n and $n + 1$.
That is, 'If the product of the outer numbers is subtracted from the square of the middle number, the result is one'.

b $a - d$, a and $a + d$ are three numbers. Show that if the product of the outer pair is subtracted from the square of the middle term, the result is d^2.

7 Expand and collect like terms.

a $(x^2 + x + 1)(x^2 - x + 1)$

b $(x + 1)(x^2 - x + 1)$

c $(x^5 - 1)(x^5 + 1)$

8 Expand and collect like terms.

a $(x - 1)(x^2 + x + 1)$

b $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

c What do you expect the result of expanding $(x - 1)(x^9 + x^8 + \ldots + 1)$ will be?

9 **a** Show that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.

b Hence write $(3^2 + 1^2)(5^2 + 2^2)$ as the sum of two squares.

10 **a** Evaluate each of the following.

i $1 \times 2 \times 3 \times 4 + 1$ **ii** $2 \times 3 \times 4 \times 5 + 1$

iii $3 \times 4 \times 5 \times 6 + 1$ **iv** $4 \times 5 \times 6 \times 7 + 1$

b Expand $(n^2 + n - 1)^2$.

c Show that $(n - 1)n(n + 1)(n + 2) + 1 = (n^2 + n - 1)^2$.

(Note that we have proved that the product of four consecutive integers plus one is a perfect square.)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

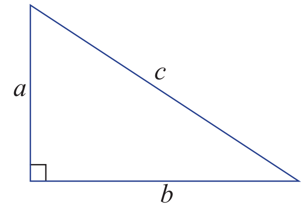
CHAPTER

2

Measurement and Geometry

Pythagoras' theorem and surds

In *ICE-EM Mathematics Year 8*, you learnt about the remarkable relationship between the lengths of the sides of a right-angled triangle. This result is known as **Pythagoras' theorem**.



The theorem states that the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. In symbols:

$$c^2 = a^2 + b^2$$

The converse of this theorem is also true. This means that if we have a triangle in which the square of one side equals the sum of the squares of the other two sides, then the triangle is right-angled, with the longest side being the hypotenuse.

Pythagoras' theorem leads to the discovery of certain irrational numbers, such as $\sqrt{2}$ and $\sqrt{3}$. These numbers are examples of **surds**. In this chapter, we investigate the arithmetic of surds.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2A Review of Pythagoras' theorem and applications

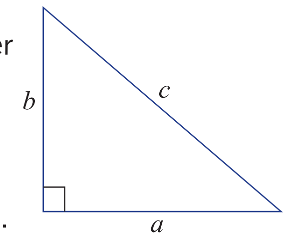
In Year 8, we used Pythagoras' theorem to solve problems related to right-angled triangles.

Pythagoras' theorem and its converse

- In any right-angled triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

  $c^2 = a^2 + b^2$

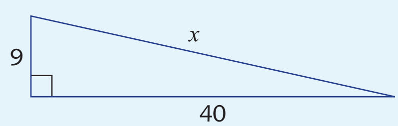
- A triangle with side lengths of a, b, c which satisfy $c^2 = a^2 + b^2$ is a right-angled triangle. The right angle is opposite the side of length c.



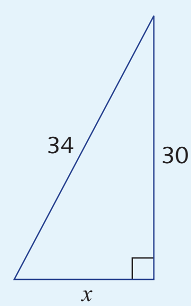
Example 1

Find the value of the unknown side in each triangle.

a



b



Solution

a
$$\begin{aligned} x^2 &= 9^2 + 40^2 \\ &= 1681 \\ x &= \sqrt{1681} \\ &= 41 \end{aligned}$$

b
$$\begin{aligned} x^2 + 30^2 &= 34^2 \\ x^2 &= 34^2 - 30^2 \\ &= 256 \\ x &= \sqrt{256} \\ &= 16 \end{aligned}$$

Calculators and rounding

In Example 1 above, the values for x^2 are perfect squares and so we could take the square root easily. This is not always the case.

In Year 8, we found the approximate square roots of numbers that are not perfect squares by looking up a table of square roots.

Instead of doing this, from now on we are going to use a calculator.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

When using a calculator to find a square root, try to have in mind a rough idea of what the answer should be. For example, $\sqrt{130}$ should be close to 11, since $11^2 = 12$ and $12^2 = 144$.

Note: $\sqrt{130} \approx 11.402$ correct to 3 decimal places.

A calculator gives the approximate value of a square root to a large number of decimal places, far more than we need. We often round off a decimal to a required number of decimal places.

The method for rounding to 2 decimal places is as follows.

- Look at the digit in the third decimal place.
- If the digit is less than 5, take the two digits to the right of the decimal point. For example, 1.764 becomes 1.76, correct to 2 decimal places.
- If the digit is more than 4, take the two digits to the right of the decimal point and increase the second of these by one. For example, 2.455 becomes 2.46, correct to 2 decimal places.

Example 2

a A calculator gives $\sqrt{3} \approx 1.732\,050\,808$. State the value of $\sqrt{3}$ correct to 2 decimal places.

b A calculator gives $\sqrt{5} \approx 2.236\,067\,977$. State the value of $\sqrt{5}$ correct to 2 decimal places.

c State the value of 1.697, correct to 2 decimal places.

Solution

a $\sqrt{3} \approx 1.73$, since the digit, 2, in the third decimal place is less than 5.

b $\sqrt{5} \approx 2.24$, since the digit, 6, in the third decimal place is more than 4.

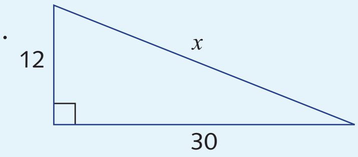
c $1.697 \approx 1.70$, since the digit, 7, in the third decimal place is more than 4 and 69 rounds to 70.

The symbol $\approx$ means ‘is approximately equal to’.

When approximating, we should state the number of decimal places to which we have rounded our answer.

Example 3

Find the length of the unknown side, correct to 2 decimal places.



Solution

$$\begin{aligned} x^2 &= 12^2 + 30^2 \\ &= 1\,044 \\ x &= \sqrt{1\,044} \\ &= 32.31 \text{ (correct to 2 decimal places)} \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

In the example on the previous page, $\sqrt{1044}$ is the exact length while 32.31 is an approximation to the length of the side.

Example 4

Determine whether or not the three side lengths given form the sides of a right-angled triangle.

a 24, 32, 40 **b** 14, 18, 23

Solution

a The square of the length of the longest side $= 40^2$
$= 1600$

The sum of the squares of the lengths of the other sides $= 24^2 + 32^2$
$= 576 + 1024$
$= 1600$

Since $40^2 = 24^2 + 32^2$, the triangle is right-angled.

b The square of the length of the longest side $= 23^2$
$= 529$

The sum of the squares of the lengths of the other sides $= 14^2 + 18^2$
$= 196 + 324$
$= 520$

Since $23^2 \neq 14^2 + 18^2$, the triangle is not right-angled.

Example 5

A door frame has height 1.7 m and width 1 m. Will a square piece of board 2 m by 2 m fit through the doorway?

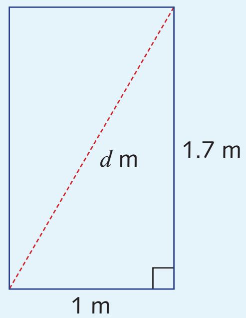
Solution

Let d m be the length of the diagonal of the doorway.

Using Pythagoras' theorem,

$$\begin{aligned} d^2 &= 1^2 + 1.7^2 \\ &= 3.89 \\ d &= \sqrt{3.89} \\ &= 1.97 \text{ (correct to 2 decimal places)} \end{aligned}$$

Hence the board will not fit through the doorway.



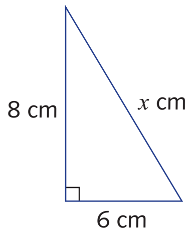
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 2A

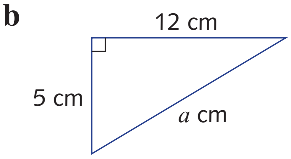
Example 1a

1 Use Pythagoras' theorem to find the value of the pronumeral.

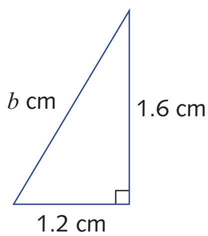
a



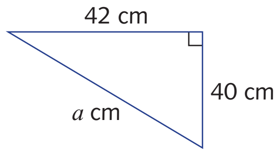
b



c



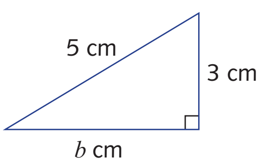
d



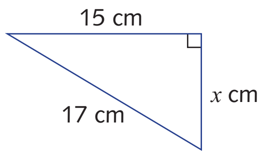
Example 1b

2 Use Pythagoras' theorem to find the value of the pronumeral.

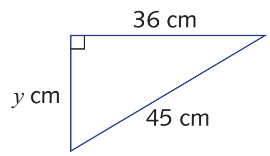
a



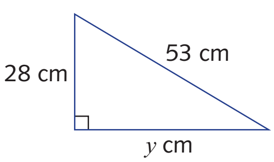
b



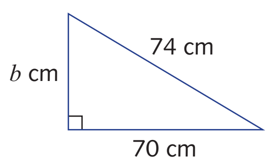
c



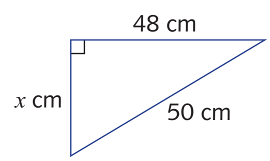
d



e



f



Example 2

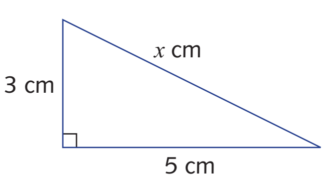
3 Use a calculator to find, correct to 2 decimal places, approximations to these numbers.

a $\sqrt{19}$ **b** $\sqrt{37}$ **c** $\sqrt{61}$ **d** $\sqrt{732}$

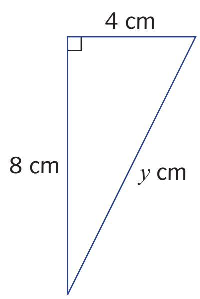
Example 3

4 Use Pythagoras' theorem to find the value of the pronumeral. Calculate your answer first as a square root and then correct to 2 decimal places.

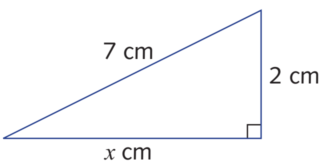
a



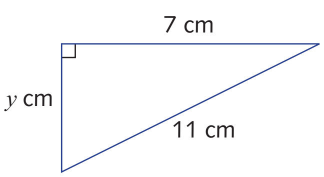
b



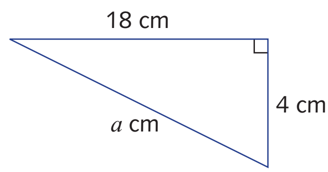
c



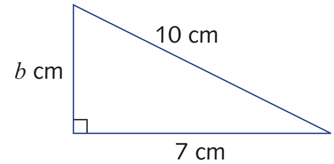
d



e

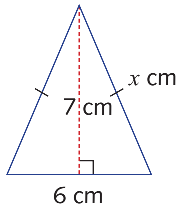


f

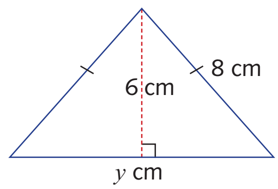


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

g



h



Example 4

5 Determine whether or not the triangle with the three side lengths given is right-angled.

a 16, 30, 34 **b** 10, 24, 26 **c** 4, 6, 7

d 4.5, 7, 7.5 **e** 6, 10, 12 **f** 20, 21, 29

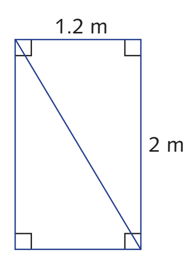
6 In the table below, the side lengths of right-angled triangles are listed. Copy and complete the table, giving answers as a whole number or square root.

	Lengths of the two shortest sides of a right-angled triangle		Length of the hypotenuse
a	3 cm	4 cm	…
b	5 cm	6 cm	…
c	4 cm	…	9 cm
d	6 cm	10 cm	…
e	…	7 cm	10 cm

Example 5

7 A door frame has height 1.8 m and width 1 m. Will a square piece of board 2.1 m wide fit through the opening?

8 A tradesman is making the wooden rectangular frame for a gate. In order to make the frame stronger and to keep it square, the tradesman will put a diagonal piece into the frame as shown in the diagram.



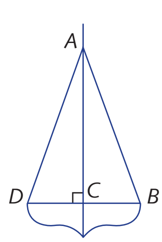
If the frame is 1.2 m wide and 2 m high, find the length of the diagonal piece of wood, in metres, correct to 3 decimal places.

9 A signwriter leans his ladder against a wall so that he can paint a sign. The wall is vertical and the ground in front of the wall is horizontal. The signwriter's ladder is 4 m long.

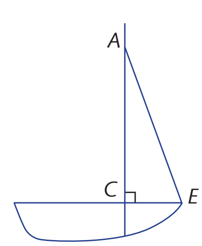
If the signwriter wants the top of the ladder to be 3.8 m above the ground when leaning against the wall, how far, correct to 1 decimal place, should the foot of the ladder be placed from the wall?

10 A boat builder needs to calculate the lengths of the stays needed to support a mast on a yacht. Two of the stays (AB and AD) will be the same length, as they go from a point A on the mast to each side of the boat, as shown in the diagram. The third stay (AE) will be different in length as it goes from the point A on the mast to the front of the boat.

Front elevation



Side elevation



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

If $AC = 5$ m, $CB = CD = 1.2$ m and $CE = 1.9$ m, find the length, to the nearest centimetre, of:

a one of the side stays, AB or AD

b the front stay, AE

c stainless steel wire needed to make the three stays

11 As part of a design, an artist draws a circle passing through the four corners (vertices) of a square.

a If the square has side lengths of 4 cm, what is the radius, to the nearest millimetre, of the circle?

b If the circle has a radius of 3 cm, what are the side lengths, to the nearest millimetre, of the square?

12 A parent is asked to make some scarves for the local Scout troop. Two scarves can be made from one square piece of material by cutting on the diagonal. If this diagonal side length is to be 100 cm long, what must be the side length of the square piece of material to the nearest mm?

13 A girl planned to swim straight across a river of width 25 m. After she had swum across the river, the girl found she had been swept 4 m downstream. How far did she actually swim? Calculate your answer, in metres, correct to 1 decimal place.

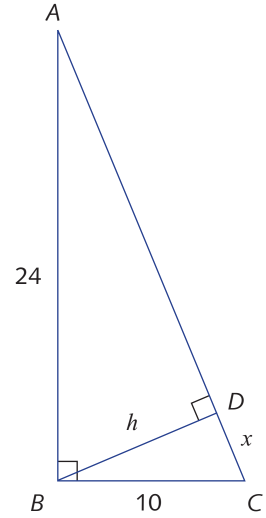
14 A yachtsman wishes to build a shed with a rectangular base to store his sailing equipment. If the shed is to be 2.6 m wide and must be able to house a 4.6 m mast, which is to be stored diagonally across the ceiling, how long must the shed be? Calculate your answer, in metres, correct to 1 decimal place.

15 In triangle ABC the line BD is drawn perpendicular to AC. h is the length of BD and x is the length of CD.

a Show that the length of AC is 26.

b Find the area of triangle ABC in two ways to show that $13h = 120$.

c Use Pythagoras' theorem to find x.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16 In diagrams 1 and 2, we have two squares with the same side length, $a + b$. The sides are divided up into lengths a and b as shown.

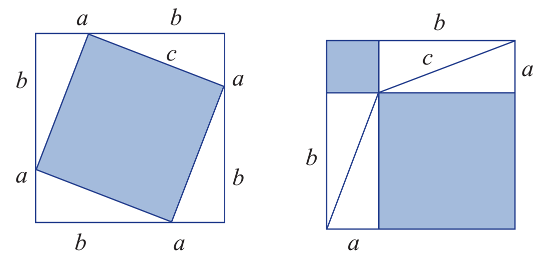


Diagram 1 Diagram 2

a Prove that the shaded part of diagram 1 is a square.

b The two shaded parts of diagram 2 are also squares. Why?

By looking carefully at the two diagrams, show that the area of the shaded square in diagram 1 is the sum of the areas of the shaded squares in diagram 2.

c Use the result of part **b** to prove Pythagoras' theorem.

2B Simplifying surds

This section deals only with surds that are square roots. If a is a positive integer which is not a perfect square then $\sqrt{a}$ is called a **surd**. We will look at surds more generally in Section 2I.

We will now review the basic rules for square roots.

If a and b are positive numbers then:

$$\begin{aligned}(\sqrt{a})^2 &= a\\ \sqrt{a^2} &= a\\ \sqrt{a} \times \sqrt{b} &= \sqrt{ab}\\ \sqrt{a} \div \sqrt{b} &= \sqrt{\frac{a}{b}}\end{aligned}$$

For example:

$$\begin{aligned}(\sqrt{11})^2 &= 11\\ \sqrt{3^2} &= 3\\ \sqrt{3} \times \sqrt{7} &= \sqrt{3 \times 7}\\ &= \sqrt{21}\\ \sqrt{35} \div \sqrt{5} &= \sqrt{35 \div 5}\\ &= \sqrt{7}\end{aligned}$$

The first two of these rules remind us that, for positive numbers, squaring and taking a square root are **inverse processes**.

When we write $2\sqrt{3}$, we mean $2 \times \sqrt{3}$. As in algebra, we usually do not explicitly write the multiplication sign.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 6

Evaluate:

a $(\sqrt{6})^2$

b $(2\sqrt{6})^2$

Solution

a $(\sqrt{6})^2 = \sqrt{6} \times \sqrt{6}$
$= 6$

b $(2\sqrt{6})^2 = 2\sqrt{6} \times 2\sqrt{6}$
$= 2 \times 2 \times \sqrt{6} \times \sqrt{6}$
$= 4 \times 6$
$= 24$

Example 7

Evaluate:

a $\sqrt{3} \times \sqrt{11}$

b $\sqrt{15} \div \sqrt{3}$

c $\dfrac{\sqrt{42}}{\sqrt{6}}$

d $\sqrt{35} \div \sqrt{10}$

Solution

a $\sqrt{3} \times \sqrt{11} = \sqrt{3 \times 11}$
$= \sqrt{33}$

b $\sqrt{15} \div \sqrt{3} = \sqrt{15 \div 3}$
$= \sqrt{5}$

c $\dfrac{\sqrt{42}}{\sqrt{6}} = \sqrt{\dfrac{42}{6}}$
$= \sqrt{7}$

d $\sqrt{35} \div \sqrt{10} = \sqrt{\dfrac{35}{10}}$
$= \sqrt{\dfrac{7}{2}}$

Example 8

Evaluate:

a $3 \times 7\sqrt{5}$

b $9\sqrt{2} \times 4$

Solution

a $3 \times 7\sqrt{5} = 21\sqrt{5}$

b $9\sqrt{2} \times 4 = 36\sqrt{2}$

Consider the surd $\sqrt{12}$. We can factor out the perfect square 4 from 12, and write:

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3} \\ &= \sqrt{4} \times \sqrt{3} \qquad \text{(using } \sqrt{ab} = \sqrt{a} \times \sqrt{b}\text{)} \\ &= 2\sqrt{3}\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Hence $\sqrt{12}$ and $2\sqrt{3}$ are equal. We will regard $2\sqrt{3}$ as a simpler form than $\sqrt{12}$ since the number under the square root sign is smaller.

To **simplify** a surd (or a multiple of a surd) we write it so that the number under the square root sign has no factors that are perfect squares other than 1. For example,

$$\sqrt{12} = 2\sqrt{3}$$

In mathematics, we are often instructed to leave our answers in **surd form**. This simply means that we should not approximate the answer using a calculator, but leave the answer, in simplest form, using square roots, cube roots, etc. This is called giving the **exact value** of the answer.

We can simplify surds directly or in stages.

Example 9

Simplify the following.

a $\sqrt{18}$

b $\sqrt{108}$

Solution

a
$$\begin{aligned}\sqrt{18} &= \sqrt{9 \times 2}\\ &= \sqrt{9} \times \sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

b
$$\begin{aligned}\sqrt{108} &= \sqrt{9 \times 4 \times 3}\\ &= \sqrt{9} \times \sqrt{4} \times \sqrt{3}\\ &= 3 \times 2\sqrt{3}\\ &= 6\sqrt{3}\end{aligned}$$

In some problems, we need to reverse this process.

Example 10

Express as the square root of a whole number.

a $3\sqrt{7}$

b $5\sqrt{3}$

Solution

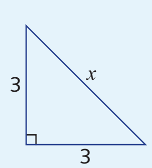
a
$$\begin{aligned}3\sqrt{7} &= \sqrt{9 \times 7}\\ &= \sqrt{63}\end{aligned}$$

b
$$\begin{aligned}5\sqrt{3} &= \sqrt{25 \times 3}\\ &= \sqrt{75}\end{aligned}$$

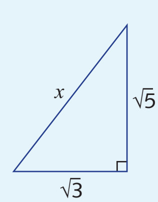
Example 11

Use Pythagoras' theorem to find the value of x. Give your answer as a surd which has been simplified.

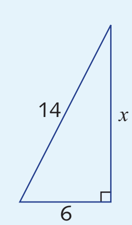
a



b



c



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a
$$\begin{aligned} x^2 &= 3^2 + 3^2 \\ &= 9 + 9 \\ &= 18 \\ x &= \sqrt{18} \\ &= \sqrt{9 \times 2} \\ &= 3\sqrt{2} \end{aligned}$$

b
$$\begin{aligned} x^2 &= (\sqrt{5})^2 + (\sqrt{3})^2 \\ &= 5 + 3 \\ &= 8 \\ x &= \sqrt{8} \\ &= \sqrt{4 \times 2} \\ &= 2\sqrt{2} \end{aligned}$$

c
$$\begin{aligned} x^2 + 6^2 &= 14^2 \\ x^2 + 36 &= 196 \\ x^2 &= 160 \\ x &= \sqrt{160} \\ &= \sqrt{160 \times 10} \\ &= 4\sqrt{10} \end{aligned}$$

The arithmetic of surds

- If a and b are positive numbers, then:

$$\begin{aligned} (\sqrt{a})^2 &= a \\ \sqrt{a^2} &= a \\ \sqrt{a} \times \sqrt{b} &= \sqrt{ab} \\ \sqrt{a} \div \sqrt{b} &= \sqrt{\frac{a}{b}} \end{aligned}$$

- A surd is in its **simplest form** if the number under the square root sign has no factors that are perfect squares, apart from 1.
- To **simplify** a surd, we write it so that the number under the square root sign has no factors that are perfect squares, apart from 1.

  For example, $\sqrt{50} = 5\sqrt{2}$.

Exercise 2B

Example 6a

1 Evaluate:

a $(\sqrt{7})^2$ **b** $(\sqrt{3})^2$ **c** $(\sqrt{11})^2$ **d** $(\sqrt{231})^2$

Example 6b

2 Evaluate:

a $(2\sqrt{3})^2$ **b** $(4\sqrt{3})^2$ **c** $(5\sqrt{2})^2$ **d** $(3\sqrt{5})^2$

e $(2\sqrt{7})^2$ **f** $(7\sqrt{3})^2$ **g** $(11\sqrt{2})^2$ **h** $(2\sqrt{11})^2$

Example 7a

3 Express as a square root of a whole number.

a $\sqrt{3} \times \sqrt{5}$ **b** $\sqrt{2} \times \sqrt{6}$ **c** $\sqrt{5} \times \sqrt{6}$ **d** $\sqrt{11} \times \sqrt{3}$

e $\sqrt{17} \times \sqrt{3}$ **f** $\sqrt{3} \times \sqrt{15}$ **g** $\sqrt{3} \times \sqrt{14}$ **h** $\sqrt{10} \times \sqrt{19}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7b, c, d

4 Express as a square root of a number.

a $\sqrt{6} \div \sqrt{2}$ **b** $\sqrt{35} \div \sqrt{7}$ **c** $\dfrac{\sqrt{39}}{\sqrt{3}}$ **d** $\dfrac{\sqrt{46}}{\sqrt{2}}$

e $\dfrac{\sqrt{77}}{\sqrt{11}}$ **f** $\sqrt{40} \div \sqrt{18}$ **g** $\sqrt{6} \div \sqrt{42}$ **h** $\dfrac{\sqrt{14}}{\sqrt{21}}$

Example 8

5 Evaluate the product.

a $2 \times 3\sqrt{2}$ **b** $6 \times 2\sqrt{5}$ **c** $11 \times 3\sqrt{7}$ **d** $6 \times 5\sqrt{7}$

e $15 \times 3\sqrt{2}$ **f** $7 \times 5\sqrt{6}$ **g** $4\sqrt{11} \times 3$ **h** $7\sqrt{3} \times 4$

6 Evaluate:

a $(\sqrt{2})^3$ **b** $(\sqrt{5})^3$ **c** $(\sqrt{5})^2 - (\sqrt{2})^2$

d $(\sqrt{7})^2 - (\sqrt{3})^2$ **e** $(\sqrt{11})^2 + (\sqrt{2})^2$ **f** $(\sqrt{5})^2 + (\sqrt{11})^2$

g $\sqrt{2} \times \sqrt{18}$ **h** $\sqrt{3} \times \sqrt{12}$ **i** $\sqrt{32} \times \sqrt{2}$

Example 9a

7 Simplify each of these surds.

a $\sqrt{8}$ **b** $\sqrt{12}$ **c** $\sqrt{45}$ **d** $\sqrt{24}$ **e** $\sqrt{27}$ **f** $\sqrt{44}$

g $\sqrt{50}$ **h** $\sqrt{54}$ **i** $\sqrt{20}$ **j** $\sqrt{98}$ **k** $\sqrt{63}$ **l** $\sqrt{60}$

m $\sqrt{126}$ **n** $\sqrt{68}$ **o** $\sqrt{75}$ **p** $\sqrt{99}$ **q** $\sqrt{28}$ **r** $\sqrt{242}$

Example 9b

8 Simplify each of these surds.

a $\sqrt{72}$ **b** $\sqrt{32}$ **c** $\sqrt{80}$ **d** $\sqrt{288}$ **e** $\sqrt{48}$ **f** $\sqrt{180}$

g $\sqrt{112}$ **h** $\sqrt{216}$ **i** $\sqrt{96}$ **j** $\sqrt{252}$ **k** $\sqrt{160}$ **l** $\sqrt{128}$

m $\sqrt{320}$ **n** $\sqrt{176}$ **o** $\sqrt{192}$ **p** $\sqrt{200}$ **q** $\sqrt{162}$ **r** $\sqrt{243}$

Example 10

9 Express each of these surds as the square root of a whole number.

a $2\sqrt{3}$ **b** $6\sqrt{3}$ **c** $7\sqrt{2}$ **d** $3\sqrt{6}$ **e** $4\sqrt{5}$ **f** $5\sqrt{7}$

g $4\sqrt{3}$ **h** $2\sqrt{13}$ **i** $6\sqrt{11}$ **j** $12\sqrt{10}$ **k** $10\sqrt{7}$ **l** $4\sqrt{11}$

10 Evaluate:

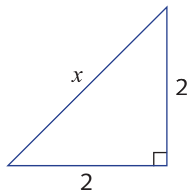
a $\left(\sqrt{\dfrac{2}{3}}\right)^2$ **b** $\sqrt{\dfrac{4}{25}}$ **c** $\sqrt{\dfrac{16}{25}}$ **d** $\sqrt{\dfrac{25}{36}}$

e $\left(\sqrt{\dfrac{5}{11}}\right)^2$ **f** $\sqrt{\dfrac{25}{121}}$ **g** $\left(\sqrt{\dfrac{7}{11}}\right)$ **h** $\sqrt{\dfrac{144}{169}}$

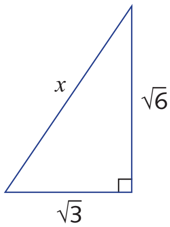
Example 11

11 Use Pythagoras' theorem to find the value of x. Give your answer as a surd which has been simplified.

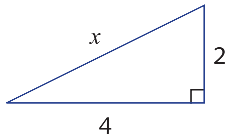
a



b

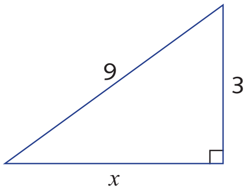


c

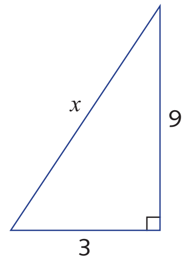


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

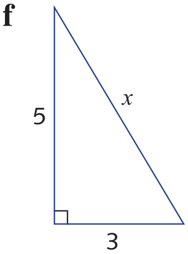
d



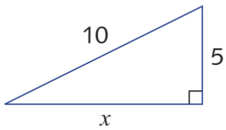
e



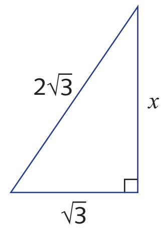
f



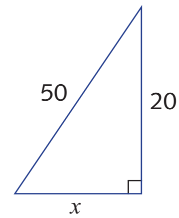
g



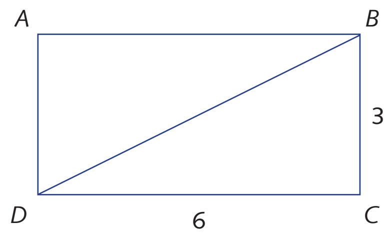
h



i



12 Find the length of the diagonal DB of the rectangle $ABCD$. Express your answer in simplest form.

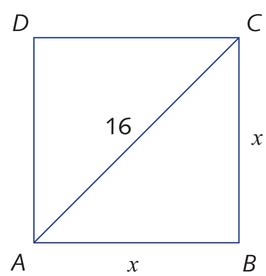


13 A square has side length $2\sqrt{3}$. Find:

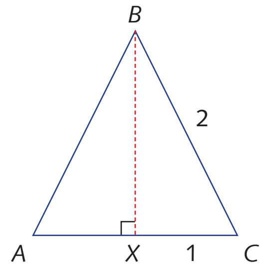
a the area of the square

b the length of the diagonal

14 $ABCD$ is a square. Find the value of x.



15 ABC is an equilateral triangle with side length 2. X is the midpoint of AC. Find the length BX.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2C Addition and subtraction of surds

We can sometimes simplify sums of surds. The sum $4\sqrt{7} + 5\sqrt{7}$ can be thought of as 4 lots of $\sqrt{7}$ plus 5 lots of $\sqrt{7}$ equals 9 lots of $\sqrt{7}$. This is very similar to algebra, where we write $4x + 5x = 9x$. We say $4\sqrt{7}$ and $5\sqrt{7}$ are **like surds** since they are both multiples of $\sqrt{7}$.

On the other hand, in algebra we cannot simplify $4x + 7y$, because $4x$ and $7y$ are not like terms. Similarly, it is not possible to write the sum of $4\sqrt{2}$ and $7\sqrt{3}$ in a simpler way. They are **unlike surds**, since one is a multiple of $\sqrt{2}$ while the other is a multiple of $\sqrt{3}$.

We can only simplify the sum or difference of like surds.

Example 12

Simplify:

a $4\sqrt{7} + 5\sqrt{7}$

b $6\sqrt{7} - 2\sqrt{7}$

c $2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2}$

d $3\sqrt{5} + 2\sqrt{7} - \sqrt{5} + 4\sqrt{7}$

Solution

a $4\sqrt{7} + 5\sqrt{7} = 9\sqrt{7}$

b $6\sqrt{7} - 2\sqrt{7} = 4\sqrt{7}$

c $2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$

d $3\sqrt{5} + 2\sqrt{7} - \sqrt{5} + 4\sqrt{7} = 2\sqrt{5} + 6\sqrt{7}$

This cannot be simplified further since $\sqrt{5}$ and $\sqrt{7}$ are unlike surds.

When dealing with expressions involving surds, we should simplify the surds first and then look for like surds.

Example 13

Simplify:

a $\sqrt{8} + 4\sqrt{2} - \sqrt{18}$

b $\sqrt{27} + 2\sqrt{5} + \sqrt{20} - 2\sqrt{3}$

Solution

a
$$\begin{aligned}\sqrt{8} + 4\sqrt{2} - \sqrt{18} &= 2\sqrt{2} + 4\sqrt{2} - 3\sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

b
$$\begin{aligned}\sqrt{27} + 2\sqrt{5} + \sqrt{20} - 2\sqrt{3} &= 3\sqrt{3} + 2\sqrt{5} + 2\sqrt{5} - 2\sqrt{3}\\ &= \sqrt{3} + 4\sqrt{5}\end{aligned}$$

This expression cannot be simplified further.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 14

Simplify:

a $\dfrac{\sqrt{3}}{2} + \dfrac{3\sqrt{3}}{5}$

b $\dfrac{\sqrt{8}}{3} - \dfrac{\sqrt{2}}{5}$

Solution

a $\dfrac{\sqrt{3}}{2} + \dfrac{3\sqrt{3}}{5} = \dfrac{5\sqrt{3}}{10} + \dfrac{6\sqrt{3}}{10}$ (Use a common denominator.)

$= \dfrac{11\sqrt{3}}{10}$

b $\dfrac{\sqrt{8}}{3} - \dfrac{\sqrt{2}}{5} = \dfrac{2\sqrt{2}}{3} - \dfrac{\sqrt{2}}{5}$ (Simplify the surds.)

$= \dfrac{10\sqrt{2}}{15} - \dfrac{3\sqrt{2}}{15}$ (Use a common denominator.)

$= \dfrac{7\sqrt{2}}{15}$

Addition and subtraction of surds

- Simplify each surd first, then look for like surds.
- We can simplify sums and differences of like surds.
- We cannot simplify sums and differences of unlike surds.

Exercise 2C

Example 12

1 Simplify:

a $\sqrt{5} - \sqrt{5}$ **b** $6\sqrt{2} + 4\sqrt{2}$ **c** $\sqrt{3} + 4\sqrt{3}$

d $8\sqrt{2} - \sqrt{2}$ **e** $10\sqrt{7} - 5\sqrt{7}$ **f** $19\sqrt{5} + 24\sqrt{5}$

g $-13\sqrt{3} + 14\sqrt{3}$ **h** $3\sqrt{11} - 5\sqrt{11}$ **i** $-45\sqrt{2} + 50\sqrt{2}$

2 Simplify:

a $2\sqrt{2} + 3\sqrt{2} + \sqrt{2}$ **b** $-4\sqrt{6} - 3\sqrt{6} - 8\sqrt{6}$

c $10\sqrt{5} - \sqrt{5} - 6\sqrt{5}$ **d** $-2\sqrt{13} - 5\sqrt{13} + 16\sqrt{13}$

e $\sqrt{7} - 8\sqrt{7} + 5\sqrt{7}$ **f** $-2\sqrt{10} + 9\sqrt{10} - 7\sqrt{10}$

g $3\sqrt{6} + 4\sqrt{2} - 2\sqrt{6} + 3\sqrt{2}$ **h** $4\sqrt{5} + 7\sqrt{3} - 2\sqrt{5} - 4\sqrt{3}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Simplify:

a $3 - \sqrt{3} + 4 - 2\sqrt{3}$

b $\sqrt{3} - 2\sqrt{2} + 2\sqrt{3} + \sqrt{2}$

c $-4\sqrt{6} + 11 + 9 - 7\sqrt{6}$

d $5\sqrt{14} + 4\sqrt{6} + \sqrt{14} + 3\sqrt{6}$

e $6\sqrt{7} - 2\sqrt{14} + 4\sqrt{14} - 7\sqrt{7}$

f $\sqrt{5} - 3\sqrt{2} - 4\sqrt{5} + 7\sqrt{2}$

Example 13

4 Simplify:

a $\sqrt{8} + \sqrt{2}$

b $3\sqrt{2} - \sqrt{8}$

c $\sqrt{50} + 3\sqrt{2}$

d $\sqrt{18} + 2\sqrt{2}$

e $\sqrt{27} + 2\sqrt{3}$

f $\sqrt{48} + 2\sqrt{3}$

g $3\sqrt{5} + \sqrt{45}$

h $\sqrt{28} + \sqrt{63}$

i $\sqrt{8} + 4\sqrt{2} + 2\sqrt{18}$

5 Simplify:

a $\sqrt{72} - \sqrt{50}$

b $\sqrt{48} + \sqrt{12}$

c $\sqrt{8} + \sqrt{2} + \sqrt{18}$

d $\sqrt{12} + 4\sqrt{3} - \sqrt{75}$

e $\sqrt{32} - \sqrt{200} + 3\sqrt{50}$

f $4\sqrt{5} - 4\sqrt{20} - \sqrt{45}$

g $\sqrt{54} + \sqrt{24}$

h $\sqrt{27} - \sqrt{48} + \sqrt{75}$

i $\sqrt{45} + \sqrt{80} - \sqrt{125}$

j $\sqrt{2} + \sqrt{32} + \sqrt{72}$

k $\sqrt{1000} - \sqrt{40} - \sqrt{90}$

l $\sqrt{512} + \sqrt{128} + \sqrt{32}$

Example 14

6 Simplify:

a $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{3}$

b $\dfrac{2\sqrt{3}}{5} + \dfrac{\sqrt{3}}{2}$

c $\dfrac{\sqrt{7}}{2} - \dfrac{\sqrt{7}}{5}$

d $\dfrac{3\sqrt{11}}{7} + \dfrac{2\sqrt{11}}{21}$

e $\dfrac{7\sqrt{2}}{6} - \dfrac{\sqrt{2}}{3}$

f $\dfrac{\sqrt{24}}{2} + \dfrac{\sqrt{6}}{5}$

g $\dfrac{\sqrt{27}}{5} - \dfrac{\sqrt{3}}{2}$

h $\dfrac{\sqrt{32}}{7} - \dfrac{2\sqrt{2}}{5}$

7 Find the value of x if:

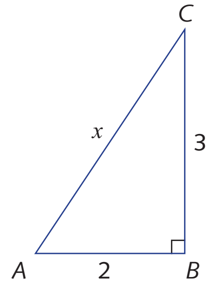
a $\sqrt{63} - \sqrt{28} = \sqrt{x}$

b $\sqrt{80} - \sqrt{45} = \sqrt{x}$

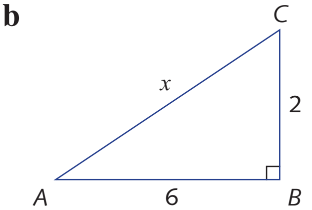
c $2\sqrt{24} - \sqrt{54} = \sqrt{x}$

8 Find the value of x and the perimeter.

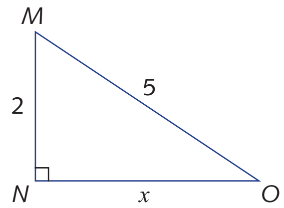
a



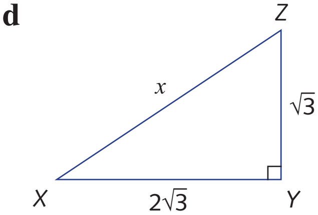
b



c



d

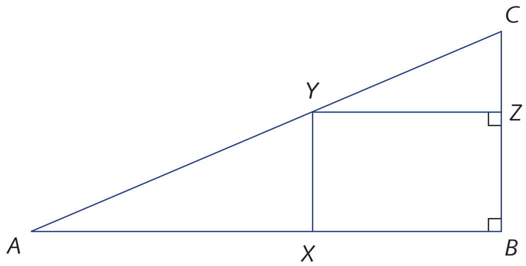


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

9 In the diagram to the right, $AB = 5\sqrt{3}$, $XB = 2\sqrt{3}$, $CB = \frac{5}{3}\sqrt{5}$, $YX = \sqrt{5}$. Find:

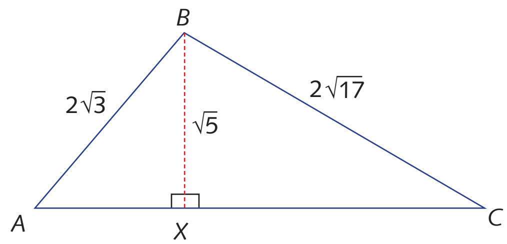
a AX **b** AY

c CZ **d** YC

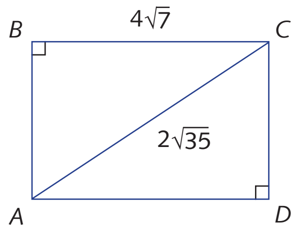


10 In the diagram to the right, find:

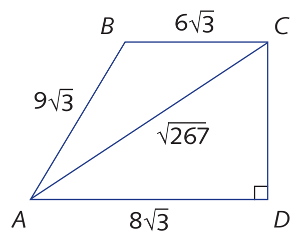
a AX **b** XC **c** AC



11 Find BA and the perimeter of the rectangle.



12 A quadrilateral $ABCD$ has side lengths $AB = 9\sqrt{3}$, $BC = 6\sqrt{3}$ and $AD = 8\sqrt{3}$. $\angle ADC = 90^\circ$ and the diagonal $AC = \sqrt{267}$. Find the perimeter of the quadrilateral.



2D Multiplication and division of surds

When we come to multiply two surds, we simply multiply the numbers outside the square root sign together, and similarly, multiply the numbers under the square root signs. A similar rule holds for division.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 15

Find $4\sqrt{7} \times 2\sqrt{2}$.

Solution

$$4\sqrt{7} \times 2\sqrt{2} = 8\sqrt{14} \qquad (4 \times 2 = 8,\ \sqrt{7} \times \sqrt{2} = \sqrt{14})$$

We can state the procedure we just used as a general rule:

$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd},$$

where b and d are positive numbers.

Example 16

Find:

a $15\sqrt{35} \div 5\sqrt{7}$

b $49\sqrt{21} \div 14\sqrt{3}$

Solution

a $15\sqrt{35} \div 5\sqrt{7} = 3\sqrt{5}$ $\qquad (15 \div 5 = 3,\ \sqrt{35} \div \sqrt{7} = \sqrt{5})$

b $49\sqrt{21} \div 14\sqrt{3} = \dfrac{7\sqrt{7}}{2}$ $\qquad \left(49 \div 14 = \dfrac{7}{2},\ \sqrt{21} \div \sqrt{3} = \sqrt{7}\right)$

We can state the procedure we just used as a general rule:

$$a\sqrt{b} \div c\sqrt{d} = \frac{a}{c}\sqrt{\frac{b}{d}},$$

where b and d are positive numbers and $c \neq 0$.

As usual, we should always give the answer in simplest form.

Example 17

Find $5\sqrt{6} \times 3\sqrt{10}$.

Solution

$$\begin{aligned} 5\sqrt{6} \times 3\sqrt{10} &= 15\sqrt{60} \\ &= 15\sqrt{4 \times 15} \\ &= 30\sqrt{15} \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

The distributive law

We can apply the distributive law to expressions involving surds, just as we do in algebra.

Example 18

Expand and simplify $2\sqrt{3}(4 + 3\sqrt{3})$.

Solution

$$\begin{aligned}2\sqrt{3}(4 + 3\sqrt{3}) &= 2\sqrt{3} \times 4 + 2\sqrt{3} \times 3\sqrt{3}\\ &= 8\sqrt{3} + 6\sqrt{9}\\ &= 8\sqrt{3} + 18\end{aligned}$$

In algebra, you learnt how to expand brackets such as $(a + b)(c + d)$. These are known as **binomial products**. You multiply each term in the second bracket by each term in the first, then add. This means you expand out $a(c + d) + b(c + d)$ to obtain $ac + ad + bc + bd$. We use this idea again when multiplying out binomial products involving surds.

Example 19

Expand and simplify:

a $(3\sqrt{7} + 1)(5\sqrt{7} - 4)$

b $(5\sqrt{2} - 3)(2\sqrt{2} - 4)$

c $(3\sqrt{2} - 4\sqrt{3})(5\sqrt{3} - \sqrt{2})$

d $(1 - \sqrt{2})(3 + 2\sqrt{2})$

Solution

a $$\begin{aligned}(3\sqrt{7} + 1)(5\sqrt{7} - 4) &= 3\sqrt{7}(5\sqrt{7} - 4) + 1(5\sqrt{7} - 4)\\ &= 15\sqrt{49} - 12\sqrt{7} + 5\sqrt{7} - 4\\ &= 105 - 7\sqrt{7} - 4\\ &= 101 - 7\sqrt{7}\end{aligned}$$

b $$\begin{aligned}(5\sqrt{2} - 3)(2\sqrt{2} - 4) &= 5\sqrt{2}(2\sqrt{2} - 4) - 3(2\sqrt{2} - 4)\\ &= 20 - 20\sqrt{2} - 6\sqrt{2} + 12\\ &= 32 - 26\sqrt{2}\end{aligned}$$

c $$\begin{aligned}(3\sqrt{2} - 4\sqrt{3})(5\sqrt{3} - \sqrt{2}) &= 3\sqrt{2}(5\sqrt{3} - \sqrt{2}) - 4\sqrt{3}(5\sqrt{3} - \sqrt{2})\\ &= 15\sqrt{6} - 6 - 60 + 4\sqrt{6}\\ &= 19\sqrt{6} - 66\end{aligned}$$

d $$\begin{aligned}(1 - \sqrt{2})(3 + 2\sqrt{2}) &= 3 + 2\sqrt{2} - 3\sqrt{2} - 4\\ &= -1 - \sqrt{2}\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Multiplication and division of surds

- For positive numbers b and d, $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$.
- For positive numbers b and d, $a\sqrt{b} \div c\sqrt{d} = \frac{a}{c}\sqrt{\frac{b}{d}}$, provided $c \neq 0$.
- Give the answers in simplest form.
- We expand binomial products involving surds just as we do in algebra:

$$(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$$

Exercise 2D

Example 15

1 Simplify:

a $\sqrt{2} \times \sqrt{3}$ **b** $\sqrt{5} \times \sqrt{2}$ **c** $\sqrt{3} \times 2\sqrt{7}$ **d** $4\sqrt{7} \times \sqrt{2}$

e $3\sqrt{2} \times 2\sqrt{3}$ **f** $4\sqrt{5} \times 2\sqrt{2}$ **g** $5\sqrt{7} \times 3\sqrt{2}$ **h** $6\sqrt{3} \times 4\sqrt{7}$

Example 16

2 Simplify:

a $\sqrt{5} \div \sqrt{5}$ **b** $8\sqrt{10} \div \sqrt{10}$ **c** $6\sqrt{3} \div 2\sqrt{3}$

d $\sqrt{21} \div \sqrt{3}$ **e** $4\sqrt{15} \div \sqrt{5}$ **f** $12\sqrt{33} \div 3\sqrt{3}$

g $\sqrt{35} \div \sqrt{7}$ **h** $3\sqrt{2} \div \sqrt{2}$ **i** $36\sqrt{15} \div 4\sqrt{3}$

j $14\sqrt{40} \div 7\sqrt{5}$ **k** $20\sqrt{30} \div 2\sqrt{6}$ **l** $7\sqrt{28} \div 4\sqrt{7}$

Example 17

3 Simplify:

a $\sqrt{6} \times \sqrt{3}$ **b** $\sqrt{6} \times \sqrt{6}$ **c** $\sqrt{7} \times 2\sqrt{7}$ **d** $4\sqrt{5} \times \sqrt{10}$

e $2\sqrt{6} \times 2\sqrt{2}$ **f** $3\sqrt{6} \times 2\sqrt{2}$ **g** $7\sqrt{10} \times 3\sqrt{2}$ **h** $3\sqrt{14} \times 2\sqrt{7}$

Example 18

4 Expand and simplify:

a $\sqrt{3}(\sqrt{6} + \sqrt{3})$ **b** $\sqrt{2}(2\sqrt{2} - \sqrt{6})$ **c** $2\sqrt{5}(\sqrt{2} - \sqrt{5})$

d $3\sqrt{3}(2\sqrt{3} - 1)$ **e** $5\sqrt{5}(4\sqrt{2} - 3)$ **f** $\sqrt{7}(2\sqrt{7} - \sqrt{14})$

g $4\sqrt{5}(2\sqrt{15} - 3\sqrt{3})$ **h** $2\sqrt{3}(3\sqrt{3} - 5)$ **i** $3\sqrt{2}(3 + 4\sqrt{3})$

j $6\sqrt{5}(\sqrt{2} + 1)$ **k** $3\sqrt{7}(2 - \sqrt{14})$ **l** $3\sqrt{5}(\sqrt{15} + 3)$

Example 19

5 Expand and simplify:

a $(4\sqrt{5} + 1)(3\sqrt{5} + 2)$ **b** $(4 + 2\sqrt{6})(2 + 5\sqrt{6})$

c $(3\sqrt{2} + 2)(3\sqrt{2} - 1)$ **d** $(1 + \sqrt{5})(7 - 6\sqrt{5})$

e $(2\sqrt{3} - 4)(3\sqrt{3} + 5)$ **f** $(3\sqrt{7} - 1)(5\sqrt{7} - 2)$

g $(7\sqrt{2} + 5)^2$ **h** $(4\sqrt{3} - 2)^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

6 Expand and simplify:

a $(3\sqrt{5} + 2)(\sqrt{2} + 3)$

b $(2\sqrt{3} - 3)(3\sqrt{2} + 1)$

c $(4 + 2\sqrt{3})(2\sqrt{7} - 5)$

d $(8 - 4\sqrt{6})(2 - \sqrt{7})$

e $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})$

f $(5\sqrt{2} - 3\sqrt{3})(2\sqrt{2} + 2\sqrt{3})$

g $(4\sqrt{7} - \sqrt{5})(2\sqrt{5} + \sqrt{7})$

h $(4\sqrt{2} - \sqrt{5})(2\sqrt{5} + \sqrt{2})$

7 If $x = \sqrt{2} - 1$ and $y = \sqrt{3} + 1$, find:

a xy

b $x + y$

c $x(x + 2)$

d $\sqrt{3}x - \sqrt{2}y$

e $2xy$

f $x + 3y$

g $(x + 1)(y + 1)$

h $\dfrac{6\sqrt{10}}{x + 1}$

i $\dfrac{1}{x} + \dfrac{1}{\sqrt{2} + 1}$

j $\dfrac{1}{x} - \dfrac{1}{\sqrt{2} + 1}$

k $x + \dfrac{1}{x}$

2E Special products

In algebra, you learned the following special expansions. They are called **identities** because they are true for all values of the pronumerals. These are especially important when dealing with surds.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)(a + b) = a^2 - b^2$$

The first two identities are called the **perfect square** identities. The last identity is known as the **difference of two squares** identity.

You will need to be very confident with these identities and be able to recognise them. Since they are true for all numbers, we can apply them to surds. Recall that $(\sqrt{a})^2 = a$ holds for any positive number a.

Example 20

Expand and simplify $(\sqrt{5} + \sqrt{3})^2$.

Solution

$$\begin{aligned}(\sqrt{5} + \sqrt{3})^2 &= (\sqrt{5})^2 + 2 \times \sqrt{5} \times \sqrt{3} + (\sqrt{3})^2 \\ &= 5 + 2\sqrt{15} + 3 \\ &= 8 + 2\sqrt{15}\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 21

Expand and simplify $(3\sqrt{2} - 2\sqrt{3})^2$.

Solution

$$\begin{aligned}(3\sqrt{2} - 2\sqrt{3})^2 &= (3\sqrt{2})^2 - 2 \times 3\sqrt{2} \times 2\sqrt{3} + (2\sqrt{3})^2 \\ &= 18 - 12\sqrt{6} + 12 \\ &= 30 - 12\sqrt{6}\end{aligned}$$

You should always express your answer in simplest form.

Example 22

Expand and simplify $(2\sqrt{3} + 4\sqrt{6})^2$.

Solution

$$\begin{aligned}(2\sqrt{3} + 4\sqrt{6})^2 &= (2\sqrt{3})^2 + 2 \times 2\sqrt{3} \times 4\sqrt{6} + (4\sqrt{6})^2 \\ &= 12 + 16\sqrt{18} + 96 \\ &= 108 + 48\sqrt{2}\end{aligned}$$

The following example shows an interesting application of the difference of two squares' identity. We will use this later in this chapter when simplifying surds with a binomial denominater.

Example 23

Expand and simplify:

a $(\sqrt{7} - 5)(\sqrt{7} + 5)$

b $(5\sqrt{6} - 2\sqrt{5})(5\sqrt{6} + 2\sqrt{5})$

Solution

a $$\begin{aligned}(\sqrt{7} - 5)(\sqrt{7} + 5) &= (\sqrt{7})^2 - 5^2 \\ &= 7 - 25 \\ &= -18\end{aligned}$$

b $$\begin{aligned}(5\sqrt{6} - 2\sqrt{5})(5\sqrt{6} + 2\sqrt{5}) &= (5\sqrt{6})^2 - (2\sqrt{5})^2 \\ &= 25 \times 6 - 4 \times 5 \\ &= 130\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Identities

The following identities are often used in calculations involving surds.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a-b)(a+b) = a^2 - b^2$$

Exercise 2E

Example 20

1 Expand and simplify:

a $(5+\sqrt{3})^2$ **b** $(\sqrt{2}+6)^2$ **c** $(4+2\sqrt{5})^2$

d $(3\sqrt{3}+1)^2$ **e** $(\sqrt{5}+\sqrt{7})^2$ **f** $(2\sqrt{3}+\sqrt{2})^2$

g $(5\sqrt{2}+3\sqrt{3})^2$ **h** $(\sqrt{2}+3\sqrt{5})^2$ **i** $(2\sqrt{5}+4\sqrt{3})^2$

j $(\sqrt{x}+\sqrt{y})^2$ **k** $(a\sqrt{x}+b\sqrt{y})^2$ **l** $(\sqrt{xy}+1)^2$

Example 21

2 Expand and simplify:

a $(\sqrt{7}-2)^2$ **b** $(4-\sqrt{3})^2$ **c** $(2\sqrt{5}-1)^2$

d $(2-3\sqrt{3})^2$ **e** $(\sqrt{5}-\sqrt{3})^2$ **f** $(2\sqrt{3}-\sqrt{2})^2$

g $(\sqrt{2}-4\sqrt{5})^2$ **h** $(4\sqrt{2}-3\sqrt{7})^2$ **i** $(\sqrt{x}-\sqrt{y})^2$

Example 22

3 Expand and simplify:

a $(2\sqrt{10}+4\sqrt{5})^2$ **b** $(5\sqrt{6}+2\sqrt{3})^2$ **c** $(\sqrt{21}+\sqrt{3})^2$

d $(2\sqrt{35}+\sqrt{5})^2$ **e** $(2\sqrt{10}-\sqrt{2})^2$ **f** $(3\sqrt{2}-\sqrt{10})^2$

g $(\sqrt{70}-3\sqrt{10})^2$ **h** $(\sqrt{50}-3\sqrt{5})^2$ **i** $(\sqrt{11}-2\sqrt{22})^2$

j $(\sqrt{10}-\sqrt{5})^2$ **k** $(2\sqrt{3}-\sqrt{6})^2$ **l** $(5\sqrt{14}-3\sqrt{21})^2$

Example 23

4 Expand and simplify:

a $(3-\sqrt{5})(3+\sqrt{5})$ **b** $(\sqrt{6}-1)(\sqrt{6}+1)$

c $(7\sqrt{2}+3)(7\sqrt{2}-3)$ **d** $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

e $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$ **f** $(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})$

g $(\sqrt{2}-3\sqrt{5})(\sqrt{2}+3\sqrt{5})$ **h** $(4\sqrt{2}-3\sqrt{5})(4\sqrt{2}+3\sqrt{5})$

i $(6\sqrt{2}-2\sqrt{7})(6\sqrt{2}+2\sqrt{7})$ **j** $(2\sqrt{3}-5\sqrt{6})(2\sqrt{3}+5\sqrt{6})$

k $(2\sqrt{5}+7\sqrt{10})(2\sqrt{5}-7\sqrt{10})$ **l** $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

5 If $x=\sqrt{2}-1$ and $y=\sqrt{2}+1$, find:

a x^2 **b** y^2 **c** x^2+y^2 **d** x^2-y^2

e xy **f** x^2y **g** y^2x **h** $\dfrac{1}{x}+\dfrac{1}{y}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Find:

a $(\sqrt{5}+\sqrt{3})^2+(\sqrt{5}-\sqrt{3})^2$

b $(\sqrt{5}+\sqrt{3})^2-(\sqrt{5}-\sqrt{3})^2$

c $(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2$

d $(\sqrt{3}+\sqrt{2})^2-(\sqrt{3}-\sqrt{2})^2$

e $(\sqrt{7}-\sqrt{2})^2+(\sqrt{7}+\sqrt{2})^2$

f $(\sqrt{6}+2)^2-(\sqrt{6}-2)^2$

7 Simplify $(a\sqrt{b}+c\sqrt{d})(a\sqrt{b}-c\sqrt{d})$.

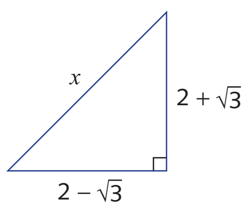
8 For the figures shown, find:

i the value of x

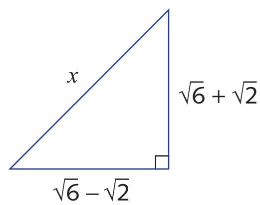
ii the area of the triangle

iii the perimeter of the triangle

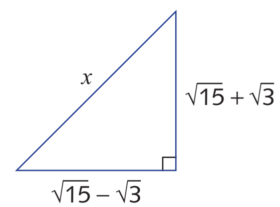
a



b



c

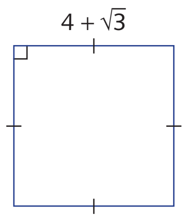


9 For the figures shown, find:

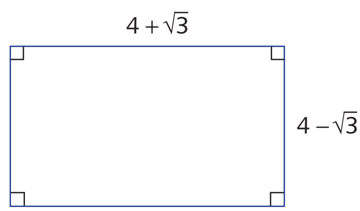
i the perimeter

ii the area

a



b



10 An equilateral triangle has sides length $1+\sqrt{3}$. Find:

a the perimeter of the triangle

b the area of the triangle

2F Rationalising the denominator

The expression $\dfrac{2}{\sqrt{3}}+\dfrac{\sqrt{3}}{2}$ is untidy. The first term has a square root in the denominator.

Fractions involving surds are easiest to deal with when they are expressed in simplest form, with a rational denominator.

When we multiply the numerator and denominator of a fraction by the same number, we form an equivalent fraction. The same happens with a quotient involving surds.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 24

Express $\dfrac{4}{\sqrt{2}}$ with a rational denominator.

Solution

$$\begin{aligned}\frac{4}{\sqrt{2}} &= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{4\sqrt{2}}{2} \\ &= 2\sqrt{2}\end{aligned}$$

(Multiply top and bottom by the same surd.)

Example 25

Write $\dfrac{9}{4\sqrt{3}}$ as a quotient with a rational denominator.

Solution

$$\begin{aligned}\frac{9}{4\sqrt{3}} &= \frac{9}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{9\sqrt{3}}{12} \\ &= \frac{3\sqrt{3}}{4}\end{aligned}$$

(Multiply top and bottom by $\sqrt{3}$.)

Example 26

Simplify $\dfrac{2}{\sqrt{3}} + \dfrac{\sqrt{3}}{2}$.

Solution

$$\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{2}$$

(Rationalise the denominator of the first term.)

$$= \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{2}$$

$$= \frac{4\sqrt{3}}{6} + \frac{3\sqrt{3}}{6}$$

(Use a common denominator.)

$$= \frac{7\sqrt{3}}{6}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Rationalising the denominator

- Rationalising the denominator means converting the denominator into a rational number.
- We use the result

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}, \text{ where } a \text{ is positive,}$$

to rationalise the denominator of a quotient involving surds.

Exercise 2F

Example 24

1 Rationalise the denominator.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{5}{\sqrt{6}}$ **c** $\frac{3}{\sqrt{3}}$ **d** $\frac{14}{\sqrt{7}}$ **e** $\frac{\sqrt{3}}{\sqrt{7}}$

f $\frac{\sqrt{18}}{\sqrt{6}}$ **g** $\frac{\sqrt{5}}{\sqrt{3}}$ **h** $\frac{\sqrt{6}}{\sqrt{3}}$ **i** $\frac{\sqrt{14}}{\sqrt{7}}$ **j** $\frac{2\sqrt{7}}{\sqrt{3}}$

Example 25

2 Rationalise the denominator.

a $\frac{1}{5\sqrt{3}}$ **b** $\frac{7}{3\sqrt{2}}$ **c** $\frac{4}{7\sqrt{2}}$ **d** $\frac{\sqrt{5}}{3\sqrt{7}}$ **e** $\frac{\sqrt{2}}{3\sqrt{10}}$

f $\frac{2\sqrt{5}}{3\sqrt{2}}$ **g** $\frac{4\sqrt{2}}{5\sqrt{7}}$ **h** $\frac{8\sqrt{18}}{2\sqrt{3}}$ **i** $\frac{\sqrt{3}}{4\sqrt{6}}$ **j** $\frac{\sqrt{15}}{3\sqrt{5}}$

3 Given that $\sqrt{2} \approx 1.412$ and $\sqrt{3} \approx 1.732$, find these values, correct to 2 decimal places, without using a calculator. (First rationalise the denominator where appropriate.)

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{3}}$ **c** $\frac{3}{\sqrt{2}}$ **d** $\frac{5}{\sqrt{3}}$

e $\sqrt{12}$ **f** $\sqrt{18}$ **g** $\frac{1}{\sqrt{12}}$ **h** $\frac{1}{\sqrt{18}}$

Example 26

4 By first rationalising the denominators, simplify each expression.

a $\frac{1}{\sqrt{2}} + \sqrt{2}$ **b** $\frac{2}{\sqrt{3}} + \frac{3}{2\sqrt{3}}$ **c** $\frac{5\sqrt{2}}{3} - \frac{1}{\sqrt{3}}$

d $\frac{4\sqrt{5}}{\sqrt{2}} - \frac{2\sqrt{5}}{\sqrt{2}}$ **e** $\frac{\sqrt{72}}{\sqrt{3}} + \frac{3}{\sqrt{2}} - \frac{2}{2\sqrt{2}}$ **f** $\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} + \frac{7}{2\sqrt{3}}$

5 If $x = 2\sqrt{14}$ and $y = 4\sqrt{2}$, find and rationalise the denominator.

a $\frac{x}{y}$ **b** $\frac{y}{x}$ **c** $\frac{2x}{y}$ **d** $\frac{\sqrt{2}x}{\sqrt{3}y}$

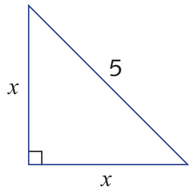
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 If $x = 2\sqrt{3}$, find and rationalise the denominator.

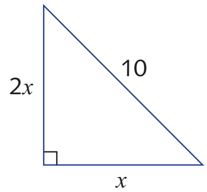
a $x + \dfrac{1}{x}$ **b** $x - \dfrac{1}{x}$ **c** $\left(x + \dfrac{1}{x}\right)^2$ **d** $\left(x - \dfrac{1}{x}\right)^2$

7 Find the value of x. Express your answer with a rational denominator.

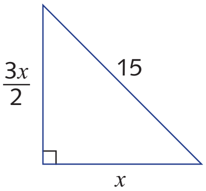
a



b

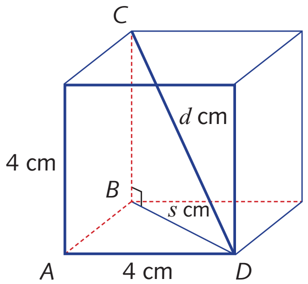


c



2G Applications of Pythagoras' theorem in three dimensions

How can we find the length of the diagonal CD of a cube whose side length is 4 cm? BD is a **face diagonal** and sometimes CD is called a **space diagonal**.



We can apply Pythagoras' theorem to triangle BAD to find the square of the length s cm of the diagonal BD.

$$\begin{aligned} s^2 &= 4^2 + 4^2 \\ &= 32 \end{aligned}$$

We can then apply Pythagoras' theorem again to triangle CBD since $\angle CBD$ is a right angle. The length d cm of the diagonal CD is given by:

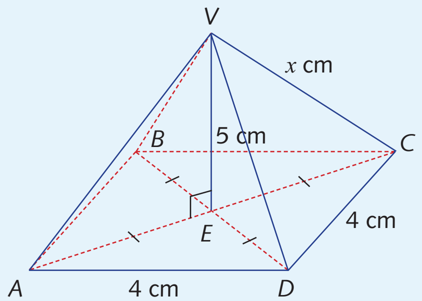
$$\begin{aligned} d^2 &= s^2 + 4^2 \\ &= 32 + 16 \\ &= 48 \\ d &= \sqrt{48} = 4\sqrt{3} \\ &\approx 6.93 \quad \text{(correct to 2 decimal places)} \end{aligned}$$

The length of the diagonal CD is 6.93 cm correct to 2 decimal places.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 27

A square pyramid has height 5 cm and square base with side length 4 cm. Find the length of the edge VC in this diagram.



Solution

The base is a square with side length 4 cm, so:

$$\begin{aligned} AC^2 &= 4^2 + 4^2 \\ &= 32 \\ AC &= \sqrt{32} \\ &= 4\sqrt{2} \end{aligned}$$

Hence $EC = 2\sqrt{2}$ (Leave in exact form to maintain accuracy.)

Triangle VEC is right-angled, so:

$$\begin{aligned} x^2 &= VE^2 + EC^2 \\ &= 52 + (2\sqrt{2})^2 \\ &= 25 + 8 \\ &= 33 \end{aligned}$$

$$\begin{aligned} x &= \sqrt{33} \\ &\approx 5.74 \end{aligned}$$ (correct to 2 decimal places)

The lenth of VC is 5.74 cm correct to 2 decimal places.

Applications of Pythagoras' theorem in three dimensions

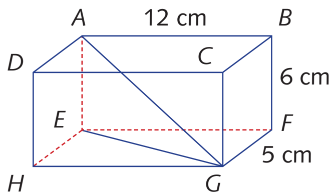
- Pythagoras' theorem can be used to find lengths in three-dimensional problems.
- Always draw a careful diagram identifying the appropriate right-angled triangle(s).
- To maintain accuracy, use exact values and only approximate using a calculator at the end of the problem if required.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 2G

Example 27

1 The rectangular prism in the diagram has a length of 12 cm, a width of 5 cm and a height of 6 cm.



a Consider triangle EFG. Find:

i EF **ii** the size of $\angle EFG$

b Find EG.

c Find AG, correct to 1 decimal place.

Note: AG is called the space diagonal of the rectangular prism.

2 Find the length of the space diagonal of the rectangular prism whose length, width and height are:

a 12 cm, 9 cm, 8 cm **b** 12 cm, 5 cm, 8 cm

c 10 cm, 4 cm, 7 cm **d** 8 cm, 6 cm, 4 cm

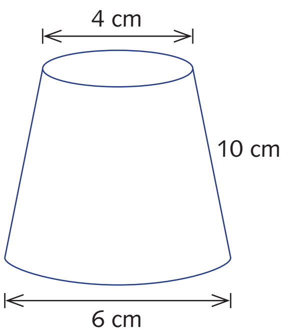
e 7 cm, 2 cm, 3 cm **f** a cm, b cm, c cm

3 Find the length of the longest pencil that can fit inside a cylindrical pencil case of length 15 cm and radius 2 cm.

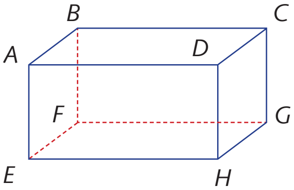
4 A bowl in the shape of a hemisphere of radius length 5 cm is partially filled with water. The surface of the water is a circle of radius 4 cm when the rim of the bowl is horizontal. Find the depth of the water.

5 A builder needs to carry lengths of timber along a corridor in order to get them to where he is working. There is a right-angled bend in the corridor along the way. The corridor is 3 m wide and the ceiling is 2.6 m above the floor. What is the longest length of timber that the builder can take around the corner in the corridor? (*Hint*: Draw a diagram.)

6 A bobbin for an industrial knitting machine is in the shape of a truncated cone. The diameter of the top is 4 cm, the diameter of the base is 6 cm and the length of the slant is 10 cm. Find the height of the bobbin.



7 For the rectangular prism shown opposite, $EH = 4$ cm and $HG = 2$ cm.



a Find the exact length of EG, giving your answer as a surd in simplest form.

b If $AE = \frac{1}{2}EG$, find the exact value of AE.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Find the length of:

i BE **ii** BH

d What type of triangle is triangle BEH?

e Show that if $EH = 2a$ cm, $HG = a$ cm and $AE = \frac{1}{2}EG$, then the sides of the triangle BEH are in the ratio 3 : 4 : 5.

2H Binomial denominators

Consider the expression $\dfrac{1}{\sqrt{7}-\sqrt{5}}$.

How can we write this as a quotient with a rational denominator?

In the section on special products, we saw that:

$$\begin{aligned}(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5}) &= \left(\sqrt{7}\right)^2 - \left(\sqrt{5}\right)^2\\ &= 7-5\\ &= 2, \text{ which is rational,}\end{aligned}$$

so

$$\begin{aligned}\frac{1}{\sqrt{7}-\sqrt{5}} &= \frac{1}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}\\ &= \frac{\sqrt{7}+\sqrt{5}}{7-5}\\ &= \frac{\sqrt{7}+\sqrt{5}}{2}\end{aligned}$$

Similarly,

$$\begin{aligned}(5\sqrt{2}-4)(5\sqrt{2}+4) &= (5\sqrt{2})^2 - 4^2\\ &= 34, \text{ which again is rational,}\end{aligned}$$

so

$$\begin{aligned}\frac{3}{5\sqrt{2}-4} &= \frac{3}{5\sqrt{2}-4} \times \frac{5\sqrt{2}+4}{5\sqrt{2}+4}\\ &= \frac{15\sqrt{2}+12}{34}\end{aligned}$$

Using the difference of two squares identity in this way is an important technique.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 28

Simplify the following:

a $\dfrac{2\sqrt{5}}{2\sqrt{5}-2}$

b $\dfrac{\sqrt{3}+\sqrt{2}}{3\sqrt{2}+2\sqrt{3}}$

Solution

a
$$\begin{aligned}\frac{2\sqrt{5}}{2\sqrt{5}-2} &= \frac{2\sqrt{5}}{2\sqrt{5}-2} \times \frac{2\sqrt{5}+2}{2\sqrt{5}+2}\\ &= \frac{20+4\sqrt{5}}{20-4}\\ &= \frac{20+4\sqrt{5}}{16}\\ &= \frac{4(5+\sqrt{5})}{16}\\ &= \frac{5+\sqrt{5}}{4}\end{aligned}$$

b
$$\begin{aligned}\frac{\sqrt{3}+\sqrt{2}}{3\sqrt{2}+2\sqrt{3}} &= \frac{\sqrt{3}+\sqrt{2}}{3\sqrt{2}+2\sqrt{3}} \times \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\\ &= \frac{(\sqrt{3}+\sqrt{2})(3\sqrt{2}-2\sqrt{3})}{18-12}\\ &= \frac{(3\sqrt{6}-6+6-2\sqrt{6})}{6}\\ &= \frac{\sqrt{6}}{6}\end{aligned}$$

Binomial denominators

To rationalise a denominator which has two terms, we use the difference of two squares identity:

- In an expression such as $\dfrac{3}{5+\sqrt{3}}$, multiply top and bottom by $5-\sqrt{3}$.
- In an expression such as $\dfrac{\sqrt{2}}{7-3\sqrt{2}}$, multiply top and bottom by $7+3\sqrt{2}$.

The surd $7-\sqrt{3}$ is called the **conjugate** of $7+\sqrt{3}$ and $7+\sqrt{3}$ is the conjugate of $7-\sqrt{3}$.

Exercise 2H

1 Simplify:

a $(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})$

b $(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})$

c $(7\sqrt{2}+4\sqrt{3})(7\sqrt{2}-4\sqrt{3})$

d $(4-\sqrt{3})(4+\sqrt{3})$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 28

2 Rationalise the denominator in each expression.

a $\dfrac{1}{\sqrt{6}+1}$ **b** $\dfrac{3}{\sqrt{2}-1}$ **c** $\dfrac{2}{3+\sqrt{5}}$

d $\dfrac{2}{3-\sqrt{5}}$ **e** $\dfrac{4}{\sqrt{5}+\sqrt{2}}$ **f** $\dfrac{1}{\sqrt{7}-\sqrt{5}}$

g $\dfrac{\sqrt{3}}{\sqrt{6}+\sqrt{5}}$ **h** $\dfrac{\sqrt{2}}{\sqrt{2}-\sqrt{3}}$ **i** $\dfrac{\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$

j $\dfrac{\sqrt{5}}{2\sqrt{5}-\sqrt{3}}$ **k** $\dfrac{\sqrt{5}}{3\sqrt{2}+4\sqrt{3}}$ **l** $\dfrac{\sqrt{3}}{2\sqrt{3}-3\sqrt{6}}$

m $\dfrac{2\sqrt{6}}{4\sqrt{2}-3\sqrt{7}}$ **n** $\dfrac{3\sqrt{2}}{6\sqrt{3}+11\sqrt{5}}$ **o** $\dfrac{2\sqrt{3}}{3\sqrt{2}+10\sqrt{3}}$

3 Rationalise the denominators in these expressions and use the decimal approximations $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.732$ to evaluate them correct to 2 decimal places.

a $\dfrac{1}{\sqrt{2}-1}$ **b** $\dfrac{1}{\sqrt{3}+2}$ **c** $\dfrac{1}{\sqrt{3}+\sqrt{2}}$ **d** $\dfrac{1}{\sqrt{3}-\sqrt{2}}$

4 Find the integers p and q such that $\dfrac{\sqrt{5}}{\sqrt{5}-2} = p + q\sqrt{5}$.

5 Simplify:

a $\dfrac{3}{\sqrt{5}-2} + \dfrac{2}{\sqrt{5}+2}$ **b** $\dfrac{2}{6-3\sqrt{3}} - \dfrac{1}{2\sqrt{3}+3}$

6 Simplify:

a $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ **b** $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ **c** $\dfrac{5}{(\sqrt{7}-\sqrt{2})^2}$

2I Irrational numbers and surds

We first recall some facts about fractions and decimals.

Fractions and decimals

We know that some fractions can be written as decimals that **terminate**. For example,

$$\frac{1}{4} = 0.25, \ \frac{3}{16} = 0.1875$$

Some fractions have decimal representations that do not terminate. For example,

$$\frac{1}{3} = 0.33333\ldots \text{ which we write as } 0.\dot{3}.$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

The dot above the 3 indicates that the digit 3 is repeated forever.

Some fractions have decimal representations that eventually repeat. For example,

$$\frac{1}{6} = 0.1666\ldots \text{ which is written as } 0.1\dot{6}.$$

Other fractions have decimal representations with more than one repeating digit. For example,

$$\frac{1}{11} = 0.090\,909\ldots \text{ which we write as } 0.\dot{0}\dot{9},$$

with a dot above both of the repeating digits.

Another example is $0.12\dot{3}\,45\dot{6} = 0.123\,456\,3456\ldots$

Converting decimals to fractions

It is easy to write terminating decimals as fractions by using a denominator which is a power of 10.

For example,

$$0.14 = \frac{14}{100} = \frac{7}{50}$$

Decimals that have a repeated sequence of digits can also be written as fractions.
For example,

$$0.\dot{1}2\dot{3} = \frac{41}{333} \text{ and } 0.67\dot{1}\dot{2} = \frac{443}{660}$$

A method for doing this is shown in the next two examples.

Example 29

Write $0.\dot{2}$ as a fraction.

Solution

Let	$S = 0.\dot{2}$	
So	$S = 0.2222\ldots$	
Then	$10S = 2.2222\ldots$	(Multiply by 10.)
	$10S = 2 + 0.222\ldots$	
Therefore	$10S = S + 2$	
Hence	$9S = 2$	
So	$S = \frac{2}{9}$	
Thus	$0.\dot{2} = \frac{2}{9}$	

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 30

Write $0.\dot{1}\dot{2}$ as a fraction.

Solution

$$\begin{aligned}
\text{Let} \quad S &= 0.\dot{1}\dot{2} \\
\text{So} \quad S &= 0.121\,212\,12\ldots \\
\text{Then} \quad 100S &= 121\,212\,12\ldots \\
\text{Therefore} \quad 100S &= 12 + S \\
\text{Hence} \quad 99S &= 12 \\
\text{So} \quad S &= \frac{12}{99} \\
&= \frac{4}{33} \\
\text{Thus} \quad 0.\dot{1}\dot{2} &= \frac{4}{33}
\end{aligned}$$

Rational numbers

A **rational number** is a number that can be written as a fraction $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

All integers are rational numbers. For example,

$$-3 = \frac{-3}{1}$$

Example 31

Explain why each of these numbers is rational.

a $5\frac{3}{7}$ **b** 2.1237 **c** $0.\dot{2}\dot{4}$

Solution

We will express each number in the form $\frac{p}{q}$.

a $5\frac{3}{7} = \frac{38}{7}$

b $2.1237 = 2\frac{1237}{10\,000}$

$= \frac{21\,237}{10\,000}$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Let
$$\begin{aligned} S &= 0.\dot{2}\dot{4} \\ &= 0.242\,42\ldots \\ 100S &= 24.242\,42\ldots \qquad \text{(Multipy by 100.)} \\ 100S &= 24 + S \\ \text{Hence} \quad 99S &= 24 \\ S &= \frac{24}{99} \\ &= \frac{8}{33} \\ \text{Thus} \quad 0.\dot{2}\dot{4} &= \frac{8}{33} \end{aligned}$$

Each number has been expressed as a fraction $\frac{p}{q}$, where p and q are integers, so each number is rational.

Irrational numbers

Mathematicians up to about 600 BCE thought that all numbers were rational. However, when we apply Pythagoras' theorem, we encounter numbers such as $\sqrt{2}$ that are not rational. The number $\sqrt{2}$ is an example of an **irrational number**.

An irrational number is one that is not rational. Hence an irrational number cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Nor can it be written as a terminating or repeating decimal.

In 300 BCE, Euclid proved that making the assumption that $\sqrt{2}$ is rational leads to a contradiction. Hence $\sqrt{2}$ was shown to be irrational. The proof is outlined here.

Assume $\sqrt{2} = \frac{p}{q}$ where p and q are integers with highest common factor 1.

Square both sides of this equation to obtain $2 = \frac{p^2}{q^2}$. We can write $p^2 = 2q^2$. Hence p^2 is even and thus p is even. We can now write, $2 = \frac{4k^2}{q^2}$ For some whole number k. From this, show q is also even which is a contradiction of our assumption that the highest common factor is 1.

The decimal expansion of $\sqrt{2}$ goes on forever but does not repeat. The value of $\sqrt{2}$ can be approximated using a calculator. The same is true of other irrational numbers such as $\sqrt{3}$, $\sqrt{14}$ and $\sqrt{91}$. Try finding these numbers on your calculator and see what you get.

Other examples of irrational numbers include:

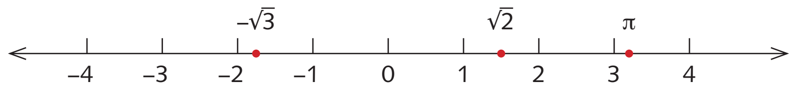
$$\sqrt{3},\ \sqrt[3]{2},\ \pi, \pi^3 \ \text{and}\ \sqrt{\pi}$$

There are infinitely many rational numbers because every whole number is rational. We can also easily write down infinitely many other fractions. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

There are infinitely many irrational numbers too. For example, $\sqrt{2}, \dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{3}, \dfrac{\sqrt{2}}{4}, \ldots$

Every number, whether rational or irrational, is represented by a point on the number line. Conversely, we can think of each point on the number line as a number.



Example 32

Arrange these irrational numbers in order of size on the number line.

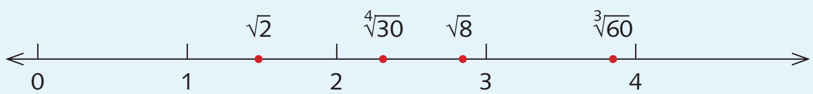
$\sqrt{8}$ $\qquad$ $\sqrt{2}$ $\qquad$ $\sqrt[3]{60}$ $\qquad$ $\sqrt[4]{30}$

Solution

Find on approximation of each number to 2 decimal places using a calculator.

$\sqrt{8} \approx 2.83$ $\qquad$ $\sqrt{2} \approx 1.14$

$\sqrt[3]{60} \approx 3.91$ $\qquad$ $\sqrt[4]{30} \approx 2.34$



Surds

In Year 8, we looked at squares, square roots, cubes and cube roots. For example:

$5^2 = 25$ $\qquad$ and $\qquad$ $\sqrt{25} = 5$

$5^3 = 125$ $\qquad$ and $\qquad$ $\sqrt[3]{125} = 5$

The use of this notation can be extended. For example:

$5^4 = 625$ $\qquad$ and $\qquad$ $\sqrt[4]{625} = 5$

This is read as 'The fourth power of 5 is 625 and the fourth root of 625 is 5'. We also have fifth roots, sixth roots and so on.

In general, we can take the nth root of any positive number a. The ***n*th root of *a*** is the positive number whose nth power is a.

The statement $\sqrt[n]{a} = b$ is equivalent to the statement $b^n = a$.

Your calculator will give you approximations to the nth root of a.

Note: For n, a positive integer, $0^n = 0$ and $\sqrt[n]{0} = 0$.

An irrational number which can be expressed as $\sqrt[n]{a}$, where a is a positive whole number, is called a **surd**.

For example, $\sqrt{2}$, $\sqrt{7}$ and $\sqrt[3]{5}$ are all examples of surds, while $\sqrt{4}$ and $\sqrt{9}$ are not surds since they are whole numbers. The number π, although it is irrational, is not a surd. This is also difficult to prove.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 33

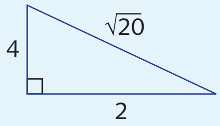
Use Pythagoras' theorem to construct a line of length $\sqrt{20}$.

Solution

Find two perfect squares that sum to 20.

$4^2 + 2^2 = 20$

Draw perpendicular line segments from a common point of lengths 2 units and 4 units. Connect their endpoints with a third line segment.



Irrational numbers and surds

- Every fraction can be written as a terminating, or eventually repeating, decimal.
- Every terminating, or eventually repeating, decimal can be written as a fraction.
- A **rational** number is a number which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$. That is, a rational number is an integer or a fraction.
- There are numbers such as π and $\sqrt{2}$ that are **irrational** (not rational).
- Each rational and each irrational number represents a point on the number line.
- Every point on the number line represents a rational or an irrational number.
- An irrational number which can be expressed as $\sqrt[n]{a}$, where n and a are positive whole numbers, is called a **surd**. π is not a surd.

Exercise 2I

Example 29

1 Write each repeating decimal as a fraction.

a $0.\dot{5}$ **b** $0.\dot{7}$ **c** $0.\dot{9}$ **d** $0.1\dot{4}$ **e** $0.2\dot{3}$ **f** $0.6\dot{2}$

Example 30

2 Write each repeating decimal as a fraction.

a $0.\dot{1}\dot{3}$ **b** $0.\dot{0}\dot{7}$ **c** $0.\dot{9}\dot{1}$ **d** $0.\dot{2}4\dot{1}$ **e** $0.\dot{6}1\dot{3}$

f $0.0\dot{1}\dot{6}$ **g** $0.3\dot{2}\dot{4}$ **h** $0.51\dot{2}\dot{6}$ **i** $0.00\dot{1}\dot{2}$

Example 31

3 Show that each number is rational by writing it as a fraction.

a $3\frac{2}{3}$ **b** 5.15 **c** $0.\dot{4}$ **d** $0.\dot{1}\dot{5}$ **e** $5\frac{1}{7}$ **f** 1.3

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

4 Which of these numbers are irrational?

a $\sqrt{7}$ **b** $\sqrt{25}$ **c** $0.\dot{6}$ **d** $\frac{\pi}{3}$

Example 32

5 By approximating correct to 2 decimal places, place these real numbers on the same number line.

a $\sqrt{7}$ **b** 2.7 **c** $\sqrt[3]{18}$ **d** 2π **e** $\sqrt{2}$

6 Which of these numbers are surds?

a 3 **b** $\sqrt{5}$ **c** 4 **d** $\sqrt{9}$ **e** $\sqrt{7}$ **f** $\sqrt{16}$

g $\sqrt{10}$ **h** $\sqrt{1}$ **i** $\sqrt{3}$ **j** $\sqrt{15}$ **k** 5 **l** $\sqrt{25}$

Example 33

7 **a** Use the fact that $1^2 + 2^2 = 5$ to construct a length $\sqrt{5}$.

b Use the fact that $4^2 + 5^2 = 41$ to construct a length $\sqrt{41}$.

8 How would you construct an interval of length:

a $\sqrt{73}$? **b** $\sqrt{12}$? **c** $\sqrt{21}$?

9 **a** Show that $\dfrac{138}{19} = 7 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{4}}}$

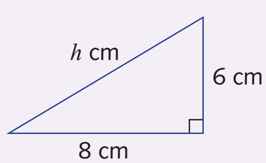
The expression on the right is called a **continued fraction**.

b Express $\dfrac{153}{11}$ as a continued fraction with all numerators 1.

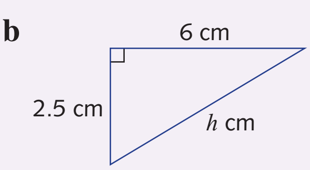
Review exercise

1 For each right-angled triangle, find the value of the pronumeral.

a

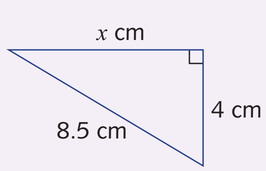


b

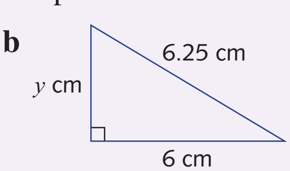


2 For each right-angled triangle, find the value of the pronumeral.

a

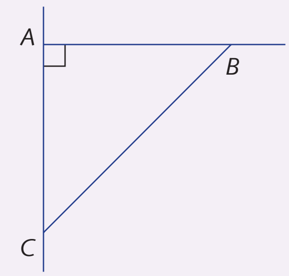


b



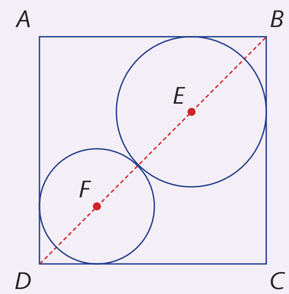
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 A support bracket is to be placed under a shelf, as shown in the diagram. If $AB = AC = 20$ cm, find, correct to the nearest millimetre, the length of BC.



4 A road runs in an east–west direction joining towns A and B, which are 40 km apart. A third town, C, is situated 20 km due north of B. A straight road is built from C, to the road between A and B and meets it at D, which is equidistant from A and C. Find the length of road CD.

5 Circles of radius 6 cm and 3 cm are placed in a square, as shown in the diagram opposite. Find, correct to 2 decimal places:



a EB **b** FD

c the length of the diagonal BD

d the side length of the square

6 Simplify:

a $3\sqrt{2} + 2\sqrt{2}$ **b** $\sqrt{32} - \sqrt{18}$

7 Simplify:

a $2\sqrt{3} \times 5\sqrt{6}$ **b** $3\sqrt{5} \times 2\sqrt{10}$

8 Simplify:

a $2\sqrt{3}(3 + \sqrt{3})$ **b** $5\sqrt{2}(3\sqrt{2} - 2)$

9 Expand and simplify:

a $(2\sqrt{2} + 1)(3\sqrt{2} - 2)$ **b** $(5\sqrt{3} - 2)(2\sqrt{3} - 1)$

10 Simplify:

a $\sqrt{80}$ **b** $\sqrt{108}$ **c** $\sqrt{125}$ **d** $\sqrt{72}$

e $\sqrt{2048}$ **f** $\sqrt{448}$ **g** $\sqrt{800}$ **h** $\sqrt{112}$

11 Simplify:

a $\sqrt{45} + 2\sqrt{5} - \sqrt{80}$ **b** $\sqrt{28} + 2\sqrt{63} - 5\sqrt{7}$

c $\sqrt{44} + \sqrt{275} - 4\sqrt{11}$ **d** $\sqrt{162} - \sqrt{200} + \sqrt{288}$

12 Express with a rational denominator.

a $\dfrac{3}{\sqrt{11}}$ **b** $\dfrac{1}{5\sqrt{15}}$ **c** $\dfrac{4}{7\sqrt{7}}$ **d** $\dfrac{3}{\sqrt{17}}$

e $\dfrac{3}{\sqrt{3}}$ **f** $\dfrac{3}{5\sqrt{15}}$ **g** $\dfrac{14}{\sqrt{7}}$ **h** $\dfrac{11}{\sqrt{3}}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

13 Express with a rational denominator.

a $\dfrac{3}{3-\sqrt{3}}$ **b** $\dfrac{22}{2+3\sqrt{5}}$ **c** $\dfrac{24}{1-\sqrt{7}}$ **d** $\dfrac{24}{1+\sqrt{17}}$

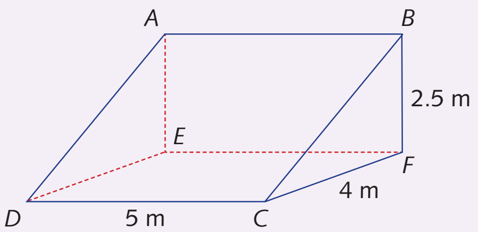
e $\dfrac{3}{2-\sqrt{3}}$ **f** $\dfrac{30}{1-\sqrt{11}}$ **g** $\dfrac{15}{2-\sqrt{7}}$ **h** $\dfrac{10}{2-\sqrt{3}}$

14 **a** Expand and simplify $(2\sqrt{5}-\sqrt{3})^2$.

b Simplify $\dfrac{2}{\sqrt{3}-2}+\dfrac{2}{\sqrt{3}}$, expressing your answer with a rational denominator.

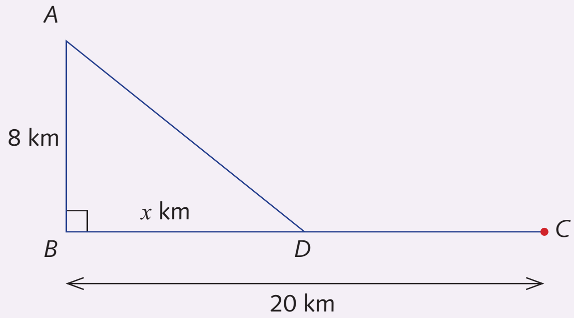
15 The diagram opposite shows part of a skate board ramp. (It is a prism whose cross-section is a right-angled triangle.) Use the information in the diagram to find:

a BC **b** AC



16 Find the length of the space diagonal of a cube with side length 5 cm.

17 A motorist departs from town B, which is 8 km due south from another town, A, and drives due east towards town C, which is 20 km from B. After driving a distance of x km, he notices that he is the same distance away from both towns A and C.

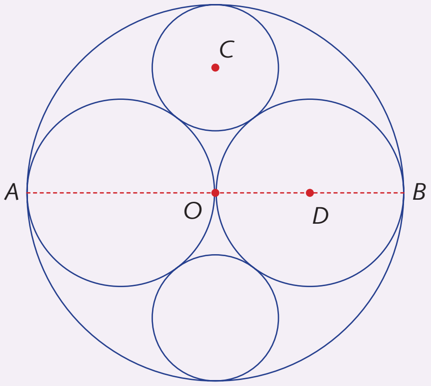


a Express the motorist's distance from A in terms of x (that is, AD).

b Express the motorist's distance from C in terms of x.

c Find the distance he has driven from B.

18 The diagram opposite shows the logo of a particular company. The large circle has centre O and radius 12 cm and AB is a diameter. D is the centre of the middle-sized circle with diameter OB. Finally, C is the centre of the smallest circle.



a What is the radius of the circle with centre D?

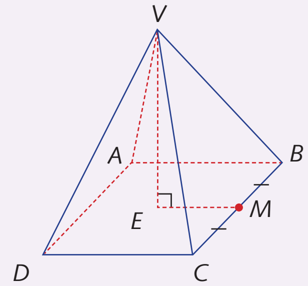
b If the radius of the circle with centre C is r cm, express these in terms of r.

i OC **ii** DC

c Find the value of r.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

19 In the diagram opposite, $VABCD$ is a square-based pyramid with $AB = BC = CD = DA = 10$ and $VA = VB = VC = VD = 10$.



a What type of triangle is triangle VBC?

b If M is the midpoint of CB, find the exact values of:

i VM

ii VE, the height of the pyramid

20 Write each repeating decimal as a fraction.

a $0.\dot{8}$ **b** $0.1\dot{5}$ **c** $0.7\dot{2}$ **d** $0.0\dot{3}$

e $0.\dot{8}\dot{1}$ **f** $0.\dot{9}\dot{6}$ **g** $0.\dot{1}0\dot{1}$ **h** $0.00\dot{1}\dot{5}$

21 Write $\frac{67}{29}$ as a continued fraction. (See Exercise 2I, Question **9**.)

22 Write $0.\dot{4}79\,28\dot{1}$ as a fraction.

23 How would you construct intervals of the following lengths? Discuss with your teacher.

a $\sqrt{3}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\sqrt{8}$ **d** $\frac{1}{\sqrt{8}}$

24 Find the length of the long diagonal of the rectangular prism whose length, width and height are:

a $3+\sqrt{2},\ 3-\sqrt{2},\ 3\sqrt{3}$ **b** $5+\sqrt{3},\ 5-\sqrt{3},\ 2\sqrt{2}$

25 For $x = \sqrt{2} = 1$ and $y = \sqrt{5} - 2$, find in simplest form.

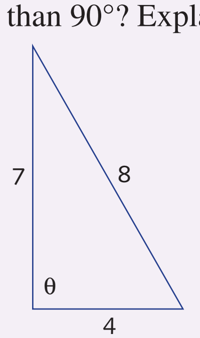
a $\frac{1}{x}$ **b** $\frac{1}{x}+\frac{1}{y}$ **c** $\frac{1}{x^2}+\frac{1}{y^2}$ **d** $\frac{1}{x}-\frac{1}{y}$

Challenge exercise

1 Prove that $\sqrt[3]{2}$ is irrational.

2 In the triangle below, $4^2 + 7^2 = 65 > 8^2$.

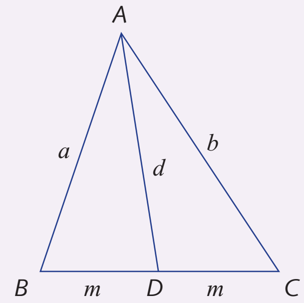
Is the angle θ greater or less than 90°? Explain your answer.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 **Apollonius' theorem** states that in any triangle, if we join a vertex to the midpoint of the opposite side, and the length of that line is d, then $a^2 + b^2 = 2(d^2 + m^2)$ where $2m = c$.

In words it says: In any triangle, the sum of the squares on two sides is equal to twice the square on half the third side together with twice the square on the median.



Prove this result as follows:

a Drop a perpendicular AE of length h from A to BC and let $ED = p$.

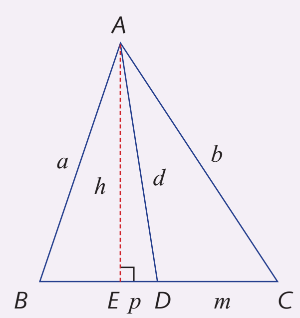
i Write BE in terms of m and p.

ii Write EC in terms of m and p.

b By applying Pythagoras' theorem to the three right-angled triangles in the diagram, complete these statements:

$a^2 = \ldots\ldots\ldots\ldots\ldots$ $\qquad$ $b^2 = \ldots\ldots\ldots\ldots\ldots$

$d^2 = \ldots\ldots\ldots\ldots\ldots$



c Add the first two equations in part **b** above and simplify.

d Use the third equation in part **b** to deduce Apollonius' theorem.

e What happens when the angle at A is a right angle?

4 A rectangle $ABCD$ has $AB = 15$ cm, $AD = 10$ cm. A point P is located inside the rectangle such that $AP = 14$ cm and $PB = 11$ cm. Find PD in exact form.

5 E is any point inside the rectangle $ABCD$. Let $AE = a$, $EB = b$, $EC = c$ and $ED = x$. Prove that $x^2 = a^2 - b^2 + c^2$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

6 We use the fact that every integer can be uniquely factorised as a product of primes to give another proof that $\sqrt{2}$ is irrational. We begin by supposing the opposite – that is, we suppose that we can write $\sqrt{2} = \frac{p}{q}$, where p and q are whole numbers with no common factors except 1.

a Explain why q^2 is a factor of p^2.

b Explain why q is a factor of p.

c Explain how this shows that $\sqrt{2}$ is irrational.

7 Prove that $\sqrt{6}$ is irrational.

8 Expand and simplify:

a $(\sqrt{2} + 1)^4$

b $(\sqrt{3} - 1)^4$

9 Prove that there are infinitely many irrational numbers between 0 and 1.

10 Prove that between any two numbers, there are infinitely many rational numbers and infinitely many irrational numbers.

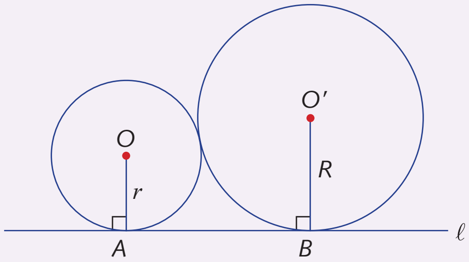
11 Rationalise the denominator of:

a $\dfrac{1}{(1 + \sqrt{2}) + \sqrt{5}}$

b $\dfrac{1}{\sqrt{7} + \sqrt{3} - 2}$

12 Show that $\sqrt[7]{7} < \sqrt[5]{5} < \sqrt{2} < \sqrt[3]{3}$ without using a calculator.

13 Two circles with centres O and O' and radii r and R meet at a single point. A and B are the points where the circles touch line ℓ. Find the distance between points A and B.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

3

Number and Algebra

Consumer arithmetic

This chapter deals with some important practical financial topics such as investing and borrowing money, income tax and GST, inflation, depreciation, profits and losses, discounts and commissions. Mention is also made of topics such as population, rainfall and the composition of materials.

When buying a car, using a credit card, or deciding how good a particular ‘bargain’ is, we all need to be competent in our understanding of consumer arithmetic. Otherwise, we run the serious risk of wasting our money or, even worse, being at the mercy of unscrupulous individuals. This is particularly important when we come to the ‘big ticket items’ such as purchasing a car or a home, or planning superannuation. It is also important when completing our tax returns.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Using a calculator and approximations

Everything in this chapter involves calculations with percentages. We are now assuming that you are using a calculator, so we have made little attempt to set questions where the numbers work out nicely. However, some of the problems can be done mentally. This chapter gives you the opportunity to learn how to efficiently use a calculator.

Nevertheless, you should always look over your work and check that the answers to your calculations are reasonable and sensible.

When the calculator displays numbers with many decimal places, you will need to round the answer in some way that is appropriate in the context of the question.

Whenever you round a number, you should be careful to use the symbol $\approx$, which means *approximately equals*, rather than the symbol $=$, which means *exactly equals*.

For example, if we are told that a grocer received $1000 in cash payments for goods that he had sold and that he banked one-third of this, we would write:

$$\begin{aligned}\text{amount banked} &= 1000 \div 3\\ &\approx \$333.33\end{aligned}$$

not

$$\begin{aligned}\text{amount banked} &= 1000 \div 3\\ &= \$333.3333\ldots\end{aligned}$$

because the grocer could not have banked fractions of a cent.

3A Review of percentages

A percentage such as 35% is a rational number and can be rewritten as a decimal or as a fraction, as follows:

$$\begin{aligned}35\% &= 35 \div 100\\ &= 0.35\end{aligned} \qquad \text{or} \qquad \begin{aligned}35\% &= \frac{35}{100}\\ &= \frac{7}{20}\end{aligned}$$

In this chapter, we encourage you to write percentages as decimals rather than fractions. Decimals are more commonly used than fractions when dealing with money.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Converting a percentage to a decimal or a fraction

To convert a percentage to a decimal or a fraction, divide by 100.

For example: $42\% = 42 \div 100$
$= 0.42$

$$12\tfrac{1}{4}\% = \frac{49}{4}\%$$
$$= \frac{49}{4} \times \frac{1}{100}$$
$$= \frac{49}{400}$$

Conversely, to convert a decimal such as 1.24, or a fraction such as $\frac{3}{8}$, to a percentage, multiply by 100%.

$$1.24 = 1.24 \times 100\% = 124\%$$

and

$$\frac{3}{8} = \frac{3}{8} \times \frac{100}{1}\% = 37\tfrac{1}{2}\%$$

Converting a decimal or a fraction to a percentage

To convert a decimal or a fraction to a percentage, multiply by 100%.

Percentages of a quantity

To calculate a percentage of a quantity:

- convert the percentage to a decimal or a fraction
- multiply the quantity by the fraction or decimal.

Example 1

The Brilliant Light Bulb Company estimates that 3.5% of its light bulbs are defective. If a shop owner buys 1250 light bulbs to light the shop, how many would he expect to be defective?

Solution

$$\begin{aligned}\text{Number of defective bulbs} &= 1250 \times 3.5\% \\ &= 1250 \times 0.035 \\ &\approx 44 \qquad \text{(Round } 43.75 \text{ to } 44.)\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Calculating a percentage

We can express one quantity as a percentage of another. Remember that both quantities must be expressed in the same unit of measurement before calculating a percentage.

Example 2

A typical computer weighs about 25 kg. When it is broken down as waste, it yields about 3 g of arsenic. What percentage of the total is this?

Solution

Using grams, the computer weighs 25 000 g and the arsenic weighs 3 g.

$$\begin{aligned}\text{Hence percentage of arsenic} &= \frac{3}{25\,000} \times 100\% \\ &= \frac{3}{250}\% \\ &= 0.012\%\end{aligned}$$

Calculating a percentage

To calculate the percentage that one quantity, a, is of another quantity, b:

- first convert both quantities to the same unit of measurement
- then form the fraction $\frac{a}{b}$ and multiply it by 100%.

Exercise 3A

1 Express each percentage as a decimal.

a 72% **b** 7.6% **c** 98% **d** 16%

e 8% **f** 6.25% **g** 175% **h** 0.6%

i $77\frac{3}{4}\%$ **j** 0.1% **k** 142.6% **l** $\frac{1}{4}\%$

2 Express each percentage as a fraction in lowest terms.

a 35% **b** 56% **c** 75% **d** $37\frac{1}{2}\%$

e $33\frac{1}{3}\%$ **f** $16\frac{2}{3}\%$ **g** 7.25% **h** 6.4%

i 210% **j** 125% **k** $112\frac{1}{2}\%$ **l** 136%

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Express each fraction or decimal as a percentage.

a $\frac{3}{5}$ **b** $\frac{3}{8}$ **c** $\frac{9}{16}$ **d** $2\frac{1}{4}$

e $\frac{7}{20}$ **f** $\frac{2}{3}$ **g** $\frac{4}{3}$ **h** $\frac{3}{400}$

i 0.43 **j** 0.225 **k** 0.04 **l** 0.015

m 1.2 **n** 2.03 **o** 1.175 **p** 0.0075

4 Copy and complete this table.

	Percentage	Fraction	Decimal
a	54%		
b		$\frac{2}{5}$	
c			0.32
d	18.5%		
e			0.06
f		$\frac{7}{8}$	
g			1.02
h	108.6%		
i		$1\frac{2}{5}$	

Example 1

5 Evaluate these amounts, correct to 2 decimal places where necessary.

a 15% of 40 **b** 57% of 1000 **c** 26% of 264

d 120% of 538 **e** 15.8% of 972 **f** 138.5% of 650

g 2.8% of 318 **h** 0.1% of 6000 **i** 150% of 846

6 Evaluate these amounts, correct to the nearest cent where necessary.

a 62% of \$10 **b** 23.7% of \$960 **c** 3.2% of \$1500

d 110% of \$1280 **e** 0.25% of \$800 **f** $6\frac{1}{2}$% of \$200

g $7\frac{3}{4}$% of \$1000 **h** $\frac{1}{4}$% of \$840 **i** 7.25% of \$1600

Example 2

7 Find what percentage the first quantity is of the second quantity, correct to 1 decimal place where necessary.

a 7 km, 50 km **b** \$4, \$200 **c** 14 kg, 400 kg

d 70 m, 50 m **e** 15 weeks, 60 weeks **f** 60 weeks, 15 weeks

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 Find what percentage the first quantity is of the second quantity, correct to 2 decimal places where necessary. You will need to express both quantities in the same unit first.

a 68 cents, \$5.00
b 3.4 cm, 2 m
c 7 g, 3 kg
d 8 hours, 2 weeks
e 15 days, 3 years
f 250 m, 4 km
g 4 km, 250 m
h 1 day, 1 year
i 1 year, 1 day
j 33 weeks, 1 century
k 56 cm, 2.4 km
l 5 apples, 16 dozen apples

9 Find what percentage the first quantity is of the second quantity, correct to 4 decimal places where necessary.

a 48 mm, 1 km
b 1.5 hours, 3 years
c 7.8 g, 60 kg
d 3.5 cents, \$1400

10 There are 740 students at a primary school, 5% of whom have red hair. Calculate the number of students in the school who have red hair.

11 A sample of a certain alloy weighs 1.6 g.

a Aluminium makes up 48% of the alloy. What is the weight of the aluminium in the sample?

b The percentage of lead in the alloy is 0.23%. What is the weight of the lead in the sample?

12 A soccer match lasted 92 minutes (including injury time). If team A was in possession for 55% of the match, for how many minutes and seconds was team A in possession?

13 Carbon dioxide makes up 0.059% of the mass of the Earth's atmosphere. The total mass of the atmosphere is about 5 million megatonnes. What is the total mass of the carbon dioxide in the atmosphere?

14 The label on a Sunnyvale tomato paste bottle says that in every 25 g serving, there are 3.6 g of carbohydrate, 0.1 g of fat, and 105 mg of sodium.

a Express as a percentage of the 25 g serving:

i the mass of carbohydrate

ii the mass of fat

iii the mass of sodium

b The Sunnyvale website claims that the percentage of protein is 3.2%. What mass of protein is that per 25 g serving?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

15 Mt Kosciusko has a height of 2228 m, while the height of Mt Everest is 8848 m. Calculate your answers to this question correct to 3 decimal places.

a What percentage is the height of Mt Everest of the height of Mt Kosciusko?

b The Earth's radius is about 6400 km. What percentage of the radius of the Earth is the height of Mt Everest?

16 In the Federal Parliament, there are 150 members in the House of Representatives, of whom 37 are from Victoria.

a Correct to 1 decimal place, what percentage of members are from Victoria?

b The population of Australia is about 22.6 million. What percentage of Australians are members of the House of Representatives?

17 The distance by air from Melbourne to Darwin is 3346 km, and from Melbourne to Singapore it is 6021 km. What percentage, correct to the nearest percent, is:

a the Melbourne–Darwin distance of the Melbourne–Singapore distance?

b the Melbourne–Singapore distance of the Melbourne–Darwin distance?

3B Using percentages

Percentages are used extensively in finance, and are common in many other practical situations. The rest of this chapter will give examples of a few well-known situations, concentrating on financial applications. If you read a newspaper for a few days, you will find a great variety of further interesting uses of percentages.

First, however, we will introduce another important method that will be used with percentages throughout this chapter.

Reversing the process to find the original amount

Suppose that 15% of the total mass of a chicken roll is actually chicken. What mass of chicken rolls can be made with 600 g of chicken?

$$\text{Mass of chicken} = \text{mass of rolls} \times 15\%$$

Reversing this:

$$\begin{aligned}\text{mass of rolls} &= \text{mass of chicken} \div 15\% \\ &= 600 \div 0.15 \\ &= 4000 \text{ g}\end{aligned}$$

Hence 4 kg of chicken rolls can be made with 600 g of chicken.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Finding the original amount

- To find, for example, 15% of a given amount, multiply by 15%.
- Conversely, to find the original amount given 15% of it, divide by 15%.

Example 3

Joshua saves 12% of his after-tax salary every week. If he saves \$90 a week, what is his after-tax salary?

Solution

Savings = after-tax salary $\times$ 12%

Reversing this:

$$\begin{aligned}\text{after-tax salary} &= \text{savings} \div 12\% \\ &= 90 \div 0.12 \\ &= \$750\end{aligned}$$

Example 4

Sterling silver is an alloy that is made up of 92.5% by mass silver and 7.5% copper.

a How much sterling silver can be made with 5 kg of silver and unlimited supplies of copper?

b How much sterling silver can be made with 5 kg of copper and unlimited supplies of silver?

Solution

a Mass of silver = mass of sterling silver $\times$ 92.5%

Reversing this:

$$\begin{aligned}\text{mass of sterling silver} &= \text{mass of silver} \div 92.5\% \\ &= 5 \div 0.925 \\ &\approx 5.405\text{ kg}\end{aligned}$$

b Mass of copper = mass of sterling silver $\times$ 7.5%

Reversing this:

$$\begin{aligned}\text{mass of sterling silver} &= \text{mass of copper} \div 7.5\% \\ &= 5 \div 0.075 \\ &\approx 66.667\text{ kg}\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Note: One reason for the choice of rounding to 3 decimal places is that there are 1000 g in a kilogram. That is, we are calculating to the nearest gram.

Commission

If a person takes a painting to a gallery to be sold, the gallery will usually charge the vendor or seller a percentage of the selling price as the fee for exhibiting, advertising and selling the painting. Such a fee is called a **commission**, and it applies whenever an agent sells goods or services such as a house or a car on behalf of someone else.

Example 5

The Eureka Gallery charges a commission of 9.2%.

a The Australian painting *Showing the Flag at Bakery Hill* was sold recently for \$180 000. How much did the Gallery receive, and how much was left for the seller?

b The Gallery received a commission of \$7912 for selling the painting *Ned at the Glen*. What was the selling price of the painting, and what did the seller actually receive?

Solution

a
$$\begin{aligned}\text{Commission} &= 180\,000 \times 9.2\% \\ &= 180\,000 \times 0.092 \\ &= \$16\,560\end{aligned}$$

$$\begin{aligned}\text{Amount received by seller} &= 180\,000 - 16\,560 \\ &= \$163\,440\end{aligned}$$

b Commission = selling price × 9.2%

Reversing this:

$$\begin{aligned}\text{selling price} &= \text{commission} \div 9.2\% \\ &= 7912 \div 0.092 \\ &= \$86\,000\end{aligned}$$

$$\begin{aligned}\text{Amount received by seller} &= 86\,000 - 7912 \\ &= \$78\,088\end{aligned}$$

Profit

Businesses aim to make a profit on their investments. A profit equation can be formulated as:

profit = total revenue (sales) – total costs

Profit and loss as percentages

Is an annual profit of \$20 000 a great performance or a poor performance? For a business with annual turnover of \$100 000, such a profit would be considered very large. For a business with annual turnover of \$100 000 000, however, it would be considered a very poor performance.

For this reason, it is often relevant to express profit or loss as a percentage of the total costs or the annual turnover.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 6

The Budget Shoe Shop spent $6600000 last year buying shoes and paying salaries and other expenses. They made a 2% profit on these costs.

a What was their profit last year?

b What was the total of their sales?

c In the previous year, their costs were $5225000 and their sales were only $5 145 000. What percentage loss did they make on their costs?

Solution

a
$$\begin{aligned}\text{Profit} &= 6600000 \times 2\% \\ &= 6600000 \times 0.02 \\ &= \$132000\end{aligned}$$

b
$$\begin{aligned}\text{Total sales} &= \text{total costs} + \text{profit} \\ &= 6600000 + 132000 \\ &= \$6732000\end{aligned}$$

c
$$\begin{aligned}\text{Last year, loss} &= \text{total costs} - \text{total sales} \\ &= 5225000 - 5145000 \\ &= \$80000\end{aligned}$$

$$\begin{aligned}\text{Percentage loss} &= \left(\frac{80000}{5225000} \times \frac{100}{1}\right)\% \\ &\approx 1.53\%\end{aligned}$$

Example 7

Joe's tile shop made a profit of 5.8% on total costs last year. If the actual profit was $83000, what were the total costs, and what were the total sales?

Solution

$$\begin{aligned}\text{Profit} &= \text{costs} \times 5.8\% \\ \text{Reversing this, costs} &= \text{profit} \div 5.8\% \\ &= 83000 \div 0.058 \\ &\approx \$1\,431034, \text{ correct to the nearest dollar.}\end{aligned}$$

$$\begin{aligned}\text{Hence, total sales} &= \text{profit} + \text{costs} \\ &\approx 83000 + 1\,431\,034 \\ &= \$1\,514\,034\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Income tax

Income tax rates are often **progressive**. This means that the more you earn, the higher the rate of tax on each extra dollar you earn.

Example 8

Income tax in the nation of Immutatia is calculated as follows.

- There is no tax on the first $\$12000$ that a person earns in any one year.
- From $\$12001$ to $\$30000$, the tax rate is $15c$ for each dollar over $\$12000$.
- From $\$30\,001$ to $\$75000$, the tax rate is $25c$ for each dollar over $\$30\,000$.
- Over $\$75000$, the tax rate is $35c$ for each dollar over $\$75000$.

Find the income tax payable by a person whose taxable income for the year is:

a $\$10600$ **b** $\$25572$ **c** $\$62300$ **d** $\$455000$

Solution

a There is no tax on an income of $\$10600$.

b Tax on first $\$12000 = \$0$

$$\begin{aligned}\text{Tax on remaining } \$13572 &= 13572 \times 0.15\\ &= \$2035.80\end{aligned}$$

This is the total tax payable.

c Tax on first $\$12000 = \$0$

$$\begin{aligned}\text{Tax on next } \$18000 &= 18000 \times 0.15\\ &= \$2700\end{aligned}$$

$$\begin{aligned}\text{Tax on remaining } \$32300 &= 32300 \times 0.25\\ &= \$8075\end{aligned}$$

$$\begin{aligned}\text{Total tax } &= 2700 + 8075\\ &= \$10\,775\end{aligned}$$

d Tax on first $\$12000 = \$0$

Tax on next $\$18000 = \$2700$ (see part **c**)

$$\begin{aligned}\text{Tax on next } \$45000 &= 45000 \times 0.25\\ &= \$11\,250\end{aligned}$$

$$\begin{aligned}\text{Tax on remaining } \$380000 &= 380000 \times 0.35\\ &= \$133000\end{aligned}$$

$$\begin{aligned}\text{Total tax } &= 2700 + 11\,250 + 133000\\ &= \$146\,950\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 3B

Note that some of the questions can be done using the unitary method. For example, Question **5** is one such question.

Example 3, 4

1 **a** Five per cent of a particular amount is \$12. Find the amount by dividing \$12 by 5% = 0.05.

b Check your answer to part **a** by taking 5% of it.

2 **a** Twenty-two per cent of a sand pile is 284 kg. Find the mass of the sand pile.

b Check your answer to part **a** by taking 22% of it.

3 Find the quantity, given that:

a 2% of it is \$12

b 6% of it is 750 g

c 30% of it is 36 minutes

d 90% of it is 54 cm

4 In each part, find the price if:

a a deposit of \$360 is 30% of the price

b a deposit of \$168 is 15% of the price

5 Find the original quantity, correct to a suitable number of decimal places, if:

a 23% of it is 100 kg

b 0.2% of it is 4 mm

c 0.92% of it is 1.86 hectares

d 97% of it is \$700

6 Cameron and Wendy together earn \$1156 per week after tax. Of this, they pay \$460 off their mortgage, \$185 for groceries, and \$260 for their car and transport, and they save \$124.

a Express each amount as a percentage of their weekly income, correct to the nearest 1%.

b Find how much is unaccounted for in the list above, and what percentage it is of their weekly income.

7 What percentage of the total cost is a deposit of:

a \$33 on a television valued at \$550?

b \$124.10 on a stove valued at \$1460?

Example 5a

8 A book dealer sells rare books and charges a commission of 8% on the selling price. Find, correct to the nearest cent, the commission charged on a book that sells for these prices, and the amount that the seller eventually receives.

a \$400 **b** \$1300 **c** \$575 **d** \$142.50

Example 5b

9 Shara the stockbroker charges 0.15% commission on all shares that she sells for clients. In each case, find the price at which a parcel of shares was sold if her commission was:

a \$30.00 **b** \$67.35 **c** \$384.75 **d** \$36.51

10 Jeff works as a salesman selling second-hand tractors. He is paid a salary of \$35 000 a year, together with a 6% commission on all the sales he makes. Find his total annual income if his sales for the year were:

a \$20 000 **b** \$1 000 000 **c** \$126 000 **d** \$3 458 000

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11 A salesperson is paid a commission on her monthly sales. What is the percentage commission if she receives a payment of:

a \$168 on sales of \$1200 **b** \$540 on sales of \$6000

c \$1530 on sales of \$18 000 **d** \$1596 on sales of \$42 000

12 Find the selling price if the commission and the commission rate are as given.

a Commission \$35, rate 7% **b** Commission \$646, rate 3.4%

c Commission \$16 586.96, rate 5.2% **d** Commission \$3518.61, rate 11.4%

13 Find, to 1 decimal place, the percentage profit or loss on costs in these situations.

a Costs \$16 000 and sales \$18 000

b Costs \$162 000 and sales \$150 000

c Costs \$2 800 000 and sales \$3 090 000

d Costs \$289 000 000 and sales \$268 000 000

Example 6

14 The Secure Locksmith Company had sales last year of \$568 000 and costs of \$521 000.

a What was their profit?

b What was the profit as a percentage of the cost price? (Calculate the percentage correct to 2 decimal places.)

c In the previous year, they made a loss of 4% of their costs of \$250 000. Find their loss and their sales.

Example 7

15 **a** A company made a profit of \$18 000, which was a 2.4% profit on its costs. Find the costs and the total sales.

b A company made a loss of \$657 000, which was a 4.5% loss on its costs. Find the costs and the total sales.

c A company made a loss of \$250 800, which was a 3.8% loss on its costs. Find the costs and the total sales.

Example 8

16 This question uses the income tax rates in the nation of Immutatia, which are as follows.

- There is no tax on the first \$12 000 that a person earns in any one year.
- From \$12 001 to \$30 000, the tax rate is 15c for each dollar over \$12 000.
- From \$30 001 to \$75 000, the tax rate is 25c for each dollar over \$30 000.
- Over \$75 000, the tax rate is 35c for each dollar over \$75 000.

a Find the income tax payable on:

i \$8000 **ii** \$14 000 **iii** \$36 000 **iv** \$200 000

b What percentage, to 2 decimal places, of each person's income was paid in income tax in parts **i–iv** of part **a**?

c Find the income if the income tax on it was:

i \$1260 **ii** \$3420 **iii** \$13 950 **iv** \$14 650

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3C Simple interest

When money is lent by a bank, whoever borrows the money normally makes a payment, called **interest**, for the use of the money.

The amount of interest paid depends on:

- the **principal**, which is the amount of money borrowed
- the **rate** at which interest is charged
- the **time** for which the money is borrowed.

Conversely, if a person invests money in a bank or elsewhere, the bank pays the person interest because the bank uses the money to finance its own investments.

This section will deal only with **simple** interest. In simple interest transactions, interest is paid on the original amount borrowed.

Formula for simple interest

How much simple interest will I pay altogether if I borrow $\$4000$ for 10 years at an interest rate of 7% per annum? (The phrase *per annum* is Latin for 'for each year'; it is often abbreviated to p.a.)

Last year you probably learned to set out the working for simple interest in two successive steps, something like this:

$$\begin{aligned}\text{Interest paid at the end of each year} &= 4000 \times 7\% \\ &= 4000 \times 0.07 \\ &= \$280\end{aligned}$$

$$\begin{aligned}\text{Total interest paid over 10 years} &= 280 \times 10 \\ &= \$2800\end{aligned}$$

This working can all be done in one step if we can develop a suitable formula.

Suppose I borrow $\$P$ for T years at an interest rate R. Using the same two-step approach as before:

$$\text{Interest paid at the end of each year} = P \times R$$

$$\begin{aligned}\text{Total interest paid over T years} &= P \times R \times T \\ &= PRT\end{aligned}$$

This gives us the well-known **simple interest formula**:

$I = PRT$ (Interest = principal × rate × time)

Using this formula, the calculation can now be set out in one step:

$$\begin{aligned}I &= PRT \\ &= 4000 \times 7\% \times 10 \qquad (\textit{Note}\text{: The interest rate } R \text{ is } 7\%, \text{ not } 7.) \\ &= 4000 \times 0.07 \times 10 \\ &= \$2800\end{aligned}$$

Note: The interest rate is given per year, so the time must also be written in years. In some books R is written as $r\%$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 9

Find the simple interest on $8000 for eight years at 9.5% p.a.

Solution

$$\begin{aligned} I &= PRT \\ &= 8000 \times 9.5\% \times 8 \\ &= 8000 \times 0.095 \times 8 \\ &= \$6080 \end{aligned}$$

Reverse use of the simple interest formula

There are four pronumerals in the formula $I = PRT$. If the values of any three are known, then substituting into the simple interest formula allows the fourth value to be found.

Example 10

Jessie borrows $3000 from her parents to help buy a car. They agree that she should only pay simple interest. Five years later she pays them back $3600, which includes simple interest on the loan. What was the interest rate?

Solution

The total interest paid was $600, the principal was $3000 and the time was 5 years.

$$\begin{aligned} I &= PRT \\ 600 &= 3000 \times R \times 5 \\ 600 &= 15\,000 \times R \\ R &= \frac{600}{15\,000} \times 100\% \qquad \text{(Interest rates are normally written as percentages.)} \\ &= 4\% \end{aligned}$$

Simple interest formula

- Suppose that a principal P is invested for T years at an interest rate R p.a. Then the total interest I is given by:

  $$I = PRT$$

  Remember that R is a percentage. If the interest rate is 5%, then $R = 0.05$.
- If the interest rate R is given per year, the time T must be given in years.
- The formula has four pronumerals. If any three are known, the fourth can be found by substitution.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 3C

Example 9

1 \$12 000 is invested at 7% p.a. simple interest for five years.

a How much interest will be earned each year?

b Find how much interest will be earned over the five-year period.

2 \$2000 is invested at 6.75% p.a. simple interest for three years.

a How much interest will be earned each year?

b Find how much interest will be earned over the three-year period.

3 Find the total simple interest earned in each of these investments.

a \$400 for three years at 6% p.a.

b \$850 for six years at 4.5% p.a.

c \$15 000 for 12 years at 8.4% p.a.

4 Find the time T for \$2000 of simple interest on a principal of \$8000 at a rate of 5% p.a.

Example 10

5 Find the rate R p.a. for \$7200 of simple interest on a principal of \$8000 over 12 years.

6 Find the principal P for \$3500 of simple interest at a rate of 7% p.a. over 10 years.

7 Calculate the missing entries for these simple interest investments.

	Principal	Rate p.a.	Time in years	Total interest
a	\$10 000	8%		\$3200
b	\$4 400 000	$7\frac{1}{2}$%		\$3 960 000
c	\$5000		6	\$1350
d	\$260 000		8	\$83 200
e		6%	5	\$900
f		3.6%	4	\$115.20

8 Sartoro invested \$80 000 in a building society that pays 6.5% p.a. simple interest. Over the years, the investment has paid him \$57 200 in interest. How many years has he had the investment?

9 Madeline has received \$168 000 in total simple interest payments on an investment of \$400 000 that she made six years ago. What rate of interest has the bank been paying?

10 An investor wishes to earn \$240 000 interest over a five-year period from an account that earns 12.5% p.a. simple interest. How much does the investor have to deposit into the account?

11 Regan has arranged to borrow \$10 000 at 9.5% p.a. for four years. She will pay simple interest to the bank every year for the loan, with the principal remaining unchanged. How much interest will Regan pay over the four years of the loan?

12 Tyler intends to live on the interest on an investment with the bank at 8.6% p.a. simple interest. She will receive \$68 000 simple interest every year from the investment.
How much money has she invested?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3D Percentage increase and decrease

When a quantity is increased or decreased, the change is often expressed as a percentage of the original amount.

This section introduces a concise method of solving problems about percentage increase and decrease. This method will be applied in various ways throughout the remaining sections of the chapter.

Percentage increase

This evening's news reported that shares in the Consolidated Nail Factory Pty Ltd were selling at $\$12.00$ yesterday, but rose 14.5% today.

Rather than calculating the price increase and adding it on, the calculation can be done in one step by using the fact that the new price is $100\% + 14.5\% = 114.5\%$ of the old price.

$$\begin{aligned}\text{New price} &= \text{old price} \times 114.5\% \\ &= 12 \times 114.5\% \\ &= 12 \times 1.145 \\ &= \$13.74\end{aligned}$$

Example 11

The number of patients admitted to St Spyridon's Hospital this year suffering from pneumonia is 56% greater than the number admitted for this condition last year.
If 245 pneumonia patients were admitted last year, how many were admitted this year?

Solution

This year's total is $100\% + 56\% = 156\%$ of last year's total.

$$\begin{aligned}\text{This year's total} &= 245 \times 156\% \\ &= 245 \times 1.56 \\ &\approx 382 \ \text{(correct to the nearest whole number)}\end{aligned}$$

Inflation

The prices of goods and services in Australia usually increase by a small amount every year.

This gradual rise in prices is called **inflation**, and is measured by taking the average percentage increase in the prices of a large range of goods and services.

Other things such as salaries and pensions are often adjusted automatically every year to take account of inflation.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 12

The war-ravaged nation of Zerbai is experiencing inflation of 35% p.a. as a result of overspending on its navy and air force. Inflation of 35% means that, on average, prices are increasing by 35% every year.

a If the price of water is adjusted in line with inflation, what will an annual bill of \$600 become in the next year?

b What should an annual salary of \$169 000 in one year increase to in the following year if it is adjusted to keep pace with inflation?

Solution

Next year's prices are $100\% + 35\% = 135\%$ of last year's prices.

a $$\text{Next year's bill} = 600 \times 1.35 = \$810$$

b $$\text{Next year's salary} = 169\,000 \times 1.35 = \$228\,150$$

Percentage decrease

The same method can be used to calculate percentage decreases. For example, suppose that 35% of a farmer's sheep station, which has an area of 7500 hectares, went under water during the recent floods.

We can calculate how much land remained above the water for his stock to graze:

$100\% - 35\% = 65\%$ of his land remained above water.

$$\begin{aligned}\text{Area remaining above water} &= 7500 \times 65\% \\ &= 7500 \times 0.65 \\ &= 4875 \text{ hectares}\end{aligned}$$

Example 13

The company that Yuri Ivanov works for is going through hard times and has decreased all its salaries by 12%. Yuri is attempting to cut every one of his expenses by the same percentage.

a His family's weekly grocery bill averages 450 roubles. What should he try to reduce the weekly price of his groceries to?

b His monthly rental is 18 000 roubles. If he moves apartments, what monthly rental should he try to find?

Solution

Yuri's new salary is $100\% - 12\% = 88\%$ of his original salary.

a $$\text{New weekly grocery bill} = 450 \times 0.88 = 396 \text{ roubles}$$

b $$\text{New monthly rental} = 18\,000 \times 0.88 = 15\,840 \text{ roubles}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Percentage increase and decrease

- To increase an amount by, for example, 15%, multiply by $1 + 0.15 = 1.15$.
- To decrease an amount by, for example, 15%, multiply by $1 - 0.15 = 0.85$.

Finding the rate of increase or decrease

Example 14

Suppose that the rainfall has increased from 480 mm p.a. to 690 mm p.a. What rate of increase is this?

Solution

Method 1

Find the actual increase by subtraction, and then express the increase as a percentage of the original rainfall.

Increase $= 210$ mm

$$\begin{aligned}\text{Percentage increase} &= \frac{\text{increase}}{\text{original rainfall}} \times 100\% \\ &= \frac{210}{480} \times 100\% \\ &= 43.75\%\end{aligned}$$

Method 2

Express the new value as a percentage of the original value, and then subtract 100%.

$$\begin{aligned}\frac{\text{new rainfall}}{\text{old rainfall}} &= \frac{690}{480} \times 100\% \\ &= 143.75\%\end{aligned}$$

So the rainfall has increased by 43.75%.

Note: Percentage decrease is sometimes represented as a negative percentage increase or, in other contexts, as a negative percentage change. This understanding is consistent with the formula, amount of change = new amount – old amount, where the new amount is less than the old amount in situations of decrease.

Reversing the process to find the original amount

Example 15

The Wind Energy Company recently announced that this year's profit of \$1400 000 constituted a 35% increase on last year's profit. What was last year's profit?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

This year's profit is $100\% + 35\% = 135\%$ of last year's profit.

$$\begin{aligned}
\text{Hence this year's profit} &= \text{last year's profit} \times 1.35 \\
\text{Reversing this, last year's profit} &= \text{this year's profit} \div 1.35 \\
&= 1400\,000 \div 1.35 \\
&\approx \$1037\,037, \text{ correct to the nearest dollar}
\end{aligned}$$

Thus, to find the original amount, we divide by 1.35, because dividing by 1.35 is the reverse of multiplying by 1.35.

Exactly the same principle applies when an amount has been decreased by a percentage.

Example 16

The price of shares in the Fountain Water Company has decreased by 15% over the last month to \$52.70. What was the price a month ago?

Solution

The new share price is $100\% - 15\% = 85\%$ of the old share price.

$$\begin{aligned}
\text{Hence} \quad \text{new price} &= \text{old price} \times 0.85 \\
\text{Reversing this,} \quad \text{old price} &= \text{new price} \div 0.85 \\
&= 52.70 \div 0.85 \\
&= \$62
\end{aligned}$$

Finding the original amount

- To find the original amount after an increase of, for example, 15%, divide by 1.15.
- To find the original amount after a decrease of, for example, 15%, divide by 0.85.

Discounts

It is very common for a shop to **discount** the price of an item. This is done to sell stock of a slow-moving item more quickly, or simply to attract customers into the shop.

Discounts are normally expressed as a percentage of the original price.

Example 17

The Elegant Shirt Shop is closing down and has discounted all its prices by 35%.

a What is the discounted price of a shirt whose original price is:

i \$120? **ii** \$75?

b What was the original price of a shirt whose discounted price is \$92.30?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a The discounted price of each item is $100\% - 35\% = 65\%$ of the old price.

i Hence discounted price $= \text{old price} \times 0.65$

$= 120 \times 0.65$

$= \$78$

ii discounted price $= 75 \times 0.65$

$= \$48.75$

b From part **a**, discounted price $= \text{old price} \times 0.65$

Reversing this, old price $= \text{discounted price} \div 0.65$

$= 92.30 \div 0.65$

$= \$142$

The GST

In 1999, the Australian Government introduced a Goods and Services Tax, or GST for short. This tax applies to nearly all goods and services in Australia.

The current rate is 10% on the pre-tax price of the goods or service.

- When GST applies, it is added to the pre-tax price. This is done by multiplying the pre-tax price by 1.10.
- Conversely, if a quoted price already includes GST, the pre-tax price is obtained by dividing the quoted price by 1.10.

Example 18

The current GST rate is 10% of the pre-tax price.

a If a domestic plumbing job costs \$630 before GST, how much will it cost after adding GST, and how much tax is paid to the Government?

b I paid \$70 for petrol recently. What was the price before adding GST, and what tax was paid to the Government?

Solution

The after-tax price is 110% of the pre-tax price.

a After-tax price $= 630 \times 1.10$

$= \$693$

Tax $= 693 - 630$

$= \$63$

Note: 10% of \$630 is \$63.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b Pre-tax price $= 70 \div 1.10$ (Divide by 1.10 to reverse the process.)

$\approx \$63.64$

Tax $\approx 70 - 63.64$

$= \$6.36$

Exercise 3D

Example 11

1 Traffic on all roads has increased by an average of 8% during the past 12 months. By multiplying by 108% = 1.08, estimate the number of vehicles now on a road given the number of vehicles the road carried a year ago was:

a 10 000 per day **b** 80 000 per day **c** 148 000 per day

Example 12

2 Prices have increased with inflation by an average of 3.8% since the same time last year. Find today's price for an item that one year ago cost:

a \$200 **b** \$1.68 **c** \$345 000 **d** \$9430

Example 13

3 Rainfall across one state has decreased over the last five years by about 24%. By multiplying by $76\% = 0.76$, estimate, correct to the nearest 10 mm, the annual rainfall this year at a place where the rainfall five years ago was:

a 1000 mm **b** 250 mm **c** 680 mm **d** 146 mm

Example 14

4 Admissions to different wards of St Luke's Hospital mostly rose from 2006 to 2007, but by quite different percentage amounts. Find the percentage increase or decrease in wards where the numbers during 2006 and 2007, respectively, were:

a 50 and 68 **b** 120 and 171 **c** 92 and 77 **d** 24 and 39

5 In another state, the percentage decrease in rainfall over the last five years has been quite variable, and in some cases, rainfall has actually increased. Find the rate of decrease or increase if the annual rainfall five years ago and now are, respectively:

a 500 mm and 410 mm **b** 920 mm and 960 mm

c 140 mm and 155 mm **d** 420 mm and 530 mm

Example 15

6 Radix Holdings Pty Ltd recently issued bonus shares to its shareholders. Each shareholder received an extra 12% of the number of shares currently held. Find the original holding of a shareholder who now holds:

a 672 shares **b** 4816 shares **c** 1000 shares **d** 40 200 shares

Example 17

7 A clothing store is offering a 15% discount on all its summer stock. What is the discounted price of an item with original price:

a \$80? **b** \$48? **c** \$680? **d** \$1.60?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 17

8 A shoe store is offering a 35% discount at its end-of-year sale. Find the original price of an item whose discounted price is:

a \$1820 **b** \$279.50 **c** \$1.56 **d** \$20.80

9 A research institute is trying to find out how much water Lake Grendel had 1000 years ago. The lake now contains 24 000 megalitres, but there are various conflicting theories about the percentage change over the last 1000 years. Find how much water the lake had 1000 years ago, correct to the nearest 10 megalitres, if in the last 1000 years the volume has:

a risen by 80% **b** fallen by 28% **c** risen by 140% **d** fallen by 4%

Example 15, 16

10 Mr Brown has a spreadsheet showing the value at which he bought his various parcels of shares, the value at 31 December last year, and the percentage increase or decrease in their value. (Decreases are shown with a negative sign.) Unfortunately, a virus has corrupted one entry in each row of his spreadsheet. Help him by calculating the missing values, correct to the nearest cent, and the missing percentages, correct to 2 decimal places.

	Value at purchase	Value at 31 December	Percentage increase
a	\$12 000		30%
b	\$28 679.26		–62%
c	\$5267.70		289.14%
d		\$72 000	20%
e		\$26 000	–22%
f		\$112 000	346.5%
g		\$15 934	–91.38%
h	\$60 000	\$81 000	
i	\$98 356.68	\$14 321.57	
j	\$14 294.12	\$2314.65	

Example 18

11 The GST is a tax on most goods and services at the rate of 10% of the pre-tax price.

a Find the after-tax price on goods or services whose pre-tax price is:

i \$170 **ii** \$4624 **iii** \$68 920 **iv** \$6.80

b Find the pre-tax price on goods or services whose after-tax price is:

i \$550 **ii** \$7821 **iii** \$192 819 **iv** \$5.28

c Find the after-tax price on goods or services on which the GST is:

i \$60 **ii** \$678.20 **iii** \$54 000 **iv** \$0.93

12 **a** A shirt originally priced at \$45 was increased in price by 100%. What percentage discount will restore it to its original price?

b The daily passenger total of the Route 58 bus was 460, and in one year, it increased by 24%. What percentage decrease next year would restore it to its original passenger total?

c Shafqat had savings of \$6000, but he spent 35% of this last year. By what percentage of the new amount must he increase his savings to restore them to their original value?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d The profit of the Arborville Gelatine Factory was $\$86\,400$, but it then decreased by 42%. By what percentage must the profit increase to restore it to its original value?

13 **a** Find, correct to 2 decimal places, the percentage decrease necessary to restore a quantity to its original value if it has been increased by:

i 10% **ii** 22% **iii** 240% **iv** 2.3%

b Find, correct to 2 decimal places, the percentage increase necessary to restore a quantity to its original value if it has been decreased by:

i 10% **ii** 22% **iii** 75% **iv** 2.3%

3E Repeated increase and decrease

The method introduced in the last section becomes very useful when two or more successive increases or decreases are applied, because the original amount can simply be multiplied successively by two or more factors. Here is a typical example.

Repeated increase

Example 19

The population of Abelsburg in the census three years ago was $46\,430$. In the three years after the census, however, its population has risen by 6.2%, 8.5% and 13.1%, respectively.

a What was its population one year after the census?

b What was its population two years after the census?

c What is its population now, three years after the census?

d What was the percentage increase in population over the three years, correct to the nearest 0.1%?

Solution

a One year after the census, the population was 106.2% of its original value.

Hence population after one year $= 46\,430 \times 1.062$

$\approx 49\,309$, correct to the nearest whole number

b Two years afterwards, the population was 108.5% of its value one year afterwards.

Hence population after two years $= (46\,430 \times 1.062) \times 1.085$

$\approx 53\,500$, correct to the nearest whole number

Note: Do not use the approximation from part **a** to calculate part **b**. Either start the calculation again, or use the unrounded value from part **a**.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Three years afterwards, the population was 113.1% of its value two years afterwards.

Hence population after three years $= (46\,430 \times 1.062 \times 1.085) \times 1.131$

$\approx 60\,508$, correct to the nearest whole number

d Population three years afterwards

$$= \text{original population} \times 1.062 \times 1.085 \times 1.131$$
$$\approx \text{original population} \times 1.303$$
$$\approx \text{original population} \times 130.3\%$$

Hence the population has increased over the three years by about 30.3%.

Note: The percentage increase of 30.3% is significantly larger than the sum of the three percentage increases,

$$6.2\% + 8.5\% + 13.1\% = 27.8\%.$$

Note: The answer to part **d** does not depend on what the original population was.

Repeated decrease

The same method can be applied just as easily to percentage decreases, as demonstrated in the next example.

Example 20

Teresa invested $\$75\,000$ from her inheritance in a mining company that has not been very successful. In the first year, she lost 55% of the money, and in the second year, she lost 72% of what remained.

a How much does she have left after one year?

b How much does she have left after two years?

c What percentage of the original inheritance has she lost over the two years?

Solution

a One year later, the percentage remaining was $100\% - 55\% = 45\%$.

Hence amount left after one year $= 75\,000 \times 0.45$

$= \$33\,750$

b Two years later, she had $100\% - 72\% = 28\%$ of what she had after one year.

Hence amount left after two years $= 75\,000 \times 0.45 \times 0.28$

$= \$9450$

c Amount left after two years $= \text{original investment} \times 0.45 \times 0.28$

$= \text{original investment} \times 0.126$

$= \text{original investment} \times 12.6\%$

So she has lost $100\% - 12.6\% = 87.4\%$ of her investment over the two years.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Combinations of increases and decreases

Some problems involve both increases and decreases. They can be solved in exactly the same way.

Example 21

The volume of water in the Welcome Dam has varied considerably over the last three years. During the first year the volume rose by 27%, then it fell 43% during the second year, and it rose 16% in the third year.

a What is the percentage increase or decrease over the three years, correct to the nearest 1%?

b If there were 366500 megalitres of water in the dam three years ago, how much water is in the dam now, correct to the nearest 500 megalitres?

Solution

a
$$\begin{aligned}\text{Final volume} &= \text{original volume} \times 1.27 \times 0.57 \times 1.16\\ &\approx \text{original volume} \times 0.84\end{aligned}$$

Since $0.84 < 1$, the volume has decreased. The percentage decrease is about $100\% - 84\% = 16\%$ over the three years.

b
$$\begin{aligned}\text{Final volume} &= \text{original volume} \times 1.27 \times 0.57 \times 1.16\\ &= 366\,500 \times 1.27 \times 0.57 \times 1.16\\ &\approx 308\,000 \text{ megalitres, correct to the nearest 500 megalitres.}\end{aligned}$$

This time the sum of the percentages is $27\% - 43\% + 16\% = 0\%$, but the volume of water has changed.

Repeated increases and decreases

- To apply successive increases of, for example, 15%, 24% and 38% to a quantity, multiply the quantity by $1.15 \times 1.24 \times 1.38$.
- To apply successive decreases of, for example, 15%, 24% and 38% to a quantity, multiply the quantity by $0.85 \times 0.76 \times 0.62$.

Reversing the process to find the original amount

As shown before, division reverses the process and allows us to find the original amount, as in the following example.

Example 22

A clothing shop discounted a shirt by 45% a month ago, and has now discounted the reduced price by 20%.

a What was the total discount on the shirt?

b If the shirt is now selling for \$61.60, what was the original price of the shirt?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a After the first discount, the price was $100\% - 45\% = 55\%$ of the original price.

After the second discount, the price was $100\% - 20\% = 80\%$ of the reduced price.

$$\begin{aligned}\text{Thus final price} &= \text{original price} \times 0.55 \times 0.80 \\ &= \text{original price} \times 0.44\end{aligned}$$

So the total discount is $100\% - 44\% = 56\%$.

b
$$\begin{aligned}\text{Reversing this, original price} &= \text{final price} \div 0.44 \\ &= 61.60 \div 0.44 \\ &= \$140\end{aligned}$$

Reversing repeated increases and decreases

- To find the original quantity after successive increases of, for example, 15%, 24% and 38%, divide the final quantity by $(1.15 \times 1.24 \times 1.38)$.
- To find the original quantity after successive decreases of, for example, 15%, 24% and 38%, divide the final quantity by $(0.85 \times 0.76 \times 0.62)$.

Successive divisions

Calculations involving brackets can be tricky to handle when using the calculator.

For example, working with the figures in the summary box above, suppose that a population has increased by 15%, 24% and 38% in three successive years and is now $50\,000$. Then:

$$\begin{aligned}\text{original population} &= 50\,000 \div (1.15 \times 1.24 \times 1.38) \\ &\approx 25\,408\end{aligned}$$

We suggest that it is easier to avoid brackets and divide $50\,000$ successively by 1.15, 1.24 and 1.38. In effect, the working then goes like this:

$$\begin{aligned}\text{original population} &= 50\,000 \div (1.15 \times 1.24 \times 1.38) \\ &= 50\,000 \div 1.15 \div 1.24 \div 1.38 \\ &\approx 25\,408\end{aligned}$$

Try the calculation both ways and see which you find more natural.

Using the power button on the calculator

When a quantity is repeatedly increased or decreased by the same percentage, the power button makes calculations quicker. Make sure that you can use it correctly by experimenting with simple calculations like $3^4 = 81$ and $2^5 = 32$.

Example 23

The drought in Paradise Valley has been getting worse for years. Each year for the last five years, the rainfall has been 8% less than the previous year's rainfall.

a What is the percentage decrease in rainfall over the five years?

b If the rainfall this year was 458 mm, what was the rainfall five years ago?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

Each year the rainfall is 92% of the previous year's rainfall.

a Final rainfall $= \text{original rainfall} \times 0.92 \times 0.92 \times 0.92 \times 0.92 \times 0.92$

$= \text{original rainfall} \times (0.92)^5$

$\approx \text{original rainfall} \times 0.659$

So rainfall has decreased by about $100\% - 65.9\% = 34.1\%$ over the five years.

b From part **a**, final rainfall $= \text{original rainfall} \times (0.92)^5$

Reversing this, original rainfall $= \text{final rainfall} \div (0.92)^5$

$= 458 \div (0.92)^5$

≈ 695 mm

Exercise 3E

Example 20

1 Oranges used to cost \$2.80 per kg, but the price has increased by 5%, 10% and 12% in three successive years. Multiply by $1.05 \times 1.1 \times 1.12$ to find their price now.

2 The dividend per share in the Electron Computer Software Company has risen over the last four years by 10%, 15%, 5% and 12%, respectively. Find the latest dividend received by a shareholder whose dividend four years ago was:

a \$1000 **b** \$1678 **c** \$28.46 **d** \$512.21

3 Rates in Bullimbamba Shire have risen 7% every year for the last seven years. Find the rates now payable by a landowner whose rates seven years ago were:

a \$1000 **b** \$346 **c** \$2566.86 **d** \$788.27

Example 20

4 A tree, whose original foliage was estimated to have a mass of 1500 kg, lost 20% of its foliage in a storm, then lost 15% of what was left in a storm the next day, then lost 40% of what was left in a third storm. Estimate the remaining mass of foliage.

5 Shares in the Metropolitan Brickworks have been falling by 23% per year for the last five years. Find the present worth of a parcel of shares whose original worth five years ago was:

a \$1000 **b** \$120 000 **c** \$25 660 **d** \$3 860 000

Example 21

6 **a** A shirt is discounted by 50% and the resulting price is then increased by 50%. By what percentage is the price increased or decreased from its original value?

b The price of a shirt is increased by 50% and the resulting price is then decreased by 50%. By what percentage is the price increased or decreased from its original value?

c Can you explain the relationship between your answers to parts **a** and **b**?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 22

7 A book shop has a 50% sale on all stock, and has a container of books with the sale price reduced by a further factor of 16%.

a What was the total discount on each book in the container?

b If a book in the container is now selling for \$10.50, what was its original price?

8 Calculate the total increase or decrease in a quantity when:

a it is increased by 20% and then decreased by 20%

b it is increased by 80% and then decreased by 80%

c it is increased by 10% and then decreased by 10%

d it is increased by 30% and then decreased by 30%

Example 23

9 The price of gemfish has been rising. The price has risen by 10%, 15% and 35% in three successive years, and they now cost \$24 per kg. Find:

a the price one year ago

b the price two years ago

c the original price three years ago

10 The crime rate in Gotham City has been rising each decade. In the last four decades the number of robberies has risen by 64%, 223%, 75% and 12%. If there are now 958 robberies per year, find how many robberies per year there were:

a one decade ago

b two decades ago

c three decades ago

d four decades ago

11 A particular strain of bacteria increases its population on a certain prepared Petri dish by 34% every hour. Calculate the size of the original population four hours ago if there are now 56 000 bacteria.

12 Flash Jim is desperate to attract customers to his used car yard. He has cut prices recently by 5%, then by 10%, then by 24%. Find, correct to the nearest \$100, the original price of a used car now priced at:

a \$10 000

b \$35 000

c \$4600

d \$76 800

13 The radioactivity of any sample of the element strontium-90 decreases by 90.75% every century. Find the percentage reduction in radioactivity over each of the periods given below. (Calculate percentages correct to 3 decimal places.)

a Two centuries

b Three centuries

c Five centuries

14 Here is a table of the annual inflation rate in Australia in the years ending 30 June 2001 to 30 June 2006 (from the Reserve Bank of Australia website).

Year	2001	2002	2003	2004	2005	2006
Inflation rate	6.0%	2.8%	2.7%	2.5%	2.5%	4.0%

Suppose the salary for certain jobs at Company X rises on the 1 July every year, in line with Australia's inflation rate for the financial year just past (ending 30 June).

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a A junior secretary earned \$40 000 from 1 July 2000 to 30 June 2001. Determine how much someone in that position would earn from:

i 1 July 2001 to 30 June 2002

ii 1 July 2006 to 30 June 2007

b A team manager was on an annual salary of \$100 000 from 1 July 2006 to 30 June 2007. Determine how much someone in that position would earn:

i in the previous financial year

ii from 1 July 2003 to 30 June 2004

15 At the start of the trading day, shares of a particular stock decrease in value by 20%. However, by the end of the day the shares 'recover' and record a 15% increase from its lowest value. Determine the percentage decrease in the value of the shares over the course of the day.

3F Compound interest

In all the examples in this section, the interest is compounded annually. This means that at the end of each year, the interest earned is added to the principal invested or borrowed. That increased amount then becomes the amount on which interest is earned in the following year. This is called **compound interest**.

Example 24

Gail has invested \$100 000 for six years with the Mountain Bank. The bank pays her interest at the rate of 7.5% p.a., compounded annually.

a How much will the investment be worth at the end of one year?

b How much will the investment be worth at the end of two years?

c How much will the investment be worth at the end of six years?

d What is the percentage increase on her original investment at the end of six years?

e What is the total interest earned over the six years?

f What would the simple interest on the investment have been, assuming the same interest rate of 7.5% p.a.?

Solution

Each year the investment is worth 107.5% of its value the previous year.

a Amount at the end of one year $= 100\,000 \times 1.075$

$= \$107\,500$

b Amount at the end of two years $= 100\,000 \times 1.075 \times 1.075$

$= 100\,000 \times (1.075)^2$

$= \$115\,562.50$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Amount at the end of six years $= 100\,000 \times (1.075)^6$

$\approx \$154\,330.15$

d Final amount $=$ original amount $\times (1.075)^6$

$\approx$ original amount $\times 1.5433$

So the total increase over six years is about 54.33%.

e Total interest $\approx 154\,330.15 - 100\,000$

$= \$54\,330.15$

f Simple interest $= PRT$

$= 100\,000 \times 0.075 \times 6$

$= \$45\,000$

Note: Compound interest for two or more years is always greater than simple interest for two or more years.

Compound interest on a loan

Exactly the same principles apply when someone borrows money from a bank and the bank charges compound interest on the loan. If no repayments are made, the amount owing compounds in the same way, and can grow quite rapidly.

This is shown in the following example.

Example 25

Hussain is setting up a plumbing business and needs to borrow \$150 000 from a bank to buy a truck and other equipment. The bank will charge him interest of 11% p.a., compounded annually. Hussain will pay the whole loan off all at once four years later.

a How much will Hussain owe the bank at the end of four years?

b What is the percentage increase in the money owed at the end of four years?

c What is the total interest that Hussain will pay on the loan?

d What would the simple interest on the loan have been, assuming the same interest rate of 11% p.a.?

Solution

Each year Hussain owes 111% of what he owed the previous year.

a Amount at the end of four years $= 150\,000 \times (1.11)^4$

$\approx \$227\,710.56$

b Final amount $=$ original amount $\times (1.11)^4$

$\approx$ original amount $\times 1.5181$

So the total increase over four years is about 51.81%.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Total interest $\approx 227\,710.56 - 150\,000$

$= \$77\,710.56$

d Simple interest $= PRT$

$= 150\,000 \times 0.11 \times 4$

$= 66\,000$

Note: Making no repayments on a loan that is accruing compound interest is a risky business practice because, as this example makes clear, the amount owing grows with increasing rapidity as time goes on. This is particularly relevant to credit card debt.

Compound interest

Suppose that an amount P is invested at an interest rate, say 7%, compounded annually. The interest is calculated each year on the new balance. The new balance is obtained by adding on 7% of the balance of the previous year. Thus:

$$\begin{aligned}\text{amount after four years} &= P \times 1.07 \times 1.07 \times 1.07 \times 1.07\\ &= P \times (1.07)^4\end{aligned}$$

Reversing the process to find the original amount

As always, division reverses the process to find the original amount, as in the following example.

Example 26

Eleni wants to borrow money for three years to start a business, and then pay all the money back, with interest, at the end of that time. The bank will not allow her final debt, including interest, to exceed \$300 000. Interest is 9% p.a., compounded annually. What is the maximum amount that Eleni can borrow?

Solution

Each year Eleni will owe 109% of what she owed the previous year.

$$\begin{aligned}\text{Hence}\qquad \text{final debt} &= \text{original debt} \times 1.09 \times 1.09 \times 1.09\\ \text{final debt} &= \text{original debt} \times (1.09)^3\\ \text{Reversing this, original debt} &= \text{final debt} \div (1.09)^3\\ &= 300\,000 \div (1.09)^3\\ &\approx \$231\,655\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 3F

Example 24

1 **a** Christine invested \$100 000 for six years at 5% p.a. interest, compounded annually.

i By multiplying by 1.05, find the value of the investment after one year.

ii By multiplying by $(1.05)^2$, find the value of the investment after two years.

iii By multiplying by $(1.05)^6$, find the value of the investment after six years.

iv Find the percentage increase in the investment over the six years.

v Find the total interest earned over the six years.

b Find the simple interest on the principal of \$100 000 over the six years at the same rate of 5% p.a.

Example 25

2 **a** Gary has borrowed \$200 000 for six years at 8% p.a. interest, compounded annually, in order to start his indoor decorating business. He intends to pay the whole amount back, plus interest, at the end of the six years.

i Find the amount owing after one year.

ii Find the amount owing after two years.

iii Find the amount owing after six years.

iv Find the percentage increase in the loan over the six years.

v Find the total interest charged over the six years.

b Find the simple interest on the principal of \$200 000 over the six years at the same rate of 8% p.a.

3 A couple take out a housing loan of \$320 000 over a period of 20 years. They make no repayments over the 20-year period of the loan. Compound interest is payable at $6\frac{1}{2}\%$ p.a., compounded annually. How much would they owe at the end of the 20-year period, and what is the total percentage increase?

4 The population of a city increases annually at a compound rate of 3.2% for five years. If the population is 21 000 initially, what is it at the end of the five-year period, and what is the total percentage increase?

5 **a** Find the compound interest on \$1000 at 5% p.a., compounded annually for 200 years.

b Find the simple interest on \$1000 at 5% p.a. for 200 years.

6 \$10 000 is borrowed for five years and compound interest at 10% p.a. is charged by the lender.

a How much money is owed to the lender after the five-year period?

b How much of this amount is interest?

Example 26

7 Money borrowed at 8% p.a. interest, compounded annually, grew to \$100 000 in four years.

a Find the total percentage increase.

b Hence find the original amount invested.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 Suzette wants to invest a sum of money now so that it will grow to \$180 000 in 10 years' time. How much should she invest now, given that the interest rate is 6% compounded annually?

9 A bank offers 8% p.a. compound interest. How much needs to be invested if the investment is to be worth \$100 000 in:

a 10 years? **b** 20 years? **c** 25 years? **d** 100 years?

10 A man now owes the bank \$56 000, after having taken out a loan five years ago. Find the original amount that he borrowed if the rate of interest per annum, compounded annually, has been:

a 3% **b** 5.6% **c** 9.25% **d** 15%

11 Mr Brown has had further difficulties with the virus that attacks his spreadsheet entries. Here is the remains of a spreadsheet that he prepared in answer to questions from business friends. The spreadsheet calculated interest compounded annually, on various amounts, at various interest rates, for various periods of time. Help him reconstruct the missing entries.

	Principal	Rate p.a.	Time in years	Final amount	Total interest
a	\$4000	6%	20		
b	\$10 000	8.2%	15		
c	\$2 000 000	4.8%	10		
d		6%	20	\$4000	
e		8.2%	15	\$10 000	
f		4.8%	10	\$2 000 000	

12 Ms Smith invested \$50 000 at 6% p.a. interest, compounded annually, for four years. The tax department wants to know exactly how much interest she earned each year. Calculate these figures for Ms Smith.

13 Mrs Robinson has taken out a loan of \$300 000 at 8% p.a. interest, compounded annually, for four years. She wants to know exactly how much interest she will be charged each year so that she can include it as a tax deduction in her income tax return. Calculate these figures for Mrs Robinson.

14 **a** Find the total percentage growth in a compound interest investment:

i at 15% for two years **ii** at 10% for three years

iii at 6% for five years **iv** at 5% for six years

v at 3% for 10 years **vi** at 2% for 15 years

b What do you observe about these results?

15 A couple took out a six-year loan to start a business. For the first three years, compound interest of 8% p.a. was charged. For the second three years, compound interest of 12% p.a. was charged. Find the total percentage increase in the amount owing.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16 One six-year loan attracts compound interest calculated at 2%, 4%, 6%, 8%, 10% and 12% in successive years. Another six-year loan attracts compound interest calculated at 12%, 10%, 8%, 6%, 4% and 2% in successive years. Find the total percentage increase in money owing in both cases, compare the two results, and explain what has happened.

17 An investment at an interest rate of 10% p.a., compounded annually, returned interest of \$40 000 after five years. Calculate the original amount invested.

3G Depreciation

Depreciation occurs when the value of an asset reduces as time passes. For example, a company may buy a car for \$40 000, but after four years the car will be worth a lot less, because the motor will be worn, the car will be out of date, the body and interior may have a few scratches, and so forth.

Accountants usually make the assumption that an asset such as a car depreciates at the same rate every year. This rate is called the **depreciation rate**. It works like compound interest. The depreciation is applied each year to the current value. In the following example, the depreciation rate is taken to be 20%.

Example 27

The Medicine Home Delivery Company bought a car four years ago for \$40 000, and assumed that the value of the car would depreciate at 20% p.a.

a What value did the car have at the end of two years?

b What value does the car have now, after four years?

c What is the percentage decrease in value over the four years?

d What is the average depreciation on the car over the four years? (Express your answer in dollars per year)

Solution

The value each year is taken to be 100% – 20% = 80% of the value in the previous year.

a Value at the end of two years
$$\begin{aligned} &= 40\,000 \times 0.80 \times 0.80 \\ &= 40\,000 \times (0.80)^2 \\ &= \$25\,600 \end{aligned}$$

b Value at the end of four years
$$\begin{aligned} &= 40\,000 \times 0.80 \times 0.80 \times 0.80 \times 0.80 \\ &= 40\,000 \times (0.80)^4 \\ &= \$16\,384 \end{aligned}$$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Final value $=$ original value $\times (0.80)^4$

$=$ original value $\times$ 0.4096

Hence the percentage decrease over four years is $100\% - 40.96\% = 59.04\%$

d Depreciation over four years $= 40\,000 - 16\,384$

$= \$23\,616$

Average depreciation per year $= 23\,616 \div 4$

$= \$5904$ per year

Depreciation

Suppose that an asset with original value P depreciates at, for example, 7% every year. To find the depreciated value, decrease the current value by 7% each year. Thus:

$$\begin{aligned}\text{value after four years} &= P \times 0.93 \times 0.93 \times 0.93 \times 0.93\\ &= P \times (0.93)^4\end{aligned}$$

Reversing the process to find the original amount

If we are given a depreciated value and the rate of depreciation, we can find the original value by division.

Example 28

A school buys new computers every four years. At the end of the four years, it offers them for sale to the students on the assumption that they have depreciated at 35% p.a. (per annum). The school is presently advertising some computers at \$400 each.

a What did each computer cost the school originally?

b What is the average depreciation on each computer, in dollars per year?

Solution

a Each year a computer is worth $100\% - 35\% = 65\%$ of its value the previous year.

Hence final value $=$ original value $\times 0.65 \times 0.65 \times 0.65 \times 0.65$

final value $=$ original value $\times (0.65)^4$

Reversing this, original value $=$ final value $\div (0.65)^4$

$= 400 \div (0.65)^4$

$\approx \$2241$

b Depreciation over four years $\approx 2241 - 400$

$= \$1841$

Average depreciation per year $\approx 1841 \div 4$

$\approx \$460$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 3G

Note: The depreciation rates in this exercise are taken from the Australian Taxation Office's Schedule of Depreciation. These are intended for income tax purposes. A company may have reasons to use different rates.

Example 27

1 The landlord of a large block of home units purchased washing machines for its units four years ago for \$400 000, and is assuming a depreciation rate of 30%.

a By multiplying by 0.70, find the value after one year.

b By multiplying by $(0.70)^2$, find the value after two years.

c By multiplying by $(0.70)^3$, find the value after three years.

d By multiplying by $(0.70)^4$, find the value after four years.

e What is the percentage decrease in value over the four years?

f What is the average depreciation on the washing machines, in dollars p.a. over the four years?

2 The Hungry Hour Cafe purchased an air-conditioning system six years ago for \$250 000, and is assuming a depreciation rate of 20%.

a Find the value after one year.

b Find the value after two years.

c Find the value after six years.

d What is the percentage decrease in value over the six years?

e What is the average depreciation, in dollars p.a., on the air-conditioning system over the six years?

3 A business spent \$560 000 installing alarms at its premises and then depreciated them at 20% p.a. Find the value after five years, and the percentage depreciation of their value.

4 The population of a sea lion colony decreases at a compound rate of 2% p.a. for 10 years. If the population is 8000 initially, what is it at the end of the 10-year period?

5 The Northern Start Bus Company bought a bus for \$480 000, depreciated it at 30% p.a., and sold it again seven years later for \$60 000. Was the price that they obtained better or worse than the depreciated value, and by how much?

6 The Backyard Rubbish Experts bought a fleet of small trucks for \$1 340 000 and depreciated them at 22.5% p.a. Five years later they sold them for \$310 000. Was the price that they obtained better or worse than the depreciated value, and by how much?

7 A landlord spent \$3400 on vacuum cleaners for his block of home units and depreciated them for taxation purposes at 25% p.a. Find their value at the end of each of the first three years, and the amount of the depreciation that the landlord could claim against his taxable income for each of those three years.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 Lara and Kate each received \$100 000 from their parents. Lara invested the money at 6.2% p.a. compounded annually, whereas Kate bought a luxury car that depreciated at a rate of 20% p.a. What were the values of their investments at the end of five years?

9 Taxis depreciate at 50% p.a., and other cars depreciate at 22.5% p.a.

a What is the total percentage depreciation on each type of vehicle after seven years?

b What is the difference in value, to the nearest dollar, after seven years of a fleet of taxis and a fleet of other cars, if both fleets cost \$1 000 000?

Example 28

10 Mr Wong's 10-year-old used car is worth \$4000, and has been depreciating at 22.5% p.a. (Calculate amounts of money in whole dollars.)

a Use division by 0.775 to find how much it was worth a year ago.

b Find how much it was worth two years ago.

c Find how much it was worth 10 years ago.

d What is the total percentage depreciation on the car over the 10-year period?

e What was the average depreciation in dollars per year over the 10-year period?

11 St Scholasticus Grammar School bought photocopying machines six years ago, which it then depreciated at 25% p.a. They are now worth \$72 000.

a How much were they worth one year ago?

b How much were they worth two years ago?

c How much were they worth six years ago?

d What is the total percentage depreciation on them over the six-year period?

e What was the average depreciation in dollars per year over the six-year period?

12 Ms Wu's seven-year-old car is worth \$5600, and has been depreciating at 22.5% p.a. Calculate your answers to the nearest dollar.

a How much was it worth four years ago?

b How much was it worth seven years ago?

c Ms Wu, however, only bought the car four years ago, at its depreciated value at that time. What has been Ms Wu's average depreciation in dollars over the four years she has owned the car?

d What was the average depreciation in dollars over the first three years of the car's life?

13 I take 900 mL of a liquid and dilute it with 100 mL of water. Then I take 900 mL of the mixture and again dilute it with 100 mL of water. I repeat this process 20 times.

a What proportion of the original liquid remains in the mixture at the end?

b How much mixture should I take if I want it to contain 20 mL of the original liquid?

14 I take a sealed glass container and remove 80% of the air. Then I remove 80% of the remaining air. I do this process six times altogether. What percentage of the original air is left in the container?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Review exercise

1 Sarah decides to spend 40% of her weekly earnings on social activities, give 15% to her mother to repay a loan, and save the rest. She earns \$84 a week.

a How much does Sarah spend each week on social activities?

b What percentage of her weekly earnings does she save?

2 Nick's share portfolio consists of shares, with value \$10 000, in the banking industry, \$3000 in mining shares and \$15 000 in the gold market.

a What percentage of his share portfolio is made up of shares in the mining sector?

b What percentage of Nick's share portfolio are not banking shares?

3 A real estate agent charges a commission of 8.9% on every property sale.

a If a house sells for \$540 000, how much commission will the real estate agent receive, and how much is left for the seller?

b If the real estate agent receives a commission of \$8455 for selling a house, what was the selling price of the house, and what did the seller actually receive?

4 It cost the owners of the Corner Newsagency \$3 500 000 to run their business last year. They recorded a profit of 4.5%.

a What was their profit last year?

b What was the total of their sales?

c In the previous year, their costs were \$2 750 000 and their sales were only \$2 635 000. What percentage loss did they make on their costs?

5 Grant earned \$1260 interest on money he had invested four years ago at a simple interest rate of 4.5% p.a. How much did Grant originally invest?

6 A country is experiencing inflation of 12% p.a.

a If the price of bread is adjusted in line with inflation, what will an annual bread bill of \$2500 become in the next year?

b If Janienne earns \$72 000 in one year and \$78 000 the next, is her salary increase keeping pace with inflation?

7 A regional country medical centre has lost the services of one of its 16 doctors due to retirement, and has been unable to replace her.

a What percentage loss is represented by the retiring doctor?

b If the medical centre treated 400 patients per day last year and need to reduce this number by the percentage found in part **a**, how many patients per day will they be able to treat in the coming year?

8 A shop made a profit of 6.2% on total costs last year. If the actual profit was \$156 000, what were the total costs and what were the total sales?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

9 In the January sales, the Best Dress shop has discounted all its prices by 18%.

a What is the discounted price of a dress with a marked price of \$240?

b What was the original price of a dress with a discounted price of \$49.20?

10 The number of books in a local library varies from year to year. Three years ago, the number fell by 25%, then it rose 41% the following year, and finally rose 8% last year.

a What is the percentage increase or decrease over the three years, correct to the nearest 1%?

b If there were 429 000 books in the library three years ago, approximately how many books are in the library now?

11 The original asking price for a farm dropped by 30% a year ago, but did not attract a buyer. The price has now been further reduced by 15%.

a By what percentage has the original asking price been reduced?

b If the farm is now for sale at \$2 677 500, what was the original asking price of the farm?

12 The height of a mature tree is measured on the same day each year. Each year for the last six years, the growth has been 9% less than the previous year's growth.

a What is the percentage decrease in growth over the six years, correct to the nearest percent?

b If the growth this year was 320 mm, what was the growth six years ago, correct to the nearest millimetre?

13 Sam's investment of \$50 000 for five years earns her interest at the rate of 6.3% p.a., compounded annually.

a How much will the investment be worth at the end of six years?

b What is the percentage increase of her original investment at the end of six years?

c What is the total interest earned over the six years?

d What would the simple interest on the investment have been, assuming the same interest rate of 6.3% p.a.?

14 A company buys new company cars every three years. At the end of the three years, it offers them for sale to the employees on the assumption that they have depreciated at 30% p.a. The company is presently advertising some cars at \$30 000 each.

a What did each car cost the company originally, correct to the nearest thousand dollars?

b What is the average depreciation in dollars p.a., correct to the nearest hundred dollars, on each car over the three-year period?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 The population of a town decreases by 15% during 2010. What percentage increase, correct to 2 decimal places, is necessary during 2011 for the population to be restored to the level it was at immediately before the decrease in 2010?

2 The length of a rectangle is increased by 15% and the width is decreased by 11%. What is the exact percentage change in the area?

3 The radius of a circular pool of water increases by 8%. What is the exact percentage change in the area of the pool of water?

4 The area of a circular pool of oil is increased by 8%. What is the percentage increase of the radius?

5 A man earns a salary of \$1440 for a 44-hour week. His weekly salary is increased by 10% and his hours are reduced by 10%. Find his new hourly salary.

6 In a particular country in 2010, 15% of the population is unemployed and 85% is employed. In 2011, 10% of the people unemployed became employed and 10% of those employed became unemployed. What percentage of the population is employed now?

7 The number of trees on Green Plateau fell by 5% every year for 10 years. Then the numbers rose by 5% every year for 20 years. What was the total percentage gain or loss of trees over the 30-year period?

8 Particular shares were released in the stock market and lost, per day, an average of 2.23% of their original value over the first four days. Over the first day, the shares increased in value by 15%, and over the second day, a further 10% increase was recorded. However, a 20% decrease in the share value occurred on the third day. What percentage decrease was recorded over the fourth day?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

4

Number and Algebra

Factorisation

The distributive law can be used to rewrite a product involving brackets as an expression without brackets. For instance, the product $3(a + 2)$ can be rewritten as $3a + 6$; this is called the expanded form of the expression, and the process is called expansion.

The process of writing an algebraic expression as a product of two or more algebraic factors is called factorisation. Factorisation is the reverse process to expansion. For example, we can write $3a + 6$ as $3(a + 2)$. This is called the factorised form.

In this chapter we will look at how to factorise expressions such as $x^2 + 7x + 12$. We recall that such an expression is called a quadratic.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4A Factorisation using common factors

If each term in the algebraic expression to be factorised contains a **common factor**, then this common factor is a factor of the entire expression. To find the other factor, divide each term by the common factor. The common factor is placed outside brackets. For this reason the procedure is sometimes called 'taking the common factor outside the brackets'.

Example 1

Factorise $4a + 12$.

Solution

4 is a common factor of $4a$ and 12.

Thus $4a + 12 = 4(a + 3)$

Notice that the answer can be checked by expanding $4(a + 3)$.

In general, take out as many common factors as possible. The common factors may involve both numbers and pronumerals. It may be easier if you first take out the common factor of the numbers and then the common factor of the pronumerals.

Example 2

Factorise: **a** $12x^2 + 3x$ **b** $36ab - 27a$

Solution

a $12x^2 + 3x = 3(4x^2 + x)$
$= 3x(4x + 1)$

b $36ab - 27a = 9(4ab - 3a)$
$= 9a(4b - 3)$

Note: It is more efficient to take out all the common factors in one step, as shown in the following examples.

Example 3

Factorise:

a $3x + 9$ **b** $2x^2 + 4x$ **c** $-7a^2 - 49$ **d** $7a^2 + 63ab$

Solution

a $3x + 9 = 3(x + 3)$

b $2x^2 + 4x = 2x(x + 2)$

c $-7a^2 - 49 = -7 \times a^2 + (-7) \times 7$
$= -7(a^2 + 7)$

$7(-a^2 - 7)$ is also correct.

d $7a^2 + 63ab = 7a(a + 9b)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 4

Factorise:

a $5pq^2 + 10p^2q + 25p^2q^2$ **b** $16ab + 10b^2 - 2a^2b$

Solution

a $5pq^2 + 10p^2q + 25p^2q^2 = 5pq(q + 2p + 5pq)$

b $16ab + 10b^2 - 2a^2b = 2b(8a + 5b - a^2)$

Exercise 4A

1 Complete each factorisation.

a $12x = 12 \times \square$ **b** $24a = 12 \times \square$ **c** $36ab = 9a \times \square$

d $15ac = 5c \times \square$ **e** $y^2 = y \times \square$ **f** $6y^2 = 3y \times \square$

g $24a^2 = 6a \times \square$ **h** $-6b^2 = 2b \times \square$ **i** $8a^2b = 2ab \times \square$

j $4x^2y = 2x \times \square$ **k** $12m^2n = 3mn \times \square$ **l** $25a^2b^2 = 5ab \times \square$

2 Fill in the blanks by finding the missing factors.

a $12a + 18 = 6 \times \square$ **b** $15p - 10 = 5 \times \square$

c $20mn - 15n = 5 \times \square$ **d** $20mn - 15n = 5n \times \square$

e $a^2 + 4a = (a + 4) \times \square$ **f** $b^2 - 10b = b \times \square$

g $6yz^2 - 18yz = 3z \times \square$ **h** $6yz^2 - 18yz = yz \times \square$

i $6yz^2 - 18yz = -3yz \times \square$ **j** $6yz^2 - 18yz = 6yz \times \square$

Example 1

3 Factorise:

a $6x + 24$ **b** $5a + 15$ **c** $ac + 5c$ **d** $a^2 + a$

e $y^2 + xy$ **f** $4x + 24$ **g** $7a - 63$ **h** $9a + 36$

i $y^2 - 3y$ **j** $-14a - 21$ **k** $-6y - 9$ **l** $-4 - 12b$

Example 2

4 Factorise:

a $4ab + 16a$ **b** $12a^2 + 8a$ **c** $18m^2n + 9mn^2$

d $15a^2b^2 + 10ab^2$ **e** $4a^2 + 6a$ **f** $8a^2 + 12ab$

5 Factorise:

a $3b - 6b^2$ **b** $4x^2 - 6xy$ **c** $9mn - 12m^2n$

d $18y - 9y^2$ **e** $4a - 6ab^2$ **f** $6xy - 4x^2$

g $14mn^2 - 21m^2n$ **h** $6pq^2 - 21qp^2$ **i** $10ab^2 - 25a^2b$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 3

6 Factorise:

a $-10b^2 + 5b$ **b** $-16a^2b - 8ab$ **c** $-x^2y - 3xy$

d $-4pq + 16p^2$ **e** $-5x^2y + 30x$ **f** $18p^2 - 4pq$

g $-8a^2b^2 - 2ab$ **h** $12xy^2 - 3x^2y$ **i** $-25m^2n^2 - 10mn^2$

7 In each part, an expression for the area of the rectangle has been given. Find an expression for the missing side length.

a $2a + 3$

Area $= 8a + 12$

b $3a$

Area $= 9a + 6ab$

c 5

Area $= 10b + 15$

d a

Area $= 2a + a^2$

e $6a$

Area $= 12a^2 + 6ab$

f $3q^2 - 2$

Area $= 24p^2q^2 - 16p^2$

Example 4

8 Factorise:

a $4a^2b - 2ab + 8ab^2$ **b** $4m^2n - 4mn + 16n^2$

c $7ab + 14a^2 + 21b$ **d** $2m^2 + 4mn + 6n$

e $5a^2b + 3ab + 4ab^2$ **f** $6a + 8ab + 10ab^2$

g $5p^2q^2 + 10pq^2 + 15p^2q$ **h** $5\ell^2 - 15\ell m - 20m^2$

4B Factorisation using the difference of two squares

You will recall from Section 1G the important identity

$$(a + b)(a - b) = a^2 - b^2,$$

which is called the difference of two squares. We can now use this result the other way around to factorise an expression that is the difference of two squares.

That is:

$$a^2 - b^2 = (a + b)(a - b)$$

That is, the factors of $a^2 - b^2$ are $a + b$ and $a - b$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 5

Factorise:

a $x^2 - 9$ **b** $25 - y^2$ **c** $4x^2 - 9$

Solution

a $x^2 - 9 = x^2 - 3^2$

$= (x + 3)(x - 3)$

Check the answer by expanding $(x + 3)(x - 3)$ to see that $x^2 - 9$ is obtained.

b $25 - y^2 = 5^2 - y^2$

$= (5 + y)(5 - y)$

c $4x^2 - 9 = (2x)^2 - 3^2$

$= (2x + 3)(2x - 3)$

Example 6

Factorise:

a $3a^2 - 27$ **b** $-16 + 9x^2$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$

$= 3(a^2 - 3^2)$

$= 3(a + 3)(a - 3)$

b $-16 + 9x^2 = 9x^2 - 16$

$= (3x)^2 - 4^2$

$= (3x - 4)(3x + 4)$

Factorisation using the difference of two squares identity

$a^2 - b^2 = (a + b)(a - b)$

Exercise 4B

Example 5

1 Factorise:

a $x^2 - 16$ **b** $x^2 - 49$ **c** $a^2 - 121$ **d** $d^2 - 400$

e $(2x)^2 - 25$ **f** $(3x)^2 - 16$ **g** $(4x)^2 - 1$ **h** $(5m)^2 - 9$

i $9x^2 - 4$ **j** $16y^2 - 49$ **k** $100a^2 - 49b^2$ **l** $64m^2 - 81p^2$

m $1 - 4a^2$ **n** $9 - 16y^2$ **o** $25a^2 - 100b^2$ **p** $-9 + x^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 6a

2 Factorise:

a $3x^2 - 48$ **b** $4x^2 - 100$ **c** $5x^2 - 45$ **d** $6x^2 - 24$

e $10x^2 - 1000$ **f** $7x^2 - 63$ **g** $8x^2 - 50$ **h** $12m^2 - 75$

i $3 - 12b^2$ **j** $20 - 5y^2$ **k** $27a^2 - 12b^2$ **l** $16x^2 - 100y^2$

m $45m^2 - 125n^2$ **n** $27a^2 - 192l^2$ **o** $-8x^2 + 32y^2$ **p** $-200p^2 + 32q^2$

Example 6b

3 Factorise:

a $-25 + x^2$ **b** $-4 + 9x^2$ **c** $-9x^2 + 4$

d $-81x^2 + 16$ **e** $-100x^2 + 9$ **f** $-18 + 50x^2$

g $-12x^2 + 27$ **h** $-36x^2 + 400$ **i** $-175 + 28x^2$

4 Use the factorisation of the difference of two squares to evaluate the following. One has been done for you.

$$\begin{aligned} 17^2 - 3^2 &= (17 + 3)(17 - 3) \\ &= 20 \times 14 \\ &= 280 \end{aligned}$$

a $23^2 - 7^2$ **b** $23^2 - 3^2$ **c** $36^2 - 6^2$

d $94^2 - 6^2$ **e** $1.8^2 - 0.2^2$ **f** $28^2 - 2.2^2$

g $11.3^2 - 8.7^2$ **h** $92.6^2 - 7.4^2$ **i** $3.214^2 - 2.214^2$

5 **a** Evaluate:

i $4^2 - 3^2$ **ii** $5^2 - 4^2$ **iii** $6^2 - 5^2$ **iv** $7^2 - 6^2$

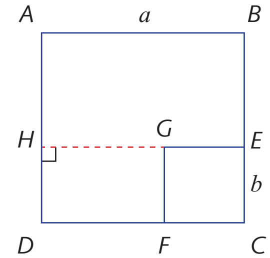
v $8^2 - 7^2$ **vi** $9^2 - 8^2$ **vii** $10^2 - 9^2$ **viii** $101^2 - 100^2$

b What do you notice?

c Use the factorisation of $(n + 1)^2 - n^2$ to prove the result of part **b**.

6 The following leads you through the geometrical proof that $a^2 - b^2 = (a - b)(a + b)$.

In the diagram opposite, square $ECFG$, of side length b, is cut out of square $ABCD$, which has side length a.



a What is the area of hexagon $ABEGFD$?

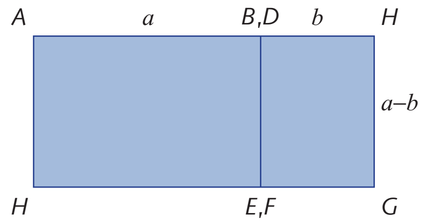
b What is the length of BE?

c What is the length of DF?

d The rectangle $HGFD$ is moved so that DF is placed on top of BE (see diagram).

i What is the area of the large shaded rectangle?

ii What have we proved?



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

4C Factorisation of simple quadratics

A **simple quadratic** expression is an expression of the form $x^2 + bx + c$, where b and c are given numbers. When we expand $(x + 3)(x + 4)$, we obtain a simple quadratic.

$$\begin{aligned} x(x+4)+3(x+4) &= x^2+4x+3x+12 \\ &= x^2+7x+12 \end{aligned}$$

We want to develop a method for reversing this process.

In the expansion $(x+3)(x+4) = x^2+7x+12$, notice that the coefficient of x is $3 + 4 = 7$. The term that is independent of x, the constant term, is $3 \times 4 = 12$. This suggests a method of factorising.

In general, when we expand $(x+p)(x+q)$, we obtain

$$x^2 + px + qx + pq = x^2 + (p+q)x + pq$$

The coefficient of x is the sum of p and q, and the constant term is the product of p and q.

Factorisation of simple quadratics

To factorise a simple quadratic, look for two numbers that add to give the coefficient of x, and that multiply together to give the constant term.

For example, to factorise $x^2+8x+15$, we look for two numbers that multiply to give 15 and add to give 8. Of the pairs that multiply to give 15 $(15 \times 1,\ 5 \times 3, -5 \times (-3)$ and $-15 \times (-1))$, only 5 and 3 add to give 8.

Therefore, $x^2+8x+15 = (x+3)(x+5)$

The result can be checked by expanding $(x + 3)(x + 5)$.

Example 7

Factorise $x^2 - 3x - 18$.

Solution

We are looking for two numbers that multiply to give -18 and add to give -3. The numbers -6 and 3 satisfy both conditions.

$x^2 - 3x - 18 = (x-6)(x+3)$

Consider factorising $x^2 - 36$.

Since the constant term is -36 and the coefficient of x is 0, we are looking for two numbers that multiply to give -36 and add to give 0. The numbers -6 and 6 satisfy both conditions.

Thus $x^2 - 36 = (x-6)(x+6)$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Note: It is not really sensible to do the example in this way – it is best to recognise $x^2 - 36$ as a difference of squares. However, it does show that the method of factorisation in this section is consistent with the earlier technique.

Example 8

Factorise $x^2 + 8x + 16$.

Solution

We are looking for two numbers that multiply to give 16 and add to give 8. The numbers 4 and 4 satisfy both conditions.

Thus $x^2 + 8x + 16 = (x + 4)(x + 4)$

$= (x + 4)^2$

Exercise 4C

Example 7

1 Factorise these quadratic expressions.

a $x^2 + 5x + 6$ **b** $x^2 + 11x + 18$

c $x^2 + 7x + 10$ **d** $x^2 + 11x + 30$

e $x^2 + 9x + 14$ **f** $x^2 + 19x + 90$

g $x^2 + 9x + 20$ **h** $x^2 + 7x + 12$

i $x^2 + 12x + 32$ **j** $x^2 + 13x + 40$

k $x^2 + 20x + 75$ **l** $x^2 + 28x + 27$

m $x^2 + 15x + 56$ **n** $x^2 + 18x + 56$

2 Factorise these quadratic expressions.

a $x^2 - 5x + 6$ **b** $x^2 - 14x + 33$

c $x^2 - 17x + 30$ **d** $x^2 - 13x + 42$

e $x^2 - 9x + 14$ **f** $x^2 - 47x + 90$

g $x^2 - 15x + 44$ **h** $x^2 - 25x + 100$

i $x^2 - 18x + 80$ **j** $x^2 - 21x + 80$

k $x^2 - 14x + 40$ **l** $x^2 - 11x + 24$

m $x^2 - 30x + 56$ **n** $x^2 - 14x + 24$

3 Factorise these quadratic expressions.

a $x^2 + x - 6$ **b** $x^2 - 8x - 33$

c $x^2 + x - 30$ **d** $x^2 - 19x - 42$

e $x^2 - 5x - 14$ **f** $x^2 - 9x - 90$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

g $x^2 - 7x - 44$
h $x^2 + 15x - 100$
i $x^2 + 2x - 80$
j $x^2 - 7x - 60$
k $x^2 + 3x - 40$
l $x^2 - 10x - 24$
m $x^2 + 4x - 21$
n $x^2 + 2x - 15$
o $x^2 - x + 56$
p $x^2 + 5x - 24$

4 Factorise these quadratic expressions.

a $x^2 - 3x + 2$
b $x^2 + 8x + 12$
c $x^2 - 3x - 10$
d $x^2 + 11x + 30$
e $x^2 - 5x - 14$
f $x^2 - 9x - 90$
g $x^2 - 5x + 4$
h $x^2 - 7x - 18$
i $x^2 - x - 12$
j $x^2 - 11x + 28$
k $x^2 + 3x - 10$
l $x^2 + x - 90$

Example 8

5 Factorise these quadratic expressions.

a $x^2 + 6x + 9$
b $x^2 + 14x + 49$
c $x^2 - 10x + 25$
d $x^2 - 18x + 81$
e $x^2 + 10x + 25$
f $x^2 + 12x + 36$
g $x^2 + 30x + 225$
h $x^2 - 16x + 64$
i $x^2 - 20x + 100$
j $x^2 - 8x + 16$

4D Factorisation using perfect squares

In Chapter 4, Example 8, we used the methods discussed previously to show that $x^2 + 8x + 16 = (x + 4)^2$. This form of quadratic factorisation is called a perfect square.

We recall from Chapter 1 that a perfect square is an expression such as $(x + 3)^2$, $(x - 5)^2$ or $(2x + 7)^2$.

The expansion of a perfect square has a special form. For example:

$$\begin{aligned}(x + 3)^2 &= (x + 3)(x + 3)\\ &= x^2 + 6x + 9\\ &= x^2 + 2 \times (3x) + 3^2\end{aligned}$$

Note that the constant term 9 is the square of half the coefficient of x.

The quadratic $x^2 + 10x + 25$ is a perfect square since the 'constant term is equal to the square of half of the coefficient of x'. So:

$$x^2 + 10x + 25 = (x + 5)^2$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

We recognise a perfect square such as this in the following way: the constant term is the square of half of the coefficient of x. For example:

$$x^2 + 12x + 36 = (x + 6)^2 \qquad x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

$$x^2 - 14x + 49 = (x - 7)^2 \qquad x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$$

Factorising using the perfect square identities

- In general, $a^2 + 2ab + b^2 = (a + b)^2$
- Similarly, $a^2 - 2ab + b^2 = (a - b)^2$

When a quadratic expression has the form of a perfect square, factorisation can occur immediately by application of the relevant identity.

Example 9

Factorise:

a $x^2 + 8x + 16$ **b** $x^2 - 10x + 25$ **c** $x^2 + 11x + \dfrac{121}{4}$

Solution

a $\begin{aligned} x^2 + 8x + 16 &= x^2 + 2 \times 4x + 4^2 \\ &= (x + 4)^2 \end{aligned}$

b $\begin{aligned} x^2 - 10x + 25 &= x^2 - 2 \times 5x + 5^2 \\ &= (x - 5)^2 \end{aligned}$

c $\begin{aligned} x^2 + 11x + \frac{121}{4} &= x^2 + 2 \times \frac{11}{2}x + \left(\frac{11}{2}\right)^2 \\ &= \left(x + \frac{11}{2}\right)^2 \end{aligned}$

Exercise 4D

Example 9

1 Factorise using the appropriate perfect square identity.

a $x^2 + 12x + 36$ **b** $x^2 - 8x + 16$ **c** $x^2 + 10x + 25$

d $a^2 - 4a + 4$ **e** $m^2 - 26m + 169$ **f** $a^2 + 28a + 196$

g $x^2 - 9x + \dfrac{81}{4}$ **h** $x^2 + 13x + \dfrac{169}{4}$ **i** $x^2 - 11x + \dfrac{121}{4}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 Copy and complete:

a $x^2 + 8x + 16 = (x + \_\_\_\_)$

b $x^2 - 10x + \_\_\_\_ = (x - 5)^2$

c $x^2 - \_\_\_\_ + \_\_\_\_ = (x - 9)^2$

d $x^2 + \_\_\_\_ + \_\_\_\_ = \left(x + \frac{7}{2}\right)^2$

e $x^2 - 9x + \_\_\_\_ = (x - \_\_\_\_)^2$

f $x^2 - \frac{5x}{2} + \_\_\_\_ = (x - \_\_\_\_)^2$

3 Identify the simple quadratic expression that cannot be factorised as a perfect square.

a **i** $x^2 + 4x + 4$ **ii** $x^2 - 6x + 12$ **iii** $x^2 - 12x + 36$ **iv** $x^2 - 10x + 25$

b **i** $x^2 + 6x + 9$ **ii** $x^2 + 5x + \frac{25}{4}$ **iii** $x^2 - 8x - 16$ **iv** $x^2 - 14x + 49$

c **i** $x^2 + \frac{2x}{3} + \frac{1}{9}$ **ii** $x^2 - 3x + \frac{9}{4}$ **iii** $x^2 - \frac{5x}{3} + \frac{25}{36}$ **iv** $x^2 + \frac{7x}{4} + \frac{49}{16}$

d **i** $x^2 - \frac{11x}{2} + \frac{121}{16}$ **ii** $x^2 - \frac{4x}{5} + \frac{4}{25}$ **iii** $x^2 - \frac{3x}{2} - \frac{9}{16}$ **iv** $x^2 - \frac{9x}{2} + \frac{81}{16}$

4 A brick company provides rectangular and square pavers.

a Draw a diagram to show how two different square pavers of side lengths a and b respectively, and two identical rectangular pavers with dimensions $a \times b$, can be arranged into a square.

b How many of each type of paver enables you to pave a square area of side length $a + 3b$? Draw a diagram to illustrate how this can be done.

4E Quadratics with common factors

Sometimes a common factor can be taken out of a quadratic expression so that the expression inside the brackets becomes a simple quadratic that can be factorised.

Example 10

Factorise:

a $3x^2 + 9x + 6$ **b** $6x^2 - 54$ **c** $-x^2 - x + 2$

Solution

a $3x^2 + 9x + 6 = 3(x^2 + 3x + 2)$ (Take out the common factor.)

$= 3(3x + 2)(3x + 1)$

b $6x^2 - 54 = 6(x^2 - 9)$

$= 6(x + 3)(x - 3)$

c $-x^2 - x + 2 = -(x^2 + x - 2)$ (Factor -1 from each term.)

$= -(x + 2)(x - 1)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 4D

Example 10a

1 Factorise:

a $2x^2 + 14x + 24$ **b** $3x^2 + 24x + 36$ **c** $3x^2 - 27x + 24$
d $4x^2 - 24x + 36$ **e** $7x^2 + 14x + 7$ **f** $5x^2 - 5x - 30$
g $4x^2 - 4x + 48$ **h** $2x^2 - 18x + 36$ **i** $5x^2 + 40x + 35$
j $3x^2 + 9x - 120$ **k** $3x^2 - 3x - 90$ **l** $5x^2 + 60x + 180$
m $2x^2 - 4x - 96$ **n** $5x^2 + 65x + 180$ **o** $3x^2 + 30x - 72$
p $3x^2 - 18x + 27$ **q** $5x^2 - 20x + 20$ **r** $3x^2 - 24x + 36$

Example 10b

2 Factorise:

a $4x^2 - 16$ **b** $2x^2 - 18$ **c** $3x^2 - 48$
d $3a^2 - 27$ **e** $6x^2 - 600$ **f** $3a^2 - 27b^2$
g $27x^2 - 3y^2$ **h** $45 - 5b^2$ **i** $12 - 3m^2$
j $128 - 2x^2$ **k** $\frac{1}{2}a^2 - 2b^2$ **l** $27x^2 - \frac{1}{3}y^2$
m $\frac{1}{4}a^2 - 9$ **n** $\frac{1}{5}x^2 - 20$ **o** $\frac{1}{4}x^2 - y^2$

Example 10c

3 Factorise:

a $-x^2 - 8x - 12$ **b** $12 - 11x - x^2$ **c** $7 - 6x - x^2$
d $9 + 8x - x^2$ **e** $-x^2 - 4x - 4$ **f** $-x^2 - 14x - 45$
g $-x^2 + 3x + 40$ **h** $42 + x - x^2$ **i** $22x - x^2 - 40$
j $11x - x^2 - 24$ **k** $-3x^2 - 30x + 72$ **l** $-56 - x^2 - 15x$
m $-16x - 63 - x^2$ **n** $-x^2 - 35 + 12x$ **o** $7x + 18 - x^2$

Review exercise

1 Factorise:

a $4x + 16$ **b** $7x - 21$ **c** $6a - 9$
d $4ab + 7a$ **e** $6pq - 11p$ **f** $5mn - 10n$
g $4uv - 8v$ **h** $a^2 + 9a$ **i** $4m^2n - 12mn$
j $a^2b - 4ab^2$ **k** $3pq - 6p^2$ **l** $6p^3q - 18pq$

2 Factorise:

a $x^2 - 9$ **b** $x^2 - 16$ **c** $9a^2 - 25$
d $16m^2 - 1$ **e** $9 - 4b^2$ **f** $100 - 81b^2$

g $16x^2 - y^2$
h $2m^2 - 50$
i $3a^2 - 27$
j $1 - 36b^2$
k $4y^2 - \frac{1}{4}$
l $p^2q^2 - 1$

3 Factorise:

a $x^2 + 8x + 12$
b $x^2 + 9x + 18$
c $x^2 + 11x + 30$
d $x^2 - 11x + 24$
e $x^2 - 10x + 24$
f $x^2 - 14x + 24$
g $x^2 - 25x + 24$
h $x^2 + x - 20$
i $x^2 - 2x - 48$
j $x^2 - 4x - 12$
k $x^2 + 3x - 40$
l $x^2 - 7x - 8$
m $x^2 - x - 132$
n $a^2 + 19a + 60$
o $x^2 - 50x + 96$

4 Factorise:

a $a^2 - 22a + 121$
b $m^2 - 14m + 49$
c $s^2 + 8s + 16$
d $a^2 + 24a + 144$
e $a^2 - 12a + 36$
f $z^2 - 40z + 400$
g $x^2 + 5x + \frac{25}{4}$
h $y^2 - \frac{2y}{3} + \frac{1}{9}$
i $a^2 + \frac{3a}{2} + \frac{9}{16}$

5 Factorise:

a $2x^2 + 18x + 40$
b $3x^2 - 30x + 63$
c $5x^2 - 50x + 120$
d $2x^2 + 8x - 90$
e $3x^2 - 6x - 105$
f $2x^2 - 6x - 260$

6 Factorise:

a $25a^2 - 16b^2$
b $a^2 + 14a + 49$
c $a^2 - a - 20$
d $1 - 36m^2$
e $4 - 9x^2y^2$
f $\frac{1}{9} - \frac{a^2}{25}$
g $m^2 - \frac{1}{4}$
h $x^3 - 49xy^2$
i $3a^2 - 75$
j $b^2 - 20b + 96$
k $n^2 - 31n + 150$
l $m^2 + 20m + 91$
m $a^2 - 7ab - 98b^2$
n $m^2 - 4m - 165$
o $x^2 + 3xy - 4y^2$
p $20m^2n - 5n^3$
q $a^2 - 2a - 63$
r $5q^2 - 5pq - 30p^2$
s $x^2 - 3x - 130$
t $42 - x - x^2$
u $x^2 + 7x - 18$

Challenge exercise

1 Factorise:

a $x^4 - 1$
b $x^4 - 16$
c $x^2 - 3$
d $x^2 - 5$
e $x^2 - 2\sqrt{2}x + 2$
f $x^2 + 2\sqrt{2}x + 2$
g $x^2 + x + \frac{1}{4}$
h $x^2 + 3x + \frac{9}{4}$
i $x^2 - x + \frac{1}{4}$
j $x^2 - 3x + \frac{9}{4}$
k $x^2 - xy - 2y^2$
l $x^2 + xy - 2y^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 Simplify:

a $\dfrac{x^2+x-2}{x^2-x-20}\times\dfrac{x^2+5x+4}{x^2-x}\div\left(\dfrac{x^2+3x+2}{x^2-2x-15}\times\dfrac{x+3}{x^2}\right)$

b $\dfrac{x^2-64}{x^2+24x+128}\times\dfrac{x^2+12x-64}{x^2-16}\div\dfrac{x^2-16x+64}{x^2-10x+16}$

c $\dfrac{x^2-18x+80}{x^2-5x-50}\times\dfrac{x^2-6x-7}{x^2-15x+56}\div\dfrac{x-1}{x+5}$

3 Factorise:

a $(x+2)^2-8x$

b $(x-3)^2+12x$

c $(x+a)^2-4ax$

d $(x-a)^2+4ax$

4 Simplify $\left(\dfrac{a}{b}+\dfrac{c}{d}\right)\div\left(\dfrac{a}{b}-\dfrac{c}{d}\right)$.

5 Factorise:

a $(x^2+1)^2-4$

b $(x^2-2)^2-4$

c $x^2+4x+4-y^2$

d $x^2+8x+16-a^2$

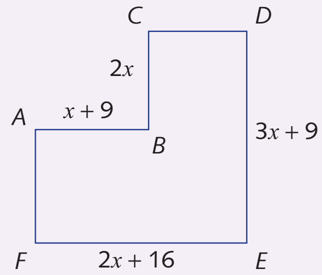
e $m^2-2m+1-n^2$

f $p^2-5p+\dfrac{25}{4}-q^2$

6 **a** By adding and subtracting $4x^2$, factorise x^4+4.

b Factorise x^4+4a^4.

7 A swimming pool is designed in an L-shape with dimensions in metres as shown.



The pool is enlarged or reduced depending on the value of x.

a Find, in terms of x, the length of:

i AF **ii** CD

b Show that the perimeter is equal to $(10x+50)$ m.

c What is the perimeter if $x=3$?

d Find the area of the swimming pool in terms of x. Expand and simplify your answer.

e It is decided that a square swimming pool would be a better use of space.

i By factorising your answer to part **d**, find the dimensions, in terms of x, of a square swimming pool with the same area as the L-shaped swimming pool.

ii What is the perimeter, in terms of x, of this square swimming pool?

8 For positive whole numbers a and b, prove that:

a if $\dfrac{a}{b}<1$ then $\dfrac{a+1}{b+1}>\dfrac{a}{b}$

b if $\dfrac{a}{b}>1$ then $\dfrac{a+1}{b+1}<\dfrac{a}{b}$

9 A right-angled triangle has a hypotenuse of length b cm and one other side of length a cm. If $b-a=1$, find the length of the third side in terms of a and b.

10 Factorise $\left(1+\dfrac{y^2+z^2-x^2}{2yz}\right)\div\left(1-\dfrac{x^2+y^2-z^2}{2xy}\right)$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

5

Number and Algebra

Linear equations and inequalities

We have seen that many real-world problems can be converted to equations. We have also seen how to solve simple equations. In this chapter, we consider a wider variety of equations and use them to solve practical problems.

Equations arise naturally when solving problems. In fact, a lot of problem-solving relies on us being able to translate a given word or real world problem into an equation, or equations, solve the equation(s) and relate the solution to the original problem. Turning a complicated problem into an equation enables us to understand and solve difficult problems. Before we reach this stage, we need to have at hand a collection of techniques for solving equations.

In earlier times, people used a number of ad hoc methods for solving equations. Only since the development of modern algebra have standard procedures and notations been introduced that enable us to solve equations quickly and efficiently.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5A Expressions

Two examples of **linear expressions** are $2x + 3$ and $x + 7$. The expression $2x + 3$ is called **linear** because the graph of $y = 2x + 3$ is a straight line, as we saw in Year 8. We begin with some revision examples and exercises.

Example 1

Brian is 6 cm taller than Geoff. Represent this information algebraically.

Solution

Since we do not know how tall Geoff is, we call his height x cm.

Then Brian is $(x + 6)$ cm tall.

Example 2

The length of a rectangle is 3 m more than its width. The width of the rectangle is w m.

a Express the length of the rectangle in terms of w.

b Express the perimeter of the rectangle in terms of w.

Solution

a Length of rectangle $= (w + 3)$ m

b Perimeter $= w + w + (w + 3) + (w + 3)$

$= (4w + 6)$ m

Exercise 5A

Example 1

1 Fiona is 5 cm taller than Tristan. Tristan's height is h cm. What is Fiona's height?

2 Seuret weighs 2 kg less than his older sister Vivian. If Vivian weighs w kg, what is Seuret's weight?

3 When Andriana's age is doubled, the number is 3 more than Helen's age. If Andriana's age is x years, what is Helen's age?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 2

4 The length of a rectangle is 5 metres more than its width.

a If w metres is the width of the rectangle, express the length of the rectangle in terms of w.

b If ℓ metres is the length of the rectangle, express the width of the rectangle in terms of ℓ.

5 In a competition, Deeksha scored 18 points more than Greta and Deirdre scored 5 points less than twice the number of points Greta scored. If a is the number of points Greta scored:

a express the number of points Deeksha scored in terms of a

b express the number of points Deirdre scored in terms of a

6 The length of a rectangular paddock is 20 m less than three times its width. If the width of the paddock is x m, express the length of the paddock in terms of x.

7 In a triathlon race, Luca ran at an average speed 5 times his average swimming speed. Also, when his average running speed was multiplied by 4, this number was 3 less than his average speed for the cycling leg.

If x km/h is Luca's average swimming speed, find expressions in terms of x for his:

a average running speed

b average cycling speed

8 Match each of the following mathematical expressions with its corresponding English expression.

a	$4 + 2x$	**i**	Six less than four times x
b	$x - 5$	**ii**	Three times one more than x
c	$2x - 4$	**iii**	Two less than one-quarter of x
d	$3(x + 1)$	**iv**	One-quarter of two less than x
e	$4x - 6$	**v**	Four less than twice x
f	$\frac{x}{4} - 2$	**vi**	Six more than half of x
g	$\frac{x - 2}{4}$	**vii**	One more than three times x
h	$x + 6$	**viii**	Five less than x
i	$3x + 1$	**ix**	Six more than x
j	$\frac{x}{2} + 6$	**x**	Four more than twice x

9 Gemma is 6 cm shorter than Gavin and 4 cm taller than Brent. If x cm represents Gemma's height, express:

a Gavin's height in terms of x

b Brent's height in terms of x

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5B Solving simple linear equations

A statement such as $x + 7 = 11$ is called an **equation** and we may **solve** the equation to find a value for x that makes the statement true. In this case, the **solution** is $x = 4$.

Reading equations

The equation $2x + 4 = 10$ can be read as 'two times a number plus 4 is equal to 10'.

The instruction 'Solve the equation $2x + 4 = 10$' can also be read as 'A number is multiplied by 2 and 4 is then added. The result is 10. Find the number.'

In this case, the number is 3. The solution is $x = 3$.

Equivalent equations

Consider these equations:

$$2x + 7 = 11 \qquad (1)$$

$$2x + 9 = 13 \qquad (2)$$

Equation (2) is obtained from equation (1) by adding 2 to each side of the equation.

So equation (1) is obtained from equation (2) by subtracting 2 from each side of the equation.

Equations (1) and (2) are said to be **equivalent equations**.

Equation (3), below, is obtained from equation (1) by subtracting 7 from each side of the equation.

$$2x = 4 \qquad (3)$$

$$x = 2 \qquad (4)$$

Equation (4) is obtained from equation (3) by dividing each side of the equation by 2. You can obtain equation (3) from equation (4) by multiplying each side of the equation by 2.

All of the above equations are **equivalent**.

Equivalent equations

- If we add the same number to, or subtract the same number from, both sides of an equation, the new equation is **equivalent** to the original equation.
- If we multiply or divide both sides of an equation by the same non-zero number, the new equation is **equivalent** to the original equation.
- Equivalent equations have exactly the same solutions.

In the following examples, equivalent equations are formed to solve the equations.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 3

Solve:

a $3x + 5 = 20$ **b** $5x - 7 = 18$ **c** $3 - 2x = 15$ **d** $-3p = \frac{2}{5}$

Solution

a $3x + 5 = 20$

$3x = 15$ (Subtract 5 from both sides.)

$x = 5$ (Divide both sides by 3.)

b $5x - 7 = 18$

$5x = 25$ (Add 7 to both sides.)

$x = 5$ (Divide both sides by 5.)

c $3 - 2x = 15$

$-2x = 12$ (Subtract 3 from both sides.)

$x = -6$ (Divide both sides by -2.)

d $-3p = \frac{2}{5}$

$p = -\frac{2}{15}$ (Divide both sides by -3.)

Example 4

Solve:

a $3x + 7 = 2x + 13$ **b** $5a - 21 = 14 - 2a$

Solution

a $3x + 7 = 2x + 13$

$3x + 7 - 2x = 2x + 13 - 2x$ (Subtract $2x$ from both sides.)

$x + 7 = 13$

$x = 6$ (Subtract 7 from both sides.)

b $5a - 21 = 14 - 2a$

$7a - 21 = 14$ (Add $2a$ to both sides.)

$7a = 35$ (Add 21 to both sides.)

$a = 5$ (Divide both sides by 7.)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 5B

1 Solve these equations.

a $a + 2 = 5$ **b** $b + 7 = 19$ **c** $c - 6 = 11$

d $d - 15 = 3$ **e** $2a = 6$ **f** $3b = 9$

g $6d = 42$ **h** $-3m = 6$ **i** $-2n = 8$

j $9q = -27$ **k** $b + 7 = 29\frac{1}{2}$ **l** $3a = \frac{2}{3}$

m $x - 6 = 5\frac{1}{3}$ **n** $-4y = \frac{8}{9}$ **o** $2x = -\frac{5}{6}$

Example 3

2 Solve these equations.

a $2a + 5 = 7$ **b** $3b + 4 = 19$ **c** $3c - 1 = 20$

d $5d - 7 = 23$ **e** $4f - 3 = 13$ **f** $3g + 17 = 5$

g $5h + 21 = 11$ **h** $6a + 17 = -1$ **i** $4a + 23 = -9$

j $3a - 16 = -31$ **k** $7b - 17 = -66$ **l** $2b + 5 = 7\frac{1}{2}$

m $2x + 11 = 7\frac{1}{4}$ **n** $4m - 9 = -13\frac{4}{5}$ **o** $-2b + 4 = 9\frac{1}{4}$

3 Solve these equations.

a $2 - 3a = 8$ **b** $3 - 4b = 15$ **c** $5 - 2c = -13$

d $3 - 5d = -22$ **e** $-6 - 7e = 15$ **f** $-4 - 3f = 14$

Example 4

4 Solve these equations for x and check your answers.

a $5x + 5 = 3x + 1$ **b** $7x + 15 = 2x + 20$ **c** $9x - 7 = 7x + 3$

d $5x - 6 = x - 2$ **e** $4x + 7 = x - 2$ **f** $3x + 1 = 9 - x$

g $4x - 3 = 18 - 3x$ **h** $2x - 3 = 7 - x$ **i** $8 - 3x = 2x - 7$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

5C Equations with brackets

In this section, we look at solving equations in which brackets are involved. In previous work in this area, you have always expanded the brackets first. In some examples, we do not do this.

Example 5

Solve:

a $3(x + 2) = 21$ **b** $2(3 - x) = 12$

Solution

a
$$\begin{aligned} 3(x + 2) &= 21 \\ x + 2 &= 7 && \text{(Divide both sides by 3.)} \\ x &= 5 \end{aligned}$$

b
$$\begin{aligned} 2(3 - x) &= 12 \\ 3 - x &= 6 && \text{(Divide both sides by 2.)} \\ -x &= 3 \\ x &= -3 \end{aligned}$$

Example 6

Solve $3(x + 5) = 31$.

Solution

Method 1

$$\begin{aligned} 3(x + 5) &= 31 \\ 3x + 15 &= 31 \\ 3x &= 16 \\ x &= \frac{16}{3} \\ x &= 5\tfrac{1}{3} \end{aligned}$$

Method 2

$$\begin{aligned} 3(x + 5) &= 31 \\ x + 5 &= \frac{31}{3} \\ x &= \frac{16}{3} \end{aligned}$$

When brackets are involved, the equation is usually solved by expanding the brackets first. In the above example, method 1 is preferable to method 2.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7

Solve:

a $2(x+1)+4(x+3)=26$ **b** $3(a+5)=2(a+6)$ **c** $3(y-3)-2(y-4)=4$

Solution

a
$$\begin{aligned} 2(x+1)+4(x+3) &= 26 \\ 2x+2+4x+12 &= 26 \quad \text{(Expand the brackets.)} \\ 6x+14 &= 26 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

b
$$\begin{aligned} 3(a+5) &= 2(a+6) \\ 3a+15 &= 2a+12 \\ a+15 &= 12 \quad \text{(Subtract } 2a \text{ from both sides.)} \\ a &= -3 \end{aligned}$$

c
$$\begin{aligned} 3(y-3)-2(y-4) &= 4 \\ 3y-9-2y+8 &= 4 \quad \text{(Expand both sets of brackets, being careful with the signs.)} \\ y-1 &= 4 \\ y &= 5 \end{aligned}$$

Exercise 5C

Example 5

1 Solve for x.

a $2(x+3)=8$ **b** $3(x-2)=15$ **c** $4(x-1)=12$

d $5(x+1)=10$ **e** $3(5-x)=9$ **f** $4(8-x)=36$

g $-3(2x-6)=12$ **h** $-4(3x-7)=-8$ **i** $-2(7x+6)=86$

Example 6

2 Solve for x.

a $2(x+3)=15$ **b** $5(x-2)=16$ **c** $7(3-x)=20$

d $4(7-x)=7$ **e** $-2(x-5)=7$ **f** $-3(2x-1)=2$

g $5(x-3)=\frac{2}{3}$ **h** $2(2x-6)=\frac{2}{6}$ **i** $-3(5x+2)=\frac{1}{4}$

Example 7

3 Solve:

a $2(a+1)+4(a+2)=22$ **b** $4(b-1)+3(b+2)=30$

c $5(c+2)-2(c+1)=17$ **d** $4(2d+1)-5(d-2)=17$

e $2(x+3)-3(x-4)=20$ **f** $5(2y-3)-3(y-5)=21$

g $5(a+3)=3(2a+1)$ **h** $5(2a-1)=2(3a+2)$

i $-2(x+4)+3(x-2)=16$ **j** $-5(x+3)-4(x+1)=17$

k $\frac{1}{2}(2x+5)+6(x-2)=4\frac{1}{2}$ **l** $\frac{1}{2}(4x+1)+2(x-2)=13$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

5D Linear equations involving fractions

When there are fractions in equations, the standard procedure is to remove the fractions by multiplying both sides of the equation by an appropriate whole number. The next step is to remove the brackets.

Example 8

Solve:

a $2x + \frac{1}{2} = \frac{2}{3}$

b $3x - \frac{1}{4} = \frac{4}{5}$

Solution

a

$$\begin{aligned} 2x + \frac{1}{2} &= \frac{2}{3} \\ 6\left(2x + \frac{1}{2}\right) &= 6 \times \frac{2}{3} \\ 12x + 3 &= 4 \\ 12x &= 1 \\ x &= \frac{1}{12} \end{aligned}$$

(Multiply both sides by 6, the lowest common multiple of the denominators.)

b

$$\begin{aligned} 3x - \frac{1}{4} &= \frac{4}{5} \\ 20\left(3x - \frac{1}{4}\right) &= 20 \times \frac{4}{5} \\ 60x - 5 &= 16 \\ 60x &= 21 \\ x &= \frac{21}{60} \\ &= \frac{7}{20} \end{aligned}$$

(Multiply both sides by 20.)

Example 9

Solve:

a $\frac{2x}{3} + \frac{1}{5} = 4$

b $10 - \frac{a+3}{4} = 6$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a

$$\begin{aligned}\frac{2x}{3} + \frac{1}{5} &= 4\\ 15\left(\frac{2x}{3} + \frac{1}{5}\right) &= 15 \times 4\\ 15 \times \frac{2x}{3} + 15 \times \frac{1}{5} &= 15 \times 4\\ 10x + 3 &= 60\\ 10x &= 57\\ x &= \frac{57}{10} \text{ or } x = 5\tfrac{7}{10}\end{aligned}$$

b

$$\begin{aligned}10 - \frac{a+3}{4} &= 6\\ 4 \times 10 - 4 \times \frac{a+3}{4} &= 4 \times 6\\ 40 - (a+3) &= 24\\ 37 - a &= 24\\ -a &= -13\\ a &= 13\end{aligned}$$

Example 10

Solve $2.1x + 3.5 = 9.4$

Solution

$$\begin{aligned}2.1x + 3.5 &= 9.4\\ 21x + 35 &= 94 \qquad \text{(Multiply both sides by 10.)}\\ 21x &= 59\\ x &= \frac{59}{21}\end{aligned}$$

Example 11

Solve $\frac{2x}{3} - 3 = x + \frac{3}{4}$.

Solution

$$\begin{aligned}\frac{2x}{3} - 3 &= x + \frac{3}{4}\\ 8x - 36 &= 12x + 9 \qquad \text{(Multiply both sides by 12.)}\\ -4x &= 45\\ x &= -\frac{45}{4}\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 12

Solve $\dfrac{a+5}{4} = \dfrac{a+3}{3}$.

Solution

$$\begin{aligned} \frac{a+5}{4} &= \frac{a+3}{3} \\ 3(a+5) &= 4(a+3) && \text{(Multiply both sides by 12.)} \\ 3a+15 &= 4a+12 \\ 15 &= a+12 \\ 3 &= a \\ a &= 3 \end{aligned}$$

Solving linear equations

To solve linear equations:

- Remove all fractions by multiplying both sides of the equation by the lowest common multiple of the denominators.
- Remove all brackets.
- Collect like terms and solve the equation.

Exercise 5D

Example 8

1 Solve:

a $2x - \dfrac{1}{2} = \dfrac{1}{4}$ **b** $3x + \dfrac{3}{2} = \dfrac{5}{3}$ **c** $\dfrac{4}{3} - 2y = \dfrac{1}{2}$

d $\dfrac{7}{2} - 3y = \dfrac{2}{3}$ **e** $\dfrac{2}{3} + 4x = \dfrac{2}{3}$ **f** $-\dfrac{3}{4} - 5y = \dfrac{2}{5}$

g $-2x + \dfrac{1}{3} = \dfrac{1}{5}$ **h** $\dfrac{2}{3} + 3x = \dfrac{1}{5}$ **i** $-3y - \dfrac{2}{5} = \dfrac{1}{4}$

Example 9a

2 Solve:

a $\dfrac{a}{3} + 5 = 3$ **b** $\dfrac{3a}{4} - \dfrac{4}{5} = \dfrac{2}{3}$ **c** $\dfrac{b}{3} - 5 = 3$

d $\dfrac{3b}{7} + 6 = 2$ **e** $2 - \dfrac{x}{3} = 6$ **f** $3 - \dfrac{3x}{4} = 6$

g $\dfrac{2}{3}(m-3) = 1$ **h** $3\left(\dfrac{m}{5} + 2\right) = 2$ **i** $-3\left(\dfrac{x}{6} + 1\right) = 4$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 9b

3 Solve:

a $\frac{2y+1}{3} + 4 = 7$ **b** $\frac{2x+5}{3} = 9$ **c** $\frac{5p-2}{4} - 1 = 6$

d $\frac{3y-1}{2} + 2 = 9$ **e** $\frac{2x-3}{5} = 3$ **f** $\frac{4a+3}{5} - 2 = 1$

g $2 + \frac{2x-1}{5} = 5$ **h** $18 - \frac{7x+2}{3} = 8$ **i** $1 - \frac{5y-3}{4} = -2$

Example 10

4 Solve:

a $e + 1.8 = 2.9$ **b** $f + 3.6 = 7.5$ **c** $g - 2.8 = 3.8$

d $1.2u = 15.6$ **e** $3.6r = 9$ **f** $-4.7x = 49.35$

g $1.2x + 4 = 10$ **h** $3.8x - 7 = 8.2$ **i** $4.6x - 2.6 = 25$

Example 11

5 Solve these equations for x and check your answers.

a $\frac{3x}{4} - 1 = \frac{x}{2} + 3$ **b** $\frac{x}{3} + 2 = \frac{4x}{3} + 3$ **c** $\frac{5x}{6} - 3 = 7 - \frac{x}{3}$

d $\frac{x}{3} - 4 = 6 - \frac{2x}{5}$ **e** $\frac{5}{3} - \frac{x}{2} = \frac{3x}{4} + \frac{7}{6}$ **f** $\frac{11}{12} - \frac{5x}{6} = \frac{3x}{4} - \frac{2}{3}$

6 Solve for x.

a $1.6x + 10 = 0.9x + 12$ **b** $5.9x - 7 = 2.4x + 35$ **c** $8.3x + 12 = 36 - 1.7x$

d $4.8 - 1.3x = 23 + 1.3x$ **e** $1.5x + 3.9 = 6.7 - 0.5x$ **f** $11.8x + 7.6 = 61.6 - 1.7x$

Example 12

7 Solve for a.

a $\frac{a+3}{2} = \frac{a+1}{5}$ **b** $\frac{a+1}{3} = \frac{2a-1}{7}$

c $\frac{2a+1}{3} = \frac{3a+1}{4}$ **d** $\frac{a+1}{2} + 1 = \frac{a-1}{5}$

e $\frac{3a+2}{4} + 2 = a$ **f** $\frac{2a+1}{2} + \frac{a}{3} = 4$

g $\frac{3a-2}{4} = \frac{a-5}{2}$ **h** $\frac{2a-1}{3} - 2 = \frac{a+3}{4}$

i $\frac{4a-1}{3} + a = 2$ **j** $\frac{a}{2} + \frac{a-1}{3} = \frac{a+1}{4}$

8 Solve:

a $5a + 9 = 24$ **b** $3b - 7 = 32$

c $\frac{c}{3} - 2 = 4$ **d** $\frac{d}{2} + 6 = 3$

e $\frac{4e}{3} + \frac{1}{2} = 2$ **f** $\frac{2f}{3} - 1 = \frac{3}{7}$

g $2g + 5 = 7g - 6$ **h** $4h - 2 = 5 - 3h$

i $2(i-1) = 5(i+6)$ **j** $3(j+2) = 2(2j-1)$

k $2(k+1) - 3(k-2) = 7$ **l** $4(\ell - 1) + 3(\ell + 2) = 8$

m $\frac{m+1}{3} = \frac{m-2}{5}$ **n** $\frac{2n-1}{3} = \frac{4n+1}{5}$

o $\frac{2q-2}{5} + 1 = 4q$ **p** $\frac{2r+1}{3} + 2 = \frac{3r-1}{4}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

5E Using linear equations to solve problems

Problem-solving often involves introducing algebra, translating the problem into an equation and then solving the equation. An important first step is to introduce an appropriate pronumeral for one of the unknown quantities.

Example 13

Three children earn weekly pocket money. Andrew earns \$2 more than Gina, and Katya earns twice the amount Gina earns. The total of the weekly pocket money is \$22.

a How much money does Gina earn?

b How much money do Andrew and Katya earn?

Solution

a Let $\$m$ be the amount of pocket money Gina earns in a week.

Then Andrew earns $\$(m + 2)$ and Katya earns $\$(2m)$,

so $\quad m + (m + 2) + 2m = 22 \quad$ (The total weekly pocket money is \$22.)

$$\begin{aligned} 4m + 2 &= 22 \\ 4m &= 20 \\ m &= 5 \end{aligned}$$

So Gina earns \$5 per week.

b Andrew earns \$7 and Katya earns \$10 per week.

Example 14

Ali and Jasmine each have a number of swap cards. Jasmine has 25 more cards than Ali, and in total the two children have 149 cards.

a How many cards does Ali have?

b How many cards does Jasmine have?

Solution

a Let x be the number of cards Ali has.

Jasmine has $(x + 25)$ cards.

Total number of cards is 149,

so $x + (x + 25) = 149$

$$\begin{aligned} 2x + 25 &= 149 \\ 2x &= 124 \\ x &= 62 \end{aligned}$$

So Ali has 62 cards.

b Jasmine has $62 + 25 = 87$ cards.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Harder examples involving rates

Speed is one of the most familiar rates. In problems involving speed, we use the relationship:

$$\textit{average speed} = \frac{\textit{distance travelled}}{\textit{time taken}}$$

or
$$\textit{time taken} = \frac{\textit{distance travelled}}{\textit{average speed}}$$

or $\textit{distance travelled} = \textit{average speed} \times \textit{time taken}$

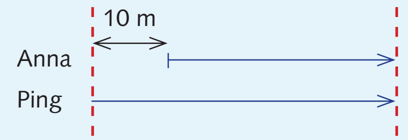
Example 15

Ping and Anna compete in a handicap sprint race. Anna starts the race 10 m ahead of Ping. Ping runs at an average speed that is 20% faster than Anna's average speed. The two sprinters will be level in the race after 9 seconds. Find the average speed of:

a Anna **b** Ping

Solution

a The diagram below represents the situation.



Let x be Anna's average speed, measured in m/s.

$$\text{Ping's average speed is 120\% of } x = \frac{120x}{100}$$
$$= \frac{6x}{5} \text{ m/s}$$

We use distance = average speed × time taken

After 9 seconds, Anna has run a distance of $9x$ m and Ping has run a distance of $\frac{6x}{5} \times 9 = \frac{54x}{5}$ m.

Anna started 10 m in front of Ping. So when they are level,

$$\begin{aligned} 9x + 10 &= \frac{54x}{5} \\ 45x + 50 &= 54x \\ 50 &= 9x \\ x &= \frac{50}{9} \\ x &= 5\tfrac{5}{9} \end{aligned}$$

So Anna runs at an average speed of $5\frac{5}{9}$ m/s.

b Ping runs at $\frac{6x}{5} = \frac{6}{5} \times \frac{50}{9} = 6\frac{2}{3}$ m/s.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 16

For a training run, a triathlete covers 50 km in $4\frac{1}{4}$ hours. She runs part of the way at a speed of 10 km/h, cycles part of the way at a speed of 40 km/h and swims the remaining distance at a speed of $2\frac{1}{2}$ km/h. The athlete runs for twice the time it takes to complete the cycle leg. How long did she take to complete the cycle leg?

Solution

Let t hours be the time for the cycle leg.

Then $2t$ hours is the time for the running leg and $(4\frac{1}{4} - t - 2t)$ hours is the time for the swim leg.

Now, distance of run + distance of cycle + distance of swim = total distance,

$$\begin{aligned}
\text{so}\quad 10 \times 2t + 40 \times t + 2\tfrac{1}{2} \times \left(4\tfrac{1}{4} - t - 2t\right) &= 50\\
20t + 40t + \frac{5}{2}\left(\frac{17}{4} - 3t\right) &= 50\\
60t + \frac{85}{8} - \frac{15t}{2} &= 50\\
480t + 85 - 60t &= 400\\
420t &= 315\\
t &= \frac{315}{420}\\
t &= \frac{3}{4}
\end{aligned}$$

The athlete takes $\frac{3}{4}$ hour, or 45 minutes, to complete the cycle leg.

Exercise 5E

In each of the following, form an equation and solve.

1 Jacques thinks of a number x. When he adds 17 to his number, the result is 32. What is the value of x?

2 When 16 is added to twice Simone's age, the answer is 44. How old is Simone?

3 When 14 is added to half of Suzette's weight in kilograms, the result is 42. How much does Suzette weigh?

4 Yolan buys 8 pens and receives 80 cents change from \$20.00. How much does a pen cost, assuming each pen costs the same amount?

5 If the sum of $2p$ and 19 is the same as the sum of $4p$ and 11, find the value of p.

6 If the sum of half of q and 6 is equal to the sum of one-third of q and 2, find the value of q.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 Derek is presently 20 years older than his daughter, Alana.

a If x represents Alana's present age, express each of the following in terms of x.

i Derek's present age

ii Alana's age in 12 years' time

iii Derek's age in 12 years' time

b If Derek's age 12 years from now is twice Alana's age 12 years from now, find their present ages.

8 Alan, Brendan and Calum each have a number of plastic toys from a fast food store. Brendan has 5 more toys than Alan, and Calum has twice as many toys as Alan.

a If x represents the number of toys Alan has, express each of the following in terms of x:

i the number of toys Brendan has

ii the number of toys Calum has

iii the total number of toys the three boys have

b If the boys have 37 toys in total, determine how many toys each boy has.

9 The length of a swimming pool is 2 m more than four times its width.

a If x metres represents the width of the pool, express the length of the pool in terms of x.

b If the perimeter of the pool is 124 m find the length and width of the pool.

10 Ms Minas earns \$3600 more than Mr Brown, and Ms Lee earns \$2000 less than Mr Brown.

a If \$$x$ represents Mr Brown's salary, express the salary of:

i Ms Minas in terms of x

ii Ms Lee in terms of x

b If the total of the three incomes is \$151 600, find the income of each person.

11 Dudley has a number of 10-cent and 20-cent coins. If their total value is \$6, how many are there of each coin if there are:

a equal numbers of each coin?

b twice as many 10-cent coins as there are 20-cent coins?

c twice as many 20-cent coins as there are 10-cent coins?

12 The distance between two towns, A and B, is 350 km. Find x if:

a the trip takes x hours at an average speed of 80 km/h

b the trip takes $3\frac{1}{2}$ hours at an average speed of x km/h

c the trip takes a total of 3 hours, travelling x hours at 120 km/h and the remaining time at 60 km/h

13 A classroom has a length 6 m shorter than its width and a perimeter of 30 m. Find the length and width of the classroom.

14 Of the total amount of money in his wallet, a man gave one-third to his daughter and one-quarter to his son. After this he only had \$250 left. How much was originally in his wallet?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

15 A student has an average mark of 68 from 10 tests. What mark must be gained in the next test to raise his average to 70?

16 Luisa travels 25 km in $3\frac{3}{4}$ hours. She walks part of the way at 4 km/h and cycles the rest at 12 km/h. How far did she walk?

17 A man is four times as old as his son. In five years he will be only three times as old as his son. What is the man's present age?

18 A bottle of cordial contains 5 litres, of which 10% is pure fruit juice. How many litres of water must be added to dilute it to 4% fruit juice?

19 A coffee blender mixes two types of coffee. Type A costs \$13 per kg and type B costs \$18 per kg. How many grams of each type of coffee should be blended so that 1 kg costs \$15?

20 A collection of coins consisting of 10-cent, 20-cent and 50-cent pieces has a value of \$4.50. The number of 20-cent pieces is twice the number of 10-cent pieces and the number of 50-cent pieces is 3 less than twice the number of 10-cent pieces. How many 10-cent, 20-cent and 50-cent pieces are there in the collection?

21 If 4 litres of laboratory alcohol is 80% pure (80% alcohol, 20% water), how many litres of water must be added so that the resulting mixture will contain 30% alcohol?

22 A rectangular garden pond has a length 60 cm more than its width. Let the width of the pond be w cm.

a Find the length of the pond in terms of w.

b If the length of the pond is 150 cm, what are the dimensions of the pond?

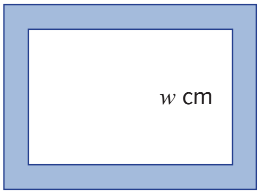
The pond has a w wide path around its perimeter.

c Express, in terms of w, the:

i length

ii width

of the outer rectangle formed by the path.



d Express, in terms of w, the area of the path.

e If the width of the pond is 120 cm, find the area of the path.

f If the area of the path is 5.6 m^2, find the width of the pond.

23 A father is concerned about his daughter's progress in mathematics. In order to encourage her, he agrees to give her 10 cents for every problem she solves correctly and to penalise her 15 cents for every problem she gets wrong. The girl completed 22 problems for homework. Let the number of questions she got right be x.

a How much money does the father have to give his daughter for getting x questions correct?

b How much does the daughter have to pay in penalties for the incorrect questions?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Using the fact that the girl made a profit of 20 cents, write down an equation involving x.

d How many questions did the girl get correct?

24 During a power failure, Sonia lights two identical candles of length 10 cm at 7 p.m. One candle burns out by 11 p.m. and the other candle burns out at 12 midnight. Assume that the length of each candle goes down by a constant rate.

a How high is each candle at 8 p.m.?

b How high is each candle at 9 p.m.?

c How high is each candle t hours after 7 p.m.?

d At what time is one candle twice as high as the other candle?

5F Literal equations

In this chapter, we have discussed methods for solving linear equations.

Sometimes equations arise in which some of the coefficients are pronumerals. These are called **literal equations**. We need to be told which pronumeral we are solving for.

Example 17

Solve:

a $ax + b = c$ for x

b $a(x + b) = c$ for x

Solution

a

$$\begin{aligned} ax + b &= c \\ ax &= c - b && \text{(Subtract } b \text{ from both sides.)} \\ x &= \frac{c - b}{a} && \text{(Divide both sides by } a\text{.)} \end{aligned}$$

b

$$\begin{aligned} a(x + b) &= c \\ ax + ab &= c \\ ax &= c - ab \\ x &= \frac{c - ab}{a} \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 18

Solve $mx - n = nx + m$ for x.

Solution

$$
\begin{aligned}
mx - n &= nx + m \\
mx - nx - n &= m && \text{(Subtract } nx \text{ from both sides.)} \\
mx - nx &= m + n && \text{(Add } n \text{ to both sides.)} \\
x(m - n) &= m + n && \text{(Factorise the left-hand side.)} \\
x &= \frac{m + n}{m - n} && \text{(Divide both sides by } m - n\text{.)}
\end{aligned}
$$

Exercise 5F

Example 17

1 Solve each of these equations for x.

a $x + b = c$

b $x - d = e$

c $p - x = q$

d $-x + m = n$

e $cx = b$

f $c - bx = e$

g $a(x + b) = c$

h $m(nx + p) = n$

i $\frac{x}{a} = b$

j $\frac{x + a}{b} = c$

k $\frac{x}{a} + b = c$

l $\frac{mx}{n} = p$

m $\frac{ax}{b} + c = d$

n $\frac{ax + b}{c} = d$

o $\frac{mx - n}{n} = m$

p $\frac{x}{f} + \frac{g}{h} = k$

q $\frac{x}{a} - \frac{a}{b} = b$

r $\frac{x}{b} - b = \frac{a}{b}$

Example 18

2 Solve each of these equations for x.

a $ax + b = cx + d$

b $mx + n = nx - m$

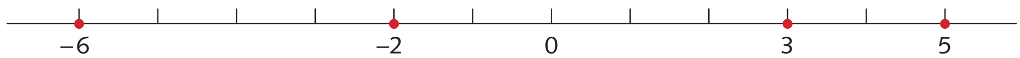
c $a(x + b) = cx + d$

d $a(x - b) = c(x - d)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5G Inequalities

The numbers -6, -2, 3 and 5 are shown on a number line.



Using the 'greater than' sign, we write $5 > 3$ because 5 is to the right of 3. Similarly, $3 > -2$ and $-2 > -6$.

We can also use the 'less than' sign and write $3 < 5, -2 < 3$ and $-6 < -2$. Notice that every negative number is less than 0. For example, $-6 < 0$.

The symbols ≤ and ≥

The symbol $\leq$ means 'is less than or equal to'.

The symbol $\geq$ means 'is greater than or equal to'.

The statement '$4 \leq 6$' means that either '$4 < 6$' or '$4 = 6$' is true. It is correct because the statement '$4 < 6$' is true. It does not matter that the second statement, '$4 = 6$', is false.

It is also correct to say that $3 \geq 3$, because the statement '$3 = 3$' is true. Once again, it does not matter that the statement '$3 > 3$' is false.

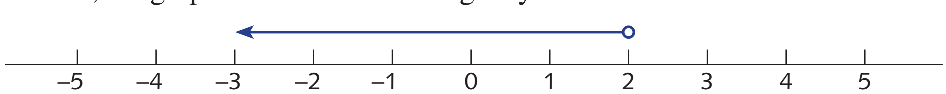
Statements such as '$-6 < 2$', '$5 > 3$', '$4 \leq 6$' and '$3 \geq 3$' are called **inequalities**.

Sets and intervals

We will need ways of describing and graphing various collections of numbers.

An example is the collection of all numbers that are less than 2.

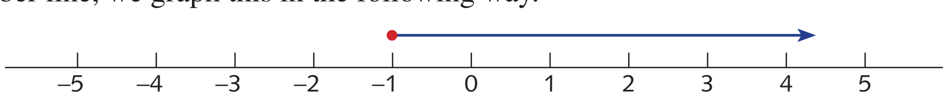
On the number line, we graph this in the following way.



The open dot, is used to indicate that 2 is not in the collection. The collection we are interested in is the interval that begins at 2 (but does not include 2) and extends indefinitely to the left, as indicated. The arrow head means 'goes on indefinitely'.

Another example is the collection of all numbers that are greater than or equal to -1.

On the number line, we graph this in the following way.



Here, the filled dot indicates that -1 is included in the interval. The interval begins at and includes -1, and extends indefinitely to the right.

The word 'set' means the same as 'collection'. Sets are indicated by curly brackets, {,}. For example, 'the set of numbers x that are less than 2' can be written as $\{x: x < 2\}$. When we write $\{x: x \geq 1\}$, it represents 'the set of all numbers x that are greater than or equal to 1'.

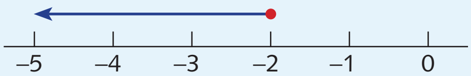
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

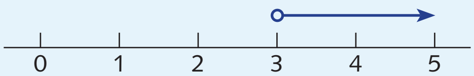
Example 19

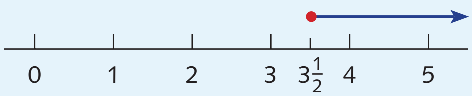
Graph each set on the number line.

a $\{x: x \le -2\}$ **b** $\{x: x > 3\}$ **c** $\left\{x: x \ge 3\frac{1}{2}\right\}$

Solution

a


b


c


Exercise 5G

1 Copy and insert > or < to make each statement true.

a 7 … 2 **b** 3 … –4 **c** –4 … –2

d –54 … –500 **e** –6 … 0 **f** –13 … –45

g 21 … 40 **h** –2 … 5 **i** 99 … –100

2 Copy and insert ≤ or ≥ to make each statement true.

a –7 … –2 **b** 5 ... –7 **c** –5 … –5

d –10 … –50 **e** 0 … 0 **f** –23 … –45

g 12 … 26 **h** 9 … 9 **i** 98 … 89

Example 19

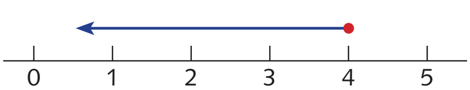
3 Graph each set on the number line.

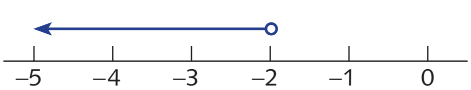
a $\{x: x > 10\}$ **b** $\{x: x \le 0\}$ **c** $\{x: x < -4\}$

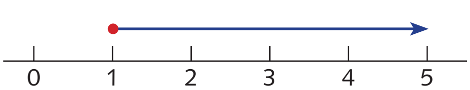
d $\{x: x \ge -2\}$ **e** $\left\{x: x < 2\frac{1}{2}\right\}$ **f** $\left\{x: x \le -1\frac{1}{2}\right\}$

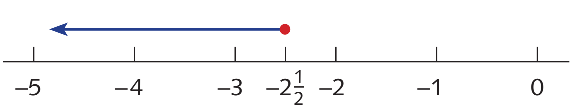
4 Use set notation to describe each interval.

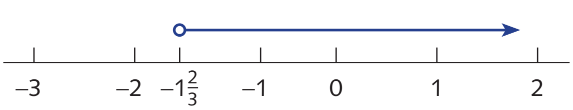
a


b


c


d


e


f


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

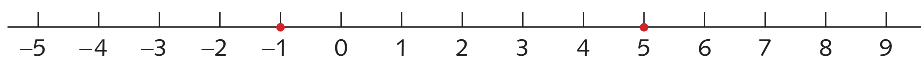
5H Solving linear inequalities

We first look at the effects of addition, subtraction, multiplication and division on inequalities.

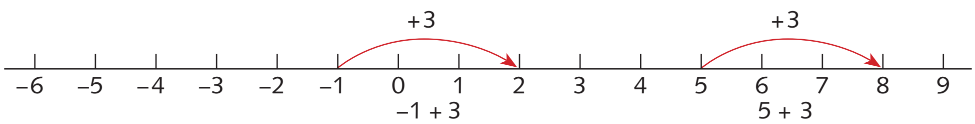
Addition and subtraction

We know that $-1 < 5$.

On the number line:

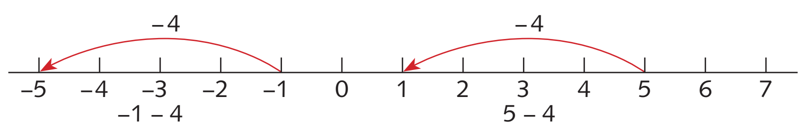


If we add 3 to each number, each moves 3 to the right.



$$-1 + 3 < 5 + 3$$

If we subtract 4 from each number, each moves 4 to the left.



$$-1 - 4 < 5 - 4$$

So we see that in each case the inequality still holds.

Addition and subtraction

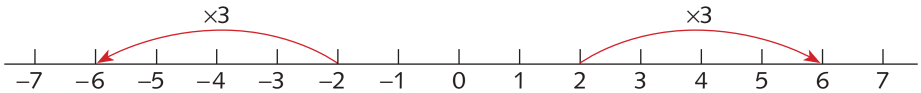
- If we add the same number to both sides of an inequality, then the resulting inequality is true.
- If we subtract the same number from both sides of an inequality, then the resulting inequality is true.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Multiplication and division

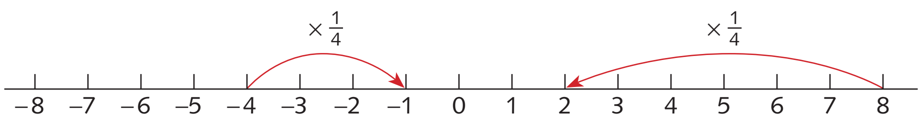
Multiplication by a positive number

If we multiply any number by 3, it moves to 3 times the distance it was **from 0** and stays on the same side of 0. The effect of multiplying both sides of an inequality by 3 is shown in a diagram.



$$-2 < 2 \text{ so } 3 \times -2 < 3 \times 2$$

If we multiply by $\frac{1}{4}$, then the new number is $\frac{1}{4}$ of the distance **from 0**, but still on the same side of 0. The effect on an inequality can be seen in a diagram.

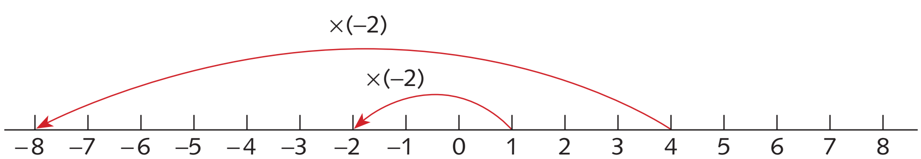


$$-4 < 8 \text{ so } \frac{1}{4} \times -4 < \frac{1}{4} \times 8$$

So the inequality remains true if we multiply both sides by a positive number.

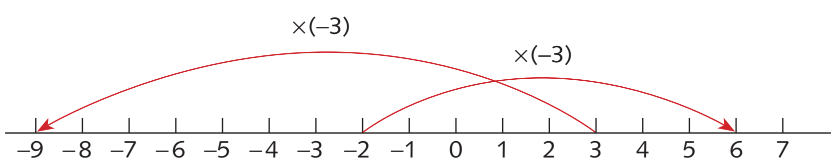
Multiplication by a negative number

We know that 1 < 4. If we multiply both numbers by −2, they move to the opposite side of 0, and are twice as far from 0 as they were before.



$$1 < 4 \text{ but } -2 \times 1 > -2 \times 4$$

Similarly,

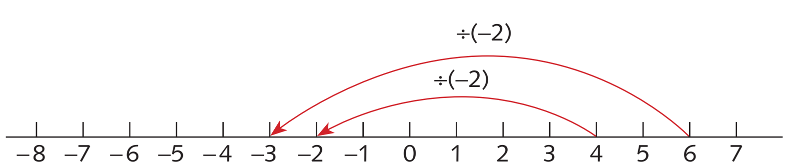


$$3 > -2 \text{ but } -3 \times 3 < -3(-2)$$

That is, in both cases the inequality sign is *reversed*.

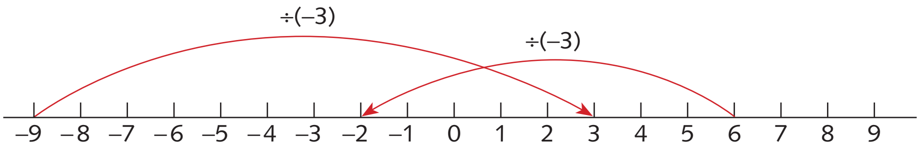
Division by a negative number

These diagrams explain what happens to an inequality when we divide both sides by a negative number.



$$4 < 6 \text{ but } 4 \div (-2) > 6 \div (-2)$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7



$6 > -9$ but $6 \div (-3) < -9 \div (-3)$

Multiplication and division

- If we multiply or divide both sides of an inequality by a positive number, then the resulting inequality is true.
- If we multiply or divide both sides of an inequality by a negative number, then we must reverse the inequality sign to make the resulting inequality true.

Linear inequalities

Solving the linear inequality $4x - 5 < 3$ means finding all of the values for x that satisfy that inequality.

It is easy to find some numbers that satisfy the inequality. For example, if $x = -2$,

$$4x - 5 = -13 < 3$$

Similarly, if $x = 0$,

$$4x - 5 = -5 < 3$$

However, $x = 4$ does not satisfy the inequality, because when $x = 4$,

$$4x - 5 = 11 > 3$$

So $x = -2$ and $x = 0$ satisfy the inequality, but $x = 4$ does not.

There is a systematic way of solving linear inequalities, as shown in the following examples. We often graph the solution set on the number line.

The method for solving linear inequalities is the same as that for solving linear equations, except for multiplying and dividing both sides by a negative number.

Example 20

a Solve the inequality $4x - 5 < 3$.
b Graph the solution set on the number line.

Solution

a If x satisfies the inequality

$$4x - 5 < 3$$

then $4x < 8$ (Add 5 to both sides.)

so $x < 2$ (Divide both sides by 4.)

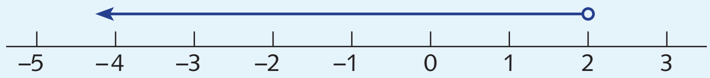
(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Conversely, if $x < 2$, then $4x < 8$ and $4x - 5 < 8 - 5 = 3$.

The solution is $\{x: x < 2\}$.

b The solution set is indicated on the number line.



Example 21

Solve each of the following inequalities.

a $-2x \le 6$

b $-\frac{x}{3} > 4$

Solution

a $-2x \le 6$

$x \ge -3$ (Divide both sides by -2 and ***reverse*** the inequality sign.)

b $-\frac{x}{3} > 4$

$x < -12$ (Multiply both sides by -3 and ***reverse*** the inequality sign.)

In Example 20, '$x < 2$', '$4x < 8$' and '$4x - 5 < 3$' are examples of **equivalent** inequalities. Equivalent inequalities have exactly the same solutions.

Two standard methods of setting out equivalent inequalities are shown in the next example.

Example 22

a Solve the inequality $2x + 3 < 3x - 4$.

b Graph the solution set on the number line.

Solution

a **Method 1**

$2x + 3 < 3x - 4$

$3 < x - 4$ (Subtract $2x$ from both sides.)

$7 < x$ (Add 4 to both sides.)

so $x > 7$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Method 2

$2x + 3 < 3x - 4$

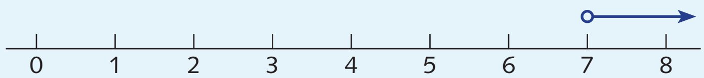
$2x < 3x - 7$ (Subtract 3 from both sides.)

$-x < -7$ (Subtract $3x$ from both sides.)

$x > 7$ (Multiply by -1 and reverse the inequality sign.)

The solution set is $\{x: x > 7\}$.

b The solution set is as shown:



Solving linear inequalities

- Multiply out brackets and multiply through by the lowest common denominator of fractions.
- Move all of the terms involving x to one side and all of the constant terms to the other side of the inequality sign.
- Simplify.
- Divide by the coefficient of x, remembering to reverse the inequality sign if the coefficient is negative.

Exercise 5H

1 **a** If $x < 6$, how many different values can x take?

b Give three values of x that satisfy $x > -1$.

c Give three non-whole numbers that satisfy $x > 2$.

Example 20

2 Solve each of these inequalities. Graph each solution set to parts **a–f** on a number line.

a $x + 3 \geq 7$ **b** $x - 2 < 3$ **c** $x + 4 \geq -8$

d $x - 10 > -6$ **e** $x - 5 > -12$ **f** $2x \geq 6$

g $3x > -15$ **h** $\frac{x}{5} \geq 4$ **i** $\frac{x}{2} \leq -3$

3 Solve:

a $2x + 1 \geq 5$ **b** $4x - 6 \leq -2$ **c** $\frac{2x}{3} > 5$

d $\frac{4x}{7} \leq -2$ **e** $3(x + 5) \geq 9$ **f** $2(5x - 2) > 5$

g $\frac{x}{3} - \frac{1}{2} \geq 1$ **h** $\frac{2x}{5} + \frac{1}{4} > 4$ **i** $2(x - 3) \leq 5$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 21

4 Solve:

a $-4x \le 20$ **b** $-10x \ge 130$ **c** $-12x > -42$

d $-\frac{x}{2} \le 5$ **e** $-\frac{x}{7} \ge 4$ **f** $-\frac{x}{5} > 4$

g $-\frac{x}{12} \ge -8$ **h** $-\frac{x}{2} \ge -8$ **i** $2 - \frac{x}{9} > 5$

5 Solve:

a $3 - 2x > 5$ **b** $2 - 5x \le -8$ **c** $4(7 - x) < 5$

d $6 - \frac{2x}{3} < 4$ **e** $4 - \frac{2x}{5} \ge 6$ **f** $8 - \frac{3x}{7} \ge 2$

Example 22

6 Solve:

a $\frac{x+3}{2} \le \frac{3-x}{2}$ **b** $\frac{2x-1}{3} - \frac{3x+2}{4} > 3$

7 Solve:

a $1.2x + 6.8 \le 15.2$ **b** $2.4 - 0.7x \le 12.9$

c $1.6(x + 7) \le 1.5(x - 3)$ **d** $2.8(x - 4) > 1.3(x + 3.5)$

8 Solve:

a $2x - 14 \le 8$ **b** $-5x + 3 \ge 78$

c $\frac{2x+1}{6} > -3$ **d** $-\frac{x+2}{3} \le 7$

e $\frac{x-8}{2} - \frac{2x}{3} \ge 3$ **f** $\frac{x}{4} > -\frac{x+12}{5}$

9 When 5 is added to twice p, the result is greater than 17. What values can p take?

10 When 16 is subtracted from half of q, the result is less than 18. What values can q take?

11 When $2p$ is subtracted from 10, the result is greater than or equal to 4. What values can p take?

12 The sum of $4d$ and 6 is greater than the sum of $2d$ and 18. What values can d take?

13 A number a is increased by 3 and this amount is then doubled. If the result of this is greater than a, what values can a take?

14 Two car hire firms offer the following deals.

Movit: \$25 plus 6 cents per km

Rentlow: \$20 plus 8 cents per km

a If a client is to drive x km, express the cost of hiring a car from:

- **i** Movit, in terms of x
- **ii** Rentlow, in terms of x

b For what distances does Movit offer the better deal?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Review exercise

1 David weighs 5 kg less than Christos. If Christos weighs w kg, express David's weight in terms of w.

2 When John's age is doubled, the number is 5 more than Kayla's age. If John is x years old, what is Kayla's age in terms of x?

3 The length of a rectangle is 10 m greater than the width of the rectangle.

a If w m is the width of the rectangle, express the length of the rectangle in terms of w.

b If the length of the rectangle is ℓ m, express the width of the rectangle in terms of ℓ.

4 Solve these equations.

a $a + 7 = 5$ **b** $-3m = 18$ **c** $4q = -27$

d $6a + 21 = -1$ **e** $7a - 26 = -35$ **f** $7b - 27 = -16$

g $-6 - 8e = 24$ **h** $-5 - 7f = 24$ **i** $5x - 4 = 11 - x$

j $18 - 7x = 3x - 15$ **k** $3 - 2x = 7 - 4x$ **l** $-5p + 8 = 6 - 7p$

5 Solve these equations.

a $5(x + 3) = 18$ **b** $5(x - 2) = 15$

c $12(x - 1) = 96$ **d** $7(x + 1) = 10$

e $7(3 - x) = 20$ **f** $6(11 - x) = 17$

g $6(a + 1) - 4(a + 2) = 24$ **h** $5(b - 1) - 8(b - 2) = 30$

i $5(a + 3) = 7(a + 11)$ **j** $15(3a - 1) = 2(6a + 7)$

k $-3(x - 2) = 2(3x - 1)$ **l** $-4(5m + 6) = -3(4 - 5m)$

6 Solve:

a $11 - 3.6c = 3.8$ **b** $12.6 - 4.5\ell = -5.4$ **c** $1.6(x + 7) = 17.6$

d $2.8(x - 4) = 16.8$ **e** $4.3(x + 11) = 53.75$ **f** $3.5(8 - x) = 29.5$

7 Solve these equations.

a $\frac{x}{3} - 2 = -7$ **b** $\frac{x}{2} + 6 = -3$

c $\frac{5p - 2}{4} + \frac{1}{2} = -2$ **d** $\frac{5p}{4} - 1 = \frac{3}{7}$

e $3x + 11 = 7x - 23$ **f** $4x - 2 = 15 - 3x$

g $-2(z - 1) = 5(z + 6)$ **h** $5(y + 2) = 2(2y - 1)$

i $12(k + 1) - 3(k - 2) = -7$ **j** $4(\ell - 1) - 3(\ell + 2) = 18$

k $\frac{m + 1}{7} = \frac{2m - 2}{7}$ **l** $\frac{2n - 1}{3} = \frac{5n + 1}{7}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

m $\dfrac{2y+1}{3}+7=11$

n $\dfrac{7-4x}{4}-1=6$

o $\dfrac{3s-2}{5}+1=6s$

p $\dfrac{2x+1}{3}+7=\dfrac{3x-1}{4}$

q $\dfrac{2a+5}{6}+a=4$

r $\dfrac{x+2}{5}+\dfrac{x-1}{2}=\dfrac{x+1}{3}$

8 The length of a rectangular lawn is 15 m more than four times its width.

a If x m is the width of the lawn, express the length of the lawn in terms of x.

b If the perimeter of the lawn is 265 m, find the length and width of the lawn.

9 Mr Guernsey earns \$14 800 more than Mr Jersey, and Mrs Mann earns \$22 000 less than Mr Jersey.

a If \$$x$ represents Mr Jersey's salary, express the salary of:

i Mr Guernsey in terms of x

ii Mrs Mann in terms of x

b If the total of the three incomes is \$351 600, find the income of each person.

10 A person has a number of 10-cent and 20-cent coins. If their total value is \$18, how many are there of each coin if there are:

a equal numbers of each coin?

b twice as many 10-cent coins as there are 20-cent coins?

c twice as many 20-cent coins as there are 10-cent coins?

11 The distance between two towns A and B is 450 km. Find x if:

a the trip takes x hours at an average speed of 90 km/h

b the trip takes $5\frac{1}{4}$ hours at an average speed of x km/h

c the trip takes 5 hours, travelling x hours at 110 km/h and the remaining time at 60 km/h

12 A room has a length 8 m shorter than its width and a perimeter of 80 m. Find the length and width of the room.

13 Solve these equations for x.

a $\dfrac{mx+n}{a}=b$

b $m(x-n)=p$

c $a(bx-c)=d$

d $mx-n=px+q$

e $\dfrac{x}{a}+\dfrac{b}{c}=d$

f $\dfrac{m(x+n)}{a}=b$

14 Copy each statement and insert the correct symbol, ≥ or ≤.

a 11 ... 5

b −8 … −12

c −14 … −16

d −8 … −7

e −11 … 5

f −10 … −100

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

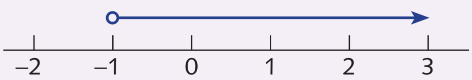
15 Graph each set on the number line.

a $\{x: x > 2\}$ **b** $\{x: x < 3\}$ **c** $\{x: x < -1\frac{1}{2}\}$

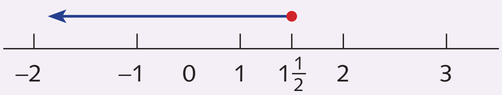
d $\{x: x \geq 4\}$ **e** $\{x: x \geq \frac{1}{3}\}$ **f** $\{x: x \leq 2\frac{1}{4}\}$

16 Use set notation to describe each interval.

a



b



17 Solve these inequalities.

a $x + 7 \geq 17$ **b** $x - 7 \leq -4$

c $-2x \geq 16$ **d** $\frac{x}{3} < -5$

e $5x + 3 \geq 13$ **f** $\frac{x}{5} - \frac{1}{2} \geq 2$

g $\frac{2x}{11} + \frac{3}{5} > 4$ **h** $6(x - 3) \leq 15$

i $4(7 - x) < 36$ **j** $7 - 4x \leq -12$

k $11 - 3x > 18$ **l** $15 - \frac{x}{18} < 14$

m $10(x + 7) \leq 15(x - 3)$ **n** $2(x - 4) > 8(x + 3.5)$

o $3(5 - x) > 8(x - 3.2)$ **p** $6(1 - x) \leq 2.4(15 - 12x)$

18 Solve these inequalities.

a $\frac{x - 3}{3} \leq \frac{3 + x}{2}$ **b** $\frac{2x + 3}{2} - \frac{x - 4}{3} > 2$

c $\frac{4 - x}{2} + \frac{3 - x}{4} < 1$ **d** $\frac{3 - 2x}{2} \geq \frac{7 - 10x}{5}$

e $\frac{5 - 2x}{3} - \frac{5 + 2x}{4} \geq -1$ **f** $\frac{2x - 1}{7} \geq \frac{2x + 3}{4}$

g $\frac{6x + 1}{3} > \frac{3x - 1}{2} + 3$ **h** $\frac{6 - 3x}{2} \geq \frac{3x - 6}{5}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 At 2 p.m., two aeroplanes leave airports 2880 km apart and fly towards one another. The average speed of one plane is twice that of the other. If they pass each other at 5 p.m., what is the average speed of each plane?

2 When a mathematics teacher was asked her age she replied, 'One-fifth of my age three years ago, when added to half my age last year, gives my age 11 years ago.' How old is she?

3 \$420 is divided between A, B and C. B receives \$20 less than A, and C receives half as much as A and B together. How much does each receive?

4 In a printing works, 75 000 leaflets are run off by two printing presses in $18\frac{3}{4}$ hours. One press delivers 200 more leaflets per hour than the other. Find the number of leaflets produced per hour by each of the machines.

5 Ten cm^3 of silver and 5 cm^3 of copper weigh 150 g. One cubic centimetre of silver weighs $1\frac{1}{6}$ times as much as 1 cm^3 of copper. Find the weight of 1 cm^3 of each of the metals.

6 A river flows at 5 km/h. A boat goes upstream half as fast as downstream. What is the speed of the boat in still water?

7 Twenty litres of milk with unknown butter-fat content from vat A is mixed with 10 litres of milk with 3% butter-fat from vat B to produce milk with $3\frac{3}{4}\%$ butter-fat content. Find the percentage of butter-fat in vat A.

8 The ends of a bar have to be machined down until the bar is 3 m long. After 5% of the length had been removed, it was found that 0.5% of the new length still had to come off. What length was removed at each machining and what was the original length of the bar?

9 A salesman makes a trip to visit a client. The traffic he encounters keeps his average speed to 40 km/h. On the return trip, he takes a route 6 km longer, but he averages 50 km/h. If he takes the same time each way, how long was the total journey?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

6

Number and Algebra

Formulas

The area, A square units, of a circle with radius r units is given by $A = \pi r^2$.

The volume, V cubic units, of a cylinder with radius r units and height h units is given by $V = \pi r^2 h$.

Einstein discovered the remarkable formula $E = mc^2$, where:

E = energy

m = mass

and c = the speed of light.

These are examples of **formulas**.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

6A Substitution into formulas

A formula relates different quantities. For instance, the formula $A = \pi r^2$ relates the radius, r units, of a circle to the area, A square units, of the circle. That is, it describes how the area of a circle depends on its radius.

Formulas with a subject

In many formulas that you see there is a single pronumeral on the left-hand side of the equal sign. This pronumeral is called the **subject** of the formula.

For instance, in the formula $E = mc^2$, E is the subject.

Substitution

If the values of all the pronumerals except the subject of a formula are known, then we can find the value of the subject by substitution.

Example 1

Find the value of the subject when the pronumerals in the formula have the values indicated.

a $F = ma$, where $a = 10$, $m = 3.5$

b $m = \dfrac{a+b}{2}$, where $a = 12$, $b = 26$

Solution

a
$$\begin{aligned} F &= ma \\ &= 10 \times 3.5 \\ &= 35 \end{aligned}$$

b
$$\begin{aligned} m &= \frac{a+b}{2} \\ &= \frac{12+26}{2} \\ &= 19 \end{aligned}$$

Example 2

The formula for the circumference C of a circle of radius r is $C = 2\pi r$.

Find the value of C when $r = 20$:

a in terms of π (that is, exactly)

b correct to 2 decimal places

Solution

a
$$\begin{aligned} C &= 2\pi r \\ &= 40\pi \end{aligned}$$

b
$$\begin{aligned} C &= 40\pi \\ &= 125.663... && \text{(using a calculator)} \\ &\approx 125.66 && \text{(correct to 2 decimal places)} \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 3

a The area of a triangle A cm^2 is given by $A = \frac{1}{2}bh$, where b cm is the base length and h cm is the height. Calculate the area of a triangle with base length 16 cm and height 11 cm.

b The simple interest payable when $\$P$ is invested at a rate of $r\%$ per year for t years is given by $I = \frac{Prt}{100}$. Calculate the simple interest payable when \$1000 is invested at 3.5% per year for 6 years.

Solution

a
$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 16 \times 11 \\ &= 88 \end{aligned}$$

The area of the triangle is 88 cm^2.

b
$$\begin{aligned} I &= \frac{Prt}{100} \\ &= \frac{1000 \times 3.5 \times 6}{100} \\ &= 210 \end{aligned}$$

The interest payable is \$210.

Substitution into a formula

When the pronumeral whose value is to be found is not the subject of the formula, it is necessary to solve an equation to find the value of this pronumeral. This is shown in the following examples.

Example 4

For a car travelling in a straight line with initial velocity u m/s and acceleration a m/s^2, the formula for the velocity v m/s at time t seconds is $v = u + at$.

a Find u if $a = 2$, $v = 15$ and $t = 7$.

b Find a if $v = 10$, $u = 6$ and $t = 3$.

Solution

a $v = u + at$

When $a = 2$, $v = 15$ and $t = 7$.

$$\begin{aligned} 15 &= u + 2 \times 7 \\ 15 &= u + 14 \\ u &= 1 \end{aligned}$$

The initial velocity is 1 m/s.

b $v = u + at$

When $v = 10$, $u = 6$ and $t = 3$.

$$\begin{aligned} 10 &= 6 + 3a \\ 4 &= 3a \\ a &= \frac{4}{3} \end{aligned}$$

The acceleration is $\frac{4}{3}$ m/s^2.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 5

The thin lens formula states that

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v},$$

where u is the distance from the object to the lens, v is the distance of the image from the lens and f is the focal length of the lens.

a Find f if $u = 2$ and $v = 5$.

b Find u if $f = 2$ and $v = 6$.

Solution

a $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

When $u = 2$ and $v = 5$,

$$\frac{1}{f} = \frac{1}{2} + \frac{1}{5}$$
$$= \frac{7}{10}$$

Taking reciprocals of both sides of the equation gives $f = \frac{10}{7}$.

b $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

If $f = 2$ and $v = 6$,

$$\frac{1}{2} = \frac{1}{u} + \frac{1}{6}$$
$$\frac{1}{u} = \frac{1}{2} - \frac{1}{6}$$
$$= \frac{1}{3}$$

Taking reciprocals of both sides of the equation gives $u = 3$.

Example 6

The area of a circle A cm^2 is given by $A = \pi r^2$, where r cm is the radius of the circle.

If $A = 20$, find r:

a exactly

b correct to 2 decimal places

Solution

a $A = \pi r^2$

When $A = 20$,

$$20 = \pi r^2$$

$$\frac{20}{\pi} = r^2 \qquad \text{(Divide both sides of equation by } \pi.)$$

$$r = \sqrt{\frac{20}{\pi}} \qquad (r \text{ is positive.})$$

b $r \approx 2.52$ (correct to 2 decimal places)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 6A

Example 1

1 For each part, find the value of the subject when the other pronumerals have the value indicated.

a $A = \ell w$, where $\ell = 5$, $w = 8$

b $s = \dfrac{d}{t}$, where $d = 120$, $t = 6$

c $A = \dfrac{1}{2}xy$, where $x = 10$, $y = 7$

d $A = \dfrac{1}{2}(a+b)h$, where $a = 4$, $b = 6$, $h = 10$

e $t = a + (n-1)d$, where $a = 30$, $n = 8$, $d = 4$

f $E = \dfrac{1}{2}mv^2$, where $m = 8, v = 4$

Example 2, 3

2 For each part, find the value of the subject when the other pronumerals have the value indicated. Calculate **a–c** correct to 3 decimal places and **d** correct to 2.

a $x = \sqrt{ab}$, where $a = 40$, $b = 50$

b $V = \pi r^2 h$, where $r = 12$, $h = 20$

c $T = 2\pi\sqrt{\dfrac{\ell}{g}}$, where $\ell = 88.2$, $g = 9.8$

d $A = P(1+R)^n$, where $P = 10\,000$, $R = 0.065, n = 10$

Example 4

3 For the formula $v = u + at$, find:

a v if $u = 6$, $a = 3$ and $t = 5$

b u if $v = 40$, $a = 5$ and $t = 2$

c a if $v = 60, u = 0$ and $t = 5$

d t if $v = 100, u = 20$ and $a = 6$

4 **a** For the formula $S = 2(a-b)$, find a if $S = 60$ and $b = 10$.

b For the formula $I = \dfrac{180n - 360}{n}$, find n if $I = 120$.

c For the formula $a = \dfrac{m+n}{2}$, find m if $a = 20$ and $n = 6$.

d For the formula $A = \dfrac{PRT}{100}$, find P if $A = 1600$, $R = 4$ and $T = 10$.

e For the formula $S = 2(\ell w + \ell h + hw)$, find h if $S = 592$, $\ell = 10$ and $w = 8$.

f For the formula $s = ut + \dfrac{1}{2}at^2$, find a if $s = 1000, u = 20$ and $t = 5$.

g For the formula $t = a + (n-1)d$, find n if $t = 58$, $d = 3$ and $a = 7$.

5 Given $v^2 = u^2 + 2ax$ and $v > 0$, find the value of v (correct to 1 decimal place) when:

a $u = 0$, $a = 5$ and $x = 10$

b $u = 2$, $a = 9.8$ and $x = 22$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 5

6 Given $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find the value of f when:

a $u = 2$ and $v = 4$ **b** $u = 6$ and $v = 9$

c $u = 12$ and $v = 11$ **d** $u = 2$ and $v = 15$

7 For the formula $s = ut + \frac{1}{2}at^2$, find the value of:

a u, when $s = 10$, $t = 20$ and $a = 2$ **b** a, when $s = 20$, $u = 5$ and $t = 2$

Example 5

8 Given $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find the value of:

a u when $f = 2$ and $v = 4$ **b** u when $f = 3$ and $v = 4$

9 Given that $P = \frac{M+m}{M-m}$, find the value of P when:

a $M = 8$ and $m = 4$ **b** $M = 26$ and $m = 17$

c $M = 19.2$ and $m = 5.9$ **d** $M = \frac{3}{4}$ and $m = \frac{2}{5}$

Example 6

10 The area A cm^2 of a square with side length x cm is given by $A = x^2$. If $A = 20$, find:

a the value of x

b the value of x correct to 2 decimal places.

11 For a rectangle of length ℓ cm and width w cm, the perimeter P cm is given by $P = 2(\ell + w)$.

Use this formula to calculate the length of a rectangle which has width 15 cm and perimeter 57 cm.

12 The formula for finding the number of degrees Fahrenheit (F) for a temperature given as a number of degrees Celsius (C) is $F = \frac{9}{5}C + 32$.

Fahrenheit temperatures are still used in the USA, but in Australia we commonly use Celsius. Calculate the Fahrenheit temperatures which people in the USA would recognise for:

a the freezing point of water, 0°C

b the boiling point of water, 100°C

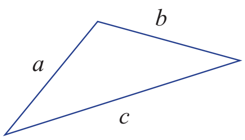
c a nice summer temperature of 25°C

Now calculate the Celsius temperatures which people in Australia would recognise for:

d 50°F **e** 104°F

13 The area A cm^2 of a triangle with side lengths a cm, b cm and c cm is given by Heron's formula

$$A^2 = s(s-a)(s-b)(s-c),$$



where $s = \frac{a+b+c}{2}$ = half the perimeter.

Find the exact areas of the triangles whose side lengths are given below.

a 6 cm, 8 cm and 10 cm **b** 5 cm, 12 cm and 13 cm

c 8 cm, 10 cm and 14 cm **d** 13 cm, 14 cm and 15 cm

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

14 Sam throws a stone down to the ground from the top of a cliff s metres high, with an initial speed of u m/s. It accelerates at a m/s^2. The stone hits the ground with a speed of v m/s given by the formula $v^2 = u^2 + 2as$. Find the speed at which the stone hits the ground, correct to 2 decimal places, if:

a $u = 0$, $a = 9.8$ and $s = 50$

b $u = 5$, $a = 9.8$ and $s = 35$

15 The distance d metres Jim's car takes to stop once the brakes are applied is given by the formula $d = 0.2v + 0.005v^2$, where v km/h is the speed of the car when the brakes are applied.

Find the distance the car takes to stop if the brakes are applied when it is travelling at each of the speeds given below. Calculate your answers correct to 3 decimal places where appropriate.

a 60 km/h

b 65 km/h

c 70 km/h

d 80 km/h

e 100 km/h

f 120 km/h

6B Changing the subject of a formula

Sometimes the pronumeral whose value is to be determined is not the subject of the formula. In this situation you have a choice. You can either:

- **rearrange** the formula to make the unknown pronumeral the subject and then substitute values for the known pronumerals (method 1), or
- **substitute** the values for the known pronumerals and then solve the resulting equation for the unknown pronumeral (method 2). This was the approach taken in Section 6A.

In either case, an equation has to be solved. In method 2, the equation involves numbers. In method 1, the equation involves pronumerals.

It is preferable to use method 1 when you are asked to find several values of a pronumeral that is not the subject of the formula.

Example 7

The cost $\$C$ of hiring Scott's car is given by the formula $C = \frac{1}{4}x + 40$, where x is the number of kilometres driven. Find the number of kilometres driven by a person who is charged $130 for hiring the car.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

Method 1

Rearrange the formula to make x the subject and then substitute the given value of C, as follows.

$$C = \frac{1}{4}x + 40$$

$$C - 40 = \frac{1}{4}x \qquad \text{(Subtract 40 from both sides of the formula.)}$$

$$x = 4C - 160 \qquad \text{(Multiply both sides of the formula by 4.)}$$

When $C = 130$, $x = 4 \times 130 - 160$
$= 360$

Thus the person drove 360 km.

Method 2

Substitute the numbers and then solve the resulting equation, giving

$$C = \frac{1}{4}x + 40$$

When $C = 130$,

$$130 = \frac{1}{4}x + 40$$

$$90 = \frac{1}{4}x \qquad \text{(Subtract 40 from both sides of the equation.)}$$

$$x = 360 \qquad \text{(Multiply both sides of the equation by 4.)}$$

Thus the person drove 360 km.

Example 8

The manager of a bed-and-breakfast guest house finds that the weekly profit $\$P$ is given by the formula

$$P = 40G - 600,$$

where G is the number of guests who stay during the week. Make G the subject of the formula and use the result to find the number of guests needed to make a profit of $800.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

$$
\begin{aligned}
P &= 40G - 600\\
P + 600 &= 40G\\
G &= \frac{P + 600}{40}
\end{aligned}
$$

When $P = 800$,

$$
\begin{aligned}
G &= \frac{800 + 600}{40}\\
&= \frac{1400}{40} = 35
\end{aligned}
$$

Thirty-five guests are required to make a profit of \$800.

Example 9

Given the formula $v^2 = u^2 + 2as$:

a rearrange the formula to make s the subject

b find the value of s when $u = 4, v = 10$ and $a = 2$

c find the value of s when $u = 4, v = 12$ and $a = 3$

Solution

a

$$
\begin{aligned}
v^2 &= u^2 + 2as\\
v^2 - u^2 &= 2as && \text{(Subtract } u^2 \text{ from both sides of the formula.)}\\
s &= \frac{v^2 - u^2}{2a} && \text{(Divide both sides of the equation by } 2a.\text{)}
\end{aligned}
$$

b When $u = 4, v = 10$ and $a = 2$.

$$
\begin{aligned}
s &= \frac{10^2 - 4^2}{2 \times 2}\\
&= \frac{100 - 16}{4}\\
&= \frac{84}{4}\\
&= 21
\end{aligned}
$$

c When $u = 4, v = 12$ and $a = 3$.

$$
\begin{aligned}
s &= \frac{12^2 - 4^2}{2 \times 3}\\
&= \frac{144 - 16}{6}\\
&= \frac{128}{6}\\
&= 21\tfrac{1}{3}
\end{aligned}
$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 10

Rearrange each of these formulas to make the pronumeral in brackets the subject.

a $E = \dfrac{p^2}{2m}$ (m)

b $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ (ℓ)

c $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ (u)

d $P = \sqrt{h+c} - a$ (h)

Solution

a
$$E = \frac{p^2}{2m}$$
$$mE = \frac{p^2}{2} \quad \text{(Multiply both sides of the equation by } m.\text{)}$$
$$m = \frac{p^2}{2E} \quad \text{(Divide both sides by } E.\text{)}$$

b
$$T = 2\pi\sqrt{\frac{\ell}{g}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{\ell}{g}} \quad \text{(Divide both sides of the equation by } 2\pi.\text{)}$$
$$\frac{\ell}{g} = \left(\frac{T}{2\pi}\right)^2 \quad \text{(Square both sides of the equation.)}$$
$$= \frac{T^2}{4\pi^2}$$
$$\ell = \frac{T^2}{4\pi^2} \times g \quad \text{(Multiply both sides of the equation by } g.\text{)}$$

That is, $\ell = \dfrac{T^2 g}{4\pi^2}$

c
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$\frac{1}{f} - \frac{1}{v} = \frac{1}{u} \quad \text{(Subtract } \tfrac{1}{v} \text{ from both sides.)}$$
$$\frac{v - f}{fv} = \frac{1}{u} \quad \text{(common denominator on LHS of equation)}$$
$$u = \frac{fv}{v - f} \quad \text{(Take reciprocals of both sides.)}$$

Note: $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ does not imply $f = u + v$.

d
$$P = \sqrt{h + c} - a$$
$$P + a = \sqrt{h + c} \quad \text{(Add } a \text{ to both sides of the equation.)}$$
$$(P + a)^2 = h + c \quad \text{(Square both sides.)}$$
$$h = (P + a)^2 - c$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Updated August 2021

The previous example shows some of the techniques that can be used to rearrange a formula.

Example 11

Make q the subject of the formula $\frac{3p}{4} - \frac{5}{q} = \frac{p^2}{3q}$.

Solution

$$\begin{aligned}
\frac{3p}{4} - \frac{5}{q} &= \frac{p^2}{3q} \\
12q\left(\frac{3p}{4} - \frac{5}{q}\right) &= 12q \times \frac{p^2}{3q} \\
3p \times 3q - 5 \times 12 &= p^2 \times 4 \qquad \text{(Multiply both sides by the lowest common denominator, } 12q.) \\
9pq - 60 &= 4p^2 \\
9pq &= 4p^2 + 60 \\
q &= \frac{4p^2 + 60}{9p}
\end{aligned}$$

Changing the subject of a formula

When rearranging a formula, the basic strategy is to move all terms involving the new subject to one side, and all the other terms to the other side. To do this:

- fractions can be eliminated by multiplying both sides of the formula by a common denominator
- all like terms should be collected
- the same operation(s) must be performed on both sides of the formula.

Exercise 6B

Example 7

1 The profit $\$P$ made each day by a store owner who sells CDs is given by the formula $P = 5n - 150$, where n is the number of CDs sold.

a What profit is made if the store owner sells 60 CDs?

b Make n the subject of the formula.

c How many CDs were sold if the store made:

i a profit of \$275? **ii** a profit of \$400?

iii a loss of \$100? **iv** no profit?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 8

2 The cost $\$C$ of hiring a reception room for a function is given by the formula $C = 12n + 250$, where n is the number of people attending the function.

a Rearrange the formula to make n the subject.

b How many people attended the function if the cost of hiring the reception room was:

i $730? **ii** $1090? **iii** $1210? **iv** $1690?

3 Given the formula $v = u + at$:

a rearrange the formula to make u the subject

b find the value of u when:

i $v = 20$, $a = 2$ and $t = 5$ **ii** $v = 40$, $a = -6$ and $t = 4$

c rearrange the formula to make a the subject

d find the value of a when:

i $v = 20$, $u = 15$ and $t = 2$ **ii** $v = -26.8$, $u = -14.4$ and $t = 2$

iii $v = \frac{1}{2}$, $u = \frac{2}{3}$ and $t = \frac{5}{6}$

e rearrange the formula to make t the subject and find t when $v = 6$, $u = 7$ and $a = -3$.

4 Given the formula $t = a + (n - 1)d$:

a rearrange the formula to make a the subject

b find the value of a when:

i $t = 11$, $n = 4$ and $d = 3$ **ii** $t = 8$, $n = 5$ and $d = -3$

c rearrange the formula to make d the subject

d find the value of d when:

i $t = 48$, $a = 3$ and $n = 16$ **ii** $t = 120$, $a = -30$ and $n = 101$

e rearrange the formula to make n the subject and find the value of n when $t = 150$, $a = 5$ and $d = 5$

Example 10

5 Rearrange each of these formulas to make the pronumeral in brackets the subject.

a $y = mx + c$ (c) **b** $y = mx + c$ (x)

c $A = \frac{1}{2}bh$ (x) **d** $C = 2\pi r$ (r)

e $P = A + 2\ell h$ (l) **f** $s = ut + \frac{1}{2}at^2$ (a)

g $A = 2\pi r^2 + 2\pi rh$ (h) **h** $V = \frac{1}{3}\pi r^2 h$ (h)

i $s = \frac{n}{2}(a + \ell)$ (a) **j** $S = \frac{n}{2}(a + \ell)$ (n)

k $V = \pi r^2 + \pi rs$ (s) **l** $E = mgh + \frac{1}{2}mv^2$ (h)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 The formula for the sum S of the interior angles in a convex n-sided polygon is $S = 180(n - 2)$. Rearrange the formula to make n the subject and use this to find the number of sides in the polygon if the sum of the interior angles is:

a $1080°$ **b** $1800°$ **c** $3240°$

Example 9

7 The kinetic energy E joules of a moving object is given by $E = \frac{1}{2}mv^2$, where m kg is the mass of the object and v m/s is its speed.

Rearrange the formula to make m the subject and use this to find the mass of the object when its energy and speed are, respectively:

a 400 joules, 10 m/s **b** 28 joules, 4 m/s **c** 57.6 joules, 2.4 m/s

8 When an object is shot up into the air with a speed of u metres per second, its height above the ground h metres and time of flight t seconds are related (ignoring air resistance) by $h = ut - 4.9t^2$.

Find the speed at which an object was fired if it reached a height of 27.5 metres after 5 seconds.

Example 11

9 Rearrange each of these formulas to make the pronumeral in brackets the subject. (All pronumerals represent positive numbers.)

a $c = a^2 + b^2$ (a) **b** $K = 3\ell^2 + 4m$ (ℓ)

c $x = \sqrt{ab}$ (b) **d** $d = 8\sqrt{\frac{h}{5}}$ (h)

e $T = \frac{2\pi}{n}$ (n) **f** $D = \frac{m}{v}$ (v)

g $E = \frac{m}{2r^2}$ (r) **h** $T = \frac{a}{4r^2}$ (r)

10 Temperatures can be measured in either degrees Fahrenheit or degrees Celsius. To convert from one scale to the other, the following formula is used: $F = \frac{9}{5}C + 32$.

a Rearrange the formula to make C the subject.

b On a particular day in Melbourne, the temperature was 28°C. What is this temperature measured in Fahrenheit?

c In Boston, USA, the minimum overnight temperature was 4°F. What is this temperature measured in Celsius?

d What number represents the same temperature in °C and °F?

e An approximate conversion formula, used frequently when converting oven temperatures, is $F = 2C + 30$. Use this to convert these temperatures:

i an oven temperature of 180°C **ii** an oven temperature of 530°F

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6C Constructing formulas

In this section we shall learn how to create formulas from given information.

Example 12

Find a formula for n, the number of cents in x dollars and y cents.

Solution

In $\$x$ there are $100x$ cents.

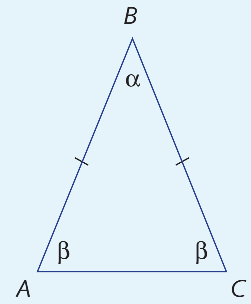
In $\$x$ and y cents there are $(100x + y)$ cents.

The formula is $n = 100x + y$.

Example 13

Here is an isosceles triangle with equal base angles marked.

Find a formula for β in terms of α.



Solution

$$\begin{aligned} \alpha + 2\beta &= 180 \qquad \text{(angle sum of triangle)} \\ 2\beta &= 180 - \alpha \\ \beta &= \frac{180 - \alpha}{2} \end{aligned}$$

Exercise 6C

Example 12

1 Construct a formula for:

a D in terms of n, where D is the number of degrees in n right angles

b c in terms of D, where c is the number of cents in $\$D$

c m in terms of h, where m is the number of minutes in h hours

d d in terms of m, where d is the number of days in m weeks

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 Construct a formula for:

a the number of centimetres n in p metres

b the number of millilitres s in t litres

c the number of centimetres q in $5p$ metres

d the number of grams x in $\frac{y}{2}$ kilograms

3 Find a formula for:

a the number of cents z in x dollars and y cents

b the number of minutes x in y minutes and z seconds

c the number of hours x in y minutes and z seconds

d the cost $\$m$ of 1 book if 20 books cost $\$c$

e the cost $\$n$ of 1 suit if 5 suits cost $\$m$

f the cost $\$m$ of 1 tyre if x tyres cost $\$y$

g the cost $\$p$ of n suits if 4 suits cost $\$k$

h the cost $\$q$ of x cars if 8 cars cost $\$b$

Example 13

4 Find a formula relating x and y for each of these statements, making y the subject.

a y is three less than x.

b y is four more than the square of x.

c y is eight times the square root of one-fifth of x.

d x and y are supplementary angles.

e A car travelled 80 km in x hours at an average speed of y km/h.

f A car used x litres of petrol on a trip of 80 km and the fuel consumption was y litres/100 km.

5 Find a formula relating the given pronumerals for each of these statements.

a The number of square cm x in y square metres

b The selling price $\$S$ of an article with an original price of $\$m$ when a discount of 20% is given

c The length c cm of the hypotenuse and the lengths a cm and b cm of the other two sides in a right-angled triangle

d The area A cm^2 of a sector of a circle with a radius of length r cm and angle θ at the centre of the circle

e The distance d km travelled by a car in t hours at an average speed of 75 km/h

f The number of hectares h in a rectangular paddock of length 400 m and width w m

6 In each part, find a formula from the information given.

a A hire car firm charges \$20 per day plus 40 cents per km. What is the total cost $\$C$ for a day in which x km was travelled?

b If there are 50 litres of petrol in the tank of a car and petrol is used at the rate of 4 litres per day, what is the number of litres y that remains after x days?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Cooking instructions for a forequarter of lamb are as follows: preheat oven to 220°C and cook for 45 min per kg plus an additional 20 min. What is the formula relating the cooking time T minutes and weight w kg?

d In a sequence of numbers the first number is 2, the second number is 4, the third is 8, the fourth is 16, etc. Assuming the doubling pattern continues, what is the formula you would use to calculate t, the nth number?

e A piece of wire of length x cm is bent into a circle of area A cm^2. What is the formula relating A and x?

7 Gareth the gardener has a large rectangular vegetable patch and he wishes to put in a path around it using concrete pavers that measure 50 cm × 50 cm. The path is to be 1 paver wide. Let n be the number of pavers required. If the vegetable patch measures x metres by y metres, find a formula for n in terms of x and y.

Review exercise

1 If $s = \frac{n}{2}(2a + (n-1)d)$:

a find the value of s when $n = 10$, $a = 6$ and $d = 3$

b find the value of a when $s = 350$, $n = 20$ and $d = 4$

c find the value of d when $s = 460$, $n = 10$ and $a = 10$

2 The formula for the geometric mean m of two positive numbers a and b is $m = \sqrt{ab}$.

a Find m if $a = 16$ and $b = 25$.

b Find a if $m = 7$ and $b = 16$.

3 If $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$:

a find x if $b = 4$, $a = 1$ and $c = -24$

b find c if $a = 1$, $x = 6$ and $b = 2$

4 A pillar is in the shape of a cylinder with a hemispherical top. If r metres is the radius of the base and h metres is the total height, the volume V cubic metres is given by the formula $V = \frac{1}{3}\pi r^2(3h - r)$.

a Rearrange the formula to make h the subject.

b Find the height of the pillar, correct to the nearest centimetre, if the radius of the pillar is 0.5 m and the volume is 10 m^3.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 Rearrange each of these formulas to make the pronumeral in brackets the subject. (All of the pronumerals represent positive numbers.)

a $A = \ell \times w$ (ℓ)

b $C = 2\pi r$ (r)

c $V = \pi r^2 h$ (h)

d $A = 2h(\ell + b)$ (b)

e $A = 4\pi r^2$ (r)

f $w = 10\sqrt{\dfrac{x}{a}}$ (x)

g $w = \sqrt{\dfrac{3V}{\pi h}}$ (V)

h $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{2}{z}$ (y)

6 If a stone is dropped off a cliff, the number of metres it has fallen after a certain number of seconds is found by multiplying the square of the number of seconds by 4.9.

a Find the formula for the distance d metres fallen by the stone in t seconds.

b Find the distance fallen in 1.5 seconds.

7 If $t = \sqrt{\dfrac{M}{M - m}}$:

a express the formula with m as the subject

b express the formula with M as the subject

c find the value of M if $m = 3$ and $t = \sqrt{2}$.

8 The total surface area S cm^2 of a cylinder is given in terms of its radius r cm and height h cm by the formula $S = 2\pi r(r + h)$.

a Express this formula with h as the subject.

b What is the height of such a cylinder if the radius is 7 cm and the total surface area is 500 cm^2? Calculate your answer in centimetres, correct to 2 decimal places.

9 The sum S of the squares of the first n whole numbers is given by the formula $S = \dfrac{n(n+1)(2n+1)}{6}$. Find the sum of the squares of:

a the first 20 whole numbers

b all the numbers from 5 to 21 inclusive

10 **a** For the formula $T = 2\pi\sqrt{\dfrac{W}{gF}}$, make F the subject.

b For the formula $P = \dfrac{\pi r x}{180} + 2r$, make x the subject.

c For the formula $D = \sqrt{\dfrac{f+x}{f-x}}$, make x the subject.

11 Cans in a supermarket are displayed in a triangular stack with one can at the top, two cans in the second row from the top, three cans in the third row from the top, and so on. What is the number of cans in the display if the number of rows is:

a 4? **b** 5? **c** n? **d** 35?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

12 Rearrange each of these formulas to make the pronumeral in brackets the subject. Assume all pronumerals take non-negative values.

a $M = \sqrt{\frac{ab}{t} + a^2}$ (t)

b $V = \frac{1}{3}a^2\sqrt{\ell^2 - \frac{a^2}{2}}$ (ℓ)

c $E = \frac{w^2 a}{(w^2 + m)b^3}$ (a)

d $T = 2\pi\sqrt{\frac{\ell + r}{g}}$ (r)

e $x = a\sqrt{\frac{k}{ph + y}}$ (h)

f $v^2 = u^2 + 2as$ (u)

13 The volume V cm^3 of metal in a tube is given by the formula $V = \pi\ell(r^2 - (r - t)^2)$ where ℓ cm is the length, r cm is the radius of the outside surface and t cm the thickness of the material.

a Find V if $\ell = 50$, $r = 5$ and $t = 0.25$.

b Make r the subject of the formula.

14 For the formula $I = \frac{180n - 360}{n}$:

a find I when $n = 6$

b make n the subject of the formula and find n when $I = 108$

15 Find the formula connecting x and y for each of these statements making y the subject.

a y is four more than twice the square of x.

b x and y are complementary angles.

c A car travelled x km in y hours at a speed of 100 km/h.

d A car travelled 100 km in y hours at a speed of x km/h.

Challenge exercise

1 Rearrange each of these formulas to make the pronumeral in brackets the subject. (All of the pronumerals represent positive numbers.)

a $P = \frac{M + m}{M - m}$ (M)

b $F = \frac{MP}{M + \ell}$ (M)

c $\frac{1}{S} = \frac{1}{R} + \frac{1}{T}$ (R)

d $E = \frac{R}{i} + P$ (i)

e $T = t - \frac{h}{\ell}$ (ℓ)

f $I = \frac{M}{4}\left(R^2 + \frac{L^2}{3}\right)$ (L)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

g $L = \ell\sqrt{1 + \frac{v^2}{c^2}}$ (v)

h $T = 2\pi\left(\frac{1+e}{1-e}\right)$ (e)

i $v = c\left(\frac{1}{r} - \frac{1}{s}\right)$ (r)

j $P = 2\pi\sqrt{\frac{h+k}{g}}$ (k)

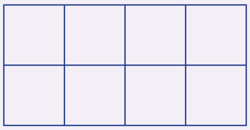
k $P = p\sqrt{1 + \frac{1}{\ell}}$ (ℓ)

l $A = \pi(R^2 - r^2)$ (R)

m $(a+b)^2 + c^2 = (a-d)^2$ (a)

n $(M - m)^2 + p^2 = M^2$ (M)

2 Sophie is playing with a box of connecting rods which can be joined together to produce a rectangular grid. The grid shown opposite is of size 4×2 (that is, length $= 4$, width $= 2$) and uses 22 rods.



a How many rods are needed to make these grids?

i 1×2 **ii** 3×2 **iii** 10×2 **iv** $n \times 2$

b How many rods are needed to make these grids?

i 1×3 **ii** 2×3 **iii** 10×3 **iv** $n \times 3$

c How many rods are needed to make a grid of size $n \times m$?

d In the 4×2 grid shown above, how many rods have been placed:

i vertically? **ii** horizontally?

e In an $n \times m$ grid, how many rods have been placed:

i vertically? **ii** horizontally? **iii** in total?

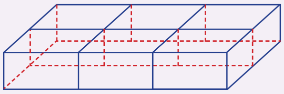
f Does your answer to part **e iii** agree with your answer to part **c**?

g Sophie makes a 4×7 grid. How many rods did she use?

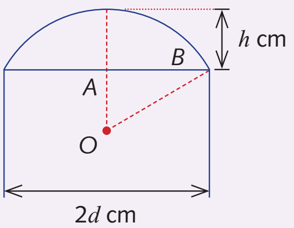
h If she used 97 rods to make a grid of width 6, what was the length of the grid?

i If she has 100 rods, can she make a grid that uses all 100 rods? If so, what size grid can she make?

j If the rods can be joined to make a three-dimensional grid, find the formula for the number of rods required to make a grid as shown of size $m \times n \times p$ (that is, length $= m$, width $= n$, height $= p$). The diagram shows a rectangular prism measuring 3 rods by 2 rods by 1 rod.



3 A builder wishes to place a circular cap of a given height above an existing window. To do this he needs to know the location of the centre of the circle (the cap is not necessarily a semicircle) and the radius of the circle. O is the centre of the required circle, the radius of the required circle is r cm, the width of the window is $2d$ cm and the height of the circular cap is h cm.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a Express each of these in terms of r, d and h.

i AB **ii** OA

b Show that $r = \dfrac{h^2 + d^2}{2h}$.

c If the window is 120 cm wide and the cap is 40 cm high, find:

i the radius of the circle

ii how far below the top of the window the centre of the circle must be placed

d If the builder used a circle of radius 50 cm and this produced a cap of height 20 cm, what was the width of the window?

4 A group of n people attend a club meeting. Before the meeting begins, they all shake hands with each other. Write a formula to find H, the number of handshakes exchanged.

5 A cyclic quadrilateral has all its vertices on a circle. Its area A is given by Brahmagupta's formula

$$A^2 = (s - a)(s - b)(s - c)(s - d)$$

where a, b, c and d are the side lengths of the quadrilateral and

$s = \dfrac{a + b + c + d}{2}$ is the 'semi-perimeter'. Find the exact area of a cyclic quadrilateral with side lengths:

a 4, 5, 6, 7 **b** 7, 4, 4, 3 **c** 8, 9, 10, 13

d 39, 52, 25, 60 **e** 51, 40, 68, 75

6 **a** The population of a town decreases by 2% each year. The population was initially P, and is Q after n years. What is the formula relating Q, P and n?

b The population of a town decreases by 5% each year. The percentage decrease over a period of n years is a%. What is the formula relating a and n?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

7

Measurement and Geometry

Congruence and special quadrilaterals

This chapter reviews work on angles, congruence tests and their applications. Congruence is an extremely useful tool in geometrical arguments. It is used here to prove properties of various types of quadrilaterals, and to develop tests for special quadrilaterals.

This chapter is a revision of all the geometry presented in Years 7 and 8.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

7A Review of angles

Most geometrical arguments rely on the relationship between different angles in the one figure. This section reviews the methods introduced so far, and the next section uses these methods to prove some interesting general results.

Describing angles by their sizes

Angles of three particular sizes have special names:

- An angle of 90° is a **right angle**.
- An angle of 180° is a **straight angle**.
- An angle of 360° is a **revolution**.

Other angles are described by being within a range of angle sizes:

- An **acute angle** is an angle between 0° and 90°.
- An **obtuse angle** is an angle between 90° and 180°.
- A **reflex angle** is an angle between 180° and 360°.

For example, 30° is an acute angle, 140° is obtuse, and 220° is reflex.

Particular pairs of angles also have special names:

- Two angles whose sum is 90° are called **complementary angles**.
- Two angles whose sum is 180° are called **supplementary angles**.

For example, 35° and 55° are complementary angles because 35° + 55° = 90°, and 70° and 110° are supplementary angles because 70° + 110° = 180°.

Angles at a point

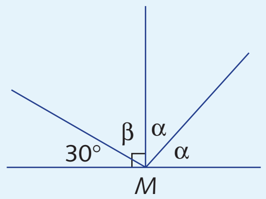
The first group of results concern angles at a point.

- Adjacent angles can be added and subtracted.
- Adjacent angles on a straight line are supplementary.
- Angles in a revolution add to 360°.
- Vertically opposite angles are equal.

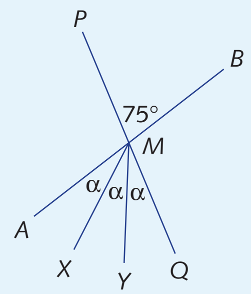
Example 1

Find the value of the pronumeral in each figure.

a



b



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $\beta = 60°$ (complementary angles)

$2\alpha = 90°$ (supplementary angles)

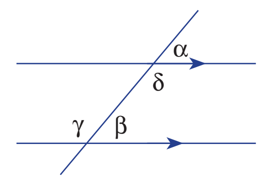
$\alpha = 45°$

b $3\alpha = 75°$ (vertically opposite angles at M)

$\alpha = 25°$

Angles across transversals

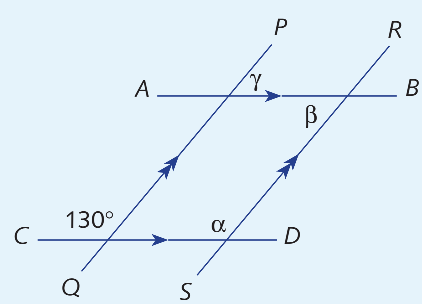
Suppose that a transversal crosses two lines.



- If the lines are parallel, then the alternate angles (for example, δ and γ) are equal.
- If the lines are parallel, then the corresponding angles (for example, α and β) are equal.
- If the lines are parallel, then the co-interior angles (for example, δ and β) are supplementary.

Example 2

Find α, β and γ in the figure to the right.



Solution

$\alpha = 130°$ (corresponding angles, $PQ \parallel RS$)

$\beta = 50°$ (co-interior angles, $AB \parallel CD$)

$\gamma = 50°$ (alternate angles, $PQ \parallel RS$)

Proving that two lines are parallel

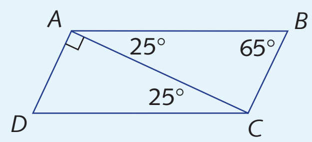
The **converses** of the previous three results are tests to prove that two lines are parallel. Suppose that a transversal crosses two lines.

- If two alternate angles are equal, then the lines are parallel.
- If two corresponding angles are equal, then the lines are parallel.
- If two co-interior angles are supplementary, then the lines are parallel.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 3

Identify every pair of parallel lines in the figure to the right.



Solution

First, $AB \parallel DC$ (alternate angles $\angle BAC$ and $\angle ACD$ are equal).

Secondly, $\angle BAD = 115°$ (adjacent angles at A).

So $AD \parallel BC$ (co-interior angles $\angle BAD$ and $\angle ABC$ are supplementary).

Angle sums of triangles and quadrilaterals

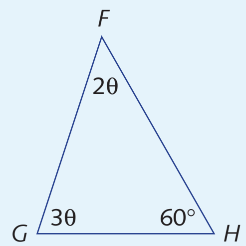
- The sum of the interior angles of a triangle is 180°.
- An exterior angle of a triangle equals the sum of the opposite interior angles.
- The sum of the interior angles of a quadrilateral is 360°.

Proofs of these results are reviewed in Exercise 7B.

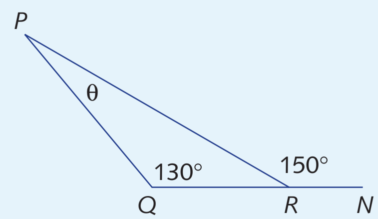
Example 4

Find θ in each diagram below.

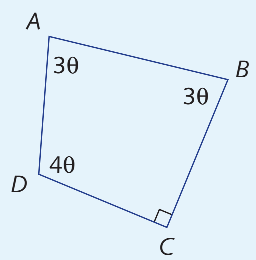
a



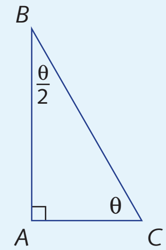
b



c



d



Solution

a $3\theta + 2\theta + 60° = 180°$ (angle sum of ΔFGH)

$5\theta = 120°$

$\theta = 24°$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b $\theta + 130° = 150°$ (exterior angle of ΔPQR)

$\theta = 20°$

c $3\theta + 3\theta + 4\theta + 90° = 360°$ (angle sum of quadrilateral $ABCD$)

$10\theta = 270°$

$\theta = 27°$

d $\theta + \frac{\theta}{2} + 90° = 180°$ (angle sum of ΔABC)

$\frac{3\theta}{2} = 90°$

$\theta = 60°$

Isosceles and equilateral triangles

A triangle is called **isosceles** if it has two equal sides; it is called **equilateral** if all three sides are equal. Thus all equilateral triangles are also isosceles triangles.

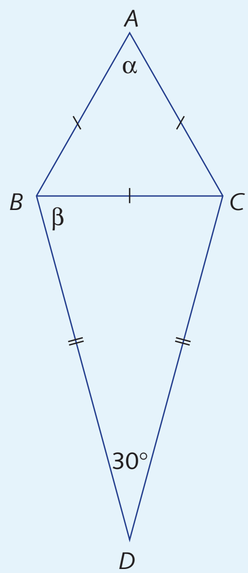
- The base angles of an isosceles triangle are equal.
- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.
- Each interior angle of an equilateral triangle is 60°.
- Conversely, if the three angles of a triangle are equal, then the triangle is equilateral.

Proofs of these results are reviewed in Exercise 7C.

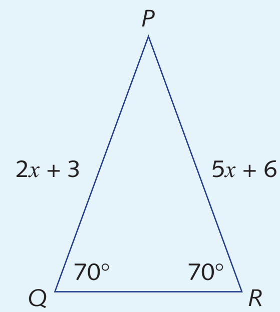
Example 5

Find α, β, γ and x in the diagrams below.

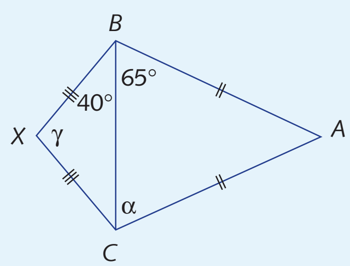
a



b



c



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a First, $\alpha = 60°$ $(\Delta ABC$ is equilateral)

Secondly, $\angle BCD = \beta$ (base angles of isosceles ΔBCD)

so $2\beta + 30° = 180°$ (angle sum of ΔBCD)

$\beta = 75°$

b $2x + 3 = 5x + 6$ (opposite angles of ΔPQR are equal)

$-3 = 3x$

$x = -1$

Thus PQ and PR each have length of 1.

c $\alpha = 65°$ (base angles of isosceles ΔABC)

$\angle BCX = 40°$ (base angles of isosceles ΔBXC)

$\gamma = 100°$ (angle sum of ΔXBC)

Exercise 7A

1 a Classify these angles using the standard terms.

i 140° **ii** 360° **iii** 33° **iv** 180° **v** 90° **vi** 350°

b Write down the complements of:

i 20° **ii** 72° **iii** 45°

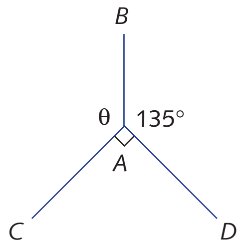
c Write down the supplements of:

i 20° **ii** 172° **iii** 90°

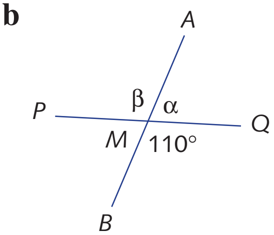
Example 1

2 This question reviews angles at a point. Find the angles marked with pronumerals, giving reasons.

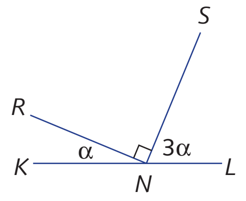
a



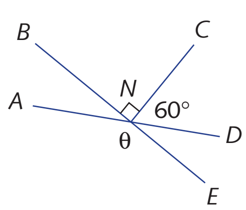
b



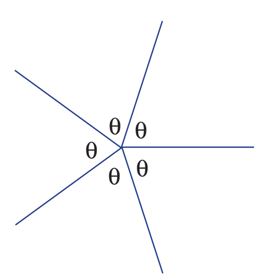
c



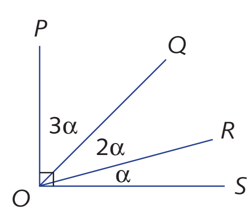
d



e



f

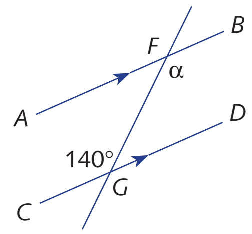


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

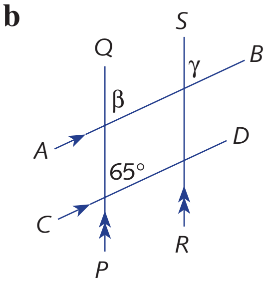
Example 2

3 This question reviews angles across transversals. Find the angles marked with pronumerals, giving reasons.

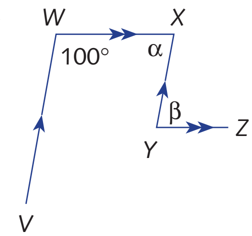
a



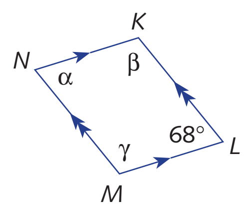
b



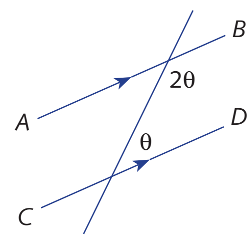
c



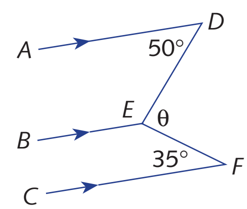
d



e



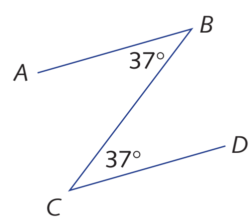
f



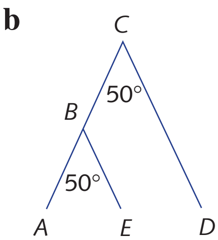
Example 3

4 This question reviews tests for two lines to be parallel. In each part, name all pairs of parallel lines, giving reasons.

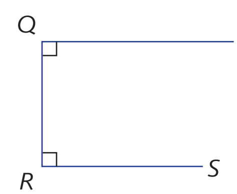
a



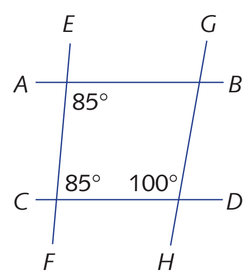
b



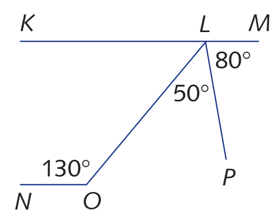
c



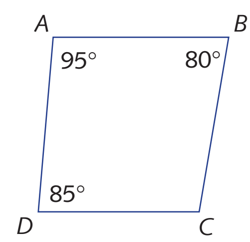
d



e



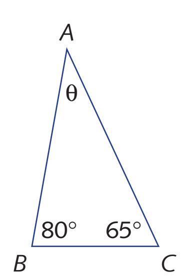
f



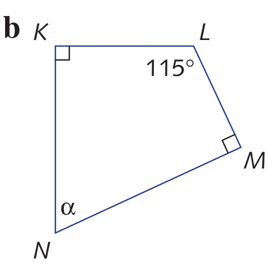
Example 4

5 This question reviews angle sums of triangles and quadrilaterals. Find the angles marked with pronumerals, giving reasons.

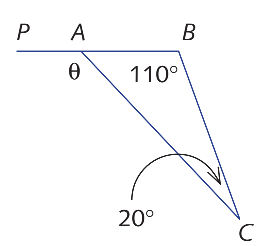
a



b



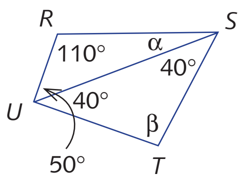
c



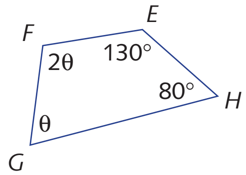
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press

Updated August 2021

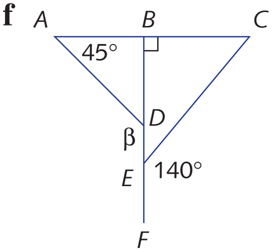
d



e



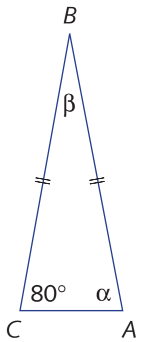
f



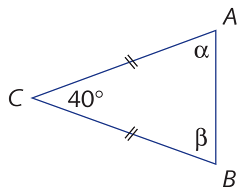
Example 5

6 This question reviews isosceles and equilateral triangles. Find the angles and sides marked with pronumerals, giving reasons.

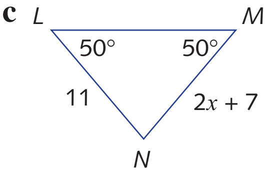
a



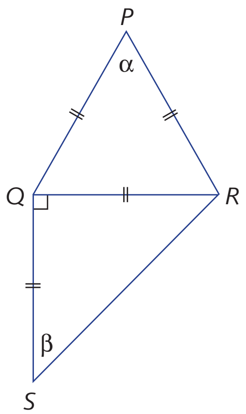
b



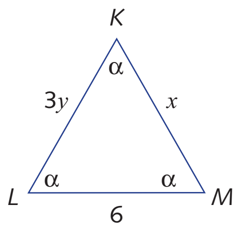
c



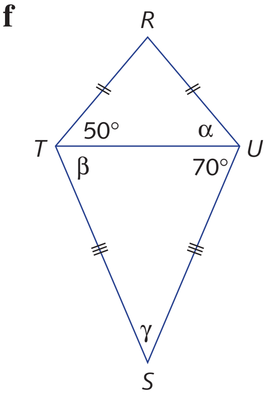
d



e

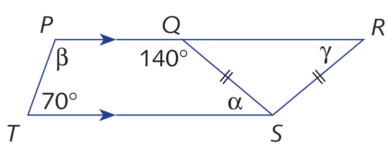


f

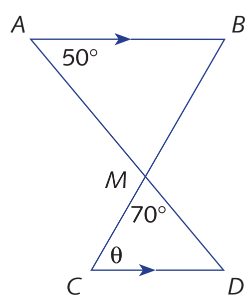


7 Find the values of the pronumerals $\alpha, \beta, \gamma, \theta$ and x. Different methods are combined in each part of this question.

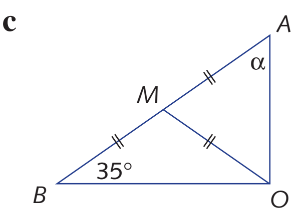
a



b

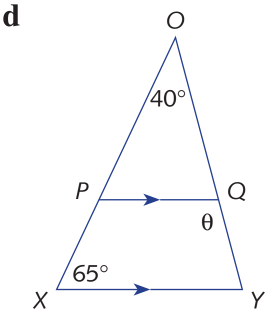


c

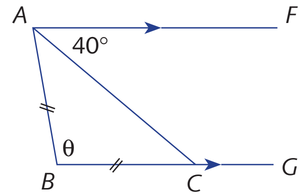


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

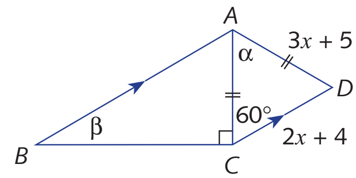
d



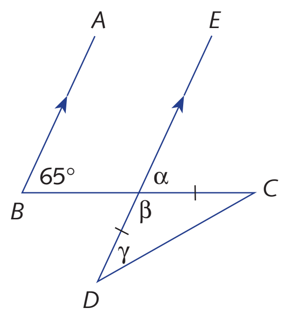
e



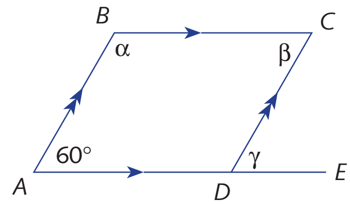
f



g



h



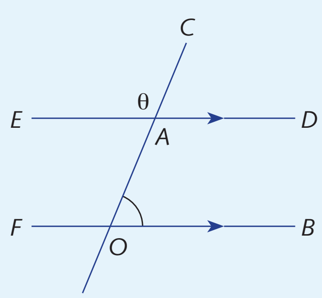
7B Reasoning with angles

This section uses the methods of Section 7A in more general situations. Construction lines are often needed.

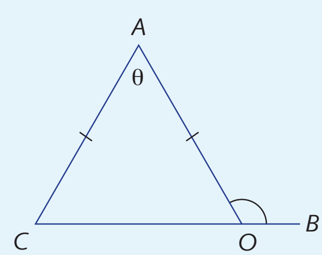
Example 6

Find the angle $\angle AOB$ in terms of θ, giving reasons.

a



b



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $\angle FOA = \theta$ (corresponding angles, $ED \parallel FB$)

$\angle AOB = 180° - \theta$ (supplementary angle)

b $\angle AOC = \dfrac{180° - \theta}{2}$ (ΔACO is isosceles)

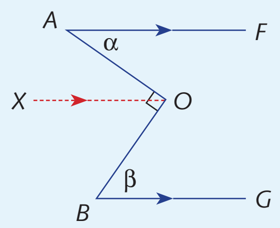
$= 90° - \dfrac{\theta}{2}$

$\angle AOB = 180° - \left(90° - \dfrac{\theta}{2}\right)$ (supplementary angles)

$= 90° + \dfrac{\theta}{2}$

Example 7

Show that $\alpha + \beta = 90°$.



Solution

Draw XO parallel to AF.

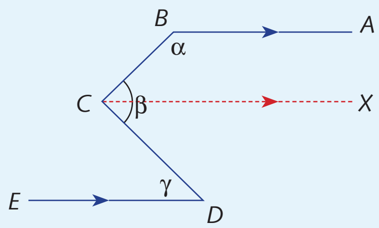
Then $\angle AOX = \alpha$ (alternate angles, $OX \parallel FA$)

and $\angle BOX = \beta$ (alternate angles, $OX \parallel GB$)

so $\alpha + \beta = 90°$ (adjacent angles)

Example 8

Prove that $\alpha + \beta - \gamma = 180°$.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

Draw CX parallel to BA.

Then $\angle BCX = 180° - \alpha$ (co-interior angles, $BA \parallel CX$)

and $\angle DCX = \gamma$ (alternate angles, $CX \parallel ED$)

Hence $\beta = 180° - \alpha + \gamma$ ($\angle BCD = \angle BCX + \angle DCX$)

so $\alpha + \beta - \gamma = 180°$

Proving theorems

The results of the previous section can be applied to prove many interesting general results, called **theorems**. For the rest of this chapter, each theorem to be proven is stated in italics. Usually the question includes a diagram, introduces the necessary pronumerals and breaks the proof down into a number of steps. Sometimes a construction is required.

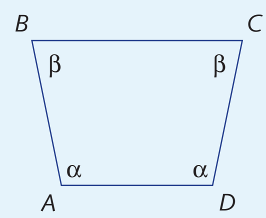
Example 9

Prove that: *A quadrilateral with two pairs of equal adjacent angles is a trapezium.*

In the quadrilateral $ABCD$ to the right, $\angle A = \angle D = \alpha$ and $\angle B = \angle C = \beta$.

a Prove that $\alpha + \beta = 180°$.

b Hence prove that $ABCD$ is a trapezium.



Solution

a $\alpha + \alpha + \beta + \beta = 360°$ (angle sum of quadrilateral $ABCD$)

so $\alpha + \beta = 180°$

b Hence $AD \parallel BC$ (co-interior angles are supplementary)

Therefore $ABCD$ is a trapezium.

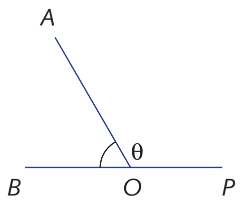
Angle sums of polygons

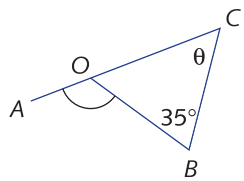
The last four questions of the following exercise deal with interior and exterior angle sums of polygons. These are interesting general results that you may want to remember as part of your known geometric facts.

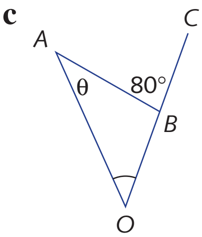
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

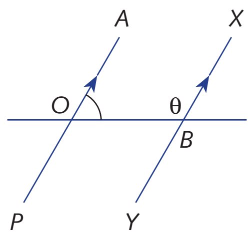
Exercise 7B

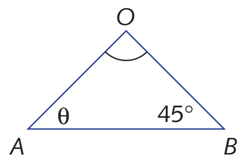
Example 6 **1** Find the marked angle $\angle AOB$ in terms of θ, giving reasons.

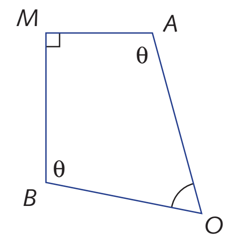
a


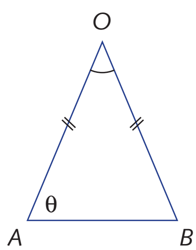
b


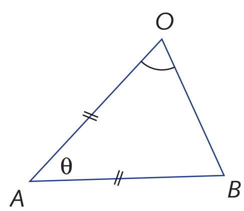
c


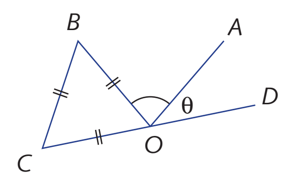
d


e


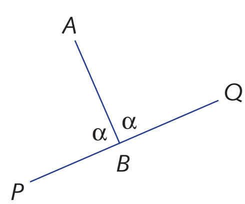
f


g


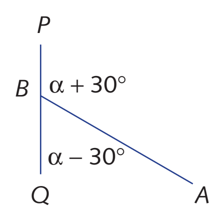
h


i


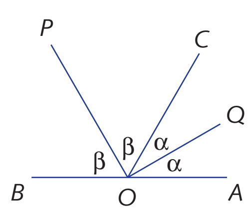
Example 7 **2** **a** Prove that $AB \perp PQ$.



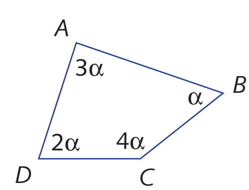
b Prove that $\angle ABP = 2 \times \angle ABQ$.



c Prove that $PO \perp QO$.

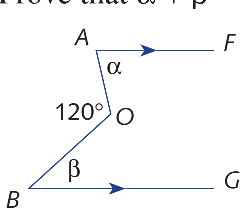


d Prove that $AB \parallel DC$.

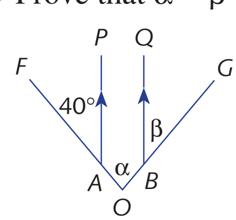


Example 8 **3** These questions may require construction lines to be drawn.

a Prove that $\alpha + \beta = 120°$.

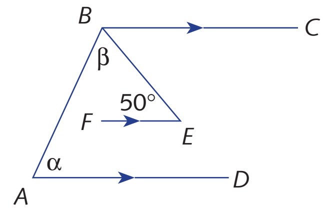


b Prove that $\alpha - \beta = 40°$.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Prove that $\alpha + \beta = 130°$.



Example 9

4 This question reviews proofs of theorems about angle sums of triangles.

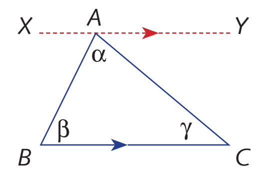
a Prove that: *The sum of the interior angles of a triangle is* 180°.

In ΔABC to the right, $\angle BAC = \alpha$, $\angle B = \beta$ and $\angle C = \gamma$.

The line XAY has been constructed parallel to BC.

i Explain why $\angle XAB = \beta$ and $\angle YAC = \gamma$.

ii Hence explain why $\alpha + \beta + \gamma = 180°$.



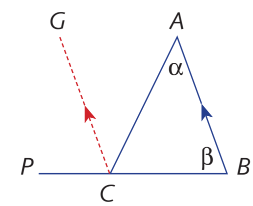
b Prove that: *An exterior angle of a triangle equals the sum of the opposite interior angles.*

In ΔABC to the right, $\angle A = \alpha$ and $\angle B = \beta$.

The line CG has been constructed parallel to BA, and the side BC has been produced to P.

i Explain why $\angle ACG = \alpha$ and $\angle PCG = \beta$.

ii Hence explain why $\angle ACP = \alpha + \beta$.

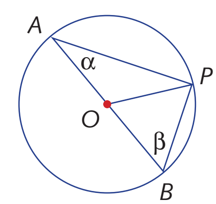


5 Prove that: *An angle in a semicircle is a right angle.*

In the circle to the right, AOB is a diameter, and P is any other point on the circle.

Let $\angle A = \alpha$ and $\angle B = \beta$.

a Prove that $\angle APB = \alpha + \beta$. **b** Hence prove that $\angle APB = 90°$.

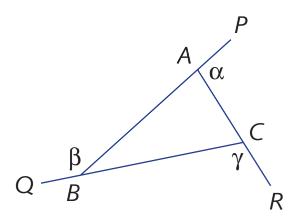


6 **a** Prove that: *The sum of the exterior angles of a triangle is* 360°.

In the diagram to the right, α, β and γ are the sizes of the three exterior angles of ΔABC.

i Find the sizes of the three interior angles of ΔABC.

ii Hence show that $\alpha + \beta + \gamma = 360°$.

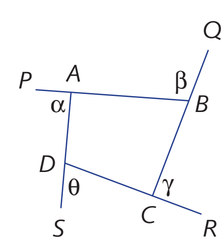


b Draw a quadrilateral $ABCD$. Use a construction line to help explain why the sum of the interior angles of $ABCD$ is 360°.

c Prove that: *The sum of the four exterior angles of a convex quadrilateral is* 360°.

In the diagram to the right, α, β, γ and θ are the sizes of the four exterior angles of the convex quadrilateral $ABCD$. (All the interior angles of a convex quadrilateral are less than 180°.)

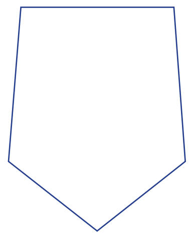
Show that $\alpha + \beta + \gamma + \theta = 360°$.

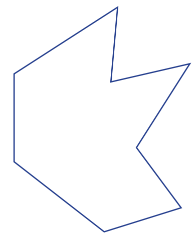


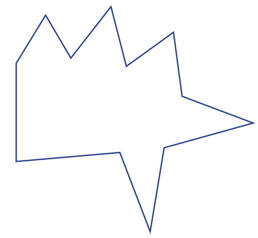
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Angle sums of polygons

7 **a** The diagrams below show a pentagon, an octagon and a dodecagon. By dividing each figure into triangles, find the sum of its interior angles.

i 

ii 

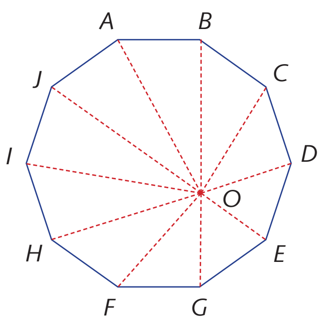
iii 

b Using a similar technique, find the sum of the interior angles of:

i a hexagon **ii** a heptagon **iii** a nonagon **iv** a decagon

c Using these examples as a guide, find a formula for the angle sum of an n-sided polygon.

8 In a convex polygon, all the interior angles are less than 180°.



a The diagram to the right shows a convex decagon, and any point O inside the decagon. Intervals are drawn from O to each of the 10 vertices, dissecting the decagon into 10 triangles.

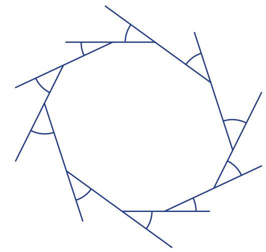
i What is the sum of the interior angles of all 10 triangles so formed?

ii Hence find the sum of the interior angles of the decagon.

b Draw a convex hexagon and repeat the process.

c Using the same technique, find a formula for the sum of the interior angles of a convex polygon with n sides. Confirm that this matches your answer to Question **7c**.

9 **a** In the convex decagon to the right, each side has been produced to form an exterior angle.



i Explain why the sum of the interior angles plus the sum of the exterior angles is 1800°.

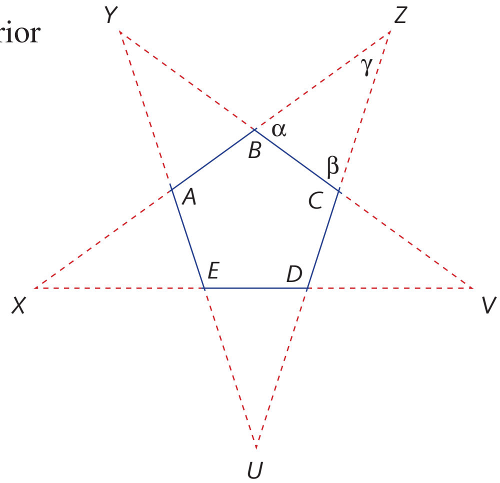
ii Hence find the sum of the 10 exterior angles.

b Draw a convex hexagon and repeat the process.

c Using the same technique, find the sum of the exterior angles of a convex polygon with n sides.

10 Find a formula for the size of each interior angle in a *regular* n-sized polygon.

11 The figure shown is a regular pentagram. It is formed by producing the sides of a regular pentagon.



a Find the size of each interior angle of the pentagon $ABCDE$.

b Find the values of α, β and γ.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7C Using congruence

In *ICE-EM Mathematics Year 8* we introduced the idea of **congruent figures**.

Congruent figures

- Two plane figures are called **congruent** if one figure can be moved on top of the other figure, by a sequence of translations, rotations and reflections, so that they coincide exactly.
- Congruent figures have exactly the same shape and size.
- When two figures are congruent, we can match every part of one figure with the corresponding part of the other, so that:
  - matching angles have the same size
  - matching paired intervals have the same length
  - matching paired regions have the same area.

The congruence arguments used in this chapter involve only congruent triangles. In Year 8 we developed four tests for two triangles to be congruent.

The four standard congruence tests for triangles

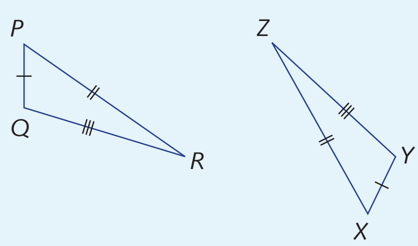
Two triangles are **congruent** if:

- **SSS:** the three sides of one triangle are respectively equal to the three sides of the other triangle, **or**
- **AAS:** two angles and one side of one triangle are respectively equal to two angles and the matching side of the other triangle, **or**
- **SAS:** two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the other triangle, **or**
- **RHS:** the hypotenuse and one side of one right-angled triangle are respectively equal to the hypotenuse and one side of the other right-angled triangle.

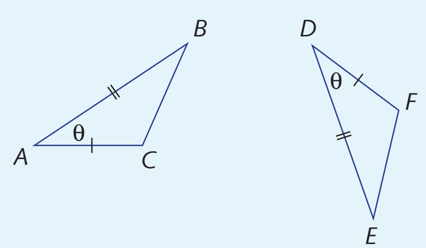
Example 10

In each part, write a congruence statement, giving a test as the reason. Make sure that you write the vertices of the two triangles in matching order.

a

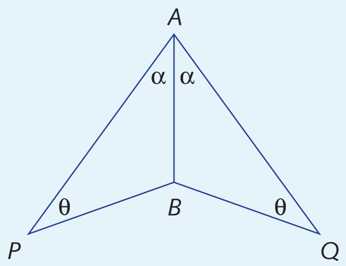


b

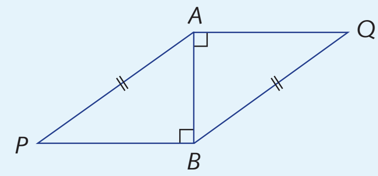


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

c



d



Solution

a $\Delta PQR \equiv \Delta XYZ$ (SSS)

b $\Delta BAC \equiv \Delta EDF$ (SAS)

c $\Delta ABP \equiv \Delta ABQ$ (AAS)

d $\Delta ABQ \equiv \Delta BAP$ (RHS)

Notice that in parts **c** and **d**, the side AB is **common** to both triangles, and thus provides one of the pairs of equal sides.

Using congruence to prove that two matching sides or angles are equal

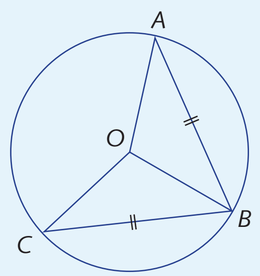
Once we have established that two triangles are congruent, we know that the remaining matching sides and angles are equal, as in the following example.

Example 11

In the diagram to the right, AB and BC are equal chords of a circle with centre O.

a Prove that $\Delta AOB \equiv \Delta COB$.

b Hence prove that $\angle AOB = \angle COB$.



Solution

a In the triangles AOB and COB:

$OA = OC$ (radii)

$OB = OB$ (common)

$AB = CB$ (given)

so $\Delta AOB \equiv \Delta COB$ (SSS)

b Hence $\angle AOB = \angle COB$ (matching angles of congruent triangles)

Using congruence to prove that two lines are parallel

In many situations, congruence is used to prove that two alternate angles, or two corresponding angles, are equal. This allows us to prove that two lines are parallel.

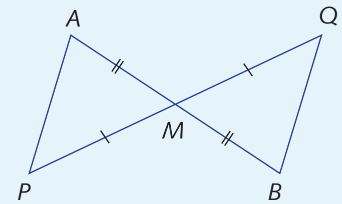
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 12

In the diagram to the right,
$AM = BM$ and $PM = QM$

a Prove that $\Delta AMP \equiv \Delta BMQ$.

b Prove that $AP \parallel BQ$.



Solution

a In the triangles AMP and BMQ:

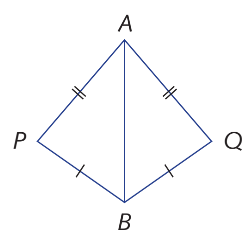
$AM = BM$	(given)
$PM = QM$	(given)
$\angle AMP = \angle BMQ$	(vertically opposite angles at M)
so $\Delta AMP \equiv \Delta BMQ$	(SAS)
b Hence $\angle APM = \angle BQM$	(matching angles of congruent triangles)
so $AP \parallel BQ$	(alternate angles are equal)

Exercise 7C

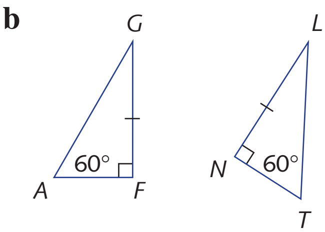
Example 10

1 In each part, write a congruence statement, giving a congruence test as the reason. Make sure that you name the vertices in matching order.

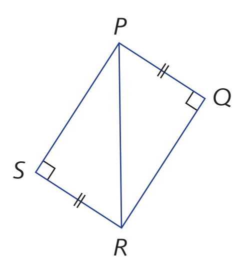
a



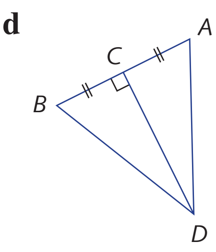
b



c

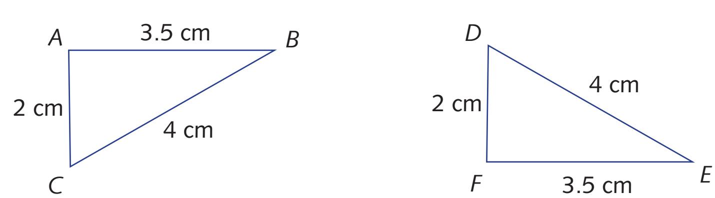


d



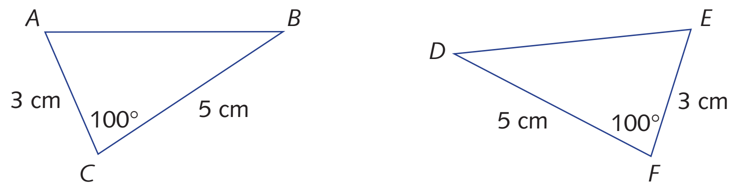
2 In each part, write a congruence statement, giving a congruence test as the reason.

a

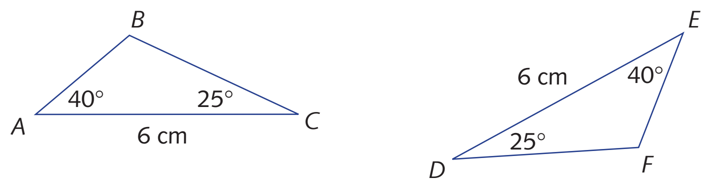


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

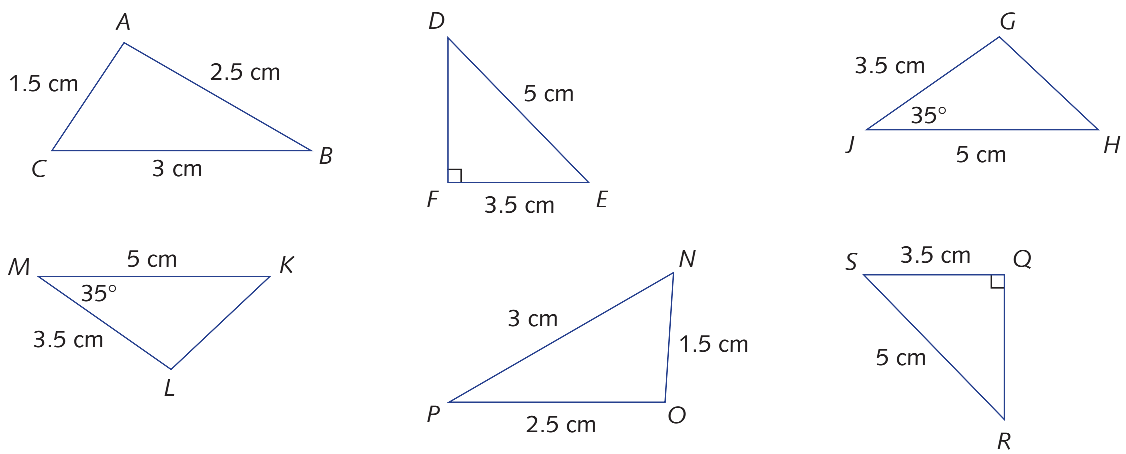
b



c

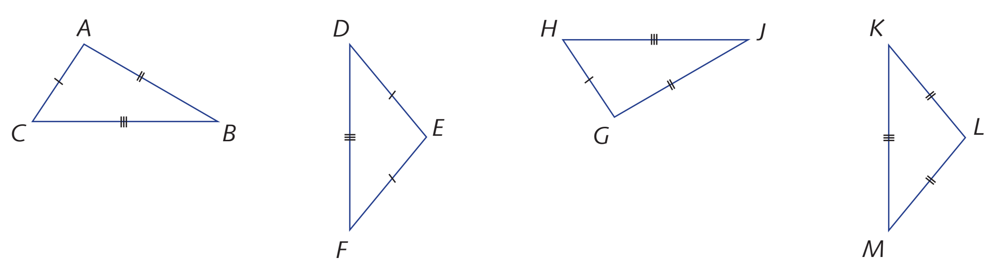


3 From the six triangles drawn, name three pairs of congruent triangles. Give reasons.

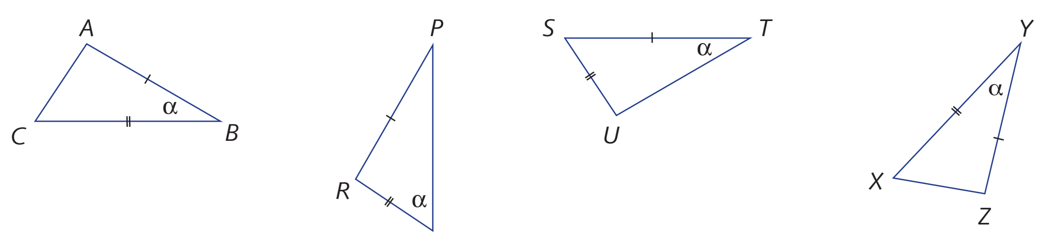


4 In each of the following, state which triangle is congruent to ΔABC, giving the appropriate congruence test as a reason.

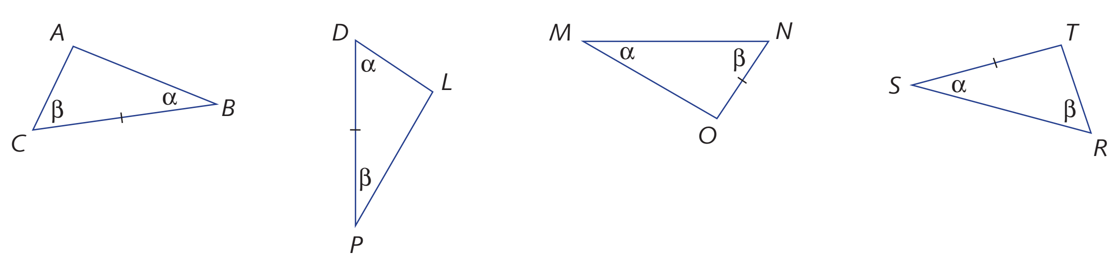
a



b

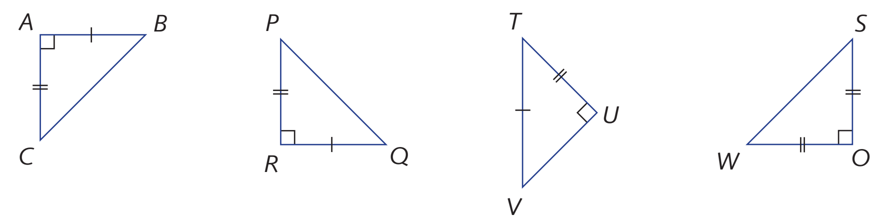


c



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d



5 $\Delta ABC \equiv \Delta PQR$ (where the vertices are in matching order).

a Which angle in ΔPQR is equal in size to $\angle ABC$?

b Which angle in ΔPQR is equal in size to $\angle CAB$?

c Which side in ΔPQR is equal in length to AC?

d Which side in ΔABC is equal in length to QR?

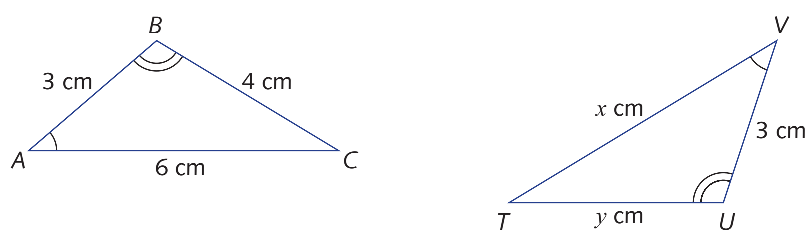
6 Find the values of the pronumerals.

a

b

c

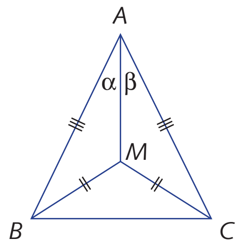
d



e

Example 11

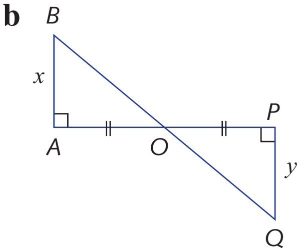
7 **a**



i Prove that $\Delta ABM \equiv \Delta ACM$.

ii Hence prove that $\alpha = \beta$.

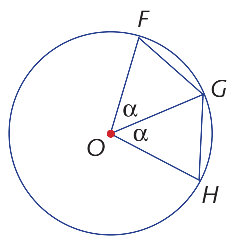
b



i Prove that $\Delta OAB \equiv \Delta OPQ$.

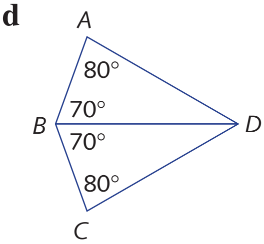
ii Hence prove that $x = y$.

c



Use congruence to prove that $FG = GH$, given that O is the centre of the circle.

d

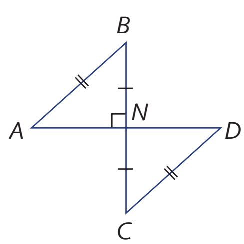


Use congruence to prove that $AD = DC$ and that $AB = BC$.

8 AB is a chord of a circle centre O and M is the midpoint of AB. Prove that OM is perpendicular to AB.

Example 12

9 **a**

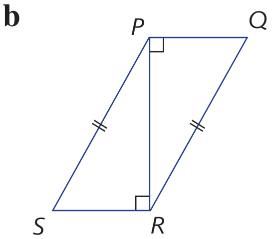


i Prove that $\Delta NAB \equiv \Delta NDC$.

ii Hence prove that $\angle A = \angle D$.

iii Hence prove that $AB \parallel CD$.

b

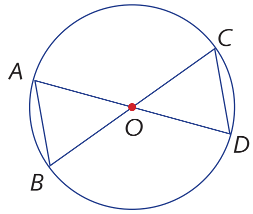


i Give a reason why $PQ \parallel SR$.

ii Prove that $\Delta PRS \equiv \Delta RPQ$.

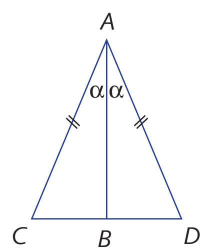
iii Hence prove that $PS \parallel QR$.

c



Use congruence to prove that $AB \parallel CD$, given that O is the centre of the circle.

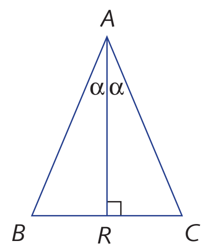
d



Prove that $\angle ABC = \angle ABD$ using congruence, and hence prove that $AB \perp CD$.

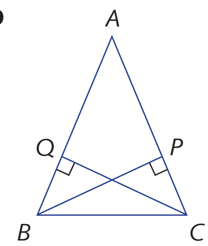
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 **a**



Use congruence to prove that ΔABC is isosceles.

b

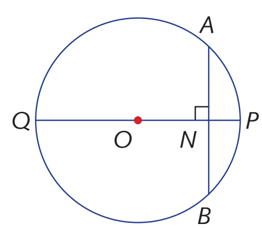


Use congruence to prove that ΔABC is isosceles, given that $BP = CQ$.

11 In the triangle ΔABC, $AB = AC$. The bisectors of angles ABC and ACB meet at X. Prove that $XB = XC$.

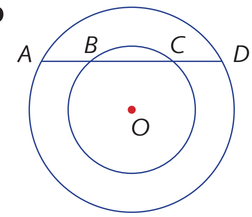
12 In each part, O is the centre of the circle or circles. You will need to construct radii and use congruence in these questions.

a



Prove that PQ bisects AB.

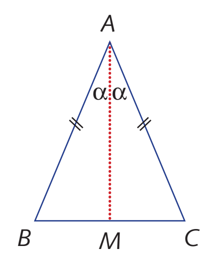
b



Prove that $AB = CD$.

13 This question reviews proofs of several theorems about isosceles and equilateral triangles.

a Prove that: *The base angles of an isosceles triangle are equal.*



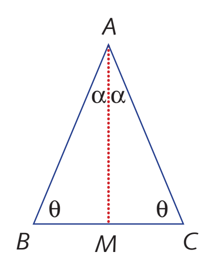
The triangle ABC to the right is isosceles, with $AB = AC$.

The bisector of $\angle BAC$ meets the base BC at M.

i Prove that $\Delta ABM \equiv \Delta ACM$.

ii Hence prove that $\angle B = \angle C$.

b Conversely, prove that: *If two angles of a triangle are equal, then the sides opposite those angles are equal.*



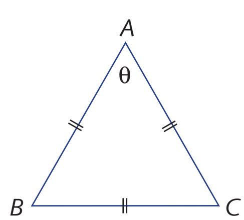
In the triangle ABC to the right, $\angle B = \angle C$.

The bisector of $\angle BAC$ meets the base BC at M.

i Prove that $\Delta ABM \equiv \Delta ACM$.

ii Hence prove that ΔABC is isosceles.

c Prove that: *Each interior angle of an equilateral triangle is* $60°$.



The triangle ABC to the right is equilateral. Let $\angle A = \theta$.

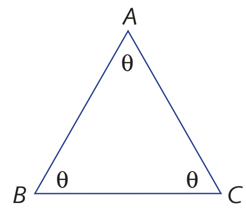
i Give a reason why $\angle B = \theta$.

ii Give a reason why $\angle C = \theta$.

iii Explain why $\theta = 60°$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d Conversely, prove that: *If all angles of a triangle are equal, then the triangle is equilateral.*



In the triangle ABC to the right, all the interior angles are equal.

Let $\angle A = \angle B = \angle C = \theta$.

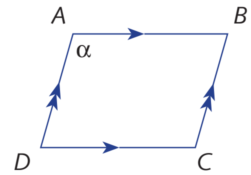
i Give a reason why $AB = AC$.

ii Give a reason why $AC = BC$.

14 This question reviews proofs by congruence of three well-known properties of parallelograms.

Recall that a **parallelogram** is a quadrilateral whose opposite sides are parallel.

a Prove that: *Opposite angles of a parallelogram are equal.*

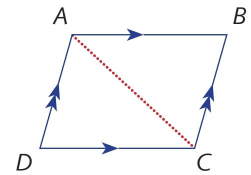


A parallelogram $ABCD$ is drawn to the right. Let $\angle A = \alpha$.

i Find the sizes of $\angle B$ and $\angle D$ in terms of α.

ii Hence find the size of $\angle C$ in terms of α.

b Prove that: *Opposite sides of a parallelogram are equal.*

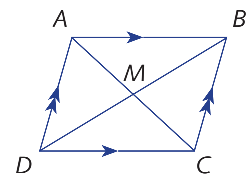


Let $ABCD$ be a parallelogram with the diagonal AC joined.

i Prove that $\Delta ABC \equiv \Delta CDA$.

ii Hence prove that $AB = DC$ and $AD = BC$.

c Prove that: *The diagonals of a parallelogram bisect each other.*



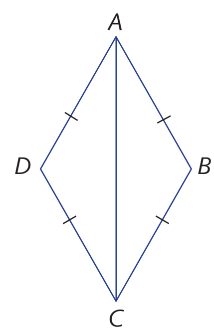
Let the diagonals of the parallelogram $ABCD$ meet at M.

i Prove that $\Delta ABM \equiv \Delta CDM$.

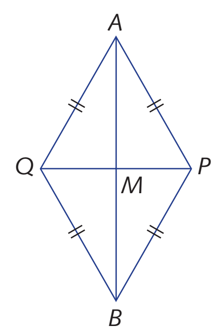
ii Hence prove that $AM = CM$ and $BM = DM$.

15 This question reviews proofs by congruence of some diagonal properties of rhombuses and rectangles.
Recall that a **rhombus** is a quadrilateral with all sides equal.

a Prove that: *A rhombus is a parallelogram.*



b Prove that: *The diagonals of a rhombus are perpendicular, and bisect the vertex angles through which they pass.*



Let the diagonals of the rhombus $APBQ$ meet at M.

i Prove that $\Delta APB \equiv \Delta AQB$.

ii Hence prove that $\angle PAM = \angle QAM$.

iii Prove also that $AM \perp PQ$.

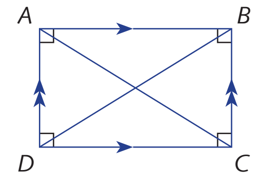
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c Prove that: *The diagonals of a rectangle are equal.*

Let $ABCD$ be a rectangle with the diagonals joined.

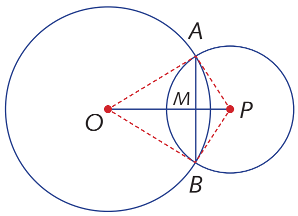
i Prove that $\Delta ADC \equiv \Delta BCD$.

ii Hence prove that $AC = BD$.



16 Prove that: *When two circles intersect, the line joining their centres is the perpendicular bisector of their common chord.*

In the diagram to the right, AB is the common chord of two intersecting circles with centres O and P. The chord AB meets the line OP at M.



a Prove that $\Delta AOP \equiv \Delta BOP$.

b Hence prove that OP bisects $\angle AOB$.

c Prove that $AOM \equiv BOM$.

d Hence prove that $AM = BM$ and $AB \perp OP$.

7D Parallelograms

In Year 8 we defined a parallelogram and discovered some of its important properties.

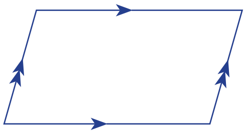
Parallelograms

Definition of a parallelogram

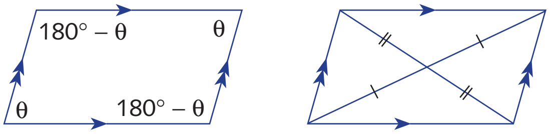
- A **parallelogram** is a quadrilateral whose opposite sides are parallel.

Properties of a parallelogram

- The opposite angles of a parallelogram are equal.
- The opposite sides of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.



The diagrams below illustrate these properties. Their proofs were reviewed in Exercise 7C, Question **13**.



The purpose of this section is to establish, and subsequently apply, four well-known tests for a quadrilateral to be a parallelogram.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Four tests for a parallelogram

- If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.
- If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.
- If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

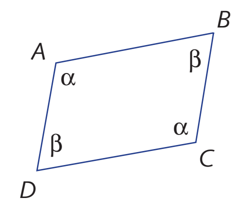
The first question in the following exercise reviews proofs of these tests.

Exercise 7D

1 This question reviews the proofs of the four tests for a quadrilateral to be a parallelogram.

a Prove that: *If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.*

Let $ABCD$ be a quadrilateral with $\angle A = \angle C = \alpha$ and $\angle B = \angle D = \beta$.

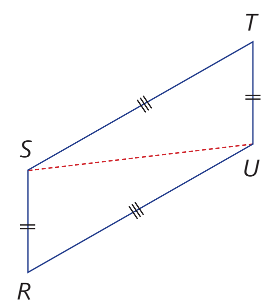


i Prove that $\alpha + \beta = 180°$.

ii Hence prove that $AB \parallel DC$ and $AD \parallel BC$.

b Prove that: *If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.*

Let $RSTU$ be a quadrilateral with $RS = UT$ and $RU = ST$.

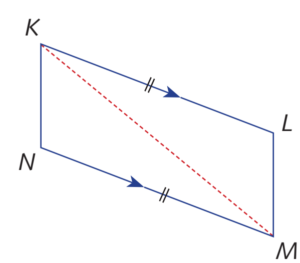


i Prove that $\Delta RSU \equiv \Delta TUS$.

ii Hence prove that $RS \parallel UT$ and $RU \parallel ST$.

c Prove that: *If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.*

Let $KLMN$ be a quadrilateral with $KL = NM$ and $KL \parallel NM$.

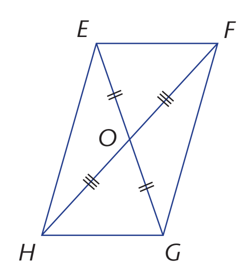


i Prove that $\Delta KLM \equiv \Delta MNK$.

ii Hence prove that $KN \parallel LM$.

d Prove that: *If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.*

Let $EFGH$ be a quadrilateral whose diagonals meet at O and bisect each other.



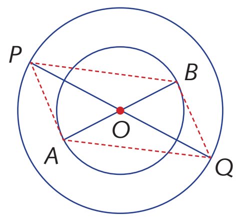
i Prove that $\Delta EHO \equiv \Delta GFO$.

ii Hence prove that $EH = FG$ and $EH \parallel FG$.

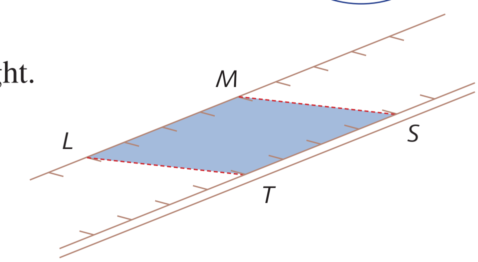
iii Explain why this proves that $EFGH$ is a parallelogram.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

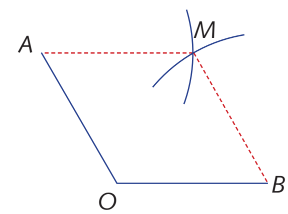
2 **a** In the diagram to the right, both circles have centre O. The interval AOB is a diameter of the smaller circle, and POQ is a diameter of the larger circle. What sort of figure is $APBQ$, and why?



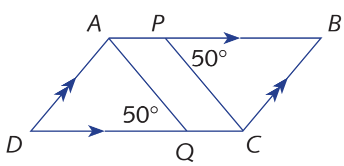
b The opposite edges of my ruler are parallel. I draw intervals LM and ST of length 4 cm on the opposite edges of the ruler, as shown in the diagram to the right. What sort of figure is $LMST$, and why?



c In the diagram to the right, $\angle AOB$ is any given angle. First I draw an arc with centre A and radius OB. Then I draw an arc with centre B and radius OA. The two arcs meet at M, and I claim that $AMBO$ is a parallelogram. Am I correct? Explain why or why not.

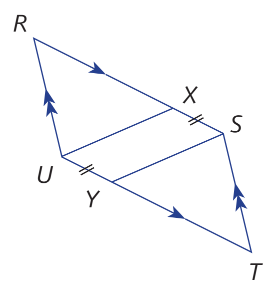


3 **a**



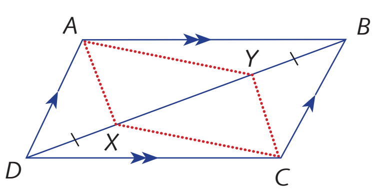
Given that $ABCD$ is a parallelogram, prove that $AP = QC$.

b



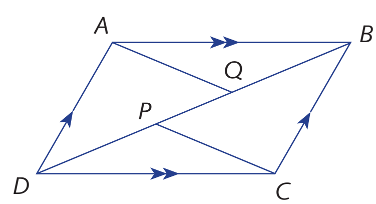
Given that $RSTU$ is a parallelogram, prove that $UX \parallel YS$.

c



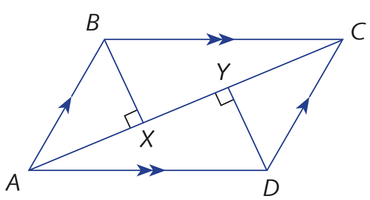
$ABCD$ is a parallelogram with X and Y points on the diagonal BD such that $BY = DX$. Prove that $AYCX$ is a parallelogram.

d



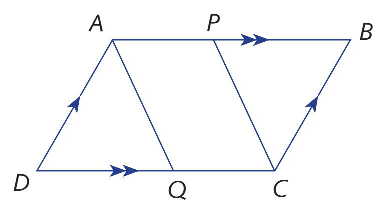
$ABCD$ is a parallelogram and $PB = DQ$. Prove that $AQ = CP$.

e



$ABCD$ is a parallelogram and BX and DY are perpendicular to AC. Prove that $BX = DY$.

f



$ABCD$ is a parallelogram with $BP = DQ$. Prove that $APCQ$ is a parallelogram.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

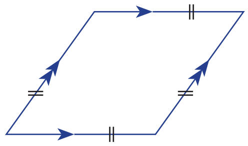
7E Tests for rhombuses, rectangles and squares

Rhombuses, rectangles and squares are special types of parallelograms. Last year we defined these figures and proved some of their properties. This section establishes some important tests for these figures.

Rhombuses

Definition of a rhombus

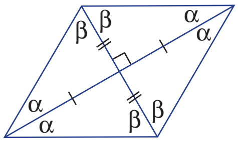
- A **rhombus** is a quadrilateral with four equal sides.

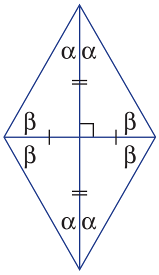


Diagonal properties of a rhombus

- The diagonals of a rhombus bisect each other at right angles.
- The diagonals of a rhombus bisect the vertex angles through which they pass.

These diagonal properties of a rhombus are far clearer when it is drawn in a diamond orientation, with its diagonals vertical and horizontal as illustrated below.





Tests for a rhombus

- If a quadrilateral is a parallelogram with two adjacent sides equal, then the parallelogram is a rhombus.
- If the diagonals of a quadrilateral bisect each other at right angles, then the quadrilateral is a rhombus.

These tests are proved in Question **1** of Exercise 7E.

Note: A rhombus is often defined to be a parallelogram with two adjacent sides equal.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Rectangles

Definition of a rectangle

- A **rectangle** is a parallelogram whose interior angles are right angles.

Diagonal properties of a rectangle

- The diagonals of a rectangle are equal and bisect each other.

Tests for a rectangle

- A parallelogram with one right angle is a rectangle.
- If all angles of a quadrilateral are equal, then the quadrilateral is a rectangle.
- If the diagonals of a quadrilateral are equal and bisect each other, then the quadrilateral is a rectangle.

The three tests for a rectangle mentioned above are proved in Questions **2a**, **2b**, and **1c**, respectively, on the next page as part of Exercise 7E.

Note: A rectangle is often defined to be a parallelogram with one right angle.

Squares

Definition of a square

- A **square** is a quadrilateral that is both a rectangle and a rhombus. Thus a square has all the properties of a rectangle and a rhombus.

Properties of a square

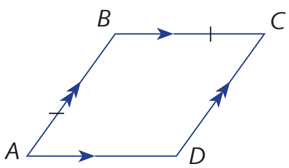
- All angles of a square are right angles.
- All sides of a square are equal.
- The diagonals of a square are equal and bisect each other at right angles.
- Each diagonal meets each side at 45°.

A square is often defined as a regular polygon with four vertices; that is, as a quadrilateral with four equal sides and four equal interior angles. Note that such a figure is a parallelogram, because its opposite sides are equal, so the two definitions agree.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

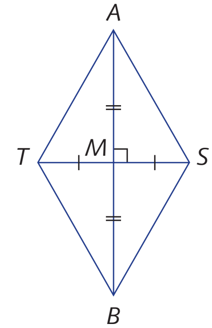
Exercise 7E

1 **a** Prove that: *If a parallelogram has two adjacent sides equal, then the parallelogram is a rhombus.*



b Prove that: *If the diagonals of a quadrilateral bisect each other at right angles, then the quadrilateral is a rhombus.*

The diagonals of the quadrilateral $ASBT$ meet at right angles at M and bisect each other.



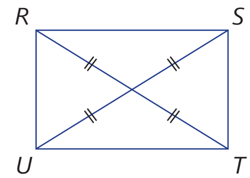
i Why is $ASBT$ a parallelogram?

ii Prove that $\Delta AMS \equiv \Delta AMT$.

iii Hence prove that $AS = AT$; that is, $ASBT$ is a rhombus.

c Prove that: *If the diagonals of a quadrilateral have equal length and bisect each other, then the quadrilateral is a rectangle.*

The diagonals of the quadrilateral $RSTU$ have equal length and bisect each other.



i Why is $RSTU$ a parallelogram?

ii Prove that $\Delta URS \equiv \Delta TSR$.

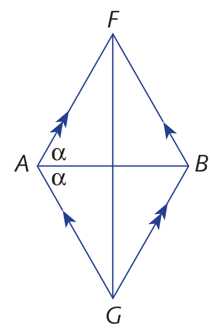
iii Hence prove that $\angle URS = \angle TSR$.

iv Hence prove that $\angle URS = 90°$.

2 **a** Prove that: *A parallelogram with one right angle is a rectangle.*

b Prove that: *If all angles of a quadrilateral are equal, then the quadrilateral is a rectangle.*

3 Prove that: *If one diagonal of a parallelogram bisects one vertex angle, then the parallelogram is a rhombus.*



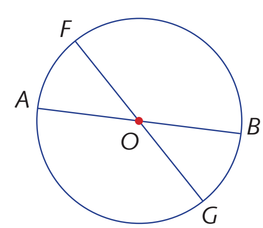
Let $AFBG$ be a parallelogram in which the diagonal AB bisects the vertex angle $\angle GAF$.

Let $\angle FAB = \angle GAB = \alpha$.

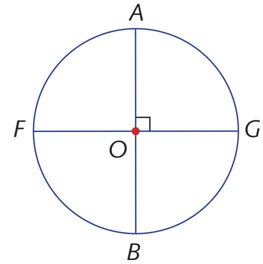
a Prove that $\angle ABF = \alpha$.

b Hence prove that $AFBG$ is a rhombus.

4 **a** In the diagram to the right, O is the centre of the circle, and AB and FG are diameters of the circle. What sort of figure is $AFBG$, and why?



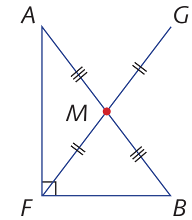
b In the diagram to the right, O is the centre of the circle, and AB and FG are perpendicular diameters of the circle. What sort of figure is $AFBG$, and why?



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 Prove that: *The circle with centre the midpoint of the hypotenuse of a right-angled triangle, and passing through the endpoints of the hypotenuse, also passes through the third vertex.*

Let ΔABF be right-angled at F, and let M be the midpoint of the hypotenuse AB. Produce FM to G so that $FM = MG$.



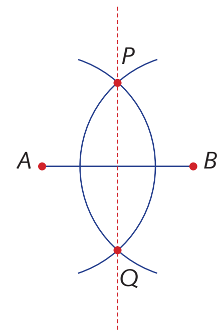
a Explain why $AFBG$ is a rectangle.

b Explain why $AM = BM = FM = GM$.

c Hence explain why the circle with centre M and radius AM passes through F.

6 This question justifies some of the constructions that you learnt in earlier years.

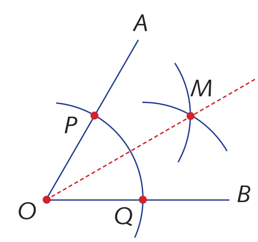
a The diagram shows the standard construction of the perpendicular bisector of a given interval AB. We draw two arcs with centres A and B respectively, equal radius, intersecting at P and Q.



i Explain why $APBQ$ is a rhombus.

ii Hence explain why PQ is the perpendicular bisector of AB.

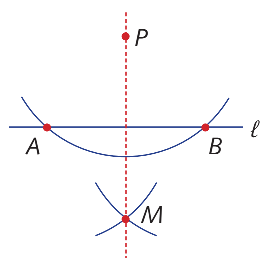
b The diagram shows the standard construction of the bisector of a given angle $\angle AOB$. We draw three arcs with centres O, P and Q respectively, and equal radius.



i Explain why $OPMQ$ is a rhombus.

ii Hence explain why OM is the bisector of $\angle AOB$.

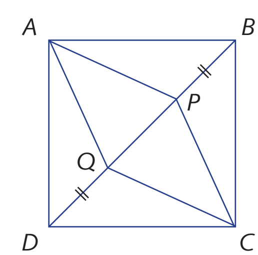
c The diagram shows the standard construction of the perpendicular from a given point P to a given line ℓ. First draw an arc with centre P cutting ℓ at A and B. Then with centres A and B and the same radius, draw arcs meeting at M.



i Explain why $APBM$ is a rhombus.

ii Hence explain why PM is perpendicular to ℓ.

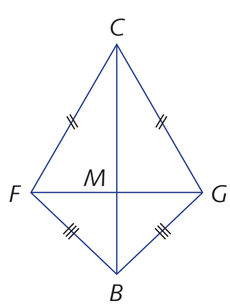
7 The points P and Q are chosen on the diagonal BD of the square $ABCD$ so that $BP = DQ$.



a Prove that the triangles ABP, CBP, ADQ and CDQ are all congruent.

b Hence prove that $APCQ$ is a rhombus.

8 A **kite** is a quadrilateral with two pairs of equal adjacent sides. As with a rhombus, the properties of a kite are best seen when it is drawn in a diamond orientation, as in the diagram.



a Let $CFBG$ be a kite with $CF = CG$ and $BF = BG$. Let the diagonals CB and FG meet at M.

i Show that $\Delta CBF \equiv \Delta CBG$.

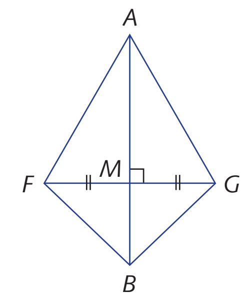
ii Hence show that CB bisects $\angle FCG$ and $\angle FBG$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

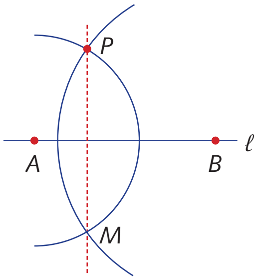
iii Show that $\Delta CMF \equiv \Delta CMG$.

iv Hence show that the diagonals are perpendicular.

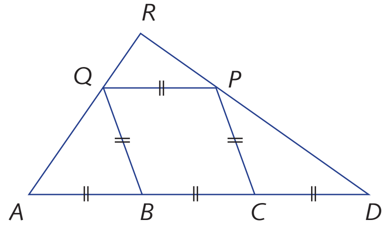
b Conversely, let $AFBG$ be a quadrilateral in which the diagonal AB bisects the diagonal FG and is perpendicular to it. Prove that $AFBG$ is a kite.



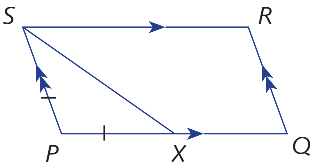
c Let ℓ be a line and P a point not on the line. Choose two points A and B on ℓ. Draw an arc with centre A and radius AP. Then draw a second arc with centre B and radius BP. Let the arcs meet at M. Explain why PM is perpendicular to ℓ.



9 $ABCD$ is a straight line such that $AB = BC = CD$ and $BCPQ$ is a rhombus. The lines produced through AQ and DP meet at R. Prove that $\angle ARD$ is a right angle.



10 $PQRS$ is a parallelogram. X is a point on PQ such that $PX = SP$. Prove that SX bisects $\angle RSP$.



11 A **trapezium** *is a quadrilateral with one pair of parallel lines.*

a The trapezium $ABCD$ has $\angle C = 70°$ and $\angle D = 50°$. Find the sizes of $\angle A$ and $\angle B$.

If the non-parallel sides of a trapezium are equal, then the figure is called an **isosceles trapezium**.

b Prove that the base angles of an isosceles trapezium are equal.

c Prove that the diagonals of an isosceles trapezium are equal in length.

d Prove that if a trapezium has equal diagonals then it is isosceles.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Review exercise

1 Classify these angles according to their size.

a 78° **b** 180° **c** 90° **d** 340° **e** 360° **f** 162°

2 Write down the complements of:

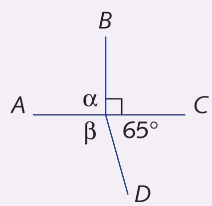
a 56° **b** 67° **c** 47° **d** 15° **e** 26° **f** 46°

3 Write down the supplements of:

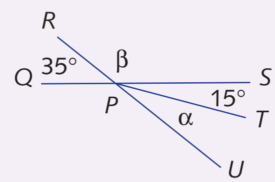
a 154° **b** 89° **c** 34° **d** 90° **e** 113° **f** 116°

4 Find the angles marked with pronumerals. Give reasons for your answers.

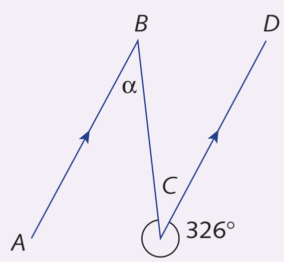
a



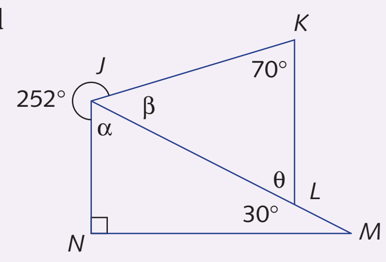
b



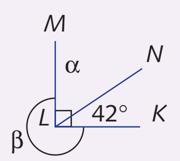
c



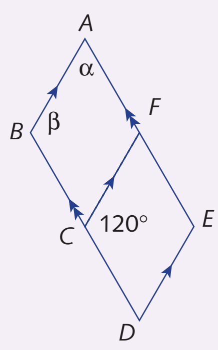
d



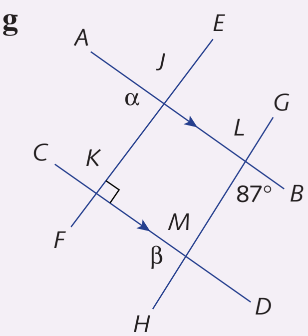
e



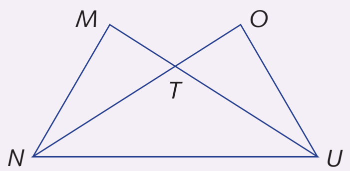
f



g



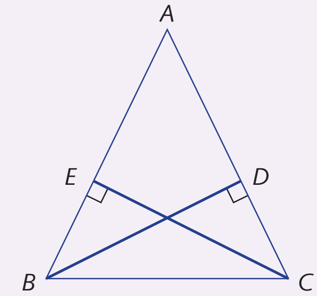
5 In the diagram to the right, $MT = OT$ and $\angle MNT = \angle OUT$. Prove that $NT = UT$.



6 In the diagram to the right, ΔABC is isosceles with $AB = AC$, $BD \perp AC$ and $CE \perp AB$.

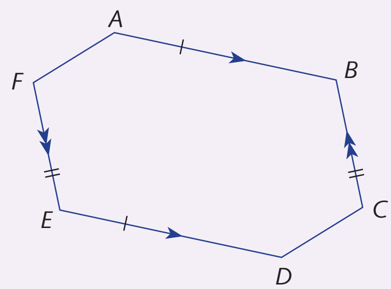
a Prove that ΔBDC is congruent to ΔBEC.

b Prove that ΔAED is isosceles.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

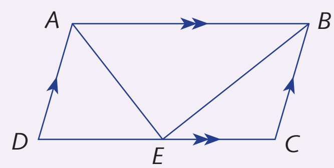
7 *ABCDEF* is a hexagon in which the interval *AB* is parallel and equal in length to the interval *ED*, and the interval *BC* is parallel and equal in length to the interval *FE*.



a Join *B* to *E* and prove that $\angle ABC = \angle FED$.

b Prove that *ACDF* is a parallelogram.

8 In a parallelogram *ABCD*, the point *E* lies on the side *DC* such that *EA* bisects $\angle BAD$ and *EB* bisects $\angle ABC$. Prove that:

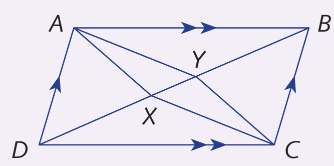


a $DE = EC$

b $DC = 2CB$

c $\angle AEB$ is a right angle

9 In the parallelogram *ABCD*, the points *X* and *Y* are on the diagonal *DB* such that $DX = DA$ and $BY = BC$. Prove that:



a $AX = CY$

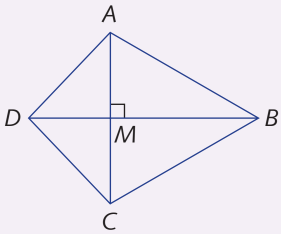
b $AY = CX$

c *AYCX* is a parallelogram

Challenge exercise

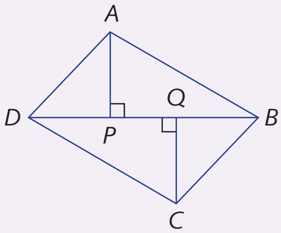
1 **a** Prove that: *If the diagonals of a quadrilateral are perpendicular, then the sums of the squares of opposite sides are equal.*

The diagonals of the convex quadrilateral *ABCD*, to the right, meet at right angles at *M*. Prove that $AB^2 + CD^2 = AD^2 + BC^2$.



b Conversely, prove that: *If the sums of the squares of opposite sides of a quadrilateral are equal, then the diagonals are perpendicular.*

In the convex quadrilateral *ABCD* to the right, $AB^2 + CD^2 = AD^2 + BC^2$. The perpendiculars to the diagonal *BD* from *A* and *C* meet *BD* at *P* and *Q* respectively. Prove that the points *P* and *Q* coincide.



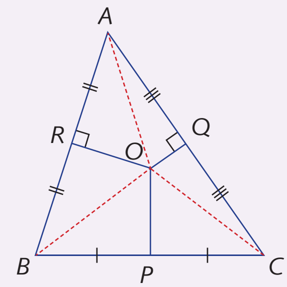
c What happens if the quadrilateral is non-convex?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 The circle passing through all three vertices of a triangle is called a **circumcircle** of the triangle.

The following theorem not only shows that every triangle has a unique circumcircle, but also shows how to construct the circumcircle.

Prove that: *The three perpendicular bisectors of a triangle are concurrent. Their point of intersection (called the circumcentre) is the centre of a circle (called a circumcircle).*

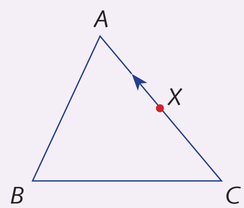


Prove this theorem using congruence. In the diagram above, the midpoints of BC, CA and AB are P, Q and R respectively.

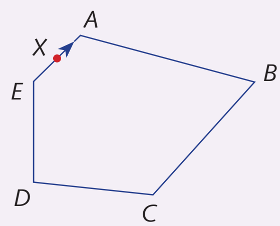
3 In Exercise 7B, you proved that the sum of the exterior angles of a convex polygon is $360°$. This question introduces another way of looking at this theorem.

a In each diagram below, imagine that you started at the point X, facing A, and walked around the figure back to X, passing through the vertices A, B, C, . . . in alphabetical order. Through how many complete revolutions would you turn during your journey? Count clockwise rotation as negative, and anticlockwise revolutions as positive.

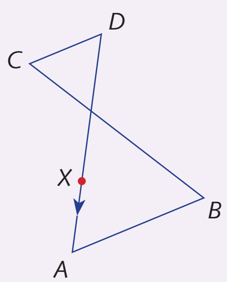
i



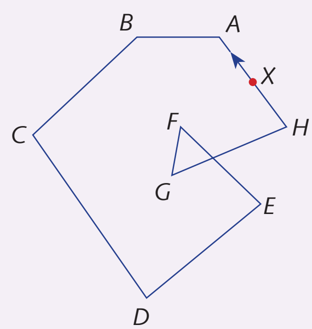
ii



iii



iv



b Use these ideas to develop a proof that the sum of the exterior angles of a convex polygon is $360°$.

4 Suppose that you make a journey on the surface of the globe. You start facing east at Singapore, which lies on the Equator. Then you walk east one-quarter of the way around the globe, then turn $90°$ north and walk to the North Pole, then turn $90°$ towards Singapore and walk back to Singapore, and finally turn facing east just as you started. Through how many revolutions have you turned during this journey, and what is the sum of the exterior angles of your triangular path?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 The diagonals of a convex quadrilateral dissect the quadrilateral into four triangles.

a Prove that: *If the quadrilateral is a trapezium, then a pair of opposite triangles have equal area.*

b Conversely, prove that: *If a pair of opposite triangles have equal area, then the quadrilateral is a trapezium.*

6 **a** Let $ABCD$ be a square. Let Q be a point on and R a point on BC, and let $AQ \perp DR$. Prove that $AQ = DR$.

b Let $ABCD$ be a square. Let ℓ and m be perpendicular lines such that ℓ intersects the sides AB and CD of the square, and m intersects the sides AD and BC. Show that the intervals cut off on ℓ and m by the square are equal in length.

7 Let $ABCD$ be a square of side length 1. Extend the sides AB to E, BC to F, CD to G and DA to H so that $BE = CF = DG = AH = 1$.

a Prove that $EFGH$ is a square, and find its area.

b Show that the centre O of $ABCD$ is also the centre of $EFGH$.

8 Prove that: *Any line through the intersection of the diagonals of a parallelogram dissects the parallelogram into two figures of equal area.*

9 Prove that: *The perpendicular bisector of the base of an isosceles triangle passes through its vertex.*

10 H and K are the midpoint of the sides AB and AC of ΔABC. Points H and K are joined and the line produced to X so that $HK = KX$. Prove that:

a CX is equal and parallel to BH.

b $HK = \frac{1}{2}BC$ and HK is parallel to BC.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

8

Number and Algebra

Index laws

In Year 7 and Year 8 we introduced powers with whole number indices and developed the index laws for these powers.

We now extend our study to rational (fractional), including negative, indices.

We live in a world of very large and very small numbers. Population sizes, government spending, intergalactic distances and the size of computer memories are examples of very large numbers. Thickness of materials, circuit diagrams and subatomic particles are examples of very small numbers. Some examples of small and large numbers are:

- the time taken by light to travel one metre is approximately 0.000 000 003 seconds
- the radius of a hydrogen atom is approximately 0.000 000 000 025 metres
- the current Big Bang model of astronomy suggests that the Universe is about 13.7 billion years old.

This chapter introduces scientific notation, which is a convenient way of writing such numbers. Significant figures are also discussed.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8A The index laws

Index notation

We recall the following from *ICE-EM Mathematics Year 8*.

- A **power** is the product of a certain number of factors, all of which are the same.

  For example,

  $2^4 = 2 \times 2 \times 2 \times 2$ is the fourth power of 2.
- The number 2 in 2^4 is called the **base**.
- The number 4 in 2^4 is called the **index** or **exponent**.
- For any number b, $b^1 = b$.
- In general, $b^n = \underbrace{b \times b \times b \times \ldots \times b}_{n}$, where there are n common factors in the product.

  Here b is the **base** and n the **index**.

Example 1

Express as a power or as a product of powers.

a $5 \times 5 \times 5$ **b** $3 \times 3 \times 7 \times 7 \times 7 \times 7$

Solution

a $5 \times 5 \times 5 = 5^3$ **b** $3 \times 3 \times 7 \times 7 \times 7 \times 7 = 3^2 \times 7^4$

Example 2

Express each number as a power of a prime.

a 81 **b** 128

Solution

a $81 = 3 \times 3 \times 3 \times 3$
$= 3^4$

b $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^7$

The *prime decomposition* of these numbers can be found by a process of repeated division (see *ICE-EM Mathematics Year 8*). This process is particularly useful when the decomposition contains more than one type of prime number.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 3

Express as a product of powers of prime numbers.

a 9000 **b** 66 150

Solution

By repeated division:

a $9000 = 2^3 \times 3^2 \times 5^3$

b $66\,150 = 1323 \times 50$

$= 2 \times 3^3 \times 5^2 \times 7^2$

The following laws for indices were discussed in Chapter 3 of *ICE-EM Mathematics Year 8* for powers with whole number indices.

In the following, a and b are integers and m and n are non-zero whole numbers.

Index laws

Index law 1

To multiply powers of the same base, add the indices.

$$a^m a^n = a^{m+n}$$

Index law 2

To divide powers of the same base, subtract the indices.

$$\frac{a^m}{a^n} = a^{m-n} \qquad \text{where } m > n \text{ and } a \neq 0$$

Index law 3

To raise a power to a power, multiply the indices.

$$(a^m)^n = a^{mn}$$

Index law 4

A power of a product is the product of the powers.

$$(ab)^m = a^m b^m$$

Index law 5

A power of a quotient is the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad \text{where } b \neq 0$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 4

Simplify, expressing the answer in index form.

a $3^2 \times 3^4$ **b** $a^3 \times a^5$

c $3x^2 \times x^3$ **d** $2a^2b^3 \times 5ab^2$

Solution

a $3^2 \times 3^4 = 3^6$

b $a^3 \times a^5 = a^8$

c $3x^2 \times x^3 = 3x^5$

d $2a^2b^3 \times 5ab^2 = 10 \times a^{2+1} \times b^{3+2}$

$= 10a^3b^5$

Example 5

Simplify in power form, and hence evaluate where appropriate.

a $\dfrac{3^5}{3^2}$ **b** $\dfrac{9^5}{9^4}$ **c** $10^6 \div 10^4$

d $a^7 \div a^4$ **e** $\dfrac{3y^4}{y}$ **f** $\dfrac{6x^5}{2x^3}$

Solution

a $\dfrac{3^5}{3^2} = 3^{5-2}$

$= 3^3$

$= 27$

b $\dfrac{9^5}{9^4} = 9^1$

$= 9$

c $10^6 \div 10^4 = 10^{6-4}$

$= 10^2$

$= 100$

d $a^7 \div a^4 = a^{7-4}$

$= a^3$

e $\dfrac{3y^4}{y} = 3 \times \dfrac{y^4}{y}$

$= 3 \times y^{4-1}$

$= 3y^3$

f $\dfrac{6x^5}{2x^3} = \dfrac{6}{2} \times \dfrac{x^5}{x^3}$

$= 3 \times x^{5-3}$

$= 3x^2$

Example 6

Simplify, expressing the answer in index form.

a $\dfrac{3x^3y^2}{4xy} \times \dfrac{6x^2y^3}{x^3y^2}$ **b** $\dfrac{8a^2b^3}{3a^3b} \div \dfrac{4ab^2}{9a^3b^5}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $\dfrac{3x^3y^2}{4xy} \times \dfrac{6x^2y^3}{x^3y^2} = \dfrac{18x^5y^5}{4x^4y^3}$

$= \dfrac{9xy^2}{2}$

b $\dfrac{8a^2b^3}{3a^3b} \div \dfrac{4ab^2}{9a^3b^5} = \dfrac{8a^2b^3}{3a^3b} \times \dfrac{9a^3b^5}{4ab^2}$

$= \dfrac{72 \times a^5 \times b^8}{12 \times a^4 \times b^3}$

$= 6ab^5$

Example 7

Simplify by expanding the brackets.

a $\left(\dfrac{2}{3}\right)^2$ **b** $\left(\dfrac{m}{n}\right)^5$ **c** $\left(\dfrac{x^3}{y^2}\right)^2 \times \left(\dfrac{y}{x}\right)^4$ **d** $\left(\dfrac{2x^2}{3}\right)^2 \div \dfrac{4x^3}{9}$

Solution

a $\left(\dfrac{2}{3}\right)^3 = \dfrac{2^3}{3^3}$

$= \dfrac{8}{27}$

b $\left(\dfrac{m}{n}\right)^5 = \dfrac{m^5}{n^5}$

c $\left(\dfrac{x^3}{y^2}\right)^2 \times \left(\dfrac{y}{x}\right)^4 = \dfrac{x^6}{y^4} \times \dfrac{y^4}{x^4}$

$= x^2$

d $\left(\dfrac{2x^2}{3}\right)^2 \div \dfrac{4x^3}{9} = \dfrac{4x^4}{9} \times \dfrac{9}{4x^3}$

$= x$

There are different possible interpretations of the word 'simplify'. There may be more than one acceptable simplified form.

A number raised to the power zero

Clearly $\dfrac{4^3}{4^3} = 1$. If the index laws are to apply then $4^{3-3} = 4^0 = 1$.

Hence we define $a^0 = 1$, for all non-zero numbers a.

Note: 0^0 is not defined. If x is a pronumeral, $x^0 = 1$ $(x \neq 0)$.

Example 8

Simplify:

a $\left(5a^3\right)^0$

b $\dfrac{6x^2y}{xy^2} \times \dfrac{y^3x}{2y^2x^2}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $(5a^3)^0 = 1$

b $\frac{6x^2y}{xy^2} \times \frac{y^3x}{2y^2x^2} = \frac{6}{2} \times \frac{x^3}{x^3} \times \frac{y^4}{y^4}$
$= 3x^0y^0$
$= 3 \times 1 \times 1$
$= 3$

Example 9

Simplify:

a $(mn^2)^0$ **b** $(a^4b^2)^3$ **c** $(2a^4)^3$ **d** $2(x^2y)^0 \times (x^2y^3)^3$

Solution

a $(mn^2)^0 = 1$

b $(a^4b^2)^3 = a^{12}b^6$

c $(2a^4)^3 = 2^3 \times a^{12}$
$= 8a^{12}$

d $2(x^2y)^0 \times (x^2y^3)^3 = 2 \times 1 \times x^6y^9$
$= 2x^6y^9$

The zero power

For all non-zero numbers a, we define $a^0 = 1$.

Exercise 8A

1 State the base and index of:

a 6^4 **b** 7^3 **c** 8^2 **d** 10^4 **e** 5 **f** 6^0

Example 2

2 Express as a power of a prime number.

a 8 **b** 27 **c** 64 **d** 243 **e** 125 **f** 81

3 Evaluate:

a 3^4 **b** 2^7 **c** 5^5 **d** 7^4 **e** $2^3 \times 3^5$ **f** $6^4 \times 3^2$

Example 3

4 Express as a product of powers of prime numbers.

a 18 **b** 24 **c** 144 **d** 90 **e** 700 **f** 84

Example 4

5 Simplify, leaving the answer as a power or a product of powers.

a $2^2 \times 2^3$ **b** $2^7 \times 2^3$ **c** $3^4 \times 3^5$ **d** $3^4 \times 3^7$

e $3^2 \times 3 \times 3^4$ **f** $3^3 \times 3^4 \times 3^5$ **g** $a^3 \times a^8$ **h** $b^7 \times b^{12}$

i $3a^2 \times a^3$ **j** $3x^2 \times 4x^3$ **k** $2y \times 3y^4$ **l** $4b^2 \times 3b^4$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Simplify:

a $a^2b^3 \times b^2$ **b** $a^3b \times a^2b^3$ **c** $x^2y \times x^3y$

d $2xy^2 \times 3x^2y$ **e** $4a^3b^2 \times a^2b^4$ **f** $5a^4b \times 2ab^3$

Example 5

7 Simplify, leaving the answer as a power or a product of powers.

a $\frac{3^7}{3^2}$ **b** $\frac{2^6}{2^2}$ **c** $5^4 \div 5$ **d** $7^5 \div 7^3$

e $10^7 \div 10^2$ **f** $\frac{10^{12}}{10^4}$ **g** $\frac{a^4}{a}$ **h** $\frac{a^5}{a^3}$

i $\frac{2x^3}{x^2}$ **j** $\frac{6x^5}{2x^2}$ **k** $\frac{10y^{12}}{5y^3}$ **l** $\frac{27p^4}{9p}$

8 Simplify:

a $\frac{a^3b^2}{a^2}$ **b** $\frac{x^3y^2}{xy}$ **c** $\frac{a^5b^3}{a^4b}$ **d** $\frac{x^4y^7}{x^3y^2}$

e $\frac{12a^6b^2}{4a^2b}$ **f** $\frac{15xy^3}{3y^2}$ **g** $\frac{16a^4b^3}{12a^2b^2}$ **h** $\frac{27x^2y^3}{18xy^2}$

Example 6

9 Simplify:

a $\frac{a^3b^2}{ab} \times \frac{a^2b}{a}$ **b** $\frac{x^3y}{xy^2} \times \frac{x^4y^5}{x^2}$ **c** $\frac{2ab^2}{3a^2b^4} \times \frac{6a^4b^5}{ab}$ **d** $\frac{12x^4y^3}{3x^2y} \times \frac{x^2y^4}{x^3y^5}$

e $\frac{6ab^2}{5a^3b} \div \frac{12ab}{15a^5b}$ **f** $\frac{7x^3y^4}{2xy^2} \div \frac{21x^2y^3}{4x^3y^2}$ **g** $\frac{14a^4b^3}{3ab^2} \div \frac{7a^5b^4}{6a^3b^5}$ **h** $\frac{12x^2y}{x^3y^4} \div \frac{6xy^2}{x^6y^7}$

10 Copy and complete.

a $a^4 \times \ldots = a^{10}$ **b** $b^7 \times \ldots = b^{16}$ **c** $4a^3 \times \ldots = 12a^7$

d $9d^5 \times \ldots = 27d^6$ **e** $a^8 \div \ldots = a^4$ **f** $x^{10} \div \ldots = x^6$

g $15d^7 \div \ldots = 3d^2$ **h** $9d^6 \div \ldots = 3d$ **i** $m^4n^5 \times \ldots = m^{10}n^7$

j $8ab^4 \times \ldots = 24a^2b^6$ **k** $a^7b^4 \div \ldots = a^2b$ **l** $14x^5y^2 \times \ldots = 42x^{10}y^5$

m $\ell^6m^7 \div \ldots = \ell^2m^5$ **n** $9m^7n^4 \div \ldots = 3m^2$ **o** $18p^2q^6 \div \ldots = 3pq$

11 Simplify each expression. Check your answer for part **a** by substituting $x = 2$ into both the original expression and the simplified expression. Repeat for $x = 3$ in part **b** and $x = -2$ in part **c**.

a $\frac{6x^2}{x}$ **b** $\frac{6x^2}{3x}$ **c** $\frac{6x^2}{6x}$ **d** $\frac{8x^4}{4x}$

e $\frac{3x^5}{x^3}$ **f** $\frac{10x^4}{5x^3}$ **g** $\frac{10x^4}{2x^4}$ **h** $\frac{12x^4}{6x^2}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 8

12 Simplify:

a a^0 **b** $2x^0$ **c** xy^0 **d** $7x^0y^0$

Example 9

13 Simplify:

a $3a^0$ **b** $6a^0$ **c** $(4a)^0$ **d** $(3b)^0$

e $4a^0 + 3b^0$ **f** $6a^0 + 7m^0$ **g** $(2a + 1)^0$ **h** $(4a + 3b)^0$

i $(4b)^0 + 2b^0$ **j** $(3b)^0 - 5d^0$ **k** $(5m^0 + 7b)^0$ **l** $(6m - 2c^0)^0$

14 Simplify, leaving the answer as a power.

a $\left(2^3\right)^4$ **b** $\left(3^2\right)^3$ **c** $(a^2)^5$ **d** $(y^5)^6$

15 Simplify:

a $(a^3)^2 \times (a^3)^4$ **b** $(x^4)^2 \times (x^3)^3$ **c** $(b^4)^2 \div (b^3)^2$

d $\dfrac{(y^3)^4}{(y^4)^2}$ **e** $2ab^2 \times 3a(b^3)^2$ **f** $\dfrac{3ab}{(b^2)^3} \times \dfrac{4b^7}{3a}$

g $\dfrac{4(x^3)^2y^4}{3x^4y^3} \times \dfrac{3x^3(y^2)^2}{8xy^5}$ **h** $\dfrac{8a^2(b^3)^2}{3ab^2} \div \dfrac{16a^5b^3}{9(a^3)^2}$ **i** $\dfrac{3(x^3y)^2}{(x^2y)^2} \div \dfrac{12x^4y^2}{(2x^3y)^2}$

16 Copy and complete (using index law 3).

a $(a^6)^{\cdots} = a^{24}$ **b** $(b^3)^{\cdots} = b^{21}$ **c** $(m^5)^{\cdots} = m^{10}$

d $(m^6)^{\cdots} = m^{30}$ **e** $(\ldots)^6 = p^{36}$ **f** $(\ldots)^5 = p^{25}$

g $(\ldots)^4 = a^8$ **h** $(\ldots)^3 = m^{15}$ **i** $(\ldots)^{\cdots} = m^{20}$

17 **a** Is it true that $(a^2)^6 = (a^6)^2$? **b** Is it true that $(b^4)^7 = (b^7)^4$?

c Generalise your result.

Example 7

18 Simplify by expanding the brackets.

a $(3a)^2$ **b** $(2x)^3$ **c** $(xy^3)^2$ **d** $(a^2b)^4$

e $\left(\dfrac{a}{5}\right)^2$ **f** $\left(\dfrac{2}{x}\right)^3$ **g** $\left(\dfrac{a}{b}\right)^5$ **h** $\left(\dfrac{x^2}{y}\right)^3$

19 Simplify:

a $(2a^2b)^2 \times 3ab^3$ **b** $(3xy^2)^3 \times (x^2y)^2$ **c** $(m^2n^3)^2 \times (mn)^3$

d $(5xy^2)^3 \times (x^2y^3)^2$ **e** $(2a^3b)^3 \times 3a^0$ **f** $(2xy^2)^0 \times (3x^2y)^3$

20 Simplify:

a $\left(\dfrac{x^2}{y}\right)^2 \times \left(\dfrac{y^2}{x}\right)^3$ **b** $\left(\dfrac{4a^2}{b}\right)^2 \times \left(\dfrac{b}{2a}\right)^3$ **c** $\left(\dfrac{x^3}{y^2}\right)^2 \div \left(\dfrac{x}{y^2}\right)^3$ **d** $\left(\dfrac{2x^4}{y}\right)^5 \div \left(\dfrac{4x^3}{y^3}\right)^2$

21 Simplify:

a $\dfrac{(3xy^2)^2 \times (2x^2y)^3}{(6x^2y)^2}$ **b** $\dfrac{3a^2b^4 \times (2ab^2)^3}{(4a^2b^3)^2}$ **c** $\dfrac{(2x^2y^3)^3 \times (5xy^2)^2}{(10x^2y)^2 \times (xy)^3}$ **d** $\dfrac{(6ab)^3 \times 2a^7b^4}{(2ab)^4 \times (3a^2b)^2}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

22 Copy and complete.

a $(\ldots)^4 = a^8b^{12}$ **b** $(\ldots)^6 = m^{30}n^{24}$ **c** $(p^3q)^{\ldots} = p^9q^3$

d $(x^4y^7)^{\ldots} = 1$ **e** $(\ldots)^4 = 16a^8$ **f** $(\ldots)^3 = 27q^9$

g $(\ldots)^2 = 49m^6$ **h** $(\ldots)^3 = 64\ell^9m^3$ **i** $(\ldots)^2 = 25m^{10}n^6$

8B Negative indices

In the last section, we defined a^n to be the product of n factors of a, where a is any number and n is a positive integer. We also defined $a^0 = 1$ for all non-zero numbers a.

We now give meaning to negative integer indices. For example, we want to give a meaning to 2^{-4}, 3^{-100} and so on. To work towards a useful definition, look at the following pattern:

$$2^5 = 32, \quad 2^4 = 16, \quad 2^3 = 8,$$
$$2^2 = 4, \quad 2^1 = 2, \quad 2^0 = 1$$

Each index is one less than the preceding one, and each number to the right of an equal sign is half the number in the previous expression.

This can be continued purely as a pattern:

$$2^{-1} = \frac{1}{2}, \qquad 2^{-2} = \frac{1}{4} = \frac{1}{2^2}, \qquad 2^{-3} = \frac{1}{8} = \frac{1}{2^3} \qquad \text{and so on.}$$

This suggests we define $2^{-n} = \dfrac{1}{2^n}$, for any positive integer n.

Negative indices

Define

$$a^{-n} = \frac{1}{a^n}$$

where a is a non-zero number and n is a positive integer.

Note that this tells us that $a^{-n} \times a^n = 1$. So $a^n = \dfrac{1}{a^{-n}}$

For example,

$$2^{10} = \frac{1}{2^{-10}}$$

So we can say that for all n, whether n is positive or negative, $a^{-n} = \dfrac{1}{a^n}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 10

Evaluate:

a 6^{-2} **b** 4^{-3} **c** 2^{-7} **d** 10^{-3}

Solution

a $6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

b $4^{-3} = \dfrac{1}{4^3} = \dfrac{1}{64}$

c $2^{-7} = \dfrac{1}{2^7} = \dfrac{1}{128}$

d $10^{-3} = \dfrac{1}{10^3} = \dfrac{1}{1000}$

Fractions and negative indices

The **reciprocal** of a fraction such as $\frac{4}{3}$ is $\frac{3}{4}$. The index -1 means 'the reciprocal of', so

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

When raising a fraction to other negative indices take the reciprocal first.

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{27}{8}$$

Example 11

Evaluate:

a $\left(\frac{1}{3}\right)^{-1}$ **b** $\left(\frac{2}{7}\right)^{-2}$ **c** $\left(4\frac{1}{4}\right)^{-2}$

Solution

a $\left(\frac{1}{3}\right)^{-1} = \frac{3}{1} = 3$

b $\left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^{2} = \frac{49}{4}$

c $\left(4\frac{1}{4}\right)^{-2} = \left(\frac{17}{4}\right)^{-2} = \left(\frac{4}{17}\right)^{2} = \frac{16}{289}$

Note that in general $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

The index laws we revised in Section 8A are also valid when using negative integer indices.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

For all integers m and n and non-zero numbers a and b the following are true.		
Zero index		$a^0 = 1$
Negative index		$a^{-n} = \dfrac{1}{a^n}$
Index law 1	Product of powers	$a^m a^n = a^{m+n}$
Index law 2	Quotient of powers	$a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$
Index law 3	Power of a power	$(a^m)^n = a^{mn}$
Index law 4	Power of a product	$(ab)^n = a^n b^n$
Index law 5	Power of a quotient	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

The results for negative indices m and n can be proved using the index laws for positive integer indices. An indication of the proof appears as a question in the Challenge exercise at the end of the chapter.

Example 12

Write as a single power and then evaluate.

a $3^4 \times 3^{-2}$ **b** $5^7 \times 5^{-8}$ **c** $13^{-8} \times 13^{15} \times 13^{-7}$ **d** $\left(\dfrac{2}{3}\right)^{-6} \times \left(\dfrac{2}{3}\right)^4$

Solution

a $3^4 \times 3^{-2} = 3^2$
$= 9$

b $5^7 \times 5^{-8} = 5^{-1}$
$= \dfrac{1}{5}$

c $13^{-8} \times 13^{15} \times 13^{-7} = 13^0$
$= 1$

d $\left(\dfrac{2}{3}\right)^{-6} \times \left(\dfrac{2}{3}\right)^4 = \left(\dfrac{2}{3}\right)^{-2}$
$= \left(\dfrac{3}{2}\right)^2$
$= \dfrac{9}{4}$

Example 13

Write as a single power and then evaluate.

a $\dfrac{2^4}{2^5}$ **b** $\dfrac{3^4}{3^7}$ **c** $\dfrac{5}{5^3}$ **d** $\dfrac{3^4}{3^6}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $\dfrac{2^4}{2^5} = 2^{-1}$

$= \dfrac{1}{2}$

b $\dfrac{3^4}{3^7} = 3^{-3}$

$= \dfrac{1}{3^3}$

$= \dfrac{1}{27}$

c $\dfrac{5}{5^3} = 5^{-2}$

$= \dfrac{1}{5^2}$

$= \dfrac{1}{25}$

d $\dfrac{3^4}{3^6} = 3^{-2}$

$= \dfrac{1}{3^2}$

$= \dfrac{1}{9}$

Example 14

Simplify, expressing the answers with positive indices.

a $a^2b^{-3} \times a^{-4}b^5$

b $\dfrac{x^2y^3}{x^3y^2}$

c $(2a^{-2}b^3)^{-2}$

d $\left(\dfrac{3m^2}{n}\right)^{-4}$

Solution

a $a^2b^{-3} \times a^{-4}b^5 = a^{2-4} \times b^{-3+5}$

$= \dfrac{1}{a^2} \times b^2$

$= \dfrac{b^2}{a^2}$

b $\dfrac{x^2y^3}{x^3y^2} = x^{-1}y^1$

$= \dfrac{1}{x} \times y$

$= \dfrac{y}{x}$

c $(2a^{-2}b^3)^{-2} = 2^{-2} \times a^4 \times b^{-6}$

$= \dfrac{1}{2^2} \times a^4 \times \dfrac{1}{b^6}$

$= \dfrac{a^4}{4b^6}$

d $\left(\dfrac{3m^2}{n}\right)^{-4} = \left(\dfrac{n}{3m^2}\right)^4$

$= \dfrac{n^4}{81m^8}$

Exercise 8B

Example 10

1 Express with a positive index and then evaluate.

a 2^{-1} **b** 5^{-1} **c** 3^{-2} **d** 6^{-2} **e** 9^{-2} **f** 10^{-2}

g 2^{-4} **h** 3^{-3} **i** 5^{-3} **j** 3^{-4} **k** 10^{-5} **l** 2^{-7}

2 Write each fraction as a power of a prime with a negative index.

a $\dfrac{1}{8}$ **b** $\dfrac{1}{9}$ **c** $\dfrac{1}{27}$ **d** $\dfrac{1}{49}$ **e** $\dfrac{1}{121}$ **f** $\dfrac{1}{125}$

g $\dfrac{1}{16}$ **h** $\dfrac{1}{64}$ **i** $\dfrac{1}{169}$ **j** $\dfrac{1}{81}$ **k** $\dfrac{1}{31}$ **l** $\dfrac{1}{32}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Express with positive indices, evaluating where possible.

a a^{-3} **b** x^{-7} **c** $3a^{-4}$ **d** $5x^{-7}$ **e** $4a^{-5}$ **f** $\dfrac{1}{x^{-3}}$

g $\dfrac{3}{a^{-4}}$ **h** $\dfrac{5}{x^{-5}}$ **i** $3^{-2}a^{-2}$ **j** $4^{-2}x^{-2}$ **k** $\dfrac{13^{-2}}{n^{-6}}$ **l** $\dfrac{y^{-4}}{x^{-3}}$

Example 11, 12

4 Simplify where possible and then evaluate.

a $\left(\dfrac{1}{4}\right)^{-1}$ **b** $\left(\dfrac{2}{5}\right)^{-2}$ **c** $\left(3\frac{1}{3}\right)^{-2}$ **d** $\left(\dfrac{2}{3}\right)^{-3}$

e $3^5 \times 3^{-2}$ **f** $5^{11} \times 5^{-8}$ **g** $7^3 \times 7^{-5}$ **h** $4^3 \times 4^{-5}$

Example 13

5 Write as a single power and then evaluate.

a $\dfrac{2^3}{2^6}$ **b** $\dfrac{4^2}{4^4}$ **c** $\dfrac{3^8}{3^9}$ **d** $\dfrac{6^5}{6^8}$ **e** $\dfrac{7^1}{7^3}$ **f** $\dfrac{5^7}{5^{10}}$

g $\dfrac{8^6}{8^7}$ **h** $\dfrac{20^4}{20^6}$ **i** $\dfrac{3^5}{3^9}$ **j** $\dfrac{2^7}{2^{13}}$ **k** $\dfrac{10^2}{10^6}$ **l** $\dfrac{12^{12}}{12^{14}}$

6 Express with negative index.

a $\dfrac{3}{x}$ **b** $\dfrac{5}{x^2}$ **c** $\dfrac{8}{x^4}$ **d** $\dfrac{3}{2x^4}$ **e** $\dfrac{4}{3x^7}$ **f** $\dfrac{2}{3x^5}$

7 Evaluate.

a $\left(\dfrac{1}{2}\right)^{-1}$ **b** $\left(\dfrac{2}{3}\right)^{-1}$ **c** $\left(\dfrac{1}{2}\right)^{-2}$ **d** $\left(\dfrac{4}{5}\right)^{-2}$ **e** $\left(2\frac{1}{4}\right)^{-2}$ **f** $\left(1\frac{1}{5}\right)^{-3}$

Example 14

8 Simplify, expressing the answer with positive indices.

a $x^{-6}y^4 \times x^2y^{-2}$ **b** $a^{-3}b^{-5} \times a^5b^{-3}$ **c** $3x^{-2}y^5 \times 5x^{-7}y^{-2}$

d $2a^{-1}b^5 \times 7ab^{-3}$ **e** $7a^3m^{-4} \times 8a^{-5}m^{-3}$ **f** $3r^2s^3 \times 4r^{-3}s^{-5}$

g $\dfrac{8a^{-4}}{2a^6}$ **h** $\dfrac{16a^{-4}}{8a^5}$ **i** $\dfrac{18a^{-4}}{4a^5}$ **j** $\dfrac{27m^{-3}}{9m^{-2}}$ **k** $\dfrac{56t^{-7}}{8t^{-2}}$

l $\dfrac{36h^{-9}}{9h^{-4}}$ **m** $\dfrac{144x^7y^5}{12x^{-3}y^4}$ **n** $\dfrac{72a^4b^{-3}}{36ab^{-2}}$ **o** $\dfrac{7a^2b^{-3}c^{-4}}{21a^5b^{-7}c^{-9}}$ **p** $\dfrac{9m^3n^4p^{-5}}{21m^{-3}n^4p^2}$

9 Copy and complete.

a $6^4 \times \ldots = 6^2$ **b** $9^5 \times \ldots = 9^4$ **c** $b^9 \times \ldots = b^7$

d $m^5 \times \ldots = m^{-6}$ **e** $a^{11} \div \ldots = a^{14}$ **f** $b^7 \div \ldots = b^{15}$

g $d^{-7} \div \ldots = d^{15}$ **h** $e^{-7} \div \ldots = e^{-5}$ **i** $(m^{-2})^{\cdots} = m^{10}$

j $(a^5)^{\cdots} = a^{-15}$ **k** $(\ldots)^{-4} = \dfrac{m^8}{16}$ **l** $(\ldots)^{-3} = \dfrac{1}{27a^9}$

m $(\ldots)^{-2} = \dfrac{m^6}{25}$ **n** $(\ldots)^{-3} = \dfrac{a^6}{b^9}$ **o** $(\ldots)^{-6} = \dfrac{m^{12}n^{18}}{p^6}$

p $(\ldots)^{-2} = p^4q^{-6}$ **q** $(\ldots)^{\cdots} = a^6b^{-4}$ **r** $(\ldots)^{\cdots} = \dfrac{m^6}{n^9}$

Write two possible alternatives for part **q** and part **r**.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 Simplify, expressing the answers with positive indices. Evaluate powers where possible.

a $(3a^2b^{-2})^3 \times (2a^4)^{-2}$

b $(5x^4y^6)^{-3} \times (5^2xy^{-1})^3$

c $(5m^2n^{-3})^{-2} \times 2(m^{-2}n^3)^2$

d $(6a^5b^{-4})^{-3} \times 2(a^3b^{-3})^2$

e $\dfrac{(x^2)^2}{y} \times \dfrac{(y^2)^{-3}}{x^3}$

f $\dfrac{(2x^3)^{-2}}{y^4} \times \dfrac{(2x^7)^2}{3y^5}$

g $\dfrac{(2a^4b^{-2})^3}{c^2} \times \dfrac{(2^2a^{-3}b^2)^{-1}}{c}$

h $\dfrac{(m^2n^3)^2}{p^{-3}} \times (mnp^{-2})^{-3}$

i $\dfrac{(a^2)^3}{b^3} \div \left(\dfrac{a}{b^2}\right)^{-2}$

j $\dfrac{(2a^4)^2}{b^7} \div \dfrac{(a^2)^{-3}}{2b}$

k $\dfrac{(4c^4d^{-3})^2}{9} \div \dfrac{3c^{-2}}{d}$

l $\dfrac{(3m^2n^3)^{-2}}{p^4} \div \dfrac{p^{-3}}{m}$

8C Fractional indices

Fractional indices with numerator one

We are now going to extend our study of indices by looking at fractional indices. We begin by considering what we mean by powers such as $3^{\frac{1}{2}}$, $2^{\frac{1}{3}}$ and $7^{\frac{1}{10}}$, in which the index is the reciprocal of a positive integer.

If a is a positive number, then $\sqrt{a}$ is the positive square root and $(\sqrt{a})^2 = a = a^1$.

We now introduce the alternative notation $a^{\frac{1}{2}}$ for $\sqrt{a}$. We do this because the third index law then continues to hold, that is:

$$\left(a^{\frac{1}{2}}\right)^2 = a^{2\times\frac{1}{2}} = a^1$$

Keep in mind that $a^{\frac{1}{2}}$ is nothing more than an alternative notation for $\sqrt{a}$.

For example, $49^{\frac{1}{2}} = 7$, $64^{\frac{1}{2}} = 8$, $100^{\frac{1}{2}} = 10$, and so on.

Every positive number a has a cube root $\sqrt[3]{a}$. It is the positive number whose cube is a.

For example:

$\sqrt[3]{8} = 2$ because $2^3 = 8$ $\qquad$ $\sqrt[3]{27} = 3$ because $3^3 = 27$

We define $a^{\frac{1}{3}}$ to be $\sqrt[3]{a}$. The third index laws continues to hold.

$$\left(a^{\frac{1}{3}}\right)^3 = a^{3\times\frac{1}{3}} = a^1$$

Similarly we define

$a^{\frac{1}{4}} = \sqrt[4]{a}$, $a^{\frac{1}{5}} = \sqrt[5]{a}$, and so on.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Fractional indices

Let a be positive or zero and let n be a positive integer.

Define $a^{\frac{1}{n}}$ to be the n^{th} root of a. That is, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

For example, $a^{\frac{1}{2}} = \sqrt{a}$ and $a^{\frac{1}{3}} = \sqrt[3]{a}$

Square roots, cube roots, fourth roots and so on are usually irrational numbers, and a calculator can be used to obtain approximations. Using a calculator, we obtain:

$$10^{\frac{1}{2}} = \sqrt{10} \approx 3.1623, \qquad 10^{\frac{1}{3}} = \sqrt[3]{10} \approx 2.1544, \qquad 10^{\frac{1}{4}} = \sqrt[4]{10} \approx 1.7783, \ldots$$

Using our new notation, here are some other numerical approximations, all recorded correct to 4 decimal places. You can use your calculator to check these.

$$2^{\frac{1}{5}} \approx 1.1487, \qquad 10^{\frac{1}{8}} \approx 1.3335, \qquad 0.2^{\frac{1}{4}} \approx 0.6687, \qquad 3.2^{\frac{1}{6}} \approx 1.2139$$

Example 15

Evaluate:

a $\sqrt[3]{27}$ **b** $\sqrt[4]{16}$ **c** $\sqrt[5]{243}$ **d** $\sqrt[3]{125}$

Solution

a $\sqrt[3]{27}$ is the number that when cubed is 27.
Thus $3^3 = 27$, because $\sqrt[3]{27} = 3$

b $\sqrt[4]{16} = 2$ because $2^4 = 16$.

c $\sqrt[5]{243} = \sqrt[5]{3^5}$
$= 3$

d $\sqrt[3]{125} = \sqrt[3]{5^3}$
$= 5$

Example 16

Evaluate:

a $9^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $16^{\frac{1}{4}}$ **d** $1\,000\,000^{\frac{1}{6}}$

Solution

a $9^{\frac{1}{2}} = 3$ (since $3^2 = 9$)

b $27^{\frac{1}{3}} = 3$ (since $3^3 = 27$)

c $16^{\frac{1}{4}} = 2$ (since $2^4 = 16$)

d $1\,000\,000^{\frac{1}{6}} = 10$ (since $10^6 = 1\,000\,000$)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Positive fractional indices

In the previous section, we defined $8^{\frac{1}{3}} = \sqrt[3]{8}$. For consistency with the index laws we now define $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$.

Positive fractional indices

If a is a positive number or zero and, p and q are positive integers, define:

$$a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p = \left(\sqrt[q]{a}\right)^p$$

Example 17

Evaluate:

a $4^{\frac{3}{2}}$ **b** $8^{\frac{2}{3}}$ **c** $81^{\frac{3}{4}}$

Solution

a $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3$

$= 3^3$ (since $2^2 = 4$)

$= 8$

b $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$

$= 2^2$ (since $2^3 = 8$)

$= 4$

c $81^{\frac{3}{4}} = \left(81^{\frac{1}{4}}\right)^3$

$= 3^3$ (since $3^4 = 81$)

$= 27$

The index laws also hold for fractional indices. A proof of these more general results is given as a question in the Challenge exercise.

Example 18

Simplify:

a $(d^{21})^{\frac{1}{7}}$ **b** $p^{\frac{2}{3}} \times p^{\frac{4}{5}}$ **c** $q^{\frac{4}{5}} \div q^{\frac{2}{3}}$

Solution

a $(d^{21})^{\frac{1}{7}} = d^{21\times\frac{1}{7}}$

$= d^3$

b $p^{\frac{2}{3}} \times p^{\frac{4}{5}} = p^{\frac{2}{3}+\frac{4}{5}}$

$= p^{\frac{10}{15}+\frac{12}{15}}$

$= p^{\frac{22}{15}}$

c $q^{\frac{4}{5}} \div q^{\frac{2}{3}} = q^{\frac{4}{5}-\frac{2}{3}}$

$= q^{\frac{12-10}{15}}$

$= q^{\frac{2}{15}}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Negative fractional indices

Since $4^{-1} = \frac{1}{4}$, we define $4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \left(\frac{1}{4}\right)^{\frac{3}{2}}$.

We can now combine the definitions of negative indices and fractional indices.

Negative fractional indices

Let a be a positive number or zero, and let p and q be positive integers. Define

$$a^{-\frac{p}{q}} = \frac{1}{a^{\frac{p}{q}}} = \left(\frac{1}{a}\right)^{\frac{p}{q}}$$

Note: If $a > 0$ and $b > 0$, $\left(\frac{a}{b}\right)^{-\frac{p}{q}} = \left(\frac{b}{a}\right)^{\frac{p}{q}}$.

Example 19

Evaluate:

a $4^{-\frac{3}{2}}$

b $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$

Solution

a $4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}}$
$= \frac{1}{8}$

b $\left(\frac{8}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{8}\right)^{\frac{1}{3}}$
$= \frac{27^{\frac{1}{3}}}{8^{\frac{1}{3}}}$
$= \frac{3}{2}$

Example 20

Simplify, writing each answer with positive indices.

a $p^{-\frac{1}{4}} \times p^{\frac{2}{3}}$ **b** $x^{\frac{2}{3}} \div x^{-\frac{1}{2}}$ **c** $(125n^{-6})^{\frac{1}{3}}$ **d** $x^{\frac{1}{5}} \div x^{\frac{1}{3}}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $p^{-\frac{1}{4}} \times p^{\frac{2}{3}} = p^{-\frac{1}{4}+\frac{2}{3}}$

$= p^{-\frac{3}{12}+\frac{8}{12}}$

$= p^{\frac{5}{12}}$

b $x^{\frac{2}{3}} \div x^{-\frac{1}{2}} = x^{\frac{2}{3}-\left(-\frac{1}{2}\right)}$

$= x^{\frac{4}{6}+\frac{3}{6}}$

$= x^{\frac{7}{6}}$

c $(125n^{-6})^{\frac{1}{3}} = 125^{\frac{1}{3}} \times n^{-6\times\frac{1}{3}}$

$= 5 \times n^{-2}$

$= \dfrac{5}{n^2}$

d $x^{\frac{1}{5}} \div x^{\frac{1}{3}} = x^{\frac{3}{15}-\frac{5}{15}}$

$= x^{-\frac{2}{15}}$

$= \dfrac{1}{x^{\frac{2}{15}}}$

Exercise 8C

Example 15

1 Evaluate:

a $\sqrt[3]{8}$ **b** $\sqrt[5]{32}$ **c** $\sqrt[3]{216}$ **d** $\sqrt[4]{81}$ **e** $\sqrt[3]{64}$ **f** $\sqrt[5]{2^{10}}$

2 Write using fractional indices. Evaluate, correct to 4 decimal places.

a $\sqrt{14}$ **b** $\sqrt[4]{64}$ **c** $\sqrt[5]{7}$ **d** $\sqrt[7]{11}$ **e** $\sqrt[3]{2^7}$

Example 16

3 Evaluate:

a $4^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $243^{\frac{1}{5}}$ **d** $81^{\frac{1}{4}}$

e $64^{\frac{1}{2}}$ **f** $25^{\frac{1}{2}}$ **g** $125^{\frac{1}{3}}$ **h** $64^{\frac{1}{3}}$

i $32^{\frac{1}{5}}$ **j** $625^{\frac{1}{4}}$ **k** $216^{\frac{1}{3}}$ **l** $49^{\frac{1}{2}}$

Example 17

4 Evaluate:

a $4^{\frac{5}{2}}$ **b** $25^{\frac{3}{2}}$ **c** $125^{\frac{2}{3}}$ **d** $64^{\frac{5}{6}}$

e $32^{\frac{2}{5}}$ **f** $81^{\frac{3}{4}}$ **g** $216^{\frac{2}{3}}$ **h** $243^{\frac{3}{5}}$

i $\left(\sqrt[4]{16}\right)^3$ **j** $\left(\sqrt[3]{27}\right)^2$ **k** $\sqrt[5]{32^4}$ **l** $\sqrt[3]{2^6}$

Example 18

5 Simplify:

a $\left(a^{\frac{1}{2}}\right)^2$ **b** $\left(b^{\frac{1}{3}}\right)^6$ **c** $(c^{12})^{\frac{1}{4}}$ **d** $(c^{10})^{\frac{1}{5}}$

e $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **f** $y^{\frac{1}{3}} \times y^{\frac{2}{3}}$ **g** $p^{\frac{3}{4}} \times p^{\frac{2}{5}}$ **h** $q^{\frac{3}{2}} \times q^{\frac{2}{3}}$

i $x^{\frac{3}{2}} \div x^{\frac{1}{2}}$ **j** $y^{\frac{2}{3}} \div y^{\frac{1}{3}}$ **k** $p^{\frac{3}{4}} \div p^{\frac{2}{5}}$ **l** $q^{\frac{3}{2}} \div q^{\frac{2}{3}}$

m $(4m^6)^{\frac{1}{2}}$ **n** $(27n^{12})^{\frac{1}{3}}$ **o** $\left(2x^{\frac{2}{3}}\right)^3$ **p** $\left(3y^{\frac{1}{2}}\right)^4$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 19

6 Evaluate:

a $4^{-\frac{1}{2}}$ **b** $25^{-\frac{1}{2}}$ **c** $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ **d** $\left(\frac{64}{27}\right)^{-\frac{1}{3}}$ **e** $32^{-\frac{2}{5}}$

f $\left(\frac{1}{81}\right)^{-\frac{1}{4}}$ **g** $81^{-\frac{1}{4}}$ **h** $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$ **i** $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$ **j** $\left(\frac{32}{243}\right)^{-\frac{1}{5}}$

Example 20

7 Simplify, expressing the answer with positive indices.

a $\left(a^{\frac{1}{2}}\right)^{-2}$ **b** $\left(b^{-\frac{2}{3}}\right)^{6}$ **c** $\left(2x^{\frac{2}{3}}\right)^{-3}$ **d** $\left(3y^{\frac{1}{2}}\right)^{-4}$ **e** $x^{\frac{1}{2}} \times x^{-\frac{3}{2}}$

f $y^{\frac{1}{3}} \times y^{-\frac{2}{3}}$ **g** $p^{\frac{3}{4}} \times p^{-\frac{2}{5}}$ **h** $q^{\frac{3}{2}} \times q^{-\frac{2}{3}}$ **i** $x^{\frac{3}{2}} \div x^{-\frac{1}{2}}$ **j** $y^{\frac{2}{3}} \div y^{-\frac{1}{3}}$

k $p^{\frac{3}{4}} \div p^{-\frac{2}{5}}$ **l** $q^{\frac{3}{2}} \div q^{-\frac{2}{3}}$ **m** $(4m^{-6})^{\frac{1}{2}}$ **n** $(27n^{-12})^{\frac{1}{3}}$ **o** $\left(2x^{-\frac{2}{5}}\right)^{5}$

8D Scientific notation

Scientific notation, or **standard form**, is a convenient way to represent very large or very small numbers. It allows such numbers to be easily read.

Here is an example with a very large number. The star Sirius A is approximately $81\,362\,000\,000\,000$ km from the Sun. This is about 81 trillion kilometres. This distance can be written neatly in scientific notation as 8.1362×10^{13} km. You can verify that if we move the decimal point 13 places to the right, inserting the necessary zeros, we arrive back at the number we started with.

We can also use this notation for very small numbers. For example, an angstrom (Å) is a unit of length equal to $0.000\,000\,0001$ m, which is the approximate diameter of a small atom. In scientific notation this is written as 1.0×10^{-10} m or 1×10^{-10} m. When we move the decimal point 10 places to the left, inserting the zeros, we arrive back at the original number. To further this example, the approximate diameter of a uranium atom is $0.000\,000\,000\,38$ m, or 3.8×10^{-10} m, or 3.8 Å.

By definition, a positive number is in scientific notation (or standard form) if it is written as $a \times 10^{b}$, where $1 \le a < 10$ and b is an integer.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

We convert a number into scientific notation by placing a decimal point after the first non-zero digit and multiplying by the appropriate power of 10.

Note: If the number to be written in scientific notation is greater than 1, then the index is positive or zero. If the number is positive and less than 1, then the index is negative.

Example 21

Write in scientific notation.

a 610 **b** 21000 **c** 46 000 000 **d** 81

e 0.0067 **f** 0.000 02 **g** 0.07 **h** 8.17

Solution

a $610 = 6.1 \times 100$
$= 6.1 \times 10^2$

b $21000 = 2.1 \times 10\,000$
$= 2.1 \times 10^4$

c $46\,000\,000 = 4.6 \times 10\,000\,000$
$= 4.6 \times 10^7$

d $81 = 8.1 \times 10$
$= 8.1 \times 10^1$

e $0.0067 = 6.7 \div 1000$
$= 6.7 \times \dfrac{1}{1000}$
$= 6.7 \times 10^{-3}$

f $0.000\,02 = 2 \div 100\,000$
$= 2 \times \dfrac{1}{100\,000}$
$= 2 \times 10^{-5}$

g $0.07 = 7 \div 100$
$= 7 \times 10^{-2}$

h $8.17 = 8.17 \times 10^0$

Example 22

Write in decimal form.

a 2.1×10^3 **b** 6.3×10^5 **c** 5×10^{-4} **d** 8.12×10^{-2}

Solution

a $2.1 \times 10^3 = 2.1 \times 1000$
$= 2100$

b $6.3 \times 10^5 = 6.3 \times 100\,000$
$= 630\,000$

c $5 \times 10^{-4} = 5 \times \dfrac{1}{10\,000}$
$= 5 \div 10\,000$
$= 0.0005$

d $8.12 \times 10^{-2} = 8.12 \times \dfrac{1}{100}$
$= 8.12 \div 100$
$= 0.0812$

Since numbers written in scientific notation involve powers, when these numbers are multiplied, divided or raised to a power, the index laws come into play.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 23

Simplify and write in scientific notation.

a $(3 \times 10^4) \times (2 \times 10^6)$

b $(9 \times 10^7) \div (3 \times 10^4)$

c $(4.1 \times 10^4)^2$

d $(2 \times 10^5)^{-2}$

Solution

a $$\begin{aligned}(3 \times 10^4) \times (2 \times 10^6) &= 3 \times 10^4 \times 2 \times 10^6\\ &= 3 \times 2 \times 10^4 \times 10^6\\ &= 6 \times 10^{10}\end{aligned}$$

b $$\begin{aligned}(9 \times 10^7) \div (3 \times 10^4) &= \frac{9 \times 10^7}{3 \times 10^4}\\ &= \frac{9}{3} \times \frac{10^7}{10^4}\\ &= 3 \times 10^3\end{aligned}$$

c $$\begin{aligned}(4.1 \times 10^4)^2 &= 4.1^2 \times \left(10^4\right)^2\\ &= 16.81 \times 10^8\\ &= 1.681 \times 10^9\end{aligned}$$

d $$\begin{aligned}(2 \times 10^5)^{-2} &= 2^{-2} \times \left(10^5\right)^{-2}\\ &= \frac{1}{2^2} \times 10^{-10}\\ &= 0.25 \times 10^{-10}\\ &= 2.5 \times 10^{-11}\end{aligned}$$

Scientific notation

- **Scientific notation**, or **standard form**, is a convenient way to represent very large and very small numbers.
- To represent a number in scientific notation, insert a decimal point after the first non-zero digit and multiply by an appropriate power of 10.

  For example:

  $81\,362\,000\,000\,000 = 8.1362 \times 10^{13}$ and $0.000\,000\,000\,38 = 3.8 \times 10^{-10}$

Some of the exercises in the rest of this chapter are best done using a calculator.

Exercise 8D

1 Write as a power of 10.

a 10 **b** 100 **c** 1000

d 10 000 **e** 1 000 000 **f** 1 000 000 000

g a googol – which is 1 followed by 100 zeros.

Note: 10^6 is a million, 10^9 is a billion and 10^{12} is a trillion.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

2 Write as a power of 10.

a $\frac{1}{10}$ **b** $\frac{1}{100}$ **c** $\frac{1}{1000}$

d 1 trillionth **e** $\frac{1}{100\,000}$ **f** 1 millionth

Example 21

3 Write in scientific notation.

a 510 **b** 5300 **c** 26 000

d 796 000 000 **e** 576 000 000 000 **f** 4 000 000 000 000

g 0.008 **h** 0.06 **i** 0.000 72

j 0.000 041 **k** 0.000 000 006 **l** 0.000 000 206

Example 22

4 Write in decimal form:

a 3.24×10^4 **b** 7.2×10^3 **c** 8.6×10^2

d 2.7×10^6 **e** 5.1×10^0 **f** 7.2×10^1

g 5.6×10^{-2} **h** 1.7×10^{-3} **i** 8.72×10^{-4}

j 2.01×10^{-3} **k** 9.7×10^{-1} **l** 2.6×10^{-7}

5 The mass of the Earth is approximately 6 000 000 000 000 000 000 000 000 kg. Write this value in scientific notation.

6 Light travels approximately 299 000 km in a second. Express this in scientific notation.

7 The mass of a copper sample is 0.0089 kg. Express this in scientific notation.

8 The distance between interconnecting lines on a silicon chip for a computer is approximately 0.000 000 04 m. Express this in scientific notation.

Example 23

9 Simplify, expressing the answer in scientific notation.

a $(4 \times 10^5) \times (2 \times 10^6)$ **b** $(2.1 \times 10^6) \times (3 \times 10^7)$

c $(4 \times 10^2) \times (5 \times 10^{-7})$ **d** $(3 \times 10^6) \times (8 \times 10^{-3})$

e $(5 \times 10^4) \div (2 \times 10^3)$ **f** $(8 \times 10^9) \div (4 \times 10^3)$

g $(6 \times 10^{-4}) \div (8 \times 10^{-5})$ **i** $(2.1 \times 10^2)^4$

h $(1.2 \times 10^6) \div (4 \times 10^7)$ **j** $(3 \times 10^{-2})^3$

k $\dfrac{(2 \times 10^5) \times (4 \times 10^4)}{1.6 \times 10^3}$ **l** $\dfrac{(8 \times 10^6) \times (4 \times 10^3)}{5 \times 10^7}$

m $(4 \times 10^{-2})^2 \times (5 \times 10^7)$ **n** $(6 \times 10^{-3}) \times (4 \times 10^7)^2$

o $\dfrac{(4 \times 10^5)^3}{(8 \times 10^4)^2}$ **p** $\dfrac{(2 \times 10^{-1})^5}{(4 \times 10^{-2})^3}$

10 If light travels at 3×10^5 km/s and our galaxy is approximately 80 000 light years across, how many kilometres is it across? (A light year is the distance light travels in a year.)

11 The mass of a hydrogen atom is approximately 1.674×10^{-27} kg and the mass of an electron is approximately 9.1×10^{-31} kg. How many electrons, correct to the nearest whole number, will have the same mass as a single hydrogen atom?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

12 If the average distance from the Earth to the Sun is 1.4951×10^8 km and light travels at 3×10^5 km/s, how long does it take light to travel from the Sun to the Earth?

13 The furthest galaxy detected by optical telescopes is approximately 4.6×10^9 light years from us. How far is this in kilometres? (Light travels at 3×10^5 km/s.)

14 In a lottery there are $\dfrac{45 \times 44 \times 43 \times 42 \times 41 \times 40}{720}$ different possible outcomes. If I mark each outcome on an entry form one at a time, and it takes me an average of 1 minute to mark each outcome, how long will it take me to cover all different possible outcomes?

8E Significant figures

Suppose that we are using a ruler marked in centimetres and millimetres to measure the length of a sheet of paper. We find that the length of the edge of the paper falls between 15.2 cm and 15.3 cm but is closer to 15.3 cm. The normal procedure is to use rounding and write the measurement as 15.3 cm.

In scientific notation, our measurement is 1.53×10^1 cm. We say that the length is 1.53×10^1 cm **correct to 3 significant figures**. This means that the length is between 1.525 cm and 1.535 cm.

All measurements involve rounding to one level of accuracy or another. For example, you may read that the mass of an electron is about 0.000 000 000 000 000 000 000 000 000 910 938 26 g. This usually means that a measurement was made and the last digit 6 is a rounding digit and is therefore not completely accurate. The other digits are accurate. Writing this in scientific notation, we say that the mass of an electron is $9.109\,3826 \times 10^{-28}$ g **correct to 8 significant figures** because there are 8 digits in the factor 9.109 3826 before the power of 10.

The mass of the Earth is about 5.9736×10^{24} kg. How many significant digits are there in this measurement?

There are 5 digits in 5.9736, so the measurement is correct to 5 significant figures.

Notice that, to use the idea of significant figures, we must first express the number in scientific notation.

People sometimes apply the same kind of ideas to shorten a given decimal number containing many digits. This need not have anything to do with measurement. It is a way of abbreviating the information you are given. The procedure is called **writing the number to a certain number of significant figures**.

To write a number to a specified number of significant figures, first write the number in scientific notation and then round to the required number of significant figures.

For example,
$$\begin{aligned} 0.000\,340\,61 &= 3.4061 \times 10^{-4} \\ &\approx 3 \times 10^{-4} && \text{(correct to 1 significant figure)} \\ &\approx 3.4 \times 10^{-4} && \text{(correct to 2 significant figures)} \\ &\approx 3.41 \times 10^{-4} && \text{(correct to 3 significant figures)} \\ &\approx 3.406 \times 10^{-4} && \text{(correct to 4 significant figures)} \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 24

Write in scientific notation and then round correct to 3 significant figures.

a 235.674 **b** 0.007 245 46

Solution

a $235.674 = 2.356\,74 \times 10^2$
$\approx 2.36 \times 10^2$

b $0.007\,245\,46 = 7.245\,46 \times 10^{-3}$
$\approx 7.25 \times 10^{-3}$

Example 25

Write in scientific notation and then round correct to 2 significant figures.

a 276 000 000 **b** 0.000 000 654

Solution

a $276\,000\,000 = 2.76 \times 10^8$
$\approx 2.8 \times 10^8$

b $0.000\,000\,654 = 6.54 \times 10^{-7}$
$\approx 6.5 \times 10^{-7}$

Significant figures

- A number may be expressed with different numbers of **significant figures**.
  For example: π is 3.1 to 2 significant figures, 3.14 to 3 significant figures, 3.142 to 4 significant figures and so on.
- To write a number correct to a specified number of significant figures, first write the number in scientific notation and then round to the required number of significant figures.

Exercise 8E

Example 24

1 Write in scientific notation, correct to 3 significant figures.

a 2.7043 **b** 634.96 **c** 8764.37
d 256 412 **e** 0.003 612 **f** 0.024 186

Example 25

2 Write in scientific notation, correct to 2 significant figures.

a 368.2 **b** 278 000 **c** 0.004 321 **d** 0.000 021 906

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Along each row, write the respective number in scientific notation, correct to the indicated number of significant figures.

	4 sig. figs	3 sig. figs	2 sig. figs	1 sig. fig.
274.62				
0.041236				
1704.28				
1.9925×10^{27}				

4 Use a calculator to evaluate the following, giving the answer in scientific notation correct to 3 significant figures.

a 3.24×0.067

b $6.24 \div 0.026$

c $4.736 \times 10^{13} \times 2.34 \times 10^{-6}$

d $(5.43 \times 10^{-6}) \div (6.24 \times 10^{-4})$

e $0.0276^2 \times \sqrt{0.723}$

f $\dfrac{17.364 \times 24.32 \times 5.4^2}{3.6 \times 7.31^2}$

g $\dfrac{6.54(5.26^2 + 3.24)}{5.4 + \sqrt{6.34}}$

h $\dfrac{6.283 \times 10^8 \times 5.24 \times 10^6}{(4.37 \times 10^7)^2}$

5 Use a calculator to evaluate, giving the answer in scientific notation correct to 4 significant figures.

a 1.234×0.1988

b $1.234 \div 0.1988$

c 1.9346^3

d $(7.919 \times 10^{21})^2$

e $\sqrt{4.863 \times 10^{-12}}$

f $\dfrac{177.41 \times 0.048}{16.23}$

g $\dfrac{7.932 \times 10^{12} \times 9.4 \times 10^{-10}}{0.000\,000\,000\,416}$

h $\dfrac{579.2 \times 0.6231}{79.05 \times 115.4}$

i $\dfrac{74\,510\,000\,000}{6.4 \times 10^{-18} \times 4.4 \times 10^{23}}$

j $\dfrac{79.99}{\sqrt{48.92} + 11.68^2}$

k $\dfrac{15.62^2(79.1 + 111.7)}{12.46 + 4.48^3}$

l $56.21 \times 12 + \dfrac{1}{2} \times 9.8 \times 12^2$

6 Estimate each of the following, correct to 1 significant figure, using appropriate units. In each case explain how you obtained your answer.

a The thickness of a sheet of paper

b The volume of your classroom

c The height of a six-storey building

d The area required for a car park for 500 cars

e The total printed area of a 600-page novel

f The volume of a warehouse that can store 300 000 pairs of shoes (still in their boxes)

g The length of a queue if every student in your school is standing in it

Discuss your answers with others in your class and with your teacher.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Review exercise

1 Evaluate:

a 4^3 **b** 2^6 **c** 8^2 **d** 10^6

2 Express as a product of powers of prime numbers.

a 120^2 **b** 900^3 **c** 315^4 **d** 490^5

3 Simplify and evaluate where possible.

a $a^6 \times a^7$ **b** $b^4 \times b^9$ **c** $3a^4 \times 5a^5$ **d** $2x^3 \times 5x^6$

e $a^7 \div a^4$ **f** $m^{12} \times m^6$ **g** $\dfrac{12b^7}{6b^2}$ **h** $\dfrac{18p^{10}}{9p}$

i $(a^4)^3$ **j** $(b^6)^5$ **k** $(2a^7)^3$ **l** $(3m^2)^4$

m a^0 **n** $3b^0$ **o** $5m^0$ **p** $(3q)^0$

4 Simplify and evaluate where possible.

a $4a^2b^3 \times 5ab^4$ **b** $2m^4n^3 \times 5m^6n^7$ **c** $\dfrac{20a^4b^2}{5a^2b}$

d $\dfrac{24m^9n^4}{18m^6n^2}$ **e** $(3a^3b)^4$ **f** $(5a^2b)^2 \times 4a^4b^3$

g $\dfrac{5a^6b^7}{4a^3b^2} \times \dfrac{12a^{10}b^9}{a^6b^7}$ **h** $\dfrac{8m^4n^2}{7m^3n} \div \dfrac{3m^3n^5}{14m^9n^{16}}$ **i** $\dfrac{(2x^2y)^3}{5x^6y^2} \times \left(\dfrac{x^3}{2y^2}\right)^3$

j $\dfrac{(4ab)^3 \times 5a^2b}{(10ab^2)^2}$ **k** $\dfrac{(3x^2y)^3 \times 2(xy^2)^3}{(3xy)^4}$ **l** $\dfrac{(4a^2b^3)^2}{3a^6b^4} \div \dfrac{ab^5}{(3a^3b^2)^3}$

5 Evaluate:

a 6^{-2} **b** 8^{-3} **c** 2^{-7} **d** 4^{-3}

e $\left(\dfrac{4}{5}\right)^{-2}$ **f** $\left(\dfrac{2}{3}\right)^{-4}$ **g** $\left(2\tfrac{1}{2}\right)^{-3}$ **h** $\left(16^{\frac{1}{4}}\right)^{-3}$

6 Simplify, expressing the answer with positive indices.

a $\dfrac{4m^2n^5p^{-6}}{16m^{-2}n^5p^3}$ **b** $(4y)^{-3}$ **c** $(2^2y^3)^{-5}$ **d** $(5^{-2}x^3)^{-5}$

e $(3^{-3}a^2b^{-1})^{-4}$ **f** $\left(\dfrac{a^3}{b^2}\right)^{-2}$ **g** $\left(\dfrac{5g^2}{h^{-3}}\right)^{-2}$ **h** $\left(\dfrac{m^{-3}}{(2n)^{-4}}\right)^{-2}$

7 Simplify, expressing the answer with positive indices.

a $4a^2 \times 5a^{-3}$ **b** $8m^2n^{-3} \times 5m^{-4}n^6$ **c** $14a^{-4} \div 7a^{-5}$

d $\dfrac{18m^2n^{-3}}{9m^4n^{-1}}$ **e** $(4m^2n^{-2})^{-3}$ **f** $\dfrac{(5m^2n)^{-2}}{10m^4n^3}$

g $(3a^4b^{-3})^{-3} \div 6a^2b^{-7}$ **h** $\dfrac{2m^3n^4}{(5m)^2} \times \dfrac{10m}{3n^{-4}}$ **i** $\dfrac{(5a^4b^{-3})^2}{a^{-2}b} \div \dfrac{5(a^{-1}b)^{-3}}{ab^{-4}}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 Evaluate:

a $49^{\frac{1}{2}}$ **b** $8^{\frac{1}{3}}$ **c** $125^{\frac{2}{3}}$ **d** $64^{\frac{2}{3}}$

e $9^{\frac{3}{2}}$ **f** $16^{-\frac{3}{2}}$ **g** $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$ **h** $\left(\frac{64}{27}\right)^{-\frac{2}{3}}$

9 Simplify, expressing the answer with positive indices.

a $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$ **b** $3b^{\frac{2}{3}} \times 4b$ **c** $p^{\frac{2}{3}} \div p^{\frac{1}{2}}$ **d** $m^{-\frac{1}{2}} \div m^{-2}$

e $\left(4x^{\frac{1}{3}}\right)^4$ **f** $\left(2x^{-\frac{1}{3}}\right)^{-2}$ **g** $\left(9a^{\frac{1}{3}}\right)^{\frac{1}{2}}$ **h** $(8p^{-2}q^3)^{\frac{1}{2}}$

10 Write in scientific notation.

a 47 000 **b** 164 000 000 **c** 0.0047

d 0.0035 **e** 840 **f** 0.840

11 Write in decimal form.

a 6.8×10^4 **b** 7.5×10^3 **c** 2.6×10^{-3}

d 9.4×10^{-2} **e** 6.7×10^0 **f** 3.2×10^{-4}

12 Simplify, writing each answer in scientific notation.

a $(3.1 \times 10^4) \times (2 \times 10^{-2})$ **b** $(5.2 \times 10^{-7}) \div (2 \times 10^3)$

c $(3 \times 10^4)^3$ **d** $(5 \times 10^{-4})^2$

e $(4 \times 10^2)^2 \times (5 \times 10^{-6})$ **f** $\dfrac{(3 \times 10^4)^3}{9 \times 10^{-2}}$

13 Write in scientific notation correct to the number of significant figures indicated in the brackets.

a 18.62 (2) **b** 18.62 (3) **c** 18.62 (1)

d 0.004 276 (2) **e** 5973.4 (2) **f** 0.473 952 (4)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party. Updated August 2021

Challenge exercise

1 The following table gives information regarding the population, approximate rate of population growth, and land area of several countries in 1996. Use the information to answer the questions below.

Country	Population (in millions)	Rate of population growth p.a.	Area of country (in km^2)
Australia	18.2	1.4%	7.6×10^6
China	1200	1.1%	9.6×10^6
Indonesia	201	1.6%	1.9×10^6
Singapore	2.9	1.1%	633
New Zealand	3.4	0.6%	2.7×10^5
United States	261	1.0%	9.2×10^6

a How many times larger is Australia than Singapore, correct to the nearest whole number?

b How many times larger is China than New Zealand, correct to 1 decimal place?

c Express Indonesia's population as a percentage of the United States' population. (Calculate your answer correct to the nearest per cent.)

d The population density of a country is defined to be the average number of people for each square kilometre of land. Calculate the population density for each country and find:

i how many times larger the population density of China is compared to Australia

ii the country with the lowest population density

iii the country with the highest population density

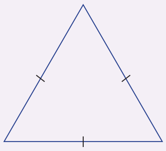
e If each country maintains its present rate of population growth, what will the population of each country be (assume compounding growth):

i 5 years from 1996? **ii** 10 years from 1996?

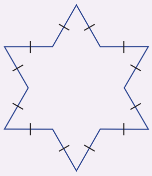
iii 100 years from 1996?

f Find the population density of each country 100 years from 1996, assuming each country maintains its present growth rate.

2 **a** Consider the equilateral triangle shown opposite. If each side of the triangle is of length 3 cm, what is the perimeter of the triangle?



b Suppose that the following procedure is performed on the triangle above: 'On the outside of each side of the triangle, draw a triangle with side lengths one-third of the side lengths of the original triangle'.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

After this procedure is done, the star shape opposite is obtained.

i What is the length of each edge?

ii What is the perimeter of the star?

iii What is the ratio of the perimeter of this star to the original perimeter?

c Suppose that the procedure of part **b** is performed on this star (that is, each of the 12 sides has a triangle with side lengths one third of the previous ones drawn on it).

i How many sides will the new star have?

ii What is the length of each edge of the new star?

iii What is the perimeter of the new star?

iv If the perimeter of the new star can be written in the form $a^2 \times 9$, find the value of a.

d Suppose that the procedure is repeated indefinitely. What will the perimeter of the shape be after the procedure has been performed:

i 3 times (expressing your answer in the form $a^3 \times 9$)?

ii 4 times (expressing your answer in a similar way)?

iii 10 times (expressing your answer using indices)?

iv n times (expressing your answer using indices)?

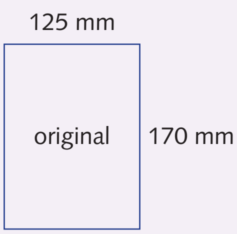
e Using your calculator, find, by trial and error, the number of times this procedure has to be done to produce a perimeter of at least:

i one metre **ii** 10 metres **iii** one kilometre

f If this procedure is performed indefinitely, what do you think happens to the perimeter? Does this surprise you?

Note: The shape produced by indefinitely repeating this procedure is called the Snowflake curve. You might like to try to draw what the snowflake looks like after each of the first few applications of the procedure.

3 A student is using a photocopy machine to reduce a picture of dimensions 170 mm × 125 mm. He sets the photocopier on 80% reduction (that is, each length in the photocopy will be 80% of its original length).



a What are the dimensions of the photocopy of the picture?

b What are the dimensions of the photocopy of the photocopy?

c If the student continues to take 80% reduction photocopies, what are the dimensions of the picture after:

i 3 photocopies? **ii** 4 photocopies? **iii** n photocopies?

In each case, express your answer in terms of a power of 0.8.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d What is the area of the original picture?

e What is the area of the picture after:

i 1 photocopy?

ii 2 photocopies?

iii 3 photocopies?

iv n photocopies?

In each case, express your answers in terms of a power of 0.8.

f If the student started with a picture of dimensions x cm $\times$ y cm, and successive photocopies are taken with reduction 80%:

i what are the dimensions of the picture after n photocopies?

ii what is the area of the picture after n photocopies?

4 Write $2^{n+1} + 2^{n+1}$ as a single power of 2.

5 Simplify:

a $\dfrac{8^n \times 3^{2n}}{2^n \times 6^n \times 9^n}$

b $\dfrac{2^{n+1}}{(2^n)^{n+1}} \div \dfrac{(2^{2n+1})^{n-1}}{4^n}$

c $\dfrac{2^{n+3} - 4 \times 2^n}{2^{2n} - 4^{n-1}}$

d $\dfrac{a^2x^m - b^2x^{m+4}}{a - bx^2}$

6 If $10^x = p$, find the following in terms of p:

a 10^{2x+1}

b 10^{1-3x}

7 Simplify $\dfrac{(yz)^{b+c}(zx)^{c+a}(xy)^{a+b}}{x^a y^b z^c}$.

8 Expand and simplify:

a $\left(x - x^{-1}\right)^2$

b $(x^2 + x^{\frac{1}{2}})^2$

c $(x^n - 3x^{-n})^2$

9 Simplify:

a $\dfrac{2^{x+1} - 2^{x-2}}{2^{x-1} - 2^{x+2}}$

b $\dfrac{x^{\frac{1}{2}}y^{-\frac{1}{2}} - x^{-\frac{1}{2}}y^{\frac{1}{2}}}{x^{-1} - y^{-1}}$

c $\dfrac{x - 1}{x - x^{\frac{1}{2}} - 2}$

d $\dfrac{x - y}{x^{\frac{1}{2}} - y^{\frac{1}{2}}}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 **a** Prove that the index laws hold for integer indices. (Use the laws for positive integer indices.)

For example, the product of powers result can be proved in the following way for negative integer indices.

Consider $a^{-p}a^{-q}$ where p and q are positive integers.

$$\begin{aligned} a^{-p}a^{-q} &= \frac{1}{a^{-p}} \times \frac{1}{a^{-q}} \\ &= \frac{1}{a^{-p}a^{-q}} \\ &= \frac{1}{a^{p+q}} \qquad \text{(index law 1 for positive integers.)} \\ &= a^{-(p+q} \\ &= a^{-p+(-q)} \end{aligned}$$

b Prove that the index laws hold for fractional indices.

For example, $a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}}$ can be proved in the following way.

$$\begin{aligned} a^{\frac{1}{n}} \times a^{\frac{1}{m}} &= a^{\frac{m}{nm}} \times a^{\frac{n}{nm}} \\ &= \sqrt[nm]{a^m} \times \sqrt[nm]{a^n} \\ &= \sqrt[nm]{a^m \times a^n} \\ &= \sqrt[nm]{a^{m+n}} \\ &= a^{\frac{m+n}{nm}} \\ &= a^{\frac{1}{n}+\frac{1}{m}} \end{aligned}$$

The result can easily be extended to $a^{\frac{p}{n}} \times a^{\frac{q}{m}} = a^{\frac{p}{n}+\frac{q}{m}}$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

9

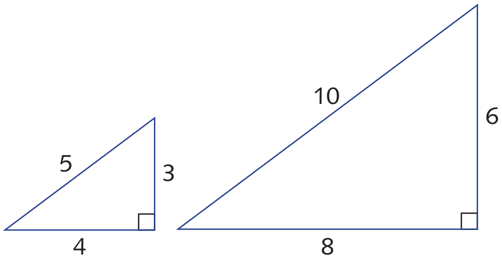
Measurement and Geometry

Enlargements and similarity

This chapter introduces a transformation called an enlargement. This allows us to develop the idea of similar figures, which is a more general idea than congruence.

For example, the two right-angled triangles shown here are similar because they have exactly the same shape, but they are not congruent because they have different side lengths.

Similarity and proportion are closely related, as we shall see. This idea has been used in art and architecture for centuries. It is now used extensively in photography and in the zoom function of a computer application.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

9A Enlargements

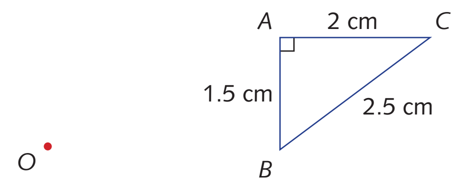
You have already met three types of transformations: translations, rotations and reflections.

This section introduces a fourth type of transformation called an **enlargement**, in which the lengths of the sides are increased or decreased by the same factor. In *ICE-EM Mathematics Year 8*, we looked at these ideas under the topic heading of **scale drawings**. Now we are going to examine these ideas a little more closely.

The figure to the right below is a right-angled triangle ABC, whose sides have length 1.5 cm, 2 cm and 2.5 cm.

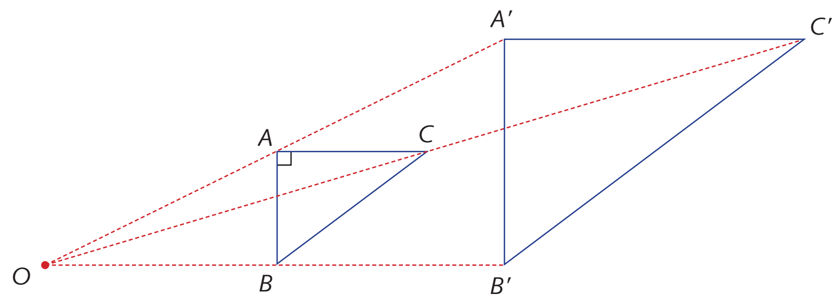
To define an enlargement of this figure, two things need to be specified.

First, we have to specify a **centre of enlargement** – in our example, it is the point O to the left of the triangle. The centre of enlargement stays fixed in the one place while the enlargement expands or shrinks everything else around it.



Secondly, we have to specify an **enlargement ratio** or **enlargement factor** – we will use an enlargement ratio of 2 here. This means that every point will double its distance from O. As a consequence, the side lengths of the triangle will double, as we saw in scale drawings.

The second diagram shows how this is done.



- The interval OA is joined, then produced to A' so that $OA' = 2 \times OA$.
- The interval OB is joined, then produced to B' so that $OB' = 2 \times OB$.
- The interval OC is joined, then produced to C' so that $OC' = 2 \times OC$.

The triangle $A'B'C'$ is then joined up, and is called the **image** of triangle ABC with enlargement factor 2 and centre of enlargement O.

The image triangle $A'B'C'$ is related to the original triangle in three important ways.

First, each side of the image triangle $A'B'C'$ is twice the length of the matching sides of the original triangle. We will usually write this using ratios:

$$\frac{A'B'}{AB} = 2 \qquad \frac{B'C'}{BC} = 2 \qquad \frac{A'C'}{AC} = 2$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Secondly, each angle of the image is equal to the matching angle of the original:

$$\angle B'A'C' = \angle BAC \qquad \angle A'C'B' = \angle ACB \qquad \angle C'B'A' = \angle CBA$$

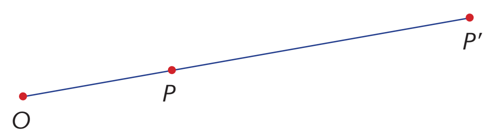
Thus the image $A'B'C'$ is a scale drawing of the original figure ABC, with ratio 2.

Thirdly, since matching angles are equal we can see that $A'B' \parallel AB$, $B'C' \parallel BC$ and $C'A' \parallel CA$.

The same kinds of results hold if we perform an enlargement by any positive factor.

Enlargement transformations

- Each enlargement has a centre of enlargement O and an enlargement factor $k > 0$.
- The enlargement moves each point P to a new point P' on the ray with vertex O passing through P.
- The distance of P' from O is k times the distance of P from O.
  That is, $OP' = k \times OP$.



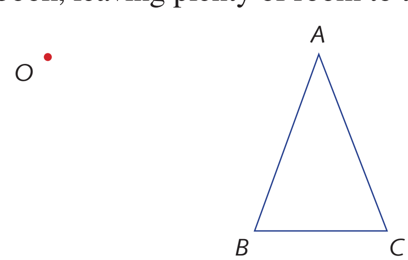
When a figure is enlarged by a factor k:

- The image of each interval has k times the length of the original interval.
- The image of an angle has the same size as the original angle.
- The image of an interval is parallel to the original interval.

The first three dot points follow from the definition of an enlargement. If $k > 1$, then the image is larger than the original figure. If $k < 1$, then the image is smaller. If $k = 1$, then the original figure is unchanged by the transformation.

Exercise 9A

1 The following diagram shows an isosceles triangle ABC and a point O. Copy or trace the diagram into your exercise book, leaving plenty of room to the right and below your diagram.



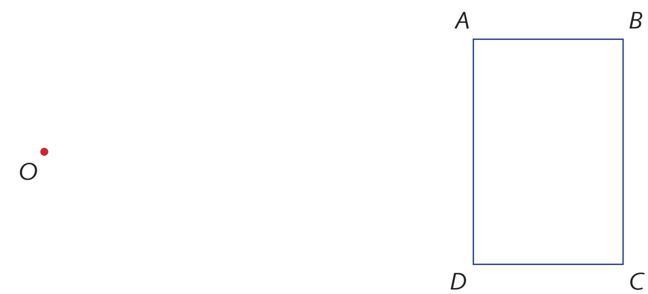
a Use ruler and compasses to apply an enlargement transformation with centre O and enlargement factor 3. Label your image $A'B'C'$.

b Verify that each side length of $\Delta A'B'C'$ is three times the matching side length of ΔABC.

c Use a protractor to verify that each angle of $\Delta A'B'C'$ is equal to the matching angle of ΔABC.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 The diagram below shows a rectangle $ABCD$ and a point O. Copy or trace the diagram into your exercise book.



a Now apply an enlargement with centre O and factor $\frac{1}{2}$ using these steps:

i Join OA and mark the midpoint A' of OA.

ii Join OB and mark the midpoint B' of OB.

iii Join OC and mark the midpoint C' of OC.

iv Join OD and mark the midpoint D' of OD.

b Use compasses to verify that each side length of $A'B'C'D'$ length of $ABCD$.

c Use a protractor to verify that each angle of $A'B'C'D'$ is a right angle, matching the angles of $ABCD$.

3 I have built a primitive slide projector by shining light from a point source through a transparent slide held up 10 cm away. The light then falls onto a wall that is parallel to the slide and distant 1 metre from it.

a Draw a labelled diagram of a triangle on the slide and its image on the wall.

b What are the centre of enlargement and the enlargement factor of the transformation?

4 A ginger cat is sitting on a thin railing 2 metres horizontally from a desk lamp, producing a shadow of the cat on a wall 3 metres behind the cat. What is the enlargement factor of the shadow with respect to the profile of the cat?

5 Ernest is making scale drawings of atoms, in which his drawing of a hydrogen atom is a circle of radius 1 cm. Here are the actual approximate radii of some of the atoms in his drawings, given in picometres (1 pm = 10^{-12} metres).

Hydrogen: 25 pm Carbon: 70 pm Gold: 135 pm Radium: 215 pm

Calculate the radii of Ernest's drawings of carbon, gold and radium atoms.

6 A map of a country is drawn to a scale of 1 : 12 500 000. Find the actual distance in kilometres between two points whose separation on the map is:

a 1 cm **b** 4 cm **c** 0.6 cm **d** 1 mm

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

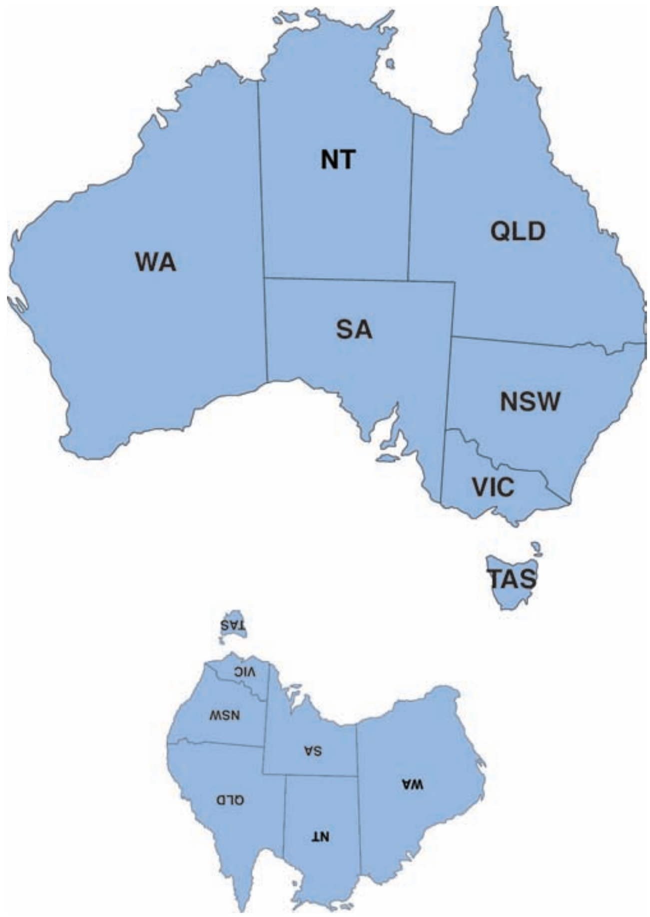
9B Using ratios in similar figures

Two figures are called **similar** if we can enlarge one figure so that its enlargement is congruent to the other figure.

In simple terms this means that by enlarging or shrinking one figure, we get the other figure perhaps translated, rotated or reflected.

Thus similar figures have the same shape, but not necessarily the same size, just as a scale drawing has the same shape as the original, but has a different size.

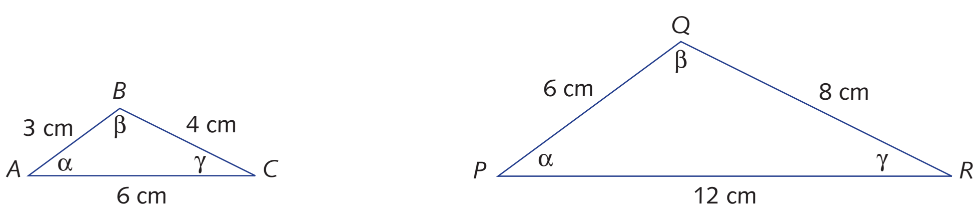
- Matching lengths of similar figures are therefore always in the same ratio, called the **similarity ratio**.
- Matching angles of similar figures are equal.



We will use the same convention for matching angles as we did for congruence.

If ΔABC is similar to ΔPQR then, $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

The two triangles in the diagram below are similar.



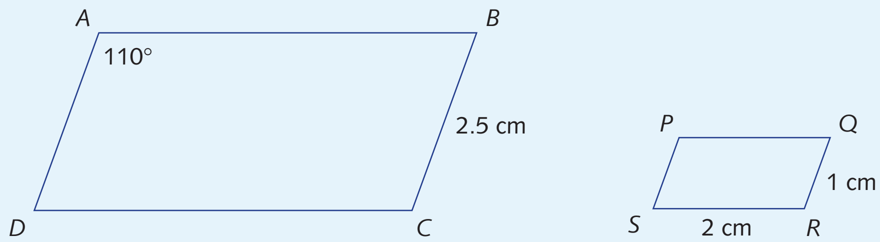
$$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = 2$$

Hence ΔABC is similar to ΔPQR and the similarity ratio is 2.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 1

These two parallelograms are known to be similar.



a What is the size of $\angle P$?

b Find the base length of the large parallelogram.

c What is the similarity ratio of the transformation taking $PQRS$ to $ABCD$?

Solution

a $\angle P = 110°$ (matching angles of similar figures)

b $\dfrac{DC}{SR} = \dfrac{BC}{QR}$ (ratio of matching sides)

$\dfrac{DC}{2} = \dfrac{2.5}{1}$

$DC = 5\text{ cm}$

c The similarity ratio is 2.5.

Note 1: In part **b**, we could have written the alternative ratio statement

$\dfrac{DC}{BC} = \dfrac{SR}{QR}$ (ratios within figures, ‘larger’ figure on the left, ‘smaller’ figure on the right)

It does not matter at all which ratio statement you choose to use. The subsequent algebra will be easier, however, if you always start by placing the unknown length on the top of the left-hand side of the equation.

Note 2: An alternative solution to part **b** would be to calculate $\dfrac{BC}{QR} = 2.5$ as the similarity ratio.

Thus $DC = 2.5 \times SR$

Similar figures

Two figures are called **similar** if an enlargement of one figure is congruent to the other figure.

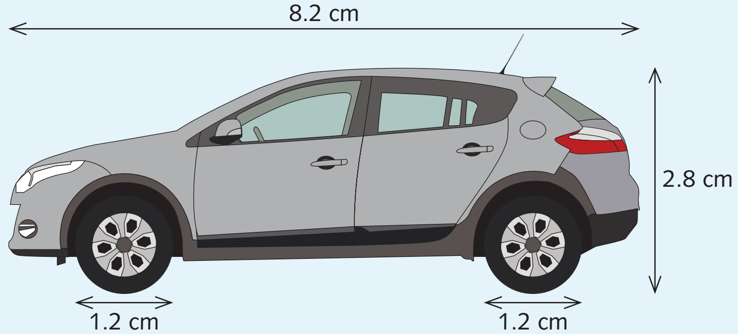
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Matching lengths and angles of similar figures

- Matching lengths in similar figures are in the same ratio, called the **similarity ratio**.
- Matching angles in similar figures are equal.

Example 2

The car shown in the photograph below is 3.8 metres long.



a Use your ruler and your calculator to find approximate ratios for the following, each correct to 1 decimal place. Your measurements are for the diagram, but the ratio is for both the diagram and the real car.

i $\dfrac{\text{height of the car}}{\text{diameter of wheel}}$ **ii** $\dfrac{\text{length of the car}}{\text{height of the car}}$

b Find the height of the real car.

Solution

a In the picture, height of the car ≈ 2.8 cm
diameter of a wheel ≈ 1.2 cm
length of the car ≈ 8.2 cm

i $\dfrac{\text{height of the car}}{\text{diameter of wheel}} \approx \dfrac{2.8}{1.2}$
≈ 2.3

ii $\dfrac{\text{length of the car}}{\text{height of the car}} \approx \dfrac{8.2}{2.8}$
≈ 2.9

b Enlargement factor $\approx \dfrac{380}{8.2} \approx 46$

The height of the car $\approx 2.8 \times 46 \approx 128$ cm $= 1.28$ m

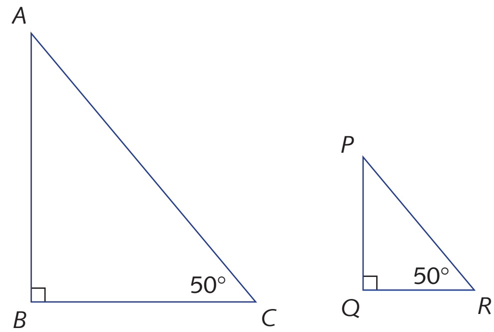
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 9B

Example 1

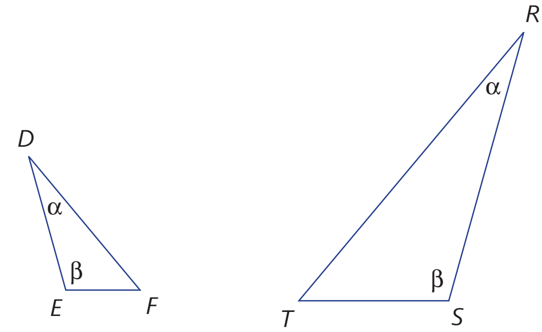
1 The diagram in each part shows two figures that are known to be similar. Copy and complete the statements below each diagram.

a



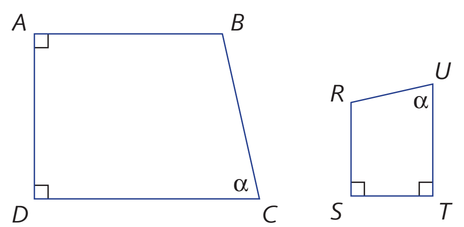
i $\frac{PR}{\dots} = \frac{PQ}{\dots}$ ii $\frac{QR}{\dots} = \frac{RP}{\dots}$

b



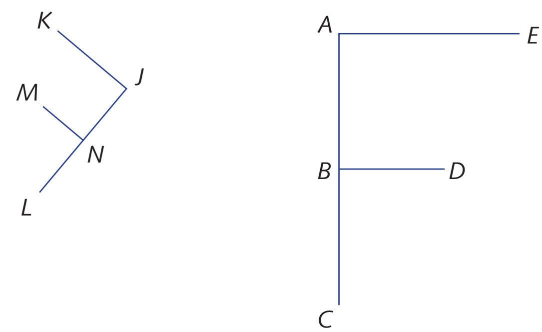
i $\frac{DE}{\dots} = \frac{DF}{\dots}$ ii $\frac{FE}{\dots} = \frac{FD}{\dots}$

c



i $\frac{AB}{\dots} = \frac{AD}{\dots}$ ii $\frac{CB}{\dots} = \frac{CD}{\dots}$

d

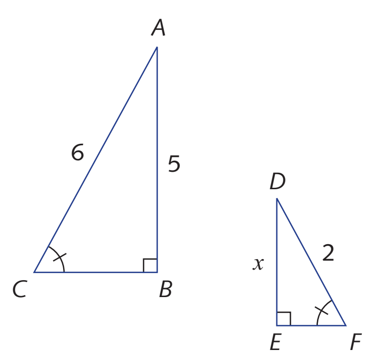


i $\frac{JK}{\dots} = \frac{MN}{\dots}$ ii $\frac{JL}{\dots} = \frac{MN}{\dots}$

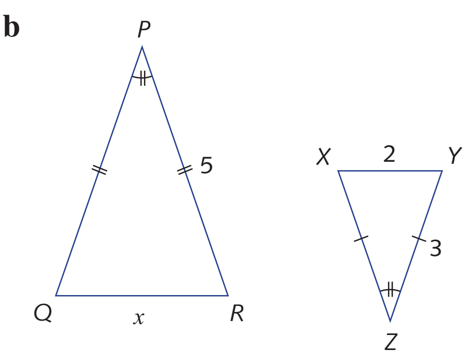
Example 1c

2 The diagram in each part shows two figures that are known to be similar. Write a ratio statement. Hence find the value of the pronumeral.

a

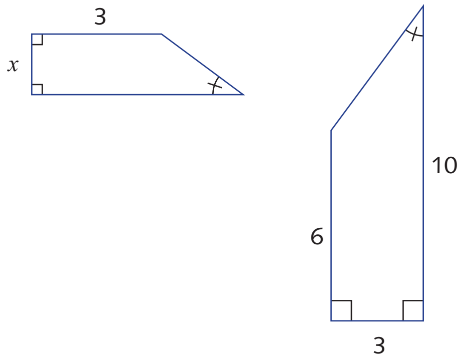


b

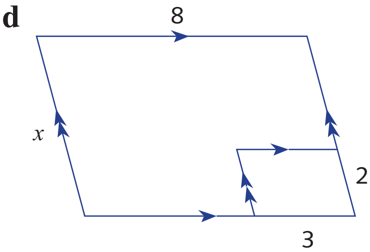


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c



d



3 The diagram below shows some of the keys of a piano.

By measuring lengths with your ruler, find approximately:

a the ratio of the lengths of the black and the white keys

b the ratio of the widths of the black and white keys at their front edges

c the ratio of the length of a white key and the width of its front edge

d the width of its front edge if the length of a white key is 13.3 cm

*In questions **4** to **6**, assume that the triangles are named in matching order.*

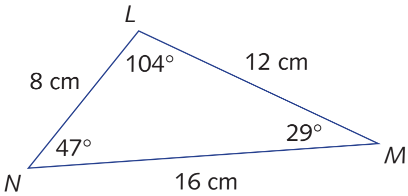
4 ΔLMN is similar to ΔTUV, where ΔLMN is shown below, and $TU = 24$ cm, find:

a $\angle TUV$

b UV

c $\angle VTU$

d VT



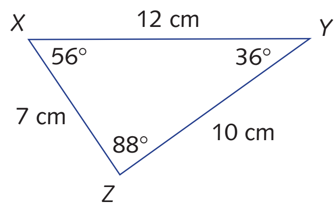
5 ΔXYZ is similar to ΔGHK, where ΔXYZ is shown below, and $HK = 18$ cm, find:

a $\angle GHK$

b $\angle GKH$

c GH

d GK



6 In two triangles ABC and LMN, $AB = 6$ cm, $LM = 12$ cm, $LN = 8$ cm and $MN = 16$ cm. What is the similarity ratio between the two triangles if:

a ΔABC is similar to ΔLMN?

b ΔABC is similar to ΔLNM?

c ΔABC is similar to ΔMNL?

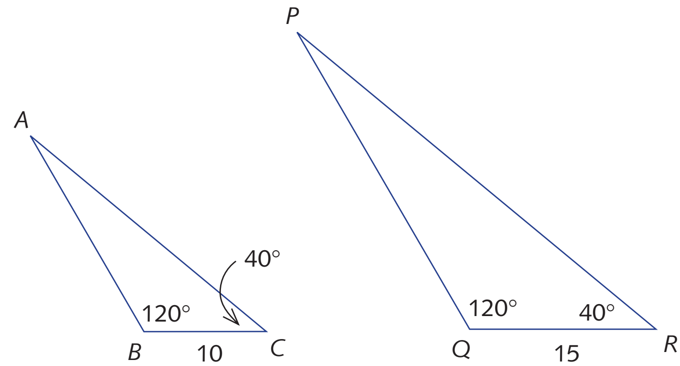
d ΔABC is similar to ΔMLN?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

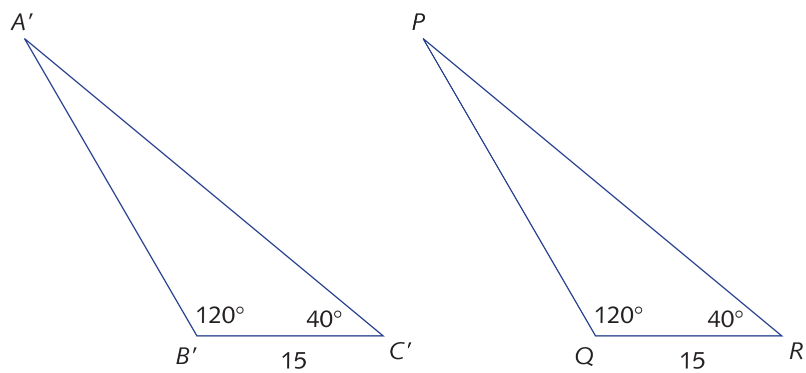
9C The AAA similarity test for triangles

As with congruence, most of the problems involving similarity come down to establishing that two triangles are similar. In this section and the next two sections, we shall establish four **similarity tests** for triangles and use them in problems.

The two triangles below have the same interior angles, but have different sizes.



Let us now enlarge the left-hand triangle by an enlargement factor of $\frac{3}{2}$ so that $B'C' = QR$, as in the diagram following. We have seen that the angles do not change, and that all three side lengths increase by the same factor.



By the AAS congruence test, the new triangle $\Delta A'B'C$ is congruent to ΔPQR.

This is because the two triangles have two pairs of angles equal, and the matching sides $B'C'$ and QR are also equal.

Hence the original triangle ΔABC is similar to ΔPQR, because its enlargement $\Delta A'B'C$ is congruent to ΔPQR.

The AAA similarity test

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

Note: When using this test, it is sufficient to prove the equality of just two pairs of angles – the third pair must then also be equal since the angle sum of any triangle is 180°. Thus the test is sometimes called 'the AA similarity test'.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Similarity statements

Arguments using similar triangles depend on a similarity statement that two triangles are similar. As with congruence, always give the similarity test in brackets afterwards, and be particularly careful to name the vertices in matching order.

Thus the conclusion of our argument above is written as:

ΔABC is similar to ΔPQR (AAA)

There are two common notations for similarity. These are:

$\Delta ABC \sim \Delta PQR$ and $\Delta ABC \parallel\!\mid \Delta PQR$

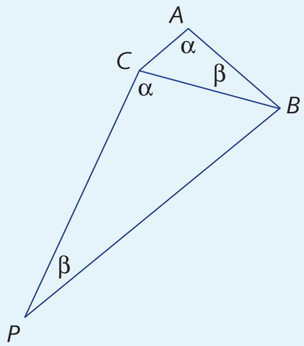
These are both read as 'triangle ABC is similar to triangle PQR'.

Example 3

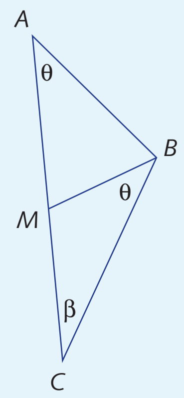
In each diagram below, write a similarity statement beginning with 'ΔABC is similar to . . .'

Be careful to name the vertices in matching order.

a



b



Solution

a ΔABC is similar ΔCPB. (AAA)
b ΔABC is similar ΔBMC. (AAA)

Notice that in part **b** the angle $\angle C$ is **common** to both triangles.

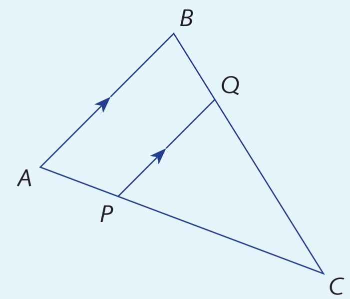
Proving similarity

In many geometric problems, we need to prove that two triangles are similar. The proofs are set out just like congruence proofs.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 4

Prove that the two triangles in the diagram to the right are similar.



Solution

In the triangles ABC and PQC:

$\angle BAC = \angle QPC$ (corresponding angles, $AB \parallel PQ$)

$\angle C = \angle C$ (common)

so ΔABC is similar to ΔPQC. (AAA)

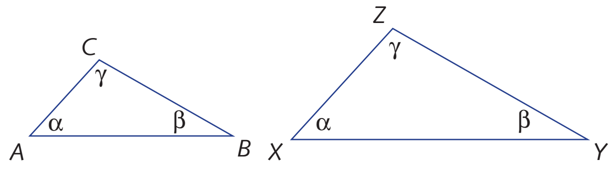
Using similar triangles to find lengths

Once we have established that two triangles are similar, we know that matching sides have the same ratio.

If ΔABC is similar to ΔXYZ, then:

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$$

This ratio is the **similarity ratio**.

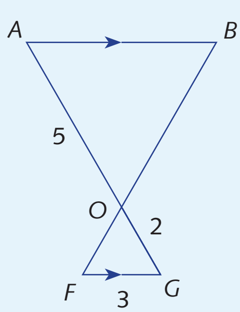


Example 5

In the diagram to the right:

a Prove that ΔABO is similar to ΔGFO.

b Hence find the length AB.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

a In the triangles ABO and GFO:

$\angle AOB = \angle GOF$ (vertically opposite at O)

$\angle A = \angle G$ (alternate angles, $AB \parallel FG$)

so ΔABO is similar to ΔGFO. (AAA)

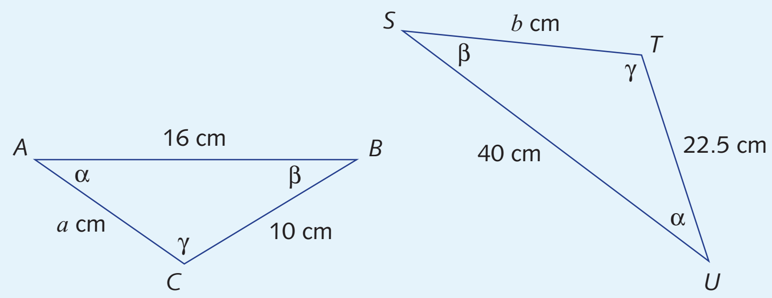
b Hence $\dfrac{AB}{3} = \dfrac{5}{2}$ (matching sides of similar triangles)

$$AB = 3 \times \frac{5}{2}$$

$$= 7\tfrac{1}{2}$$

Example 6

Find the values of a and b in the following diagrams.



Solution

ΔABC is similar to ΔUST. (AAA)

Hence $\dfrac{ST}{BC} = \dfrac{SU}{BA}$ (matching sides of similar triangles)

$$\frac{b}{10} = \frac{40}{16}$$

$$b = 25$$

Also $\dfrac{AC}{UT} = \dfrac{BA}{SU}$ (matching sides of similar triangles)

$$\frac{a}{22.5} = \frac{16}{40}$$

So $a = 9$

Alternatively, since $\dfrac{SU}{BA} = \dfrac{40}{16} = 2.5$, the similarity ratio from ΔABC to ΔUST is 2.5; that is, every length in ΔUST will be 2.5 times the matching length in ΔABC.

Thus $ST = 2.5 \times BC$ and $UT = 2.5 \times AC$

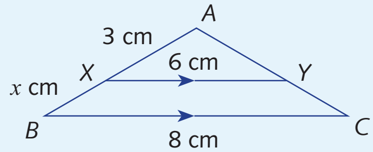
$b = 2.5 \times 10$ $\quad$ $22.5 = 2.5 \times a$

$= 25$ $\quad$ $a = 9$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7

Find the value of x.



Solution

In the triangles ABC and AXY:

$\angle A = \angle A$ (common)

$\angle ABC = \angle AXY$ (corresponding angles, $XY \parallel BC$)

so ΔABC is similar to ΔAXY (AAA).

Hence $\dfrac{AB}{AX} = \dfrac{BC}{XY}$ (matching sides of similar triangles)

$$\frac{x+3}{3} = \frac{8}{6} = \frac{4}{3}$$

$$x + 3 = 4$$

$$x = 1$$

Example 8

A tall vertical flagpole throws a 5 metre shadow. At the same time of day, a fencepost of height 2 metres throws a 30 cm shadow.

a Draw a diagram, identifying two similar triangles and proving that they are similar.

b Hence find the height of the flagpole.

Solution

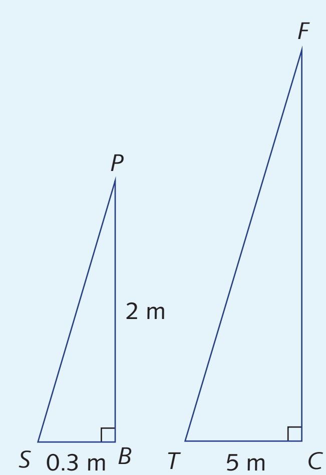
a In the diagram, FC is the flagpole, PB is the fencepost, and T and S mark the end of the shadows.

In the triangles FCT and PBS:

$\angle C = \angle B = 90°$ (pole and post are vertical)

$\angle T = \angle S$ (both are the angle of elevation of the Sun)

so ΔFCT is similar to ΔPBS. (AAA)



b Hence $\dfrac{FC}{2} = \dfrac{5}{0.3}$ (matching sides of similar triangles)

$$FC = 33\tfrac{1}{3} \text{ metres}$$

Thus the height of the flagpole is $33\frac{1}{3}$ metres.

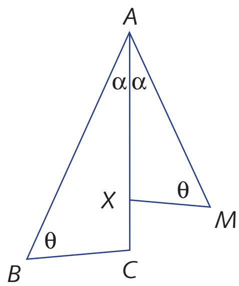
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Exercise 9C

Example 3

1 Copy and complete each similarity statement, giving the similarity test and naming the vertices in matching order. Then copy and complete the ratio statement.

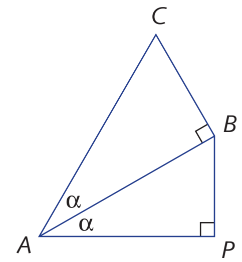
a



ΔABC is similar to … (…)

$\frac{AB}{\ldots} = \frac{BC}{\ldots}$

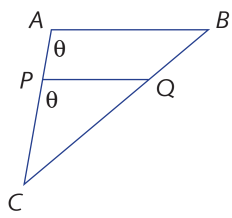
b



ΔABC is similar to … (…)

$\frac{AB}{\ldots} = \frac{BC}{\ldots}$

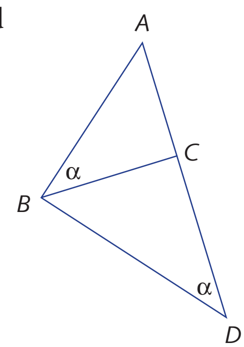
c



ΔABC is similar to … (…)

$\frac{AB}{\ldots} = \frac{BC}{\ldots}$

d



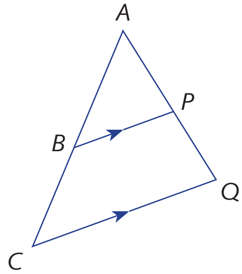
ΔABC is similar to … (…)

$\frac{AB}{\ldots} = \frac{BC}{\ldots}$

Example 4

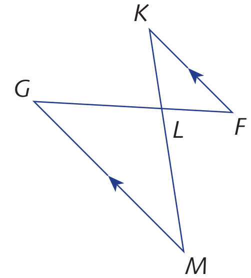
2 Prove that the two triangles in each diagram are similar, naming the vertices in matching order. Then copy and complete the ratio statement.

a



$\frac{AB}{\ldots} = \frac{AP}{\ldots}$

b



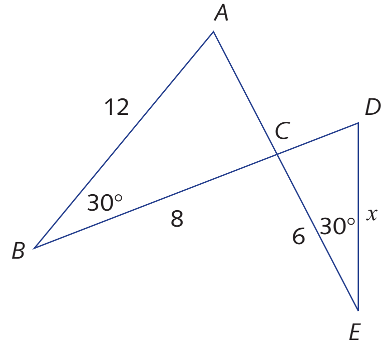
$\frac{GL}{\ldots} = \frac{GM}{\ldots}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

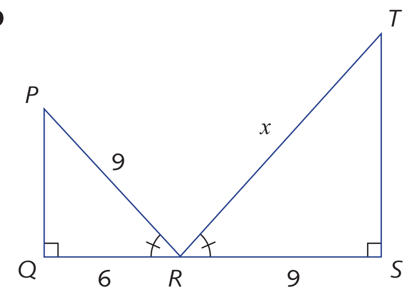
Example 5, 6, 7

3 Prove that the two triangles in each diagram are similar. Then write a ratio statement and find the value of the pronumeral.

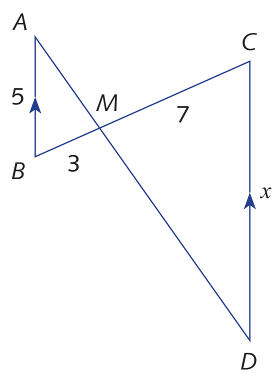
a



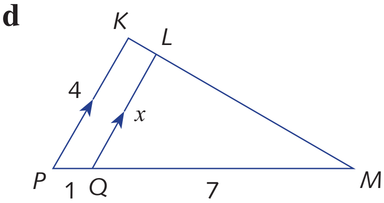
b



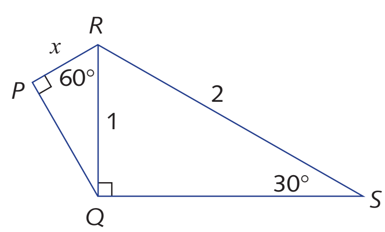
c



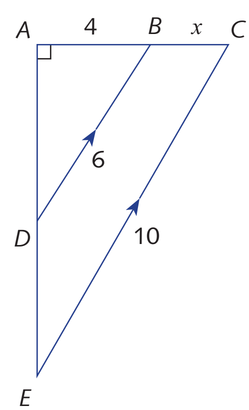
d



e



f



Example 8

4 In each part, you will need to draw a diagram and prove that two triangles are similar.

a A stick 1.5 metres long leans against a wall and reaches 1 metre up the wall. How far up the wall will a ladder 10 metres long reach if it is parallel to the stick?

b A 3 metre high road-sign casts a shadow 1.4 metres long. How high is a telegraph pole that casts a shadow 3.5 metres long at the same time of day?

c A steep road rises 1.2 metres for every 6 metres travelled along the road surface. How far will a car have to travel to rise 600 metres?

d The front of an A-frame hut is an isosceles triangle of height 5 metres and base 8 metres. How wide will a model of the hut be if its height is 0.8 metres?

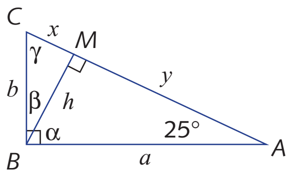
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 **a** Find the sizes of α, β and γ in the diagram on the right.

b The three triangles in the diagram are similar.

Copy and complete the statement:

ΔABC is similar to … which is similar to … (AAA).

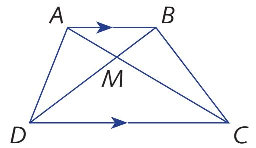


c Hence copy and complete:

i $\dfrac{a}{b} = \dfrac{\cdots}{\cdots} = \dfrac{\cdots}{\cdots}$ **ii** $\dfrac{h}{a} = \dfrac{\cdots}{\cdots} = \dfrac{\cdots}{\cdots}$

6 Prove that: *The diagonals of a trapezium dissect the trapezium into four triangles, two of which are similar.*

Let the diagonals of the trapezium $ABCD$ meet at M. Identify the two similar triangles and prove that they are similar.

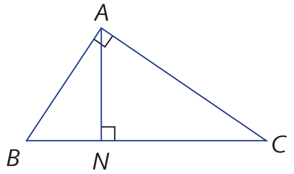


7 Prove that: *The altitude to the hypotenuse of a right-angled triangle divides the hypotenuse into two intervals whose product equals the square of the altitude.*

Let AN be the altitude to the hypotenuse BC of the right-angled triangle ΔABC.

a Prove that ΔABN is similar to ΔCAN.

b Hence prove that $AN^2 = BN \times CN$.

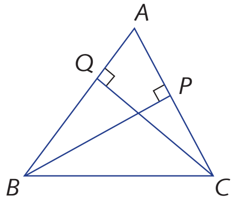


8 Let ABC be a triangle, and let BP and CQ be altitudes.

a Prove that ΔABP is similar to ΔACQ.

b Hence prove that $\dfrac{BP}{CQ} = \dfrac{AB}{AC}$.

c As an alternative approach, explain why $\dfrac{1}{2} \times AC \times BP$ and $\dfrac{1}{2} \times AB \times CQ$ are both equal to the area of ΔABC.



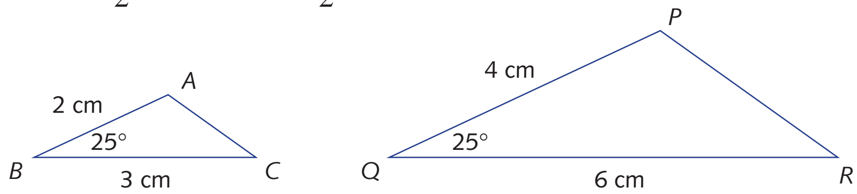
d Hence prove that $\dfrac{BP}{CQ} = \dfrac{AB}{AC}$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

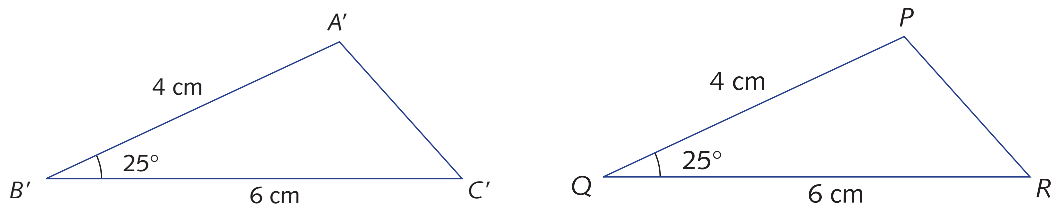
The SAS similarity test for triangles

The two triangles below have a pair of equal angles, and the sides including this angle are in the same ratio 1 : 2.

$$\angle B = \angle Q,\ AB = \frac{1}{2}PQ \text{ and } BC = \frac{1}{2}QR.$$



Using the same procedure as before, we can enlarge the first triangle by a factor of 2 to produce a new triangle $\Delta A'B'C'$, and $\Delta A'B'C'$ is congruent to ΔPQR by the SAS congruence test.



Hence the original triangle ΔABC is similar to ΔPQR.

This gives us our second similarity test.

The SAS similarity test

If the ratio of the lengths of two pairs of matching sides are equal and the included angles are equal then the two triangles are similar.

The statement that the two triangles above are similar is thus written as

ΔABC is similar to ΔPQR (SAS).

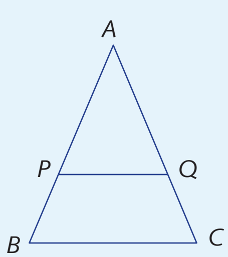
Example 9

In triangle ABC,

$AB = AC = 10$ and $AP = AQ = 7$.

a Prove that ΔABC is similar to ΔAPQ.

b Prove that PQ is parallel to BC.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a In the triangles ABC and APQ:

$$\frac{AB}{AP} = \frac{10}{7} \qquad \text{(given)}$$

$$\frac{AC}{AQ} = \frac{10}{7} \qquad \text{(given)}$$

$$\angle A = \angle A \qquad \text{(common)}$$

so ΔABC is similar to ΔAPQ (SAS).

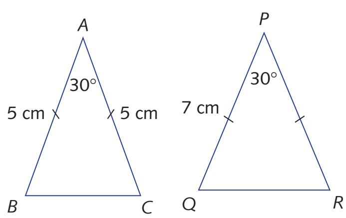
b $\angle ABC = \angle APQ$ (matching angles of similar triangles)

So $BC \parallel PQ$. (corresponding angles are equal)

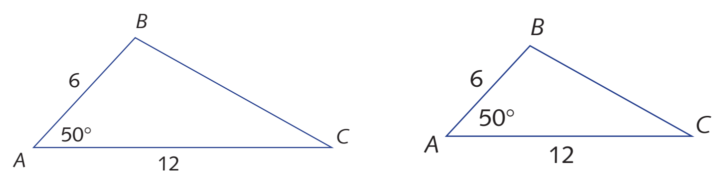
Exercise 9D

1 Prove that the two triangles in each diagram are similar, and write the similarity statement and the similarity factor (left to right).

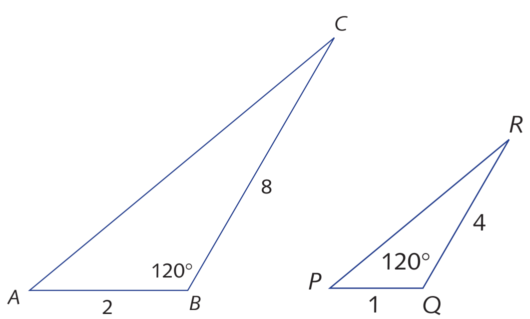
a



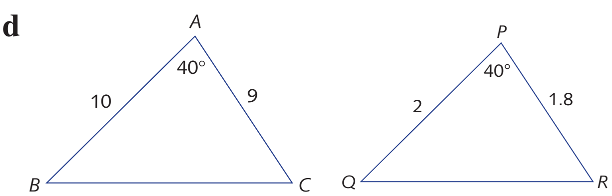
b



c



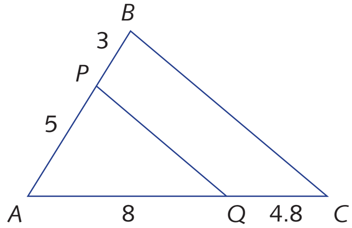
d



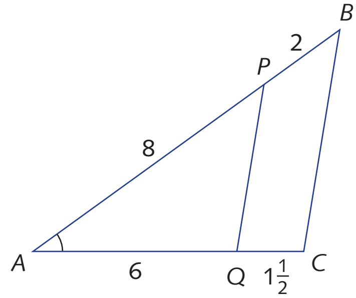
Example 9

2 In each part below, prove that ΔABC is similar to ΔAPQ. Then prove that PQ is parallel to BC.

a

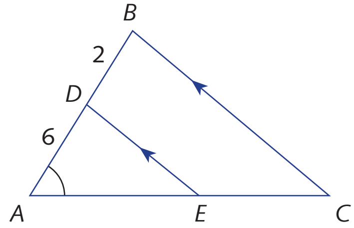


b



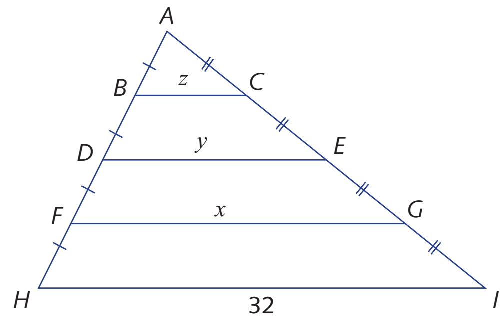
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Prove that ΔABC is similar to ΔADE and copy and complete the ratio statement.



$$\frac{AB}{\ldots} = \frac{AC}{\ldots} = \ldots$$

4 Prove that triangles ABC, ADE, AFG, and AHI are similar and find x, y and z.



5 In triangles PQR and XYZ, $\angle P = \angle Z$ and $PQ \times YZ = PR \times XZ$. Name the other pairs of equal angles in the triangles.

6 Prove that if $ABCD$ is a quadrilateral such that $\angle A = \angle C$ and $\dfrac{DA}{AB} = \dfrac{BC}{CD}$, then $ABCD$ is a parallelogram.

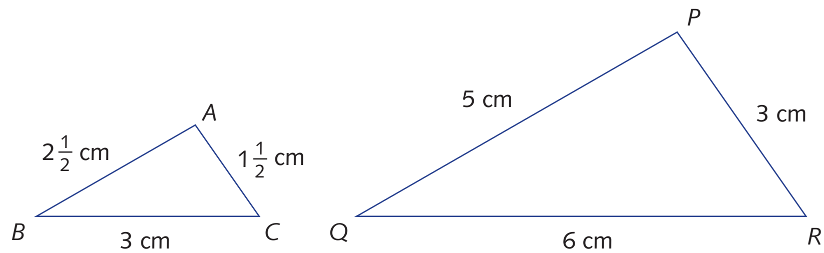
9E The SSS and RHS similarity tests

We now introduce two further similarity tests, giving four tests altogether.

The SSS similarity test

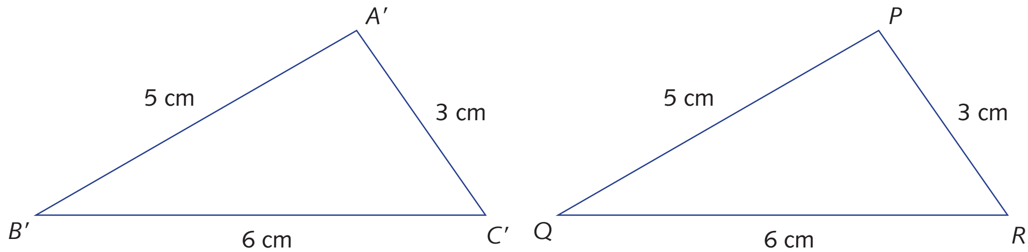
The two triangles below have different sizes, but their sides can be paired up so that matching sides have the same ratio 1 : 2.

$$AB = \frac{1}{2}PQ,\ BC = \frac{1}{2}QR \text{ and } CA = \frac{1}{2}RP$$



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Let us now enlarge the first triangle by a factor 2, so that the side lengths all double.



By the SSS congruence test, the new triangle $\Delta A'B'C'$ is congruent to ΔPQR.

Hence the original triangle ΔABC is similar to ΔPQR.

The statement that the two triangles above are similar is thus written as

ΔABC is similar to ΔPQR (SSS).

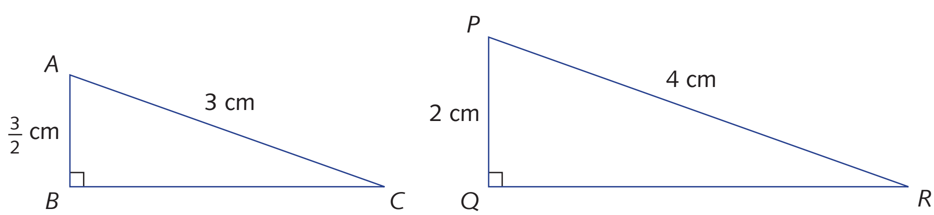
As an interesting example, the Pythagorean triples $3, 4, 5; 6, 8, 10; 9, 12, 15; \ldots$ give an infinite family of similar right-angled triangles.

The SSS similarity test

If we can match up the sides of one triangle with the sides of the other so that the ratio of matching lengths is constant, then the triangles are similar.

The RHS similarity test

The two triangles below are both right-angled, and the hypotenuse and one side are in the same ratio $3 : 4$.



Once again, we can enlarge the first triangle by a factor of $\frac{4}{3}$ to produce a triangle $\Delta A'B'C'$, and $\Delta A'B'C'$ is congruent to ΔPQR by the RHS congruence test.

Hence the original triangle ΔABC is similar to ΔPQR.

This gives us our fourth and last similarity test.

The RHS similarity test

- If the ratio of the hypotenuses of two right-angled triangles equals the ratio of another pair of sides, then the two triangles are similar.

The statement that the two triangles above are similar is thus written as

ΔABC is similar to ΔPQR (RHS).

Recall that given the hypotenuse and one side of a right-angled triangle, the third side can be found by Pythagoras' theorem.

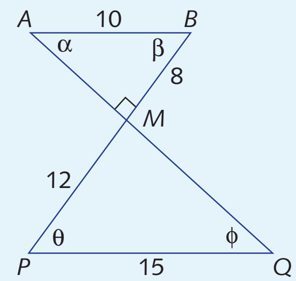
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 10

a Prove that the two triangles in the diagram to the right are similar.

b Identify the equal angles in two triangles.

c Prove that $AB \parallel PQ$.



Solution

a In the triangles ABM and QPM

$\angle AMB = \angle QMP = 90°$ (vertically opposite)

$\dfrac{AB}{QP} = \dfrac{2}{3} = \dfrac{BM}{PM}$ (given)

so ΔABM is similar to ΔQPM (RHS similarity).

b Hence $\alpha = \phi$ and $\beta = \theta$ (matching angles of similar triangles).

c Hence $AB \parallel PQ$ (alternate angles are equal).

Using the four similarity tests in problems

Here are examples using the AAA, SSS, SAS and RHS similarity tests in problems.

Example 11

a Prove that the two triangles in the diagram are similar.

b Which of the marked angles are equal?

Solution

a In the triangles ΔABC and ΔCBD:

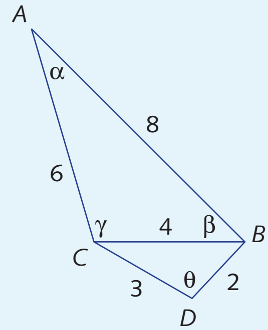
$AB = 2 \times CB$ (given)

$BC = 2 \times BD$ (given)

$CA = 2 \times DC$ (given)

so ΔABC is similar to ΔCBD (SSS).

b Hence $\gamma = \theta$ (matching angles of similar triangles).



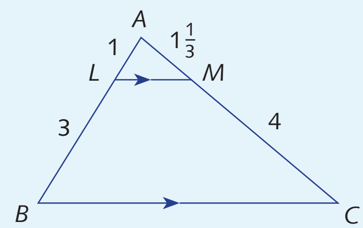
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 12

a Prove that the two triangles in the diagram are similar.

b Hence prove that $LM \parallel BC$.

Solution



a In the triangles ΔABC and ΔALM:

$AB = 4 \times AL$

$AC = 4 \times AM$

$\angle BAC = \angle LAM$ (common)

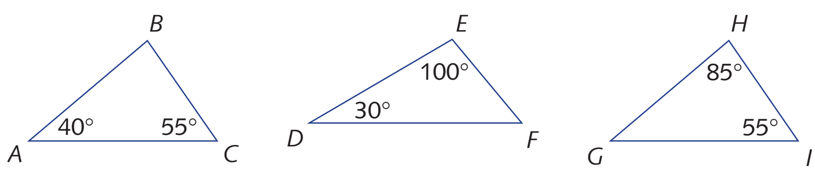
so ΔBAC is similar to ΔLAM (SAS).

b Hence $\angle ABC = \angle ALM$ (matching angles of similar triangles) and so $LM \parallel BC$ (corresponding angles are equal).

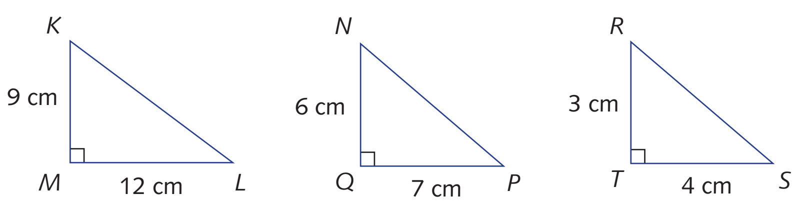
Exercise 9E

1 Name the pair of similar triangles in each part, naming the vertices in matching order and stating the similarity test.

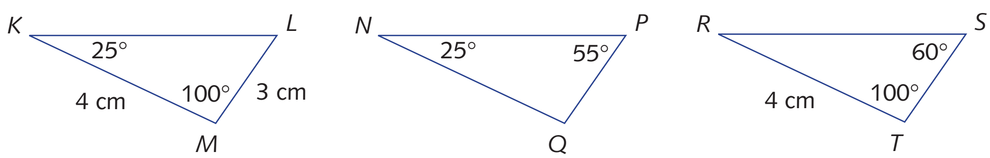
a



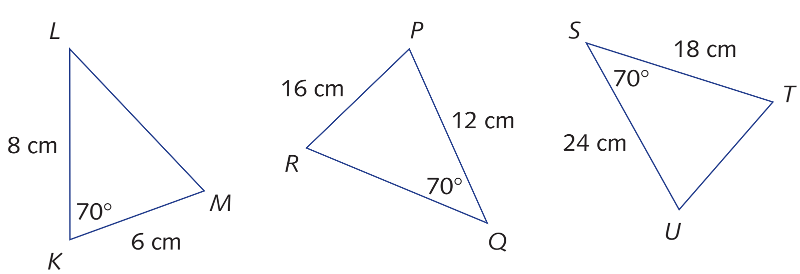
b



c

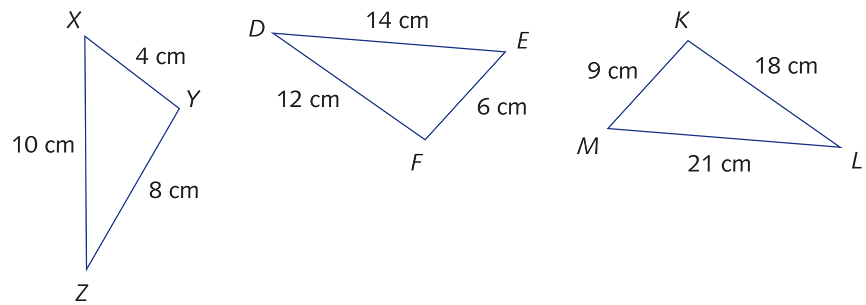


d



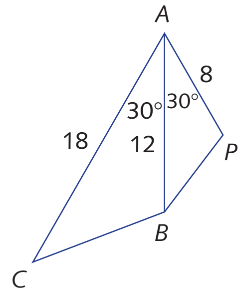
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

e



2 Copy and complete each similarity statement, stating the similarity test, and naming the vertices in matching order. They copy and complete the ratio statement.

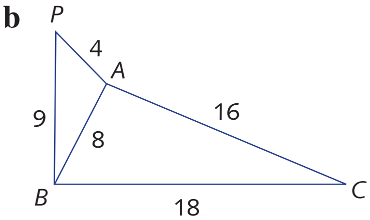
a



ΔABC is similar to … (…)

$\frac{AB}{\dots} = \frac{BC}{\dots}$

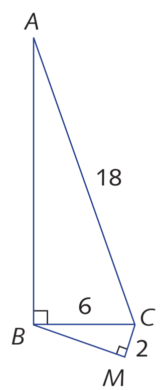
b



ΔABC is similar to … (…)

$\frac{AB}{\dots} = \frac{BC}{\dots}$

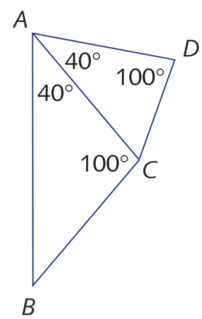
c



ΔABC is similar to … (…)

$\frac{AB}{\dots} = \frac{BC}{\dots}$

d

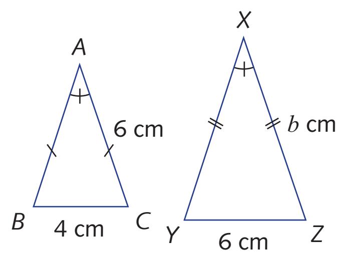


ΔABC is similar to … (…)

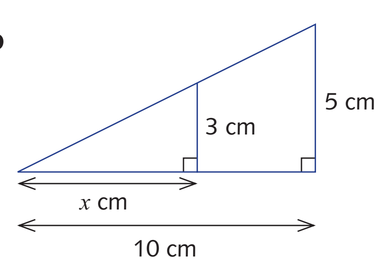
$\frac{AB}{\dots} = \frac{BC}{\dots}$

3 Find the values of the pronumerals in each part.

a

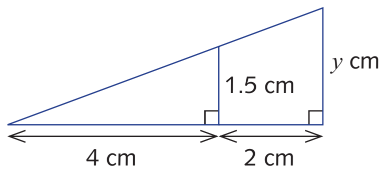


b

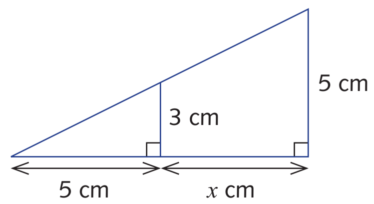


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

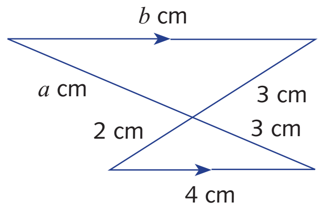
c



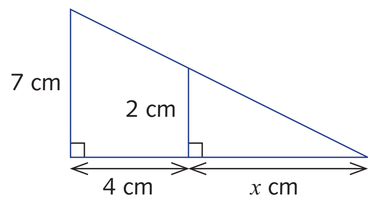
d



e



f



4 **a** Find the size of:

i $\angle ABD$

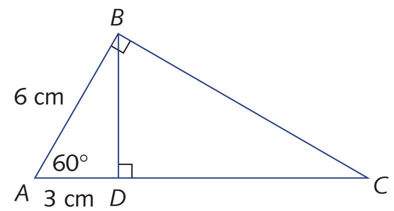
ii $\angle ACB$

b Justify the statement that ΔABC is similar to $\angle ADB$.

c Find AC.

d Show that $BD = 3\sqrt{3}$ cm.

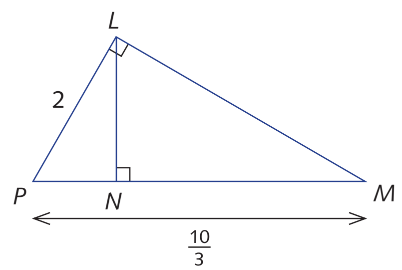
e Find BC.



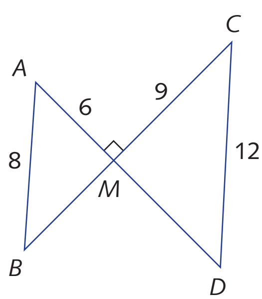
5 **a** Find LM.

b Find LN.

c Find MN.



6 **a**

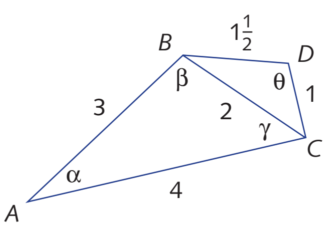


i Prove that ΔABM is similar to $\angle CDM$.

ii Hence prove that $\angle B = \angle D$.

iii Is it true that $AB \parallel CD$?

b

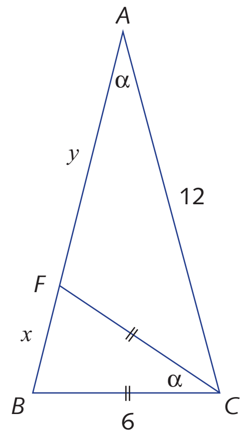


i Prove that ΔABC is similar to $\Delta\ldots$

ii Which of α, β and γ is equal to θ?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

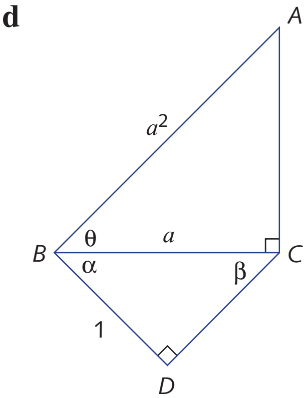
c



i Prove that ΔABC is similar to ΔCBF.

ii Hence find x and y.

d



i Prove that ΔABC is similar to $\Delta\ldots$

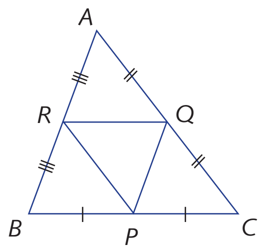
ii Is α, or β equal to θ?

7 **a** Prove that: *The line interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length.*

b Prove that: *The line through the midpoint of one side of a triangle parallel to another side of the triangle meets the third side at its midpoint.*

c Prove that: *The midpoints of the sides of a quadrilateral form the vertices of a parallelogram*. Let $ABCD$ be a quadrilateral, and let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively. Construct diagonals AC and BD.

8 Prove that: *The intervals joining the midpoints of the sides of a triangle dissect the triangle into four congruent triangles each similar to the original triangle.*



9 If you think the statement is true, justify your answer using a similarity test. If you think it is false, draw two triangles that provide a counter-example.

a Any two equilateral triangles are similar.

b Any two isosceles triangles are similar.

c Any two isosceles triangles with equal apex angles are similar.

d Any two isosceles triangles with equal length bases are similar.

e Any two right-angled triangles are similar.

f Any two right-angled triangles with equal length hypotenuses are similar.

g Any two isosceles right-angled triangles are similar.

h Any two right-angled triangles in which one other angle is 40° are similar.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 If you think the statement is true, justify your answer. If you think it is false, draw two figures that provide a counterexample.

a Any two squares are similar.

b Any two rectangles are similar.

c Any two rectangles in which one side is three times another are similar.

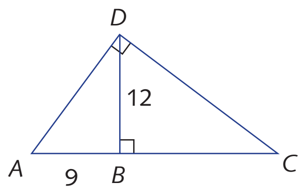
d Any two rectangles whose diagonals meet at 30° are similar.

e Any two rectangles whose diagonals have length 6 cm are smilar.

f Any two rhombuses are similar.

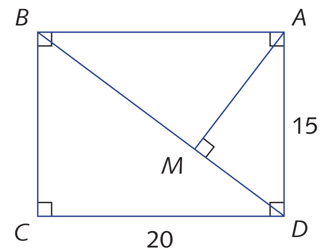
g Any two rhombuses with vertex angles 50° and 130° are similar.

11 **a**



Find AD, DC and BC.

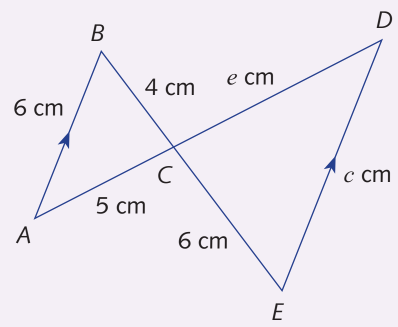
b



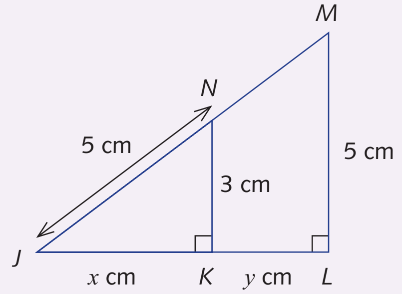
Find AM, BM and DM.

Review exercise

1 **a** Prove that triangle ABC is similar to triangle DEC and find the value of e and c.



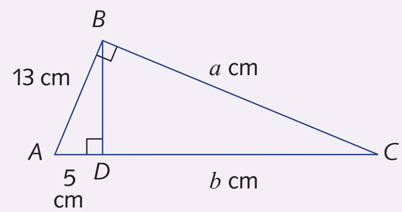
b Prove that triangle JNK is similar to triangle JML and find the values of x and y.



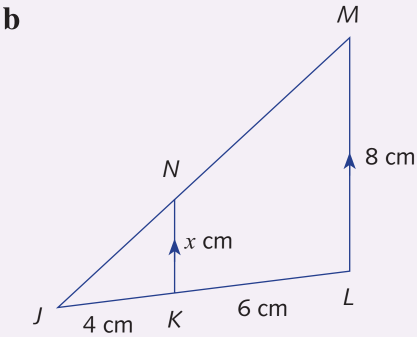
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 Write down a pair of similar triangles and find the value of each pronumeral.

a

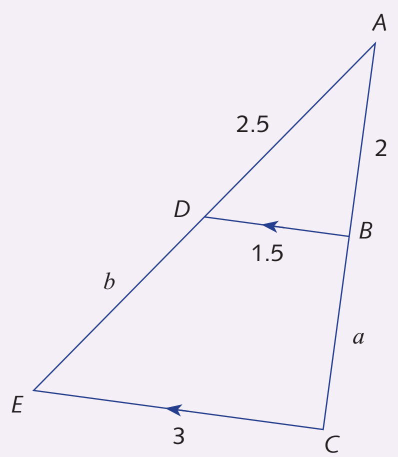


b

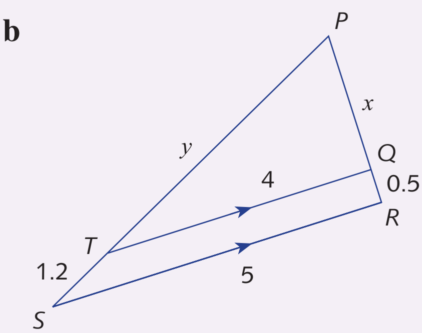


3 Write down a pair of similar triangles and find the value of each pronumeral.

a

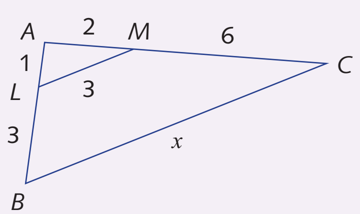


b



4 A vertical stick of length 30 cm casts a shadow of length 5 cm at 12 p.m. Find the length, in centimetres, of the shadow cast by a 1 metre ruler placed in the same position at the same time of the day.

5 **a**

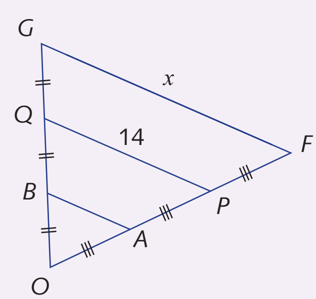


i Prove that ΔALM is similar to ΔABC.

ii Hence prove that $LM \parallel BC$.

iii Find x.

b



i Prove that ΔOPQ is similar to ΔOFG.

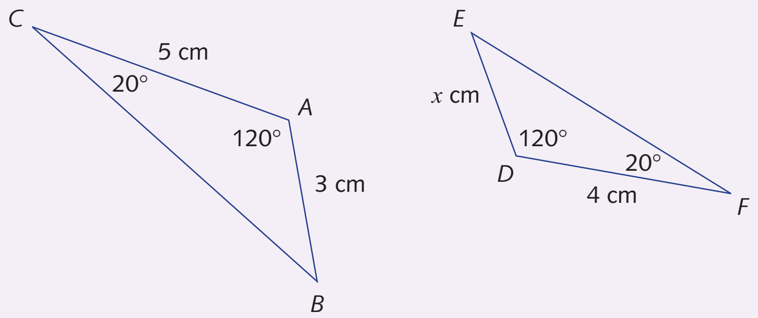
ii Hence prove that $PQ \parallel FG$.

iii Find x.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Prove that ΔABC is similar to ΔDEF.

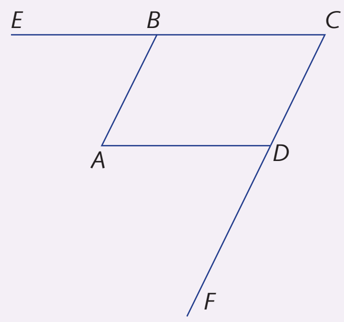
Find x.



7 A tower standing on level ground casts a shadow 40 m long. A vertical stick 3 m high is placed at the tip of the shadow. The stick is found to cast a shadow 6 m long. Find the height of the tower.

8 A line from the top of a church steeple to the ground just passes over the top of a pole 2.5 m high and meets the ground at a point A, 1.5 m from the base of the pole. If the distance from A to a point directly below the church steeple is 30 m, find the height of the steeple.

9 $ABCD$ is a parallelogram with $\angle BAD$ acute. The point E lies on the ray CB such that ΔABE is isosceles with $AB = AE$. The point F lies on the ray CD such that ΔADF is isosceles with $AD = AF$.



a Prove that ΔABE is similar to ΔADF.

b Prove that $DE = BF$.

10 ABC is a triangle, M is a point in the interval AB such that $AB = 3AM$ and N is a point in the line segment AC such that $AC = 3AN$.

a Prove that $BC \parallel MN$.

b If BN and CM intersect at P, prove that $BP = 3PN$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 You proved in Exercise 9E, Question **8** that the midpoints of the sides of a quadrilateral form the vertices of a parallelogram. Now prove that this parallelogram has area half the area of the original quadrilateral.

2 Let $ABCD$ be a trapezium with $AB \parallel CD$. Let the diagonals AC and BD meet at O.

Let the line through O parallel to AB meet AD at P and BC at Q.

By considering the pairs of triangles ADB and POD, BDC and QOB, and AOB and COD, prove that O is the midpoint of PQ.

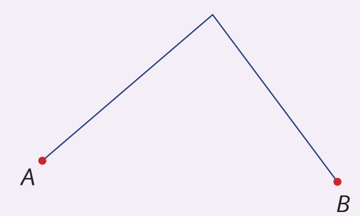
3 Let $ABCD$ be a rectangle, and let K and L be the midpoints of AB and CD respectively. Let AC meet KL at M.

a Show that M is the midpoint of KL.

b Let DK meet AC at X. Find the ratio $AX : XC$.

c Find the ratio of the area of ΔKXA to the area of the rectangle $ABCD$.

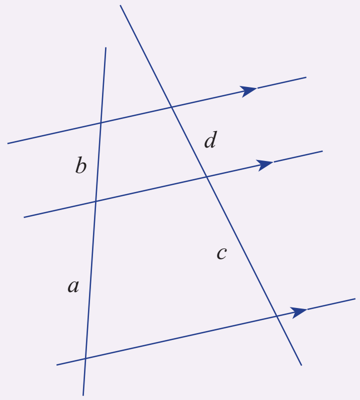
4 Take any two points A and B in the plane and a piece of string longer than the distance AB. Imagine fixing one end of the string at A and the other at B, and pulling the string taut by using a pencil.



Now move the pencil around keeping the string taut and letting the string slide around the point of the pencil. In this way we trace out a curve called an **ellipse**.

Prove that an enlargement transformation takes an ellipse to another ellipse.

5 Take three parallel lines and any two transversals that are not parallel.

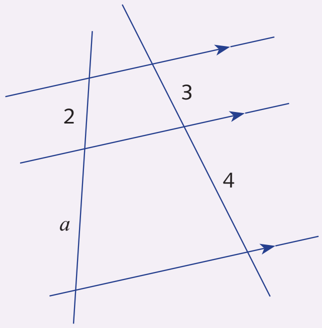


a Prove that the ratio of the lengths cut off by the parallel lines is the same for both transversals.

That is, prove that $\frac{a}{b} = \frac{c}{d}$ in the diagram opposite.

b Is the result still true if the transversals are parallel?

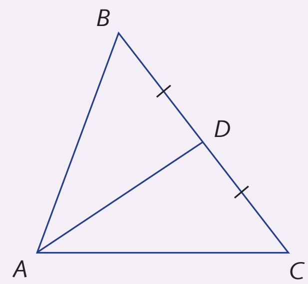
c In the diagram below, find a.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

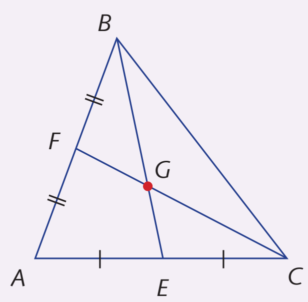
6 An **altitude** of a triangle is an interval from a vertex meeting the opposite side at right angles. Prove that if two altitudes of a triangle are equal, then the triangle is isosceles.

7 In this question we will prove that: *The medians of a triangle are concurrent and trisect each other*.



A **median** of a triangle is the line which joins a vertex to the middle of the opposite side.

a Let the medians BE and CF meet at G. Show that $BG = 2GE$ and $CG = 2GF$.



b Deduce that the third median AD also passes through G. The point G is called the **centroid** of the triangle ABC.

c Prove that if two of the medians of a triangle have equal length, then the triangle is isosceles.

8 **Theorem**

Let ABC and PQR be equiangular triangles, with $\angle A = \angle P = \alpha$, $\angle B = \angle Q = \beta$, $\angle C = \angle R = \gamma$. Let $BC = a$, $CA = b$ and $AB = c$, and let $QR = ka$, where k is a positive rational number. Then $RP = kb$ and $PQ = kc$.

Use congruence (not similarity) to prove the result for $k = 2$ and then $k = 3$.

This is a beginning of the proof that the four similarity tests can be proven from the congruence tests without enlargements when the similarity factor is a rational number.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

10

Review and problem-solving

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

10A Review

Chapter 1: Algebra

1 Evaluate $3x + 2y^2$ when:

a $x = 2$ and $y = 3$ **b** $x = 5$ and $y = 2$

c $x = -23$ and $y = -3$ **d** $x = \frac{1}{2}$ and $y = \frac{-3}{5}$

2 Simply each of these expressions by collecting like terms.

a $3a + 2b\text{-}a + 4b$ **b** $5x^2y - 3xy + 7xy - x^2y$

c $7m + 12n^2 + 2n^2 - 9m$ **d** $p^2 - 6p - p + 15$

3 Simplify:

a $7ab \times 2a$ **b** $-3x \times -2y$ **c** $\frac{20xy}{5x}$ **d** $25a \div 5 \times 3$

4 Write each expression as a single fraction.

a $\frac{a}{5} - \frac{2a}{3}$ **b** $\frac{3x}{8} - \frac{2x}{5}$ **c** $\frac{a}{5} \times \frac{2a}{3}$

d $\frac{a}{2b} \times \frac{2ab}{7}$ **e** $\frac{3x}{4} \div \frac{6x}{7}$ **f** $\frac{ab}{3} \div \frac{6b}{b}$

5 Expand:

a $3(a + 4)$ **b** $6(x - 1)$ **c** $2(3b + 2)$ **d** $5(4d - 1)$

e $-3(3d - 2)$ **f** $-2(5\ell - 4)$ **g** $-2x(3x + 1)$ **h** $4x(2x + 3)$

6 Expand and collect like terms for each of these expressions.

a $3(a + 2) + 4(a + 5)$ **b** $4(2x - 1) + 3(3x + 2)$

c $5(3d - 2) + 4(2d - 7)$ **d** $8(4e + 3) - 5(e - 1)$

e $6(f - 2) - 3(2f - 5)$ **f** $2x(x + 4) + 3(x - 2)$

g $x(3x + 2) - 4x(2x - 3)$ **h** $2x(5x + 4) - 6x(3x - 7)$

7 Simplify:

a $\frac{x+1}{4} + \frac{x+3}{3}$ **b** $\frac{x-2}{2} + \frac{x-1}{3}$ **c** $\frac{2x+1}{3} - \frac{x+1}{4}$ **d** $\frac{3x-1}{4} - \frac{2x-1}{6}$

8 Expand and simplify:

a $(x + 3)(x + 5)$ **b** $(x + 7)(x - 3)$ **c** $(x - 3)(x + 8)$

d $(2x + 1)(3x - 2)$ **e** $(4x + 3)(3x + 5)$ **f** $(5x - 2)(2x + 3)$

g $(x + 5)(x - 5)$ **h** $(2x + 3)(2x - 3)$ **i** $(3x - 5)(3x + 5)$

j $(x + 7)^2$ **k** $(2x - 5)^2$ **l** $(3x - 4)^2$

m $(x + 2)^2 - (x - 4)^2$ **n** $(2x + 3)^2 - (2x - 3)^2$

o $(x + 1)(2x + 3) + (2x - 1)(3x + 2)$ **p** $(x + 2)(2x - 5) - (3x + 1)(2x - 4)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

9 Copy and complete:

a $(x + 3)(x + \ldots) = x^2 + 10x + \ldots$

b $(x + 2)(x + \ldots) = x^2 - x - \ldots$

c $(x + 6)(\ldots + \ldots) = x^2 + 11x + 30$

d $(x + 4)(\ldots + \ldots) = x^2 + 10x + 24$

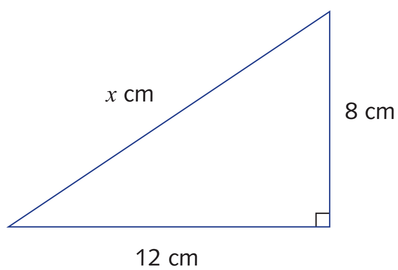
e $(2x - 1)(x + \ldots) = 2x^2 + \ldots - 6$

f $(3x + 2)(\ldots + \ldots) = 6x^2 + \ldots + 14$

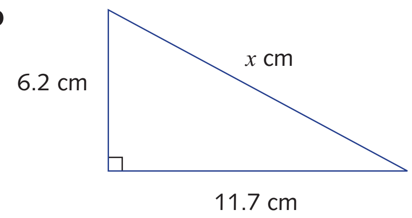
Chapter 2: Pythagoras' theorem and surds

1 For each of these right-angled triangles, find the value of the pronumeral, correct to 1 decimal place.

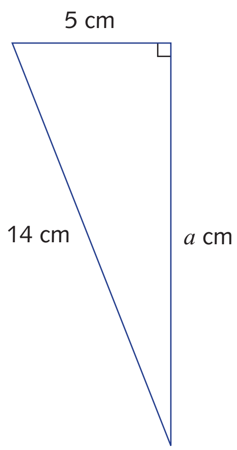
a



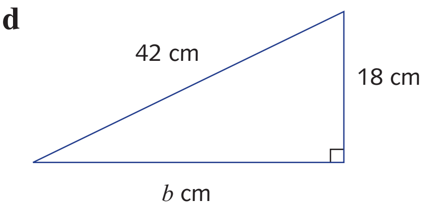
b



c



d



2 The lengths of the sides of a triangle are 8.2 cm, 11.6 cm and 14.3 cm. Is the triangle right-angled?

3 In each part below, the two shorter side lengths of a right-angled triangle are given. State the length of the hypotenuse.

a 3 cm, 4 cm

b 5 cm, 12 cm

c 4 cm, 7.5 cm

d 0.3 cm, 0.4 cm

e 1 cm, 2.4 cm

f 12 cm, 22.5 cm

4 A gardener is designing a rectangular lawn $ABCD$. If $AB = 4.2$ m and $BC = 3.15$ m, how far apart should A and C be to ensure $\angle ABC = 90°$?

5 A plane takes off and after climbing on a straight line path for a distance of 1 km, it has flown a horizontal distance of 900 m. What is the plane's altitude, correct to the nearest metre?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Simplify each of these surds.

a $\sqrt{20}$ **b** $\sqrt{75}$ **c** $2\sqrt{18}$ **d** $4\sqrt{50}$ **e** $5\sqrt{108}$ **f** $9\sqrt{27}$

7 Write each number as the square root of a whole number.

a $2\sqrt{3}$ **b** $3\sqrt{2}$ **c** $10\sqrt{5}$ **d** $4\sqrt{7}$

8 Simplify:

a $4\sqrt{2} + 7\sqrt{2}$ **b** $8\sqrt{3} - 5\sqrt{3}$ **c** $4\sqrt{2} \times 5\sqrt{3}$ **d** $3\sqrt{5} \times 4\sqrt{7}$

e $\sqrt{18} + \sqrt{32}$ **f** $\sqrt{27} - \sqrt{12}$ **g** $4\sqrt{12} + 3\sqrt{75}$ **h** $8\sqrt{50} - 2\sqrt{98}$

9 Expand and simplify:

a $\sqrt{2}(\sqrt{3} + \sqrt{10})$ **b** $\sqrt{3}(4\sqrt{3} - 5)$ **c** $3\sqrt{5}(2\sqrt{2} - 4\sqrt{5})$

d $2\sqrt{2}(3\sqrt{3} + 4\sqrt{2})$ **e** $(2\sqrt{3} + 1)(3\sqrt{3} - 2)$ **f** $(4\sqrt{2} + 3)(5\sqrt{2} - 7)$

g $(3\sqrt{2} - 1)^2$ **h** $(\sqrt{5} + 1)^2$ **i** $(2\sqrt{5} + 7\sqrt{2})(2\sqrt{5} - 7\sqrt{2})$

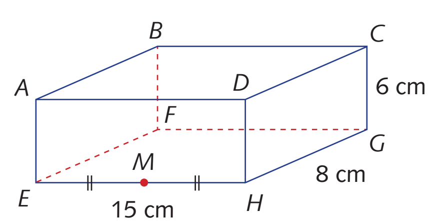
10 Express each number with a rational denominator.

a $\dfrac{3}{\sqrt{3}}$ **b** $\dfrac{2\sqrt{5}}{\sqrt{5}}$ **c** $\dfrac{2}{4\sqrt{3}}$ **d** $\dfrac{5\sqrt{3}}{3\sqrt{2}}$

11 Express each number with a rational denominator.

a $\dfrac{3\sqrt{2}}{\sqrt{5} + 2}$ **b** $\dfrac{\sqrt{3}}{2\sqrt{3} - 1}$ **c** $\dfrac{3\sqrt{2} + 1}{\sqrt{5} + 2}$ **d** $\dfrac{\sqrt{2} + 1}{\sqrt{3} + \sqrt{2}}$

12 For the rectangular prism to right, calculate the length of each of these intervals. Give your answers as surds in simplest form.



a EG **b** EC **c** HC

d GM **e** CM **f** AM

13 A vase is in the shape of a cylinder with base radius 4 cm and height 10 cm. What is the length, correct to 1 decimal place, of the longest flower stem that can just fit in the vase?

Chapter 3: Consumer arithmetic

1 Express each percentage as a fraction in its simplest form.

a 18% **b** 64% **c** 2.6% **d** 8.5% **e** $37\frac{1}{2}\%$ **f** $6\frac{2}{3}\%$

2 Express each percentage as a decimal.

a 8% **b** 27% **c** 9.6% **d** 45.8% **e** $12\frac{1}{4}\%$ **f** $38\frac{1}{2}\%$

3 Express each rational number as a percentage.

a $\dfrac{2}{5}$ **b** $\dfrac{5}{8}$ **c** 0.61 **d** 0.02 **e** $\dfrac{4}{7}$ **f** $\dfrac{5}{9}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Copy and complete the following table.

	Percentage	Fraction	Decimal
a	25%		
b		$\frac{3}{10}$	
c			0.26
d		$\frac{2}{3}$	
e	8%		
f			0.075

5 Calculate:

a 8% of 120 **b** 16% of 54 **c** 85% of \$400 **d** $9\frac{1}{2}$% of \$6000

6 There are 650 students at a high school, 54% of whom are boys. How many boys are at the school?

7 Netball is played by 6% of Australians. If the population of Australia is 22 500 000, how many Australians play netball?

8 In a class of 25 students, 8 travel to school by train. What percentage of the class travel to school by train?

9 In a survey of 1200 adults, it was discovered that 114 of them were unemployed. What percentage of the adults surveyed were unemployed?

10 Find the new value if:

a 80 is increased by 40% **b** 150 is increased by 6%

c 240 is decreased by 12% **d** 160 is decreased by 4%

11 During a sale, the price of a sofa bed is reduced by 20%. If the original price of the bed was \$650, what is its sale price?

12 A salesperson is given a salary increase of 4%. If her existing weekly salary is \$640, what will her new weekly salary be?

13 Joe's Electrical Store is having an 8% discount sale. The sale price of some items is given below. Calculate the price of the items before they were reduced.

a Heater \$276 **b** Vacuum cleaner \$138

c Dishwasher \$690 **d** Microwave \$132.80

14 The enrolment of a school increased from 680 to 740. Calculate the percentage increase, correct to 2 decimal places.

15 During a sale the price of a suit is reduced from \$420 to \$370. Calculate the percentage discount, correct to 1 decimal place.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

16 What single percentage change, correct to 2 decimal places, is equivalent to each of these multiple changes?

a A 6% increase followed by a 12% increase

b A 10% increase followed by a 10% decrease

c A 16% decrease followed by a 8% decrease

d A 12% decrease followed by a 14% increase

17 Over the course of a year an employee is given successive salary increases of 4%, 6% and 5%.

a If the employee's original monthly salary was \$2600, what is the employee's salary after the three increases?

b What single percentage change is equivalent to the three successive salary increases?

18 To obtain a bonus, a salesperson's sales must increase by 20% in a two-month period. If the salesperson's sales increase by 8% in the first month, by what percentage must they increase in the second month to ensure the bonus is obtained?

19 Mia invests \$6000 in the bank. How much will she have in her account after three years if the bank pays:

a 8% simple interest p.a. **b** 4% compound interest p.a.

20 The value of a new car depreciates at a compound rate of 6% each year. If the car has an initial value of \$19 960, calculate its value after:

a one year **b** five years **c** 10 years

Chapter 4: Factorisation

1 Factorise:

a $5a + 10$ **b** $6c - 8$ **c** $9d - 24$ **d** $3e^2 + 9e$

e $6f^2 + 10f$ **f** $-3h^2 - 15h$ **g** $4a^2b + 6ab^2$ **h** $9mn^2 + 12mn$

2 Factorise:

a $x^2 + 7x + 12$ **b** $x^2 - 9x + 18$ **c** $x^2 - 5x - 6$ **d** $x^2 + 3x - 28$

e $x^2 - 11x + 30$ **f** $x^2 - 14x + 24$ **g** $x^2 - 6x - 55$ **h** $3x^2 + 6x + 9$

i $4x^2 - 8x + 12$ **j** $x^2 - 100$ **k** $9x^2 - 16y^2$ **l** $1 - 16a^2$

3 Write each expression as a simplified single fraction.

a $\dfrac{1}{(x-1)^2} \div \dfrac{1}{x^2 - 1}$ **b** $\dfrac{x-4}{x^2 + 2x + 1} \times \dfrac{x+1}{x^2 - 16}$

c $\dfrac{m-2}{4m} \times \dfrac{m}{m-2}$ **d** $\dfrac{p+1}{8(p-1)} \times \dfrac{4(p-1)}{(p+1)(p+2)}$

e $\dfrac{4}{a} \div \dfrac{2}{a^2}$ **f** $\dfrac{5a-7}{2a+4} \times \dfrac{12}{10a-14}$

g $\dfrac{x^2 + 3x - 4}{2x - 2} \times \dfrac{6x - 12}{x - 1}$ **h** $\dfrac{x^3}{y^2} \div \dfrac{x}{2y^3}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Chapter 5: Linear equations and inequalities

1 Write each of the following statements using symbols.

a two more than x
b four less than b
c half of c
d three times d
e one more than twice e
f two less than one-third of f
g twice three more than s
h half of one less than h

2 Solve these equations.

a $4a = 36$
b $\frac{b}{4} = 2$
c $5x - 3 = 42$
d $3x + 19 = 46$
e $7 - 2y = -15$
f $8 - 3m = 2$
g $4(2x - 3) = 84$
h $7(1 - x) = 42$
i $5x - \frac{1}{2} = 17$
j $\frac{2x}{3} + \frac{1}{4} = 1$
k $3 - \frac{x}{5} = 4$
l $2 - \frac{3x}{4} = 5$

3 Solve the following equations.

a $2x + 7 = x + 9$
b $3x + 11 = 4x + 27$
c $2x - 9 = 3x + 15$
d $4x - 5 = 8x - 13$
e $5(x + 1) = 2(3x + 1)$
f $4(2x - 3) = 3(x + 4)$
g $2(x + 1) + 3(x + 2) = 17$
h $4(x - 3) - 3(2x + 1) = 18$
i $\frac{2a + 1}{3} = \frac{a - 1}{4}$
j $\frac{a - 3}{2} + 1 = \frac{a - 1}{4}$
k $\frac{3a}{2} + 1 = \frac{a + 1}{3}$
l $\frac{a - 1}{5} = \frac{2a + 3}{4}$

4 When $1\frac{1}{2}$ is added to twice a certain number, the result is $4\frac{3}{4}$. What is the number?

5 Gina is 4 years older than her sister. When their ages are added together the result is 22. How old is Gina?

6 Frankie buys four pencils and two rulers at the newsagent at a total cost of \$5.62. If each ruler costs 35 cents more than each pencil, what is the cost of a pencil?

7 A woman is three times as old as her daughter. In four years' time she will be only two and a half times as old as her daughter. What is the woman's present age?

8 In an 80 km biathlon, a competitor completes the bicycle leg in 2 hours and the running leg in 40 minutes. If the competitor cycles 20 km/h faster than he runs, at what speed does he cycle?

9 Solve each equation for x.

a $7x + a = 3$
b $ax - b = c$
c $\frac{x}{a} + b = c$
d $a - x = b$
e $a - \frac{x}{b} = c$
f $x(a + b) = c$
g $a(x + b) = cx$
h $\frac{x}{a} + \frac{b}{c} = 0$
i $\frac{x}{a} - \frac{b}{c} = d$
j $\frac{x}{a} + b = \frac{cx}{d} + e$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 If $ax + b = c$:

a solve the equation for x

b use the formula you found in part **a** to solve for x:

i $2x + 1 = 5$ **ii** $3x - 5 = 7$ **iii** $-3x - 1 = 5$

iv $\frac{x}{2} + 1 = 3$ **v** $\frac{x}{3} - 2 = -3$ **vi** $\frac{x}{5} - \frac{1}{2} = 2$

11 If $\frac{x}{a} + b = \frac{c}{d}$:

a solve the equation for x

b use the formula you found in part **a** to solve for x:

i $\frac{x}{2} + 3 = \frac{3}{7}$ **ii** $\frac{x}{2} - \frac{1}{2} = \frac{1}{2}$ **iii** $3x - \frac{1}{3} = 3$

12 Solve each equation for x.

a $(x + a)(x - a) = (x - b)^2$ **b** $\frac{bx}{1 + bx} + \frac{ax}{a + ax} = 2$

13 Solve these inequalities.

a $2x + 9 < 23$ **b** $1 - 5x \geq 21$ **c** $3d + 5 \leq 5d + 7$

d $2(3q + 1) \leq 18$ **e** $\frac{\ell + 2}{3} \leq \frac{\ell - 5}{7}$ **f** $\frac{2(m + 1)}{3} + 1 \leq \frac{m - 4}{2}$

Chapter 6: Formulas

1 Find the value of the subject of each of the following formulas when the pronumerals have the values indicated.

a $V = \ell bh$, when $\ell = 20$, $b = 4.6$ and $h = 2.8$

b $s = ut + \frac{1}{2}at^2$, when $u = 4.6$, $a = 9.8$ and $t = 4$

c $E = \frac{p^2}{2m}$, when $p = 14.6$ and $m = 8$

d $A = P\left(1 + \frac{r}{100}\right)^n$, when $p = 1000$, $r = 8$ and $n = 4$

2 Rewrite each formula so that the pronumeral shown in the box is the subject.

a $A = \pi ab$ $\boxed{a}$ **b** $A = \pi r(r + \ell)$ $\boxed{\ell}$

c $S = \frac{n(a - 1)}{2}$ $\boxed{a}$ **d** $\ell = a + (n - 1)d$ $\boxed{n}$

e $v = at + bt$ $\boxed{t}$ **f** $ap = b^2 - bp$ $\boxed{p}$

g $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ $\boxed{b}$ **h** $a = \frac{h}{R - r}$ $\boxed{r}$

i $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\boxed{\ell}$ **j** $m = \frac{3x}{may^2}$ $\boxed{y}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 **a** Given that $v = u + at$, find the value of a when $v = 26$, $u = 4$ and $t = 11$.

b Given that $F = \dfrac{GMm}{V^2}$, find the value of G when $F = 200$, $M = 10$, $m = 4$ and $V = 2$.

c Given that $P = \dfrac{1}{r} + \dfrac{1}{q}$, find the value of r when $P = 8.6$ and $q = 0.4$.

4 **a** Conversion of a given temperature from the Fahrenheit scale to the Centigrade scale is done by means of the formula

$$C = \frac{5(F - 32)}{9}$$

i The melting point of gold is 2280°F. Convert this into Celsius (correct to the nearest degree).

ii A healthy human being's temperature is approximately 36.9°C. What is the equivalent on the Fahrenheit scale, correct to 1 decimal place?

b A formula used in life insurance is $Q = \dfrac{2m}{2 + m}$.

i Calculate Q if $m = -0.7$ (correct to 4 significant figures).

ii Calculate m if $Q = 3$.

5 For $x = -\dfrac{a}{2} - \dfrac{b}{c}$, find the exact value of x if:

a $a = 2,\ b = -2,\ c = -1$

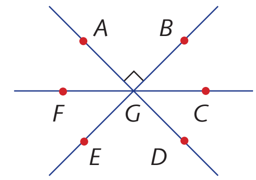
b $a = \sqrt{2},\ b = \sqrt{2},\ c = 4$

c $a = 0,\ b = 0,\ c = 5$

d $a = \pi,\ b = -\pi,\ c = 7$

Chapter 7: Congruence and special quadrilaterals

1 In the diagram opposite, state whether the following angles are acute, obtuse, right or straight.



a $\angle BGC$ **b** $\angle AGE$ **c** $\angle FGD$

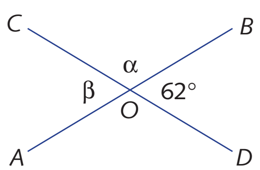
d $\angle EGB$ **e** $\angle CGD$ **f** $\angle EGD$

2 Given that $ABC = 46°$, $\angle PQR = 134°$, $\angle LMN = 43°$, $\angle DEF = 44°$ and $\angle XYZ = 43°$, which two angles are:

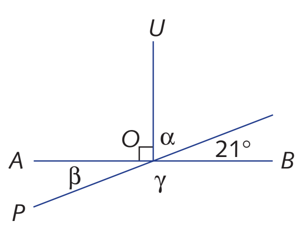
a complementary? **b** supplementary?

3 Calculate the values of the pronumerals.

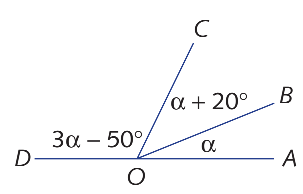
a



b



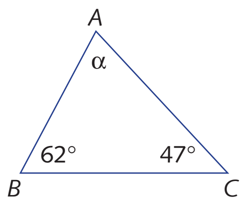
c



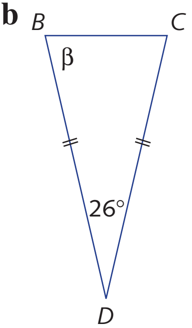
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Calculate the values of the pronumerals.

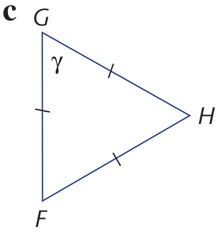
a



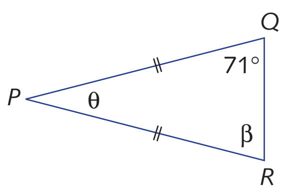
b



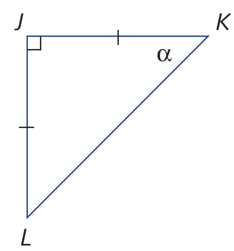
c



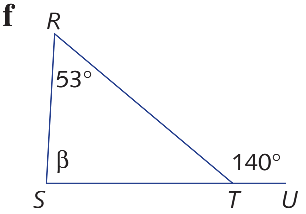
d



e

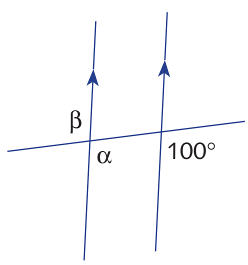


f

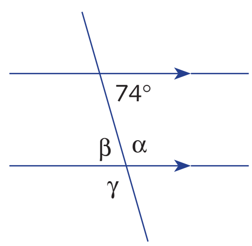


5 Find the values of the pronumerals.

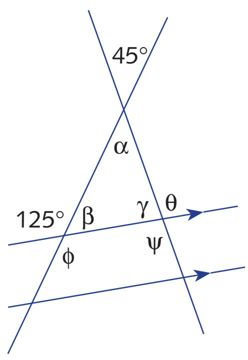
a



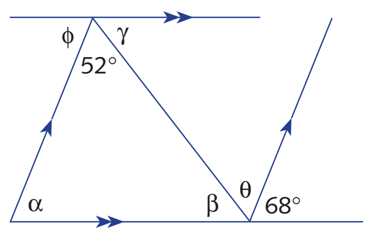
b



c



d



6 In the diagram opposite, which angle is:

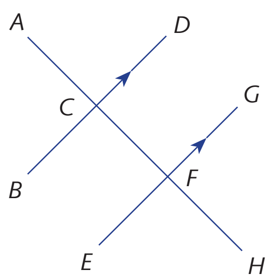
a corresponding to $\angle ACB$?

b vertically opposite $\angle BCF$?

c co-interior to $\angle CFE$?

d corresponding to $\angle CFG$?

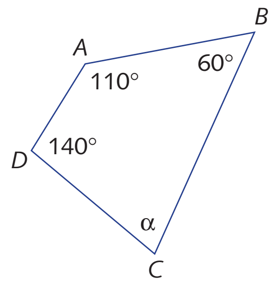
e alternate to $\angle CFE$?



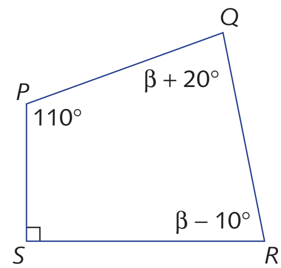
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

7 Find the values of the pronumerals.

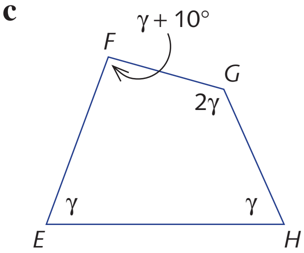
a



b

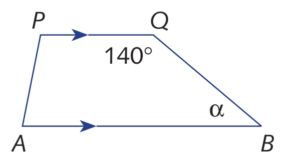


c

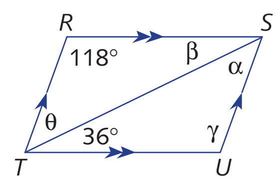


8 Find the values of the pronumerals.

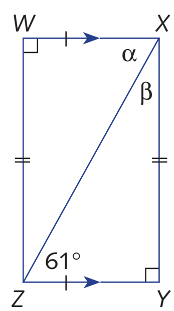
a



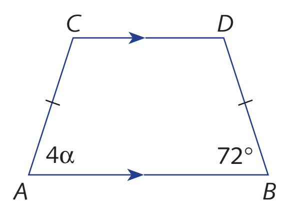
b



c

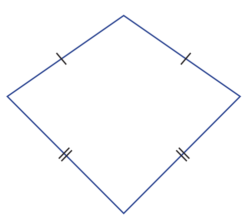


d

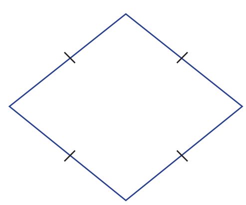


9 State the type of quadrilateral for each of the following.

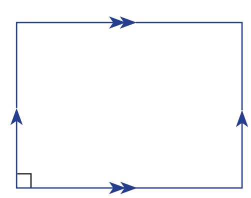
a



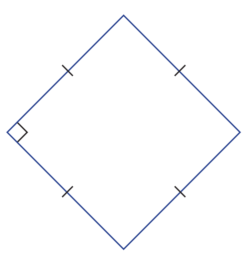
b



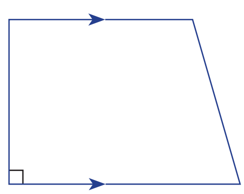
c



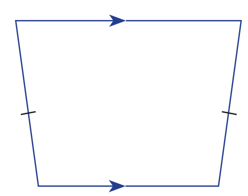
d



e

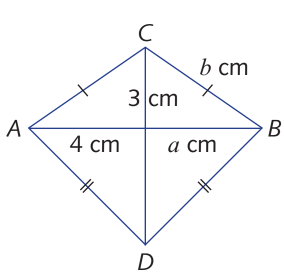


f

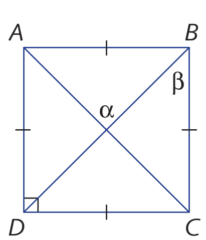


10 Find the values of the pronumerals.

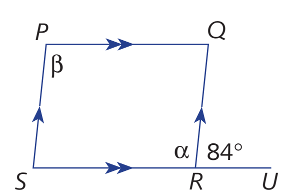
a



b

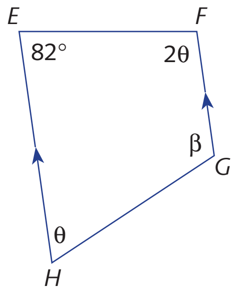


c

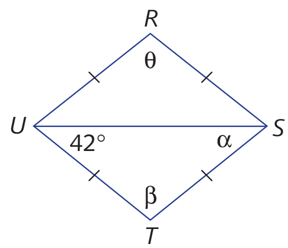


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

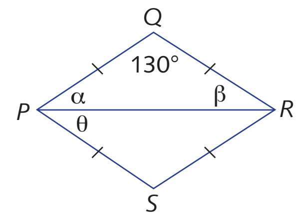
d



e



f



11 From the list – trapezium, isosceles trapezium, parallelogram, rhombus, rectangle, square, kite – name the quadrilaterals that have:

a opposite angles equal

b diagonals of equal length

c diagonals that intersect at 90°

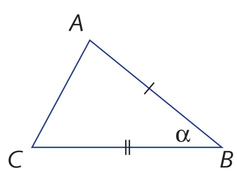
d both diagonals as angle bisectors

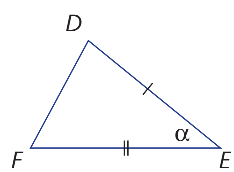
e opposite sides parallel

f adjacent angles supplementary

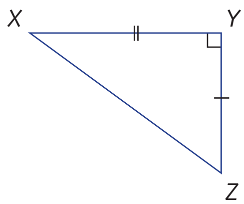
12 Each pair of triangles is congruent. State the congruence test and name the two triangles with the vertices in correct order.

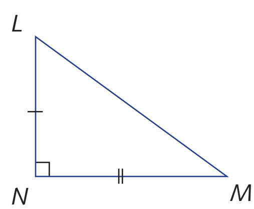
a



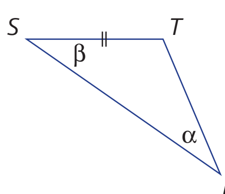


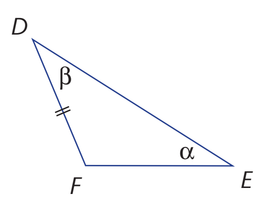
b





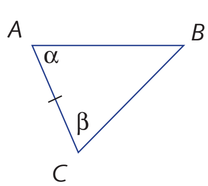
c

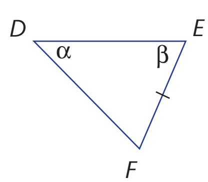


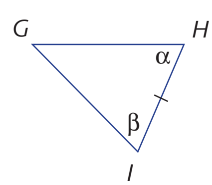


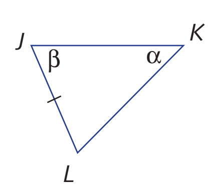
13 In each part, name the triangle congruent to ΔABC. State the congruence test used.

a

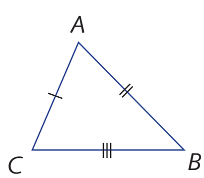


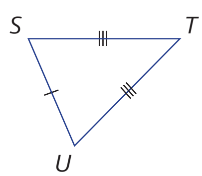


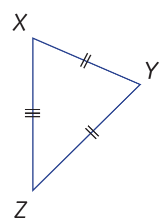


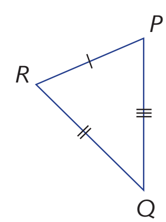


b



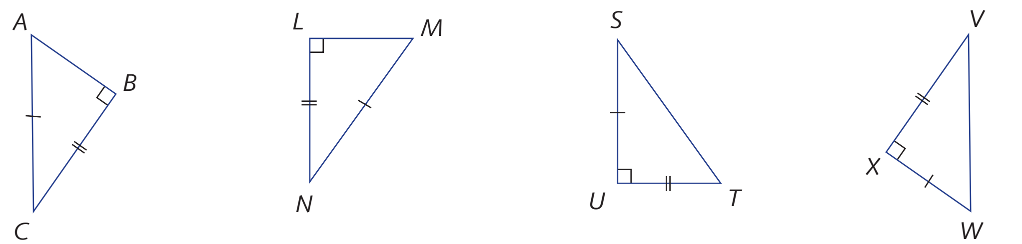




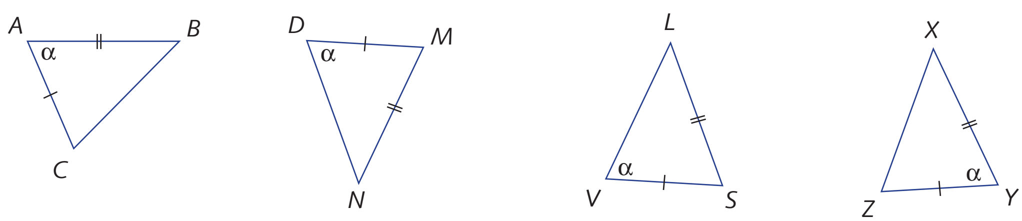


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c

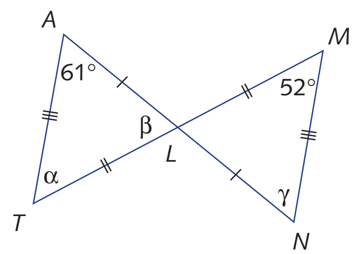


d

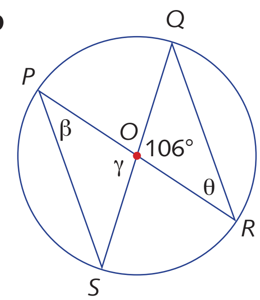


14 Each pair of triangles is congruent. Name the two triangles with the vertices in the correct order and then find the value of each pronumeral.

a

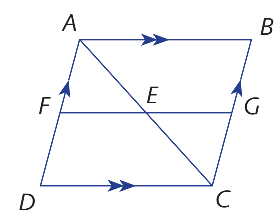


b

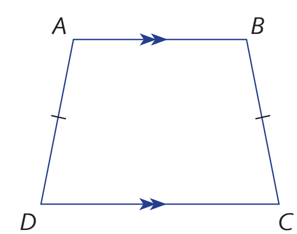


(O is the centre of the circle.)

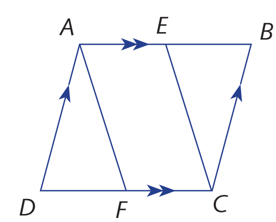
15 In the parallelogram $ABCD$, F is the midpoint of AD and G is the midpoint of BC. If AC and FG intersect at E, prove that E is the midpoint of AC.



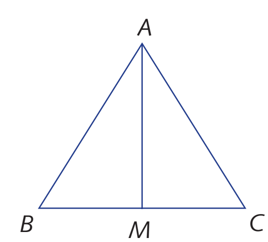
16 In the isosceles trapezium $ABCD$, E is the midpoint of AB and F is the midpoint of CD. Prove that $EF \perp DC$.
(*Hint*: Join AF and BF.)



17 In the parallelogram $ABCD$, E is the midpoint of AB and F is the midpoint of CD. Prove that $AECF$ is a parallelogram.

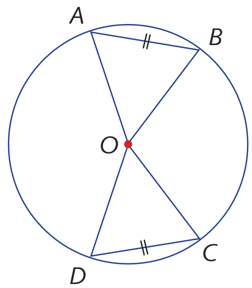


18 In the diagram to the right, ABC is isosceles with $AB = AC$ and M is the midpoint of the interval BC. Prove that $\angle ABC = \angle ACB$.
(Only congruence is to be used.)

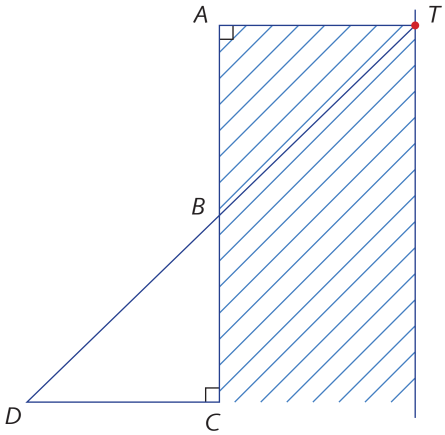


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

19 AB and CD are chords of a circle with centre O. If $AB = CD$, prove that $\angle AOB = \angle COD$.



20 A scout wants to estimate the width of a river. On the opposite river bank there is a tree T. The scout marks a point A on his side of the river directly opposite T. He then goes 6 paces along the river bank perpendicular to AT and marks a point B, then continues a further 6 paces to mark a point C. The scout then walks away from the river, at a right angle to BC, to a point D where B and T are in a line. From C to D is 7 paces.



a Approximately how many paces wide is the river?

b If the scout's pace is 0.8 m long, approximately how wide is the river in metres?

Chapter 8: Index laws

1 Express in index form.

a $3 \times 3 \times 3 \times 3$ **b** $a \times a \times a \times a \times a \times a$

2 Evaluate:

a 6^3 **b** 5^4 **c** 2^7 **d** 4^5

3 Evaluate:

a 2.6^3 **b** 1.7^2 **c** 8.9^4 **d** 0.047^2

4 Simplify, writing each answer in index form.

a $5^6 \times 5^7$ **b** $3^5 \times 3^7$ **c** $2x^4 \times 5x^2$ **d** $3m^4 \times 5m^6$

e $\dfrac{5^7}{5^2}$ **f** $\dfrac{24a^6}{16a^2}$ **g** $\dfrac{15n^{10}}{10n^7}$ **h** $(5^3)^2$

i $(2n^6)^3$ **j** $2a^0$ **k** $(3ab^2)^3$ **l** $(5x^2y)^0$

5 Simplify:

a $\dfrac{m^5n^4}{m^2n} \times \dfrac{m^6n^7}{m^4n^8}$ **b** $\dfrac{p^4q^5}{pq^2}$ **c** $(3m^2n)^3 \times 2m^4n$

d $\dfrac{(2m^4n)^3}{4m^5n}$ **e** $\dfrac{16q^4r^2}{3q^2} \div \dfrac{8qr^3}{pq^5r^4}$ **f** $\dfrac{7x^2y^4}{2xy} \div \dfrac{21xy^5}{4x^3y^6}$

6 Express with a positive index and evaluate as a fraction.

a 4^{-2} **b** 10^{-4} **c** 8^{-1} **d** $\left(\dfrac{2}{3}\right)^{-2}$ **e** $\left(\dfrac{3}{4}\right)^{-3}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 Express with positive indices.

a a^{-2} **b** b^{-6} **c** $a^{-2}b^{-4}$ **d** $m^{-1}n^{-3}$ **e** $\frac{1}{x^{-4}}$

f $\frac{2}{y^{-3}}$ **g** $3m^{-2}$ **h** $5m^{-2}$ **i** $\frac{m^{-4}}{n^{-6}}$ **j** $\frac{3p^{-1}}{q^{-2}}$

8 Simplify, expressing your answer with positive indices.

a $x^{-4}y^{-2} \times x^2y^{-6}$ **b** $p^{-5}q^{-3} \times pq^{-2}$ **c** $4n^{-2}b \times 3a^4b^{-6}$

d $2\ell^2m^{-1} \times 5\ell^{-4}m^{-2}$ **e** $\frac{56t^{-4}}{8t^2}$ **f** $\frac{25\ell^{-2}}{15\ell^{-6}}$

g $\frac{28x^6y^2}{21x^{10}y}$ **h** $\frac{16u^{-2}v}{12u^{-6}v^{-7}}$ **i** $(2mn^{-1})^{-2}$

j $(3m^{-1}n^2)^{-4}$ **k** $(2a^{-4}b^{-1})^{-3} \times 3a^6b^{-2}$ **l** $(a^{-1}b^2)^{-4} \times (a^2b^{-3})^3$

m $\frac{(3a^{-2}b^{-1})^{-3}}{a^2b^{-4}} \times \frac{a^{-3}b^4}{12a^6b^{-2}}$ **n** $\frac{(a^{-1}b^4)^{-3}}{(a^2b^{-1})^4} \times \frac{a^{-2}b}{(ab^{-1})^3}$ **o** $\frac{(a^2)^4}{b^4} \div \frac{ab^{-2}}{(b^{-1})^3}$

p $\frac{ab^{-1}}{4a^2} \div \frac{(2a^{-3}b)^{-2}}{12a^{-1}b^3}$ **q** $\frac{(4xy)^0}{(4xy)^{-1}} \times \frac{3x^2y^{-6}}{(x^{-2}y^3)^{-4}}$ **r** $\frac{2m^{-3}n^{-2}}{(2mn^4)^{-2}} \times \frac{3m^{-3}}{(mn^2)^0}$

9 Evaluate without using a calculator.

a $9^{\frac{1}{2}}$ **b** $8^{\frac{1}{3}}$ **c** $16^{\frac{1}{4}}$ **d** $243^{\frac{1}{5}}$

e $4^{\frac{3}{2}}$ **f** $27^{\frac{2}{3}}$ **g** $49^{\frac{3}{2}}$ **h** $8^{\frac{4}{3}}$

10 Simplify, expressing the answer with positive indices.

a $\sqrt[3]{8a^2}$ **b** $\sqrt[5]{32a^5b}$ **c** $\sqrt[4]{16a^2}$ **d** $\sqrt[3]{5x^5} \times \sqrt[3]{25x^3}$

e $3x^{\frac{3}{2}} \times 2x^{\frac{5}{2}}$ **f** $\left(32a^{10}b^5\right)^{\frac{2}{5}}$ **g** $\frac{\left(12x^3y^5\right)^{\frac{2}{3}}}{(4y)^{\frac{1}{3}}}$ **h** $6a^{\frac{3}{2}}b^{\frac{1}{3}} \times 2a^{\frac{1}{2}}b^{\frac{5}{3}}$

11 Write in scientific notation (standard form).

a 21 000 **b** 410 **c** 61 000 000 000

d 2400 **e** 0.0062 **f** 0.0471

g 0.000 0007 **h** 0.000 38 **i** 46 **j** 2.9

12 Write as a decimal.

a 7.2×10^4 **b** 3.8×10^2 **c** 9.7×10^{-1} **d** 2.06×10^{-2}

e 1.52×10^2 **f** 4.07×10^3 **g** 1.6×10^{-4} **h** 8.7×10^{-2}

13 Simplify, giving your answer in standard form.

a $\left(5 \times 10^2\right) \times \left(4 \times 10^6\right)$ **b** $\left(6 \times 10^{-4}\right) \div \left(2 \times 10^{-3}\right)$

c $\left(4.2 \times 10^3\right)^2$ **d** $\frac{(3 \times 10^{-2})^2}{1.8 \times 10^{-4}}$

14 Write in standard form, correct to the number of significant figures indicated in the brackets.

a 241 000 (2) **b** 627 (1) **c** 0.000 649 26 (3)

d 0.008 716 (2) **e** 27 654 321 (4) **f** 0.267 83 (3)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

15 Give a 1 significant figure estimate for:

a $210\,000 \times 0.00613$

b $470\,000\,000 \div 216\,000$

c $\dfrac{0.000\,473}{0.00991}$

d $(0.003\,14)^2$

16 Evaluate, giving your answer to the number of significant figures indicated in the brackets.

a 4.671×1.304 (2)

b $\dfrac{18.6 \times 1.75}{0.26}$ (1)

c $\dfrac{87.01 \times 0.003}{2.765}$ (2)

d $\dfrac{\sqrt{47.8}}{2.73}$ (4)

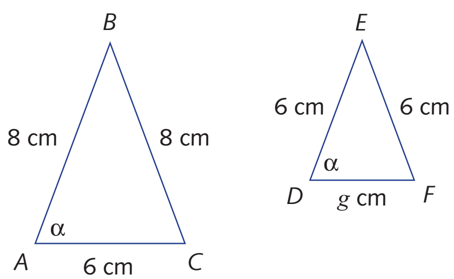
e $\dfrac{18.7^2 + 9.21^2}{18.7^2 - 9.21^2}$ (4)

f $\dfrac{0.03^2}{\sqrt{0.03}}$ (2)

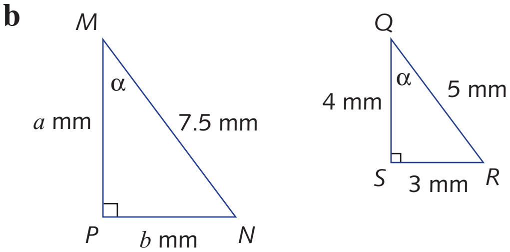
Chapter 9: Enlargements and similarity

1 For each part, write a similarity statement with the vertices in the correct order. Then find the value of each unknown side length.

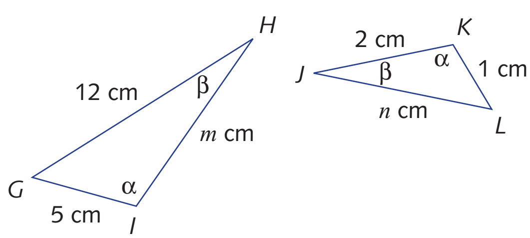
a



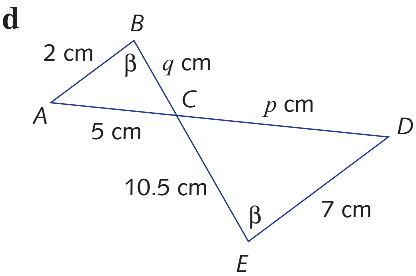
b



c

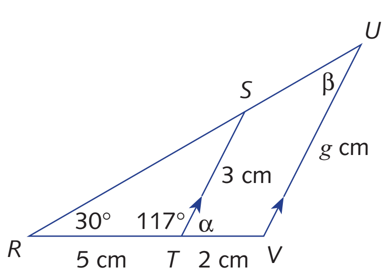


d

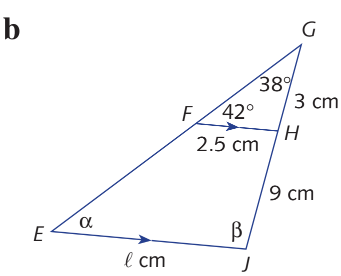


2 Name each pair of similar triangles, giving the similarity test used, and then find the value of each pronumeral.

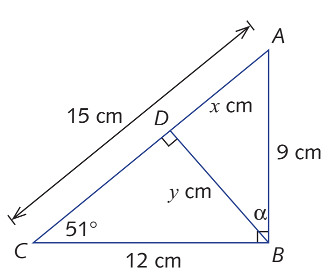
a



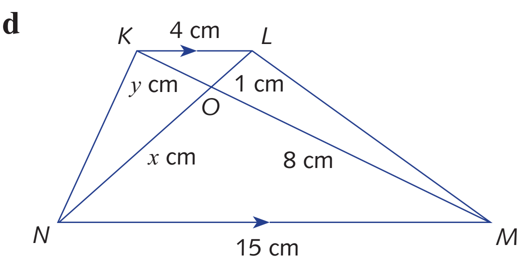
b



c



d



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 At a particular time of the day, a tree casts a shadow of length 5.2 m and a 30 cm ruler casts a shadow of length 16 cm. Find the height of the tree.

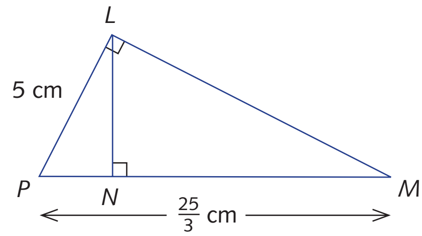
4 A girl is walking along a footpath and notices that the end of her shadow coincides with the end of the shadow of a lamp post. If the girl is 1.4 m tall and she is standing 5 metres from the lamp post, find the height of the lamp post if the length of her shadow is 80 cm.

5 In the diagram to the right:

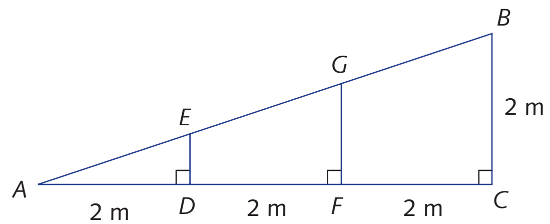
a find LM

b find LN

c find MN



6 A builder is constructing a ramp AB, as drawn in the diagram, with $BC = 2$ metres and $AC = 6$ metres. Two vertical supports ED and GF need to be placed at 2-metre intervals.



a Find:

i DE **ii** FG

b Find AB.

c The builder needs to construct two timber frames (as drawn in the diagram) to support the ramp.

i What length of timber, to the nearest metre, will he need?

ii Timber costs \$4.60 per metre and is only sold in lengths which are a multiple of one metre. How much will the timber cost the builder?

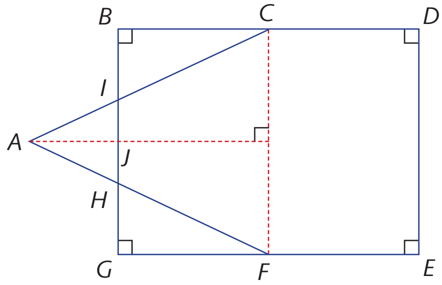
7 A ladder leans against a vertical wall. The base of the ladder is 2.4 metres from the wall and the top of the ladder is 3.2 metres above the ground. A man of height 1.7 metres climbs 1 metre up the ladder and stands in a vertical position.

a How long is the ladder?

b How far off the ground is the top of the man's head?

c How far from the wall is the man?

8 An engineer is building a trailer. The tray $BDEG$ of the trailer is 2 metres long and 1.5 metres wide – that is, $BD = 2$ m and $DE = 1.5$ m. The intervals AC and AF represent the draw-bar of the trailer, where $AC = AF$, C is the midpoint of BD and F is the midpoint of GE.



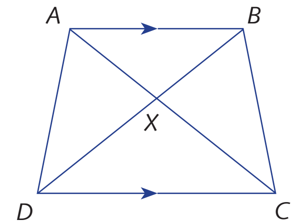
a As part of the design, $AJ = 1$ metre. Find, correct to the nearest millimetre:

i AC **ii** BI

b The engineer changes the distance of A from BG so that $BI : IJ = 7 : 8$. Find, correct to the nearest millimetre:

i BI **ii** AJ **iii** AC

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

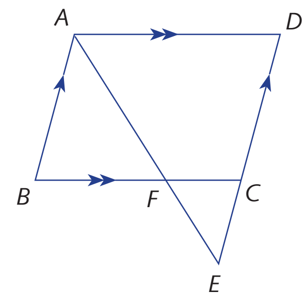


9 $ABCD$ is an isosceles trapezium with $AB \parallel CD$ and $AD = BC$. The diagonals of $ABCD$ intersect at X.

a Prove that ΔABX is similar to ΔCDX.

b Given that $AB = 4$ cm, $BX = 2.5$ cm and $DC = 8$ cm, calculate:

i AX **ii** DX



10 **a** Prove that ΔAFB is similar to ΔEFC.

b Given that $AD = 10$ cm, $CD = 4$ cm and $CE = 1$ cm, calculate FC.

c Name two other pairs of similar triangles.

11 AB and DC are the parallel sides of a trapezium and are of such length that when diagonal DB is constructed, $\angle DAB = \angle DBC$. Prove that $AB \times BC = DB \times AD$.

12 $ABCD$ is a parallelogram with E on BC such that $BE = 2EC$. The line AE and the line DC intersect at F.

a Prove that $AB = 2CF$. **b** Prove that $AF = 3EF$.

10B Problem-solving

1 Mahmoud wants to buy a new car, which is valued at \$45 000. He could buy the car through an agreement with the car dealer or by borrowing the money from a bank.

a At the bank, Mahmoud asks for a loan of \$45 000. The bank offers a loan at 7% p.a. simple interest to be repaid in equal monthly instalments for 5 years. Calculate:

i the amount of interest Mahmoud would pay

ii the total amount to be paid by Mahmoud

iii the amount to be paid each month

b At 'BJ's Car Sales', Mahmoud is offered the following deal: pay \$5000 in cash now and then 60 monthly instalments of \$934, starting in a month's time.

Calculate:

i the total amount it would cost Mahmoud to buy the car this way

ii the rate of interest charged per annum, at simple interest

c Who offers the better deal, the car dealer or the bank?

2 The workers at a production factory are negotiating a pay rise with the management. The union wants a pay increase of 2% for the first year followed by a further pay increase of 4% in the second year, with no further increase for two more years. The management offers

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a 5% pay increase for the first year with no further increase for three more years. Riley is currently paid \$30 000 a year.

a Under the union's claim:

i what would Riley's salary be in the first year?

ii what would Riley's salary be in the second year?

iii how much would Riley earn during the four years?

b Under the management's offer:

i what would Riley's salary be in the first year?

ii how much would Riley earn during the four years?

c Which package would be better for Riley?

3 A container holds 1.25 litres of a fruit juice drink. This drink is 25% fruit juice and the remainder is water.

a If 0.5 litres of the drink is poured out and replaced by 50% fruit juice drink, find the percentage of the contents of the container that is now fruit juice.

b If x litres of the drink is poured out and replaced by 50% fruit juice drink, find the percentage of the drink in the container that is now fruit juice.

c How many litres of the fruit juice drink has to be poured out and replaced by the 50% fruit juice drink for the percentage to increase to 41%?

4 Two cars leave Roma on the Warrego Highway travelling in the same direction. One car leaves the town 30 minutes before the other. The first car averages 80 km/h while the second car averages 96 km/h.

a How far apart are the cars when the first car is:

i 50 km from Roma?

ii x km from Roma, assuming that the first car is still in front of the second car, and that the second car has left Roma?

b How far from Roma does the second car catch up to the first?

5 The pilot of an ultra-light aircraft has determined that the plane averages 75 km/h in still air.

a If she flies out for 20 km and then returns by the same route, how long will she take?

b If she flies out for 1 hr 42 min and back for 1 hr 42 min, how far does she travel in total?

c If there is a 40 km/h headwind as the pilot flies out and a 40 km/h tailwind on return, what is the longest time the pilot can fly out to be able to return in 4 hours? (Answer in hours and minutes.)

6 The cash price of a car is advertised as \$32 000. Alternatively you can buy the car with a deposit of 40% and repayments of \$377.60 per month for 5 years to a finance company.

a What is the price paid for the car under the alternative scheme?

b What is the amount of interest paid to the finance company under the alternative scheme?

c What is the rate of simple interest p.a. that will apply in the alternative scheme?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d The car depreciates in value by 12% per year.

i Find the value of the car at the end of the 5 years.

ii Calculate your average total cost per month over the 5-year period if you use the Finance Company option. Include interest and depreciation, and assume that it costs \$1000 per year for comprehensive insurance and registration, \$200 per month for petrol and \$800 per year for servicing.

7 **a** To fill my tank with petrol costs \$55.00 at \$1.10 per litre. What is the price in dollars per litre if, after a price rise, it costs \$62.50 to fill the tank with the same amount of petrol?

b A person purchases 50 litres at \$1.12 per litre. Some time later the price has increased to \$1.40 per litre. How many litres can now be purchased for the same amount of money?

c Let \$$C$ be the cost of buying V litres of petrol at R dollars per litre. Find the formula relating C, V and R.

8 George runs twice as fast as he walks. When going to school one day, he walks for twice the time that he runs and takes a total of 20 minutes to get to school.

a If x km/h is George's walking speed, find the distance that he travels to school in terms of x.

b The next day George runs for twice the time that he walks.

i If t hours is the time it takes George to get to school, find the distance to school in terms of x and t.

ii Hence find how many minutes it takes him to get to school on the second day.

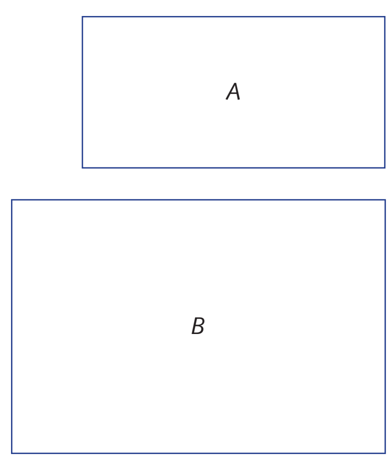
9 A car can travel 100 km on 11 litres of petrol when its speed is 50 km/h, but when its speed is 90 km/h it consumes 15 litres per 100 km.

a If the car travels 28 km at 50 km/h and an additional 130 km at 90 km/h, how many litres of petrol will it consume?

b The car travels x km at 50 km/h and $5x$ km at 90 km/h. Find a formula for y, the number of litres of petrol it consumes on the journey.

10 In rectangle A, the length is 5 m more than the width.

Rectangle B is obtained from rectangle A by increasing the width of A by 50% and increasing the length of A by 20%.



a If the width of A is x m, find, in terms of x:

i the length of A

ii the length and width of B

iii the perimeter of A, expressed in simplest form

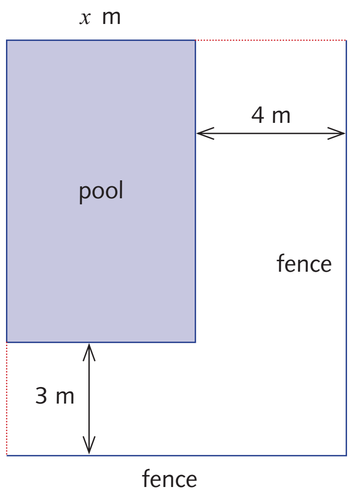
iv the perimeter of B, expressed in simplest form

v the difference between the perimeters of A and B, expressed in simplest form

b If the difference between the perimeters of A and B is 23 m, what is the value of x?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11 Andrew has been contracted by a family to build a fence around two sides of their swimming pool. The swimming pool is in the shape of a rectangle and it is twice as long as it is wide. Council regulations require the fence to be a certain distance from the pool, as shown in the diagram.



a If the width of the pool is x m, express each of the following in terms of x.

i The length of the pool

ii The length of the fence that is parallel to the width of the pool

iii The length of the fence that is parallel to the length of the pool

iv The total length of the fence

Andrew gives the family a quote for the job. He says that it will cost \$6.00 per metre to build the section of the fence that is parallel to the pool's width, and the section parallel to the pool's length will cost \$8.00 per metre.

b Express the total cost \$$C$ of the fence in terms of x.

The family intend to pave the area between the pool and the fence. They need to calculate the area between the pool edge and the fence to determine the cost of the paving.

c **i** Express the area covered by the pool and the fence enclosure in terms of x.

ii Show that the area to be paved is equal to $(11x + 12)$ m^2.

iii If the width of the pool is 4 m, find the area to be paved.

iv If the area to be paved is 61.5 m^2, calculate the width and length of the pool.

12 Two trains travel between towns A and B. They leave at the same time with one train travelling from A to B and the other from B to A. They arrive at their destinations one hour and four hours respectively, after passing one another at point P. The slower train travels at 35 km/h.

a How far does the slower train travel after they pass?

b If the faster train travels at x km/h, how far, in terms of x, does the faster train travel after they pass?

c Hence find the number of hours, in terms of x, that the faster train takes to reach point P. Repeat for the slower train to find a different expression, in terms of x, for the same time.

d Hence find the speed of the faster train.

13 Cam purchased a number of identical items, each costing \$$x$. The total cost was \$40.

a How many items were purchased?

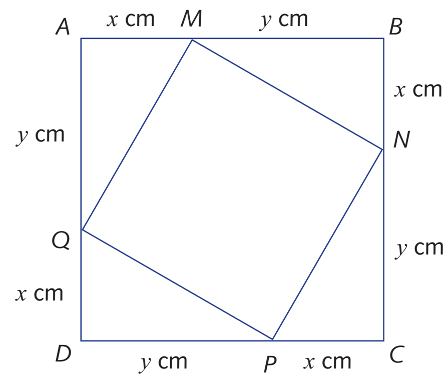
b How many items could be purchased for \$40 if the price of each individual item increased by 40 cents? (Calculate your answer in terms of x.)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c If the number of items obtained for \$40 at the cheaper price was five more than the number obtained at the dearer price, write down an equation involving x.

d Hence find x and state the original cost of each item.

14 $ABCD$ is a square of side length 21 cm. $AM = x$ cm and $MB = y$ cm.



a Find y in terms of x.

Triangles AMQ, BNM, CPN and DQP are congruent, right-angled triangles.

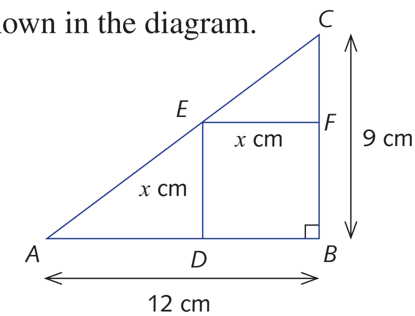
b Find the area of $MNPQ$ in terms of x.

c i If the area of $MNPQ$ is $225\ \text{cm}^2$, find x.

ii In this case, find the dimensions of the triangles.

d If the area of the square $MNPQ$ is half the area of the square $ABCD$, find the value of x.

15 a In the right-angled triangle ABC is a square $BDEF$ as shown in the diagram.

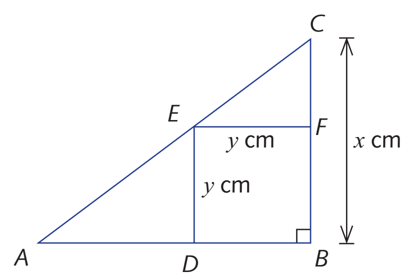


i Why is ΔEFC similar to ΔABC?

ii Hence, find x.

iii Hence, find the area of the square $BDEF$ as a percentage of the area of ΔABC, correct to 1 decimal place.

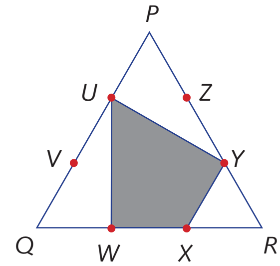
b In this diagram, the square $BDEF$ is inside ΔABC as shown. If $BC = x$ cm, $EF = y$ cm and $AB = 2BC$:



i find the relationship between x and y

ii hence find the area of the square $BDEF$ as a percentage of the area of ΔABC, correct to 1 decimal place

16 Let PQR be an equilateral triangle with sides of length 3 units. U, V, W, X, Y and Z divide the sides into unit lengths.



a Prove that ΔQUW is congruent to ΔPYU.

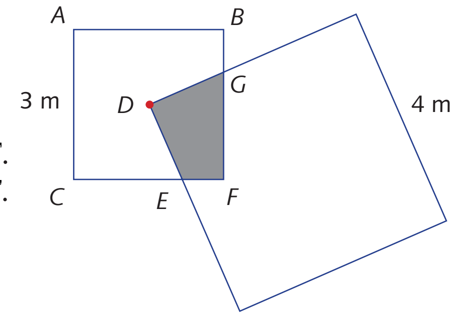
b Show area $\Delta UQW = \dfrac{2}{9} \times$ area ΔPQR.

c Show area $\Delta XYR = \dfrac{1}{9} \times$ area ΔPQR.

d Find the area of the shaded region in terms of the area of ΔPQR.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

17 A 3-metre square and a 4-metre square overlap as shown in the diagram. D is the centre of the 3-metre square.



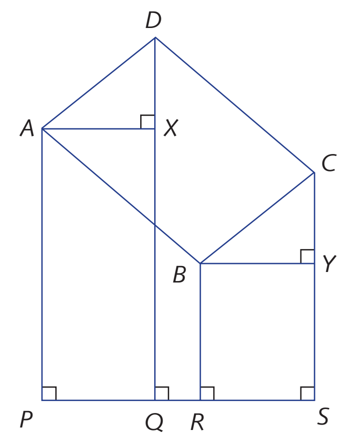
a Draw the interval DY where Y is on GF and $DY \perp GF$.
Draw the interval DX where X is on CF and $DX \perp CF$.

b Show that $\Delta DXE \equiv \Delta DYG$.

c Explain why area $DEFG$ = area $DXFY$.

d Find the area of the shaded region.

18 $ABCD$ is a parallelogram where $AP = 12$, $DQ = 16$, $CS = 10$, $PQ = 5$ and $QR = 2$.



a Prove that $\Delta ADX \equiv \Delta BCY$.

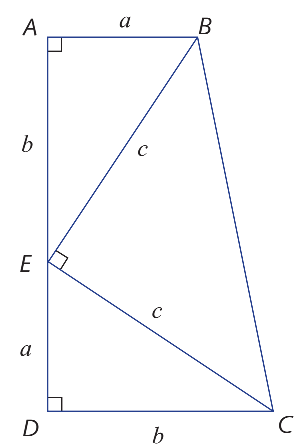
b Explain why $BY = 5$.

c Find the length of the interval BR.

d Find the area of $BRSC$.

e Find the area of the parallelogram $ABCD$.

19 Triangles ABE and DEC are congruent, right-angled triangles with side lengths as shown.



a **i** Find the area of triangle ABE.

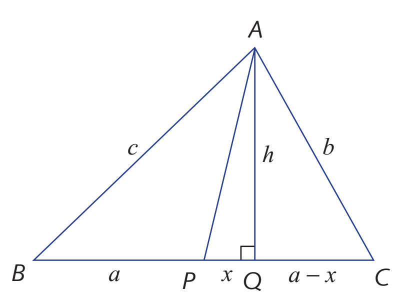
ii Find the area of triangle BEC.

iii Use the results of **i** and **ii** to find the area of the trapezium $ABCD$.

b Use the formula to find the area of trapezium $ABCD$.

c Use your results from parts **a** and **b** to show that $a^2 + b^2 = c^2$, hence proving Pythagoras' theorem.

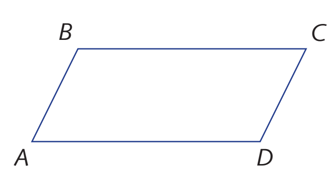
20 **a** ABC is a triangle and AQ is an altitude where Q lies between B and C. P is the midpoint of BC.



i Use Pythagoras' theorem in triangles ABQ and ACQ to find relationships for c^2 and b^2 in terms of a, x and h.

ii Prove that
$AB^2 + AC^2 = 2BP^2 + 2AP^2$

b Use part **a** to prove that for any parallelogram $ABCD$
$2AB^2 + 2BC^2 = AC^2 + BD^2$



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

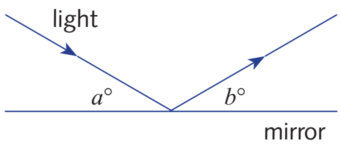
How does a sextant work?

A sextant is a navigational device used by sailors to determine the angle between the horizon and a reference point, usually a star. Knowing this angle allows the sailor to determine the latitude of the vessel's position.

A sextant uses the idea of double reflection to measure angles. The concept was first conceived by Sir Isaac Newton in 1699, but the first sextants were not commercially produced for about another 20 years.

In this investigation, you will discover how a sextant works and how it can be used to measure the angle between the horizon and an object in the sky. To do this, the following important result about reflection is needed.

When light is reflected by a mirror, the angle of the incoming ray of light is equal to the outgoing angle of the ray of light. In the diagram opposite, $a = b$.



A sextant uses double reflection to measure angles.

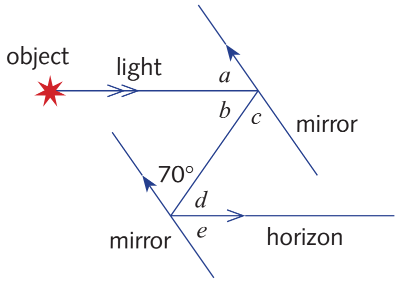
1 In the diagram opposite, the two mirrors are parallel.

a Find the value of each pronumeral.

b Does $b = d$?

c By changing the angle of size $70°$ to other angles and finding b and d in each case, is it always true that $b = d$?

d What does the fact that $b = d$ tell you about the line from the object and the horizon?



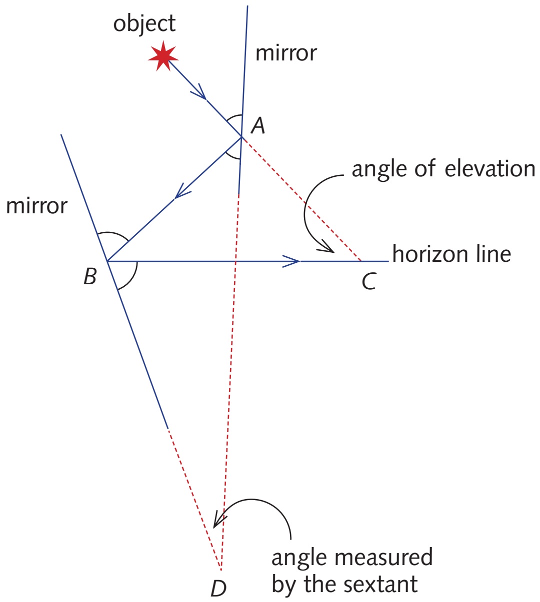
2 Suppose that the two mirrors are no longer parallel. The angle measured by the sextant is the angle between the two mirrors ($\angle ADB$ in this diagram).

a Find the angle of elevation, given that $\angle CBD = 70°$ and $\angle ADB = 25°$.

b Find the angle of elevation, given that $\angle CBD = 64°$ and $\angle ADB = 39°$.

c By investigating other values of $\angle CBD$ and $\angle ADB$, can you guess a connection between the angle measured on the sextant and the angle of elevation?

d Find the angle of elevation if $\angle CBD = y°$ and $\angle ADB = x°$.

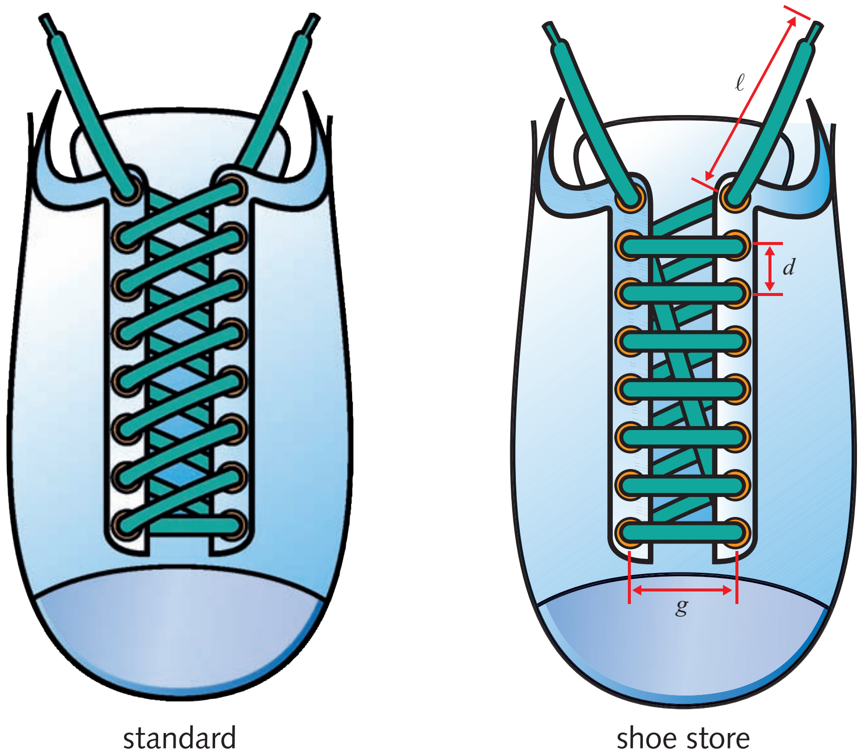


e Does this general result agree with your conjecture of part **iv**?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

How long is a piece of string?

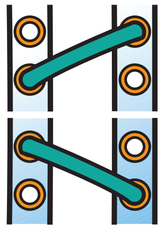
A shoe manufacturer is trying to calculate how long a shoe lace needs to be for a pair of shoes. There are at least two ways to lace shoes (as shown below).



The length of shoe lace required will depend on:

- the number n of pairs of eyelets
- the distance d between successive eyelets
- the length g of the gap between corresponding left and right eyelets
- the length ℓ of the end of each shoe lace.

1 Consider the standard lacing pattern.



a How many diagonal segments (as shown in these diagrams) are there if n is equal to:

i 3? **ii** 4? **iii** 5?

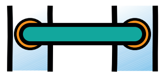
b How is the number of diagonal segments related to n?

c How long is each diagonal segment? Calculate your answer in terms of d and g.

d What is the total length of lacing required for the standard lacing pattern? Calculate your answer in terms of n, d, ℓ and g.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

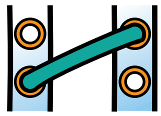
2 Consider the shoe store lacing pattern.



a How many horizontal segments (as shown here) are there if n is equal to:

i 3? **ii** 4? **iii** 5?

b How is the number of horizontal segments related to n?



c How many diagonal segments like the one shown in this diagram are there, if n is equal to:

i 3? **ii** 4? **iii** 5?

d How is the number of diagonal segments related to n?

e What is the length of each horizontal segment?

f What is the length of each diagonal segment? Give your answer in terms of d and g.

g How far is it from the bottom pair of eyelets to the top pair of eyelets if n equals:

i 3? **ii** 4? **iii** 5?

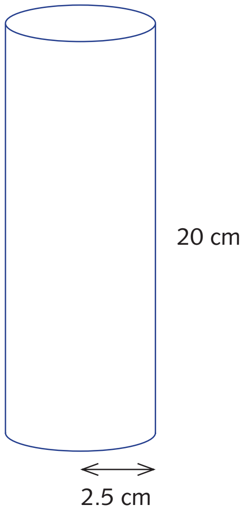
h What is the distance from the bottom pair of eyelets to the top pair in terms of d and n?

i What is the length of the long diagonal segment? Give your answer in terms of n, d, ℓ and g.

j What is the total length of lacing required for the shoe store lacing pattern? Give your answer in terms of n, d, ℓ and g.

3 By investigating the total length of lace required for each lacing pattern for different values of n, g, ℓ and d, determine which pattern requires less shoe lace.

Packaging

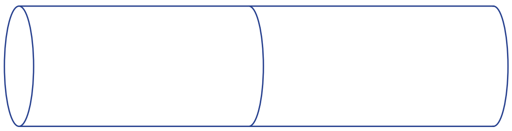


In a factory, balls are packed in canisters, which are then wrapped in bundles of six with shrink-wrap film. To make the bundle as secure as possible, the *total volume* of the pack, including any air space, needs to be as small as possible. If the volume was not as small as possible, the shrink-wrap film could be pulled tighter, thus decreasing the volume and making the package more secure.

In this project, you will investigate how cylindrical canisters can be packed so as to minimise the volume of the pack.

You will need the following information. The canisters are cylindrical, with a base radius of 2.5 cm and a height of 20 cm.

The final package can be no longer than 20 cm – that is, the cans cannot be stacked end-to-end like this.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

In the following, we consider cross-sections of the bundles of canisters. When canisters are bundled, two types of region are important. These are shown below by the shading. The radius of each circle is 2.5 cm.

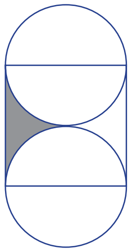
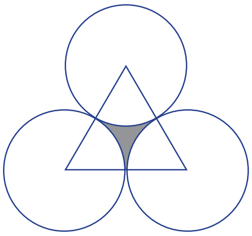
Diagram 1 

Diagram 2 

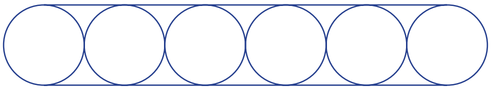
1 a Calculate the exact area of the shaded region in diagram 1.

b i What is the side length of the triangle shown in diagram 2?

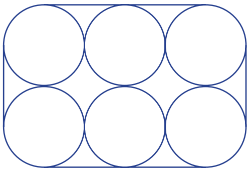
ii What is the height of the triangle? Calculate an exact answer.

iii What is the exact area of the shaded region in diagram 2?

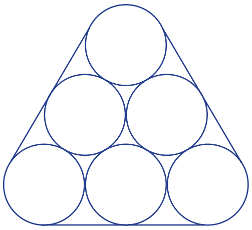
2 a Suppose that the six canisters are packed in a row. What is the total volume of the package? Calculate your answer to 3 decimal places.



b Suppose that the six canisters are packed in two rows of three. What is the total volume of the package? Calculate your answer to 3 decimal places.



c Suppose that the six canisters are packed as shown in the diagram opposite. What is the total volume of the package? Calculate your answer to 3 decimal places.



d Which is the best packaging: **a**, **b** or **c**?

3 Instead of being packaged in bundles of six, the company now decides to package the canisters in bundles of seven. How should the canisters be packed so as to minimise the volume?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

CHAPTER

11

Measurement and Geometry

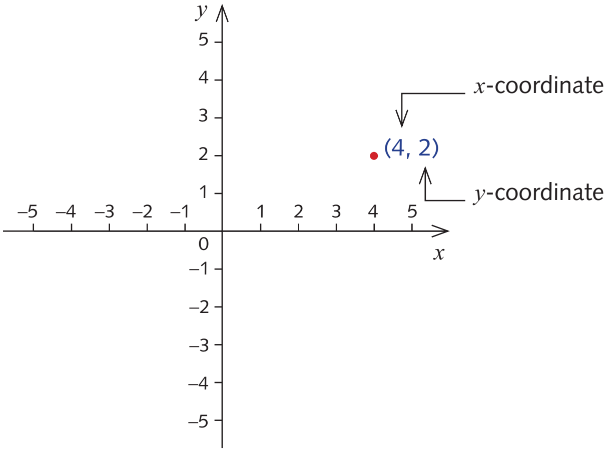
Coordinate geometry

The Cartesian or number plane is divided into four **quadrants** by two perpendicular axes called the ***x*-axis** (a horizontal line) and the ***y*-axis** (a vertical line). These axes intersect at a point called the **origin**. The position of any point in the plane can be represented by an ordered pair of numbers (x, y). These numbers are the **coordinates** of the point.

The point with coordinates (4, 2) has been plotted on the Cartesian plane shown. The coordinates of the origin are (0, 0).

René Descartes (1596–1650) introduced coordinates to show how algebra could be used to solve geometric problems and we therefore use the adjective 'Cartesian' for the number plane.

In coordinate geometry a point is represented by a pair of numbers, and a line is represented by a linear equation. In this chapter we graph points and lines and find out how to determine whether lines are perpendicular or parallel. We will learn how to calculate the distance between two points and the coordinates of the midpoint of an interval.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11A Distance between two points

Once the coordinates of two points are known, the distance between them can easily be found.

Example 1

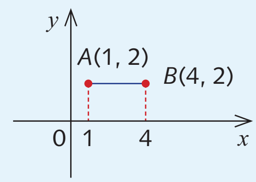
Find the distance between each pair of points.

a $A(1, 2)$ and $B(4, 2)$ **b** $A(1, -2)$ and $B(1, 3)$

Solution

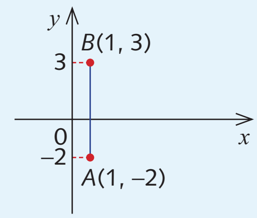
a The distance $AB = 4 - 1 = 3$

Note: The distance AB is the positive difference of the x-coordinates of the two points.



b The distance $AB = 3 - (-2) = 5$

Note: The distance AB is the positive difference of the y-coordinates of the two points.



The above example involves the special cases when the interval AB is horizontal or vertical. To calculate the distance between two points when the interval between them is neither vertical nor horizontal, we use Pythagoras' theorem.

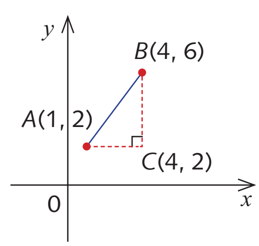
The distance between the points $A(1, 2)$ and $B(4, 6)$ is calculated below.

$AC = 4 - 1 = 3$ and $BC = 6 - 2 = 4$

By Pythagoras' theorem

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

so $AB = \sqrt{25} = 5$



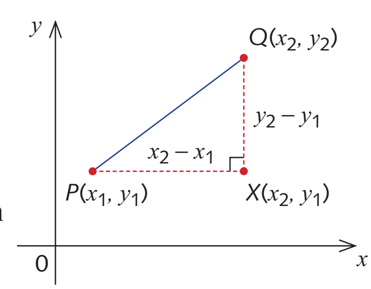
The general case

We can obtain a formula for the length of any interval.

Suppose that $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points, as shown opposite.

Form the right-angled triangle PQX, where X is the point (x_2, y_1). Then

$$PX = x_2 - x_1$$
$$QX = y_2 - y_1$$



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

By Pythagoras' theorem

$$PQ^2 = PX^2 + QX^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

so $$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In practice, we sometimes work out the square of PQ and then take the square root.

Note: You will notice that our diagram assumes $x_2 - x_1$ and $y_2 - y_1$ are positive. If either or both of these are negative, it is not necessary to change the formula since they are squared.

Example 2

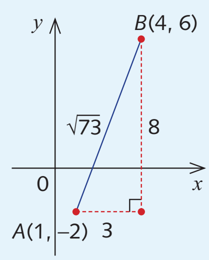
Find the distance between each pair of points.

a $A(1, -2)$ and $B(4, 6)$

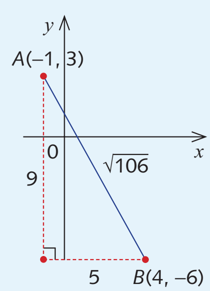
b $A(-1, 3)$ and $B(4, -6)$

Solution

a
$$\begin{aligned} AB^2 &= (4-1)^2 + (6-(-2))^2 \\ &= 3^2 + 8^2 \\ &= 9 + 64 \\ &= 73 \\ AB &= \sqrt{73} \end{aligned}$$



b
$$\begin{aligned} AB^2 &= (-1-4)^2 + (3-(-6)) \\ &= (-5)^2 + 9^2 \\ &= 25 + 81 \\ &= 106 \\ AB &= \sqrt{106} \end{aligned}$$



Example 3

Use the distance formula to find the distance between the points.

a $A(-4, -3)$ and $B(5, 7)$

b $A(7, -3)$ and $B(0, -7)$

Solution

a Let $x_1 = -4$, $x_2 = 5$, $y_1 = -3$ and $y_2 = 7$

$$\begin{aligned} AB^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (5-(-4))^2 + (7-(-3))^2 \\ &= 9^2 + 10^2 \\ &= 181 \\ AB &= \sqrt{181} \end{aligned}$$

b
$$\begin{aligned} AB^2 &= (0-7)^2 + (-7-(-3))^2 \\ &= 49 + 16 \\ &= 65 \\ AB &= \sqrt{65} \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Distance between two points

The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

or

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is just Pythagoras' theorem.

Exercise 11A

Example 1

1 Find the distance between each pair of points.

a (2, 3) and (8, 3)
b (5, 7) and (−1, 7)
c (−1, 2) and (−1, 12)
d (−2, −7) and (−2, 8)

Example 2, 3

2 Find the distance between each pair of points.

a (2, 3) and (8, 11)
b (5, 7) and (8, 3)
c (−1, 0) and (4, 12)
d (−2, −7) and (6, 8)
e (−1, −3) and (1, −1)
f (−3, 3) and (3, 0)
g (2, −3) and (6, −5)
h (−6, −3) and (1, −1)

3 How far are the following points from the origin?

a (2, 3)
b (−5, 7)
c (−1, 4)
d (−4, −5)

4 Which of the two points $M(3, 6)$ and $N(6, -4)$ is closer to $P(-2, -1)$?

5 Show that the point $A(8, 4)$ is equidistant (that is, the same distance) from points $B(-4, -1)$ and $C(13, 16)$.

6 Given the three points $A(0, 0)$, $B(3, 4)$ and $C(6, 0)$:

a calculate the distance AB
b calculate the distance BC
c calculate the distance AC
d identify the type of triangle ABC

7 The points $A(5, 3)$, $B(-17, -8)$ and $C(-6, -19)$ are joined to form a triangle. Prove that the triangle is isosceles.

8 The points $A(-2, 1)$, $B(1, 3)$ and $C(7, -6)$ are joined to form a triangle. Prove that the triangle is right-angled.

9 Calculater the perimeter of ΔPQR with vertices $P(3, 1)$, $Q(8, 6)$ and $R(10, 0)$.

10 Show that the points $A(0, -5)$, $B(5, 0)$, $C(6, 7)$ and $D(1, 2)$ are the vertices of a rhombus.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

11B The midpoint of an interval

The coordinates of the midpoint of an interval can be found by averaging the coordinates of its endpoints.

Example 4

Find the coordinates of the midpoint of the interval AB, given:

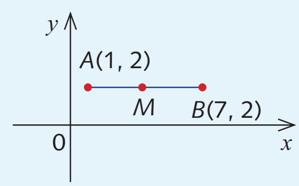
a $A(1,\ 2)$ and $B(7,\ 2)$ **b** $A(1, -2)$ and $B(1,\ 3)$

Solution

a AB is a horizontal interval since the y-coordinates of A and B are equal.

The coordinates of the midpoint of AB are (4, 2).

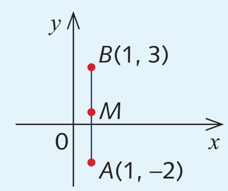
Note: 4 is the *average* of 1 and 7; that is, $4 = \dfrac{1+7}{2}$



b The midpoint of AB has coordinates $\left(1, \dfrac{1}{2}\right)$.

Note: $\dfrac{1}{2}$ is the *average* of 3 and –2;

that is, $\dfrac{1}{2} = \dfrac{3+(-2)}{2}$



What happens when the interval is not parallel to one of the axes?

The general case

Let $P(x_1,\ y_1)$ and $Q(x_2,\ y_2)$ be two points and let $M(x,\ y)$ be the midpoint of the interval PQ.

The triangles PMS and MQT are congruent (AAS), so $PS = MT$ and $MS = QT$.

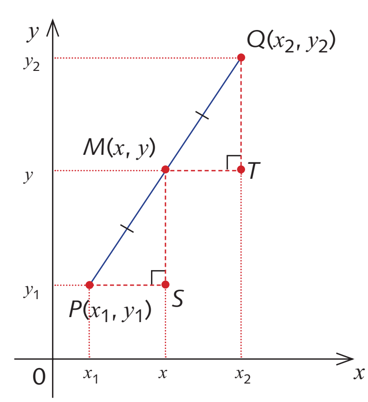
Hence the x-coordinate of M is the average of the x-value of the two points, P and Q.

$$x = \frac{x_1 + x_2}{2}$$

And the y-coordinate of M is the average of the y-values of the two points, P and Q.

$$y = \frac{y_1 + y_2}{2}$$

This provides a formula for the midpoint of any interval.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Midpoint of an interval

The midpoint of the interval with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The coordinates are found by calculating the average of x_1 and x_2 and the average of y_1 and y_2.

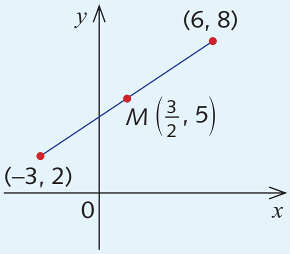
Example 5

Find the coordinates of the midpoint of the interval joining the points (6, 8) and (–3, 2).

Solution

The midpoint M has coordinates

$$\left(\frac{6 + (-3)}{2}, \frac{8 + 2}{2}\right) = \left(\frac{3}{2}, 5\right)$$



Finding an endpoint given the midpoint and the other endpoint

Example 6

If $M(3, 6)$ is the midpoint of the interval AB and A has coordinates (–1, 1), find the coordinates of B.

Solution

The coordinates of A are (–1, 1).

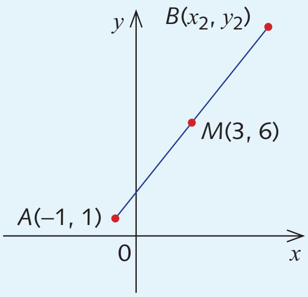
Let the coordinates of B be (x_2, y_2).

$$\frac{x_2 + (-1)}{2} = 3 \quad \text{and} \quad \frac{y_2 + 1}{2} = 6$$

$$x_2 - 1 = 6 \quad \text{and} \quad y_2 + 1 = 12$$

$$x_2 = 7 \quad \text{and} \quad y_2 = 11$$

Thus B has coordinates (7, 11).



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 11B

Example 4, 5

1 Find the coordinates of the midpoint of the interval AB with endpoints:

a (2, 3) and (8, 3)
b (5, 7) and (−1, 7)
c (−1, 0) and (4, 12)
d (−2, −7) and (6, 8)
e (−1, −3) and (1, −1)
f (−3, 3) and (3, 0)
g (2, −3) and (6, −5)
h (−6, −3) and (0, −1)

Example 6

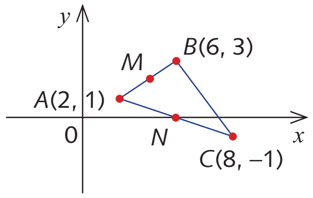
2 The point M is the midpoint of the interval AB. Find the coordinates of B given that the coordinates of A and M are:

a $A(1, 6)$ and $M(10, 6)$
b $A(1, 6)$ and $M(-10, 6)$
c $A(-1, 4)$ and $M(-3, 2)$
d $A(2, -6)$ and $M(1, 8)$
e $A(3, -1)$ and $M(-2, 10)$
f $A(5, -6)$ and $M(-2, -4)$

3 In each of the following, M is the midpoint of the interval AB. Fill in the missing entries.

	A	*B*	*M*
a	(1, 6)	(5, 10)	
b		(3, 6)	(9, 12)
c	(−3, −4)		(1, −1)
d	(2.3, 6.1)	(8.5, 3.2)	
e		(−1.6, −2.4)	(0.4, −1.9)
f	(−3.6, 4)		(0, 3)

4 Consider the three points $A(2, 1), B(6, 3)$ and $C(8, -1)$. Let M and N be the midpoints of AB and AC respectively.



a Find the coordinates of M.
b Find the coordinates of N.
c Calculate the distance BC.
d Calculate the distance MN.
e Compare the distance BC with the distance MN.
f What is the relation between ΔABC and ΔAMN?

5 The parallelogram $PQRS$ has vertices $P(1, 6)$, $Q(5, 12)$, $R(3, 3)$ and $S(-1, -3)$.

a Find the coordinates of the midpoint of the diagonal PR.
b Find the coordinates of the midpoint of the diagonal SQ.
c What well-known property of a parallelogram does this demonstrate?

6 Use the midpoint formula to find the coordinates of three more points that lie on the line passing through the points (0, 0) and (2, 3).

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 The triangle ABC has vertices $A(-2,\ 1)$, $B(-1,\ 3)$ and $C(7,\ -1)$.

a Find the coordinates of M, the midpoint of AC.

b Find the distance BM.

c Show that M is equidistant from A, B and C.

8 The triangle ABC has vertices $A(2,\ 1)$, $B(4,\ 5)$ and $C(6,\ 1)$.

a Find the coordinates of the midpoints M and N of sides AB and BC respectively.

b $P(4,\ 1)$ is the midpoint of side AC. Show that $BC = 2MP$ and $AB = 2PN$.

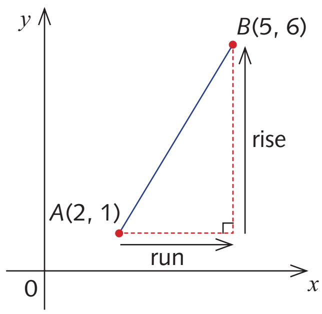
9 The triangle ABC has vertices $A(1,\ 6)$, $B(1,\ 10)$ and $C(a,\ 8)$. The area of ΔABC is 16. Find the possible values of a.

10 The quadrilateral $ABCD$ has vertices $A(1, 0)$, $B(4, 8)$, $C(11, 12)$ and $D(-1, 10)$. M, N, O and P are the midpoints of the sides AB, BC, CD and DA respectively. Find the lengths of the sides of quadrilateral $MNOP$ and describe this quadrilateral.

11C The gradient of a line

Gradient of an interval

The **gradient** of an interval AB is defined as $\frac{\text{rise}}{\text{run}}$, where the **rise** is the change in the y values as you move from A to B and the **run** is the change in the x values as you move from A to B.

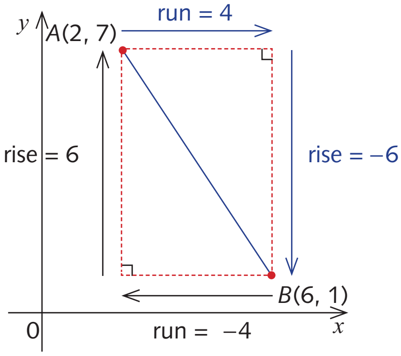


For the points $A(2,\ 1)$ and $B(5,\ 6)$,

$$\begin{aligned}\text{gradient of interval } AB &= \frac{\text{rise}}{\text{run}}\\ &= \frac{6-1}{5-2}\\ &= \frac{5}{3}\end{aligned}$$

Notice that as you move from A to B along the interval, the y value increases as the x value increases. This means the gradient is **positive**. In the diagram below,

$$\begin{aligned}\text{gradient of interval } AB &= \frac{\text{rise}}{\text{run}}\\ &= \frac{1-7}{6-2}\\ &= \frac{-6}{4}\\ &= -\frac{3}{2}\end{aligned}$$



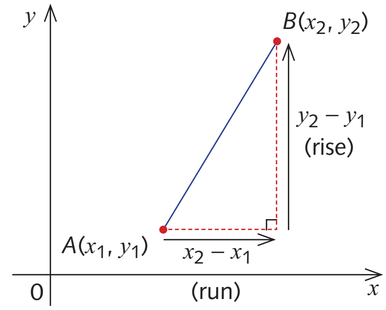
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Notice that in this case the y value decreases as the x value increases. This means the gradient is **negative**.

Notice also that the gradient of AB = the gradient of BA $\left(\frac{\text{rise}}{\text{run}} = \frac{6}{-4} = -\frac{3}{2}\text{ for the gradient of }BA\right)$.

In general, provided $x_2 \neq x_1$,

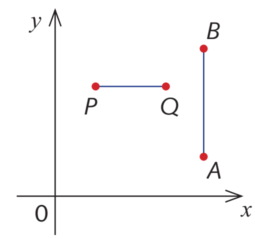
$$\text{gradient of interval } AB = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Since $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$, it does not matter which point we take as the first and which point we take as the second.

If the rise is zero, the interval is horizontal, as shown by the interval PQ. The gradient of the interval is zero.

If the run is zero, the interval is vertical, as shown by the interval AB. This interval does not have a gradient.



The gradient of PQ is zero.
The gradient of AB is not defined.

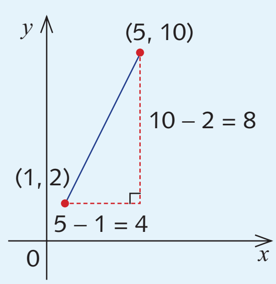
Example 7

Find the gradient of the interval joining the points (1, 2) and (5, 10).

Solution

For the points (1, 2) and (5, 10),

$$\begin{aligned}\text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{10 - 2}{5 - 1}\\ &= \frac{8}{4}\\ &= 2\end{aligned}$$



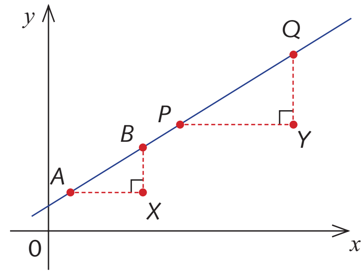
Gradient of a line

The **gradient of a line** is defined as the gradient of any interval within the line.

For this to make sense, we must show that any two intervals on a line have the same gradient.

Suppose that AB and PQ are two intervals on the same straight line. Draw right-angled triangles ABX and PQY, with sides AX and PY parallel to the x-axis and sides BX and QY parallel to the y-axis.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7



Triangle ABX is similar to triangle PQY since the corresponding angles are equal. Therefore,

$$\frac{QY}{PY} = \frac{BX}{AX} \qquad \text{(ratios of sides in similar triangles)}$$

That is, the intervals have the same gradient.

Example 8

A line passes through the point (5, 7) and has gradient $\frac{2}{3}$. Find:

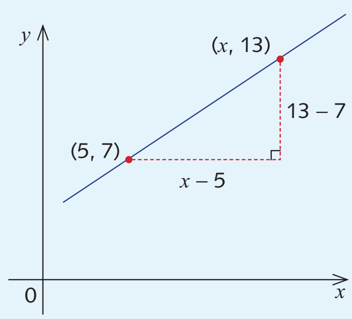
a the x-coordinate of the point on the line with y-coordinate 13

b the y-coordinate of the point on the line with x-coordinate -1

Solution

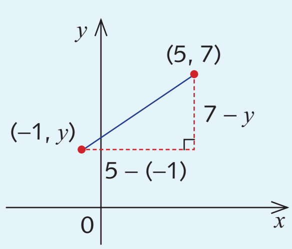
a

$$\begin{aligned} \frac{13-7}{x-5} &= \frac{2}{5} \\ \frac{6}{x-5} &= \frac{2}{3} \\ 18 &= 2(x-5) \\ 9 &= x-5 \\ x &= 14 \end{aligned}$$



b

$$\begin{aligned} \frac{7-y}{5-(-1)} &= \frac{2}{3} \\ 3(7-y) &= 12 \\ 7-y &= 4 \\ y &= 3 \end{aligned}$$



The ***x*-intercept** of a line is the x value of the point at which the line cuts the x-axis.

The ***y*-intercept** of a line is the y value of the point at which the line cuts the y-axis.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 9

a Find the gradient of the line with:

i x-intercept 3 and y-intercept 6

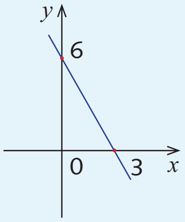
ii x-intercept -4 and y-intercept 8

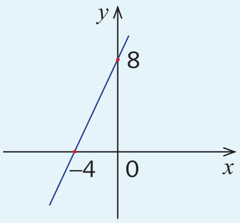
b Find the y-intercept of the line with gradient -2 and x-intercept -4.

Solution

a i Since (0, 6) and (3, 0) lie on the line,

$$\text{gradient} = \frac{6-0}{0-3} = -2$$





ii Since (0, 8) and (−4, 0) lie on the line,

$$\text{gradient} = \frac{8-0}{0-(-4)} = 2$$

b Gradient is −2,

so $\dfrac{\text{rise}}{\text{run}} = -2$

Let the y-intercept be b

$$\frac{b-0}{0-(-4)} = -2$$

So $b = -8$

Gradient of a line

- The **gradient of an interval** AB joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$\frac{y_2-y_1}{x_2-x_1}, \qquad \text{provided that } x_2 \neq x_1$$

- The **gradient of a line** is defined as the gradient of any interval within the line.
- A vertical line does not have a gradient.
- A horizontal line has gradient zero.

Exercise 11C

Example 7

1 Find the gradient of each interval AB.

a $A(6, 3), B(2, 0)$

b $A(-2, 6), B(0, 10)$

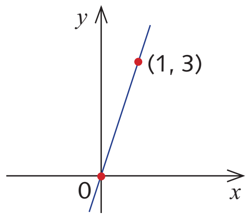
c $A(-1, 10), B(6, -4)$

d $A(2, 3), B(-4, 5)$

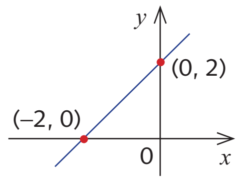
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

2 Find the gradient of each line.

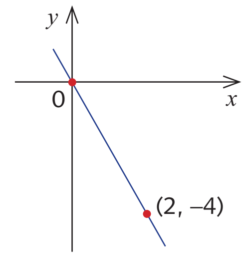
a



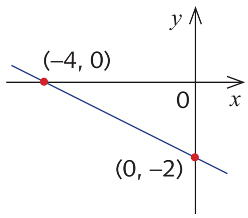
b



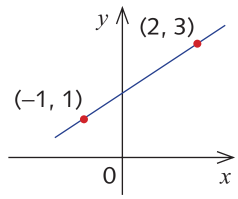
c



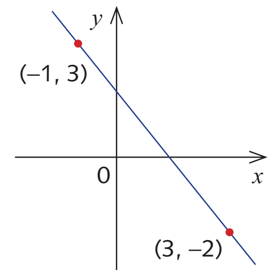
d



e



f



Example 7

3 Find the gradient of the line passing through each pair of points.

a $(0, 0)$ and $(6, -2)$ **b** $(0,\ 0)$ and $(-2,\ -3)$

c $(-2, -5)$ and $(-4,\ -1)$ **d** $(-2,\ 9)$ and $(3,\ -1)$

Example 8

4 **a** A line passes through the point $(4,\ 8)$ and has gradient $\frac{1}{2}$. Find the y-coordinate of the point on the line when $x = 8$.

b A line passes through the point $(-1,\ 6)$ and has gradient -1. Find the y-coordinate of the point on the line when $x = 4$.

5 A line passes through the point $(1,\ 3)$ and has gradient 3. Find:

a the x-coordinate of the point on the line when $y = 12$

b the y-coordinate of the point on the line when $x = 3$

c the x-coordinate of the point on the line when $y = 0$

d the y-coordinate of the point on the line when $x = -2$

6 **a** A line passes through $(0, 0)$ and has gradient 2. Copy and complete the table of values.

x	–2		1	
y		–2		6

b A line passes through the point $(2, 1)$ and has gradient $\frac{3}{4}$. Copy and complete the following table of values.

x		–2	2	6	
y	–5		1		10

c A line passes through the point $(2, -6)$ and has gradient -2. Copy and complete the following table of values.

x		–4		0	2
y	10		0		–6

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 9

7 Copy and complete the following table. (Each part refers to a straight line. You may need to draw a diagram for each part.)

	x-intercept	y-intercept	Gradient
a	-1	2	
b	-2		3
c		4	$\frac{1}{2}$
d		-2	$\frac{2}{3}$
e	-5		-1
f		8	-2
g	10	5	

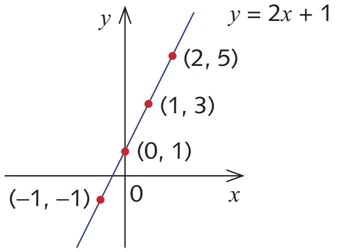
8 A line passes through the point $(3,\ 6)$ and has gradient 2. Find where the line crosses the x-axis and the y-axis.

9 A line passes through the point $(2,\ 6)$ and crosses the y-axis at the point $(0, 4)$. At what point does it cross the x-axis?

10 A line passes through $(0, b)$ and $(a, 0)$. Find the gradient of the line.

11D The equation of a straight line

When we plot points that satisfy the equation $y = 2x + 1$, we find that they lie in a straight line. When we deal with lines in coordinate geomentry, we will be dealing with their equations.

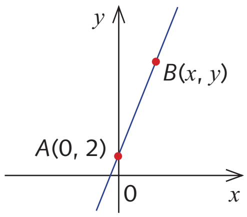


Can we find the equation of the line given suitable geometric information about the line? The following shows that this can be done given the gradient of the line and the y-intercept.

The line $y = 3x + 2$

Consider the line with gradient 3 and y-intercept 2. This passes through the point $A(0,\ 2)$. Let $B(x, y)$ be any point on this line.

$$\begin{aligned}\text{Gradient of interval } AB &= \frac{\text{rise}}{\text{run}}\\ &= \frac{y-2}{x-0}\\ &= \frac{y-2}{x}\end{aligned}$$



We know the gradient of the line is 3.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Therefore
$$\begin{aligned}\frac{y-2}{x} &= 3\\ y-2 &= 3x\\ y &= 3x+2\end{aligned}$$

So the coordinates (x, y) of B satisfy the equation $y = 3x + 2$.
Conversely, suppose that the point $B(x, y)$ in the plane satisfies $y = 3x + 2$. Then

$$\begin{aligned}y-2 &= 3x\\ \frac{y-2}{x-0} &= 3\end{aligned}$$

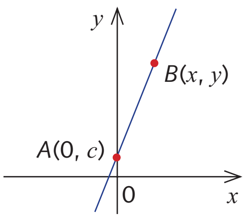
Thus we have shown that the interval joining (x, y) with $(0,\ 2)$ has gradient 3.

So B lies on the line with gradient 3 and y-intercept 2.

We summarise this by saying that the **equation of the line** is $y = 3x + 2$.

The line $y = mx + c$

Consider the line with gradient m and y-intercept c. It passes through the point $A(0, c)$. Let $B(x, y)$ be any point on this line.



$$\begin{aligned}\text{Gradient of interval } AB &= \frac{y-c}{x-0}\\ &= \frac{y-c}{x}\end{aligned}$$

We know the gradient of the line is m.

Therefore
$$\begin{aligned}\frac{y-c}{x} &= m\\ y-c &= mx\\ y &= mx+c\end{aligned}$$

That is, the line in the Cartesian plane with gradient m and y-intercept c has equation $y = mx + c$.
Conversely, the points whose coordinates satisfy the equation $y = mx + c$ always lie on the line with gradient m and y-intercept c.

Example 10

The gradient of a line is -6 and the y-intercept is 2. Find the equation of the line.

Solution

The equation of a straight line can be written as $y = mx + c$.

Thus the equation of the line is $y = -6x + 2$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 11

Write down the gradient and y-intercept of the line with equation $y = 3x - 4$.

Solution

The gradient of the line is 3 and the y-intercept is –4.

Horizontal lines

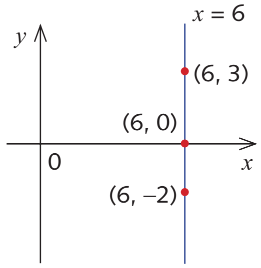
All points on a horizontal line have the same y-coordinate, but the x-coordinate can take any value. In general, the equation of the horizontal line through $P(a, b)$ is $y = b$.

Vertical lines

All points on a vertical line have the same x-coordinate, but the y-coordinate can take any value. For example, the equation of the vertical line through the point $(6,\ 0)$ is $x = 6$.

In general, the equation of the vertical line through $P(a, b)$ is $x = a$.

Because a vertical line does not have a gradient, its equation does not fit the form $y = mx + c$.



Checking whether a point lies on the graph

We check whether the coordinates of the point satisfy the equation of the line.

Example 12

Check whether or not each of the following points lie on the line with equation $y = 2x + 3$.

a $(3,\ 9)$ **b** $(-2,\ 7)$ **c** $(4,\ 11)$

Solution

a Substitute $(3, 9)$ into the equation.

LHS = 9, RHS = 2(3) + 3 = 9

The point (3, 9) lies on the line $y = 2x + 3$.

b Substitute (–2, 7) into the equation.

LHS = 7, RHS = $2(-2) + 3 = -1$

The point (–2, 7) does not lie on the line $y = 2x + 3$.

c Substitute (4, 11) into the equation.

LHS = 1, RHS = $2(4) + 3 = 11$

The point (4, 11) lies on the line $y = 2x + 3$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Finding the coordinates of a point on a line by substitution

We can find the unknown coordinate of a point on a line by substituting into the equation of the line.

Example 13

a Find the y-coordinate for the point on the line $y = 6x - 7$ with the x-coordinate:

i -1 **ii** 0 **iii** 20

b Find the x-coordinate for the point on the line $y = 6x - 7$ with the y-coordinate:

i 11 **ii** 0 **iii** 23

Solution

a i For $x = -1$,

$$\begin{aligned} y &= 6 \times (-1) - 7 \\ &= -6 - 7 \\ &= -13 \end{aligned}$$

The y-coordinate is -13.

ii For $x = 0$,

$$\begin{aligned} y &= 6 \times 0 - 7 \\ &= -7 \end{aligned}$$

The y-coordinate is -7.

iii For $x = 20$,

$$\begin{aligned} y &= 6 \times 20 - 7 \\ &= 113 \end{aligned}$$

The y-coordinate is 113.

b i For $y = 11$,

$$\begin{aligned} 6x - 7 &= 11 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

The x-coordinate is 3.

ii For $y = 0$,

$$\begin{aligned} 6x - 7 &= 0 \\ 6x &= 7 \\ x &= \frac{7}{6} \end{aligned}$$

The x-coordinate is $\frac{7}{6}$.

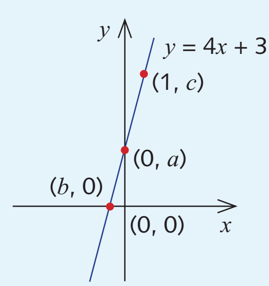
iii For $y = 23$,

$$\begin{aligned} 6x - 7 &= 23 \\ 6x &= 30 \\ x &= 5 \end{aligned}$$

The x-coordinate is 5.

Example 14

The graph of $y = 4x + 3$ is shown opposite. Find the values of a, b and c.



Solution

When the x-coordinate is 1, the y-coordinate is c.

$$\begin{aligned} c &= 4 \times 1 + 3 \\ &= 7 \end{aligned}$$

When the x-coordinate is 0, the y-coordinate is a.

$$\begin{aligned} a &= 4 \times 0 + 3 \\ &= 3 \end{aligned}$$

(*continued over page*)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

When the y-coordinate is 0, the x-coordinate is b.

$$4b + 3 = 0$$
$$4b = -3$$
$$b = -\frac{3}{4}$$

That is, $a = 3$, $b = -\frac{3}{4}$ and $c = 7$

Equation of a straight line

- The equation of the vertical line through $P(a, b)$ is $x = a$.
- Every non-vertical line in the Cartesian plane has equation $y = mx + c$, where m is the gradient of the line and c is the y-intercept.
- Conversely, the points whose coordinates satisfy the equation $y = mx + c$ all lie on the line with gradient m and y-intercept c.

Example 15

Rewrite each equation in the form $y = mx + c$. Hence find the value of the gradient and y-intercept of the line.

a $2x + 3y = 6$

b $-2x + 8y = 15$

Solution

a We rearrange the equation to make y the subject.

$$2x + 3y = 6$$

so $$3y = 6 - 2x$$
$$y = 2 - \frac{2x}{3}$$
$$y = -\frac{2x}{3} + 2$$

The gradient of the line is $-\frac{2}{3}$ and the y-intercept is 2.

b $-2x + 8y = 15$

$$8y = 15 + 2x$$
$$y = \frac{2x}{8} + \frac{15}{8}$$
$$y = \frac{x}{4} + \frac{15}{8}$$

The gradient of the line is $\frac{1}{4}$ and the y-intercept is $\frac{15}{8}$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Exercise 11D

Example 10

1 Write down the equation of the line that has:

a gradient 2 and y-intercept 3 **b** gradient 3 and y-intercept 4

c gradient -2 and y-intercept 1 **d** gradient -1 and y-intercept 3

e gradient $\frac{2}{3}$ and y-intercept 1 **f** gradient $-\frac{3}{4}$ and y-intercept 0

Example 11

2 Write down the gradient and y-intercept of each line. Draw a graph of each line by plotting two points.

a $y = 2x + 1$ **b** $y = 3x + 4$ **c** $y = -2x + 5$

d $y = -2x - 6$ **e** $y = \frac{2}{3}x + 1$ **f** $y = -2x$

g $y = -4x$ **h** $y = 1 - 3x$ **i** $y = 2 - 5x$

Example 12

3 Check whether or not each of these points lies on the line with equation $y = -2x + 3$.

a $(3, 9)$ **b** $(-2, 7)$ **c** $(-1, 5)$ **d** $(4, -5)$

4 Check whether or not each of these points lies on the line with equation $y = -6x$.

a $(0, 0)$ **b** $(1, 6)$ **c** $(-1, 6)$ **d** $(4, -10)$

Example 13a

5 Find the y-coordinate of the point on the line $y = 3x - 4$ with x-coordinate:

a 2 **b** 0 **c** -2

6 Find the y-coordinate of the point on the line $y = -3x + 4$ with x-coordinate:

a 5 **b** -2 **c** 0

Example 13b

7 Find the x-coordinate of the point on the line $y = 2x + 6$ with y-coordinate:

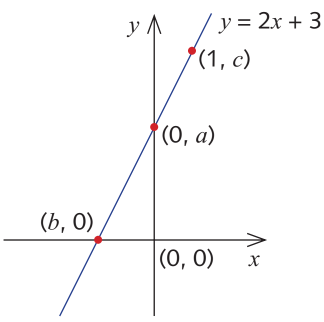
a 10 **b** 0 **c** -4

8 Find the x-coordinate of the point on the line $y = -2x - 8$ with y-coordinate:

a 10 **b** 0 **c** -3

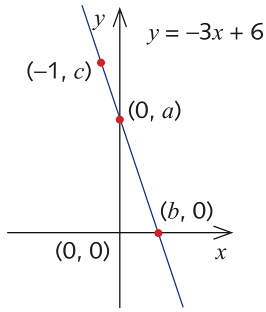
Example 14

9 The graph of $y = 2x + 3$ is shown below. Find the values of a, b and c.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 The graph of $y = -3x + 6$ is shown below. Find the values of a, b and c.



Example 15

11 Rewrite in the form $y = mx + c$ and then write down the gradient and y-intercept.

a $2x + y = 10$ **b** $10x + 2y = 4$ **c** $3x - 2y = 6$

d $4x - 3y = 12$ **e** $5y - 2x = 9$ **f** $3x - 4y = 6$

g $x = 2y - 4$ **h** $x = 3y + 1$ **i** $x = -2y$

j $x = -4y$ **k** $y + 3x = 0$ **l** $x - 2y = 0$

12 **a** Express the equation $ax + by = d$, where a, b and d are constants ($b \neq 0$), in the form $y = mx + c$.

b Write down the gradient and y-intercept of the line whose equation is $ax + by = d$.

13 Show that the line passing through $(a, 0)$ and $(0, b)$ has equation $\frac{x}{a} + \frac{y}{b} = 1$.

11E Graphing straight lines

Two-point method

Two points determine a straight line. To draw a line we use the equation to find the coordinates of two points on the line.

Example 16

Draw the graph of

a $y = 2x + 3$ **b** $y = 3x$

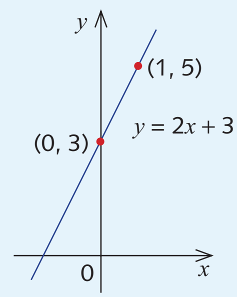
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a Substitute $x = 0$, so $y = 2 \times 0 + 3$
$= 3$

Substitute $x = 1$, so $y = 2 \times 1 + 3$
$= 5$

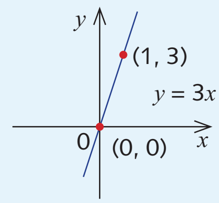
Two points on the line are (0, 3) and (1, 5).



b When $x = 0$, $y = 0$

When $x = 1$, $y = 3$

Two points on the line are (0, 0) and (1, 3).



Two-intercept method

The points where the line crosses the axes are usually of particular interest. The ***x*-intercept** is found by substituting $y = 0$ and the ***y*-intercept** is found by substituting $x = 0$.

This method does not work if the line is parallel to an axis, or passes through the origin, since such lines have only one intercept.

Example 17

Using the two-intercept method, sketch the graph of:

a $y = 3x - 4$ **b** $2x + 3y + 6 = 0$

Solution

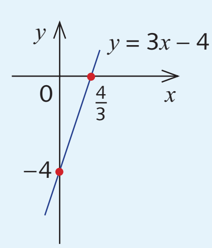
a When $x = 0, y = -4$

When $y = 0$, $3x - 4 = 0$

$$3x = 4$$

$$x = \frac{4}{3}$$

The two points $(0, -4)$ and $\left(\frac{4}{3}, 0\right)$ lie on the line.



b When $x = 0$, $3y + 6 = 0$

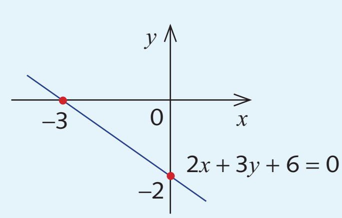
$$3y = -6$$

$$y = -2$$

When $y = 0$, $2x + 6 = 0$

$$2x = -6$$

$$x = -3$$

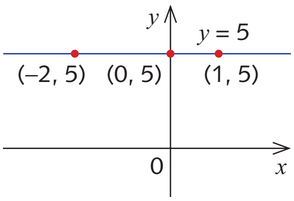


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Horizontal lines

On a horizontal line, all points have the same y-coordinate, but the x-coordinate can take any value. For example, the equation of the horizontal line through the point $(0, 5)$ is $y = 5$.

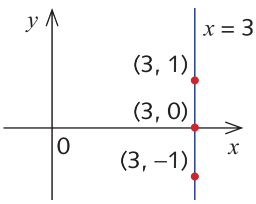
The equation of the horizontal line through the point (a, b) is $y = b$.



Vertical lines

Recall that, on a vertical line, all points have the same x-coordinate, but the y-coordinate can take any value. For example, the equation of the vertical line through the point $(3, 0)$ is $x = 3$.

The equation of the vertical line through the point (a, b) is $x = a$.



Graphing straight lines

- Two points determine a straight line. To draw a line we use the equation of the line to find the coordinates of two points on the line.
- If the line is not parallel to one of the axes and does not pass through the origin, the x-intercept and y-intercept can be found and the graph drawn.
- The equation of the horizontal line through the point (a, b) is $y = b$.
- The equation of the vertical line through the point (a, b) is $x = a$.

Exercise 11E

Example 16a

1 Sketch the graph of each line by calculating the y-intercept and finding one other point.

a $y = 2x + 1$ **b** $y = 3x + 2$ **c** $y = 3x - 2$

d $y = 4x + 5$ **e** $y = 4 - 2x$ **f** $y = 3 - x$

g $y = \frac{1}{2}x + 1$ **h** $y = \frac{2}{3}x + 2$ **i** $y = -2x + 3$

Example 16b

2 Sketch the graph of:

a $y = 4x$ **b** $y = 2x$ **c** $y = \frac{1}{2}x$

d $y = -2x$ **e** $y = -x$ **f** $x + 2y = 0$

g $2y - 3x = 0$ **h** $3x + y = 0$ **i** $\frac{x}{3} - \frac{y}{2} = 0$

Example 17

3 Sketch the graph of each line by calculating the coordinates of the x- and y-intercepts.

a $2x + y = 4$ **b** $x + 3y = 6$ **c** $2x + 3y = 6$

d $3x + y = 2$ **e** $x - y = 4$ **f** $3x - y = 3$

g $x - 2y = 4$ **h** $3y - x = 4$ **i** $2y - 3x = 4$

j $\frac{x}{2} + \frac{y}{3} = 1$ **k** $\frac{x}{3} - \frac{y}{4} = 1$ **l** $\frac{2x}{3} - \frac{3y}{4} = 2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Sketch the graph of:

a $y = 3$ **b** $x = -1$ **c** $x + 2 = 0$

d $y - 5 = 0$ **e** $4 - y = 0$ **f** $7 + x = 0$

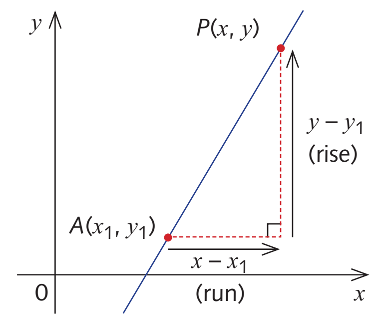
11F The equation of a line using the gradient and a point

Let $A(x_1, y_1)$ be a given point. Consider the line through A with gradient m. Let $P(x, y)$ be a general point on the line $x \neq x_1$.

Then $$m = \frac{y - y_1}{x - x_1}$$

and $$y - y_1 = m(x - x_1)$$

This is the equation of the line that passes through the point $A(x_1, y_1)$ and has gradient m.



Example 18

Find the equation of the line that passes through the point $(-2, 3)$ and has gradient -4.

Solution

We use $y - y_1 = m(x - x_1)$.

The equation of this line is

$$\begin{aligned} y - 3 &= -4(x - (-2)) \\ y - 3 &= -4x - 8 \\ y &= -4x - 5 \end{aligned}$$

The equation of a line through two given points

The equation of a line can be found if the coordinates of two points on the line are known by first calculating the gradient and then using the method given above.

Example 19

Find the equation of the line that passes through the point $A(1, 3)$ and $B(4, 8)$.

Solution

$$\begin{aligned} \text{Gradient of interval } AB &= \frac{8-3}{4-1} \\ &= \frac{5}{3} \end{aligned}$$

(*continued over page*)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Now, using $y - y_1 = m(x - x_1)$, where $(x_1, y_1) = (1,\ 3)$ and $m = \dfrac{5}{3}$:

$$y - 3 = \frac{5}{3}(x - 1)$$

$$y - 3 = \frac{5}{3}x - \frac{5}{3}$$

$$y = \frac{5}{3}x + \frac{4}{3}$$

Note: The point $B(4,\ 8)$ could be used instead of $A(1,\ 3)$.

Equation of a line using the gradient and a point

- The equation of the line that passes through a point $A(x_1, y_1)$ and has gradient m is
$$y - y_1 = m(x - x_1)$$
- The equation of the line is determined if the coordinates of two points on the line are known. For the points $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_2 \neq x_1$
  - find the gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
  - the equation is then $y - y_1 = m(x - x_1)$.
- If $x_1 = x_2$ then the equation is $x = x_1$.

Exercise 11F

Example 18

1 Find the equation of the line that passes through:

a the point $(1,\ 3)$ and has gradient 1

b the point $(2,\ 4)$ and has gradient 3

c the point $(-1,\ 1)$ and has gradient 4

d the point $(2, -2)$ and has gradient $\dfrac{2}{3}$

e the point $(-1,\ 0)$ and has gradient $-\dfrac{1}{3}$

f the point $(-1, -4)$ and has gradient $-\dfrac{2}{5}$

2 The line ℓ has equation $y = 3x + 5$.

a What is the y-intercept of ℓ?

b What is the gradient of ℓ?

c Find the equation of the line that has the same y-intercept as ℓ but has a gradient of $\dfrac{1}{2}$.

d Find the equation of the line that has the same gradient as ℓ but has a y-intercept of -2.

3 A line passes through the points $A(1, 3)$ and $B(4, 12)$.

a Find the gradient of AB.

b Using the gradient and the point A, find the equation of line AB.

c Using the gradient and the point B, find the equation of line AB.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 19

4 Find the equation of the line through each pair of points A and B.

a $A(5, 3)$ and $B(2, -1)$

b $A(4, 1)$ and $B(6, 7)$

c $A(1, 2)$ and $B(2, 4)$

d $A(-1, 6)$ and $B(2, -3)$

e $A(-2, 4)$ and $B(1, -6)$

f $A(-1, -2)$ and $B(3, 4)$

In the following questions, draw a sketch showing all relevant points and lines.

5 Consider the interval AB with endpoints $A(-1, -4)$ and $B(3, 8)$.

a Find the gradient of AB.

b Find the coordinates of C, the midpoint of interval AB.

c Find the equation of the line that passes through C with gradient $-\frac{1}{2}$.

d What are the coordinates of the point D, where the line intersects the y-axis?

e How far is the point C from the point D?

6 The quadrilateral $ABCD$ has vertices $A(1, 2)$, $B(5, 6)$, $C(8, 0)$ and $D(6, -2)$.

a Find the coordinates of M, the midpoint of AB.

b Find the coordinates of N, the midpoint of CD.

c Find the gradient of MN.

d Find the equation of the line MN.

e Find the coordinates of P, the midpoint of AC.

f Find the equation of the line that passes through P with gradient 3.

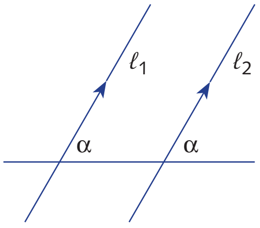
11G Parallel and perpendicular straight lines

Parallel lines

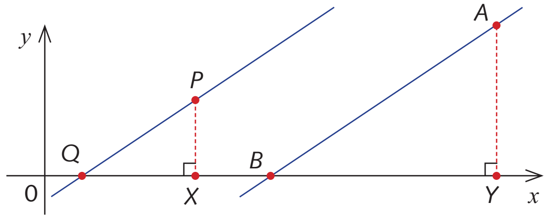
If two lines ℓ_1 and ℓ_2 are parallel, then the corresponding angles are equal.

Conversely, if the corresponding angles are equal, then the lines are parallel.

We are now going to show that two lines are parallel if they have the same gradient and, conversely, if they have the same gradient, they are parallel.



In the diagram below, the line through P meets the x-axis at Q. The line through A meets the x-axis at B. XP and AY are perpendicular to the x-axis.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Proof that lines are parallel implies equal gradients

If the lines are parallel, $\angle PQX = \angle ABY$ (corresponding angles).

Triangles QPX and BAY are similar by the AAA test.

Therefore $\dfrac{PX}{QX} = \dfrac{AY}{BY}$ (matching sides in similar triangles).

So, the gradients are equal.

Proof that equal gradients implies lines are parallel

If the gradients are equal, $\dfrac{PX}{QX} = \dfrac{AY}{BY}$

Thus $\dfrac{PX}{AY} = \dfrac{QX}{BY}$

So the triangles are similar by the SAS test.

Hence the corresponding angles PQX and ABY are equal and the lines are parallel.

Example 20

Show that the line passing through the points $A(6,\ 4)$ and $B(7,\ 11)$ is parallel to the line passing through $P(0,\ 0)$ and Q(1, 7)

Solution

$$\begin{aligned}\text{Gradient of } AB &= \frac{11-4}{7-6}\\ &= 7\\ \text{Gradient of } PQ &= \frac{7-0}{1-0}\\ &= 7\end{aligned}$$

The two lines have the same gradient and so they are parallel.

Example 21

Find the equation of the line that is parallel to the line with equation $y = -2x + 6$ and that passes through the point $A(1,\ 10)$.

Solution

The gradient of the line $y = -2x + 6$ is -2.

Therefore the line through the point $A(1,\ 10)$ parallel to $y = -2x + 6$ also has gradient -2 and hence the equation is

$$\begin{aligned} y - y_1 &= m(x - x_1)\\ y - 10 &= -2(x - 1)\\ y - 10 &= -2x + 2\\ y &= -2x + 12\end{aligned}$$

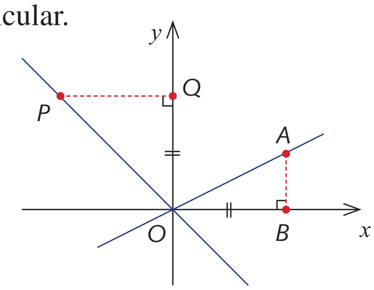
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Perpendicular lines

We are now going to show that if two lines are **perpendicular**, then the product of their gradients is -1 (or one is vertical and the other horizontal). The converse is also true. That is, if the product of the gradients of two lines is -1, then they are perpendicular.

We first consider the case where both lines pass through the origin.

Draw two lines passing through the origin, with one of the lines having positive gradient and the other negative gradient.



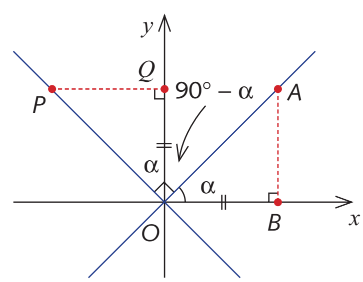
Form right-angled triangles OPQ and OAB with $OQ = OB$.

Gradient of the line $OA = \dfrac{AB}{BO}$

Gradient of the line $OP = -\dfrac{OQ}{PQ}$

$$\begin{aligned}\text{Product of the gradient} &= -\frac{OQ}{PQ} \times \frac{AB}{BO}\\ &= -\frac{OQ}{PQ} \times \frac{AB}{OQ} \quad (\text{since } OQ = OB)\\ &= -\frac{AB}{PQ}\end{aligned}$$

Proof that if two lines are perpendicular, then the product of their gradients is –1



If the lines are perpendicular, $\angle POQ = \angle AOB$ because when each of these angles is added to $\angle AOQ$, the result is 90°.

Therefore triangles OPQ and OAB are congruent (AAS).

So $PQ = AB$ and the product of the gradients, $-\dfrac{AB}{PQ}$, is -1.

Proof that if the product of the gradients is –1, then the lines are perpendicular

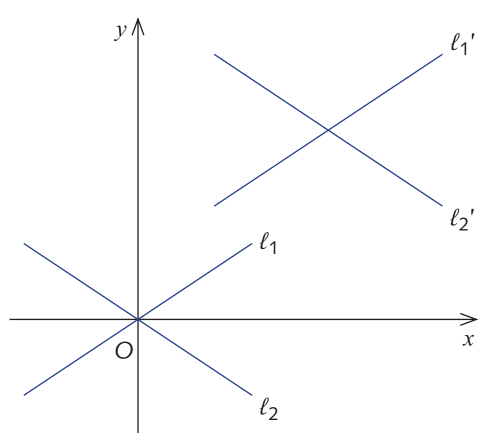
If the product is -1, $AB = PQ$ since the product of the gradients $= -\dfrac{AB}{PQ}$.

So the triangles OBA and OQP are congruent (SAS).

Therefore $\angle POQ = \angle AOB$ and $\angle AOP = 90° - \alpha + \alpha = 90°$

Hence the lines are perpendicular.

Lines that do not meet at the origin



If we are given two lines anywhere in the plane, we can draw lines through the origin parallel to the original two lines.

In the given diagram, $\ell_1 \parallel \ell_1'$ and $\ell_2 \parallel \ell_2'$. Therefore, lines ℓ_1 and ℓ_1' have the same gradient, and lines ℓ_2 and ℓ_2' have the same gradient.

Hence, any relationship between the gradients of ℓ_1 and ℓ_2 is shared by ℓ_1' and ℓ_2' and vice-versa. This means the above result established for lines through the origin holds for any pair of lines (not parallel with the axes).

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 22

Show that the line through the points $A(6,\ 0)$ and $B(0,\ 12)$ is perpendicular to the line through $P(8,\ 10)$ and $Q(4,\ 8)$.

Solution

$$\begin{aligned}\text{Gradient of } AB &= \frac{12-0}{0-6}\\ &= -2\\ \text{Gradient of } PQ &= \frac{10-8}{8-4}\\ &= \frac{2}{4}\\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}(\text{Gradient of } AB)\times(\text{gradient of } PQ) &= -2\times\frac{1}{2}\\ &= -1\end{aligned}$$

Hence the line AB is perpendicular to the line PQ, which we can write as $AB \perp PQ$.

Parallel and perpendicular lines

- Two non-vertical lines are **parallel** if they have the same gradient. Conversely, if two non-vertical lines are parallel, then they have the same gradient.
- Two lines are **perpendicular** if the product of their gradients is –1 (or if one is vertical and the other is horizontal). Conversely, if two lines are perpendicular, then the product of their gradients is –1 (or one is vertical and the other is horizontal).

Example 23

Find the equation of the line that passes through the point (1, 3) and is perpendicular to the line whose equation is $y = 2x + 1$.

Solution

The gradient of the line $y = 2x + 1$ is 2.

Hence the gradient of a line perpendicular to this line is $-\frac{1}{2}$.

The required equation is:

$$\begin{aligned}y-3 &= -\frac{1}{2}(x-1)\\ 2(y-3) &= -(x-1)\\ 2y+x &= 1+6\\ 2y+x &= 7\end{aligned}$$

Thus the equation of the required line is $2y + x = 7$ or $y = \frac{1}{2}x + \frac{7}{2}$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 11G

1 The equations of eight lines are given below. State which lines are parallel.

a $y = 2x - 3$ **b** $3x + y = 7$ **c** $y = 4 - 2x$

d $x = \frac{1}{2}y + 1$ **e** $y = \frac{1}{3}x - 3$ **f** $2y - x = 7$

g $3y + x = 8$ **h** $y = -2x + 5$

Example 21

2 Find the equation of the line that:

a is parallel to the line $y = 2x - 3$ and passes through the point $(1, 5)$

b is parallel to the line $y = 4 - x$ and passes through the point $(-2, -1)$

c is parallel to the line $y - 3x = 4$ and passes through the point $(0, -3)$

d is parallel to the line $2y + x = 3$ and passes through the point $(4, -2)$

3 If $y = (2a - 3)x + 1$ is parallel to $y = 3x - 4$, find the value of a.

4 If $y = (3a + 2)x - 1$ is parallel to $y = ax - 4$, find the value of a.

5 In each part, lines ℓ_1 and ℓ_2 are perpendicular. In the table, the gradient of ℓ_1 is given. Find the gradient ℓ_2.

	Gradient of ℓ_1	Gradient of ℓ_2
a	$\frac{1}{2}$	
b	2	
c	$\frac{4}{3}$	
d	3	
e	$-\frac{3}{4}$	
f	-5	
g	$-\frac{1}{2}$	
h	$\frac{2}{3}$	

6 The equations of eight lines are given below. State which lines are perpendicular.

a $y = 2x - 4$ **b** $4y + 3x = 7$ **c** $y = x + 2$

d $3y - x = 5$ **e** $y = 4 - x$ **f** $2y + x = 7$

g $3y - 4x = 8$ **h** $y = -3x + 5$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 23

7 Find the equation of the line that:

a is perpendicular to the line $y = 2x - 3$ and passes through the point $(1,\ 4)$

b is perpendicular to the line $y = 4 - 2x$ and passes through the point $(-2, -1)$

c is perpendicular to the line $x + 3y = 7$ and passes through the point $(2, -3)$

d is perpendicular to the line $y - 3x = 4$ and passes through the point $(1, -3)$

8 $ABCD$ is a quadrilateral with vertices $A(1,\ 2), B(3,\ 5), C(7, -1)$ and $D(5, -5)$. Draw a diagram showing all points and lines.

a Find the coordinates of M, the midpoint of AB.

b Find the coordinates of N, the midpoint of BC.

c Calculate the gradient of MN.

d If P is the midpoint of CD and Q is the midpoint of DA, find the gradient of PQ.

e What can be concluded about the intervals MN and PQ?

f Find the gradients of QM and PN.

g What type of quadrilateral is $MNPQ$?

9 If $y = (a + 1)x + 7$ is perpendicular to $y = 2x - 4$, find the value of a.

10 If $y = (2a + 3)x + 1$ is perpendicular to $y = 2x - 4$, find the value of a.

11 Let AB be the interval with endpoints $A(2,\ 3)$ and $B(6, 11)$

a Find the gradient of AB.

b Find the coordinates of C, the midpoint of AB.

c Find the equation of the line that is perpendicular to AB and passes through C. This line is the **perpendicular bisector** of AB.

d What is the y-intercept of this perpendicular bisector?

12 Triangle ABC has vertices $A(-1, -3), B(4,\ 2)$ and $C(12, -6)$. By calculating gradients, show that ΔABC is right-angled.

13 Quadrilateral $ABCD$ has vertices $A(2, -2), B(5,\ 2), C(9, -1)$ and $D(6, -5)$. Show that the diagonals AC and BD are perpendicular.

14 The coordinates of the vertices of ΔPQR are $P(1, -2), Q(3,\ 6)$ and $R(7,0)$. Find:

a the coordinates of S, the midpoint of PQ

b the gradient of SR

c the equation of the line SR (This line is called a **median**. It passes through a vertex and the midpoint of the side opposite that vertex.)

d the equations of the other two medians of the triangle

15 In triangle ABC, the **altitude** through A is the line through A perpendicular to BC. The coordinates of the vertices of the triangle are $A(0,\ 1), B(4,\ 7)$ and $C(6, -1)$. Find the equation of the altitude through:

a C **b** B **c** A

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16 $A(5, 6)$ and $B(6, 7)$ are adjacent vertices of a square $ABCD$.

a Find the length of each side of the square.

b Find the gradient of AB.

c Find the gradient of BC.

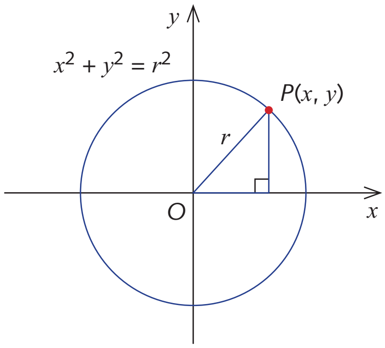
d The coordinates of C are $(7, c)$. Find the value of c.

e Find the coordinates of D.

11H The equation of a circle

Circles with centre the origin

Consider a circle in the coordinate plane with centre the origin and radius r.



If $P(x, y)$ is a point on the circle, its distance from the origin is r and, by Pythagoras' theorem, $x^2 + y^2 = r^2$.

Conversely, if a point $P(x, y)$ in the plane satisfies the equation $x^2 + y^2 = r^2$, its distance from $O(0, 0)$ is r, so it lies on a circle with centre the origin and radius r.

The circle with centre $O(0, 0)$ and radius r has equation

$$x^2 + y^2 = r^2$$

Example 24

Sketch the graph of the circle $x^2 + y^2 = 25$ and test whether the points $(3, -4)$, $(-3, 4)$, $(1, 4)$ and $(3, 2)$ lie on the circle.

Solution

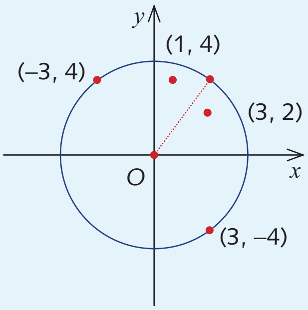
The circle has centre the origin and radius 5.

The point $(3, -4)$ lies on the circle as $3^2 + (-4)^2 = 5^2$

The point $(-3, 4)$ lies on the circle as $(-3)^2 + 4^2 = 5^2$

The point $(1, 4)$ does not lie on the circle as $1^2 + 4^2 = 17 \neq 5^2$

The point $(3, 2)$ does not lie on the circle as $3^2 + 2^2 = 13 \neq 5^2$



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

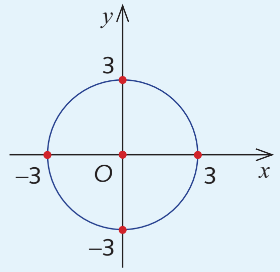
Example 25

Sketch the graphs of the circles with the following equations.

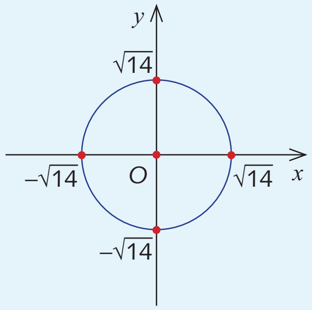
a $x^2 + y^2 = 9$ **b** $x^2 + y^2 = 14$

Solution

a $x^2 + y^2 = 3^2$ is the equation of a circle with centre the origin and radius 3.



b $x^2 + y^2 = 14$ is the equation of a circle with centre the origin and radius $\sqrt{14}$.



Exercise 11H

Example 24

1 Test whether or not the point with the given coordinates is on the circle $x^2 + y^2 = 16$.

a $(4,\ 0)$ **b** $(4,\ 4)$ **c** $(0, -4)$

d $(2\sqrt{2}, 2\sqrt{2})$ **e** $(2,\ 2)$ **f** $(2\sqrt{2}, -2\sqrt{2})$

g $(-2,\ 2)$ **h** $(2, 2\sqrt{3})$ **i** $(\sqrt{5}, \sqrt{11})$

2 Test whether or not the point with the given coordinates is on the circle $x^2 + y^2 = 75$.

a $(5\sqrt{3}, 0)$ **b** $(5, 5\sqrt{2})$ **c** $(-5\sqrt{2}, 5)$

d $(0, -5\sqrt{3})$ **e** $(6, \sqrt{39})$ **f** $(5,\ 5)$

g $(25,\ 3)$ **h** $(8, \sqrt{11})$ **i** $(-\sqrt{11}, 8)$

Example 25

3 Sketch the graph of the circle, labelling the x- and y-intercepts.

a $x^2 + y^2 = 16$ **b** $x^2 + y^2 = 3$ **c** $x^2 + y^2 = 25$

d $x^2 + y^2 = 20$ **e** $x^2 + y^2 = 10$ **f** $x^2 + y^2 = 36$

g $y^2 = 8 - x^2$ **h** $x^2 = 15 - y^2$ **i** $y^2 = 11 - x^2$

4 Write down the equation of the circle with centre the origin and radius:

a 11 **b** $\sqrt{7}$ **c** $2\sqrt{3}$ **d** $5\sqrt{3}$

5 Find the equation of the circle with centre the origin passing through the point:

a $(1,\ 1)$ **b** $(2,\ 1)$ **c** $(1,\ 7)$ **d** $(\sqrt{2}, \sqrt{2})$

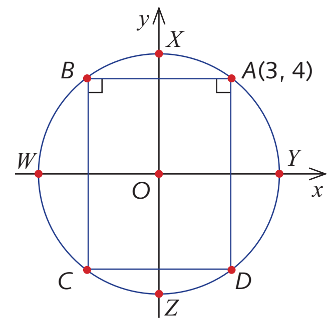
e $(-1,\ 6)$ **f** $(-2, -7)$ **g** $(-12,\ 5)$ **h** $(2\sqrt{3}, 2\sqrt{3})$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Find the equation of the circle with centre the origin and diameter:

a 6 **b** 12 **c** 7 **d** 11 **e** $\frac{3}{4}$

7 The point $A(3, 4)$ lies on the circle $x^2 + y^2 = 25$. The rectangle $ABCD$ has axes of symmetry the x- and y-axes.



a Find the coordinates of B, C and D.

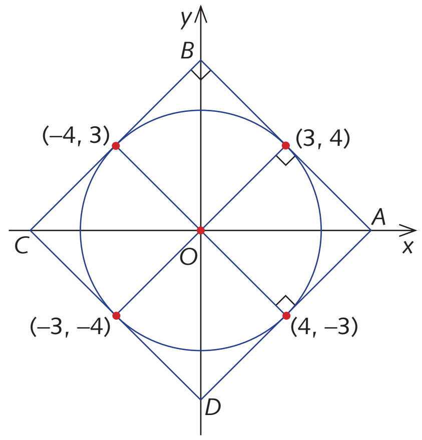
W and Y are the x-intercepts and X, Z are the y-intercepts of the circle.

b Find the distance XY.

c Find the area of the square $WXYZ$.

d Determine which has a greater area: $WXYZ$ or $ABCD$? By how much?

8 The circle to the right has equation $x^2 + y^2 = 25$.



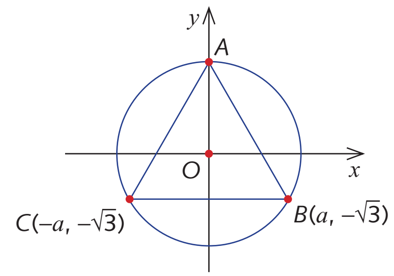
a Find the gradient AB.

b Find the equation of the line AB.

c Find the distance AB.

d Find the area of the square $ABCD$.

9 In the diagram to the right, ΔABC is inscribed in the circle $x^2 + y^2 = 12$.



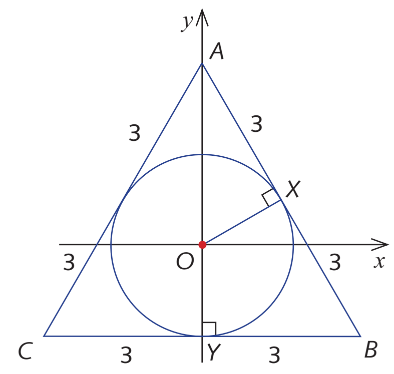
a Find the value of a.

b Find AB.

c Find BC.

d What type of triangle is ΔABC?

10 A circle is inscribed in the equilateral triangle ABC.



a Find AY.

b Give reasons why ΔAXO is similar to ΔAYB.

c Find OX and hence find the equation of the circle.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Review exercise

1 Find the distance between the two points.

a $(-3,\ 4)$ and $(-3,\ 13)$ **b** $(-5,\ 2)$ and $(1,\ 2)$

c $(-2, -6)$ and $(3,\ 6)$ **d** $(-2, -7)$ and $(13,\ 1)$

2 Find the coordinates of the midpoint of the interval AB with endpoints:

a $(-3,\ 4)$ and $(-3,\ 13)$ **b** $(-5,\ 2)$ and $(1,\ 2)$

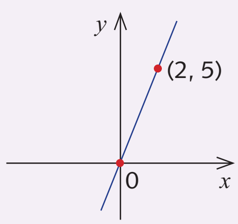
c $(-2, -6)$ and $(3,\ 6)$ **d** $(-2, -7)$ and $(13,\ 1)$

3 Find the gradient of each interval AB.

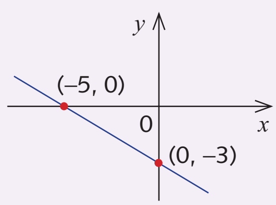
a $A(5,\ 4), B(1,\ 0)$ **b** $A(-5,\ 3), B(0,\ 12)$

4 Find the gradient of each of the following lines.

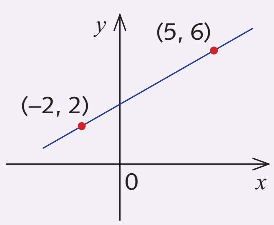
a



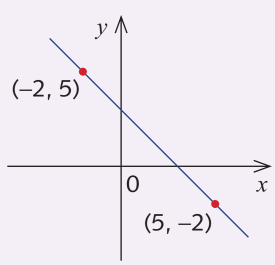
b



c



d



5 Find the gradient of the line passing through the two points.

a $(0,\ 0)$ and $(4, -3)$ **b** $(-3, -2)$ and $(-5, -6)$

6 A line passes through the point $(2,\ 3)$ and has gradient 4.

a Find the value of x for the point on the line with $y = 11$.

b Find the value of y for the point on the line with $x = 5$.

7 A line passes through the point $(4,\ 12)$ and has gradient 2. Find where the line crosses the x- and y-axes.

8 A line passes through the point $(1,\ 5)$ and crosses the y-axis at the point $(0,\ 3)$. At what point does it cross the x-axis?

9 Write down the gradient and y-intercept of each line.

a $y = 3x + 2$ **b** $y = -3x + 4$ **c** $y = \frac{1}{4}x - 7$

d $y = -\frac{2}{5}x + 6$ **e** $y = -8x$ **f** $y = 2 - 9x$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 Write down the equation of the line that has:

a gradient 3 and y-intercept 5

b gradient -1 and y-intercept 4

c gradient $\frac{3}{4}$ and y-intercept -2

d gradient $-\frac{1}{7}$ and y-intercept 0

11 Rewrite in the form $y = mx + c$ and then write down the gradient and y-intercept.

a $3x + y = 12$ **b** $9x + 4y = 6$ **c** $2x - 3y = 8$ **d** $4y - 3x = 9$

e $x = 7y - 2$ **f** $x = -9y$ **g** $y + 2x = 0$ **h** $x - 11y = 0$

12 Find the equation of the line that:

a passes through the point (2, 4) and has gradient 1

b passes through the point $(-2, 0)$ and has gradient 4

c passes through the point $(3, -1)$ and has gradient $\frac{1}{2}$

d passes through the point $(-2, -5)$ and has gradient $-\frac{2}{5}$

13 Sketch the graph of each of the following lines by using the y-intercept and finding another point.

a $y = 4x - 3$ **b** $y = 5 - 6x$ **c** $y = \frac{1}{3}x + 2$

14 Sketch the graph of each of the following lines by finding the coordinates of the x- and y-intercepts.

a $3x + y = 4$ **b** $x + 2y = 5$ **c** $3x + 4y = 6$

d $x - y = 5$ **e** $5x - y = 9$ **f** $2y - x = 8$

g $3y - 4x = 5$ **h** $\frac{x}{3} + \frac{y}{4} = 1$ **i** $\frac{2x}{5} - \frac{3y}{7} = 2$

15 Sketch the graph of:

a $y = 2$ **b** $x = -5$ **c** $x + 7 = 0$

d $y - 4 = 0$ **e** $9 - y = 0$ **f** $6 + x = 0$

16 The equations of six lines are given below. Which pairs of lines are parallel?

a $4y + x = 8$ **b** $-3x + y = 6$ **c** $y = 5 - 3x$

d $x = \frac{1}{3}y + 2$ **e** $y = 5 - \frac{1}{4}x$ **f** $y = -3x + 10$

17 Find the equation of the line parallel to the line $y = 3x - 4$ and passing through the point (2, 6).

18 The equations of six lines are given below. State which pairs of lines are perpendicular.

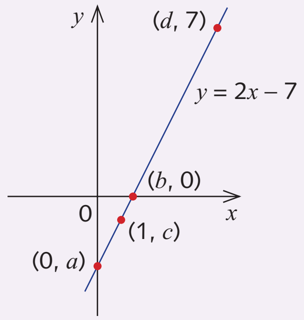
a $y = 5x - 1$ **b** $5y + 2x = 7$ **c** $5y - 2x = 8$

d $2y - x = 5$ **e** $y = -2x + 5$ **f** $5y + x = -2$

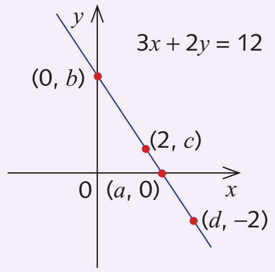
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

19 Find the equation of the line that is perpendicular to the line with equation $y = 2x - 4$ and passes through the point $(3, 8)$.

20 The graph of $y = 2x - 7$ is shown opposite.
Find the values of a, b, c and d.



21 The graph of $3x + 2y = 12$ is shown opposite.
Find the values of a, b, c and d.



22 The interval AB has endpoints $A(1, 7)$ and $B(-1, -11)$.

a Find the gradient AB.

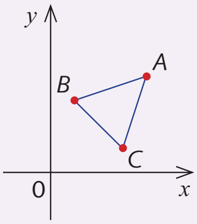
b Find the distance between points A and B.

c Find the equation of the line that passes through A and B.

d Find the coordinates of the midpoint of interval AB.

e Find the equation of the perpendicular bisector of AB.

23 ΔABC is isosceles with $AC = BC$. The coordinates of C, B and A are $(3, 1), (1, 3)$ and $(4, a)$ respectively. Find the value of a.



24 The line through the points $A(0, 7)$ and $B(11, -6)$ is parallel to the line through the points $C(6, 12)$ and $D(-11, d)$. Find the value of d.

25 Find the equation of the line with x-intercept 6 and y-intercept 11.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 For the points $A(-1, 3)$ and $B(4, 2)$, find the coordinates of the point P on the interval AB such that $AP : PB$ equals:

a $1 : 1$

b $2 : 1$ (P is closer to B.)

c $2 : 3$ (P is closer to A.)

2 Show that the points $(1,-1)$, $(-1, 1)$ and $(-\sqrt{3},-\sqrt{3})$ are the vertices of an equilateral triangle.

3 Show that the points $A(1,-1)$, $B(7, 3)$, $C(3, 5)$ and $D(-3, 1)$ are the vertices of a parallelogram and find the length of its diagonals.

4 If $(3, -1)$, $(-4, 3)$ and $(1, 5)$ are three vertices of a parallelogram, find the coordinates of the fourth vertex if it lies in the first quadrant.

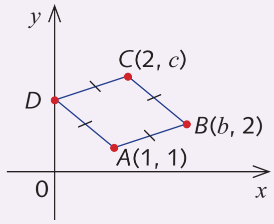
5 Find the equation of the line whose intercepts are twice those of the line with equation $2x - 3y - 6 = 0$.

6 In the rhombus $ABCD$, A has coordinates $(1, 1)$ and the coordinates of B are $(b, 2)$.

The gradient of AB is $\frac{1}{2}$.

a Find the value of b.

b Find the length of AB.



The point C has coordinates $(2, c)$, where c is a positive integer.

c Find the value of c.

d Find the gradient of BC.

e State the gradient of:

i CD **ii** AD

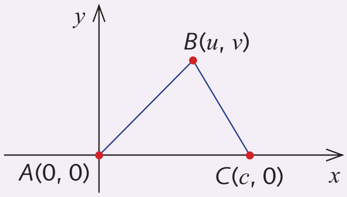
f Find the coordinates of D.

7 Let ABC be any triangle. Take the x-axis to be along BC and the y-axis through the midpoint O of BC perpendicular to BC. Prove that $AB^2 + AC^2 = 2AO^2 + 2OC^2$. (This result is known as Apollonius' theorem.)

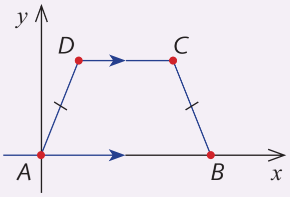
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Updated August 2021

8 Use coordinate geometry to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length. Take the base of the triangle to be on the x-axis with one vertex at the origin.

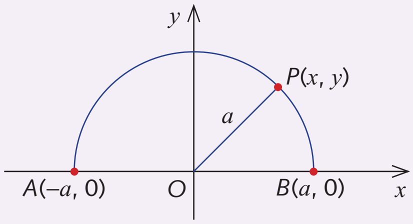


9 Use coordinate geometry to prove that the diagonals of an isosceles trapezium $ABCD$ are equal in length. Choose coordinates appropriately.



10 Use coordinate geometry to prove that the medians of any triangle are concurrent. *Hint*: On each median find the coordinates of the point that divides the median in the ratio $2:1$.

11 The semicircle shown below is drawn with centre O at the origin and radius a. $P(x, y)$ is any point on the semicircle.



a Show that $x^2 + y^2 = a^2$.

b Show that PA is perpendicular to the line PB.

c Show that $PA^2 + PB^2 = 4a^2$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

12

Statistics and Probability

Probability

Probability deals with how likely it is that something will happen. It is an area of mathematics with many diverse applications. Probability is used in weather forecasting and in insurance to calculate risk factors and premiums. It is also used in science to predict the risks of new medical treatments and to forecast the effects of climate change.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

12A An introduction to probability

Probability measures the likelihood of an event occurring on a scale from 0 to 1, inclusive.

In this section we look at methods for determining probabilities. We recall some ideas from *ICE-EM Mathematics Year* 8.

Sample space

A box contains 12 identical marbles numbered from 1 to 12. The box is shaken and a marble is randomly taken from it and its number noted. This is an example of doing a random **experiment**. The numbers $1, 2, \ldots\ldots, 12$ are called the **outcomes** of this experiment. The outcomes for this experiment are **equally likely**. The probability of each outcome is $\frac{1}{12}$. The complete set of possible outcomes (or sample points) for any experiment is called the **sample space** of that experiment. For example, we can write down the sample space ξ for this experiment as:

$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

In this chapter, all of the experiments have finite sample spaces with equally likely outcomes. For a sample space with n equally likely outcomes, the probability of each outcome is $\frac{1}{n}$.

Events

An **event** is a collection of outcomes. It is a subset of the sample space.

Suppose that for the experiment above we are interested in getting a prime number. In this case 'the number is prime' is the event that interests us. Some of the outcomes will give rise to this event. For instance, if the outcome of the experiment is 2, the event 'the number is prime' takes place. We say that the outcome 2 is **favourable to the event** 'the number is prime'. If the outcome is 4, the event 'the number is prime' does not occur. The outcome 4 is **not favourable to the event**.

Of the 12 outcomes, these are the ones that are favourable to the event 'the number is prime':

$$\{2, 3, 5, 7, 11\}$$

In many situations, 'success' means 'favourable to the event' and 'failure' means 'not favourable to the event'.

Events are named by capital letters. For instance, we talk about:

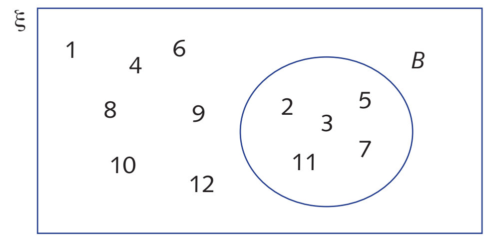
B is the event 'a prime is obtained' from the experiment described previously.
C is the event 'obtaining an even number' when a die is tossed.

An outcome is favourable to an event if it is a member of that event. For example:

5 is a member of B and 6 is a member of C.

These sample spaces and events can be illustrated with **Venn diagrams**. Venn diagrams were used in *ICE-EM Mathematics Year 8* and were introduced in *ICE-EM Mathematics Year 7*.

Here is the sample space ξ and the event B. In this context ξ is the universal set for the experiment of withdrawing a marble and observing the number on it, as described previously.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Probability of an event

The probability of the event A is written $P(A)$.

Since the probability of an event is a number between 0 and 1 inclusive, we can assert:

$$0 \le P(A) \le 1 \text{ for all events } A$$

For an experiment in which all of the outcomes are equally likely:

$$\text{Probability of an event} = \frac{\text{number of outcomes favourable to that event}}{\text{total number of outcomes}}$$

For the event B described above, $P(B) = \dfrac{5}{12}$.

In general, the probability of an event is the sum of the probabilities of the outcomes that are favourable to that event.

The total probability is 1

The sum of the probabilities of the outcomes of an experiment is 1.

For the previous experiment of taking a marble, each outcome has probability $\dfrac{1}{12}$. The sum of these probabilities is 1.

The words 'random' and 'randomly'

In probability, we frequently hear these words, as in the following situation.

A classroom contains 23 students. A teacher comes into the room and chooses a student at random to answer a question about history.

What does this mean? It means that the teacher chooses the student as if the teacher knows nothing at all about the students. Another way of interpreting this is to imagine that the teacher had her eyes closed and had no idea who was in the class when she choose a student. Each student is equally likely to be chosen. The probability of a particular student being chosen is $\dfrac{1}{23}$.

Example 1

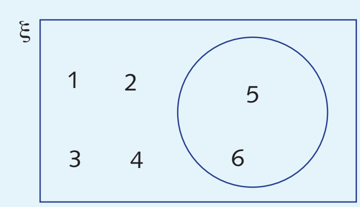
A die is rolled once. Draw a Venn diagram for the experiment and circle the outcomes favourable to the event 'the number is greater than 4'. What is the probability of a number greater than 4 being obtained?

Solution

$\xi = \{1, 2, 3, 4, 5, 6\}$

The outcomes favourable to the event are a 5 or a 6 appearing.

$$P(\text{number greater than } 4) = \frac{2}{6} = \frac{1}{3}$$



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 2

A standard pack of playing cards consists of four suits: Hearts, Diamonds, Clubs and Spades. Each suit has 13 cards consisting of an Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The pack is shuffled and a card is drawn. What is the probability of drawing:

a a King

b a heart?

Solution

The sample space ξ is the 52 playing cards.

a Let K be the event 'a King is drawn'.

$$P(K) = \frac{4}{52} = \frac{1}{13}$$

b Let H be the event 'a heart is drawn'.

$$P(H) = \frac{13}{52} = \frac{1}{4}$$

Exercise 12A

Example 1

1 One letter is chosen at random from the word 'SALE'. What is the probability that it is L?

2 What is the probability of choosing a prime number from the numbers 5, 6, 7, 8, 9, 10, 11, 12, 13?

3 What is the probability of randomly picking the most expensive car from a range of eight new and differently priced cars in a showroom?

4 What is the probability of choosing an integer that is exactly divisible by 5 from the set $\{5, 6, 7, 8, 9, 10, 11, 12\}$?

5 In a raffle, 500 tickets are sold. If you have bought one ticket, what is the probability that you will win first prize?

6 One card is chosen at random from a pack of 52 ordinary playing cards. What is the probability that it is the King of Hearts?

7 A number is chosen from the first 25 positive whole numbers. What is the probability that it is exactly divisible by both 3 and 4?

Example 2

8 One card is drawn at random from a pack of playing cards. What is the probability that it is:

a a King?

b a red card?

c a Spade?

d a picture card? (A picture card is a King, Queen or Jack.)

9 A book of 420 pages has pictures on 60 of its pages. If one page is chosen at random, what is the probability that it has a picture on it?

10 One counter is picked at random from a bag containing 20 red counters, 5 white counters and 15 yellow counters. What is the probability that the counter removed is:

a red?

b yellow?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11 On a spinning wheel, the numbers go from 0 to 36. The wheel is spun. What is the probability that when it stops the resulting number will be:

a an even number? **b** an odd number?

c a number less than 15, excluding 0?

12 A number is chosen at random from the first 25 positive whole numbers. What is the probability that it is not a prime number?

12B The complement, union and intersection

The complement of an event

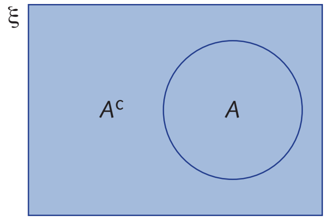
In some problems the outcomes in the event A can be difficult to count, whereas the event 'not A' may be easier to deal with.

The event 'not A' consists of every possible outcome in the sample space ξ that is not in A. The set 'not A' is called the **complement** of A and is denoted by A^c.

Every outcome of a sample space is contained in exactly one of A or A^c.

Therefore $P(A) + P(A^c) = 1$ *and* $P(A^c) = 1 - P(A)$.

This can be illustrated with a Venn diagram, as shown.



Example 3

A card is drawn from a standard pack. What is the probability that it is not the King of Hearts?

Solution

Let A be the event 'the King of Hearts is drawn'.

Then A^c is the event 'the King of Hearts is not drawn'.

$$P(A) = \frac{1}{52}$$

$$P(A^c) = 1 - P(A)$$

$$= \frac{51}{52}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Union and intersection

Sometimes, rather than just considering a single event, we want to look at two or more events.

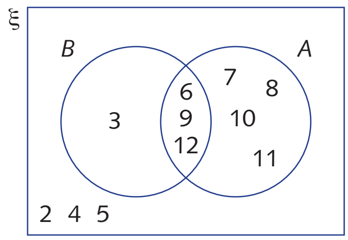
A bowl contains 11 marbles numbered from 2 to 12. One marble is withdrawn.

The sample space $\xi = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

Let A be the event 'a number greater than 5 is chosen'.

Let B be the event 'a number divisible by 3 is chosen'.

The Venn diagram illustrating these events is as shown.

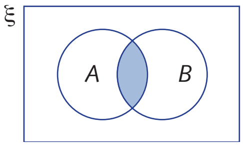


The outcomes favourable to the event 'the number is divisible by 3 and greater than 5' is the **intersection** of the sets A and B. That is, $A \cap B$. The event $A \cap B$ is often called 'A and B'.

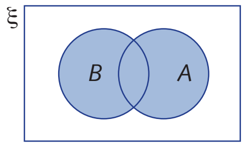
In this example, $A \cap B = \{6, 9, 12\}$.

The outcomes favourable to the event 'the number is divisible by 3 or greater than 5' is the **union** of the sets A and B. That is, $A \cup B$. The event $A \cup B$ is often called 'A or B'.

In this example, $A \cup B = \{3, 6, 7, 8, 9, 10, 11, 12\}$



$A \cap B$ is shaded



$A \cup B$ is shaded

For an outcome to be in the event $A \cup B$, it must be in *either* the set of outcomes for A or the set of outcomes for B. Of course, it could be in both sets.

For an outcome to be in the event '$A \cap B$', it must be in *both* the sets of outcomes for A and the set of outcomes for B.

We recall the **addition rule** for probability. For any two events, A and B:

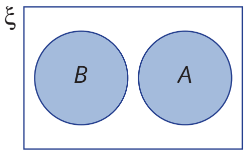
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

We subtract $P(A \cap B)$ from $P(A) + P(B)$ because $A \cap B$ is a subset of both A and B and would be counted twice otherwise.

Two events are **mutually exclusive** if they have no outcomes in common. That is:

$$A \cap B = \emptyset$$

where $\emptyset$ is the empty set.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

The addition rule becomes $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive.

Here are some examples using these ideas.

Example 4

Twenty plastic discs numbered from 1 to 20 are placed in a bowl. A disc is randomly removed, its number noted and then replaced.

a Find the probability of a disc with an even number *or* a number greater than or equal to 15 being obtained.

b Find the probability of a disc with a number that is even *and* divisible by 5 being obtained.

Solution

The sample space is:

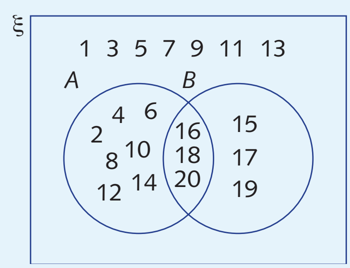
$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

Let A be the event 'an even number'.

Let B be the event 'a number greater than or equal to 15'.

Let C be the event 'a number is divisible by 5'.

a $A \cup B = \{2, 4, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20\}$

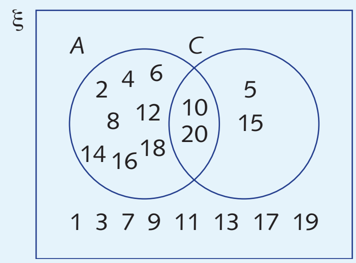


The number of outcomes in $A \cup B$ is 13.

$$P(A \cup B) = \frac{13}{20}$$

b $A \cap C = \{10, 20\}$

The number of outcomes in $A \cap C$ is 2.



$$P(A \cap C) = \frac{2}{20} = \frac{1}{10}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 5

There are 250 students in Year 9 in a school: 60 study music, 90 study French, and 23 study both French and music.

If a student is chosen at random, what is the probability that they study:

a both music and French?

b music or French?

c music but not French?

d French but not music?

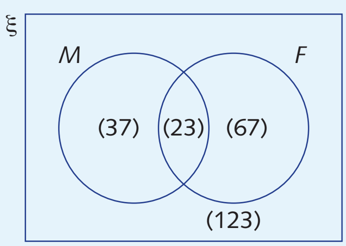
e music or French but not both?

Solution

Let M be the event ‘study music’.

Let F be the event ‘study French’.

The brackets in the Venn diagram are used to indicate that it is the number of elements in the region.



a From the Venn diagram, 23 students study French and music.

$$P(M \cap F) = \frac{23}{250}$$

b From the Venn diagram, 127 students study French or music.

$$P(M \cup F) = \frac{127}{250}$$

c From the Venn diagram, 37 students study music but not French.

‘Not French’ is the event F^c.

$$P(M \cap F^c) = \frac{37}{250}$$

d From the Venn diagram, 67 students study French but not music.

‘Not music’ is the event M^c.

$$P(M^c \cap F) = \frac{67}{250}$$

e From the Venn diagram, 104 students study French or music but not both.

The events $M \cap F^c$ and $M^c \cap F$ are mutually exclusive.

Probability of a student studying French or music but not both:

$$\frac{37}{250} + \frac{67}{250} = \frac{52}{125}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 6

The eye colour and gender of 150 people were recorded. The results are shown in the table below.

Gender \ Eye colour	Blue	Brown	Green	Grey
Male	20	25	5	10
Female	40	35	5	10

What is the probability that a person chosen at random from the sample:

a has blue eyes **b** is male **c** is male and has green eyes

d is female and does not have blue eyes **e** has blue eyes or is female?

Solution

The sample space ξ has 150 outcomes, or the size of $\xi = 150$.

Let: A be the event 'has blue eyes'

B be the event 'has brown eyes'

G be the event 'has green eyes'

M be the event 'is male'

F be the event 'is female'.

a The probability that a person has blue eyes is

$$P(A) = \frac{20 + 40}{150} = \frac{60}{150} = \frac{2}{5}$$

b The probability that a person is male is

$$P(M) = \frac{20 + 25 + 5 + 10}{150} = \frac{60}{150} = \frac{2}{5}$$

c The probability that a person is male and has green eyes is

$$P(M \cap G) = \frac{5}{150} = \frac{1}{30}$$

d The probability that a person is female and does not have blue eyes is

$$P(F \cap A^c) = \frac{35 + 5 + 10}{150} = \frac{50}{150} = \frac{1}{3}$$

e The probability that a person has blue eyes or is female is

$$P(A \cup F) = \frac{20 + 40 + 35 + 5 + 10}{150} = \frac{110}{150} = \frac{11}{15}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Complement, or and and

- The event 'not A' includes every possible outcome of the sample space ξ that is not in A
  The event 'not A' is called the **complement** of A and is denoted by A^c.
  $P(A^c) = 1 - P(A)$
- An outcome that is in the event '$A \cup B$' is in either A *or* B, or both.
- An outcome that is in the event '$A \cap B$' is in both A *and* B.
- For any two events, A and B
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Two events A and B are **mutually exclusive** if $A \cap B = \varnothing$ and
  $P(A \cup B) = P(A) + P(B)$

Exercise 12B

1 A number is chosen at random from the first 15 positive whole numbers. What is the probability that it is not a prime number?

Example 3

2 A card is drawn at random from an ordinary pack of 52 playing cards. What is the probability that it is not a King?

3 A number is chosen at random from the first 30 positive whole numbers. What is the probability that it is not divisible by 7?

4 In a raffle, 1000 tickets are sold. If you buy 50 tickets, what is the probability that you will not win the first prize?

5 A letter is chosen at random from the 10 letters of the word 'COMMISSION'. What is the probability that the letter is:

a N? **b** S? **c** a vowel? **d** not S?

Example 4

6 A card is drawn at random from a well-shuffled pack of playing cards. Find the probability that the card chosen:

a is a Club

b is a court card (that is, Ace, King, Queen or Jack)

c has a face value between 2 and 9 inclusive

d is a Club and a court card

e is a Club or a court card

f has a face value between 2 and 5 inclusive and is a court card

g has a face value between 2 and 5 inclusive or is a court card

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 A six-sided die is thrown and the uppermost number is noted. Find the probability that the number is:

a even and a 6
b even or a 6
c less than or equal to 4 and a 6
d less than or equal to 3 or a 6
e even and less than or equal to 4
f odd or less than or equal to 3

Example 6

8 A survey of 200 people was carried out to determine hair and eye colour. The results are shown in the table below.

Eyecolour \ Hair colour	Fair	Brown	Red	Black
Blue	25	9	6	18
Brown	16	16	18	22
Green	15	17	22	16

What is the probability that a person chosen at random from this group has:

a blue eyes?
b red hair?
c fair or brown hair?
d blue or brown eyes?
e red hair and green eyes?
f eyes that are not green?
g hair that is not red?
h eyes that are not blue or hair that is not fair?

Use an appropriate Venn diagram in the following questions.

Example 5

9 In a group of 100 students, 60 study mathematics, 70 study physics and 30 study both mathematics and physics.

a Represent this information on a Venn diagram.

b One student is selected at random from the group. What is the probability that the student studies:

i mathematics and not physics?

ii physics but not mathematics?

iii neither physics nor mathematics?

10 In a group of 40 students, 26 play tennis and 19 play soccer. Assuming that each of the 40 students plays at least one of soccer and tennis, find the probability that a student chosen at random from this group plays:

a both tennis and soccer
b only tennis
c only one sport
d only soccer

11 In a group of 65 students, 30 students study geography, 42 study history and 20 study both history and geography. If a student is chosen at random from the group of 65 students, find the probability that the student studies:

a history or geography
b neither history nor geography
c history but not geography
d only one of history or geography

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

12C Relative frequency

When we toss a coin, we can assume that the two outcomes are equally likely, so each has a probability of $\frac{1}{2}$ of occurring. This is reasonable, provided that the coin is well made and evenly balanced. We then make a 'probability model' consisting of $\{H, T\}$, where H and T both have a probability of $\frac{1}{2}$. This is what we have done so far in this chapter.

Estimates of probability

If we find a coin on the street that is badly worn and bent by the weather and passing traffic, it would not be reasonable to assume that a Head and a Tail are equally likely to occur when we toss the coin. So we would have no idea of what the probability of a Head would be.

One way of attempting to get around the difficulty is to toss the coin a large number of times and record how many Heads occur. If we tossed the coin 100 times, we might get these results:

Heads: 40 Tails: 60

Then we could divide to get $\frac{40}{100} = \frac{2}{5}$ as an estimate of the probability of getting a Head.

This is called **relative frequency**.

The relative frequency of getting a Tail is therefore $\frac{3}{5}$.

We are going to use relative frequency as an estimate of the probability for future experiments. So in this case, our estimate for the probability of getting a Head is $\frac{2}{5}$.

Limitations

If we think about relative frequency, we will see there are some tricky issues. For example:

- Would we get the same relative frequency if we performed another 100 coin tosses?
- If we toss a perfectly new coin from the mint 100 times, would the relative frequency of a Head be $\frac{1}{2}$?

The answer to both questions is: 'No, unless you are very lucky'. There will always be **random variation** between the outcomes of repeated trials like these.

The idea of relative frequency is useful and important. Statisticians use it very effectively by combining it with techniques of **sampling**. This is a sophisticated area of probability and statistics. Random sampling was considered in *ICE-EM Mathematics Year 8*. We are going to use relative frequency in simple ways in this section.

Relative frequency

The relative frequency of the event A is defined to be

$$\frac{\text{number of times } A \text{ occurs}}{\text{total number of times the experiment is performed}}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7

Clara has a die. She does not know if the values 1, 2, 3, 4, 5 and 6 are equally likely. She rolls the die 2000 times and obtains the following results.

Outcome	1	2	3	4	5	6
Frequency	290	370	330	280	340	390

Use the table to calculate the relative frequency to estimate the probability that the next roll will be:

a a 2 **b** an even number **c** a number greater than 4

Solution

a The relative frequency of obtaining a 2 is $\frac{370}{2000} = \frac{37}{200}$

This can be taken as an estimate of the probability of obtaining a 2.

b Of the 2000 rolls, $370 + 280 + 390 = 1040$ yielded an even number.

The relative frequency of obtaining an even number $= \frac{1040}{2000}$

$= \frac{13}{25}$

This can be taken as an estimate of the probability of obtaining an even number.

c Of the 2000 rolls, $340 + 390 = 730$ yielded a number greater than 4.

The relative frequency of obtaining a number greater than 4 $= \frac{730}{2000}$

$= \frac{73}{200}$

This can be taken as an estimate of the probability of obtaining a number greater than 4.

'or', 'and' and relative frequency

We can use a two-way table to help determine relative frequencies.

Example 8

A survey of 200 people was carried out to determine hair and eye colour. The results are shown in the table below.

Eyecolour \ Hair colour	Fair	Brown	Red	Black
Blue	25	9	6	18
Brown	16	16	18	22
Green	15	17	22	16

What is the relative frequency of:

a fair or brown hair? **b** green eyes and black hair?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a Relative frequency of fair or brown hair $= \dfrac{25 + 16 + 15 + 9 + 16 + 17}{200}$

$= \dfrac{49}{100}$

b Relative frequency of green eyes and black hair $= \dfrac{16}{200} = \dfrac{2}{25}$

Exercise 12C

1 A market researcher interviews 200 people and discovers that 120 of them use a particular brand of toothpaste. Using this data, what is the relative frequency of a person:

a using the particular brand of toothpaste

b not using the particular brand of toothpaste?

2 A quality-control inspector discovers that, of 1500 electrical components tested, 25 are faulty. Using this data, what is an estimate for probability that the next component tested is:

a faulty? **b** not faulty?

3 A meteorologist's records indicate that it has rained on 30% of the occasions when a particular weather pattern occurred. What is the estimated probability that it will rain the next time the particular weather pattern occurs?

4 A bag contains a number of counters, each of which is coloured blue, yellow, red or green. A student chooses a counter from the bag at random, notes its colour and returns it to the bag. She repeats this process 100 times. The results of the experiment are in the following table.

Colour	Blue	Yellow	Red	Green
Frequency	18	21	53	8

a What is the estimated probability that the next counter she withdraws is:

i blue? **ii** red? **iii** not green?

b If the bag contains 10 counters, how many counters of each colour do you think are in the bag?

5 Abdul chose a card at random from a not-necessarily standard pack of cards, noted its suit and returned it to the pack. He repeated this process 500 times. The results of his experiment are in the following table.

Suit	Spades	Hearts	Diamond	Clubs
Frequency	112	119	135	

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a Fill in the missing entry in the table.

b What is the estimate peobability that next card Abdul withdraw is:

i a Spade? **ii** a Heart? **iii** not a Club? **iv** a red card?

6 One thousand people were surveyed about their favourite sport. That results of the survey are given in the table below.

Sport	AFL	Rugby Union	Rugby League	Soccer	Baseket ball
Frequency	188	160	145		210

a Fill in the missing entry in the table.

b What is the relative frequency of:

i AFL?

ii Rugby Union?

iii a sport other than soccer?

7 A survey of 600 people was carried out to determine hair and eye colour. The results are shown in the table below.

Eyecolour \ Hair colour	Fair	Brown	Red	Black
Blue	125	30	15	45
Brown	40	40	45	55
Green	68	42	55	40

What is the relative frequency of:

a fair or red hair? **b** blue or green eyes?

c red hair and green eyes? **d** green eyes and brown hair?

8 The 700 subjects volunteering for a medical study are classified by blood pressure (high, normal and low) and gender.

Gender \ Blood pressure	High	Normal	Low
Male	352	100	52
Female	60	88	48

Find the relative frequency of:

a male and high blood pressure **b** male or low blood pressure

c female and low blood pressure **d** male or normal blood pressure

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

12D Multi-stage experiments

Rolling two dice, tossing three coins and drawing six lottery numbers are examples of multi-stage experiments.

Tossing a coin twice

The possible outcomes of this experiment are:

HH HT TH TT

where H stands for 'heads' and T stands for 'tails', and the order of the letters indicates the outcomes of the two separate tosses. The four outcomes of the experiment are equally likely, so each has a probability of $\frac{1}{4}$.

Example 9

A coin is tossed twice. What is the probability of getting:

a one head and one tail in any order?

b at least one tail?

Solution

a Here is the sample space, with circles around the favourable outcomes.

HH HT TH TT

$$P(\text{one head and one tail}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

b Here is the sample space, with the favourable outcomes circled.

HH HT TH TT

$$P(\text{at least one tail}) = \frac{3}{4}$$

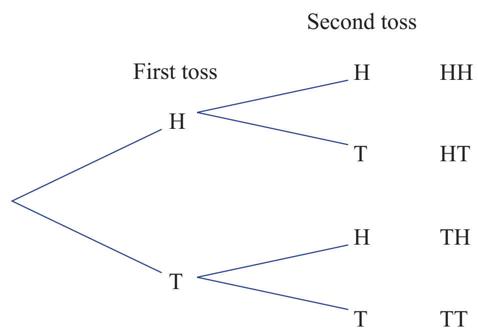
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Tree diagrams

Another way of illustrating the outcomes of the experiment of tossing a coin twice is a **tree diagram**.

Each path, starting from the far left and ending with H or T on the far right, represents a possible outcome.

The sample space is $\xi = \{HH, HT, TH, TT\}$.

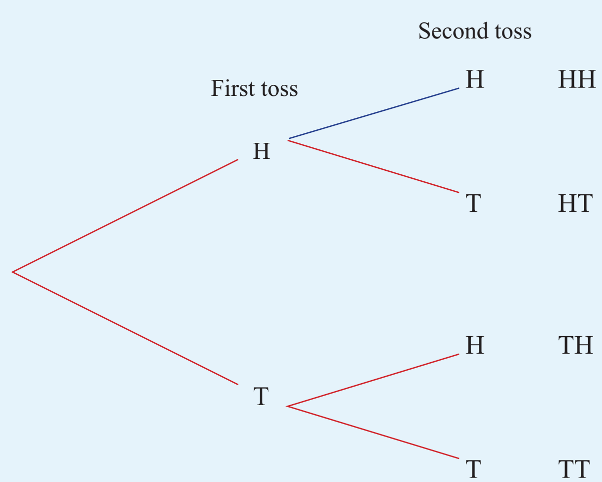


Example 10

Using a tree diagram, calculate the probability of getting at least one tail when a coin is tossed twice.

Solution

The tree diagram, with the favourable paths indicated with red lines, is shown below.



There are 3 paths that give at least one tail and 4 $(= 2 \times 2)$ possible paths in total, so the event 'at least one tail' is $\{HT, TH, TT\}$ and $P(\text{at least one tail}) = \frac{3}{4}$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Tossing a coin three times

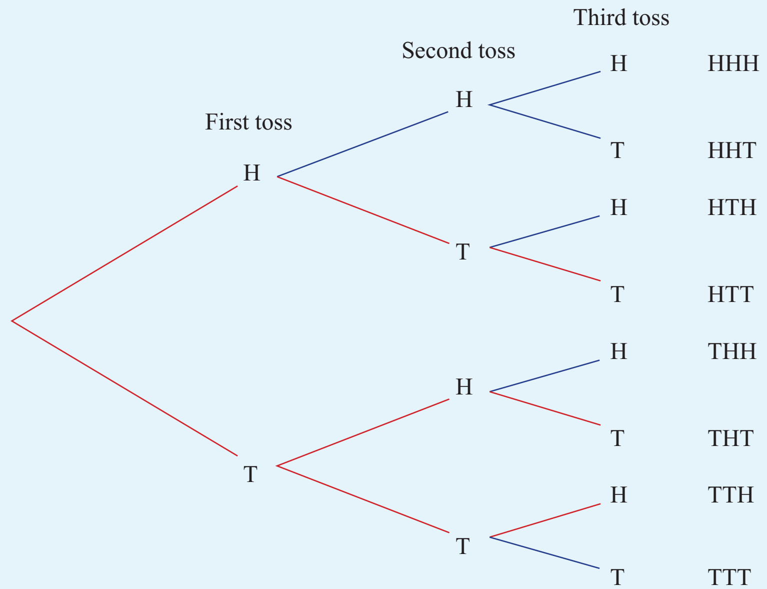
Tree diagrams are especially useful when the experiment consists of more than two stages.

Example 11

A coin is tossed three times. What is the probability of getting exactly one head?

Solution

The paths that contain exactly one head are indicated with red lines in the tree diagram below.



There are 3 of these and 8 $(= 2 \times 2 \times 2)$ possible paths in total, so the event 'exactly one head' $= \{$HTT, THT, TTH$\}$ and P (exactly one head) $= \dfrac{3}{8}$.

Tossing a die twice

If a die is tossed twice, there are $6 \times 6 = 36$ outcomes. It would be possible to draw the corresponding tree diagram, but that would be cumbersome. A better approach is to use an **array diagram**.

Each die has six faces, marked with dots from 1 to 6. The outcomes of the experiment can be set out in an array, as shown below.

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Each entry in the array represents a pair of possible results, the first from die 1 and the second from die 2.

For example, the entry $(3,\ 4)$ in the array corresponds to the outcome:

die $1 = 3$ die $2 = 4$

That is, the first die shows a 3, the second a 4.

Each outcome is equally likely, and there are $6 \times 6 = 36$ of them in total, so:

$$\text{probability of each outcome} = \frac{1}{36}$$

Example 12

Two dice are tossed. What is the probability of getting a 6 on at least one of the dice?

Solution

The favourable outcomes are shaded in the array diagram below.

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$$P(\text{at least one } 6) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$
$$= \frac{11}{36}$$

How could a solution to the above problem be provided by the addition rule?

Example 13

Two dice are tossed. What is the probability that the sum of their faces lies between 4 and 7 inclusive?

Solution

Here we can take a shortcut by making a diagram showing the sums of the faces. The sums for the favourable outcomes are shaded.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are 18 favourable outcomes out of a total of 36, so:

$$P(\text{sum lies between 4 and 7 inclusive}) = \frac{18}{36} = \frac{1}{2}$$

Multi-stage experiments

The outcomes of a multi-stage experiment can be represented by:

- systematic listing
- a tree diagram
- an array (only suitable for two-stage experiments).

Exercise 12D

1 A 10-cent coin is tossed and an ordinary six-sided die is rolled. Copy and complete the following array, which shows the possible outcomes.

10-cent coin \ Die	1	2	3	4	5	6
H		(H, 2)				
T				(T, 4)		

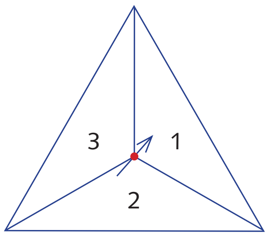
2 One bag contains 1 red counter, 2 yellow counters and 1 blue counter. Another bag contains 1 yellow counter, 2 red counters and 1 blue counter. One counter is taken at random from each bag. Copy and complete the following array to show all the possible outcomes for this experiment.

1st bag \ 2nd bag	R	R	Y	B
R	(R, R)			
Y				(Y, B)
Y				
B	(B, R)			

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 A spinner like the one in the diagram is spun twice. Copy and complete the array to show all the possible outcomes for this experiment.

1st spin \ 2nd spin	1	2	3
1			
2			
3			



Example 12

4 A coin and a six-sided die are tossed. Construct an array that shows all the equally likely possible outcomes of this random experiment, and use it to find the probability of obtaining:

a a head and a six

b a head and an even number

c a tail

d a four

5 One bag of coins contains one 10-cent coin and one 50-cent coin. Another bag contains three 10-cent coins and two 50-cent coins. One coin is removed at random from each bag. Construct an array to show all the equally likely outcomes of this experiment, and use it to find the probability that:

a a 50-cent coin is taken from each bag

b a 10-cent coin is taken from each bag

c the coins are of different value

6 One bookshelf contains 3 storybooks and 1 textbook, while the next shelf holds 2 storybooks and 3 textbooks. Draw an array showing the various ways in which you could pick a pair of books, one from each shelf. Use this array to find the probability that:

a both books are storybooks

b both books are textbooks

7 The four Aces and the four Kings are removed from a pack of playing cards. One card is taken from the set of four Aces and one card is taken from the set of four Kings. Construct an array for the outcomes of this experiment, and use it to find the probability that the two cards chosen:

a are both black

b are both Spades

c include at least one black card

d are both the same suit

Example 13

8 Two fair six-sided dice are tossed and the uppermost numbers are noted. Construct an array for the possible outcomes (or use the table given in Example 13) to find the probability of obtaining:

a two sixes

b a total of 7

c the same number on both dice

d a total less than or equal to 10

9 A special six-sided die has two blank faces and the other faces numbered 1, 3, 4 and 6. This die is rolled along with an ordinary six-sided die with faces numbered 1, 2, 3, 4, 5 and 6. Make an array showing the values of the pairs of uppermost faces of the dice, and use it to find the probability of getting a total score of:

a 6

b 10

c 1

d at least 6

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 10 **10** Two coins are tossed.

a Draw a tree diagram showing all the outcomes.

b What is the probability that 'a head and a tail' are showing?

Example 11 **11** A coin is tossed three times.

a Draw a tree diagram showing all the outcomes.

b What is the probability that three heads are obtained?

c What is the probability that exactly one head is obtained?

d What is the probability that at least one head is obtained?

12 A painter is going to paint a child's bedroom. He has two choices of colour for the ceiling, blue or white; he has three choices of colour for the walls, blue, grey or white; and he has two choices of colour for the skirting boards, grey or white. He has to choose the colour for the ceiling, the walls and the skirting boards.

a Draw a tree diagram showing all the outcomes.

b If he chooses at random, what is the probability that the ceiling, the walls and the skirting boards are painted different colours?

13 A family has three children. By drawing a tree diagram that shows all the possible combinations of children, find the probability that:

a all three children are girls

b only one child is a boy

c at least two of the children are girls

12E Two-step experiments involving replacement

With replacement

A bag contains three red balls, R_1, R_2 *and* R_3, and two black balls, B_1 and B_2. A ball is chosen at random and its colour recorded. It is then put back in the bag, the balls are mixed thoroughly and a second ball is chosen. Its colour is also noted. The sample space is shown in the array below.

First ball \ Second ball	R_1	R_2	R_3	B_1	B_2
R_1	(R_1, R_1)	(R_1, R_2)	(R_1, R_3)	(R_1, B_1)	(R_1, B_2)
R_2	(R_2, R_1)	(R_2, R_2)	(R_2, R_3)	(R_2, B_1)	(R_2, B_2)
R_3	(R_3, R_1)	(R_3, R_2)	(R_3, R_3)	(R_3, B_1)	(R_3, B_2)
B_1	(B_1, R_1)	(B_1, R_2)	(B_1, R_3)	(B_1, B_1)	(B_1, B_2)
B_2	(B_2, R_1)	(B_2, R_2)	(B_2, R_3)	(B_2, B_1)	(B_2, B_2)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

The sample space contains 25 pairs. They are equally likely. Each outcome has the probability $\frac{1}{25}$ of occurring.

Let A be the event 'both balls are red':

$$A = \{(R_1, R_1), (R_1, R_2), (R_1, R_3), (R_2, R_1), (R_2, R_2), (R_2, R_3), (R_3, R_1), (R_3, R_2), (R_3, R_3)\}$$

Therefore $P(A) = \frac{9}{25}$

Without replacement

Again, we start with a bag containing three red and two black balls. A ball is chosen at random and its colour recorded. The ball is *not* put back in the bag. A second ball is chosen at random from the remaining balls and its colour recorded.

The sample space is listed in the array below. There are 5×4 outcomes in the sample space.

Second ball / First ball	R_1	R_2	R_3	B_1	B_2
R_1	–	(R_1, R_2)	(R_1, R_3)	(R_1, B_1)	(R_1, B_2)
R_2	(R_2, R_1)	–	(R_2, R_3)	(R_2, B_1)	(R_2, B_2)
R_3	(R_3, R_1)	(R_3, R_2)	–	(R_3, B_1)	(R_3, B_2)
B_1	(B_1, R_1)	(B_1, R_2)	(B_1, R_3)	–	(B_1, B_2)
B_2	(B_2, R_1)	(B_2, R_2)	(B_2, R_3)	(B_2, B_1)	–

The '–' indicates that the pair cannot occur.

Let A be the event 'both balls are red':

$$A = \{(R_1, R_2), (R_1, R_3), (R_2, R_1), (R_2, R_3), (R_3, R_1), (R_3, R_2)\}$$

Therefore $P(A) = \frac{6}{20} = \frac{3}{10}$

Exercise 12E

1 The letters O, R, I, G, I, N are printed on plastic squares and placed in a hat.

a A square is randomly taken from the hat, observed and replaced. Then a second square is removed.

i What is the probability of obtaining two 'I's?

ii What is the probability of obtaining an 'I' on the first and an 'R' on the second?

iii What is the probability of an 'R' on the first and an 'I' on the second?

b A square is randomly taken from the hat, observed and not replaced. Then a second square is removed.

i What is the probability of obtaining two 'I's?

ii What is the probability of obtaining an 'I' on the first and an 'R' on the second?

iii What is the probability of an 'R' on the first and an 'I' on the second?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 A fishpond has 2 gold fish and 3 black fish. Two fish are taken without replacement, one after another. What is the probability of:

a two gold fish?

b two black fish?

c a black and a gold in any order?

3 A jar contains 5 blue and 3 red jelly beans. Joe randomly takes one out and eats it. Leanne then takes one out. What is the probability that:

a Joe obtains a blue jelly bean and Leanne a red?

b Joe obtains a red jelly bean and Leanne a red?

c Joe obtains a blue jelly bean and Leanne a blue?

d Leanne obtains a red jelly bean?

4 A two-digit number is randomly created from the set of digits: 3, 4, 5, 6 and 7.

a If each digit is allowed to be used more than once, what is the probability that it will be:

i an even number greater than 50?

ii an odd number less than 60?

b If each digit is allowed to be used only once, what is the probability that it will be:

i an even number greater than 50?

ii divisible by 11?

Review exercise

1 There are 10 marbles, numbered 1 to 10, in a bowl. A marble is randomly taken out. What is the probability of getting:

a a 6?

b an even number?

c a number divisible by 5?

d a number greater than 4?

2 Twenty cards are put in a hat. Fifteen of the cards are blue and numbered 1 to 15. The other five cards are red and numbered 16 to 20. A card is taken randomly from the hat. Find the probability of taking out:

a a blue card

b a red card

c a red card with an odd number

d a number greater than 16

e an odd number

f a number divisible by 3

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 A ball is drawn from a box containing 6 red balls, 4 white balls and 10 blue balls. Find the probability of:

a getting a blue ball or a red ball

b not getting a red ball

4 Clare and David made a record of the colour of handbags carried by customers in a restaurant. This is what they recorded.

Colour	White	Red	Yellow	Green	Blue	Black
Frequency	20	8	6	5	12	16

a What was the total number of handbags recorded?

b If a person with a handbag is selected at random, what is the probability that the handbag is:

i red? **ii** blue? **iii** white? **iv** not green?

5 The names Arthur, Con, Christine, Anna, Donald and Enid are written separately on cards and put in a hat. One card is drawn out. What is the probability that:

a a girl is chosen?

b a boy is chosen?

c Christine in chosen?

d Con is not chosen?

6 All students at a school were asked what their method of transport to school was. The results were: 40% walk to school, 20% come by car, 12% use public transport and the rest ride a bicycle.

A student is chosen at random. Find the probability that this student:

a walks to school

b rides a bicycle to school

c does not use public transport

7 If a letter of the alphabet is chosen at random, what is the probability that it is:

a a vowel?

b a consonant?

c one of the letters of AMSI?

d one of the letters of ICE-EM?

e one of the consonants of ICE-EM?

f one of the vowels of ICE-EM?

8 A coin is tossed three times. What is the probability of getting:

a three heads **b** two heads **c** one head **d** no heads?

9 A die is rolled and a coin is tossed. The pair of results is written down.

a Draw a tree diagram for the experiment.

b Draw an array diagram for the experiment.

c What is the probability of obtaining a tail and a 5?

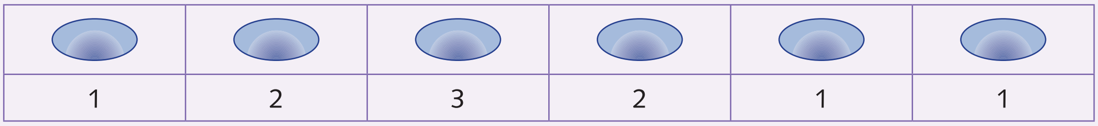
d List the favourable outcomes for the event ‘obtain a head following an even number’.

e What is the probability of obtaining a head following an even number?

f What is the probability of obtaining a tail and a number greater than 3?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 A ball is dropped at random into one of six holes, numbered as shown in the diagram below. The number under each hole gives the score obtained when the ball drops into that hole.



a State the probability of scoring 1.

b If the ball is dropped twice and the scores are added, find the probability of obtaining:

i a total of 5 **ii** a total of 4

11 A green die and a blue die are rolled and the results showing on the top faces are noted. What is the probability of getting:

a a 1 on the blue and a 2 on the green

b a 1 and a 2

c a 5 on the green and an odd number on the blue

d at least one 5

e an odd number on the blue and an even number on the green

12 Josephine has written 3 different letters and she correctly addresses 3 different envelopes. She is not at all careful in putting the letters in the envelopes, to the extent that it can be considered totally random. There are 6 ways of putting the letters in the envelopes. The 6 possibilities may be shown in a table, as below. In the second column, one possibility has been completed for you. It represents that letter 2 has been put in envelope 1, letter 3 has been put in envelope 2, and letter 1 has been put in envelope 3.

E1	L2					
E2	L3					
E3	L1					

a Complete the table.

b What is the probability that all letters are in the correct envelopes?

c What is the probability that exactly one letter is in the correct envelope?

d What is the probability that no letter is in the correct envelope?

13 Two dice are rolled and the results are noted. What is the probability that:

a the sum of the numbers is odd?

b the product of the numbers is even?

c the difference of the two numbers is 2?

d the sum of the numbers is greater than 9?

e one of the numbers is 3 and the sum of the numbers is less than 6?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

14 A game consists of choosing a card from a set of 5 cards and tossing a coin. The 5 cards are labelled from 'A' to 'E'.

a Draw a tree diagram to show the different outcomes of the experiment.

b Find the probability of getting:

i a head and an 'A'

ii a tail and an 'E'

iii a head and a vowel

iv a tail and a consonant

Challenge exercise

1 a A coin is tossed twice. By drawing a tree diagram, calculate the probability that at least one head is obtained.

b A coin is tossed three times. By drawing a tree diagram, calculate the probability that at least one head is obtained.

c A coin is tossed four times.

i How many outcomes are there? (*Hint*: Try to picture what the tree diagram would look like without actually drawing it.)

ii What is the probability that at least one head is obtained?

d A coin is tossed n times.

i How many outcomes are there?

ii What is the probability that at least one head is obtained?

2 a If a die is rolled, what is the probability that a six is not obtained?

b If two dice are rolled, what is the probability that no sixes are obtained?

3 For each event state the following:

i How many outcomes are there?

ii What is the probability that no sixes are obtained?

iii What is the probability that at least one six is obtained?

a Three dice are rolled.

b Four dice are rolled.

c n dice are rolled.

4 In a certain town there are 100 000 cars, each having a licence plate number from 1 to 100 000. What is the probability that the first car you see on a given day does not have a 6 or 7 in its number plate?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

CHAPTER

13

Measurement and Geometry

Trigonometry

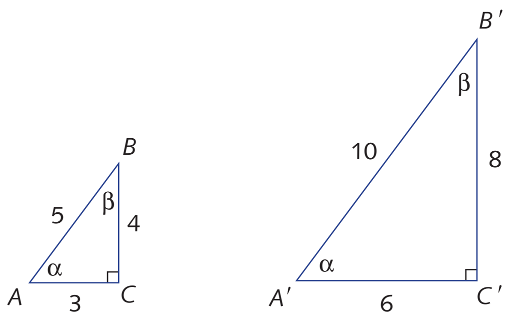
In this chapter we will begin studying a very important branch of mathematics called trigonometry, which starts with the ratios of side lengths in right-angled triangles. This topic is of great practical importance to surveyors, architects, engineers and builders, and also has many other applications.

The Greeks had a form of trigonometry based on using lengths of chords of circles. This was further developed by Arab mathematicians. German scholar Regiomontanus (1436–76) wrote the first book on what we might call 'modern' trigonometry to help him in his study of astronomy.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press

13A Introduction

Suppose we have two similar right-angled triangles as shown on the right. The angles of one match up with the angles of the other, and their matching sides are in the same ratio.



Since the matching sides are in the same ratio, then

$$\frac{BC}{B'C'} = \frac{AB}{A'B'} = \frac{1}{2}$$

Notice that this can be written as

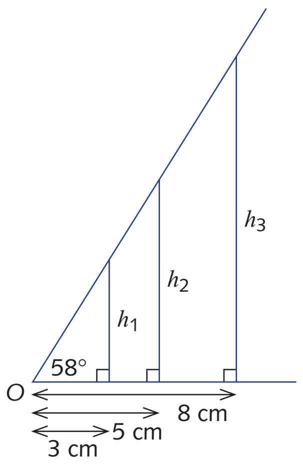
$$\frac{BC}{AB} = \frac{B'C'}{A'B'} = \frac{4}{5}$$

This tells us that in a right-angled triangle, once the angles are fixed, the ratios of sides are constant.

Trigonometry is concerned with this idea.

Activity: Similar triangles

Use your protractor to draw an angle of 58°. Place markers at distances of 3 cm, 5 cm, and 8 cm from O and draw perpendiculars as shown.



Measure the heights h_1, h_2 and h_3.

You should find that $h_1 \approx 4.8$ cm, $h_2 \approx 8.0$ cm and $h_3 \approx 12.8$ cm.

Now look at the ratio of the heights to the bases $\frac{h_1}{3}$, $\frac{h_2}{5}$ and $\frac{h_3}{8}$. These ratios are all (approximately) 1.6.

The ratios are the same since the triangles are similar.

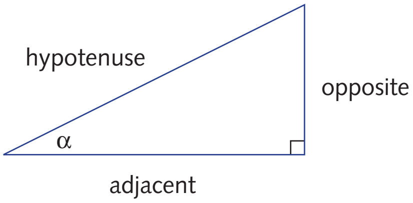
Naming the sides

To be more specific about the ratios of sides, we need to introduce some names.

You already know that the longest side in a right-angled triangle, which is the side opposite the right angle, is called the **hypotenuse**.

We now choose one of the two acute angles and give it a name. One of the Greek letters α, β, γ or θ is generally used. This angle is sometimes called the **reference angle**.

The side opposite the reference angle is called the **opposite side**, often abbreviated to the **opposite**; the remaining side, which is between the reference angle and the right angle, is called the **adjacent side** or simply **adjacent**. The word 'adjacent' means 'near' or 'next to'. The side named the **adjacent** is next to the reference angle.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

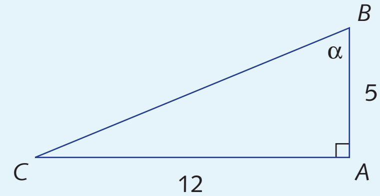
Example 1

a Label the three sides of the triangle relative to the marked reference angle.

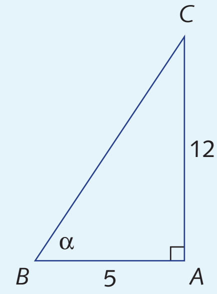
b Let $AB = 5$ and $AC = 12$.

i Find the length of BC.

ii Write down the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}}$.

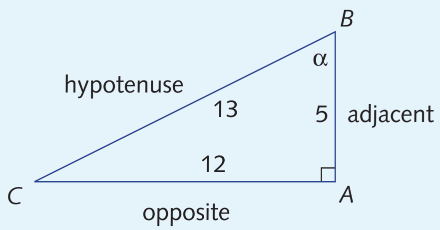


iii Write down the ratio $\dfrac{\text{opposite}}{\text{adjacent}}$.



Solution

a



b i By Pythagoras' theorem:

$$BC = \sqrt{5^2 + 12^2}$$
$$= 13$$

ii $\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{12}{13}$

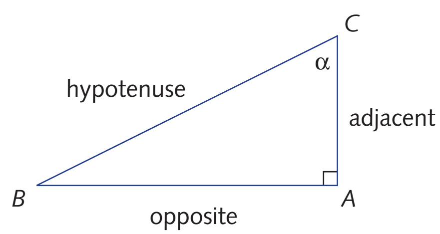
iii $\dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{12}{5}$

In this chapter we will generally leave ratios as improper fractions.

For example, we will write $\dfrac{12}{5}$ rather than $2\frac{2}{5}$.

Introduction to trigonometry

- For a right-angled triangle, once the angles are fixed, the ratios of sides are constant.
- For α, the reference angle, the sides of a right-angled triangle are named as shown on the right



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

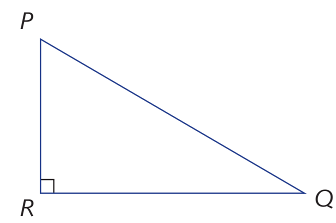
Exercise 13A

Example 1a

1 For the triangle opposite, name the opposite side, the adjacent side and the hypotenuse, using:

a $\angle PQR$ as reference angle

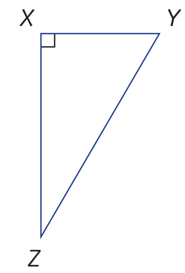
b $\angle RPQ$ as reference angle



2 For the triangle opposite, name the opposite side, the adjacent side and the hypotenuse, using:

a $\angle XYZ$ as reference angle

b $\angle XZY$ as reference angle

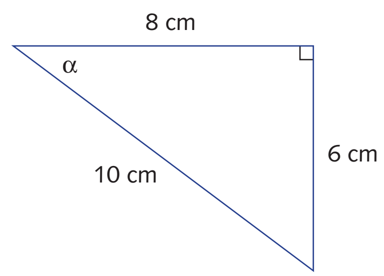


3 For the triangle opposite, relative to the reference angle α, what is the length of:

a the hypotenuse?

b the opposite side?

c the adjacent side?

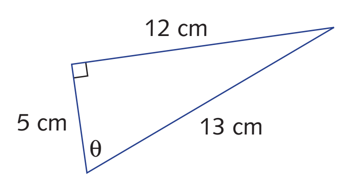


4 For the triangle opposite, relative to the reference angle θ, what is the length of:

a the hypotenuse?

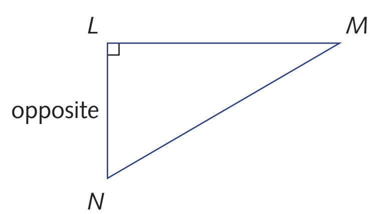
b the opposite side?

c the adjacent side?

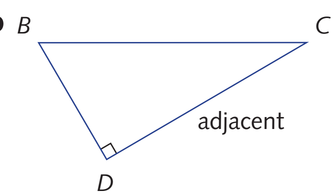


5 Name the angle that has been used as the reference angle for the naming of the sides.

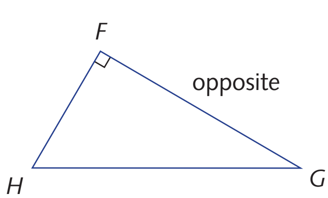
a



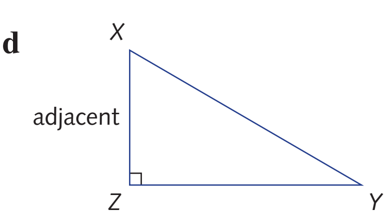
b



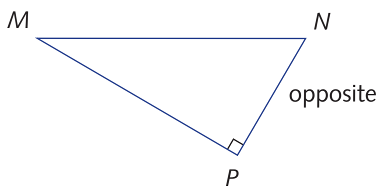
c

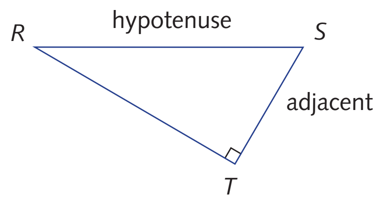


d



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

e 

f 

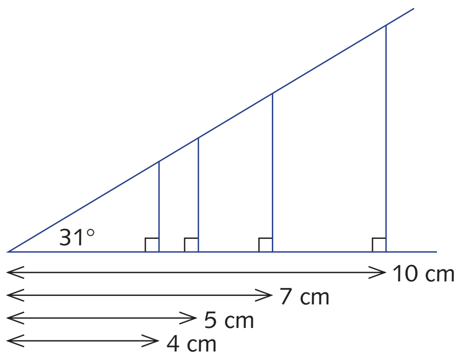
6 Draw diagrams of right-angled triangles with:

a reference angle 30° and the opposite side of length 8 cm

b reference angle 42° and the adjacent side of length of 5 cm

c reference angle 40° and the hypotenuse of length of 10 cm

7 Use your protractor to draw an angle of 31°. Mark off distances of 4 cm, 5 cm, 7 cm and 10 cm. Draw a perpendicular at each point. Measure the heights and find, approximately, the ratio $\frac{\text{opposite}}{\text{adjacent}}$ in each triangle.

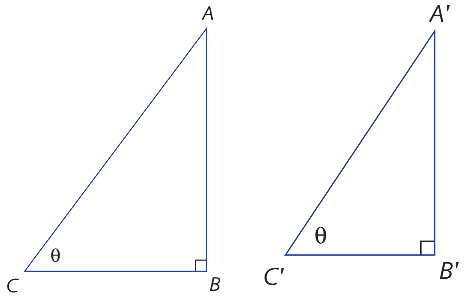


13B The three basic trigonometric ratios

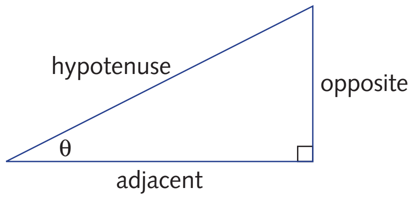
We saw in the previous section that once we fix an acute angle in a right-angled triangle, the ratios of the various sides remain the same, irrespective of the size of the triangle. This happens because the triangles are similar.

So in the diagrams on the right:

$$\frac{AB}{AC} = \frac{A'B'}{A'C'} \qquad \frac{BC}{AC} = \frac{B'C'}{A'C'} \qquad \frac{AB}{BC} = \frac{A'B'}{B'C'}$$



The above ratios have special names based on their relationship to the reference θ angle. These names are sine, cosine and tangent respectively, and in the context of their application to θ, we write:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Note: Collectively, the sine, cosine and tangent rations are known as the **basic trigonometric** ratios. Their names stem from Latin words that reflect a relationship they have with circles.

You will need to learn the three ratios for sine, cosine and tangent carefully. A simple mnemonic is

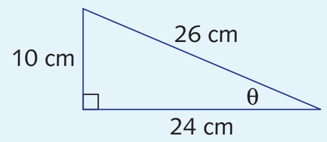
SOH CAH TOA

Sine – **O**pposite / **H**ypotenuse
Cosine – **A**djacent / **H**ypotenuse
Tangent – **O**pposite / **A**djacent

Example 2

For the triangle shown, write down the value of:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



Solution

a $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{10}{26} = \dfrac{5}{13}$

b $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{24}{26} = \dfrac{12}{13}$

c $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{10}{24} = \dfrac{5}{12}$

Example 3

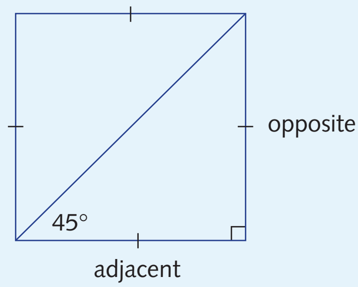
Draw the diagonal of a square and find the value of tan 45°.

Solution

Two right-angled triangles are formed, each with an angle of 45°. Each triangle is isosceles (angle sum of a triangle).

Hence the length of the opposite equals the length of the adjacent, so

$$\tan 45^\circ = \frac{\text{opposite}}{\text{adjacent}} = 1$$



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

To find the tangent ratio for other angles, we could draw a triangle and measure lengths. This is what we did in the activity 'Similar triangles' on page 369 when we found the ratio of the height to the base in the triangle containing 58°. Hence, $\tan 58° \approx 1.6$.

Finding the values of trigonometric ratios in this way can only be approximate. Fortunately, your calculator, using clever mathematics, gives you these values with good accuracy. For example, your calculator gives $\tan 58° \approx 1.600\,334\,529$, which is approximately 1.6.

Make sure your calculator is in *degree* mode. Try entering tan 45 into your calculator and check that you get 1.

Example 4

Use your calculator to find, correct to 3 decimal places:

a $\tan 15°$ **b** $\tan 63°$

Solution

a $\tan 15° \approx 0.268$ **b** $\tan 63° \approx 1.963$

Values of the trigonometric ratios

Notice that since the hypotenuse is the longest side in a right-angled triangle, the ratios $\dfrac{\text{opposite}}{\text{hypotenuse}}$ and $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ are always less than 1.

That is, if $0° < \theta < 90°$, then

$$0 < \sin\theta < 1 \quad \text{and} \quad 0 < \cos\theta < 1$$

Example 5

Use a calculator to find, correct to 3 decimal places:

a $\sin 15°$ **b** $\cos 63°$ **c** $\sin 23\frac{1}{2}°$

Solution

A calculator gives:

a $\sin 15° \approx 0.259$ **b** $\cos 63° \approx 0.454$ **c** $\sin 23\frac{1}{2}° \approx 0.399$

Finding the lengths of sides

We can use sine, cosine and tangent to find the side lengths of right-angled triangles.

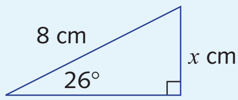
Depending on which sides are given, we choose the appropriate ratio: either sine, cosine or tangent. You should always write down the ratio since the unknown side may be in the denominator.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

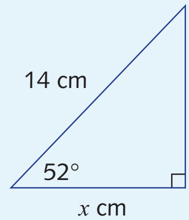
Example 6

Find the value of x, correct to 4 decimal places.

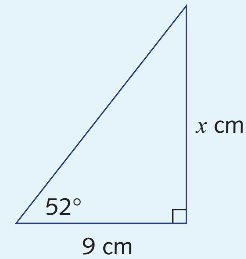
a



b



c



Solution

a This problem involves opposite and hypotenuse, so use sine.

$$\begin{aligned} \frac{x}{8} &= \sin 26^{\circ} \\ x &= 8 \times \sin 26^{\circ} \\ &\approx 3.5070 \end{aligned}$$

b This problem involves adjacent and hypotenuse, so use cosine.

$$\begin{aligned} \frac{x}{14} &= \cos 52^{\circ} \\ x &= 14 \times \cos 52^{\circ} \\ &\approx 8.6193 \end{aligned}$$

c This problem involves opposite and adjacent, so use tangent.

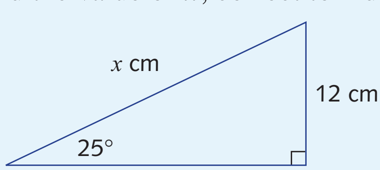
$$\begin{aligned} \frac{x}{9} &= \tan 52^{\circ} \\ x &= 9 \times \tan 52^{\circ} \\ &\approx 11.5195 \end{aligned}$$

If the unknown is the length of the hypotenuse, an extra step is needed in the calculation.

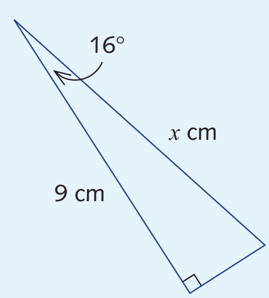
Example 7

Find the value of x, correct to 4 decimal places.

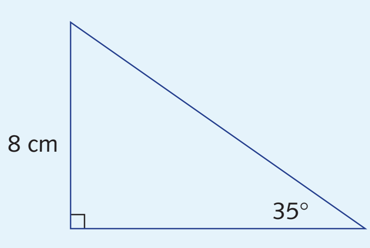
a



b



c



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

a This problem involves opposite and hypotenuse, so use sine.

$$\sin 25° = \frac{12}{x}$$
$$x \sin 25° = 12$$
$$x = \frac{12}{\sin 25°}$$
$$\approx 28.3944$$

b This problem involves adjacent and hypotenuse, so use cosine.

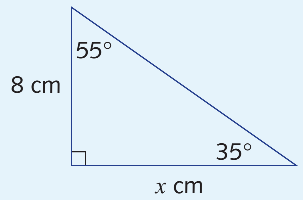
$$\cos 16° = \frac{9}{x}$$
$$x \cos 16° = 9$$
$$x = \frac{9}{\cos 16°}$$
$$\approx 9.3627$$

c This problem involves adjacent and opposite, so use tangent.

$$\tan 35° = \frac{8}{x}$$
$$x \tan 35° = 8$$
$$x = \frac{8}{\tan 35°}$$
$$\approx 11.4252$$

or

$$\frac{x}{8} = \tan 55° \text{ (}55° \text{ is the complement of } 35°\text{)}$$
$$x = 8 \tan 55°$$



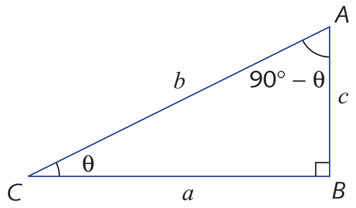
Complementary angles

In the triangle ABC, $\sin\theta = \frac{c}{b}$ and $\cos(90° - \theta) = \frac{c}{b}$. Hence

$\sin\theta = \cos(90° - \theta)$

Also, $\cos\theta = \frac{a}{b}$ and $\sin(90° - \theta) = \frac{a}{b}$. Hence

$\cos\theta = \sin(90° - \theta)$



For example:

$\sin 70° = \cos 20°$ $\quad$ $\cos 40° = \sin 50°$

$\cos 15° = \sin 75°$ $\quad$ $\sin 2° = \cos 88°$

The three basic trigonometric ratios

Let $0° < \theta < 90°$. Then:

- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\quad \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
- $\sin\theta = \cos(90° - \theta)$, $\cos\theta = \sin(90° - \theta)$
- $0 < \sin\theta < 1$ $\quad 0 < \cos\theta < 1$ $\quad \tan\theta > 0$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

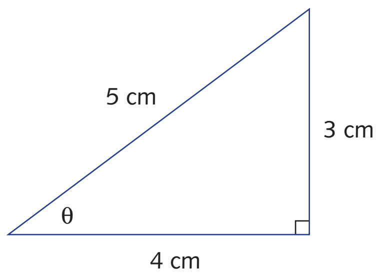
Exercise 13B

Example 2

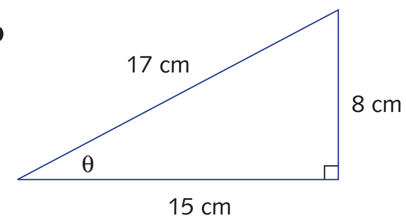
1 Write down the value, as a fraction in simplest form, of:

i sin θ **ii** cos θ **iii** tan θ

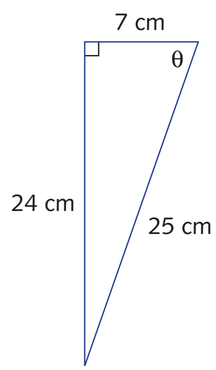
a



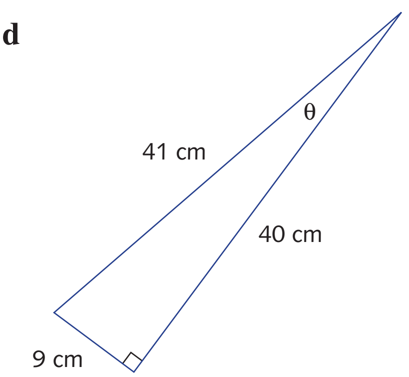
b



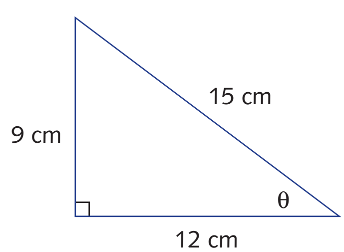
c



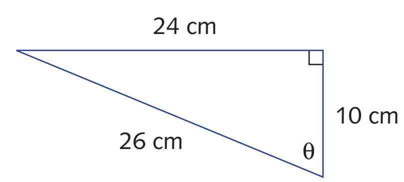
d



e



f



Example 4, 5

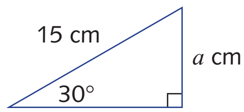
2 Using your calculator, give the value, to 4 decimal places, of:

a sin 10° **b** sin 20° **c** sin 30° **d** cos 46°
e cos 75° **f** tan 49° **g** tan 81° **h** sin 1°
i cos 37° **j** cos 88° **k** tan 3° **l** tan 45°

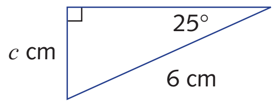
Example 6

3 Find the value of each pronumeral, giving your answer correct to 2 decimal places.

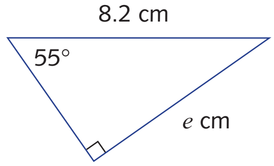
a



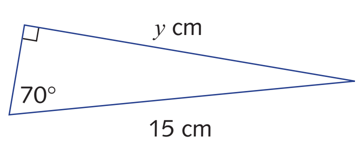
b



c



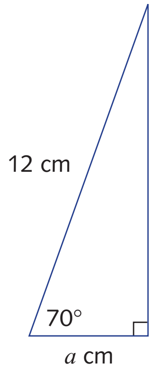
d



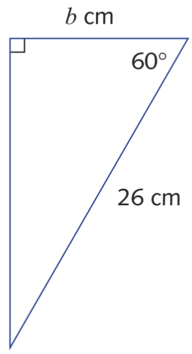
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Find the value of each pronumeral, giving your answer correct to 3 decimal places.

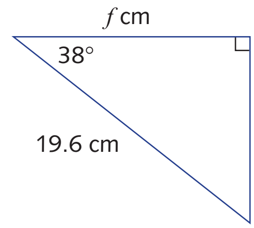
a



b

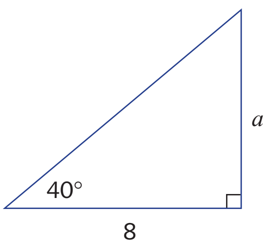


c

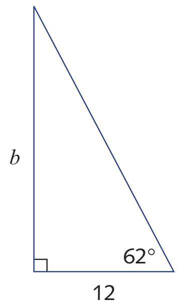


5 Find the value of each pronumeral, giving your answer correct to 4 decimal places.

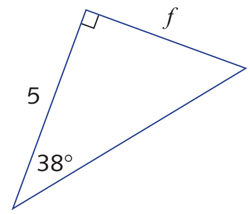
a



b

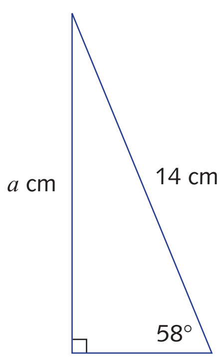


c

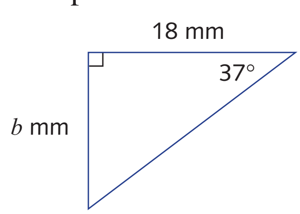


6 Find the value of each pronumeral, correct to 4 decimal places.

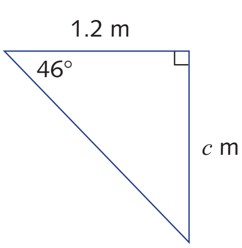
a



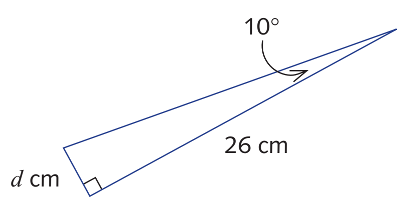
b



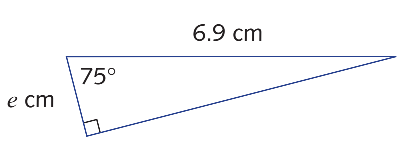
c



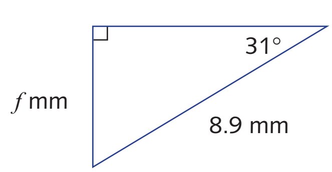
d



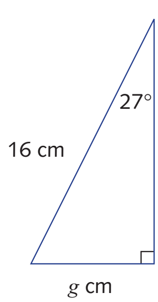
e



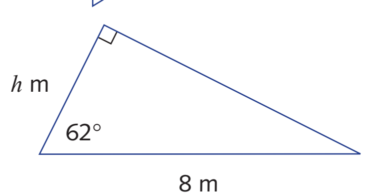
f



g

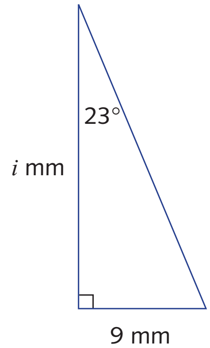


h

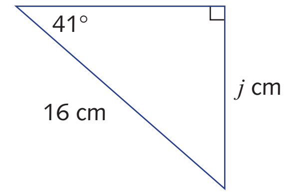


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

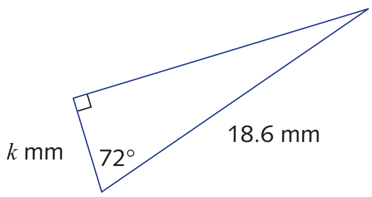
i



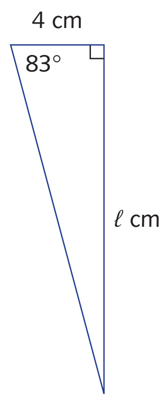
j



k



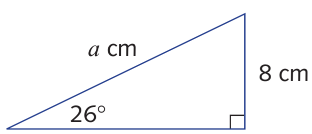
l



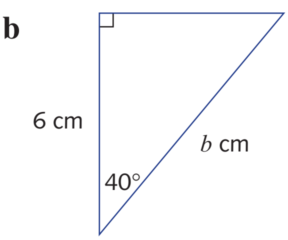
Example 7

7 Find the value of each pronumeral, correct to 4 decimal places.

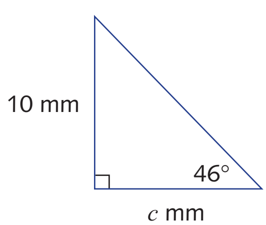
a



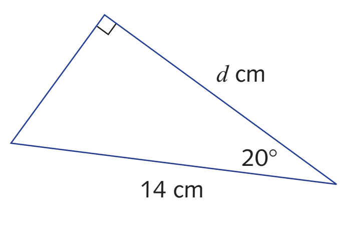
b



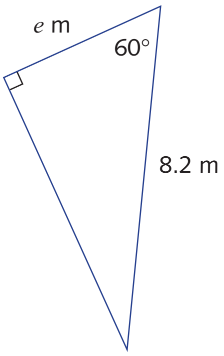
c



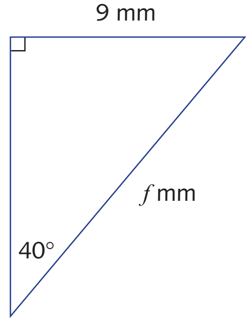
d



e

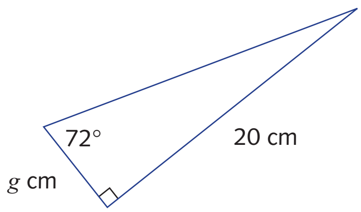


f

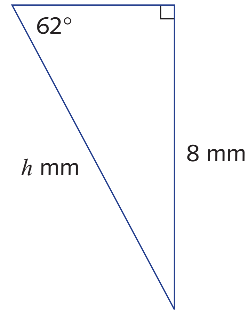


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

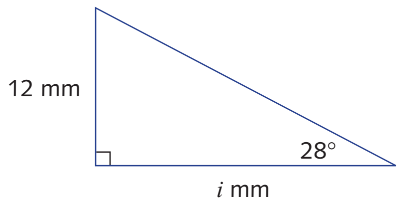
g



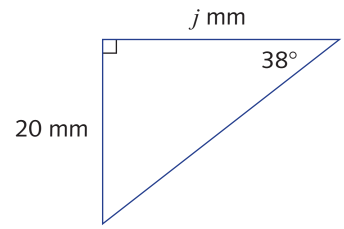
h



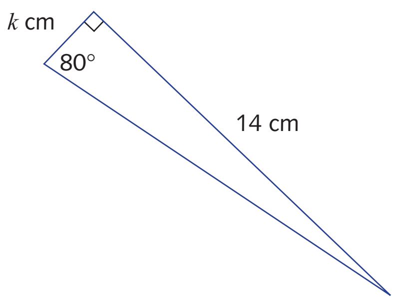
i



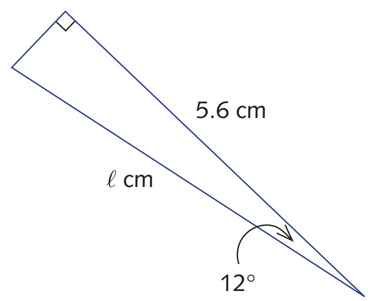
j



k

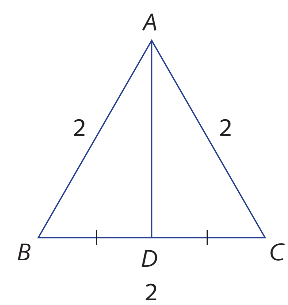


l



8 Draw a right-angled triangle with an angle of 45°. If one of the shorter sides has length 1, write down the lengths of the other sides without using trigonometry. Hence find the exact values of sin 45°, cos 45° and tan 45°.

9 ABC is an equilateral triangle, each of whose sides is 2 units. D is the midpoint of BC.



a What are the angles $\angle ABD, \angle BAD$ and $\angle ADB$?

b What is the length of BD?

c Find the length of AD in surd form.

d Use triangle ABD to complete the following table with exact values (in surd form where necessary).

θ	30°	60°
sin θ		
cos θ		
tan θ		

10 Use complementary angles to complete the following.

a sin 5° = cos …

b cos 72° = sin …

c sin 65° = cos …

d cos 46° = sin …

11 Draw a diagram and explain why $\tan(90° - \theta) = \dfrac{1}{\tan\theta}$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

In the following questions, $0° < \theta < 90°$.

12 Find a value of θ for which $\sin\theta = \cos\theta$.

13 **a** Which is the largest side of a right-angled triangle?

b Is it possible for values of $\sin\theta$ to be greater than 1? Why?

c Is it possible for values of $\cos\theta$ to be greater than 1? Why?

d Is it possible for values of $\tan\theta$ to be greater than 1? Why?

14 **a** What happens to the value of $\sin\theta$ as θ gets closer and closer to $0°$?

b What happens to the value of $\sin\theta$ as θ gets closer and closer to $90°$?

c Can you explain why the value of $\sin\theta$ behaves like this?

15 Repeat Question **13** for:

i $\cos\theta$ **ii** $\tan\theta$.

13C Finding angles

We have seen that in any right-angled triangle with reference angle θ, there are three basic ratios associated with that angle and that the value of each ratio can be found using the calculator. In order to use the trigonometric ratios to find angles in a right-angled triangle, we need to reverse the process. Thus, given the sine, cosine or tangent of some angle between $0°$ and $90°$, we want to find the angle with the given ratio.

We have seen that $\tan 45° = 1$. We say that $45°$ is the **inverse tangent** of 1.

This is written as $\tan^{-1} 1 = 45°$

We use the calculator to find $\tan^{-1} x$. For example, $\tan 37° = 0.735\ldots$ and $\tan^{-1} 0.735$ is approximately equal to $37°$, correct to 2 decimal places.

Similarly, $\sin 30° = \frac{1}{2}$. We say $30°$ is the **inverse sine** of $\frac{1}{2}$ and write $\sin^{-1} 0.5 = 30°$

This notation is standard, but can be rather misleading. The index -1 does NOT mean 'one over' as it normally does in algebra. To help you avoid confusion, you should always read $\sin^{-1} x$ as *inverse sine of x* and $\tan^{-1} x$ as *inverse tan of x* and so on.

Similarly, a calculator gives $\cos 35° \approx 0.8192$ and $\cos^{-1} 0.8192 \approx 35°$ (read this as **inverse cosine** of 0.8192).

Finding angles in right-angled triangles

- If $x > 0$, then $\tan^{-1} x$ is the angle whose tangent is x.
- If $0 < x < 1$, then $\sin^{-1} x$ is the angle whose sine is x.
- If $0 < x < 1$, then $\cos^{-1} x$ is the angle whose cosine is x.
- Tangent, sine and cosine are applied to a reference angle to determine a specific ratio of side lengths in a right-angled triangle.
- Inverse tangent, inverse sine and inverse cosine are applied to a ratio to determine the size of a specific reference angle.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 8

Find, correct to the nearest degree:

a $\sin^{-1} 0.6$ **b** $\cos^{-1} 0.412$ **c** $\tan^{-1} 2$

d the angle θ for which $\sin\theta = 0.8$

e the angle θ for which $\cos\theta = 0.2$ **f** $\cos^{-1} 2$

Solution

a By calculator

$$\begin{aligned}\sin^{-1} 0.6 &= 36.869\,89\ldots^\circ \\ &\approx 37^\circ\end{aligned}$$

b $$\begin{aligned}\cos^{-1} 0.412 &= 65.669\,46\ldots^\circ \\ &\approx 66^\circ\end{aligned}$$

c $$\begin{aligned}\tan^{-1} 2 &= 63.434\,94\ldots^\circ \\ &\approx 63^\circ\end{aligned}$$

d if $\sin\theta = 0.8$

then $$\begin{aligned}\theta &= \sin^{-1} 0.8 \\ &= 53.13\ldots^\circ \\ &\approx 53^\circ\end{aligned}$$

e If $\cos\theta = 0.2$

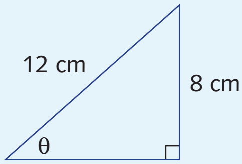
then $$\begin{aligned}\theta &= \cos^{-1} 0.2 \\ &= 78.463\ldots^\circ \\ &\approx 78^\circ\end{aligned}$$

f $2 > 1$, so $\cos^{-1} 2$ is not defined.

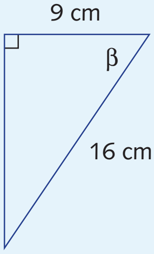
Example 9

Find each angle, correct to the nearest degree.

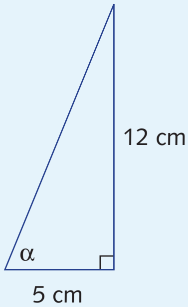
a



b



c



Solution

a $\sin\theta = \dfrac{8}{12} = \dfrac{2}{3}$

so $\theta = \sin^{-1}\dfrac{2}{3}$

$\approx 42^\circ$

b $\cos\beta = \dfrac{9}{16}$

so $\beta = \cos^{-1}\dfrac{9}{16}$

$\approx 56^\circ$

c $\tan\alpha = \dfrac{12}{5}$

so $\alpha = \tan^{-1}\dfrac{12}{5}$

$\approx 67^\circ$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 13C

Example 8a, b, c

1 Find, correct to the nearest degree:

a $\sin^{-1} 0.7$ **b** $\sin^{-1} 0.732$ **c** $\cos^{-1} 0.713$

d $\cos^{-1} 0.1234$ **e** $\tan^{-1} 0.1$ **f** $\tan^{-1} 12$

Example 8d, e

2 Find θ, correct to the nearest degree.

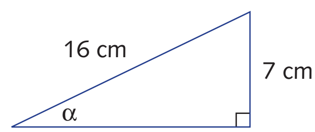
a $\sin\theta = 0.4067$ **b** $\sin\theta = 0.8480$ **c** $\cos\theta = 0.9816$

d $\cos\theta = 0.5299$ **e** $\tan\theta = 1.5399$ **f** $\tan\theta = 4.0108$

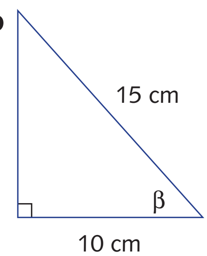
Example 9

3 Find the size of each marked angle, correct to the nearest degree.

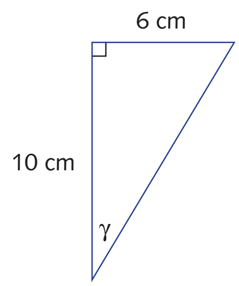
a



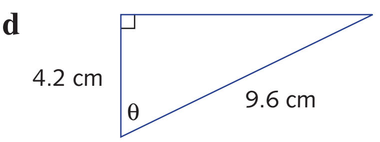
b



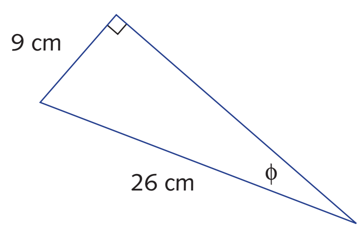
c



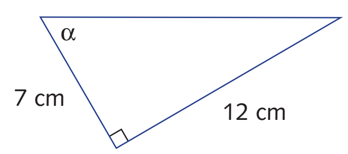
d



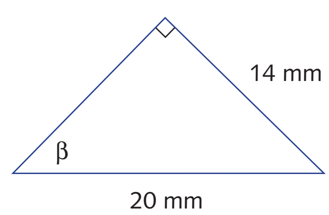
e



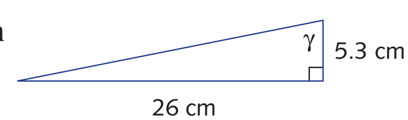
f



g



h



4 What are the sizes of the angles, correct to the nearest degree, in a:

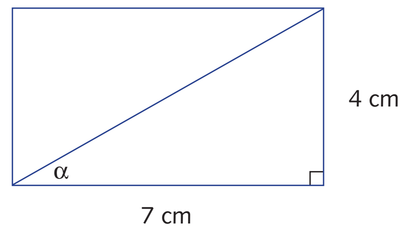
a 3, 4, 5 right-angled triangle?

b 5, 12, 13 right-angled triangle?

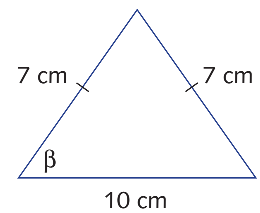
c 7, 24, 25 right-angled triangle?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

5 Find the angle between the diagonal and the base of the rectangle, correct to 2 decimal places.



6 Find the base angle in the isosceles triangle, correct to 2 decimal places.

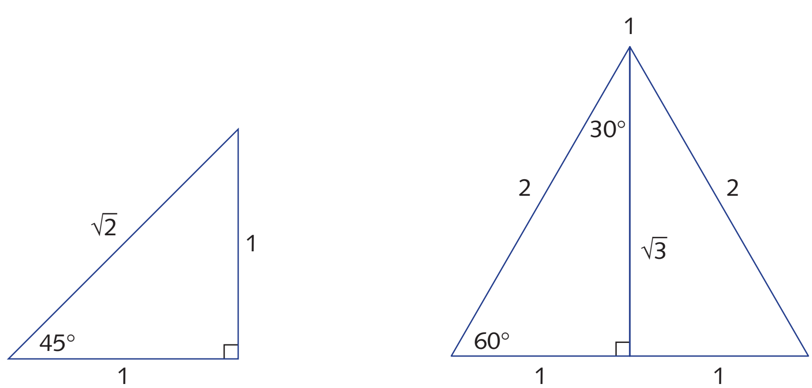


7 The diagonals of a rhombus have lengths 7 cm and 10 cm. Find the angles of the rhombus correct to 2 decimal places.

13D Miscellaneous exercises

The following exercise reinforces the techniques introduced in the previous sections.

You will have found the exact values of sine, cosine and tangent for 30º, 45º and 60º in previous exercises. These values are exemplified in the following triangles. One is a right-angled isosceles triangle with shorter sides of 1 unit, and the other is an equilateral triangle with side lengths 2 and with an angle bisector drawn.



These trigonometric ratios are given in the table below.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

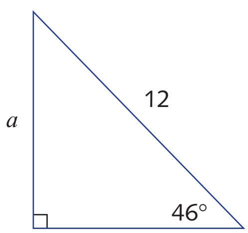
Check the details in the triangles and the entries in the table. We can either learn the table or remember the diagrams to construct the table. Note that, by complementary angles, $\sin 30° = \cos 60°$, $\sin 60° = \cos 30°$ and $\sin 45° = \cos 45°$.

Knowing that $\cos 60° = \frac{1}{2}$ and $\tan 45° = 1$, we can quickly reconstruct the triangles to find all the values.

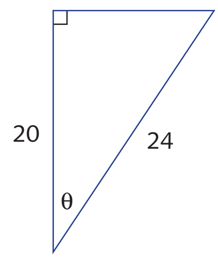
Exercise 13D

1 Find the value of the pronumerals, giving angles to the nearest degree and lengths correct to 2 decimal places.

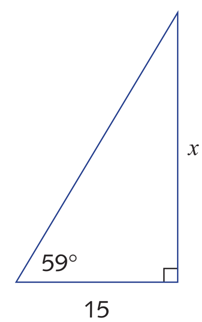
a



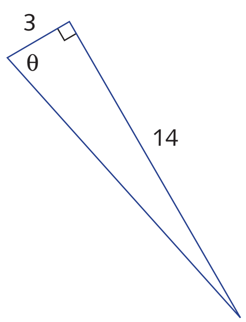
b



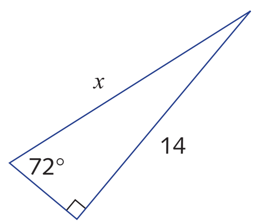
c



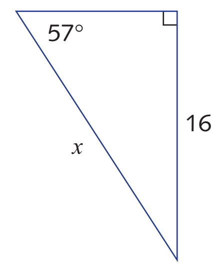
d



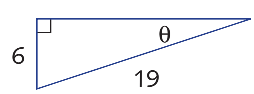
e



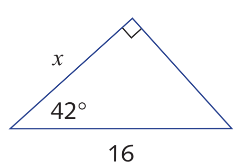
f



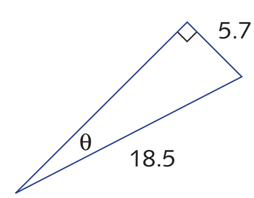
g



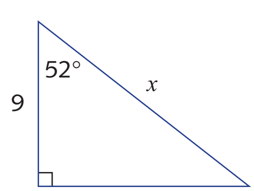
h



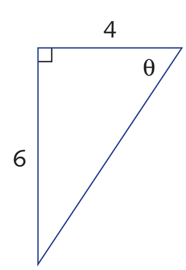
i



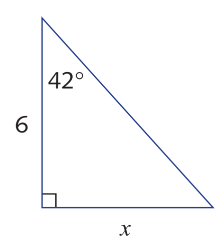
j



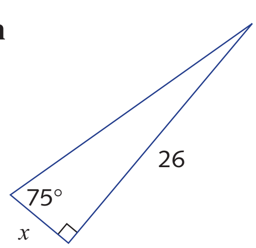
k



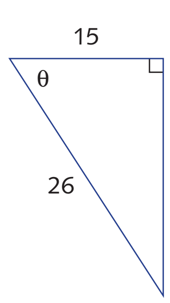
l



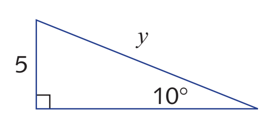
m



n



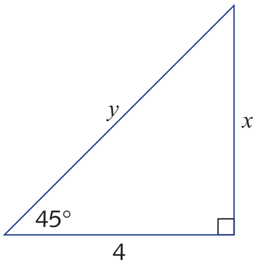
o



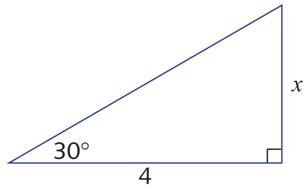
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 Find the exact value of the pronumerals.

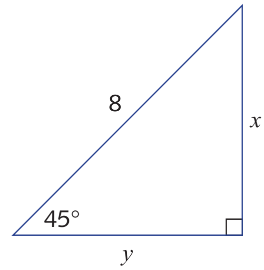
a



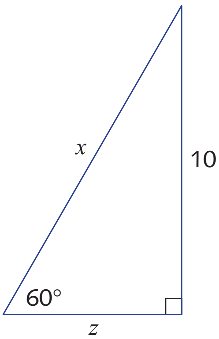
b



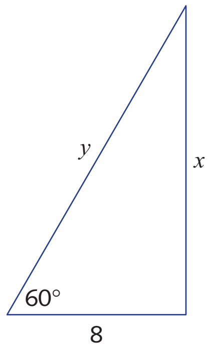
c



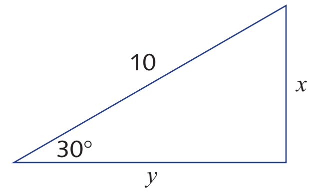
d



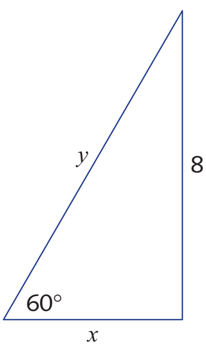
e



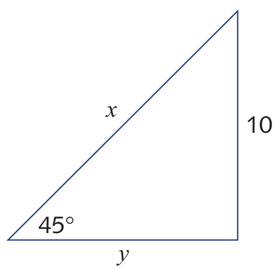
f



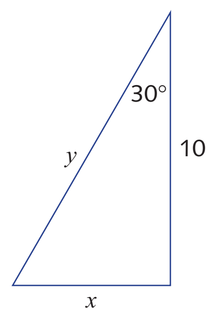
g



h



i



13E Solving problems using trigonometry

The results of the previous sections can be applied to practical problems.

In tackling each problem you should draw a diagram and clearly identify the side or angle you are asked to find.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

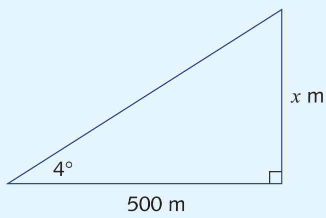
Example 10

A plane climbs at an angle of 4° after take-off. Find its altitude when it has flown a horizontal distance of 500 m.

Solution

Let the altitude be x m.

$$\text{Then } \tan 4° = \frac{x}{500}$$
$$x = 500 \tan 4°$$
$$\approx 35$$



Thus, the plane's altitude is 35 m (to the nearest metre).

Example 11

A rope, 4 m long, is attached to a vertical pole. The rope, held taut, is pegged into the ground 2 m from the base of the pole.

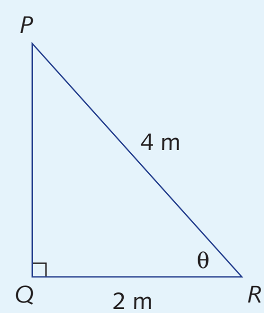
a Find the angle the rope makes with the ground.

b Find the angle the rope makes with the pole.

Solution

a Let PQ be the pole and θ be the angle between the rope and the ground.

$$\text{Then } \cos\theta = \frac{2}{4}$$
$$= \frac{1}{2}$$
$$\text{Hence} \quad \theta = \cos^{-1}\frac{1}{2}$$
$$= 60°$$



Thus, the rope makes an angle of 60° with the ground.

b $\angle QPR = 90° - 60°$

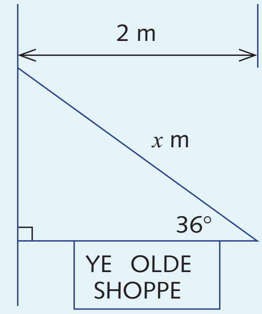
$= 30°$

Thus, the rope makes an angle of 30° with the pole.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 12

A shop sign is supported by two rods. One rod is horizontal and is of length 2 m. The other rod is of length x m and makes an angle of 36° with the horizontal. Find its length correct to 2 decimal places.



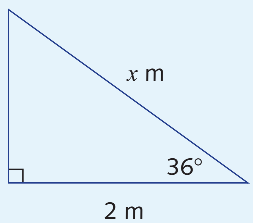
Solution

From the diagram $\cos 36° = \dfrac{2}{x}$

$$x \cos 36° = 2$$

Hence $x = \dfrac{2}{\cos 36°}$

$$\approx 2.47$$



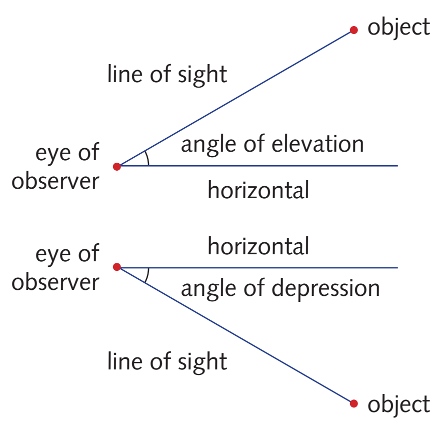
Therefore the rod is of length 2.47 m (correct to 2 decimal places).

Angles of elevation and depression

When a person looks at an object that is higher than the person, the angle between the line of sight and the horizontal is called the **angle of elevation**.

On the other hand, when the object is lower than the person, the angle between the horizontal and the line of sight is called the **angle of depression**.

Angles of elevation and angles of depression are always measured from the horizontal.



Example 13

From a point P, 30 m from a building, the angle of elevation of the top of the building is 41°. Find the height of the building, correct to the nearest metre.

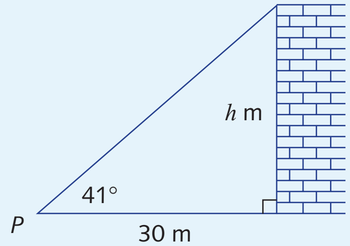
Solution

Let h m be the height of the building.

Now $\tan 41° = \dfrac{h}{30}$

hence $h = 30 \tan 41°$

$$\approx 26$$



So the building is 26 m high (correct to the nearest metre).

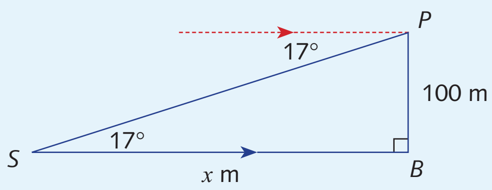
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 14

From the top of a cliff, 100 m above sea level, the angle of depression to a ship sailing below is 17°. How far is the ship from the base of the cliff, to the nearest metre?

Solution

The diagram to the right shows the top of the cliff P, the ship S and the base of the cliff B. Let $SB = x$ m be the distance of the ship from the cliff. By alternate angles, $\angle PSB = 17°$.



Hence $\tan 17° = \dfrac{100}{x}$

so $x = \dfrac{100}{\tan 17°}$

$= 327.085$

The distance is 327 m (correct to the nearest metre).

Compound problems

Example 15

From a point A, 10 m from the base of a tree of height h m, the angle of elevation of the top of a tree is 36°. From a point B, x m further away from the base of the tree, the angle of elevation is 20°.

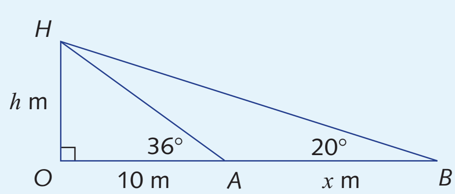
a Draw a diagram to illustrate the information given.

b Find the height of the tree, correct to the nearest tenth of a metre.

c Find the distance x m, correct to the nearest tenth of a metre.

Solution

a



b In ΔOHA $\quad \tan 36° = \dfrac{h}{10}$

so $h = 10 \tan 36°$

$= 7.2654...$ (leave this answer in your calculator)

≈ 7.3 (correct to the nearest tenth of a metre)

(*Continued over page*)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party. Updated August 2021

c In ΔHOB $\quad \dfrac{h}{OB} = \tan 20°$

Hence $\quad OB = \dfrac{h}{\tan 20°}$

$= \dfrac{10 \tan 36°}{\tan 20°}$

$= 19.96...$

so $\quad x = AB$

$= OB - OA$

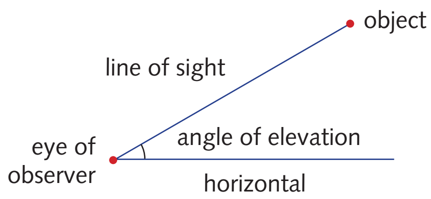
$= 9.96$

≈ 10.0 (correct to the nearest tenth of a metre)

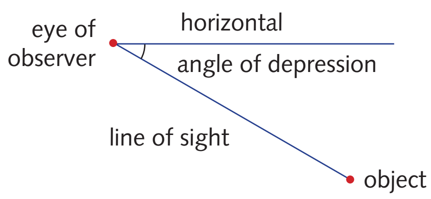
Note: Do not re-enter approximations. Keep values in your calculator.

Angles of elevation and depression

- When the object is higher than the person, the angle between the line of sight and the horizontal is called the angle of elevation.



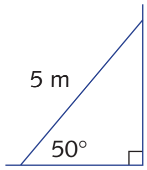
- When the object is lower than the person, the angle between the line of sight and the horizontal is called the angle of depression.



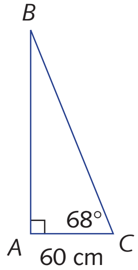
Exercise 13E

Example 10

1 A 5 m ladder leans against a wall, making an angle of 50° with the floor. How far up the wall (correct to the nearest centimetre) is the top of the ladder?

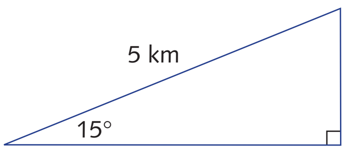


2 A girl's shadow is 60 cm long. If the line joining the top of the girl's head and the end of the shadow makes an angle of 68° with the ground, how tall is the girl (correct to the nearest centimetre)?



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Gregor drives up a hill that is inclined at an angle of 15° to the horizontal. If he drives for 5 km, through what vertical distance has he risen? Give your answer correct to the nearest metre.



4 A tower is held in place by a rope of length 25 m. The rope is attached to the top of the tower and makes an angle of 76° with the ground. Find:

a the height of the tower, correct to the nearest centimetre

b how far, to the nearest centimetre, the base of the tower is from the anchor point

5 A ladder of length 4 m is leaning against a wall so that the top of the ladder is at the top of the wall. If the ladder makes an angle of 7° with the wall, find, correct to the nearest centimetre:

a the height of the wall

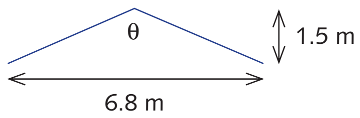
b how far the base of the ladder is from the wall

Example 11

6 A skateboard ramp is 5 m long and rises a vertical distance of 2 m. What angle does the ramp make with the horizontal ground? Give your answer correct to the nearest degree.

7 An isosceles triangle has base length 10 cm and the other two sides are each of length 6 cm. Find the angles in the triangle, giving your answers correct to the nearest degree.

8 A roof truss is 6.8 m wide and 1.5 m high. Calculate the angle θ between the two beams, correct to the nearest degree.



9 A horizontal awning that has a depth of 80 cm is attached to the top of a window of height 2 m. What is the least angle that a ray of sunlight can make with the window if no light is to shine directly into it? Give your answer correct to the nearest tenth of a degree.

Example 13

10 The angle of elevation from a point on a horizontal surface to the top of a vertical pole 20 m away is 35°. Find the height of the pole, correct to the nearest metre.

Example 14

11 From a point on top of a tower of height 5.8 m, the angle of depression to a swimmer is 16°. How far from the base of the tower is the swimmer? Give your answer correct to the nearest tenth of a metre.

12 Two points in space 25 m apart are such that the vertical distance between them is 18 m. Find the angle of elevation or depression between the two points. Give your answer correct to the nearest degree.

13 Two bushwalkers, Gore and Tess, are testing new electronic measuring devices. Gore is on a mountain side at the 2300 m contour line. Tess is at the 2150 m contour line on a different mountain. Their measuring devices say that the direct distance between them is 1.35 km. Find the angle of elevation, correct to 2 decimal places, from Tess to Gore.

14 Carlos is standing 16 m from the base of a tree that is 14 m high. Carlos's eye is 1.6 m from the ground.

a What is the angle of elevation from Carlos's eye to the top of the tree, correct to the nearest degree?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b If Carlos looks up at an angle of 26°, how far up the tree will he see? Give your answer correct to the nearest centimetre.

c What is the angle of elevation from Carlos's eye to a point halfway up the tree, correct to the nearest degree?

15 From a point 100 m from the base of a building, the angle of elevation to the top of the building is 31°.

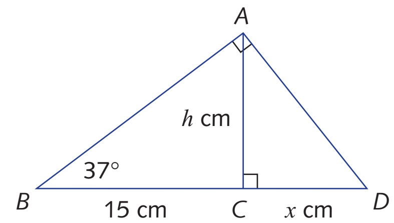
a How high is the building, correct to the nearest tenth of a metre?

b If there is a vertical flag pole of height 15 m on top of the building, what is the angle of elevation of the top of the flag pole? Give your answer correct to the nearest degree.

16 For the diagram on the right:

a find the value of h, correct to 2 decimal places

b use this value to find x, correct to 2 decimal places

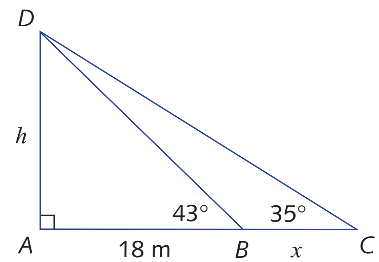


Example 15 **17** For the diagram on the right:

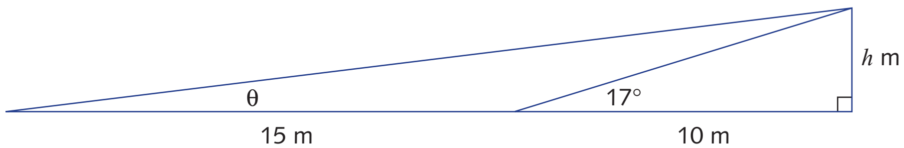
a find the value of h, correct to the nearest centimetre

b use this value to find the length AC, correct to the nearest centimetre

c hence find x, correct to the nearest centimetre



18 For the diagram below:



a find the value of h, correct to the nearest centimetre

b hence find the value of θ, correct to the nearest degree

19 The top of a 25 m tower is observed from two points, A and B, in line with the base of the tower, but on opposite sides of it. The angle of elevation of the top of the tower is 26° from A and 34° from B.

a Draw a diagram to represent the information above.

b How far apart are the points A and B, correct to the nearest centimetre?

20 At a point A on the ground, 12 m from the base of a building, the angle of elevation of the top of the building is 51°. From a point B on the ground x m further out from A, in line with the base of the building, the angle of elevation is 40°.

a Draw a diagram to represent the information above.

b Find the height of the building to the nearest centimetre.

c Find the value of x correct to the nearest centimetre.

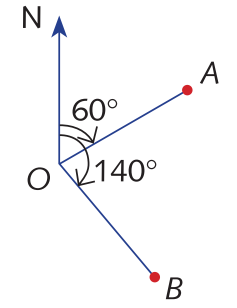
21 The line $y = 3x + 6$ crosses the x-axis at an angle of θ with the positive direction of the x-axis. Find the value of θ, correct to 1 decimal place.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

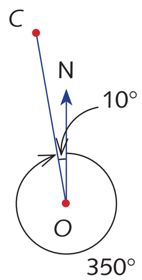
13F True bearings

Bearings are used to indicate the direction of an object from a fixed reference point. **True bearings** give the angle $\theta°$ from the north, measured clockwise. We write a true bearing of $\theta°$ as $\theta°$T, where $\theta°$ is an angle between 0° and 360°. It is customary to write the angle using three digits, so 0°T is written 000°T, 15°T is written 015°T and so on.

For example, the true bearing of A from O is 060°T, and the true bearing of B from O is 140°T.

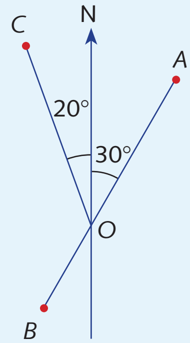


The true bearing of C from O is 350°T.



Example 16

Write down the true bearings of A, B and C from O.



Solution

The true bearing of A from O is 030°T.

For B, $30° + 180° = 210°$, so the true bearing of B from O is 210°T.

For C, $360° - 20° = 340°$, so the true bearing of C from O is 340°T.

Example 17

Anthony walks for 490 m on a true bearing of 140°T from point A to point B.

Find, correct to the nearest metre:

a how far east the point B is from point A

b how far south the point B is from point A

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a Distance east $= 490 \sin 40°$

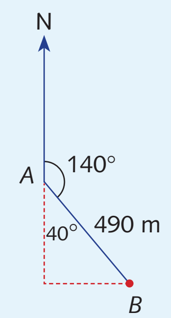
$= 314.96...$

≈ 315 m

b Distance south $= 490 \cos 40°$

$= 375.36...$

≈ 375 m



Example 18

The bearing of B from A is 075°. Find the bearing of A from B.

Solution

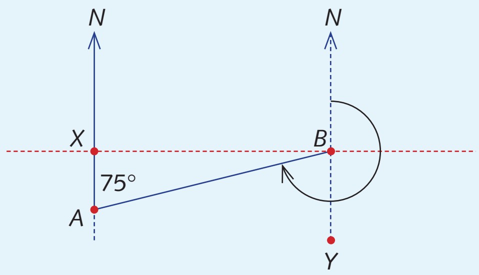
Mark the direction of B from A with respect to the direction of north, then re-draw a compass cross with origin at B.

$\angle ABY = 75°$ (alternate)

$\angle ABX = 15°$ (sum of angles in triangle ABX)

$180° + 75° = 255°$ (or $270° - 15° = 255°$)

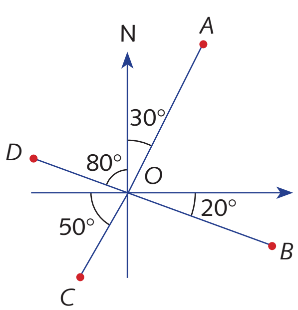
So the true bearing of A from B is 255°.



Exercise 13F

Example 16

1 For the diagram opposite, write down the true bearing of points A, B, C and D from O.



2 Draw a diagram to illustrate the fact that:

a the bearing from O to Z is 160°T

b the bearing from P to Q is 340°T

c to get from C to D, you travel on a bearing of 210°T

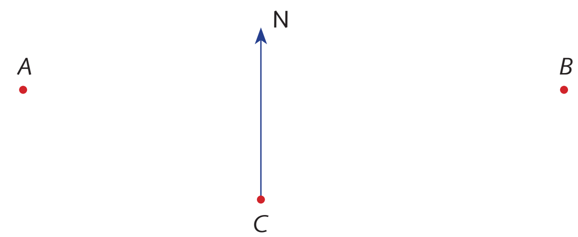
d to get from A to Y, you travel on a bearing of 170°T

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Using a protractor, measure the bearing of:

a A from C

b B from C



Example 17

4 Jo hikes for 369 m on a bearing of 053°T from point A to point B. Find, correct to the nearest metre:

a how far east point B is from point A

b how far north point B is from point A

5 A ship leaves port and sails on a bearing of 160°T for 200 km. Find, correct to the nearest tenth of a kilometre:

a how far south the ship is from the port

b how far east the ship is from the port

6 During an orienteering competition, Fran runs 1600 m on a bearing of 316°T from checkpoint 1 to checkpoint 2.

a How far west of checkpoint 1 is checkpoint 2, correct to the nearest metre?

b How far north of checkpoint 1 is checkpoint 2, correct to the nearest metre?

Example 18

7 Alan is standing 100 m due east of a landmark. Benny is standing 250 m due south of the landmark. What is the bearing, correct to the nearest tenth of a degree, of:

a Benny from Alan?

b Alan from Benny?

8 Christos is 200 m from a tree and Danielle is 400 m from the same tree. What is the bearing of Danielle from Christos, to the nearest degree, if:

a Christos is due north of the tree and Danielle is due east of the tree?

b Christos is due west of the tree and Danielle is due south of the tree?

c Christos is due east of the tree and Danielle is due south of the tree?

9 A hiker starts at point O and travels for 6 km on a bearing of 070°T to a point A. The hiker wishes to travel from A to a point B, which is 15 km due east of O. Give answers correct to the nearest hundredth of a kilometre and to the nearest degree.

a How far east of O is A?

b How far north of O is A?

c How far east of A is B?

d How far south of A is B?

e How far is B from A?

f On what bearing does the hiker travel if she walks directly from A to B?

10 A car travels on a bearing of 280°T on a very long straight road at 85 km/h, starting from an initial point at 12 p.m. Find how far north the car is from its original point at 12:30 p.m., correct to one-tenth of a kilometre.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

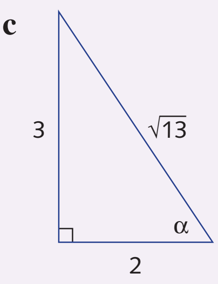
Review exercise

1 Write down the sine, cosine and tangent ratio of the indicated angle. Leave answers in exact form.

a

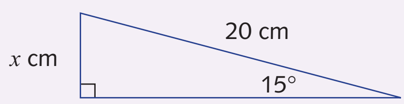
b

c

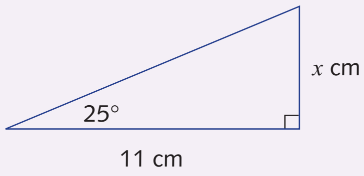


2 Find the value of each pronumeral, correct to 2 decimal places.

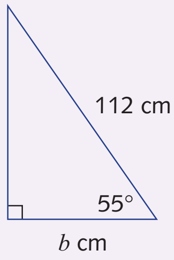
a



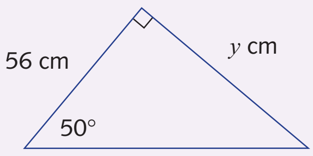
b



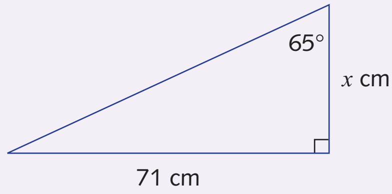
c



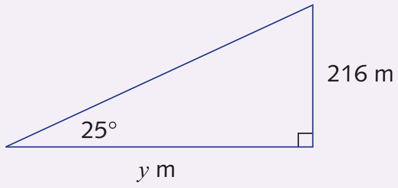
d



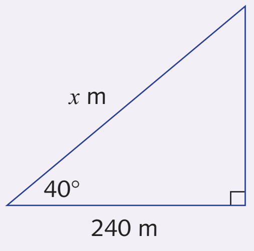
e



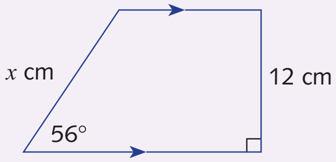
f



g



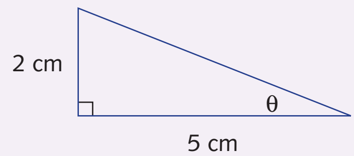
h



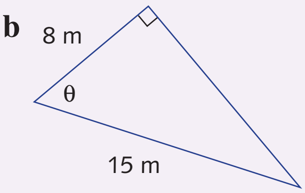
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Find the value of each pronumeral, correct to the nearest degree.

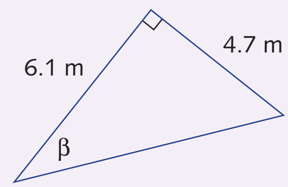
a



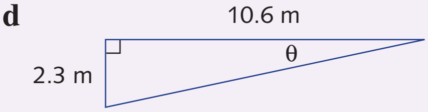
b



c



d



4 A ladder 9 m long is resting against a wall, making an angle of 55° with the ground.

a How high up the wall does the ladder reach, correct to the nearest centimetre?

b How far away from the wall is the foot of the ladder, correct to the nearest centimetre?

5 A ship is travelling due north at 15 km/h. The navigator sees a light due east, and eight minutes later the light has a bearing of 125°T from the ship. What is the distance from the ship to the light each time the navigator sees it, correct to the nearest metre?

6 The sides of a rectangle are 4 cm and 5 cm in length.

a Find the angles that a diagonal makes with the sides, correct to the nearest degree.

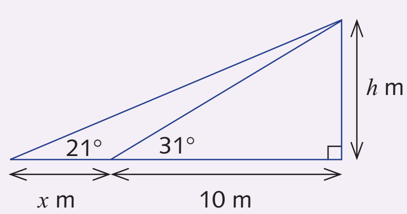
b What are the angles of intersection of the diagonals, correct to the nearest degree?

7 The angle of elevation of the top of a vertical cliff is observed to be 41° from a boat 450 m from the base of the cliff. What is the height of the cliff, correct to the nearest metre?

8 Find the angle of depression, to the nearest degree, of a boat 500 m out to sea as observed from the top of a cliff of height 90 m.

9 A ship sails on a bearing of 156°T until it is 45 km south of its starting point. How far has it travelled, correct to 2 decimal places?

10



a Find the height h, correct to the nearest centimetre.

b Find the value of x, correct to the nearest centimetre.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 The 'good ship' is at A, 1 km due north of the 'enemy ship' at 12 p.m. After this time the good ship moves on a bearing of 045°T while the enemy ship remains stationary.

a The good ship has travelled 1.5 km from A. Find, to the nearest metre:

i how far east it is from its position at 12 p.m.

ii how far north it is from its position at 12 p.m.

iii how far it is from the enemy ship

b Suppose that the good ship travels from A on the same bearing for x km. Find the distance between the ships d km, in terms of x.

c Finally, suppose that the good ship travels from A at a speed of 10 km/h. How far apart are the ships at 2:45 p.m. that day, correct to 2 decimal places?

2 A police helicopter, hovering at an altitude of 1000 m, observes a car travelling along a straight highway. At that instant, the angle of depression of the car from the helicopter is 27°. Five seconds later, the angle of depression of the car from the helicopter is 26°.

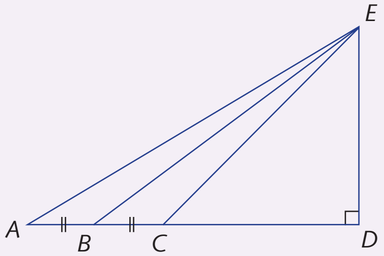
a What is the horizontal distance the car is from the helicopter when it is first sighted, giving your answer correct to the nearest tenth of a metre?

b Calculate the horizontal distance the car is from the helicopter 5 seconds after it is first sighted, giving your answer correct to the nearest tenth of a metre.

c Find the distance the car has travelled in the 5 seconds, giving your answer correct to the nearest tenth of metre.

d What is the speed at which the car is travelling in metres per second, correct to 1 decimal place?

3 Consider the diagram shown below.



a Prove that $BD = \frac{1}{2}(AD + CD)$.

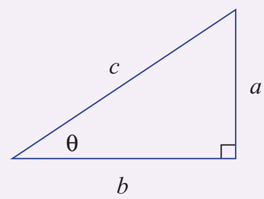
b Hence, deduce that $\tan DEB = \frac{1}{2}\tan\angle AED + \frac{1}{2}\tan\angle CED$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

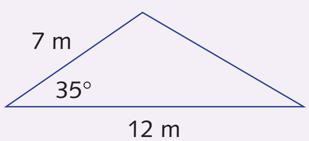
4 Consider the diagram shown below.

a Write down the values of $\sin\theta$ and $\cos\theta$.

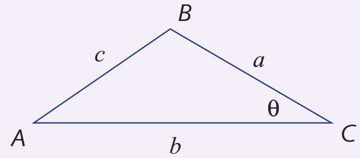
b Show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$.



5 **a** Find the area in square metres of the triangle shown below, correct to 2 decimal places.

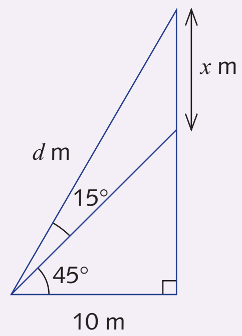


b Prove that the area of triangle ABC is $\frac{1}{2}ab\sin\theta$.



6 **a** For the diagram shown below, find the value of d.

b Show that $x = 10(\sqrt{3} - 1)$.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

14

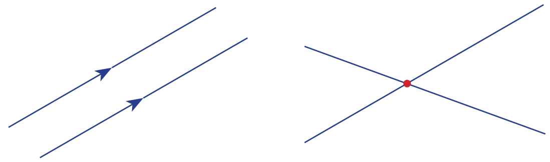
Number and Algebra

Simultaneous linear equations

The pair of equations $x - y = 4$ and $x + y = 6$ are called simultaneous equations. We wish to find the numbers x and y that satisfy both equations simultaneously. In this case $x = 5$ and $y = 1$.

We know that the x- and y-coordinates of any point on a straight line satisfy the linear equation of the line. Hence the solution $x = 5$, $y = 1$ to the linear equations $x - y = 4$ and $x + y = 6$ is the point where the two lines meet.

In this chapter we will be solving simultaneous linear equations. Since the two linear equations correspond to two distinct lines, the geometry of the situation is very simple – the lines are either parallel or they meet at a single point.

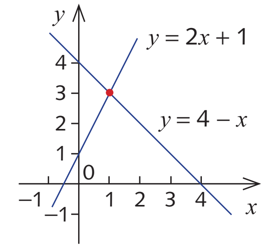


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

14A Solving simultaneous equations by drawing graphs

This introductory section shows how it is possible to solve simultaneous equations by drawing graphs. Care must be taken to make the graphs accurate, and it is important to check that the values found for x and y correspond to a point lying on both lines.

For example, the graph of $y = 2x + 1$ shown opposite consists of all points (x, y) that satisfy $y = 2x + 1$.



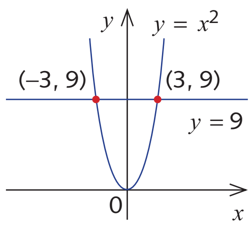
The graph of $y = 4 - x$ is drawn on the same set of axes. We see that the two lines intersect at the point $(1, 3)$. Since $(1, 3)$ is on both lines, $x = 1$ and $y = 3$ satisfy both equations. We can verify this by substitution.

We say that $x = 1$ and $y = 3$ satisfy the pair of equation **simultaneously**. $\begin{cases} y = 2x + 1 \\ y = 4 - x \end{cases}$

Another way of saying this is that $x = 1$, $y = 3$ is the **solution** to the pair of **simultaneous equations** $\begin{cases} y = 2x + 1 \\ y = 4 - x \end{cases}$

In this chapter we will only be considering linear equations, but note what happens when we look at the simultaneous equations $y = x^2$ and $y = 9$. One solution is $x = 3$ and $y = 9$, and another solution is $x = -3$ and $y = 9$.

So in general, two simultaneous equations may have more than one solution.



Example 1

Solve the simultaneous equations

$y = x + 3$

$y = 2x + 1$

by drawing the graphs of the corresponding lines on the one set of axes.

Solution

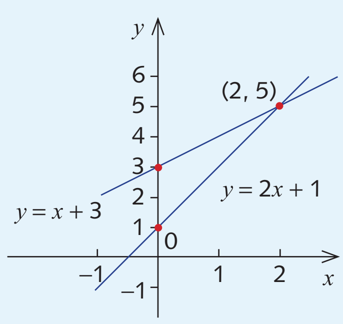
To sketch the graph of a line, we plot two points and join them (we can plot a third point as a check).

For $y = x + 3$:

x	–1	0	1
y	2	3	4

For $y = 2x + 1$:

x	–1	0	1
y	–1	1	3



We see from the diagram that the two lines intersect at $(2,5)$.

That is, the solution to the simultaneous equations is $x = 2$ and $y = 5$.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

We must check by substitution that $x = 2$ and $y = 5$ satisfy both equations.

$y = x + 3$ gives $5 = 2 + 3$ ✓

$y = 2x + 1$ gives $5 = 2 \times 2 + 1$ ✓

Note: If the check does not work, then the graphs are not drawn accurately enough.

Example 2

Solve the simultaneous equations

$x + 2y = 7$

$y = x - 1$

by drawing the graphs of the corresponding lines on the one set of axes.

Solution

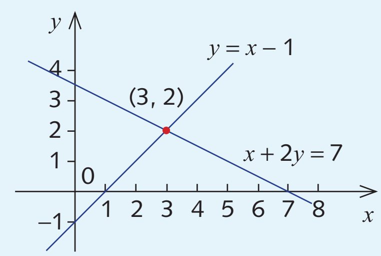
This time we draw the graphs by finding the two intercepts of each line.

If $x + 2y = 7$ and $x = 0$, then $y = 3\frac{1}{2}$

If $x + 2y = 7$ and $y = 0$, then $x = 7$

Hence $\left(0, 3\frac{1}{2}\right)$ and $(7, 0)$ are two points on the line.

Similarly, $(0, -1)$ and $(1, 0)$ determine the line $y = x - 1$.



From the graph, the lines meet at $(3, 2)$, so the solution to the simultaneous equations is $x = 3$ and $y = 2$.

Check by substitution: $x + 2y = 7$ gives $3 + 2 \times 2 = 7$ ✓

$y = x - 1$ gives $2 = 3 - 1$ ✓

Exercise 14A

In this exercise it is important that you sketch the graphs accurately. The use of grid paper is recommended.

1 **a** Determine by substitution whether (1, 2) is a solution of the simultaneous equations $x + y = 3$ and $y = x + 1$.

b Determine by substitution whether $(-1, 6)$ is a solution of the simultaneous equations $x + y = 3$ and $y = x + 7$.

Example 1

2 **a** On the same set of axes, plot the graphs of:

i $y = x$ **ii** $y = 2x + 1$ **iii** $x + y = 4$

b Use graphs to solve the following pairs of equations simultaneously for x and y.

i $y = x$, $y = 2x + 1$

ii $y = x$, $x + y = 4$

iii $y = 2x + 1$, $x + y = 4$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 2

3 By drawing graphs, solve simultaneously for x and y. Check each answer by substitution.

a $y = x + 2$
$y = 2x + 3$

b $y = 2x - 3$
$y = 3x - 5$

c $y = x + 2$
$y = 6 - x$

d $x + y = 4$
$2x + y = 6$

e $x - y = 1$
$3x + y = 3$

f $y = -2x + 3$
$y = x$

g $y = 2x + 1$
$y = 4 - x$

h $x - y = 2$
$x = 2y - 3$

i $x + 3y = 5$
$2x - y = -4$

4 On the same set of axes, draw the lines $y = 2x + 1$ and $y = 2x + 3$.

a What do you notice about these two lines?

b How many solutions do the simultaneous equations have?

5 On the same set of axes, draw the lines $x + y = 3$ and $2x + 2y = 6$.

a What do you notice about these two lines?

b How many solutions do the simultaneous equations have?

6 How many solutions do the simultaneous equations $y = 2x + 3$ and $2x - y = 6$ have?

7 **a** By drawing graphs, solve simultaneously for x and y.

i $y = x + 2$
$y = 3x + 1$

ii $y = 4x - 1$
$y = x + 4$

iii $3x + 2y = 6$
$2x - y = 4$

b Can you see any limitations in using graphs to solve a pair of simultaneous equations?

14B Substitution

In the previous section we learnt how to solve simultaneous linear equations by graphical means. This method clearly has its limitations and is very slow and tedious in practice. It is also difficult to draw sufficiently accurate graphs, particularly when the solutions are not integers. In the next two sections we shall introduce two algebraic methods called **substitution** and **elimination** for solving simultaneous equations.

Example 3

Solve $y = 2x + 1 \quad (1)$

$y = 3x - 2 \quad (2)$

for x and y.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

Using equation (1), $2x + 1$ can be substituted for y in equation (2).

$$\begin{aligned} 2x + 1 &= 3x - 2 \\ 1 &= x - 2 \\ x &= 3 \end{aligned}$$

Substituting $x = 3$ into (1):

$$\begin{aligned} y &= 2 \times 3 + 1 \\ &= 7 \end{aligned}$$

Check by substituting into (2):

$y = 3x - 2$ gives $7 = 3 \times 3 - 2$ ✓

Hence the solution is $x = 3$ and $y = 7$.

Note: This method is called the **substitution** method since one of the pronumerals is replaced (or substituted) by an expression involving the other pronumeral.

Example 4

Solve this pair of equations for x and y.

$y = 2x + 1$ (1)

$3x + 2y = 9$ (2)

Solution

Using equation (1), $2x + 1$ can be substituted for y in equation (2), giving:

$$\begin{aligned} 3x + 2(2x + 1) &= 9 \\ 3x + 4x + 2 &= 9 \\ 7x + 2 &= 9 \\ 7x &= 7 \\ x &= 1 \end{aligned}$$

Substituting $x = 1$ in equation (1) gives:

$$\begin{aligned} y &= 2 \times 1 + 1 \\ y &= 3 \end{aligned}$$

Check that (1, 3) satisfies both equations.

Hence the solution is $x = 1$ and $y = 3$.

Example 5

Solve the following simultaneous equations.

$x = 2y - 1$ (1)

$3y - 2x = 12$ (2)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

$3y - 2(2y - 1) = 12$ Substitute the expression for x from (1) into (2).

$$\begin{aligned} 3y - 4y + 2 &= 12 \\ -y + 2 &= 12 \\ -y &= 10 \\ y &= -10 \end{aligned}$$

Substituting $y = -10$ into (1) gives:

$$\begin{aligned} x &= 2(-10) - 1 \\ &= -21 \end{aligned}$$

Check that $(-21, -10)$ satisfies both equations.

Hence the solution is $x = -21$ and $y = -10$.

Note that Example 5 would be difficult to solve graphically.

Harder examples using substitution

If neither pronumeral is the subject of an equation, the method can still be used, but more algebra is involved. We make one of the pronumerals the subject of one of the equations.

Example 6

Solve $3x - 2y = 1$ (1)

$7y - 5x = 2$ (2)

for x and y.

Solution

We make x the subject of equation (1).

$$3x = 1 + 2y$$

$$x = \frac{1}{3}(1 + 2y) \qquad (3)$$

Substitute from (3) into (2).

$$7y - 5\left(\frac{1}{3}(1 + 2y)\right) = 2$$

$$\begin{aligned} 21y - 5 - 10y &= 6 \qquad \text{(Multiply both sides by 3)} \\ 11y &= 11 \\ y &= 1 \end{aligned}$$

Substituting $y = 1$ into (1) gives $x = 1$.

Check that $(1, 1)$ satisfies equations (1) and (2).

Hence the solution is $x = 1$ and $y = 1$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solving simultaneous linear equations by substitution

- Make x or y the subject of one equation and substitute into the other equation.
- Solve the resulting equation.
- Substitute this value back into one of the original equations to find the value of the other pronumeral.
- Check that your solution satisfies the original equations.

Exercise 14B

Example 3

1 Solve the simultaneous equations using the substitution method.

a $y = x + 2$, $y = 2x + 3$

b $y = 2x - 1$, $y = 3x + 4$

c $y = 2x + 6$, $y = 4x - 2x$

d $y = x - 1$, $y = 3x - 5$

e $y = 1 - x$, $y = 3 - 2x$

f $y = 3x - 5$, $y = 6x - 11$

g $y = 4 - 3x$, $y = 5x - 2x$

h $y = 2x$, $y - 5x - 2x$

Example 4

2 Solve the simultaneous equations using the substitution method.

a $y = x + 1$, $x + y = 3$

b $y = x - 2$, $3x + y = 14$

c $y = 2x + 1$, $x + 2y = 12$

d $y = 1 - 2x$, $x + y = 2$

e $y = 3 - x$, $x - 3y = -5$

f $y = x - 4$, $x + y = 6$

g $y = x + 3$, $2x + 3y = 14$

h $y = 3x - 2$, $3x + y = 22$

i $y = 3 - 2x$, $x + 2y = 9$

j $y = 2x - 6$, $x - 2y = -3$

k $y = 3 - 2x$, $x + y = 0$

l $y = 5 - 2x$, $3x + 2y = 8$

Example 5

3 Solve the simultaneous equations using the substitution method.

a $x = 2y + 1$, $y = x - 2$

b $x = 3y - 1$, $2x + y = 12$

c $x = y - 1$, $x = 3y + 7$

d $y = 3x - 5$, $x = 2y + 6$

e $a = b + 3$, $2a + b = 18$

f $a + 3b = 6$, $a = 2b + 1$

g $a = 7 - 2b$, $a = b + 4$

h $3a - 2b = 2$, $a = b - 1$

Example 6

4 Solve the simultaneous equations using the substitution method.

a $3x + 4y = 11$, $7x - 2y = 3$

b $5x - 3y = 12$, $2x + 5y = 11$

c $2a + 7b = 13$, $3a + 4b = 9$

d $7a + 9b = -13$, $-2a - 6b = 11$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

14C Elimination

The other standard method of solving simultaneous equations is called **elimination**. This method creates an equation involving just one pronumeral, which can then be solved. Substitution then gives the value of the other pronumeral.

Consider the true statements:

$$6 + 3 = 9 \qquad (1)$$

$$5 + 7 = 12 \qquad (2)$$

If we add the left-hand sides of statements (1) and (2) and the right-hand sides of the statements we obtain:

$$(6 + 3) + (5 + 7) = 9 + 12$$

The resulting statement is true because statements (1) and (2) are true. We use this idea in the solving of simultaneous equations using the elimination method.

Example 7

Solve for x and y:

$$2x + y = 8 \qquad (1)$$

$$x - y = 1 \qquad (2)$$

Solution

We can eliminate the pronumeral y by adding the left-hand sides and the right-hand sides of the two equations.

$$\begin{aligned} (1) + (2): \qquad (2x + y) + (x - y) &= 8 + 1 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

Substituting $x = 3$ into (1):

$$\begin{aligned} 6 + y &= 8 \\ y &= 2 \end{aligned}$$

Check mentally that (3, 2) satisfies equations (1) and (2).

Hence the solution is $x = 3$ and $y = 2$.

For the above example, substitution is almost as easy as elimination. In the next example, elimination is much easier.

Example 8

Solve for x and y:

$$3x - 4y = 6 \qquad (1)$$

$$2x + 4y = 4 \qquad (2)$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

We eliminate the terms involving y by adding (1) and (2).

$$\begin{aligned}(1) + (2):\quad (3x - 4y) + (2x + 4y) &= 6 + 4\\ 5x &= 10\\ x &= 2\end{aligned}$$

Substituting $x = 2$ into (1) gives $y = 0$.

Check that (2, 0) satisfies both equations.

Hence the solution is $x = 2$ and $y = 0$.

Example 9

Solve: $x + 3y = 7$ (1)

$x + y = 5$ (2)

Solution

Subtracting one equation from the other eliminates x.

$$\begin{aligned}(1) - (2):\quad (x + 3y) - (x + y) &= 7 - 5\\ 2y &= 2\\ y &= 1\end{aligned}$$

Substituting $y = 1$ into (1), gives $x = 4$.

Check that (4, 1) satisfies both equations.

Hence the solution is $x = 4$ and $y = 1$.

Example 10

Solve: $2x - 3y = 4$ (1)

$5x - 3y = 19$ (2)

Solution

Clearly (1) – (2) eliminates y, but (2) – (1) is better as it avoids negative coefficients.

$$\begin{aligned}(2) - (1):\quad (5x - 3y) - (2x - 3y) &= 19 - 4\\ 3x &= 15\\ x &= 5\end{aligned}$$

Substituting $x = 5$ into (1) gives: $10 - 3y = 4$

$$\begin{aligned}6 &= 3y\\ y &= 2\end{aligned}$$

Check that (5, 2) satisfies both equations.

Hence the solution is $x = 5$ and $y = 2$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Harder examples using elimination

If we multiply both sides of an equation by a non-zero number, we obtain an **equivalent equation**. For example, $x + y = 5$ is equivalent to $2x + 2y = 10$.

Often, adding the equations, or subtracting one equation from the other, does not eliminate either pronumeral. In this situation we need to multiply one or both equations by a number to make the coefficients match.

Examples 11 and 12 show how to do this.

Example 11

Solve the following simultaneous equations.

$x + y = 5 \qquad (1)$

$2x + 3y = 14 \qquad (2)$

Solution

If we multiply equation (1) by 2, we can eliminate x.

(1) × 2: $\quad 2x + 2y = 10 \qquad (3)$

(2) − (3): $\quad (2x + 3y) - (2x + 2y) = 14 - 10$

$$y = 4$$

Substituting $y = 4$ into (1) gives $x = 1$.

Check that $(1,4)$ satisfies equations (1) and (2).

Hence the solution is $x = 1$ and $y = 4$.

We could also have eliminated the pronumeral y by multiplying (1) by 3 and subtracting.

Example 12

Solve: $\quad 3x - 2y = 5 \qquad (1)$

$\qquad 4x + 3y = 18 \qquad (2)$

Solution

Multiply both sides of equation (1) by 3 and both sides of equation (2) by 2 so that we can eliminate y.

(1) × 3: $\quad 9x - 6y = 15 \qquad (3)$

(2) × 2: $\quad 8x + 6y = 36 \qquad (4)$

(3) + (4): $\quad 17x = 51$

$$x = 3$$

Substituting into (1): $9 - 2y = 5$

$$4 = 2y$$

$$y = 2$$

Check that $(3, 2)$ satisfies the original equations.

Hence the solution is $x = 3$ and $y = 2$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

General principles involved in solving by elimination

In solving simultaneous equations we have used two ideas repeatedly.

First, if $a = b$ and $c = d$, then:

$$a + c = b + d \qquad \text{and} \qquad a - c = b - d$$

Second, if $a = b$ is an equation and m is a non-zero constant, then $am = bm$.

This is called 'multiplying by the constant m'.

The skill is in choosing the order in which to perform these steps.

Keeping track of the steps used, such as (3) = (1) × 3, makes calculation and checking easier.

Elimination method for solving simultaneous equations

- Check if adding or subtracting the given equations will result in the elimination of a pronumeral. If so, then perform that operation to both sides of the equations.
- Otherwise, select a pronumeral to eliminate and take note of the respective coefficients. Identify the least common multiple of the two coefficients and create equivalent equations with the magnitude of this value represented in each. Add or subtract these equivalent equations so that elimination of the chosen pronumeral occurs.
- Solve the resulting equation.
- Find the value of the other pronumeral by substitution into one of the two equations (choose the simpler one where possible).
- Check your solution satisfies the original equations.

To decide which method is better to solve simultaneous equations, the following rule of thumb may be used.

Choosing the method (rule of thumb)

If one (or both) of the equations is of the form $y = \ldots$ or $x = \ldots$, use **substitution**; otherwise use **elimination**.

Exercise 14C

Example 7

1 Solve the simultaneous equations using the elimination method.

a $x - y = 2$ $3x + y = 14$	**b** $2x - 3y = 3$ $x + 3y = 6$	**c** $3x + y = 4$ $-3x + 2y = -10$
d $2x - y = 1$ $5x + y = 13$	**e** $3x + 2y = 14$ $4x - 2y = 14$	**f** $-x - y = 3$ $x + 5y = -11$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 8, 9, 10

2 Solve the simultaneous equations using the elimination method.

a $x + y = 5$
$3x + y = 9$

b $3x + 4y = 15$
$x + 4y = 13$

c $2x - y = 7$
$4x - y = 15$

d $3x - 2y = 1$
$5x - 2y = 7$

e $2x + y = 6$
$2x - 3y = -4$

f $3x - 5y = 10$
$3x + 6y = 21$

g $2x + 3y = 1$
$-x + 3y = 4$

h $x - 2y = -8$
$3x - 2y = -12$

i $-2x - 5y = -3$
$4x - 5y = 21$

3 Solve the simultaneous equations using the elimination method.

a $2x + 3y = 11$
$2x - y = 7$

b $3x - y = -3$
$2x + y = -7$

c $4x - 3y = 17$
$-4x + y = -19$

d $3x - 5y = -27$
$2x - 5y = -23$

e $3x - y = -7$
$3x + 5y = -1$

f $4x - 3y = 8$
$2x + 3y = 4$

4 Solve the simultaneous equations using the elimination method.

a $x + y = 2$
$3x + 4y = 7$

b $x - y = 2$
$2x + 3y = 9$

c $4x + y = 1$
$3x - 2y = -13$

d $x - 5y = 9$
$3x + 2y = 10$

e $5x - 3y = -2$
$-x + 2y = -1$

f $3x - y = 11$
$5x + 2y = 14$

g $2x + 5y = 10$
$x - 3y = 5$

h $x - 3y = 0$
$2x - 4y = 2$

i $-3x - 2y = 4$
$x + 7y = 5$

Example 11, 12

5 Solve the simultaneous equations using the elimination method.

a $2x + 3y = 12$
$3x - 4y = -1$

b $5x - 2y = 17$
$4x + 3y = 9$

c $2x + 3y = 1$
$5x + 4y = -1$

d $3x - 5y = -21$
$5x + 3y = -19$

e $4x - y = -3$
$5x + 3y = -8$

f $6x - 5y = 8$
$3x + 2y = 13$

g $4a + 3b = 22$
$2a - 5b = -2$

h $3a - 4b = 3$
$7a - 3b = 26$

i $5a + 4b = 16$
$3a + 7b = 5$

j $4a - b = 1$
$3a + 7b = -38$

k $2a - 3b = -13$
$11a + 4b = 31$

l $8a - 3b = 14$
$5a + 7b = 62$

6 Solve the simultaneous equations. Use the most appropriate method.

a $2x + 3y = 8$
$3x - y = 1$

b $2x - 3y = 4$
$3x - 2y = 11$

c $y = 5 - 2x$
$2x - 3y = 17$

d $y = 2x + 8$
$y = -3x - 2$

e $x = 1 - 3y$
$3y - 2x = 4$

f $x = 2y + 1$
$x = 3y - 2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

g $2x - 4y = 13$
$3x - 5y = 17$

h $4x + 5y = 4$
$2x + 4y = 3$

i $y = 5 - 9x$
$5x - 3y = 1$

j $y = 4x - 2$
$y = 6x - \frac{7}{2}$

k $x = 3y + 3$
$x = 6y + 1$

l $x = 1 - y$
$7x - y = 2$

14D Problems involving simultaneous linear equations

In this section we consider problems where it is natural to introduce two pronumerals, write down two simultaneous equations and solve them.

Example 13

At a hi-fi store, Benjamin bought a number of CDs costing \$10 each and a number of DVDs costing \$20 each. If he spent \$120 in total and bought twice as many CDs as DVDs, how many of each did he buy?

Solution

Method 1

Let x be the number of CDs Benjamin bought.

Let y be the number of DVDs he bought.

Then: $10x + 20y = 120$ (1) (total cost in dollars)

$x = 2y$ (2) (twice as many CDs as DVDs)

This pair of equations can be solved simultaneously.

Substituting from equation (2) into equation (1) gives:

$$
\begin{aligned}
10 \times 2y + 20y &= 120 \\
20y + 20y &= 120 \\
y &= 3
\end{aligned}
$$

Substituting $y = 3$ into equation (2) gives:

$$x = 6$$

Therefore Benjamin bought 6 CDs and 3 DVDs.

You should check that your answer satisfies the given information.

Method 2

This question can be done using just one pronumeral.

Benjamin bought twice as many CDs as DVDs.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Let y be the number of DVDs he bought. He bought $2y$ CDs.

$$\begin{aligned} \text{Cost of DVDs} &= \$20y \\ \text{Cost of CDs} &= \$2y \times 10 \\ &= \$20y \\ \text{Hence} \quad 40y &= 120 \\ y &= 3 \end{aligned}$$

Benjamin bought 3 DVDs and 6 CDs.

Note: Many of the problems in Exercise 14D can be solved in both ways.

Example 14

A farmyard contains some pigs and some chickens. Altogether there are 60 heads and 200 legs. How many pigs are in the yard?

Solution

Let x be the number of pigs in the yard.

Let y be the number of chickens in the yard.

Then:
$$\begin{aligned} x + y &= 60 \qquad (1) \qquad \text{(number of heads)} \\ 4x + 2y &= 200 \qquad (2) \qquad \text{(number of legs)} \end{aligned}$$

Since only x is required, we eliminate y.

$(1) \times 2$: $\quad 2x + 2y = 120 \qquad (3)$

$(2) - (3)$: $\quad 2x = 80$

$$x = 40$$

Therefore there are 40 pigs in the yard.

As a check, show that there are 20 chickens, 60 heads and 200 legs.

Exercise 14D

Solve each of the following problems by introducing two pronumerals and forming two simultaneous equations, even though some of them can be done more easily using one pronumeral.

1 Two numbers have a sum of 36 and a difference of 2. Find the numbers.

Example 13

2 Jane and Jacqui each think of a number. The number Jane thinks of is twice Jacqui's number and the sum of the two numbers is 39. What numbers did they think of?

3 Two numbers have a sum of 58. One number is two more than the other number. Find the two numbers.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press Updated August 2021

4 The sum of two numbers is 19. Three times one number minus twice the other number is 2. Find the two numbers.

5 At present, Martha is 24 years older than her daughter. In eight years' time she will be three times her daughter's age. What are Martha's and her daughter's present ages?

6 The equal sides of an isosceles triangle are 5 cm longer than the base. If the perimeter of the triangle is 34 cm, find the side lengths of the triangle.

7 Oscar is currently three times his son's age. In 14 years' time, he will be twice his son's age. What is Oscar's current age?

Example 14

8 Georgina has \$8.00 to spend. She discovers that she can buy either 4 cans of soft drink and 4 chocolate bars or 2 cans of soft drink and 7 chocolate bars. How much does each item cost?

9 Romeo spends \$15.60 and buys 3 hamburgers and 5 buckets of chips. Manni spends \$12.40 and buys 4 hamburgers and 2 buckets of chips. How much does a hamburger cost?

10 An army transport vehicle has a carrying capacity of 5 tonnes. It can be used to transport either 20 men and 17 pieces of equipment or 35 men and 11 pieces of equipment. How many kilograms does each man weigh? (Assume that all men have the same weight and all pieces of equipment have the same weight.)

11 At a clothing store, trousers cost twice as much as shirts. A businessman buys 4 shirts and 2 pairs of trousers for a total cost of \$240. How much does each shirt cost?

12 The length of a rectangular paddock is 5 m longer than twice its width. If the perimeter of the paddock is 154 m, find the dimensions of the paddock.

13 Four years ago Jie was twice Tan's age. In four years' time, Jie will be one and a quarter times Tan's age. How old are Jie and Tan now?

14 Toni's piggy bank contains \$19.20, made up of 10-cent pieces and 20-cent pieces. If there are 114 coins in the piggy bank, how many coins of each denomination are there?

15 Sammi thinks of a two-digit number. The sum of the digits is 8. If she reverses the digits, the new number is 36 greater than her original number. What was Sammi's original number?

16 Jocelyn thinks of a fraction. If she adds 4 to both the numerator and the denominator, the new fraction is equal to $\frac{4}{5}$. If she subtracts 5 from both the numerator and the denominator, the new fraction is equal to $\frac{1}{2}$. What fraction did she think of?

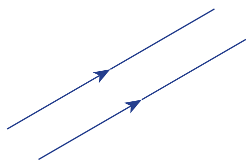
17 Find a fraction that is equal to $\frac{7}{9}$ if 10 is added to both the numerator and the denominator, and is equal to $\frac{1}{2}$ if 5 is subtracted from both the numerator and the denominator.

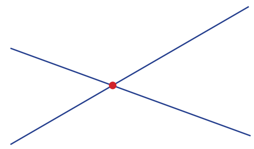
18 A secretary buys a number of 45-cent and 60-cent stamps for a total cost of \$22.50. If he interchanges the numbers of the two kinds of stamps, the total cost would have been \$23.70. How many of each kind of stamp did he originally buy?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

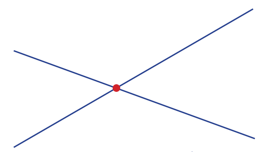
14E Geometry and simultaneous equations

Two distinct lines are either parallel or meet at a point.





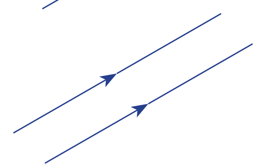
There are three cases with a system of two linear equations.



Case 1 — **There is one solution**

$x + y = 5$
$x - y = 3$

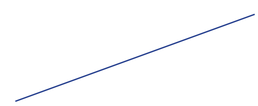
The solution to these equations is $x = 4$ and $y = 1$.
The two corresponding lines intersect at a single point.



Case 2 — **There are no solutions**

$x + y = 5$
$x + y = 7$

$x + y = 5$, so $x + y$ cannot equal 7.
The two equations are parallel.



Case 3 — **There are infinitely many solutions**

$x + y = 5$
$2x + 2y = 10$

The two equations are equivalent.
The two equations represent the same line. Every point on this line represents a solution.

In case 1, the lines have different gradients and therefore intersect at a point. In case 2 and case 3, the gradients are equal and therefore the lines are either parallel or coincide.

Equivalent equations were discussed in Chapter 5.

Example 15

Determine whether the simultaneous equations have no solutions, one solution or infinitely many solutions. Illustrate these graphically.

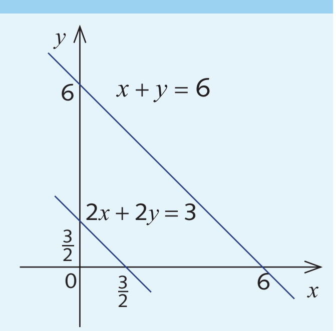
a $x + y = 6$
$2x + 2y = 3$

b $x + y = 8$
$x - y = 2$

c $x - y = 2$
$3x - 3y = 6$

Solution

a The equation $x + y = 6$ can be written as $y = -x + 6$, so the gradient of the corresponding line is -1.
The equation $2x + 2y = 3$ can be written as $y = -x + \frac{3}{2}$, so the gradient is -1.
The lines are parallel but have different y-intercepts. Therefore there are no solutions to the simultaneous equations.



(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b $x + y = 8 \qquad (1)$

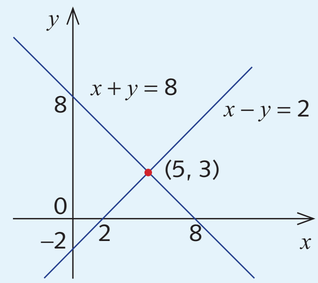
$x - y = 2 \qquad (2)$

Add (1) and (2): $2x = 10$

$x = 5$

Substituting in (1) gives $y = 3$.

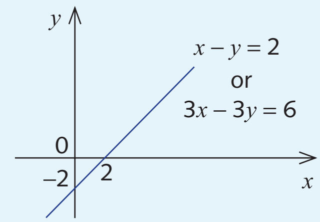
There is one solution: $x = 5$ and $y = 3$.



c $x - y = 2 \qquad (1)$

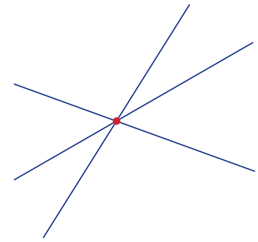
$3x - 3y = 6 \qquad (2)$

These are equivalent equations. They determine the same straight line. All the points on the line satisfy both equations. There are infinitely many solutions.



Concurrence

Three or more lines are said to be **concurrent** if they intersect at a single point.



Example 16

Show that the lines $y = 3x + 2, y = 2x + 3$ and $y = -3x + 8$ are concurrent.

Solution

Let: $y = 3x + 2 \qquad (1)$

$y = 2x + 3 \qquad (2)$

$y = -3x + 8 \qquad (3)$

We solve equations (1) and (2) simultaneously.

Substitute for y from (1) into (2):

$$3x + 2 = 2x + 3$$

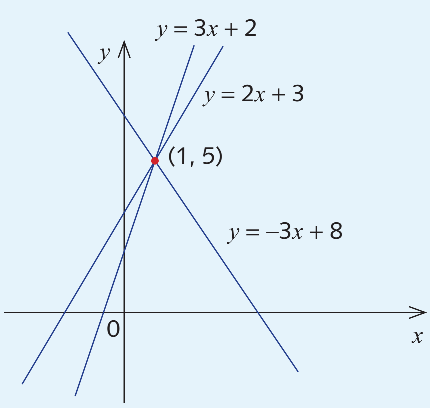
$$x = 1$$

Substituting $x = 1$ into equation (1) gives $y = 5$.

Finally check whether (1, 5) satisfies equation (3).

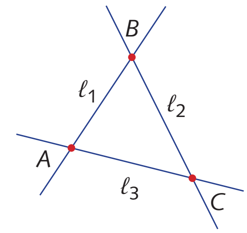
$$5 = -3 \times 1 + 8 \quad ✓$$

(1, 5) satisfies all three equations, hence the lines are concurrent.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Areas of triangles



If we are given the equations of three lines ℓ_1, ℓ_2 and ℓ_3 that form a triangle, we can use the techniques developed in this chapter to find the coordinates of the vertices A, B and C of the triangle.

We can determine the area of a triangle given the coordinates of the vertices. In this section we will deal only with cases in which one line is parallel to one of the coordinate axes. The general case is dealt with in the Challenge exercise.

Example 17

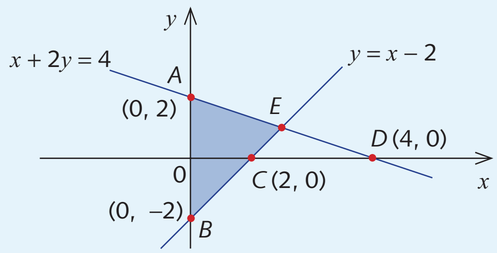
Find the area of the triangle formed by the lines $x + 2y = 4, y = x - 2$ and the y-axis.

Solution

It is essential to draw a diagram.

The line $y = x - 2$ meets the axes at $C(2,0)$ and $B(0,-2)$.

The line $x + 2y = 4$ meets the axes at $D(4,0)$ and $A(0,2)$.



We want the area of triangle ABE.

To find the coordinates of E, we solve the simultaneous equation:

$$y = x - 2 \qquad (1)$$

$$x + 2y = 4 \qquad (2)$$

Substituting (1) into (2):

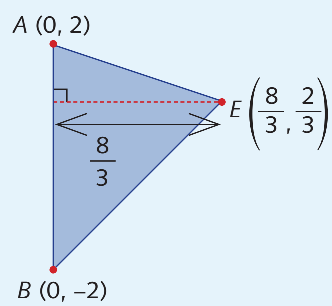
$$x + 2(x - 2) = 4$$

$$3x - 4 = 4$$

$$x = \frac{8}{3}$$

And from (1):

$$y = \frac{2}{3}$$



The length of AB is 4.

The distance from E to AB = x-coordinate of E

$$= \frac{8}{3}$$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

$$\begin{aligned}\text{So area of triangle } ABE &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 4 \times \frac{8}{3}\\ &= \frac{16}{3}\end{aligned}$$

Geometry and simultaneous equations

- There are three cases with a system of two linear equations:

  Case 1 **There is one solution**. The two corresponding lines meet at a single point.

  Case 2 **There are no solutions**. The two corresponding lines are parallel.

  Case 3 **There are infinitely many solutions**. The two equations are equivalent. They represent the same line.

- Three or more lines are said to be **concurrent** if they intersect at a single point.

Exercise 14E

1 For each pair of equations, find the coordinates of the point where the lines intersect.

a $2x + y = 4$, $3x - y = 1$

b $2x - 2y = 7$, $3x - 2y = 11$

c $y = 5 + x$, $2x - y = 3$

d $y = 2x - 12$, $y = -3x - 2$

e $x - 4y = 13$, $3x - 5y = 18$

f $2x + y = 4$, $3x - 4y = 6$

g $2x - 4y = 13$, $3x - 5y = 17$

h $y = 5 - 9x$, $5x - 3y = 1$

i $x - 3y + 3 = 0$, $x - 6y + 1 = 0$

Example 15

2 For each pair of equations, draw the two straight lines and determine whether there are no solutions, one solution or infinitely many solutions.

a $y = 2x + 3$ and $y = -x + 3$

b $x + 2y = 6$ and $y = 4x + 3$

c $y = 2x + 6$ and $y - 2x = 3$

d $x + y = 6$ and $y = -x + 6$

e $x + y = 6$ and $x + y = 8$

f $3x + 3y = 12$ and $y = -x + 4$

Example 16

3 Show that the three lines are concurrent and give the coordinates of their point of intersection.

a $y = 3x + 2,\ y = -x + 2,\ y = 2x + 2$

b $x + y = 2,\ y = 4x - 8,\ 2x + 3y = 4$

c $2x + y = 0,\ y = -x,\ y - 5x = 0$

d $y = 2x + 1,\ x + y = 4,\ 2x - y = -1$

4 Show that the lines $y = 2x + 1$, $y = 3x - 2$ and $y = 4x + 6$ are not concurrent.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 17

5 Find the area of the triangle formed by the lines $x + y = 4, y = x$ and the:

a x-axis **b** y-axis

6 Find the area of the triangle formed by the lines $x + 2y = 4, y = x + 1$ and the:

a x-axis **b** y-axis

7 Find the area of the triangle formed by the lines $3x - 2y = 12, y = x - 5$ and the:

a x-axis **b** y-axis

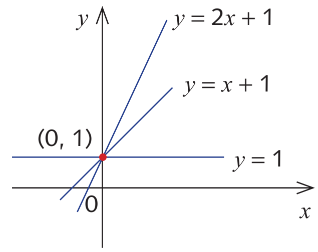
8 Find the area of the triangle formed by the lines $x - y = 4, y = -x$ and the line:

a $x = 5$ **b** $y = 3$

14F Families of straight lines

We have drawn on the right the lines with the following equations:

$$y = 1$$
$$y = x + 1$$
$$y = 2x + 1$$



These are examples of lines of the form $y = mx + 1$. All of these lines pass through $(0, 1)$.

Conversely, every non-vertical line passing through $(0, 1)$ has equation $y = mx + 1$ for some value of m.

We can describe this family geometrically as the family of all non-vertical lines through $(0, 1)$.

Example 18

The family of lines $y = mx + 1$ with varying gradient m all pass through the point $(0,1)$.

a For what value of m does the line $y = mx + 1$ not intersect the line $y = 2x + 5$?

b For what values of m does the line $y = mx + 1$ intersect the line $y = 2x + 5$?

c If the line $y = 2x + 5$ intersects the line $y = mx + 1$ at $(-1, 3)$, find m.

Solution

a The y-intercepts of the two lines are 5 and 1 respectively, so the lines are different. If the lines do not intersect they must be parallel. So $m = 2$.

b When $m \neq 2$ the lines will intersect.

c Substituting $x = -1$ and $y = 3$ into $y = mx + 1$

$$3 = -m + 1$$

Hence $m = -2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 14F

1 **a** Sketch the lines $y = x + 1$, $y = x + 2$ and $y = x - 3$, and geometrically describe the family of lines $y = x + b$.

b Sketch the lines $y = 2x + 1$, $y = -x + 1$ and $y = -2x + 1$, and geometrically describe the family of lines $y = mx + 1$.

c Sketch the lines $y = 2x + 2$, $y = -x + 2$ and $y = 3x + 2$, and geometrically describe the family of lines $y = mx + 2$.

Example 18

2 The equations of two lines are $y = x + 3$ and $y = mx - 2$.

a For what values of m will the two lines not intersect?

b For what values of m will the two lines intersect?

c Given that the lines intersect at $(5, 8)$, find m.

3 The equations of two lines are $y = x - 1$ and $mx - 2y = 4$.

a For what values of m will the two lines not intersect?

b For what values of m will the two lines intersect?

c Given that the lines intersect at $(2, 1)$, find m.

4 The equations of two lines are $y = mx + 5$ and $6x + 3y = 15$.

a For what values of m will the two lines not intersect?

b For what values of m will the two lines intersect?

c For the values of m found in part **b**, comment on the intersection of the lines.

5 The equations of two lines are $x + 2y = 7$ and $2x + 4y = k$.

a For what values of k will the two lines not intersect?

b For what values of k will the two lines intersect?

c For the values of k found in part **b**, comment on the intersection of the lines.

6 The lines with equations $y = x + k$ and $y = mx + 3$ intersect at $(1, 2)$. Find m and k.

7 The lines with equations $y = nx + 2$ and $y = mx + 3$ intersect at $(3, -1)$. Find m and n.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Review exercise

1 Solve the simultaneous equations using the substitution method.

a $y = x + 4$
$y = 2x + 3$

b $y = 3x - 2$
$y = 2x - 1$

c $y = 2x - 3$
$y = 3x - 5$

d $y = x + 1$
$y = 4 - 2x$

2 Solve the simultaneous equations using the substitution method.

a $y = 2x + 1$
$x + y = 4$

b $y = 3x - 2$
$3x + 2y = 5$

c $y = 2x - 3$
$x + 2y = 9$

d $y = 7 - 2x$
$x + y = 5$

3 Solve the simultaneous equations using the substitution method.

a $x = 3y + 2$
$y = x - 6$

b $x = 2y - 1$
$2x + y = 11$

c $x = y - 3$
$x = 3y - 13$

d $y = 3x - 2$
$x = 2y + 3$

4 Solve the simultaneous equations using the elimination method.

a $x + y = 2$
$3x - y = 10$

b $2x - 3y = 1$
$3x + 3y = 9$

c $3x + y = 4$
$-3x + 2y = -10$

d $2x - y = 1$
$x + y = 5$

5 Solve the simultaneous equations using the elimination method.

a $x + y = 2$
$2x + y = 5$

b $3x + 2y = 13$
$x + 2y = 9$

c $2x - y = 9$
$3x - y = 11$

d $2x - 4y = 1$
$5x - 4y = 8$

6 Solve the simultaneous equations using the elimination method.

a $3x + 2y = 10$
$3x - y = 7$

b $2x - y = -3$
$5x + y = -11$

c $x - 3y = 5$
$-x + y = -19$

d $3x - 7y = -17$
$2x - 7y = -13$

7 Solve the simultaneous equations using the elimination method.

a $x + y = 2$
$2x + 3y = 7$

b $x - y = 2$
$3x + 4y = 6$

c $2x + y = 3$
$5x - 2y = -15$

d $x - 5y = 9$
$2x + 3y = 10$

8 Solve the simultaneous equations using the elimination method.

a $2x + 5y = 12$
$3x - 4y = -5$

b $3x - 2y = 17$
$4x + 3y = 0$

c $2x + 3y = -4$
$5x + 4y = -3$

d $3x - 5y = -21$
$4x - 3y = 8$

9 For each pair of equations, solve for x and y using the most appropriate method.

a $x + 2y = 6$
$3x - y = 2$

b $2x - 5y = 10$
$5x - 2y = 12$

c $y = 3 - 2x$
$2x - 5y = 12$

d $y = 2x + 5$
$y = -3x - 7$

e $x = 1 - 2y$
$5y - 2x = 10$

f $x = 5y + 3$
$x = 3y - 2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 For each pair of lines, find the coordinates of the point where the graphs intersect.

a $5x + y = 7$
$3x - y = 1$

b $x - 3y = 5$
$4x - 3y = 13$

c $y = 3 + x$
$3x - 2y = 6$

d $y = 2x - 10$
$y = -3x + 4$

e $x - 3y = 13$
$4x - 5y = 18$

f $5x + y = 8$
$2x - 3y = 10$

11 The equations of two lines are $y = 2x + 5$ and $mx - 2y = 4$.

a For what values of m will the two lines not intersect?

b For what values of m will the two lines intersect?

c Given that the lines intersect at $(-1, 3)$, find m.

12 The equations of two lines are $mx + y = 2$ and $2x + 5y = 10$.

a For what values of m will the two lines not intersect?

b For what values of m will the two lines intersect?

c For the values of m found in part **b**, comment on the intersection of the lines.

13 The lines with equations $y = 2x + k$ and $y = mx - 5$ intersect at $(15, 40)$. Find m and k.

14 Find the area of the triangle formed by the lines $x + 3y = 9, y = 7 - x$ and the:

a x-axis

b y-axis

15 Tickets to the circus cost \$30 for adults and \$12 for children. For a matinee performance, 960 people attended and \$19 080 was collected in ticket sales. Find the number of adults and children who attended the performance.

16 The difference of two numbers is 5. The sum of three times one number and two times the other is 25. Find the two numbers.

17 Find the value of m for which the lines $y = 3x + 2, y = 2x + 3$ and $y = mx$ are concurrent.

18 **a** Find the point of intersection of the lines $y = 3x + 6$ and $y = 2 - x$.

b Find the equation of the line that passes through this point of intersection and has y-intercept 6.

19 Find the value of c for which the lines $x + y = 8, x - y = 12$and $3x + 2y = c$ are concurrent.

20 The line $ax + by = 6$ is concurrent with the lines $4x - 3y = 17$ and $4x + y = -19$ and is parallel to the line $y = 6x$. Find the values of a and b.

21 Solve the simultaneous equations.

a $3x + y = 5x + 4$ and $3x + y = 2y$

b $3x + 2y = x - 3y$ and $4x + 6y = 3x - 7$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

22 For a certain fraction, if 1 is added to the numerator and 1 is subtracted from the denominator the result is 1 For the same fraction, if 1 is subtracted from the numerator and 1 is added to the denominator the result is $\frac{3}{5}$. Find the fraction.

23 Solve the simultaneous equations.

a $\frac{x}{3} + \frac{y}{4} = 1$

$\frac{3x}{2} - y = 4$

b $\frac{2x}{7} + \frac{y}{8} = 0$

$\frac{3x}{4} - \frac{y}{3} = 0$

24 Two lines have gradients 3, 4 respectively and y-intercepts 1,–2 respectively. Find the coordinates of the point of intersection of the two lines.

25 The triangle ABC has a perimeter of 4 m. Find the length of each side if $AB + BC = 2AC$ and $AB + AC = 3BC$.

26 The line $y = 3x + c$ intersects the line $3x - 2y = 6$ at the point (–4,–9). Find the value of c.

27 The lines $y = ax + 11$ and $y = \frac{1}{2}x - 3$ are perpendicular.

a Find the value of a.

b Find the coordinates of the point of intersection of the lines.

28 Points A, B and C have coordinates $(0,\ 1)$, $(5,\ 11)$ and $(1,\ 8)$ respectively. The line from C, which is perpendicular to AB, meets AB at the point N.

a Find the equations of AB and CN.

b Find the coordinates of N.

29 The lines $y = mx + 6$ and $y = nx - 3$ meet at a point where $x = 9$. Find the values of m and n if $m + n = 7$.

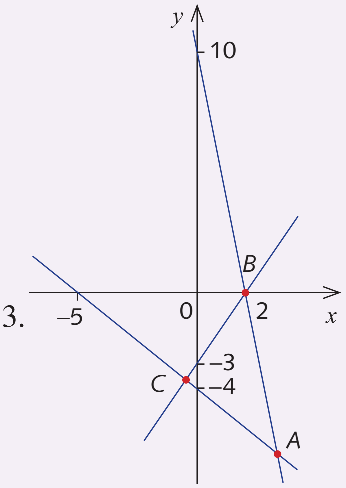
30 The equations of the lines that define ΔABC are:

AB: $5x + y = 10$

BC: $3x - 2y = 6$

CA: $4x + 5y = -20$

Find the coordinates of A, B and C.



31 In the rectangle $ABCD$,
$AB = p + 2q, BC = 3p - 10, CD = 5(p - q)$ and $DA = 2q + 3$.
Find the perimeter of the rectangle.

32 The length of the sides of a triangle are $2x - 4, y - 7$ and $2y - 4x$. If the triangle is equilateral, find the length of a side.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 Assume that m is positive throughout this question.

a Find the point of intersection of the lines $x + y = 4$ and $y = mx + 1$ in terms of m.

b Find the area of the region enclosed by the lines $x + y = 4$, $y = mx + 1$ and the x-axis in terms of m.

c Given the area of the triangle formed by the lines $x + y = 4$, $y = mx + 1$ and the x-axis is 8 square units, find the value of m.

2 a Find the point of intersection of the lines $2x + y = 2$ and $y = x + c$ in terms of c.

b Find the area of the region enclosed by the lines $2x + y = 2$, $y = x + c$ and the y-axis in terms of c.

c Given that the area of the region enclosed by the lines $2x + y = 2$, $y = x + c$ and the y-axis is 6 square units, find the value of c.

3 Solve the simultaneous equations for x and y. Assume that a and b are non-zero numbers.

a $ax - by = b$

$bx + ay = a$

b $2bx - a^2y = ab$

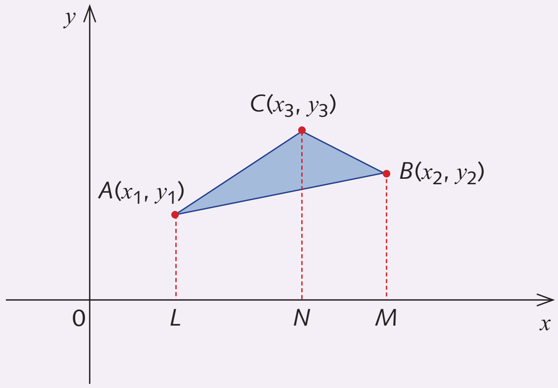
$ax + aby = a^2 + b^2$

c $\dfrac{x}{2a} + \dfrac{y}{2b} = a + b$

$x - y = -b(a + b)$

4 In the diagram shown below:

Area of triangle ABC = area of trapezium $ALNC$ + area of trapezium $CNMB$ − area of trapezium $LABM$



a Use this observation to prove that:

Area of triangle $ABC = \dfrac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

b Use the formula to calculate the area of the triangle ABC where A, B and C have coordinates $(2, 3)$, $(4, 1)$ and $(3, 6)$, respectively.

c Find the area of the triangle DEF where D, E and F have coordinates $(2, 5)$, $(4, 6)$ and $(3, 2)$ respectively. Explain the answer.

d If the area determined by this formula is zero, what can be said about the points?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 Solve the simultaneous equations.

a $x + 2y - 3z = 10$

$y + z = 7$

$y - z = 3$

b $x + y + z = 6$

$2x + 3y + 4z = 20$

$3x - 2y + 2z = 5$

6 Solve the simultaneous equations.

a $\frac{1}{x} - \frac{1}{y} = 4$

$\frac{1}{y} - \frac{1}{z} = 5$

$\frac{1}{x} + \frac{1}{z} = 1$

b $\frac{2}{x} - \frac{3}{y} + 4z = 16$

$\frac{1}{x} + \frac{6}{y} - z = 7$

$\frac{3}{x} - \frac{1}{y} + \frac{z}{3} = 15$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

15

Number and Algebra

Further factorisation

Consider the identity $(x-3)(x-4) = x^2 - 7x + 12$. Going from left to right is called expansion. Going from right to left is called factorisation. Once a quadratic is factorised, it is easy to write down the solutions to the corresponding equation.

In this chapter we review methods of factorising, including taking out common factors, factorising monic quadratics and recognising the difference of two squares or perfect squares.

We then introduce two more methods of factorising: factorising by grouping and extending the method of factorising simple quadratics to the general case $ax^2 + bx + c$. We then apply these methods in the sections on algebraic fractions.

In the last section we introduce the new technique of 'completing the square', which applies to all quadratics.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

15A Grouping

When factorising an expression with four terms, it sometimes helps to group the terms in pairs. It may happen that after factorising each pair, the resulting expression has a common factor. We may have to rearrange the four terms first.

Example 1

Factorise $3ax - a + 12x - 4$.

Solution

$$\begin{aligned} 3ax - a + 12x - 4 &= (3ax - a) + (12x - 4) \\ &= a(3x - 1) + 4(3x - 1) \\ &= (3x - 1)(a + 4) \end{aligned}$$

The rearrangement of the expression can take place in more than one way to give the result.

Example 2

Factorise $ab - 6 + 2a - 3b$.

Solution

$$\begin{aligned} ab - 6 + 2a - 3b &= ab + 2a - 3b - 6 \\ &= a(b + 2) - 3(b + 2) \\ &= (b + 2)(a - 3) \end{aligned}$$

Alternatively:

$$\begin{aligned} ab - 6 + 2a - 3b &= ab - 3b - 6 + 2a \\ &= b(a - 3) + 2(a - 3) \\ &= (a - 3)(b + 2) \end{aligned}$$

Example 3

Factorise $6x^2 - x - 12$ by writing it in the form $6x^2 - 9x + 8x - 12$.

Solution

We group the terms into pairs and factorise each pair.

$$\begin{aligned} 6x^2 - x - 12 &= (6x^2 - 9x) + (8x - 12) \\ &= 3x(2x - 3) + 4(2x - 3) \\ &= (2x - 3)(3x + 4) \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 15A

1 Factorise by grouping in pairs.

a $x(x+2)+5(x+2)$

b $2x(3-x)+5(3-x)$

c $4x(2x+3)+6(2x+3)$

d $2x(4-x)-5(4-x)$

e $4x(2x-3)-6(2x-3)$

f $4x(x-2)-7(x-2)$

g $2x(5-2x)+5(5-2x)$

h $7x(11x+5)+(11x+5)$

Example 1

2 Factorise by grouping in pairs.

a $mx+2m+3x+6$

b $ax+2a+4x+8$

c $mx+3m+2x+6$

d $5xy+10x+5y+10$

e $8ab-16a+b-2$

f $x-2+4xy-8y$

g $ab-7a-9b+63$

h $3ab+6a+5b+10$

i $2ab-12a-5b+30$

j $12x-8xy-15+10y$

Example 2, 3

3 Factorise by grouping in pairs.

a $x^2-3x-4x+12$

b $2x^2-2x+5x-5$

c $6x^2-4xy+3xy-2y^2$

d $10x^2+15xy-4xy-6y^2$

e $x^2-3x+4x-12$

f $4x^2+20x+x+5$

g $3x^2+12x+x+4$

h $6x^2-9x-8x+12$

i $6x^2+15x-4x-10$

j $2x^2-2x+x-1$

4 Copy and complete by grouping and factorising.

a $x^2+5x+6 = x^2+2x+3x+6$
$= \ldots$

b $x^2+5x-14 = x^2+7x-2x-14$
$= \ldots$

c $x^2-7x-18 = x^2-9x+2x-18$
$= \ldots$

d $x^2-5x-36 = x^2-9x+4x-36$
$= \ldots$

e $x^2+11x+30 = x^2+6x+5x+30$
$= \ldots$

f $x^2+29x+100 = x^2+25x+4x+100$
$= \ldots$

15B Factorising the general quadratic ax^2+bx+c

A quadratic expression is an expression of the form ax^2+bx+c, where a, b and c are given numbers (and $a \neq 0$). In Chapter 4 we introduced a technique for factorising **monic quadratic** expressions where $a=1$. This was extended to the class of quadratic expressions where $a=1$, but where a was a common factor of b and c. For example:

$$-2x^2+28x-96 = -2\left(x^2-14x+48\right) \quad \text{(factor out } -2\text{)}$$
$$= 2(x-6)(x-8) \quad \text{(find factors of 48 that sum to } -14\text{: } -6 \text{ and } -8\text{)}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Consider the quadratic expression $4x^2 + 5x - 6$. The coefficient of x^2 is not 1 and the coefficients have no common factor. Here is a method for factorising such quadratics.

Step 1: Multiply the coefficient of x^2, 4, by the constant term, −6.

$$4 \times (-6) = -24$$

Step 2: Find two numbers whose product is −24 and whose sum is 5, the coefficient of x. The numbers are 8 and −3.

$$8 \times (-3) = -24 \text{ and } 8 + (-3) = 5$$

Step 3: Split the x term using these two numbers.

$$5x = 8x - 3x$$

Step 4: Next use grouping to factorise the quadratic.

$$\begin{aligned} 4x^2 + 5x - 6 &= 4x^2 + 8x - 3x - 6 \\ &= 4x(x + 2) - 3(x + 2) \\ &= (x + 2)(4x - 3) \end{aligned}$$

Note: Grouping in a different way also works.

$$\begin{aligned} 4x^2 - 3x + 8x - 6 &= x(4x - 3) + 2(4x - 3) \\ &= (4x - 3)(x + 2) \end{aligned}$$

That is, $5x = 8x - 3x$ and $5x = -3x + 8x$ both work.

Factorising the general quadratic

To factorise a quadratic of the form $ax^2 + bx + c$, find two numbers α and β whose sum is b and whose product is ac. Split the middle term as $\alpha x + \beta x$ and factorise by grouping.

Note: This method also works for monic quadratics.

Example 4

Factorise $2x^2 + 9x + 4$.

Solution

Multiply 2 by 4 to obtain 8. Look for two numbers whose product is 8 and whose sum is 9. The numbers are 8 and 1.

Hence
$$\begin{aligned} 2x^2 + 9x + 4 &= 2x^2 + 8x + x + 4 \\ &= 2x(x + 4) + 1(x + 4) \\ &= (2x + 1)(x + 4) \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 5

Factorise:

a $6x^2 - 19x + 10$

b $3x^2 - 5x - 28$

Solution

a Multiply 6 by 10 to obtain 60. Look for two numbers whose product is 60 and whose sum is −19. The numbers are −15 and −4.

Hence
$$\begin{aligned}6x^2 - 19x + 10 &= 6x^2 - 15x - 4x + 10\\ &= 3x(2x - 5) - 2(2x - 5)\\ &= (2x - 5)(3x - 2)\end{aligned}$$

b Multiply 3 by −28 to obtain −84. Look for two numbers whose product is −84 and whose sum is −5. The numbers are −12 and 7.

$$\begin{aligned}3x^2 - 5x - 28 &= 3x^2 - 12x + 7x - 28\\ &= 3x(x - 4) + 7(x - 4)\\ &= (x - 4)(3x + 7)\end{aligned}$$

There are other methods for factorising quadratics in which the coefficient of x^2 is not 1, but this method is recommended.

Exercise 15B

Example 4

1 Factorise the quadratic expressions.

a $2x^2 + 7x + 5$ **b** $2x^2 + 11x + 5$ **c** $2x^2 + 13x + 21$

d $2x^2 + 17x + 21$ **e** $3x^2 + 14x + 8$ **f** $3x^2 + 10x + 8$

g $4x^2 + 16x + 15$ **h** $4x^2 + 23x + 15$ **i** $2x^2 + 31x + 15$

j $2x^2 + 17x + 15$ **k** $4x^2 + 32x + 15$ **l** $6x^2 + 19x + 10$

m $12x^2 + 32x + 5$ **n** $12x^2 + 16x + 5$ **o** $8x^2 + 34x + 21$

Example 5

2 Factorise the quadratic expressions.

a $3x^2 - 14x + 8$ **b** $3x^2 - 10x + 8$ **c** $4x^2 - 21x + 5$

d $4x^2 - 9x + 5$ **e** $6x^2 - 11x + 3$ **f** $6x^2 - 7x + 2$

g $4x^2 - 12x + 5$ **h** $4x^2 - 16x + 15$ **i** $6x^2 - 19x + 10$

j $6x^2 - 25x + 4$ **k** $6x^2 - 11x + 5$ **l** $6x^2 - 17x + 10$

m $4x^2 - 39x + 27$ **n** $4x^2 - 24x + 27$ **o** $10x^2 - 7x + 1$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Factorise the quadratic expressions.

a $2x^2 - 9x - 5$ **b** $2x^2 - 3x - 5$ **c** $2x^2 + 9x - 5$

d $2x^2 + 9x - 18$ **e** $3x^2 + x - 10$ **f** $3x^2 - 13x - 10$

g $4x^2 - 4x - 15$ **h** $4x^2 + 17x - 15$ **i** $4x^2 - 7x - 15$

j $4x^2 - 28x - 15$ **k** $12x^2 + 17x - 5$ **l** $12x^2 + 11x - 5$

m $6x^2 - 11x - 10$ **n** $10x^2 - 31x - 14$ **o** $20x^2 - 7x - 3$

4 Find the missing factor and check by expansion.

a $x^2 + 7x + 10 = (x + 5)\ldots\ldots$ **b** $x^2 - 9 = (x - 3)\ldots\ldots$

c $x^2 + 8x + 16 = (x + 4)\ldots\ldots$ **d** $a^2 + 2a + ab + 2b = (a + 2)\ldots\ldots$

e $9x^2 - 16y^2 = (3x + 4y)\ldots\ldots$ **f** $2x^2 - 7x - 4 = (2x + 1)\ldots\ldots$

g $6x^2 + 13x + 6 = (3x + 2)\ldots\ldots$ **h** $6x^2 - 17x - 3 = (6x + 1)\ldots\ldots$

i $6x^2 + 35x - 6 = (x + 6)\ldots\ldots$ **j** $4x^2 + 12x + 9 = (2x + 3)\ldots\ldots$

5 Copy and complete.

a $x^2 + 7x + \ldots = (x + 5)(x + \ldots)$ **b** $x^2 - 6x \ldots = (x - 4)(x \ldots)$

c $x^2 + \ldots x + 15 = (x + 3)(x + \ldots)$ **d** $x^2 \ldots x - 24 = (x - 4)(x \ldots)$

6 **a** If $ax^2 + bx + c = (px + q)(rx + s)$, show that $a = pr$, $b = ps + qr$ and $c = qs$.

b Hence show that if $ax^2 + bx + c$ can be factorised, then b can be written as the sum of two numbers whose product is ac. (This shows why our method of factorisation works.)

15C Simplifying, multiplying and dividing algebraic fractions

We can sometimes use factorisation techniques to simplify algebraic fractions. In this section, we show how some quotients and products of quotients can be simplified by first factorising and then cancelling common factors.

Example 6

Simplify:

a $\dfrac{x^2 + xy}{x^2 - y^2}$ **b** $\dfrac{8n^2 - 50m^2}{4n^3 + 10mn^2}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

a $\dfrac{x^2 + xy}{x^2 - y^2} = \dfrac{x\cancel{(x+y)}}{\cancel{(x+y)}(x-y)}$

$= \dfrac{x}{x-y}$

b $\dfrac{8n^2 - 50m^2}{4n^3 + 10mn^2} = \dfrac{\cancel{2}(4n^2 - 25m^2)}{\cancel{2}n^2(2n+5m)}$

$= \dfrac{\cancel{(2n+5m)}(2n-5m)}{n^2\cancel{(2n+5m)}}$

$= \dfrac{2n-5m}{n^2}$

Example 7

Simplify:

a $\dfrac{x^2 - x - 12}{x^2 - 12x + 32}$ **b** $\dfrac{2x^2 + 9x - 18}{2x^2 - 11x + 12}$

Solution

a $\dfrac{x^2 - x - 12}{x^2 - 12x + 32} = \dfrac{(x-4)(x+3)}{(x-4)(x-8)} = \dfrac{(x+3)}{(x-8)}$

b $2x^2 + 9x - 18 = 2x^2 + 12x - 3x - 18$ $(2 \times (-18) = -36 = 12 \times (-3);$ $12 + (-3) = 9)$

$= 2x(x+6) - 3(x+6)$

$= (x+6)(2x-3)$

and $2x^2 - 11x + 12 = 2x^2 - 8x - 3x + 12$ $(2 \times 12 = 24 = (-8) \times (-3);$ $-8 + (-3) = -11)$

$= 2x(x-4) - 3(x-4)$

$= (x-4)(2x-3)$

so $\dfrac{2x^2 + 9x - 18}{2x^2 - 11x + 12} = \dfrac{(x+6)\cancel{(2x-3)}}{(x-4)\cancel{(2x-3)}}$

$= \dfrac{x+6}{x-4}$

Example 8

Simplify:

a $\dfrac{x^2 - 3x - 10}{x^2 - 5x} \times \dfrac{2x^2 - 2x}{x^2 - x - 6}$ **b** $\dfrac{4x^2 - 1}{4x^2 + 4x + 1} \div \dfrac{4x^2 - 4x + 1}{2x^2 - 3x + 1}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a $$\frac{x^2-3x-10}{x^2-5x} \times \frac{2x^2-2x}{x^2-x-6} = \frac{\cancel{(x-5)}\,\cancel{(x+2)}}{\cancel{x}\,\cancel{(x-5)}} \times \frac{2\cancel{x}(x-1)}{(x-3)\cancel{(x+2)}}$$
$$= \frac{2(x-1)}{(x-3)}$$

b $$\frac{4x^2-1}{4x^2+4x+1} \div \frac{4x^2-4x+1}{2x^2-3x+1} = \frac{4x^2-1}{4x^2+4x+1} \times \frac{2x^2-3x+1}{4x^2-4x+1}$$
$$= \frac{\cancel{(2x+1)}\,\cancel{(2x-1)}}{(2x+1)\cancel{(2x+1)}} \times \frac{\cancel{(2x-1)}(x-1)}{\cancel{(2x-1)}\,\cancel{(2x-1)}}$$
$$= \frac{x-1}{2x+1}$$

Exercise 15C

Example 6

1 Simplify:

a $\frac{n-m}{m-n}$ **b** $\frac{k^2-k\ell}{\ell-k}$ **c** $\frac{6x^2+6xy}{y^2-x^2}$

d $\frac{18k^2-8\ell^2}{6k-4\ell}$ **e** $\frac{12p^2-27q^2}{6p^2-9pq}$ **f** $\frac{pq}{p^2q^2+pq}$

g $\frac{m^2+mn}{n^2+nm}$ **h** $\frac{s^2-t^2}{t-s}$ **i** $\frac{3x-3y}{xy-x^2}$

j $\frac{3p^3-3pq^2}{p^2+pq}$ **k** $\frac{x^3-xy^2}{2x+2y}$ **l** $\frac{m^3-m^2n}{n-m}$

Example 7a

2 Simplify:

a $\frac{x^2-1}{x^2-x-2}$ **b** $\frac{x^2-4}{x^2-3x-10}$ **c** $\frac{x^2+4x-5}{x^2-2x+1}$

d $\frac{x^2+x-12}{x^2-3x}$ **e** $\frac{x^2-x-20}{x^2+x-12}$ **f** $\frac{x^2+6x+9}{x^2+x-6}$

g $\frac{2x^2-18}{3x^2+3x-18}$ **h** $\frac{x^2-5x+6}{x^2-4x+4}$ **i** $\frac{3x^2+3x-36}{x^2+8x+16}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7b

3 Simplify:

a $\dfrac{(2x-3)(x+1)}{(2x-3)(x+2)}$ **b** $\dfrac{(5x+6)(x+1)}{(2x-1)(5x+6)}$ **c** $\dfrac{(7x+1)(x-3)}{(x-3)(7x+1)}$

d $\dfrac{2x^2-3x+1}{x^2-2x+1}$ **e** $\dfrac{4x^2-1}{4x-2}$ **f** $\dfrac{5x^2+9x-2}{x^2-4}$

g $\dfrac{2x^2+3x-2}{2x^2-7x+3}$ **h** $\dfrac{2x^2-5x+3}{2x^2-x-3}$ **i** $\dfrac{2x^2-7x+3}{2x^2-5x-3}$

Example 8a

4 Simplify:

a $\dfrac{3x^2}{2x+1}\times\dfrac{4x^2-1}{x^3+5x^2}$ **b** $\dfrac{x^2-x-2}{x^2+3x}\times\dfrac{5x}{x-2}$

c $\dfrac{2x^2+3x-2}{x^2+2x}\times\dfrac{x^2-4x}{x^2-5x-4}$ **d** $\dfrac{3x^2-10x+3}{3x^2-7x+2}\times\dfrac{3x^2-6x}{4x^2-11x-3}$

Example 8b

5 Simplify:

a $\dfrac{3x^2-3x}{x+1}\div\dfrac{1-x}{x^2+x}$ **b** $\dfrac{x^2-4}{2x^2-6x}\div\dfrac{6-x-x^2}{9-x^2}$

c $\dfrac{2x^2+5x-3}{x^2-1}\div\dfrac{x^2-x-12}{x^2-3x-4}$ **d** $\dfrac{16x^2+8x+1}{8x^2+14x+3}\div\dfrac{4x+1}{4x^2+4x-3}$

e $\dfrac{2x^2-3x-2}{x^2+3x}\div\dfrac{2x^2+x}{x^2+2x-3}\div\dfrac{x^2-3x+2}{x^2}$

6 Simplify:

a $\dfrac{6x^2+x-2}{10x^2-9x+2}\times\dfrac{10x^2+x-2}{6x^2+7x+2}$ **b** $\dfrac{x^2-9}{2x^2-7x+3}\div\dfrac{2x^2-3x+1}{x^2-1}$

c $\dfrac{x^2}{2x^2-7x+3}\times\dfrac{2x^2-11x+15}{2x^2-5x}$ **d** $\dfrac{2x^2-x-3}{x^2-1}\div\dfrac{2x^2-5x+3}{x^2+2x+1}$

e $\dfrac{p^2-q^2}{2p^2+p}\times\dfrac{2pq+q}{p^2-pq}\div(p+q)$

f $\dfrac{2a^2-32}{a^2+7a+12}\div\dfrac{4a^2-4a-48}{12a^2+15a}\div\dfrac{(a-1)(4a+5)}{a}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

15D Adding and subtracting algebraic fractions

Recall how we add two fractions with different denominators:

$$\begin{aligned}\frac{3}{4}+\frac{7}{10}&=\frac{15}{70}+\frac{49}{70} \quad \text{(Express fractions with a common denominator.)}\\&=\frac{64}{70}\\&=\frac{32}{35}\end{aligned}$$

Notice that we use the lowest common multiple of 14 and 10, which is 70, as the common denominator.

We use the same procedures to add and subtract algebraic fractions.

Example 9

Express as a single fraction.

a $\dfrac{2x}{5}+\dfrac{x}{3}$ **b** $-\dfrac{m}{3}+\dfrac{2m}{9}$

Solution

a $$\begin{aligned}\frac{2x}{5}+\frac{x}{3}&=\frac{6x}{15}+\frac{5x}{15}\\&=\frac{11x}{15}\end{aligned}$$

b $$\begin{aligned}-\frac{m}{3}+\frac{2m}{9}&=-\frac{3m}{9}+\frac{2m}{9}\\&=-\frac{m}{9}\end{aligned}$$

When the denominator is an algebraic expression, the procedure is the same.

Example 10

Express as a single fraction.

a $\dfrac{4}{x}+\dfrac{2}{x}$ **b** $\dfrac{4}{x}+\dfrac{2}{3x}$ **c** $\dfrac{5}{x^2}+\dfrac{4}{7x}$ **d** $\dfrac{4}{x-1}+\dfrac{2}{x+1}$

Solution

a $$\frac{4}{x}+\frac{2}{x}=\frac{6}{x}$$

b $$\begin{aligned}\frac{4}{x}+\frac{2}{3x}&=\frac{12}{3x}+\frac{2}{3x}\\&=\frac{14}{3x}\end{aligned}$$

c $$\begin{aligned}\frac{5}{x^2}+\frac{4}{7x}&=\frac{35}{7x^2}+\frac{4x}{7x^2}\\&=\frac{4x+35}{7x^2}\end{aligned}$$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d $$\begin{aligned}\frac{4}{x-1}+\frac{2}{x+1} &= \frac{4(x+1)}{(x-1)(x+1)}+\frac{2(x-1)}{(x+1)(x-1)}\\ &= \frac{4(x+1)+2(x-1)}{(x+1)(x-1)}\\ &= \frac{4x+4+2x-2}{(x+1)(x-1)}\\ &= \frac{6x+2}{(x+1)(x-1)} \quad \text{or} \quad \frac{6x+2}{x^2-1}\end{aligned}$$

Either answer is acceptable.

Example 11

Simplify $\dfrac{4}{x-1}+\dfrac{2}{1-x}$.

Solution

Note that $$\frac{2}{1-x}=\frac{-2}{x-1}$$

Hence $$\begin{aligned}\frac{4}{x-1}+\frac{2}{1-x} &= \frac{4}{x-1}+\frac{-2}{x-1}\\ &= \frac{2}{x-1}\end{aligned}$$

Example 12

Simplify $\dfrac{5}{(x-3)(x+2)}-\dfrac{4}{(x+2)(x-1)}$.

Solution

The common denominator of the two expressions is $(x-3)(x+2)(x-1)$.

$$\begin{aligned}\frac{5}{(x-3)(x+2)}-\frac{4}{(x+2)(x-1)} &= \frac{5(x-1)-4(x-3)}{(x-3)(x+2)(x-1)}\\ &= \frac{5x-5-4x+12}{(x-3)(x+2)(x-1)}\\ &= \frac{x+7}{(x-3)(x+2)(x-1)}\end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 13

Simplify $\dfrac{4}{x^2+5x+6}+\dfrac{3}{x^2+4x+3}$.

Solution

Note that $x^2+5x+6=(x+3)(x+2)$ and $x^2+4x+3=(x+3)(x+1)$.

The common denominator of the two fractions is $(x+1)(x+2)(x+3)$.

$$\begin{aligned}\text{So } \frac{4}{x^2+5x+6}+\frac{3}{x^2+4x+3} &= \frac{4}{(x+3)(x+2)}+\frac{3}{(x+3)(x+1)}\\ &= \frac{4(x+1)}{(x+3)(x+2)(x+1)}+\frac{3(x+2)}{(x+3)(x+2)(x+1)}\\ &= \frac{4x+4+3x+6}{(x+3)(x+2)(x+1)}\\ &= \frac{7x+10}{(x+3)(x+2)(x+1)}\end{aligned}$$

Exercise 15D

Example 9

1 Express as a single fraction.

a $\dfrac{3x}{8}+\dfrac{x}{8}$ **b** $\dfrac{4x}{5}-\dfrac{2x}{5}$ **c** $\dfrac{2x}{7}-\dfrac{5x}{7}$ **d** $\dfrac{4x}{5}+\dfrac{x}{2}$

e $\dfrac{6x}{5}-\dfrac{x}{4}$ **f** $\dfrac{2x}{7}-\dfrac{2x}{3}$ **g** $\dfrac{2x}{3}+\dfrac{x}{2}-\dfrac{3x}{4}$ **h** $\dfrac{3x}{5}-\dfrac{2x}{3}$

Example 10a, b

2 Express as a single fraction.

a $\dfrac{4}{x}+\dfrac{3}{x}$ **b** $\dfrac{5}{x}+\dfrac{6}{x}$ **c** $\dfrac{3}{x}+\dfrac{4}{2x}$

d $\dfrac{5}{x^2}+\dfrac{3}{x}$ **e** $\dfrac{5}{x}-\dfrac{3}{x}$ **f** $\dfrac{7}{x^2}-\dfrac{3}{x}$

g $\dfrac{4}{x}+\dfrac{3}{2x}$ **h** $\dfrac{7}{2x}-\dfrac{4}{3x}$ **i** $\dfrac{11}{3x}-\dfrac{5}{2x}$

Example 10c, 11

3 Simplify:

a $\dfrac{2}{x+1}+\dfrac{1}{x+2}$ **b** $\dfrac{3}{x+4}+\dfrac{2}{x-1}$ **c** $\dfrac{5}{x+3}+\dfrac{4}{x-2}$

d $\dfrac{3}{x+1}-\dfrac{2}{x+3}$ **e** $\dfrac{7}{x+2}-\dfrac{4}{x-1}$ **f** $\dfrac{3}{2x-1}-\dfrac{1}{x+2}$

g $\dfrac{2}{2a+1}+\dfrac{3}{a-4}$ **h** $\dfrac{4}{b+3}-\dfrac{2}{2b-1}$ **i** $\dfrac{4}{b-1}-\dfrac{3}{1-2b}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Simplify:

a $\dfrac{1}{x-5}+\dfrac{3}{x-5}$ **b** $\dfrac{4}{a+5}+\dfrac{2}{a-5}$ **c** $\dfrac{x}{x-3}+\dfrac{1}{x+4}$

d $\dfrac{2x}{(x-2)^2}+\dfrac{2}{x-2}$ **e** $\dfrac{1}{a-5}-\dfrac{2}{a-5}$ **f** $\dfrac{2a}{a-4}+\dfrac{3a}{a+4}$

g $\dfrac{1}{(x-7)^2}-\dfrac{2}{x-7}$ **h** $\dfrac{2}{(a-1)^2}-\dfrac{2a}{(a-1)^2}$ **i** $\dfrac{2x+3}{x-4}-\dfrac{2x-4}{x-4}$

Example 12, 13

5 Simplify:

a $\dfrac{1}{(x+1)(x+2)}+\dfrac{2}{(x+2)(x+3)}$ **b** $\dfrac{2}{(x-1)(x+3)}+\dfrac{1}{(x-1)(x-2)}$

c $\dfrac{2}{(x-3)(x-2)}-\dfrac{4}{(x-1)(x-2)}$ **d** $\dfrac{4}{(x+1)(x-3)}-\dfrac{3}{x+1}$

e $\dfrac{3}{x^2-x-2}+\dfrac{4}{x^2-3x+2}$ **f** $\dfrac{5}{x^2-1}+\dfrac{3}{x^2-2x-3}$

g $\dfrac{3}{(x+1)(x-2)}+\dfrac{1}{(x+3)(2-x)}$ **h** $\dfrac{4}{x^2+2x-3}-\dfrac{1}{1-x^2}$

i $\dfrac{3}{x(x+2)}+\dfrac{4}{(x+2)(x-1)}$ **j** $\dfrac{1}{(x-2)(x+4)}-\dfrac{3}{(x+4)(x-3)}$

k $\dfrac{3}{(x+1)(x+3)}+\dfrac{2}{x(x+3)}$ **l** $\dfrac{4}{(x+1)(x+2)}+\dfrac{3}{x+1}$

m $\dfrac{1}{x^2-2x-8}+\dfrac{3}{x^2+3x+2}$ **n** $\dfrac{2}{x^2+2x}+\dfrac{3}{x^2-4}$

o $\dfrac{2}{x(3-x)}+\dfrac{4}{(x+1)(x-3)}$ **p** $\dfrac{3}{4-x^2}+\dfrac{4}{x(x-2)}$

6 Simplify:

a $\dfrac{2}{x+1}+\dfrac{3}{x}$ **b** $\dfrac{3}{2m-n}+\dfrac{5}{3m+n}$ **c** $\dfrac{m}{m-n}-\dfrac{m}{m+n}$

d $\dfrac{x}{3x+1}-\dfrac{5}{x^2}$ **e** $\dfrac{x-1}{x+1}-\dfrac{x-2}{x+2}$ **f** $\dfrac{3k+1}{k^2-1}+\dfrac{3k-1}{k+1}$

g $\dfrac{5}{2p+1}+\dfrac{1}{p}$ **h** $\dfrac{1}{1+q}+\dfrac{1}{1-q}$ **i** $\dfrac{7}{2k+\ell}+\dfrac{5}{k-2\ell}$

j $\dfrac{2p}{p+1}+\dfrac{3p}{p-1}$ **k** $\dfrac{p+2}{p-1}+\dfrac{p-1}{p+2}$ **l** $\dfrac{7m+n}{2m-n}+\dfrac{3m-n}{2m+n}$

m $\dfrac{1}{x+2}+\dfrac{2}{x-2}+\dfrac{4}{x^2-4}$ **n** $\dfrac{1}{x+3}+\dfrac{6}{x^2-9}+\dfrac{1}{3-x}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

o $\dfrac{3}{x+3}+\dfrac{2}{x-4}-\dfrac{5x-6}{x^2-x-12}$

p $\dfrac{2}{k+5}+\dfrac{1}{k-4}-\dfrac{9}{k^2+k-20}$

q $\dfrac{1}{x+1}+\dfrac{1}{x-1}+\dfrac{2x-2}{1-x^2}$

r $\dfrac{4}{2+x}-\dfrac{3}{2-x}-\dfrac{7x}{x^2-4}$

s $\dfrac{5}{3+x}+\dfrac{2}{x-3}-\dfrac{6-6x}{9-x^2}$

t $\dfrac{2\ell}{(1-2\ell)^2}-\dfrac{1-\ell}{1-5\ell+6\ell^2}+\dfrac{2}{1-3\ell}$

u $\dfrac{p}{p+q}+\dfrac{q}{p-q}+\dfrac{p^2+q^2}{q^2-p^2}$

v $\dfrac{1}{p^2-4p+3}-\dfrac{1}{p^2-3p+2}-\dfrac{1}{p^2-5p+6}$

15E Completing the square

In this section a general technique for factorising quadratics is introduced.

What number must be added to x^2+6x to make a perfect square?

It is 9, which is the square of half of the coefficient of x. We get

$$x^2+6x+9=(x+3)^2$$

This idea is the basis for an important technique called **completing the square**.

The key step is to add and subtract the square of half the coefficient of x. For example, to complete the square of $x^2+10x-6$, we add and subtract 25, which is the square of half of 10.

Focus on x^2+10x. The related perfect square is $x^2+10x+25$.

$$\begin{aligned} x^2+10x-6 &= x^2+10x+25-25-6 \quad \text{(Add and subtract 25.)}\\ &= (x^2+10x+25)-31\\ &= (x+5)^2-31 \end{aligned}$$

Completing the square

- Add the square of half of the coefficient of x and then take it away.
- Write the first part of the quadratic as a perfect square.

For example, $$\begin{aligned} x^2-10x-6 &= (x^2-10x+25)-25-6\\ &= (x-5)^2-31 \end{aligned}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 14

Complete the square.

a $x^2 + 4x + 2$ **b** $x^2 + 7x - 4$ **c** $x^2 - 12x - 10$

Solution

a $x^2 + 4x + 2 = (x^2 + 4x + 4) - 4 + 2$

$= (x + 2)^2 - 2$

b $x^2 + 7x - 4 = \left(x^2 + 7x + \frac{49}{4}\right) - \frac{49}{4} - 4$

$= \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} - \frac{16}{4}$

$= \left(x + \frac{7}{2}\right)^2 - \frac{65}{4}$

c $x^2 - 12x - 10 = (x^2 - 12x + 36) - 36 - 10$

$= (x - 6)^2 - 46$

Example 15

Factorise by completing the square.

a $x^2 - 2x - 8$ **b** $x^2 - 4x - 3$ **c** $x^2 + 7x + 1$

Solution

a $x^2 - 2x - 8 = (x^2 - 2x + 1) - 1 - 8$

$= (x - 1)^2 - 9$ ($9 = 3^2$. Use the 'difference of two squares' identity.)

$= (x - 1 - 3)(x - 1 + 3)$

$= (x - 4)(x + 2)$

b $x^2 - 4x - 3 = (x^2 - 4x + 4) - 4 - 3$

$= (x - 2)^2 - 7$

$= (x - 2)^2 - (\sqrt{7})^2$

$= (x - 2 - \sqrt{7})(x - 2 + \sqrt{7})$

c $x^2 + 7x + 1 = x^2 + 7x + \frac{49}{4} - \frac{49}{4} + 1$

$= \left(x + \frac{7}{2}\right)^2 - \frac{45}{4}$

$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{\sqrt{45}}{2}\right)^2$

$= \left(x + \frac{7}{2} - \frac{3\sqrt{5}}{2}\right)\left(x + \frac{7}{2} + \frac{3\sqrt{5}}{2}\right)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 15E

1 Copy and complete the following.

a $x^2 + 6x + \ldots = (x + 3)^2$

b $x^2 + 12x + \ldots = (x + 6)^2$

c $x^2 + 5x + \ldots = \left(x + \frac{5}{2}\right)^2$

d $x^2 + 7x + \ldots = \left(x + \frac{7}{2}\right)^2$

e $x^2 - 12x + \ldots = (x - 6)^2$

f $x^2 - 100x + \ldots = (x - \ldots)^2$

g $x^2 - \ldots x + \ldots = (x - 5)^2$

h $x^2 - 80x + \ldots = (x - \ldots)^2$

Example 14

2 Complete the square.

a $x^2 + 2x - 5$ **b** $x^2 + 2x + 7$ **c** $x^2 + 4x + 1$

d $x^2 + 6x + 2$ **e** $x^2 + 6x - 3$ **f** $x^2 - 6x + 6$

g $x^2 - 8x - 5$ **h** $x^2 + 8x + 25$ **i** $x^2 + 12x - 11$

j $x^2 + 3x - 2$ **k** $x^2 + 11x + 7$ **l** $x^2 - 10x - 3$

m $x^2 + 4x + 10$ **n** $x^2 - 8x + 20$ **o** $x^2 - 6x + 9$

p $x^2 + 5x + 8$ **q** $x^2 + 10x - 4$ **r** $x^2 + 5x - 10$

Example 15

3 Factorise by first completing the square and then using the difference of two squares identity.

a $x^2 + 4x + 3$ **b** $x^2 + 5x + 4$ **c** $x^2 + 5x + 6$

d $x^2 + 6x + 5$ **e** $x^2 + 6x - 3$ **f** $x^2 - 10x - 3$

g $x^2 + 12x - 5$ **h** $x^2 - 12x + 5$ **i** $x^2 + 8x - 5$

j $x^2 - 10x - 6$ **k** $x^2 - 11x + 1$ **l** $x^2 + 9x - 8$

m $x^2 - 7x + 2$ **n** $x^2 - 7x - 2$ **o** $x^2 + 13x - 5$

Review exercise

1 Factorise:

a $9x - 63$ **b** $24x^2 + 8$ **c** $-36x + 42x^2$

d $8y - 16y^2$ **e** $-64b - 32$ **f** $21m^2 + 7mn$

2 Factorise:

a $a(a + 5) + 4(a + 5)$

b $2p(6p - 1) - 7q(6p - 1)$

c $4x(3y + 5) - (3y + 5)$

d $2m + 9 + 2n(2m + 9)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 Factorise:

a $a^2 - 121$ **b** $9x^2 - 36$ **c** $64m^2 - 169n^2$

d $7x^2 - 63$ **e** $x^2 - 36$ **f** $x^2 - 16y^2$

g $3m^2 - 27$ **h** $6x^2 - 24$ **i** $4b^2 - 100$

4 Factorise the quadratic expressions.

a $x^2 + 5x - 24$ **b** $x^2 - 15x + 36$ **c** $x^2 - 5x - 14$

d $4a^2 + 24a + 20$ **e** $3r^2 - 6r - 24$ **f** $5a^2 - 60a + 180$

g $5a^2 - 10a + 5$ **h** $-2p^2 + 16p - 32$ **i** $-b^2 - 3b + 4$

5 Factorise:

a $bx + 3b + 4x + 12$ **b** $x^2 - 5x - 6x + 30$

c $6x - 12xy - 4 + 8y$ **d** $x^3 - 3x + 5x^2 - 15$

6 Factorise the quadratic expression.

a $4x^2 + 25x + 6$ **b** $6x^2 - 7x + 2$ **c** $12x^2 - 17x - 5$

d $2t^2 + 5t - 3$ **e** $10a^2 - a - 2$ **f** $28x^2 - 85x + 63$

g $39x^2 + 131x - 44$ **h** $-96x^2 + 76x - 15$ **i** $15x^2 + 4x - 35$

7 Find the missing factor.

a $x^2 + 9x + 20 = (x + 4)\ldots\ldots$ **b** $x^2 - 16 = (x - 4)\ldots\ldots$

c $x^2 + 10x + 25 = (x + 5)\ldots\ldots$ **d** $m^2 + 3m + mn + 3n = (m + 3)\ldots\ldots$

e $25a^2 - 9b^2 = (5a + 3b)\ldots\ldots$ **f** $3x^2 - 10x - 8 = (3x + 2)\ldots\ldots$

8 Simplify:

a $\dfrac{(x+5)(x+3)}{(x+5)}$ **b** $\dfrac{x^2 - x - 6}{x^2 - 8x + 15}$ **c** $\dfrac{x^2 + 7x + 10}{x^2 + 6x + 8}$ **d** $\dfrac{(x+3)(3x+1)}{x^2 + 6x + 9}$

e $\dfrac{3x^2 - 7x + 2}{x^2 - 4x + 4}$ **f** $\dfrac{9x^2 - 4}{12x - 8}$ **g** $\dfrac{8x^2 + 5x - 3}{x^2 - 1}$ **h** $\dfrac{3x^2 + x - 2}{3x^2 + 4x - 4}$

9 Simplify:

a $\dfrac{a^2 + ab}{a^2 - ab} \times \dfrac{ab^2 + b^2}{a^3 + a^2b}$ **b** $\dfrac{a^2}{a^2 - 4} \times \dfrac{a^2 - 5a + 6}{3a - a^2}$

c $\dfrac{m^2 - m - 6}{m^2 - 9} \times \dfrac{m^2}{m^2 + 2m}$ **d** $\dfrac{3x^2 + x - 2}{x^2 - 1} \times \dfrac{x^2 - 2x + 1}{x^2 - x - 2}$

e $\dfrac{4x^2 + x - 3}{x^2 + 2x} \div \dfrac{x^2 - 4x - 5}{x^2 - 3x - 10}$ **f** $\dfrac{2x^2 + x - 3}{x^2 + 2x} \div \dfrac{x^2 - 4x + 3}{x^2} \div \dfrac{2x^2 + 3x}{x^2 - x - 6}$

g $\dfrac{x^2 - 3x - 4}{x^2 - 4x} \div \dfrac{x^2 - 4x + 4}{x^2 - 4}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 Simplify:

a $\dfrac{3x}{10}+\dfrac{x}{10}$

b $\dfrac{7x}{6}-\dfrac{3x}{4}$

c $\dfrac{x}{2}+\dfrac{3x}{4}-\dfrac{2x}{3}$

d $\dfrac{3}{x+1}+\dfrac{1}{x+3}$

e $\dfrac{5}{x+2}+\dfrac{4}{x-1}$

f $\dfrac{5}{x-2}+\dfrac{1}{1-3x}$

g $\dfrac{2}{x+4}+\dfrac{3}{x-4}$

h $\dfrac{2}{(x-3)^2}+\dfrac{4}{x-3}$

i $\dfrac{3x}{(x-5)^2}+\dfrac{1}{x-5}$

11 Simplify:

a $\dfrac{1}{(x-2)(x+1)}+\dfrac{2}{(x-2)(x-3)}$

b $\dfrac{3}{(x+3)(x+1)}+\dfrac{6}{x+3}$

c $\dfrac{2}{x^2+x-2}+\dfrac{5}{x^2-3x+2}$

d $\dfrac{3}{x^2-4}+\dfrac{6}{x^2-2x-8}$

e $\dfrac{2}{(x+7)(x-3)}+\dfrac{3}{(x+4)(3-x)}$

f $\dfrac{6}{x(x+3)}+\dfrac{2}{(x+3)(x-4)}$

12 Simplify:

a $\dfrac{b}{a-b}-\dfrac{a}{a+b}$

b $\dfrac{3x+5}{x^2-9}+\dfrac{3x-1}{x+3}$

c $\dfrac{3}{4p+q}+\dfrac{3}{p-2q}$

d $\dfrac{2}{x+5}+\dfrac{1}{x-5}+\dfrac{5}{x^2-25}$

e $\dfrac{2}{x+4}+\dfrac{3}{x-5}-\dfrac{4x-1}{x^2-x-20}$

f $\dfrac{a}{a-b}+\dfrac{b}{a+b}-\dfrac{a^2+b^2}{a^2-b^2}$

g $\dfrac{1}{a^2-6a+8}-\dfrac{1}{a^2-5a+6}-\dfrac{1}{a^2-7a+12}$

13 Complete the square.

a x^2+4x-4 **b** x^2-6x+7 **c** x^2-8x-6 **d** x^2+3x-1

14 Factorise:

a $15x^2+5x-10$

b $12x^2+23x+10$

c $12x^2-7x-10$

d $9x^2-36x+35$

e $3x^2-26x+55$

f $3x^2+16x-35$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 Simplify:

a $\dfrac{1}{c-d}+\dfrac{1}{d-c}$

b $\dfrac{p}{p-q}+\dfrac{q}{q-p}$

c $\dfrac{1}{a+\dfrac{1}{b}}+\dfrac{1}{b+\dfrac{1}{a}}-\dfrac{1}{\dfrac{1}{2}a+\dfrac{1}{2}b}$

d $\dfrac{b+c}{b-c}+\dfrac{b-c}{b+c}+\dfrac{4bc}{c^2-b^2}$

e $\dfrac{b+c}{(b-a)(c-a)}+\dfrac{c+a}{(c-b)(a-b)}+\dfrac{a+b}{(a-c)(b-c)}$

2 Use the method of completing the square to show that:

a $x^2+4x+15 \geq 11$

b $x^2+2x+15 \geq 14$

3 **a** Show that the area, A cm^2, of a rectangle of perimeter 20 cm is given by the formula $A = w(10-w)$, where w cm is the width.

b Complete the square for the quadratic expression $10w - w^2$ and hence show $A \leq 25$.

c What value of w makes the area equal to 25 m^2?

d Complete: Of all the rectangles with perimeter 20 m, the one with the largest area is ...

4 By considering $\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)$, prove that 'the sum of a positive number and its reciprocal is greater than or equal to 2'.

5 For positive numbers a and b:

a Show that $\dfrac{a+b}{2} \geq \sqrt{ab}$.

b When are $\dfrac{a+b}{2}$ and $\sqrt{ab}$ equal?

c Deduce that $\dfrac{a}{b}+\dfrac{b}{a} \geq 2$.

d Show that $\dfrac{2}{\dfrac{1}{a}+\dfrac{1}{b}} \leq \sqrt{ab} \leq \dfrac{a+b}{2}$.

Mean

Each of these numbers has a name. If a and b are positive then:

- $\dfrac{a+b}{2}$ is called the **arithmetic mean** of a and b.
- $\dfrac{2}{\dfrac{1}{a}+\dfrac{1}{b}}$ is called the **harmonic mean** of a and b.
- $\sqrt{ab}$ is called the **geometric mean** of a and b.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 Prove that a positive number plus one-quarter of its reciprocal is always greater than 1.

7 If $(a - b)^2 + 10ab = 120$, find the maximum possible value of ab.

8 If a and b are positive numbers, prove that $a^3 + b^3 \geq a^2b + ab^2$.

9 **a** Find all integer solutions of the equation $xy + 2x + 3y = 8$ by first writing it in the form $(x + a)(y + b) = c$.

b Find all integer solutions of $xy - 5x + 2y = 21$.

10 By factorising the left-hand side of the equation, find all integer solutions of the equation $2x^2 + xy - 15y^2 = 51$.

11 Given that $x^2 + y^2 = 28$ and $xy = 14$, find the value of $x^2 - y^2$.

12 Prove that for any numbers a, b, c, d:

a $2abcd \leq a^2b^2 + c^2d^2$

b $6abcd \leq a^2b^2 + a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 + c^2d^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

16

Measurement and Geometry

Measurement – areas, volumes and time

In this chapter we review earlier material on the areas of plane figures and the surface areas and volumes of solids. The detailed and careful development of this material was undertaken in *ICE-EM Mathematics Year 8*.

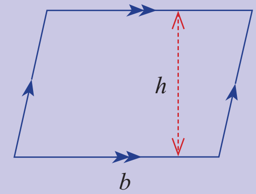
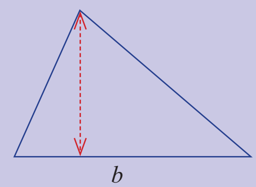
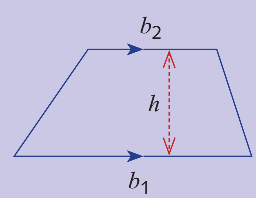
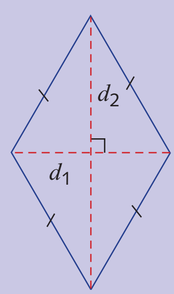
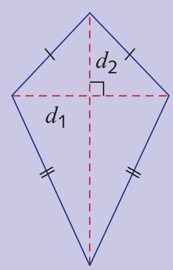
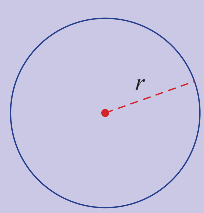
Calculating areas, volumes and surface areas is a very practical and important skill. We have mixed many of the different types of figures and solids together, so in each problem you will need to recall which formula is appropriate. We also deal with composite solids, for which more than one formula is needed.

In the final two sections of this chapter, conversion of metric units is reviewed and extended and very small and very large measurements and their units are considered.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16A Review of area

The formulas for the areas of the standard or basic plane figures are summarised below.

Name of figure	Diagram	Area (A)
Parallelogram		$A = bh$
Triangle		$A = \frac{1}{2}bh$
Trapezium		$A = \frac{1}{2}(b_1 + b_2)h$
Rhombus		$A = \frac{1}{2}d_1\,d_2$ d_1 and d_2 are the lengths of the diagonals.
Kite		$A = \frac{1}{2}d_1\,d_2$ d_1 and d_2 are the lengths of the diagonals.
Circle		$A = \pi r^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

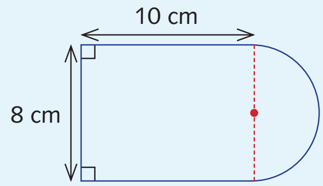
A **composite figure** involves two or more of these basic figures. Its area is usually obtained by adding or subtracting the areas of basic figures.

In all of the diagrams in this chapter, assume that all quadrilaterals are rectangles unless the context indicates otherwise.

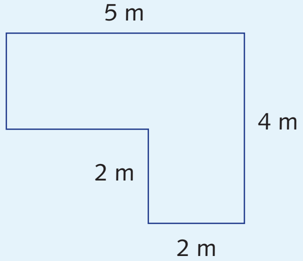
Example 1

Calculate the area, correct to 2 decimal places, of these plane figures.

a



b



Solution

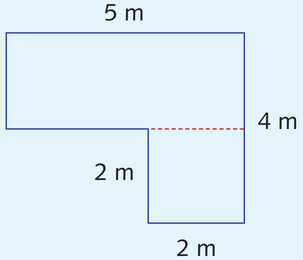
a The figure consists of a rectangle with dimensions 8 cm by 10 cm, and a semicircle of radius 4 cm.

So $A = 10 \times 8 + \frac{1}{2} \times \pi \times 4^2$

$= (80 + 8\pi)\ \text{cm}^2$

$\approx 105.13\ \text{cm}^2$

b The figure consists of a rectangle with dimensions 5 m by 2 m, and a square of side length 2 m.



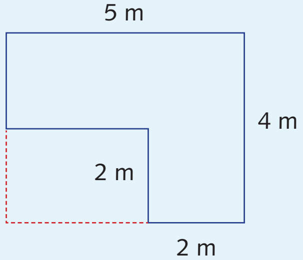
So $A = 5 \times 2 + 2 \times 2$

$= 10 + 4$

$= 14\ \text{m}^2$

Alternative solution

The figure can also be thought of as a rectangle with dimensions 5 m by 4 m, with a rectangle with dimensions 3 m by 2 m cut out.



So $A = 5 \times 4 - 3 \times 2$

$= 20 - 6$

$= 14\ \text{m}^2$, as before

Note: We read 14 m^2 as '14 square metres', not '14 metres squared'.

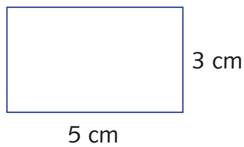
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 16A

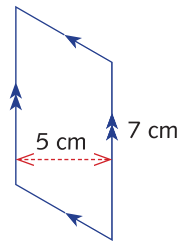
Unless otherwise specified, give the answer to each problem in exact form. If the answer involves π, also give the answers correct to 2 decimal places.

1 Calculate the area of each figure.

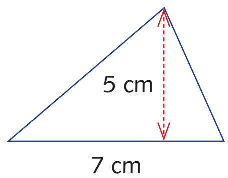
a



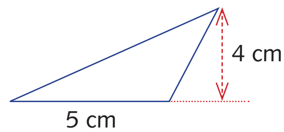
b



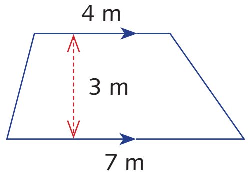
c



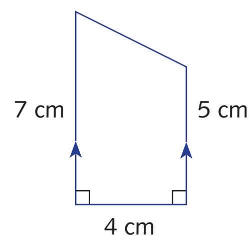
d



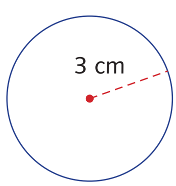
e



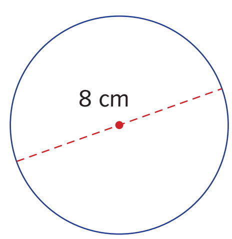
f



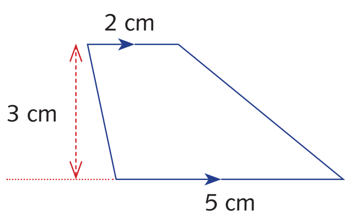
g



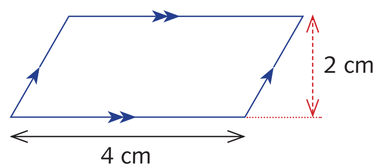
h



i

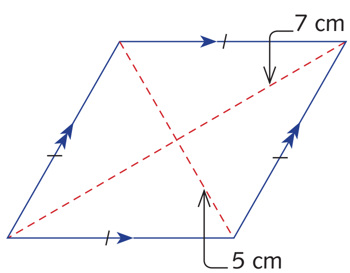


j

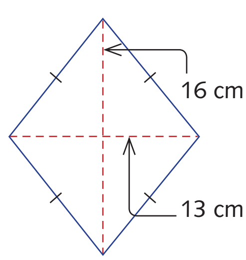


2 Find the area of each figure. Labels indicate diagonal lengths.

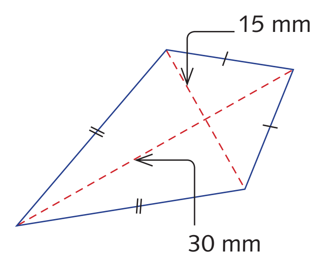
a



b



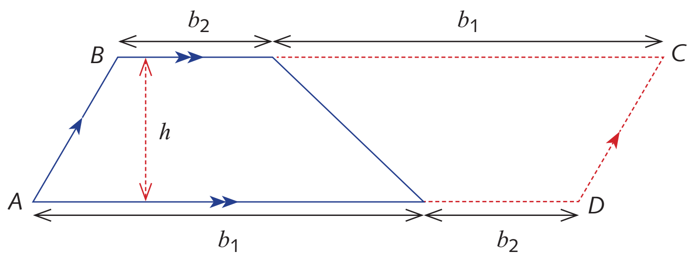
c



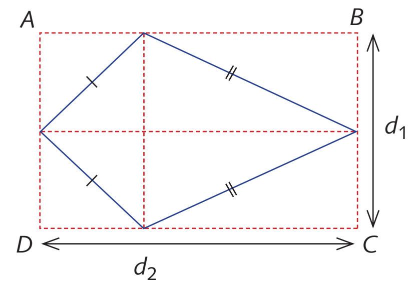
3 Find the area of a square with diagonals of length 5 cm.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

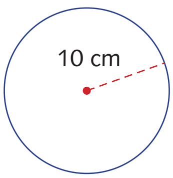
4 Use the diagram below to derive the formula for the area of the trapezium *ABCD*.



5 Use the diagram opposite to derive the formula for the area of the kite *ABCD*.

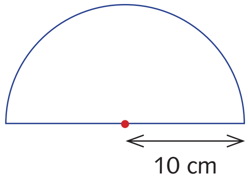


6 **a** Find the area of the circle shown below.

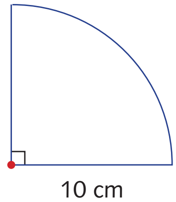


b Find the area of each sector. (The centre of the circle is marked with a dot.)

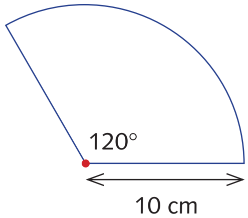
i



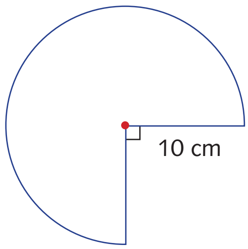
ii



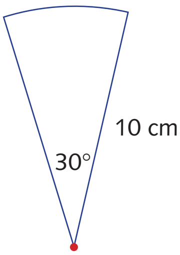
iii



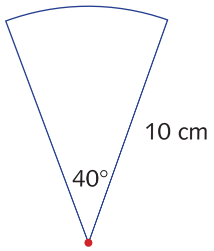
iv



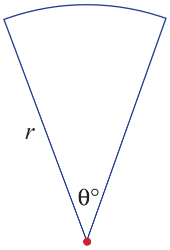
v



vi



c Find the area of the sector of a circle in terms of the radius r and the angle θ at the centre.



7 Calculate the radius of the circle with area:

a $12\ \text{mm}^2$ **b** $50\ \text{cm}^2$ **c** $9\pi\ \text{m}^2$ **d** $25\pi\ \text{cm}^2$

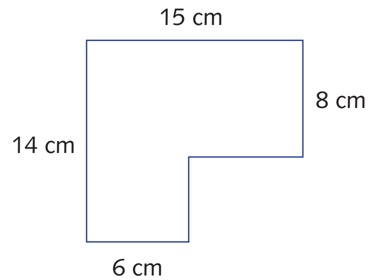
8 Calculate the radius of a circle whose area is half that of a circle with radius 6 cm.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

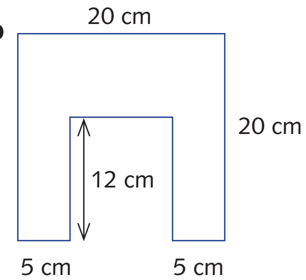
Example 1

9 Calculate the area of each region.

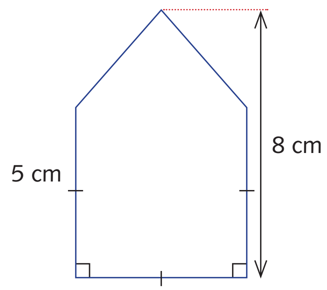
a



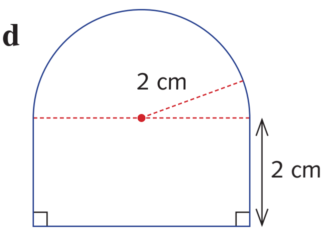
b



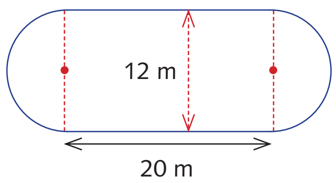
c



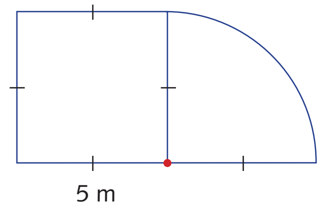
d



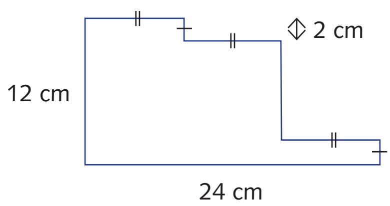
e



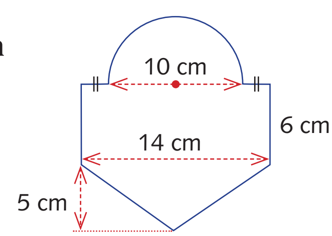
f



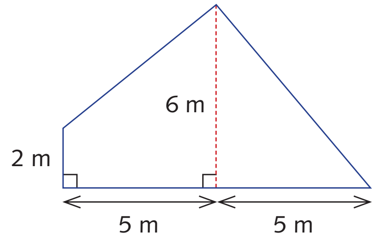
g



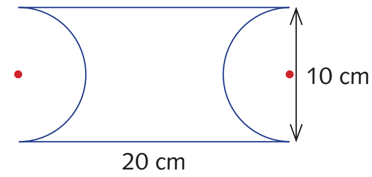
h



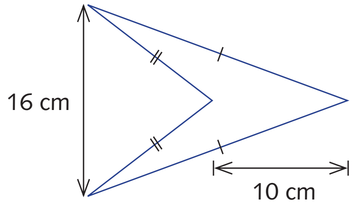
i



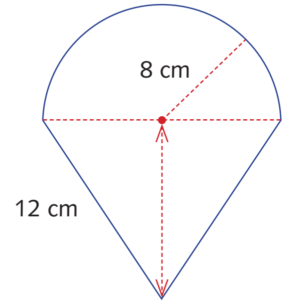
j



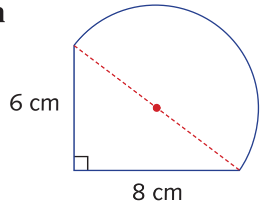
k



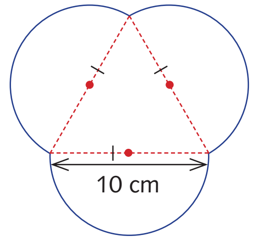
l



m



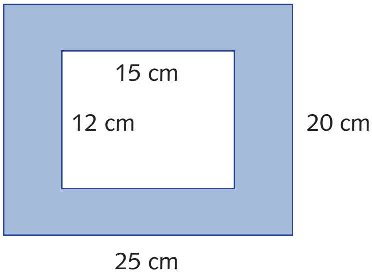
n



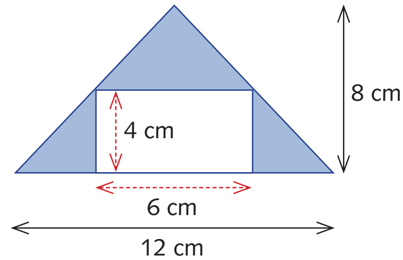
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 Calculate the area of the shaded region in each diagram.

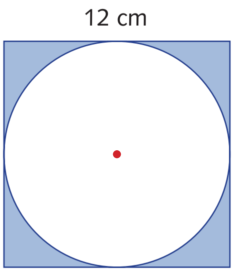
a



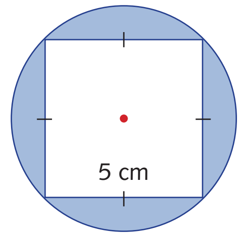
b



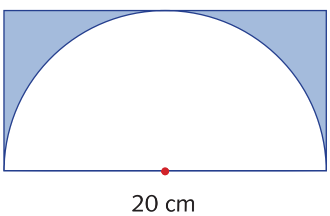
c



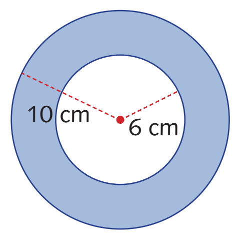
d



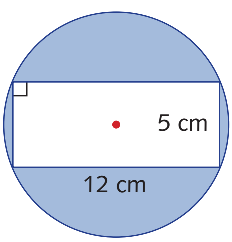
e



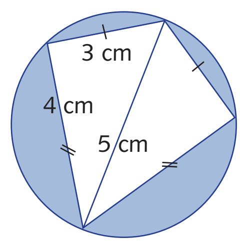
f



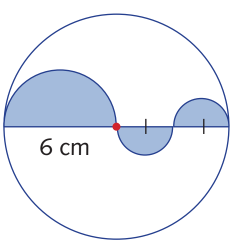
g



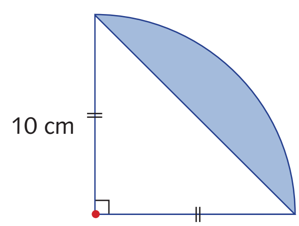
h



i



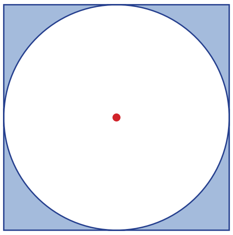
j



11 In the diagram opposite:

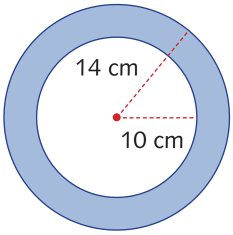
a Find the area of the shaded region, correct to 2 decimal places, if the square has side length 10 cm.

b Find the side length of the square, correct to 2 decimal places, if the area of the shaded region is 100 cm^2.



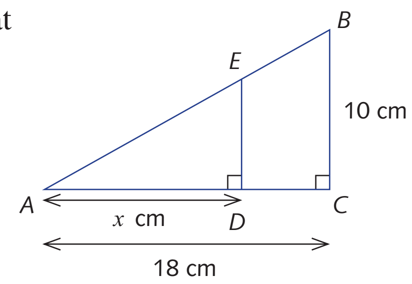
12 **a** Find the area of the shaded region in the diagram opposite.

b A third circle is drawn with the same centre as the other two circles. Find the radius of this circle if it cuts the shaded region into halves.

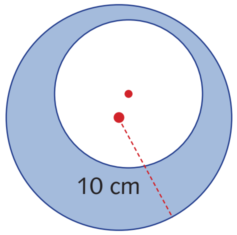


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

13 In the diagram opposite, calculate the exact value of x so that DE divides ΔABC into two regions of equal area.



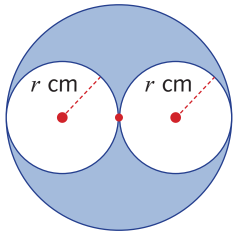
14 In the diagram opposite, the outer circle has radius 10 cm and the area of the shaded region is equal to the area of the unshaded region. Find the radius of the small circle.



15 In the diagram opposite:

a Find the area of the shaded region if $r = 6$.

b Find the value of r if the area of the shaded region is 200 cm^2.



16B Review of surface area of a prism

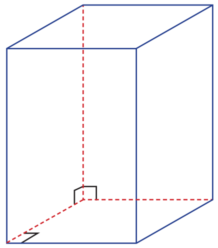
In the previous section, we reviewed the area of plane figures. Such figures are two–dimensional because they can be drawn on a flat piece of paper. The three-dimensional objects that we will now consider are called **prisms**. They have both a surface area and a volume. We begin by looking at surface areas.

A **polyhedron** is a solid bounded by polygons.

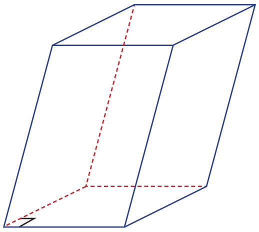
A **prism** is a polyhedron that has two congruent and parallel faces and all its remaining faces are parallelograms.

A **right prism** is a prism in which the top and bottom polygons are vertically above each other, and the vertical polygons connecting their sides are rectangles. A prism that is not a right prism is often called an **oblique prism**.

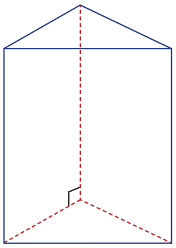
Some examples of prisms are shown below.



Right rectangular prism



Oblique rectangular prism

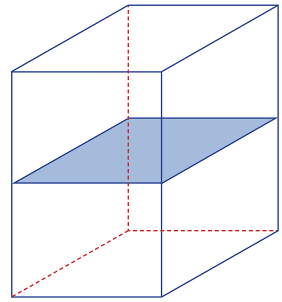


Right triangular prism

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

In this chapter, we will work only with right prisms. The word 'right' will usually be omitted.

A prism with a rectangular base is called a **rectangular prism**, while a **triangular prism** has a triangular base.

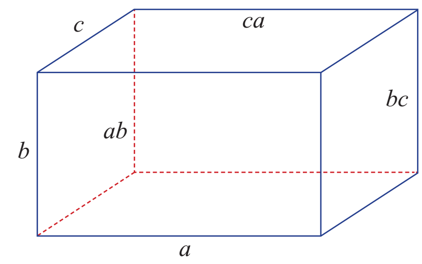


You will notice that if we slice a prism by a plane parallel to its base, the cross-section is congruent to its base and so has the same area as the base.

In this chapter, we will use the pronumeral S for the surface area of a solid and V for its volume.

Surface area of a prism

The **surface area of a prism** is the sum of the areas of its faces.



A rectangular prism with dimensions a, b and c has six faces. These occur in pairs, faces with areas ab, bc and ca each occurring twice.

We can therefore write:

surface area of a rectangular prism $= S = 2(ab + bc + ca)$

We do not need to learn this formula. We can simply find the area of each face and take the sum of the areas. The same idea applies to all other types of prisms.

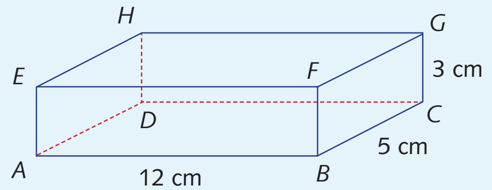
Surface area of a prism

To find the surface area of a prism, find the area of each face and add up the areas.

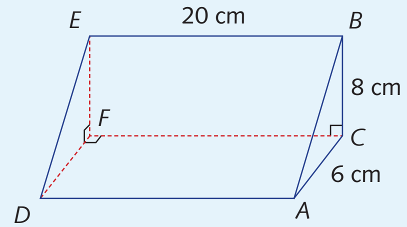
Example 2

Find the surface area of:

a the rectangular prism



b the triangular prism



Solution

a Area of the six rectangular faces $= 2(12 \times 5 + 12 \times 3 + 5 \times 3)$

$= 222 \text{ cm}^2$

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b By Pythagoras' theorem, $AB^2 = 6^2 + 8^2$

$AB = 10$ cm

Area of the three rectangular faces $= 8 \times 20 + 6 \times 20 + 10 \times 20$

$= 480 \text{ cm}^2$

Area of the two rectangular faces $= 2 \times \frac{1}{2} \times 6 \times 8$

$= 48 \text{ cm}^2$

Hence Surface area $= 480 + 48$

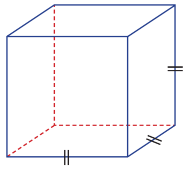
$= 528 \text{ cm}^2$

Exercise 16B

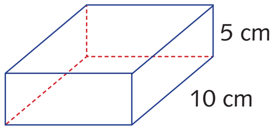
Example 2a

1 Calculate the surface area of each prism.

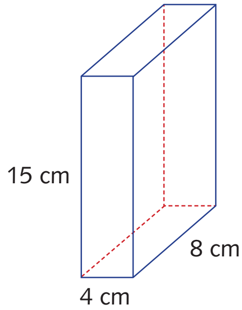
a



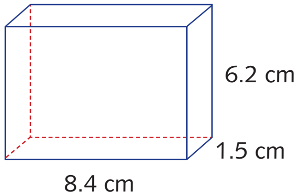
b



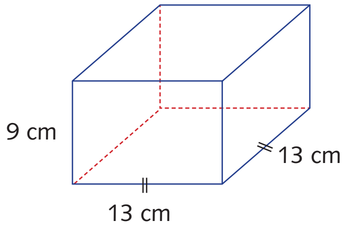
c



d



2 An ice-cream container, open at the top, has dimensions as shown in the diagram opposite. Find the surface area of the outside of the container.

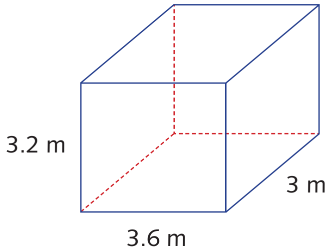


3 A shoe box, open at the top, has length 25 cm, width 18 cm and height 9.8 cm. Find the surface area of the outside.

4 Dianne is going to paint her room. The dimensions of the room are shown opposite. What area must Dianne paint if she paints:

a only the walls?

b the walls and the ceiling?

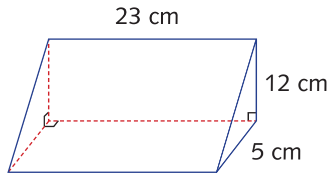


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

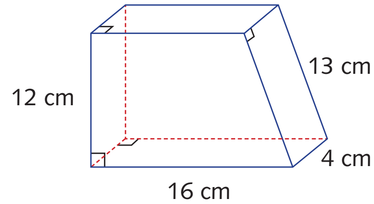
Example 2b

5 Calculate the surface area of each solid.

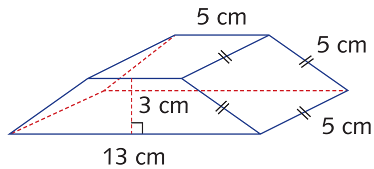
a



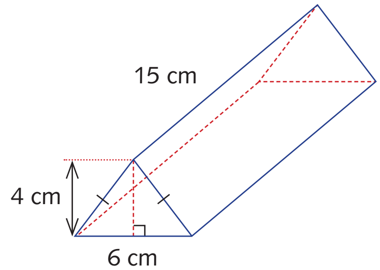
b



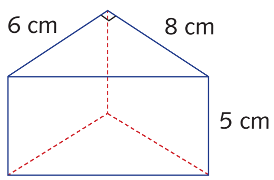
c



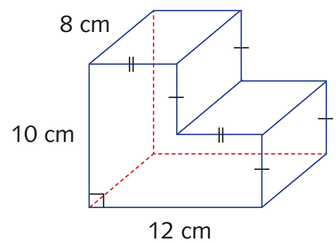
d



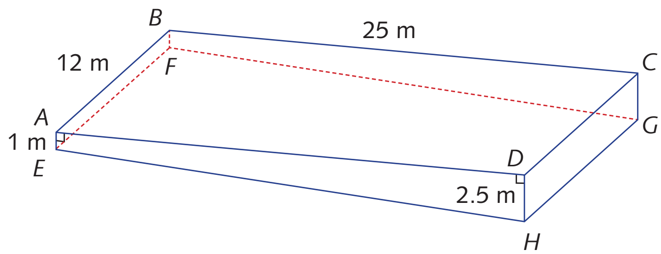
e



f



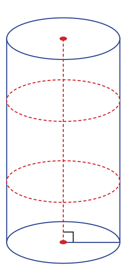
6 A swimming pool has dimensions as shown in the diagram below.



a Find the length *EH*, correct to the nearest millimetre.

b Find the approximate number of tiles, each of size 15 cm × 15 cm, required to tile the sides and bottom of the pool. (The tiler may cut the tiles and piece them together.)

16C Surface area of a cylinder



A **cylinder** is a solid that has parallel circular discs of equal radius at the top and the bottom. Each cross-section parallel to the top is a circle with the same radius, and the centres of these circular cross-sections lie on a straight line, called the **axis of the cylinder**.

We will use a dot (•) to indicate the centre of the circular base or top.

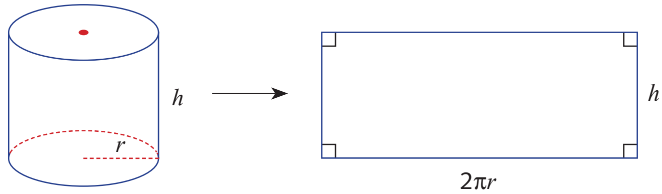
As we did with prisms, we find the surface area of a cylinder by adding up the area of the curved section of the cylinder, and the area of the two circles.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Area of the curved surface

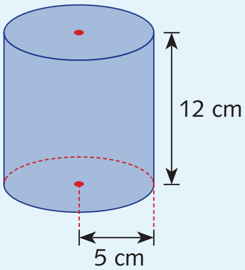
Suppose that we have a cylinder with base radius r and height h. If we roll it along a flat surface through one revolution, as shown in the diagram, the curved surface traces out a rectangle. The height of the rectangle is the height of the cylinder, while the length of the rectangle is the circumference of the circle, which is $2\pi r$, so the area of the curved part is $2\pi rh$.

Area of curved surface of cylinder $= 2\pi rh$



Example 3

Calculate the total surface area, correct to 2 decimal places, of a cylinder with base radius 5 cm and height 12 cm.



Solution

$$\begin{aligned}\text{Area of curved surface} &= 2\pi rh\\ &= 2\pi \times 5 \times 12\\ &= 120\pi \text{ cm}^2\\ \text{Area of each circular end} &= \pi \times 5^2\\ &= 25\pi \text{ cm}^2\\ \text{Surface area of the cylinder} &= 120\pi + 25\pi + 25\pi\\ &= 170\pi \text{ cm}^2\\ &\approx 534.07 \text{ cm}^2\end{aligned}$$

You should generally work in terms of π and only approximate in the last step.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

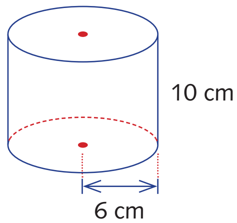
Exercise 16C

Unless otherwise specified, give the answer to each problem in exact form and also correct to 2 decimal places.

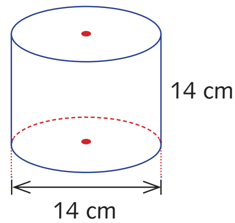
Example 3

1 Calculate the surface area of each cylinder.

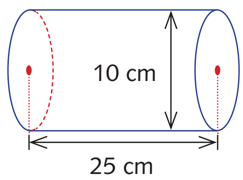
a



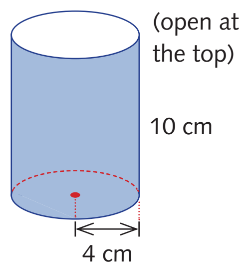
b



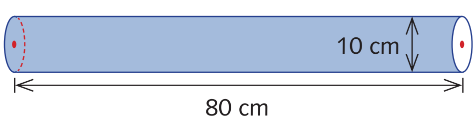
c



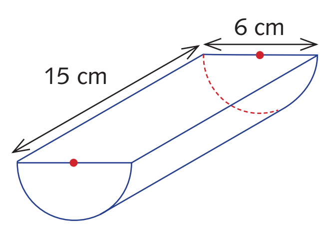
d



2 The outside surface of a piece of open pipe with diameter 10 cm and length 80 cm is to be painted. Calculate the area to be painted.

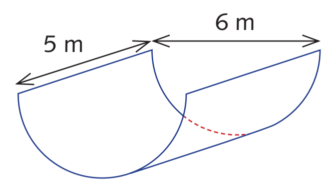


3 For the solid half-cylinder opposite, find:

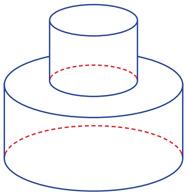


a the area of the top
b the area of the two ends
c the curved surface area
d the total surface area

4 A skating ramp is in the shape of a half-cylinder. If the two sides are 6 m apart and the length of the ramp is 5 m, find the area of the ramp.



5 A cylinder of radius 3 cm and height 4 cm is mounted on a cylinder of radius 6 cm and height 5 cm. Find the area of the visible surfaces, if the larger cylinder is sitting on a table.

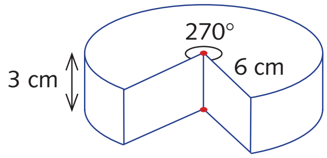


6 The surface area of a cylinder is 25.13 m^2 and the radius of the base is 1 m. Find the height of the cylinder, correct to 2 decimal places.

7 **a** The radius of a cylinder is equal to its height, and it has a total surface area of 500 cm^2. Find its height.

b The height of a cylinder is equal to its diameter, and it has a total surface area of 500 cm^2. Find the radius.

8 A sector of cheese is cut from a cylindrical block of cheese. Find the surface area of the remaining piece, correct to 2 decimal places.



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16D Volumes

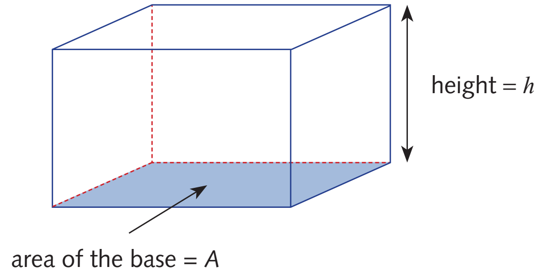
Volumes of right prisms

Recall that a right prism is a polyhedron that has two congruent and parallel faces, and all its remaining faces are rectangles. A prism has a uniform cross-section.

Volume of a rectangular prism

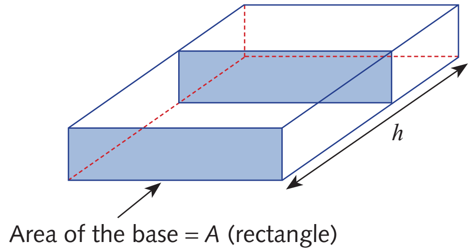
We have seen earlier that the volume of a right rectangular prism is given by:

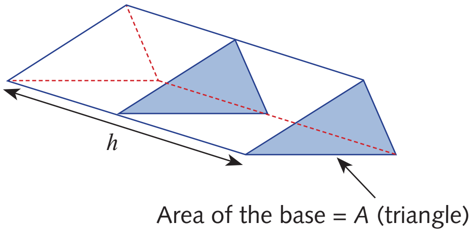
$$\begin{aligned}\text{volume of a right rectangular prism} &= \text{area of base} \times \text{height}\\ &= A \times h\\ V &= Ah\end{aligned}$$



The same formula holds for any prism. If A is the area of the cross-section and h is the height, then:

$$\text{Volume} = Ah$$

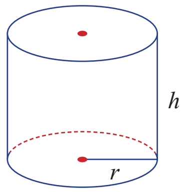


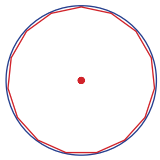


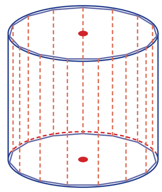
Note: In the last two diagrams, the prisms are not resting on their bases. Regardless of the orientation, 'height' (h) always represents the distance between the congruent faces of the right prism aligning with the uniform cross-section. When dealing with triangular prisms, don't confuse the 'h' in the formula for the area of a triangle with the 'height' in the formula, $V = Ah$.

Volume of a cylinder

A cylinder has a circular base. Suppose that we now draw a polygon with a large number of sides inside the base circle, with the vertices on the circle. The volume of the corresponding prism will be the area of the base multiplied by the height, so it seems reasonable that the volume of the cylinder should also be the area of its base multiplied by the height.







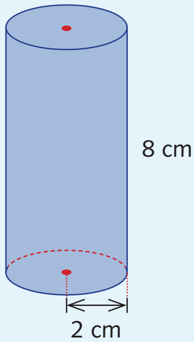
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Thus the volume of a cylinder with radius r and height h is equal to the area of the base, πr^2, multiplied by the height h:

$$\text{volume of a cylinder} = \pi r^2 h$$

Example 4

Calculate the volume of a cylinder with radius 2 cm and height 8 cm, correct to 2 decimal places.



Solution

$$\begin{aligned} V &= \text{area of base} \times \text{height} \\ &= \pi \times 2^2 \times 8 \\ &= 32\pi \text{ cm}^3 \\ &\approx 100.53 \text{ cm}^3 \end{aligned}$$

Note: This is read as '100.53 cubic centimetres', not '100.53 centimetres cubed'.

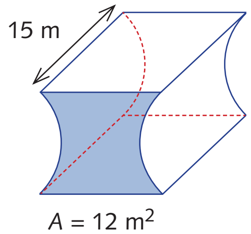
Some related solids

Prisms and cylinders are solids with constant cross-sectional areas. The volume, V, of any solid with a constant cross-sectional area is given by exactly the same formula

$$V = \text{area of base} \times \text{height}$$

For example, for the solid to the right:

$$\begin{aligned} V &= \text{area of base} \times \text{height} \\ &= 12 \times 15 \\ &= 180 \text{ m}^3 \end{aligned}$$



Volumes

- Volume of a solid with constant cross-sectional area A and height h is Ah.
- Volume of a cylinder = area of base × height:

$$V = \pi r^2 h$$

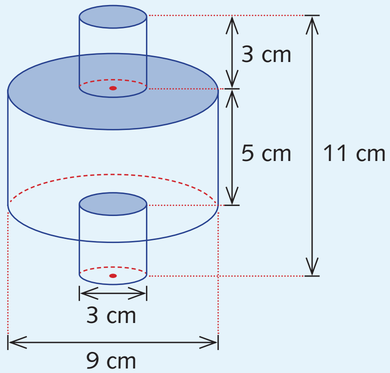
A composite solid involves two or more of the basic solids. Its volume is usually obtained by adding or subtracting the volumes of basic solids.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

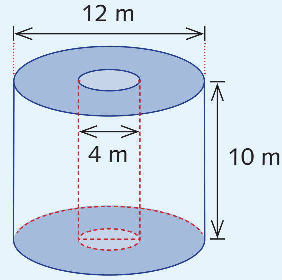
Example 5

Find the volume of each solid in terms of π and then correct to 2 decimal places.

a



b



Solution

a Volume of large cylinder $= \pi \times 4.5^2 \times 5$

$= 101.25\pi \text{ cm}^3$

Volume of each small cylinder $= \pi \times 1.5^2 \times 3$

$= 6.75\pi \text{ cm}^3$

Hence volume of solid $= 101.25\pi + 6.75\pi + 6.75\pi$

$= 114.75\pi \text{ cm}^3$

$\approx 360.50 \text{ cm}^3$

b Volume = volume of large cylinder − volume of small cylinder

$= \pi \times 6^2 \times 10 - \pi \times 2^2 \times 10$

$= 360\pi - 40\pi$

$= 320\pi \text{ m}^3$

$\approx 1005.31 \text{ m}^3$

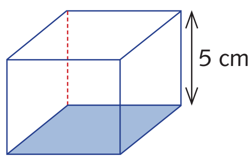
Exercise 16D

Unless otherwise specified, give the answer to each problem in exact form and also correct to 2 decimal places.

Example 4

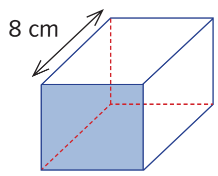
1 Calculate the volume of each solid. The area, A, of the shaded face is given.

a



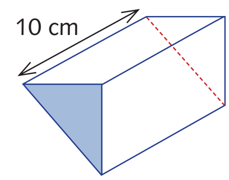
$A = 24 \text{ cm}^2$

b



$A = 30 \text{ cm}^2$

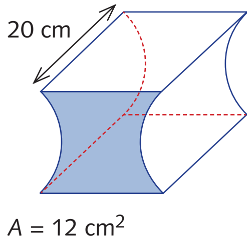
c



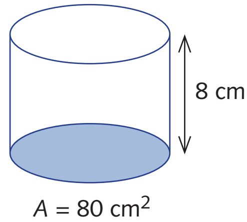
$A = 18 \text{ cm}^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

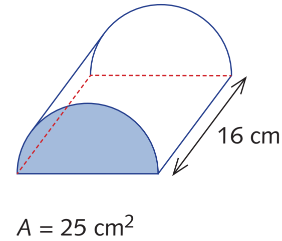
d



e



f



2 Use the result

volume = area of base × height

to express the volume of each solid, in terms of the given pronumerals.

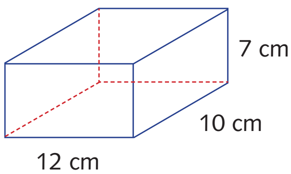
a The volume of a cube of side length a

b The volume of a rectangular prism with length a, width b and height c

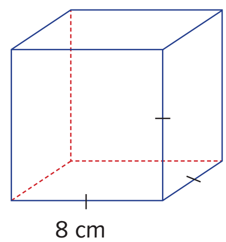
c The volume of a cylinder with radius a and height b

3 Calculate the volume of each prism.

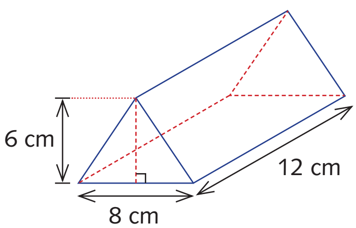
a



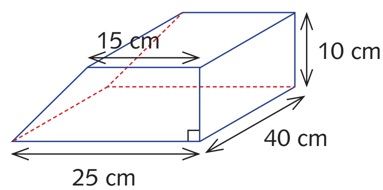
b



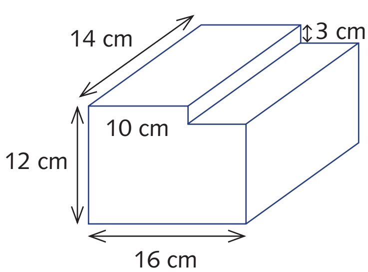
c



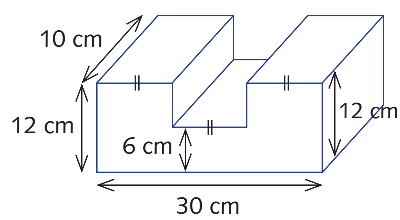
d



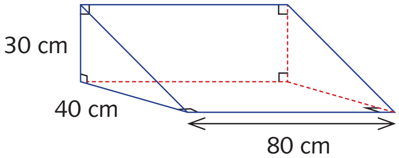
e



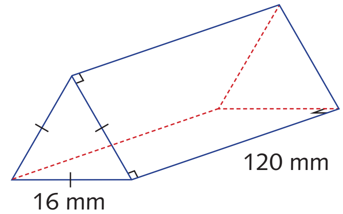
f



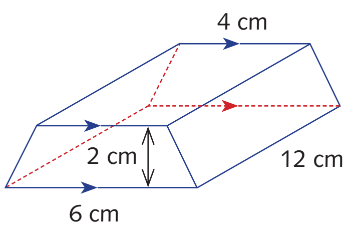
g



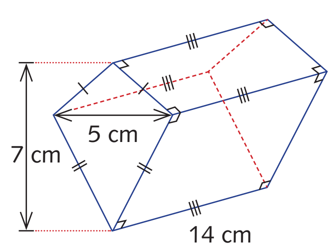
h



i



j



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 **a** Find the area of the base of a prism whose volume is 240 cm^3, given that its height is 8 cm.

b Find the height of a rectangular prism if its square base has sides of length 8.5 cm and its volume is 255 cm^3.

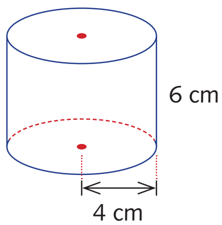
c Find the length of the side of a cube whose volume is 15.625 m^3.

d Find the dimensions of a square-based prism if its height is twice the side length of the square and the volume is 1228.25 mm^3.

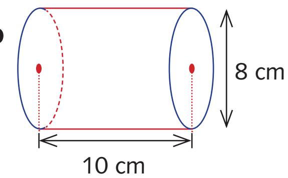
Example 4, 5

5 Find the volume of each solid.

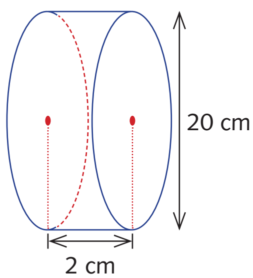
a



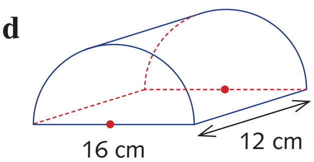
b



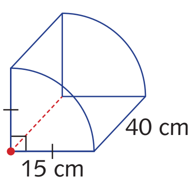
c



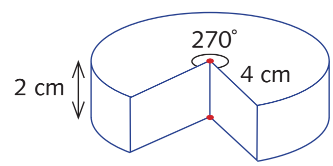
d



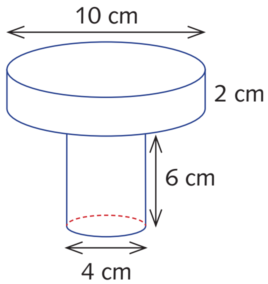
e



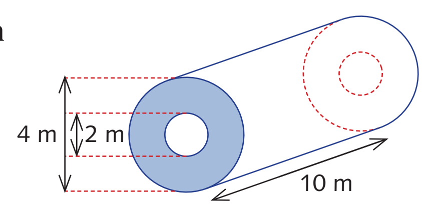
f



g



h



6 **a** A cylinder with base radius 4 cm has a volume of 200 cm^3. Find its height.

b A cylinder of height 8 cm has a volume of 375 cm^3. Find its base radius.

c A cylinder of volume 1000 cm^3 is such that its base radius is equal to its height. Find its height.

d A cylinder of volume 500 cm^3 is such that its diameter is equal to its height. Find its height.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16E Conversion of units

We often need to convert from one unit into another. We recall:

$1 \text{ cm} = 10 \text{ mm} \qquad 1 \text{ m} = 100 \text{ cm} \qquad 1 \text{ km} = 1000 \text{ m}$

Just as lengths can be converted from one unit into another, so can areas and volumes. We can use the basic length conversions above to obtain area and volume conversions. The underpinning reasoning comes from the fact that area is measured in squares and volume in cubes. A detailed discussion of this can be found in *ICE-EM Mathematics Year 8*.

Area conversions

To obtain the conversion factor for areas, we square the corresponding conversion factor for lengths.

Example 6

Convert each measurement into square centimetres.

a 2.5 m^2

b 3600 mm^2

Solution

a

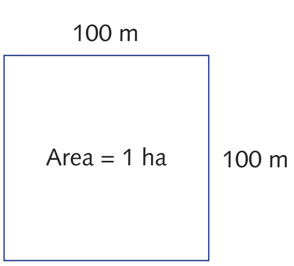
$$\begin{aligned} 1\text{m}^2 &= 100^2 \text{ cm}^2 \\ &= 10\,000 \text{ cm}^2 \\ \text{so } 2.5\text{m}^2 &= 2.5 \times 10\,000 \text{ cm}^2 \\ &= 25\,000 \text{ cm}^2 \end{aligned}$$

b

$$\begin{aligned} 1 \text{ cm}^2 &= 10^2 \text{ mm}^2 \\ &= 100 \text{ mm}^2 \\ \text{so } 3600 \text{ mm}^2 &= \frac{3600}{100} \text{ cm}^2 \\ &= 36 \text{ cm}^2 \end{aligned}$$

Hectares

A **hectare** (ha) is the area enclosed by a square with a side length of 100 m.
That is,



$$\begin{aligned} 1 \text{ ha} &= 100 \times 100 \text{ m}^2 \\ &= 10\,000 \text{ m}^2 \\ &= 10^4 \text{ m}^2 \\ 100 \text{ ha} &= 100 \times 10^4 \text{ m}^2 \\ &= 10^6 \text{ m}^2 \\ &= 1 \text{ km}^2 \end{aligned}$$

The metric system was introduced into Europe by Napoleon in the early nineteenth century, and was adopted by Australia in the 1960s. Until that time the Imperial system was used and land was measured in acres. In the metric system the hectare is the unit used for land measurement.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7

The area of a cattle station in outback Australia is 200 000 ha. Calculate:

a the area in square metres

b the area in square kilometres

c the dimensions of the station, if it is a square, correct to 1 decimal place

d the dimensions of the station, if it is a rectangle and one side length is 50 km

Solution

a
$$\begin{aligned} 1 \text{ ha} &= 100 \times 100 \text{ m}^2 \\ &= 10\,000 \text{ m}^2 \end{aligned}$$

so
$$\begin{aligned} 200\,000 \text{ ha} &= 200\,000 \times 10\,000 \text{ m}^2 \\ &= 2\,000\,000\,000 \text{ m}^2 \\ &= 2 \times 10^9 \text{ m}^2 \end{aligned}$$

b
$$1 \text{ km}^2 = 100 \text{ ha}$$

so
$$200\,000 \text{ ha} = 2000 \text{ km}^2$$

c Let x km be the side length of the square.

so $\quad x^2 = 2000$

hence $\quad x \approx 44.7$

Thus the station is approximately 44.7 km by 44.7 km.

d Area $=$ length $\times$ width

$2000 = 50 \times$ width

Width $= 40$ km

Thus the station is 40 km by 50 km.

Volume conversions

To obtain a conversion factor for volumes, we cube the corresponding conversion factor for lengths.

Example 8

Convert each measurement into the units indicated in brackets.

a $2760 \text{ mm}^3 \left(\text{cm}^3\right)$ **b** $0.27 \text{ m}^3 \left(\text{cm}^3\right)$ **c** $256\,000 \text{ cm}^3 \left(\text{m}^3\right)$ **d** $0.59 \text{ cm}^3 \left(\text{mm}^3\right)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

a
$$\begin{aligned} 10\text{ m} &= 1\text{ cm} \\ \text{so}\quad 10^3\text{ mm}^3 &= 1\text{ cm}^3 \\ \text{hence}\quad 2700\text{ mm}^3 &= \frac{2760}{1000}\text{ cm}^3 \\ &= 2.76\text{ cm}^3 \end{aligned}$$

b
$$\begin{aligned} 1\text{ m} &= 100\text{ cm} \\ \text{so}\quad 1\text{ m}^3 &= 100^3\text{ cm}^3 = 10^6\text{ cm}^3 \\ \text{hence}\quad 0.27\text{ m}^3 &= 0.27 \times 10^6\text{ cm}^3 \\ &= 270\,000\text{ cm}^3 \end{aligned}$$

c From part **b**: $10^6\text{ cm}^3 = 1\text{ m}^3$
$$\begin{aligned} \text{hence}\quad 256\,000\text{ cm}^3 &= \frac{256\,000}{1\,000\,000}\text{ m}^3 \\ &= 0.256\text{ m}^3 \end{aligned}$$

d From part **a**: $1\text{ cm}^3 = 1000\text{ mm}^3$
$$\begin{aligned} \text{hence}\quad 0.59\text{ cm}^3 &= 0.59 \times 1000\text{ mm}^3 \\ &= 590\text{ mm}^3 \end{aligned}$$

Liquids

In many situations, particularly when liquids are involved, litres are used as the unit of volume.

A **litre** (1 L) is equal to 1000 cm^3.

Thus, a litre is the volume of a cube of side length 10 cm. Also, a cubic metre is 1000 L.

$$\begin{aligned} 1\text{ L} &= 1000\text{ cm}^3 \\ 1\text{ m}^3 &= 10^6\text{ cm}^3 \\ &= 1000\text{ L} \end{aligned}$$

A **millilitre** (1 mL) is equal to $\frac{1}{1000}$ of a litre and is thus equal to 1 cm^3.

$$1\text{ mL} = \frac{1}{1000}\text{ L} = 1\text{ cm}^3$$

Example 9

A large water trough in the shape of a rectangular prism has internal dimensions of 3 m by 0.6 m by 0.5 m. How many litres of water does the trough hold when full?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

Since $1000 \text{ cm}^3 = 1 \text{ L}$, the volume is best calculated in cubic centimetres.

$$\begin{aligned} V &= 300 \text{ cm} \times 60 \text{ cm} \times 50 \text{ cm} \\ &= 900\,000 \text{ cm}^3 \\ &= 900 \text{ L} \end{aligned}$$

Hence the water trough holds 900 L.

Conversion of units

Length

$$\begin{tikzcd} \text{mm} \arrow[r, bend left, "\div 10"] & \text{cm} \arrow[l, bend left, "\times 10"] \arrow[r, bend left, "\div 100"] & \text{m} \arrow[l, bend left, "\times 100"] \arrow[r, bend left, "\div 1000"] & \text{km} \arrow[l, bend left, "\times 1000"] \end{tikzcd}$$

Area

$$\begin{tikzcd} \text{mm}^2 \arrow[r, bend left, "\div 10^2"] & \text{cm}^2 \arrow[l, bend left, "\times 10^2"] \arrow[r, bend left, "\div 100^2\ (= 10^4)"] & \text{m}^2 \arrow[l, bend left, "\times 10^4"] \arrow[r, bend left, "\div 1000^2\ (= 10^6)"] & \text{km}^2 \arrow[l, bend left, "\times 10^6"] \end{tikzcd}$$

Volume

$$\begin{tikzcd} \text{mm}^3 \arrow[r, bend left, "\div 10^3"] & \text{cm}^3 \arrow[l, bend left, "\times 10^3"] \arrow[r, bend left, "\div 100^3\ (= 10^6)"] & \text{m}^3 \arrow[l, bend left, "\times 10^6"] \arrow[r, bend left, "\div 1000^3\ (= 10^9)"] & \text{km}^3 \arrow[l, bend left, "\times 10^9"] \end{tikzcd}$$

Area of land

$1 \text{ ha} = 10^4 \text{ m}^2$ $\quad$ $1 \text{ km}^2 = 100 \text{ ha}$

Litres

$1 \text{ L} = 1000 \text{ mL}$ $\quad$ $1 \text{ mL} = 1 \text{ cm}^3$ $\quad$ $1 \text{ m}^3 = 1000 \text{ L}$

Exercise 16E

Example 6

1 Convert each measurement into the unit indicated in brackets.

a 300 mm^2 (cm^2) **b** 3.1 m^2 (cm^2) **c** 0.5 m^2 (mm^2)

d 0.6 cm^2 (mm^2) **e** 0.36 km^2 (m^2) **f** 2800 cm^2 (m^2)

Example 7

2 A rectangular piece of land measures 260 m by 430 m. Calculate the area of the land in:

a square metres **b** hectares **c** square kilometres

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 A rectangular piece of land has an area of 2.7 ha. If the block of land is 135 m wide, how long is the block of land?

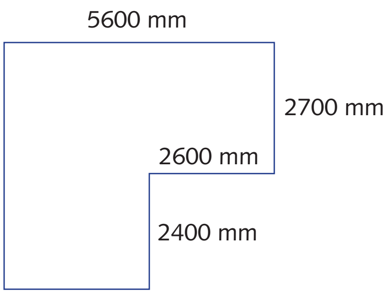
4 A cattle station has an area of 260 km^2. What is its area in hectares?

5 One acre is approximately 0.4 ha. A rectangular block of land measures $50 \text{ m} \times 150 \text{ m}$. Calculate the area of the block of land in:

a hectares

b acres (approximately)

6 Robyn intends to paint the ceiling of her living room. A plan of the room is drawn with measurements. One litre of paint will cover 12 m^2. Calculate:



a the area of the ceiling in square metres

b the amount of paint needed to put one coat of paint on the ceiling

Example 8

7 Convert each measurement into the units indicated in brackets.

a $5760 \text{ mm}^3 \left(\text{cm}^3\right)$

b $0.56 \text{ m}^3 \left(\text{cm}^3\right)$

c $0.62 \text{ m}^3 (\text{L})$

d $2600 \text{ cm}^3 (\text{L})$

e $52000 \text{ mm}^3 (\text{mL})$

f $960 \text{ L} \left(\text{m}^3\right)$

Example 9

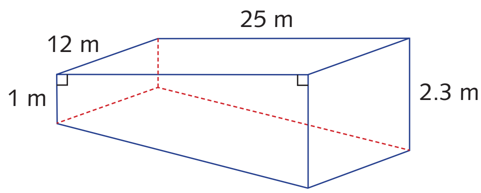
8 A large tank in the shape of a rectangular prism has internal dimensions $3 \text{ m} \times 2 \text{ m} \times 1.5 \text{ m}$. How many litres does the tank hold when full?

9 A cylindrical water tank has a diameter of 3 m and a height of 2 m. Calculate, to the nearest 100 mL, the volume of the tank in:

a cubic metres

b litres

10 A school swimming pool has dimensions as shown in the diagram opposite. How long would it take to fill this pool if the pump can deliver 1500 L of water a minute?



11 A medicine bottle is in the shape of a cylinder with internal base diameter 50 mm and internal height 80 mm. If the normal dosage of medicine is 15 mL, how many dosages can be obtained from the bottle? (Assume that initially the bottle is completely full.)

12 A swimming pool is in the shape of a rectangular prism. The pool is 12 m long, 4 m wide and 1.5 m deep. The pool is to be lined with tiles.

a How many tiles are needed to line the pool if each tile is $200 \text{ mm} \times 200 \text{ mm}$.

b A path 1 m wide is to be laid around the pool. How many pavers are needed to make the path if the size of each of the pavers is $500 \text{ mm} \times 500 \text{ mm}$?

c When the pool is completed, it is filled at 400 L/h. How long will it take to fill the pool to 10 cm below the top?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16F Small and large units of measurement

In Chapter 8 we introduced scientific notation as a useful way of expressing very small or very large numbers. Our metric system has prefixes that allow us to describe small or large amounts succinctly. You have already seen how the metric system uses powers of 10.

In the metric system:

- the prefix 'kilo' corresponds to the power 10^3
- the prefix 'milli' corresponds the power 10^{-3}

Thus the units of length, mass and volume:

- 1 kilogram = 10^3 grams, 1 kilometer = 10^3 metres and 1 kilolitre = 10^3 litres
- 1 milligram = 10^{-3} grams, 1 millimetre = 10^{-3} metres and 1 millilitre = 10^{-3} litres

The prefixes and the associated powers of 10 in the table below are used with our metric system to help to express the size of very small or very large quantities.

Prefix	Power	Symbol	Prefix	Power	Symbol
kilo	10^3	K	milli	10^{-3}	m
mega	10^6	M	micro	10^{-6}	μ
giga	10^9	G	nano	10^{-9}	n
tera	10^{12}	T	pico	10^{-12}	p
peta	10^{15}	P	femto	10^{-15}	f

The following facts illustrate the use of some of these prefixes:

- A large dam has capacity 392 813 megalitres. This is written as 392 813 ML.
- Human hair ranges from 18 micrometres (18 μm) to 180 micrometres (180 μm) in width.
- An average piece of paper is 90 micrometres (90 μm) thick.
- Atoms are between 62 (62 pm) and 520 picometres (520 pm) in diameter.
- The subatomic particle, the proton, is estimated to have a radius of 0.8768 femtometres (0.8768 fm).
- The minimum distance between the Earth and Jupiter is 591 gigametres (591 Gm).
- The average distance between Saturn and the Sun is 1.079 terametres (1.079 Tm).
- The angstrom (Å) is often used in the natural sciences to express the sizes of atoms, the lengths of chemical bonds and the wavelengths of electromagnetic radiation:

  1 angstrom = 10^{-1} nanometres = 10^{-10} metres

Computer memory

Prefixes are used to describe digital memory storage capacity. The **byte** is the fundamental unit used to measure storage capacity.

In computer science, storage capacity occurs in powers of 2, but there has been some attempt to make this binary conversion conform to the usual use of the prefixes.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Storage capacity	Closest power of 10 bytes	Actual number of bytes
kilobyte (kB)	10^3	$2^{10} = 1024 = 1.024 \times 10^3$
megabyte (MB)	10^6	$2^{20} = 1\,048\,576 = 1.048\,576 \times 10^6$
gigabyte (GB)	10^9	$2^{30} = 1\,073\,741\,824 = 1.073\,741\,824 \times 10^9$
terabyte (TB)	10^{12}	$2^{40} = 1\,099\,511\,627\,776 = 1.099\,511\,627\,776 \times 10^{12}$

Time

Seconds are part of the metric system and can be combined with the prefixes given above:

$$1 \text{ microsecond} = 10^{-6} \text{ seconds}$$
$$1 \text{ nanosecond} = 10^{-9} \text{ seconds}$$
$$1 \text{ picosecond} = 10^{-12} \text{ seconds}$$

Some examples include:

- A camera flash illuminates for approximately 1000 microseconds.
- 2.68 microseconds were subtracted from the Earth day after the 2004 Indian Ocean earthquake.
- The time for fusion reaction in a hydrogen bomb is 20 to 24 nanoseconds.
- The time for light to travel 1 millimetre in a vacuum is approximately 3.3365 picoseconds.

Light years

- A light year is the distance that light travels in one year ($365\frac{1}{4}$ days). It is most commonly used in astronomy for measuring distances to stars and other galaxies.
- A light year is about $9\,460\,000\,000\,000\,000$ metres $= 9.46 \times 10^{15}$ metres $=$ 9.46 petametres
- The nearest star to our solar system is Proxima Centuri, which is about 4.2 light years away. The centre of our galaxy is about 30 000 light years away.

Exercise 16F

1 Convert each measurement into metres, expressing your answer in scientific form.

a 23 millimetres **b** 67 picometres **c** 456 picometres

d 25 micrometres **e** 93 nanometres **f** 651 nanometres

2 Convert each measurement into microseconds.

a One day **b** One week **c** 14 hours 25 seconds **d** 0.65 seconds

3 Convert each measurement into nanoseconds.

a One day **b** One week **c** 14 hours 35 seconds **d** 672 microseconds

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 A cube holds 1 kilolitre of water. What is the length of a side of the cube? (Remember: 1 litre = 1000 cm^3)

5 A reservoir holds 765 910 megalitres of water. How many litres is this? Express your answer in scientific form.

6 The half-life of a copernicum-277 atom is 240 microseconds. Express this in seconds using scientific notation.

7 The following was stated in a newspaper: 'One million megalitres of pollution spewed onto the Great Barrier Reef.'

a How many litres is this?

b How many gigalitres is this?

8 The volume of a dam on a farm is estimated using the following formula.

$$\text{Volume } (\text{m}^3) = 0.4 \times \text{surface area} \times \text{depth}$$

0.4 is a conversion factor that takes into account the slope of the sides of dams.

If the shape of the dam surface is a circle of diameter 40 m and the depth is 4 m, calculate the volume of water held in the dam in megalitres.

9 Light travels 1 millimetre in 3.3 picoseconds.

a How long does it take for light to travel 1 metre?

b How long does it take for light to travel 1 kilometre?

c How far does light travel in 1 nanosecond?

Review exercise

1 Convert each measurement to the given units.

a 1.2 cm into m

b 0.23 m^2 into cm^2

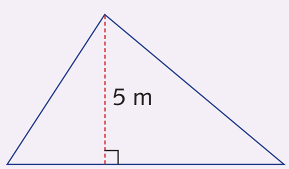
c 0.55 km^2 into ha

d 0.35 cm^3 to mm^3

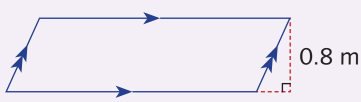
e 840 cm^3 to L

2 Find the area of each figure.

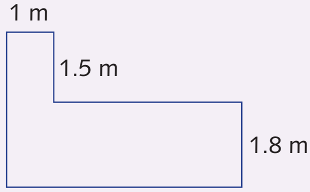
a



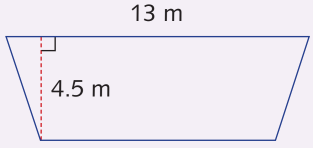
b



c



d



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

3 Convert each measurement to the units shown in brackets.

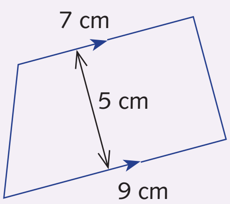
a 20 cm (mm) **b** 3200 mm (m) **c** 5 ha (m^2)

d 2000 cm^2 (m^2) **e** 3 cm^2 (mm^2) **f** 3.2 L (cm^3)

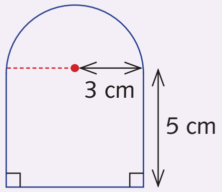
g 0.5 m^3 (cm^3) **h** 2 m^3 (L) **i** 25 000 m^2 (ha)

4 Calculate the area of each figure.

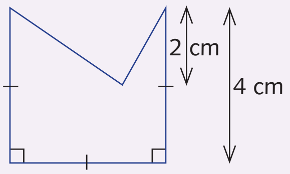
a



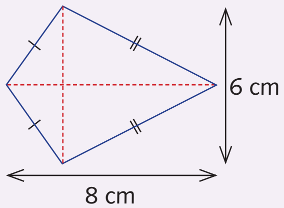
b



c



d

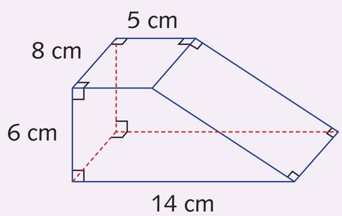


5 For each of the following solids, calculate:

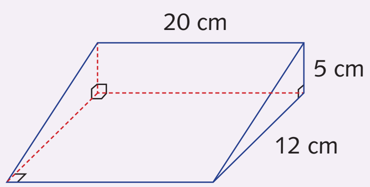
i the surface area

ii the volume

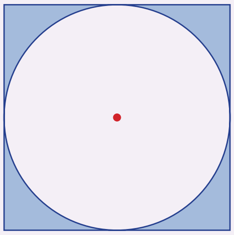
a



b



6 If the square in the shape opposite has an area of 64 cm^2, find the area of the shaded region.



7 A rectangular garden plot 8 m by 3 m is surrounded by a concrete path 1.2 m wide and laid to a depth of 8 m.

a Find the area of the path.

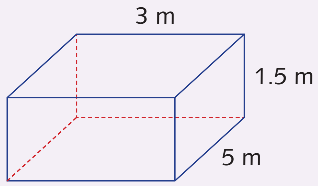
b Find the volume of concrete required.

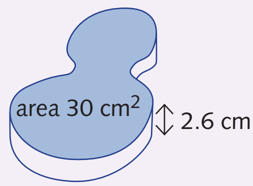
c Find the cost of the path if the concrete costs $75 per cubic metre.

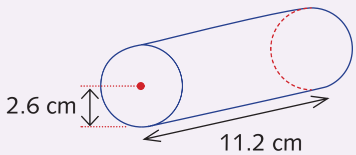
8 Find the total surface area of a solid cylinder of radius 5 cm and height 10 cm.

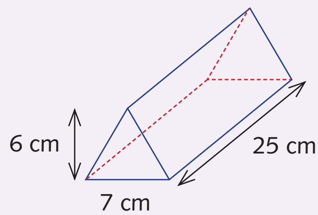
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

9 Find the volume of each solid.

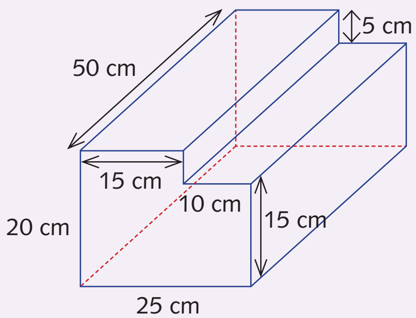
a


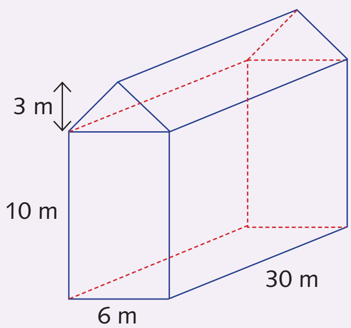
b


c


d


10 Find the volume of each solid.

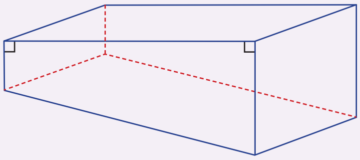
a


b


Challenge exercise

1 A rectangular house that measures 25 m by 15 m has a water tap at one corner of the house. A hose 20 m long is connected to the tap. What is the largest area of lawn over which the hose can reach?

2 A swimming pool is in the shape of a trapezoidal prism, as shown. The pool is 10 m long, 5 m wide, 1 m deep at one end and 2 m deep at the other end. The pool is filled with water until it is 10 cm from the top.

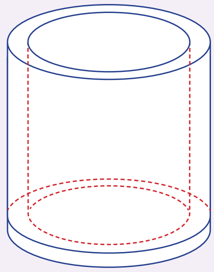


a Calculate, to the nearest litre, the volume of water in the pool.

b After a group of children played in the pool, the level of the water is 0.15 m below the top. How much water, correct to the nearest litre, splashed out of the pool?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 A foam drink-can holder is in the shape of a cylinder. The inside radius is 5.5 cm and the inside height is 11 cm. The foam is 1 cm thick and forms the curved surface of the cylinder. The base is cut out of foam 1 cm thick.

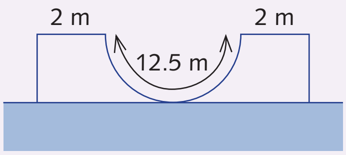


a Calculate the volume inside the cylinder in cubic centimetres.

b Calculate the volume of foam in the drink-can holder in cubic centimetres.

c If a cylindrical drink can perfectly fills the holder, find the total surface area of the exposed foam in square centimetres.

4 A skateboard ramp, a cross-section of which is shown opposite, consists of a half-cylinder with platforms of width 2 m. The curved part of the ramp has length 12.5 cm.

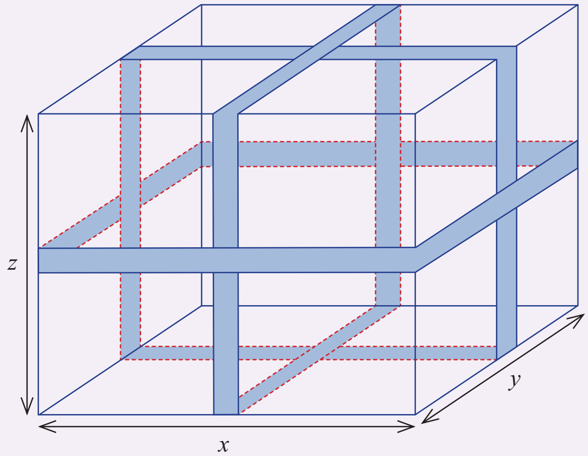


a Find the radius of the cylinder, correct to 2 decimal places.

b Find, correct to 2 decimal places, the total width of the ramp, including the platform.

c If the ramp is 8 m long, find the surface area of the curved part of the ramp.

5 **a** A rectangular prism has faces of area 66, 48 and 88. Find the volume of the prism.

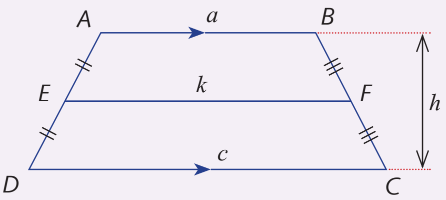


b A box in the shape of a rectangular prism is strapped as shown. The length of the straps are 100, 60 and 80.

i Find the dimensions of the box.

ii Find the volume of the box.

6 Let $ABCD$ be a trapezium with parallel sides AB and CD. Let E and F be the midpoints of AD and BC respectively. Let k be the length of EF. Let $AB = a$ and $CD = c$.



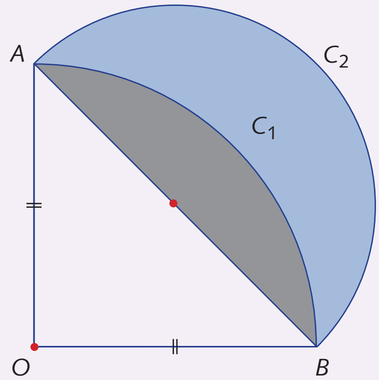
a Prove that $k = \dfrac{a+c}{2}$.

b Deduce that the area of the trapezium is kh, where h is the distance between the parallel sides.

7 **a** Let $ABCD$ be a convex quadrilateral. A line through the vertex A, parallel to the diagonal BD, meets CD (extended) at E. Show that the area of triangle BCE equals the area of $ABCD$.

b Let $ABCDE$ be a convex pentagon. Construct a triangle with the same area as this pentagon.

8 OA and OB are radii of a circle C_1. A circle C_2 is drawn with diameter AB as shown. Find the ratio of the area to the area .



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

CHAPTER

17

Number and Algebra

Quadratic equations

We have been solving linear equations for some time. We will now learn how to apply factorisation to solve quadratic equations.

Quadratic equations turn up routinely in mathematics. Being able to solve them is a fundamental skill. For example, finding the distance a rocket or cricket ball will travel involves quadratic equations.

The ancient Babylonians were solving quadratic equations more than 5000 years ago!

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

17A Solution of simple quadratic equations

In Chapter 15 we factorised quadratic expressions. For example:

$x^2 - 2x = x(x - 2)$ $\qquad$ $x^2 - 16 = (x - 4)(x + 4)$

$x^2 - x - 12 = (x - 4)(x + 3)$ $\qquad$ $x^2 - 4x + 4 = (x - 2)^2$

$2x^2 + x - 6 = (2x - 3)(x + 2)$

The following are examples of **quadratic equations**.

$x(x - 2) = 0 \quad x^2 - x - 12 = 0 \quad x^2 + 6x = 8$

Solving a quadratic equation means finding the values of x that satisfy the equation. One important method for solving a quadratic equation is factorising and then using the following simple idea:

If the product of two numbers is zero, at least one of the numbers is zero.

In symbols:

If $ab = 0$, then $a = 0$ or $b = 0$. (Both a and b may equal zero.)

To solve a quadratic equation:

- Move all the terms to the left-hand side of the equation, leaving zero on the right-hand side.
- Factorise the expression as a product of two factors.
- Equate each factor to zero and solve each linear equation.

The solutions to these linear equations are the solutions to the quadratic equation.

In Example 1 the quadratic equations are given in factorised form.

Example 1

Solve each equation.

a $(x - 3)(x + 4) = 0$ $\qquad$ **b** $x(x + 5) = 0$

c $(3x + 4)(x + 7) = 0$ $\qquad$ **d** $(3 - 2x)(4x + 5) = 0$

Solution

a
$$(x - 3)(x + 4) = 0$$
$$x - 3 = 0 \text{ or } x + 4 = 0$$
$$x = 3 \text{ or } x = -4$$

b
$$x(x + 5) = 0$$
$$x = 0 \text{ or } x + 5 = 0$$
$$x = 0 \text{ or } x = -5$$

c
$$(3x + 4)(x + 7) = 0$$
$$3x + 4 = 0 \text{ or } x + 7 = 0$$
$$x = -\frac{4}{3} \text{ or } x = -7$$

d
$$(3 - 2x)(4x + 5) = 0$$
$$3 - 2x = 0 \text{ or } 4x + 5 = 0$$
$$x = \frac{3}{2} \text{ or } x = -\frac{5}{4}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 2

Solve each equation.

a $x^2 - 2x = 0$

b $x^2 - 2x = 48$

c $x^2 = 3x + 10$

d $x^2 - 7x = 18$

Solution

In each case we move all terms to the left-hand side, and factorise the quadratic expression.

a
$$\begin{aligned} x^2 - 2x &= 0 \\ x(x - 2) &= 0 \\ x = 0 \text{ or } x - 2 &= 0 \\ x = 0 \text{ or } x &= 2 \end{aligned}$$

b
$$\begin{aligned} x^2 - 2x &= 48 \\ x^2 - 2x - 48 &= 0 \\ (x - 8)(x + 6) &= 0 \\ x = 8 \text{ or } x &= -6 \end{aligned}$$

c
$$\begin{aligned} x^2 &= 3x + 10 \\ x^2 - 3x - 10 &= 0 \\ (x - 5)(x + 2) &= 0 \\ x = 5 \text{ or } x &= -2 \end{aligned}$$

d
$$\begin{aligned} x^2 - 7x &= 18 \\ x^2 - 7x - 18 &= 0 \\ (x - 9)(x + 2) &= 0 \\ x = 9 \text{ or } x &= -2 \end{aligned}$$

Notice that in these examples we obtain two solutions to the original equation. We can check they are solutions by substitution. For example, for part **c** above:

If $x = 5$: $\quad$ LHS $= 5^2 = 25$

RHS $= 3 \times 5 + 10$

$= 25$

Therefore LHS = RHS

If $x = -2$: $\quad$ LHS $= (-2)^2 = 4$

RHS $= 3 \times -2 + 10$

$= 4$

Therefore LHS = RHS

We have checked that $x = 5$ and $x = -2$ are solutions to the quadratic equation $x^2 = 3x + 10$.

Numerical common factor

If there is a **numerical factor** common to all of the coefficients in the equation, we can divide both sides of the equation by that common factor.

Example 3

Solve each equation.

a $6x^2 = 18x$

b $2x^2 - 10x + 12 = 0$

c $-x^2 + 9x - 18 = 0$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution

a
$$
\begin{aligned}
6x^2 &= 18x \\
x^2 &= 3x \qquad \text{(Divide both sides of the equation by 6.)} \\
x^2 - 3x &= 0 \\
x(x-3) &= 0 \\
x &= 0 \text{ or } x - 3 = 0 \\
x &= 0 \text{ or } x = 3
\end{aligned}
$$

Note: In part **a** we cannot divide both sides of the equation by x since we would lose the solution $x = 0$.

b
$$
\begin{aligned}
2x^2 - 10x + 12 &= 0 \qquad \text{(Divide both sides of the equation by 2.)} \\
2(x^2 - 5x + 6) &= 0 \\
x^2 - 5x + 6 &= 0 \\
(x-3)(x-2) &= 0 \\
x - 3 &= 0 \text{ or } x - 2 = 0 \\
x &= 3 \text{ or } x = 2
\end{aligned}
$$

c
$$
\begin{aligned}
-x^2 + 9x - 18 &= 0 \\
-1(x^2 - 9x + 18) &= 0 \\
x^2 - 9x + 18 &= 0 \qquad \text{(Divide both sides of the equation by }-1\text{.)} \\
(x-3)(x-6) &= 0 \\
x - 3 &= 0 \text{ or } x - 6 = 0 \\
x &= 3 \text{ or } x = 6
\end{aligned}
$$

Equations involving a difference of squares

Example 4

Solve $18 - 2y^2 = 0$.

Solution

$$
\begin{aligned}
18 - 2y^2 &= 0 \\
9 - y^2 &= 0 \qquad \text{(Divide both sides of the equation by }-2\text{.)} \\
(y-3)(y+3) &= 0 \\
y &= 3 \text{ or } y = -3
\end{aligned}
$$

This problem could also be solved as follows:

$$
\begin{aligned}
18 - 2y^2 &= 0 \\
18 &= 2y^2 \\
y^2 &= 9 \\
y &= 3 \text{ or } y = -3
\end{aligned}
$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution of simple quadratic equations

- To solve a quadratic equation:
  - If there is a numerical common factor, divide both sides of the equation by the factor.
  - Move all terms to the left-hand side of the equation, leaving zero on the right-hand side.
  - If possible, factorise the resulting expression as a product of two factors.
  - Equate each factor to zero and solve each equation.
- Solutions to the equations of the form $x^2 = a$, $a \geq 0$ are $x = \sqrt{a}$ or $x = -\sqrt{a}$.
- Not all quadratic equations have solutions. For example, $x^2 + 5 = 0$ has no solutions since $x^2 + 5 \geq 5$ for all values of x. Hence $x^2 + 5$ will never equal zero.

Exercise 17A

Example 1

1 Solve the equations.

a $x(x-2) = 0$ **b** $(x+1)(x-1) = 0$ **c** $2x(x+1) = 0$

d $(3x-1)(2x+5) = 0$ **e** $(1-x)(3-2x) = 0$ **f** $(x+1)^2 = 0$

g $x(x-3) = 0$ **h** $(x-3)(x-4) = 0$ **i** $x(x+5) = 0$

j $(3x-2)(x+2) = 0$ **k** $(2+x)(3-x) = 0$ **l** $(x-3)^2 = 0$

Example 2

2 Solve each quadratic equation. Check your solutions in parts **c**, **e**, **f** and **l**.

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 7x = -12$ **c** $y^2 - 7y + 10 = 0$

d $x^2 + 13x - 30 = 0$ **e** $b^2 - 13b - 14 = 0$ **f** $z^2 - 27z - 90 = 0$

g $x^2 = -5x - 4$ **h** $x^2 = 19x - 18$ **i** $x^2 + x - 12 = 0$

j $x^2 - 16x + 28 = 0$ **k** $x^2 + 3x - 10 = 0$ **l** $x^2 + x = 90$

3 Solve each quadratic equation.

a $x^2 - 5x = 0$ **b** $x^2 + 7x = 0$ **c** $3x^2 - 6x = 0$

d $5x^2 - x = 0$ **e** $4x^2 = -10x$ **f** $x^2 + 4x = 0$

g $x^2 = 6x$ **h** $4x^2 + 12x = 0$ **i** $3x - 2x^2 = 0$

j $5x - 15x^2 = 0$ **k** $9x^2 - 18x = 0$ **l** $49x^2 = 7x$

Example 3

4 Solve the quadratic equations.

a $5x^2 - 10x = 0$ **b** $2x^2 + 5x = 0$ **c** $4x^2 = -12x$

d $3x = 2x^2$ **e** $5x = 15x^2$ **f** $3a^2 + 15a + 18 = 0$

g $4r^2 - 4r - 24 = 0$ **h** $5c^2 + 40c + 60 = 0$ **i** $3a^2 - 48a + 192 = 0$

j $2n^2 - 22n + 60 = 0$ **k** $4s^2 = 36s + 280$ **l** $-x^2 + 24x + 25 = 0$

m $-x + 42 = x^2$ **n** $-2x^2 + 10x + 48 = 0$ **o** $3x^2 - 48x = -84$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 4

5 Solve each quadratic equation.

a $x^2 - 1 = 0$ **b** $x^2 - 25 = 0$ **c** $4x^2 - 1 = 0$

d $2x^2 - 50 = 0$ **e** $x^2 + 6x + 9 = 0$ **f** $x^2 + 10x + 25 = 0$

g $x^2 - 12x + 36 = 0$ **h** $x^2 - 16 = 0$ **i** $16x^2 - 25 = 0$

j $9 - x^2 = 0$ **k** $72 - 2y^2 = 0$ **l** $x^2 - 4x + 4 = 0$

m $x^2 - 2x + 1 = 0$ **n** $x^2 = 6x + 27$ **o** $x^2 = 8x + 33$

17B Solution of quadratic equations when the coefficient of x^2 is not 1

In Chapter 15 we factorised quadratics for which the coefficient of x^2 is not 1. We will use this technique of factorisation in the following example.

Example 5

Solve each quadratic equation.

a $6x^2 + 7x + 2 = 0$ **b** $12x^2 = 23x - 5$

Solution

a $6x^2 + 7x + 2 = 0$

Find two numbers whose product is $6 \times 2 = 12$ and whose sum is 7. They are 4 and 3.

$$(6x^2 + 4x) + (3x + 2) = 0 \quad \text{(Split the middle term.)}$$
$$2x(3x + 2) + 1(3x + 2) = 0 \quad \text{(Use grouping.)}$$
$$(3x + 2)(2x + 1) = 0$$
$$3x + 2 = 0 \text{ or } 2x + 1 = 0$$
$$x = -\frac{2}{3} \text{ or } x = -\frac{1}{2}$$

Note: It does not matter in which order we split the middle term. In the above example, we could write $4x + 3x$ or $3x + 4x$, and factorise in pairs. Try it for yourself.

b
$$12x^2 = 23x - 5$$
$$12x^2 - 23x + 5 = 0$$

Find two numbers whose product is $12 \times 5 = 60$ and whose sum is -23. The numbers are -20 and -3.

$$12x^2 - 20x - 3x + 5 = 0$$
$$4x(3x - 5) - 1(3x - 5) = 0$$
$$(3x - 5)(4x - 1) = 0$$
$$3x - 5 = 0 \text{ or } 4x - 1 = 0$$
$$x = \frac{5}{3} \text{ or } x = \frac{1}{4}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Quadratic equations of the form $ax^2 + bx + c = 0$

- If the coefficients have a numerical common factor, divide both sides of the equation by that factor.
- To factorise a quadratic expression such as $ax^2 + bx + c$, find two numbers α and β whose product is ac and whose sum is b. Write the middle term as $\alpha x + \beta x$ and factorise by grouping.

Exercise 17B

1 Solve the quadratic equations.

a $4x(6x + 12) + 8(6x + 12) = 0$
b $3x(2x - 4) + 6(2x - 4) = 0$
c $5x(3x - 6) + 10(3x - 6) = 0$
d $2x(x - 5) + 12(x - 5) = 0$

Example 5a

2 Solve the quadratic equations.

a $2x^2 - 5x - 3 = 0$
b $6x^2 + x - 2 = 0$
c $8x^2 - 14x + 3 = 0$
d $3x^2 + 5x - 2 = 0$
e $6x^2 - 11x - 10 = 0$
f $10x^2 + 11x - 6 = 0$
g $6x^2 + 5x + 1 = 0$
h $6x^2 - 7x - 10 = 0$
i $12x^2 + 5x - 2 = 0$
j $10x^2 + 31x + 15 = 0$
k $12x^2 - 8x - 15 = 0$
l $15x^2 + 13x + 2 = 0$

Example 5b

3 Solve the quadratic equations.

a $7x^2 - 16x = 15$
b $2x^2 + 3x = 2$
c $6x^2 = 7x + 3$
d $6x^2 + 5x = 6$
e $7x^2 = 78x - 11$
f $4x^2 = 3x + 1$
g $3x^2 = 8x + 3$
h $5x^2 - 23x = 84$
i $4x^2 - 15 = 4x$
j $12x^2 = 4x + 5$
k $3x^2 - 4 = 4x$
l $6x = 2x^2 - 8$

4 Solve the quadratic equations.

a $9x^2 + 4x - 5 = 0$
b $5x^2 + 7x - 12 = 0$
c $10x^2 - 22x + 4 = 0$
d $-8a^2 - 24a + 14 = 0$
e $-4a^2 + 8a + 21 = 0$
f $6a^2 - 56a + 50 = 0$
g $5x^2 + 26x + 24 = 0$
h $6x^2 + 11x - 10 = 0$
i $3x^2 - 41x - 60 = 0$
j $-8x^2 + 4x + 12 = 0$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

17C Quadratics in disguise

In many mathematical problems and applications, equations arise that do not initially appear to be quadratic equations. We often need to rearrange an equation to put it into the standard form $ax^2 + bx + c = 0$.

We start with two simple examples.

Example 6

Solve each equation.

a $x(x + 5) = 6$

b $\dfrac{x(x - 3)}{2} + x = 1$

Solution

a
$$\begin{aligned} x(x + 5) &= 6 \\ x^2 + 5x - 6 &= 0 \\ (x + 6)(x - 1) &= 0 \\ x + 6 = 0 \text{ or } x - 1 &= 0 \\ x = -6 \text{ or } x &= 1 \end{aligned}$$

We can check that these are the correct solutions by substitution.

If $x = -6$, LHS $= -6(-6 + 5) = 6 =$ RHS

If $x = 1$, LHS $= 1(1 + 5) = 6 =$ RHS

b
$$\begin{aligned} \frac{x(x - 3)}{2} + x &= 1 \\ x(x - 3) + 2x &= 2 \quad \text{(Multiply both sides by 2.)} \\ x^2 - 3x + 2x &= 2 \\ x^2 - x - 2 &= 0 \\ (x + 1)(x - 2) &= 0 \\ x + 1 = 0 \text{ or } x - 2 &= 0 \\ x = -1 \text{ or } x &= 2 \end{aligned}$$

Some equations involve fractions in which the pronumeral appears in the denominator. We always assume that the pronumeral cannot take a value that makes the denominator equal to zero. It is important to check that your answers are in fact correct solutions to the given equation.

Example 7

Solve:

a $x = \dfrac{3x - 2}{x}$

b $\dfrac{x - 2}{3} = \dfrac{5}{x}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

a

$$x = \frac{3x - 2}{x}$$

$$x^2 = 3x - 2 \quad \text{(Multiply both sides by } x.\text{)}$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x - 1 = 0 \text{ or } x - 2 = 0$$

$$x = 1 \text{ or } x = 2$$

b **Method 1**

$$\frac{x-2}{3} = \frac{5}{x}$$

$$3x \times \frac{x-2}{3} = 3x \times \frac{5}{x}$$

(Multiply both sides of the equation by $3x$.)

$$x(x - 2) = 15$$

$$x^2 - 2x = 15$$

$$x^2 - 2x - 15 = 0$$

$$(x + 3)(x - 5) = 0$$

$$x + 3 = 0 \text{ or } x - 5 = 0$$

$$x = -3 \quad \text{or} \quad x = 5$$

Method 2

$$\frac{x-2}{3} \times \frac{5}{x}$$

$$x(x - 2) = 3 \times 5$$

(This is called cross-multiplication.)

We must check mentally that these solutions satisfy the original equation.

Note: **Cross-multiplication** is an effective method in the solution of such equations. It is justified by Method 1 in the above example.

Example 8

Solve $(x - 2)(2x + 5) = 2x + 5$.

Solution

Method 1

$$(x - 2)(2x + 5) = 2x + 5$$

$$2x^2 - 4x + 5x - 10 = 2x + 5$$

$$2x^2 + x - 10 = 2x + 5$$

$$2x^2 - x - 15 = 0$$

$$(2x + 5)(x - 3) = 0$$

$$2x + 5 = 0 \text{ or } x - 3 = 0$$

$$x = -\frac{5}{2} \text{ or } x = 3$$

Method 2

$$(x - 2)(2x + 5) = 2x + 5$$

$$(x - 2)(2x + 5) - (2x + 5) = 0$$

$$(2x + 5)(x - 2 - 1) = 0$$

$$(2x + 5)(x - 3) = 0$$

$$2x + 5 = 0 \text{ or } x - 3 = 0$$

$$x = -\frac{5}{2} \text{ or } x = 3$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 9

Solve the equation $\dfrac{x+4}{2} - \dfrac{3}{x-3} = 1$.

Solution

$$\frac{x+4}{2} - \frac{3}{x-3} = 1$$

$$(x+4)(x-3) - 6 = 2x - 6 \qquad \text{(Multiply both sides by } 2(x-3).)$$

$$x^2 + x - 12 - 6 = 2x - 6$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \text{ or } x = -3$$

Exercise 17C

Example 6a

1 Solve the equations.

a $x(x-6) = -8$ **b** $x(x-7) = -10$ **c** $x(x-5) = -4$

d $x(x-8) = -15$ **e** $x(x-3) = 4$ **f** $x(x-6) = 27$

g $(x-3)(x-4) = 12$ **h** $(x+7)(x-2) + 14 = 0$

i $(x+2)(x-6) - 65 = 0$ **j** $(x-1)(x-2) = 20$

Example 6b, 7

2 Solve the quadratic equations.

a $x + 5 = \dfrac{14}{x}$ **b** $\dfrac{15}{x} = x - 2$ **c** $\dfrac{6}{x} - x = 1$

d $x + \dfrac{6}{x} = 7$ **e** $x + \dfrac{32}{x} = 18$ **f** $\dfrac{x+1}{3} = \dfrac{10}{x}$

g $\dfrac{x+1}{4} = \dfrac{5}{x}$ **h** $x - \dfrac{14}{x} = 5$ **i** $\dfrac{x^2 - 4x}{12} = \dfrac{x-4}{2}$

j $\dfrac{1}{x+1} = \dfrac{5x-6}{6x}$ **k** $\dfrac{2x+1}{3} = \dfrac{-(51+32x)}{3(2x-3)}$ **l** $x + \dfrac{1}{x} = \dfrac{13}{6}$

m $\dfrac{3}{x-1} = \dfrac{3x-2}{x(x-2)}$ **n** $\dfrac{35-4x}{5} = \dfrac{36-5x}{5x}$

Example 8

3 Solve the quadratic equations.

a $x(x+3) = x(2x-4)$ **b** $(x+3)(2x-1) = (2x-1)(3x-4)$

c $3x(x+3) = 2x(2x+5)$ **d** $4(x+3)(2x-3) = 2(x+3)(5x+8)$

e $(5x+2)(x-7) = (x-7)(2x+1)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 Solve the quadratic equations.

a $x + 4 = \dfrac{12}{x}$ **b** $x - 3 = \dfrac{10}{x}$ **c** $x - \dfrac{35}{x} = 2$

d $x = \dfrac{18}{x} - 7$ **e** $(x + 2)(x + 4) = 3$ **f** $(x - 2)(x + 5) = 8$

g $(2x + 1)(x + 5) = 18$ **h** $(2x - 3)(x + 2) = 4$ **i** $x = \dfrac{5}{x} - 4$

j $x = \dfrac{2}{x - 1}$ **k** $x = \dfrac{7}{2x + 3} + 1$ **l** $x = \dfrac{13}{3x - 1} - 3$

Example 9

5 Solve the quadratic equations.

a $\dfrac{2x}{3} + \dfrac{x - 1}{15} = \dfrac{2}{3} + \dfrac{22}{x}$ **b** $\dfrac{x}{2} + \dfrac{x - 4}{x - 4} = \dfrac{x}{3}$

c $\dfrac{x + 1}{7} + \dfrac{8}{x - 2} = 3$ **d** $\dfrac{x - 1}{x + 2} - \dfrac{x - 3}{x - 4} = -\dfrac{2}{3}$

e $x(x + 9) + 3 = x(2 - 3x)$ **f** $3x(x - 2) = x(x + 1)$

17D Applications of quadratic equations

When we apply mathematics to practical problems, we often arrive at equations that we have to solve. In many cases these equations are quadratic equations.

Some of the solutions to the quadratics may not be relevant to the problem. For example, a quadratic equation may yield negative or fractional solutions, which may not make sense as solutions to the original problem. This point is illustrated in the next two examples.

Example 10

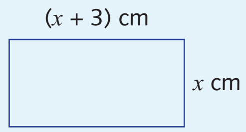
A rectangle has one side 3 cm longer than the other. The rectangle has area 28 cm^2. How long is the shorter side?

Solution

Let x cm be the length of the shorter side. The other side has length $(x + 3)$ cm.

Area of rectangle is:

$$\begin{aligned} x(x + 3) &= 28 \\ x^2 + 3x - 28 &= 0 \\ (x - 4)(x + 7) &= 0 \\ x - 4 = 0 \text{ or } x + 7 &= 0 \\ x = 4 \text{ or } x &= -7 \end{aligned}$$



Since length is positive, $x = -7$ makes no sense. Hence the shorter side has length 4 cm and the longer side is 7 cm.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 11

Each term in the sequence 5, 9, 13, 17, … is obtained by adding 4 to the previous number. The sum, S, of the first n terms in this sequence is given by $S = 2n^2 + 3n$. How many terms must we add to make a sum of 90?

Solution

$S = 90$, so $\quad 2n^2 + 3n = 90$

$$2n^2 + 3n - 90 = 0$$

(Find two numbers with product $2 \times (-90) = -180$ and sum 3. The numbers are 15 and -12.)

$$\begin{aligned} 2n^2 + 15n - 12n - 90 &= 0 \\ n(2n + 15) - 6(2n + 15) &= 0 \\ (2n + 15)(n - 6) &= 0 \\ 2n + 15 = 0 \text{ or } n - 6 &= 0 \\ n = -\frac{15}{2} \text{ or } n &= 6 \end{aligned}$$

Since n is a positive whole number, the solution $n = -\frac{15}{2}$ does not make sense here.
So $n = 6$.

Check: $5 + 9 + 13 + 17 + 21 + 25 = 90$

Exercise 17D

Although some of these problems can be solved by inspection, in each case, introduce a pronumeral, write down a quadratic equation and solve it.

Example 10

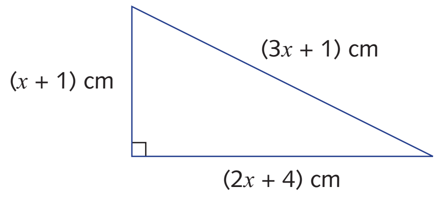
1 The product of a certain whole number and four more than that number is 140. Find the number.

2 The product of a certain whole number and three less than that number is 108. Find the number.

3 The product of a certain integer and nine more than that integer is 10. What can the integer be?

4 The product of a certain integer and twice that integer is 32. What can the integer be?

5 Find the value of x in the diagram opposite.

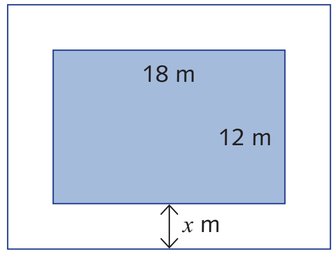


Example 11

6 The sum of n terms in the sequence 23, 19, 15, 11, … is given by the rule $S = 25n - 2n^2$. How many terms must we add to make a sum of 72?

7 A metal sheet is 50 cm wide and 60 cm long. It has squares cut out of the corners so that it can be folded to form a box with a base area of 1200 cm^2. Find the length of the side of the squares.

8 A rectangular lawn is 18 m long and 12 m broad. The lawn is surrounded by a path of width x m. The area of the path is equal to the area of the lawn. Find x.



9 A man travelled from town A to town B at a speed of V km/h. The distance from A to B is 120 km. The man returned from B to A at a speed of $(V-2)$ km/h. He took two hours longer on the return journey. Find V.

10 Sally rides to school each morning. She discovers that if she were to ride the 10 km to school at a speed 10 km/h faster than she normally rides, she would get to school 10 minutes earlier. Find Sally's normal riding speed. (Let V km/h be Sally's normal speed.)

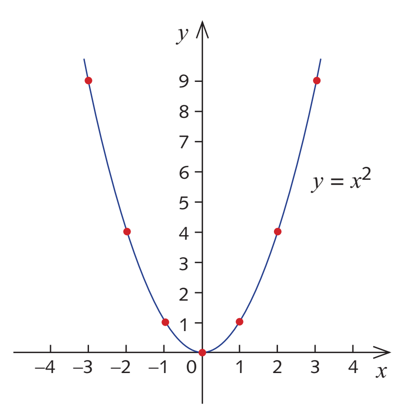
11 Giorgia travelled 108 km and found that she could have made the journey in four and a half hours less time had she travelled at a speed 2 km/h faster. What was Giorgia's speed?

12 A, B and C do a piece of work together. A could have done it alone in six hours longer, B in 15 hours longer and C in twice the time. How long did it take all three to do the work together?

17E Graphs of quadratics

In this section we plot graphs of quadratics. First, we complete a table of values for $y = x^2$ for x between 3 and 3.

x	3	2	1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9



If we plot the corresponding points and join them with a smooth curve, we obtain the graph to the right.

This graph is called a **parabola**. The point $(0, 0)$ is called the **vertex** or **turning point** of the parabola. The vertex corresponds to the minimum value of y, since x^2 is positive except when $x = 0$.

The graph is symmetrical about the y-axis (the line $x = 0$). This line is called the **axis of symmetry** of the parabola.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

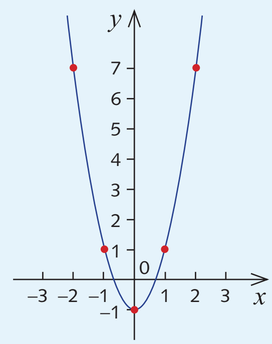
Example 12

Plot the graph of $y = 2x^2 - 1$ for $-2 \le x \le 2$. Find the axis of symmetry and the coordinates of the vertex.

Solution

A suitable table of values is:

x	−2	−1	0	1	2
$y = 2x^2 - 1$	7	1	−1	1	7



Notice that the y values are the same for $x = -1$ and $x = 1$.

The equation of the axis of symmetry is $x = 0$. When $x = 0$, $y = -1$, so the vertex has coordinates $(0, -1)$.

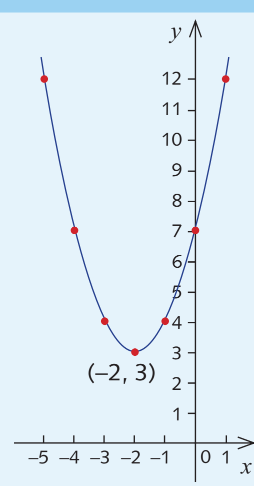
Example 13

Plot the graph of $y = x^2 + 4x + 7$ for $-5 \le x \le 1$.

Solution

A suitable table of values is:

x	−5	−4	−3	−2	−1	0	1
$y = x^2 + 4x = 7$	12	7	4	3	4	7	12



Plot the points and join them with a smooth curve. The graph shown opposite is obtained.

This parabola is just like the parabola $y = x^2$, except that it has been translated.

The equation of the axes of symmetry is $x = -2$.

The vertex has coordinates $(-2, 3)$.

Graphs of quadratics

- The graph of $y = x^2$ is called a parabola.
- The point $(0, 0)$ is called the vertex.
- The graph is symmetrical about the y-axis.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 17E

Example 12

1 Plot the graph of each of the following for $-3 \le x \le 3$. In each case, give the coordinates of the vertex.

a $y = x^2 + 1$ **b** $y = x^2 - 1$ **c** $y = x^2 - 4$

d $y = x^2 + 2$ **e** $y = x^2 - 3$ **f** $y = x^2 + 3$

Example 13

2 Plot each of the following graphs for the given x values.

a $y = x^2 - 4x + 3,\ 0 \le x \le 4$ **b** $y = x^2 + 2x - 3, -4 \le x \le 2$

c $y = 2x^2 - 2, -2 \le x \le 2$ **d** $y = x^2 + x - 2, -3 \le x \le 2$

e $y = x^2 - 2x - 8, -3 \le x \le 5$ **f** $y = x^2 - 2x, -2 \le x \le 4$

g $y = x^2 + 3x, -4 \le x \le 1$ **h** $y = 2x^2 - 2x, -1 \le x \le 2$

3 Plot each of the following graphs for the given x-values.

a $y = -x^2, -3 \le x \le 3$ **b** $y = 2 - x^2, -3 \le x \le 3$

c $y = x - x^2, -2 \le x \le 3$ **d** $y = 2x - x^2, -1 \le x \le 3$

4 **a** Plot the graph of $y = x^2 - x - 2$ for $-2 \le x \le 3$.

b Solve the equation $x^2 - x - 2 = 0$.

c How would you read the solution to the equation $x^2 - x - 2 = 0$ from the graph?

5 **a** Plot the graph of $y = x^2 - x - 6$ for $-3 \le x \le 4$.

b Solve the equation $x^2 - x - 6 = 0$.

c How would you read the solution to the equation $x^2 - x - 6 = 0$ from the graph?

6 **a** Plot the graph of $y = x^2 + 2x - 3$ for $-4 \le x \le 2$.

b Solve the equation $x^2 + 2x - 3 = 0$.

c How would you read the solution to the equation $x^2 + 2x - 3 = 0$ from the graph?

17F Solving quadratic equations by completing the square

In all the examples so far, the quadratic expressions factorised nicely and gave us integer or rational solutions. This is not always the case. For example, $x^2 - 7 = 0$ has solutions $x = \sqrt{7}$ and $x = -\sqrt{7}$.

Here are the steps for solving the quadratic $x^2 + 4x - 9 = 0$ using the method of completing the square.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

We first complete the square on the left-hand side.

$$
\begin{aligned}
x^2 + 4x + 4 - 4 - 9 &= 0 \quad \text{(Add and subtract the square of half the coefficient of } x.\text{)}\\
(x+2)^2 - 13 &= 0\\
(x+2)^2 &= 13\\
x + 2 = \sqrt{13} \text{ or } \quad x + 2 &= -\sqrt{13}\\
x = -2 + \sqrt{13} \text{ or } x &= -2 - \sqrt{13}
\end{aligned}
$$

These two numbers are the solutions to the original equation as each step is reversible. Checking by substitution is hard. It is more efficient to check your calculation.

- Quadratic equations with integer coefficients can have:
  - integer or rational solutions; for example, $4x^2 - 1 = 0$ has solutions $\frac{1}{2}$ and $-\frac{1}{2}$.
  - solutions involving square roots; for example, $x^2 - 7 = 0$ has solutions $-\sqrt{7}$ and $\sqrt{7}$.
  - no solution; for example, $x^2 + 1 = 0$ has no solutions.
- The method of completing the square enables us to solve equations whose solutions involve square roots.

Example 14

Solve:

a $x^2 - 6x - 1 = 0$

b $x^2 + 8x + 2 = 0$

Solution

a

$$
\begin{aligned}
x^2 - 6x - 1 &= 0\\
x^2 - 6x + 9 - 9 - 1 &= 0\\
(x-3)^2 - 10 &= 0\\
(x-3)^2 &= 10\\
x - 3 = \sqrt{10} \text{ or } x - 3 &= -\sqrt{10}\\
x = 3 + \sqrt{10} \text{ or } x &= 3 - \sqrt{10}
\end{aligned}
$$

b

$$
\begin{aligned}
x^2 + 8x + 2 &= 0\\
x^2 + 8x + 16 - 16 + 2 &= 0\\
(x+4)^2 - 14 &= 0\\
(x+4)^2 &= 14\\
x + 4 = \sqrt{14} \text{ or } x + 4 &= -\sqrt{14}\\
x = -4 + \sqrt{14} \text{ or } x &= 4 - \sqrt{14}
\end{aligned}
$$

It is not always the case that the coefficient of x is an even number. If the coefficient of x is odd, fractions will arise, but the method is the same.

Example 15

Solve $x^2 - 5x - 3 = 0$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

$$x^2 - 5x - 3 = 0$$

$$x^2 - 5x + \frac{25}{4} - \frac{25}{4} - 3 = 0 \qquad \text{(Add and subtract the square of half the coefficient of } x.)$$

$$\left(x^2 - 5x + \frac{25}{4}\right) - \frac{37}{4} = 0$$

$$x^2 - 5x + \frac{25}{4} = \frac{37}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{37}{4}$$

$$x - \frac{5}{2} = \frac{\sqrt{37}}{2} \text{ or } x - \frac{5}{2} = -\frac{\sqrt{37}}{2}$$

$$x = \frac{5 + \sqrt{37}}{2} \text{ or } x = \frac{5 - \sqrt{37}}{2}$$

The method of completing the square works just as well when the roots are rational.

Example 16

Solve $x^2 - 5x + 6 = 0$.

Solution

$$x^2 - 5x + 6 = 0$$

$$\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + 6 = 0$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{1}{4} = 0$$

$$x - \frac{5}{2} = \sqrt{\frac{1}{4}} \text{ or } x - \frac{5}{2} = -\sqrt{\frac{1}{4}}$$

$$x = \frac{5}{2} + \frac{1}{2} \text{ or } x = \frac{5}{2} - \frac{1}{2}$$

$$x = 3 \text{ or } x = 2$$

Alternatively

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x - 3 = 0 \text{ or } x - 2 = 0$$

$$x = 3 \text{ or } x = 2$$

There are quadratic equations that have no solutions. Consider, for example, $x^2 - 6x + 12 = 0$:

$$x^2 - 6x + 12 = 0$$

$$(x^2 - 6x + 9) - 9 + 12 = 0$$

$$(x - 3)^2 + 3 = 0$$

$$(x - 3)^2 = -3$$

Since $(x - 3)^2 \geq 0$ for all values of x, there is no solution to the equation $(x - 3)^2 = -3$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Solution of quadratic equations by completing the square

- To solve a quadratic equation with the coefficient of x^2 equal to 1 by completing the square, we:
  - add and subtract the square of half the coefficient of x
  - complete the square
  - solve for x.
- The solutions of a quadratic equation often involve surds.

Exercise 17F

1 Solve the equations.

a $x^2 - 5 = 0$ **b** $x^2 - 11 = 0$ **c** $x^2 - 12 = 0$

d $2x^2 - 6 = 0$ **e** $50 - 5x^2 = 0$ **f** $40 - 8x^2 = 0$

g $(x-2)^2 = 5$ **h** $(x+3)^2 = 6$ **i** $(x+2)^2 = 8$

j $\left(x+\frac{7}{2}\right)^2 = 9$ **k** $(x-2)^2 = 16$ **l** $(x+4)^2 - 7 = 0$

m $\left(x-\frac{11}{2}\right)^2 - \frac{7}{4} = 0$ **n** $\left(x-\frac{1}{2}\right)^2 = \frac{11}{4}$ **o** $\left(x+\frac{15}{4}\right)^2 - \frac{7}{4} = 0$

Example 14

2 Solve the equations by completing the square.

a $x^2 + 2x - 1 = 0$ **b** $x^2 + 4x + 1 = 0$ **c** $x^2 - 12x + 23 = 0$

d $x^2 + 6x + 7 = 0$ **e** $x^2 - 8x - 1 = 0$ **f** $x^2 + 10x + 12 = 0$

g $x^2 + 8x + 6 = 0$ **h** $x^2 + 6x - 1 = 0$ **i** $x^2 + 6x + 1 = 0$

j $x^2 + 8x - 2 = 0$ **k** $x^2 + 8x + 2 = 0$ **l** $x^2 + 20x - 20 = 0$

Example 15

3 Solve the equations by completing the square.

a $x^2 + x - 1 = 0$ **b** $x^2 - 3x + 1 = 0$ **c** $x^2 - 5x - 1 = 0$

d $x^2 + 3x - 2 = 0$ **e** $x^2 + 5x + 1 = 0$ **f** $x^2 - 6x + 2 = 0$

g $x^2 + 10x + 23 = 0$ **h** $x^2 + 9x + 4 = 0$ **i** $x^2 + 3x - 11 = 0$

j $x^2 + 11x + 10 = 0$ **k** $x^2 + 7x + 10 = 0$ **l** $x^2 + 7x - 5 = 0$

4 For each equation, complete the square and show that the equation has no solution.

a $x^2 + 8x + 18 = 0$ **b** $x^2 - 4x + 6 = 0$ **c** $x^2 + 2x + 4 = 0$

d $x^2 + x + 2 = 0$ **e** $x^2 + 2x + 2 = 0$ **f** $x^2 + 4x + 5 = 0$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

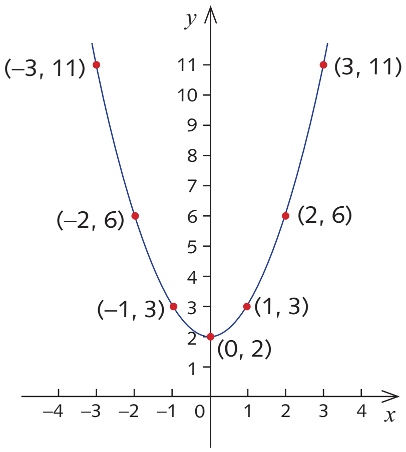
17G Sketching graphs of the form $y = x^2 + bx + c$

In Section 17E we plotted parabolas. Important features of a parabola include the vertex or turning point and the axis of symmetry. The graphs were plotted using a set of suggested x-values. In all cases the vertex occurred midway between the greatest x-value and the least x-value. What can we do when these hints are not provided?

Consider the following graphs. A connection exists between the equation and the location of the vertex of its corresponding graph.

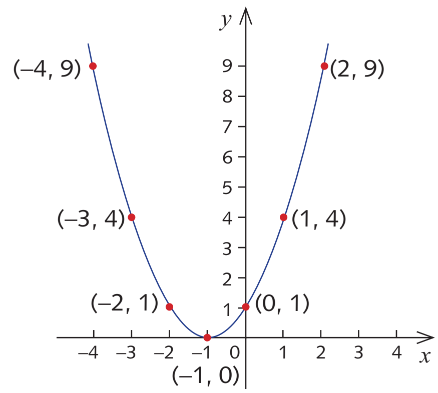
$y = x^2 + 2$

x	–3	–2	–1	0	1	2	3
y	11	6	3	2	3	6	11



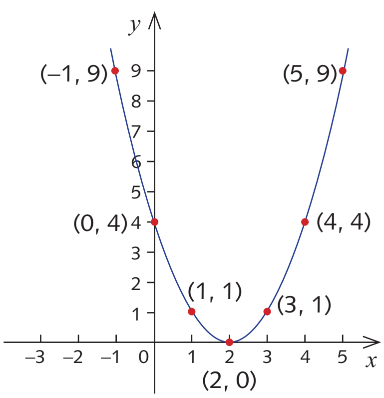
$y = (x + 1)^2$

x	–4	–3	–2	–1	0	1	2
y	9	4	1	0	1	4	9



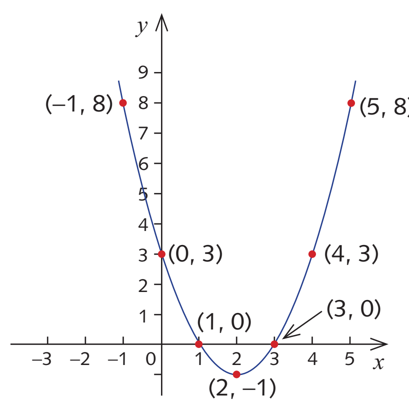
$y = (x - 2)^2$

x	–1	0	1	2	3	4	5
y	9	4	1	0	1	4	9



$y = (x - 2)^2 - 1$

x	–1	0	1	2	3	4	5
y	8	3	0	–1	0	3	8



The connection between rule and graph, evident above, is summarised below.

Graphs of the form $y = (x - h)^2 + k$ are parabolas with a vertex, or turning point, at (h, k) and axis of symmetry, $x = h$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

These graphs are congruent to the shape of the basic parabola $y = x^2$, but have been subjected to a horizontal shift of h units and vertical shift of k units. These types of graph transformations are called 'translations'. A formal treatment of translations, and other graph transformations, is given in *ICE-EM Mathematics Year 10*.

Example 17

Find the turning point and axis of symmetry for the following quadratic graphs.

a $y = (x + 3)^2$

b $y = x^2 - 1$

c $y = (x - 1)^2 - 2$

d $y = (x + 1)^2 + 3$

Solution

a $y = (x + 3)^2$

$= (x - (-3))^2 + 0$

Turning point is $(-3, 0)$.
Axis of symmetry is $x = -3$.

b $y = x^2 - 1$

$= (x - 0)^2 - 1$

Turning point is $(0, -1)$.
Axis of symmetry is $x = 0$ (the y-axis).

c $y = (x - 1)^2 - 2$

Turning point is $(1, -2)$.
Axis of symmetry is $x = 1$.

d $y = (x + 1)^2 + 3$

$= (x - (-1))^2 + 3$

Turning point is $(-1, 3)$.
Axis of symmetry is $x = -1$.

By completing the square, any quadratic equation of the form $y = x^2 + bx + c$ can be converted to the form $y = (x - h)^2 + k$.

We begin the process by adding then subtracting $\left(\frac{b}{2}\right)^2$.

Example 18

Convert the following to the form $y = (x - h)^2 + k$.

a $y = x^2 + 4x + 1$

b $y = x^2 - 3x + 4$

Solution

a $y = x^2 + 4x + 1$

$= \left(x^2 + 4x + \left(\frac{4}{2}\right)^2\right) - \left(\frac{4}{2}\right)^2 + 1$

$= (x^2 + 4x + 4) - 4 + 1$

$= (x + 2)^2 - 3$

b $y = x^2 - 3x + 4$

$= \left(x^2 - 3x + \left(\frac{-3}{2}\right)^2\right) - \left(\frac{-3}{2}\right)^2 + 4$

$= \left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4} + 4$

$= \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Parabola sketching

Any quadratic graph should be sketched with the following features labelled:

1 the vertex (or the turning point)
2 the y-intercept
3 the x-intercept(s), where they exist.

In Chapter 11, we found the axis intercepts of linear graphs. The y-intercept was found by substituting $x = 0$ into the rule and solving for y. The x-intercept was found by substituting $y = 0$ into the rule and solving for x. This is no different for quadratic rules, or indeed the equation for any other type of graph.

Example 19

Sketch the following quadratic graphs.

a $y = (x + 2)^2 + 3$ **b** $y = (x - 1)^2 - 3$ **c** $x^2 + 4x - 5$

Solution

a $y = (x + 2)^2 + 3$

The turning point is $(-2, 3)$.

When $x = 0$,

$y = (0 + 2)^2 + 3$
$= 4 + 3$
$= 7$

$(0, 7)$ is the y-intercept.

There are no x-intercepts.

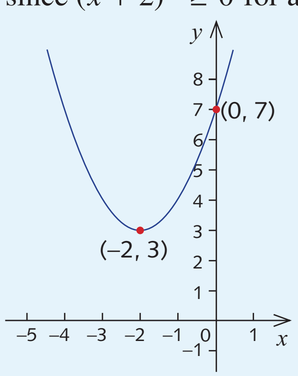
This can be confirmed algebraically.

When $y = 0$

$0 = (x + 2)^2 + 3$

$(x + 2)^2 = -3$

But this equation has no solutions since $(x + 2)^2 \geq 0$ for all x.



b $y = (x - 1)^2 - 3$

The turning point is $(1, -3)$.

When $x = 0$,

$y = (0 - 1)^2 - 3$
$= 1 - 3$
$= -2$

$(0, -2)$ is the y-intercept.

When $y = 0$,

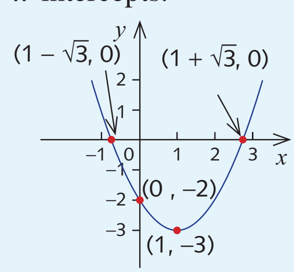
$0 = (x - 1)^2 - 3$

$(x - 1)^2 = 3$

$\therefore x - 1 = \sqrt{3}$ or $x - 1 = -\sqrt{3}$

$\therefore x = 1 + \sqrt{3}$ or $x = 1 - \sqrt{3}$

$(1 + \sqrt{3}, 0)$ and $(1 - \sqrt{3}, 0)$ are x-intercepts.



Note: $1 - \sqrt{3} \approx -0.7$. Use your calculator to find the approximate values for any x-intercepts involving surds.

(continued over page)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

c $y = x^2 + 4x - 5$ (Convert to the form $y = (x - h)^2 + k$.)

$$y = x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5$$

$$= (x^2 + 4x + 4) - 4 - 5$$

$$= (x + 2)^2 - 9$$

The turning point is $(-2,\ -9)$.

When $x = 0$, $y = -5$.

The y-intercept is $(0,\ -5)$.

When $y = 0$,

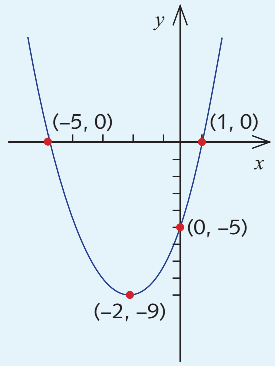
$0 = (x + 2)^2 - 9$

$\therefore (x + 2)^2 = 9$

$\therefore x + 2 = 3$ or $x + 2 = -3$

$\therefore x = 1$ or $x = -5$

The x-intercepts are $(1,\ 0)$ and $(-5,\ 0)$.



Exercise 17G

Example 17

1 Find the turning point and axis of symmetry for the following quadratic graphs.

a $y = x^2 - 1$ **b** $y = (x - 1)^2$ **c** $y = (x + 2)^2 - 1$

d $y = x^2 + 4$ **e** $y = (x + 3)^2$ **f** $y = (x + 1)^2 + 2$

g $y = (x - 2)^2 - 2$ **h** $y = 3 + (x - 1)^2$ **i** $y = -2 + (x - 3)^2$

Example 18

2 Convert the following to the form $y = (x - h)^2 + k$.

a $y = x^2 - 4x + 2$ **b** $y = x^2 + 8x + 21$ **c** $y = x^2 - 6x - 15$

d $y = x^2 + 5x - 1$ **e** $y = x^2 - 3x + 5$ **f** $y = x^2 - 7x - 10$

3 Solve the following equations, where solution(s) exist.

a $x^2 - 4 = 0$ **b** $x^2 - 11 = 0$ **c** $(x + 1)^2 - 4 = 0$

d $(x + 3)^2 + 5 = 0$ **e** $(x + 2)^2 - 9 = 0$ **f** $(x - 3)^2 - 5 = 0$

g $(x + 1)^2 - 12 = 0$ **h** $(x - 7)^2 + 1 = 0$ **i** $(x + 4)^2 - 18 = 0$

j $x^2 + 3 = 0$ **k** $25 - (x - 3)^2 = 0$ **l** $\left(x - \frac{7}{2}\right)^2 - \frac{25}{4} = 0$

m $\left(x + \frac{3}{2}\right)^2 - \frac{7}{4} = 0$ **n** $\left(x - \frac{5}{3}\right)^2 - \frac{40}{9} = 0$ **o** $\left(x + \frac{3}{4}\right)^2 + \frac{9}{4} = 0$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 19a, b

4 Sketch the following parabolas, labelling the turning point and any intercepts.

a $y = (x + 1)^2 + 4$ **b** $y = (x - 3)^2$ **c** $y = x^2 - 5$

d $y = x^2 + 1$ **e** $y = (x - 3)^2 + 1$ **f** $y = (x - 2)^2 - 1$

g $y = (x + 2)^2 - 4$ **h** $y = (x + 1)^2 - 9$ **i** $y = (x + 1)^2 - 3$

j $y = (x + 4)^2$ **k** $y = \left(x - \frac{3}{2}\right)^2 - \frac{25}{4}$ **l** $y = \left(x + \frac{7}{3}\right)^2 - \frac{16}{9}$

Example 19c

5 Sketch the following parabolas, labelling the turning points and any intercepts.

a $y = x^2 + 2x + 6$ **b** $y = x^2 - 6x + 10$ **c** $y = x^2 - 4x + 7$

d $y = x^2 + 2x - 3$ **e** $y = x^2 + 4x + 4$ **f** $y = x^2 - 8x - 20$

g $y = x^2 + 2x - 1$ **h** $y = x^2 - 6x - 3$ **i** $y = x^2 + 10x + 20$

j $y = x^2 + 5x - 6$ **k** $y = x^2 - 3x - 10$ **l** $y = x^2 - x + 1$

m $y = x^2 + 3x + 4$ **n** $y = x^2 - 5x + 3$ **o** $y = x^2 + 7x + 2$

Review exercise

1 Solve the equations.

a $(x - 5)(x + 3) = 0$ **b** $x(x - 5) = 0$ **c** $x(x + 7) = 0$

d $(2x - 8)(x - 8) = 0$ **e** $(3x + 8)(x + 13) = 0$ **f** $(7 - 2x)(11x - 7) = 0$

2 Solve the equations.

a $x^2 - 3x + 2 = 0$ **b** $x^2 - 7x + 12 = 0$ **c** $x^2 - 11x + 10 = 0$

d $x^2 + 29x - 30 = 0$ **e** $x^2 - 5x - 14 = 0$ **f** $x^2 - x - 90 = 0$

g $x^2 - 5x - 24 = 0$ **h** $x^2 - 11x + 18 = 0$ **i** $x^2 - x - 12 = 0$

j $x^2 - 11x + 28 = 0$ **k** $x^2 + 9x - 10 = 0$ **l** $x^2 + x - 110 = 0$

3 Solve the equations.

a $x^2 + 18x + 81 = 0$ **b** $9x^2 - 16 = 0$ **c** $4x - x^2 = 0$

d $3x^2 - 12x - 36 = 0$ **e** $x^2 - 8x + 16 = 0$ **f** $9f^2 - 36f + 11 = 0$

g $12y^2 + 21 = -32y$ **h** $7x^2 = 28$ **i** $x^2 - 64 = 0$

4 Solve the equations.

a $5d^2 - 10 = 0$ **b** $\frac{2y^2}{3} - 5 = 0$ **c** $\frac{3(x - 10)^2}{5} - 12 = 0$

d $y^2 - 8y + 3 = 0$ **e** $1 = m^2 - m$ **f** $3n + 3 = n^2$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 Plot each graph for the x-values specified.

a $y = x^2 - 2x - 3, \ -2 \le x \le 4$

b $y = x^2 + 2x - 3, \ -4 \le x \le 2$

c $y = x^2 - 3, \ -2 \le x \le 2$

d $y = x^2 + 2x - 8, \ -5 \le x \le 3$

6 Solve the equations.

a $x^2 - \dfrac{8x}{3} = 1$

b $x + \dfrac{8x}{3} - \dfrac{13}{6} = 0$

c $\dfrac{7}{3x - 4} - \dfrac{2}{x + 2} = 1$

7 Solve the equations by completing the square.

a $x^2 + 6x - 2 = 0$

b $x^2 - 4x - 4 = 0$

c $x^2 - 10x + 20 = 0$

d $x^2 + 5x + 3 = 0$

e $x^2 - 7x + 5 = 0$

f $x^2 + 3x - 7 = 0$

8 Solve the equations.

a $x^2 + 3x - 40 = 0$

b $x^2 - 3x - 40 = 0$

c $x^2 + 4x - 12 = 0$

d $x^2 + 4x + 3 = 0$

e $x^2 - 12x + 32 = 0$

f $x^2 - 14x + 32 = 0$

g $x^2 + 8x + 15 = 0$

h $x^2 + 8x + 14 = 0$

i $x^2 - 12x - 27 = 0$

j $x^2 - 12x - 20 = 0$

k $x^2 + 7x + 5 = 0$

l $x^2 + 5x - 10 = 0$

9 Solve the equations.

a $\dfrac{3x + 1}{4x + 7} = 1 - \dfrac{6}{x + 7}$

b $\dfrac{4}{x - 1} - \dfrac{5}{x + 2} = \dfrac{3}{x}$

c $\dfrac{x + 4}{x - 4} + \dfrac{x - 3}{x + 3} = \dfrac{16}{3}$

d $\dfrac{x + 3}{2x - 7} = \dfrac{2x - 1}{x - 3}$

10 Sketch the graphs, labelling all key features.

a $y = x^2 - 3$

b $y = (x + 1)^2$

c $y = (x - 1)^2 - 9$

d $y = x^2 - 2x + 4$

e $y = x^2 - 4x + 3$

f $y = x^2 + 6x + 4$

11 Find two numbers, the sum of whose squares is 74 and whose sum is 12.

12 The perimeter of a rectangular field is 500 m and its area is 14 400 m^2. Find the lengths of the sides.

13 The base and height of a triangle are $x + 3$ and $2x - 5$. If the area of the triangle is 20, find x.

14 Two positive numbers differ by 7 and the sum of their squares is 169. Find the numbers.

15 A rectangular field, 70 m long and 50 m wide, has a path of uniform width around it. If the area of the path is 1024 m^2, find the width of the path.

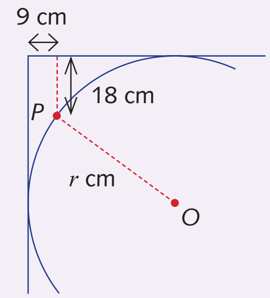
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 Solve the following:

a $\dfrac{x-\sqrt{x+1}}{x+\sqrt{x+1}}=\dfrac{11}{5}$ **b** $\dfrac{1}{2x-1}=\dfrac{2x+1}{2x^2+5x-3}$

2 A table with a circular top is placed in the corner of a rectangular room so that it touches the two walls. A point, P, on the edge of the table, as shown in the opposite diagram, is 18 cm from one wall and 9 cm from the other wall.



a If the radius of the table top is r cm, show that r satisfies the equation $r^2-54r+405=0$.

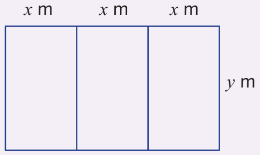
b Find the radius of the table.

c How far is the centre of the table from the corner of the room?

3 While wrapping his son's Christmas present, which is in the shape of a rectangular box, a parent notices that if the length of the box was increased by 2 cm, the width increased by 3 cm and the height increased by 4 cm, the box would become a cube and its volume would be increased by 1207 cm^3.

Find the length of the edge of the resulting cube and the dimensions of the box.

4 A gardener decides to subdivide a rectangular garden bed of area 30 m^2 into three equal sections.

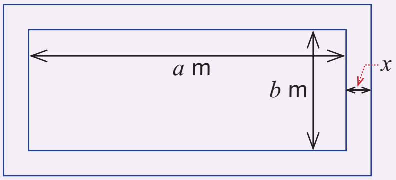


He places edging along the outside of the garden bed and as dividers between each section. It takes 32 m of edging. Each section of garden has length y metres (which is also the length of each divider) and width x metres.

a Express the area of the entire garden bed in terms of x.

b Find the dimensions of the garden bed.

5 A rectangular lawn a metres long and b metres wide has a path of uniform width x metres around it.



a Find the area of the path in terms of a, b and x.

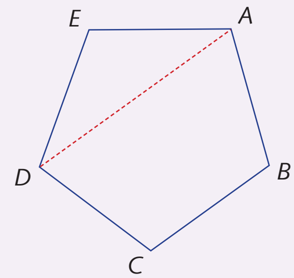
b Let $a=28$ and $b=50$.

i Find the area of the path in terms of x.

ii If the area of the path is 160 m^2, find the value of x.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 A diagonal of a polygon is a line segment joining a vertex to a non-adjacent vertex. In the diagram opposite, AD is a diagonal of the polygon $ABCDE$.



a How many diagonals has:

i a quadrilateral? **ii** a pentagon? **iii** a hexagon?

iv a decagon? **v** a 20-sided polygon?

b Explain why the number of diagonals in an n-sided polygon is $\dfrac{n(n-3)}{2}$.

c A particular polygon has 405 diagonals. How many sides does it have?

d What polygon has between 1900 and 2000 diagonals?

7 At a party, each guest shakes hands with each other guest.

a How many hand shakes are there if there are:

i 2 guests? **ii** 3 guests? **iii** 4 guests? **iv** 10 guests?

b At a party with n guests, explain why there are $\dfrac{n(n-1)}{2}$ hand shakes.

c At Bill's party there were 190 hand shakes. How many people were at the party?

8 A right-angled triangle has a hypotenuse of length 41 cm and an area of 180 cm^2. What are the lengths of the other two sides?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

18

Number and Algebra

Rates and direct proportion

Speed and the rate of flow of water or other liquids are important examples of rates. We encounter many different kinds of rates in everyday life.

Simple rates provide examples of direct proportion. The distance travelled by a body moving at constant speed is directly proportional to the time it travels. Another familiar example from science is that, for a body moving with constant acceleration, the distance travelled is proportional to the square of the time travelling.

There is a huge variety of applications of proportion, and this will become evident through the many examples in this chapter.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

18A Rates

Rates were introduced in *ICE-EM Mathematics Year 8*. They are a measure of how one quantity changes for every unit of another quantity. For example:

50 km/h means that at car travels 50 km in 1 hour.

20 L/min means 20 L of water flows in 1 minute.

30 km/L means a vehicle travels 30 km on 1L (of fuel).

In each of these examples we are describing a constant rate of change or an average rate of change.

Speed

Speed is one of the most familiar rates. It is a measure of how fast something is travelling. Many of the techniques introduced here can be applied in other rate situations.

Constant speed

If the speed of an object does not change over time, we say that the object is travelling with **constant speed**.

Three quantities are associated with questions that involve constant speed. These are distance, time and, of course, constant speed.

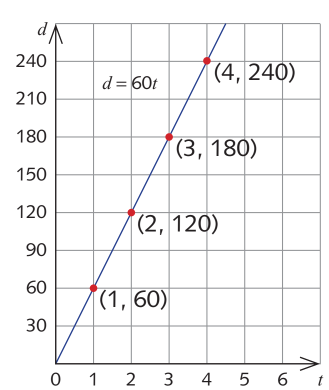
Finding the distance given a fixed speed

A car travels at a constant speed of 60 km/h.

- The car travels 60 km in 1 hour.
- The car travels 120 km in 2 hours.
- The car travels $60t$ km in t hours.

We can complete a table of values for the distance d (in kilometres) travelled by the car after t hours (see next page). The graph can be drawn by first plotting any two of the points.

t (h)	0	1	2	3	4
d (km)	0	60	120	180	240



The graph has a gradient of 60 and a d-axis intercept of 0. The gradient is the speed of the car in kilometres per hour.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 1

Maurice jogs at 6 km/h for 40 minutes.

a What is Maurice's speed in:

i metres per minute (m/min)?

ii metres per second (m/s)?

b How far does Maurice jog? Give your answers in:

i kilometres

ii metres

Solution

a i $6 \text{ km/h} = 6000 \text{ m/h} = \dfrac{6000}{60} = 100 \text{ m/min}$

ii $6 \text{ km/h} = 100 \text{ m/min} = \dfrac{100}{60} = \dfrac{5}{3} \text{ m/s}$

b i $\text{Distance travelled in 40 min} = 6 \times \dfrac{40}{60}$

$= 4 \text{ km}$

ii $\text{Distance travelled in 40 min} = 4 \times 1000$

$= 4000 \text{ m}$

Example 2

A car is travelling at 100 km/h.

a What is the formula for the distance d (in kilometres) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

Solution

a In 1 hour, the car travels 100 km.
In 2 hours, the car travels 200 km.
The formula is $d = 100t$.

b The gradient of the graph of d against t is 100.

Average speed

When we drive a car or ride a bike, it is very rare for our speed to remain the same for a long period of time. Most of the time, especially in the city, we are slowing down or speeding up, so our speed is not constant. If we travel 20 km in 1 hour, we say that our **average speed** is 20 km/h.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 3

A car travels 140 km in 1 hour 45 minutes. What is the average speed of the car?

Solution

$$1 \text{ hour } 45 \text{ minutes} = 1\tfrac{3}{4} \text{ hours} = \frac{7}{4} \text{ hours}$$

$$\begin{aligned}\text{Average speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= 140 \div \frac{7}{4} \\ &= 140 \times \frac{4}{7} \\ &= 80 \text{ km/h}\end{aligned}$$

Constant rate

Every question involving a constant rate gives rise to a straight-line graph. The gradient of the line is the constant rate.

Example 4

A cylindrical tank can hold a maximum of 40 L of water. It has 10 L of water in it to start with. Water is flowing slowly in at a rate of 5 L per minute.

a Prepare a table of values showing how much water is in the container at 1-minute intervals from 0 up to 6 minutes.

b Plot the graph of the volume V (in litres) of water in the tank against time t (in minutes) since the start.

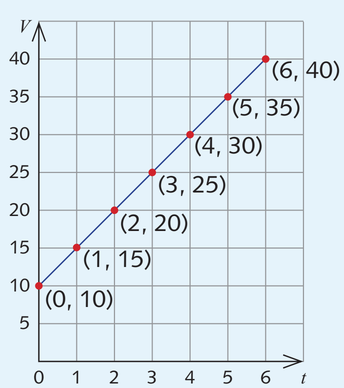
c Give the formula for V in terms of t.

Solution

a

t (min)	0	1	2	3	4	5	6
V (L)	10	15	20	25	30	35	40

b



c From the graph, the gradient is 5 and the V-axis intercept is 10 L. The formula is $V = 5t + 10$, where t takes values from 0 to 6 minutes inclusively.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 5

A man is walking home at 6 km/h. He starts at a point 18 km from his home. Draw a graph representing his trip home. State the gradient and vertical axis intercept, and give a formula that describes the trip.

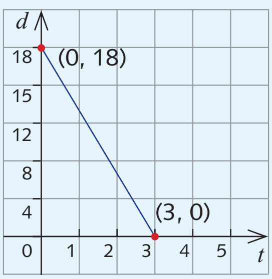
Solution

Let d km be the distance from home after travelling for t hours.

When $t = 0,\ d = 18$.

When $t = 3,\ d = 0$.

The graph can be drawn now that we have two points.



The d-axis intercept is 18.

The gradient is –6.

Thus, the formula is $d = -6t + 18,\ 0 \le t \le 3$.

Exercise 18A

1 Doug walks at 5 km/h for 45 minutes. How far does he walk? Give your answer in metres.

2 Tranh runs at 8 m/s for 10.5 seconds. How far does he run?

Example 1

3 **a** Convert 50 km/h into metres per second.

b Convert 10 m/s into kilometres per hour.

c Convert 9.5 m/s into kilometres per hour.

Example 3

4 **a** A plane travels 800 km in 1 hour 15 minutes. What is the average speed of the plane?

b A car travels 84 km in 50 minutes. What is the average speed of the car? Give your answer in kilometres per hour.

c Yusef walks the 2.1 km to the beach in 22 minutes. What is Yusef’s average speed? Give your answer in metres per second.

Example 2

5 A car is travelling at 80 km/h.

a What is the formula for the distance d (in kilometres) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 4

6 Maria decides to drive to the next town, which is 100 km away. She drives at 80 km/h.

a What is Maria's speed in kilometres per minute?

b Let t denote the number of minutes that have elapsed since Maria set out. Prepare a table of values showing how far (d km) she is from her starting point at 15-minute intervals.

c Plot the graph of d against t.

d Give the formula for d in terms of t.

e Use the formula to find how far Maria has driven after 44 minutes.

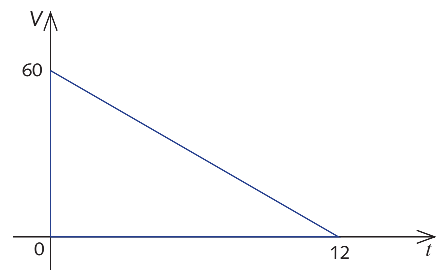
7 Water is flowing from a tank at a constant rate. The graph shows the volume of water (V litres) in the tank after t hours.

a What is the volume of water in the tank initially?

b At what rate is water flowing from the tank?

c Give the formula for V in terms of t.

d How many litres of water will be in the tank after 7 hours?



e i What would be the formula for V in terms of t if there were initially 120L of water in the tank and water flowed out at 6 L per hour?

ii How long would it take the tank to empty under these conditions?

Example 5

8 A man is walking to a town at 6.5 km/h. He starts at a point 20 km from the town.

a Draw a graph of distance travelled against time taken.

b State the gradient and vertical axis intercept.

c Give a formula for the distance travelled d km after t hours.

d If the man starts walking at 3 p.m. and sunset is at 5:45 p.m., will he arrive during daylight?

18B Direct proportion

In the previous section we looked at questions involving constant rates. Constant rates provide examples of direct proportion. We introduce direct proportion with a constant speed situation.

David drives from his home at a constant speed of 100 km/h. The formula for the distance d km travelled in t hours is $d = 100t$. David will go twice as far in twice the time, three times as far in three times the time and so on.

We say that d is **directly proportional** to t and the number 100 is called the **constant of proportionality**.

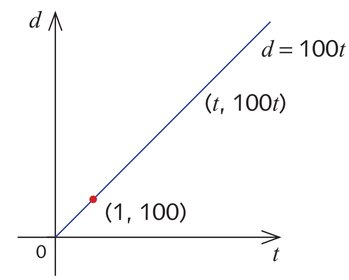
The statement 'd is directly proportional to t' is written as $d \propto t$.

The graph of d against t is a straight line passing through the origin.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

For any pair of values on the graph, (t_1, d_1), $\frac{d_1}{t_1} = 100$.

This is not just the gradient of the graph, $d = 100t$, but also the constant of proportionality in the relationship $d \propto t$.

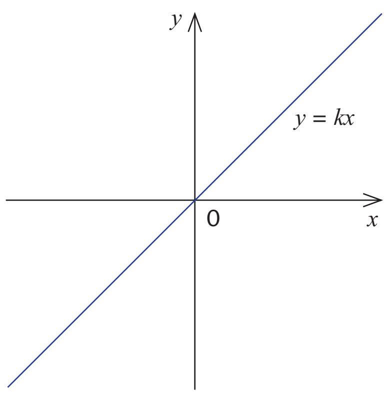


In general:

- The variable y is said to be **directly proportional** to x if $y = kx$ for some non-zero constant k.
- The constant k is called the **constant of proportionality**.
- The statement 'y is directly proportional to x' is written symbolically as

$$y \propto x$$

We know that the graph of $y = kx$ is a straight line passing through the origin. Its gradient k is the constant of proportionality. (The values that x can take are often the positive real numbers, but this is not always the case.)



And since the graph passes through the origin, only one pair of values is needed to find k.

Example 6

The cost $\$C$ of carpet 3 m wide is directly proportional to the length of carpet, ℓ metres. If 15 m of carpet cost \$1650, find:

a the formula for C in terms of ℓ

b the cost of 22 m of carpet

Solution

a It is given that: $C \propto \ell$

Therefore $C = k\ell$, for some constant k

$C = 1650$ when $\ell = 15$

so $1650 = k \times 15$

$k = 110$

Thus $C = 110\ell$

b When $\ell = 22$ $\quad C = 110 \times 22$

$= 2420$

22 metres of carpet costs \$2420.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 7

In an electrical wire, the resistance (R ohms) varies directly with the length (L m) of the wire.

a If a wire 6 m long has a resistance of 5 ohms, what would be the resistance in a wire of length 4.5 m?

b How long is a wire for which the resistance is 3.8 ohms?

Solution

First, find the constant of proportionality.

$$R = kL$$

When $L = 6$ and $R = 5$

$$5 = 6k$$

So $k = \frac{5}{6}$

Thus $R = \frac{5L}{6}$

a when $L = 4.5$, $R = \frac{5 \times 4.5}{6}$

$$R = 3.75$$

The resistance of a wire of length 4.5 m is 3.75 ohms.

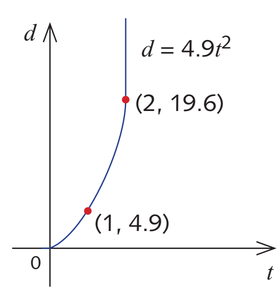
b When $R = 3.8$,

$$3.8 = \frac{5L}{6}$$

$$L = 4.56$$

The length of a wire of resistance 5 ohms is 4.56 m.

Change of variable



A metal ball is dropped from the top of a tall building and the distance it falls is recorded each second.

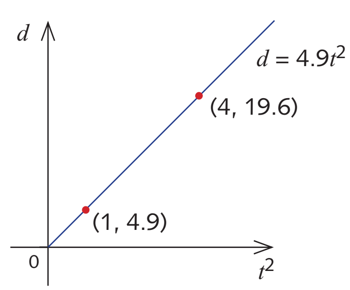
From physics, the formula for d metres, the distance the ball has fallen in t seconds, is given by $d = 4.9t^2$.

In this case, we say that d is directly proportional to the square of t.

We plot the graph of d against t. Note that, since t is positive, the graph is half a parabola.

We can also draw the graph of d against t^2.

t	0	1	2	3
t^2	0	1	4	9
d	0	4.9	19.6	44.1



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

This is a straight line passing through the origin. The gradient of this line is 4.9.

d is directly proportional to t^2, which is written as $d \propto t^2$.

This means that for any two values t_1 and t_2 with corresponding values d_1 and d_2:

$$\frac{d_1}{t_1^2} = \frac{d_2}{t_2^2} = 4.9$$

So once again the gradient of the line is the constant of proportionality.

Example 8

From physics, the energy E microjoules (abbreviated as E mJ) of a body in motion is directly proportional to the square of its speed, v m/s. If a body travelling at a speed of 10 m/s has energy 400 mJ, find:

a the constant of proportionality

b the formula for E in terms of v

c the energy of the body when it travels at a speed of 15 m/s

d the speed if the moving body has energy 500 mJ

Solution

a Energy is directly proportional to the square of the speed:

$$E \propto v^2$$

So $\quad E = kv^2$, for some constant k

Now $\quad E = 400$ when $v = 10$

So $\quad 400 = 100k$

$$k = 4$$

b From part **a**, $E = 4v^2$

c When $v = 15$

$$\begin{aligned} E &= 4 \times 15^2 \\ &= 900 \end{aligned}$$

Therefore the body travelling at speed 15 m/s has energy of 900 mJ.

d When $E = 500$

$$\begin{aligned} 500 &= 4 \times v^2 \\ v^2 &= 125 \\ v &= \sqrt{125} \quad (v > 0) \\ &= 5\sqrt{5} \\ &\approx 11.18 \text{ m/s (correct to 2 decimal places)} \end{aligned}$$

Therefore the body has energy 500 mJ when travelling at a speed of 11.18 m/s.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 9

The mass w grams of a plastic material required to mould a solid ball is directly proportional to the cube of the radius r centimetres of the ball. If 40 g of plastic is needed to make a ball of radius 2.5 cm, what size ball can be made from 200 g of the same type of plastic?

Solution

$$w \propto r^3$$
$$w = kr^3$$
$$w = 40 \text{ when } r = 2.5$$

Thus $40 = k \times (2.5)^3$

$$k = 2.56$$

So the formula is $w = 2.56r^3$

When $w = 200$, $200 = 2.56r^3$

$$r^3 = 78.125$$
$$r = \sqrt[3]{78.125}$$
$$r \approx 4.27 \qquad \text{(correct to 2 decimal places)}$$

Increase and decrease

If one quantity is proportional to another, we can investigate what happens to one of the quantities when the other is changed.

Suppose that $a \propto b$. Then $a = kb$ for a positive constant k.

If the value of b is doubled, the value of a is doubled. For example, if $b = 1$, then $a = k$. So $b = 2$ gives $a = 2k$. Similarly, if the value of b is tripled, the value of a is tripled.

Example 10

Given that $a \propto b^3$, what is the change in a when b is:

a doubled **b** halved?

Solution

Since $a \propto b^3$, $a = kb^3$ for some positive constant k.

a To see the effect of doubling b, choose $b = 1$.

(Any value can be chosen, but $b = 1$ is the easiest to deal with.)

When $b = 1$, $a = k$

When $b = 2$, $a = 8k$

When b is doubled, a is multiplied by 8.

b When $b = 1$, $a = k$

When $b = \frac{1}{2}$, $a = \frac{k}{8}$

When b is halved, a is divided by 8.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 11

Given that $y \propto \sqrt{x}$, what is the percentage change in:

a y when x is increased by 20%

b x when y is decreased by 30%?

Solution

Since $y \propto \sqrt{x}$, $y = k\sqrt{x}$

a When $x = 1, y = k$

If x is increased by 20%, $x = 1.2$

$$y = k\sqrt{1.2}$$
$$\approx 1.095k$$

y is approximately 109.5% of its previous value.

So y has increased by approximately 9.5%.

b If $y = k\sqrt{x}, y^2 = k^2x$ and $x = \frac{y^2}{k^2}$ (making x the subject)

When $y = 1$, $x = \frac{1}{k^2}$

If y is decreased by 30%, $y = 0.7$ and $x = \frac{0.49}{k^2}$

x is 49% of its previous value.

So x has decreased by 51%.

Direct proportion

- y is **directly proportional** to x if there is a positive constant k such that $y = kx$.
- The symbol used for 'is proportional to' is $\propto$. We write $y \propto x$.
- The constant k is called the **constant of proportionality**.
- If y is directly proportional to x, the graph of y against x is a straight line through the origin. The gradient of the line is the constant of proportionality.

Exercise 18B

All variables take positive values only.

Example 6

1 **a** Given that $a \propto b$ and $b = 0.5$ when $a = 1$, find the formula for a in terms of b.

b Given that $m \propto n$ and $m = 9.6$ when $n = 3$, find the formula for m in terms of n.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 Consider the following table of values.

x	0	1	2	3	4	5
y	0	2	8	18	32	50

a Set up a new table of values for y and x^2.

b Plot the graph of y against x^2. What type of graph do you obtain?

c Find the gradient of the graph of y against x^2.

d Assuming that there is a simple relationship between the two variables, find a formula for y in terms of x.

3 Consider the following table of values.

p	0	1	4	9	16
q	0	3	6	9	12
$\sqrt{p}$					

a Plot the graph of q against p.

b Complete the table of values and calculate $\dfrac{q}{\sqrt{p}}$ for each pair $(q, \sqrt{p})$.

c Assuming that there is a simple relationship between the two variables, find a formula for q in terms of p.

4 Write each of the following in symbols.

a The distance d kilometres travelled by a motorist is directly proportional to t hours of travel.

b The volume V of a sphere is directly proportional to the cube of its radius r.

c The distance d kilometres to the visible horizon is directly proportional to the square root of the height h metres above sea level.

5 Write each of the following in words.

a $P \propto Q$ **b** $\ell \propto m^2$ **c** $a^2 \propto \sqrt{b}$ **d** $p^3 \propto \ell^2$

Example 7

6 **a** Given that $p \propto q$ and $p = 9$ when $q = 1.5$, find the formula for p in terms of q and the exact value of:

i p when $q = 4$ **ii** q when $p = 27$

b Given that $m \propto n^2$ and $m = 10$ when $n = 2$, find the formula for m in terms of n and the exact value of:

i m when $n = 5$ **ii** n when $m = 12$

7 In each of the following, find the formula connecting the pronumerals.

a $R \propto s$ and $s = 7$ when $R = 14$.

b a is directly proportional to the square root of b and $a = 3$ when $b = 4$.

c V is directly proportional to r^3 and $V = 216$ when $r = 3$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 In each of the following tables $y \propto x$. Find the constant of proportionality in each case and complete the table.

a

x	0	1	2	3
y	0	7		

b

x	2	8	12	18
y	1			

c

x		3	6	15
y	24		72	

d

x	2		6	15
y		9.5	19	

9 On a particular road map, a distance of 0.5 cm on the map represents an actual distance of 8 km. What actual distance would a distance of 6.5 cm on the map represent?

10 The estimated cost $\$C$ of building a brick veneer house on a concrete slab is directly proportional to the area A of floor space in square metres. If it costs \$80 000 for 150 m^2, how much floor space could you expect for \$126 400?

Example 8

11 The mass m kilograms of a steel beam of uniform cross-section is directly proportional to its length ℓ metres. If a 6 m section of the beam has a mass of 400 kg, what will be the mass, to the nearest kilogram, of a section 5 m long?

Example 9

12 The power p kilowatts needed to run a boat varies as the cube of its speeds metres per second. If 400 kW will run a boat at 3 m/s, what power, to the nearest kilowatt, is needed to run the same boat at 5 m/s?

Example 10

13 If air resistance is neglected, the distance d metres that an object falls from rest is directly proportional to the square of the time t seconds of the fall. An object falls 9.6 m in 1.4 seconds. How far will the object fall in 2.8 seconds?

14 Given that $y \propto x^2$, what is the effect on y when x is:

a doubled **b** multiplied by 4 **c** divided by 5?

15 The surface area of a sphere, A cm^2, is directly proportional to the square of the radius, r centimetres. What is the effect on:

a the surface area when the radius is doubled

b the radius when the surface area is doubled?

16 Given that $m \propto n^4$, what is the effect on:

a m when n is doubled **b** m when n is halved

c n when m is multiplied by 16 **d** n when m is divided by 4?

Example 11

17 Given that $a \propto \sqrt{b}$, what is the effect, to 2 decimal places, on a when b is:

a increased by 21% **b** decreased by 12%?

18 Given that $p \propto \sqrt[3]{q}$, what is the effect on:

a p when q is increased by 10% **b** p when q is decreased by 10%

c q when p is increased by 20% **d** q when p is decreased by 20%?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Review exercise

1 Andrew walks at 5 km/h for 1 hour and 45 minutes. How far does he walk? Give your answer in metres.

2 Lisbeth runs at 7.5 m/s for 12 seconds. How far does she run?

3 **a** Convert 80 km/h into metres per second.

b Convert 25 m/s into kilometres per hour.

4 **a** A plane travels 1000 km in 1 hour 20 minutes. What is the average speed of the plane?

b A car travels 125 km in 1 hour 20 minutes. What is the average speed of the car? Give your answer in kilometres per hour.

5 A car is travelling at 95 km/h.

a What is the formula for the distance d (in kilometres) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

6 Write each of the following in words.

a $x \propto y$ **b** $p \propto n^2$ **c** $a \propto \sqrt{b}$ **d** $p \propto q^3$

7 **a** Given that $p \propto q$ and $p = 12$ when $q = 1.5$, find the exact value of:

i p when $q = 6$ **ii** q when $p = 81$

b Given that $a \propto b^2$ and $a = 20$ when $b = 4$, find the formula for a in terms of b and find the exact value of:

i a when $b = 5$ **ii** a when $b = 12$

8 In each of the following tables $y \propto x$. Find the constant of proportionality in each case and complete the table.

a

x	0	1	2	3
y	0	12		

b

x	2	8	12	18
y	3			

9 Given that $y \propto x^3$, what is the effect on y when x is:

a doubled? **b** multiplied by 3? **c** divided by 4?

10 Given that $a \propto b^2$, what is the effect on a when b is:

a increased by 5%? **b** decreased by 8%?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Challenge exercise

1 If $a \propto c$ and $b \propto c$, prove that $a + b$, $a - b$ and $\sqrt{ab}$ are directly proportional to c.

2 It is known that $a \propto x$, $b \propto \frac{1}{x^2}$ and $y = a + b$. If $y = 30$ when $x = 2$ or $x = 3$, find an expression for y in terms of x.

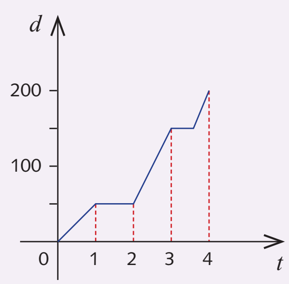
3 If x and y are positive numbers $x^2 + y^2$ varies directly as $x + y$ and $y = 2$ when $x = 2$, find the value of y when $x = \frac{4}{5}$.

4 For stones of the same quality, the value of a diamond is proportional to the square of its weight. Find the loss incurred by cutting a diamond worth $\$C$ into two pieces whose weights are in the ratio $a : b$.

5 If $a + b \propto a - b$, prove that $a^2 + b^2 \propto ab$.

6 One car travelling at 80 km/h leaves Melbourne at 8 a.m. It is followed at 10 a.m. by another car travelling on the same road at 110 km/h. At what time will the second car overtake the first?

7 A salesman travelled from town A to town B, which is a distance of 200 km. The graph shows his distance (d kilometres) from town A, t hours after noon.



From the graph, find:

a the distance travelled:

i in the first hour

ii in the third hour

b the speed at which the salesman travelled during:

i the first hour

ii the third hour

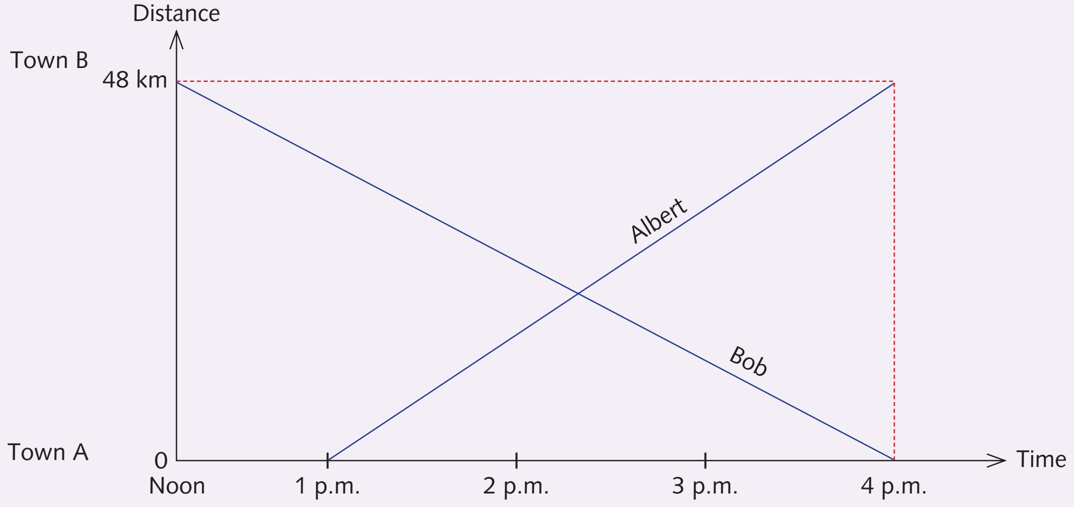
c how far from town A the salesman was after 2 hours of travelling

d at what time the salesman was first 50 km from town B

8 Two aeroplanes pass each other in flight while travelling in opposite directions. Each of the planes continues on its flight for 45 minutes, after which the planes are 840 km apart. The speed of the first aeroplane is $\frac{3}{4}$ the speed of the other aeroplane. Calculate the average speed of each aeroplane.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

9 The following graph shows the distance from town A against time for two cyclists, Albert and Bob. Town B is 48 km from town A.



a How far did Albert travel?

b What was Albert's speed?

c What was Bob's speed?

d After how many hours do the two cyclists pass each other?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

19

Statistics and Probability

Statistics

Statistics is concerned with the collection and analysis of **data**. We encounter statistics in many aspects of everyday life. It would be hard to find an area of business or industry that does not use some form of statistics in its decision-making processes.

Questions such as 'Is the Government's economic policy having an effect on youth unemployment?', 'Is the State Road Authority's advertising campaign having an effect on the road toll?', 'How does Australia's standard of living compare with that of other countries?' or 'Is there a relationship between marks in English and marks in Mathematics?' all require data to be collected and analysed before they can be answered in an intelligent and justifiable way.

The **data** has to be **collected**, **recorded** and **represented** in a way that is appropriate to the questions we wish to answer. We then **analyse** the data and draw conclusions. This process is discussed at length in the final section of this chapter.

In this chapter we will deal only with small data sets. This will enable us to do the statistical calculations and graphical representations by hand to allow us to understand the ideas clearly. It is important to be aware that when larger data sets are involved, statistical software is often used.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

19A Stem-and-leaf plots and back-to-back stem-and-leaf plots

In this chapter we provide a summary of all of the representations that you have met so far in your study of statistics, and then introduce other representations.

We first revise stem-and-leaf plots.

Stem-and-leaf plots

Numerical data (quantitative data) consists of values in which there is a definite numerical order. For example, scores in a test or heights of students in a class.

A **stem-and-leaf plot** can be used to represent numerical data.

Stem-and-leaf plots

- A stem-and-leaf plot represents the values in a data set in the form of a stem and a leaf.
- The **stem** is the first digit of a two-digit number, or the first two digits of a three-digit number, and so on.
- The **leaf** is usually the last digit of the value. For example:
  - For the number 47, 4 is the stem and 7 is the leaf.
  - For the number 251, 25 is the stem and 1 is the leaf.
  - For the number 11.6, 11 is the stem and 6 the leaf.

Stem-and-leaf plots are useful for displaying the shape of the data and giving the reader a quick overview. They retain most of the raw numerical data. They are also useful for highlighting outliers and finding the mode. However, stem-and-leaf plots are only useful for data sets between 15 and values.

Example 1

The marks out of 50 obtained by 16 students in a Mathematics test are:

43 24 29 19 11 14 25 17 32 27 29 7 14 19 39 49

Represent this information on a stem-and-leaf plot.

Solution

We use the first digit of each mark as the stem, writing 7 as 07.

0	7
1	1 4 4 7 9 9
2	4 5 7 9 9
3	2 9
3	3 9

3 | 2 represents a mark of 32

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 2

The approximate heights of 20 students, in centimetres, are given below.

164	158	152	167	146	149	167	171	181	154
167	158	164	172	176	180	178	165	159	153

Represent this information on a stem-and-leaf plot and give the frequency of each group.

Solution

We use the first two digits of each height as the stem.

Stem	Leaf	Frequency
14	6 9	2
15	2 3 4 8 8 9	6
16	4 4 5 7 7 7	6
17	1 2 6 8	4
18	0 1	2

17 | 2 means 172

Back-to-back stem-and-leaf plots

Back-to-back stem-and-leaf plots are used to compare two similar sets of numerical data.

Example 3

A class of 25 students sit for two Mathematics tests, each out of 100. Their results are recorded in the following back-to-back stem-and-leaf plot.

Test 1		Test 2
5 5	4	
9 6 6 4	5	5 6 6
9 8 8 7 6 5 5 4 3 3	6	5 7
8 8 5 2 2 2 1	7	5 9
1	8	0 0 4 4 5 5 5 6 7 9 9
0	9	0 0 1 3 5 7 7

a How many students scored 70 or more for each test?

b How many students scored less than 50 for each test?

Solution

a Nine students scored 70 or more in test 1 and 20 students scored more than 70 in test 2.

b Two students scored less than 50 in test 1 and no student scored less than 50 in test 2.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 4

The back-to-back stem-and-leaf plot shows the city and country highway fuel consumption of 16 cars. The fuel consumption is measured in litres per 100 km.

City		Country
	5	8 9
	6	5 5 6 7
8 7 7 5	7	4 5 5 6 7
7 6 3	8	0 0 6 8 8
	9	
5 5 4 4	10	
5 3	11	
8 7 6	12	

5 |7| 4 means 7.5 L per 100 km in the city and 7.4 L per 100 km in the country.

a Find the maximum and minimum fuel consumption (in litres per 100 km) for:

i country driving **ii** city driving

b Find the percentage of cars tested with fuel consumption less than 8 litres per 100 km for:

i country driving **ii** city driving

Solution

a i 8.8 L per 100 km and 5.8 L per 100 km are the maximum and minimum fuel consumptions for country driving respectively.

ii 12.8 L per 100 km and 7.5 L per 100 km are the maximum and minimum fuel consumptions for city driving respectively.

b i $\left(\frac{11}{16}\right) \times 100\% = 68.75\%$

68.75% of cars tested for country driving have a fuel consumption that is less than 8 litres per 100 km.

ii $\left(\frac{4}{16}\right) \times 100\% = 25\%$

25% of cars tested for city driving have a fuel consumption that is less than 8 litres per 100 km.

Exercise 19A

Example 1

1 The marks for a mathematics test done by a particular class of 25 students were:

48 43 29 36 37 21 15 24 35 44 37 35 25
39 28 25 46 37 24 26 42 45 33 47 29

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a Present this information on a stem-and-leaf plot.

b Which 10-mark group contains the most students?

Example 2

2 The approximate weights in kilograms of 30 people are:

85	78	94	86	104	93	76	84	95	91	106	89	94	97	91
82	76	93	84	86	79	96	94	81	77	87	82	96	102	86

a Present this information on a stem-and-leaf plot.

b How many of the 30 people weigh:

i more than 85 kg?

ii less than 90 kg?

iii strictly between 80 and 95 kg?

3 The stem-and-leaf plot below displays the approximate time (in minutes) taken to travel to school for a class of students.

0	8 9
1	2 4 8
2	2 3 3 6
3	1 4 5 5 8
4	2 4 4 6
5	0 2 4

1 | 4 means 14

a How many students are in the class?

b How many students take:

i less than 25 minutes?

ii more than 40 minutes?

iii between 20 and 40 minutes?

c The two students who take the longest and shortest time to get to school arrive at the same time. How much later does one leave home than the other?

4 The stem-and-leaf plot opposite displays the approximate percentage inflation rate for a number of countries, correct to 1 decimal place.

2	8 9
3	1 4 4 7
4	2 4 5 5 7 9
5	0 1 1 4 5 7 9
6	2 3 5 6 6
7	2 4

2 | 4 means 2.8%

a How many countries are included in the sample?

b How many countries in the sample have an inflation rate that is:

i greater than 5.5%? **ii** less than 3.4%?

c If countries with an inflation rate greater than 6.4% are classified as having an unstable economy, what percentage of countries have an unstable economy

Example 3

5 A class of 16 students sits for two Mathematics tests. Their results are recorded in the following back-to-back stem-and-leaf plot.

Test 1		Test 2
9 8 5 5	4	
7 5 3 2	5	8
9 8 8 7 6 6 6 5	6	1
	7	5
	8	3 3 3 4 4 5 5 5 6 7 9
	9	4 4

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

a What percentage of students scored 50 or more for:

i test 1? **ii** test 2?

b What percentage students scored less than 50 for:

i test 1? **ii** test 2?

c What percentage of students scored between 60 and 70 (inclusively) for:

i test 1? **ii** test 2?

Example 4

6 Two walking clubs record the ages of their members, as shown below. They both have a membership of 22 people.

a Draw a back-to-back stem-and-leaf plot to represent their ages.

Club 1: 82, 82, 78, 78, 78, 73, 73, 73, 73, 72, 69, 67, 67, 65, 34, 25, 24, 23, 16, 13, 12, 11

Club 2: 37, 38, 39, 42, 43, 55, 57, 65, 65, 66, 66, 66, 67, 68, 68, 69, 71, 72, 72, 72, 73

b Compare the ages of the members of the two clubs.

7 The following data gives the number of kicks by members of the Yellowlegs and Blackarm teams in their Australian Rules country grand final.

Blackarm: 8, 8, 10, 10, 11, 11, 12, 12, 12, 12, 13, 13, 14, 15, 16, 17, 18, 19, 19, 20, 21, 26

Yellowlegs: 0, 4, 5, 6, 7, 8, 8, 9, 9, 10, 10, 13, 13, 14, 15, 16, 16, 17, 18, 19, 19, 25

a Prepare a back-to-back stem-and-leaf plot, using two rows per stem.

It is started for you here.

Blackarm		Yellowlegs
	0	0 4
	0	5
	1	0 0
6 5	1	
	2	
	2	

b Compare the number of players of both clubs who had:

i fewer than 10 kicks **ii** 15 or more kicks

8 The weights of 40 male and 40 female students are compared in the back-to-back stem-and-leaf plot shown below. The weights are given correct to the nearest kilogram.

Females		Males
9 8	3	
9 8 8 7 7 7 7 6 6	4	
9 8 7 7 7 5 4 1 1 0 0 0	5	2 8
5 4 4 4 0 0 0 0	6	1 1 1 3 3 4 4 5 5 5 7 7 8 9
5 2 1 0 0	7	0 0 1 3 4 4 5 6 6
1 0 0	8	0 0 1 2 3 3 5 5 7 8
	9	0 7
	10	0 0 9

6 | 1 means 61 kg

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a **i** Find the percentage of males who weigh more than 79 kg.

ii Find the percentage of females who weigh less than 50 kg.

b **i** What are the weights of the 20th and 21st heaviest males?

ii What are the weights of the 20th and 21st heaviest females?

c **i** What is the difference in weight between the heaviest and lightest males?

ii What is the difference in weight between the heaviest and lightest females?

9 Collect data in your class comparing differences between boys and girls using back-to-back stem-and-leaf plots. Height, arm length or hours of television watched are just some examples you may chose.

19B Grouped data

Without a lot of practice, little useful information can be gained from looking at unstructured data, such as in Example 5 below.

To understand a data set with a large number of values, it is helpful both to divide the range of values of the data into intervals, called **class intervals** or classes, and to record the **frequency** of each class interval – that is, the number of data items in each class. The resulting summary of the data is called a grouped frequency table or simply a **frequency table**.

Example 5

The marks out of 50 for a mathematics test done by a class of 25 students are:

48	43	29	36	37	21	15	24	35	44	37	35	25
29	39	28	25	46	37	24	26	42	45	33	47	

Present this information in a frequency table, using groupings of 15–19, 20–24, 25–29 and so on, up to 45–49.

Solution

Grouping the data produces the following frequency table.

Mark	Tally	Frequency
15–19	I	1
20–24	III	3
25–29	~~IIII~~ I	6
30–34	I	1
35–39	~~IIII~~ II	7
40–44	IIII	3
45–49	IIII	4

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

Choice of classes

In examples involving grouped data, the following points should be considered:

- Interval sizes should be chosen so that a sensible number of classes results. Between 5 and 10 classes are commonly used.
- Classes should be of equal width, except occasionally for the first or last class. For example, if an exam is marked out of 50, and 10 classes are used, the last class would normally go from 45 to 50.
- It is best to use classes such as 0–4, 5–9, 10–14, … or 0–9, 10–19, 20–29, … or 0–99, 100–199 since they match our decimal number system. Avoid using classes such as 12–16, 17–21, 22–26, even if the smallest value is 12. Apart from being systematic, this allows classes to be easily created from a stem-and-leaf plot.

The frequency table below shows the amount of sodium in 100 gram samples of different foods.

Amount of sodium (mg)	Tally	Frequency
0–49	𝍸 𝍸 III	13
50–99	IIII	4
100–149	𝍸 𝍸 𝍸	15
150–199	𝍸 𝍸 𝍸 𝍸	20
200–249	𝍸 𝍸 𝍸 𝍸 III	23
250–299	𝍸 𝍸 𝍸 IIII	19
300–349	𝍸 I	6

The amount of sodium is a continuous variable, and it can take any value from 0 to 350 for the foods considered. The amounts here have been stated correct to the nearest milligram.

Consider the class of foods that contain 200–249 mg of sodium (in each 100 g sample) of which there are 23.

This means that there are 23 values from 199.5 up to, but not including, 249.5. The real endpoints – 199.5 and 249.5 – are called **class boundaries**.

Exercise 19B

Example 5

1 The number of runs scored in each innings by a batsman throughout a cricket season was:

42	18	5	73	97	61	47	31	8	1
14	26	71	58	27	11	26	51	4	1
15	92	18	37	40	65	72	3	5	18

a Present this information in a frequency table using class intervals

0–9, 10–19, 20–29, ...

b On how many occasions did the batsman score:

i more than 49 runs?

ii fewer than 10 runs?

iii fewer than 80 runs?

iv between 20 and 59 runs?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

2 The approximate times taken to run 100 m by 25 students were as follows:

13.9	11.1	12.6	14.1	13.8	13.2	14.4	12.8	12.1
12.5	11.8	13.4	14.2	12.6	11.9	12.8	14.7	14.2
13.6	13.9	14.5	13.1	12.7	12.9	13.6		

a Present this information in a frequency table using class intervals 11.0–11.4, 11.5–11.9, 12.0–12.4 and so on.

b Present this information in a frequency table using class intervals 11.0–11.9, 12.0–12.9, 13.0–13.9 and so on.

3 The ages of employees in a factory range from 22 to 64.

a Explain how you might break up the age range into five classes.

b Do the same for nine classes.

4 On a particular Saturday, the prices of houses sold in Geelong, Victoria, ranged from \$294 000 to \$618 000. Show how to group the selling prices of the houses into eight classes.

5 The maximum daily temperature in Sydney was recorded each day for a month, correct to 0.1°C. The results are given in the table.

a On how many days was the recorded maximum temperature:

i greater than 25.7°C?

ii less than 24.6°C?

iii between 24.3°C and 25.7°C (inclusive)?

iv between 24.9°C and 26.0°C (inclusive)?

Maximum temperature (°C)	Frequency
24.0–24.2	8
24.3–24.5	6
24.6–24.8	4
24.9–25.1	4
25.2–25.4	3
25.5–25.7	3
25.8–26.0	2

b Why is it impossible to tell on how many days the maximum temperature was above 25°C?

6 The following set of raw data shows the lengths, recorded to the nearest millimetre, of 40 leaves taken from a particular tree.

42	56	27	52	60	47	49	51	32	30
54	33	54	43	49	46	48	41	43	61
51	40	45	50	45	45	42	53	42	58
33	55	46	39	37	39	34	40	48	38

a Construct a frequency table with classes 25–29, 30–34 and so on.

b In which class is a leaf measured as 29.7 mm included?

c In which class is a leaf measured as 34.3 mm included?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

19C Histograms and summary of representations

Histograms

Numerical data can be displayed in a **histogram**. Data grouped into classes is often displayed this way. A histogram is a type of graph in which the frequencies of the data are displayed by touching columns. They are particularly useful with data sets containing a large number of values.

Example 6

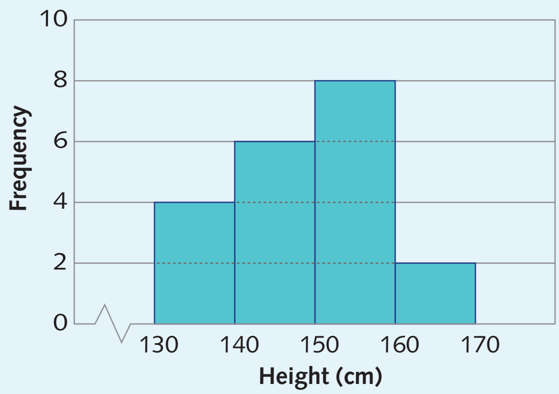
The heights of 20 Year 9 students were measured and the results are shown in the frequency table opposite. Represent this information in a histogram. The heights were measured correct to the nearest centimetre.

Height (in cm)	Frequency
130 – 139	4
140 – 149	6
150 – 159	8
160 – 169	2

Solution

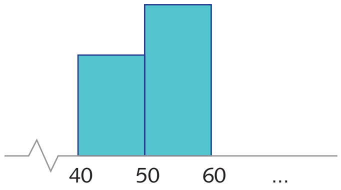
In this graph, the left-most column represents the class 130–139; the second, 140–149; and so on. All our examples follow a similar convention.

From the discussion in the previous section, this means that a height of 139.4 cm would be included in the 130–139 class and that a height of 139.8 cm would be included in the 140–149 class.



Note:

- The values on the horizontal axis in Example 6 indicate, for example, that the first class consists of heights, measured to the nearest centimetre, at least 130 cm and less than 140 cm.
- If the variable is integer-valued, such as a mark, then in the diagram shown the first class is 40–49, the second class is 50–59 and so on.



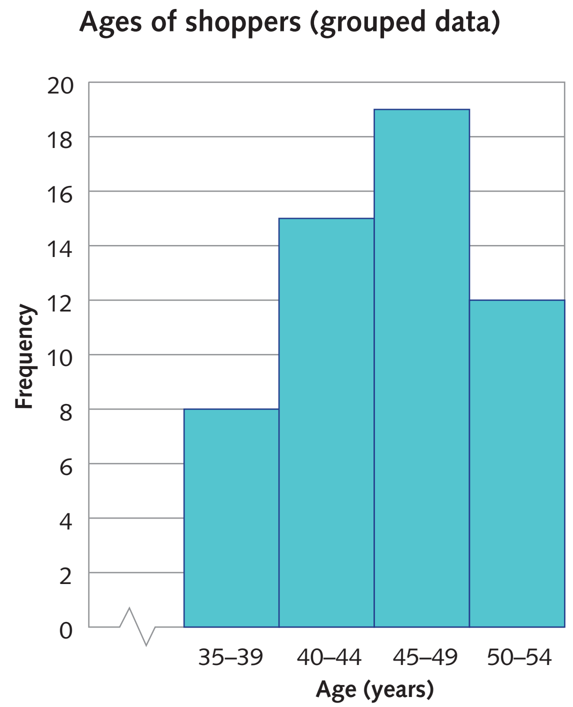
- When working with histograms, another name for class is **bin**.

A number of shoppers were asked to record their ages as they left a department store. The data was recorded as follows.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Age (years)	35	36	37	38	39	40	41	42	43	44
Tally	I	~~IIII~~ I		I		~~IIII~~ II	II	II		IIII
Frequency	1	6	0	1	0	7	2	2	0	4
Age (years)	45	46	47	48	49	50	51	52	53	54
Tally	II	III	~~IIII~~	~~IIII~~	IIII	IIII	III	II	I	II
Frequency	2	3	5	5	4	4	3	2	1	2

Some aspects of the information become clearer after grouping. The final histogram is shown below.



Now we can read off facts such as:

- The 45–49 age range contains the most shoppers.
- Nineteen shoppers fall in the 45–49 age range.

Classes are usually chosen so that the entire data set is broken up into at most 10 classes. The advantage of grouping data is that trends are often easier to recognise.

A disadvantage of grouping data into classes is that we cannot see the individual values that were recorded.

Types of representations

So far we have been considering only numerical data, such as age, height or test scores. However, we are often also interested in data concerning the preferences or attitudes in a community, or the number of items with certain attributes. Such data is called **categorical**.

The following representations have been introduced in this book or previous books.

The table on the next page provides some guidelines for selecting which representation to use.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Type of data	Representation	Qualifications on use
Categorical	Pictograph	Usually fewer categories
	Column graph	Not too many categories
Numerical (or quantitative)	Dot plots	Best for small data sets
	Histogram	Best for medium to large data sets
	Stem-and-leaf plot	Best for small to medium data sets

Pictographs, column graphs and dot plots were introduced in earlier books. We provide an example of each.

Pictograph

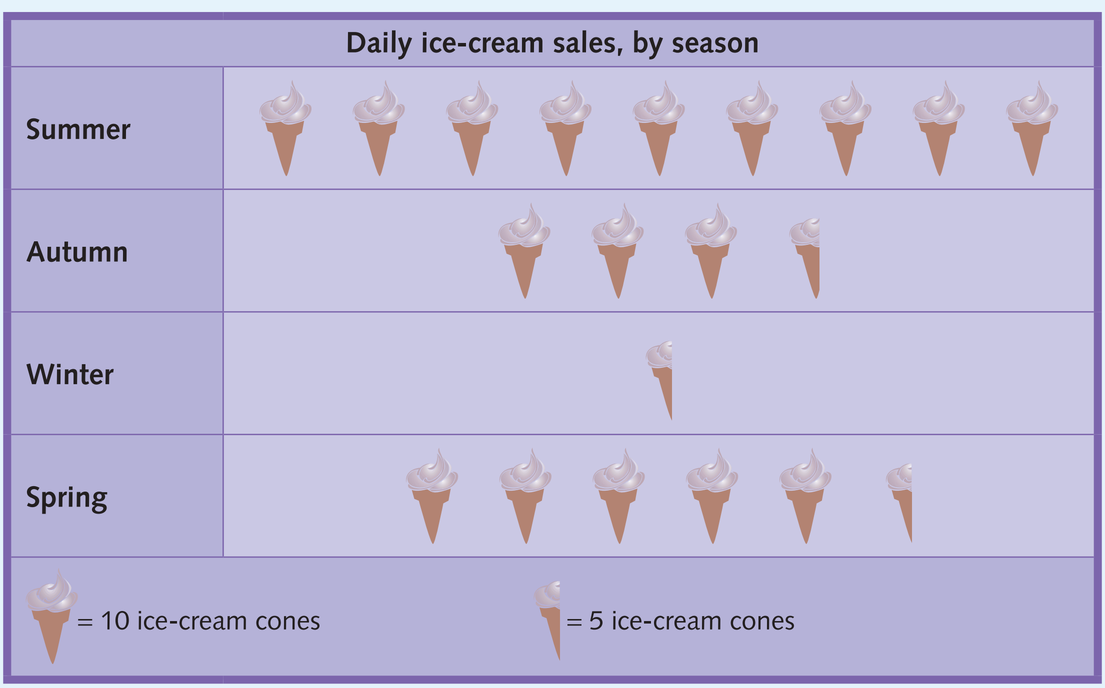
Example 7

The table below shows the average daily sales of ice-cream cones from a kiosk in each season over a 12-month period.

Season	Summer	Autumn	Winter	Spring
Ice-cream sales	90	35	5	55

Use a pictograph to represent these data.

Solution



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Column graph

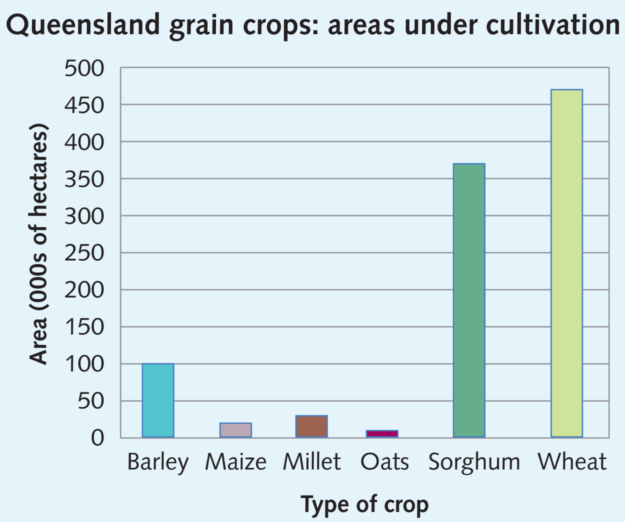
Example 8

The table below gives the approximate areas under cultivation, in thousands of hectares, for different grain crops in a region of Queensland.

Grain crop	Area under cultivation (in thousands of hectares)
Barley	100
Maize	20
Millet	30
Oats	10
Sorghum	370
Wheat	470

Draw a column graph representing the data.

Solution



Dot plot

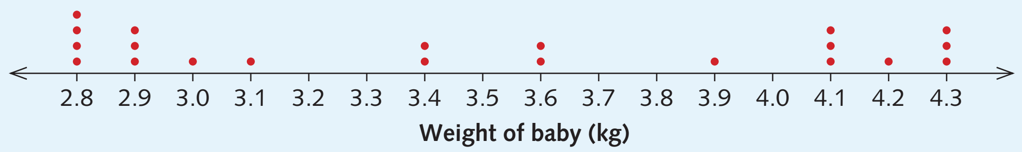
Example 9

Twenty-one babies were born at a hospital on one weekend last year. Their approximate birth weights, in kilograms, are given here.

2.8 2.8 2.8 2.8 2.9 2.9 2.9 3.0 3.1 3.4 3.4
3.6 3.6 3.9 4.1 4.1 4.1 4.2 4.3 4.3 4.3

Draw a dot plot for the data.

Solution



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 19C

The first five questions are review questions.

Example 8

1 In a class of 25 students, 10 have green eyes, 8 have blue eyes, 4 have brown eyes and 3 have grey eyes. Present this information in a column graph.

2 The table opposite lists the percentage of viewers watching each television channel on a particular night. Present this information in a column graph.

Channel	Percentage of viewers
1	26
2	34
3	25
4	15

Example 9

3 A group of 30 people in the building trade were asked how many times in the last week they had visited a particular hardware shop. Their responses were:

0 2 2 1 0 0 3 4 1 1 0 0 0 3 1 1 1 2 3 0 0 1 1 2 2 1 3 0 0 0

Draw a dot plot for the data.

4 A group of students were asked to select their favourite fast food. The results are in the table below.

Food type	Number of students
Hamburgers	8
Chicken	8
Fish and chips	15
Pizza	22

Draw a column graph to illustrate the results.

5 The ages in years of the 15 members of a sporting team are:

23 19 18 19 23 25 22 22 19 22 18 23 24 21

Construct a dot plot.

Questions on histograms follow.

Example 6

6 The frequency table shown opposite gives information regarding the test results of a group of 23 students. Present this information in a histogram.

Score	Frequency
15–19	1
20–24	3
25–29	5
30–34	2
35–39	6
40–44	3
45–49	3

7 In a medical test, a group of students had the distance from hip to heel measured. The measurements were made correct to the nearest centimetre. The results were as follows.

85	86	91	87	77	88	83	86	74	89	85	85	80
94	82	84	89	84	94	84	76	93	86	84	94	84

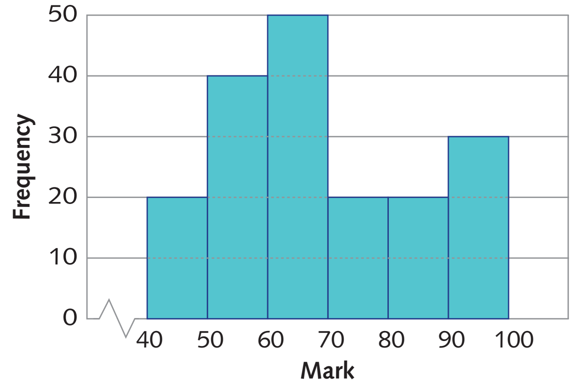
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Present this information in a histogram using the classes:

a 70–79, 80–89, 90–99

b 70–74, 75–79, 80–84, 85–89, 90–94

8 The histogram below gives information about the results of Year 9 students in a history examination.



a How many students sat for the examination?

b If the pass mark was 60, how many students passed?

c What percentage of students obtained 90 or more?

d What percentage of students obtained less than 70?

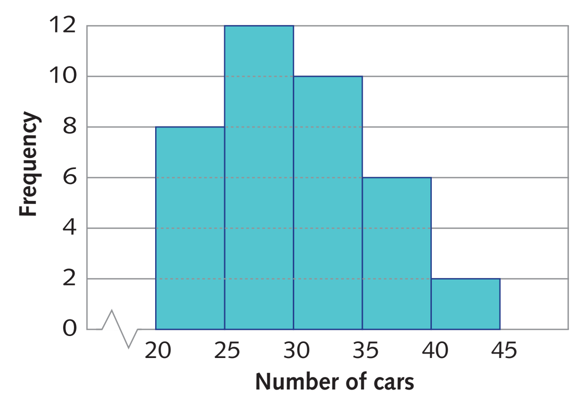
9 **a** Present the information given in the stem-and-leaf plot opposite using a histogram.

b Why is it impossible to present the information in a histogram using a stem-and-leaf plot?

0	2 3 4
1	1 5 7 9 9
2	3 4 6
3	
4	5 7 8 8
5	2 5

2 | 4 means 24

10 A student counted the number of cars that passed through an intersection on each cycle of the traffic lights. The results of her investigation are shown in the histogram opposite.



a For how many cycles did the student record the number of cars passing through the intersection?

b In how many cycles did:

i fewer than 30 cars pass through the intersection

ii at least 35 cars pass through the intersection?

c Why is it impossible to determine from the histogram how many cars in total passed through the intersection during the survey period?

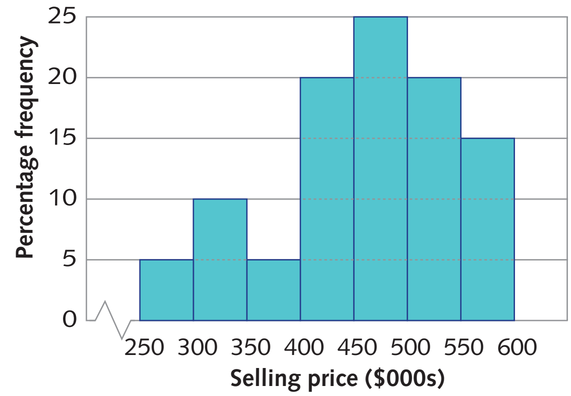
d Determine:

i the minimum number of cars that could have passed through the intersection during the survey period

ii the maximum number of cars that could have passed through the intersection in the survey period

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11 The percentage frequency histogram opposite gives the auction prices of houses sold in Wollongong on a particular day.



a What percentage of houses sold for:

i less than \$500 000

ii at least \$400 000

iii between \$300 000 and \$550 000?

b If 420 houses were sold, how many houses sold for:

i less than \$350 000

ii at least \$300 000

iii between \$500 000 and \$600 000?

12 Thirty calls were made by an employee of a telephone call centre. The lengths, correct to the nearest minute, are summarised in the following table.

Length of call	0–5	6–10	11–15	16–20
Number of calls	15	8	5	2

a If a call went for 5 minutes and 40 seconds, in which class is it recorded?

b Draw a histogram of the data.

Activity

Reaction times (left and right)

Group 1 (possibly 20 students)

1 Stand next to a wall with another student.

2 Hold a ruler against a wall with the 0 cm mark at the bottom.

3 Ask the other student to close their left eye and align their index finger of their right hand with the 0 cm mark. (Their finger is to be 5 cm from the wall.)

4 Explain that you will let go of the ruler without warning, and they must try to pin it against the wall with their index finger of their right hand.

5 Measure and record the distance dropped.

Group 2 (possibly 20 students)

1 Repeat steps 1–5 above, closing the right eye and using the index finger of the left hand.

2 Draw a stem-and-leaf plot diagram for both sets of data and compare.

3 Draw two histograms.

4 Think of other ways of carrying out this activity.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

19D Mean, median, mode and range

Mean

The **mean** of a numerical data set is a measure of its centre. It is calculated by first adding together all the data values and then dividing the sum by the number of data values.

Mean

We use the following formula to find the mean of a set of data:

$$\text{mean} = \frac{\text{sum of values}}{\text{number of values}}$$

The more common name for mean is 'average'.

Example 10

Allen obtained the following marks in 8 tests:

43 35 41 29 33 39 47 42

Calculate the mean.

Solution

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of values}}{\text{number of values}}\\ &= \frac{43 + 35 + 41 + 29 + 33 + 39 + 47 + 42}{8}\\ &= \frac{309}{8}\\ &= 38.625\end{aligned}$$

Note that the mean was not a mark obtained by Allen in any of his tests.

Example 11

The following table gives the number of children in each of 20 families. Calculate the mean number of children per family.

Number of children	0	1	2	3
Frequency	4	5	7	4

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Solution

There are 4 families with 0 children. This gives a total of 0 children.

There are 5 families with 1 child. This gives a total of $1 \times 5 = 5$ children.

Continuing in this way, we have:

$$\begin{aligned}\text{average} &= \frac{0 \times 4 + 1 \times 5 + 2 \times 7 + 3 \times 4}{4 + 5 + 7 + 4}\\ &= \frac{31}{20}\\ &= 1.55\end{aligned}$$

It is obviously impossible for a family to have 1.55 children. In general, the average of a data set is not one of the original values.

Median

We often see the median value used to describe the housing market in a city. The **median** is the middle value when all values are arranged in numerical order. Here are some numbers arranged in numerical order.

$$2 \quad 2 \quad 3 \quad 3 \quad 3 \quad 4 \quad \boxed{5} \quad 11 \quad 13 \quad 18 \quad 18 \quad 19 \quad 21$$

Median

This data set has an odd number of values. The middle value is 5 since it has the same number of values on either side of it. Hence the median of this data set is 5.

Here is another set of numbers arranged in numerical order.

$$1 \quad 3 \quad 4 \quad 4 \quad 5 \quad 7 \mid 9 \quad 11 \quad 13 \quad 13 \quad 19 \quad 21$$

Median

The above data set has an even number of values. The middle values are 7 and 9. We take the average of 7 and 9 to calculate the median.

$$\text{Median} = \frac{7 + 9}{2} = 8$$

Hence the median of this data set is 8. Note that this value does not occur in the data set.

Median

- When the data set has an *odd* number of values and they are arranged in numerical order, the median is the *middle value*.
- When the number of values is *even* and they are arranged in numerical order, the median is the *average of the two middle values*.
- In general, if there are n items in the *ordered* data set, the median lies in the $\left(\frac{n+1}{2}\right)$th position.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 12

Calculate the median of the following data sets.

a 43, 35, 41, 29, 33, 39, 42 **b** 4, 6, 8, 5, 12, 10

Solution

a To locate the median, first put the values in numerical order:

29 33 35 $\boxed{39}$ 41 42 43

Median = 39

b Again, place the values in numerical order.

4 5 6 ⋮ 8 10 12

$$\text{Median} = \frac{6+8}{2} = 7$$

Since the data values need to be written in numerical order to locate the median, a stem-and-leaf plot helps find the median.

Example 13

The stem-and-leaf plot below shows the approximate weights of the students in a class. Determine the median weight of the students.

Stem	Leaf	Frequency
4	4 6 7 7 8	5
5	2 4 5 5 6 $\boxed{8}$ 8 9	8
6	0 2 4 5 7 9	6
7	0 2	2

6 | 4 means 64 kg

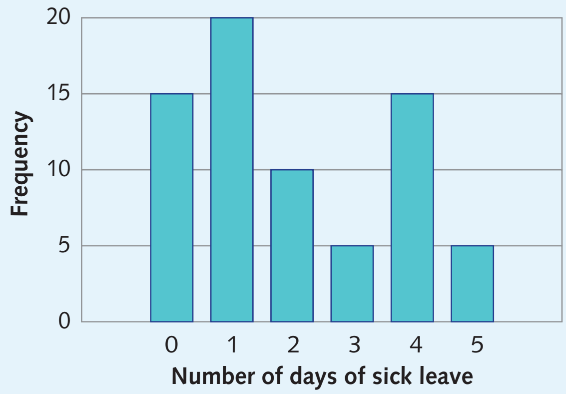
Solution

From the stem-and-leaf plot, 21 weights have been recorded. So the median will be the weight in position 11 (giving 10 weights on either side). This will be the sixth value in the second class. Hence the median weight is 58 kg.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Example 14

The column graph below gives the number of days of sick leave taken by employees at a factory during a particular month. Locate the median.



Solution

From the column graph, there are $15 + 20 + 10 + 5 + 15 + 5 = 70$ data values, so the median will the average of the 35th and 36th positions.

The 35th position is a 1 (there are 15 zeroes and 20 ones) and the 36th position is a 2, so

$$\text{median} = \frac{1+2}{2} = 1.5 \text{ days of sick leave}$$

Mode

One of the questions we often use statistics to answer is 'Which is the most popular?' The most popular or most common value will be the most frequently occurring value in a data set. A value with the highest frequency is called the **mode**. There may be more than one mode.

For example, in a survey of 'favourite sports' the following results were obtained:

swimming, golf, golf, badminton, swimming, cricket, golf, swimming, cricket, golf, cricket, golf, badminton, swimming, swimming, cricket, cricket, golf, swimming, cricket, swimming, swimming

We can arrange the data into a frequency table.

Favourite sport	Tally	Frequency
Golf	~~IIII~~ I	6
Cricket	~~IIII~~ I	6
Swimming	~~IIII~~ III	8
Badminton	II	2

In the survey above, the sport with the highest frequency is swimming. Hence the mode is swimming. It should be noted that in this example of categorical data it is senseless to refer to mean or median.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 15

The number of emails Annie sent each day was recorded for 30 days. The results are shown below. Find the mean, median mode.

Number of emails	Frequency
12	8
13	6
14	4
15	8
16	4

Solution

i Mean $= \dfrac{12 \times 8 + 13 \times 6 + 14 \times 4 + 15 \times 8 + 16 \times 4}{30} = \dfrac{414}{30} = 13.8$

ii The median lies in the $\dfrac{30+1}{2} = 15.5$th position (between 15th and 16th). Median $= 14$

iii Mode $= 12$ and 15 (both have the highest frequency)

There are two modes for this data, so the data are said to be **bimodal**.

Mode

- The **mode** is the value (or values) with the highest frequency.

Range

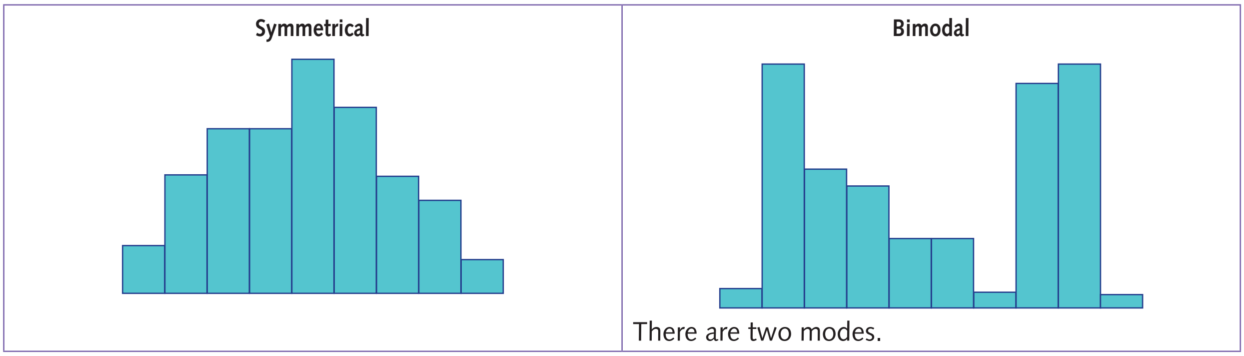
The range is one of the simplest, and easiest to calculate, measures of the spread of a data set.

The **range** of a set of numerical data is the difference between the largest data value and the smallest data value.

For example, the range of data in Example 15 is $16 - 12 = 4$ emails.

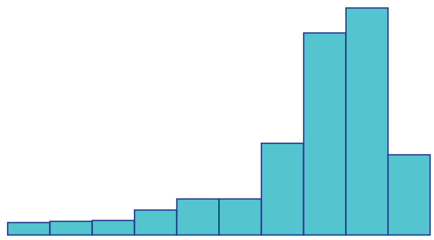
Comparing data displays

Compare the following histograms.



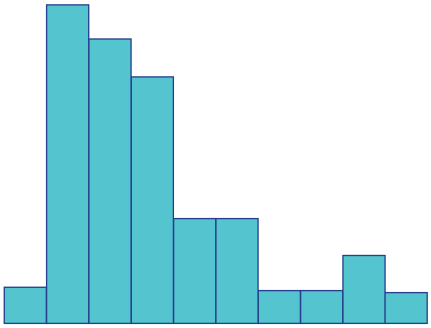
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Negatively skewed



It has a long tail to the left and a very short tail to the right.

Positively skewed



It has a long tail to the right and a very short tail to the left.

These properties can also be seen through other representations, such as with the stem-and-leaf plot given here. The stem-and-leaf plot below displays a positively skewed distribution of data.

1	0 0 1 1 2 2
2	4 6 6
3	1 2 2 3 3 4 5 5 5 7
4	
5	6 6
6	7
7	
8	9
18	7
19	2

5 | 6 means 56

Sometimes there are one or two values very different from the others. These values are called **outliers**. For example, the values 187 and 192 are outliers in the data set above.

The mean, median and mode convey different information and each has its advantages and disadvantages. Consider the data set below, giving monthly salaries of the 16 employees in a small firm.

\$7000	\$7000	\$7000	\$7000	\$7000	\$7700	\$7700	\$8400	\$8400
\$9800	\$9800	\$11200	\$11200	\$13900	\$20000	\$42000		

Median = \$8400

Mode = \$7000

Mean = \$11568.75

The mean is much higher than both the median and mode. Normally, the median is a preferable measure of 'centre' to the mean in situati ons such as this. The values \$20 000 and \$42 000 are much larger than the others and have a large effect on the mean.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Exercise 19D

All answers are to be given correct to 1 decimal place, unless otherwise specified.

Example 10

1 Calculate the mean and mode of each data set.

a 1, 1, 3, 5, 5, 5, 10 **b** 4, 4, 4, 7, 8, 8, 10, 11

2 In a football season, the number of kicks obtained by a player week by week was:

22 16 18 31 10 8 19 16 18 12 10 9 16

Calculate the mean number of kicks per week obtained by the player.

3 Twelve students sat for a test and their results are displayed in the stem-and-leaf plot opposite.

Stem	Leaf
1	8 9
2	2 4 5 6
3	1 4 9
4	2 3 6

4 | 3 means 43

a Calculate the mean of the marks.

b How many students obtained a mark higher than the mean?

c If a 13th student obtained a mark of 32 for the test, would the mean for the 13 students be higher or lower than the answer for part **a**?

4 The number of strokes scored on the 18th hole of a golf course was recorded for a number of golfers. The results are shown opposite.

Number of strokes	Number of players
2	1
3	6
4	27
5	20
6	10

a How many players had their score recorded?

b What is the average score?

c How many players took fewer strokes than the average?

d What number of strokes is the mode?

5 Five people have an average weight of 67 kg. If a child of weight 25 kg is added to the group, what is the average weight?

6 Part-way through a cricket season, a batsman has had scores of 15, 76, 42 and 27. Assume that the batsman is dismissed in each innings.

a Calculate the batsman's average.

b If his average after the next innings is 42, how many runs did he score in that innings?

c The batsman has 12 innings in a season. He wants to have an average of 50 at the end of the season. How many runs does he need to score in the remaining 7 innings?

7 During a term, a student has an average of 46 after the first 4 tests and his average for the next 6 tests is 38. What is his average for the 10 tests?

8 A data set has a mean of 15. What will happen to the mean (that is, will it decrease or increase) if:

a a data value of 24 is added to the set?

b a data value of 15 is added to the set?

c data values of 6 and 25 are added to the set?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

Example 12

9 Find the median for each of the following data sets.

a 8, 6, 12, 4, 1, 9, 15, 3

b 18, 26, 47, 13, 18

c 1.6, 1.9, 2.4, 1.8, 3.7, 0.9, 2.6, 1.7

d 647, 326, 849, 586, 710, 694

10 Copy and complete the following table, which relates the number of data values to the position of the median. (A position of 5.5 means that the median is the average of the 5th and 6th data values.)

Number of data values	5	11	21	10	20	100				
Position of median	3			5.5			15	22	13.5	24.5

Example 13

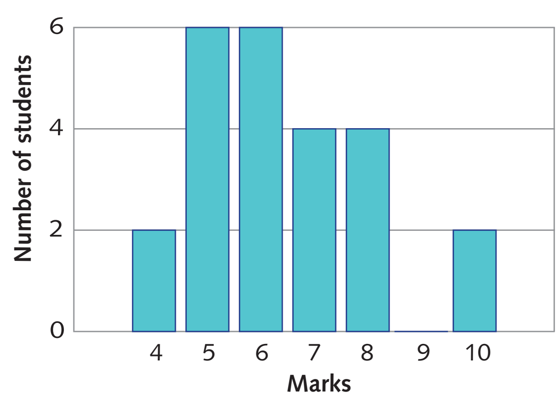
11 Fill in the frequency column, and hence find the median of the following data set.

		Frequency
5	4 5 6 8 9	
6	2 2 4 5 7 9	
7	1 3 5 5 8	
8	2 4 7 9	
9	3 5	

7 | 1 means 71

Example 14

12 The marks obtained for a quiz by a group of students are displayed in the column graph opposite.



a How many students had their marks recorded?

b What is the median of the marks?

c What is the mode of the marks?

13 The distinct values a, b, c, d, e and f are arranged here in numerical order, with a having the least value. Describe the effect on the median and the mean if:

a f is increased by 12

b the value a is deleted from the list

c a is decreased by 6 and f is increased by 6

d a is decreased by 16 and f is increased by 6

e b and e are both increased (but b is still less than c)

f c is increased by 4 (c is still less than d) and b is decreased by 4

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

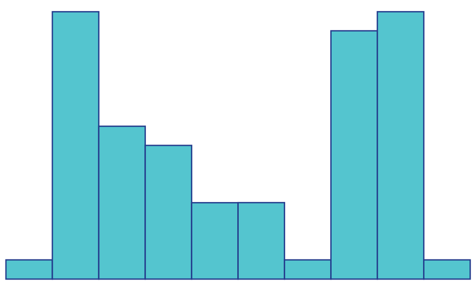
14 The following statement in a newspaper seems to be incorrect.

According to latest enrolment figures, half the student population of the university are over the age of 24. The 'average' student is 29 years old.

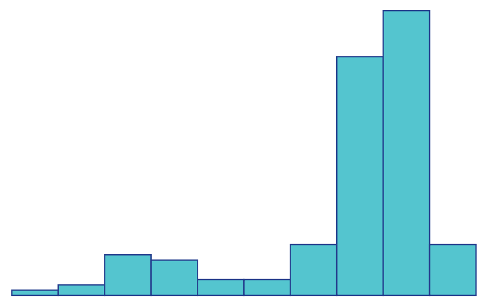
Can you give an example of a set of 4 students with mean age 24 and median age of 29?

15 Describe each set of data.

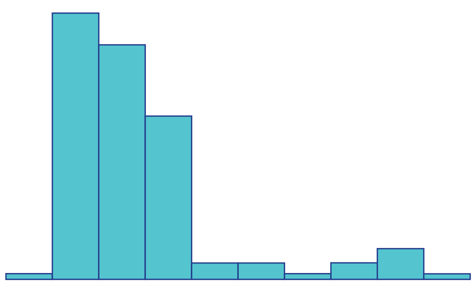
a



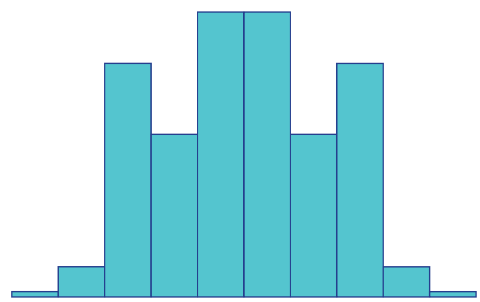
b



c



d



16 For the data in each stem-and-leaf plot, find the range, median and mean.

a

0	
1	0 0 1 1 2 2
2	4 6 6
3	1 2 2 3 3 4 5 5 5 7
4	
5	6 6
6	5 7

5 | 6 means 56

b

0	8
1	0 0
2	1 1
3	2 4
4	2 2
5	2 2 4 5 6 8 8
6	3 4 4 5 5 7 8 9

3 | 1 means 31

c Describe the distribution in part **b**.

17 Two professional baseball teams in the United States have a total of 63 players on their lists of players. The median salary of these players is \$1500 000, and the mean salary is \$3382 202. The range of salaries is \$31506 000. What would you expect the histogram of this data to look like?

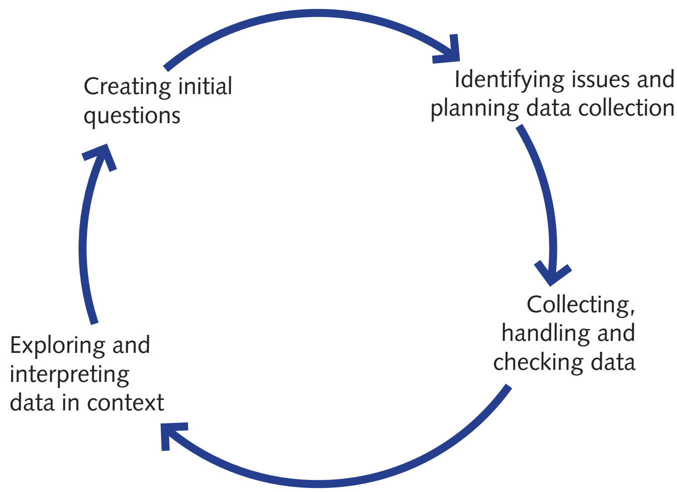
19E The statistical investigation process

The best way to understand how data is collected and presented is to do data-collection and presentation activities yourself. Some of the different ways that you can organise, present and discuss data have been explained throughout this chapter.

In this section we discuss the statistical data investigation process. Although there are many ways to collect and organise data, we will use the following steps:

- creating initial questions
- identifying issues and planning data collection
- collecting, handling and checking data
- exploring and interpreting data in context.

The statistical data investigation process is illustrated in the following diagram.



You can see that the statistical process never really ends. Once we have interpreted the data, we might discover that there are more questions we want to ask and more data to collect.

As an example, consider this process in the context of the suggested Activity, 'Reaction times (left and right)'.

Initial questions	• How long is the reaction time? • Is there a difference between left and right side reaction times? • Are left-handed students consistently different to right-handed students in these recorded times?
Issues and planning	• What height should the ruler be held in relation to the person? • Should several attempts be recorded and the average taken? • How accurate is the measurement of reaction distance?
Collecting, handling and checking data	• Should all data be recorded? • What should be recorded if the participant fails to stop the ruler?
Exploring and interpreting data	• Display with back-to-back stem-and-leaf plots. • Interpret results by measuring mean, median and range. • Address initial questions.

Further suggestions for statistical investigations appear on the AMSI website.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 19E

1 Here are some questions to initiate a discussion on techniques for collecting data.

a What proportion of Australian households recorded a television program during the past week?

b What percentage of a large shipment of components of a machine can stand up to a stress test?

c Is a particular component of a particular model of car safe? One hundred and fifty thousand of the cars have been distributed worldwide.

2 Here are some issues for which statistical information would help. Discuss how to go about collecting it.

a A coffee-making machine manufacturer wants to study people's colour preference for their machines.

b A council wants to determine the best resources for their library to hold.

c An employer of 10 000 people wants to determine their employees attitudes towards the company

Review exercise

1 A maths test was given to two different classes and results were recorded in the back-to-back stem-and-leaf plot below.

Class A		Class B
	3	2 4
3	4	5 5 8
9 6 6 3	5	4 9
8 6 5 4 1	6	3
8 7 5 4 2 2 0	7	2 3 3 8
7 6 3 0	8	0 1 4 4 6 7
4 1	9	1 3 6
	10	0

Note: 0 | 7 | 2 means 70% in class A and 72% in class B.

a What percentage of students scored at least 80 in:

i class A? ii class B?

b What percentage of students scored less than 50 in:

i class A? ii class B?

c What was the range of marks in:

i class A? ii class B?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d What was the median mark in:

  i class A? **ii** class B?

e What was the mean mark in:

  i class A? **ii** class B?

f Describe the distribution of data in:

  i class A? **ii** class B?

2 T-shirts sold in a shop were tallied over a week with their sizes noted.

T-shirt size	Tally	Frequency
S	𝍸 𝍸 𝍸 III	
M	𝍸 𝍸 𝍸 𝍸 I	
L	𝍸 𝍸 II	
XL	𝍸 II	
XXL	III	

a Complete the frequency column of the table.

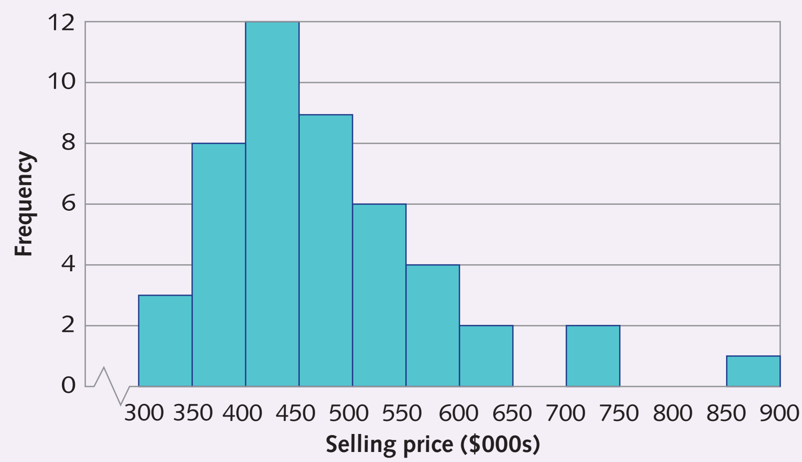
b Determine the mode of the size of the T-shirts sold.

c Determine the percentage of T-shirts sold that were small (S) or medium (M).

d Create a bar chart of the data.

e Explain why calculations of mean or median makes no sense in this situation.

3 Auction prices of houses sold in Blackpool over the month of September 2016 are shown in the histogram below.



a Determine the total number of houses sold in Blackpool over the month of September 2016.

b Determine the percentage of houses, sold in Blackpool over the month of September 2016, that are cheaper than $500 000.

c Describe the distribution of data.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d Determine the class of prices in which the median occurred.

e Identify whether the mean or median house price would be greater and explain why.

4 A student received the following marks for five tests: 75, 83, 74, 66 and 90.

a Calculate the mean.

b Determine the minimum mark this student will need to receive on her sixth test in order to average at least 80.

c If the student's median mark is 78 after six tests, determine the mark she received on her sixth test.

Challenge exercise

Locating a country's centre of population

The **population centre** of a country is defined as the point whose latitude and longitude are the average of the latitudes and longitudes of all people in the country. By calculating the population centre of a country, trends about population movement and growth can be obtained.

The following questions investigate the population centre of Australia in 1990. To make the calculations easier, it might be useful to use the statistics facility of a graphics calculator, a spreadsheet or a statistical software package. (Give your answers correct to 1 decimal place.)

1 The city of Melbourne has latitude 38°S and longitude 145°E, and the city of Brisbane has latitude 27°S and longitude 153°E.

a Suppose that one person lives in Melbourne and another person lives in Brisbane. What is the average of the two people's:

i latitude **ii** longitude?

b Find the average of the latitudes and longitudes if:

i 10 people lived in Melbourne and 5 people lived in Brisbane

ii 20 people lived in Melbourne and 30 people lived in Brisbane

iii 1000 people lived in Melbourne and 800 people lived in Brisbane

2 Consider the following table, which gives the latitude, longitude (both rounded to the nearest whole number) and the population in millions (rounded to the nearest million) of the five largest Australian capital cities in 1990.

City	Latitude	Longitude	Population (in millions)
Melbourne	38°S	145°E	3
Sydney	34°S	150°E	4
Perth	32°S	116°E	1
Adelaide	35°S	139°E	1
Brisbane	27°S	153°E	1

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

a Calculate the population centre of the cities Melbourne and Sydney taken together (that is, the average of the 7 million latitudes for these two cities and the 7 million longitudes for these two cities).

b Calculate the population centre of Melbourne and Perth taken together.

c Calculate the population centre of Melbourne, Perth and Sydney taken together.

d Calculate the population centre of Melbourne, Adelaide and Brisbane taken together.

e Calculate the population centre of the five capital cities taken together. Locate the population centre on a map of Australia.

3 The following table gives the population (in 1000s) of the eight Australian capital cities for a number of years throughout the 20th century.

Year	Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
1901	497	502	121	162	71	36	1	–
1920	885	763	206	255	152	50	1	–
1930	1191	1000	280	311	212	59	2	7
1940	1294	1083	336	330	230	69	2	12
1950	1557	1302	445	434	313	84	5	22
1960	2133	1831	578	577	409	112	12	50
1970	2752	2448	847	827	672	151	33	129
1980	3232	2760	1029	934	902	170	51	246
1990	3657	3081	1302	1050	1193	184	73	310

Using the information about latitude and longitude given in Question **2**, together with the fact that Hobart has latitude 43°S and longitude 147°E, Darwin has latitude 12°S and longitude 131°E, and Canberra has latitude 35°S and longitude 149°E, calculate the population centre of the eight capital cities for each year given above.

Use your calculations to answer the following questions.

a Describe the direction in which the Australian population centre has moved since 1901. Can you give some reasons for this?

b Estimate where the population centre for Australia was in the year 2005.

c Why is it important to recognise these shifts in population? What groups in the community need to know such information?

Note that a more realistic model would include population centres outside the capital cities.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

CHAPTER

20

Review and problem-solving

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

20A Review

Chapter 11: Coordinate geometry

1 Find the distance between the points with the given coordinates.

a $(3, 2)$ and $(5, 10)$ **b** $(2, 3)$ and $(5, 12)$ **c** $(-1, 3)$ and $(0, 6)$

d $(3, -2)$ and $(5, 0)$ **e** $(-2, 9)$ and $(3, -1)$ **f** $(2, 4)$ and $(5, -2)$

2 Find the coordinates of the midpoint of the interval with the given endpoints.

a $(3, 2)$ and $(5, 10)$ **b** $(2, 3)$ and $(5, 12)$ **c** $(-1, 3)$ and $(0, 6)$

d $(3, -2)$ and $(5, 0)$ **e** $(-2, 9)$ and $(3, -1)$ **f** $(2, 4)$ and $(5, -2)$

3 Find the gradient of the line that passes through each pair of points.

a $(3, 2)$ and $(5, 10)$ **b** $(2, 3)$ and $(5, 12)$ **c** $(-1, 3)$ and $(0, 6)$

d $(3, -2)$ and $(5, 0)$ **e** $(-2, 9)$ and $(3, -1)$ **f** $(2, 4)$ and $(5, -2)$

4 A line has gradient 2 and passes through the point $(1, 3)$.

a Use this information to fill in the table of values below for points (x, y) on the line.

x	0	1	2	3	4
y		3			

b Find the equation of the line.

5 A line has gradient $-\frac{1}{2}$ and passes through the point $(2, 6)$.

a Use this information to fill in the table of values below.

x	0	1	2	3	4
y			6		

b Find the equation of the line.

6 A line has gradient 2 and passes through the point $(-1, 6)$. Find the equation of the line.

7 A line passes through the points $(2, 4)$ and $(5, 6)$.

a Find the gradient of the line.

b Find the equation of the line.

8 Sketch the graph and label the x- and y-intercepts for the following lines.

a $y = 2x - 1$ **b** $y = 3x + 2$ **c** $2x - 3y = 6$ **d** $x + 4y = 5$

9 Find the gradient, x-intercept and y-intercept of the following lines.

a $y = 5x - 3$ **b** $y = 4 - x$ **c** $3x - 4y = 12$ **d** $5x + 2y = 4$

10 a Find the equation of the line through the point $(3, 2)$ parallel to the line with equation $y = 2x + 1$.

b Find the equation of the line through the point $(3, 2)$ perpendicular to the line with equation $y = 2x + 1$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Chapter 12: Probability

1 A letter is chosen at random from the word RANDOM.

a List the sample space for this experiment.

b i If B is the event ‘the letter chosen is a vowel’, write down the outcomes favourable to B.

ii What is the probability of B occurring?

c Find the probability that the letter chosen:

i is a consonant

ii comes after K in the alphabet

iii comes before H in the alphabet

iv has an axis of symmetry

2 The numbers 1 to 25 are written on 25 cards. If a card is selected at random, find the probability that the number on the card is:

a 14

b greater than 20

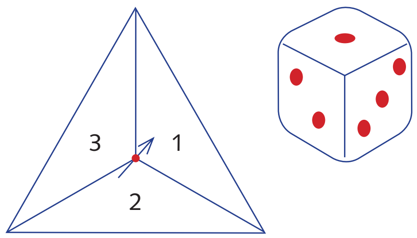
c even

d a multiple of 5

e odd and a multiple of 3

f neither even nor a multiple of 5

3 In a game, a three-sided spinner is spun and a die is rolled. The two numbers obtained are added together.



a Draw an array to show all the possible scores.

b If the spinner is spun and the dice is rolled, find the probability that the score obtained is:

i 7

ii greater than 5

iii odd

iv a multiple of 3

4 Two students, Annie and Bianca, each choose a number between 1 and 5 inclusive. By listing all the possible outcomes, find the probability that:

a they both choose an even number

b Bianca chooses a larger number than Annie

c the product of their numbers is greater than 10

d the sum of their numbers is less than 6

5 Two fair dice are rolled and the uppermost numbers are noted.

a Represent the sample space by drawing an array and listing all the possible outcomes.

b Find the probability of obtaining:

i a double 6

ii at least one 6

iii a double of any number

iv a total of 10 or more

6 A fair coin is tossed four times.

a Draw a tree diagram to represent the outcomes of this experiment and list the sample space.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b Find the probability of getting:

i four tails **ii** three tails and one head **iii** fewer than three tails

7 In a group of 200 students, 60 study geography, 80 study economics and 70 study neither.

a Represent this information in a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i geography? **ii** geography and economics?

iii at least one of these subjects?

8 A girl performs the following experiment. She draws a marble from a bag containing several different coloured marbles and notes its colour. She then replaces the marble. She repeats the experiment 50 times and keeps tally of the number of marbles of each colour. Her results are given below.

Colour	Red	Blue	Yellow	Green
Number drawn	6	17	19	8

a Using the table, estimate the probability that the next marble drawn is:

i red **ii** blue **iii** not green **iv** yellow or blue

b Which of the following statements do you think are true? Justify your answer.

i The bag contains 50 marbles.

ii The bag contains 4 marbles.

iii The bag contains twice as many blue marbles as green marbles.

iv The bag contains equal numbers of blue and yellow marbles.

v The bag contains only red, blue, yellow and green marbles.

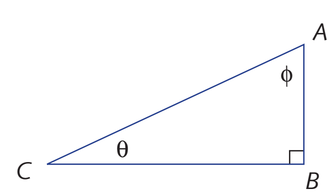
Chapter 13: Trigonometry

1 In the triangle opposite, name:

a the hypotenuse **b** the side opposite θ

c the side opposite ϕ **d** the side adjacent to θ

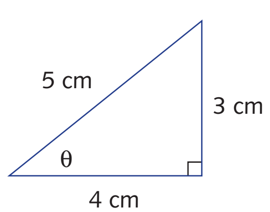
e the side adjacent to ϕ



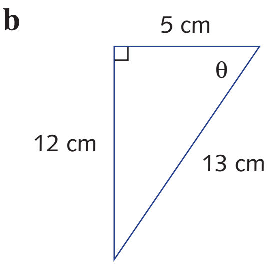
2 In each of the following triangles, write down the value of:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

a

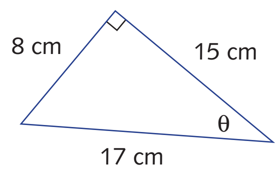


b

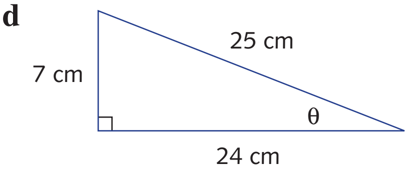


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c



d



3 Find, correct to 4 decimal places:

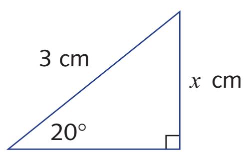
a $\cos 15°$ **b** $\sin 86°$ **c** $\tan 64°$

4 Find θ, to the nearest degree:

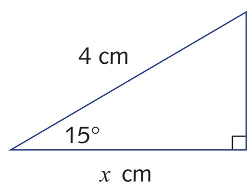
a $\tan \theta = 2$ **b** $\sin \theta = 0.2$ **c** $\cos \theta = 0.34$

5 Find the values of the pronumerals, correct to 2 decimal places.

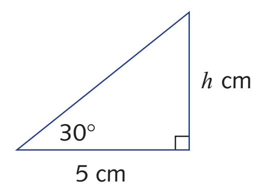
a



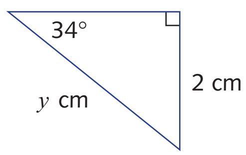
b



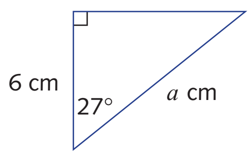
c



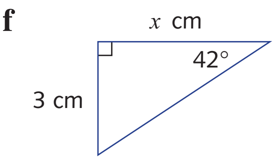
d



e

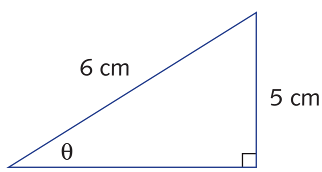


f

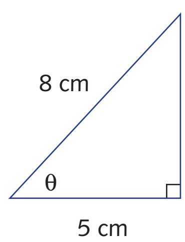


6 Find θ, correct to the nearest degree.

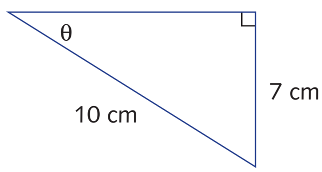
a



b



c



7 A road makes an angle of $6°$ with the horizontal. How much does it rise over a distance of 800 m along the road? Give your answer to the nearest metre.

8 A straight slide is 3.5 m long and has a vertical ladder 2.1 m high. Find the angle the slide makes with the ground, giving your answer to the nearest degree.

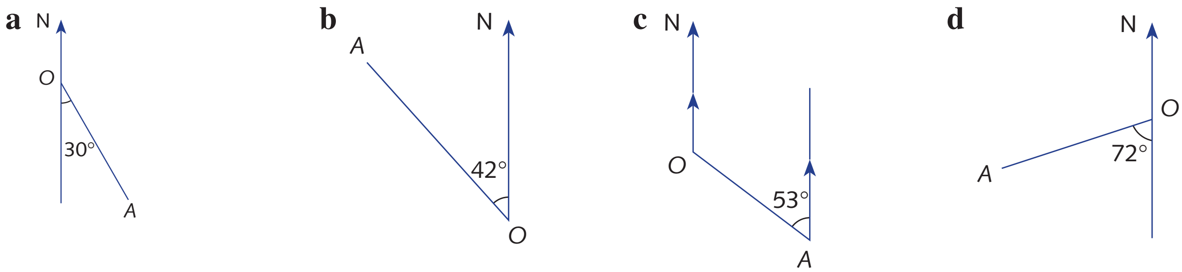
9 A ladder of length 4.5 m is leaning against a vertical wall. If the top of the ladder is 3 m above the ground, find:

a the angle the ladder makes with the wall, giving your answer to the nearest degree

b the distance the foot of the ladder is from the wall, giving your answer to the nearest centimetre.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 Give the true bearing of A from O.



11 A plane flies on a bearing of 142°T for 400 km. How far south of its starting point is the plane now? (Give your answer in kilometres, correct to 1 decimal place.)

12 A ship travels on a bearing of 320°T until it is 50 km west of its starting point.

a How far did it travel? Give your answer correct to 1 decimal place.

b How far north of its starting point is it? Give your answer correct to 1 decimal place.

13 A man is watching a ship from the top of a scenic lookout. His eye level is 30 m above sea level. If the angle of depression from the man to the ship is 8°, how far from the foot of the lookout is the ship? Give your answer to the nearest metre.

14 A bird-watcher is lying on the ground 15 m from a vertical tree that contains a bird's nest. If the nest is 8 m up the tree, what is the angle of elevation from the watcher to the nest? Give your answer to the nearest degree.

15 The angle of depression of a yacht from the top of the cliff is 44°. If the yacht sails 200 m further away from the cliff, the angle of depression is now 38°. How high is the cliff? Give your answer correct to the nearest metre.

Chapter 14: Simultaneous linear equations

1 Find the coordinates of the point of intersection of the lines $y = 2x - 4$ and $x + y = 7$. Sketch the graphs, labelling the point of intersection and the x- and y-intercepts of each line.

2 Use the substitution method to solve the following pairs of simultaneous equations.

a $y = 3x - 5$
$x + y = 3$

b $x - 2y = 7$
$x = 3y + 1$

c $y = 3 - 5x$
$y = 4x - 3$

3 Use the elimination method to solve the following pairs of simultaneous equations.

a $2x + y = 4$
$x - y = 5$

b $5x - 2y = 3$
$x - y = 2$

c $2x + 3y = 8$
$3x - 4y = 10$

d $5x - 4y = 13$
$2x - 7y = 6$

4 The equations of two lines are $y = mx + 7$ and $2x - 5y = 10$.

a For what value of m do the two lines not intersect?

b For what values of m do the two lines intersect?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Cambridge University Press

Updated August 2021

5 The lines with equations $y = 3x - k$ and $y = mx - 2$ intersect at $(2, 3)$. Find m and k.

6 Find the area of the triangle formed by the lines $x + 3y = 6$, $y = 5 - x$ and:

a the x-axis **b** the y-axis

7 In a farmyard containing pigs and chickens, there are 40 heads and 144 legs. How many pigs are in the yard?

8 Tibor is currently 7 times his son's age. In 8 years' time, he will be 3 times his son's age. What is Tibor's current age?

9 The lines with equations $y = x + 2$, $y = 4 - x$ and $y = 2x + c$ are concurrent. Find the point of concurrency and hence find c.

10 Solve the following simultaneous equations.

a $ax + by = c$ $\quad (a^2 + b^2 \neq 0)$
$bx - ay = d$

b $2ax + y = c$
$by + x = d$ $\quad \left(ab \neq \frac{1}{2}\right)$

Chapter 15: Further factorisation

1 Factorise:

a $4x - 100$ **b** $7x^2 - 49$ **c** $2y - 16y^2$ **d** $10a - 16a^2$

2 Factorise:

a $x^2 - 100$ **b** $x^2 - 49$ **c** $9x^2 - 16y^2$ **d** $1 - 16a^2$

3 Factorise:

a $3x^2 - 27$ **b** $2x^2 - 8$ **c** $18x^2 - 32y^2$ **d** $2 - 18a^2$

4 Factorise:

a $x^2y^2 - 4$ **b** $x^2 - 64y^2$ **c** $8y^2 - 32$ **d** $2a^2 - 32$

5 Factorise:

a $(x + 2)^2 - 4$ **b** $(y + 2)^2 - 9$ **c** $3(x + 2)^2 - 27$ **d** $(a - 1)^2 - 1$

6 Factorise:

a $x^2 + 7x + 12$ **b** $x^2 + 9x + 18$ **c** $x^2 - 5x - 6$
d $x^2 + 3x - 28$ **e** $x^2 - 11x + 30$ **f** $x^2 - 14x + 24$
g $x^2 + 3x - 70$ **h** $x^2 - 6x - 55$ **i** $3x^2 + 6x + 9$
j $4x^2 - 8x + 12$ **k** $-x^2 - x + 6$ **l** $-2x^2 - 3x + 44$

7 Factorise by grouping:

a $8xy + 2y + 12x + 3$ **b** $6ax + 15x - 4a - 10$
c $6ac + 2bd - 3bc - 4ad$ **d** $2m^2 - mp + 6mn - 3np$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 Factorise:

a $2x^2 + 9x + 10$ **b** $6x^2 + 19x + 10$ **c** $6x^2 + 13x - 5$

d $8x^2 - 10x - 3$ **e** $21x^2 - 53x - 8$ **f** $6x^2 - 7x - 20$

g $4x^2 - 12x + 9$ **h** $9x^2 + 24x + 16$ **i** $4x^2 - 8x - 12$

9 Simplify:

a $\dfrac{1}{(x-1)^2} \div \dfrac{1}{x^2-1}$ **b** $\dfrac{x-4}{x^2+2x+1} \times \dfrac{x+1}{x^2-16}$

c $\dfrac{m-2}{4m} \times \dfrac{m}{m-2}$ **d** $\dfrac{p+1}{8(p-1)} \times \dfrac{4(p-1)}{(p+1)(p+2)}$

e $\dfrac{4}{a} \div \dfrac{2}{a^2}$ **f** $\dfrac{5a-7}{2a+4} \times \dfrac{12}{10a-14}$

g $\dfrac{2x^2+3x-2}{(2x-1)^2}$ **h** $\dfrac{(x+3)(x-2)}{x+6} \div \dfrac{x+3}{x+6}$

i $\dfrac{x^2+3x-4}{2x-4} \times \dfrac{6x-12}{x-1}$ **j** $\dfrac{2x^2-6x}{x^2-1} \div \dfrac{x^2-x-6}{x^2-x-2}$

10 Express each of the following as a single fraction.

a $\dfrac{x+1}{4} + \dfrac{x+3}{3}$ **b** $\dfrac{x-2}{2} + \dfrac{x-1}{3}$ **c** $\dfrac{2x+1}{3} - \dfrac{x+1}{4}$ **d** $\dfrac{3x-1}{4} - \dfrac{2x-1}{6}$

11 Express the following as a single fraction.

a $\dfrac{2}{x} + \dfrac{1}{x+1}$ **b** $\dfrac{4}{x-1} + \dfrac{2}{x+2}$ **c** $\dfrac{x}{x-1} + \dfrac{2x}{x+3}$

d $\dfrac{x+2}{x+3} - \dfrac{x-1}{x-2}$ **e** $\dfrac{2}{2x+1} - \dfrac{x-4}{x-3}$ **f** $\dfrac{x-1}{3x+2} - \dfrac{x-2}{2x-1}$

12 Express the following as a single fraction.

a $\dfrac{4}{x} + \dfrac{3}{x^2}$ **b** $\dfrac{5}{x+1} - \dfrac{2}{(x+1)(x+2)}$

c $\dfrac{2}{x+3} + \dfrac{x+2}{x^2-9}$ **d** $\dfrac{3}{x(x-1)} + \dfrac{2}{x(x+1)}$

e $\dfrac{4}{x^2+4x+4} - \dfrac{2}{x^2+x-2}$ **f** $\dfrac{3}{x^2-x-6} + \dfrac{4}{x^2-4x+3}$

g $\dfrac{x}{2-x} + \dfrac{2}{x^2-4}$ **h** $\dfrac{3}{x^2-1} - \dfrac{2}{1-x}$

13 Complete the square, expressing each of the following in the form $(x+h)^2 + k$.

a $x^2 + 4x + 2$ **b** $x^2 + 6x + 6$ **c** $x^2 - 2x - 6$ **d** $x^2 + 3x - 8$

e $x^2 - x - 1$ **f** $x^2 - 10x + 20$ **g** $x^2 + 3x - 7$ **h** $x^2 - 6x - 5$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

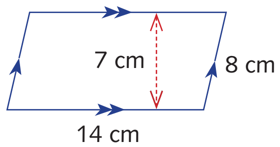
14 Factorise by first competing the square (surds could be involved in the final expression).

a $x^2 + 4x - 1$ **b** $x^2 + 2x - 2$ **c** $x^2 - 6x + 1$ **d** $x^2 + 5x + 2$

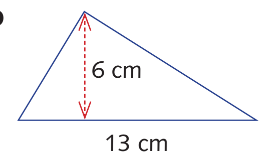
Chapter 16: Measurement – areas, volumes and time

1 Calculate the area of the following shapes.

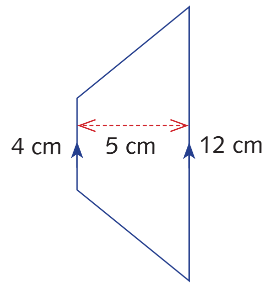
a



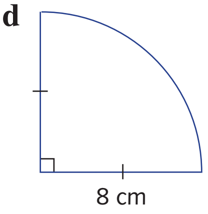
b



c

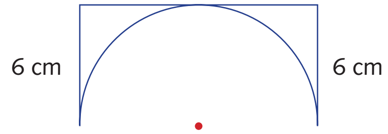


d

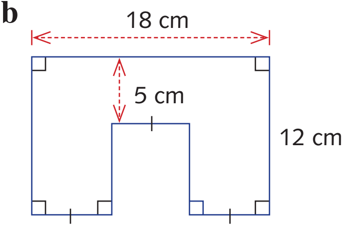


2 Calculate the area of the following shapes.

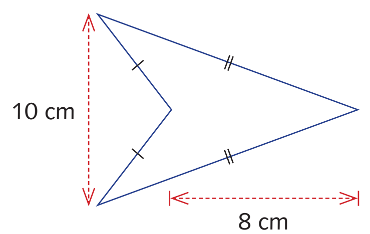
a



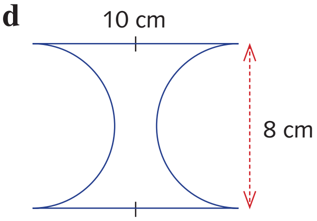
b



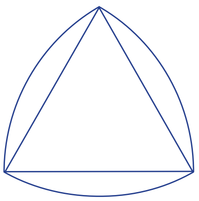
c



d



3 A company logo consists of an equilateral triangle with an arc of a circle drawn on each side using the opposite vertex as centre. If the length of each side of the equilateral triangle is 6 cm, find:



a the height of the triangle (giving an exact answer)

b the area of the logo, correct to 2 decimal places

4 Calculate the surface area of:

a a cube with side length 1.2 m

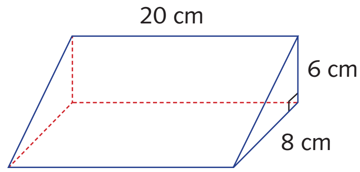
b a rectangular prism with side lengths 5 cm, 4 cm and 2 cm

c a cylinder with radius 4 cm and height 5 cm, correct to 1 decimal place

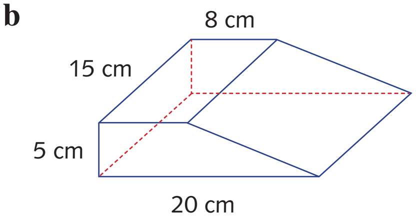
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 Calculate the surface area of the following shapes.

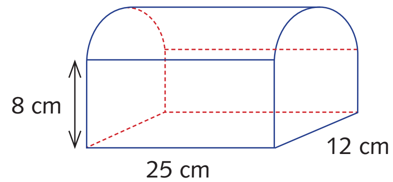
a



b

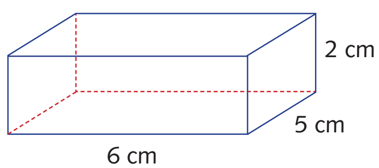


6 A tool box has the shape shown below. Calculate its surface area, correct to 1 decimal place.

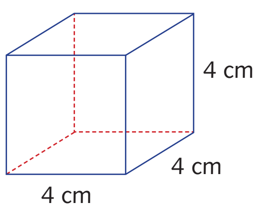


7 Calculate the volume of each of the following solids.

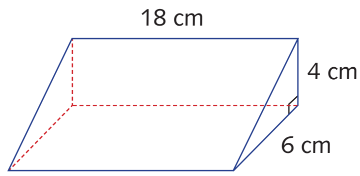
a



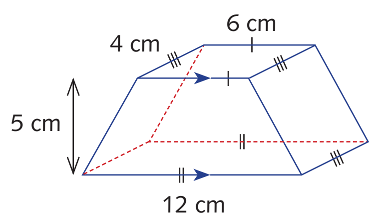
b



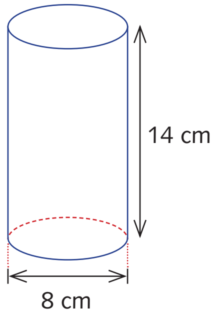
c



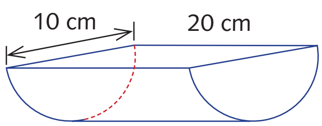
d



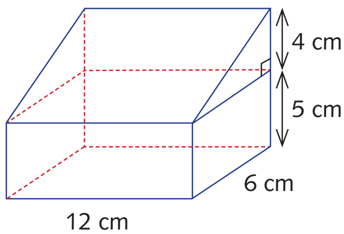
e



f



g



8 A child's pool is in the shape of a cylinder with radius 1.2 m and depth 20 cm. Calculate the volume of the pool in litres, correct to the nearest litre.

9 Convert each of the following measurements into the units shown in brackets.

a $14\ \text{cm}^2$ (mm^2) **b** $1600\ \text{mm}^2$ (cm^2) **c** $2.1\ \text{m}^2$ (cm^2)

d $46\,000\ \text{cm}^2$ (m^2) **e** $64\,000\ \text{m}^2$ (km^2) **f** $8000\ \text{m}^2$ (ha)

g $41000\ \text{mm}^2$ (m^2) **h** $2.6\ \text{km}^2$ (cm^2)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

10 Convert the following measurements into the units shown in brackets.

a 410 mm^3 (cm^3) **b** 0.2 cm^3 (mm^3)

c $64\,000\text{ cm}^3$ (m^3) **d** 8000 cm^3 (litres)

e $8\,600\,000\text{ mL}$ (m^3) **f** 0.006 m^3 (mL)

g $9\,000\,000\,000\,000\text{ m}^3$ (km^3) **h** 0.26 km^3 (m^3)

i 4.1 km^3 (L) **j** 8.6 m^3 (mm^3)

Chapter 17: Quadratic equations

1 Solve:

a $x(x-3)=0$ **b** $3x(2x+1)=0$

c $(3m+1)(2m-5)=0$ **d** $(4p+3)(5p-7)=0$

2 Solve:

a $x^2-5x=0$ **b** $2x^2-3x=0$ **c** $8x-x^2=0$

d $x^2-16=0$ **e** $4x^2-9=0$ **f** $2x^2-72=0$

g $3x^2-75=0$ **h** $x^2+7x+6=0$ **i** $x^2+9x+18=0$

j $x^2-14x+33=0$ **k** $x^2=6x-8$ **l** $-x^2+3x+40=0$

m $x^2+5x=14$ **n** $-2x^2+7x+15=0$ **o** $6x^2+7x+2=0$

p $4x^2+17x-15=0$ **q** $6x^2-11x-2=0$ **r** $15x^2=x+2$

3 Solve:

a $x^2=8x-12$ **b** $x^2=3x+28$ **c** $x(x-1)=12$

d $x(x+2)=15$ **e** $(x+2)(x+3)=2$ **f** $(x+1)(x-3)=21$

g $x=\dfrac{12}{x-4}$ **h** $x=\dfrac{18}{x+7}$ **i** $x=\dfrac{10}{x}-3$ **j** $x=\dfrac{12}{x}+4$

4 One more than a number is the same as 42 divided by the number. What can the number be?

5 Twenty-one less than a number is equal to 100 divided by the number. What can the number be?

6 The product of a number and four less than the number is equal to 16 more than twice the number. Find the number.

7 Draw up a table of values for each of the following and use your table to plot the graph. In each case, state the x-intercepts.

a $y=x^2-x-6,\ -3\le x\le 4$ **b** $y=x^2+x-2,\ -3\le x\le 2$

c $y=x^2+3x-4,\ -5\le x\le 2$ **d** $y=3-2x-x^2,\ -4\le x\le 2$

8 Solve the equations.

a $x^2-7=0$ **b** $2x^2-16=0$ **c** $8-x^2=0$ **d** $30-6x^2=0$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 © Brown et al. 2017 Cambridge University Press

9 Solve each equation by completing the square.

a $x^2 + 2x - 2 = 0$ b $x^2 - 6x + 6 = 0$ c $x^2 + 4x + 2 = 0$

d $x^2 + 3x + 1 = 0$ e $x^2 - 5x + 3 = 0$ f $x^2 - x - 1 = 0$

10 Sketch the parabolas.

a $y = (x - 1)^2$ b $y = (x + 2)^2 - 1$ c $y = x^2 + 3x + 3$

Chapter 18: Rates and direct proportion

1 Drew runs at 8 m/s for 12 seconds. How far does he run?

2 a Convert 180 km/h into metres per second.

b Convert 12 m/s to kilometres per hour.

3 a A plane travels 2000 km in 2 hours 30 minutes. What is the average speed of the plane?

b A car travels 200 km in 1 hour 50 minutes. What is the average speed of the car? Give your answer in kilometres per hour (km/h).

4 A car is travelling at 80 km/h.

a What is the formula for the distance d (in kilometres) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

5 Write each of the following in words.

a $x \propto z$ b $y \propto x^3$ c $p \propto \sqrt{n}$ d $x^3 \propto z^4$

6 In each of the following $q \propto p$. Find the constant of proportionality and complete the table.

a

p	2	4		9
q	14		35	

b

p	1	4	7	
q		2		8

c

p	2	6	8	
q		8		24

7 Given $y \propto x^3$ and $y = 40$ when $x = 2$, find the formula for y in terms of x and the exact value of:

a y when $x = 10$ b y when $x = 3.5$ c x when $y = 135$

8 Given $d \propto t^2$ and $d = 18.75$ when $t = 5$, find the formula for d in terms of t and the exact value of:

a d when $t = 9$ b d when $t = 12$ c t when $d = 48$

9 The surface area of a sphere varies as the square of the radius. If the surface area of a spherical ball of radius 7 cm is 196π cm^2, find the corresponding surface area of a sphere of radius 3.5 cm.

10 Given that $q \propto p^3$, what is the effect on q when p is doubled?

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Chapter 19: Statistics

1 In a class of 20 students, 8 travel to school by train, 6 catch a bus, 4 are driven to school and 2 walk. Represent this information in a column graph.

2 Twenty students were asked how long they spend doing homework each night. Their responses, in minutes, are given below:

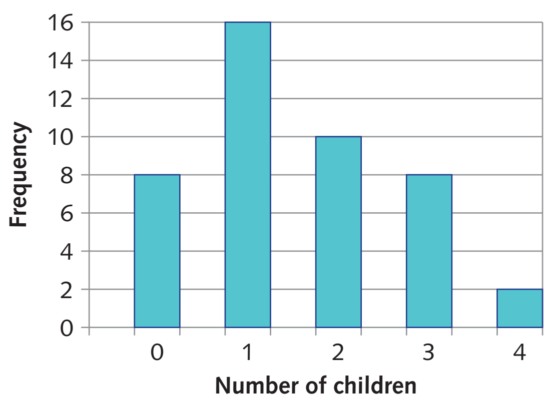
36 42 51 31 35 28 36 34 37 45

36 31 27 38 39 44 45 40 37 44

a Represent this information in a stem-and-leaf plot.

b How many students spent more than 40 minutes on homework?

3 The column graph below shows the number of children in families that completed a survey.



a How many families were surveyed?

b How many of the families have more than two children?

c What percentage of the families have no child or one child, correct to the nearest 1%?

d In total, how many children are there?

4 Over the course of a year, Ken obtained the following test results in Mathematics.

46 55 44 62 75 83 75 68 49 59 72 75 61

a Calculate the range for Ken's test results.

b Calculate the mean of his results, correct to 1 decimal place.

c If Ken does one more test and raises his average to 64, what was his mark on that test?

5 The average of 7 numbers is 12. An 8th number is added, and the average becomes 12.5. What number was added?

6 The average of 8 numbers is 24. What is the average of the same 8 numbers plus the two numbers 22 and 32?

7 The times, in seconds, it takes for nine students to run 100 m are:

12.6 14.2 13.1 12.9 15.1 14.6 13.7 14.2 14.8

a Calculate the mean, correct to 1 decimal place.

b Calculate the median.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 In an AFL football season, the goals scored by a player week by week are:

2 5 1 3 2 4 2 3 5 2 0 1 0 4 6 2 3 5 4 2 1 3

a Identify the mode in this data set.

b Calculate the mean number of goals scored in a week.

c Calculate the median number of goals scored in a week.

d In how many weeks did the player score more goals than the mean?

9 The numbers of Tasmanian devils that crossed a particular road over a number of weeks are recorded in the table below.

Week number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Number Of Tasmanian devils	24	41	44	26	27	38	35	39	40	41	23	46	33	36	38	41	24	22	31

a Draw a stem-and-leaf plot of the data.

b Find the mean, median and range of this data.

10 The heights of 15 children, to the nearest centimetre, are given below.

87 96 94 102 105 89 96 92 110 104 99 107 95 107

a Represent this information on a stem-and-leaf plot.

b Represent this information on a histogram using the classes:

i 85–89, 90–94, 95–99, 100–104, 105–109, 110–114

ii 80–89, 90–99, 100–109, 110–119

20B Problem-solving

1 Marbles of several different colours are placed in a bag. Sarah randomly draws one of the marbles from the bag, notes its colour and then replaces the marble.

a After doing this 10 times, her results are as follows.

Colour	Red	Blue	Yellow
Number of marbles	2	4	4

Decide whether each of the following statements is definitely true, could be true or is definitely false.

i There are 10 marbles in the bag.

ii There are twice as many blue marbles in the bag as there are red marbles.

iii There are equal numbers of blue and yellow marbles in the bag.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

iv The bag contains only 2 marbles and both are red.

v The bag contains marbles coloured red, blue or yellow only.

b Repeat part **a** if the results after Sarah has drawn 1000 marbles are as follows.

Colour	Red	Blue	Yellow
Number of marbles	201	701	98

2 Goran walks from O to A, 10 km away, on a true bearing of 300°T. He then walks 24 km from A to B on a true bearing of 30°T.

a Draw a diagram to represent Goran's walk and include the given information.

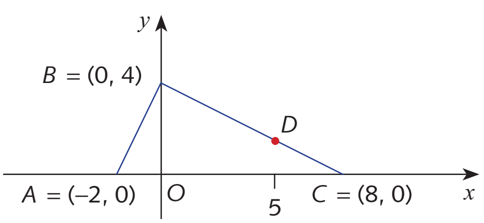
b State the size of $\angle OAB$.

c Calculate the distance between his start and finish points (that is, OB).

d Calculate the size of $\angle AOB$, correct to 2 decimal places.

e Hence calculate the bearing of O from B correct to the nearest degree.

3 Points A, B, C have coordinates as shown in the diagram. D is the point on line BC with an x-coordinate of 5.



a i Calculate the gradient of line AB.

ii Show that the equation of the line BC is $y = -\frac{1}{2}x + 4$. Find the coordinates of D.

iii Calculate the ratios $\frac{OB}{OA}$ and $\frac{OC}{OB}$.

iv Explain why ΔAOB is similar to ΔBOC.

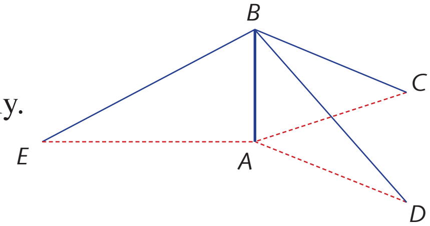
b Let the size of $\angle OAB = \alpha$.

i In terms of α, what is the size of $\angle OBA$?

ii In ΔOBC, find the size of $\angle OBC$ in terms of α.

iii Find the size of $\angle ABC$.

4 Students at an adventure camp erect a vertical mast AB, as shown in the diagram. BC, BD and BE are straight wires that are fixed to the ground at C, D and E respectively.



a In ΔEBA, $\angle EAB$ is a right angle and $\angle BEA = 30°$. If $EB = 40$ m, use trigonometry to show that the height of the mast AB is 20 m.

b In ΔEBA, use Pythagoras' theorem to find the length EA, giving your answer in simplest surd form.

c In ΔCBA, $\angle CAB$ is a right angle and the length AC is 11.547 m. Find the size of $\angle BCA$, giving your answer correct to the nearest degree.

d In ΔDBA, $\angle DAB$ is a right angle and $\angle BDA = 40°$. Calculate the length AD, giving your answer as a number of metres correct to 3 decimal places.

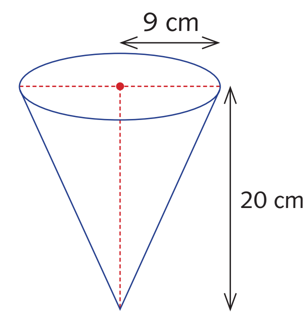
e Calculate the combined distance $(EA + CA + DA)$, giving your answer correct to 2 decimal places.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 In this question you will need to use the following formula:

$$\text{volume of a cone, } V = \frac{1}{3}\pi r^2 h$$

A right circular cone has height 20 cm and radius 9 cm, as shown in the diagram.



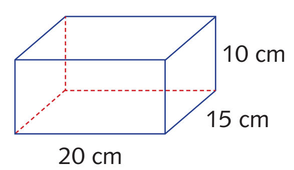
a Water is poured into the inverted cone until the depth of the water is 14 cm.

i If r centimetres is the radius of the water's surface, use similar triangles to calculate the value of r.

ii Use your answer to calculate the area of the water's surface, giving your answer in decimal form correct to 2 decimal places.

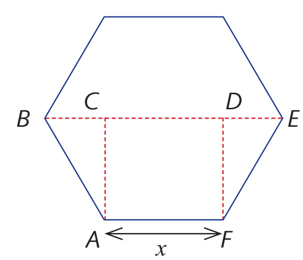
b The cone is now filled right to the top with water. What is the volume of the water in the cone, correct to the nearest 0.1 cm^3?

c The water is then poured from the cone into the rectangular tank shown.



Calculate the depth of water in the tank, correct to 2 decimal places.

6 A gardener is building a vegetable garden in the shape of a regular hexagon. In the given diagram $ACDF$ is a rectangle. Let x metres be the length AF.



a Find the size of $\angle BAC$.

b Find the following in terms of x.

i BC **ii** AC

c Show that the area (A m^2) of the hexagonal vegetable garden is given by $A = \dfrac{3\sqrt{3}x^2}{2}$.

d Find, correct to 2 decimal places, the area (A m^2) if $x = 2.5$ m.

To fit in the gardener's backyard, the distance between opposite sides of the vegetable garden must be less than or equal to 7 m.

e Find the largest value of x, correct to 2 decimal places, such that the vegetable garden, perhaps rotated, will fit in the gardener's backyard.

f Using this value, find correct to 2 decimal places the largest possible area of the vegetable garden.

7 **a** A card is drawn at random from a pack of 52 playing cards. Find the probability that the card will be:

i a Heart **ii** a Queen **iii** not a Queen

iv a Queen of Hearts **v** a Queen or a Heart

A new experiment involves drawing two cards from a pack of 52 playing cards. A card is drawn at random and the suit is noted – Diamond (D), Heart (H), Spade (S), Club (C). The card is returned to the pack and the cards are shuffled. A second card is drawn at random from the pack and its suit is also noted.

b Draw an array that lists all the possible outcomes, specifically, all the possible suit pairs that can be randomly drawn.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c List the set of outcomes in which at least one Heart is obtained.

d Find the probability that the following is obtained:

i at least one Heart

ii exactly one Heart

iii a Heart and a Spade (in any order)

iv two cards of the same suit

e The experiment described above (involving drawing two cards with replacement) is repeated 100 times and the number of pairs that have the same suit are counted. On average, how many pairs would you expect to have the same suit?

8 Substance A (initially 30 g) evaporates at a rate of $1\frac{1}{2}$ g per minute while substance B (initially 20 g) evaporates at a rate of $\frac{2}{3}$ g per minute. y_A and y_B grams are the amounts of substances A and B respectively at time t minutes after the substances begin to evaporate.

a Write down formulas for y_A and y_B in terms of t.

b Find the time it takes for substance A to evaporate to 0 g.

c After how long do both substances have the same weight?

Consider now that substance A initially weighs a grams and evaporates at a rate of b grams per minute, and substance B initially weighs c grams and decays at a rate of d grams per minute.

d Write down rules for y_A and y_B in terms of t.

e Find in terms of a, b, c and d:

i the time when substance A and B are of the same weight

ii the weight of each substance A and B when they are of the same weight, expressing your answer as a fraction.

9 The length of a rectangular lawn is 3 m shorter than twice the width. There is a path around the lawn 0.75 m wide. Let the width of the lawn be x metres.

a Express the length of the lawn in terms of x.

b Express the area of the lawn in terms of x.

c **i** Show that the area A cm^2 of the path in terms of x is $A = \dfrac{9(2x-1)}{4}$.

ii If the area of the path is 20.25 m^2, find x.

iii Find the area of the lawn using the value of x obtained in part **ii**.

d If the area of the lawn is 90 m^2, find the corresponding value of x.

10 A jumbo jet does a return trip from Perth to Melbourne flying on the same course. On the trip from Perth to Melbourne, there is a tailwind and the jet averages 700 km/h. On the return trip, the jet averages 560 km/h The total travel time is 11 hours, including a two-hour stop in Melbourne. Let d kilometres be the distance between Melbourne and Perth.

a Express the time for the trip from Perth to Melbourne in terms of d.

b Express the time for the trip from Melbourne to Perth in terms of d.

c Express the total time for the return trip from Perth to Melbourne in terms of d.

d Hence calculate the total distance travelled.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

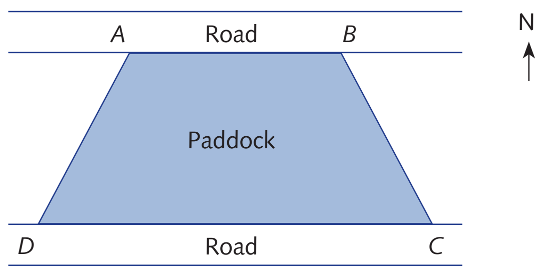
11 Jan travels at an average speed of x km/h and Len travels at an average speed of $(x + 4)$ km/h on a trip of 120 km.

a How long does it take Jan to complete the trip?

b How long does it take Len to complete the trip?

c Jan actually takes $1\frac{1}{2}$ hours longer than Len to complete the trip. Find Jan's average speed.

12 A farmer owns a paddock marked $ABCD$ in the diagram, which is located between two east–west roads. A is north-east of D. The fence AB is 1200 m, the fence AD is 1500 m and the fence from DC is 3200 m.



Calculate:

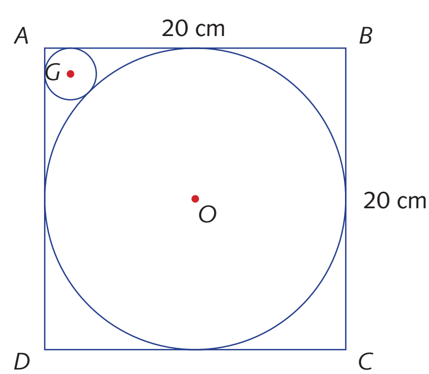
a the distance between the roads, to the nearest metre

b the area of the paddock in hectares, to the nearest hectare

c the bearing of the corner post B from the corner post C, correct to the nearest degree

13 A circle of radius 10 cm and centre O is placed inside the square $ABCD$ of side length 20 cm. A small circle with centre G is placed between a corner of the square and the circle so that it touches the circle and the two sides of the square.

Note: You may assume that the points O, G and A lie on the same line.



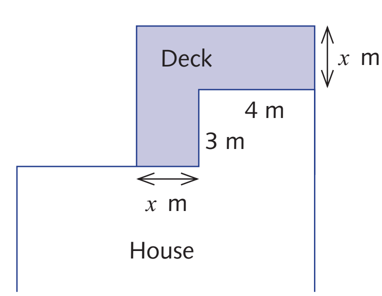
a Find the exact length of OA.

b If the radius of the small circle is r centimetres, find the length of GA in terms of r.

c Using $OA = OG + GA$, find OA in terms of r.

d Find the exact value of r.

14 A timber deck is built at the back of a house, as shown in the diagram. The measurements along the existing walls are 3 m and 4 m, as shown. The width of the deck is x metres, as shown.



a Express the area A m^2 of the deck in terms of x.

b If $A = 30$, find x.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Answers

Chapter 1 answers

Exercise 1A

1 21 **2** -60

3 **a** 1 **b** 34 **c** -15 **d** -13

4 **a** 7 **b** -13 **c** 1 **d** 19 **e** -19 **f** -1

5 **a** $-1\frac{1}{4}$ **b** $-1\frac{1}{6}$ **c** $1\frac{11}{20}$ **d** $\frac{11}{12}$

6 **a** 0.7 **b** -3.3 **c** 0.1 **d** 14.2

7 **a** 5 **b** 53 **c** 45 **d** 53 **e** $-1\frac{5}{9}$ **f** $-\frac{17}{50}$

8 **a** -72 **b** 68 **c** 14
d -18 **e** $-\frac{1}{2}$ **f** $-\frac{7}{9}$

9 **a** $3\frac{2}{3}$ **b** $-\frac{1}{3}$ **c** $\frac{1}{3}$ **d** $-3\frac{2}{3}$

10 **a** $2\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{12}$ **d** $-1\frac{5}{6}$

Exercise 1B

1 a, b, c, d, g, h, k, l, o, p, s, t

2 **a** $9a$ **b** $11b$ **c** $5c$ **d** $7d$ **e** $10x^2$
f $3a^2$ **g** $3a$ **h** $13f$ **i** $-7m$ **j** p
k $2a^2$ **l** $11m^2$ **m** $3ab$ **n** $-2a^2d$

3 **a** $5a$ **b** $3b$ **c** $4mn$ **d** $5pq$ **e** $3x^2$
f $5m^2$ **g** $10ab$ **h** $11m^2n$ **i** $8a^2b$ **j** $7\ell m$

4 **a** $13c + 4d$ **b** $12m + n$ **c** $11p + 4$
d $14 + 5m$ **e** $m - 8n$ **f** $13ab - b$
g $6a^2 + 7ab - 11b$ **h** $-5x^2 + 12x$ **i** $p^2 - 11p$

5 **a** $21xy$ **b** $16xy^2$ **c** $-x^2 - 10y$
d $13v^2z - 25z$ **e** $16yz + 4x$ **f** $7x^3 - 4y^3 + 5x^2$
g $13y^2 - 4x^2$ **h** $3x^2 + 2xy$ **i** $5a^2b - 8ab^2$

6 **a** $\frac{7x}{12}$ **b** $\frac{7a}{10}$ **c** $\frac{13c}{42}$
d $\frac{4z}{3}$ **e** $\frac{3c}{10}$ **f** $\frac{5x}{4}$

7 **a** $\frac{z}{6}$ **b** $\frac{2z}{15}$ **c** $\frac{x}{56}$
d $\frac{2x}{3}$ **e** $\frac{x}{8}$ **f** $\frac{6c}{7}$

8 **a** $\frac{11x}{12}$ **b** $\frac{22x}{21}$ **c** $\frac{5x}{4}$
d $\frac{13x}{6}$ **e** $\frac{3x}{22}$ **f** $\frac{x}{6}$
g $-\frac{7x}{33}$ **h** $\frac{43x}{12}$ **i** $\frac{x}{4}$

9 **b**

$9m + 8n$
$3m + 4n$ | $6m + 4n$
$m + 3n$ | $2m + n$ | $4m + 3n$

c

$15p - 3q$
$6p - 2q$ | $9p - q$
$2p - 3q$ | $4p + q$ | $5p - 2q$

d

$11a + 5b$
$5a + 3b$ | $6a + 2b$
$3a + 2b$ | $2a + b$ | $4a + b$

e

$11x + 2y$
$5x + 5y$ | $6x - 3y$
$4x - y$ | $x + 6y$ | $5x - 9y$

f

$7d + e$
$3d + 6e$ | $4d - 5e$
$-2d + 3e$ | $5d + 3e$ | $-d - 8e$

Exercise 1C

1 **a** $6a$ **b** $20b$ **c** $12ab$ **d** $10cd$
e $-20fg$ **f** $-12mn$ **g** $6pq$ **h** $30\ell n$
i a^2 **j** m^2 **k** $8a^2$ **l** $8m^2$
m $56a^2b$ **n** $6p^2q$ **o** $-10mn^2$ **p** $12cd^2e$

2 **a** 2 **b** 2 **c** $2b$ **d** $3n$
e $4a$ **f** $5p$ **g** $-2b$ **h** -8ℓ
i $3mn$ **j** $7de$ **k** $-5bc$ **l** $-5df$

3 **a** $3x$ **b** $9y$ **c** $3a^2$ **d** $2m$
e $4t$ **f** $5p$ **g** $7t$ **h** $9x$
i $6y$ **j** $-2x^2$ **k** $4a^2$ **l** $3x^2$

4 **a** $\frac{2x}{3}$ **b** $\frac{a}{3}$ **c** $-\frac{2m}{3}$ **d** $\frac{2p}{3}$
e $\frac{2x^2}{3}$ **f** $\frac{3xy}{4}$ **g** $-3a$ **h** $3b$
i $\frac{1}{3}$ **j** $-\frac{y}{2}$

5 **a**

$160a^2b^2$
$8ab$ | $20ab$
$2a$ | $4b$ | $5a$

b

$128mn^2p$
$16mn$ | $8np$
$4m$ | $4n$ | $2p$

c

$48a^2b$
$24a$ | $2ab$
24 | a | $2b$

d

$96m^2n^2q$
$16mn^2$ | $6mq$
$8n^2$ | $2m$ | $3q$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

6 a $\frac{3b}{2}$ b $\frac{2x}{5}$ c $\frac{y^2}{10}$
d $\frac{b^2}{6}$ e $\frac{1}{10a^2}$ f $\frac{3p^2}{8q^2}$
g $\frac{8}{y^2}$ h $\frac{2mn}{3}$ i $\frac{3y}{10z}$

7 a $\frac{3b}{2}$ b $\frac{4x}{5}$ c $\frac{8}{5}$ d $\frac{2}{27}$
e $3\frac{1}{3}$ or $\frac{10}{3}$ f $\frac{3}{8}$ g $\frac{2x}{5}$ h $\frac{y}{4}$
i $\frac{3}{10}$ j $\frac{7}{15n^2}$ k $-\frac{1}{4}$ l $-\frac{12y}{x^2}$

Exercise 1D

1 a $5x + 15$ b $2b + 14$ c $6a + 3$
d $12d + 30$ e $4a - 28$ f $3b - 15$
g $6d - 12$ h $8f - 32$ i $8f - 10$
j $6g - 18$ k $15p - 10$ l $30q - 6$

2 a $-2a - 8$ b $-3b - 18$ c $-12a - 42$
d $-12b - 27$ e $-6a + 2$ f $-24b + 42$
g $-10b + 35$ h $-21b + 14$ i $-12b + 20$
j $-20b + 35$ k $-27x - 18$ l $-48y + 60$

3 a $x + 3$ b $3x + 6$ c $8p + 4$
d $15q + 30$ e $3x - 12$ f $-5\ell + 3$
g $-4p + 5$ h $-6m + 10$ i $-20n + 80$

4 a $4x + \frac{1}{2}$ b $\frac{5y}{2} + \frac{1}{5}$ c $\frac{x}{10} + \frac{1}{5}$
d $\frac{3x}{14} + \frac{5}{21}$ e $\frac{y}{6} - \frac{1}{4}$ f $-\frac{a}{5} + \frac{2}{5}$

5 a $a^2 + 4a$ b $b^2 + 7b$ c $c^2 - 5c$
d $6g^2 - 10g$ e $20h^2 - 28h$ f $10i^2 + 14i$
g $12j^2 + 21j$ h $-5k^2 + 4k$ i $-3\ell^2 + \ell$
j $-10m^2 + 8m$ k $-15n^2 - 21n$ l $-12x^2 + 20x$

6 a $6a^2 + 3ab$ b $8c^2 - 4cd$ c $10d^2 - 20de$
d $6pq - 10pr$ e $-6x^2 - 15xy$ f $-6z^2 + 8yz$
g $6a + 8a^2b$ h $10m^2 - 20mn$ i $10x^2y + 15x$
j $6p - 15p^2q$ k $-12xy + 18y^2$ l $-30ab + 70b^2$

7 a Did not multiply the 6 by 4; $4(a + 6) = 4a + 24$
b Added 1 and 5 instead of multiplying; $5(a + 1) = 5a + 5$
c $-3 \times -5 = 15$, not -15; $-3(p - 5) = 3p + 15$
d $a \times a = a^2$, not $2a$; $a(a + b) = a^2 + ab$
e $2m \times 3m = 6m^2$, not $6m$; $2m(3m + 5) = 6m^2 + 10m$
f $4a \times 3a = 12a^2$, not $7a^2$; $4a(3a + 5) = 12a^2 + 20a$
g Did not multiply the 7 by $3a$; $3a(4a - 7) = 12a^2 - 21a$
h $-6 \times x = -6x$, not $6x$; $-6(x - 5) = -6x + 30$
i $3x \times -7y = -21xy$, not $-21y$; $3x(2x - 7y) = 6x^2 - 21xy$

8 a $8a + 23$ b $5b + 25$ c $2c + 5$
d $13g + 10$ e $7h + 4$ f $3i - 30$
g $j - 6$ h $8a^2 + 13a$ i $10b^2 - 9b$
j $15a^2 + 6a$ k $2b^2 - 15b$ l 10
m $4 + 3q$ n $4ab$ o $28m^2 + 9mn$

9 a $\frac{5x + 12}{6}$ b $\frac{7x + 6}{12}$ c $\frac{19x - 9}{15}$
d $\frac{19x - 40}{12}$ e $\frac{34x + 45}{21}$ f $\frac{24x - 5}{10}$
g $\frac{14x + 9}{12}$ h $\frac{-19x - 10}{10}$ i $\frac{24 - 35x}{12}$

10 a $5y + 14$ b $7x + 9$ c $16b - 9$
d $a + 1$ e $b - 22$ f -5
g $x^2 + x - 6$ h $2p^2 - 3p - 5$ i $12y^2 - 29y + 15$
j $4x^2 - 5x - 7$ k $6p^2 - 6p - 4$ l $10z$
m $4y^2 - 16y$ n $15z^2 - 10z$

11 a i $12 \times 99 = 12 \times (100 - 1) = 12 \times 100 - 12 \times 1$
$= 1200 - 12 = 1188$
ii $14 \times 53 = 14 \times (50 + 3) = 14 \times 50 + 14 \times 3$
$= 700 + 42 = 742$
b i $14 \times 21 = 14 \times (20 + 1) = 14 \times 20 + 14 \times 1$
$= 280 + 14 = 294$
ii $17 \times 101 = 17 \times (100 + 1) = 17 \times 100 + 17 \times 1$
$= 1700 + 17 = 1717$
iii $70 \times 29 = 70 \times (30 - 1) = 70 \times 30 - 70 \times 1$
$= 2100 - 70 = 2030$
iv $8 \times 121 = 8 \times (100 + 21) = 8 \times 100 + 8 \times 21$
$= 800 + 168 = 968$
v $13 \times 72 = 13 \times (70 + 2) = 13 \times 70 + 13 \times 2$
$= 910 + 26 = 936$
vi $17 \times 201 = 17 \times (200 + 1) = 17 \times 200 + 17 \times 1$
$= 3400 + 17 = 3417$

12 a $x^3 + 3x$ b $x^3 + 2x^2 + x$ c $2x^3 - 6x^2$
d $6x^2 - 2x^3$ e $15a^2 + 5a$ f $6a^2 + 12a^3 - 6a^4$

Exercise 1E

1 a $x^2 + 7x + 12$ b $a^2 + 13a + 40$ c $a^2 + 12a + 27$
d $x^2 + 8x + 15$ e $x^2 + 3x + 2$ f $a^2 + 17a + 72$

2 a $x^2 - 6x + 5$ b $x^2 - 5x + 6$ c $p^2 - 2p - 24$
d $x^2 - x - 42$ e $x^2 + 8x - 33$ f $x^2 - 5x - 24$
g $x^2 - 3x - 54$ h $x^2 - 18x + 77$ i $x^2 + 3x - 28$

3 a $6x^2 + 17x + 12$ b $15x^2 + 22x + 8$ c $5x^2 + 11x + 2$
d $2a^2 - 11a + 15$ e $6ab - 2b + 15a - 5$ f $8m^2 + 2m - 3$
g $6p^2 + 11p - 10$ h $15x^2 - 46x + 16$ i $6x^2 - 23x + 7$
j $6x^2 + 7x - 10$ k $7x^2 - 26x - 45$ l $8b^2 + 8b - 6$

4 a $2a^2 + 7ab + 3b^2$ b $2m^2 + 7mn + 3n^2$
c $8c^2 - 10cd - 3d^2$ d $8x^2 + 6xy - 5y^2$
e $3x^2 + 13ax - 10a^2$ f $6x^2 + 13xy - 5y^2$
g $3b^2 + 5ab - 2a^2$ h $15q^2 - 4pq - 4p^2$
i $16pq - 4p^2 - 15q^2$

5 a $x^2 + 7x + 10$ b $x^2 - 4x - 21$ c $x^2 - 10x + 24$
d $2x^2 + 7x + 3$ e $3x^2 + 17x + 10$ f $12x^2 - x - 1$
g $6x^2 - 19x + 15$ h $15x^2 + 29x - 14$ i $8x^2 + 2x - 3$
j $12x^2 - x - 35$ k $4x^2 - 1$ l $2x^2 - 3x - 20$

6 a $x + 1$ b $x + 3$ c $x - 2$ d $x - 5$
e $3x - 2$ f $x - 1$ g $2x - 7$ h $5x - 6$

7 a 12, 35 b 3, 18 c 6, 24 d 2, 6
e 3, 13 f 2, 7, 5 g 2, 1, 5 h 4, 3, 2

8 a $x^2 - 9$ b $x^2 + 6x + 9$ c $x^2 - 10x + 25$
d $49x^2 - 1$ e $2x^2 + x - 15$ f $49x^2 + 14x + 1$
g $4 - x^2$ h $4x^2 + 12x + 9$ i $25a^2 - 1$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

9 a $\frac{a^2}{6}+\frac{7a}{6}+2$ b $\frac{2b^2}{15}-\frac{14b}{15}-4$ c $\frac{2x^2}{25}-\frac{7x}{10}-1$
d $\frac{y^2}{12}+\frac{13y}{16}-\frac{9}{4}$ e $\frac{m^2}{2}-\frac{19m}{12}-3$ f $\frac{b^2}{4}-\frac{117b}{200}-\frac{1}{10}$

Exercise 1F

1 a x^2+2x+1 b $x^2+10x+25$ c $x^2+12x+36$
d $x^2+40x+400$ e $a^2+16a+64$ f x^2+4x+4
g $x^2+18x+81$ h $x^2+2ax+a^2$

2 a $x^2-8x+16$ b $x^2-14x+49$ c $x^2-12x+36$
d x^2-2x+1 e $x^2-10x+25$ f $x^2-40x+400$
g $x^2-22x+121$ h $x^2-2ax+a^2$

3 a $9x^2+12x+4$ b $4a^2+4ab+b^2$
c $4a^2+12ab+9b^2$ d $9a^2+24ab+16b^2$
e $4x^2+12xy+9y^2$ f $4a^2+12a+9$
g $9x^2+6ax+a^2$ h $25x^2+40xy+16y^2$

4 a $9x^2-12x+4$ b $16x^2-24x+9$
c $4a^2-4ab+b^2$ d $4a^2-12ab+9b^2$
e $9a^2-24ab+16b^2$ f $4x^2-12xy+9y^2$
g $9c^2-6bc+b^2$ h $16x^2-40x+25$

5 a $\frac{x^2}{4}+3x+9$ b $\frac{x^2}{9}-\frac{4x}{3}+4$
c $\frac{4x^2}{25}-\frac{4x}{5}+1$ d $\frac{9x^2}{16}+x+\frac{4}{9}$

6 a no
c two 6 cm × 10 cm rectangles of paper

7 $(6+4)^2$ does not equal 6^2+4^2;
$(6+4)^2=6^2+2\times4\times6+4^2$

8 a The large square has side length $(a+b)$, so its area is $(a+b)^2$. The large square consists of four rectangles and a smaller square. Each rectangle has an area of ab and the smaller square has an area of $(a-b)^2$. So the area of the large square may also be written as $(a-b)^2+4ab$.
b $(a-b)^2+4ab=a^2-2ab+b^2+4ab=a^2+2ab+b^2$ and $(a+b)^2=a^2+2ab+b^2$

9 a 961 b 361 c 1764 d 324
e 2601 f 10 201 g 9801 h 40 401
i 90 601 j 39 601

10 a 1.0201 b 0.9801 c 16.0801 d 16.1604
e 0.998 001 f 1.0404 g 9.0601 h 0.9604

11 a $2x^2-12x+20$ b $2x^2+8$ c $5x^2-14x+10$
d $8x^2+50$ e $2x^2+2x+41$ f $5x^2-22x+25$

12 a $4x^2+12x+14$ b $4x^2-12x+14$ c $4x-6$

13 a $\frac{x^2}{2}+2$ b $\frac{5x^2}{9}+\frac{10x}{3}+10$
c $\frac{13x^2}{16}+6x+13$ d $\frac{x^2}{5}+\frac{5}{16}$

14 c and d

Exercise 1G

1 a x^2-16 b x^2-49 c a^2-1
d a^2-81 e c^2-9 f d^2-4
g z^2-49 h $100-x^2$ i x^2-25

2 a $4x^2-1$ b $9x^2-4$ c $16a^2-25$
d $9x^2-25$ e $4x^2-49$ f $25a^2-4$
g $4r^2-9s^2$ h $4x^2-9y^2$ i $25a^2-4b^2$

3 a $\frac{4x^2}{9}-1$ b $\frac{x^2}{4}-9$ c $\frac{x^2}{9}-\frac{1}{4}$ d $\frac{4x^2}{25}-\frac{9}{16}$

4 **a**, **c**, **d** are difference of squares expansions; **b**, **e**, **f** are perfect square expansions.

5 a 399 b 899 c 396 d 896
e 391 f 3599 g 9999 h 9996

6 a 0.9999 b 24.9999 c 63.9996 d 0.9975
e 99.9999 f 399.99

7 a $a-b$ b $a+b$ c $(a-b)(a+b)$
e $(a-b)(a+b)=a^2+ab-ab-b^2=a^2-b^2$

Exercise 1H

1 a $-4x-12$ b $2x+11$ c $32-8a$
d -24 e $2x^2-4$ f $7b^2+20$
g $a^2+ab-3b$ h $2y^2-4xy$

2 a 6 b -242 c $-\frac{3}{2}$ d $-\frac{35}{9}$

3 a $\frac{11}{8}$ b $\frac{8}{9}$ c 1 d $\frac{1}{2}$

4 a $\frac{7}{8}$ b $-\frac{3}{8}$ c $\frac{41}{12}$ d $\frac{77}{18}$

5 a 4 b 4 c $\frac{196}{81}$ d 0 e 1

6 a $\frac{23a^2}{21}$ b $\frac{11a}{2b}$ c $\frac{13x}{3y^3}$ d $\frac{6m}{n^2}$

7 a $\frac{a^2b}{10}$ b $\frac{x^2}{3y}$ c $\frac{2m}{n^2}$ d $\frac{3p^3}{2n^2}$

8 a 5 b $9a^2$ c $4b^2$
d $a^2+2ab+b^2$ e $9m^2-12mn+4n^2$

9 a $9a^2+9a$ b $4b^2-6b$
c $a^2-2ab+b^2+3a-3b$ d $4m^2+4mn+n^2+6m+3n$

10 a $9a^2-6a+1$ b $4b^2+4b+1$
c $4a^2$ d $16b^2-8b+1$

11 a $2x^3+3x^2+4x$ b $3a^3-12a^2+3a$
c m^3+5m^2+8m+4 d $2p^3-p^2-12p-9$
e x^3-1 f x^3+1

12 $3+\sqrt{15}$

13 a $x^2-y^2+2yz-z^2$ b $a^2-b^2+2bc-c^2$
c $4x^2-y^2+2yz-z^2$ d $x^2+2xz+z^2+2x+2z+1$
e $x^2+y^2+z^2+2xy+2xz+2yz$
f $x^2+y^2+z^2+2xy-2xz-2yz$

14 a $\frac{12}{5}$ b $\frac{50}{3}$ c $\frac{2}{5}$ d $\frac{2}{7}$
e $\frac{12}{17}$ f $\frac{2}{x+y}$

15 a i $\frac{a}{c}=3,\ \frac{a-b}{b-c}=3$ ii $\frac{a}{c}=\frac{3}{2},\ \frac{a-b}{b-c}=\frac{3}{2}$
iii $\frac{a}{c}=3,\ \frac{a-b}{b-c}=3$

16 a $a+b-c$ b $b-a-6c$ c a
d $a-b-c$ e $3x+4y$

17 a $6a^2-2bc$ b $2a^3$ c $\frac{1}{a^4}-\frac{2}{a^2b^2}+\frac{1}{b^4}$
d $a^4-2+\frac{1}{a^4}$ e $3a^2+6$ f $ab-\frac{1}{ab}$
g 0

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Review exercise

1 a 9 b 31 c −14 d −18

2 a 15 b −15 c 3 d 27
e −27 f −3

3 a 3 b 8 c 63 d $-\frac{8}{9}$ e $\frac{16}{9}$ f 3.41

4 a $9a$ b $11b$ c $3a$ d $6a$
e $13x$ f $-2a^2$

5 a $9a$ b $2b$ c $13mn$ d $6pq$
e $13ab$ f $11m^2n$ g $13a^2b$ h $11\ell m$
i $21xy^2$

6 a $3x+6$ b $4b+24$ c $15b-10$
d $-12d-30$ e $8x-30$ f $12g-18$
g $-6p-6$ h $-15b+25$ i $-20b+35$
j $7g+30$ k $11h+5$ l $3x-16$
m $y-2$ n $10a^2+8a$ o $-10b^2+31b$
p $6z-8y+2$ q z^2+2z-6 r $12y^2-40y+25$

7 a x^2+6x+8 b $a^2-15a+44$ c $a^2-10a+25$
d $p^2-3p-28$ e $10x^2+14x+4$ f $8x^2-14x+5$
g $44x^2-62x-6$ h $8x^2+2x-3$ i $49a^2+14a-3$

8 a $x^2+22x+121$ b $x^2+12x+36$
c $x^2-30x+225$ d $x^2-20x+100$
e $x^2-4xy+4y^2$ f $4a^2+20ab+25b^2$
g $25x^2+20x+4$ h $25x^2-60x+36$

9 a x^2-36 b z^2-49 c p^2-1
d $25x^2-1$ e $49x^2-25$ f $100-9a^2$
g $25a^2-4b^2$ h $144x^2-y^2$ i $64x^2-9a^2$

10 a $6x^2-xy-y^2$ b $9x^2-6ax-8a^2$
c $15c^2+32bc+16b^2$ d $9x^2+30xy+25y^2$
e $a^2-4ab+4b^2$ f $25\ell^2+20\ell m+4m^2$
g $9x^2-y^2$ h $25m^2-4n^2$
i $9x^2+30ax+25a^2$

11 a $\frac{a^2}{6}-\frac{2a}{3}-2$ b $\frac{2x^2}{3}+\frac{10x}{3}-24$
c $\frac{a^2}{4}-1$ d $\frac{6x^2}{25}+\frac{7x}{5}-3$
e $\frac{5b^2}{6}-\frac{4b}{3}-6$ f $\frac{a^2}{3}+2a-24$

Challenge exercise

1 a $2(x)+2(x+3)=4x+6$ b 14 cm c 7.5
d $(x^2+3x)\text{ cm}^2$ e 18 cm^2

2 a $(3x+9)$ cm
b $\left(\frac{3}{2}x+6\right)$ cm
c $\left(\frac{9x^2}{2}+\frac{63x}{2}+54\right)\text{ cm}^2$
d $\left(\frac{9x^2}{2}+\frac{63x}{2}+54\right)-(2x+6)(x+4)=\frac{5}{2}x^2+\frac{35}{2}x+30$
e $\frac{5}{4}$

3 $ad+ae+af+bd+be+bf+cd+ce+cf$

	a	*b*	*c*
d	*da*	*db*	*dc*
e	*ea*	*eb*	*ec*
f	*fa*	*fb*	*fc*

4 a i 88 ii 90
b product of inner pair − product of outer pair = 2
c $n+1,\ n+2,\ n+3$
d i n^2+3n ii n^2+3n+2
iii $n^2+3n+2-(n^2+3n)=2$

5 a i 72 ii 80
b $n+2,\ n+4,\ n+6$
c i n^2+6n ii n^2+6n+8 iii 8

7 a x^4+x^2+1 b x^3+1 c $x^{10}-1$

8 a x^3-1 b x^5-1 c $x^{10}-1$

9 b 13^2+11^2

10 a i 25 ii 121 iii 361 iv 841
b $n^4+2n^3-n^2-2n+1$

Chapter 2 answers

Exercise 2A

1 a $x=10$ b $a=13$ c $b=2$ d $a=58$

2 a $b=4$ b $x=8$ c $y=27$
d $y=45$ e $b=24$ f $x=14$

3 a 4.36 b 6.08 c 7.81 d 27.06

4 a $x=\sqrt{34}\approx 5.83$ b $y=\sqrt{80}\approx 8.94$
c $x=\sqrt{45}\approx 6.71$ d $y=\sqrt{72}\approx 8.49$
e $a=\sqrt{340}\approx 18.44$ f $b=\sqrt{51}\approx 7.14$
g $x=\sqrt{58}\approx 7.62$ h $y=2\sqrt{28}\approx 10.58$

5 a yes b yes c no
d no e no f yes

6 a 5 cm b $\sqrt{61}$ cm c $\sqrt{65}$ cm
d $\sqrt{136}$ cm e $\sqrt{51}$ cm

7 no 8 2.332 m 9 1.2 m

10 a 5.14 m b 5.35 m c 15.63 m

11 a 2.8 cm b 4.2 cm

12 70.7 cm 13 25.3 m 14 3.8 m

15 b $A=\frac{1}{2}\times 24\times 10=120=\frac{1}{2}\times 26\times h=13h$
c $x=\frac{50}{13}$

Exercise 2B

1 a 7 b 3 c 11 d 231

2 a 12 b 48 c 50 d 45 e 28
f 147 g 242 h 44

3 a $\sqrt{15}$ b $\sqrt{12}$ c $\sqrt{30}$ d $\sqrt{33}$
e $\sqrt{51}$ f $\sqrt{45}$ g $\sqrt{42}$ h $\sqrt{190}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 **a** $\sqrt{3}$ **b** $\sqrt{5}$ **c** $\sqrt{13}$
d $\sqrt{23}$ **e** $\sqrt{7}$ **f** $\sqrt{\frac{20}{9}} = \frac{2\sqrt{5}}{3}$
g $\sqrt{\frac{1}{7}}$ **h** $\sqrt{\frac{2}{3}}$

5 **a** $6\sqrt{2}$ **b** $12\sqrt{5}$ **c** $33\sqrt{7}$ **d** $30\sqrt{7}$
e $45\sqrt{2}$ **f** $35\sqrt{6}$ **g** $12\sqrt{11}$ **h** $28\sqrt{3}$

6 **a** $2\sqrt{2}$ **b** $5\sqrt{5}$ **c** 3
d 4 **e** 13 **f** 16
g 6 **h** 6 **i** 8

7 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $3\sqrt{5}$
d $2\sqrt{6}$ **e** $3\sqrt{3}$ **f** $2\sqrt{11}$
g $5\sqrt{2}$ **h** $3\sqrt{6}$ **i** $2\sqrt{5}$
j $7\sqrt{2}$ **k** $3\sqrt{7}$ **l** $2\sqrt{15}$
m $3\sqrt{14}$ **n** $2\sqrt{17}$ **o** $5\sqrt{3}$
p $3\sqrt{11}$ **q** $2\sqrt{7}$ **r** $11\sqrt{2}$

8 **a** $6\sqrt{2}$ **b** $4\sqrt{2}$ **c** $4\sqrt{5}$
d $12\sqrt{2}$ **e** $4\sqrt{3}$ **f** $6\sqrt{5}$
g $4\sqrt{7}$ **h** $6\sqrt{6}$ **i** $4\sqrt{6}$
j $6\sqrt{7}$ **k** $4\sqrt{10}$ **l** $8\sqrt{2}$
m $8\sqrt{5}$ **n** $4\sqrt{11}$ **o** $8\sqrt{3}$
p $10\sqrt{2}$ **q** $9\sqrt{2}$ **r** $9\sqrt{3}$

9 **a** $\sqrt{12}$ **b** $\sqrt{108}$ **c** $\sqrt{98}$ **d** $\sqrt{54}$
e $\sqrt{80}$ **f** $\sqrt{175}$ **g** $\sqrt{48}$ **h** $\sqrt{52}$
i $\sqrt{396}$ **j** $\sqrt{1440}$ **k** $\sqrt{700}$ **l** $\sqrt{176}$

10 **a** $\frac{2}{3}$ **b** $\frac{2}{5}$ **c** $\frac{4}{5}$ **d** $\frac{5}{6}$
e $\frac{5}{11}$ **f** $\frac{5}{11}$ **g** $\frac{7}{11}$ **h** $\frac{12}{13}$

11 **a** $2\sqrt{2}$ **b** 3 **c** $2\sqrt{5}$
d $6\sqrt{2}$ **e** $3\sqrt{10}$ **f** $\sqrt{34}$
g $5\sqrt{3}$ **h** 3 **i** $10\sqrt{21}$

12 $3\sqrt{5}$

13 **a** 12 **b** $2\sqrt{6}$

14 $8\sqrt{2}$ **15** $\sqrt{3}$

Exercise 2C

1 **a** 0 **b** $10\sqrt{2}$ **c** $5\sqrt{3}$
d $7\sqrt{2}$ **e** $5\sqrt{7}$ **f** $43\sqrt{5}$
g $\sqrt{3}$ **h** $-2\sqrt{11}$ **i** $5\sqrt{2}$

2 **a** $6\sqrt{2}$ **b** $-15\sqrt{6}$ **c** $3\sqrt{5}$ **d** $9\sqrt{13}$
e $-2\sqrt{7}$ **f** 0 **g** $\sqrt{6}+7\sqrt{2}$ **h** $2\sqrt{5}+3\sqrt{3}$

3 **a** $7-3\sqrt{3}$ **b** $3\sqrt{3}-\sqrt{2}$ **c** $-11\sqrt{6}+20$
d $6\sqrt{14}+7\sqrt{6}$ **e** $-\sqrt{7}+2\sqrt{14}$ **f** $-3\sqrt{5}+4\sqrt{2}$

4 **a** $3\sqrt{2}$ **b** $\sqrt{2}$ **c** $8\sqrt{2}$
d $5\sqrt{2}$ **e** $5\sqrt{3}$ **f** $6\sqrt{3}$
g $6\sqrt{5}$ **h** $5\sqrt{7}$ **i** $12\sqrt{2}$

5 **a** $\sqrt{2}$ **b** $6\sqrt{3}$ **c** $6\sqrt{2}$
d $\sqrt{3}$ **e** $9\sqrt{2}$ **f** $-7\sqrt{5}$
g $5\sqrt{6}$ **h** $4\sqrt{3}$ **i** $2\sqrt{5}$
j $11\sqrt{2}$ **k** $5\sqrt{10}$ **l** $28\sqrt{2}$

6 **a** $\frac{5\sqrt{2}}{6}$ **b** $\frac{9\sqrt{3}}{10}$ **c** $\frac{3\sqrt{7}}{10}$ **d** $\frac{11\sqrt{11}}{21}$
e $\frac{5\sqrt{2}}{6}$ **f** $\frac{6\sqrt{6}}{5}$ **g** $\frac{\sqrt{3}}{10}$ **h** $\frac{6\sqrt{2}}{35}$

7 **a** 7 **b** 5 **c** 6

8 **a** $\sqrt{13}; 5+\sqrt{13}$ **b** $2\sqrt{10}; 2\sqrt{10}+8$
c $\sqrt{21}; 7+\sqrt{21}$ **d** $\sqrt{15}; \sqrt{15}+3\sqrt{3}$

9 **a** $3\sqrt{3}$ **b** $4\sqrt{2}$ **c** $\frac{2\sqrt{5}}{3}$ **d** $\frac{8\sqrt{2}}{3}$

10 **a** $\sqrt{7}$ **b** $3\sqrt{7}$ **c** $4\sqrt{7}$

11 $BA = 2\sqrt{7}$; perimeter $= 12\sqrt{7}$ **12** $28\sqrt{3}$

Exercise 2D

1 **a** $\sqrt{6}$ **b** $\sqrt{10}$ **c** $2\sqrt{21}$ **d** $4\sqrt{14}$
e $6\sqrt{6}$ **f** $8\sqrt{10}$ **g** $15\sqrt{14}$ **h** $24\sqrt{21}$

2 **a** 1 **b** 8 **c** 3 **d** $\sqrt{7}$
e $4\sqrt{3}$ **f** $4\sqrt{11}$ **g** $\sqrt{5}$ **h** 3
i $9\sqrt{5}$ **j** $4\sqrt{2}$ **k** $10\sqrt{5}$ **l** $\frac{7}{2}$

3 **a** $3\sqrt{2}$ **b** 6 **c** 14 **d** $20\sqrt{2}$
e $8\sqrt{3}$ **f** $12\sqrt{3}$ **g** $42\sqrt{5}$ **h** $42\sqrt{2}$

4 **a** $3\sqrt{2}+3$ **b** $4-2\sqrt{3}$ **c** $2\sqrt{10}-10$
d $18-3\sqrt{3}$ **e** $20\sqrt{10}-15\sqrt{5}$ **f** $14-7\sqrt{2}$
g $40\sqrt{3}-12\sqrt{15}$ **h** $18-10\sqrt{3}$ **i** $9\sqrt{2}+12\sqrt{6}$
j $6\sqrt{10}+6\sqrt{5}$ **k** $6\sqrt{7}-21\sqrt{2}$ **l** $15\sqrt{3}+9\sqrt{5}$

5 **a** $62+11\sqrt{5}$ **b** $68+24\sqrt{6}$ **c** $16+3\sqrt{2}$
d $-23+\sqrt{5}$ **e** $-2-2\sqrt{3}$ **f** $107-11\sqrt{7}$
g $123+70\sqrt{2}$ **h** $52-16\sqrt{3}$

6 **a** $3\sqrt{10}+2\sqrt{2}+9\sqrt{5}+6$ **b** $6\sqrt{6}-9\sqrt{2}+2\sqrt{3}-3$
c $8\sqrt{7}-20+4\sqrt{21}-10\sqrt{3}$ **d** $16-8\sqrt{7}-8\sqrt{6}+4\sqrt{42}$
e 2 **f** $4\sqrt{6}+2$
g $7\sqrt{35}+18$ **h** $7\sqrt{10}-2$

7 **a** $\sqrt{6}-\sqrt{3}+\sqrt{2}-1$ **b** $\sqrt{2}+\sqrt{3}$
c 1 **d** $-\sqrt{3}-\sqrt{2}$
e $2\sqrt{6}+2\sqrt{2}-2\sqrt{3}-2$ **f** $\sqrt{2}+3\sqrt{3}+2$
g $\sqrt{6}+2\sqrt{2}$ **h** $6\sqrt{5}$
i $2\sqrt{2}$ **j** 2
k $2\sqrt{2}$

Exercise 2E

1 **a** $10\sqrt{3}+28$ **b** $12\sqrt{2}+38$
c $16\sqrt{5}+36$ **d** $6\sqrt{3}+28$
e $2\sqrt{35}+12$ **f** $4\sqrt{6}+14$
g $30\sqrt{6}+77$ **h** $6\sqrt{10}+47$
i $16\sqrt{15}+68$ **j** $x+2\sqrt{xy}+y$
k $a^2x+2ab\sqrt{xy}+b^2y$ **l** $xy+2\sqrt{xy}+1$

2 **a** $11-4\sqrt{7}$ **b** $19-8\sqrt{3}$ **c** $21-4\sqrt{5}$
d $31-12\sqrt{3}$ **e** $8-2\sqrt{15}$ **f** $14-4\sqrt{6}$
g $82-8\sqrt{10}$ **h** $95-24\sqrt{14}$ **i** $x-2\sqrt{xy}+y$

3 **a** $80\sqrt{2}+120$ **b** $60\sqrt{2}+162$ **c** $6\sqrt{7}+24$
d $20\sqrt{7}+145$ **e** $42-8\sqrt{5}$ **f** $28-12\sqrt{5}$
g $160-60\sqrt{7}$ **h** $95-30\sqrt{10}$ **i** $99-44\sqrt{2}$
j $15-10\sqrt{2}$ **k** $18-12\sqrt{2}$ **l** $539-210\sqrt{6}$

4 **a** 4 **b** 5 **c** 89 **d** 1
e 2 **f** 10 **g** -43 **h** -13
i 44 **j** -138 **k** -470 **l** $x-y$

5 **a** $3-2\sqrt{2}$ **b** $3+2\sqrt{2}$ **c** 6 **d** $-4\sqrt{2}$
e 1 **f** $\sqrt{2}-1$ **g** $\sqrt{2}+1$ **h** $2\sqrt{2}$

6 **a** 16 **b** $4\sqrt{15}$ **c** 10
d $4\sqrt{6}$ **e** 18 **f** $8\sqrt{6}$

7 a^2b-c^2d

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Updated August 2021

8 **a** **i** $\sqrt{14}$ **ii** $\frac{1}{2}$ **iii** $4+\sqrt{14}$

b **i** 4 **ii** 2 **iii** $4+2\sqrt{6}$

c **i** 6 **ii** 6 **iii** $2\sqrt{15}+6$

9 **a** **i** $16+4\sqrt{3}$ **ii** $19+8\sqrt{3}$

b **i** 16 **ii** 13

10 **a** $3+3\sqrt{3}$ **b** $\sqrt{3}+\frac{3}{2}$

Exercise 2F

1 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{5\sqrt{6}}{6}$ **c** $\sqrt{3}$ **d** $2\sqrt{7}$

e $\frac{\sqrt{21}}{7}$ **f** $\sqrt{3}$ **g** $\frac{\sqrt{15}}{3}$ **h** $\sqrt{2}$

i $\sqrt{2}$ **j** $\frac{2\sqrt{21}}{3}$

2 **a** $\frac{\sqrt{3}}{15}$ **b** $\frac{7\sqrt{2}}{6}$ **c** $\frac{2\sqrt{2}}{7}$ **d** $\frac{\sqrt{35}}{21}$

e $\frac{\sqrt{5}}{15}$ **f** $\frac{\sqrt{10}}{3}$ **g** $\frac{4\sqrt{14}}{35}$

h $4\sqrt{6}$ **i** $\frac{\sqrt{2}}{8}$ **j** $\frac{\sqrt{3}}{3}$

3 **a** 0.71 **b** 0.58 **c** 2.12

d 2.89 **e** 3.46 **f** 4.24

g 0.29 **h** 0.24

4 **a** $\frac{3\sqrt{2}}{2}$ **b** $\frac{7\sqrt{3}}{6}$ **c** $\frac{5\sqrt{2}-\sqrt{3}}{3}$

d $\sqrt{10}$ **e** $2\sqrt{6}+\sqrt{2}$ **f** $\frac{13\sqrt{3}}{6}$

5 **a** $\frac{\sqrt{7}}{2}$ **b** $\frac{2\sqrt{7}}{7}$ **c** $\sqrt{7}$ **d** $\frac{\sqrt{42}}{6}$

6 **a** $\frac{13\sqrt{3}}{6}$ **b** $\frac{11\sqrt{3}}{6}$ **c** $\frac{169}{12}$ **d** $\frac{121}{12}$

7 **a** $\frac{5\sqrt{2}}{2}$ **b** $2\sqrt{5}$ **c** $\frac{30\sqrt{13}}{13}$

Exercise 2G

1 **a** **i** 12 cm **ii** 90°

b 13 cm **c** 14.3 cm

2 **a** 17 cm **b** $\sqrt{233}\approx 15.3$ cm

c $\sqrt{165}\approx 12.8$ cm **d** $\sqrt{116}\approx 10.8$ cm

e $\sqrt{62}\approx 7.9$ cm **f** $\sqrt{a^2+b^2+c^2}$ cm

3 $\sqrt{241}\approx 15.5$ cm **4** 2 cm

5 $\frac{\sqrt{1969}}{5}\approx 8.87$ m **6** $\sqrt{99}\approx 9.95$ cm

7 **a** $2\sqrt{5}$ cm **b** $\sqrt{5}$ cm

c **i** 3 cm **ii** 5 cm

d right-angled triangle

e $BE=\frac{3a}{2}$ cm, $EH=2a=\frac{4a}{2}$ cm, $BH=\frac{5a}{2}$ cm

Exercise 2H

1 **a** 2 **b** 17 **c** 50 **d** 13

2 **a** $\frac{\sqrt{6}-1}{5}$ **b** $3\sqrt{2}+3$ **c** $\frac{3-\sqrt{5}}{2}$

d $\frac{\sqrt{5}+3}{2}$ **e** $\frac{4\sqrt{5}-4\sqrt{2}}{3}$ **f** $\frac{\sqrt{7}+\sqrt{5}}{2}$

g $3\sqrt{2}-\sqrt{15}$ **h** $-2-\sqrt{6}$ **i** $\frac{\sqrt{10}-1}{9}$

j $\frac{\sqrt{15}+10}{17}$ **k** $\frac{4\sqrt{15}-3\sqrt{10}}{30}$ **l** $\frac{-2-3\sqrt{2}}{14}$

m $\frac{-6\sqrt{42}-16\sqrt{3}}{31}$ **n** $\frac{33\sqrt{10}-18\sqrt{6}}{497}$ **o** $\frac{10-\sqrt{6}}{47}$

3 **a** 2.41 **b** 0.27 **c** 0.32 **d** 3.15

4 $p=5,\ q=2$

5 **a** $5\sqrt{5}+2$ **b** $\frac{7}{3}$

6 **a** 8 **b** $-2\sqrt{15}$ **c** $\frac{9+2\sqrt{14}}{5}$

Exercise 2I

1 **a** $\frac{5}{9}$ **b** $\frac{7}{9}$ **c** 1 **d** $\frac{13}{90}$ **e** $\frac{7}{30}$ **f** $\frac{28}{45}$

2 **a** $\frac{13}{99}$ **b** $\frac{7}{99}$ **c** $\frac{91}{99}$

d $\frac{241}{999}$ **e** $\frac{613}{999}$ **f** $\frac{8}{495}$

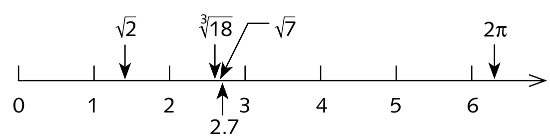
g $\frac{107}{330}$ **h** $\frac{203}{396}$ **i** $\frac{1}{825}$

3 **a** $\frac{11}{3}$ **b** $\frac{103}{20}$ **c** $\frac{4}{9}$

d $\frac{5}{33}$ **e** $\frac{36}{7}$ **f** $\frac{13}{10}$

4 **a** yes **b** no **c** no **d** yes

5



To 2 decimal places:

$\sqrt{7}\approx 2.65$, $\sqrt[3]{18}\approx 2.62$, $\sqrt{2}\approx 1.41$, $2\pi\approx 6.28$

6 **a** no **b** yes **c** no **d** no

e yes **f** no **g** yes **h** no

i yes **j** yes **k** no **l** no

8 **a** A triangle with sides 8, 3, $\sqrt{73}$

b A triangle with sides 2, 4, $\sqrt{12}$

c A triangle with sides 2, $\sqrt{21}$, 5

9 **b** $13+\cfrac{1}{1+\cfrac{1}{10}}$

Review exercise

1 **a** $h=10$ **b** $h=6.5$

2 **a** $x=7.5$ **b** $y=1.75$

3 28.3 cm **4** 25 km

5 **a** 8.49 cm **b** 4.24 cm **c** 21.73 cm **d** 15.36 cm

6 **a** $5\sqrt{2}$ **b** $\sqrt{2}$

7 **a** $30\sqrt{2}$ **b** $30\sqrt{2}$

8 **a** $6\sqrt{3}+6$ **b** $30-10\sqrt{2}$

9 **a** $10-\sqrt{2}$ **b** $32-9\sqrt{3}$

10 **a** $4\sqrt{5}$ **b** $6\sqrt{3}$ **c** $5\sqrt{5}$ **d** $6\sqrt{2}$

e $32\sqrt{2}$ **f** $8\sqrt{7}$ **g** $20\sqrt{2}$ **h** $4\sqrt{7}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11 **a** $\sqrt{5}$ **b** $3\sqrt{7}$ **c** $3\sqrt{11}$ **d** $11\sqrt{2}$

12 **a** $\frac{3\sqrt{11}}{11}$ **b** $\frac{\sqrt{15}}{75}$ **c** $\frac{4\sqrt{7}}{49}$ **d** $\frac{3\sqrt{17}}{17}$
e $\sqrt{3}$ **f** $\frac{\sqrt{15}}{25}$ **g** $2\sqrt{7}$ **h** $\frac{11\sqrt{3}}{3}$

13 **a** $\frac{\sqrt{3}+3}{2}$ **b** $\frac{22(3\sqrt{5}-2)}{41}$ **c** $-4\sqrt{7}-4$
d $\frac{3\sqrt{17}-3}{2}$ **e** $3\sqrt{3}+6$ **f** $-3\sqrt{11}-3$
g $-5\sqrt{7}-10$ **h** $20+10\sqrt{3}$

14 **a** $23-4\sqrt{15}$ **b** $-\frac{4\sqrt{3}}{3}-4$

15 **a** $\frac{\sqrt{89}}{2}$ m **b** $\frac{3\sqrt{21}}{2}$ m

16 $5\sqrt{3}$ cm

17 **a** $\sqrt{64+x^2}$ **b** $20-x$ **c** 8.4 km

18 **a** 6 cm **b i** $(12-r)$ cm **ii** $(6+r)$ cm
c $r=4$

19 **a** equilateral **b i** $5\sqrt{3}$ cm **ii** $5\sqrt{2}$ cm

20 **a** $\frac{8}{9}$ **b** $\frac{7}{45}$ **c** $\frac{13}{18}$ **d** $\frac{1}{30}$
e $\frac{9}{11}$ **f** $\frac{32}{33}$ **g** $\frac{101}{999}$ **h** $\frac{1}{660}$

21 $2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{2}}}$

22 $\frac{479\,281}{999\,999}=\frac{43\,571}{90\,909}$

24 **a** 7 **b** 8

25 **a** $\sqrt{2}-1$ **b** $\sqrt{2}+\sqrt{5}+1$
c $12+4\sqrt{5}-2\sqrt{2}$ **d** $\sqrt{2}-\sqrt{5}-3$

Challenge exercise

2 $\theta < 90°$

3 **a i** $m-p$ **ii** $m+p$
b $a^2=h^2+(m-p)^2,\ b^2=h^2+(m+p)^2,\ d^2=h^2+p^2$
c $a^2+b^2=2h^2+2m^2+2p^2$
d By Pythagoras' theorem, $4m^2=a^2+b^2$. Hence by Apollonius' theorem $m=d$.

4 $2\sqrt{74-20\sqrt{6}}$ cm

7 Suppose $\sqrt{6}=\frac{p}{q}$ where $\frac{p}{q}$ is a fraction in simplest form. Then $6=\frac{p^2}{q^2}$, $p^2=6q^2$. So p^2 is even. Hence p is even. So $p=2k$ where k is an integer. Therefore $(2k)^2=6q^2$ $4k^2=6q^2$. Therefore $3q^2$ is an even number. So q^2 is even and q is even. This is a contradiction.

8 **a** $17+12\sqrt{2}$ **b** $28-16\sqrt{3}$

11 **a** $\frac{3+2\sqrt{2}-\sqrt{5}-\sqrt{10}}{2}$ **b** $\frac{2\sqrt{3}+\sqrt{21}-3}{12}$

12 Raise to a suitable integer power

13 $2\sqrt{rR}$

Chapter 3 answers

Exercise 3A

1 **a** 0.72 **b** 0.076 **c** 0.98 **d** 0.16
e 0.08 **f** 0.0625 **g** 1.75 **h** 0.006
i 0.7775 **j** 0.001 **k** 1.426 **l** 0.0025

2 **a** $\frac{7}{20}$ **b** $\frac{14}{25}$ **c** $\frac{3}{4}$ **d** $\frac{3}{8}$
e $\frac{1}{3}$ **f** $\frac{1}{6}$ **g** $\frac{29}{400}$ **h** $\frac{8}{125}$
i $2\frac{1}{10}$ **j** $1\frac{1}{4}$ **k** $1\frac{1}{8}$ **l** $1\frac{9}{25}$

3 **a** 60% **b** $37\frac{1}{2}\%$ **c** $56\frac{1}{4}\%$ **d** 225%
e 35% **f** $66\frac{2}{3}\%$ **g** $133\frac{1}{3}\%$ **h** 0.75%
i 43% **j** $22\frac{1}{2}\%$ **k** 4% **l** $1\frac{1}{2}\%$
m 120% **n** 203% **o** $117\frac{1}{2}\%$ **p** 0.75%

4 **a** $\frac{27}{50}$, 0.54 **b** 40%, 0.4 **c** 32%, $\frac{8}{25}$
d $\frac{37}{200}$, 0.185 **e** 6%, $\frac{3}{50}$ **f** 87.5%, 0.875
g 102%, $1\frac{1}{50}$ **h** $1\frac{43}{500}$, 1.086 **i** 140%, 1.4

5 **a** 6 **b** 570 **c** 68.64
d 645.6 **e** 153.58 **f** 900.25
g 8.90 **h** 6 **i** 1269

6 **a** \$6.20 **b** \$227.52 **c** \$48
d \$1408 **e** \$2 **f** \$13
g \$77.50 **h** \$2.10 **i** \$116

7 **a** 14% **b** 2% **c** 3.5%
d 140% **e** 25% **f** 400%

8 **a** 13.6% **b** 1.7% **c** 0.23%
d 2.38% **e** 1.37% **f** 6.25%
g 1600% **h** 0.27% **i** 36 500%
j 0.63% **k** 0.02% **l** 2.60%

9 **a** 0.0048% **b** 0.0057%
c 0.013% **d** 0.0025%

10 37

11 **a** 0.768 g **b** 0.00368 g

12 50 minutes 36 seconds

13 2950 megatonnes

14 **a i** 14.4% **ii** 0.4% **iii** 0.42%
b 0.8 g

15 **a** 397.713% **b** 0.138%

16 **a** 24.7% **b** 0.000 664%

17 **a** 56% **b** 180%

Exercise 3B

1 **a** \$240 **b** \$12

2 **a** 1290.91 kg **b** 284 kg

3 **a** \$600 **b** 12.5 kg **c** 2 h **d** 60 cm

4 **a** \$1200 **b** \$1120

5 **a** 434.782 kg **b** 2 m or 200 cm
c 202.17 ha **d** \$721.65

6 **a** mortgage: 40%, groceries: 16%, car/transport: 22%, savings: 11%
b \$127, 11%

7 **a** 6% **b** 8.5%

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 **a** $32, $368 **b** $104, $1196 **c** $46, $529 **d** $11.40, $131.10

9 **a** $20 000 **b** $44 900 **c** $256 500 **d** $24 340

10 **a** $36 200 **b** $95 000 **c** $42 560 **d** $242 480

11 **a** 14% **b** 9% **c** 8.5% **d** 3.8%

12 **a** $500 **b** $19 000 **c** $318 980 **d** $30 865

13 **a** 12.5% profit **b** 7.4% loss **c** 10.4% profit **d** 7.3% loss

14 **a** $47 000 **b** 9.02% **c** $10 000, $240 000

15 **a** costs: $750 000, total: $768 000
b costs: $14 600 000, total: $13 943 000
c costs: $6 600 000, total: $6 349 200

16 **a i** $0 **ii** $300 **iii** $4200 **iv** $57 700
b i 0% **ii** 2.14% **iii** 11.67% **iv** 28.85%
c i $20 400 **ii** $32 880 **iii** $75 000 **iv** $77 000

Exercise 3C

1 **a** $840 **b** $4200

2 **a** $135 **b** $405

3 **a** $72 **b** $229.50 **c** $15 120

4 5 years **5** 7.5% **6** $5000

7 **a** 4 years **b** 12 years **c** 4.5% **d** 4% **e** $3000 **f** $800

8 11 years **9** 7% **10** $384 000

11 $3800 **12** $790 697.67

Exercise 3D

1 **a** 10 800 per day **b** 86 400 per day **c** 159 840 per day

2 **a** $207.60 **b** $1.74 **c** $358 110 **d** $9788.34

3 **a** 760 mm **b** 190 mm **c** 520 mm **d** 110 mm

4 **a** 36% increase **b** 42.5% increase **c** 16.3% decrease **d** 62.5% increase

5 **a** 18% decrease **b** 4.3% increase **c** 10.7% increase **d** 26.2% increase

6 **a** 600 **b** 4300 **c** 893 **d** 35 893

7 **a** $68 **b** $40.80 **c** $578 **d** $1.36

8 **a** $2800 **b** $430 **c** $2.40 **d** $32

9 **a** 13 330 megalitres **b** 33 330 megalitres **c** 10 000 megalitres **d** 25 000 megalitres

10 **a** $15 600 **b** $10 898.12 **c** $20 498.73 **d** $60 000 **e** $33 333.33 **f** $25 083.99 **g** $184 849.19 **h** 35% **i** −85.44% **j** −83.81%

11 **a i** $187 **ii** $5086.40 **iii** $75 812 **iv** $7.48
b i $500 **ii** $7110 **iii** $175 290 **iv** $4.80
c i $660 **ii** $7460.20 **iii** $594 000 **iv** $10.23

12 **a** 50% **b** 19.36% **c** 53.85% **d** 72.41%

13 **a i** 9.09% **ii** 18.03% **iii** 70.59% **iv** 2.25%
b i 11.11% **ii** 28.21% **iii** 300% **iv** 2.35%

Exercise 3E

1 $3.62

2 **a** $1487.64 **b** $2496.26 **c** $42.34 **d** $761.98

3 **a** $1605.78 **b** $555.60 **c** $4121.82 **d** $1265.79

4 612 kg

5 **a** $270.68 **b** $32 481.41 **c** $6945.61 **d** $1 044 818.69

6 **a** 25% decrease **b** 25% decrease
c same since $\left(\times\frac{3}{2}\times\frac{1}{2}\right)$ is the same as $\left(\times\frac{1}{2}\times\frac{3}{2}\right)$

7 **a** 58% **b** $25

8 **a** 4% decrease **b** 64% decrease **c** 1% decrease **d** 9% decrease

9 **a** $17.78 per kg **b** $15.46 per kg **c** $14.05 per kg

10 **a** 855 **b** 489 **c** 151 **d** 92

11 17 369

12 **a** $15 400 **b** $53 900 **c** $7100 **d** $118 200

13 **a** 99.144% **b** 99.921% **c** 99.999%

14 **a i** $42 400 **ii** $48 911.44
b i $96 153.85 **ii** $91 520.62

15 8% decrease

Exercise 3F

1 **a i** $105 000 **ii** $110 250 **iii** $134 009.56 **iv** $34.01% **v** $34 009.56
b $30 000

2 **a i** $216 000 **ii** $233 280 **iii** $317 374.86 **iv** 58.69% **v** $117 374.86
b $96 000

3 $1 127 566.42, 252% **4** 24 582, 17.06%

5 **a** $17 291 580.82 **b** $10 000

6 **a** $16 105.10 **b** $6105.10

7 **a** 36.05% **b** $73 502.99

8 $100 511.06

9 **a** $46 319.35 **b** $21 454.82 **c** $14 601.79 **d** $45.46

10 **a** $48 306.09 **b** $42 645.03 **c** $35 981.63 **d** $27 841.90

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

11 **a** \$12 828.54, \$8828.54
b \$32 614.36, \$22 614.36
c \$3 196 265.32, \$1 196 265.32
d \$1247.22, \$2752.78
e \$3066.13, \$6933.87
f \$1 251 460.57, \$748 539.43

12 \$3000, \$3180, \$3370.80, \$3573.05

13 \$24 000, \$25 920, \$27 993.60, \$30 233.09

14 **a** **i** 32.25% **ii** 33.1% **iii** 33.82% **iv** 34.01% **v** 34.39% **vi** 34.59%
b The investments have similar outcomes compared to the simple interest of 30% for one year.

15 76.98%

16 49.61% for both loans since multiplication is commutative

17 \$65 518.99

Exercise 3G

1 **a** \$280 000 **b** \$196 000 **c** \$137 200 **d** \$96 040 **e** 75.99% **f** \$75 990 p.a.

2 **a** \$200 000 **b** \$160 000 **c** \$65 536 **d** 73.79% **e** \$30 744 p.a.

3 \$183 500.80, 67.23% **4** 6537

5 better by \$20 469.94 **6** worse by \$64 639.28

7 year 1: \$2550 depreciation: \$850, year 2: \$1912.50 depreciation: \$637.50, year 3: \$1434.38 depreciation: \$478.13

8 Lara: \$135 089.81, Kate: \$32 768

9 **a** taxi: 99.2%, car: 83.2%
b taxi: \$7813, car: \$167 924, difference \$160 111

10 **a** \$5161 **b** \$6660 **c** \$51173 **d** 92.2% **e** \$4717 p.a.

11 **a** \$96 000 **b** \$128 000 **c** \$404 543 **d** 82.2% **e** \$55 424 p.a.

12 **a** \$15523 **b** \$33348 **c** \$2481 per year **d** \$5942 per year

13 **a** 12.16% **b** 164.5 mL

14 0.0064%

Review exercise

1 **a** \$33.60 **b** 45%

2 **a** 10.7% **b** 64.3%

3 **a** \$48 060, \$491940 **b** \$95 000, \$86 545

4 **a** \$157 500 **b** \$3 657 500 **c** 4.2%

5 \$7000

6 **a** \$2800
b no, salary should be \$80 640

7 **a** 6.25% **b** 375

8 \$2 516 129; \$2 672 129

9 **a** \$196.80 **b** \$60

10 **a** 14% increase **b** 489 961

11 **a** 40.5% **b** \$4 500 000

12 **a** 43% **b** 564 mm

13 **a** \$72 138.91 **b** 44.3% **c** \$22 138.91 **d** \$18 900

14 **a** \$87 000 **b** \$19 200 p.a.

Challenge exercise

1 17.65 **2** 2.35% increase

3 16.64% increase **4** ≈3.9%

5 \$40 per hour **6** 78%

7 ≈58.9% gain **8** ≈10%

Chapter 4 answers

Exercise 4A

1 **a** x **b** $2a$ **c** $4b$ **d** $3a$ **e** y **f** $2y$ **g** $4a$ **h** $-3b$ **i** $4a$ **j** $2xy$ **k** $4m$ **l** $5ab$

2 **a** $2a + 3$ **b** $3p - 2$ **c** $4mn - 3n$ **d** $4m - 3$ **e** a **f** $b - 10$ **g** $2yz - 6y$ **h** $6z - 18$ **i** $6 - 2z$ **j** $z - 3$

3 **a** $6(x + 4)$ **b** $5(a + 3)$ **c** $c(a + 5)$ **d** $a(a + 1)$ **e** $y(y + x)$ **f** $4(x + 6)$ **g** $7(a - 9)$ **h** $9(a + 4)$ **i** $y(y - 3)$ **j** $-7(2a + 3)$ **k** $-3(2y + 3)$ **l** $-4(1 + 3b)$

4 **a** $4a(b + 4)$ **b** $4a(3a + 2)$ **c** $9mn(2m + n)$ **d** $5ab^2(3a + 2)$ **e** $2a(2a + 3)$ **f** $4a(2a + 3b)$

5 **a** $3b(1 - 2b)$ **b** $2x(2x - 3y)$ **c** $3mn(3 - 4m)$ **d** $9y(2 - y)$ **e** $2a(2 - 3b^2)$ **f** $2x(3y - 2x)$ **g** $7mn(2n - 3m)$ **h** $3pq(2q - 7p)$ **i** $5ab(2b - 5a)$

6 **a** $5b(1 - 2b)$ **b** $-8ab(2a + 1)$ **c** $-xy(x + 3)$ **d** $4p(4p - q)$ **e** $5x(6 - xy)$ **f** $2p(9p - 2q)$ **g** $-2ab(4ab + 1)$ **h** $3xy(4y - x)$ **i** $5mn^2(5m + 2)$

7 **a** 4 **b** $2b + 3$ **c** $2b + 3$ **d** $a + 2$ **e** $2a + b$ **f** $8p^2$

8 **a** $2ab(2a - 1 + 4b)$ **b** $4n(m^2 - m + 4n)$ **c** $7(ab + 2a^2 + 3b)$ **d** $2(m^2 + 2mn + 3n)$ **e** $ab(5a + 3 + 4b)$ **f** $2a(3a + 4b + 5b^2)$ **g** $5pq(pq + 2q + 3p)$ **h** $5(\ell^2 - 3\ell m - 4m^2)$

Exercise 4B

1 **a** $(x - 4)(x + 4)$ **b** $(x - 7)(x + 7)$ **c** $(a - 11)(a + 11)$ **d** $(d - 20)(d + 20)$ **e** $(2x - 5)(2x + 5)$ **f** $(3x - 4)(3x + 4)$ **g** $(4x - 1)(4x + 1)$ **h** $(5m - 3)(5m + 3)$ **i** $(3x - 2)(3x + 2)$ **j** $(4y - 7)(4y + 7)$ **k** $(10a - 7b)(10a + 7b)$ **l** $(8m - 9p)(8m + 9p)$ **m** $(1 - 2a)(1 + 2a)$ **n** $(3 - 4y)(3 + 4y)$
o $(5a - 10b)(5a + 10b) = 25(a - 2b)(a + 2b)$
p $(x - 3)(x + 3)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 **a** $3(x-4)(x+4)$ **b** $4(x-5)(x+5)$
c $5(x-3)(x+3)$ **d** $6(x-2)(x+2)$
e $10(x-10)(x+10)$ **f** $7(x-3)(x+3)$
g $2(2x-5)(2x+5)$ **h** $3(2m-5)(2m+5)$
i $3(1-2b)(1+2b)$ **j** $5(2-y)(2+y)$
k $3(3a-2b)(3a+2b)$ **l** $4(2x-5y)(2x+5y)$
m $5(3m-5n)(3m+5n)$ **n** $3(3a-8\ell)(3a+8\ell)$
o $8(2y-x)(2y+x)$ **p** $8(2q-5p)(2q+5p)$

3 **a** $(x-5)(x+5)$ **b** $(3x-2)(3x+2)$
c $(2-3x)(2+3x)$ **d** $(4-9x)(4+9x)$
e $(3-10x)(3+10x)$ **f** $2(5x-3)(5x+3)$
g $3(3-2x)(3+2x)$ **h** $4(10-3x)(10+3x)$
i $7(2x-5)(2x+5)$

4 **a** 480 **b** 520 **c** 1260
d 8800 **e** 3.2 **f** 3
g 52 **h** 8520 **i** 5.428

5 **a i** 7 **ii** 9 **iii** 11 **iv** 13
v 15 **vi** 17 **vii** 19 **viii** 201
b When considering the difference of the squares of consecutive whole numbers, take the sum of the two numbers.
c $(n+1)^2 - n^2 = (n+1-n)(n+1+n) = n+n+1 = 2n+1$

6 **a** $a^2 - b^2$ **b** $a-b$ **c** $a-b$
d i $(a-b)(a+b)$ **ii** $a^2 - b^2 = (a-b)(a+b)$

Exercise 4C

1 **a** $(x+3)(x+2)$ **b** $(x+9)(x+2)$ **c** $(x+5)(x+2)$
d $(x+6)(x+5)$ **e** $(x+7)(x+2)$ **f** $(x+10)(x+9)$
g $(x+4)(x+5)$ **h** $(x+3)(x+4)$ **i** $(x+8)(x+4)$
j $(x+8)(x+5)$ **k** $(x+15)(x+5)$ **l** $(x+27)(x+1)$
m $(x+7)(x+8)$ **n** $(x+14)(x+4)$

2 **a** $(x-3)(x-2)$ **b** $(x-11)(x-3)$ **c** $(x-15)(x-2)$
d $(x-6)(x-7)$ **e** $(x-7)(x-2)$ **f** $(x-45)(x-2)$
g $(x-11)(x-4)$ **h** $(x-20)(x-5)$ **i** $(x-8)(x-10)$
j $(x-15)(x-6)$ **k** $(x-10)(x-4)$ **l** $(x-8)(x-3)$
m $(x-28)(x-2)$ **n** $(x-12)(x-2)$

3 **a** $(x+3)(x-2)$ **b** $(x-11)(x+3)$ **c** $(x+6)(x-5)$
d $(x-21)(x+2)$ **e** $(x-7)(x+2)$ **f** $(x-15)(x+6)$
g $(x-11)(x+4)$ **h** $(x+20)(x-5)$ **i** $(x+10)(x-8)$
j $(x-12)(x+5)$ **k** $(x+8)(x-5)$ **l** $(x-12)(x+2)$
m $(x+7)(x-3)$ **n** $(x+5)(x-3)$ **o** $(x-8)(x+7)$
p $(x+8)(x-3)$

4 **a** $(x-2)(x-1)$ **b** $(x+6)(x+2)$ **c** $(x-5)(x+2)$
d $(x+6)(x+5)$ **e** $(x-7)(x+2)$ **f** $(x-15)(x+6)$
g $(x-1)(x-4)$ **h** $(x-9)(x+2)$ **i** $(x-4)(x+3)$
j $(x-7)(x-4)$ **k** $(x+5)(x-2)$ **l** $(x+10)(x-9)$

5 **a** $(x+3)^2$ **b** $(x+7)^2$ **c** $(x-5)^2$
d $(x-9)^2$ **e** $(x+5)^2$ **f** $(x+6)^2$
g $(x+15)^2$ **h** $(x-8)^2$ **i** $(x-10)^2$
j $(x-4)^2$

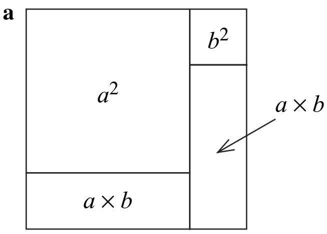
Exercise 4D

1 **a** $(x+6)^2$ **b** $(x-4)^2$ **c** $(x+5)^2$
d $(a-2)^2$ **e** $(m-13)^2$ **f** $(a+14)^2$
g $\left(x-\frac{9}{2}\right)^2$ **h** $\left(x+\frac{13}{2}\right)^2$ **i** $\left(x-\frac{11}{2}\right)^2$

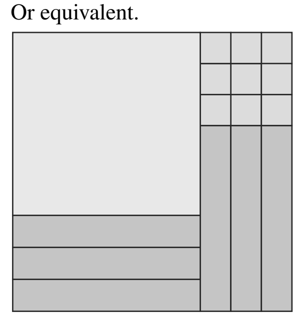
2 **a** $x^2+8x+16=(x+4)^2$ **b** $x^2-10x+25=(x-5)^2$
c $x^2-18x+81=(x-9)^2$ **d** $x^2+7x+\frac{49}{4}=\left(x+\frac{7}{2}\right)^2$
e $x^2-9x+\frac{81}{4}=\left(x-\frac{9}{2}\right)^2$ **f** $x^2-\frac{5x}{2}+\frac{25}{16}=\left(x-\frac{5}{4}\right)^2$

3 **a** ii **b** iii **c** iv **d** iii

4 **a**



b 1 paver $a \times a$, 9 pavers $b \times b$, 6 rectangular pavers $a \times b$. Or equivalent.



Exercise 4E

1 **a** $2(x+3)(x+4)$ **b** $3(x+2)(x+6)$
c $3(x-1)(x-8)$ **d** $4(x-3)^2$
e $7(x+1)^2$ **f** $5(x+2)(x-3)$
g $4(x-4)(x+3)$ **h** $2(x-3)(x-6)$
i $5(x+1)(x+7)$ **j** $3(x-5)(x+8)$
k $3(x+5)(x-6)$ **l** $5(x+6)^2$
m $2(x+6)(x-8)$ **n** $5(x+4)(x+9)$
o $3(x+12)(x-2)$ **p** $3(x-3)^2$
q $5(x-2)^2$ **r** $3(x-2)(x-6)$

2 **a** $4(x+2)(x-2)$ **b** $2(x+3)(x-3)$
c $3(x+4)(x-4)$ **d** $3(a+3)(a-3)$
e $6(x+10)(x-10)$ **f** $3(a+3b)(a-3b)$
g $3(3x+y)(3x-y)$ **h** $5(3+b)(3-b)$
i $3(2+m)(2-m)$ **j** $2(8+x)(8-x)$
k $\frac{1}{2}(a+2b)(a-2b)$ **l** $\frac{1}{3}(9x+y)(9x-y)$
m $\frac{1}{4}(a+6)(a-6)$ **n** $\frac{1}{5}(x+10)(x-10)$
o $\frac{1}{4}(x+2y)(x-2y)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 a $-(x+6)(x+2)$ b $-(x-1)(x+12)$
c $-(x-1)(x+7)$ d $-(x-9)(x+1)$
e $-(x+2)^2$ f $-(x+9)(x+5)$
g $-(x+5)(x-8)$ h $-(x-7)(x+6)$
i $-(x-2)(x-20)$ j $-(x-3)(x-8)$
k $-3(x-2)(x+12)$ l $-(x+7)(x+8)$
m $-(x+7)(x+9)$ n $-(x-7)(x-5)$
o $-(x+2)(x-9)$

Review exercise

1 a $4(x+4)$ b $7(x-3)$ c $3(2a-3)$
d $a(4b+7)$ e $p(6q-11)$ f $5n(m-2)$
g $4v(u-2)$ h $a(a+9)$ i $4mn(m-3)$
j $ab(a-4b)$ k $3p(q-2p)$ l $6pq(p^2-3)$

2 a $(x-3)(x+3)$ b $(x-4)(x+4)$
c $(3a-5)(3a+5)$ d $(4m-1)(4m+1)$
e $(3-2b)(3+2b)$ f $(10-9b)(10+9b)$
g $(4x-y)(4x+y)$ h $2(m-5)(m+5)$
i $3(a-3)(a+3)$ j $(1-6b)(1+6b)$
k $\frac{1}{4}(4y-1)(4y+1)$ l $(pq-1)(pq+1)$

3 a $(x+2)(x+6)$ b $(x+3)(x+6)$
c $(x+5)(x+6)$ d $(x-8)(x-3)$
e $(x-6)(x-4)$ f $(x-2)(x-12)$
g $(x-1)(x-24)$ h $(x+5)(x-4)$
i $(x-8)(x+6)$ j $(x-6)(x+2)$
k $(x+8)(x-5)$ l $(x-8)(x+1)$
m $(x-12)(x+11)$ n $(a+4)(a+15)$
o $(x-2)(x-48)$

4 a $(a-11)^2$ b $(m-7)^2$ c $(s+4)^2$
d $(a+12)^2$ e $(a-6)^2$ f $(z-20)^2$
g $\left(x+\frac{5}{2}\right)^x$ h $\left(y-\frac{1}{3}\right)^2$ i $\left(a-\frac{3}{4}\right)^2$

5 a $2(x+4)(x+5)$ b $3(x-3)(x-7)$
c $5(x-4)(x-6)$ d $2(x+9)(x-5)$
e $3(x-7)(x+5)$ f $2(x-13)(x+10)$

6 a $(5a-4b)(5a+4b)$ b $(a+7)^2$
c $(a-5)(a+4)$ d $(1-6m)(1+6m)$
e $(2-3xy)(2+3xy)$ f $\left(\frac{1}{3}-\frac{a}{5}\right)\left(\frac{1}{3}+\frac{a}{5}\right)$
g $\left(m-\frac{1}{2}\right)\left(m+\frac{1}{2}\right)$ h $x(x-7y)(x+7y)$
i $3(a-5)(a+5)$ j $(b-8)(b-12)$
k $(n-25)(n-6)$ l $(m+7)(m+13)$
m $(a-14b)(a+7b)$ n $(m-15)(m+11)$
o $(x+4y)(x-y)$ p $5n(2m-n)(2m+n)$
q $(a-9)(a+7)$ r $5(q-3p)(q+2p)$
s $(x-13)(x+10)$ t $-(x+7)(x-6)$
u $(x+9)(x-2)$

Challenge exercise

1 a $(x^2+1)(x-1)(x+1)$ b $(x^2+4)(x-2)(x+2)$
c $(x-\sqrt{3})(x+\sqrt{3})$ d $(x-\sqrt{5})(x+\sqrt{5})$
e $(x-\sqrt{2})^2$ f $(x+\sqrt{2})^2$
g $\left(x+\frac{1}{2}\right)^2$ h $\left(x+\frac{3}{2}\right)^2$
i $\left(x-\frac{1}{2}\right)^2$ j $\left(x-\frac{3}{2}\right)^2$
k $(x-2y)(x+y)$ l $(x+2y)(x-y)$

2 a x b $\dfrac{x-2}{x+4}$ c $\dfrac{x+1}{x-1}$

3 a $(x-2)^2$ b $(x+3)^2$
c $(x-a)^2$ d $(x+a)^2$

4 $\dfrac{ad+bc}{ad-bc}$

5 a $(x-1)(x+1)(x^2+3)$ b $x^2(x-2)(x+2)$
c $(x+2-y)(x+2+y)$ d $(x+4-a)(x+4+a)$
e $(m-1-n)(m-1+n)$ f $\left(p-\frac{5}{2}-q\right)\left(p-\frac{5}{2}+q\right)$

6 a $(x^2+2-2x)(x^2+2+2x)$
b $(x^2+2a^2-2ax)(x^2+2a^2+2ax)$

7 a i $(x+9)$ m ii $(x+7)$ m
b $(10x+50)$ m c 80 m
d $(4x^2+48x+144)$ m^2
e i side length $(2x+12)$ m
ii $(8x+48)$ m

9 $\sqrt{a+b}$ 10 $\dfrac{x(x+y+z)}{z(x-y+z)}$

Chapter 5 answers

Exercise 5A

1 $(h+5)$ cm 2 $(w-2)$ kg 3 $2x-3$

4 a $(w+5)$ m b $(\ell-5)$ m

5 a $a+18$ b $2a-5$

6 $(3x-20)$ m

7 a $5x$ km/h b $(20x+3)$ km/h

8 a x b viii c v d ii
e i f iii g iv h ix
i vii j vi

9 a $(x+6)$ cm b $(x-4)$ cm

Exercise 5B

1 a $a=3$ b $b=12$ c $c=17$ d $d=18$
e $a=3$ f $b=3$ g $d=7$ h $m=-2$
i $n=-4$ j $q=-3$ k $b=22\frac{1}{2}$ l $a=\frac{2}{9}$
m $x=11\frac{1}{3}$ n $y=-\frac{2}{9}$ o $x=-\frac{5}{12}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 **a** $a = 1$ **b** $b = 5$ **c** $c = 7$ **d** $d = 6$
e $f = 4$ **f** $g = -4$ **g** $h = -2$ **h** $a = -3$
i $a = -8$ **j** $a = -5$ **k** $b = -7$ **l** $b = 1\frac{1}{4}$
m $x = -1\frac{7}{8}$ **n** $m = -1\frac{1}{5}$ **o** $b = -2\frac{5}{8}$

3 **a** $a = -2$ **b** $b = -3$ **c** $c = 9$
d $d = 5$ **e** $e = -3$ **f** $f = -6$

4 **a** -2 **b** 1 **c** 5
d 1 **e** -3 **f** 2
g 3 **h** $\frac{10}{3}$ **i** 3

Exercise 5C

1 **a** 1 **b** 7 **c** 4
d 1 **e** 2 **f** -1
g 1 **h** 3 **i** -7

2 **a** $4\frac{1}{2}$ **b** $5\frac{1}{5}$ **c** $\frac{1}{7}$
d $5\frac{1}{4}$ **e** $1\frac{1}{2}$ **f** $\frac{1}{6}$
g $3\frac{2}{15}$ **h** $3\frac{1}{10}$ **i** $-\frac{5}{12}$

3 **a** $a = 2$ **b** $b = 4$ **c** $c = 3$
d $d = 1$ **e** $x = -2$ **f** $y = 3$
g $a = 12$ **h** $a = \frac{9}{4}$ **i** $x = 30$
j $x = -4$ **k** $x = 2$ **l** $x = 4\frac{1}{8}$

Exercise 5D

1 **a** $x = \frac{3}{8}$ **b** $x = \frac{1}{18}$ **c** $y = \frac{5}{12}$
d $y = \frac{17}{18}$ **e** $x = -\frac{1}{60}$ **f** $y = -\frac{23}{100}$
g $x = \frac{1}{15}$ **h** $x = \frac{13}{45}$ **i** $y = -\frac{13}{60}$

2 **a** $a = -6$ **b** $a = \frac{88}{45}$ **c** $b = 24$
d $b = -\frac{28}{3}$ **e** $x = -6$ **f** $x = -4$
g $m = \frac{9}{2}$ **h** $m = -\frac{20}{3}$ **i** $x = -14$

3 **a** $y = 4$ **b** $x = 11$ **c** $p = 6$
d $y = 5$ **e** $x = 9$ **f** $a = 3$
g $x = 8$ **h** $x = 4$ **i** $y = 3$

4 **a** $e = 1.1$ **b** $f = 3.9$ **c** $g = 6.6$
d $u = 13$ **e** $r = 2.5$ **f** $x = -10.5$
g $x = 5$ **h** $x = 4$ **i** $x = 6$

5 **a** 16 **b** -1 **c** $\frac{60}{7}$
d $\frac{150}{11}$ **e** $\frac{2}{5}$ **f** 1

6 **a** $\frac{20}{7}$ **b** 12 **c** 2.4
d -7 **e** 1.4 **f** 4

7 **a** $-\frac{13}{3}$ **b** -10 **c** 1 **d** $-\frac{17}{3}$
e 10 **f** $\frac{21}{8}$ **g** -8 **h** $\frac{37}{5}$
i 1 **j** 1

8 **a** $a = 3$ **b** $b = 13$ **c** $c = 18$
d $d = -6$ **e** $e = \frac{9}{8}$ **f** $f = \frac{15}{7}$
g $g = \frac{11}{5}$ **h** $h = 1$ **i** $i = -\frac{32}{3}$
j $j = 8$ **k** $k = 1$ **l** $\ell = \frac{6}{7}$
m $m = -\frac{11}{2}$ **n** $n = -4$ **o** $q = \frac{1}{6}$
p $r = 31$

Exercise 5E

1 $x = 15$ **2** 14 **3** 56 kg

4 \$2.40 **5** $p = 4$ **6** $q = -24$

7 **a i** $x + 20$ **ii** $x + 12$ **iii** $x + 32$
b Alana is 8 years old and Derek is 28 years old.

8 **a i** $x + 5$ **ii** $2x$ **iii** $4x + 5$
b Alan has 8 toys, Brendan has 13 toys and Calum has 16 toys.

9 **a** $(4x + 2)$ m
b length 50 m, width 12 m

10 **a i** $\$(x + 3600)$ **ii** $\$(x - 2000)$
b Ms Minas earns \$53 600, Mr Brown earns \$50 000 and Ms Lee earns \$48 000.

11 **a** 20 of each
b fifteen 20-cent coins and thirty 10-cent coins
c twenty-four 20-cent coins and twelve 10-cent coins

12 **a** $4\frac{3}{8}$ **b** 100 **c** $\frac{17}{6}$

13 length 4.5 m, width 10.5 m **14** \$600

15 90 **16** 10 km **17** 40 **18** 7.5 L

19 600 g of A and 400 g of B

20 four 10-cent coins, eight 20-cent coins, five 50-cent coins

21 $6\frac{2}{3}$ L of water

22 **a** $(w + 60)$ cm **b** 150 cm × 90 cm
c i $(w + 160)$ cm **ii** $(w + 100)$ cm
d $(200w + 16000)$ cm^2 **e** 40 000 cm^2
f 200 cm

23 **a** $10x$ cents **b** $15(22 - x)$ cents
c $10x - 15(22 - x) = 20$; that is, $25x - 330 = 20$
d 14

24 **a** 7.5 cm, 8 cm **b** 5 cm, 6 cm
c $10 - 25t$ cm, $10 - 2t$ cm **d** 10:20 p.m.

Exercise 5F

1 **a** $c - b$ **b** $d + e$ **c** $p - q$
d $m - n$ **e** $\frac{b}{c}$ **f** $\frac{c - e}{b}$
g $\frac{c - ab}{a}$ **h** $\frac{n - mp}{mn}$ **i** ab
j $bc - a$ **k** $a(c - b)$ **l** $\frac{np}{m}$
m $\frac{b(d - c)}{a}$ **n** $\frac{cd - b}{a}$ **o** $\frac{mn + n}{m}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

p $\dfrac{fhk - fg}{h}$ **q** $\dfrac{a^2 + ab^2}{b}$ **r** $a + b^2$

2 **a** $\dfrac{d - b}{a - c}$ or $\dfrac{b - d}{c - a}$ **b** $\dfrac{m + n}{n - m}$

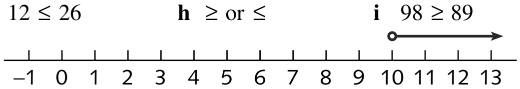
c $\dfrac{d - ab}{a - c}$ or $\dfrac{ab - d}{c - a}$ **d** $\dfrac{ab - cd}{a - c}$ or $\dfrac{cd - ab}{c - a}$

Exercise 5G

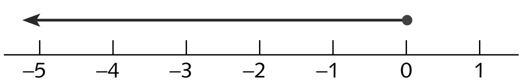
1 **a** $7 > 2$ **b** $3 > -4$ **c** $-4 < -2$
d $-54 > -500$ **e** $-6 < 0$ **f** $-13 > -45$
g $21 < 40$ **h** $-2 < 5$ **i** $99 > -100$

2 **a** $-7 \le -2$ **b** $5 \ge -7$ **c** $\ge$ or $\le$
d $-10 \ge -50$ **e** $\ge$ or $\le$ **f** $-23 \ge -45$
g $12 \le 26$ **h** $\ge$ or $\le$ **i** $98 \ge 89$

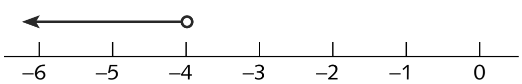
3 **a**



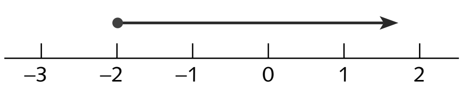
b



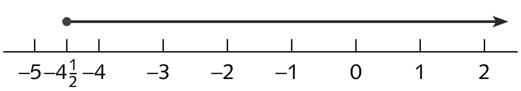
c



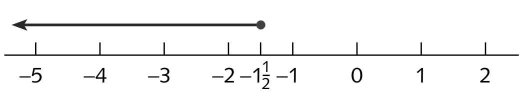
d



e



f

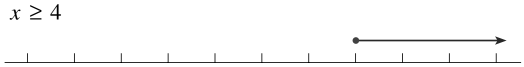


4 **a** $\{x: x > -2\}$ **b** $\{x: x \le 4\}$
c $\{x: x < -2\}$ **d** $\{x: x \ge 1\}$
e $\left\{x: x \le -2\tfrac{1}{2}\right\}$ **f** $\left\{x: x > -1\tfrac{2}{3}\right\}$

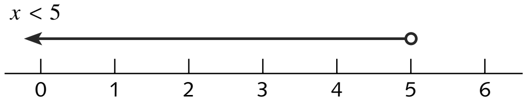
Exercise 5H

1 **a** infinitely many **b** $-\dfrac{1}{2}$, 0, 2.6, etc.
c 2.7, 11.8, $14\dfrac{1}{2}$, etc.

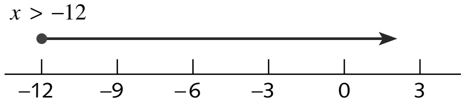
2 **a** $x \ge 4$



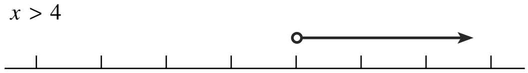
b $x < 5$



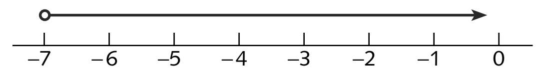
c $x > -12$



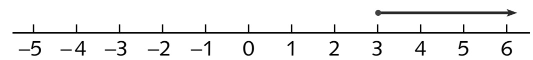
d $x > 4$



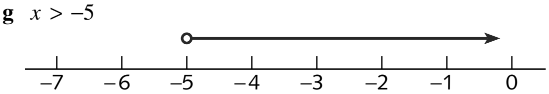
e $x > -7$



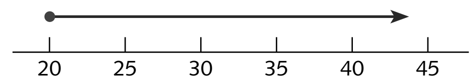
f $x \ge 3$



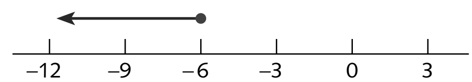
g $x > -5$



h $x \ge 20$



i $x \le -6$



3 **a** $x \ge 2$ **b** $x \le 1$ **c** $x > 7\frac{1}{2}$
d $x \le -3\frac{1}{2}$ **e** $x \ge -2$ **f** $x > \dfrac{9}{10}$
g $x \ge 4\frac{1}{2}$ **h** $x > 9\frac{3}{8}$ **i** $x \le 5\frac{1}{2}$

4 **a** $x \ge -5$ **b** $x \le -13$ **c** $x < 3\frac{1}{2}$
d $x \ge -10$ **e** $x \le -28$ **f** $x < -20$
g $x > -48$ **h** $x \le 16$ **i** $x < -27$

5 **a** $x < -1$ **b** $x \ge 2$ **c** $x > 5\frac{3}{4}$
d $x > 3$ **e** $x \le -5$ **f** $x \le 14$

6 **a** $x \le 0$ **b** $x < -46$

7 **a** $x \le 7$ **b** $x \ge -15$ **c** $x \le -157$ **d** $x > 10.5$

8 **a** $x \le 11$ **b** $x \le -15$ **c** $x > -9\frac{1}{2}$
d $x \ge -23$ **e** $x \le -42$ **f** $x > -5\frac{1}{3}$

9 $p > 6$ **10** $q < 68$ **11** $p \le 3$

12 $d > 6$ **13** $a > -6$

14 **a** **i** $\$(25 + 0.06x)$ **ii** $\$(20 + 0.08x)$
b $x > 250$ km

Review exercise

1 $(w - 5)$ kg **2** $2x - 5$

3 **a** $(w + 10)$ m **b** $(\ell - 10)$ m

4 **a** $a = -2$ **b** $m = -6$ **c** $q = -\dfrac{27}{4}$
d $a = -\dfrac{11}{3}$ **e** $a = -\dfrac{9}{7}$ **f** $b = \dfrac{11}{7}$
g $e = -\dfrac{15}{4}$ **h** $f = -\dfrac{29}{7}$ **i** $x = \dfrac{5}{2}$
j $x = \dfrac{33}{10}$ **k** $x = 2$ **l** $p = -1$

5 **a** $x = \dfrac{3}{5}$ **b** $x = 5$ **c** $x = 9$
d $x = \dfrac{3}{7}$ **e** $x = \dfrac{1}{7}$ **f** $x = \dfrac{49}{6}$
g $a = 13$ **h** $b = -\dfrac{19}{3}$ **i** $a = -31$
j $a = \dfrac{29}{33}$ **k** $x = \dfrac{8}{9}$ **l** $m = -\dfrac{12}{35}$

6 **a** $c = 2$ **b** $\ell = 4$ **c** $x = 4$
d $x = 10$ **e** $x = 1.5$ **f** $x = -\dfrac{3}{7}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 **a** $x = -15$ **b** $x = -18$ **c** $p = -\frac{8}{5}$
d $p = \frac{8}{7}$ **e** $x = \frac{17}{2}$ **f** $x = \frac{17}{7}$
g $z = -4$ **h** $y = -12$ **i** $k = -\frac{25}{9}$
j $\ell = 28$ **k** $m = 3$ **l** $n = -10$
m $y = \frac{11}{2}$ **n** $x = -\frac{21}{4}$ **o** $s = \frac{1}{9}$
p $x = 91$ **q** $a = \frac{19}{8}$ **r** $x = \frac{13}{11}$

8 **a** $(4x + 15)$ m **b** width 23.5 m, length 109 m

9 **a** **i** $\$(x + 14800)$ **ii** $\$(x - 22000)$
b Mr Jersey's income \$119 600, Mr Guersney's income \$134 400, Mrs Mann's income \$97 600

10 **a** 60 of each coin
b 45 20-cent coins and 90 10-cent coins
c 72 20-cent coins and 36 10-cent coins

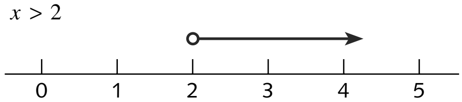
11 **a** 5 **b** $85\frac{5}{7}$ **c** 3

12 24 m by 16 m

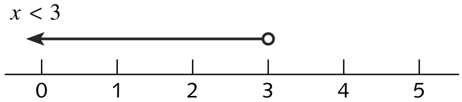
13 **a** $\frac{ab - n}{m}$ **b** $\frac{p + mn}{m}$
c $\frac{ac + d}{ab}$ **d** $\frac{n + q}{m - p}$
e $\frac{acd - ab}{c}$ **f** $\frac{ab - mn}{m}$

14 **a** $\geq$ **b** $\geq$ **c** $\geq$
d $\leq$ **e** $\leq$ **f** $\geq$

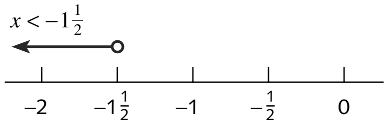
15 **a** $x > 2$



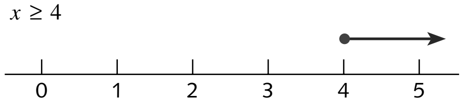
b $x < 3$



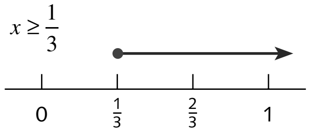
c $x < -1\frac{1}{2}$



d $x \geq 4$



e $x \geq \frac{1}{3}$



f $x \leq 2\frac{1}{4}$

16 **a** $\{x: x > -1\}$ **b** $\left\{x: x \leq 1\frac{1}{2}\right\}$

17 **a** $x \geq 10$ **b** $x \leq 3$ **c** $x \leq -8$
d $x < -15$ **e** $x \geq 2$ **f** $x \geq \frac{25}{2}$
g $x > 18.7$ **h** $x \leq \frac{11}{2}$ **i** $x > -2$
j $x \geq \frac{19}{4}$ **k** $x < -\frac{7}{3}$ **l** $x > 18$
m $x \geq 23$ **n** $x < -6$ **o** $x < \frac{203}{55}$
p $x \leq \frac{25}{19}$

18 **a** $x \geq -15$ **b** $x > -\frac{5}{4}$ **c** $x > \frac{7}{3}$
d $x \geq -\frac{1}{10}$ **e** $x \leq \frac{17}{14}$ **f** $x \leq -\frac{25}{6}$
g $x > \frac{13}{3}$ **h** $x \leq 2$

Challenge exercise

1 320 km/h, 640 km/h **2** 33

3 A receives \$150, B receives \$130, C receives \$140.

4 1900 and 2100 leaflets/h

5 1 cm^3 of silver weighs 10.5 g and 1 cm^3 of copper weighs 9 g.

6 15 km/h **7** $4\frac{1}{8}\%$

8 $\frac{600}{3781}$ m, $\frac{3}{199}$ m, $3\frac{657}{3781}$ m **9** 54 km

Chapter 6 answers

Exercise 6A

1 **a** 40 **b** 20 **c** 35
d 50 **e** 58 **f** 64

2 **a** 44.721 **b** 9047.787
c 18.850 **d** 18 771.37

3 **a** 21 **b** 30
c 12 **d** $13\frac{1}{3}$

4 **a** 40 **b** 6 **c** 34
d 4000 **e** 12 **f** 72
g 18

5 **a** 10 **b** 20.9

6 **a** $1\frac{1}{3}$ **b** $3\frac{3}{5}$ **c** $5\frac{17}{23}$ **d** $1\frac{13}{17}$

7 **a** -19.5 **b** 5

8 **a** 4 **b** 12

9 **a** 3 **b** $\frac{43}{9}$ **c** $\frac{251}{133}$ **d** $\frac{23}{7}$

10 **a** $2\sqrt{5}$ **b** 4.47

11 13.5 cm

12 **a** 32°F **b** 212°F
c 77°F **d** 10°C
e 40°C

13 **a** 24 cm^2 **b** 30 cm^2
c $16\sqrt{6}$ cm^2 **d** 84 cm^2

14 **a** 31.30 m/s **b** 26.66 m/s

15 **a** 30 m **b** 34.125 m
c 38.5 m **d** 48 m
e 70 m **f** 96 m

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 6B

1 **a** \$150 **b** $n = \frac{P+150}{5}$

c **i** 85 **ii** 110 **iii** 10 **iv** 30

2 **a** $n = \frac{C-250}{12}$

b **i** 40 **ii** 70 **iii** 80 **iv** 120

3 **a** $u = v - at$ **b** **i** 10 **ii** 64

c $a = \frac{v-u}{t}$

d **i** 2.5 **ii** −6.2 **iii** −0.2

e $t = \frac{v-u}{a}, \frac{1}{3}$

4 **a** $a = t - (n-1)d$ **b** **i** 2 **ii** 20

c $d = \frac{t-a}{n-1}$ **d** **i** 3 **ii** 1.5

e $n = \frac{t-a}{d} + 1$, 30

5 **a** $c = y - mx$ **b** $x = \frac{y-c}{m}$ **c** $b = \frac{2A}{h}$

d $r = \frac{C}{2\pi}$ **e** $\ell = \frac{P-A}{2h}$ **f** $a = \frac{2(s-ut)}{t^2}$

g $h = \frac{A - 2\pi r^2}{2\pi r}$ **h** $h = \frac{3V}{\pi r^2}$ **i** $a = \frac{2S}{n} - \ell$

j $n = \frac{2S}{a+\ell}$ **k** $s = \frac{V - \pi r^2}{\pi r}$ **l** $h = \frac{2E - mv^2}{2mg}$

6 $n = \frac{S}{180} + 2$

a 8 **b** 12 **c** 20

7 $m = \frac{2E}{v^2}$

a 8 kg **b** 3.5 kg **c** 20 kg

8 30 m/s

9 **a** $a = \sqrt{c - b^2}$ **b** $\ell = \sqrt{\frac{K-4m}{3}}$ **c** $b = \frac{x^2}{a}$

d $h = \frac{5d^2}{64}$ **e** $n = \frac{2\pi}{T}$ **f** $v = \frac{m}{D}$

g $r = \sqrt{\frac{m}{2E}}$ **h** $r = \frac{1}{2}\sqrt{\frac{a}{T}}$

10 **a** $C = \frac{5(F-32)}{9}$ **b** $82\frac{2}{5}$°F **c** $-15\frac{5}{9}$°C

d −40°F = −40°C **e** **i** 390°F **ii** 250°C

Exercise 6C

1 **a** $D = 90n$ **b** $c = 100D$ **c** $m = 60h$ **d** $d = 7m$

2 **a** $n = 100p$ **b** $s = 1000t$ **c** $q = 500p$ **d** $x = 500y$

3 **a** $z = 100x + y$ **b** $x = y + \frac{z}{60}$ **c** $x = \frac{60y + z}{3600}$

d $m = \frac{c}{20}$ **e** $n = \frac{m}{5}$ **f** $m = \frac{y}{x}$

g $p = \frac{nk}{4}$ **h** $q = \frac{xb}{8}$

4 **a** $y = x - 3$ **b** $y = x^2 + 4$ **c** $y = 8\sqrt{\frac{x}{5}}$

d $y = 180 - x$ **e** $y = \frac{80}{x}$ **f** $y = \frac{5x}{4}$

5 **a** $x = 10\,000y$ **b** $S = 0.8m$ **c** $c^2 = a^2 + b^2$

d $A = \frac{\pi r^2 \theta}{360}$ **e** $d = 75t$ **f** $h = \frac{w}{25}$

6 **a** $C = 20 + 0.4x$ **b** $y = 50 - 4x$ **c** $T = 20 + 45w$

d $t = 2^n$ **e** $A = \frac{x^2}{4\pi}$

7 $n = 4(x + y + 1)$

Review exercise

1 **a** 195 **b** −20.5 **c** 8

2 **a** 20 **b** $\frac{49}{16}$

3 **a** $-2 + 2\sqrt{7}$ **b** −48

4 **a** $h = \frac{V}{\pi r^2} + \frac{r}{3}$ **b** 1290 cm

5 **a** $\ell = \frac{A}{w}$ **b** $r = \frac{C}{2\pi}$

c $h = \frac{V}{\pi r^2}$ **d** $b = \frac{A}{2h} - \ell$

e $r = \sqrt{\frac{A}{4\pi}}$ **f** $x = \frac{aw^2}{100}$

g $V = \frac{w^2 \pi h}{3}$ **h** $y = \frac{xz}{2x - z}$

6 **a** $d = 4.9t^2$ **b** 11.025 m

7 **a** $m = M - \frac{M}{t^2} = \frac{M(t^2-1)}{t^2}$

b $M = \frac{t^2 m}{t^2 - 1}$ **c** 6

8 **a** $h = \frac{S}{2\pi r} - r$ **b** 4.37 cm

9 **a** 2870 **b** 3281

10 **a** $F = \left(\frac{2\pi}{T}\right)^2 \frac{W}{g}$ **b** $x = \frac{180(P - 2r)}{\pi r}$

c $x = \frac{f(D^2 - 1)}{D^2 + 1}$

11 **a** 10 **b** 15

c $\frac{n(n+1)}{2}$ **d** 630

12 **a** $t = \frac{ab}{M^2 - a^2}$ **b** $\ell = \sqrt{\frac{9V^2}{a^4} + \frac{a^2}{2}}$

c $a = \frac{Eb^3(w^2 + m)}{w^2}$ **d** $r = g\left(\frac{T}{2\pi}\right)^2 - \ell$

e $h = \frac{ka^2 - yx^2}{px^2}$ **f** $u = \sqrt{v^2 - 2as}$

13 **a** $V = \frac{975\pi}{8}$ **b** $r = \frac{1}{2t}\left(\frac{V}{\pi\ell} + t^2\right)$

14 **a** 120 **b** $n = \frac{360}{180 - I}$, 5

15 **a** $y = 2x^2 + 4$ **b** $y = 90 - x$

c $y = \frac{x}{100}$ **d** $y = \frac{100}{x}$

Challenge exercise

1 **a** $M = \frac{m(P+1)}{(P-1)}$ **b** $M = \frac{F\ell}{P - F}$

c $R = \frac{ST}{T - S}$ **d** $i = \frac{R}{E - P}$

e $\ell = \frac{h}{t - T}$ **f** $L = \sqrt{\frac{12I}{M} - 3R^2}$

g $L = \frac{c}{\ell}\sqrt{L^2 - \ell^2}$ **h** $e = \frac{T - 2\pi}{T + 2\pi}$

i $r = \frac{cs}{vs + c}$ **j** $k = g\left(\frac{P}{2\pi}\right)^2 - h$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

k $\ell = \dfrac{p^2}{P^2 - p^2}$ **l** $R = \sqrt{\dfrac{A}{\pi} + r^2}$

m $a = \dfrac{d^2 - b^2 - c^2}{2(b+d)}$ **n** $M = \dfrac{m^2 + p^2}{2m}$

2 **a** **i** 7 **ii** 17 **iii** 52 **iv** $5n+2$

b **i** 10 **ii** 17 **iii** 73 **iv** $7n+3$

c $2mn + n + m$

d **i** 10 **ii** 12

e **i** $(n+1)m$ **ii** $n(m+1)$

iii $m(n+1)+n(m+1)$

f yes **g** 67 **h** 7 **i** 33×1

j $m(n+1)(p+1)+n(m+1)(p+1)+p(m+1)(n+1)$

3 **a** **i** d cm **ii** $(r-h)$ cm

c **i** 65 cm **ii** 25 cm

d 80 cm

4 $H = \dfrac{n}{2}(n-1)$

5 **a** $2\sqrt{210}$ **b** $10\sqrt{3}$ **c** $2\sqrt{2310}$

d 1764 **e** 3234

6 **a** $Q = 0.98^n P$ **b** $a = 100\left(1 - 0.95^n\right)$

Chapter 7 answers

Exercise 7A

1 **a** **i** obtuse angle **ii** revolution **iii** acute angle

iv straight angle **v** right angle **vi** reflex angle

b **i** 70° **ii** 18° **iii** 45°

c **i** 160° **ii** 8° **iii** 90°

2 **a** $\theta = 135°$ (revolution at A)

b $\beta = 110°$ (vertically opposite angles at M), $\alpha = 70°$ (supplementary angles)

c $\alpha = 22\frac{1}{2}°$ (supplementary angles)

d $\theta = 150°$ (vertically opposite angles)

e $\theta = 72°$ ($5\theta = 360°$, angles in a revolution)

f $\alpha = 15°, 6\theta = 90°$

3 **a** $\alpha = 140°$ (alternate angles, $AB \parallel CD$)

b $\beta = 65°$ (corresponding angles, $AB \parallel CD$),

$\gamma = 65°$ (corresponding angles, $PQ \parallel RS$)

c $\alpha = 80°$ (co-interior angles, $VW \parallel XY$),

$\beta = 80°$ (alternate angles, $WX \parallel YZ$)

d $\alpha = 112°$ (co-interior angles, $KN \parallel LM$),

$\beta = 68°$ (co-interior angles, $LK \parallel MN$),

$\gamma = 112°$ (co-interior angles, $NK \parallel LM$)

e $\theta = 60°$ ($\theta + 2\theta = 180°$, co-interior angles, $AB \parallel CD$)

f $\theta = 85°$, $\angle BED = 130°$ (co-interior angles, $AD \parallel BE$), $\angle BEF = 145°$ (co-interior angles, $BE \parallel CF$), $\theta = 85°$ (angle at a point E)

4 **a** $AB \parallel CD$ (alternate angles are equal)

b $BE \parallel CD$ (corresponding angles are equal)

c $QP \parallel RS$ (co-interior angles are supplementary)

d no parallel lines

e $NO \parallel KM$ (alternate angles are equal)

f $AB \parallel DC$ (co-interior angles are supplementary)

5 **a** $\theta = 35°$ (angle sum of ΔABC)

b $\alpha = 65°$ (angle sum of quadrilateral $KLMN$)

c $\theta = 130°$ (exterior angle of ΔABC)

d $\alpha = 20°$ (angle sum of ΔURS), $\beta = 100°$ (angle sum of ΔUST)

e $\theta = 50°$ ($3\theta + 210° = 360°$, angle sum of quadrilateral $EFGH$)

f $\beta = 135°$ (exterior angle of ΔABD)

6 **a** $\alpha = 80$ (base angles of isosceles ΔABC), $\beta = 20°$ (angle sum of ΔABC)

b $\alpha = \beta = 70°$ (base angles of isosceles ΔABC)

c $x = 2$ (opposite sides of ΔLMN are equal)

d $\alpha = 60°$ (ΔPQR is equilateral), $\beta = 45°$ (base angle of right isosceles ΔQRS)

e $\alpha = 60°, x = 6, y = 2$ (ΔKLM is equilateral)

f $\alpha = 50°$ (base angles of isosceles ΔTRU), $\alpha = 70°$ (base angles of isosceles ΔSTU), $\alpha = 40°$ (angle sum of ΔSTU)

7 **a** $\alpha = 40°$ (co-interior angles, $PQ \parallel TS$),

$\beta = 110°$ (angle sum of quadrilateral $PQST$),

$\gamma = 40°$ (base angle of isosceles ΔQRS, supplementary to $\angle PQS$)

b $\theta = 60°$, $\angle AMB = 70°$ (vertically opposite), $\angle ABM = 60°$ (angle sum of ΔABM), $\theta = 60°$, (alternate angles, $AB \parallel CD$)

c $\alpha = 55°$, $\angle BOM = 35°$ (BMO isosceles), $\angle OMA = 70°$ (exterior angle of ΔBMO), $\angle MOA = \alpha$ (ΔMOA isosceles), $a + \alpha + 70° = 180°$ (angle sum ΔAOM), $\alpha = 55°$

d $\angle OPQ = 65°$ (corresponding angles $PQ \parallel XY$),

$\theta = 105°$ (exterior angle ΔOPQ)

e $\angle ACB = 40°$ (alternate angles, $AF \parallel BG$), $\angle BAC = 40°$ (ΔABC isosceles), $\theta = 100°$ (angle sum ΔABC)

f $\alpha = 60°$ (ΔADC is equilateral), $\beta = 30°$ (co-interior angles, $BA \parallel CD$), $x = -1$ (ΔADC is equilateral)

g $\alpha = 65°$ (corresponding angles, $AB \parallel ED$), $\beta = 115°$ (supplementary angles), $\gamma = 32\frac{1}{2}°$ ($2\gamma = \alpha$, exterior angle)

h $\alpha = 120°$ (co-interior angles, $AD \parallel BC$), $\beta = 60°$ (co-interior angles, $AB \parallel CD$), $\gamma = 60°$ (corresponding angles, $AB \parallel CD$)

Exercise 7B

1 **a** $\angle AOB = 180° - \theta$ (supplementary angles at O)

b $\angle AOB = 35° + \theta$ (exterior angle of ΔOBC)

c $\angle AOB = 80° - \theta$ (exterior angle of ΔAOB)

d $\angle AOB = 180° - \theta$ (co-interior angles are supplementary, $AP \parallel XY$)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

e $\angle AOB = 135° - \theta$ (angle sum of ΔAOB)

f $\angle AOB = 270° - 2\theta$ (angle sum of quadrilateral $AOBM$)

g $\angle AOB = 180° - 2\theta$ (angle sum of isosceles ΔAOB)

h $\angle AOB = 90° - \frac{1}{2}\theta$ (angle sum of isosceles ΔAOB)

i $\angle AOB = 120° - \theta$ (adjacent supplementary angles, $\angle BOC = 60°$, ΔBOC is equilateral)

2 **a** $2\alpha = 180°$ (supplementary angles) so $\alpha = 90°$. Hence, $AB \perp PQ$.

b $2\alpha = 180°$ (supplementary angles), so $\angle ABP = 120°$ and $\angle ABQ = 60°$. Hence $\angle ABP = 2 \times \angle ABQ$.

c $2\alpha + 2\beta = 180°$ (supplementary angles) so $\alpha + \beta = 90°$. Hence, $PO \perp QO$.

d $10\alpha = 360°$ (angle sum of quadrilateral $ABCD$) so $\alpha = 36°$.

$\angle DAB + \angle ADC = 5\alpha = 180°$ (or $\angle ABC + \angle BCD = 5\alpha = 180°$)

Hence $AB \parallel DC$ (co-interior angles are supplementary)

3 **a** Construct OP parallel to AF (P to the left of O). Then $\angle AOP = \alpha$ (alternate angles) and $\angle BOP = \beta$ (alternate angles). Hence $\angle AOB = 120° = \alpha + \beta$.

b Construct AC parallel to OG. Then $\angle FAC = \alpha$ (corresponding angles) and $\angle PAC = \beta$ (corresponding angles). $\angle FAP = \angle FAC - \angle PAC = \alpha - \beta = 40°$.

c $\angle CBE = 50°$ (alternate angles, $FE \parallel BC$),

$\alpha + \beta + 50° = 180°$ (suppementary angles, $AD \parallel BC$), $\alpha + \beta = 130°$.

4 **a** **i** $\angle XAB = \beta$ because alternate angles are equal, $XY \parallel BC$.

$\angle YAC = y$ because alternate angles are equal, $XY \parallel BC$.

ii $\alpha + \beta + \gamma = 180°$ because of straight angle at A.

b **i** $\angle ACG = \alpha$ because alternate angles are equal, $AB \parallel GC$.

$\angle PCG = \beta$ because corresponding angles are equal, $AB \parallel GC$.

ii $\angle ACP = \angle PCG + \angle ACG = \alpha + \beta$ (adjacent angles).

5 **a** $\angle APO = \alpha$ since ΔAOP is isosceles (OA and OP are radii). $\angle BPO = \beta$ since ΔBOP is isosceles (OB and OP are radii). $\angle APB = \angle APO + \angle BPO = \alpha + \beta$

b $2\alpha + 2\beta = 180°$ (angle sum of ΔAPB). Hence $\alpha + \beta = 90°$.

6 **a** **i** $\angle BAC = 180° - \alpha, \angle ABC = 180° - \beta$ and $\angle ACB = 180° - \gamma$ (supplementary angles).

ii $\angle BAC + \angle ABC + \angle ACB = 180° - \alpha + 180° - \beta - 180° - \gamma = 180°$ (angle sum of ΔABC). Hence $\alpha + \beta + \gamma = 360°$.

b Join the diagonal AC. The interior angles of ΔABC add to 180° and the interior angles of ΔACD add to 180°. Since the sum of the interior angles of quadrilateral $ABCD$ is the sum of the angles of ΔABC and ΔACD, then the interior angles of $ABCD$ add to 360°.

c $\angle DAB = 180° - \alpha, \angle ABC = 180° - \beta$, $\angle BCD = 180° - \gamma$ and $\angle CDA = 180° - \gamma$ (supplementary angles).
$\angle DAB + \angle ABC + \angle BCD + \angle CDA = 720° - (\alpha + \beta + \gamma + \theta) = 360°$. Hence $\alpha + \beta + \gamma + \theta = 360°$.

7 **a** **i** 540° **ii** 1080° **iii** 1800°

b **i** 720° **ii** 900° **iii** 1260° **iv** 1440°

c angle sum $= 180(n - 2)°$

8 **a** **i** 1800° **ii** $1800° - 360° = 1440°$

b **i** 1080° **ii** 720°

c angle sum $= (180n - 360)°$

9 **a** **i** Each pair of interior and exterior angles is supplementary, and as there are 10 pairs, the sum of the exterior angles plus the sum of the interior angles is $10 \times 180° = 1800°$.

ii The sum of the interior angles is equal to $(180° \times 10) - 360° = 1440°$ so the sum of the exterior angles is equal to $1800° - 1440° = 360°$.

b The sum of the exterior angles plus the interior angles is 1080°. The sum of the interior angles is 720° so the sum of the exterior angles is 360°.

c 360°

10 interior angle $= \dfrac{180°n - 360°}{n} = 180° - \dfrac{360°}{n}$

11 **a** interior angle $= \left(180° - \dfrac{360°}{5}\right) = 108°$

b $\alpha = 72°$, $\beta = 72°$, $\gamma = 36°$

Exercise 7C

1 **a** $\Delta ABP \equiv \Delta ABQ$ (SSS) **b** $\Delta FAG \equiv \Delta NTL$ (AAS)

c $\Delta PQR \equiv \Delta RSP$ (RHS) **d** $\Delta ACD \equiv \Delta BCD$ (SAS)

2 **a** $\Delta BAC \equiv \Delta EFD$ (SSS) **b** $\Delta ACB \equiv \Delta EFD$ (SAS)

c $\Delta ABC \equiv \Delta EFD$ (AAS)

3 $\Delta ABC \equiv \Delta OPN$ (SSS); $\Delta DEF \equiv \Delta RSQ$ (RHS)
$\Delta GJH \equiv \Delta LMK$ (SAS)

4 **a** $\Delta ABC \equiv \Delta GJH$ (SSS) **b** $\Delta ABC \equiv \Delta ZYK$ (SAS)

c $\Delta ABC \equiv \Delta LDP$ (AAS) **d** $\Delta ABC \equiv \Delta RQP$ (SAS)

5 **a** $\angle PQR$ **b** $\angle RPQ$ **c** PR **d** BC

6 **a** $\alpha = 30°$, $\beta = 100°$

b $\alpha = 40°$, $\beta = 80°$

c $a = 12$, $b = 12$, $\alpha = 67°$, $\beta = 23°$

d $x = 6$, $y = 4$

e $\alpha = 83°$, $\beta = 55°$, $\gamma = 42°$, $\delta = 83°$

7 **a** **i** $AB = AC$ (given), $AM = AM$ (common), $BM = CM$ (given) so $\Delta ABM = \Delta ACM$ (SSS).

ii Hence $\alpha = \beta$ (matching angles of congruent triangles)

b **i** $OA = OP$ (given), $\angle AOB = \angle POQ$ (vertically opposite angles), $\angle OAB = \angle OPQ$ (given) so $\Delta OAB = \Delta OPQ$ (AAS).

ii Hence $x = y$ (matching sides of congruent triangles).

c $OG = OG$ (common), $OF = OH$ (radii), $\angle FOG = \angle HOG$ (given), so $\Delta GOF = \Delta GOH$ (SAS). Hence $FG = GH$ (matching sides of congruent triangles).

d $BD = BD$ (common), $\angle ABD = \angle CBD = 70°$ (given), $\angle DAB = \angle DCB = 80°$ (given) so $\Delta DAB \equiv \Delta DCB$ (AAS). Hence $AD = DC$ and $AB = BC$ (matching sides of congruent triangles).

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 $OA = OB$ (radii of a circle), $OM = OM$ (common side), $AM = BM$ (given) $\Delta OAM = \Delta OBM$ (SSS), $\angle OMA = \angle OMB = 90°$ (supplementary angles)

9 **a** **i** $AB = DC$ (given), $\angle ANB = \angle DNC = 90°$ (vertically opposite angles) and $BN = CN$ (given) so $\Delta ABN \pm \Delta DCN$ (RHS).

ii Hence $\angle A = \angle D$ (matching angles of congruent triangles).

iii Hence $AB \parallel CD$ (alternate angles are equal).

b **i** $PQ \parallel SR$ because $\angle QPR = \angle SRP$ (alternate angles are equal).

ii $PR = PR$ (common), $QR = SP$ (given), and $\angle QPR = \angle SRP$ so $\Delta PRS \equiv \Delta RPQ$ (RHS)

iii $\angle QRP = \angle SPR$ (matching angles in congruent triangles). Hence $PS \parallel QR$ (alternate angles are equal).

c $\angle AOB = \angle COD$ (vertically opposite angles), $OA = OB = OC = OD$ (radii). So $\Delta AOB \equiv \Delta DOC$ (SAS). Hence $\angle ABO = \angle DCO$ (matching angles in congruent triangles). Hence $AB \parallel CD$ (alternate angles equal).

d $AB = AB$ (common), $AC = AD$ (given), $\angle BAC = \angle BAD$ (given) so $\Delta BAC \equiv \Delta BAD$ (SAS). $\angle ABC = \angle ABD$ (matching angles in congruent triangles), so $\angle ABC = \angle ABD = 90°$ (supplementary angles). Hence $AB \perp CD$.

10 **a** $\angle BAR = \angle CAR$ (given), $AR = AR$ (common), $\angle ARC = \angle ARB$ (supplementary angles) so $\Delta ARB = \Delta ARC$ (AAS). $AB = AC$ *(matching sides in congruent triangles)* and hence ΔABC is isosceles.

b $BP = CQ$ (given), $\angle BQC = \angle CPB$ (given) and BC is common so $\Delta BQC \equiv \Delta CPB$ (RHS). Hence $\angle QBC = \angle PCB$ *(matching angles in congruent triangles)* so ΔABC is isosceles.

11 Let Z be the point where the bisector of $\angle ABC$ meets AC. Let W be the point where the bisector of $\angle ACB$ meets AB. $AB = AC$ (ΔABC isosceles). $\angle ABZ = \angle ACW$ (bisector of base angles of ΔABC). $\angle BAZ = \angle CAW, \Delta ABZ \equiv \Delta ACW$ (AAS). $AZ = AW$ (matching sides of congruent triangles). Therefore, $WB = ZC, \Delta WXB \equiv \Delta ZXC$ (AAS). $BX = CX$ (matching sides).

12 **a** Construct OA and OB. $OA = OB$ (radii), ON is common, $\angle ANO = \angle BNO = 90°$, so $\Delta AON \equiv \Delta BON$ (RHS), so $AN = BN$ (matching sides of congruent triangles), so PQ bisects AB.

b Construct OA, OB, OC and OD. $OA = OD$ (radii of larger circle), so $\angle DAO = \angle ADO$ (base angles of isosceles ΔADO). $OB = OC$ (radii of smaller circle), so $\angle CBO = \angle BCO$ (base angles of isosceles ΔBCO). $\angle ABO = 180° - \angle CBO$ (supplementary at B) $= 180° - \angle BCO = \angle DCO$ (supplementary at B), so $\angle AOB = \angle DOC$. So $ABO = DCO$ (AAS). So $AB = CD$ (matching sides of congruent triangles).

13 **a** **i** $AB = AC$ (given), $\angle BAM = \angle CAM$ (given), AM is common. So $\Delta BAM \equiv \Delta CAM$ (SAS).

ii Hence $\angle B = \angle C$ (matching angles of congruent triangles).

b **i** $\angle ABM = \angle ACM$ (given), $\angle MAB = \angle MAC$ (given) and AM is common so $\Delta ABM \equiv \Delta ACM$ (AAS).

ii Hence $AB = AC$ (matching sides of congruent triangles).

c **i** $AC = BC$ so $\angle B = \theta$ (the base angles of an isosceles triangle are equal).

ii $AB = BC$ so $\angle C = \theta$ (the base angles of an isosceles triangle are equal).

iii The angle sum of a triangle is 180°. $\angle A + \angle B + \angle C = 3\theta = 180°$ so $\theta = 60°$.

d **i** $AB = AC$ (opposite angles are equal).

ii $AC = BC$ (opposite angles are equal.

14 **a** **i** $\angle B = 180° - \alpha$ (co-interior angles are supplementary, $AD \parallel BC$), $\angle D = 180° - \alpha$ (co-interior angles are supplementary, $AB \parallel DC$).

ii $\angle C = \alpha$ (angle sum of quadrilateral $ABCD$).

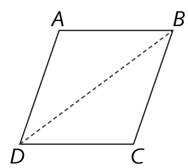
b **i** AC is common, $\angle DAC = \angle BCA$ (alternate angles are equal) $\angle BAC = \angle DCA$ (alternate angles are equal) so $\Delta ACD \equiv \Delta CAB$ (AAS).

ii Hence $AB = DC$ and $AD = BC$ (matching sides of congruent triangles).

c **i** $\angle AMB = \angle CMD$ (vertically opposite angles), $AB = CD$ (opposite sides of a parallelogram are equal), $\angle ABM = \angle CDM$ (alternate angles are equal) so $\Delta ABM \equiv \Delta CDM$ (AAS).

ii Hence $AM = CM$ and $BM = DM$ (matching sides of congruent triangles).

15 **a** $ABCD$ is a rhombus. Therefore $AB = BC = CD = DA$ so $\Delta BD \equiv \Delta CDB$ (SSS). Hence $\angle ABD = \angle CDB$ (matching angles of congruent triangles). Thus $AB \parallel CD$ (alternate angles are equal). Similarly $AD \parallel CB$. Thus $ABCD$ is a parallelogram.



b **i** $AP = PB = AQ = BQ$ (given), AB is common. So $\Delta APB \equiv \Delta AQB$ (SSS).

ii Hence $\angle PAM = \angle QAM$ (matching angles of congruent triangles).

iii $AP = AQ$ (given), $\angle QAM = \angle PAM$ (from **ii**), AM is common, so $\Delta QAM \equiv \Delta PAM$ (SAS). Hence $\angle QMA = \angle PMA$ (matching angles in congruent triangles) so $\angle QMA = \angle PMA = 90°$ (supplementary angles). Hence $AM \perp PQ$.

c **i** DC is common, $\angle ADC = \angle BCD$ (given), $AD = BC$ (opposite sides of a parallelogram). So $\Delta ADC \equiv \Delta BCD$ (SAS).

ii Hence $AC = BD$ (matching sides of congruent triangles).

16 **a** $AO = BO$ (radii), $AP = BP$ (radii), PO is common, so $\Delta AOP \equiv \Delta BOP$ (SSS).

b $\angle AOP = \angle BOP$ (matching angles of congruent triangles). Hence OP bisects $\angle AOB$.

c OM is common, $\angle AOM = \angle BOM$ (from **b**), $OA = OB$ (radii) so $\Delta AOM \equiv \Delta BOM$ (SAS).

d Hence $AM = BM$ (matching sides of congruent triangles). $\angle AMO = \angle BMO$ (matching angles of congruent triangles) and $\angle AMO$ and $\angle BMO$ are supplementary so $\angle AMO = \angle BMO = 90°$. Hence $AB \perp OP$.

Exercise 7D

1 **a** **i** $2\alpha + 2\beta = 360°$ (angle sum of quadrilateral $ABCD$) so $\alpha + \beta = 180°$.

ii $\alpha + \beta = 180°$ so $AB \parallel DC$ (co-interior angles are supplementary) and $AD \parallel BC$ (co-interior angles are supplementary).

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b i $RS = UT$ (given), $RU = ST$ (given), $SU = SU$ (common), so $\Delta RSU \equiv \Delta TUS$ (SSS).

ii Hence $\angle RSU = \angle TUS$ (matching angles of congruent triangles), so $RS \parallel UT$ (alternate angles are equal). Similarly $\angle RUS = \angle TSU$(matching angles of congruent triangles), so $RU \parallel ST$ (alternate angles are equal).

c i $KL = NM$ (given), $\angle LKM = \angle NMK$ (alternate angles, $KL \parallel NM$), $KM = KM$ (common), so $\Delta LKM \equiv \Delta NMK$ (SAS).

ii Hence $\angle LMK = \angle NKM$ (matching angles of congruent triangles), so $KN \parallel LM$ (alternate angles are equal).

d i $EO = OG$ (given), $OH = FO$ (given), $\angle EOH = \angle GOF$ (vertically opposite angles at O), so $\Delta EHO \equiv \Delta GFO$ (SAS).

ii $EH = FG$ (matching sides of congruent triangles), $\angle HEO = \angle FGO$ (matching angles of congruent triangles), so $EH \parallel FG$ (alternate angles are equal).

iii If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

2 a Since its diagonals bisect each other, $APBQ$ is a parallelogram.

b Since one pair of opposite sides, namely LM and ST, are equal and parallel, $LMST$ is a parallelogram.

c Correct. Since $AM = OB$ and $OA = BM$, $AMBO$ is a parallelogram.

3 a $\angle PAQ = \angle AQD = 50°$ (alternate angles, $CD \parallel AB$), $PC \parallel AQ$ (corresponding angles are equal, $\angle BPC = \angle PAQ = 50°$), $AP \parallel QC$ (given), so $APCQ$ is a parallelogram. So $AP = QC$.

b $XS \parallel UY$ (given), $XS = UY$ (given), so $SYUX$ is a parallelogram. So $UX \parallel YS$.

c $AD = CB$ (opposite sides of a parallelogram), $BY = DX$ (given), $\angle ADX = \angle CBY$ (alternate angles, $AD \parallel BC$), $\Delta AXD \equiv \Delta CYB$ (SAS), $AX = CY$ (matching sides). Similarly $AY = XC$, so $AXCY$ is a parallelogram.

d $\angle ADQ = \angle CBP$ (alternate angles, $AD \parallel BC$), $AD = BC$ (opposite angles of a parallelogram, $DQ = BP$ (given), $\Delta BPC \equiv \Delta DQA$ (SAS). $AQ = CP$ (matching sides).

e $AB = CD$ (opposite sides of a parallelogram), $\angle BAX = \angle DCY$ (alternate angles, $AB \parallel CD$), $\angle BXA = \angle DYC$ (given), $\Delta CYD \equiv \Delta AXB$ (AAS), $BX = DY$ (matching angles of congruent triangles).

f $DA = BC$ (opposite sides of a parallelogram), $\angle ADQ = \angle PBC$ (opposite angles of a parallelogram), $BP = DQ$ (given), $\Delta ADQ \equiv \Delta CBP$ (SAS). $AQ = PC$ (matching sides of congruent triangles).

Also $\angle BPC = \angle DQA$ (matching angles of congruent triangles) and $\angle BPC = \angle PCD$ (alternate angles, $AB \parallel DC$).

Hence $\angle DQA = \angle PCD$. Thus $AQ \parallel PC$ and $ABCQ$ is a parallelogram.

Exercise 7E

1 a Opposite sides of a parallelogram are equal. Therefore all sides are equal.

b i Since the diagonals of the quadrilateral bisect each other, $ASBT$ is a parallelogram.

ii $SM = TM$ (given), $\angle AMS = \angle AMT = 90°$ (supplementary at M), $AM = AM$ (common), so $\Delta AMS \equiv \Delta AMT$ (SAS).

iii $AS = AT$ (matching sides of congruent triangles)

c i Since the diagonals of the quadrilateral bisect each other, $RSTU$ is a parallelogram.

ii $US = RT$ (given), $RS = RS$ (common), $RU = ST$ (opposite sides of parallelogram equal), so $\Delta URS \equiv \Delta TSR$ (SSS).

iii $\angle URS = \angle TSR$ (matching angles of congruent triangles)

iv $\angle URS + \angle TSR = 180°$ (co-interior angles, $UR \parallel ST$), $2\angle URS = 180°$ ($\angle URS = \angle TSR$), $\angle URS = 90°$ and $URST$ is a rectangle.

2 a Given parallelogram $ABCD$:

if $\angle ABC = 90°$, then $\angle BCD = 90°$ (co-interior angles $AB \parallel CD$), and $\angle DAB = 90°$ (co-interior angles $DA \parallel CB$), $\angle CDA = 90°$ (angle sum of a quadrilateral is 360°). Since all interior angles are right angles, $ABCD$ is a rectangle.

b Suppose parallelogram $ABCD$ has $\angle ABC = \angle BCD = \angle CDA = \angle DAC$. Since angle sum of a quadrilateral is 360°, and all angles are equal, then each interior angles is $360° \div 4 = 90°$, so all interior angles are right angles, and $ABCD$ is a rectangle.

3 a $\angle ABF = \angle GAB = \alpha$ (alternate angles, $AG \parallel BF$)

b $AF = BF$ (equal sides of isosceles AFB). So 2 adjacent sides are equal in a parallelogram. Hence $AFBG$ is a rhombus.

4 a Since AB and FG are equal and bisect each other, $AFBG$ is a rectangle.

b Since AB and FG are equal and bisect each other at right angles, $AFBG$ is a square.

5 a Since AB and FG bisect each other, $AFBG$ is a parallelogram. Since $\angle AFB$ is a right angle, $AFBG$ is a rectangle.

b Since the diagonals of a rectangle are equal and bisect each other, $AM = BM = FM =$ radius of the circle.

6 a i $AP = AQ = BP = BQ$, so $APBQ$ is a rhombus.

ii Since the diagonals of a rhombus bisect each other at right angles, PQ is the perpendicular bisector of AB.

b i $OP = OQ = PM = QM$, so $OPMQ$ is a rhombus.

ii Since the diagonals of a rhombus bisect the vertex angles through which they pass, OM is the bisector of $\angle AOB$.

c i $PA = PB = AM = BM$, so $APBM$ is a rhombus.

ii Since the diagonals of a rhombus bisect each other at right angles, PM is perpendicular to ℓ.

7 a $\angle ABP = \angle CBP = \angle ADQ = \angle CDQ = 45°$ (diagonals of square $ABCD$ meet each side at 45°), $BP = DQ$ (given), $AB = BC = CD = DA$ (equal sides of square $ABCD$), so $\Delta ABP \equiv \Delta CBP \equiv \Delta ADQ \equiv \Delta CDQ$ (SAS).

b $AP = CP = AQ = CQ$ (matching sides of congruent triangles), so $APCQ$ is a rhombus.

8 a i $CF = CG$ (given), $BF = BG$ (given), $CB = CB$ (common), so $\Delta CBF \equiv \Delta CBG$ (SSS).

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

ii $\angle FCM = \angle GCM$ (matching angles of congruent triangles), so CB bisects $\angle FCG$. $\angle FBM = \angle GBM$ (matching angles of congruent triangles), so CB bisects $\angle FBG$.

iii $CF = CG$ (given), $CM = CM$ (common), $\angle FCM = \angle GCM$ (matching angles of congruent triangles CBF and CBG), so $\Delta CMF \equiv \Delta CMG$ (SAS).

iv $\angle CMF = \angle CMG$ (matching angles of congruent triangles), and $\angle CMF + \angle CMG = 180°$ (straight angle at M), $2\angle CMF = 180°$ ($\angle CMF = \angle CMG$), $\angle CMF = 90°$ so $CB \perp FG$.

b $FM = GM$ (given), $\angle AMF = \angle AMG = 90°$ $AM = AM$ (common), so $\Delta AMF \equiv \Delta AMG$ (SAS), so $AF = AG$ (matching sides of congruent triangles). $\angle BMF = \angle BMG = 90°$ (straight angle at M), $BM = BM$ (common), so $\Delta BMF \equiv \Delta BMG$ (SAS), so $BF = BG$ (matching sides of congruent triangles). Since $AF = AG$ and $BF = BG$, $AFBG$ is a kite.

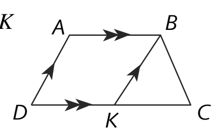
c $AP = AM$ (radii of circle centre A), $BP = BM$ (radii of circle centre B), so $APBM$ is a kite. Since the diagonals of a kite are perpendicular, $PM \perp \ell$.

9 Draw diagonals BP and CQ of rhombus $BCPQ$. $\angle AQB = \angle QAB$ (ΔABQ isosceles), $\angle QBC = 2\angle QAB$ (exterior angle), $\angle QAB = \angle PBC$ (diagonal of rhombus bisects angle), $BP \parallel AQ$ (corresponding angles equal), similarly $QC \parallel PD$. QC and BP meet at right angles at X (diagonals of a rhombus are perpendicular). AR is perpendicular to DR.

10 $\angle XSP = \angle SXP$ (ΔSPX isosceles), $\angle RSX = \angle PXS$ (alternate angles, $SR \parallel PQ$), $\angle PSR = 2\angle PSX$, SX bisects $\angle RSP$

11 **a** $\angle A = 130°$ and $\angle B = 110°$

b $BC = BC$ and $AB \parallel DC$. Draw BK parallel to AD.



$ABKD$ is a parallelogram. $AD = BK$ (opposite sides of a parallelogram)

Thus $BK = BC$ and hence $DKBC$ is isosceles.

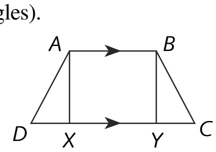
Therefore $\angle BKC = \angle BCK$.

$\angle BCK = \angle ADK$ (corresponding angles $AD \parallel BK$).

Therefore $\angle BCD = \angle ADC$.

c $\Delta DBC \equiv \Delta CAD$ (SAS) since $\angle BCD = \angle ADC$, $DC = CD$ (common) and $AD = BC$ (given). Thus $DB = CA$ (matching sides of congruent triangles).

d Draw a perpendicular from A to meet line DC at X and from B to meet DC at Y. $ABCD$ is a rectangle and so $AX = BY$.



$\Delta DBY \equiv \Delta CAX$ (RHS). Thus $\angle BDY = \angle ACY$.

Therefore $\Delta DBC \equiv \Delta CAD$ (SAS) and $AD = BC$ (matching sides of congruent triangles).

Review exercise

1 **a** acute **b** straight angle
c right angle **d** reflex angle
e revolution **f** obtuse angle

2 **a** 34° **b** 23° **c** 43°
d 75° **e** 64° **f** 44°

3 **a** 26° **b** 91° **c** 146°
d 90° **e** 67° **f** 64°

4 **a** $\alpha = 90°$ (supplementary); $\beta = 115°$ (supplementary)

b $\alpha + 15° = 35°$ (vertically opposite), $\alpha = 20°$, $\beta = 145°$ (supplementary)

c $\angle BCD = 34°$ (revolution), $\alpha = 34°$ (alternate angles, $AB \parallel CD$)

d $\alpha = 60°$ (angle sum of triangle), $\beta = 48°$ (revolution), $\theta = 62°$ (angle sum of triangle)

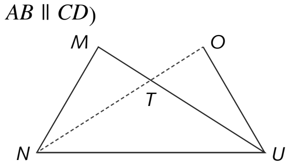
e $\theta = 48°$ (complementary), $\beta = 270°$ (revolution)

f $\beta = 120°$ (corresponding $AB \parallel FC$), $\alpha = 60°$ (co-interior, $AE \parallel BD$)

g $\angle LMK = 87°$ (alternate angles, $CD \parallel AB$), $\beta = 93°$, $\alpha = 90°$ (alternate angles $AB \parallel CD$)

5 $MT = OT$ (given) and $\angle MNT = \angle OUT$ (given) $\Delta NMT \equiv \Delta UOT$ (AAS).

Therefore $NT = UT$.



6 ΔABC is isosceles with $AB = AC$. $BD \perp AC$, $CE \perp AB$.

a $\angle DCB = \angle EBC$ (ΔABC is isosceles)

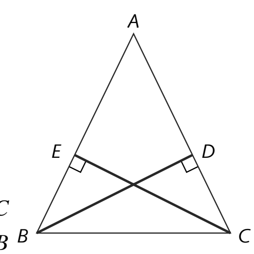
$\Delta BDC \equiv \Delta BEC$ (AAS)

b $\Delta EDB \equiv \Delta DEC$ (SSS)

$\angle EDB = \angle DEC$ (matching angles)

$\angle AED = 180° - 90° - \angle DEC$

$\angle ADE = 180° - 90° - \angle EDB$



Therefore $\angle AED = \angle ADE$.

7 **a** $\angle ABE = \angle BED$ (alternate angles, $AB \parallel ED$)

$\angle CBE = \angle FEB$ (alternate angles, $BC \parallel FE$)

$\angle ABC = \angle ABE + \angle CBE = \angle BED + \angle BEF = \angle FED$

b $\Delta ABC \equiv \Delta DEF$ (SAS)

$AC = FD$. Let BE meet AC at X and FD at Y. Thus $\angle AXB = \angle DYE$ (angle sum of triangles) and $\angle DYE = \angle FYB$ (vertically opposite).

$ACDF$ is a parallelogram.

8 **a** $\angle BAE = \angle AED$ (alternate angles, $AB \parallel CD$)

$\angle ABE = \angle CEB$ (alternate angles, $AB \parallel CD$)

$\angle DAE = \angle BAE$ (given)

ΔAED is isosceles and ΔBEC is isosceles.

$AD = BC$ (opposite sides of parallelogram)

$AD = DE$ and $BC = EC$ (ΔAED is isosceles and ΔBEC is isosceles.)

Therefore $DE = EC$.

b ΔBEC is isosceles, so $EC = BC$

ΔAED is isosceles, so $AD = DE$

$AD = BC$

so $DE = EC$

Therefore $DC = 2CB$.

c $2\angle BAE + 2\angle ABE = 180°$, and therefore

$\angle BAE + \angle ABE = 90°$

$\angle AEB = 180° - (\angle BAE + \angle ABE) = 90°$.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

9 **a** ΔAXD is isosceles and ΔBYC is isosceles.

$\angle ADX = \angle CBY$ (alternate angles, $AD \parallel BC$)

$\Delta AXD \equiv CYB$ (SAS)

$AX = CY$ (matching sides)

b $\angle XDC = \angle YBA$ (alternate angles, $AB \parallel CD$)

$DX = BY$ (part **a**)

$\Delta AYB \equiv \Delta CXD$ (SAS)

$AY = CX$ (matching sides of congruent triangles)

c Opposite sides are of equal length.

Challenge exercise

1 **a** Let $AM = a$, $BM = b$, $CM = c$ and $DM = d$. By Pythagoras' theorem,

$AB^2 = a^2 + b^2$, $BC^2 = b^2 + c^2$, $CD^2 = c^2 + d^2$ and $AD^2 = a^2 + d^2$.

So $AB^2 + CD^2 = a^2 + b^2 + c^2 + d^2$, and $AD^2 + BC^2 = a^2 + d^2 + b^2 + c^2$,

so $AB^2 + CD^2 = AD^2 + BC^2$.

b Let $AP = a$, $BQ = b$, $CQ = c$ and $DP = d$. By Pythagoras' theorem,

$AB^2 = a^2 + (b + PQ)^2$, $BC^2 = b^2 + c^2$,

$CD^2 = c^2 + (d + PQ)^2$ and $AD^2 = a^2 + d^2$.

Now $AB^2 + CD^2 = AD^2 + BC^2$,

so $a^2 + (b + PQ)^2 + c^2 + (d + PQ)^2 = a^2 + b^2 + c^2 + d^2$,

and $\quad 2PQ^2 + 2b \times PQ + 2d \times PQ = 0$

$$2PQ(PQ + b + d) = 0$$

$$PQ = 0$$

So P and Q coincide, and the diagonals of the quadrilateral are perpendicular.

c Parts **a** and **b** still hold.

2 *To prove*: $OA = OB = OC$

$AR = BR$ (given), $\angle ARO = \angle BRO = 90°$ (given R), $OR = OR$ (common),

so $\Delta ARO \equiv \Delta BRO$ (SAS). So $OA = OB$ (matching sides of congruent triangles).

$AQ = CQ$ (given), $\angle AQO = \angle CQO = 90°$ (given), $OQ = OQ$ (common),

so $\Delta AQO \equiv \Delta CQO$ (SAS). So $OA = OC$ (matching sides of congruent triangles).

So $OA = OB = OC$.

To prove: $OP \perp BC$

$BP = CP$ (given), $OB = OC$ (from above), $OP = OP$ (common),

so $\Delta BPO \equiv \Delta CPO$ (SSS). So $\angle BPO = \angle CPO$ (matching angles of congruent triangles) = 90° (straight angle at P). So $OP \perp BC$.

3 **a i** 1 positive **ii** 1 negative

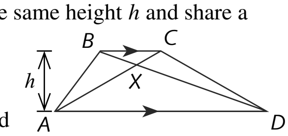
iii 0 **iv** 2 positive

b For a convex polygon must complete one revolution. Angle sum of exterior angles is 360°.

4 $\frac{3}{4}$ of a positive revolution = 270°

Sum of exterior angles = 270°

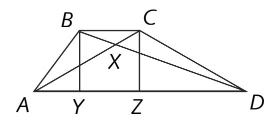
5 **a** ΔABD and ΔACD have the same height h and share a common base AD.



Therefore, area of ΔABD = area of ΔACD and area of ΔABX + area of ΔAXD = area of ΔDXC + area of ΔAXD.

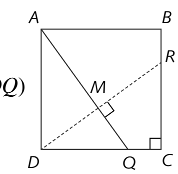
Hence area of ΔABX = area of ΔDCX.

b If area of ΔABX = area of ΔDCX, then area of ΔABD = area of ΔACD.



These two triangles have a common base AD and therefore their heights BY and CZ are equal and BC is parallel to AD. So $ABCD$ is a trapezium.

6 **a** $DC = DA$ (sides of a square)



Let $\angle DAQ = \alpha$

$\angle AQD = 90° - \alpha$ (angle sum of ΔADQ)

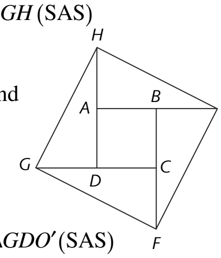
$\angle MDQ = 90° - (90° - \alpha) = \alpha$ (angle sum of ΔDMQ)

$\Delta DAQ \equiv \Delta CDR$ (AAS)

$AQ = DR$ (matching sides of congruent triangles)

b Consider suitable translations.

7 **a** $\Delta AHE \equiv \Delta BEF \equiv \Delta CFG \equiv \Delta DGH$ (SAS)



By Pythagoras' theorem, $EF = FG = GH = HE = \sqrt{5}$, and the four angles are 90°. Area of square = 5

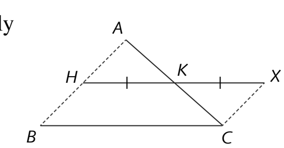
b Draw diagonals FH and EG to intersect at O'. Show $\Delta HAO' \equiv \Delta EBO' \equiv \Delta FCO' \equiv \Delta GDO'$ (SAS)

This shows that that O' is an equal distance from each vertex of the original square and hence is the centre.

8 The line through the intersection cuts opposite sides and two trapezia are formed. The opposite sides of a parallelogram being of equal length enables you to show that these trapezia are of equal area.

9 Construct the line from the vertex to the midpoint of the base, and note the two triangles formed are congruent. Hence the line is the perpendicular bisector. Now use the fact that there is only one perpendicular bisector of the base.

10 **a** $HK = XK$ (given)



$\angle AKH = \angle CKX$ (vertically opposite)

$AK = KC$ (K is the midpoint of AC)

$\Delta AKM \equiv \Delta CKX$ (SAS)

$XC = AH$ (matching sides)

b $CXHB$ is a parallelogram and so $HX = BC$.

$HK = \frac{1}{2}\ HX = \frac{1}{2}\ BC$ and $BC \parallel HK$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Chapter 8 answers

Exercise 8A

1

	a	b	c	d	e	f
Base	6	7	8	10	5	6
Exponent	4	3	2	4	1	0

2 **a** 2^3 **b** 3^3 **c** 2^6 **d** 3^5 **e** 5^3 **f** 3^4

3 **a** 81 **b** 128 **c** 3125 **d** 2401 **e** 1944 **f** 11 664

4 **a** 2×3^2 **b** $2^3 \times 3$ **c** $2^4 \times 3^2$ **d** $2 \times 3^2 \times 5$ **e** $2^2 \times 5^2 \times 7$ **f** $2^2 \times 3 \times 7$

5 **a** 2^5 **b** 2^{10} **c** 3^9 **d** 3^{11} **e** 3^7 **f** 3^{12} **g** a^{11} **h** b^{19} **i** $3a^5$ **j** $12x^5$ **k** $6y^5$ **l** $12b^6$

6 **a** a^2b^5 **b** a^5b^4 **c** x^5y^2 **d** $6x^3y^3$ **e** $4a^5b^6$ **f** $10a^5b^4$

7 **a** 3^5 **b** 2^4 **c** 5^3 **d** 7^2 **e** 10^5 **f** 10^8 **g** a^3 **h** a^2 **i** $2x$ **j** $3x^3$ **k** $2y^9$ **l** $3p^3$

8 **a** ab^2 **b** x^2y **c** ab^2 **d** xy^5 **e** $3a^4b$ **f** $5xy$ **g** $\frac{4a^2b}{3}$ **h** $\frac{3xy}{2}$

9 **a** a^3b^2 **b** x^4y^4 **c** $4a^2b^2$ **d** $4xy$ **e** $\frac{3a^2b}{2}$ **f** $\frac{2x^3y}{3}$ **g** $4ab^2$ **h** $2x^4y^2$

10 **a** a^6 **b** b^9 **c** $3a^4$ **d** $3d$ **e** a^4 **f** x^4 **g** $5d^5$ **h** $3d^5$ **i** m^6n^2 **j** $3ab^2$ **k** a^5b^3 **l** $3x^5y^3$ **m** ℓ^4m^2 **n** $3m^5n^4$ **o** $6pq^5$

11 **a** $6x$ **b** $2x$ **c** x **d** $2x^3$ **e** $3x^2$ **f** $2x$ **g** 5 **h** $2x^2$

12 **a** 1 **b** 2 **c** x **d** 7

13 **a** 3 **b** 6 **c** 1 **d** 1 **e** 7 **f** 13 **g** 1 **h** 1 **i** 3 **j** –4 **k** 1 **l** 1

14 **a** 2^{12} **b** 3^6 **c** a^{10} **d** y^{30}

15 **a** a^{18} **b** x^{17} **c** b^2 **d** y^4 **e** $6a^2b^8$ **f** $4b^2$ **g** $\frac{x^4}{2}$ **h** $\frac{3a^2b}{2}$ **i** x^4

16 **a** 4 **b** 7 **c** 2 **d** 5 **e** p^6 **f** p^5 **g** a^2 **h** m^5 **i** $(m^1)^{20}$ or $(m^2)^{10}$ or $(m^4)^5$ or $(m^5)^4$ or $(m^{10})^2$ or $(m^{20})^1$

17 **a** yes **b** yes **c** $(am)^n = (an)^m$

18 **a** $9a^2$ **b** $8x^3$ **c** x^2y^6 **d** a^8b^4 **e** $\frac{a^2}{25}$ **f** $\frac{8}{x^3}$ **g** $\frac{a^5}{b^5}$ **h** $\frac{x^6}{y^3}$

19 **a** $12a^5b^5$ **b** $27x^7y^8$ **c** m^7n^9 **d** $125x^7y^{12}$ **e** $24a^9b^3$ **f** $27x^6y^3$

20 **a** xy^4 **b** $2ab$ **c** x^3y^2 **d** $2x^{14}y$

21 **a** $2x^4y^5$ **b** $\frac{3ab^4}{2}$ **c** $2xy^8$ **d** $3a^2b$

22 **a** a^2b^3 **b** m^5n^4 **c** 3 **d** 0 **e** $2a^2$ **f** $3q^3$ **g** $7m^3$ **h** $4\ell^3m$ **i** $5m^5n^3$

Exercise 8B

1 **a** $\frac{1}{2^1} = \frac{1}{2}$ **b** $\frac{1}{5^1} = \frac{1}{5}$ **c** $\frac{1}{3^2} = \frac{1}{9}$ **d** $\frac{1}{6^2} = \frac{1}{36}$ **e** $\frac{1}{9^2} = \frac{1}{81}$ **f** $\frac{1}{10^2} = \frac{1}{100}$ **g** $\frac{1}{2^4} = \frac{1}{16}$ **h** $\frac{1}{3^3} = \frac{1}{27}$ **i** $\frac{1}{5^3} = \frac{1}{125}$ **j** $\frac{1}{3^4} = \frac{1}{81}$ **k** $\frac{1}{10^5} = \frac{1}{100000}$ **l** $\frac{1}{2^7} = \frac{1}{128}$

2 **a** 2^{-3} **b** 3^{-2} **c** 3^{-3} **d** 7^{-2} **e** 11^{-2} **f** 5^{-3} **g** 2^{-4} **h** 2^{-6} **i** 13^{-2} **j** 3^{-4} **k** 31^{-1} **l** 2^{-5}

3 **a** $\frac{1}{a^3}$ **b** $\frac{1}{x^7}$ **c** $\frac{3}{a^4}$ **d** $\frac{5}{x^7}$ **e** $\frac{4}{a^5}$ **f** x^3 **g** $3a^4$ **h** $5x^5$ **i** $\frac{1}{9a^2}$ **j** $\frac{1}{16x^2}$ **k** $\frac{n^6}{169}$ **l** $\frac{x^3}{y^4}$

4 **a** 4 **b** $\frac{25}{4}$ **c** $\frac{9}{100}$ **d** $\frac{27}{8}$ **e** 27 **f** 125 **g** $\frac{1}{49}$ **h** $\frac{1}{16}$

5 **a** $2^{-3} = \frac{1}{8}$ **b** $4^{-2} = \frac{1}{16}$ **c** $3^{-1} = \frac{1}{3}$ **d** $6^{-3} = \frac{1}{216}$ **e** $7^{-2} = \frac{1}{49}$ **f** $5^{-3} = \frac{1}{125}$ **g** $8^{-1} = \frac{1}{8}$ **h** $20^{-2} = \frac{1}{400}$ **i** $3^{-4} = \frac{1}{81}$ **j** $2^{-6} = \frac{1}{64}$ **k** $10^{-4} = \frac{1}{10000}$ **l** $12^{-2} = \frac{1}{144}$

6 **a** $3x^{-1}$ **b** $5x^{-2}$ **c** $8x^{-4}$ **d** $\frac{3}{2}x^{-4}$ **e** $\frac{4}{3}x^{-7}$ **f** $\frac{2x^{-5}}{3}$

7 **a** 2 **b** $\frac{3}{2}$ **c** 4 **d** $\frac{25}{16}$ **e** $\frac{16}{81}$ **f** $\frac{125}{216}$

8 **a** $\frac{y^2}{x^4}$ **b** $\frac{a^2}{b^8}$ **c** $\frac{15y^3}{x^9}$ **d** $14b^2$ **e** $\frac{56}{a^2m^7}$ **f** $\frac{12}{rs^2}$ **g** $\frac{4}{a^{10}}$ **h** $\frac{2}{a^9}$ **i** $\frac{9}{2a^9}$ **j** $\frac{3}{m}$ **k** $\frac{7}{t^5}$ **l** $\frac{4}{h^5}$ **m** $12x^{10}y$ **n** $\frac{2a^3}{b}$ **o** $\frac{b^4c^5}{3a^3}$ **p** $\frac{3m^6}{7p^7}$

9 **a** 6^{-2} **b** 9^{-1} **c** b^{-2} **d** m^{-11} **e** a^{-3} **f** b^{-8} **g** d^{-22} **h** e^{-2}

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

i -5 **j** -3 **k** $\dfrac{2}{m^2}$ **l** $3a^3$

m $5m^{-3}$ **n** $\dfrac{b^3}{a^2}$ **o** $\dfrac{p}{m^2n^3}$ **p** $p^{-2}q^3$

q $(a^3b^{-2})^2$ or $(b^2a^3)^{-2}$ **r** $\left(\dfrac{m^2}{n^3}\right)^3$ or $\left(\dfrac{n^3}{m^2}\right)^{-3}$

There is more than one answer for parts **q** and **r**.

10 **a** $\dfrac{27}{4a^2b^6}$ **b** $\dfrac{125}{x^9y^{21}}$ **c** $\dfrac{2n^{12}}{25m^8}$ **d** $\dfrac{b^6}{108a^9}$
e $\dfrac{x}{y^7}$ **f** $\dfrac{x^8}{3y^9}$ **g** $\dfrac{2a^{15}}{b^8c^3}$ **h** mn^3p^9
i $\dfrac{a^8}{b^7}$ **j** $\dfrac{8a^{14}}{b^6}$ **k** $\dfrac{16c^{10}}{27d^5}$ **l** $\dfrac{1}{9m^3n^6p}$

Exercise 8C

1 **a** 2 **b** 2 **c** 6 **d** 3 **e** 4 **f** 4

2 **a** $14^{\frac{1}{2}}$; 3.7417 **b** $64^{\frac{1}{4}} = 8^{\frac{1}{2}}$; 2.8284
c $7^{\frac{1}{5}}$; 1.4758 **d** $11^{\frac{1}{7}}$; 1.4085
e $2^{\frac{7}{3}}$; 5.0397

3 **a** 2 **b** 3 **c** 3 **d** 3
e 8 **f** 5 **g** 5 **h** 4
i 2 **j** 5 **k** 6 **l** 7

4 **a** 32 **b** 125 **c** 25 **d** 32
e 4 **f** 27 **g** 36 **h** 27
i 8 **j** 9 **k** 16 **l** 4

5 **a** a **b** b^2 **c** c^3 **d** c^2
e x^2 **f** y **g** $p^{\frac{23}{20}}$ **h** $q^{\frac{13}{6}}$
i x **j** $y^{\frac{1}{3}}$ **k** $p^{\frac{7}{20}}$ **l** $q^{\frac{5}{6}}$
m $2m^3$ **n** $3n^4$ **o** $8x^2$ **p** $81y^2$

6 **a** $\dfrac{1}{2}$ **b** $\dfrac{1}{5}$ **c** $\dfrac{5}{2}$ **d** $\dfrac{3}{4}$
e $\dfrac{1}{4}$ **f** 3 **g** $\dfrac{1}{3}$ **h** 5
i $\dfrac{3}{2}$ **j** $\dfrac{3}{2}$

7 **a** $\dfrac{1}{a}$ **b** $\dfrac{1}{b^4}$ **c** $\dfrac{1}{8x^2}$ **d** $\dfrac{1}{81y^2}$
e $\dfrac{1}{x}$ **f** $\dfrac{1}{y^{\frac{1}{3}}}$ **g** $p^{\frac{7}{20}}$ **h** $q^{\frac{5}{6}}$
i x^2 **j** y **k** $p^{\frac{23}{20}}$ **l** $q^{\frac{13}{6}}$
m $\dfrac{2}{m^3}$ **n** $\dfrac{3}{n^4}$ **o** $\dfrac{32}{x^2}$

Exercise 8D

1 **a** 10^1 **b** 10^2 **c** 10^3 **d** 10^4
e 10^6 **f** 10^9 **g** 10^{100}

2 **a** 10^{-1} **b** 10^{-2} **c** 10^{-3} **d** 10^{12}
e 10^{-5} **f** 10^{-6}

3 **a** 5.10×10^2 **b** 5.3×10^3 **c** 2.6×10^4
d 7.96×10^8 **e** 5.76×10^{11} **f** 4×10^{12}
g 8×10^{-3} **h** 6×10^{-2} **i** 7.2×10^{-4}
j 4.1×10^{-5} **k** 6×10^{-9} **l** 2.06×10^{-7}

4 **a** 32 400 **b** 7200 **c** 860
d 2 700 000 **e** 5.1 **f** 72
g 0.056 **h** 0.0017 **i** 0.000 872
j 0.002 01 **k** 0.97 **l** 0.000 000 26

5 6×10^{24} kg **6** 2.99×10^5 km/s

7 8.9×10^{-3} kg **8** 4×10^{-8} m

9 **a** 8×10^{11} **b** 6.3×10^{13}
c 2×10^{-4} **d** 2.4×10^4
e 2.5×10^1 **f** 2×10^6
g $\approx 7.5 \times 10^0$ **h** 3×10^{-2}
i 1.94481×10^9 **j** 2.7×10^{-5}
k 5×10^6 **l** 6.4×10^2
m 8×10^4 **n** 9.6×10^{12}
o 1×10^7 **p** 5×10^0

10 $\approx 7.57 \times 10^{17}$ km **11** 1840

12 8.3 minutes ($\approx$ 8 minutes 18 seconds)

13 $\approx 4.35 \times 10^{22}$

14 5656 days and 7 hours

Exercise 8E

1 **a** 2.70×10^0 **b** 6.35×10^2 **c** 8.76×10^3
d 2.56×10^5 **e** 3.61×10^{-3} **f** 2.42×10^{-2}

2 **a** 3.7×10^2 **b** 2.8×10^5
c 4.3×10^{-3} **d** 2.2×10^{-5}

3

2.746×10^2	2.75×10^2	2.75×10^2	3×10^2
4.124×10^{-2}	4.12×10^{-2}	4.1×10^{-2}	4×10^{-2}
1.704×10^3	1.70×10^3	1.7×10^3	2×10^3
1.993×10^{27}	1.99×10^{27}	2.0×10^{27}	2×10^{27}

4 **a** 2.17×10^{-1} **b** 2.40×10^2 **c** 1.11×10^8
d 8.70×10^{-3} **e** 6.48×10^{-4} **f** 6.40×10^1
g 2.55×10^1 **h** 1.72×10^0

5 **a** 2.453×10^{-1} **b** 6.207×10^0 **c** 7.241×10^0
d 6.271×10^{43} **e** 2.205×10^{-6} **f** 5.247×10^{-1}
g 1.792×10^{13} **h** 3.956×10^{-2} **i** 2.646×10^4
j 5.577×10^{-1} **k** 4.547×10^2 **l** 1.380×10^3

Review exercise

1 **a** 64 **b** 64 **c** 64 **d** 1 000 000

2 **a** $2^6 \times 3^2 \times 5^2$ **b** $2^6 \times 3^6 \times 5^6$
c $3^8 \times 5^4 \times 7^4$ **d** $2^5 \times 5^5 \times 7^{10}$

3 **a** a^{13} **b** b^{13} **c** $15a^9$ **d** $10x^9$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Cambridge University Press

Updated August 2021

e a^3 f m^{18} g $2b^5$ h $2p^9$
i a^{12} j b^{30} k $8a^{21}$ l $81m^8$
m 1 n 3 o 5 p 1

4 a $20a^3b^7$ b $10m^{10}n^{10}$ c $4a^2b$
d $\frac{4}{3}m^3n^2$ e $81a^{12}b^4$ f $100a^8b^5$
g $15a^7b^7$ h $\frac{16}{3}m^7n^{12}$ i $\frac{x^9}{5y^5}$
j $\frac{16a^3}{5}$ k $\frac{2x^5y^5}{3}$ l $144a^6b^3$

5 a $\frac{1}{36}$ b $\frac{1}{512}$ c $\frac{1}{128}$ d $\frac{1}{64}$
e $\frac{25}{16}$ f $\frac{81}{16}$ g $\frac{1}{2\sqrt{2}}$ h $\frac{1}{8}$

6 a $\frac{m^4}{4p^9}$ b $\frac{1}{64y^3}$ c $\frac{1}{2^{10}y^{15}}$
d $\frac{5^{10}}{x^{15}}$ e $\frac{3^{12}b^4}{a^8}$ f $\frac{b^4}{a^6}$
g $\frac{1}{25g^4h^6}$ h $\frac{m^6}{2^8n^8}$

7 a $\frac{20}{a}$ b $\frac{40n^3}{m^2}$ c $2a$
d $\frac{2}{m^2n^2}$ e $\frac{n^6}{64m^6}$ f $\frac{1}{250m^8n^5}$
g $\frac{b^{16}}{162a^{14}}$ h $\frac{4m^2n^8}{15}$ i $\frac{5a^8}{b^8}$

8 a 7 b 2 c 25 d 16
e 27 f $\frac{1}{64}$ g 4 h $\frac{9}{16}$

9 a $a^{\frac{5}{6}}$ b $12b^{\frac{5}{3}}$ c $p^{\frac{1}{6}}$
d $m^{\frac{3}{2}}$ e $256x^{\frac{4}{3}}$ f $\frac{x^{\frac{2}{3}}}{4}$
g $3a^{\frac{1}{6}}$ h $\frac{2^{\frac{3}{2}}q^{\frac{3}{2}}}{p}$

10 a 4.7×10^4 b 1.64×10^8 c 4.7×10^{-3}
d 3.5×10^{-3} e 8.4×10^2 f 8.40×10^{-1}

11 a 68 000 b 7500 c 0.0026
d 0.094 e 6.7 f 0.000 32

12 a 6.2×10^2 b 2.6×10^{-10}
c 2.7×10^{13} d 2.5×10^{-7}
e 8×10^{-1} f 3×10^{14}

13 a 1.9×10^1 b 1.86×10^1
c 2×10^1 d 4.3×10^{-3}
e 6.0×10^3 f 4.740×10^{-1}

Challenge exercise

1 a 12 006 b 35.6 c 77%
d 2.4, 125.0, 105.8, 4581.4, 12.6, 28.4
i 52.2 i Australia ii Singapore
e i 1.95×10^7, 1.27×10^9, 2.18×10^8, 3.06×10^6, 3.50×10^6, 2.74×10^8
i 2.09×10^7, 1.34×10^9, 2.36×10^8, 3.24×10^6, 3.61×10^6, 2.88×10^8
ii 7.31×10^7, 3.58×10^9, 9.83×10^8, 8.66×10^6, 6.18×10^6, 7.06×10^8
f 9.6, 3.7×10^2, 5.2×10^2, 1.4×10^4, 2.3×10^1, 7.7×10^1

2 a 9 cm
b i 1 cm ii 12 cm iii $1\frac{1}{3}$
c i 48 ii $\frac{1}{3}$ cm iii 16 cm iv $a = \frac{4}{3}$
d i $\left(\frac{4}{3}\right)^3 \times 9$ ii $\left(\frac{4}{3}\right)^4 \times 9$
iii $\left(\frac{4}{3}\right)^{10} \times 9$ iv $\left(\frac{4}{3}\right)^n \times 9$
e i 9 ii 17 iii 33
f becomes very large

3 a 100 mm × 136 mm b 80 mm × 108.8 mm
c i 125×0.8^3 mm, 170×0.8^3 mm
ii 125×0.8^4 mm, 170×0.8^4 mm
iii 125×0.8^n mm, 170×0.8^n mm
d 21 250 mm^2
e i $21\,250 \times 0.8^2$ mm^2 ii $21\,250 \times 0.8^4$ mm^2
iii $21\,250 \times 0.8^6$ mm^2 iv $21\,250 \times 0.8^{2n}$ mm^2
f i $x \times 0.8^n$ cm, $y \times 0.8^n$ cm
ii $xy \times 0.8^{2n}$ cm^2

4 2^{n+2}

5 a $\left(\frac{2}{3}\right)^n$ b 2^{-3n^2+4n+2}
c $\frac{16}{3 \times 2^n}$ d $x^m(a + bx^2)$

6 a $10p^2$ b $\frac{10}{p^3}$ 7 $(xyz)^{a+b+c}$

8 a $x^2 - 2 + \frac{1}{x^2}$ b $x^4 + 2x^{\frac{5}{2}} + x$ c $x^{2n} - 6 + \frac{9}{x^{2n}}$

9 a $-\frac{1}{2}$ b $-(xy)^{\frac{1}{2}}$ c $\frac{x^{\frac{1}{2}} - 1}{x^{\frac{1}{2}} - 2}$ d $x^{\frac{1}{2}} + y^{\frac{1}{2}}$

Chapter 9 Answers

Exercise 9A

3 b centre: the light; enlargement factor: 11
4 enlargement factor: $\frac{5}{2}$
5 carbon 2.8 cm; gold 5.4 cm; radium 8.6 cm
6 a 125 km b 500 km c 75 km d 12.5 km

Exercise 9B

1 a i $\frac{PR}{AC} = \frac{PQ}{AB}$ ii $\frac{QR}{BC} = \frac{RP}{CA}$
b i $\frac{DE}{RS} = \frac{DF}{RT}$ ii $\frac{FE}{TS} = \frac{FD}{TR}$
c i $\frac{AB}{SR} = \frac{AD}{ST}$ ii $\frac{CB}{UR} = \frac{CD}{UT}$
d i $\frac{JK}{AE} = \frac{MN}{DB}$ ii $\frac{JL}{AC} = \frac{MN}{DB}$

2 a $\frac{x}{5} = \frac{2}{6}$; $x = \frac{5}{3}$ b $\frac{x}{2} = \frac{5}{3}$; $x = \frac{10}{3}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

c $\frac{x}{3}=\frac{3}{6}; x=\frac{3}{2}$ d $\frac{x}{2}=\frac{8}{3}; x=\frac{16}{3}$

3 a 0.7 b 0.6 c 6 d 2.2 cm

4 a 29° b 32 cm c 104° d 16 cm

5 a 36° b 88° c 21.6 cm d 12.6 cm

6 a 2 b $\frac{4}{3}$ c $\frac{8}{3}$ d 2

Exercise 9C

1 a ΔABC is similar to ΔAMX (AAA); $\frac{AB}{AM}=\frac{BC}{MX}$

b ΔABC is similar to ΔAPB (AAA); $\frac{AB}{AP}=\frac{BC}{PB}$

c ΔABC is similar to ΔPQC (AAA); $\frac{AB}{PQ}=\frac{BC}{QC}$

d ΔABC is similar to ΔADB (AAA); $\frac{AB}{AD}=\frac{BC}{DB}$

2 a ΔACQ is similar to $\Delta ABP(AAA)$; $\angle BAP=\angle CAQ$ (common);

$\angle QCA=\angle PBA$ (corresponding angles, $BP \parallel CQ$); $\frac{AB}{AC}=\frac{AP}{AQ}$

b ΔKFL is similar to ΔMGL (AAA); $\angle KLF=\angle GLM$ (vertically opposite);

$\angle FKL=\angle GML$ (alternate angles, $KF \parallel GM$); $\frac{GL}{FL}=\frac{GM}{FK}$

3 a $\angle ABC=\angle DEC$ (given); $\angle ACB=\angle DCE$ (vertically opposite at C);

ΔBCA is similar to $ECD(AAA)$; $\frac{x}{12}=\frac{6}{8}; x=9$

b $\angle PRQ=\angle TRS$ (given); $\angle PQR=\angle TSR=90°$;

ΔPQR is similar to $TSR(AAA)$; $\frac{x}{9}=\frac{9}{6}; x=13\frac{1}{2}$

c $\angle CMD=\angle BMA$ (vertically opposite at M);

$\angle DCM=\angle ABM$ (alternate angles, $AB \parallel CD$);

ΔAMB is similar to DMC (AAA); $\frac{x}{5}=\frac{7}{3}; x=11\frac{2}{3}$

d $\angle KMP=\angle LMQ$ (common); $\angle KPM=\angle LQM$ (corresponding angles, $KP \parallel LQ$);

ΔKPM is similar to ΔLQM (AAA); $\frac{x}{4}=\frac{7}{8}; x=3\frac{1}{2}$

e $\angle QRS=60°$ (angle sum of triangle);

$\angle PRQ=\angle QRS=60°$;

$\angle RQS=\angle RPQ=90°$; ΔPRQ is similar to ΔQRS (AAA); $\frac{x}{1}=\frac{1}{2}; x=\frac{1}{2}$

f $\angle DAB=\angle EAC$ (common); $\angle AEC=\angle ADB$ (corresponding angles, $DB \parallel EC$);

ΔACE is similar to ΔABD (AAA);

$\frac{x+4}{4}=\frac{10}{6}; x=2\frac{2}{3}$

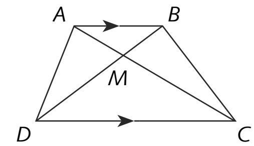
4 a $6\frac{2}{3}$ m b $7\frac{1}{2}$ m c 3000 m d 1.28 m

5 a $\alpha=65°, \beta=25°, \gamma=65°$

b ΔABC is similar to ΔAMB, which is similar to ΔBMC.

c i $\frac{a}{b}=\frac{y}{h}=\frac{h}{x}$ ii $\frac{h}{a}=\frac{b}{x+y}=\frac{x}{b}$

6

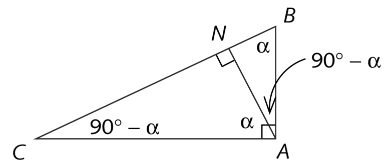


$\angle BMA=\angle DMC$ (vertically opposite at M)

$\angle ABM=\angle CDM$ (alternate angles, $AB \parallel DC$)

ΔABM is similar to ΔCDM (AAA).

7



a Let $\angle NBA=\alpha$.

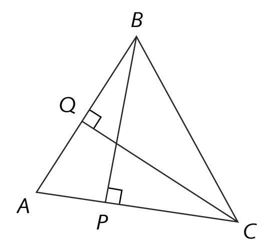
$\angle BAN=\angle ACN=90°-\alpha$ (angle sum of triangle)

$\angle ANB=\angle CNA=90°$ (given)

ΔABN is similar to ΔCAN (AAA).

b $\frac{AN}{CN}=\frac{BN}{AN}$, $AN^2=BN \times CN$

8



a $\angle BAP=\angle CAQ$ (common);

$\angle APB=\angle AQC=90°$;

ΔAPB is similar to ΔAQC (AAA).

b $\frac{BP}{CQ}=\frac{AB}{AC}$ (matching sides of similar triangles)

c $\frac{1}{2}\times AC \times BP=\frac{1}{2}\times AB \times CQ$

(both $\frac{1}{2}\times$ base $\times$ height = area of same triangle ABC)

d So $\frac{BP}{CQ}=\frac{AB}{AC}$

Exercise 9D

1 a $QP=PR=7$cm (ΔPQR is isosceles), $\angle BAC=\angle QPR$ and $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{5}{7}$. Thus ΔPQR is similar to ΔABC (SAS).

b $\angle BAC=\angle EDF=50°$ and $\frac{BA}{ED}=\frac{AC}{DF}=2$.

Thus ΔBAC is similar to ΔEDF (SAS).

c $\angle ABC=\angle PQR=120°$ and $\frac{CB}{RQ}=\frac{AB}{PQ}=2$.

Thus ΔBAC is similar to ΔQPR (SAS).

d $\angle BAC=\angle QPR=40°$ and $\frac{AB}{PQ}=\frac{AC}{PR}=5$.

Thus ΔBAC is similar to ΔQPR (SAS).

2 a $AB=8$ and $AC=12.8$. $\angle BAC=\angle PAQ$.

$\frac{AB}{AP}=\frac{AC}{AQ}=\frac{8}{5}$. Thus ΔBAC is similar to ΔPAQ (SAS).

Hence $\angle APQ=\angle ABC$ (matching angles in similar

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

triangles). Thus $PQ \parallel BC$ (corresponding angles equal).

b $AB = 10$ and $AC = 7.5$; $\angle BAC = \angle PAQ$;

$\dfrac{AB}{AP} = \dfrac{AC}{AQ} = \dfrac{5}{4}$. Thus ΔBAC is similar to ΔPAQ (SAS).

Hence $\angle APQ = \angle ABC$ (matching angles in similar triangles). Thus $PQ \parallel BC$ (corresponding angles equal).

3 $\angle BAC = \angle DAE, \angle ADE = \angle ABC$ (corresponding angles, $DE \vee BC$), $\angle AED = \angle ACB$ (corresponding angles, $DE \parallel BC$). Thus ΔBAC is similar to ΔDAE (AAA).

$\dfrac{AB}{AD} = \dfrac{AC}{AE} = \dfrac{4}{3}$

4 $\angle HAI$ is common to all triangles. Then, for example $\dfrac{AD}{AB} = \dfrac{AE}{AC} = 2$. Thus ΔABC is similar to ΔADE (SAS).

The other two triangles are similar to these two triangles.

$z = 8, y = 16$ and $x = 24$.

5 $\angle P = \angle Z$ and $\dfrac{PQ}{XZ} = \dfrac{PR}{YZ}$. Therefore ΔRPQ is similar to ΔYZX (SAS).

$\angle Q = \angle X, \angle R = \angle Y$

6 ΔABD is similar to ΔCDB (SAS). Therefore $\angle CBD = \angle ADB$ and $\angle CDB = \angle ABD$ (matching angles of similar triangles). Hence $BC \parallel AD$ and $CD \parallel AB$ and $ABCD$ is a parallelogram.

Exercise 9E

1 **a** ΔABC is similar to ΔGHI (AAA).

b ΔKML is similar to ΔRTS (SAS).

c ΔKLM is similar to ΔNPQ (AAA).

d ΔLKM is similar to ΔUST (SAS).

e ΔDEF is similar to ΔLMK (SSS).

2 **a** ΔABC is similar to ΔAPB (SAS); $\dfrac{AB}{AP} = \dfrac{BC}{PB}$

b ΔABC is similar to ΔAPB (SSS); $\dfrac{AB}{AP} = \dfrac{BC}{PB}$

c ΔABC is similar to ΔBMC (RHS); $\dfrac{AB}{BM} = \dfrac{BC}{MC}$

d ΔABC is similar to ΔACD (AAA); $\dfrac{AB}{AC} = \dfrac{BC}{CD}$

3 **a** $b = 9$ **b** $x = 6$ **c** $y = 2.25$

d $x = \dfrac{10}{3}$ **e** $a = \dfrac{9}{2}, b = 6$ **f** $x = \dfrac{8}{5}$

4 **a i** 30° **ii** 30°

b AAA **c** $6\sqrt{3}$ cm **e** $3\sqrt{3}$ cm

5 **a** $\dfrac{8}{3}$ **b** $\dfrac{8}{5}$ **c** $\dfrac{32}{15}$

6 **a i** $\dfrac{CM}{MA} = \dfrac{CD}{AB}$ and $\angle CMD = \angle AMB = 90°$; ΔAMB is similar to ΔCMD (RHS).

ii $\angle ABM = \angle CDM$ (matching angles of similar triangles).

iii no (alternate angles not equal)

b i BDC; $\dfrac{BD}{BA} = \dfrac{DC}{BC} = \dfrac{BC}{AC}$; ΔBDC is similar to ΔABC (SSS).

ii $\beta = \theta$

c i $\angle BAC = \angle FCB$ (given) and $\angle FBC = \angle ABC$ (common); ΔBAC is similar to ΔFCB (AAA).

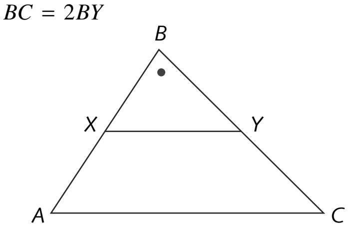
ii $x = 3, y = 9$ (ΔABC isosceles)

d i CBD; $\dfrac{BA}{BC} = \dfrac{BC}{BD} = a$. $\angle ACB = \angle BDC = 90°$. ΔABC is similar to ΔCBD (RHS).

ii α

7 **a** Let X and Y be the midpoints of AB and BC respectively.

$BA = 2BX$ and $BC = 2BY$



Therefore $\dfrac{BA}{BX} = 2$ and $\dfrac{BC}{BY} = 2$

$\angle ABC = \angle XBY$ (common angle)

ΔABC is similar to ΔXBY.

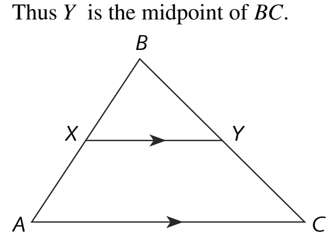
Therefore $XY = \dfrac{1}{2}AC$ (matching sides of similar triangles) and $\angle BXY = \angle BAC$ (matching angles of similar triangles).

Thus $AC \parallel XY$ (corresponding angles equal).

b Let X be the midpoint of AB and Y a point on BC such that $XY \parallel AC$. $\angle BXY = \angle BAC$ (corresponding angles, $XY \parallel AC$). Therefore ΔABC is similar to ΔXBY

$\dfrac{BY}{BC} = \dfrac{BX}{BA} = \dfrac{1}{2}$ (matching sides of similar triangles).

Thus Y is the midpoint of BC.



c Draw diagonals AC and BD. ΔABC and $\Delta ADC, PQ \parallel AC \parallel SR$ and similarly $PS \parallel BD \parallel QR$. Opposite sides of a quadrilateral are parallel implies that $PQRS$ is a parallelogram.

8 We know the interval joining midpoints of two sides of a triangle is parallel to the third side. Therefore $PR \parallel AC$ and $PQ \parallel AB$ and $RQ \parallel BC$ and $RP = \dfrac{1}{2}AC, PQ = \dfrac{1}{2}AB$ and $RQ = \dfrac{1}{2}BC$.

$\Delta ARQ = \Delta\ RBP = \Delta\ QPC$ (SSS) (Each side length is equal to half the length of a side of ΔABC.)

ΔABC is similar to ΔPQR (AAA).

9 **a** True, AAA (all angles are 60°)

b False. For example, an isosceles triangle with base angles of 30° is not similar to an isosceles triangle with base angles 60°.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c True, AAA. If the apex angle is $a°$, each of the two base angles $= 90° - \frac{a}{2}$.

d False. Draw two isosceles triangles with common base but different base angles.

e False. The right-angled triangle with sides 3, 4, 5 is not similar to the right-angled triangle with sides 5, 12, 13.

f False. Draw a circle with the fixed hypotenuse as diameter. Draw the triangles with the third vertex on the circle.

g True (SAS) **h** True (AAA)

10 **a** true **b** false **c** true **d** true
e false **f** false **g** true

11 **a** $AD = 15, DC = 20, BC = 16$
b $AM = 12, BM = 16, DM = 9$

Review exercise

1 **a** $\angle BAC = \angle CDE$ (alternate angles) $AB \parallel ED$;
$\angle BCA = \angle ECD$ (vertically opposite angles at C);
ΔBCA is similar to ECD(AAA); $e = 7\frac{1}{2}$ and $c = 9$

b $\angle NJK = \angle MJL$ (common), $\angle NKJ = \angle MLK = 90°$;
ΔJNK is similar to ΔJML (AAA); $x = 4$ and $y = 2\frac{2}{3}$

2 **a** ΔBAD is similar to ΔCAB is similar to ΔCBD; $a = 31\frac{1}{5}$ and $b = 28\frac{4}{5}$

b ΔMJL is similar to ΔNJK; $x = 3\frac{1}{5}$

3 **a** ΔABD is similar to ΔACE, $b = 2.5$, $a = 2$
b $x = 2$ and $y = 4.8$, ΔSPR is similar to ΔTPQ

4 $16\frac{2}{3}$ cm

5 **a** **i** $\frac{AB}{AL} = \frac{AC}{AM}$ and $\angle BAC = \angle LAM$ (common); ΔBAC is similar to ΔLAM (SAS)

ii $\angle ABC = \angle ALM$ (matching angles of similar triangles). Therefore $LM \parallel BC$ (corresponding angles equal).

iii $\frac{x}{3} = \frac{4}{1}, x = 12$

b **i** $\angle QOP = \angle GOF$ (common); $\frac{OG}{OQ} = \frac{OF}{OP} = \frac{3}{2}$;
ΔOPQ is similar to ΔOFG (SAS).

ii $\angle OPQ = \angle OFG$ (matching angles of similar triangles). Therefore $PQ \parallel FG$ (corresponding angles equal).

iii $x = 21$

6 $\angle ACB = \angle DFE = 20°$
$\angle CAB = \angle FDE = 120°$
ΔABC is similar to ΔDEF (AAA); $2\frac{2}{5}$ cm

7 20 m **8** 50 m

9 **a** $\angle BAD = \angle ABE$ (alternate angles, $AD \parallel BC$)
$\angle AEB = \angle ABE$ (isosceles)
$\angle ADF = \angle BAD$ (alternate angles, $AB \parallel CD$)
$\angle AFD = \angle ADF$ (isosceles)
ΔAEB is similar to ΔADF (AAA).

b In ΔABF and ΔAED, $AB = AE$ (given); $AD = AF$ (given);
$\angle EAD = \angle BAD + \angle EAB$ (adjacent angles) and
$\angle FAB = \angle BAD + \angle DAF$ (adjacent angles);
$\angle DAF = \angle EAB$ (matching angles of similar triangles);
$\Delta ABF = \Delta AED$ (SAS) and $DE = BF$ (matching sides of congruent triangles).

10 **a** $\frac{BA}{AM} = \frac{CA}{AN}$, so ΔAMN is similar to ΔABC (SAS).
Therefore $BC \parallel MN$

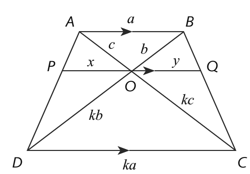
b $\angle PBC = \angle PNM$ (alternate angles, $BC \parallel MN$);
$\angle MPN = \angle BPC$ (vertically opposite).
So ΔMPN is similar to ΔCPB; $\frac{BP}{PN} = \frac{BC}{MN} = 3$

Challenge exercise

Only outlines are given for some proofs in this section.

1 *Hint*: Draw the diagonals of the quadrilateral.

2 By similar triangle, $\frac{x}{ka} = \frac{c}{c+kc}$

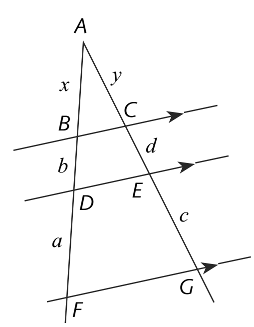


$$x = \frac{ka}{1+k}$$
$$\frac{y}{ka} = \frac{b}{kb+b}$$
$$y = \frac{ak}{1+k}$$

Therefore $x = y$

3 **a** $\Delta AKM \equiv \Delta CLM$ **b** 1 : 2 **c** 1 : 12

5 **a** ΔABC is similar to ΔADE is similar to ΔAFG (AAA).

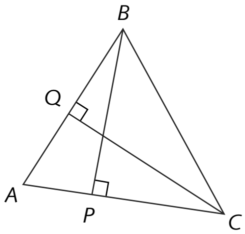


$\frac{x}{y} = \frac{x+b}{y+d}$ and $\frac{x}{y} = \frac{x+a+b}{y+c+d}$

$xy + xd = yx + yb$ and $xy + xc + xd = yx + ya + yb$
$xd = yb$ $\quad$ $xc + xd = ya + yb$
$\frac{x}{y} = \frac{b}{d}$ and $x(c+d) = y(a+b)$
$$\frac{x}{y} = \frac{a+b}{c+d} = \frac{b}{d}$$
$$ad + bd = bc + bd$$
Therefore $\frac{a}{b} = \frac{c}{d}$

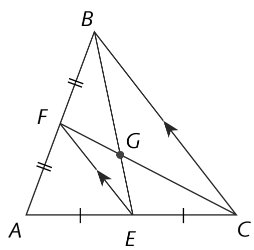
b yes **c** $\frac{8}{3}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

6 

ΔAPB is similar to ΔAQC. $\dfrac{BA}{CA} = \dfrac{PB}{CQ}$

If $PB = CQ$ then $BA = CA$. The triangle is isosceles.

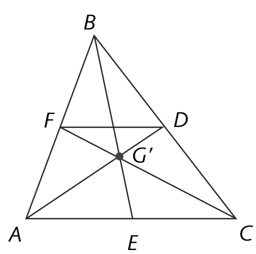
7 a 

Draw FE.

$FE \parallel BC$ (E and F are midpoints of AB and AC respectively)

$BC = 2FE$ (E and F are midpoints of AB and AC respectively)

ΔFGE is similar to ΔCGB. $CG = 2GF$ and $BG = 2GE$

b 

Let D be the midpoint of BC. AD is the median from A.

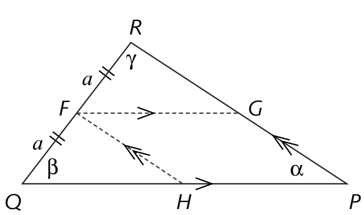
$FD \parallel AC$ (F and D are midpoints)

$AC = 2FD$ (D and F are midpoints)

$\Delta AG'C$ is similar to $\Delta DG'F$. $G'C = 2G'F$.

Hence $G' = G$

c If $BE = CF$, then $BG = CG$ ($BG = \dfrac{2}{3}BE$ and $CG = \dfrac{2}{3}CF$), ΔBGC is isosceles and therefore $\angle BCG = \angle CBG$, $\Delta CFB \equiv \Delta BEC$ (SAS), $BF = CE$ (matching sides), $AB = AC$ and triangle ABC is isosceles.

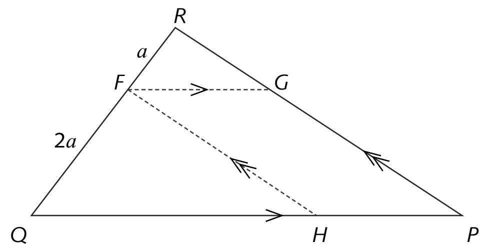
8 

First, we prove the result when $k = 2$.

Let F be the midpoint of QR. Then $QF = FR = a$.

Construct G on RP and H on QP so that $FG \parallel QP$ and $FH \parallel RP$. Omitting the details of the angle-chasing. $\Delta RFG \equiv \Delta FQH$ (*AAS* congruence test).

Using opposite sides of parallelograms and matching sides of congruent triangles. $HP = FG = QH$ and $RG = FH = GP$, as required.



Secondly, we prove the result when $k = 3$.

Let F be the divide RQ in the ratio $1 : 2$. Then $FR = \mathrm{a}$ and $FQ = 2a$. Construct G on RP and H on QP so that $FG \parallel QP$ and $FH \parallel RP$.

Omitting the details of the angle-chasing. ΔRFG is similar to ΔFQH (AAA similarity test with ratio $1 : 2$).

Using opposite sides of parallelograms and matching sides of congruent triangles.

$HP = FG = \dfrac{1}{2}QH$ and $RG = \dfrac{1}{2}\ FH = \dfrac{1}{2}GP$,

as required.

Chapter 10 answers

Chapter 10A review

Chapter 1: Algebra

1 **a** 24 **b** 23 **c** -51 **d** $\dfrac{111}{50}$

2 **a** $2a + 6b$ **b** $4x^2y + 4xy$
c $14n^2 - 2m$ **d** $p^2 - 7p + 15$

3 **a** $14a^2b$ **b** $6xy$ **c** $4y$ **d** $15a$

4 **a** $\dfrac{-7a}{15}$ **b** $\dfrac{-x}{40}$ **c** $\dfrac{2a^2}{15}$
d $\dfrac{a^2}{7}$ **e** $\dfrac{7}{8}$ **f** $\dfrac{ab}{18}$

5 **a** $3a + 12$ **b** $6x - 6$ **c** $6b + 4$
d $20d - 5$ **e** $-9d + 6$ **f** $-10\ell + 8$
g $-6x^2 - 2x$ **h** $8x^2 + 12x$

6 **a** $7a + 26$ **b** $17x + 2$
c $23d - 38$ **d** $27e + 29$
e 3 **f** $2x^2 + 11x - 6$
g $-5x^2 + 14x$ **h** $-8x^2 + 50x$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 a $\frac{7x+15}{12}$ b $\frac{5x-8}{6}$
c $\frac{5x+1}{12}$ d $\frac{5x-1}{12}$

8 a $x^2+8x+15$ b $x^2+4x-21$
c $x^2+5x-24$ d $6x^2-x-2$
e $12x^2+29x+15$ f $10x^2+11x-6$
g x^2-25 h $4x^2-9$
i $9x^2-25$ j $x^2+14x+49$
k $4x^2-20x+25$ l $9x^2-24x+16$
m $12x-12$ n $24x$
o $8x^2+6x+1$ p $-4x^2+9x-6$

9 a 7, 21 b 3, 6 c x, 5
d x, 6 e 6, $11x$ f $2x$, 7, $25x$

Chapter 2: Pythagoras' theorem and surds

1 a 14.4 b 13.2
c 13.1 d 37.9

2 no

3 a 5 cm b 13 cm c 8.5 cm
d 0.5 cm e 2.6 cm f 25.5 cm

4 5.25 m 5 436 m

6 a $2\sqrt{5}$ b $5\sqrt{3}$ c $6\sqrt{2}$
d $20\sqrt{2}$ e $30\sqrt{3}$ f $27\sqrt{3}$

7 a $\sqrt{12}$ b $\sqrt{18}$ c $\sqrt{500}$ d $\sqrt{112}$

8 a $11\sqrt{2}$ b $3\sqrt{3}$ c $20\sqrt{6}$
d $12\sqrt{35}$ e $7\sqrt{2}$ f $\sqrt{3}$
g $23\sqrt{3}$ h $26\sqrt{2}$

9 a $\sqrt{6}+2\sqrt{5}$ b $12-5\sqrt{3}$ c $6\sqrt{10}-60$
d $6\sqrt{6}+16$ e $16-\sqrt{3}$ f $19-13\sqrt{2}$
g $19-6\sqrt{2}$ h $6+2\sqrt{5}$ i -78

10 a $\sqrt{3}$ b 2 c $\frac{\sqrt{3}}{6}$ d $\frac{5\sqrt{6}}{6}$

11 a $3\sqrt{10}-6\sqrt{2}$ b $\frac{6+\sqrt{3}}{11}$
c $3\sqrt{10}+\sqrt{5}-6\sqrt{2}-2$ d $\sqrt{6}+\sqrt{3}-2-\sqrt{2}$

12 a 17 cm b $5\sqrt{13}$ cm c 10 cm
d $\frac{\sqrt{481}}{2}$ cm e 12.5 cm f $\frac{3\sqrt{41}}{2}$ cm

13 12.8 cm

Chapter 3: Consumer arithmetic

1 a $\frac{9}{50}$ b $\frac{16}{25}$ c $\frac{13}{500}$
d $\frac{17}{200}$ e $\frac{3}{8}$ f $\frac{1}{15}$

2 a 0.08 b 0.27 c 0.096
d 0.458 e 0.1225 f 0.385

3 a 40% b 62.5% c 61%
d 2% e $57\frac{1}{7}\%$ f $55\frac{5}{9}\%$

4 a $\frac{1}{4}$, 0.25 b 30%, 0.3 c 26%, $\frac{13}{50}$
d $66\frac{2}{3}\%$, 0.6 e $\frac{2}{25}$, 0.08 f 7.5%, $\frac{3}{40}$

5 a 9.6 b 8.64 c \$340 d \$570

6 351 7 1 350 000

8 32% 9 9.5%

10 a 112 b 159 c 211.2 d 153.6

11 \$520 12 \$665.60

13 a \$300 b \$150 c \$750 d \$144.35

14 8.82% 15 11.9%

16 a 18.72% increase b 1% decrease
c 22.72% decrease d 0.32% increase

17 a \$3009.55 b 15.752% increase

18 11.11%

19 a \$7440 b \$6749.18

20 a \$18 762.40 b \$14 648.72 c \$10 750.76

Chapter 4: Factorisation

1 a $5(a+2)$ b $2(3c-4)$
c $3(3d-8)$ d $3e(e+3)$
e $2f(3f+5)$ f $-3h(h+5)$
g $2ab(2a+3b)$ h $3mn(3n+4)$

2 a $(x+3)(x+4)$ b $(x+6)(x+3)$
c $(x-6)(x+1)$ d $(x+7)(x-4)$
e $(x-5)(x-6)$ f $(x-2)(x-12)$
g $(x-11)(x+5)$ h $3(x^2+2x+3)$
i $4(x^2-2x+3)$ j $(x-10)(x+10)$
k $(3x-4y)(3x+4y)$ l $(1-4a)(1+4a)$

3 a $\frac{x+1}{x-1}$ b $\frac{1}{(x+1)(x+4)}$ c $\frac{1}{4}$
d $\frac{1}{2(p+2)}$ e $2a$ f $\frac{3}{a+2}$
g $\frac{3(x-2)(x+4)}{(x-1)}$ h $2x^2y$

Chapter 5: Linear equations and inequalities

1 a $x+2$ b $b-4$ c $\frac{c}{2}$
d $3d$ e $2e+1$ f $\frac{f}{3}-2$
g $2(s+3)$ h $\frac{1}{2}(h-1)$

2 a $a=9$ b $b=8$ c $x=9$ d $x=9$
e $y=11$ f $m=2$ g $x=12$ h $x=-5$
i $x=\frac{7}{2}$ j $x=\frac{9}{8}$ k $x=-5$ l $x=-4$

3 a $x=2$ b $x=-16$ c $x=-24$
d $x=2$ e $x=3$ f $x=\frac{24}{5}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

g $x = \frac{9}{5}$ **h** $x = -\frac{33}{2}$ **i** $a = -\frac{7}{5}$

j $a = 1$ **k** $a = -\frac{4}{7}$ **l** $a = -\frac{19}{6}$

4 $\frac{13}{8}$ **5** 13 **6** 82 cents

7 36 **8** 35 km/h

9 **a** $x = \frac{3-a}{7}$ **b** $x = \frac{b+c}{a}$

c $x = a(c-b)$ **d** $x = a - b$

e $x = ab - bc = b(a-c)$ **f** $x = \frac{c}{a+b}$

g $x = \frac{ab}{c-a}$ **h** $x = \frac{-ab}{c}$

i $x = \frac{ab}{c} + ad = \frac{ab + cad}{c}$ **j** $x = \frac{ad(e-b)}{d-ac}$

10 **a** $x = \frac{c-b}{a}$

b **i** 2 **ii** 4 **iii** −2

iv 4 **v** −3 **vi** $12\frac{1}{2}$

11 **a** $x = \frac{ac}{d} - ab$

b **i** $-\frac{36}{7}$ **ii** 2 **iii** $\frac{10}{9}$

12 **a** $\frac{a^2 + b^2}{2b}$ **b** $\frac{-2}{a+b}$

13 **a** $x < 7$ **b** $x \le -4$ **c** $d \ge -1$

d $q \le \frac{8}{3}$ **e** $\ell \le -\frac{29}{4}$ **f** $m \le -22$

Chapter 6: Formulas

1 **a** 257.6 **b** 96.8 **c** 13.3225 **d** 1360.5

2 **a** $a = \frac{A}{\pi b}$ **b** $\ell = \frac{A - \pi r^2}{\pi r}$ **c** $a = \frac{2S + n}{n}$

d $n = \frac{\ell + d - a}{d}$ **e** $t = \frac{v}{a+b}$ **f** $p = \frac{b^2}{a+b}$

g $b = \frac{ac}{c-a}$ **h** $r = R - \frac{h}{a}$ **i** $\ell = \frac{gT^2}{4\pi^2}$ **j** $y = \sqrt{\frac{3x}{m^2 a}}$

3 **a** 2 **b** 20 **c** $\frac{10}{61}$

4 **a** **i** 1249°C **ii** 98.4°F

b **i** −1.077 **ii** −6

5 **a** −3 **b** $-\frac{3\sqrt{2}}{4}$ **c** 0 **d** $-\frac{5\pi}{14}$

Chapter 7: Congruence and special quadrilaterals

1 **a** acute **b** right **c** obtuse

d straight **e** acute **f** right

2 **a** $\angle ABC$ and $\angle DEF$ **b** $\angle ABC$ and $\angle PQR$

3 **a** $\alpha = 118°$, $\beta = 62°$

b $\alpha = 69°$, $\beta = 21°$, $\gamma = 159°$ **c** $\alpha = 42°$

4 **a** $\alpha = 71°$ **b** $\beta = 77°$ **c** $\gamma = 60°$

d $\beta = 71°, \theta = 38°$ **e** $\alpha = 45°$ **f** $\beta = 87°$

5 **a** $\alpha = 100°$, $\beta = 100°$ **b** $\alpha = 106°$, $\beta = 74°$, $\gamma = 106°$

c $\beta = 55°$, $\phi = 125°$, $\alpha = 45°$, $\gamma = 80°$, $\theta = 100°$, $\Psi = 100°$

d $\alpha = 68°$, $\beta = 60°$, $\theta = 52°$, $\gamma = 60°$, $\phi = 68°$

6 **a** $\angle AFE$ **b** $\angle ACD$ **c** $\angle BCF$

d $\angle ACD$ **e** $\angle DCF$

7 **a** $\alpha = 50°$ **b** $\beta = 75°$ **c** $\gamma = 70°$

8 **a** $\alpha = 40°$

b $\beta = 36°$, $\alpha = 26°$, $\theta = 26°$, $\gamma = 118°$

c $\alpha = 61°$, $\beta = 29°$ **d** $\alpha = 18$

9 **a** kite **b** rhombus **c** rectangle

d square **e** trapezium **f** isosceles trapezium

10 **a** $a = 4$, $b = 5$ **b** $\alpha = 90°$, $\beta = 45°$

c $\alpha = 96°$, $\beta = 96°$ **d** $\theta = 49°$, $\beta = 131°$

e $\alpha = 42°, \beta = 96°, \theta = 96°$ **f** $\alpha = \beta = \theta = 25°$

11 **a** parallelogram, rhombus, rectangle, square, kite (one pair only)

b isosceles trapezium, rectangle, square

c rhombus, square, kite

d rhombus, square

e parallelogram, rhombus, rectangle, square, trapezium and isosceles trapezium (one pair only)

f parallelogram, rhombus, rectangle, square, trapezium (two pairs only)

12 **a** $\Delta ABC \equiv \Delta DEF$ (SAS) **b** $\Delta XYZ \equiv \Delta MNL$ (SAS)

c $\Delta STU \equiv \Delta DFE$ (ASA)

13 **a** $\Delta ABC \equiv \Delta HGI$ (ASA) **b** $\Delta ABC \equiv \Delta RQP$ (SSS)

c $\Delta ABC \equiv \Delta MLN$ (RHS) **d** $\Delta ABC \equiv \Delta YXZ$ (SAS)

14 **a** $\Delta ALT \equiv \Delta NLM$ (SSS), $\alpha = 52°$, $\beta = 67°$, $\gamma = 61°$

b $\Delta POS \equiv \Delta ROQ$ (SAS), $\gamma = 106°$, $\beta = 37°$, $\theta = 37°$

15 $ABCD$ is a parallelogram. Therefore $AD = BC$.

Hence $AF = GC$, $\angle FAE = \angle GCE$ (alternate angles, $AB \equiv DC$),

$\angle AEF = \angle CEG$ (vertically opposite). Hence

$\Delta AEF \equiv \Delta CEG$ (AAS),

$AE \equiv EC$ (matching sides of congruent triangles)

16 Let $BX \parallel AD$.

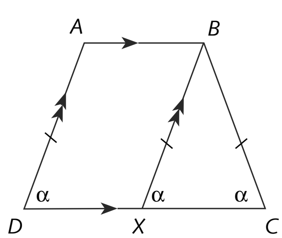
Therefore $ABXD$ is a parallelogram.

Therefore $BX = AD = BC$.

Therefore BXC is isosceles.

$\angle ADC = \angle BXC$ (corresponding angles),

$= \angle BCD$ (isosceles triangle)

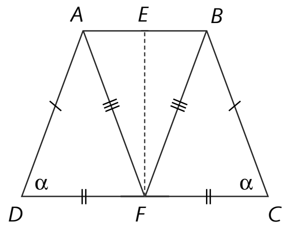


Hence $\Delta ADF \equiv \Delta BCF$ (SAS),

$\therefore$ $AF \equiv BF$ (matching sides),

$\therefore$ $EF \perp DC$ (property of isosceles triangle)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021



17 AE and FC are equal in length and parallel.
Hence $AECF$ is a parallelogram.

18 $\Delta ABM \equiv \Delta ACM$ (SSS), $\angle ABC = \angle ACB$ (matching angles)

19 $\Delta AOB \equiv \Delta COD$ (SSS), $\angle AOB = \angle COD$ (matching angles)

20 **a** 7 **b** 5.6 m

Chapter 8: Index laws

1 **a** 3^4 **b** a^6

2 **a** 216 **b** 625 **c** 128 **d** 1024

3 **a** 17.576 **b** 2.89
c 6274.2241 **d** 0.002 209

4 **a** 5^{13} **b** 3^{12} **c** $10x^6$ **d** $15m^{10}$
e 5^5 **f** $\frac{3a^4}{2}$ **g** $\frac{3n^3}{2}$ **h** 5^6
i $8n^{18}$ **j** 2 **k** $27a^3b^6$ **l** 1

5 **a** m^5n^2 **b** p^3q^3 **c** $54m^{10}n^4$
d $2m^7n^2$ **e** $\frac{2pq^6r^3}{3}$ **f** $\frac{2x^3y^4}{3}$

6 **a** $\frac{1}{4^2} = \frac{1}{16}$ **b** $\frac{1}{10^4} = \frac{1}{10\,000}$ **c** $\frac{1}{8}$
d $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ **e** $\left(\frac{4}{3}\right)^3 = \frac{64}{27}$

7 **a** $\frac{1}{a^2}$ **b** $\frac{1}{b^6}$ **c** $\frac{1}{a^2b^4}$ **d** $\frac{1}{mn^3}$ **e** x^4
f $2y^3$ **g** $\frac{3}{m^2}$ **h** $\frac{5}{m^6}$ **i** $\frac{n^6}{m^4}$ **j** $\frac{3q^2}{p}$

8 **a** $\frac{1}{x^2y^8}$ **b** $\frac{1}{p^4q^5}$ **c** $\frac{12a^4}{n^2b^5}$ **d** $\frac{10}{\ell^2m^3}$ **e** $\frac{7}{t^6}$
f $\frac{5\ell^4}{3}$ **g** $\frac{4y}{3x^4}$ **h** $\frac{4u^4v^8}{3}$ **i** $\frac{n^2}{4m^2}$ **j** $\frac{m^4}{81n^8}$
k $\frac{3a^{18}b}{8}$ **l** $\frac{a^{10}}{b^{17}}$ **m** $\frac{b^{13}}{324a^5}$ **n** $\frac{1}{a^{10}b^4}$ **o** $\frac{a^7}{b^5}$
p $\frac{12b^4}{a^8}$ **q** $\frac{12y^7}{x^5}$ **r** $\frac{24n^6}{m^4}$

9 **a** 3 **b** 2 **c** 2 **d** 3
e 8 **f** 9 **g** 343 **h** 16

10 **a** $2a^{\frac{2}{3}}$ **b** $2ab^{\frac{1}{5}}$ **c** $2a^{\frac{1}{2}}$ **d** $5x^{\frac{8}{3}}$
e $6x^4$ **f** $4a^4b^2$ **g** $6^{\frac{2}{3}}x^2y^3$ **h** $12a^2b^2$

11 **a** 2.1×10^4 **b** 4.1×10^2 **c** 6.1×10^{10}
d 2.4×10^3 **e** 6.2×10^3 **f** 4.71×10^{-2}
g 7×10^7 **h** 3.8×10^4 **i** 4.6×10^1
j 2.9×10^0

12 **a** 72 000 **b** 380 **c** 0.97
d 0.0206 **e** 152 **f** 4070
g 0.000 16 **h** 0.087

13 **a** 2×10^9 **b** 3×10^{-1} **c** 1.764×10^7 **d** 5×10^0

14 **a** 2.4×10^5 **b** 6×10^2 **c** 6.49×10^{-4}
d 8.7×10^{-3} **e** $2.76^5 \times 10^7$ **f** 2.68×10^{-1}

15 **a** 1×10^3 **b** 2×10^3 **c** 5×10^{-2} **d** 1×10^{-5}

16 **a** 6.1 **b** 100 **c** 0.094
d 2.533 **e** 1.641 **f** 0.0052

Chapter 9: Enlargements and similarity

1 **a** ΔABC is similar to ΔDEF (AAA), $g = \frac{9}{2}$
b ΔMNP is similar to ΔQRS (AAA), $a = 6$, $b = 4.5$
c ΔGHI is similar to ΔLJK (AAA), $m = 10$, $n = 2.4$
d ΔABC is similar to ΔDEC, $p = \frac{35}{2}$, $q = 3$

2 **a** ΔRST is similar to ΔRUV (AAA), $\alpha = 63°$, $\beta = 33°$, $g = \frac{21}{5}$
b ΔGHF is similar to ΔGJE (AAA), $\alpha = 42°$, $\beta = 100°$, $\ell = 10$
c ΔADB is similar to ΔABC (AAA), $\alpha = 51°$, $x = \frac{27}{5}$, $y = \frac{36}{5}$
d ΔKLO is similar to ΔMNO (AAA), $x = \frac{15}{4}$, $y = \frac{32}{15}$

3 9.75 m **4** 10.15 m

5 **a** $\frac{20}{3}$ cm **b** 4 cm **c** $\frac{16}{3}$ cm

6 **a i** $\frac{2}{3}$ m **ii** $\frac{4}{3}$ m **b** $2\sqrt{10}$ m
c i 33 m **ii** \$151.80

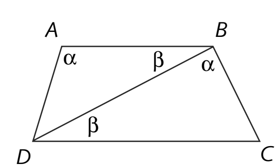
7 **a** 4 m **b** 2.5 m **c** 1.8 m

8 **a i** 2136 mm **ii** 375 mm
b i 350 mm **ii** 1143 mm **iii** 2270 mm

9 **b i** 2.5 cm **ii** 5 cm

10 **b** 2 cm
c ΔDAE is similar to ΔCFE; ΔDAE is similar to ΔBFA.

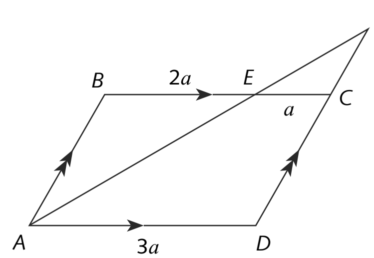
11 $\angle ABD = \angle BDC$ (alternate angles, $AB \parallel DC$),
ΔABD is similar to ΔBDC (AAA), $\frac{AB}{BD} = \frac{AD}{BC}$ (matching sides of similar triangles)
$AB \times BC = DB \times AD$



12 **a** ΔBEA is similar to ΔCEF (AAA).
$\frac{AB}{CF} = \frac{BE}{CE} = 2$ $\therefore AB = 2CF$
b ΔAFD is similar to ΔEFC.
$\frac{AF}{EF} = \frac{AD}{EC} = 3$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

$\therefore AF = 3EF$



10B Problem-solving

1 a i \$15 750 ii \$60 750 iii \$1012.50
b i \$61 040 ii 7.13% c the bank

2 a i \$30 600 ii \$31 824 iii \$126 072
b i \$31500 ii \$126 000 c the union's claim

3 a 35% b $(25 + 20x)\%$ c 0.8 litres

4 a i 38 km ii $48 - \frac{x}{5}$ km b 240 km

5 a 32 minutes b 255 km c 3 hours 4 minutes

6 a \$35 456 b \$3456 c 2.16%
d i \$16 887 ii \$659.48

7 a \$1.25/litre b 40 litres c $C = VR$

8 a $\frac{4x}{9}$ km b i distance $= \frac{5xt}{3}$ ii 16 minutes

9 a 22.58 litres b $y = 0.86x$

10 a i $(x + 5)$ m ii $1.2(x + 5)$ m, $1.5x$ m
iii $(4x + 10)$ m iv $(5.4x + 12)$ m v $(1.4x + 2)$ m
b 15

11 a i $2x$ m ii $(x + 4)$ m iii $(2x + 3)$ m iv $(3x + 7)$ m
b $C = 22x + 48$
c i $2x^2 + 11x + 12$ iii 56 m^2 iv 4.5 m, 9 m

12 a 140 km b x km
c $\frac{x}{35}$ hours, $\frac{140}{x}$ hours d 70 km/h

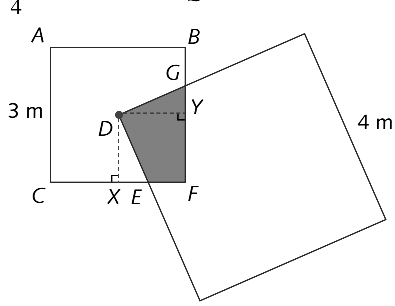
13 a $\frac{40}{x}$ items b $\frac{40}{x + 0.4}$ items
c $\frac{40}{x} = \frac{40}{x + 0.4} + 5$ d \$1.60

14 a $y = 21 - x$ b $(2x^2 - 42x + 441)$ cm^2
c i 9 or 12 ii 9 cm, 12 cm, 15 cm
d $x = 10.5$

15 a i AAA ii $5\frac{1}{7}$ iii 49.0%
b i $x = \frac{3y}{2}$ ii 44.4%

16 d $\frac{9}{4}$ of the area of ΔPQR

17 a



b ASA

c area $DEFG$ = area $DEFY$ + area ΔDYG
= area $DEFY$ + area ΔDXE
= area $DXFY$

d $\frac{9}{4}$ m^2

18 b $BY = AX$ (matching sides of congruent triangles)
$= PQ$ (opposite sides of rectangle $AXPQ$) = 5
c 6 d 40 e 58

19 a i $\frac{1}{2}ab$ ii $\frac{1}{2}c^2$ iii $ab + \frac{1}{2}c^2$
b $\frac{1}{2}(a + b)^2$ c Equate the two expressions for area $ABCD$.

20 a i $c^2 = (a + x)^2 + h^2$, $b^2 = (a - x)^2 + h^2$

How does a sextant work?

1 a $a = c = e = 70°$, $b = d = 40°$
b yes c yes d They are parallel.

2 a 50° b 78°
c The angle of elevation is twice the angle measured by the sextant.
d $(2x)°$ e yes

How long is a piece of string?

1 a i 4 ii 6 iii 8
b $2(n - 1)$ c $\sqrt{d^2 + g^2}$
d $2(n - 1)\sqrt{d^2 + g^2} + g + 2\ell$

2 a i 2 ii 3 iii 4
b $n - 1$ c i 2 ii 3 iii 4
d $n - 1$ e g f $\sqrt{d^2 + g^2}$
g i $2d$ ii $3d$ iii $4d$
h $(n - 1)d$ i $\sqrt{g^2 + (n - 1)^2 d^2}$
j $(n - 1)(g + \sqrt{d^2 + g^2}) + \sqrt{g^2 + (n - 1)^2 d^2} + 2\ell$

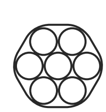
3 Standard pattern uses less for $n > 2$.

Packaging

1 a $\frac{1}{2}\left(25 - \frac{25\pi}{4}\right)$ cm^2
b i 5 cm ii $\frac{5\sqrt{3}}{2}$ cm iii $\left(\frac{25\sqrt{3}}{4} - \frac{25\pi}{8}\right)$ cm^2

2 a 2892.699 cm^3 b 2892.699 cm^3
c 2758.724 cm^3 d Packaging **c** is the best.

3



ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

Chapter 11 answers

Exercise 11A

1 **a** 6 **b** 6 **c** 10 **d** 15

2 **a** 10 **b** 5 **c** 13 **d** 17
e $2\sqrt{2}$ **f** $3\sqrt{5}$ **g** $2\sqrt{5}$ **h** $2\sqrt{10}$

3 **a** $\sqrt{13}$ **b** $\sqrt{74}$ **c** $\sqrt{17}$ **d** $\sqrt{41}$

4 $MP = \sqrt{74}$, $NP = \sqrt{73}$, so N is closer to P.

5 $AB = AC = 13$

6 **a** 5 **b** 5 **c** 6
d isosceles

7 $AB = 11\sqrt{5}$, $AC = 11\sqrt{5}$, $BC = 11\sqrt{2}$, so the triangle is isosceles.

8 $AB = \sqrt{13}$, $BC = 3\sqrt{13}$, $AC = \sqrt{130}$, $AB^2 + BC^2 = 13 + 117 = 130$ and $AC^2 = 130$. So $AB^2 + BC^2 = AC^2$, and by the converse of Pythagoras' theorem the triangle is right-angled.

9 $10\sqrt{2} + 2\sqrt{10}$

10 $ABCD$ is a rhombus since $AB = BC = CD = DA = 5\sqrt{2}$.

Exercise 11B

1 **a** (5, 3) **b** (2, 7) **c** (1.5, 6) **d** (2, 0.5)
e (0, −2) **f** (0, 1.5) **g** (4, −4) **h** (−3, −2)

2 **a** (19, 6) **b** (−21, 6) **c** (−5, 0) **d** (0, 22)
e (−7, 21) **f** (−9, −2)

3 **a** (3, 8) **b** (15, 18) **c** (5, 2) **d** (5.4, 4.65)
e (2.4, −1.4) **f** (3.6, 2)

4 **a** (4, 2) **b** (5, 0) **c** $2\sqrt{5}$ **d** $\sqrt{5}$
e $BC = 2MN$ **f** similar

5 **a** (2, 4.5) **b** (2, 4.5)
c The diagonals of a parallelogram bisect each other.

6 (1, 1.5) midpoint, (4, 6) where (2, 3) is the midpoint of (0, 0) and (4, 6); (−2, −3) where (0, 0) is the midpoint of (2, 3) and (−2, −3); other answers are possible.

7 **a** (2.5, 0) **b** $\frac{\sqrt{85}}{2}$
c M is midpoint of AC, so $AM = CM = \frac{\sqrt{85}}{2}$ and $BM = \frac{\sqrt{85}}{2}$.

8 **a** $M(3, 3)$, $N(5, 3)$
b $MP = \sqrt{5}$, $BC = 2\sqrt{5}$; $PN = \sqrt{5}$, $AB = 2\sqrt{5}$

9 **a** $a = 9$ or -7

10 $M\left(\frac{5}{2}, 4\right)$, $N\left(\frac{15}{2}, 10\right)$, $O(5, 11)$, $P(0, 5)$; $OP = MN = \sqrt{61}$; $NO = MP = \frac{\sqrt{29}}{2}$; $MNOP$ is a parallelogram.

Exercise 11C

1 **a** $\frac{3}{4}$ **b** 2 **c** −2 . **d** $-\frac{1}{3}$

2 **a** 3 **b** 1 **c** −2 **d** $-\frac{1}{2}$
e $\frac{2}{3}$ **f** $-\frac{5}{4}$

3 **a** $-\frac{1}{3}$ **b** $\frac{3}{2}$ **c** −2 **d** −2

4 **a** 10 **b** 1

5 **a** 4 **b** 9 **c** 0 **d** −6

6 **a**

x	−2	−1	1	3
y	−4	−2	2	6

b

x	−6	−2	2	6	14
y	−5	−2	1	4	10

c

x	−6	−4	−1	0	2
y	10	6	0	−2	−6

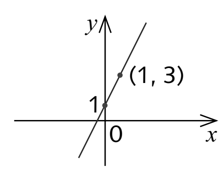
7 **a** 2 **b** 6 **c** −8 **d** 3
e −5 **f** 4 **g** $-\frac{1}{2}$

8 (0, 0) **9** (−4, 0) **10** $-\frac{b}{a}$

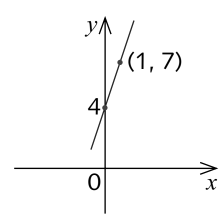
Exercise 11D

1 **a** $y = 2x + 3$ **b** $y = 3x + 4$ **c** $y = -2x + 1$
d $y = 3 - x$ **e** $y = \frac{2}{3}x + 1$ **f** $y = -\frac{3}{4}x$

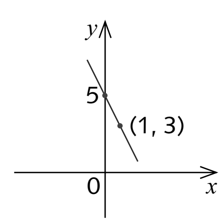
2 **a** Gradient is 2, y-intercept is 1.



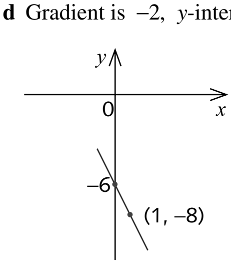
b Gradient is 3, y-intercept is 4.



c Gradient is −2, y-intercept is 5.

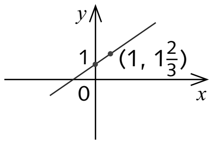


d Gradient is −2, y-intercept is −6.

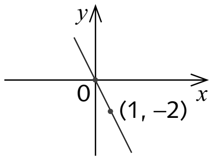


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

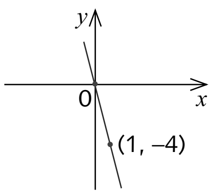
e Gradient is $\frac{2}{3}$, y-intercept is 1.



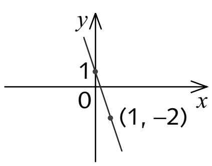
f Gradient is -2, y-intercept is 0.



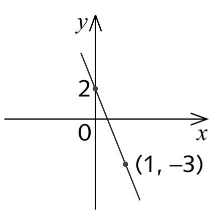
g Gradient is -4, y-intercept is 0.



h Gradient is -3, y-intercept is 1.



i Gradient is -5, y-intercept is 2.



3	**a** no	**b** yes	**c** yes	**d** yes
4	**a** yes	**b** no	**c** yes	**d** no
5	**a** 2	**b** -4	**c** -10	
6	**a** -11	**b** 10	**c** 4	
7	**a** 2	**b** -3	**c** -5	
8	**a** -9	**b** -4	**c** $-\frac{5}{2}$	

9 $a = 3, b = -\frac{3}{2}, c = 5$ **10** $a = 6, b = 2, c = 9$

11 **a** $y = -2x + 10$, gradient is -2, y-intercept is 10

b $y = -5x + 2$, gradient is -5, y-intercept is 2

c $y = \frac{3}{2}x - 3$, gradient is $\frac{3}{2}$, y-intercept is -3

d $y = \frac{4}{3}x - 4$, gradient is $\frac{4}{3}$, y-intercept is -4

e $y = \frac{2}{5}x + \frac{9}{5}$, gradient is $\frac{2}{5}$, y-intercept is $\frac{9}{5}$

f $y = \frac{3}{4}x - \frac{3}{2}$, gradient is $\frac{3}{4}$, y-intercept is $-\frac{3}{2}$

g $y = \frac{1}{2}x + 2$, gradient is $\frac{1}{2}$, y-intercept is 2

h $y = \frac{1}{3}x - \frac{1}{3}$, gradient is $\frac{1}{3}$, y-intercept is $-\frac{1}{3}$

i $y = -\frac{1}{2}x$, gradient is $-\frac{1}{2}$, y-intercept is 0

j $y = -\frac{1}{4}x$, gradient is $-\frac{1}{4}$, y-intercept is 0

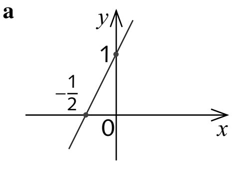
k $y = -3x$, gradient is -3, y-intercept is 0

l $y = \frac{1}{2}x$, gradient is $\frac{1}{2}$, y-intercept is 0

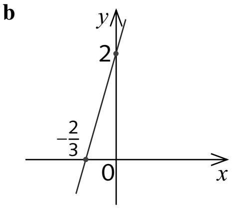
12 **a** $y = -\frac{a}{b}x + \frac{d}{b}$ **b** Gradient is $-\frac{a}{b}$, y-intercept is $\frac{d}{b}$.

Exercise 11E

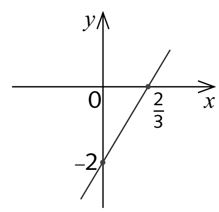
1 **a**



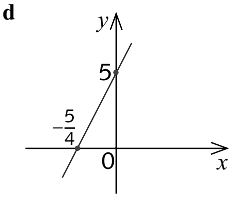
b



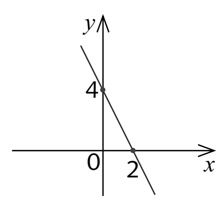
c



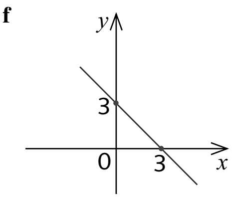
d



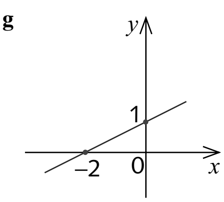
e



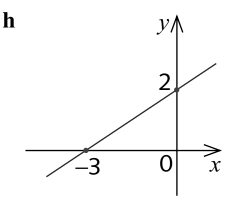
f



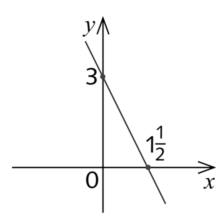
g



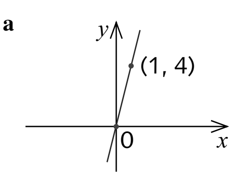
h



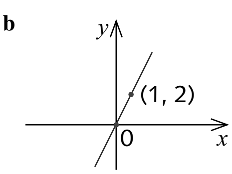
i



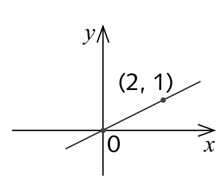
2 **a**



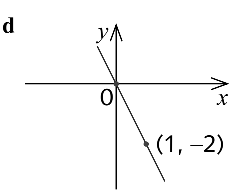
b



c

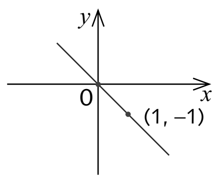


d

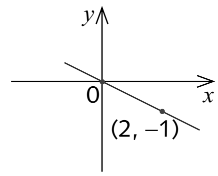


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

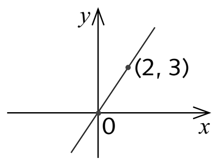
e



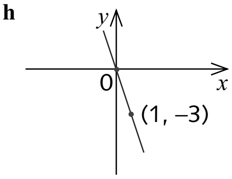
f



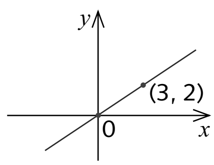
g



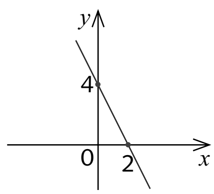
h



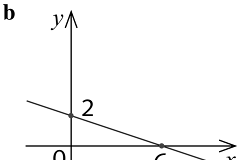
i



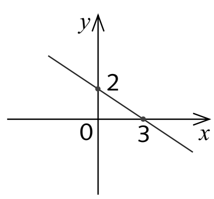
3 **a**



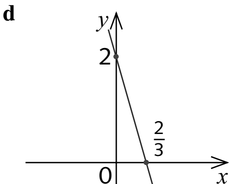
b



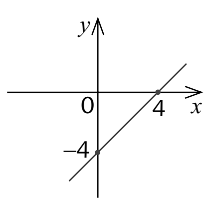
c



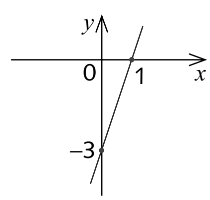
d



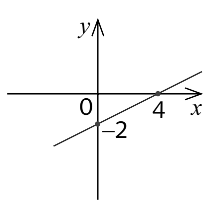
e



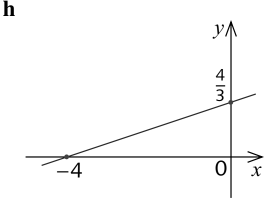
f



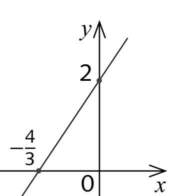
g



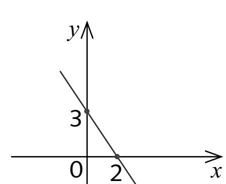
h



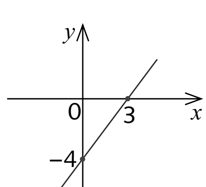
i



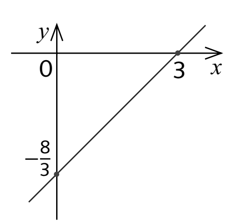
j



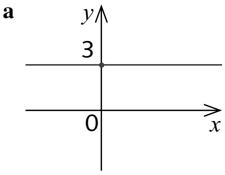
k



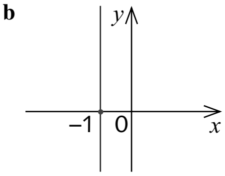
l



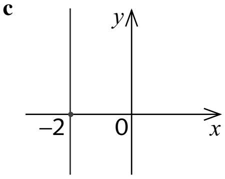
4 **a**



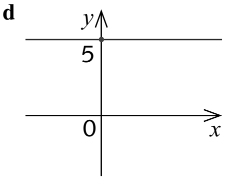
b



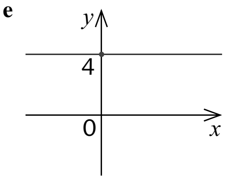
c



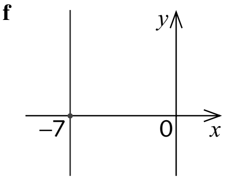
d



e



f



Exercise 11F

1 **a** $y = x + 2$ **b** $y = 3x - 2$ **c** $y = 4x + 5$

d $y = \frac{2x}{3} - \frac{10}{3}$ **e** $y = -\frac{x}{3} - \frac{1}{3}$ **f** $y = -\frac{2x}{5} - \frac{22}{5}$

2 **a** 5 **b** 3 **c** $y = \frac{1}{2}x + 5$ **d** $y = 3x - 2$

3 **a** 3 **b** $y = 3x$ **c** $y = 3x$

4 **a** $y = \frac{4x}{3} - \frac{11}{3}$ **b** $y = 3x - 11$ **c** $y = 2x$

d $y = -3x + 3$ **e** $y = -\frac{10x}{3} - \frac{8}{3}$ **f** $y = \frac{3}{2}x - \frac{1}{2}$

5 **a** 3 **b** (1, 2) **c** $y = -\frac{1}{2}x + \frac{5}{2}$

d $\left(0, \frac{5}{2}\right)$ **e** $\frac{\sqrt{5}}{2}$

6 **a** (3, 4) **b** (7, – 1) **c** $-\frac{5}{4}$

d $y = -\frac{5x}{4} + \frac{31}{4}$ **e** (4.5, 1) **f** $y = 3x - 12.5$

Exercise 11G

1 a and b, c and h

2 **a** $y = 2x + 3$ **b** $x + y = -3$

c $y = 3x - 3$ **d** $y = -\frac{1}{2}x$

3 $a = 3$ **4** $a = -1$

5 **a** –2 **b** $-\frac{1}{2}$ **c** $-\frac{3}{4}$ **d** $-\frac{1}{3}$

e $\frac{4}{3}$ **f** $\frac{1}{5}$ **g** 2 **h** $-\frac{3}{2}$

6 a and f, b and g, c and e, d and h

7 **a** $x + 2y - 9 = 0$ **b** $x - 2y = 0$

c $y = 3x - 9$ **d** $x + 3y + 8 = 0$

8 **a** (2,3.5) **b** (5,2) **c** $-\frac{1}{2}$ **d** $-\frac{1}{2}$

e They are parallel.

f Gradient of $QM = -5$, gradient of $PN = -5$

g parallelogram

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

9 $a = -\frac{3}{2}$ 10 $a = -\frac{7}{4}$

11 **a** 2 **b** (4,7) **c** $x + 2y - 18 = 0$ **d** 9

12 Gradient of $AB = 1$, gradient of $BC = -1$, gradient of $AC = -\frac{3}{13}$.
Since (gradient of AB) × (gradient of BC) = −1, $\angle ABC$ is a right angle.

13 Gradient of $AC = \frac{1}{7}$, gradient of $BD = -7$.
Since (gradient of AC) × (gradient of BD) = −1, $AC \perp BD$.

14 **a** (2,2) **b** $-\frac{2}{5}$ **c** $2x + 5y - 14 = 0$
d $5x - 4y - 13 = 0$, $y = -7x + 27$

15 **a** $2x + 3y - 9 = 0$ **b** $y = 3x - 5$ **c** $x - 4y + 4 = 0$

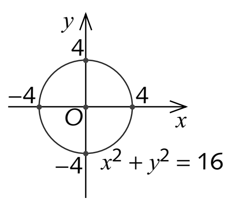
16 **a** $\sqrt{2}$ **b** 1 **c** −1
d $c = 6$ **e** (6, 5)

Exercise 11H

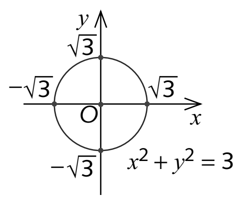
1 **a** on the circle **b** not on the circle **c** on the circle
d on the circle **e** not on the circle **f** on the circle
g not on the circle **h** on the circle **i** on the circle

2 **a** on the circle **b** on the circle **c** on the circle
d on the circle **e** on the circle **f** not on the circle
g not on the circle **h** on the circle **i** on the circle

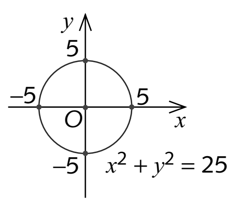
3 **a**



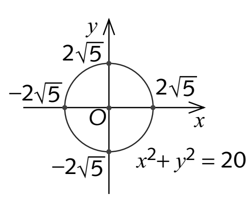
b



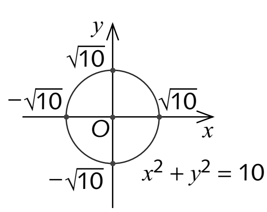
c



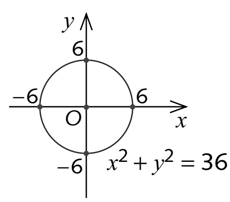
d



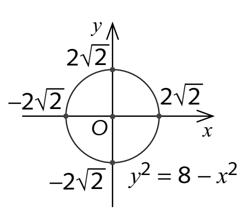
e



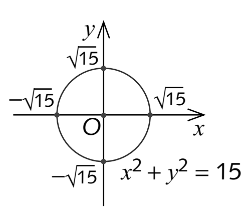
f



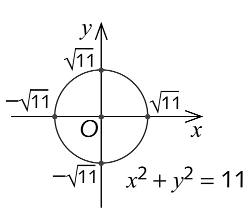
g



h



i



4 **a** $x^2 + y^2 = 121$ **b** $x^2 + y^2 = 7$
c $x^2 + y^2 = 12$ **d** $x^2 + y^2 = 75$

5 **a** $x^2 + y^2 = 2$ **b** $x^2 + y^2 = 5$ **c** $x^2 + y^2 = 50$
d $x^2 + y^2 = 4$ **e** $x^2 + y^2 = 37$ **f** $x^2 + y^2 = 53$
g $x^2 + y^2 = 169$ **h** $x^2 + y^2 = 24$

6 **a** $x^2 + y^2 = 9$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = \frac{49}{4}$
d $x^2 + y^2 = \frac{121}{4}$ **e** $x^2 + y^2 = \frac{9}{64}$

7 **a** $B(-3, 4)$, $C(-3, -4)$, $D(3, -4)$
b $5\sqrt{2}$ **c** 50 **d** $XYZW$, by 2

8 **a** $-\frac{3}{4}$ **b** $y = -\frac{3}{4}x + \frac{25}{4}$
c 10 **d** 100

9 **a** $a = 3$ **b** 6 **c** 6 **d** equilateral

10 **a** $3\sqrt{3}$ **b** ΔAXO is similar to ΔAYB (AAA).
c $OX = \sqrt{3}$; $x^2 + y^2 = 3$

Review exercise

1 **a** 9 **b** 6 **c** 13 **d** 17

2 **a** $\left(-3, \frac{17}{2}\right)$ **b** (−2, 2) **c** $\left(\frac{1}{2}, 0\right)$ **d** $\left(5\frac{1}{2}, -3\right)$

3 **a** 1 **b** $\frac{9}{5}$

4 **a** $\frac{5}{2}$ **b** $-\frac{3}{5}$ **c** $\frac{4}{7}$ **d** −1

5 **a** $-\frac{3}{4}$ **b** 2

6 **a** 4 **b** 15

7 (−2, 0), (0, 4) 8 $\left(-1\frac{1}{2}, 0\right)$

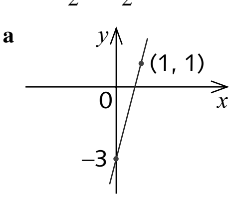
9 **a** 3, 2 **b** −3, 4 **c** $\frac{1}{4}, -7$
d $-\frac{2}{5}, 6$ **e** −8, 0 **f** −9, 2

10 **a** $y = 3x + 5$ **b** $y = -x + 4$
c $y = \frac{3}{4}x - 2$ **d** $y = -\frac{1}{7}x$

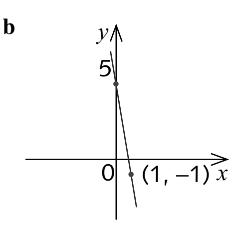
11 **a** $y = -3x + 12$; −3, 12 **b** $y = -\frac{9}{4}x + \frac{3}{2}$; $-\frac{9}{4}, \frac{3}{2}$
c $y = \frac{2}{3}x - \frac{8}{3}$; $\frac{2}{3}, -\frac{8}{3}$ **d** $y = \frac{3}{4}x + \frac{9}{4}$; $\frac{3}{4}, \frac{9}{4}$
e $y = \frac{1}{7}x + \frac{2}{7}$; $\frac{1}{7}, \frac{2}{7}$ **f** $y = -\frac{1}{9}x$; $-\frac{1}{9}, 0$
g $y = 2x$; −2, 0 **h** $y = \frac{1}{11}x$; $\frac{1}{11}, 0$

12 **a** $y = x + 2$ **b** $y = 4x + 8$
c $y = \frac{1}{2}x - \frac{5}{2}$ **d** $y = -\frac{2}{5}x - \frac{29}{5}$

13 **a**

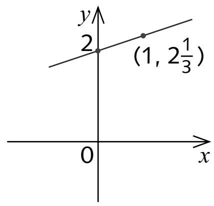


b

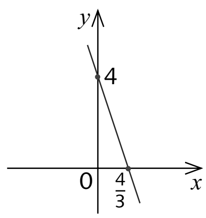


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

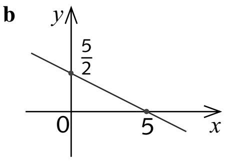
c



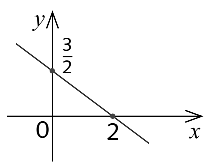
14 a



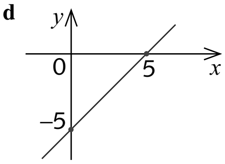
b



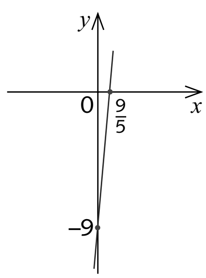
c



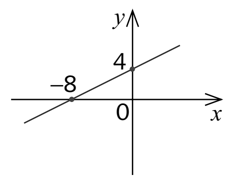
d



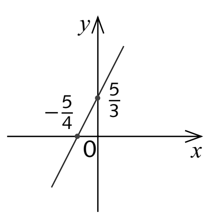
e



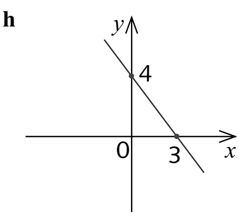
f



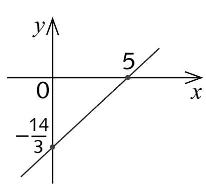
g



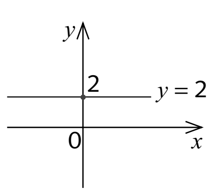
h



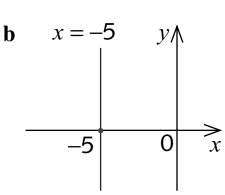
i



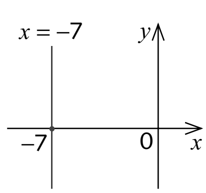
15 a



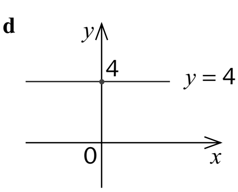
b



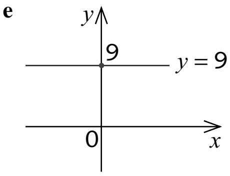
c



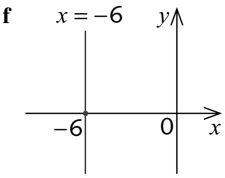
d



e



f



16 b and d, c and f, a and e **17** $y = 3x$

18 a and f, d and e **19** $y = -\frac{1}{2}x + \frac{19}{2}$

20 $a = -7$, $b = \frac{7}{2}$, $c = -5$, $d = 7$

21 $a = 4$, $b = 6$, $c = 3$, $d = \frac{16}{3}$

22 **a** 9 **b** $2\sqrt{82}$ **c** $y = 9x - 2$

d $(0, -2)$ **e** $y = -\frac{1}{9}x - 2$

23 $a = 1 + \sqrt{7}$ **24** $\frac{353}{11}$ **25** $11x + 6y = 66$

Challenge exercise

1 **a** (1.5, 2.5) **b** $\left(\frac{7}{3}, \frac{7}{3}\right)$ **c** $\left(1, \frac{13}{5}\right)$

2 $AB = BC = AC = 2\sqrt{2}$

3 gradient AB = gradient $CD = \frac{2}{3}$

gradient BC = gradient $AD = -\frac{1}{2}$; $AC = 2\sqrt{10}$, $BD = 2\sqrt{26}$

4 (8, 1) **5** $2x - 3y - 12 = 0$

6 **a** 3 **b** $\sqrt{5}$

c 4 **d** −2

e i $\frac{1}{2}$ **ii** −2 **f** (0, 3)

7 Let $A = (a, b)$ and $C = (c, 0)$. So $B = (-c, 0)$.
$AB^2 = (a + c)^2 + b^2$, $AC^2 = (c - a)^2 + b^2$, $AO^2 = a^2 + b^2$, $OC^2 = c^2$

$AB^2 + AC^2 = (a + c)^2 + b^2 + (c - a)^2 + b^2 = 2(a^2 + b^2 + c^2) = 2(AO^2 + OC^2) = 2AO^2 + 2OC^2$

8 Let M be the midpoint of line interval AB and let N be the midpoint of line interval BC.

$M = \left(\frac{u}{2}, \frac{v}{2}\right)$, $N = \left(\frac{u+c}{2}, \frac{v}{2}\right)$

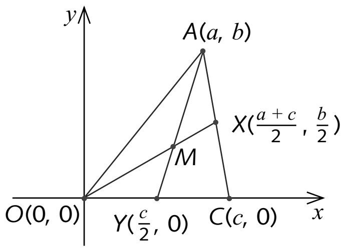
gradient $AC = 0$ = gradient MN, $\therefore AC \parallel MN$

length $= MN = \frac{c}{2} = \frac{1}{2} \times$ length of AC

9 Let $B = (b, 0)$ and $C = (c, d)$, so $D = (b - c, d)$. Then, $AC = BD = \sqrt{c^2 + d^2}$

10 Let Y be the midpoint of OC and let M be the point on AY such that $AM : MY = 2 : 1$. M has coordinates $\left(\frac{a+c}{3}, \frac{b}{3}\right)$.
Let X be the midpoint of AC and let N be the point on OX such that $ON : NX = 2 : 1$. N has coordinates $\left(\frac{a+c}{3}, \frac{b}{3}\right)$, and similarly for the third median.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7



11 **a** P has coordinate (x, y). By Pythagoras' theorem, $x^2 + y^2 = a^2$

b gradient of $PA = \frac{y}{x+a}$; gradient of $PB = \frac{y}{x-a}$

Product of the gradients $= \frac{y^2}{x^2 - a^2} = \frac{a^2 - x^2}{x^2 - a^2} = -1$.

PA is perpendicular to PB.

c Pythagoras' theorem applied to ΔAPB.

Chapter 12 answers

Exercise 12A

1 $\frac{1}{4}$ **2** $\frac{4}{9}$ **3** $\frac{1}{8}$ **4** $\frac{1}{4}$

5 $\frac{1}{500}$ **6** $\frac{1}{52}$ **7** $\frac{2}{25}$

8 **a** $\frac{1}{13}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$ **d** $\frac{3}{13}$

9 $\frac{1}{7}$ **10** **a** $\frac{1}{2}$ **b** $\frac{3}{8}$

11 **a** $\frac{19}{37}$ **b** $\frac{18}{37}$ **c** $\frac{14}{37}$ **12** $\frac{16}{25}$

Exercise 12B

1 $\frac{3}{5}$ **2** $\frac{12}{13}$ **3** $\frac{13}{15}$ **4** $\frac{19}{20}$

5 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{4}{5}$

6 **a** $\frac{1}{4}$ **b** $\frac{4}{13}$ **c** $\frac{8}{13}$ **d** $\frac{1}{13}$

e $\frac{25}{52}$ **f** 0 **g** $\frac{8}{13}$

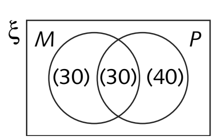
7 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** 0 **d** $\frac{2}{3}$

e $\frac{1}{3}$ **f** $\frac{2}{3}$

8 **a** $\frac{29}{100}$ **b** $\frac{23}{100}$ **c** $\frac{49}{100}$ **d** $\frac{13}{20}$

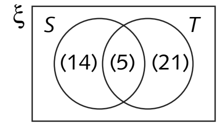
e $\frac{11}{100}$ **f** $\frac{13}{20}$ **g** $\frac{77}{100}$ **h** $\frac{7}{8}$

9 **a**



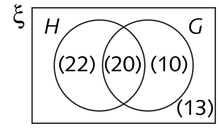
b **i** $\frac{3}{10}$ **ii** $\frac{2}{5}$ **iii** 0

10



a $\frac{1}{8}$ **b** $\frac{21}{40}$ **c** $\frac{7}{8}$ **d** $\frac{7}{20}$

11



a $\frac{4}{5}$ **b** $\frac{1}{5}$ **c** $\frac{22}{65}$ **d** $\frac{32}{65}$

Exercise 12C

1 **a** $\frac{3}{5}$ **b** $\frac{2}{5}$ **2** **a** $\frac{1}{60}$ **b** $\frac{59}{60}$

3 $\frac{3}{10}$

4 **a** **i** $\frac{9}{50}$ **ii** $\frac{53}{100}$ **iii** $\frac{23}{25}$

b 2 blue, 2 yellow, 5 red, 1 green

5 **a** 134

b **i** $\frac{28}{125}$ **ii** $\frac{119}{500}$

iii $\frac{183}{250}$ **iv** $\frac{127}{250}$

6 **a** 297

b **i** $\frac{47}{250}$ **ii** $\frac{4}{25}$ **iii** $\frac{703}{1000}$

7 **a** $\frac{29}{50}$ **b** $\frac{7}{10}$ **c** $\frac{11}{120}$ **d** $\frac{7}{100}$

8 **a** $\frac{88}{175}$ **b** $\frac{138}{175}$ **c** $\frac{12}{175}$ **d** $\frac{148}{175}$

Exercise 12D

1

		Die					
		1	**2**	**3**	**4**	**5**	**6**
10c coin	**H**	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
	T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

2

		2nd bag			
		R	**R**	**Y**	**B**
1st bag	**R**	(R, R)	(R, R)	(R, Y)	(R, B)
	Y	(Y, R)	(Y, R)	(Y, Y)	(Y, B)
	Y	(Y, R)	(Y, R)	(Y, Y)	(Y, B)
	B	(B, R)	(B, R)	(B, Y)	(B, B)

3

		2nd spin		
		1	**2**	**3**
1st spin	**1**	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 **a** $\frac{1}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$

5

		Bag 2				
		T	**T**	**T**	**F**	**F**
Bag 1	**F**	(F, T)	(F, T)	(F, T)	(F, F)	(F, F)
	T	(T, T)	(T, T)	(T, T)	(T, F)	(T, F)

a $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{1}{2}$

6

		2nd shelf				
		S	**S**	**T**	**T**	**T**
1st shelf	**S**	(S, S)	(S, S)	(S, T)	(S, T)	(S, T)
	S	(S, S)	(S, S)	(S, T)	(S, T)	(S, T)
	S	(S, S)	(S, S)	(S, T)	(S, T)	(S, T)
	T	(T, S)	(T, S)	(T, T)	(T, T)	(T, T)

a $\frac{3}{10}$ **b** $\frac{3}{20}$

7

		King			
		S	**H**	**D**	**C**
Ace	**S**	(S, S)	(S, H)	(S, D)	(S, C)
	H	(H, S)	(H, H)	(H, D)	(H, C)
	D	(D, S)	(D, H)	(D, D)	(D, C)
	C	(C, S)	(C, H)	(C, D)	(C, C)

a $\frac{1}{4}$ **b** $\frac{1}{16}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$

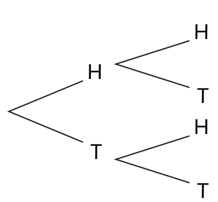
8 **a** $\frac{1}{36}$ **b** $\frac{1}{6}$ **c** $\frac{1}{6}$ **d** $\frac{11}{12}$

9

		2nd die					
		–	**–**	**1**	**3**	**4**	**6**
1st die	**1**	(1, –)	(1, –)	(1, 1)	(1, 3)	(1, 4)	(1, 6)
	2	(2, –)	(2, –)	(2, 1)	(2, 3)	(2, 4)	(2, 6)
	3	(3, –)	(3, –)	(3, 1)	(3, 3)	(3, 4)	(3, 6)
	4	(4, –)	(4, –)	(4, 1)	(4, 3)	(4, 4)	(4, 6)
	5	(5, –)	(5, –)	(5, 1)	(5, 3)	(5, 4)	(5, 6)
	6	(6, –)	(6, –)	(6, 1)	(6, 3)	(6, 4)	(6, 6)

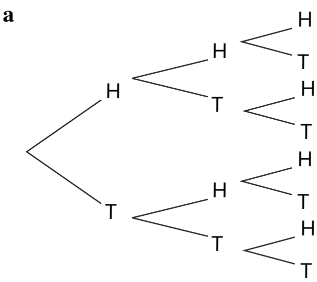
a $\frac{5}{36}$ **b** $\frac{1}{18}$ **c** $\frac{1}{18}$ **d** $\frac{19}{36}$

10 **a**



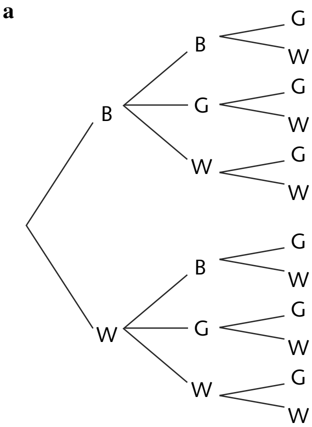
b $\frac{1}{2}$

11 **a**



b $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{7}{8}$

12 **a**



b $\frac{1}{4}$

13 **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$

Exercise 12E

1 **a i** $\frac{1}{9}$ **ii** $\frac{1}{18}$ **iii** $\frac{1}{18}$

b i $\frac{1}{15}$ **ii** $\frac{1}{15}$ **iii** $\frac{1}{15}$

2 **a** $\frac{1}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{5}$

3 **a** $\frac{15}{56}$ **b** $\frac{3}{28}$ **c** $\frac{5}{14}$ **d** $\frac{3}{8}$

4 **a i** $\frac{6}{25}$ **ii** $\frac{9}{25}$

b i $\frac{1}{4}$ **ii** 0

Review exercise

1 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{5}$ **d** $\frac{3}{5}$

2 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{10}$ **d** $\frac{1}{5}$

e $\frac{1}{2}$ **f** $\frac{3}{10}$

3 **a** $\frac{4}{5}$ **b** $\frac{7}{10}$

4 **a** 67

b i $\frac{8}{67}$ **ii** $\frac{12}{67}$ **iii** $\frac{20}{67}$ **iv** $\frac{62}{67}$

5 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{6}$ **d** $\frac{5}{6}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

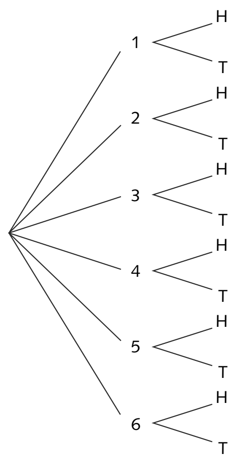
6 **a** $\frac{9}{20}$ **b** $\frac{23}{100}$ **c** $\frac{22}{25}$

7 **a** $\frac{5}{26}$ **b** $\frac{21}{26}$ **c** $\frac{2}{13}$ **d** $\frac{2}{13}$

e $\frac{1}{13}$ **f** $\frac{1}{13}$

8 **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{3}{8}$ **d** $\frac{1}{8}$

9 **a**



b

Die / Coin	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

c $\frac{1}{12}$ **d** (H, 2) (H, 4) (H, 6) **e** $\frac{1}{4}$ **f** $\frac{1}{4}$

10 **a** $\frac{1}{2}$ **b** **i** $\frac{1}{9}$ **ii** $\frac{5}{18}$

11 **a** $\frac{1}{36}$ **b** $\frac{1}{18}$ **c** $\frac{1}{12}$ **d** $\frac{11}{36}$ **e** $\frac{1}{4}$

12 **a**

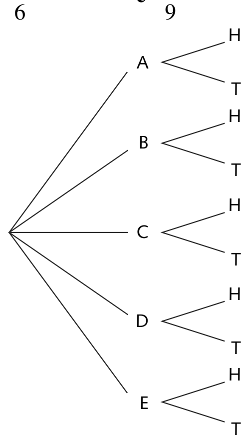
E1	L2	L2	L1	L1	L3	L3
E2	L3	L1	L2	L3	L1	L2
E2	L1	L3	L3	L2	L2	L1

b $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

13 **a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{2}{9}$

d $\frac{1}{6}$ **e** $\frac{1}{9}$

14 **a**



b **i** $\frac{1}{10}$ **ii** $\frac{1}{10}$ **iii** $\frac{1}{5}$ **iv** $\frac{3}{10}$

Challenge exercise

1 **a** $\frac{3}{4}$ **b** $\frac{7}{8}$

c **i** 16 **ii** $\frac{15}{16}$ **d** **i** 2^n **ii** $\frac{2^n - 1}{2^n}$

2 **a** $\frac{5}{6}$ **b** $\frac{25}{36}$

3 **a** **i** 216 **ii** $\frac{125}{216}$ **iii** $\frac{91}{216}$

b **i** 1296 **ii** $\frac{625}{1296}$ **iii** $\frac{671}{1296}$

c **i** 6^n **ii** $\left(\frac{5}{6}\right)^n$ **iii** $1-\left(\frac{5}{6}\right)^n$

4 $\frac{8^5}{10^5} = \frac{1024}{3125}$

Chapter 13 answers

Exercise 13A

1 **a** Opposite side is PR, adjacent side is QR, hypotenuse is PQ.

b Opposite side is QR, adjacent side is PR, hypotenuse is PQ.

2 **a** Opposite side is XZ, adjacent side is XY, hypotenuse is YZ.

b Opposite side is XY, adjacent side is XZ, hypotenuse is YZ.

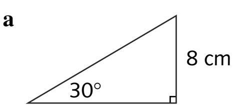
3 **a** 10 cm **b** 6 cm **c** 8 cm

4 **a** 13 cm **b** 12 cm **c** 5 cm

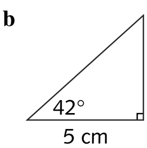
5 **a** $\angle LMN$ **b** $\angle BCD$ **c** $\angle FHG$

d $\angle YXZ$ **e** $\angle NMP$ **f** $\angle RST$

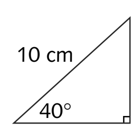
6 **a**



b



c



7 $\frac{\text{opposite}}{\text{adjacent}} \approx 0.60$

Exercise 13B

1 **a** **i** $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$

b **i** $\frac{8}{17}$ **ii** $\frac{15}{17}$ **iii** $\frac{8}{15}$

c **i** $\frac{24}{25}$ **ii** $\frac{7}{25}$ **iii** $\frac{24}{7}$

d **i** $\frac{9}{41}$ **ii** $\frac{40}{41}$ **iii** $\frac{9}{40}$

e **i** $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$

f **i** $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

2 **a** 0.1736 **b** 0.3420 **c** 0.5 **d** 0.6947
e 0.2588 **f** 1.1504 **g** 6.3138 **h** 0.0175
i 0.7986 **j** 0.0349 **k** 0.0524 **l** 1

3 **a** 7.50 **b** 2.54 **c** 6.72 **d** 14.10

4 **a** 4.104 **b** 13 **c** 15.445

5 **a** 6.7128 **b** 22.5687 **c** 3.9064

6 **a** 11.8727 **b** 13.5640 **c** 1.2426 **d** 4.5845
e 1.7859 **f** 4.5838 **g** 7.2638 **h** 3.7558
i 21.2027 **j** 10.4969 **k** 5.7477 **l** 32.5774

7 **a** 18.2494 **b** 7.8324 **c** 9.6569 **d** 13.1557
e 4.1 **f** 14.0015 **g** 6.4984 **h** 9.0606
i 22.5687 **j** 25.5988 **k** 2.4686 **l** 5.7251

8 The other sides are of length 1 unit and $\sqrt{2}$ units;
$\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

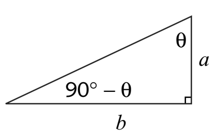
9 **a** 60°, 30°, 90° **b** 1 **c** $\sqrt{3}$

d

θ	30°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$

10 **a** 85° **b** 18° **c** 25° **d** 44°

11 From the diagram, $\tan\theta = \frac{b}{a}$ and $\tan(90° - \theta) = \frac{a}{b}$,
so $\tan(90° - \theta) = \frac{1}{\tan\theta}$



12 45°

13 **a** hypotenuse
b No, the opposite side cannot be longer than the hypotenuse.
c No, the adjacent side cannot be longer than the hypotenuse.
d Yes, the opposite side can be longer than the adjacent side.

14 **a** $\sin\theta$ tends to 0.
b $\sin\theta$ tends to 1.
c In a triangle with a fixed hypotenuse length, as θ gets larger, so does the opposite side length and approaches the length of the hypotenuse.

15 **i** **a** cos θ tends to 1.
b cos θ tends to 0.
c In a triangle with a fixed hypotenuse length of 1 as θ decreases the adjacent side increases and approaches the length of the hypotenuse.

ii **a** tan θ tends to 0 as θ tends to 0
b Increases without bound as θ approaches 90°
c In a triangle with a fixed hypotenuse length, as θ tends to 90° the opposite side length tends to the hypotenuse side length and the adjacent side length tends to 0, so the ratio of $\frac{\text{opposite}}{\text{adjacent}}$ increases without bound.

Exercise 13C

1 **a** 44° **b** 47° **c** 45°
d 83° **e** 6° **f** 85°

2 **a** 24° **b** 58° **c** 11°
d 58° **e** 57° **f** 76°

3 **a** 26° **b** 48° **c** 31° **d** 64°
e 20° **f** 60° **g** 44° **h** 78°

4 **a** 37°, 53°, 90° **b** 23°, 67°, 90° **c** 16°, 74°, 90°

5 29.74° **6** 44.42° **7** 110.02° and 69.98°

Exercise 13D

1 **a** 8.63 **b** 34° **c** 24.96 **d** 78°
e 14.72 **f** 19.08 **g** 18° **h** 11.89
i 18° **j** 14.62 **k** 56° **l** 5.40
m 6.97 **n** 55° **o** 28.79

2 **a** $x = 4$, $y = 4\sqrt{2}$ **b** $x = \frac{4\sqrt{3}}{3}$
c $x = 4\sqrt{2}$, $y = 4\sqrt{2}$ **d** $z = \frac{10\sqrt{3}}{3}$, $x = \frac{20\sqrt{3}}{3}$
e $y = 16$, $x = 8\sqrt{3}$ **f** $x = 5$, $y = 5\sqrt{3}$
g $x = \frac{8\sqrt{3}}{3}$, $y = \frac{16\sqrt{3}}{3}$ **h** $y = 10$, $x = 10\sqrt{2}$
i $x = \frac{10\sqrt{3}}{3}$, $y = \frac{20\sqrt{3}}{3}$

Exercise 13E

1 3.83 m **2** 149 cm **3** 1294 m

4 **a** 24.26 m **b** 6.05 m

5 **a** 3.97 m **b** 0.49 m

6 24°

7 34°, 113°, 34° **8** 132°

9 21.8° **10** 14 m **11** 20.2 m

12 46° **13** 6.38°

14 **a** 38° **b** 9.40 m **c** 19°

15 **a** 60.1 m **b** 37°

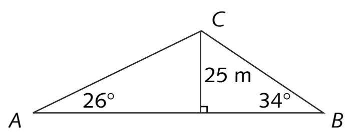
ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

16 **a** 11.30 **b** 8.52

17 **a** 16.79 m **b** 23.97 m **c** 5.97 m

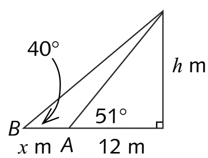
18 **a** 3.06 m **b** 7°

19 **a**



b 88.32 m

20 **a**



b 14.82 m

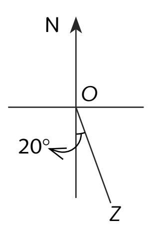
c 5.66 m

21 71.6°

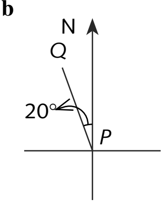
Exercise 13F

1 030°T, 110°T, 220°T, 280°T

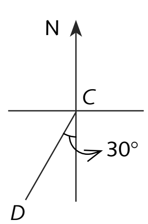
2 **a**



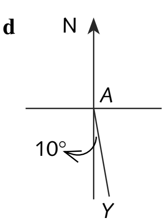
b



c



d



3 **a** 295°T **b** 070°T

4 **a** 295 m **b** 222 m

5 **a** 187.9 km **b** 68.4 km

6 **a** 1111 m **b** 1151 m

7 **a** 201.8°T **b** 21.8°T

8 **a** 117°T **b** 153°T **c** 207°T

9 **a** 5.64 km **b** 2.05 km **c** 9.36 km
d 2.05 km **e** 9.58 km **f** 102°T

10 7.4 km

Review exercise

1 **a** $\sin\alpha = \frac{24}{25}, \cos\alpha = \frac{7}{25}, \tan\alpha = \frac{24}{7}$

b $\sin\theta = \frac{40}{41}, \cos\theta = \frac{9}{41}, \tan\theta = \frac{40}{9}$

c $\sin\alpha = \frac{3\sqrt{13}}{13}, \cos\alpha = \frac{2\sqrt{13}}{13}, \tan\alpha = \frac{3}{2}$

2 **a** 5.18 cm **b** 5.13 cm **c** 64.24 cm
d 66.74 cm **e** 33.11 cm **f** 463.21 m
g 313.30 m **h** 14.47 cm

3 **a** 22° **b** 58° **c** 38° **d** 12°

4 **a** 7.37 m **b** 5.16 m

5 2856 m, 3487 m

6 **a** 39° and 51° **b** 77° and 103°

7 391 m **8** 10° **9** 49.26 km

10 **a** $h = 6.01$ **b** $x = 5.65$

Challenge exercise

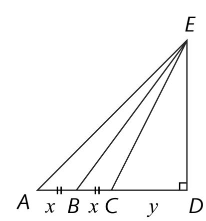
1 **a i** 1.061 km **ii** 1.061 km **iii** 2.318 km

b $d = \sqrt{x^2 + \sqrt{2}x + 1}$ **c** 28.22 km

2 **a** 1962.6 m **b** 2050.3 m
c 87.7 m **d** 17.5 m/s

3 **a**

$$\begin{aligned} BD &= x + y \\ 2BD &= x + x + y + y \\ &= (x + x + y) + y \\ &= AD + CD \\ BD &= \frac{1}{2}(AD + CD) \end{aligned}$$



b $\tan\angle DEB = \frac{BD}{ED}$

$\tan\angle AED = \frac{AD}{ED}$ $\qquad \frac{BD}{ED} = \frac{1}{2}\left(\frac{AD}{ED} + \frac{CD}{ED}\right)$

$\tan\angle CED = \frac{CD}{ED}$

so $\tan\angle DEB = \frac{1}{2}\tan\angle AED + \frac{1}{2}\tan\angle CED$

4 **a** $\sin\theta = \frac{a}{c},\ \cos\theta = \frac{b}{c}$

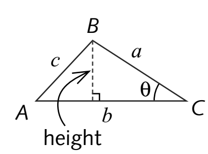
b $a^2 + b^2 = c^2$, so $\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1, (\sin\theta)^2 + (\cos\theta)^2 = 1$

5 **a** 24.09 m^2

b height $= a\sin\theta$

area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2}ba\sin\theta$



6 **a** 20

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

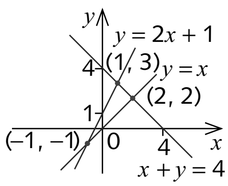
Chapter 14 answers

Exercise 14A

1 **a** $1+2=3$ and $2=1+1$, so $(1,\ 2)$ is a solution.

b $-1+6=5\neq 3$, so $(-1,\ 6)$ does not satisfy $x+y=3$.

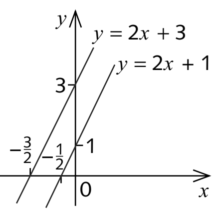
2 **a**



b **i** $x=-1, y=-1$ **ii** $x=2, y=2$ **iii** $x=1, y=3$

3 **a** $x=-1, y=1$ **b** $x=2, y=1$ **c** $x=2, y=4$ **d** $x=2, y=2$ **e** $x=1, y=0$ **f** $x=1, y=1$ **g** $x=1, y=3$ **h** $x=7, y=5$ **i** $x=-1, y=2$

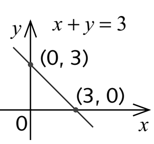
4



a The lines are parallel, so they will not intersect.

b no solutions

5



a The two lines are the same line.

b There are an infinite number of solutions.

6 None; the lines are parallel.

7 **a** **i** $x=\frac{1}{2}, y=2\frac{1}{2}$

ii $x=\frac{5}{3}, y=\frac{17}{3}$

iii $x=2,\ y=0$

b time-consuming and not always accurate

Exercise 14B

1 **a** $x=-1, y=1$ **b** $x=-5, y=-11$ **c** $x=-\frac{1}{2}, y=5$ **d** $x=\ 2, y=\ 1$ **e** $x=2, y=-1$ **f** $x=2, y=1$ **g** $x=-1, y=7$ **h** $x=\frac{5}{4}, y=\frac{5}{2}$

2 **a** $x=1, y=2$ **b** $x=4, y=2$ **c** $x=2, y=5$ **d** $x=-1, y=3$ **e** $x=1, y=2$ **f** $x=5, y=1$ **g** $x=1, y=4$ **h** $x=4, y=10$ **i** $x=-1, y=5$ **j** $x=5, y=4$ **k** $x=3, y=-3$ **l** $x=2, y=1$

3 **a** $x=3, y=1$ **b** $x=5, y=2$ **c** $x=-5, y=-4$ **d** $x=\frac{4}{5}, y=-\frac{13}{5}$ **e** $a=7, b=4$ **f** $a=3, b=1$ **g** $a=5, b=1$ **h** $a=4, b=5$

4 **a** $x=1, y=2$ **b** $x=3, y=1$ **c** $a=\frac{11}{13}, b=\frac{21}{13}$ **d** $a=\frac{7}{8}, b=-\frac{17}{8}$

Exercise 14C

1 **a** $x=4, y=2$ **b** $x=3, y=1$ **c** $x=2, y=-2$ **d** $x=2, y=3$ **e** $x=4, y=1$ **f** $x=-1, y=-2$

2 **a** $x=2, y=3$ **b** $x=1, y=3$ **c** $x=4, y=1$ **d** $x=3, y=4$ **e** $x=\frac{7}{4}, y=\frac{5}{2}$ **f** $x=5, y=1$ **g** $x=-1, y=1$ **h** $x=-2, y=3$ **i** $x=4, y=-1$

3 **a** $x=4, y=1$ **b** $x=-2, y=-3$ **c** $x=5, y=1$ **d** $x=-4, y=3$ **e** $x=-2, y=1$ **f** $x=2, y=0$

4 **a** $x=1, y=1$ **b** $x=3, y=1$ **c** $x=-1, y=5$ **d** $x=4, y=-1$ **e** $x=-1, y=-1$ **f** $x=\frac{36}{11}, y=-\frac{13}{11}$ **g** $x=5, y=0$ **h** $x=3, y=1$ **i** $x=-2, y=1$

5 **a** $x=\frac{45}{17}, y=\frac{38}{17}$ **b** $x=3, y=-1$ **c** $x=-1, y=1$ **d** $x=-\frac{79}{17}, y=\frac{24}{17}$ **e** $x=-1, y=-1$ **f** $x=3, y=2$ **g** $a=4, b=2$ **h** $a=5, b=3$ **i** $a=4, b=-1$ **j** $a=-1, b=-5$ **k** $a=1, b=5$ **l** $a=4, b=6$

6 **a** $x=1, y=2$ **b** $x=5, y=2$ **c** $x=4, y=-3$ **d** $x=-2, y=4$ **e** $x=-1, y=\frac{2}{3}$ **f** $x=7, y=3$ **g** $x=\frac{3}{2}, y=-\frac{5}{2}$ **h** $x=\frac{1}{6}, y=\frac{2}{3}$ **i** $x=\frac{1}{2}, y=\frac{1}{2}$ **j** $x=\frac{3}{4}, y=1$ **k** $x=5, y=\frac{2}{3}$ **l** $x=\frac{3}{8}, y=\frac{5}{8}$

Exercise 14D

1 19, 17 **2** Jacqui 13, Jane 26

3 28, 30 **4** 8, 11

5 Martha is 28 and her daughter is 4.

6 8 cm, 13 cm, 13 cm **7** 42

8 soft drink \$1.20, chocolate bar 80c

9 \$2.20 **10** 80 kg **11** \$30 **12** 53 m × 24 m

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

13 Jie is $9\frac{1}{3}$ years and Tan is $6\frac{2}{3}$ years.

14 Thirty-six 10c pieces, seventy-eight 20c pieces

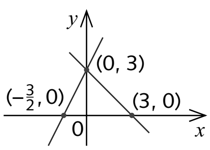
15 26   16 $\frac{8}{11}$ 17 $\frac{11}{17}$

18 Twenty-six 45c stamps, eighteen 60c stamps

Exercise 14E

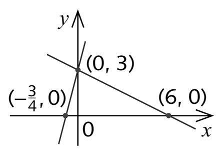
1 **a** $(1,2)$ **b** $\left(4,\frac{1}{2}\right)$ **c** $(8,13)$
d $(2,-8)$ **e** $(1,-3)$ **f** $(2,0)$
g $\left(\frac{3}{2},-\frac{5}{2}\right)$ **h** $\left(\frac{1}{2},\frac{1}{2}\right)$ **i** $\left(-5,-\frac{2}{3}\right)$

2 **a**



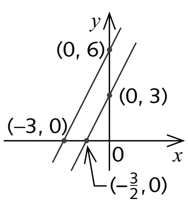
One solution $(0,3)$

b



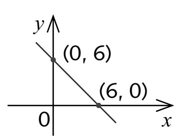
One solution $(0,3)$

c



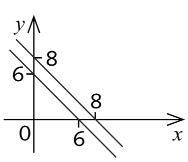
No solutions; lines are parallel

d



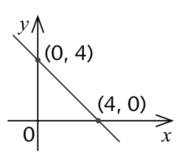
Infinitely many solutions; line is repeated

e



No solutions; lines are parallel

f



Infinitely many solutions; line is repeated

3 **a** $y = 3x + 2$ and $y = -x + 2$ intersect at $(0, 2)$ and $(0, 2)$ lies on the line $y = 2x + 2$. Lines intersect at $(0, 2)$.

b $x + y = 2$ and $y = 4x - 8$ intersect at $(2, 0)$ and $(2, 0)$ lies on the line $2x + 3y = 4$. Lines intersect at $(2, 0)$.

c $2x+y= 0$ and $y = -x$ intersect at $(0, 0)$ and $(0, 0)$ lies on the line $y - 5x = 0$. Lines intersect at $(0, 0)$.

d $x + y = 4$ and $y = 2x + 1$ intersect at $(1, 3)$ and $(1, 3)$ lies on the line $2x - y = -1$. Lines intersect at $(1, 3)$.

4 Lines $y = 2x + 1$ and $y = 3x - 2$ intersect at $(3,7)$. This point does not lie on the line $y = 4x + 6$.

5 **a** 4 square units **b** 4 square units

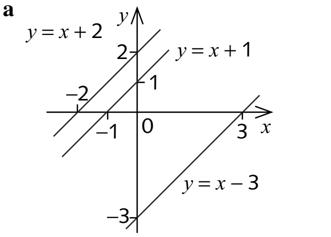
6 **a** $\frac{25}{6}$ square units **b** $\frac{1}{3}$ square units

7 **a** $\frac{3}{2}$ square units **b** 1 square unit

8 **a** 9 square units **b** 25 square units

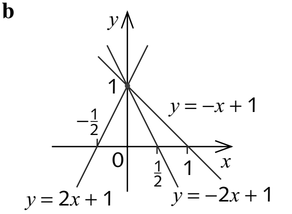
Exercise 14F

1 **a**



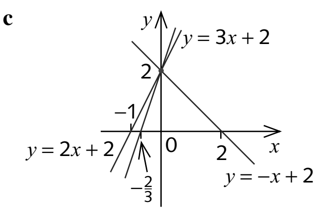
The lines are all parallel to $y = x$.

b



The lines all pass through (0, 1).

c



The lines all pass through $(0, 2)$.

2 **a** $m = 1$ **b** $m \neq 1$ **c** $m = 2$

3 **a** $m = 2$ **b** $m \neq 2$ **c** $m = 3$

4 **a** There are none. **b** m can be any real number.

c The lines will intersect at $(0,5)$ when $m \neq -2$. When $m = -2$, the lines are the same.

5 **a** $k \neq 14$ **b** $k = 14$

c If $k = 14$, the lines are the same.

6 $k = 1, m = -1$ 7 $n = -1, m = -\frac{4}{3}$

Review exercise

1 **a** $x = 1, y = 5$ **b** $x = 1, y = 1$
c $x = 2, y = 1$ **d** $x = 1, y = 2$

2 **a** $x = 1, y = 3$ **b** $x = 1, y = 1$
c $x = 3, y = 3$ **d** $x = 2, y = 3$

3 **a** $x = 8, y = 2$ **b** $x = \frac{21}{5}, y = \frac{13}{5}$
c $x = 2, y = 5$ **d** $x = \frac{1}{5}, y = -\frac{7}{5}$

4 **a** $x = 3, y = -1$ **b** $x = 2, y = 1$
c $x = 2, y = -2$ **d** $x = 2, y = 3$

5 **a** $x = 3, y = -1$ **b** $x = 2, y = \frac{7}{2}$
c $x = 2, y = -5$ **d** $x = \frac{7}{3}, y = \frac{11}{12}$

6 **a** $x = \frac{8}{3}, y = 1$ **b** $x = -2, y = -1$
c $x = 26, y = 7$ **d** $x = -4, y = \frac{5}{7}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 **a** $x = -1, y = 3$ **b** $x = 2, y = 0$

c $x = -1, y = 5$ **d** $x = \frac{77}{13}, y = -\frac{8}{13}$

8 **a** $x = 1, y = 2$ **b** $x = 3, y = -4$

c $x = 1, y = -2$ **d** $x = \frac{103}{11}, y = \frac{108}{11}$

9 **a** $x = \frac{10}{7}, y = \frac{16}{7}$ **b** $x = \frac{40}{21}, y = -\frac{26}{21}$

c $x = \frac{9}{4}, y = -\frac{3}{2}$ **d** $x = -\frac{12}{5}, y = \frac{1}{5}$

e $x = -\frac{5}{3}, y = \frac{4}{3}$ **f** $x = -\frac{19}{2}, y = -\frac{5}{2}$

10 **a** $(1, 2)$ **b** $\left(\frac{8}{3}, -\frac{7}{9}\right)$

c $(12, 15)$ **d** $\left(\frac{14}{5}, -\frac{22}{5}\right)$

e $\left(-\frac{11}{7}, -\frac{34}{7}\right)$ **f** $(2, -2)$

11 **a** $m = 4$ **b** $m \neq 4$ **c** $m = -10$

12 **a** no such m **b** any m

c The lines are the same if $m = \frac{2}{5}$, otherwise they intersect at $(0, 2)$.

13 $m = 3, k = 10$

14 **a** 1 square unit **b** 12 square units

15 420 adults and 540 children

16 7, 2 or 8, 3 **17** $m = 5$

18 **a** $(-1, 3)$ **b** $y = 3x + 6$

19 $c = 26$ **20** $a = -6, b = 1$

21 **a** $x = 4, y = 12$ **b** $x = 5, y = -2$

22 $\frac{7}{9}$

23 **a** $x = \frac{48}{17}, y = \frac{4}{17}$ **b** $x = 0, y = 0$

24 $(3, 10)$ **25** $AB = \frac{5}{3}$ m, $BC = 1$ m, $AC = \frac{4}{3}$ m

26 $c = 3$ **27** **a** -2 **b** $\left(\frac{28}{5}, -\frac{1}{5}\right)$

28 **a** AB: $y = 2x + 1$; CN: $y = -\frac{1}{2}x + \frac{17}{2}$ **b** $(3, 7)$

29 $m = 3, n = 4$

30 $A\left(\frac{10}{3}, -\frac{20}{3}\right)$; $B(2, 0)$; $C\left(-\frac{10}{23}, -\frac{84}{23}\right)$

31 52 **32** 6

Challenge exercise

1 **a** $\left(\frac{3}{m+1}, \frac{4m+1}{m+1}\right)$ **b** $\frac{(4m+1)^2}{2m(m+1)}$ square units

c $m = \frac{1}{8}$

2 **a** $\left(\frac{2-c}{3}, \frac{2(c+1)}{3}\right)$ **b** $\frac{(c-2)^2}{6}$ square units

c $c = 8$ or $c = -4$

3 **a** $x = \frac{2ab}{a^2 + b^2}$, $y = \frac{a^2 - b^2}{a^2 + b^2}$

b $x = a$, $y = \frac{b}{a}$ **c** $x = ab$, $y = b(2a + b)$

4 **a** area of triangle ABC

$= $ area of trapezium $ALNC$ + area of trapezium $CNMB$ − area of trapezium $LABM$

$= \frac{1}{2}(y_1 + y_3)(x_3 - x_1) + \frac{1}{2}(y_2 + y_3)(x_2 - x_3) - \frac{1}{2}(y_1 + y_2)(x_2 - x_1)$

$= \frac{1}{2}(x_1(-y_1 - y_3 + y_1 + y_2) + x_2(y_2 + y_3 - y_1 - y_2) + x_3(y_1 + y_3 - y_2 - y_3))$

$= \frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

b 4

c The formula gives $-\frac{7}{2}$. The formula gives a positive answer when the point C is above the interval AB and a negative answer when C is below the interval AB. So the area is $\frac{7}{2}$.

d The points are collinear.

5 **a** $x = 6, y = 5, z = 2$ **b** $x = 1, y = 2, z = 3$

6 **a** $x = \frac{1}{5}, y = 1, z = -\frac{1}{4}$ **b** $x = \frac{1}{5}, y = \frac{3}{2}, z = 2$

Chapter 15 answers

Exercise 15A

1 **a** $(x + 2)(x + 5)$

b $(2x + 5)(3 - x) = -(2x + 5)(x - 3)$

c $(2x + 3)(4x + 6) = 2(2x + 3)(2x + 3) = 2(2x + 3)^2$

d $(2x - 5)(4 - x) = -(2x - 5)(x - 4)$

e $(4x - 6)(2x - 3) = 2(2x - 3)^2$

f $f(4x - 7)(x - 2)$

g $(2x + 5)(5 - 2x) = -(2x + 5)(2x - 5)$

h $(7x + 1)(11x + 5)$

2 **a** $(x + 2)(m + 3)$ **b** $(x + 2)(a + 4)$

c $(x + 3)(m + 2)$ **d** $5(x + 1)(y + 2)$

e $(8a + 1)(b - 2)$ **f** $(x - 2)(4y + 1)$

g $(b - 7)(a - 9)$ **h** $(b + 2)(3a + 5)$

i $(b - 6)(2a - 5)$ **j** $(3 - 2y)(4x - 5)$

3 **a** $(x - 3)(x - 4)$ **b** $(x - 1)(2x + 5)$

c $(3x - 2y)(2x + y)$ **d** $(2x + 3y)(5x - 2y)$

e $(x - 3)(x + 4)$ **f** $(4x + 1)(x + 5)$

g $(x + 4)(3x + 1)$ **h** $(3x - 4)(2x - 3)$

i $(3x - 2)(2x + 5)$ **j** $(2x + 1)(x - 1)$

4 **a** $(x + 3)(x + 2)$ **b** $(x + 7)(x - 2)$

c $(x - 9)(x + 2)$ **d** $(x - 9)(x + 4)$

e $(x + 6)(x + 5)$ **f** $(x + 25)(x + 4)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 15B

1 **a** $(2x+5)(x+1)$ **b** $(2x+1)(x+5)$
c $(2x+7)(x+3)$ **d** $(2x+3)(x+7)$
e $(3x+2)(x+4)$ **f** $(3x+4)(x+2)$
g $(2x+3)(2x+5)$ **h** $(4x+3)(x+5)$
i $(2x+1)(x+15)$ **j** $(2x+15)(x+1)$
k $(2x+1)(x+15)$ **l** $(3x+2)(2x+5)$
m $(6x+1)(2x+5)$ **n** $(6x+5)(2x+1)$
o $(4x+3)(2x+7)$

2 **a** $(3x-2)(x-4)$ **b** $(3x-4)(x-2)$
c $(4x-1)(x-5)$ **d** $(4x-5)(x-1)$
e $(3x-1)(2x-3)$ **f** $(3x-2)(2x-1)$
g $(2x-1)(2x-5)$ **h** $(2x-3)(2x-5)$
i $(2x-5)(2x-2)$ **j** $(6x-1)(x-4)$
k $(6x-5)(x-1)$ **l** $(6x-5)(x-2)$
m $(4x-3)(x-9)$ **n** $(2x-3)(2x-9)$
o $(5x-1)(2x-1)$

3 **a** $(2x+1)(x-5)$ **b** $(2x-5)(x+1)$
c $(2x-1)(x+5)$ **d** $(2x-3)(x+6)$
e $(3x-5)(x+2)$ **f** $(3x+2)(x-5)$
g $(2x+3)(2x-5)$ **h** $(4x-3)(x+5)$
i $(4x+5)(x-3)$ **j** $(2x+1)(x-15)$
k $(4x-1)(3x+5)$ **l** $(4x+5)(x-3)$
m $(2x-5)(3x+2)$ **n** $(2x-7)(5x+2)$
o $(4x+1)(5x-3)$

4 **a** $(x+2)$ **b** $(x+3)$ **c** $(x+4)$
d $(a+b)$ **e** $(3x-4y)$ **f** $(x-4)$
g $(2x+3)$ **h** $(x-3)$ **i** $(6x-1)$
j $(2x+3)$

5 **a** $x^2+7x+10=(x+5)(x+2)$
b $x^2-6x+8=(x-4)(x-2)$
c $x^2+8x+15=(x+3)(x+5)$
d $x^2+2x-24=(x-4)(x+6)$

6 **a** Expand the right-hand side: $prx^2+(qr+sp)x+qs$
b ax^2+bx+c can be factorised if there are numbers p, q, r and s such that $ac=(pr)(qs)=(ps)(qr)$ and $b=(ps+qr)$.

Exercise 15C

1 **a** -1 **b** $-k$ **c** $\frac{6x}{y-x}$
d $3k+2\ell$ **e** $\frac{2p-3q}{p}$ **f** $\frac{1}{pq+1}$
g $\frac{m}{n}$ **h** $-(s+t)$ **i** $-\frac{3}{x}$
j $3(p-q)$ **k** $\frac{x(x-y)}{2}$ **l** $-m^2$

2 **a** $\frac{x-1}{x-2}$ **b** $\frac{x-2}{x-5}$ **c** $\frac{x+5}{x-1}$
d $\frac{x+4}{x}$ **e** $\frac{x-5}{x-3}$ **f** $\frac{x+3}{x-2}$
g $\frac{2(x-3)}{3(x-2)}$ **h** $\frac{x-3}{x-2}$ **i** $\frac{3(x-3)}{x+4}$

3 **a** $\frac{x+1}{x+2}$ **b** $\frac{x+1}{2x-1}$ **c** 1
d $\frac{2x-1}{x-1}$ **e** $\frac{2x+1}{2}$ **f** $\frac{5x-1}{x-2}$
g $\frac{x-2}{x-3}$ **h** $\frac{x-1}{x+1}$ **i** $\frac{2x-1}{2x+1}$

4 **a** $\frac{3(2x-1)}{x+5}$ **b** $\frac{5(x+1)}{x+3}$
c $\frac{2x-1}{(x-1)}$ **d** $\frac{3x}{4x+1}$

5 **a** $-3x^2$ **b** $\frac{x+2}{2x}$ **c** $\frac{2x+1}{x-1}$
d $2x-1$ **e** 1

6 **a** 1 **b** $\frac{(x+3)(x+1)}{(2x-1)^2}$ **c** $\frac{x}{2x-1}$
d $\frac{(x+1)^2}{(x-1)^2}$ **e** $\frac{q}{p^2}$ **f** $\frac{3a^2}{2(a-1)(a+3)^2}$

Exercise 15D

1 **a** $\frac{x}{2}$ **b** $\frac{2x}{5}$ **c** $-\frac{3x}{7}$
d $\frac{13x}{10}$ **e** $\frac{19x}{20}$ **f** $-\frac{81}{21}$
g $\frac{5x}{12}$ **h** $-\frac{x}{15}$

2 **a** $\frac{7}{x}$ **b** $\frac{11}{x}$ **c** $\frac{5}{x}$
d $\frac{5+3x}{x^2}$ **e** $\frac{2}{x}$ **f** $\frac{7-3x}{x^2}$
g $\frac{11}{2x}$ **h** $\frac{13}{6x}$ **i** $\frac{7}{6x}$

3 **a** $\frac{3x+5}{(x+1)(x+2)}$ **b** $\frac{5(x+1)}{(x+4)(x-1)}$
c $\frac{9x+2}{(x+3)(x-2)}$ **d** $\frac{x+7}{(x+1)(x+3)}$
e $\frac{3(x-5)}{(x+2)(x-1)}$ **f** $\frac{x+7}{(2x-1)(x+2)}$
g $\frac{8a-5}{(2a+1)(a-4)}$ **h** $\frac{2(3b-5)}{(b+3)(2b-1)}$
i $\frac{11b-7}{(b-1)(2b-1)}$

4 **a** $\frac{4}{x-5}$ **b** $\frac{2(3a-5)}{(a-5)(a+5)}$
c $\frac{x^2+5x-3}{(x-3)(x+4)}$ **d** $\frac{4(x-1)}{(x-2)^2}$
e $\frac{1}{5-a}=\frac{-1}{a-5}$ **f** $\frac{5a^2-4a}{(a-4)(a+4)}$
g $\frac{15-2x}{(x-7)^2}$ **h** $\frac{2}{1-a}$
i $\frac{7}{x-4}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 a $\frac{3x+5}{(x+1)(x+2)(x+3)}$ b $\frac{3x-1}{(x-1)(x+3)(x-2)}$

c $\frac{2(5-x)}{(x-3)(x-2)(x-1)}$ d $\frac{13-3x}{(x+1)(x-3)}$

e $\frac{7x+1}{(x+1)(x-1)(x-2)}$ f $\frac{2(4x-9)}{(x-1)(x+1)(x-3)}$

g $\frac{2(x+4)}{(x+1)(x-2)(x+3)}$ h $\frac{5x+7}{(x+3)(x-1)(x+1)}$

i $\frac{7x-3}{x(x+2)(x-1)}$ j $\frac{3-2x}{(x-2)(x+4)(x-3)}$

k $\frac{5x+2}{x(x+1)(x+3)}$ l $\frac{3x+10}{(x+1)(x+2)}$

m $\frac{4x-11}{(x-4)(x+2)(x+1)}$ n $\frac{5x+4}{x(x-2)(x+2)}$

o $\frac{2(x-1)}{x(x-3)(x+1)}$ p $\frac{x+8}{x(x-2)(x+2)}$

6 a $\frac{5x+3}{x(x+1)}$ b $\frac{19m-2n}{(2m-n)(3m+n)}$

c $\frac{2mn}{(m-n)(m+n)}$ d $\frac{x^3-15x-5}{x^2(3x+1)}$

e $\frac{2x}{(x+1)(x+2)}$ f $\frac{3k^2-k+2}{(k-1)(k+1)}$

g $\frac{7p+1}{p(2p+1)}$ h $\frac{2}{(1+q)(1-q)}$

i $\frac{17k-9\ell}{(2k+\ell)(k-2\ell)}$ j $\frac{p(5p+1)}{(p-1)(p+1)}$

k $\frac{2p^2+2p+5}{(p-1)(p+2)}$ l $\frac{2(10m^2+2mn+n^2)}{(2m-n)(2m+n)}$

m $\frac{3}{x-2}$ n 0 o 0 p $\frac{3}{k+5}$

q $\frac{2}{(x+1)(x-1)}$ r $\frac{-2}{(x-2)(2+x)}$ s $\frac{1}{x+3}$

t $\frac{1}{(1-2\ell)^2}$ u 0 v $\frac{-1}{(p-1)(p-3)}$

Exercise 15E

1 Complete the square.

a $x^2+6x+9=(x+3)^2$

b $x^2+12x+36=(x+6)^2$

c $x^2+5x+\frac{25}{4}=\left(x+\frac{5}{2}\right)^2$

d $x^2+7x+\frac{49}{4}=\left(x+\frac{7}{2}\right)^2$

e $x^2-12x+36=(x-6)^2$

f $x^2-100x+2500=(x-50)^2$

g $x^2-10x+25=(x-5)^2$

h $x^2-80x+1600=(x-40)^2$

2 a $(x+1)^2-6$ b $(x+1)^2+6$

c $(x+2)^2-3$ d $(x+3)^2-7$

e $(x+3)^2-12$ f $(x-3)^2-3$

g $(x-4)^2-21$ h $(x+4)^2+9$

i $(x+6)^2-47$ j $\left(x+\frac{3}{2}\right)^2-\frac{17}{4}$

k $\left(x+\frac{11}{2}\right)^2-\frac{93}{4}$ l $(x-5)^2-28$

m $(x+2)^2+6$ n $(x-4)^2+4$

o $(x-3)^2$ p $\left(x+\frac{5}{2}\right)^2+\frac{7}{4}$

q $(x+5)^2-29$ r $\left(x+\frac{5}{2}\right)^2-\frac{65}{4}$

3 a $(x+3)(x+1)$ b $(x+4)(x+1)$

c $(x+3)(x+2)$ d $(x+5)(x+1)$

e $(x+3+2\sqrt{3})(x+3-2\sqrt{3})$

f $(x-5+2\sqrt{7})(x-5-2\sqrt{7})$

g $(x+6+\sqrt{41})(x+6-\sqrt{41})$

h $(x-6-\sqrt{31})(x-6+\sqrt{31})$

i $(x+4+\sqrt{21})(x+4-\sqrt{21})$

j $(x-5+\sqrt{31})(x-5-\sqrt{31})$

k $\left(x-\frac{11-3\sqrt{13}}{2}\right)\left(x-\frac{11+3\sqrt{13}}{2}\right)$

l $\left(x+\frac{9+\sqrt{113}}{2}\right)\left(x+\frac{9-\sqrt{113}}{2}\right)$

m $\left(x-\frac{7+\sqrt{41}}{2}\right)\left(x-\frac{7-\sqrt{41}}{2}\right)$

n $\left(x-\frac{7-\sqrt{57}}{2}\right)\left(x-\frac{7+\sqrt{57}}{2}\right)$

o $\left(x+\frac{13+3\sqrt{21}}{2}\right)\left(x+\frac{13-3\sqrt{21}}{2}\right)$

Review exercise

1 a $9(x-7)$ b $8(3x^2+1)$

c $6x(7x-6)$ d $8y(1-2y)$

e $-32(2b+1)$ f $7m(3m+n)$

2 a $(a+5)(a+4)$ b $(6p-1)(2p-7q)$

c $(4x-1)(3y+5)$ d $(2m+9)(2n+1)$

3 a $(a+11)(a-11)$ b $9(x+2)(x-2)$

c $(8m+13n)(8m-13n)$ d $7(x+3)(x-3)$

e $(x+6)(x-6)$ f $(x-4y)(x+4y)$

g $3(m-3)(m+3)$ h $6(x-2)(x+2)$

i $4(b-5)(b+5)$

4 a $(x-3)(x+8)$ b $(x-3)(x-12)$

c $(x-7)(x+2)$ d $4(a+1)(a+5)$

e $3(r-4)(r+2)$ f $5(a-6)^2$

g $5(a-1)^2$ h $-2(p-4)^2$

i $-(b+4)(b-1)$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 **a** $(x+3)(b+4)$ **b** $(x-5)(x-6)$
c $2(3x-2)(1-2y)$ **d** $(x+5)(x^2-3)$

6 **a** $(4x+1)(x+6)$ **b** $(2x-1)(3x-2)$
c $(4x+1)(3x-5)$ **d** $(t+3)(2t-1)$
e $(5a+2)(2a-1)$ **f** $(4x-7)(7x-9)$
g $(3x+11)(13x-4)$ **h** $-(8x-3)(12x-5)$
i $(3x+5)(5x-7)$

7 **a** $(x+5)$ **b** $(x+4)$ **c** $(x+5)$
d $(m+n)$ **e** $(5a-3b)$ **f** $(x-4)$

8 **a** $x+3$ **b** $\dfrac{x+2}{x-5}$ **c** $\dfrac{x+5}{x+4}$
d $\dfrac{3x+1}{x+3}$ **e** $\dfrac{3x-1}{x-2}$ **f** $\dfrac{3x+2}{4}$
g $\dfrac{8x-3}{x-1}$ **h** $\dfrac{x+1}{x+2}$

9 **a** $\dfrac{b^2(a+1)}{a^2(a-b)}$ **b** $-\dfrac{a}{a+2}$ **c** $\dfrac{m}{m+3}$
d $\dfrac{(3x-2)(x-1)}{(x-2)(x+1)}$ **e** $\dfrac{4x-3}{x}$ **f** 1
g $\dfrac{(x+1)(x+2)}{x(x-2)}$

10 **a** $\dfrac{2x}{5}$ **b** $\dfrac{5x}{12}$ **c** $\dfrac{7x}{12}$
d $\dfrac{2(2x+5)}{(x+3)(x+1)}$ **e** $\dfrac{3(3x+1)}{(x+2)(x-1)}$
f $\dfrac{14x-3}{(x-2)(3x-1)}$ **g** $\dfrac{5x+4}{(x+4)(x-4)}$
h $\dfrac{2(2x-5)}{(x-3)^2}$ **i** $\dfrac{4x-5}{(x-5)^2}$

11 **a** $\dfrac{3x-1}{(x-3)(x+1)(x-2)}$ **b** $\dfrac{3(2x+3)}{(x+3)(x+1)}$
c $\dfrac{7x+6}{(x+2)(x-1)(x-2)}$ **d** $\dfrac{3(3x-8)}{(x-4)(x+2)(x-2)}$
e $\dfrac{-x-13}{(x-3)(x+4)(x+7)}$ **f** $\dfrac{8(x-3)}{x(x+3)(x-4)}$

12 **a** $\dfrac{2ab-a^2+b^2}{(a-b)(a+b)}$ **b** $\dfrac{3x^2-7x+8}{(x-3)(x+3)}$
c $\dfrac{15p-3q}{(4p+q)(p-2p)}$ **d** $\dfrac{3x}{(x-5)(x+5)}$
e $\dfrac{x+3}{(x+4)(x-5)}$ **f** $\dfrac{2b}{a+b}$
g $\dfrac{-1}{(a-2)(a-4)}$

13 **a** $(x+2)^2-8$ **b** $(x-3)^2-2$
c $(x-4)^2-22$ **d** $\left(x+\dfrac{3}{2}\right)^2-\dfrac{13}{4}$

14 **a** $5(3x-2)(x+1)$ **b** $(3x+2)(4x+5)$
c $(3x+2)(4x-5)$ **d** $(3x-7)(3x-5)$
e $(3x-11)(x-5)$ **f** $(3x-5)(x+7)$

Challenge exercise

1 **a** 0 **b** 1
c $\dfrac{a^2+b^2-2}{(ab+1)(a+b)}$ **d** $\dfrac{2(b-c)}{b+c}$ **e** 0

2 **a** $x^2+4x+15=(x+2)^2+11\ge 11$ as $(x+2)^2\ge 0$
b $x^2+2x+15=(x+1)^2+14\ge 14$ as $(x+1)^2\ge 0$

3 **a** Let ℓ cm be the length of the rectangle.
$2\ell+2w=20,\ \ell=10-w,\ A=w\times\ell=w(10-w)$
b $A=25-(w-5)^2$ so $A\le 25$ since $(w-5)^2\ge 0$
c $w=5$ **d** the square

4 $x>0$ for $\sqrt{x}$ and $\dfrac{1}{\sqrt{x}}$ to both exist.
$\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^2\ge 0,\ x-2+\dfrac{1}{x}\ge 0,\ x+\dfrac{1}{x}\ge 2$

5 **a** $(a-b)^2\ge 0,\ a^2-2ab+b^2\ge 0,\ a^2+2ab+b^2\ge 4ab,$
$(a+b)^2\ge 4ab,\ a+b\ge 2\sqrt{ab},\ \dfrac{a+b}{2}\ge\sqrt{ab}$
b when $a=b$
c Let $A=\dfrac{a}{b}$ and $B=\dfrac{b}{a}$; then $\dfrac{A+B}{2}\ge\sqrt{AB}$,
$A+B\ge 2\sqrt{AB},\ \dfrac{a}{b}+\dfrac{b}{a}\ge 2\sqrt{\dfrac{a}{b}\times\dfrac{b}{a}},\ \dfrac{a}{b}+\dfrac{b}{a}\ge 2$
d From part **a**, $\dfrac{a+b}{2}\ge\sqrt{ab}$ and $\dfrac{A+B}{2}\ge\sqrt{AB}$
where $A=\dfrac{1}{a}$ and $B=\dfrac{1}{b}$, so $\dfrac{\frac{1}{a}+\frac{1}{b}}{2}\ge\sqrt{\dfrac{1}{ab}}$, so
$\dfrac{2}{\frac{1}{a}+\frac{1}{b}}\le\sqrt{ab}$

6 $(2x-1)^2\ge 0,\ 4x^2-4x+1\ge 0,\ 4x^2+1\ge 4x,$
$x+\dfrac{1}{4x}\ge 1$ for $x>0$

7 12 when $a=b=\sqrt{12}$

8 $(a+b)(a-b)^2\ge 0$ for a and b positive,
$a^3-a^2b-ab^2+b^3\ge 0,\ a^3+b^3\ge a^2b+ab^2$

9 **a** $x=4,\ y=0;\ x=-1,\ y=5;\ x=11,\ y=-1;$
$x=-2,\ y=12;\ x=-4,\ y=-16;\ x=-5,\ y=-9;$
$x=-10,\ y=-4;\ x=-17,\ y=-3$
b $x=9,\ y=6;\ x=-1,\ y=16;\ x=-13,\ y=4;$
$x=-3,\ y=-6$

10 $x=6,\ y=-1;\ x=-6;\ y=1$

11 0

12 **a** $(ab-cd)^2\ge 0,\ a^2b^2-2abcd+c^2d^2\ge 0,$
$2abcd\le a^2b^2+c^2d^2$
b Expand $(ac-bd)^2$, $(ad-bc)^2$ to obtain
$2abcd\le a^2c^2+b^2d^2$ and $2abcd\le a^2d^2+b^2c^2$.
Add the three inequalities to obtain the result.

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Chapter 16 answers

Exercise 16A

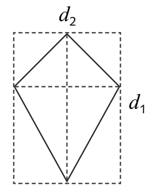
1 **a** 15 cm^2 **b** 35 cm^2 **c** 17.5 cm^2
d 10 cm^2 **e** 16.5 m^2 **f** 24 cm^2
g $9\pi \approx 28.27$ cm^2 **h** $16\pi \approx 50.27$ cm^2 **i** 10.5 cm^2
j 8 cm^2

2 **a** 17.5 cm^2 **b** 104 cm^2 **c** 225 mm^2

3 12.5 cm^2

4 $2A = h(b_1 + b_2)$. Therefore $A = \frac{h}{2}(b_1 + b_2)$

5 Area of rectangle = $d_1 \times d_2$
So area of kite = $\frac{1}{2} \times d_1 \times d_2$



6 **a** $100\pi \approx 314.16$ cm^2
b **i** $50\pi \approx 157.08$ cm^2 **ii** $25\pi \approx 78.54$ cm^2
iii $\frac{100}{3}\pi \approx 104.72$ cm^2 **iv** $75\pi \approx 235.62$ cm^2
v $\frac{25}{3}\pi \approx 26.81$ cm^2 **vi** $\frac{100}{3}\pi \approx 104.72$ cm^2
c $\frac{\pi r^2\theta}{360}$

7 **a** $\sqrt{\frac{12}{\pi}} \approx 1.95$ mm **b** $\sqrt{\frac{50}{\pi}} \approx 3.99$ cm
c 3 m **d** 5 cm

8 $3\sqrt{2}$ cm ≈ 4.24 cm

9 **a** 156 cm^2 **b** 280 cm^2
c 32.5 cm^2 **d** $8+2\pi \approx 14.28$ cm^2
e $240 + 36\pi \approx 353.10$ cm^2 **f** $25 + 6.25\pi \approx 44.63$ cm^2
g 192 cm^2 **h** $119 + 12.5\pi \approx 158.27$ cm^2
i 35 m^2 **j** $200 - 25\pi \approx 121.46$ cm^2
k 80 cm^2 **l** $96 + 32\pi \approx 196.53$ cm^2
m $24 + 12.5\pi \approx 63.27$ cm^2 **n** $25\sqrt{3} + 37.5\pi \approx 161.11$ cm^2

10 **a** 320 cm^2 **b** 24 cm^2
c $144 - 36\pi \approx 30.90$ cm^2 **d** $12.5\pi - 25 \approx 14.27$ cm^2
e $200 - 50\pi \approx 42.92$ cm^2 **f** $64\pi \approx 201.06$ cm^2
g $\frac{169\pi}{4} - 60 \approx 72.73$ cm^2 **h** $\frac{25\pi}{4} - 12 \approx 7.63$ cm^2
i $\frac{27\pi}{4} \approx 21.21$ cm^2 **j** $25\pi - 50 \approx 28.54$ cm^2

11 **a** 21.46 cm^2 **b** 21.59 cm

12 **a** $96\pi \approx 301.59$ cm^2 **b** $\sqrt{148} \approx 12.17$ cm

13 $x = 9\sqrt{2}$

14 $5\sqrt{2} \approx 7.07$ cm

15 **a** $72\pi \approx 226.19$ cm^2 **b** $r = \frac{10}{\sqrt{\pi}} \approx 5.64$ cm

Exercise 16B

1 **a** 150 cm^2 **b** 460 cm^2
c 424 cm^2 **d** 147.96 cm^2

2 637 cm^2

3 1292.8 cm^2

4 **a** 42.24 m^2 **b** 53.04 m^2

5 **a** 750 cm^2 **b** 532 cm^2 **c** 194 cm^2
d 264 cm^2 **e** 168 cm^2 **f** 532 cm^2

6 **a** 25.045 m **b** 19 113

Exercise 16C

1 **a** $192\pi \approx 603.19$ cm^2 **b** $294\pi \approx 923.63$ cm^2
c $300\pi \approx 942.48$ cm^2 **d** $96\pi \approx 301.59$ cm^2

2 800π cm^2 ≈ 2513.27 cm^2

3 **a** 90 cm^2 **b** $9\pi \approx 28.27$ cm^2
c $45\pi \approx 141.37$ cm^2
d $(90 + 54\pi)$ cm^2 ≈ 259.65 cm^2

4 $15\pi \approx 47.12$ m^2

5 $120\pi \approx 377.00$ cm^2

6 3.00 m

7 **a** $\sqrt{\frac{125}{\pi}} \approx 6.31$ cm **b** $\sqrt{\frac{250}{3\pi}} \approx 5.15$ cm

8 $36 + 81\pi \approx 290.47$ cm^2

Exercise 16D

1 **a** 120 cm^3 **b** 240 cm^3 **c** 180 cm^3
d 240 cm^3 **e** 640 cm^3 **f** 400 cm^3

2 **a** $V = a \times a^2 = a^3$ **b** $V = abc = ab \times c$
c $V = \pi a^2 \times b = \pi a^2 b$

3 **a** 840 cm^3 **b** 512 cm^3
c 288 cm^3 **d** 8000 cm^3
e 2436 cm^3 **f** 3000 cm^3
g 48 000 cm^3 **h** $7680\sqrt{3}$ mm^3 ≈ 13302.15 mm^3
i 120 cm^3 **j** 245 cm^3

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

4 a 30 cm^2 b $\frac{60}{17}\text{cm} \approx 3.53\text{ cm}$
c 2.5 m d $8.5\text{ mm} \times 8.5\text{ mm} \times 17\text{ mm}$

5 a $96\pi \approx 301.59\text{ cm}^3$ b $160\pi \approx 502.65\text{ cm}^3$
c $200\pi \approx 628.32\text{ cm}^3$ d $384\pi \approx 1206.37\text{ cm}^3$
e $2250\pi \approx 7068.58\text{ cm}^3$ f $24\pi \approx 75.40\text{ cm}^3$
g $74\pi \approx 232.48\text{ cm}^3$ h $30\pi \approx 94.25\text{ m}^3$

6 a $\frac{25}{2\pi} \approx 3.98\text{ cm}$ b $\sqrt{\frac{375}{8\pi}} \approx 3.86\text{ cm}$
c $\frac{10}{\sqrt[3]{\pi}} \approx 6.83\text{ cm}$ d $\sqrt[3]{\frac{2000}{\pi}} \approx 8.60\text{ cm}$

Exercise 16E

1 a 3 cm^2 b 31000 cm^2 c 500000 mm^2
d 60 mm^2 e 360000 m^2 f 0.28 m^2

2 a 111800 m^2 b 11.18 ha c 0.1118 km^2

3 200 m

4 26 000 ha

5 a 0.75 ha b 1.875 acres

6 a 22.32 m^2 b 1.86 L

7 a 5.76 cm^3 b $560\,000\text{ cm}^3$ c 620 L
d 2.6 L e 52 mL f 0.96 m^3

8 9000 L

9 a $4.5\pi \approx 14.1372\text{ m}^3$ b 14 137.2 L

10 330 minutes

11 10 dosages (0.47 of a dose left over)

12 a 2400 b 144 c 168 hours

Exercise 16F

1 a 2.3×10^{-2} m b 6.7×10^{-11} m
c 4.56×10^{-10} m d 2.5×10^{-5} m
e 9.3×10^{-8} m f 6.51×10^{-7} m

2 a $8.64 \times 10^{10}\,\mu\text{s}$ b $6.048 \times 10^{11}\,\mu\text{s}$
c $5.043 \times 10^{10}\,\mu\text{s}$ d $6.5 \times 10^{5}\,\mu\text{s}$

3 a 8.63×10^{13} ns b 6.048×10^{14} ns
c 5.0435×10^{13} ns d 6.72×10^{5} ns

4 1 m

5 7.6591×10^{11} L

6 2.4×10^{-4} s

7 a 10^{12} L b 1×10^{3} GL

8 2.01 ML (correct to 2 decimal places)

9 a 3.3 ns b 3.3 μs
c 303 mm (correct to 3 significant figures)

Review exercise

1 a 0.012 m b 2300 cm^2 c 55 ha
d 350 mm^3 e 0.84 L

2 a 23 m^2 b 2.16 m^2
c 10.5 m^2 d 51.75 m^2

3 a 200 mm b 3.2 m c $50\,000\text{ m}^2$
d 0.2 m^2 e 300 mm^2 f 3200 cm^3
g $500\,000\text{ cm}^3$ h 2000 L i 2.5 ha

4 a 40 cm^2 b $\left(\frac{9\pi}{2} + 30\right) \approx 44.14\text{ cm}^2$
c 12 cm^2 d 24 cm^2

5 a i $(24\sqrt{13} + 314) \approx 400.53\text{ cm}^2$ ii 660 cm^2
b i 456 cm^3 ii 600 cm^3

6 $(64 - 16\pi) \approx 13.73\text{ cm}^2$

7 a 32.16 m^2 b 2.5728 m^3 c \$192.96

8 $150\pi \approx 471.24\text{ cm}^2$

9 a 22.5 m^3 b 78 cm^3
c $75.712\pi \approx 237.86\text{ cm}^3$ d 525 cm^3

10 a $22\,500\text{ cm}^3$
b 2070 m^3

Challenge exercise

1 $306.25\pi \approx 962.11\text{ m}^2$

2 a 70 000 L b 2500 L

3 a $332.75\pi \approx 1045.365\text{ cm}^3$ b $174.25\pi \approx 547.42\text{ cm}^3$
c $210.25\pi \approx 660.52\text{ cm}^2$

4 a $\frac{12.5}{\pi} \approx 3.98$ m b 11.96 m c 100 m^2

5 a $528 = \sqrt{66 \times 48 \times 88}$
b i 20, 30, 10 ii 6000

8 $1 : \frac{\pi}{2} - 1$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Chapter 17 answers

Exercise 17A

1 **a** $x = 0$ or $x = 2$ **b** $x = -1$ or $x = 1$
c $x = 0$ or $x = -1$ **d** $x = \frac{1}{3}$ or $x = -\frac{5}{2}$
e $x = 1$ or $x = \frac{3}{2}$ **f** $x = -1$
g $x = 0$ or $x = 3$ **h** $x = 3$ or $x = 4$
i $x = 0$ or $x = -5$ **j** $x = \frac{2}{3}$ or $x = -2$
k $x = -2$ or $x = 3$ **l** $x = 3$

2 **a** $x = -1$ or $x = -2$ **b** $x = -3$ or $x = -4$
c $y = 2$ or $y = 5$ **d** $x = -15$ or $x = 2$
e $b = -1$ or $b = 14$ **f** $z = -3$ or $z = 30$
g $x = -1$ or $x = -4$ **h** $x = 1$ or $x = 18$
i $x = -4$ or $x = 3$ **j** $x = 2$ or $x = 14$
k $x = -5$ or $x = 2$ **l** $x = -10$ or $x = 9$

3 **a** $x = 0$ or $x = 5$ **b** $x = 0$ or $x = -7$
c $x = 0$ or $x = 2$ **d** $x = 0$ or $x = \frac{1}{5}$
e $x = 0$ or $x = -\frac{5}{2}$ **f** $x = 0$ or $x = -4$
g $x = 0$ or $x = 6$ **h** $x = 0$ or $x = -3$
i $x = 0$ or $x = \frac{3}{2}$ **j** $x = 0$ or $x = \frac{1}{3}$
k $x = 0$ or $x = 2$ **l** $x = 0$ or $x = \frac{1}{7}$

4 **a** $x = 0$ or $x = 2$ **b** $x = 0$ or $x = -\frac{5}{2}$
c $x = 0$ or $x = -3$ **d** $x = 0$ or $x = \frac{3}{2}$
e $x = 0$ or $x = \frac{1}{3}$ **f** $a = -3$ or $a = -2$
g $r = 3$ or $r = -2$ **h** $c = -6$ or $c = -2$
i $a = 8$ **j** $n = 5$ or $n = 6$
k $s = 14$ or $s = -5$ **l** $x = -1$ or $x = 25$
m $x = -7$ or $x = 6$ **n** $x = 8$ or $x = -3$
o $x = 14$ or $x = 2$

5 **a** $x = -1$ or $x = 1$ **b** $x = -5$ or $x = 5$
c $x = -\frac{1}{2}$ or $x = \frac{1}{2}$ **d** $x = -5$ or $x = 5$
e $x = -3$ **f** $x = -5$
g $x = 6$ **h** $x = -4$ or $x = 4$
i $x = \frac{5}{4}$ or $x = \frac{-5}{4}$ **j** $x = -3$ or $x = 3$
k $y = -6$ or $y = 6$ **l** $x = 2$
m $x = 1$ **n** $x = -3$ or $x = 9$
o $x = -3$ or $x = 11$

Exercise 17B

1 **a** $x = -2$ **b** $x = 2$ or $x = -2$
c $x = -2$ or $x = 2$ **d** $x = -6$ or $x = 5$

2 **a** $x = -\frac{1}{2}$ or $x = 3$ **b** $x = \frac{1}{2}$ or $x = -\frac{2}{3}$
c $x = \frac{1}{4}$ or $x = \frac{3}{2}$ **d** $x = \frac{1}{3}$ or $x = -2$
e $x = -\frac{2}{3}$ or $x = \frac{5}{2}$ **f** $x = \frac{2}{5}$ or $x = -\frac{3}{2}$
g $x = -\frac{1}{3}$ or $x = -\frac{1}{2}$ **h** $x = -\frac{5}{6}$ or $x = 2$
i $x = \frac{1}{4}$ or $x = -\frac{2}{3}$ **j** $x = -\frac{3}{5}$ or $x = -\frac{5}{2}$
k $x = -\frac{5}{6}$ or $x = \frac{3}{2}$ **l** $x = -\frac{1}{5}$ or $x = -\frac{2}{3}$

3 **a** $x = -\frac{5}{7}$ or $x = 3$ **b** $x = -2$ or $x = \frac{1}{2}$
c $x = -\frac{1}{3}$ or $x = \frac{3}{2}$ **d** $x = -\frac{3}{2}$ or $x = \frac{2}{3}$
e $x = \frac{1}{7}$ or $x = 11$ **f** $x = -\frac{1}{4}$ or $x = 1$
g $x = -\frac{1}{3}$ or $x = 3$ **h** $x = -\frac{12}{5}$ or $x = 7$
i $x = -\frac{3}{2}$ or $x = \frac{5}{2}$ **j** $x = -\frac{1}{2}$ or $x = \frac{5}{6}$
k $x = -\frac{2}{3}$ or $x = 2$ **l** $x = -1$ or $x = 4$

4 **a** $x = \frac{5}{9}$ or $x = -1$ **b** $x = -\frac{12}{5}$ or $x = 1$
c $x = \frac{1}{5}$ or $x = 2$ **d** $a = \frac{7}{2}$ or $a = -\frac{1}{2}$
e $a = \frac{7}{2}$ or $a = -\frac{3}{2}$ **f** $a = \frac{25}{3}$ or $a = 1$
g $x = -\frac{6}{5}$ or $x = -4$ **h** $x = \frac{2}{3}$ or $x = -\frac{5}{2}$
i $x = -\frac{4}{3}$ or $x = 15$ **j** $x = \frac{3}{2}$ or $x = -1$

Exercise 17C

1 **a** $x = 2$ or $x = 4$ **b** $x = 2$ or $x = 5$
c $x = 1$ or $x = 4$ **d** $x = 3$ or $x = 5$
e $x = -1$ or $x = 4$ **f** $x = -3$ or $x = 9$
g $x = 0$ or $x = 7$ **h** $x = -5$ or $x = 0$
i $x = -7$ or $x = 11$ **j** $x = 6$ or $x = -3$

2 **a** $x = -7$ or $x = 2$ **b** $x = -3$ or $x = 5$
c $x = -3$ or $x = 2$ **d** $x = 1$ or $x = 6$
e $x = 2$ or $x = 16$ **f** $x = -6$ or $x = 5$
g $x = -5$ or $x = 4$ **h** $x = -2$ or $x = 7$
i $x = 4$ or $x = 6$ **j** $x = -\frac{3}{5}$ or $x = 2$
k $x = -4$ or $x = -3$ **l** $x = \frac{2}{3}$ or $x = \frac{3}{2}$
m $x = -2$ **n** $x = 1$ or $x = 9$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

3 **a** $x = 0$ or $x = 7$ **b** $x = \frac{1}{2}$ or $x = \frac{7}{2}$

c $x = 0$ or $x = -1$ **d** $x = -14$ or $x = -3$

e $x = 7$ or $x = -\frac{1}{3}$

4 **a** $x = -6$ or $x = 2$ **b** $x = -2$ or $x = 5$

c $x = -5$ or $x = 7$ **d** $x = -9$ or $x = 2$

e $x = -1$ or $x = -5$ **f** $x = -6$ or $x = 3$

g $x = -\frac{13}{2}$ or $x = 1$ **h** $x = -\frac{5}{2}$ or $x = 2$

i $x = -5$ or $x = 1$ **j** $x = -1$ or $x = 2$

k $x = -\frac{5}{2}$ or $x = 2$ **l** $x = \frac{4}{3}$ or $x = -4$

5 **a** $x = -5$ or $x = 6$ **b** $x = -6$

c $x = 6$ or $x = 16$ **d** $x = 1$ or $x = 7$

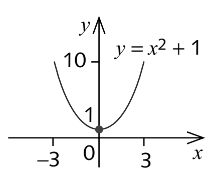
e $x = -1$ or $x = -\frac{3}{4}$ **f** $x = 0$ or $x = \frac{7}{2}$

Exercise 17D

1	10	**2**	12	**3**	1 or -10
4	4 or -4	**5**	$x = 4$	**6**	8
7	10 cm	**8**	$x = 3$	**9**	$V = 12$
10	20 km/h	**11**	6 km/h	**12**	3 hours

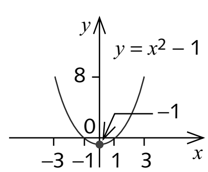
Exercise 17E

1 **a**



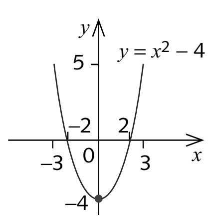
Vertex $(0, 1)$

b



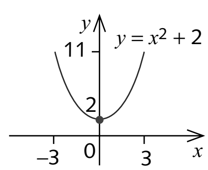
Vertex $(0, -1)$

c



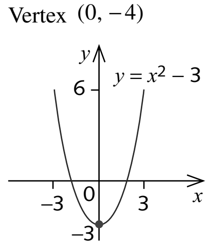
Vertex $(0, -4)$

d



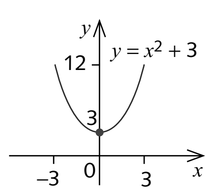
Vertex $(0, 2)$

e



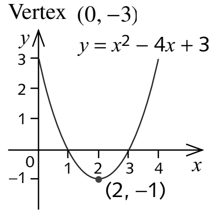
Vertex $(0, -3)$

f

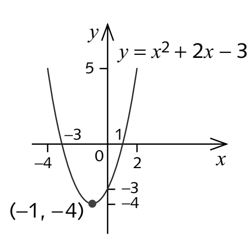


Vertex $(0, 3)$

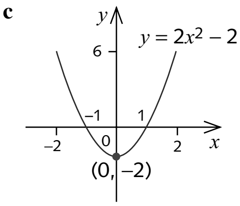
2 **a**



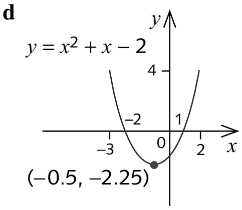
b



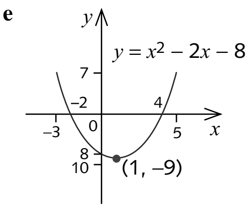
c



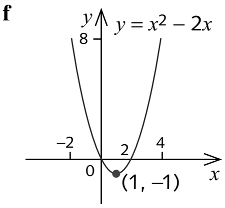
d



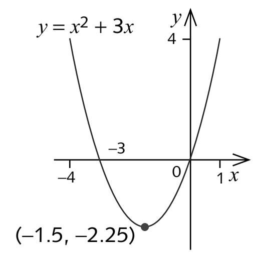
e



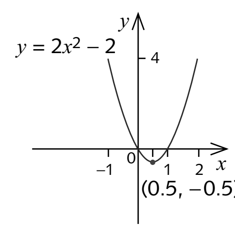
f



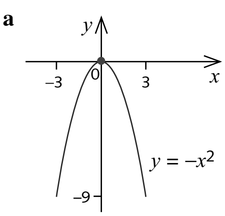
g



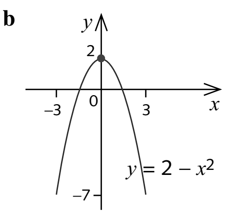
h



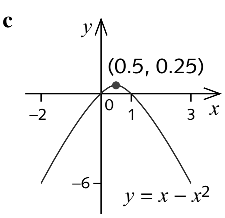
3 **a**



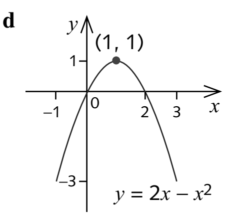
b



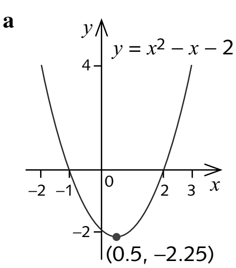
c



d



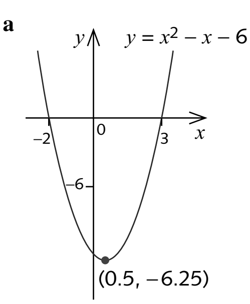
4 **a**



b $x = -1$ or $x = 2$

c The solutions to $x^2 - x - 2 = 0$ are where the graph crosses the x-axis as this is where $y = 0$.

5 **a**

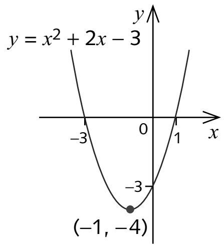


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b $x = 3$ or $x = -2$

c The solutions to $x^2 - x - 6 = 0$ are where the graph crosses the x-axis as this is where $y = 0$.

6 **a**



b $x = -3$ or $x = 1$

c The solutions to $x^2 + 2x - 3 = 0$ are where the graph crosses the x-axis as this is where $y = 0$.

Exercise 17F

1 **a** $x = \sqrt{5}$ or $x = -\sqrt{5}$ **b** $x = \sqrt{11}$ or $x = -\sqrt{11}$

c $x = 2\sqrt{3}$ or $x = -2\sqrt{3}$ **d** $x = \sqrt{3}$ or $x = -\sqrt{3}$

e $x = \sqrt{10}$ or $x = -\sqrt{10}$ **f** $x = \sqrt{5}$ or $x = -\sqrt{5}$

g $x = 2 + \sqrt{5}$ or $x = 2 - \sqrt{5}$

h $x = -3 + \sqrt{6}$ or $x = -3 - \sqrt{6}$

i $x = -2 + \sqrt{8}$ or $x = -2 + \sqrt{8}$
$(x = -2 + 2\sqrt{2}$ or $x = -2 - 2\sqrt{2})$

j $x = -\dfrac{1}{2}$ or $x = -\dfrac{13}{2}$

k $x = 6$ or $x = -2$

l $x = -4 + \sqrt{7}$ or $x =- 4\sqrt{7}$

m $x = \dfrac{11 + \sqrt{7}}{2}$ or $x = \dfrac{11 - \sqrt{7}}{2}$

n $x = \dfrac{1 - \sqrt{11}}{2}$ or $x = \dfrac{1 + \sqrt{11}}{2}$

o $x = \dfrac{-15}{4} + \dfrac{\sqrt{7}}{2}$ or $x = \dfrac{-15}{4} - \dfrac{\sqrt{7}}{2}$

2 **a** $x = -\sqrt{2} - 1$ or $x = \sqrt{2} - 1$

b $x = -\sqrt{3} - 2$ or $x = \sqrt{3} - 2$

c $x = 6 - \sqrt{13}$ or $x = 6 + \sqrt{13}$

d $x = -\sqrt{2} - 3$ or $x = \sqrt{2} - 3$

e $x = 4 - \sqrt{17}$ or $x = 4 + \sqrt{17}$

f $x = -\sqrt{13} - 5$ or $x = \sqrt{13} - 5$

g $x = -4 + \sqrt{10}$ or $x = -4 - \sqrt{10}$

h $x = -3 - \sqrt{10}$ or $x = -3 + \sqrt{10}$

i $x = -3 - 2\sqrt{2}$ or $x = -3 + 2\sqrt{2}$

j $x = -4 - 3\sqrt{2}$ or $x = -4 + 3\sqrt{2}$

k $x = -4 - \sqrt{14}$ or $x = -4 + \sqrt{14}$

l $x = -10 - 2\sqrt{30}$ or $x = -10 + 2\sqrt{30}$

3 **a** $x = \dfrac{-\sqrt{5} - 1}{2}$ or $x = \dfrac{\sqrt{5} - 1}{2}$

b $x = \dfrac{3 - \sqrt{5}}{2}$ or $x = \dfrac{3 + \sqrt{5}}{2}$

c $x = \dfrac{5 - \sqrt{29}}{2}$ or $x = \dfrac{5 + \sqrt{29}}{2}$

d $x = \dfrac{-\sqrt{17} - 3}{2}$ or $x = \dfrac{\sqrt{17} - 3}{2}$

e $x = \dfrac{-\sqrt{21} - 5}{2}$ or $x = \dfrac{\sqrt{21} - 5}{2}$

f $x = 3 - \sqrt{7}$ or $x = 3 + \sqrt{7}$

g $x = -\sqrt{2} - 5$ or $x = \sqrt{2} - 5$

h $x = \dfrac{-9 - \sqrt{65}}{2}$ or $x = \dfrac{-9 + \sqrt{65}}{2}$

i $x = \dfrac{-3 - \sqrt{53}}{2}$ or $x = \dfrac{-3 + \sqrt{53}}{2}$

j $x = -10$ or $x = -1$

k $x = -5$ or $x = -2$

l $x = \dfrac{-7 - \sqrt{69}}{2}$ or $x = \dfrac{-7 + \sqrt{69}}{2}$

4 **a** $x^2 + 8x + 18 = (x + 4)^2 + 2 \geq 2$ for all x

b $x^2 - 4x + 6 = (x - 2)^2 + 2 \geq 2$ for all x

c $x^2 + 2x + 4 = (x + 1)^2 + 3 \geq 3$ for all x

d $x^2 + x + 2 = \left(x + \dfrac{1}{2}\right)^2 + \dfrac{7}{4} \geq \dfrac{7}{4}$ for all x

e $x^2 + 2x + 2 = (x + 1)^2 + 1 \geq 1$ for all x

f $x^2 + 4x + 5 = (x + 2)^2 + 1 \geq 1$ for all x

Exercise 17G

1 **a** $(0, -1), x = 0$ **b** $(1, 0), x = 1$

c $(-2, -1), x = -2$ **d** $(0, 4), x = 0$

e $(-3, 0), x = -3$ **f** $(-1, 2), x = -1$

g $(2, -2), x = 2$ **h** $(1, 3), x = 1$

i $(3, -2), x = 3$

2 **a** $y = (x - 2)^2 - 2$ **b** $y = (x + 4)^2 + 5$

c $y = (x - 3)^2 - 24$ **d** $y = \left(x + \dfrac{5}{2}\right)^2 - \dfrac{29}{4}$

e $y = \left(x - \dfrac{3}{2}\right)^2 + \dfrac{11}{4}$ **f** $y = \left(x - \dfrac{7}{2}\right)^2 - \dfrac{89}{4}$

3 **a** $x = 2$ or -2 **b** $x = \sqrt{11}$ or $-\sqrt{11}$

c $x = 1$ or -3 **d** no solution

e $x = 1$ or -5

f $x = 3 + \sqrt{5}$ or $3 - \sqrt{5}$

g $x = -1 + 2\sqrt{3}$ or $-1 - 2\sqrt{3}$

h no solution

i $x = -4 - 3\sqrt{2}$ or $-4 + 3\sqrt{2}$

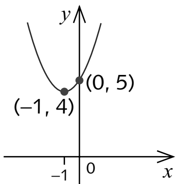
j no solution **k** $x = -2$ or 8 **l** $x = 1$ or 6

m $x = \dfrac{-3 + \sqrt{7}}{2}$ or $\dfrac{-3 - \sqrt{7}}{2}$

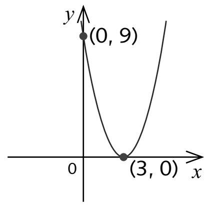
n $x = \dfrac{5 + 2\sqrt{10}}{3}$ or $\dfrac{5 - 2\sqrt{10}}{2}$ **o** no solution

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

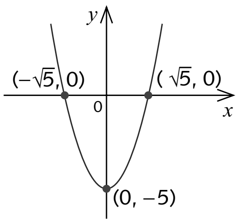
4 **a**



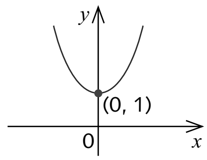
b



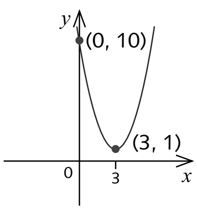
c



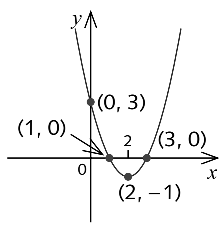
d



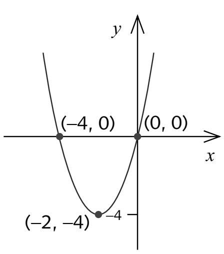
e



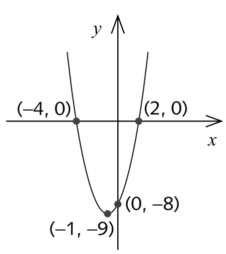
f



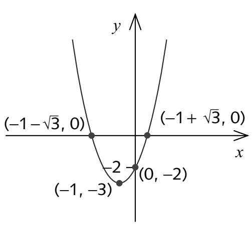
g



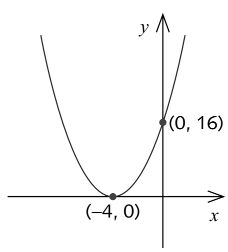
h



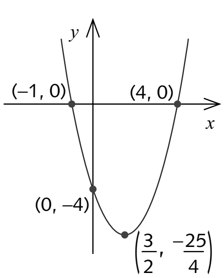
i



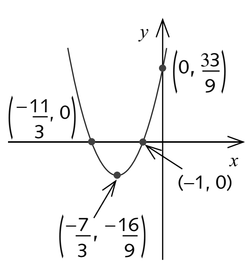
j



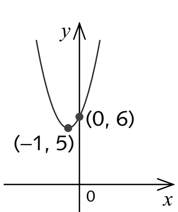
k



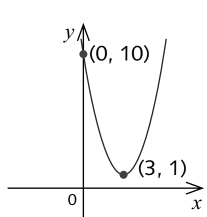
l



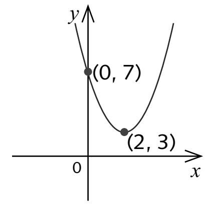
5 **a**



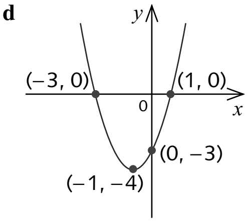
b



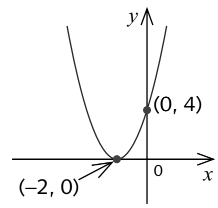
c



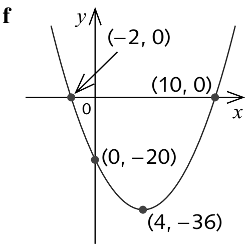
d



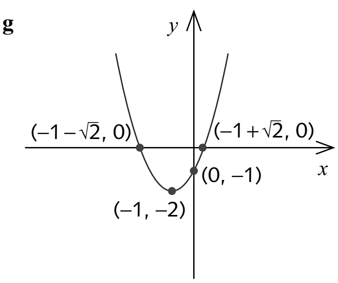
e



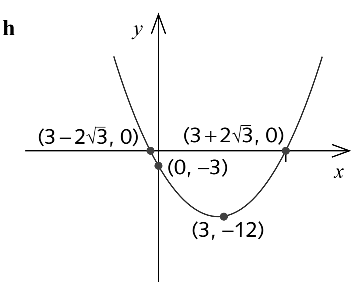
f



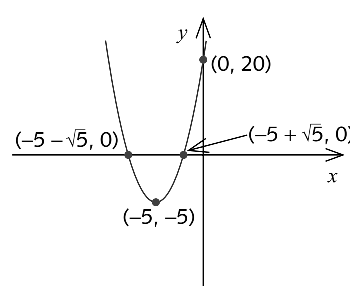
g



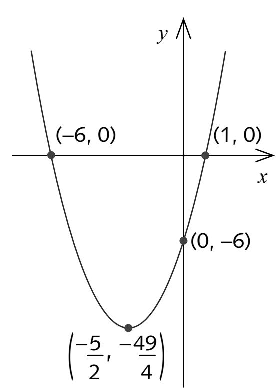
h



i

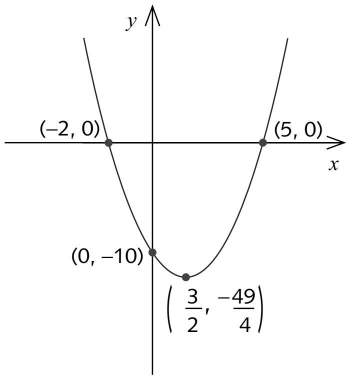


j

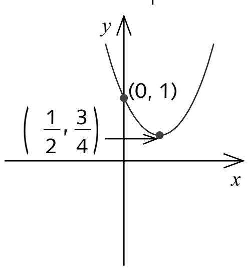


ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

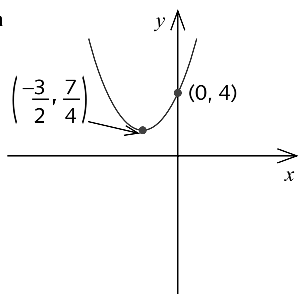
k



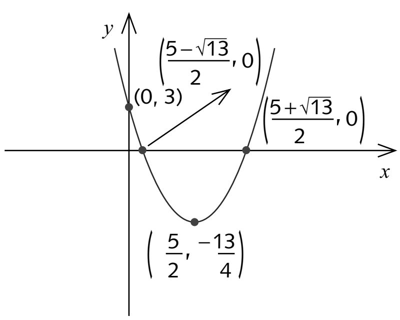
l



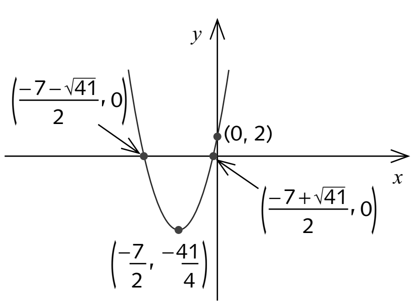
m



n



o



Review exercise

1 **a** $x = -3$ or $x = 5$ **b** $x = 5$ or $x = 0$

c $x = -7$ or $x = 0$ **d** $x = 4$ or $x = 8$

e $x = -13$ or $x = -\frac{8}{3}$ **f** $x = \frac{7}{2}$ or $x = \frac{7}{11}$

2 **a** $x = 1$ or $x = 2$ **b** $x = 3$ or $x = 4$

c $x = 1$ or $x = 10$ **d** $x = -30$ or $x = 1$

e $x = -2$ or $x = 7$ **f** $x = -9$ or $x = 10$

g $x = -3$ or $x = 8$ **h** $x = 2$ or $x = 9$

i $x = -3$ or $x = 4$ **j** $x = 4$ or $x = 7$

k $x = -10$ or $x = 1$ **l** $x = -11$ or $x = 10$

3 **a** $x = -9$ **b** $x = -\frac{4}{3}$ or $x = \frac{4}{3}$

c $x = 4$ or $x = 0$ **d** $x = 6$ or $x = -2$

e $x = 4$ **f** $f = \frac{1}{3}$ or $f = \frac{11}{3}$

g $y = -\frac{3}{2}$ or $y = -\frac{7}{6}$ **h** $x = 2$ or $x = -2$

i $x = -8$ or $x = 8$

4 **a** $d = \sqrt{2}$ or $d = -\sqrt{2}$ **b** $y = \sqrt{\frac{15}{2}}$ or $y = -\sqrt{\frac{15}{2}}$

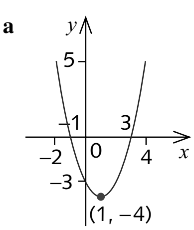
c $x = 10 + 2\sqrt{5}$ or $x = 10 - 2\sqrt{5}$

d $y = 4 + \sqrt{13}$ or $y = 4 - \sqrt{13}$

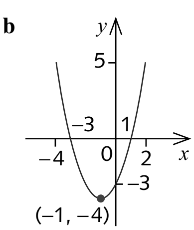
e $m = \frac{1+\sqrt{5}}{2}$ or $m = \frac{1-\sqrt{5}}{2}$

f $n = \frac{3+\sqrt{21}}{2}$ or $n = \frac{3-\sqrt{21}}{2}$

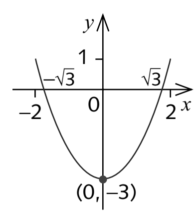
5 **a**



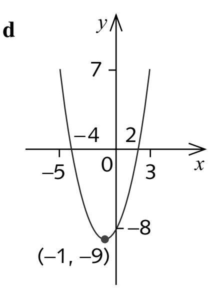
b



c



d



6 **a** $x = -\frac{1}{3}$ or $x = 3$

b $x = \frac{2}{3}$ or $x = \frac{3}{2}$

c $x = -\frac{10}{3}$ or $x = 3$

7 **a** $x = -\sqrt{11} - 3$ or $x = \sqrt{11} - 3$

b $x = 2 - 2\sqrt{2}$ or $x = 2 + 2\sqrt{2}$

c $x = 5 + \sqrt{5}$ or $x = 5 - \sqrt{5}$

d $x = \frac{-\sqrt{13} - 5}{2}$ or $x = \frac{\sqrt{13} - 5}{2}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

e $x = \dfrac{7+\sqrt{29}}{2}$ or $x = \dfrac{7-\sqrt{29}}{2}$

f $x = \dfrac{-\sqrt{37}-3}{2}$ or $x = \dfrac{\sqrt{37}-3}{2}$

8 a $x = -8$ or $x = 5$ b $x = -5$ or $x = 8$

c $x = -6$ or $x = 2$ d $x = -1$ or $x = -3$

e $x = 8$ or $x = 4$

f $x = 7 - \sqrt{17}$ or $x = 7 + \sqrt{17}$

g $x = -5$ or $x = -3$

h $x = -4 - \sqrt{2}$ or $x = -4 + \sqrt{2}$

i $x = 6 - 3\sqrt{7}$ or $x = 6 + 3\sqrt{7}$

j $x = 6 - 2\sqrt{14}$ or $x = 6 + 2\sqrt{14}$

k $x = \dfrac{-7-\sqrt{29}}{2}$ or $x = \dfrac{-7+\sqrt{29}}{2}$

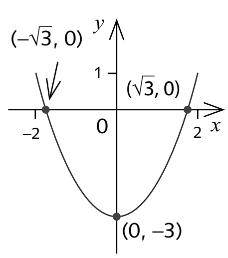
l $x = \dfrac{-5-\sqrt{65}}{2}$ or $x = \dfrac{-5+\sqrt{65}}{2}$

9 a $x = 0$ or $x = 11$ b $x = -\dfrac{1}{2}$ or $x = 3$

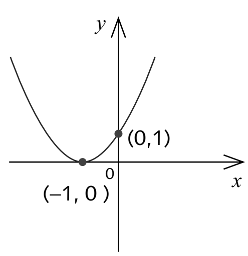
c $x = 6$ or $x = -\dfrac{22}{5}$ d $x = 4$ or $x = \dfrac{4}{3}$

10

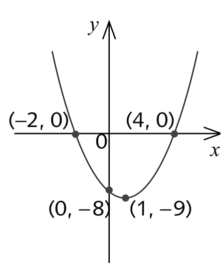
a



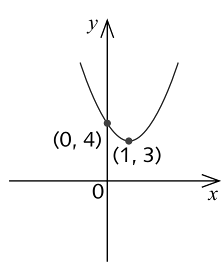
b



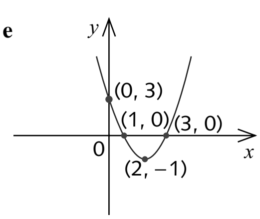
c



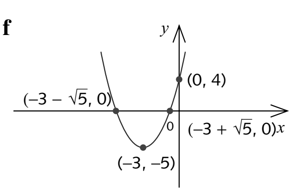
d



e



f



11 7, 5

12 90 m, 160 m

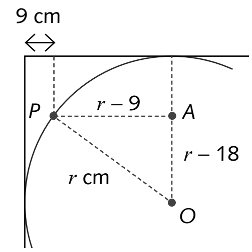
13 $x = 5$ **14** 5, 12

15 4 m

Challenge exercise

1 a $x = -\dfrac{8}{9}$

b $x = 2$ (x cannot equal $\dfrac{1}{2}$)



2 a $AO = (r - 18)$ cm, $AP = (r - 9)$ cm, $OP = r$ cm

Use Pythagoras' theorem in triangle OAP.

b 45 cm c $45\sqrt{2}$ cm

3 cube side length = 13 cm, 11 cm × 10 cm × 9 cm

4 a $\dfrac{1}{2}(48x - 9x^2)\ \text{m}^2$

b 10 m × 3 m or 6 m × 5 m

5 a $2ax + 2bx + 4x^2$

b i $4x^2 + 156x$ ii $x = 1$

6 a i 2 ii 5 iii 9 iv 35 v 170

c 30 d 64-gon

7 a i 1 ii 3 iii 6 iv 45 c 20

8 9 cm, 40 cm

Chapter 18 answers

Exercise 18A

1 3750 m **2** 84 m

3 a $\dfrac{125}{9} = 13\frac{8}{9}$ m/s b 36 km/h c 34.2 km/h

4 a 640 km/h b 100.8 km/h c $\dfrac{35}{22} = 1\frac{13}{22}$ m/s

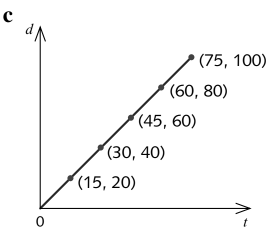
5 a $d = 80t$ b gradient = 80

6 a $\dfrac{4}{3}$ km/min

b

***t* (min)**	15	30	45	60	75
***d* (km)**	20	40	60	80	100

c



d $d = \dfrac{4t}{3}$, $0 \le t \le 75$ e $\dfrac{176}{3}$ km $= 58\frac{2}{3}$ km

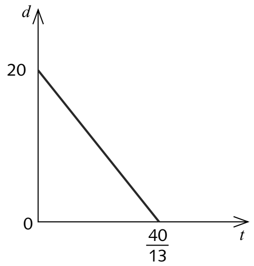
7 a 60 L b 5 L/h

c $V = 60 - 5t$, $0 \le t \le 12$ d 25 L

e i $V = 120 - 6t$, $0 \le t \le 20$ ii 20 hours

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

8 a



b Gradient = -6.5 km/h; vertical axis intercept = 20.

c $d = 20 - 6.5t$, $0 \le t \le 3\frac{1}{13}$

d No. He arrives at roughly 6:05 p.m.

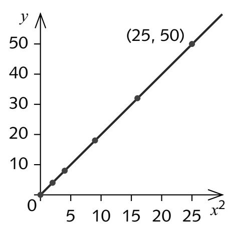
Exercise 18B

1 a $a = 2b$ b $m = 3.2n$

2 a

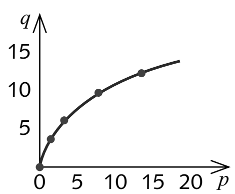
x^2	0	1	4	9	16	25
y	0	2	8	18	32	50

b



c Gradient of the graph of y versus x^2 is 2. d $y = 2x^2$

3 a



b

$\sqrt{p}$	0	1	2	3	4
q	0	3	6	9	12

$\frac{q}{\sqrt{p}} = 3$, for each pair (p, q)

c $q = 3\sqrt{p}$

4 a $d \propto t$ b $V \propto r^3$ c $d \propto \sqrt{h}$

5 a P varies directly as Q.
b ℓ varies directly as the square of m.
c The square of a is directly proportional to the square root of b.
d The cube of p varies directly as the square of ℓ.

6 a $p = 6q$ i 24 ii 4.5
b $m = 2.5n^2$ i 62.5 ii $\sqrt{4.8} \approx 2.191$

7 a $R = 2s$ b $a = 1.5\sqrt{b}$ c $V = 8r^3$

8 a $y = 7x$

x	0	1	2	3
y	0	7	14	21

b $y = \frac{x}{2}$

x	2	8	12	18
y	1	4	6	9

c $y = 12x$

x	2	3	6	15
y	24	36	72	180

d $y = \frac{19x}{6}$

x	2	3	6	15
y	$\frac{19}{3}$	9.5	19	47.5

9 104 km 10 237 m^2 11 333 kg 12 1852 kW

13 38.4 m

14 a y is multiplied by 4.
b y is multiplied by 16.
c y is divided by 25.

15 a The surface area is multiplied 4.
b The radius is multiplied by $\sqrt{2}$.

16 a m is multiplied by 16.
b m is multiplied by $\frac{1}{16}$.
c n is doubled.
d n is divided by $\sqrt{2}$.

17 a a is increased by 10%.
b a is decreased by approximately 6.19%.

18 a p is increased by about 3.23%.
b p is decreased by about 3.45%.
c q is increased by 72.8%.
d q is decreased by 48.8%.

Review exercise

1 8750 m 2 90 m

3 a $\frac{200}{9}$ m/s b 90 km/h

4 a 750 km/h b 93.75 km/h

5 a $d = 95t$ b gradient = 95

6 a x is directly proportional to y.
b p is directly proportional to the square of n.
c a is directly proportional to the square root of b.
d p is directly proportional to the cube of q.

7 a i $p = 48$ ii $q = \frac{81}{8}$
b i $a = \frac{125}{4}$ ii $a = 180$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Updated August 2021

8 **a** $k = 12, y = 12x$

x	0	1	2	3
y	0	12	24	36

b $k = 1.5, y = 1.5x$

x	2	8	12	18
y	3	12	18	27

9 **a** y is multiplied by 8. **b** y is multiplied by 27. **c** y is divided by 64.

10 **a** a is increased by 10.25%. **b** a is decreased by 15.36%.

Challenge exercise

1 $a = kc$ and $b = Kc$

$a + b = (k + K)c$ so $(a + b) \propto c$

$a - b = (k - K)c$ so $(a - b) \propto c$

$\sqrt{ab} = \sqrt{kc \times Kc} = \sqrt{kK}c$ so $\sqrt{ab} \propto c$

2 $y = \dfrac{150x}{19} + \dfrac{1080}{19x^2}$

3 $y = \dfrac{12}{5}$

4 $\dfrac{2abC}{(a+b)^2}$

5 $a + b = k(a - b)$

$a^2 + 2ab + b^2 = k^2(a^2 - 2ab + b^2)$

$2abk^2 + 2ab = k^2a^2 - a^2 + k^2b^2 - b^2$

$2ab(k^2 + 1) = (k^2 - 1)(a^2 + b^2)$

$a^2 + b^2 = \dfrac{k^2 + 1}{k^2 - 1} \times 2ab$

$a^2 + b^2 \propto ab$

6 3:20 p.m.

7 **a i** 50 km **ii** 100 km
b i 50 km/h **ii** 100 km/h
c 50 km **d** 3:00 p.m.

8 640 km/h and 480 km/h

9 **a** 48 km **b** 16 km/h
c 12 km/h **d** $\dfrac{16}{7}$ h

Chapter 19 answers

Exercise 19A

1 **a**

1	5
2	1 4 4 5 5 6 8 9 9
3	3 5 5 6 7 7 7 9
4	2 3 4 5 6 7 8

3 | 5 means 35

b 20–29

2 **a**

7	6 6 7 8 9
8	1 2 2 4 4 5 6 6 6 7 9
9	1 1 3 3 4 4 4 5 6 6 7
10	2 4 6

8 | 1 means 81 kg

b i 19 **ii** 16 **iii** 18

3 **a** 21
b i 8 **ii** 7 **iii** 9
c 46 minutes

4 **a** 26
b i 9 **ii** 3 **c** $19\frac{3}{13}\%$

5 **a i** 75% **ii** 100%
b i 25% **ii** 0%
c i 50% **ii** 6.25%

6 **a**

Club 2		Club 1
	1	1 2 3 6
	2	3 4 5
9 8 7	3	4
3 2	4	
7 5	5	
9 8 8 7 6 6 6 5 5	6	5 7 7 9
3 2 2 2 1	7	2 3 3 3 3 8 8 8
	8	2 2

b Club 1 has teenage members and members in their twenties; while, for club 2, the youngest member is aged 37, with most members being in their sixties. Club 1 has two members aged 82 and 8 members in their seventies, whereas club 2 has 5 members in their seventies.

7 **a**

Blackarm		Yellowlegs
	0	0 4
8 8	0	9 9 8 8 7 6 5
0 0 1 1 2 2 2 2 3 3 4	1	4 3 3 0 0
5 6 6 7 8 9 9	1	9 9 8 7 6 6 5
0 1	2	
6	2	5

b i Yellowlegs had 9 team members who had less than 10 kicks, whereas the Blackarms only had two.
ii The teams had the same number of players who had 15 or more kicks.

8 **a i** 37.5% **ii** 27.5%
b i 74 kg and 73 kg **ii** 51 kg and 51 kg
c i 57 kg **ii** 43 kg

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

Exercise 19B

1 **a**

Runs	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	7	6	3	2	3	2	2	3	0	2

b **i** 9 **ii** 7 **iii** 28 **iv** 10

2 **a**

Time	11.0–11.4	11.5–11.9	12.0–12.4	12.5–12.9	13.0–13.4	13.5–13.9	14.0–14.4	14.5–14.9
Frequency	1	6	1	7	3	5	4	2

b

Time	11.0–11.9	12.0–12.9	13.0–13.9	14.0–14.9
Frequency	3	8	8	6

3 **a** 20–29, 30–39, 40–49, 50–59, 60–69
b 20–24, 25–29, 30–34, 35–39, 40–44, 45–49, 50–54, 55–59, 60–64

4 250 000–299 999, 300 000–349 999, 350 000–399 999, 400 000–449 999
450 000–499 999, 500 000–549 999, 550 000–599 999, 600 000–649 999

5 **a** **i** 2 **ii** 14 **iii** 20 **iv** 12
b because 25 is in the middle of a class interval

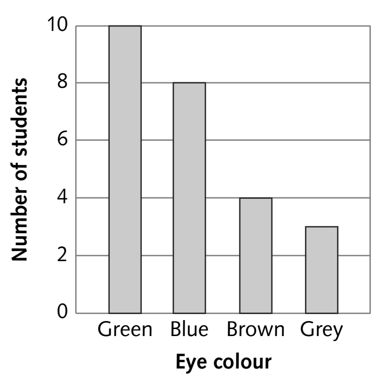
6 **a**

Length (mm)	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64
Frequency	1	5	4	8	10	7	3	2

b 30–34 **c** 30–34

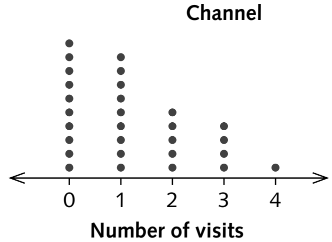
Exercise 19C

1

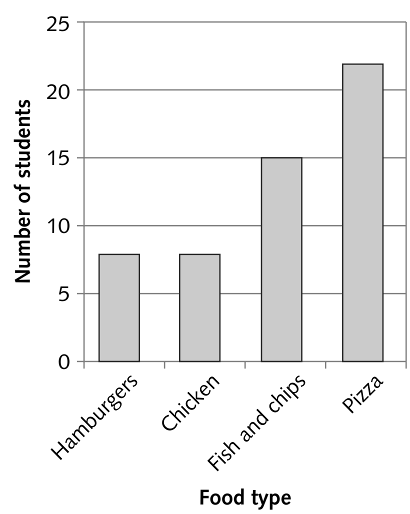


2

3



4

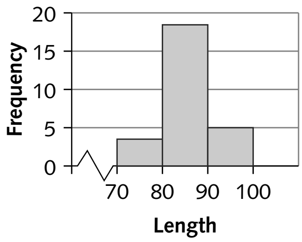


5

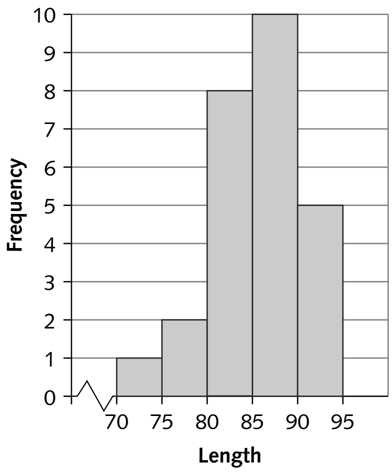
6

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

7 **a**

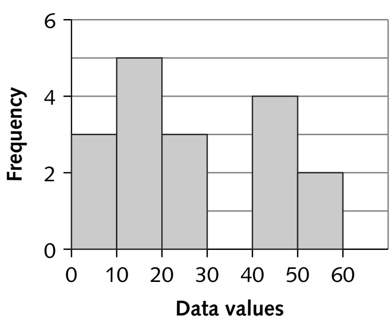


b



8 **a** 180 **b** 120 **c** $16\frac{2}{3}\%$ **d** $61\frac{1}{9}\%$

9 **a**

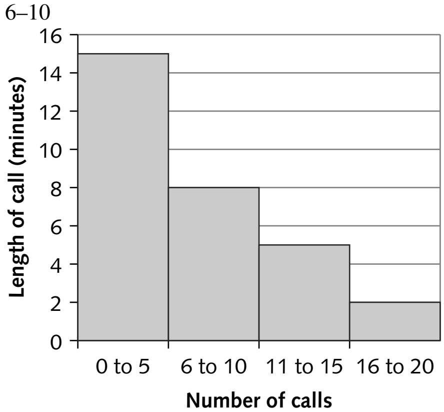


b because we don't know the individual values

10 **a** 38
b **i** 20 **ii** 8
c because we do not know any of the actual values
d **i** 1050 **ii** 1202

11 **a** **i** 65% **ii** 80% **iii** 80%
b **i** 63 **ii** 399 **iii** 147

12 **a** 6–10
b



Exercise 19D

1 **a** mean = 4.3, mode = 5 **b** mean = 7, mode = 4

2 15.8

3 **a** 30.8 **b** 6 **c** higher

4 **a** 64 **b** 4.5 **c** 34 **d** 4

5 **a** 60 kg

6 **a** 40 **b** 50 **c** 390

7 41.2

8 **a** increase **b** no change **c** increase

9 **a** 7 **b** 18 **c** 1.85 **d** 670.5

10

Number of data values	5	11	21	10	20	100	29	43	26	48
Position of median	3	6	11	5.5	10.5	50.5	15	22	13.5	24.5

11 frequency 5, 6, 5, 4, 2; median = 70

12 **a** 24 **b** 6 **c** 5 and 6

13 **a** median unchanged, mean increased
b median becomes d, mean increased
c median unchanged, mean unchanged
d median unchanged, mean decreased
e median unchanged, mean increased
f median increased, mean unchanged

14 29, 29, 9, 29 or 7, 29, 29, 31 or 7, 27, 31, 31 – there are many possible answers.

15 **a** bimodal **b** negatively skewed
c positively skewed **d** symmetrical

16 **a** range = 57, median = 32, mean = 31.4
b range = 61, median = 54.5, mean = 47.1
c negatively skewed

17 Half of the values are less than $1500 000. The histogram would probably be positively skewed. The range tells us that there is at least one salary far above the median.

Review exercise

1 **a** **i** ≈26% **ii** ≈45%
b **i** ≈4% **ii** ≈23%
c **i** 51 **ii** 68
d **i** 72 **ii** 75.5
e **i** ≈71.9% **ii** ≈71.8%
f **i** symmetrical
ii bimodal (non-symmetric spread with two class peaks)

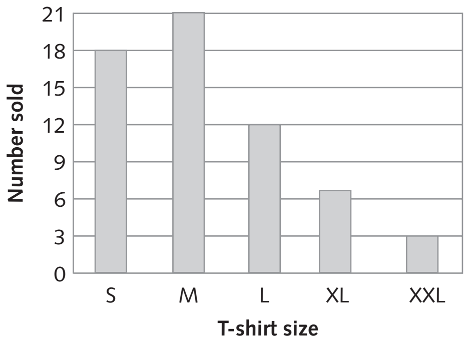
2 **a**

T-shirt size	Frequency
S	18
M	21
L	12
XL	7
XXL	3

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

b medium **c** ≈64%

d



e The data is categorical, not numerical.

3 **a** 47 **b** ≈68% **c** positively skewed
d \$450 000 – <500 000
e The mean would be higher than the median since a few houses sold for a substantially greater amount than the others. The house sold for \$850 000 to \$900 000 is an outlier.

4 **a** 77.6 **b** 92 **c** 81

Challenge exercise

1 **a i** 32.5°S **i** 149°E
b i 34.3°S, 147.7°E **ii** 31.4°S, 149.8°E
iii 33.1°S, 148.6°E

2 **a** 35.7°S, 147.9°E **b** 36.5°S, 137.8°E
c 35.3°S, 143.9°E **d** 35.2°S, 145.4°E
e 34.4°S, 144.3°E, near Griffith, NSW

3 1901: 35.1°S, 145.3°E, 1920: 34.9°S, 145.1°E,
1930: 34.8°S, 145.1°E, 1940: 34.7°S, 145.2°E
1950: 34.6°S, 145°E, 1960: 34.7°S, 145°E,
1970: 34.5°S, 144.5°E, 1980: 34.4°S, 144.3°E
1990: 34.2°S, 143.9°E

a It has moved north and west. In 1901 most of the population was in Melbourne and Sydney. Since then growth of population in other states has occurred.
b From 1901 to 1990 the population centre has moved 0.9° north and 1.4° west. This is approximately $\frac{0.9°}{89}$ north per year and $\frac{1.4°}{89}$ west per year.

So in 2005 the population centre was approximately $35.1° - \frac{0.9°}{89} \times 104 = 34°$S and

$145.3° - \frac{1.4°}{89} \times 104 = 143.7°$E

c Governments need to be aware of population shifts so that they can provide essential services to meet the needs of the population

Chapter 20 answers

20A Review

Chapter 11: Coordinate geometry

1 **a** $2\sqrt{17}$ **b** $3\sqrt{10}$ **c** $\sqrt{10}$
d $2\sqrt{2}$ **e** $5\sqrt{5}$ **f** $3\sqrt{5}$

2 **a** (4, 6) **b** $\left(\frac{7}{2}, \frac{15}{2}\right)$ **c** $\left(-\frac{1}{2}, \frac{9}{2}\right)$
d (4, –1) **e** $\left(\frac{1}{2}, 4\right)$ **f** $\left(\frac{7}{2}, 1\right)$

3 **a** 4 **b** 3 **c** 3
d 1 **e** –2 **f** –2

4 **a**

x	0	1	2	3	4
y	1	3	5	7	9

b $y = 2x + 1$

5 **a**

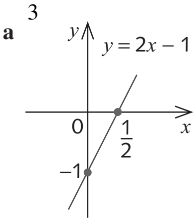
x	0	1	2	3	4
y	7	$6\frac{1}{2}$	6	$5\frac{1}{2}$	5

b $y = -\frac{1}{2}x + 7$

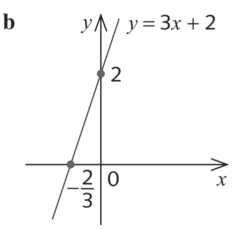
6 $y = 2x + 8$

7 **a** $\frac{2}{3}$ **b** $3y - 2x = 8$

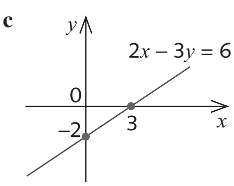
8 **a**



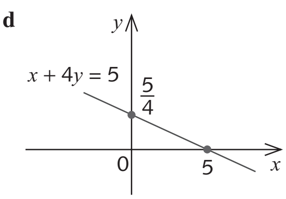
b



c



d



9 **a** gradient = 5, x-intercept = $\frac{3}{5}$, y-intercept = –3
b gradient = –1, x-intercept = 4, y-intercept = 4
c gradient = $\frac{3}{4}$, x-intercept = 4, y-intercept = –3
d gradient = $-\frac{5}{2}$, x-intercept = $\frac{4}{5}$, y-intercept = 2

10 **a** $y = -2x - 4$ **b** $x + 2y = 7$

Chapter 12: Probability

1 **a** = {R, A, N, D, O, M}
b i {A, O} **ii** $P(B) = \frac{1}{3}$
c i $\frac{2}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{2}{3}$

2 **a** $\frac{1}{25}$ **b** $\frac{1}{5}$ **c** $\frac{12}{25}$
d $\frac{1}{5}$ **e** $\frac{4}{25}$ **f** $\frac{5}{6}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7 Cambridge University Press
 Updated August 2021

3 **a**

Spinner \ Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9

b **i** $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{3}$

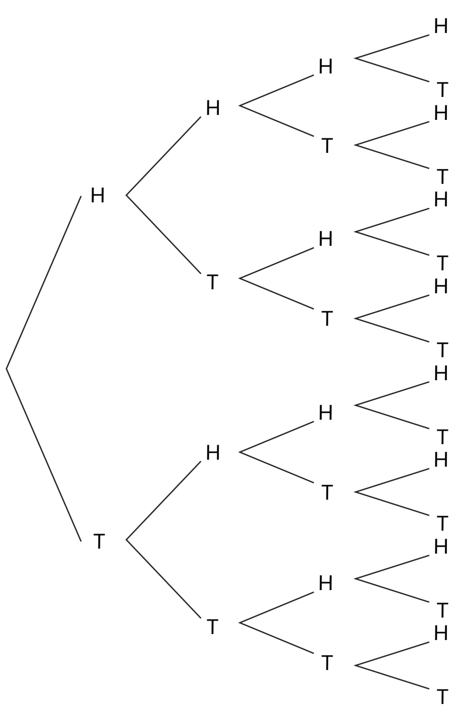
4 **a** $\frac{4}{25}$ **b** $\frac{2}{5}$ **c** $\frac{8}{25}$ **d** $\frac{2}{5}$

5 **a**

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b **i** $\frac{1}{36}$ **ii** $\frac{11}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{6}$

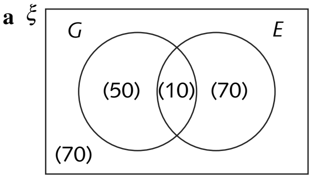
6 **a**



ξ = {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

b **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{11}{16}$

7 **a**



b **i** $\frac{3}{10}$ **ii** $\frac{1}{20}$ **iii** $\frac{13}{20}$

8 **a** **i** $\frac{3}{25}$ **ii** $\frac{17}{50}$ **iii** $\frac{21}{25}$ **iv** $\frac{18}{25}$

b None of the statements have to be true, although iii, iv and v are more likely than i and ii. None have to be false.

Chapter 13: Trigonometry

1 **a** AC **b** AB **c** BC **d** BC **e** AB

2 **a** **i** $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$

b **i** $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$

c **i** $\frac{8}{17}$ **ii** $\frac{15}{17}$ **iii** $\frac{8}{15}$

d **i** $\frac{7}{25}$ **ii** $\frac{24}{25}$ **iii** $\frac{7}{24}$

3 **a** 0.9659 **b** 0.9976 **c** 2.0503

4 **a** 63° **b** 21° **c** 70°

5 **a** 1.03 **b** 3.86 **c** 2.89
d 3.58 **e** 6.73 **f** 3.33

6 **a** 56° **b** 51° **c** 44°

7 84 m **8** 37°

9 **a** 48° **b** 3.35 m

10 **a** 150°T **b** 318°T **c** 127°T **d** 252°T

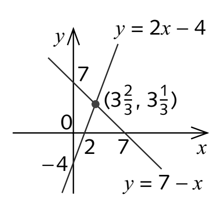
11 315.2 km

12 **a** 77.8 km **b** 59.6 km

13 213 m **14** 28° **15** 818 m

Chapter 14: Simultaneous linear equations

1 $\left(3\frac{2}{3}, 3\frac{1}{3}\right)$



2 **a** $x = 2,\ y = 1$ **b** $x = 19,\ y = 6$

c $x = \frac{2}{3}, y = -\frac{1}{3}$

3 **a** $x = 3,\ y = -2$ **b** $x = -\frac{1}{3}, y = -\frac{7}{3}$

c $x = \frac{62}{17}, y = \frac{4}{17}$ **d** $x = \frac{67}{27}, y = -\frac{4}{27}$

4 **a** $\frac{2}{5}$ **b** all numbers, except $\frac{2}{5}$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

5 $m = \frac{5}{2}, k = 3$

6 a $\frac{1}{4}$ square units b $\frac{27}{4}$ square units

7 32 pigs 8 28 years 9 (1, 3); $c = 1$

10 a $x = \frac{ac + bd}{a^2 + b^2}, y = \frac{bc - ad}{a^2 + b^2}$

b $x = \frac{bc - d}{2ab - 1}, y = \frac{2ad - c}{2ab - 1}$

Chapter 15: Factorisation

1 a $4(x - 25)$ b $7x(x - 7)$
c $2y(1 - 8y)$ d $2a(5 - 8a)$

2 a $(x - 10)(x + 10)$ b $(x - 7)(x + 7)$
c $(3x - 4y)(3x + 4y)$ d $(1 - 4a)(1 + 4a)$

3 a $3(x - 3)(x + 3)$ b $2(x - 2)(x + 2)$
c $2(3x - 4y)(3x + 4y)$ d $2(1 - 3a)(1 + 3a)$

4 a $(xy - 2)(xy + 2)$ b $(x - 8y)(x + 8y)$
c $8(y - 2)(y + 2)$ d $2(a - 4)(a + 4)$

5 a $x(x + 4)$ b $(y + 5)(y - 1)$
c $3(x - 1)(x + 5)$ d $a(a - 2)$

6 a $(x + 3)(x + 4)$ b $(x + 6)(x + 3)$
c $(x - 6)(x + 1)$ d $(x + 7)(x - 4)$
e $(x - 5)(x - 6)$ f $(x - 2)(x - 12)$
g $(x + 10)(x - 7)$ h $(x - 11)(x + 5)$
i $3(x^2 + 2x + 3)$ j $4(x^2 - 2x + 3)$
k $-(x + 3)(x - 2)$ l $-(2x + 11)(x - 4)$

7 a $(2y + 3)(4x + 1)$ b $(3x - 2)(2a + 5)$
c $(3c - 2d)(2a - b)$ d $(2m - p)(m + 3n)$

8 a $(2x + 5)(x + 2)$ b $(2x + 5)(3x + 2)$
c $(3x - 1)(2x + 5)$ d $(4x + 1)(2x - 3)$
e $(7x + 1)(3x - 8)$ f $(3x + 4)(2x - 5)$
g $(2x - 3)^2$ h $(3x + 4)^2$
i $4(x - 3)(x + 1)$

9 a $\frac{x+1}{x-1}$ b $\frac{1}{(x+1)(x+4)}$ c $\frac{1}{4}$
d $\frac{1}{2(p+2)}$ e $2a$ f $\frac{3}{a+2}$
g $\frac{x+2}{2x-1}$ h $x - 2$ i $3(x + 4)$
j $\frac{2x(x-2)}{(x-1)(x+2)}$

10 a $\frac{7x+15}{12}$ b $\frac{5x-8}{6}$
c $\frac{5x+1}{12}$ d $\frac{5x-1}{12}$

11 a $\frac{3x+2}{x(x+1)}$ b $\frac{6(x+1)}{(x-1)(x+2)}$
c $\frac{x(3x+1)}{(x-1)(x+3)}$ d $\frac{-2x-1}{(x-2)(x+3)}$
e $\frac{-2x^2+9x-2}{(x-3)(2x+1)}$ f $\frac{-x^2+x+5}{(2x-1)(3x+2)}$

12 a $\frac{4x+3}{x^2}$ b $\frac{5x+8}{(x+1)(x+2)}$
c $\frac{3x-4}{(x-3)(x+3)}$ d $\frac{5x+1}{x(x-1)(x+1)}$
e $\frac{2(x-4)}{(x-1)(x+2)^2}$ f $\frac{7x+5}{(x-3)(x-1)(x+2)}$
g $\frac{2-2x-x^2}{(x-2)(x+2)}$ h $\frac{2x+5}{(x-1)(x+1)}$

13 a $(x + 2)^2 - 2$ b $(x + 3)^2 - 3$
c $(x - 1)^2 - 7$ d $\left(x + \frac{3}{2}\right)^2 - \frac{41}{4}$
e $\left(x - \frac{1}{2}\right)^2 - \frac{5}{4}$ f $(x - 5)^2 - 5$
g $\left(x + \frac{3}{2}\right)^2 - \frac{37}{4}$ h $(x - 3)^2 - 14$

14 a $(x + 2 + \sqrt{5})(x + 2 - \sqrt{5})$ b $(x + 1 + \sqrt{3})(x + 1 - \sqrt{3})$
c $(x - 3 + 2\sqrt{2})(x - 3 - 2\sqrt{2})$
d $\left(x + \frac{5}{2} - \frac{\sqrt{17}}{2}\right)\left(x + \frac{5}{2} + \frac{\sqrt{17}}{2}\right)$

Chapter 16: Measurement – areas, volumes and time

1 a 98 cm^2 b 39 cm^2
c 40 cm^2 d $16\pi \approx 50.27$ cm^2

2 a 72 $18\pi \approx 15.45$ cm b 174 cm^2
c 40 cm^2 d $80 - 16\pi = 29.73$ cm^2

3 a $3\sqrt{3}$ cm b $18\pi - 18\sqrt{3} \approx 25.37$ cm^2

4 a 8.64 m^2 b 76 cm^2 c $72\pi \approx 226.2$ cm^2

5 a 528 cm^2 b 830 cm^2

6 $892 + 150\pi \approx 1363.2$ cm^2

7 a 60 cm^3 b 64 cm^3 c 216 cm^3 d 180 cm^3
e $224\pi \approx 703.72$ cm^3 f $250\pi \approx 785.40$ cm^3
g 504 cm^3

8 905 L

9 a 1400 mm^2 b 16 cm^2 c 21000 cm^2
d 4.6 m^2 e 0.064 km^2 f 0.8 ha
g 0.041 m^2 h 2.6×10^{10} cm^2

10 a 0.41 cm^3 b 200 mm^3 c 0.064 m^3
d 8 L e 8.6 m^3 f 6000 mL
g 9000 km^3 h 260 000 000 m^3 i 4.1×10^{12} L
j 8.6×10^9 mm^3

Chapter 17: Quadratic equations

1 a $x = 0, 3$ b $x = 0, -\frac{1}{2}$
c $m = -\frac{1}{3}, \frac{5}{2}$ d $p = -\frac{3}{4}, \frac{7}{5}$

2 a $x = 0, 5$ b $x = 0, \frac{3}{2}$ c $x = 0, 8$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

d $x = -4, 4$ **e** $x = -\frac{3}{2}, \frac{3}{2}$ **f** $x = -6, 6$

g $x = -5, 5$ **h** $x = -1, -6$ **i** $x = -3, -6$

j $x = 3, 11$ **k** $x = 2, 4$ **l** $x = -5, 8$

m $x = -7, 2$ **n** $x = -\frac{3}{2}, 5$ **o** $x = -\frac{1}{2}, -\frac{2}{3}$

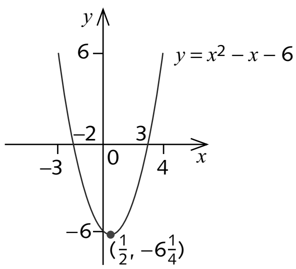
p $x = -5, \frac{3}{4}$ **q** $x = -\frac{1}{6}, 2$ **r** $x = -\frac{1}{3}, \frac{2}{5}$

3 **a** $x = 2, 6$ **b** $x = -4, 7$ **c** $x = -3, 4$
d $x = -5, 3$ **e** $x = -4, -1$ **f** $x = -4, 6$
g $x = -2, 6$ **h** $x = -9, 2$ **i** $x = -5, 2$
j $x = -2, 6$

4 −7 or 6 **5** 25 or −4 **6** −2 or 8

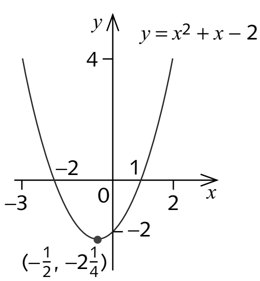
7 **a**

x	y
−3	6
−2	0
−1	−4
0	−6
1	−6
2	−4
3	0
4	6



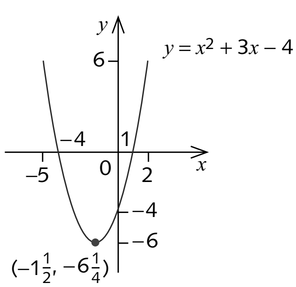
b

x	y
−3	4
−2	0
−1	−2
0	−2
1	0
2	4



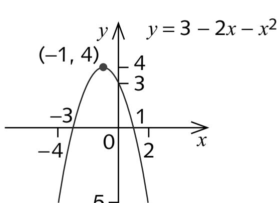
c

x	y
−5	6
−4	0
−3	−4
−2	−6
−1	−6
0	−4
1	0
2	6



d

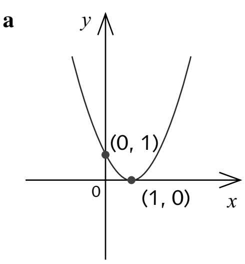
x	y
−4	−5
−3	0
−2	3
−1	4
0	3
1	0
2	−5



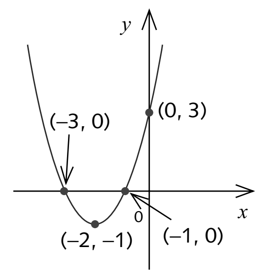
8 **a** $x = \sqrt{7}$ or $x = -\sqrt{7}$ **b** $x = 2\sqrt{2}$ or $x = -2\sqrt{2}$
c $x = 2\sqrt{2}$ or $x = -2\sqrt{2}$ **d** $x = \sqrt{5}$ or $x = -\sqrt{5}$

9 **a** $x = -1 + \sqrt{3}$ or $x = -1 - \sqrt{3}$
b $x = 3 + \sqrt{3}$ or $x = 3 - \sqrt{3}$
c $x = -2 + \sqrt{2}$ or $x = -2 - \sqrt{2}$
d $x = \frac{-3 + \sqrt{5}}{2}$ or $x = \frac{-3 - \sqrt{5}}{2}$
e $x = \frac{5 - \sqrt{13}}{2}$ or $x = \frac{5 + \sqrt{13}}{2}$
f $x = \frac{1 - \sqrt{5}}{2}$ or $x = \frac{1 + \sqrt{5}}{2}$

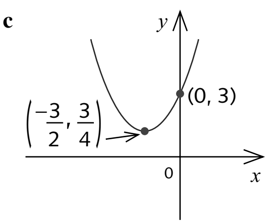
10 **a**



b



c



Chapter 18: Rates and direct proportion

1 96 m

2 **a** 50 m/s **b** 43.2 km/h

3 **a** 800 km/h **b** $109\frac{1}{11}$ km/h ≈ 109.1 km/h

4 **a** $d = 80t$ **b** 80

5 **a** x is directly proportional to z.
b y is directly proportional to x^3.
c p is directly proportion to $\sqrt{n}$.
d x^3 is directly proportional to z^4.

6 **a** The constant of proportionality is 7.

p	2	4	5	9
q	14	28	35	63

b The constant of proportionality is $\frac{1}{2}$.

p	1	4	7	16
q	$\frac{1}{2}$	2	$\frac{7}{2}$	8

c The constant of proportionality is $\frac{4}{3}$.

p	2	6	8	18
q	$\frac{8}{3}$	8	$\frac{32}{3}$	24

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

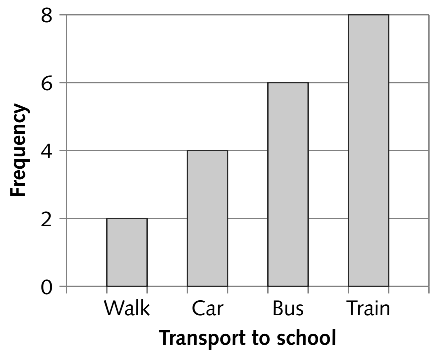
7 $y = 5x^3$
a $y = 5000$ **b** $y = 214.375$ **c** $x = 3$

8 $d = 0.75t^2$
a $d = 60.75$ **b** $d = 108$ **c** $t = 8$

9 49π **10** q is multiplied by 8.

Chapter 19: Statistics

1 **a**



2 **a** **b** 6

2	7 8
3	1 1 4 5 6 6 6 7 7 8 9
4	0 2 4 4 5 5
5	1

3 **a** 44 **b** 10 **c** 55% **d** 68

4 **a** 39 **b** 63.4 **c** 72

5 16 **6** 24.6

7 **a** 13.9 **b** 14.2

8 **a** 2 **b** $\frac{30}{11} \approx 2.72$ **c** 2.5 **d** 11

9 **a**

Stem	Leaves
2	2 3 4 4 6 7
3	1 3 5 6 8 8 9
4	0 1 1 1 4 6

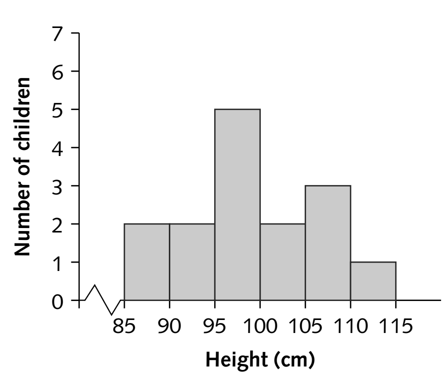
b mean = 34.16; median = 36; range = 24

10 **a**

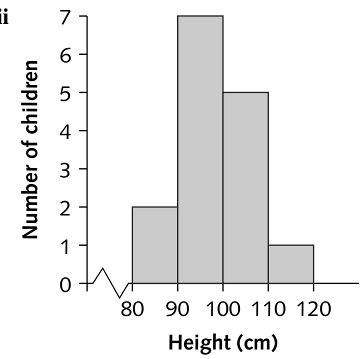
8	7 9
9	2 4 5 6 6 8 9
10	2 4 5 7 7
11	0

10 | 4 means 104 cm

b **i**



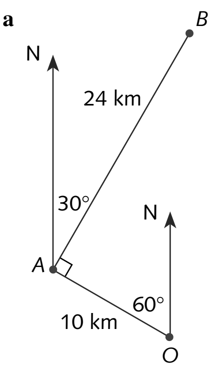
ii



20B Problem-solving

1 **a** **i** could be true **ii** could be true
iii could be true **iv** definitely false
v could be true
b **i** could be true **ii** could be true
iii could be true **iv** definitely false
v could be true

2 **a**



b 90° **c** 26 km **d** 67.38° **e** 007°T

3 **a** **i** 2 **ii** $\left(5, \frac{3}{2}\right)$

iii $\frac{OB}{OA} = \frac{OC}{OB} = 2$ **iv** SAS

b **i** $90° - \alpha$ **ii** α **iii** 90°

4 **b** $20\sqrt{3}$ m **c** 60°
d 23.835 m **e** 70.02 m

5 **a** **i** $r = 6.3$ **ii** 124.69 cm^2
b 1696.5 cm^3 **c** 5.65 cm

6 **a** 30°

b **i** $\frac{x}{2}$ m **ii** $\frac{\sqrt{3}x}{2}$ m

d 16.24 m^2 **e** $\frac{7}{\sqrt{3}} \approx 4.04$ m **f** 42.44 m^2

7 **a** **i** $\frac{1}{4}$ **ii** $\frac{1}{13}$ **iii** $\frac{12}{13}$ **iv** $\frac{1}{52}$ **v** $\frac{4}{13}$

b

	D	*H*	*S*	*C*
D	(*D*, *D*)	(*D*, *H*)	(*D*, *S*)	(*D*, *C*)
H	(*H*, *D*)	(*H*, *H*)	(*H*, *S*)	(*H*, *C*)
S	(*S*, *D*)	(*S*, *H*)	(*S*, *S*)	(*S*, *C*)
C	(*C*, *D*)	(*C*, *H*)	(*C*, *S*)	(*C*, *C*)

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

c $\{(H, D), (D, H), (H, H), (S, H), (C, H), (H, S), (H, C)\}$

d i $\frac{7}{16}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$

iv $\frac{1}{4}$ **e** 25

8 **a** $y_A = 30 - \frac{3}{2}t$, $y_B = 20 - \frac{2}{3}t$

b 20 min **c** 12 min

d $y_A = a - bt$, $y_B = c - dt$

e i $t = \frac{c - a}{d - b}$ **ii** $\frac{ad - bc}{d - b}$

9 **a** $(2x - 3)$ m **b** $x(2x - 3)$ m^2

c ii $x = 5$ **iii** 35 m^2 **d** $x = \frac{15}{2}$

10 **a** $\frac{d}{700}$ h **b** $\frac{d}{560}$ h

c $\left(\frac{d}{700} + \frac{d}{560} + 2\right)$ h **d** 5600 km

11 **a** $\frac{120}{x}$ h **b** $\frac{120}{x + 4}$ h **c** 16 km/h

12 **a** $\frac{1500}{\sqrt{2}} \approx 1061$ m **b** 233 ha **a** 318°T

13 **a** $10\sqrt{2}$ cm **b** $\sqrt{2}r$ cm

c $\sqrt{2}r + r + 10$ **d** $r = 30 - 20\sqrt{2}$

14 **a** $A = x^2 + 7x$ **b** $x = 3$

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7