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MATHEMATICS

Year 9

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Contents

Chapter 9: Pythagoras' theorem and trigonometry 553

Statistics and probability 623

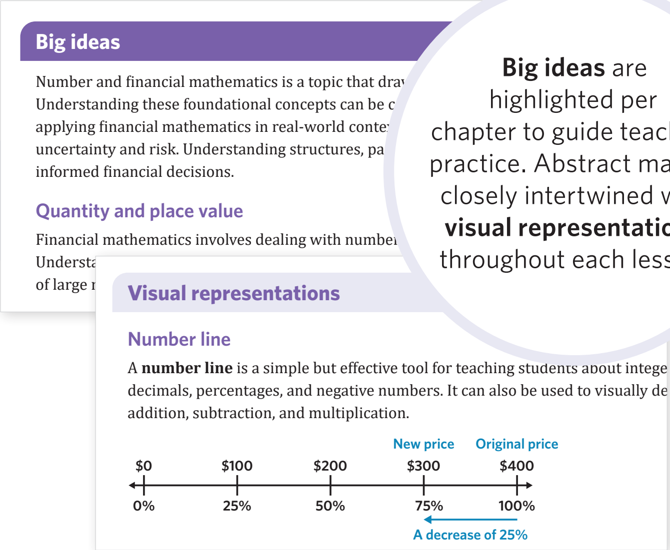
Chapter 10: Probability 623

Chapter 11: Statistics 681

Back to contents

Guide to Edrolo Year 9 Mathematics

This resource deepens students' conceptual understanding to develop a strong foundation of knowledge through print and online video lessons.



Big ideas

Number and financial mathematics is a topic that drav
Understanding these foundational concepts can be c
applying financial mathematics in real-world conte
uncertainty and risk. Understanding structures, pa
informed financial decisions.

Quantity and place value

Financial mathematics involves dealing with number
Understa
of large r

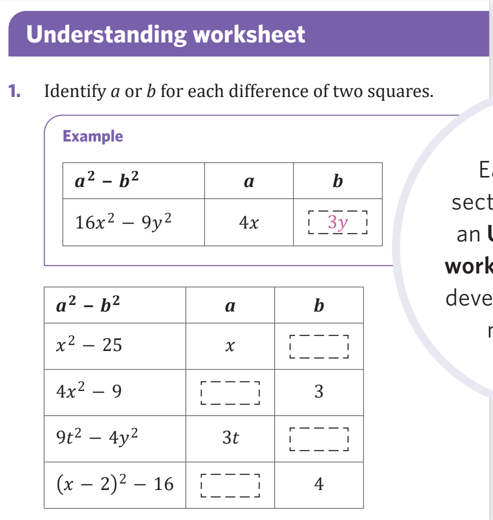
Visual representations

Number line

A **number line** is a simple but effective tool for teaching students about intege
decimals, percentages, and negative numbers. It can also be used to visually de
addition, subtraction, and multiplication.

Big ideas are highlighted per chapter to guide teaching practice. Abstract math is closely intertwined with **visual representations** throughout each lesson.

Each question section begins with an **Understanding worksheet** to aid the development of core maths skills.



Understanding worksheet

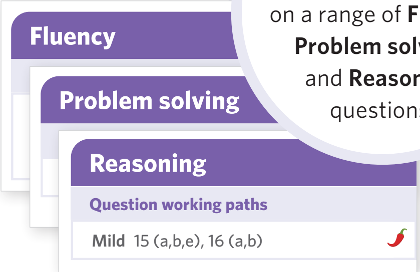
1. Identify a or b for each difference of two squares.

Example

$a^2 - b^2$	a	b
$16x^2 - 9y^2$	$4x$	$3y$

$a^2 - b^2$	a	b
$x^2 - 25$	x	
$4x^2 - 9$		3
$9t^2 - 4y^2$	$3t$	
$(x - 2)^2 - 16$		4

Students practice their knowledge and skills on a range of **Fluency**, **Problem solving**, and **Reasoning** questions.



Fluency

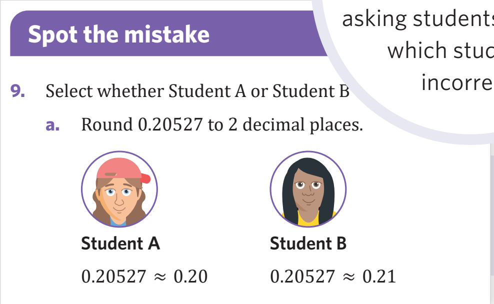
Problem solving

Reasoning

Question working paths

Mild 15 (a,b,e), 16 (a,b) 🌶

Spot the mistake targets misconceptions and provides non-examples asking students to locate which student is incorrect.



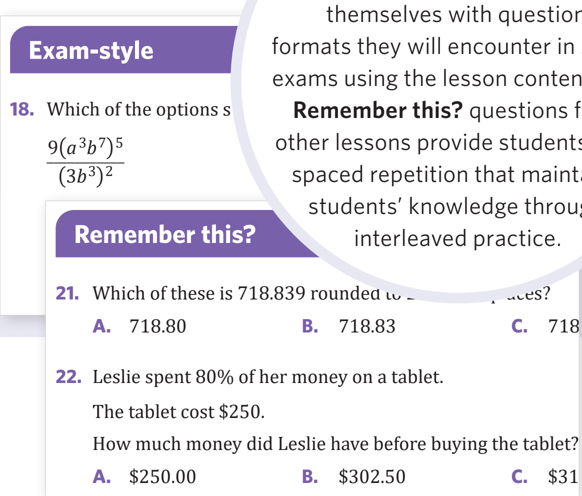
Spot the mistake

9. Select whether Student A or Student B

a. Round 0.20527 to 2 decimal places.

Student A	Student B
$0.20527 \approx 0.20$	$0.20527 \approx 0.21$

Exam-style questions provide students' practice to familarise themselves with question formats they will encounter in senior exams using the lesson content, and **Remember this?** questions from other lessons provide students with spaced repetition that maintains students' knowledge through interleaved practice.



Exam-style

18. Which of the options s

$$\frac{9(a^3b^7)^5}{(3b^3)^2}$$

Remember this?

21. Which of these is 718.839 rounded to
ces?

A. 718.80 **B.** 718.83 **C.** 718

22. Leslie spent 80% of her money on a tablet.

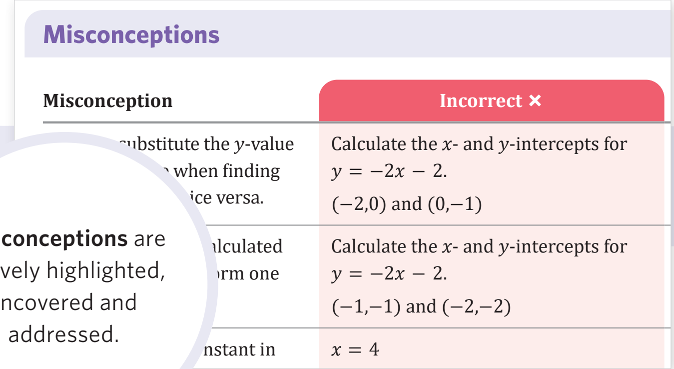
The tablet cost $250.

How much money did Leslie have before buying the tablet?

A. $250.00 **B.** $302.50 **C.** $31

Visit Edrolo.com.au to access all the digital materials.

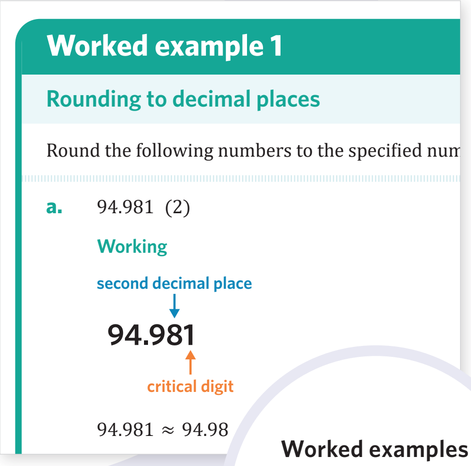
Back to contents



Misconceptions

Misconception	Incorrect ✕
ubstitute the y-value when finding ice versa.	Calculate the x- and y-intercepts for $y = -2x - 2$. $(-2,0)$ and $(0,-1)$
lculated rm one	Calculate the x- and y-intercepts for $y = -2x - 2$. $(-1,-1)$ and $(-2,-2)$
nstant in	$x = 4$

Misconceptions are actively highlighted, uncovered and addressed.



Worked example 1

Rounding to decimal places

Round the following numbers to the specified num

a. 94.981 (2)

Working

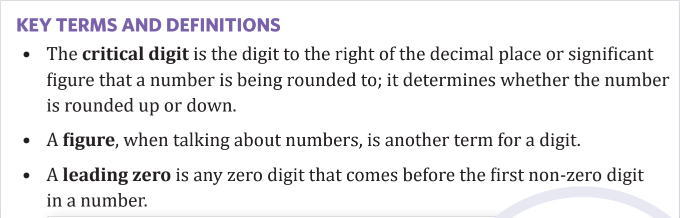
second decimal place

94.981

critical digit

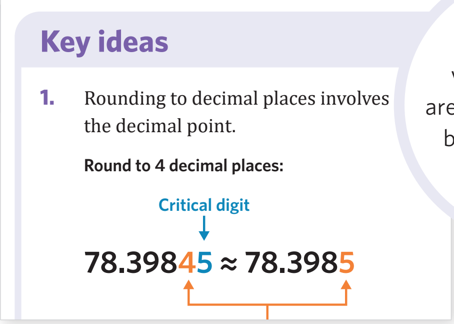
94.981 ≈ 94.98

Worked examples model the thinking and working for specific skills outlined in each lesson.



KEY TERMS AND DEFINITIONS

- The **critical digit** is the digit to the right of the decimal place or significant figure that a number is being rounded to; it determines whether the number is rounded up or down.
- A **figure**, when talking about numbers, is another term for a digit.
- A **leading zero** is any zero digit that comes before the first non-zero digit in a number.



Key ideas

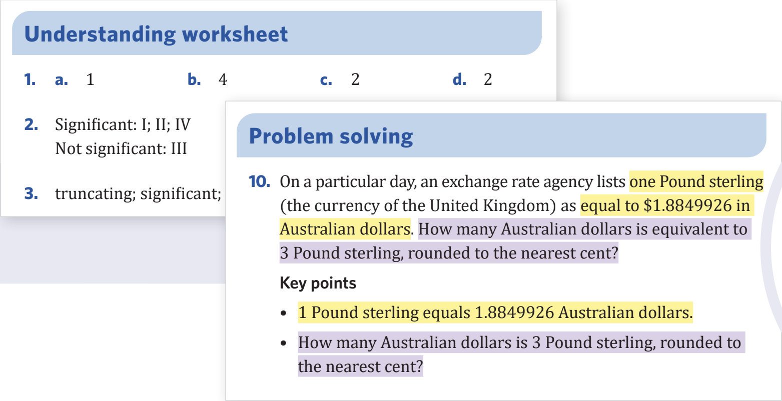
1. Rounding to decimal places involves the decimal point.

Round to 4 decimal places:

Critical digit

78.39845 ≈ 78.3985

New ideas and vocabulary used are highlighted at the beginning of each lesson.



Understanding worksheet

1. **a.** 1	**b.** 4	**c.** 2	**d.** 2

2. Significant: I; II; IV
Not significant: III

3. truncating; significant;

Problem solving

10. On a particular day, an exchange rate agency lists one Pound sterling (the currency of the United Kingdom) as equal to $1.8849926 in Australian dollars. How many Australian dollars is equivalent to 3 Pound sterling, rounded to the nearest cent?

Key points

- 1 Pound sterling equals 1.8849926 Australian dollars.
- How many Australian dollars is 3 Pound sterling, rounded to the nearest cent?

Answers are given in the back of the book. Full worked problem solving solutions are stepped out in detail to understand the why behind the answer. Detailed solutions are available online.

Back to contents

Back to contents

Chapter 1

Number and financial mathematics

Number and algebra

Calculator skills

See online in additional materials for using CAS calculator guides.

1D Taxation

1E Simple interest

1F Compound interest and depreciation

Back to contents

Chapter 1 research summary

Number and financial mathematics

Big ideas

Number and financial mathematics is a topic that draws on several big ideas in mathematics. Understanding these foundational concepts can be crucial in effectively engaging with and applying financial mathematics in real-world contexts. Financial decisions often involve uncertainty and risk. Understanding structures, patterns, and relationships can help make informed financial decisions.

Quantity and place value

Financial mathematics involves dealing with numbers that represent monetary values. Understanding the place value system is crucial in this context, as it helps in making sense of large numbers that are often encountered in financial transactions and calculations.

Structure and patterns

Recognising patterns and structures is important in financial mathematics. For example, understanding interest rates, investment growth and the cost of a loan are all reliant on recognising patterns of how interest rates affect growth and the cost of a loan. Spotting trends and patterns can also help in making informed financial decisions.

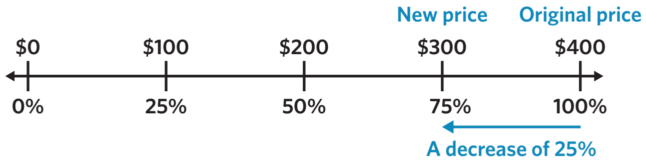
Proportional reasoning

In financial mathematics, quantifying monetary values, costs, profits, and interest rates all require the understanding and use of proportional relationships. Understanding the concept of percentage as a proportion and ratio can help students to understand more difficult financial concepts like simple and compound interest. Understanding how the quantities can change in relation to each other is an important skill in understanding investments and life long skills.

Visual representations

Number line

A **number line** is a simple but effective tool for teaching students about integers, fractions, decimals, percentages, and negative numbers. It can also be used to visually demonstrate addition, subtraction, and multiplication.



Bar model

A **bar model** is a powerful visual tool that can be utilised to represent various mathematical and financial concepts. A bar model can be drawn to represent to scale quantities and proportions in financial mathematics. Using different colours helps students visually differentiate between various parts of the data. Bar models can be used to teach students to understand percentages and ratios. For instance, if a student saves 25% of their allowance, represent the total allowance as a full bar and shade one-fourth of it to represent the savings. Bar models can also be used to visualise how to calculate simple and compound interest.

Back to contents

Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students assume the number of significant figures is the same as the number of decimal places in a number.	0.**07302** 5 significant figures	0.0**7302** 4 significant figures	1A
Students identify the leading zero after the decimal point as significant.	0.**07302**	0.0**7302**	1A
Students do not count the zeros in between non-zero digits as significant.	**4**0.06	**40.06**	1A
Students find the proportion of the wrong quantity when calculating percentages.	Calculate the percentage profit or loss on a car that cost \$12 000 to make and sold for \$9750. $\text{Loss} = 12\,000 - 9750$ $= 2250$ $\%\text{ loss} = \frac{2250}{9750} \times 100$ $= 23.076...$ $\approx 23\%$	Calculate the percentage profit or loss on a car that cost \$12 000 to make and sold for \$9750. $\text{Loss} = 12\,000 - 9750$ $= 2250$ $\%\text{ loss} = \frac{2250}{12\,000} \times 100$ $= 18.75$ $\approx 19\%$	1B
Students add and subtract percentage changes.	$100\% - 7\% = 93\%$ $\text{New price} = \$325 - 93$ $= \$232$	$100\% - 7\% = 93\%$ $\text{New price} = 93\% \text{ of } \$325$ $= 0.93 \times \$325$ $= \$302.25$	1B
Students apply identical but opposite percentage changes to the new amount to calculate the old amount.	Calculate the original price of a bicycle that had a mark-up of 20% and is now priced at \$599.99. $100\% - 20\% = 80\%$ $\text{Original price} = 80\% \text{ of } \$599.99$ $= \$479.99$	Calculate the original price of a bicycle that had a mark-up of 20% and is now priced at \$599.99. $\text{Original price} = \frac{599.99}{1.20}$ $= \$499.99$	1B
Students apply an overtime rate to a whole day's hours instead of just the hours worked overtime.	A baker worked 11.04 hours, including 1.04 hours of overtime, at a base rate of \$27.54 per hour. How much income was earned? $11.04 \times 27.54 \times 1.5 = \$456.0624$ $\approx \$456.06$	A baker worked 11.04 hours, including 1.04 hours of overtime, at a base rate of \$27.54 per hour. How much income was earned? $11.04 - 1.04 = 10$ $10 \times 27.54 = \$275.40$ $1.04 \times 27.54 \times 1.5 = \$42.9624$ $\approx \$42.96$ $275.40 + 42.96 = \$318.36$	1C
Students add to instead of multiplying the base rate for weekend and overtime pays.	Calculate the income earned for 7.56 hours worked on a Sunday at a base rate of \$30.24. Sunday pay is paid at 1.75 times the base rate. $7.56 \times 30.24 + 1.75 = \$230.3644$ $\approx \$230.36$	Calculate the income earned for 7.56 hours worked on a Sunday at a base rate of \$30.24. Sunday pay is paid at 1.75 times the base rate. $7.56 \times 30.24 \times 1.75 = \$400.0752$ $\approx \$400.08$	1C

Continues →

Back to contents

Misconception	Incorrect ✕	Correct ✓	Lesson
Students calculate income tax on the whole income instead of the amount over the threshold.	Reed's gross income for a year is \$42 053. How much income tax do they owe? $\text{Income tax} = 42\,053 \times 0.19$ $= \$7990.07$	Reed's gross income for a year is \$42 053. How much income tax do they owe? $42\,053 - 18\,200 = \$23\,853$ $\text{Income tax} = 23\,853 \times 0.19$ $= \$4532.07$	1D
Students subtract deductions from income tax instead of taxable income.	Alex's gross income for a year is \$63 500. He can claim \$680 of his expenses as deductions. How much income tax does he owe? $63\,500 - 45\,000 = \$18\,500$ $5092 + 0.325 \times 18\,500 = \$11\,104.50$ $\text{Income tax} = 11\,104.50 - 680$ $= \$10\,424.50$	Alex's gross income for a year is \$63 500. He can claim \$680 of his expenses as deductions. How much income tax does he owe? $63\,500 - 680 = \$62\,820$ $62\,820 - 45\,000 = \$17\,820$ $\text{Income tax} = 5092 + 0.325 \times 17\,820$ $= \$10\,883.50$	1D
Students confuse withheld tax with income tax.	Calculate the payable income tax on a gross income of \$123 234 with \$5023 worth of deductions, rounded to the nearest cent. \$33 176.00	Calculate the payable income tax on a gross income of \$123 234 with \$5023 worth of deductions, rounded to the nearest cent. \$28 885.58	1D
Students do not convert an interest rate when calculating simple interest for non-annual time periods.	$P = 2500$ $I = 102.08$ $t = 14$ months $102.08 = \frac{2500 \times r \times 14}{100}$ $102.08 \times 100 = 35\,000r$ $r = 0.29\%$ per year	$P = 2500$ $I = 102.08$ $t = 14$ months $102.08 = \frac{2500 \times r \times 14}{100}$ $102.08 \times 100 = 35\,000r$ $r = 0.29\%$ per month $r = 0.29\% \times 12$ $= 3.48\%$ per year	1E
Students add the interest rate instead of finding the percentage of the principal.	$A = 600 + 15.4 \times 7$ $= \$707.80$	$I = \frac{600 \times 15.4 \times 7}{100}$ $= \$646.80$ $A = 600 + 646.80$ $= \$1246.80$	1E
Students confuse simple interest with compound interest.	A \$20 000 investment attracts interest at 2.4% p.a. compounding monthly. How much interest will there be after 8 months, rounded to the nearest cent? $I = \frac{20\,000 \times \frac{24}{12} \times 8}{100}$ $= \$320$	A \$20 000 investment attracts interest at 2.4% p.a. compounding monthly. How much interest will there be after 8 months, rounded to the nearest cent? $A = 20\,000 \times \left(1 + \frac{\frac{24}{12}}{100}\right)^{8}$ $= 20\,322.24898$ $\approx \$20\,322.25$ $20\,322.25 - 20\,000 = \$322.25$	1F

Continues →

Misconception	Incorrect ✗	Correct ✓	Lesson
Students mistake the value of the investment/loan for the interest.	$P = 1500$ $r = 2.4$ $n = 3$ $A = 1500\left(1 + \frac{2.4}{100}\right)^{3}$ $\approx \$1610.61$ $I = 1610.61 - 1500$ $I = 110.61$	$P = 1500$ $r = 2.4$ $n = 3$ $I = 1500\left(1 + \frac{2.4}{100}\right)^{3}$ $\approx \$1610.61$	1F
Students do not convert an interest rate when calculating compound interest for non-annual time periods.	$12 500 at 3.6% p.a. depreciating monthly for 4.5 years. $P = 12\,500$ $r = 3.6$ $n = 4.5 \times 12 = 54$ $A = 12\,500\left(1 - \frac{3.6}{100}\right)^{54}$ $\approx \$1726.12$	$12 500 at 3.6% p.a. depreciating monthly for 4.5 years. $P = 12\,500$ $r = \frac{3.6}{12}$ $n = 4.5 \times 12 = 54$ $A = 12\,500\left(1 - \frac{\frac{3.6}{12}}{100}\right)^{54}$ $\approx \$10\,627.93$	1F

Back to contents

1A Decimals and significant figures

LEARNING INTENTIONS

Students will be able to:

- round numbers to a specified number of decimal places
- identify the significant figures in a number
- round numbers to a specified number of significant figures.

Numbers are rounded so that they are easier to read and interpret. However, rounded numbers are less accurate as rounding removes digits. Numbers can be rounded to a specified number of decimal places or significant figures, depending on the scenario.

KEY TERMS AND DEFINITIONS

- The **critical digit** is the digit to the right of the decimal place or significant figure that a number is being rounded to; it determines whether the number is rounded up or down.
- A **figure**, when talking about numbers, is another term for a digit.
- A **leading zero** is any zero digit that comes before the first non-zero digit in a number.
- A **trailing zero** is any zero digit that comes after the last non-zero digit in a number.
- **Truncating** means shortening a number, usually through rounding.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Bankoo/Shutterstock.com

In finance, currencies are usually rounded to two decimal places. In science, it is common to see numbers rounded to three significant figures.

Key ideas

1. Rounding to decimal places involves truncating or approximating a number to a specified number of digits after the decimal point.

Round to 4 decimal places:

Critical digit

$$78.39845 \approx 78.3985$$

The digit is increased as the critical digit is 5 or more.

Round to 3 decimal places:

Critical digit

$$78.39846 \approx 78.398$$

The digit is unchanged as the critical digit is 4 or less.

2. The significant figures (sig fig or s.f.) of a number contribute to the accuracy of a measured or calculated quantity. There are rules for determining whether a figure is significant. Whole numbers and decimals can be rounded to a specified number of significant figures.

Rule	Example
All non-zero digits are significant.	**923**
Zeros between non-zero digits are significant.	**201** or **40.06**
Leading zeros are not significant.	0.0**7302**
Zeros to the right of the last non-zero digit are always significant if the number is a decimal.	**24**00 or **2400.0**

Continues →

Back to contents

3. The results of multiplications and divisions involving values that have been measured are usually rounded to the smallest number of significant figures.

$$\overset{\text{4 s.f.}}{35.21} \times \overset{\text{2 s.f.}}{2.7} = 95.067$$

$$\approx 95 \longleftarrow \text{Rounded to 2 s.f.}$$

Worked example 1

Rounding to decimal places

Round the following numbers to the specified number of decimal places.

a. 94.981 (2)

WE1a

Working

$$94.9\overset{\substack{\text{second decimal place}\\ \downarrow}}{8}\underset{\substack{\uparrow\\ \text{critical digit}}}{1}$$

94.981 ≈ 94.98

Thinking

Step 1: Identify the digit being rounded (in the second decimal place) and the critical digit.

Step 2: The critical digit is less than 5, so the digit being rounded stays the same.

Visual support

$$94.9\overset{\substack{\text{digit to be rounded}\\ \downarrow}}{8}\underset{\substack{\uparrow\\ \text{critical digit}}}{1} \approx \overset{\substack{\text{2 decimal places}\\ \downarrow}}{94.98}$$

b. 3.2975 (3)

WE1b

Working

$$3.29\overset{\substack{\text{third decimal place}\\ \downarrow}}{7}\underset{\substack{\uparrow\\ \text{critical digit}}}{5}$$

3.2975 ≈ 3.298

Thinking

Step 1: Identify the digit being rounded (in the third decimal place) and the critical digit.

Step 2: The critical digit is 5, so the digit being rounded increases.

Student practice

Round the following numbers to the specified number of decimal places.

a. 70.411 (1) **b.** 3882.878 (2)

Worked example 2

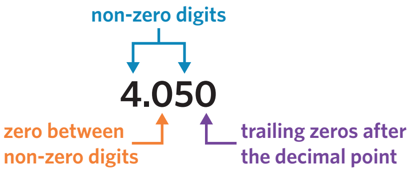
Identifying and rounding to significant figures

For the following:

a. Determine the number of significant figures in 4.050. WE2a

Working	Thinking
4.0**5**0	**Step 1:** Identify any non-zero digits. They are significant.
4.**0**50	**Step 2:** Identify any zeros between non-zero digits. They are significant.
4.05**0**	**Step 3:** Identify any trailing zeros after the decimal point. They are significant.
4	**Step 4:** State the number of significant figures.

Visual support



b. Determine the number of significant figures in 140 500. WE2b

Working	Thinking
140 **5**00	**Step 1:** Identify any non-zero digits. They are significant.
14**0** 500	**Step 2:** Identify any zeros between non-zero digits. They are significant.
4	**Step 3:** State the number of significant figures.

c. Round 0.0708 to two significant figures. WE2c

Working	Thinking
0.0**708**	**Step 1:** Identify the significant figures.
	**Step 2:** Identify the digit being rounded (the second significant figure) and the critical digit.
$0.0708 \approx 0.071$	**Step 3:** The critical digit is more than 5, so the digit being rounded increases.

Student practice

For the following:

a. Determine the number of significant figures in 2.5510.

b. Determine the number of significant figures in 5600.

c. Round 68.052 to two significant figures.

Back to contents

Worked example 3

Rounding results of calculations

Complete the following calculations and round each result to the appropriate number of significant figures.

a. 5.1 × 6.90

WE3a

Working	Thinking
5.1 has 2 significant figures. 6.90 has 3 significant figures.	**Step 1:** Identify the number of significant figures in each number.
5.1 × 6.90 = 35.19	**Step 2:** Complete the calculation.
35.19 ≈ 35 (2 s.f.)	**Step 3:** Round the result to the smallest number of significant figures identified in Step 1.

Visual support

$$\overset{2\text{ s.f.}}{5.1} \times \overset{3\text{ s.f.}}{6.90} = 35.19$$

$$\approx 35 \longleftarrow \text{Rounded to 2 s.f.}$$

b. 1230 ÷ 0.06327

WE3b

Working	Thinking
1230 has 3 significant figures. 0.06327 has 4 significant figures.	**Step 1:** Identify the number of significant figures in each number.
1230 ÷ 0.06327 = 19440.493...	**Step 2:** Complete the calculation.
19440.493... ≈ 19 400 (3 s.f.)	**Step 3:** Round the result to the smallest number of significant figures identified in Step 1.

Student practice

Complete the following calculations and round each result to the appropriate number of significant figures.

a. 4.05 × 0.21 **b.** 501 ÷ 1.4

Back to contents

1A Questions

Understanding worksheet

1. In each of the following, the critical digit is highlighted. To how many decimal places will each number be rounded?

Example

7.71171 will be rounded to [3] decimal places.

a. 3.25305 will be rounded to [ ] decimal places.

b. 5.04049 will be rounded to [ ] decimal places.

c. 2094.285 will be rounded to [ ] decimal places.

d. 78.6015 will be rounded to [ ] decimal places.

2. Determine whether the bold digit is significant or not significant.

Example

Number	Significant	Not significant
0.004**5**6	☑	☐

Number	Significant	Not significant
I. 1.4**0**3	☐	☐
II. 3**5**.7	☐	☐
III. 0.**0**052	☐	☐
IV. 5**2**27	☐	☐

3. Fill in the blanks by using the words provided.

leading | **significant** | **not significant** | **truncating**

Rounding numbers to a certain number of decimal places or significant figures involves [ ] the number. There are rules for determining whether a figure is significant or not. In the number 5040, the zero between the 5 and 4 is [ ] whereas the zero after the 4 is [ ] . A zero that comes before the first non-zero digit is a [ ] zero, and they are always not significant.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Round the following numbers to the specified number of decimal places. WE1

a. 3.142 (2)

b. 158.356 (1)

c. 10.50437 (4)

d. 35.7435 (3)

e. 0.04573 (1)

f. 423.6666 (2)

g. 1308.44642578 (5)

h. 989.986 (1)

5. Determine the number of significant digits in the following numbers. WE2a,b

a. 6.128
b. 6604.09
c. 34 000
d. 0.630
e. 1800.0
f. 1.0000
g. 0.00502
h. 320 001

---

6. Round the following numbers to the specified number of significant figures. WE2c

a. 85.183 (3)
b. 159.875 (4)
c. 6.0103 (3)
d. 83 000 (1)
e. 0.004816 (2)
f. 78 530 (3)
g. 13.42050 (1)
h. 524 005 (5)

---

7. Complete the following calculations and round each result to the appropriate number of significant figures. WE3

a. 21.5×6.3
b. 12.44×0.31
c. $3.642 \div 0.17$
d. 35.05×0.070
e. 51.835×82.3
f. $1270 \div 4.002$
g. 8400×2.05313
h. $74.234 \div 3.17$

---

8. Which of the following correctly shows 0.00706 rounded to 2 significant figures?

A. 0.0
B. 0.007
C. 0.0071
D. 0.01
E. 0.72

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Round 0.20527 to 2 decimal places.

Student A

$0.20527 \approx 0.20$

Student B

$0.20527 \approx 0.21$

b. Round 2031.3526 to three significant figures.

Student A

$2031.3526 \approx 2030$

Student B

$2031.3526 \approx 203$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. On a particular day, an exchange rate agency lists one pound sterling (the currency of the United Kingdom) as equal to $1.8849926 in Australian dollars. How many Australian dollars is equivalent to 3 pound sterling, rounded to the nearest cent?

11. The distance between the Earth and the Sun is called 1 AU (astronomical unit). 1 AU is equal to 150 780 000 km. How many significant figures are there in this value?

12. Cecilia measured the length of a table using a metre ruler with centimetre markings. She measured it to be 1.5 m. She then used a centimetre ruler with millimetre markings to measure the same table. She measured it to be 154.3 cm.

Which ruler measures the length of the table to the greatest number of significant figures?

13. Jerone wants to figure out how much faster his new bike will be. He measured the diameter of one wheel of his new bike as 41.2 cm.

What is the circumference of the wheels, given the circumference can be calculated using $C = \pi d$, rounded to an appropriate number of significant figures?

14. The time (t, in hours) a car travels given particular speed (s, in km/h) and distance (d, in km) can be calculated using $t = \frac{d}{s}$. The speed of a car was recorded to be $50.\dot{3}$ km/h and the total distance travelled was 206 km. What is the difference between the times calculated using speed rounded to two and three significant figures? Round the answer to the nearest minute.

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. The following table shows the times it took to complete the race of four Formula 1 drivers at the Australian Grand Prix in 2023. A time of 1:17.308 corresponds to 1 minute and 17.308 seconds or 77.308 seconds. The order of the drivers represents the positions they started the race in (the starting grid).

Starting position	Driver	Finish time
1	Max Verstappen	1:16.732
3	Lewis Hamilton	1:17.104
6	Fernando Alonso	1:17.139
10	Nico Hulkenberg	1:17.675

a. State the number of significant figures and decimal places used to record the finish times after the times have been converted to seconds.

b. Round Max Verstappen's finish time to 1 decimal place.

c. Round Nico Hulkenberg's finish time to 2 significant figures.

d. What is the minimum number of significant figures that Lewis Hamilton's and Fernando Alonso's race times can be rounded to so that their finish order can be determined?

e. Compared to Formula 1 racing, give an example of a sport which measures finishing times with fewer significant figures. Briefly explain your answer.

16. The length and width of a wall were measured to be 3.552 m and 1.342 m respectively.

a. Round both measurements to 2 significant figures and then calculate the area of the wall using the rounded values. Round to an appropriate number of significant figures.

b. Round both measurements to 3 significant figures and then calculate the area of the wall using the rounded values. Round to an appropriate number of significant figures.

c. Calculate the area of the wall using the unrounded measurements and compare this to the answers to parts **a** and **b**. What can you conclude about what happens to the accuracy of a number when it is rounded?

Exam-style

17. A graph was made that shows the relationship between average temperature (y) and latitude (x) recorded at seven different weather stations. The equation of the graph is shown below. (1 MARK)

$y = 42.9842 - 0.877447x$

When the numbers in this equation are correctly rounded to three significant figures, the equation will be

A. $y = 42.984 - 0.877x$

B. $y = 42.984 - 0.878x$

C. $y = 43.0 - 0.878x$

D. $y = 42.9 - 0.878x$

E. $y = 43.0 - 0.877x$

18. The following equation is for a straight line graph. (2 MARKS)

$y = 0.0391 + 5.2756x$

a. Round the value for the intercept to two decimal places. 1 MARK

b. Round the value of the slope to two significant figures. 1 MARK

19. Leela measured the dimensions of a triangular shaped window to calculate the area of the fly screen she requires. The formula to calculate the area of a triangle is $A = \frac{1}{2}bh$. She measures the base as 0.78 m and the height as 2.14 m. (2 MARKS)

Calculate the area of the fly screen required, rounded to an appropriate number of significant figures.

20. Grant needs to round the values 9.235946... and 1.002493 to four significant figures. (2 MARKS)

He obtains 9.235 and 1.0025. Did Grant round these values correctly? Justify your answer.

Remember this?

21. Which of these is 718.839 rounded to 2 decimal places?

A. 718.80 **B.** 718.83 **C.** 718.84 **D.** 718.89 **E.** 719.00

22. Leslie spent 80% of her money on a tablet.

The tablet cost $250.

How much money did Leslie have before buying the tablet? Round to the nearest cent.

A. $250.00 **B.** $302.50 **C.** $312.50 **D.** $313.00 **E.** $350.00

23. The following table shows attendance at a festival over three nights.

Night	Number of people
Friday	31 200
Saturday	32 430
Sunday	29 500

The cost of each ticket is $5.

What was the mean amount of money collected from ticket sales per night, rounded to the nearest dollar.

A. $145 265 **B.** $155 215 **C.** $155 216 **D.** $155 217 **E.** $165 245

Back to contents

1B Profits and discounts

LEARNING INTENTIONS

Students will be able to:

- apply percentages to calculate mark-ups and discounts
- calculate the original price before a discount or mark-up
- calculate the percentage profit or loss made on a sale.

Percentages have many applications. They are particularly useful in financial contexts for both consumers and businesses. This lesson will involve calculating selling prices after mark-ups and discounts are applied, determining prices before mark-ups and discounts, and calculating percentage profits made on sales.

KEY TERMS AND DEFINITIONS

- The **sale price** is how much a product or service is sold for.
- The **cost price** is how much a product or service costs to produce or provide.
- A **profit** occurs when a product or service is sold for more than it cost.
- A **loss** occurs when a product or service is sold for less than it cost.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Dmytro Ostapenko/Shutterstock.com

Percentages are used in financial contexts to help make informed decisions. Discounts and mark-ups encourage customers to buy more products, or increase the profit earned on products. Percentage profits and losses allow a business to determine how well they are performing.

Key ideas

1. **Mark-ups** and **discounts** are percentage increases and decreases respectively applied to the original sale price of a product or service. The percentage mark-ups and discounts, and the original price after a mark-up or discount can also be calculated.

$$\text{Amount to mark-up or discount by} = \%\ \text{Increase or decrease} \times \text{Original price}$$

$$\%\ \text{mark-up or discount} = \frac{\text{Change in price}}{\text{Original price}} \times 100$$

2. **Profits** and **losses** are usually reported as a percentage of the cost price of a product or service.

$$\text{profit} = \text{selling price} - \text{cost}$$

$$\text{loss} = \text{cost} - \text{selling price}$$

$$\%\ \text{profit or loss} = \frac{\text{profit or loss}}{\text{cost price}} \times 100$$

Worked example 1

Calculating mark-ups and discounts

For each of the following, calculate the new price for the given mark-up or discount.

a. $400 decreases by 25%

WE1a

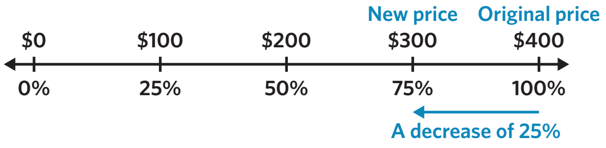
Method 1

Working	Thinking
$25\% \text{ of } \$400 = 0.25 \times 400$ $= \$100$	**Step 1:** Calculate the dollar amount of the decrease.
$\text{New price} = \$400 - \$100$ $= \$300$	**Step 2:** Subtract the dollar amount from the original amount.

Method 2

Working	Thinking
$100\% - 25\% = 75\%$	**Step 1:** Calculate the percentage decrease.
$\text{New price} = 75\% \text{ of } \$400$ $= 0.75 \times 400$ $= \$300$	**Step 2:** Calculate the new price after the percentage increase.

Visual support



b. $80 increases by 33%

WE1b

Method 1

Working	Thinking
$33\% \text{ of } \$80 = 0.33 \times 80$ $= \$26.40$	**Step 1:** Calculate the dollar amount of the increase.
$\text{New price} = \$80 + \$26.4$ $= \$106.40$	**Step 2:** Add the dollar amount to the original amount.

Method 2

Working	Thinking
$100\% + 33\% = 133\%$	**Step 1:** Calculate the percentage increase.
$\text{New price} = 133\% \text{ of } \$80$ $= 1.33 \times 80$ $= \$106.40$	**Step 2:** Calculate the new price after the percentage increase.

Student practice

For each of the following, calculate the new price for the given mark-up or discount.

a. $200 decreases by 26%.

b. $50 increases by 40%.

Back to contents

Worked example 2

Calculating percentage changes and original prices

For the following:

a. Calculate the percentage mark-up on an item whose price increased from $10.50 to $15.25. Round to the nearest percent.

WE2a

Working	**Thinking**
$15.25 - 10.50 = 4.75$	**Step 1:** Calculate the change in price.
$\text{Mark-up} = \frac{4.75}{10.50} \times 100$ $= 45.238...\%$ $\approx 45\%$	**Step 2:** Calculate the percentage mark-up.

Visual support

$$\text{Mark-up} = \frac{\overset{\text{Change in price}}{4.75}}{\underset{\text{Original price}}{10.50}} \times 100$$

$$\approx 45\%$$

b. Calculate the original price of an item if the new price is $75 after a 25% mark-up.

WE2b

Working	**Thinking**
$100\% + 25\% = 125\%$ $= 1.25$	**Step 1:** Calculate the percentage increase.
$\text{Original price} = \frac{75}{1.25}$ $= \$60$	**Step 2:** Calculate the original price before the mark-up.

c. Calculate the original price of an item if the new price is $53.50 after a 15% discount. Round to the nearest cent.

WE2c

Working	**Thinking**
$100\% - 15\% = 85\%$ $= 0.85$	**Step 1:** Calculate the percentage decrease.
$\text{Original price} = \frac{53.50}{0.85}$ $= \$62.941...$ $\approx \$62.94$	**Step 2:** Calculate the original price before the discount.

Student practice

For the following:

a. Calculate the discount percentage on an item whose price decreased from $150 to $110. Round to the nearest percent.

b. Calculate the original price of an item if the new price is $23.50 after a 20% mark-up. Round to the nearest cent.

c. Calculate the original price of an item if the new price is $450 after a 32% discount. Round to the nearest cent.

Back to contents

Worked example 3

Calculating percentage profits and losses

Calculate the percentage profit or loss for each of the following. Round to the nearest percent as needed.

a. A book that cost \$5 to make is sold for \$13.

WE3a

Working

$$\begin{aligned} \text{profit} &= \text{selling price} - \text{cost price} \\ &= 13 - 5 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \%\ \text{profit} &= \frac{8}{5} \times 100 \\ &= 160\%\ \text{profit} \end{aligned}$$

Thinking

Step 1: Calculate the profit made.

Step 2: Calculate the profit as a percentage of the cost price.

Visual support

$$\%\ \text{profit} = \frac{\overset{\text{Profit}}{8}}{\underset{\text{Cost price}}{5}} \times 100$$

$$= 160\%$$

b. A TV that cost \$3000 to make is sold for \$2500.

WE3b

Working

$$\begin{aligned} \text{loss} &= \text{cost price} - \text{selling price} \\ &= 3000 - 2500 \\ &= 500 \end{aligned}$$

$$\begin{aligned} \%\ \text{loss} &= \frac{500}{3000} \times 100 \\ &= 16.\dot{6}\% \\ &\approx 17\%\ \text{loss} \end{aligned}$$

Thinking

Step 1: Calculate the loss made.

Step 2: Calculate the loss as a percentage of the cost price.

Student practice

Calculate the percentage profit or loss for each of the following. Round to the nearest percent as needed.

a. A water bottle that cost \$2.50 to make is sold for \$8.

b. A phone that cost \$350 to make is sold for \$200.

1B Questions

Understanding worksheet

1. Calculate the profits and losses for each of the following.

Example

Cost: $350
Sale price: $299
Loss: $51

a. Cost: $99
Sale price: $120
Profit: ______

b. Cost: $820
Sale price: $749
Loss: ______

c. Cost: $899
Sale price: $1999
Profit: ______

d. Cost: $1502
Sale price: $1050
Loss: ______

2. Convert the following percentages increases and decreases to decimals by filling in the boxes.

Example

Increase by 15%: 100% + 15% = 1. 15

a. Increase by 20%: 100% + 20% = 1. ______

b. Decrease by 30%: 100% − 30% = 0. ______

c. Increase by 2%: 100% + 2% = ______

d. Decrease by 3%: 100% − 3% = ______

3. Fill in the blanks by using the words provided.

mark-up **cost price** **loss** **sale price**

A business may decide to apply a ______ on a product. This results in its ______ increasing. When a product is sold for less than its cost price, a ______ is made. Dividing a profit amount by the ______ calculates the percentage profit of a product.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. For each of the following, calculate the new price for the given mark-up or discount. Round to the nearest cent as needed. WE1

a. $100 increases by 15%.

b. $250 decreases by 20%.

c. $80 increases by 4%.

d. $350 decreases by 8%.

e. $600 increases by 5.5%.

f. $45 decreases by 14.7%.

g. $1200 increases by 126%.

h. $9000 decreases by 42.52%.

Back to contents

5. Calculate the percentage mark-ups and discounts for each of the following. Round to the nearest percent as needed. WE2a

a. $200 increases to $230.

b. $250 decreases to $150.

c. $382.50 increases to $450.

d. $350 decreases to $245

e. $420 increases to $840.

f. $437.90 decreases to $378.15.

g. $35 increases to $140.

h. $902.10 decreases to $490.50.

---

6. Calculate the original price of each of the following. Round to the nearest cent as needed. WE2b

a. A TV that had a mark-up of 20% and is now priced at $1200.

b. A book that had a discount of 15% and is now priced at $10.20.

c. A tennis racquet that had a mark-up of 5% and is now priced at $249.99.

d. A graphic novel that had a discount of 50% and is now priced at $12.

e. A pair of wireless headphones that had a mark-up of 8.5% and is now priced at $40.

f. A PS5 that had a discount of 27.75% and is now priced at $799.99.

g. A smartphone that had a mark-up of 10.5% and is now priced at $399.99.

h. A vacation package that had a discount of 15.75% and is now priced at $2000.

---

7. Calculate the percentage profit or loss for each of the following. Round to the nearest percent as needed. WE3

a. A remote-controlled drone that cost $40 to make sells for $50.

b. An artwork that cost $200 to make is sold for $100.

c. A musical instrument that cost $50 to make sells for $70.

d. A set of art pencils that cost $30 to make sells for $20.

e. A movie ticket that is worth $21.95 sells for $24.50

f. A sports watch that cost $350 to make sells for $299.

g. A chess board that cost $499 to make sells for $542.95

h. A laptop that cost $2442.02 to make sells for $1099.95

---

8. Calculate the original price of a bicycle that had a mark-up of 20% and is now priced at $599.99, rounded to the nearest cent.

A. $120.00
B. $479.99
C. $499.99
D. $579.99
E. $719.99

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the new price of a $325 item that is discounted by 7%. Round to the nearest dollar as needed.

Student A

Method 1

100% − 7% = 93%

New price = $325 − 93

= $232

Student B

Method 1

100% − 7% = 93%

New price = 93% of $325

= 0.93 × $325

= $302.25

≈ $302

Back to contents

b. Calculate the percentage profit or loss on a car that cost \$12 000 to make and sold for \$9750. Round to the nearest percent as needed.

Student A

$$\begin{aligned}\text{Loss} &= 12\,000 - 9750 \\ &= 2250 \\ \%\ \text{loss} &= \frac{2250}{9750} \times 100 \\ &= 23.076... \\ &\approx 23\%\end{aligned}$$

Student B

$$\begin{aligned}\text{Loss} &= 12\,000 - 9750 \\ &= 2250 \\ \%\ \text{loss} &= \frac{2250}{12\,000} \times 100 \\ &= 18.75 \\ &\approx 19\%\end{aligned}$$

Problem solving

Question working paths

Mild 10, 11, 12	**Medium** 11, 12, 13	**Spicy** 12, 13, 14

10. M&H, a clothing store, offers a 15% birthday discount on the price of any single item. It is Quinn's birthday and she is considering buying a shirt that costs \$35.45. After the discount, how much would Quinn pay?

11. Patrick is shopping for dress shoes and sees a pair of Julius Marlow shoes on sale. He looks at the sticker and sees that the shoes have been reduced from \$159.99 to \$100. Calculate the percentage discount on the shoes, rounded to the nearest percent.

12. Sam and Alex bought a house in 2018 for \$650 000 and sold it three years later for \$796 278. What was the percentage profit made on the sale of the house, rounded to the nearest percent?

13. Olivia saw on social media that her favourite bubble tea shop is increasing the prices of their bubble teas by 8%. When she next visits the shop, the new price is \$5.20. Determine the original price of the bubble tea before the increase in price, to the nearest cent.

14. Gareth is analysing the profits and costs of his video game business over the last three weeks. The following table shows the amounts, in dollars, of sales and costs in each week.

Week	**1**	**2**	**3**
Sales	\$13 032	\$8571	\$20 102
Costs	\$10 023	\$7053	\$18 281

In which week did Gareth achieve the highest percentage profit?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	**Medium** 15 (a,b,c,e), 16 (a,b)	**Spicy** All

15. Salima runs a specialty bakery. Her most popular item is a gluten-free croissant, which she sells 300 of per week for \$7.50. She also sells vegan pain-au-chocolates. However, her operating costs have increased recently and she needs to increase her prices to keep the business profitable. She decides to apply a store-wide mark-up of 12%.

a. How much will the gluten-free croissant cost after the mark-up?

b. If the new price of her vegan pain-au-chocolates is \$6.50, what was the original price? Round the answer to the nearest cent.

Back to contents

c. What percentage profit does she expect to make on the gluten-free croissants after the price increase, given they cost \$4.85 to make? Round the answer to the nearest percent.

d. One week after increasing her prices, she sells 250 gluten-free croissants. What is the difference in profit compared to the week before the mark-up? Assume the cost of making croissants before the mark-up was \$4.33.

e. Apart from increasing her prices, what are two other changes Salima could do to reduce her operating costs?

16. An item at a supermarket costs \$10.

a. The item is marked-up by 20% then discounted by 20%. What is the new price?

b. The original price of the item is multiplied by the product of 1.2×0.8. What is the result of this calculation?

c. Using your answers from parts **a** and **b**, explain how the results of repeated percentage increases and decreases can be calculated in one step.

Exam-style

17. Justin makes and sells electrical circuit boards. (1 MARK)

He has one fixed cost of \$420 each week.

Each circuit board costs \$15 to make.

The selling price of each circuit board is \$27.

The weekly profit if Justin makes and sells 200 circuit boards per week is

A. \$1980 **B.** \$2400 **C.** \$2820 **D.** \$4980 **E.** \$5400

18. Amy makes and sells quilts. (2 MARKS)

Each quilt sells for \$60 and costs \$36 to make.

a. Calculate the percentage profit, rounded to the nearest percent. 1 MARK

b. If Amy discounts the price of the quilt by 10%, how much is the new price of the quilt? 1 MARK

19. A manufacturer makes and sells heaters. (2 MARKS)

The fixed cost to manufacture the heaters is \$16 000 per month.

Each heater costs \$52 to produce.

The selling price of each heater is \$280.

What is the minimum number of heaters needed to be sold per month in order to make a profit?

20. In 2021, it cost \$2.45 for a car to travel on a toll road. Due to inflation, this amount increased to \$2.52 in 2022. (3 MARKS)

If the price of fuel increases by the same percentage amount, calculate the price of fuel in 2022 if it costs \$1.56 per litre in 2021. Round to the nearest cent.

Remember this?

21. $2.3 \times \star = 1.38$

What value of ★ makes this number sentence correct?

A. −0.6 **B.** −0.4 **C.** 0.4 **D.** 0.6 **E.** 1.3

Back to contents

22. James and Tamara saved money for a new Xbox game over a month.

If Tamara had collected $20 more, she would have collected exactly twice as much as James.

Which row in the table shows how much money they both could have collected?

	James	**Tamara**
A.	$15	$35
B.	$24	$28
C.	$36	$72
D.	$49	$94
E.	$50	$101

23. A group of 115 people were asked if they need to wear glasses or not.

This table shows the results.

	Need glasses	**Do not need glasses**	**Total**
Men	10	45	55
Women	25	35	60
Total	35	80	115

A woman was selected at random.

What is the probability she does not need glasses, rounded to two decimal places

A. 0.30 **B.** 0.42 **C.** 0.52 **D.** 0.58 **E.** 0.70

Back to contents

1C Income

LEARNING INTENTIONS

Students will be able to:

- calculate wages with and without overtime and weekend pay
- convert salaries between weekly, fortnightly, monthly and annual rates
- understand and calculate income from commissions, piece work and royalties.

This lesson will involve calculating wages for hours worked during a week, including any overtime pay accrued. For workers on salaries, they are paid fixed amounts each paycheck. It is possible to calculate how much income is earned daily, fortnightly and monthly to assist with budgeting. Some jobs also involve being paid on commission, piece work or royalties.

KEY TERMS AND DEFINITIONS

- **Overtime** is any time worked in excess of reasonable work hours and usually attracts time-and-a-half or double pay.
- Hours worked at **time-and-a-half** pay is equal to 1.5 times the hourly rate.
- Hours worked at **double pay** is equal to 2 times the hourly rate.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Grusho Anna/Shutterstock.com

If you have a job, you will likely receive a pay slip. It shows you how much you earned as a salary or wage, including any overtime and weekend pay. Knowing how to calculate your wages yourself can help you budget your spending and living costs.

Key ideas

1. **Salaries** are fixed yearly amounts, usually paid to full or part-time employees monthly or fortnightly. **Wages** are incomes earned from fixed hourly rates for hours worked, usually paid to casual or contractor workers. Casual and weekend work may attract pay at increased rates.

2. There are alternative forms of incomes that apply to certain types of employment.

Type of income	Description	Calculation
Commission payments	Payments based on how much an employee sells.	Flat fee or percentage of each sale
Piece work	Payments made based on the amount an employee picks, packs or makes.	Pay = piece rate × number of pieces
Royalty payments (or **royalties**)	Payments made to a person for the rights to use another person's work.	Rate or percentage of ongoing sales

3. Salaries and wages can be converted between annual (yearly), monthly, fortnightly, weekly and daily rates.

1 year = 12 months = 26 fortnights = 52 weeks = 365/366 days

1 fortnight = 2 weeks = 14 days

1 week = 7 days

Back to contents

Worked example 1

Calculating wages

Calculate the income earned in the following scenarios. Round to the nearest cent as needed.

Assume overtime and Saturday pay are paid at time-and-a-half, and Sunday pay is 1.75 times the base rate.

a. 5.5 hours worked at a rate of \$21.38 per hour.

WE1a

Working	Thinking
$5.5 \times 21.38 = \$117.59$	Multiply the hours worked by the rate.

Visual support

b. 6.89 hours worked on a Sunday at a base rate of \$27.42.

WE1b

Working	Thinking
$6.89 \times 27.42 \times 1.75 = \$330.616...$ $= \$330.62$	Multiply the hours worked by the base rate and the Sunday rate.

c. 10.45 hours, including 0.45 hours of overtime, worked at a base rate of \$25.65 per hour.

WE1c

Working	Thinking
$10.45 - 0.45 = 10$	**Step 1:** Calculate the normal working hours.
$10 \times 25.65 = \$256.50$	**Step 2:** Multiply the normal working hours by the base rate.
$0.45 \times 25.65 \times 1.5 = \$17.313...$	**Step 3:** Multiply the overtime hours by base the rate and 1.5.
$256.50 + 17.313... = \$273.813...$ $\approx \$273.81$	**Step 4:** Calculate the total wage earned.

Student practice

For each of the following, calculate the income earned. Round to the nearest cent as needed.
Assume overtime and Saturday pay are paid at time-and-a-half, and Sunday pay is 1.75 times the base rate.

a. 8.35 hours worked at a rate of \$27.43 per hour.

b. 4.25 hours worked on a Saturday at a base rate of \$29.42.

c. 12.53 hours, including 0.53 hours of overtime, at a rate of \$24.63 per hour.

Back to contents

Worked example 2

Calculating wages from alternative sources of income

For each of the following, calculate the income earned. Round to the nearest cent as needed.

a. A travel agent earns 7% commission on a $2300 sale. WE2a

Working

$$7\% \text{ of } \$2300 = \frac{7}{100} \times 2300$$
$$= \$161$$

Thinking

Calculate the dollar amount of the commission.

b. A fruit picker picks 20 kg of fruit and is paid $13 per kg. WE2b

Working

$13 \times 20 = \$260$

Thinking

Multiply the picking rate by the amount picked.

c. A YouTuber has 54 232 views on a video and receives royalties of $0.00418 per view. WE2c

Working

$$54\ 232 \times 0.00418 = \$226.689\ldots$$
$$\approx \$226.69$$

Thinking

Multiply the number of views by the amount of dollars per view.

Student practice

For each of the following, calculate the income earned. Round to the nearest cent as needed.

a. A book seller earns 5% commission on a $40 sale.

b. A photographer takes 40 photos and is paid $0.50 per photo.

c. A video presenter receives royalties at a rate of $0.05 per subscriber, and has 500 subscribers.

Worked example 3

Converting rates

Convert the following salaries to the given rates. Round to the nearest cent as needed.

a. $1754 per fortnight (annual) WE3a

Working

$1754 \times 26 = \$45\ 604$

Thinking

Multiply the fortnightly salary by the number of fortnights in a year (26).

Visual support

Continues →

b. $64 032 per year (weekly)

WE3b

Working	**Thinking**
$64\ 032 \div 52 = 1231.384...$ $\approx \$1231.38$	Divide the yearly salary by the number of weeks in a year (52).

c. $900 over 5 days, 7.5 hours worked per day (hourly)

WE3c

Working	**Thinking**
$900 \div 5 = \$180$ per day	**Step 1:** Divide the weekly salary by the number of days worked.
$\$180 \div 7.5 = \$24$ per hour	**Step 2:** Divide the daily salary by the hours worked each day.

Student practice

Convert the following salaries to the given rates. Round to the nearest cent as needed.

a. $1342 per fortnight (annual)

b. $76 324 per year (weekly)

c. $1025 over 4 days, 8 hours worked per day (hourly)

1C Questions

Understanding worksheet

1. For each of the following rates and calculations, write in the box what new rate is being calculated (weekly, fortnightly, monthly or annual).

Example

Rate	Calculation	New rate
$200 per day	× 7	Weekly

Rate	Calculation	New rate
$1000 per week	× 52	
$3000 per month	÷ 2	
$4500 per month	× 12	
$45 000 per year	÷ 26	

2. Match the following scenarios to the type of income earned.

Scenario			Type of income
Spotify pays a musician $0.003 per song streamed.	●	●	Commission
A real estate agent earns 2% of the sale price of a property.	●	●	Wage
A barista earns $23.63 per hour.	●	●	Piece work
A pear picker earns $35 per bin of fruit picked	●	●	Royalty

3. Fill in the blanks by using the words provided.

royalty **salary** **piece work** **percentage**

There are many different ways to earn income. When a worker earns a fixed amount of pay regularly, they earn a . When a worker is paid for the rights to use their work, they earn a . When a worker earns commission, it is usually calculated as a of the sales they made. When a worker earns a wage based on a number of tasks completed, they are completing .

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f), 7 (e,f,g,h), 8

4. Calculate the income earned in the following scenarios. Round to the nearest cent as needed. **WE1a**

- **a.** 4 hours worked at a rate of $21.50 per hour.
- **b.** 2.5 hours worked at a rate of $22.75 per hour.
- **c.** 5.75 hours worked at a rate of $26.90 per hour.
- **d.** $6\frac{1}{2}$ hours worked at $43.54 per hour.
- **e.** 7.83 hours worked at $29.44 per hour.
- **f.** 8 hours and 20 minutes worked at a rate of $32.42 per hour.
- **g.** 6 hours and 45 minutes worked at a rate of $27.25 per hour.
- **h.** 7 hours and 23 minutes worked at a rate of $38.34 per hour.

---

5. Calculate the income earned in the following scenarios. Round to the nearest cent as needed. Assume overtime and Saturday pay are paid at time-and-a-half, and Sunday pay is 1.75 times the base rate. **WE1b,c**

- **a.** 8 hours worked on a Saturday with a base rate of $24.50 per hour.
- **b.** 4.5 hours worked on a Sunday at a base rate of $23.25 per hour.
- **c.** 11 hours worked, including 1 hour of overtime, at a base rate of $26.30 per hour.
- **d.** 6.32 hours worked on a Saturday at a base rate of $25.74 per hour.
- **e.** $3\frac{3}{4}$ hours worked on a Sunday at a base rate of $32.52 per hour.
- **f.** 9 hours and 35 minutes worked on a Saturday at a base rate of $23.42 per hour.
- **g.** 3 hours and 54 minutes worked on a Sunday at a base rate of $36.42 per hour.
- **h.** 12.54 hours worked, including 2.54 hours of overtime, on a Sunday at a base rate of $24.63 per hour.

---

6. Calculate the income earned in the following scenarios. Round to the nearest cent as needed. **WE2**

- **a.** A shoe store clerk receives 10% commission on $450 worth of sales.
- **b.** A supermarket store picker packs 5 bags and is paid $5 per bag.
- **c.** A sound effects artist receives 2% royalties on the sale of a $40 sound effects package.
- **d.** A mortgage broker receives 0.70% commission on a $500 000 home loan.
- **e.** A newspaper delivery person receives $20 for every 40 newspapers delivered, and delivers 160.
- **f.** An animator receives 5% royalties on the first 1000 views of their work, and 8% thereafter. They had 1680 views and each view is worth $0.50.

---

7. Convert the following salaries to the given rates. Round to the nearest cent as needed. **WE3**

- **a.** $120 per day (weekly)
- **b.** $50 000 per year (monthly)
- **c.** $1500 per fortnight (yearly)
- **d.** $2753 over 12 days, 6 hours worked per day (hourly)
- **e.** $264 per day (yearly (365 days))
- **f.** $65 230 per year, 7 hours worked per day 5 days per week (hourly)
- **g.** $1032 per week (half-yearly)
- **h.** $10 985 per quarter (weekly)

Back to contents

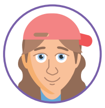
8. Calculate the income earned for 7.56 hours worked on a Sunday at a base rate of \$30.24. Sunday pay is paid at 1.75 times the base rate.

A. \$228.61 **B.** \$230.36 **C.** \$241.84 **D.** \$281.53 **E.** \$400.08

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. A baker worked 11.04 hours, including 1.04 hours of overtime, at a base rate of \$27.54 per hour. How much income was earned?



Student A

$11.04 - 10 = 10$

$10 \times 27.54 = \$270.54$

$1.04 \times 27.54 \times 1.5 = \$42.962\ldots$

$270.54 + 42.962\ldots = \$313.502\ldots$

$\approx \$313.50$

Student B

$11.04 \times 27.54 = \$304.041\ldots$

$1.04 \times 27.54 \times 1.5 = \$42.962\ldots$

$304.041\ldots + 42.962\ldots = \$347.004$

$\approx \$347.00$

b. A delivery driver who earns \$30 for every 10 packages delivered. Calculate how much income he earns delivering 85 packages.

Student A

$1 \text{ package} = \dfrac{30}{10}$

$= \$3$

$\$3 \times 85 = \$255$

Student B

$\$30 \times 85 = \$2550$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Maddie picked up a Saturday shift at the cafe she works at. She will work from 10 am until 4 pm. Her base rate is \$26.72 per hour, but she gets time-and-a-half pay on Saturdays. Determine the amount of money she expects to earn for this shift.

11. Cal is a retail worker who earns \$27.91 per hour, as well as 5% commission on any sales made. On a particular day, he worked 6.54 hours and sold \$680 worth of goods. How much did he make on this day?

12. Jane has a side-gig at an e-commerce warehouse as a picker and packer. She earns a packing rate for each item she picks and packs for delivery. The packing rate is \$1.25 per item. She usually packs 105 items per day. If she works 3 times a week, calculate how much she expects to earn in one week.

13. Salima is an author of two best selling books. She receives royalties of 7.5% for her first book, and 10% for her second book. Both books retail at \$19.95. In one year, 1500 copies of her first book and 3400 copies of her second book were sold. How much income from royalties will she earn?

14. Alex works as a hotel receptionist. In one week, they worked 20 hours, which included 7 hours on a public holiday and 5 hours on Sunday. They received \$1263.52 in their bank account.

What is their hourly wage? Assume they earn 1.75 times their base rate for Sunday work, and double-pay for public holiday work.

Back to contents

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Tobi just finished his first week at a new casual job at a computer hardware store. His hourly rate is \$19.76 and he receives overtime pay at a time-and-a-half rate for hours worked above 10 in a single shift. If he works on a weekend, he is paid 1.75 times his hourly rate for the whole shift. He also earns 8% commission on any sales made. The table below shows the hours he worked during the week:

	Mon	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
Hours worked	5:00 pm – 11:00 pm		7:00 am – 12:15 pm		7:30 am – 6:30 pm		7:30 am – 3:30 pm

a. How much did Tobi earn working on Wednesday?

b. On average, he sells \$850 worth of hardware each day he works. How much commission income did he earn for the week?

c. Calculate Tobi's total earnings from his wages for the whole week.

d. Tobi is saving up to go on holiday. He estimates he'll need \$6000. He is able to put 20% of his earnings per week into savings. Assuming he works the same hours and earns the same commission each week, after how many weeks will he be able to afford his holiday?

e. What are two ways that Tobi could reduce the amount of time he needs to spend saving?

16. A worker whose base hourly rate is \$35.83 can choose to work on one of the days described in parts **a** and **b**. Calculate their total pay for each given option.

a. 6 hours of work at 1.5 times the base hourly rate.

b. 4 hours of work at 1.75 times the base hourly rate.

c. Compare the two results from parts **a** and **b** and decide which of the given days results in better pay. How can this be determined by comparing the reduction in hours and the increase in pay rates only?

Exam-style

17. The income earned in 7.5 hours, including 2 hours of overtime at time-and-a-half rate, when the base hourly pay is \$27.85, is closest to (1 MARK)

A. \$212 **B.** \$220 **C.** \$237 **D.** \$282 **E.** \$313

18. A computer store clerk's hourly base pay rate is \$30 and a normal work day is 7 hours long. Pay is doubled on a public holiday. (3 MARKS)

a. Calculate the store clerk's income after a normal work day. 1 MARK

b. Calculate the store clerk's income after a work day that falls on a public holiday. 1 MARK

c. Calculate the clerk's income after a work day that falls on a public holiday on which they also make 10% commission from a computer that sold for \$1300. 1 MARK

19. The given table shows Darryl's roster for the week. On Sundays, he gets paid 1.75 times his base hourly pay rate of \$22.45, while on Saturdays he gets 1.5 times his base hourly pay rate. (3 MARKS)

Sunday	**Tuesday**	**Thursday**	**Friday**	**Saturday**
11 am to 4 pm	9 am to 4 pm	9 am to 4 pm	12 pm to 8 pm	10 am to 4 pm

Calculate Darryl's total income for the week shown in the roster. Round to the nearest cent.

Back to contents

20. A newly appointed doctor has a yearly salary of \$122 508. They work an average of 40 hours every week for 47 weeks of the year. After all the tax margins have been applied, around 27% of their total earnings are taxed. Calculate the hourly take home pay of a doctor working under the described conditions. (3 MARKS)

Remember this?

21. Preston has a LEGO scale model of a castle that is 30 cm wide and 20 cm high. He wants to enlarge it so that it has a width of 60 cm.

What will be the new height of the model?

A. 10 cm **B.** 20 cm **C.** 40 cm **D.** 50 cm **E.** 80 cm

22. Yindi is buying a fitness watch. Some of the watches are on sale.

Which fitness watch will be the cheapest?

A. \$150
B. \$160, take 5% off
C. \$180, take 10% off
D. \$200, take 30% off
E. \$325, take 50% off

23. At the dessert store, a jumbo cookie and a deluxe donut each cost \$5.

The store owner uses a formula to determine the amount of money collected from the sale of x jumbo cookies and y deluxe donuts daily.

Which formula could the store owner use to determine the correct amount of money collected daily, in dollars.

A. $5xy$ **B.** $10xy$ **C.** $5x + y$ **D.** $10(x + y)$ **E.** $5(x + y)$

Back to contents

1D Taxation

LEARNING INTENTIONS

Students will be able to:

- calculate withheld tax using an online tax calculator
- understand and calculate net income
- understand and calculate taxable income, income tax and simple tax returns.

Income tax is a crucial aspect of personal finance that affects everyone who earns an income. Every pay cycle, employees have tax withheld from their gross earnings. At the end of the year, the total income tax owed is calculated and compared to the tax already paid. Understanding how this process occurs will allow you to estimate how much tax you may owe or be owed.

KEY TERMS AND DEFINITIONS

- **Gross income** is the money earned from a salary or wage before tax.
- The **Australian Taxation Office (ATO)** manages taxation on behalf of the government.
- **Withheld tax** is money taken from gross income by an employer and paid to the ATO.
- **Tax deductions** are certain expenses that reduce taxable income, and therefore the amount of income tax paid.
- **PAYG** stands for Pay-As-You-Go and means that tax is withheld each pay cycle rather than paid at the end of the financial year.
  In Australia, the **financial year** is between July of one year to June of the next year.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Andrey_Popov/Shutterstock.com

Some people and businesses employ accountants to assist them with their finances. Part of an accountant's job may be to submit tax returns. They need to understand the taxation system to ensure their clients are paying their fair share of tax.

Key ideas

1. In a pay cycle, **net income** is the money received after withholding tax from the **gross income**. The amount of withheld tax depends on the weekly earnings.

Net income = gross income − withheld tax

2. **Taxable income** is the difference between gross income for a whole year and any **tax deductions**. This is used to calculate the **income tax** owed.

Taxable income = gross income − tax deductions

Taxable income range	Income tax
$0–$18 200	Nil
$18 201–$45 000	19 cents for each $1 over $18 200
$45 001–$120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001–$180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

Resident tax rates for 2022–2023, excluding the Medicare levy. (https://www.ato.gov.au/rates/individual-income-tax-rates/)

Continues →

3. The amount of tax owed to or owed by the ATO is the difference between the calculated income tax, and the sum of all PAYG tax withheld every pay cycle.

Tax refund or tax owed = income tax − total withheld tax

Worked example 1

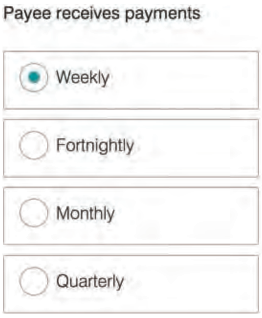
Calculating withheld tax

For each of the following earnings, calculate the amount of withheld tax using the online tax calculator.
(https://www.ato.gov.au/Calculators-and-tools/Host/?anchor=TWC&anchor=TWC/questions#TWC/questions)

a. $942 per week

WE1a

Working



Payee receives payments

● Weekly

○ Fortnightly

○ Monthly

○ Quarterly

Payee's gross weekly earnings *

$ 942 .00

Withheld tax = $142

Thinking

Step 1: Select 'Weekly'.

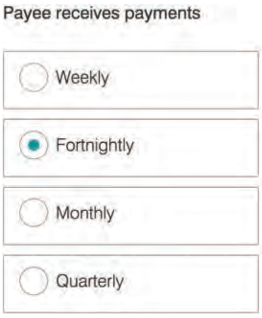
Step 2: Enter the gross weekly earnings.

Step 3: Press 'Calculate'.

b. $2452 per fortnight

WE1b

Working



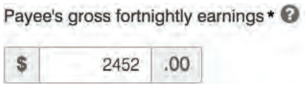
Payee receives payments

○ Weekly

● Fortnightly

○ Monthly

○ Quarterly



Payee's gross fortnightly earnings *

$ 2452 .00

Withheld tax = $480

Thinking

Step 1: Select 'Fortnightly'.

Step 2: Enter the gross fortnightly earnings.

Step 3: Press 'Calculate'.

Student practice

For each of the following earnings, calculate the amount of withheld tax using the online tax calculator.

a. $1504 per week **b.** $4635 per fortnight

Back to contents

Worked example 2

Calculating income tax

Consider the following scenarios.

a. A worker's taxable income is \$54 035. Calculate the income tax owed, rounded to the nearest cent. **WE2a**

Working	Thinking
\$5092 plus 32.5 cents for each \$1 over \$45 000	**Step 1:** Identify the tax rate for this income.
$54\,035 - 45\,000 = \$9035$	**Step 2:** Calculate the portion of the taxable income above the threshold.
$\text{Income tax} = 5092 + 0.325 \times 9035$ $= \$8029.38$	**Step 3:** Calculate the income tax.

b. A worker's income is \$46 032. They can claim \$3521 worth of tax deductions. Calculate the income tax owed. Rounded to the nearest cent. **WE2b**

Working	Thinking
$46\,032 - 3521 = \$42\,511$	**Step 1:** Subtract the tax deductions from the income.
19 cents for each \$1 over \$18 200	**Step 2:** Identify the tax rate for this income.
$42\,511 - 18\,200 = \$24\,311$	**Step 3:** Calculate the portion of the taxable income above the threshold.
$\text{Income tax} = 24\,311 \times 0.19$ $= \$4519.09$	**Step 4:** Calculate the income tax.

Student practice

Consider the following scenarios.

a. A worker's taxable income is \$36 974. Calculate the income tax owed, rounded to the nearest cent.

b. A worker's income is \$125 531. They can claim \$7832 worth of tax deductions.
Calculate the income tax owed. Rounded to the nearest cent.

1D Questions

Understanding worksheet

1. For each of the following, calculate the missing values.

Example

Gross earnings	Withheld tax	Net income
$5923	$1252	$4671

Gross earnings	Withheld tax	Net income
$850	$112	
	$136	$1164
$3750		$3252
	$1182	$3300

2. Match each given taxable income to the corresponding row in the tax table.

Taxable income

- $54 000 ●
- $12 500 ●
- $93 200 ●
- $123 800 ●

	Taxable income range	Tax on this income
●	$0–$18 200	Nil
●	$18 201–$45 000	19 cents for each $1 over $18 200
●	$45 001–$120 000	$5092 plus 32.5 cents for each $1 over $45 000
●	$120 001–$180 000	$29 467 plus 37 cents for each $1 over $120 000
●	$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

3. Fill in the blanks by using the words provided.

withheld tax **taxable income** **income tax** **deductions**

Every pay cycle, tax is withheld. At the end of the financial year, ______ is calculated using a workers ______ and this is compared to the total ______.

Taxable income can be reduced using ______.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7

4. For each of the following earnings, calculate the amount of withheld tax using the online tax calculator from the ATO.gov.au website. WE1

a. $680 per week **b.** $1450 per fortnight **c.** $6500 per month **d.** $1220 per fortnight

e. $852 per week **f.** $9124.53 per month **g.** $1795 per fortnight **h.** $2430.51 per month

Back to contents

5. For each of the following gross incomes, calculate the payable income tax. Where necessary, rounded to the nearest cent. WE2a

a. \$40 000 **b.** \$54 500 **c.** \$132 300 **d.** \$16 520
e. \$86 923 **f.** \$93 685 **g.** \$10 853 **h.** \$236 821

6. For each of the following gross incomes with deductions, calculate the payable income tax. WE2b

a. Gross income: \$38 000, deductions: \$200
b. Gross income: \$67 500, deductions: \$350
c. Gross income: \$125 490, deductions: \$1532
d. Gross income: \$45 320, deductions: \$450
e. Gross income: \$18 632, deductions: \$325
f. Gross income: \$243 024, deductions: \$10 402

7. Calculate the payable income tax on a gross income of \$123 234 with \$5023 worth of deductions, rounded to the nearest cent.

A. \$25 640.58 **B.** \$28 885.58 **C.** \$30 663.58 **D.** \$33 176.00 **E.** \$38 415.65

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Reed's gross income for a year is \$42 053. How much income tax do they owe?

Student A

$\text{Income tax} = 42\,053 \times 0.19$

$= \$7990.07$

Student B

$42\,053 - 18\,200 = \$23\,853$

$\text{Income tax} = 23\,853 \times 0.19$

$= \$4532.07$

b. Alex's gross income for a year is \$63 500. He can claim \$680 of his expenses as deductions. How much income tax does he owe?

Student A

$63\,500 - 45\,000 = \$18\,500$

$5092 + 0.325 \times 18\,500 = \$11\,104.50$

$\text{Income tax} = 11\,104.50 - 680$

$= \$10\,424.50$

Student B

$63\,500 - 680 = \$62\,820$

$62\,820 - 45\,000 = \$17\,820$

$\text{Income tax} = 5092 + 0.325 \times 17\,820$

$= \$10\,883.50$

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. Bonnie checks her fortnightly pay slip and sees that \$398 in tax is withheld from her gross pay of \$2352. Determine her net income over a month.

10. Lincoln earns a \$5000 pay rise after a performance review. Before the review, his gross salary was \$43 550. Calculate the difference in income tax he owes after the pay rise.

11. Surae's net income each fortnightly pay cycle is \$1840. Her gross income is \$58 500 per year. Calculate how much tax she owes or is owed at the end of the year.

12. Mako's annual gross income is $56 505. After she submitted her tax return, she discovered she could claim $600 worth of deductions. How much less income tax would she pay if she claimed those deductions?

13. The total amount of income tax Makhim owes is $11 982. What is his gross income? Assume he has no deductions.

Reasoning

Question working paths

Mild 14 (a,b,c,e) 🌶	**Medium** 14 (a,b,c,e), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. Nick has finished his first year as an English teacher. His gross pay each fortnight has been $2923. There are a number of deductions he can claim: donations ($50), teacher registration fees ($170) and cost for a first aid course ($200).

- **a.** What is his annual gross income?
- **b.** By first calculating his withheld tax, calculate his annual net income.
- **c.** Determine his taxable income, given his gross income and deductions.
- **d.** By first calculating his income tax, determine how much money he will owe or be owed by the tax office at the end of the year.
- **e.** Nick could hire an accountant to help with his taxes. List at least one advantage and one disadvantage of doing this.

15. In 2019, the income tax ranges were as shown in the table.

Taxable income range	Income tax
$0–$18 200	Nil
$18 201–$37 000	19 cents for each $1 over 18 200
$37 001–$90 000	$3572 plus 32.5 cents for each $1 over $37 000
$90 001–$180 000	$20 797 plus 37 cents for each $1 over $90 000
$180 001 and over	$54 097 plus 45 cents for each $1 over $180 000

- **a.** Calculate the income tax on a gross income of $54 000 using the table in Key Idea 2.
- **b.** Calculate the income tax on a gross income of $54 000 using the table from 2019.
- **c.** Compare your answers in parts **a** and **b** and the two tables, and comment on the effect of changing the income tax ranges.

Exam-style

16. The amount of withheld tax on Sally's fortnightly salary is $168. (1 MARK)

If this represents 12% of her gross income, what is her gross income?

- **A.** $180
- **B.** $188
- **C.** $1400
- **D.** $1568
- **E.** $2016

17. Last year, Luke's taxable income was \$87 000 and the tax payable on this income was \$19 822. (3 MARKS)

This year, Luke's taxable income has increased by \$16 800. The following table shows the income tax rates for these years.

Taxable income range	Tax on this income
\$0–\$18 200	Nil
\$18 201–\$37 000	19 cents for each \$1 over 18 200
\$37 001–\$87 000	\$3572 plus 32.5 cents for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37 cents for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45 cents for each \$1 over \$180 000

a. Use the table to calculate the tax payable by Luke this year. 2 MARKS

b. How much extra tax will Luke have to pay as a result of the increase in his taxable income? 1 MARK

18. Peter wants to know whether more or less tax is being withheld each pay cycle compared to how much income tax he actually owes at the end of the year. Calculate the difference between tax withheld and income tax each month. His gross annual income is \$75 350, and each monthly pay cycle his net pay is \$4905.17. (2 MARKS)

19. The table shows the income tax rates for the 2017–2018 financial year. (3 MARKS)

Taxable income range	Tax on this income
\$0–\$18 200	Nil
\$18 201–\$37 000	19 cents for each \$1 over 18 200
\$37 001–\$87 000	\$3572 plus 32.5 cents for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37 cents for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45 cents for each \$1 over \$180 000

A Medicare levy is also added and is calculated as 2% of taxable income.

For the 2017–2018 financial year, Charlie pays a Medicare levy of \$1934.80.

Calculate the tax payable on Charlie's taxable income.

Remember this?

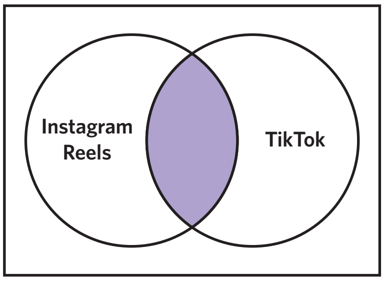
20. One night playing 2K23, Sam won 16 games and lost 5 games. Approximately what percentage of the games were lost?

A. 5%
B. 24%
C. 31%
D. 69%
E. 76%

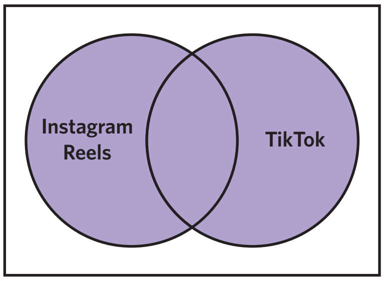
21. Nina wants to find the number of students who use either Instagram Reels or TikTok but not both.

Which shaded region in the Venn diagrams represents the information Nina is looking for?

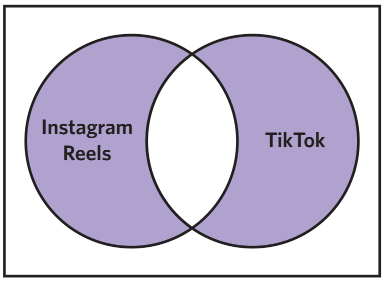
A.



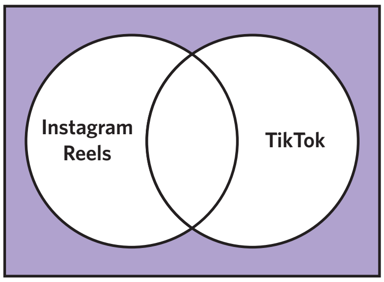
B.



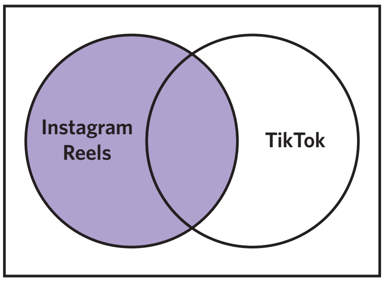
C.



D.

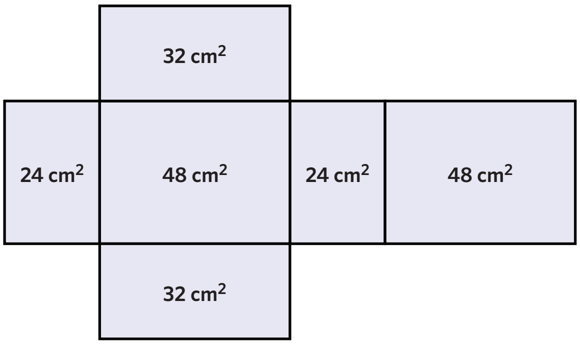


E.



22. Tao makes a rectangular prism from cardboard.

He cuts out 6 rectangles and joins them together as shown.



What is the volume of the rectangular prism?

A. 18 cm^3 **B.** 96 cm^3 **C.** 144 cm^3 **D.** 192 cm^3 **E.** 208 cm^3

Back to contents

1E Simple interest

LEARNING INTENTIONS

Students will be able to:

- calculate simple interest
- determine the value of a loan or investment
- calculate simple interest for different time periods
- calculate the principal, rate or time period of a loan or investment.

Simple interest is the most basic form of interest. Interest is calculated as a percentage of an amount of money that is invested or borrowed. This lesson will introduce calculating interest for different time periods, as well as calculating the principal, interest rate or time period in different scenarios.

KEY TERMS AND DEFINITIONS

- A **loan** is a sum of money borrowed and paid back with interest.
- An **investment** is a sum of money that generates income through interest.
- The **principal** is the initial amount loaned out or invested.
- **Per annum** (or **p.a.**) means every year, and is commonly used to refer to interest rates.
- A **lender** is a financial institution that loans money.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Westlight/Shutterstock.com

When a loan is taken out from a bank or other lender, it will attract interest. Interest represents the cost to borrow other people's money. Money that is stored in savings accounts as investments are paid interest so that the bank can use that money for loans.

Key ideas

1. Simple interest is calculated as a percentage (the interest rate) of the initial value of a loan or investment. This is multiplied by the number of time periods.

$I = \frac{Prt}{100}$ where

- I is the amount of interest earned or paid, in dollars
- P is the principal amount, in dollars
- r is the interest rate, as a percentage
- t is the number of time periods

Principal (P)	**Interest rate p.a (r)**	**Time (t)**	**Interest = $\frac{Prt}{100}$**
\$2500	3%	1 year	$\frac{2500 \times 3 \times 1}{100} = \$75$
\$2500	3%	2 years	$\frac{2500 \times 3 \times 2}{100} = \$150$
\$2500	3%	3 years	$\frac{2500 \times 3 \times 3}{100} = \$225$
\$2500	3%	4 years	$\frac{2500 \times 3 \times 4}{100} = \$300$

Continues →

Back to contents

2. The value, or amount, of an investment or loan is the sum of the principal and interest accrued.

$A = P + I$ where

- A is the total value of the loan or investment
- P is the principal amount
- I is the simple interest accrued

Principal (P)	**Interest rate p.a (r)**	**Time (t)**	**Interest = $\frac{Prt}{100}$**	**Amount = $P + I$**
\$2500	3%	1 year	$\frac{2500 \times 3 \times 1}{100} = \$75$	$A = 2500 + 75$ $= \$2575$
\$2500	3%	2 years	$\frac{2500 \times 3 \times 2}{100} = \$150$	$A = 2575 + 75$ $= \$2650$
\$2500	3%	3 years	$\frac{2500 \times 3 \times 3}{100} = \$225$	$A = 2650 + 75$ $= \$2725$
\$2500	3%	4 years	$\frac{2500 \times 3 \times 4}{100} = \$300$	$A = 2725 + 75$ $= \$2800$

Worked example 1

Calculating simple interest

For each of the following loans and investments, calculate the simple interest earned. Round to the nearest cent as needed.

a. \$2000 at 5% p.a. for 2 years.

WE1a

Working

$P = 2000$

$r = 5$

$t = 2$

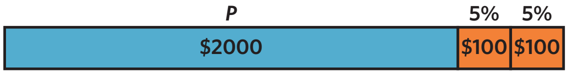
$$I = \frac{2000 \times 5 \times 2}{100}$$
$$= \$200$$

Thinking

Step 1: Identify the principal, interest rate and time period.

Step 2: Substitute the values into the simple interest formula.

Visual support



b. \$500 at 4.5% p.a. for 8 months.

WE1b

Working

$P = 500$

$t = 8$

$r = \frac{4.5}{12}$ per month

$$I = \frac{500 \times \frac{4.5}{12} \times 8}{100}$$
$$= \$15$$

Thinking

Step 1: Identify the principal and time period.

Step 2: Convert the interest rate to match the time period.

Step 3: Substitute the values into the simple interest formula.

Continues →

Back to contents

c. $1532 at 3.4% p.a. for 85 weeks.

WE1c

Working

$P = 1532$

$t = 85$

$r = \frac{3.4}{52}$ per week

$I = \dfrac{1532 \times \frac{3.4}{52} \times 85}{100}$

$= \$85.143...$

$\approx \$85.14$

Thinking

Step 1: Identify the principal and interest rate.

Step 2: Convert the interest rate to match the time period.

Step 3: Substitute the values into the simple interest formula.

Student practice

For each of the following loans and investments, calculate the simple interest earned. Round to the nearest cent as needed.

a. $300 at 5% p.a. for 3 years.

b. $1200 at 3.5% p.a. for 6 months.

c. $864 at 6.8% p.a. for 73 weeks.

Worked example 2

Calculating the rate, principal and time for simple interest

For each of the following loans and investments, calculate the missing value (principal, interest rate or time period). Round to two decimal places as needed.

a. $250 interest earned at 3% p.a. over 4 years.

WE2a

Working

$I = 250$

$r = 3$

$t = 4$

$250 = \dfrac{P \times 3 \times 4}{100}$

$250 \times 100 = 12 \times P$

$\frac{25\,000}{12} = P$

$P = \$2083.333...$

$\approx \$2083.33$

Thinking

Step 1: Identify the known information.

Step 2: Substitute the known values into the simple interest formula.

Step 3: Solve for the missing value.

Continues →

b. $600 interest earned on a $4000 loan at 6% p.a.

WE2b

Working	Thinking
$I = 600$ $P = 4000$ $r = 6$	**Step 1:** Identify the known information.
$600 = \dfrac{4000 \times 6 \times t}{100}$	**Step 2:** Substitute the known values into the simple interest formula.
$600 \times 100 = 24\,000 \times t$ $\dfrac{60\,000}{24\,000} = t$ $t = 2.5$ years	**Step 3:** Solve for the missing value.

c. $150 interest earned at 7.5% p.a. over 18 months.

WE2c

Working	Thinking
$I = 150$ $r = \dfrac{7.5}{12}$ per month $t = 18$	**Step 1:** Identify the known information, and convert the rate to match the time period.
$150 = \dfrac{P \times \frac{7.5}{12} \times 18}{100}$	**Step 2:** Substitute the known values into the simple interest formula.
$15\,000 = P \times 11.25$ $\dfrac{15\,000}{11.25} = P$ $P = \$1333.333...$ $P \approx \$1333.33$	**Step 3:** Solve for the missing value.

Student practice

For each of the following loans and investments, calculate the missing value (principal, interest rate or time period). Round to two decimal places as needed.

a. $150 interest earned at 4% p.a. over 2 years.

b. $660 interest earned on $5500 investment at 2% p.a.

c. $150 interest earned at 7.5% p.a. over 9 months.

Back to contents

1E Questions

Understanding worksheet

1. Match the interest rates to the corresponding principal and annual interest amounts (assume only one year of interest).

Interest rate

1% ●

5% ●

10% ●

15% ●

	Principal	**Interest**
●	$80	$4
●	$100	$10
●	$40	$6
●	$200	$2

2. Fill in the blanks for each of the following.

Example

Principal	**Interest**	**Amount**
$1000	$60	$1060

Principal	**Interest**	**Amount**
$500	$15	
	$6	$306
$120		$124.80
	$20	$810

3. Fill in the blanks by using the words provided.

interest rate | **principal** | **investment** | **loan**

When money is borrowed from a lender, this is called a ______ . When money is given to a bank in a savings account, it is called an ______ . Both attract interest which is calculated as a percentage of the ______ of a loan or investment. The amount of interest is determined by the ______ .

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. For each of the following loans and investments, calculate the simple interest earned. Round to the nearest cent as needed. WE1a

a. $1000 at 3% p.a. for 1 year.

b. $5000 at 4.5% p.a. for 3 years.

c. $2500 at 6% p.a. for 5 years.

d. $3750 at 2.75% p.a. for 4 years.

e. $15 842 at 7.25% p.a. for 2 years.

f. $8421.42 at 4.21% p.a. for 1 year.

g. $1205.67 at 5.57% p.a. for 12 years.

h. $6934.52 at 3.892% p.a. for 15 years.

5. For each of the following loans and investments, calculate the simple interest earned. Round to the nearest cent as needed. WE1b,c

a. \$2000 at 4.5% p.a. for 6 months.
b. \$1500 at 6% p.a. for 26 weeks.
c. \$30 at 2.75% p.a. for 3 months.
d. \$540 at 7.25% p.a. for 14 weeks.
e. \$2520.50 at 4.2% p.a. for 14 months.
f. \$1504.50 at 7.64% p.a. for 4 fortnights.
g. \$6734.64 at 3.81% p.a. for 1 year and 6 months.
h. \$46 053 at 5.502% p.a. for 3 years and 4 weeks.

---

6. For each of the following loans and investments, calculate the missing value (principal, annual interest rate or time period in years). Round to two decimal places as needed. WE2

a. \$125 interest earned at 2% p.a. over 3 years.
b. \$800 earns \$96 in interest over 5 years.
c. \$500 earns \$75 in interest at 7.5% p.a.
d. \$240 in interest earned over 6 years with an interest rate of 5% p.a.
e. A loan of \$600 earns \$105 in interest at a rate of 4.375%.
f. Over 15 months, \$563 is earned with an initial investment of \$7035.
g. A loan earns \$160 in interest over 38 weeks with an interest rate of 6.42% p.a.
h. \$80 in interest is earned on a \$1245 investment over 2 years and 5 months.

---

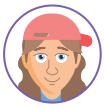
7. Calculate the value of an investment with a principal of \$4500 that attracts simple interest at 4.23% p.a. for 11 months.

A. \$174.49 **B.** \$4325.51 **C.** \$4546.53 **D.** \$4674.49 **E.** \$6593.85

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. \$600 is borrowed at a rate of 15.4% p.a. What is the value of the loan after 7 years?



Student A

$I = \dfrac{600 \times 15.4 \times 7}{100}$

$= \$646.80$

$A = 600 + 646.80$

$= \$1246.80$

Student B

$A = 600 + 15.4 \times 7$

$= \$707.8$

b. An investment of \$2500 earns \$102.08 interest over 14 months. What is the interest rate per annum?

Student A

$P = 2500$

$I = 102.08$

$t = 14$ months

$102.08 = \dfrac{2500 \times r \times 14}{100}$

$102.08 \times 100 = 35\,000r$

$r = 0.29\%$ per month

$r = 0.29\% \times 12$

$= 3.48\%$ per year

Student B

$P = 2500$

$I = 102.08$

$t = 14$ months

$102.08 = \dfrac{2500 \times r \times 14}{100}$

$102.08 \times 100 = 35\,000r$

$r = 0.29\%$ per year

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. Ryan receives $200 for his birthday. He decides to invest it in a bond that offers an annual interest rate of 4%. Determine the amount of interest he will earn after 2 years.

10. Alex is looking at buying a $1200 electric guitar. He currently has $1000 and he decides to put it in a savings account that will accrue simple interest. He sets a goal of buying the guitar in 16 months. What is the minimum annual interest rate required to afford the guitar in this time?

11. Emily wants to start her own small business in 6 months time. She decides to open a simple interest savings account that offers an annual interest rate of 6%. If she needs $600 more, calculate the amount she will need to invest initially.

12. Maki is planning to buy his first car. The car he has set his sights on costs $10 000, but he only has $3000 saved up. He decides to borrow the remaining amount as a personal loan, and pay it off when its value reaches $9000. If the bank charges simple interest at 7.16% p.a., how long, to the nearest year, will he wait to pay off the loan?

13. Shari is a student who wants to travel abroad for her studies. She borrows $2500 from a lender. The lender states that the amount will attract interest for 16 months at a rate of 8.33% p.a. in which time no repayments can be made by the borrower. After this, the loan stops attracting interest and she'll need to start paying off the value of the loan, including interest. If she wants to pay off her loan in 3 years, how much will her monthly repayments be?

Reasoning

Question working paths

Mild 14 (a,b,d) 🌶	**Medium** 14 (a,b,d), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. Paulo receives some money from selling his vintage Nintendo 64. He wants to invest the money in a savings account to earn interest. After some research, he finds two banks who offer the following savings options:

- Account 1: a maximum of $2000 invested at 2.5% p.a.
- Account 2: a maximum of $1500 invested at 5.5% p.a.

Assume Paulo invests the maximum amount into his accounts.

a. How much interest is earned on account 1 after 5 years?

b. What is the value of his investment using account 2 after 8 years?

c. After how many years does account 2 become the best value savings account?

d. List one thing that Paulo should consider when deciding which bank to choose.

15. Consider a loan with a principal of $500 and an interest rate of 5% p.a.

a. Calculate the value of the loan after one year.

b. Calculate the value of the loan the second year using the value from part **a** as the principal.

c. By comparing how much interest is earned in part **a** to part **b**, explain which method earns the greater amount of interest.

Exam-style

16. The value of a simple interest loan, in dollars, after 2 years was $102 000. The principal of the investment was $100 000. (1 MARK)
The interest rate, per annum, of this investment is

A. 0.01% **B.** 0.98% **C.** 1.00% **D.** 1.96% **E.** 2.00%

17. Alex sends a bill to his customers after repairs are completed. If a customer does not pay the bill by the due date, interest is charged. Alex charges simple interest after the due date at the rate of 1.5% per month on the amount of the original bill. (3 MARKS)

a. Alex sent Marcus a bill of $200 for repairs to his car. Marcus paid the full amount one month after the due date. Calculate how much Marcus paid. 1 MARK

b. Alex was paid $318 for repairs that originally cost $300. How many months late was the bill for these repairs? 2 MARKS

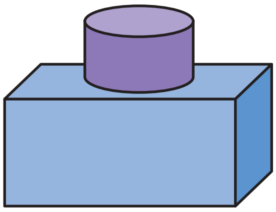
18. Julie deposits $12 000 into a savings account that will pay simple interest every month. The interest rate is 6.2% p.a. After how many months would the value of the savings first exceed $16 000? (2 MARKS)

19. Samuel opens a savings account with $5000 that earns simple interest on the original deposit amount each month at a rate of 3.6% p.a. He decides to deposit an additional $50 at the end of each month. How much extra money would be in the savings account after one year, on top of his original deposit? (3 MARKS)

Remember this?

20. This two-dimensional shape is made from

A. a cylinder and a cube.
B. a circle and a rectangle.
C. a sphere and a rectangle.
D. a sphere and a rectangular prism.
E. a cylinder and a rectangular prism.



21. Delilah has 32 cousins. There are 8 more male cousins than female cousins. How many female cousins does Delilah have?

A. 8 **B.** 10 **C.** 12 **D.** 14 **E.** 16

22. The given table shows how the volume of barley relates to the mass of barley.

Volume (m^3)	Mass (tonnes)
50	30
100	60
150	90
200	120

A farmer has three trailers that can each contain 85 m^3 of barley. How many tonnes of barley can their trailers contain all together?

A. 51 tonnes
B. 85 tonnes
C. 95 tonnes
D. 120 tonnes
E. 153 tonnes

Back to contents

1F Compound interest and depreciation

LEARNING INTENTIONS

Students will be able to:

- understand how compound interest works
- calculate compound interest using the compound interest formula
- calculate the principal using the compound interest formula.

Compound interest and depreciation are both special applications of simple interest. Instead of interest or depreciation being calculated only on the principal of a loan, investment, or value of an asset, it is calculated on the accumulated amount, which includes both the principal and the interest earned/charged or depreciation incurred, in the previous time period. While compound interest results in the growth of savings or investments, compound depreciation refers to the diminishing value of an asset over time.

KEY TERMS AND DEFINITIONS

- The **compound period** of a loan or investment is how often interest is calculated.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

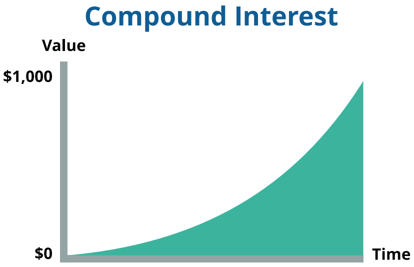
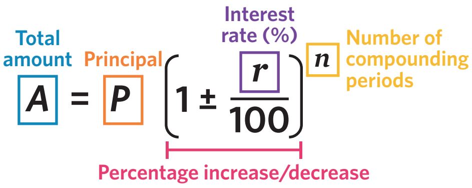


Image: Kanin_a/Shutterstock.com

When money is deposited into a savings account, the interest earned on the initial deposit compounds over time, allowing the savings to grow faster than if the money was held as cash. The interest is usually calculated at the end of each month (compounded monthly), and the interest rate can also change over time.

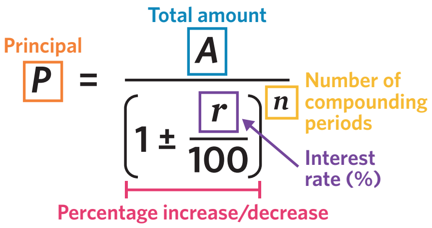
Key ideas

1. Compound interest and depreciation can be calculated using the compound interest and depreciation formulas, which take into account the principal, interest rate, and time period.



$$A = P\left(1 \pm \frac{r}{100}\right)^{n}$$

2. The principal amount can be calculated using the compound interest or depreciation formula by rearranging the formula to solve for the principal.



$$P = \frac{A}{\left(1 \pm \frac{r}{100}\right)^{n}}$$

3. Compound interest and depreciation can be calculated for different time periods. The compounding periods (n) and the rate (r) are adjusted accordingly.

Compounding periods	Time periods
Compounding annually	1
Compounding semi-annually	2
Compounding quarterly	4
Compounding monthly	12

Back to contents

Worked example 1

Calculating total amounts with compound interest

For each of the following, calculate the value of the investment. Round to the nearest cent as needed.

a. $5000 at 4% p.a. compounded annually for 3 years.

WE1a

Working

$P = 5000$

$r = 4$

$n = 3$

$$A = 5000 \times \left(1 + \frac{4}{100}\right)^3$$
$$= \$5624.32$$

Thinking

Step 1: Identify the principal, interest rate, and number of compounding periods.

Step 2: Substitute the values into the compound interest formula.

Visual support

$$\overset{\text{Total amount}}{\boxed{A}} = \overset{\text{Principal}}{\boxed{5000}} \times \underbrace{\left(1 + \frac{\overset{\text{Interest rate (\%)}}{\boxed{4}}}{100}\right)}_{\text{Percentage increase}}^{\boxed{3}\ \text{Number of compounding periods}}$$

b. $400 at 5% p.a. compounded monthly for 2 years.

WE1b

Working

$P = 400$

$n = 2 \times 12 = 24$

$r = \frac{5}{12}$ per month

$$A = 400 \times \left(1 + \frac{\frac{5}{12}}{100}\right)^{24}$$
$$\approx \$441.98$$

Thinking

Step 1: Identify the principal and number of compounding periods.

Step 2: Convert the interest rate to match the compounding period.

Step 3: Substitute the values into the compound interest formula.

Student practice

For each of the following, calculate the value of the investment. Round to the nearest cent as needed.

a. $1000 at 8% p.a. compounded annually for 2 years.

b. $800 at 9% p.a. compounded monthly for 9 months.

Back to contents

Worked example 2

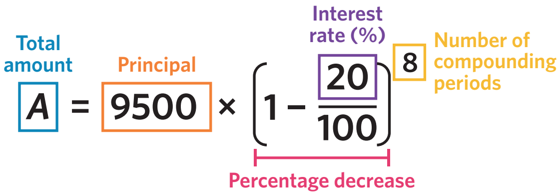
Calculating total amounts using depreciation

For each of the following, calculate the depreciated value of the asset. Round to the nearest cent as needed.

a. $9500 at 20% p.a. depreciating annually for 8 years. WE2a

Working	Thinking
$P = 9500$ $r = 20$ $n = 8$	**Step 1:** Identify the principal, interest rate, and number of compounding periods.
$A = 9500 \times \left(1 - \frac{20}{100}\right)^{8}$ $= \$1593.84$	**Step 2:** Substitute the values into the compound depreciation formula.

Visual support



b. $1000 at 3% p.a. depreciating quarterly for 3 years. WE2b

Working	Thinking
$P = 1000$ $n = 3 \times 4 = 12$	**Step 1:** Identify the principal and number of compounding periods.
$r = \frac{3}{4}$ per month	**Step 2:** Convert the interest rate to match the compounding period.
$A = 1000 \times \left(1 - \frac{\frac{3}{4}}{100}\right)^{12}$ $\approx \$913.62$	**Step 3:** Substitute the values into the compound depreciation formula.

Student practice

For each of the following, calculate the depreciated value of the asset. Round to the nearest cent as needed.

a. $7500 at 4% p.a. depreciated annually for 3 years.

b. $700 at 2% p.a. depreciated monthly for 2 years.

Back to contents

1F Questions

Understanding worksheet

1. Fill in the blanks in the following table.

Example

Total amount (*A*)	Principal (*P*)	Interest = (Total amount − principal)
$2160	$2000	$160

Total amount (*A*)	Principal (*P*)	Interest = (Total amount − principal)
	$5000	$100
$5202		$102
$5306.04	$5202	
	$5306.04	$106.12

2. Fill in the blanks.

Example

A $500 loan attracts 10% interest compounded annually for 10 years.

$$A = 500 \times \left(1 + \frac{10}{100}\right)^{\boxed{10}}$$

a. A $1000 investment earns interest at 7% p.a. compounded annually for 3 years.

$$A = \boxed{} \times \left(1 + \frac{7}{100}\right)^{3}$$

b. An asset depreciates by 15% p.a. annually for 7 years.

$$A = 8000 \times \left(1 - \frac{\boxed{}}{100}\right)^{7}$$

c. A savings account worth $6500 has an interest rate of 5.4% p.a. with interest compounding monthly for 2 years.

$$A = 6500 \times \left(1 + \frac{\frac{5.4}{\boxed{}}}{100}\right)^{24}$$

d. A $20 000 car loan compounds monthly for 1 year at 12% p.a.

$$A = 20\,000 \times \left(1 + \frac{\frac{12}{12}}{100}\right)^{\boxed{}}$$

3. Fill in the blanks by using the words provided.

percentage **sum** **simple** **compound**

Compared to ________ interest, ________ interest is calculated using the ________ of the principal and the total interest. To calculate the value over time, apply repeated ________ increases for the number of compounding periods.

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. For each of the following, calculate the value of the investment. Round to the nearest cent as needed.

WE1

- **a.** \$10 000 at 5% p.a. compounded annually for 1 year.
- **b.** \$130 at 10% p.a. compounded annually for 3 years.
- **c.** \$2500 at 4.5% p.a. compounded annually for 2 years.
- **d.** \$10 502.50 at 9.23% p.a. compounded annually for 8 years.
- **e.** \$6000 at 7% p.a. compounded monthly for 2 months.
- **f.** \$570 at 14% p.a. compounded monthly for 3 years.
- **g.** \$5025.35 at 24.3% p.a. compounded monthly for 3 years.
- **h.** \$2044.75 at 0.76% p.a. compounded monthly for 4 years and 9 months.

5. For each of the following, calculate the depreciated value of the asset. Round to the nearest cent as needed.

WE2

- **a.** \$5000 at 10% p.a. depreciating annually for 2 years.
- **b.** \$6000 at 12% p.a. depreciating annually for 3 years.
- **c.** \$700 at 14% p.a. depreciating annually for 4 years.
- **d.** \$1000 at 3% p.a. depreciating annually for 2 years.
- **e.** \$1200 at 4% p.a. depreciating monthly for 3 years.
- **f.** \$800 at 2% p.a. depreciating quarterly for 1 year.
- **g.** \$2245.25 at 5% p.a. depreciating quarterly for 2 years.
- **h.** \$1545.70 at 6% p.a. depreciating monthly for 3 years.

6. For each of the following investment values, calculate the principal. Round to the nearest cent as needed.

- **a.** \$14 131 at 4% p.a. compounded annually for 3 years.
- **b.** \$9030 at 6.5% p.a. compounded annually for 8 years.
- **c.** \$3053.50 at 3.48% p.a. compounded annually for 9 years.
- **d.** \$5211.65 at 1.42% p.a. compounded monthly for 10 months.
- **e.** \$20 242.81 at 10.2% p.a. compounded monthly for 14 years.
- **f.** \$45 764.44 at 10.23% p.a. compounded weekly for 94 weeks.
- **g.** \$605.32 at 4.97% p.a. compounded annually for 4 years.
- **h.** \$3704.24 at 0.58% p.a. compounded weekly for 7 years and 21 weeks.

7. A \$20 000 loan attracts interest at 2.4% p.a. compounding monthly. How much interest will there be after 8 months, rounded to the nearest cent?

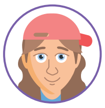
- **A.** \$320
- **B.** \$322.25
- **C.** \$4228.76
- **D.** \$20 322.25
- **E.** \$65 997.34

Back to contents

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Calculate the total amount of an investment if the initial amount of $34 832 compounds monthly for 3 years at 4.2% p.a.



Student A

$P = 34\,832$

$r = \frac{4.2}{12}$

$n = 3$

$$A = 34\,832\left(1 + \frac{\frac{4.2}{12}}{100}\right)^{3}$$

$\approx \$39\,407.74$

Student B

$P = 34\,832$

$r = \frac{4.2}{12}$

$n = 3 \times 12 = 36$

$$A = 34\,832\left(1 + \frac{\frac{4.2}{12}}{100}\right)^{36}$$

$\approx \$39\,500.63$

b. Calculate the total amount of an asset if the initial amount of $12 500 depreciates monthly for 4 years and 6 months at 3.6% p.a.

Student A

$P = 12\,500$

$r = \frac{3.6}{12}$

$n = 4.5 \times 12$

$$A = 12\,500\left(1 - \frac{\frac{3.6}{12}}{100}\right)^{54}$$

$\approx \$10\,627.93$

Student B

$P = 12\,500$

$r = 3.6$

$n = 4.5 \times 12$

$$A = 12\,500\left(1 - \frac{3.6}{100}\right)^{54}$$

$\approx \$1726.12$

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. John has purchased a new photocopier for $3500, which depreciates by 25% p.a. Calculate the value of the photocopier after 5 years. Round to the nearest cent.

10. Jenny invested some money in a savings account that offers a yearly interest rate of 2.7%. After 4 years, the balance in her account has grown to $12 300. If the interest is compounded annually, what was the initial amount of money she invested in the account?

11. Elon's electric car has a value of $50 607.68, rounded to the nearest cent, after 5 years. The car's value depreciated by 3.4% p.a. during that time. Calculate the initial value (P) of Elon's car.

12. The interest in Liam's savings account compounds monthly. He receives a notification that the interest rate on his savings account will rise from 3.45% to 3.70% p.a. If his current balance is $4560, calculate the difference in interest accrued between the two interest rates over three months.

13. The balance on credit cards typically attracts interest that compounds daily after a certain number of days, known as the interest-free period. Shak's credit card has an interest-free period of 25 days and the interest rate is 9.95% p.a. If the balance of Shak's card is $583, how much more does he pay if he pays the balance after 40 days?

Back to contents

Reasoning

Question working paths

Mild 14 (a,b,c,e) 🌶	**Medium** 14 (a,b,c,e), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. Kahreem has $50 000 and decides to invest this in a term deposit. A term deposit earns interest at a fixed interest rate for a certain period of time (the term). The value of the investment cannot be withdrawn until the end of the term.

- Option 1: 5.1% p.a. compounding annually with a term of 24 months.
- Option 2: 2.35% p.a. compounding monthly with a term of 27 months.

a. What is the value of option 1 at the end of the term?

b. Calculate the difference in interest accrued between option 1 and option 2 at the end of the term and identify the investment option with the best value.

c. If Kahreem wants to use option 1 to have an investment value of $150 000 at the end of the term, how much more will he need to invest initially?

d. If there were no restrictions on the term Kahreem could deposit the money, calculate how long it would take to double his investment using option 1. Round to the nearest number of years.

e. What is an advantage or disadvantage of using a term deposit compared to a savings account?

15. For the principal amount of $5000:

a. Calculate the new amount if it accrues monthly compound interest at 10% p.a. for 5 years.

b. Calculate the new amount if it accrues annual compound interest at 10% p.a. for 5 years.

c. Compare how much the amount increased by in parts **a** to **b**. Comment on the amount of money earned using monthly or annual compounding periods.

Exam-style

16. Peter currently earns $21.50 per hour. His hourly wage will increase by 2.1% compounded each year for the next four years. (1 MARK)

What will his hourly wage be after four years?

A. $21.50 \times \frac{2.1}{100}$

B. $21.50 \times \frac{2.1}{100}^{4}$

C. $21.50 \times \left(1 + \frac{2.1}{100}\right)^{4}$

D. $21.50 + 21.50 \times 0.21 \times 4$

E. $21.50 + 21.50 \times 0.021 \times 4$

17. Julie invests $12 500 in a compound interest savings account. Interest is calculated using a fixed monthly rate on the amount in the account at the beginning of the month. The annual interest rate is 1.8%. Julie has created a spreadsheet to show the activity in her savings account. The details for the first month are shown. (3 MARKS)

Month	**Total amount in the account at the beginning of month**	**Monthly interest**	**Total amount in the account at end of month**
1	$12 500.00	$18.75	$12 518.75
2			
3			
4			

a. Determine the total amount in the account at the beginning of the second month. 1 MARK

b. Determine the total amount in the account at the end of the fourth month. 2 MARKS

18. Daniel borrows \$5000, which he intends to repay fully in a lump sum after one year. (2 MARKS)

The annual interest rate and compounding period for two different loans are given below:

- Loan 1: 12.6% per annum, compounding annually
- Loan 2: 12.6% per annum, compounding monthly

When fully repaid, explain which loan will cost Daniel the least amount of money?

19. Nisa has a credit card on which interest at 17% per annum, compounded daily, is charged on the amount owing. At the beginning of the month, Nisa owes \$500 on her credit card. She makes no other purchases using the credit card. (3 MARKS)

Assuming that interest is charged for 15 days and there are 365 days in a year, calculate the amount owing on the credit card.

Remember this?

20. Sara wants to buy 8 packets of chips. One packet of chips is \$3.75. How much is 8 packets of chips?

A. \$24 **B.** \$24.75 **C.** \$26.25 **D.** \$26.75 **E.** \$30

21. The distance between Jupiter and Neptune is 3 722 670 000 km. What is the distance in scientific notation?

A. $372\,267 \times 10^{2}$
B. $372\,267 \times 10^{4}$
C. $372\,267 \times 10^{9}$
D. 3.72267×10^{9}
E. 3.72267×10^{10}

22. $\frac{7}{16} + 3\frac{2}{5} = \frac{9}{2} - \frac{53}{x}$

What does x represent?

A. $x = 16$ **B.** $x = 32$ **C.** $x = 80$ **D.** $x = 82$ **E.** $x = 160$

Back to contents

Chapter 1 extended application

1. Billy owns a food truck out of which he sells ice creams and pies 280 days out of the year. During the summer season, Billy parks his truck near the beach, which makes up 30% of the total time he stays open.

a. On one day Billy sold 102 ice creams, 35 pies, and 150 bottles of water. What was Billy's gross profit for the day?

b. At the start of the summer season Billy employs a part time adult worker for 16 hours per week at an hourly rate of $23.45. Use an online tax calculator from the ATO.gov.au website to calculate the employee's net pay for a week to the nearest dollar.

c. Calculate Billy's employee's total net pay for the entire summer season.

d. Over the summer season, Billy sold 6000 ice creams, 550 pies, and 8050 bottles of water. After he paid his part time employee's gross wages, the remaining amount was taxed at 25%. Calculate Billy's total net profit amount for the summer season to the nearest dollar.

e. Identify an advantage or a disadvantage of having a summer job while you are still at school.

Menu

Ice cream
$5.50

Pie
$6.00

Bottle of water
$3.75

2. Kyle is a mechanic who is calculating the total cost of a job for their friend, John. They have listed each item on an invoice which is to be given to the customer.

a. Calculate the total cost of all parts listed in the invoice, rounded to the nearest cent.

b. Kyle's shop is offering a 15% discount on John's service. Calculate the discounted cost of the full service, rounded to the nearest cent.

c. The hourly rate of labour listed on John's invoice was paid as overtime at 1.5 times the regular hourly rate. What is the regular hourly rate of the mechanic who completed the job, rounded to the nearest cent?

d. Kyle must add 10% GST to the total cost of John's job. Using Kyle's discount, calculate the total cost, including GST, as payable by the customer, rounded to the nearest cent.

e. Identify an advantage or a disadvantage of working overtime.

KYLE'S CARS

Parts:
- Spark plugs ×4 @$13.95 each
- Brake pads ×2 @$51 each

Full service $190

Labour ×6 hours @$57.55 per hour

3. Lynfield High School has been gifted $1 000 000 by an anonymous donor. The donor has only asked that the given conditions are met.

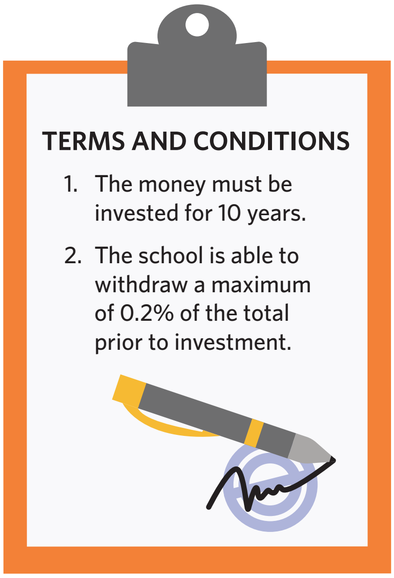
a. Calculate the maximum amount the school can withdraw before investing the money.

b. The school's financial adviser proposed that the whole amount should be invested at a simple interest rate of 1.5% p.a. for 10 years. Calculate the value of the investment after 10 years using this option.

c. The student committee wants to withdraw the maximum allowable amount for school improvements first, then invest the money at an interest rate of 1.5% p.a. compounding annually for 10 years. Calculate the value of the investment after 10 years using this option, rounded to the nearest dollar.

d. Identify which of the investment options from parts **b** and **c** has a greater value after 10 years and express the total return amount as a percentage of the principal investment to the nearest percent.

e. What kind of improvements would you want to see at your school if a large amount of money was donated to them?



TERMS AND CONDITIONS

1. The money must be invested for 10 years.
2. The school is able to withdraw a maximum of 0.2% of the total prior to investment.

Back to contents

Chapter 1 review

Multiple choice

1. Which of the bolded digits is not significant? 1A

A. 0.**9**83 **B.** **0**.938474 **C.** 42.**0**0092 **D.** 91.7**4**5 **E.** 432.3**0**

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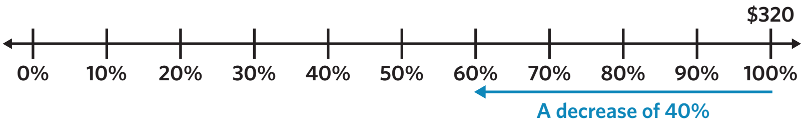
2. What is the new price in the percentage decrease shown on the given number line? 1B



A. $60 **B.** $160 **C.** $192 **D.** $280 **E.** $320

---

3. Which of the options shows an incorrect rate conversion? 1C

	Rate	Calculation	New rate
A.	$1200 per week	× 52	Annual or yearly
B.	$4500 per month	× 12	Annual or yearly
C.	$5350 per month	÷ 12	Weekly
D.	$45 890 per year	÷ 26	Fortnightly
E.	$67 280 per year	÷ 26	Fortnightly

---

4. Which of the options shows the correct net income? 1D

	Gross earnings	Withheld tax	Net income
A.	$4500	$120	$4620
B.	$8900	$350	$8505
C.	$10 300	$568	$10 868
D.	$13 580	$603	$12 977
E.	$15 800	$732	$15 074

---

5. Which of the options depicts an incorrect calculation of interest and the subsequent final amount rounded to the nearest dollar? 1E

	Principal (P)	Interest rate p.a (r)	Time (t)	Interest $= \frac{Prt}{100}$	Amount $= P + I$
A.	$3350	2%	1 year	$\frac{3350 \times 2 \times 1}{100} = \$67$	$A = 3350 + 67$ $A = \$3417$
B.	$4802	4%	2 years	$\frac{4802 \times 4 \times 2}{100} = \$384$	$A = 4802 + 384$ $A = \$5186$
C.	$4678	3%	2 years	$\frac{4678 \times 3 \times 2}{100} = \$281$	$A = 4678 + 281$ $A = \$4959$
D.	$5025	0.5%	5 years	$\frac{5025 \times 5 \times 5}{100} = \$1256$	$A = 5025 + 1256$ $A = \$6281$
E.	$5342	1.2%	3 years	$\frac{5342 \times 1.2 \times 3}{100} = \$192$	$A = 5342 + 192$ $A = \$5534$

Fluency

6. Complete the following calculations and round each result to the appropriate number of significant figures. 1A

   a. 17.1×4.9 **b.** $11.189 \div 0.23$ **c.** 172.236×0.019 **d.** $53.414 \div 6.14572$

7. Calculate the original price of each of the following. Round to the nearest cent as needed. 1B

   a. A laptop that had a discount of 50% and is now priced at $850.

   b. An iPhone that had a mark-up of 7% and is now priced at $1799.99.

   c. A croissant that had a discount of 9.25% and is now priced at $3.79.

   d. A plane ticket that had a mark-up of 37.75% and is now priced at $2159.99.

8. Calculate the income earned in the following scenarios. Round to the nearest cent as needed. Assume overtime and Saturday pay are paid at time-and-a-half, and Sunday pay is 1.75 times the base rate. 1C

   a. 6.5 hours worked on a Sunday with a base rate of $23.23 per hour.

   b. 4 hours worked, including 0.3 hours of overtime, at a base rate of $28.46 per hour.

   c. $7\frac{5}{7}$ hours worked on a Sunday at a base rate of $31.25 per hour.

   d. 12.54 hours worked, including 2.54 hours worked on a Sunday, at a base rate of $24.63 per hour.

9. For each of the following earnings, calculate the amount of withheld tax using the online tax calculator from the ATO.gov.au website. 1D

   a. $880 per week **b.** $5400 per month

   c. $475.25 per week **d.** $3191.75 per fortnight

10. For each of the following gross incomes with deductions, calculate the payable income tax. 1D

    a. Gross income: $18 000; deductions: $500 **b.** Gross income: $39 700; deductions: $750

    c. Gross income: $85 470; deductions: $660 **d.** Gross income: $117 320; deductions: $912

11. For each of the following loans and investments, calculate the simple interest earned. Round to the nearest cent as needed. 1E

    a. $5000 at 5% p.a. for 1 year. **b.** $470.50 at 4.15% p.a. for 5 months.

    c. $2971.55 at 4.65% p.a. for 11 fortnights. **d.** $216 479 at 6.015% p.a. for 2 years and 18 weeks.

12. For each of the following loans and investments, calculate the missing value (principal, annual interest rate or time period in years). Round to two decimal places as needed. 1E

    a. $520 interest earned at 4% p.a. over 7 years.

    b. $1942 interest earned over 6 years with an interest rate of 5% p.a.

    c. Over 18 months, $1206 is earned from an initial investment of $15 730.

    d. A loan of $119 780 earns $10 486 in interest at a rate of 8.025% p.a.

13. For each of the following, calculate the value of the investment. Round to the nearest cent as needed. 1F

    a. $40 000 at 9% p.a. compounded annually for 1 year.

    b. $5072.25 at 4.13% p.a. compounded annually for 11 years.

    c. $100 031.45 at 6.105% p.a. compounded monthly for 2.5 years.

    d. $5.76 at 1.27% p.a. compounded monthly for 80 years and 11 months.

Back to contents

14. For each of the following, calculate the depreciated value of the asset. Round to the nearest cent as needed. 1F

a. \$6000 at 10% p.a. depreciating annually for 3 years.

b. \$700 at 12% p.a. depreciating annually for 4 years.

c. \$920 at 3% p.a. depreciating quarterly for 1 year.

d. \$2654.90 at 5.5% p.a. depreciating monthly for 5 years.

Problem solving

15. Beccy is running a marathon in the United States, where they express distance using miles. Given that one mile is 1.60934 km, how far did Beccy run in kilometres if the marathon was 26.2188 miles? Round the answer to three decimal places. 1A

---

16. OSPM is a retailer for eyewear. The cost price is \$102 for a pair of prescription glasses and \$85 for a pair of sunglasses. OSPM sells a pair of prescription glasses and a pair of sunglasses for \$495, making a 105% profit on the pair of prescription glasses. What is the percentage profit for the pair of sunglasses, correct to two decimal places? 1B

---

17. Alexis has been paid \$975.95 for her work during the past two weeks. She knows that she has worked 21.25 regular hours and 4.35 hours overtime. Given that her overtime pay is 150% of her normal pay, calculate Alexis' regular pay rate per hour. 1C

---

18. Victoria works for a charity where she is offered a \$15 590 tax deduction per year. She also purchases a private insurance policy which is equivalent to an additional \$1218.40 tax deduction per year. In the end, Victoria's income is within the range of \$45 001–\$120 000. Calculate the possible minimum and maximum of Victoria's gross income prior to tax deductions. 1D

---

19. Dhanya opened a Goalsaver account at the start of April. The Goalsaver account pays Dhanya 4.65% simple interest monthly given that the account balance at the end of month is higher than at the beginning of the month; otherwise the interest paid monthly will only be 0.02%. The below table shows Dhanya's account balance over three months. The simple interest is paid on the first day of each month. 1E

Month	**Balance on first day of the month**	**Balance on the last day of month**	$I = \frac{Prt}{100}$
April	\$1278.90	\$1525.15	$I = \frac{1525.15 \times \frac{4.65}{12} \times 1}{100} \approx 5.91$
May	$1525.15 + 5.91 = \$1531.06$	\$1448.03	
June		\$1918.28	

Using the filled-in example from the table, complete the rest of the table. Calculate and round to the nearest cent.

---

20. Kashifa wants to invest \$49 750.15 for one year, where she has a choice of whether interest is compounded weekly, monthly or yearly. If the interest compounds weekly, the interest rate offered will be 2.4935% p.a. If the interest compounds monthly, the interest rate offered will be 2.5635% p.a. If the interest compounds yearly, the interest rate offered will be 2.7901% p.a. Which product should Kashifa choose to maximise her earnings? 1F

Back to contents

Reasoning

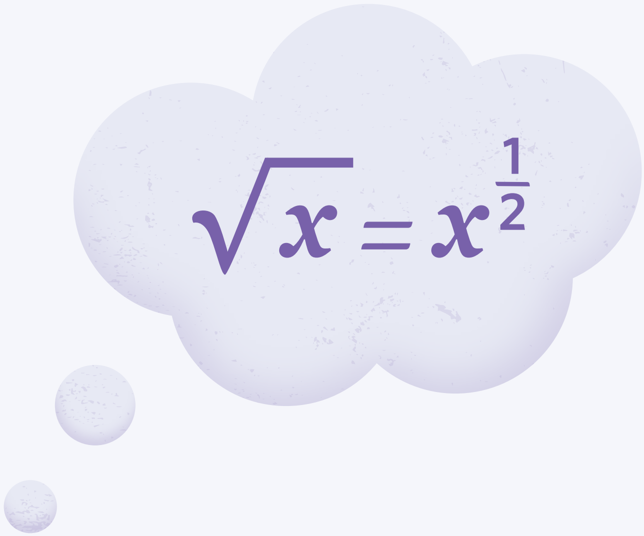
21. Jess is the accountant for a small business called Candle & Co. Jess charges $250 per hour for her services.

- **a.** Candle & Co increase the price of their $35 candle bundles by 30%. What is the new price of the bundles?
- **b.** By the end of the financial year, Jess spent a total of 90 regular hours and 6 overtime hours doing Candle & Co's accounts. Assuming that overtime is paid at 1.75 times the base pay rate, calculate Jess' gross income from doing these accounts.
- **c.** At the end of the financial year, Jess' total gross income comes to $191 500. There are a number of deductions Jess can claim: stationery ($80), office furniture ($485) and utilities ($650). Calculate the income tax on Jess' taxable annual income.
- **d.** Candle & Co took out a loan when they first started their business. In January 2018, they borrowed $6500 compounding monthly at an interest rate of 4.5% p.a. Assuming that no repayments were made until March 2021, calculate the value of the loan when repayments began.
- **e.** Identify a way that Candle & Co can improve their profits without increasing prices and potentially losing customers.

22. For a principal amount of $11 000:

- **a.** Calculate the amount after 3 years if it compounds annually at an interest rate of 5% p.a.
- **b.** Calculate the amount after 3 years if it compounds monthly at an interest rate of 5% p.a.
- **c.** Comparing the new amounts calculated in parts **a** and **b**, comment on the amount of money earned using monthly or annual compounding periods.

Back to contents



Chapter 2

Indices and surds

Number and algebra

Calculator skills

See online in additional materials for using CAS calculator guides.

2C Negative indices
2D Scientific notation
2E Fractional indices

Back to contents

Chapter 2 research summary

Indices and surds

Big ideas

Indices, also known as exponents or powers, are a way of expressing repeated multiplication. The exact value of surds can only be expressed in square root, cube root or other root forms; therefore, they are irrational numbers. The big ideas proportional reasoning, generalising and patterning, spatial awareness, interconnectedness, reasoning and proof underpin the concept of indices and surds.

Proportional reasoning

When we increase the power by one, we are multiplying the base by itself one more time. This is an application of multiplicative thinking which is a key aspect of proportional reasoning.

Generalising

Indices and surds involve patterns and generalisation. For example the index laws such as $a^n \times a^m = a^{n+m}$ are general rules that apply to all indices. The process of simplifying surds also involves identifying and applying patterns.

Spatial awareness

Understanding the geometric representation of squares, cubes and their inverses of square roots, cube roots and other roots can help connect surds to spatial concepts. For example, in order to determine the length of the side of a square with a given area, a student would understand that the square root of the area is equal to a side length.

Interconnectedness

Indices and surds are interconnected with many other areas of mathematics. For example, they are used in algebra for solving equations, geometry in the Pythagorean theorem, and further into calculus in determining derivatives of power functions.

Reasoning

Understanding and using indices and surds involves mathematical reasoning. For example, to simplify a surd students must identify the largest perfect square factor of the number under the root. This requires logical thinking and problem-solving skills.

Visual representations

Repeated multiplication

While not a traditional visual, repeated multiplication is a good way to visually explain the concept of indices.

$$4^3 \times 4^7 = (4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4)$$
$$= 4^{3+7}$$
$$= 4^{10}$$

$$x^{-5} = \frac{1}{\underbrace{x \times x \times x \times x \times x}} = \frac{1}{x^5}$$

The negative power of 5 shows there is a repeated division by x five times

The number must be multiplied by 10 eight times

$$3.27 \times 10^8 = 3.27 \overbrace{\times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}$$
$$= 327\,000\,000$$

Back to contents

Misconceptions

Misconception	Incorrect ✕	Correct ✓	Lesson
Students apply index laws to terms with different bases.	$x^p \times y^q = x^{p+q}$	$x^p \times x^q = x^{p+q}$	2A
Students multiply or divide bases when applying the first two index laws.	$\frac{x^p}{x^q} = 1^{p-q}$	$\frac{x^p}{x^q} = x^{p-q}$	2A
Students multiply or divide exponents when applying the first two index laws.	$x^p \times x^q = x^{p \times q}$	$x^p \times x^q = x^{p+q}$	2A
The coefficient inside brackets is not raised to a power on the outside of the brackets.	$(3x^2)^5 = 3 \times x^{2\times5} = 3x^{10}$	$(3x^2)^5 = 3^5 \times x^{2\times5} = 3^5x^{10}$	2B
The coefficient on the outside of brackets is raised to a power on the outside of the brackets.	$5(x^2)^{10} = 5^{10}x^{20}$	$5(x^2)^{10} = 5x^{20}$	2B
The student adds the powers together when raising powers.	$(x^2)^{10} = x^{12}$	$(x^2)^{10} = x^{20}$	2B
Rules for the addition or subtraction of negative numbers are not followed when simplifying negative indices using index laws.	$a^{-8+2} = a^{-10}$	$a^{-8+2} = a^{-6}$	2C
A negative index is treated as a factor affecting the direction of the term.	$\frac{2x^{-7}y^{-1}}{16xy} = -8x^8y^2$	$\frac{2x^{-7}y^{-1}}{16xy} = 8x^{-8}y^{-2}$	2C
Miscounting the number of place values when converting to or from scientific notation.	$\underline{6}8\,100\,000 \rightarrow a = \underline{6}.8\,100\,000$ $n = 5$ $68\,075\,000 \approx 68\,100\,000$ $= 6.81 \times 10^5$	$\underline{6}8\,100\,000 \rightarrow a = \underline{6}.8\,100\,000$ $n = 7$ $68\,075\,000 \approx 68\,100\,000$ $= 6.81 \times 10^7$	2D
Leaving more or less than one significant figure to the left of the decimal point when converting to scientific notation.	$0.00\underline{2}93 \rightarrow a = \underline{0}.293$ $n = -2$ $0.00293 = 0.293 \times 10^{-2}$	$0.00\underline{2}93 \rightarrow a = \underline{2}.93$ $n = -3$ $0.00293 = 2.93 \times 10^{-3}$	2D
Using positive powers of 10 for very small numbers and negative powers of 10 for very large numbers when converting to or from scientific notation.	$0.00293 = 2.93 \times 10 \times 10 \times 10$ $= 2.93 \times 10^3$	$0.00293 = 2.93 \div 10 \div 10 \div 10$ $= \frac{2.93}{10^3}$ $= 2.93 \times 10^{-3}$	2D
Using the numerator of the fractional index to represent the root and the denominator to represent the power.	$x^{\frac{p}{n}} = \sqrt[p]{x^n}$	$x^{\frac{p}{n}} = \sqrt[n]{x^p}$	2E

Continues →

Back to contents

Misconception	Incorrect ✕	Correct ✓	Lesson
Dividing the radicand by the order of the root to evaluate the root.	$100\,000^{\frac{1}{5}} = \frac{100\,000}{5}$ $= 20\,000$	$100\,000^{\frac{1}{5}} = \sqrt[5]{100\,000}$ $= 10$	2E
Adding or subtracting unlike surds.	$\sqrt{2} + 3\sqrt{5} + 2\sqrt{2}$ $= \sqrt{2} + 2\sqrt{2} + 3\sqrt{5}$ $= (1 + 2 + 3)\sqrt{2} + \sqrt{5}$ $= 6\sqrt{2} + \sqrt{5}$	$\sqrt{2} + 3\sqrt{5} + 2\sqrt{2}$ $= \sqrt{2} + 2\sqrt{2} + 3\sqrt{5}$ $= (1 + 2)\sqrt{2} + 3\sqrt{5}$ $= 3\sqrt{2} + 3\sqrt{5}$	2F
Ignoring the sign of the surd when simplifying.	$5\sqrt{7} - \sqrt{6} - 3\sqrt{7}$ $= 5\sqrt{7} - 3\sqrt{7} - \sqrt{6}$ $= (5 + 3)\sqrt{7} - \sqrt{6}$ $= 8\sqrt{7} - \sqrt{6}$	$5\sqrt{7} - \sqrt{6} - 3\sqrt{7}$ $= 5\sqrt{7} - 3\sqrt{7} - \sqrt{6}$ $= (5 - 3)\sqrt{7} - \sqrt{6}$ $= 2\sqrt{7} - \sqrt{6}$	2F
Thinking that a non-whole number radicand will always produce a surd.	$-\sqrt{0.2916} = -0.54$ is a decimal $\therefore -\sqrt{0.2916}$ is a surd.	$-\sqrt{0.2916} = -0.54$ is not an irrational number $\therefore -\sqrt{0.2916}$ is not a surd.	2F

Back to contents

2A The First and Second index laws

LEARNING INTENTIONS

Students will be able to:

- multiply and divide terms with the same base in index form
- simplify expressions using the First and Second index laws
- simplify expressions involving the zero index.

Indices are used to represent repeated factors. These factors can be numeric or algebraic terms or expressions. The First and Second index laws are applied when multiplying and dividing indices, whereas the zero index is a special property of all non-zero bases, including those that are algebraic.

KEY TERMS AND DEFINITIONS

- **Index notation** (or **form**) is a way of representing repeated factors of the same number.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

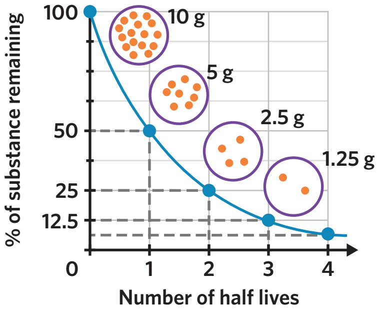


Image: zizou7/Shutterstock.com

Radioactive substances decay at an exponential rate, halving in amount after periods of time specific to individual elements. This is called the half life of an element, with some taking much longer to decay than others. These kinds of relationships are exponential due to the repeated multiplication or division in the formulas that describe them.

Key ideas

1. The First Index Law states that when multiplying terms with the same base in index form their indices are added.

$$x^p \times x^q = x^{p+q}$$

2. The Second Index Law states that when dividing terms with the same base in index form the index of the divisor (or denominator) is subtracted from the index of the dividend (or numerator).

$$\frac{x^p}{x^q} = x^{p-q}$$

3. Any non-zero number raised to the power of zero has the value 1.

$$\frac{x^r}{x^r} = x^{r-r} = x^0 = 1$$

Back to contents

Worked example 1

Using the First and Second index laws on numeric expressions

Simplify the following.

a. $4^3 \times 4^7$ WE1a

Working

$4^3 \times 4^7 = 4^{3+7} = 4^{10}$

Thinking

Identify the common base of the terms being multiplied. Add the indices of the terms to calculate the index of the product.

Visual support

$$4^{\boxed{3}} \times 4^{\boxed{7}} = (4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4)$$

$$= 4^{3+7}$$

$$= 4^{10}$$

b. $\frac{5^8}{5^4}$ WE1b

Working

$\frac{5^8}{5^4} = 5^{8-4} = 5^4$

Thinking

Identify the common base of the terms being divided. Subtract the indices of the terms to calculate the index of the quotient.

Student practice

Simplify the following.

a. $2^3 \times 2^8$

b. $\frac{3^9}{3^4}$

Worked example 2

Using the First and Second index laws on algebraic expressions

Simplify the following.

a. $x^2 \times x^5$ WE2a

Working

$x^2 \times x^5 = x^{2+5} = x^7$

Thinking

Identify the common base of the terms being multiplied. Add the indices of the terms to get the index of the product.

Continues →

b. $\frac{y^9}{y^3}$

WE2b

Working

$$\frac{y^9}{y^3} = y^{9-3} = y^6$$

Thinking

Identify the common base of the terms being divided. Subtract the indices of the terms to calculate the index of the quotient.

Visual support

$$\frac{y^9}{y^3} = \frac{\cancel{y}\times\cancel{y}\times\cancel{y}\times y\times y\times y\times y\times y\times y}{\cancel{y}\times\cancel{y}\times\cancel{y}}$$

$$= y^{9-3}$$

$$= y^6$$

Student practice

Simplify the following.

a. $t^7 \times t^5$

b. $\frac{p^{10}}{p^4}$

Worked example 3

Simplifying expressions using the First and Second index laws

Simplify the following.

a. $2x^{10} \times 4xy^4$

WE3a

Working

$$2x^{10} \times 4xy^4 = 2 \times 4 \times x^{10+1} \times y^4$$

$$= 8x^{11}y^4$$

Thinking

Step 1: Multiply the coefficients together. Identify the terms with common bases and add their indices.

Step 2: Fully simplify the expression.

Visual support

$$2x^{10} \times 4xy^4 = 2 \times 4 \times x^{10+1} \times y^4$$

$$= 8x^{11}y^4$$

b. $\frac{16r^7t^8}{4t^2r^6}$

WE3b

Working

$$\frac{16r^7t^8}{4t^2r^6} = (16 \div 4)r^{7-6}t^{8-2}$$

$$= 4r^1t^6 = 4rt^6$$

Thinking

Step 1: Divide the coefficients. Identify the terms with common bases and subtract their indices.

Step 2: Fully simplify the expression.

Continues →

Back to contents

c. $\dfrac{6a^2b^5 \times 2b^2a^0}{3ab^7}$

WE3c

Working

$$6a^2b^5 \times 2b^2a^0 = 6 \times 2 \times a^{2+0} \times b^{5+2}$$
$$= 12a^2b^7$$

$$\frac{6a^2b^5 \times 2b^2a^0}{3ab^7} = \frac{12a^2b^7}{3ab^7}$$
$$= (12 \div 3)a^{2-1}b^{7-7}$$
$$= 4a^1b^0$$
$$= 4a \times 1$$
$$= 4a$$

Thinking

Step 1: Simplify the numerator.

Step 2: Rewrite the fraction with the numerator simplified.

Step 3: Identify the terms with common bases, subtract their indices and fully simplify the expression.

Student practice

Simplify the following.

a. $3p^7 \times 2r^3p^6$

b. $\dfrac{12x^5y^{10}}{4y^4x^3}$

c. $\dfrac{2t^0r^7 \times 4r^4t^8}{2t^3r^{11}}$

Back to contents

2A Questions

Understanding worksheet

1. Fill in the blanks.

Example

$$2y^7 \times y^5 = 2y^{7+5} = 2y^{\boxed{12}}$$

a. $x^3 \times 3x^6 = 3x^{3+6} = 3x^{\boxed{}}$

b. $5y \times y^7 = 5y^{1+7} = 5y^{\boxed{}}$

c. $6t^5 \times 7t^0 = 42t^{5+0} = 42t^{\boxed{}}$

d. $10p^2 \times p^4 \times 2p^8 = 20p^{2+4+8} = 20p^{\boxed{}}$

2. Fill in the blanks.

Example

$$\frac{18t^9}{3t^5} = 6t^{9-5} = 6t^{\boxed{4}}$$

a. $\frac{3x^5}{x^3} = 3x^{5-3} = 3x^{\boxed{}}$

b. $\frac{8a^7}{2a} = 4a^{7-1} = 4a^{\boxed{}}$

c. $\frac{16b^8}{20b^3} = \frac{4b^{8-3}}{5} = \frac{4b^{\boxed{}}}{5}$

d. $\frac{25c^{11} \times c}{10c^8} = \frac{5c^{11+1-8}}{2} = \frac{5c^{\boxed{}}}{2}$

3. Fill in the blanks by using the words provided.

one	form	identical	laws

Multiplication and division of numbers in index ______ requires the application of the First and Second index laws. These laws may only be applied to indices with ______ bases.

A combination of both ______ is used to simplify expressions involving mixed operations.

Any non-zero number or variable with a power of zero is always equal to ______ .

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Simplify the following. **WE1**

a. 5×5^9

b. $\frac{7^{11}}{7}$

c. $3^{12} \times 3^4$

d. $\frac{8^{14}}{8^6}$

e. $2 \times 2^3 \times 2^5$

f. $6^7 \times 6^2 \times 6^0$

g. $\frac{4^2 \times 4^8}{4^3}$

h. $\frac{10^{15}}{10^6 \times 10^9}$

Back to contents

5. Simplify the following. WE2

a. $y^2 \times y^6$ **b.** $\frac{x^8}{x^6}$ **c.** $c^0 \times c^9$ **d.** $\frac{r^{10}}{r^{10}}$

e. $t \times t^7 \times t^6$ **f.** $\frac{p^{16} \times p^0}{p^9}$ **g.** $\frac{x^4 \times x^{13}}{x^{14}}$ **h.** $\frac{y^{21}}{y \times y^6 \times y^4}$

6. Simplify the following. WE3a

a. $y \times x^2y^3$ **b.** $3p^4 \times t^7p^3$

c. $6vw^5 \times w^2v^6$ **d.** $4a^5b^8 \times 2b^5a^7$

e. $2c^0d^{13} \times 7c^6d^0$ **f.** $7pq \times 8q^2p^5 \times q$

g. $2x^2y^4 \times xy \times 3xy^5$ **h.** $4t^4r^9 \times 2t^2r^4 \times t^6r^5$

7. Simplify the following. WE3b,c

a. $\frac{x^8y^7}{yx^5}$ **b.** $\frac{4a^2b^{10}}{2ab^4}$ **c.** $\frac{12p^6q^{11}}{3q^8p^6}$ **d.** $\frac{3xy^2 \times x^4y^3}{x^2y^4}$

e. $\frac{2v^{12}w^7 \times 3w^3v}{2v^2w^5}$ **f.** $\frac{4m^6n^4 \times 5n^3m^7}{10m^{13}n^2}$ **g.** $\frac{5t^0y^{11} \times 6t^6y^3}{25t^4y^0 \times y}$ **h.** $\frac{p^9r^8 \times pr \times 8r^6p^2}{6pr \times r^7}$

8. Which of the options shows the given expression fully simplified?

$\frac{x^{15}y^9}{y^3x^5}$

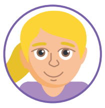
A. x^3y^3 **B.** xy^{16} **C.** $x^{10}y^6$ **D.** $x^{12}y^4$ **E.** $x^{75}y^{27}$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Simplify the following.

$2a^5b^3 \times 7b^2a^8$



Student A

$2a^5b^3 \times 7b^2a^8 = 2 \times 7 \times a^{5+8} \times b^{3+2}$

$= 14a^{13}b^5$

Student B

$2a^5b^3 \times 7b^2a^8 = 2 \times 7 \times a^{5+2} \times b^{3+8}$

$= 14a^7b^{11}$

b. Simplify the following.

$\frac{21x^{14}y^9}{7x^2y^9}$

Student A

$\frac{21x^{14}y^9}{7x^2y^9} = (21 \div 7)x^{14 \div 2}y^{9 \div 9}$

$= 3x^7y$

Student B

$\frac{21x^{14}y^9}{7x^2y^9} = (21 \div 7)x^{14-2}y^{9-9}$

$= 3x^{12}y^0$

$= 3x^{12}$

Back to contents

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. In a racing computer game, a player can double their score every five seconds if they continuously avoid any obstacles. What is the final score of a player with 7200 points who continuously avoids obstacles for the last 35 seconds of a race?

11. Every day for a week, the number of people who have seen a video online increases by a factor of four. If initially, only 300 people saw the video, how many saw it by the end of that week?

12. The height of a 2 m tall bamboo plant increases by a factor of four every week. How many days will it take the bamboo to grow to 32 m?

13. Each person at a wedding can choose a combination of dishes from a set menu to make up their own five-course meal. A guest can choose one of three types of dishes for each course. How many different combinations of five-course meals are available?

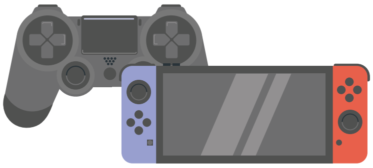
14. Garth wants to crochet a quilt made of identical squares with a total area of 1.44 m^2. If Garth wants the quilt to consist of exactly 6^2 squares, how long should he make one side of each individual square, in cm?

Reasoning

Question working paths

Mild 15 (a,b,d) 🌶	**Medium** 15 (a,b,d), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Random access memory (or RAM) is a measure, in memory units, of the memory capacity of a computer chip. It can affect the speed at which a computer processes information, as well as the amount of data that can be stored. Gaming consoles are specially designed computers that are affected by RAM.



Memory units

1 kilobyte (KB) = 2^{10} bytes (B)

1 megabyte (MB) = 2^{10} KB

1 gigabyte (GB) = 2^{10} MB

a. An early gaming console from the 1990s had 2 MB of RAM. How many KB does this equal? Express the answer in index form using powers of 2.

b. Convert 1 GB to KB and express the answer in index form using powers of 2.

c. A modern console comes with at least 1 terabyte (TB) of RAM. If 1 TB $= 2^{10}$ GB, how many times has the memory capacity of a gaming console doubled since the 1990s?

d. Name a device or object that you find useful which requires the use of a computer chip.

16. Consider the following expression where n can be any integer.

2^n

a. Substitute $n = 3$ into the expression and evaluate.

b. Substitute $n = 4$ into the expression and evaluate.

c. Use the values from parts **a** and **b**, together with the First Index Law, to evaluate 2^7. How can the law for multiplying numbers in index form help with the calculation of larger indices?

Back to contents

Exam-style

17. Determine which option is equivalent to the given expression. (1 MARK)

$$\frac{x^7y \times 4y^3x^4}{12x^8y^4}$$

A. $\frac{3}{x^3}$ **B.** $\frac{x^3}{3}$ **C.** $\frac{x^3y}{3}$ **D.** $3x^3$ **E.** $3x^3y$

18. Consider the given equation. (2 MARKS)

$96 = 3 \times 2^x$

a. What is the value of 2^x? 1 MARK

b. What is the value of x? 1 MARK

19. When the radius of a sphere is doubled, its volume increases by a factor of 2 three times. If the volume of a sphere with a radius of 10 cm is 4189 cm^3, what is the volume of a sphere with a radius of 40 cm? (2 MARKS)

20. Consider the given formula modelling the growth of cells in a sample. (3 MARKS)

$P = 257 \times 3^t$

where P is the number of cells and t is time in hours since the initial observation.

What time and day did observation of the sample begin if the number of cells is 187 353 at 3:37 am on Friday?

Remember this?

21. This list shows the time taken, in minutes, by ten students to complete a puzzle.

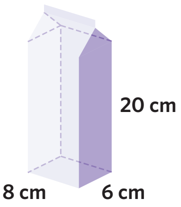
31, 18, 42, 43, 37, 49, 32, 31, 34, 44

What is the range?

A. 18 minutes **B.** 26 minutes **C.** 31 minutes **D.** 36 minutes **E.** 49 minutes

22. The given diagram is a drawing of a milk carton. It has the shape of a prism.

The shaded side has an area of 129 square centimetres.

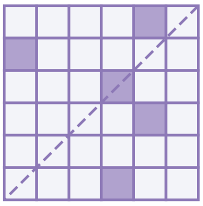


The capacity of one cubic centimetre is 1 millilitre.

What is the capacity of the carton in millilitres?

A. 774 mL **B.** 960 mL **C.** 1000 mL **D.** 1032 mL **E.** 2580 mL

23. Chelsea is shading squares in a grid to make a pattern.



She needs to shade extra squares so that the dotted line will be a line of symmetry.

What is the smallest number of extra squares she needs to shade?

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

Back to contents

2B The Third, Fourth and Fifth index laws

LEARNING INTENTIONS

Students will be able to:

- raise a term in index form to a power
- raise a product or quotient to a power
- simplify expressions using the index laws.

The Third Index Law is used for raising values in index form to a power. The Fourth and Fifth index laws are applied to powers of products and quotients respectively. These last two laws can be derived by using a combination of the first three laws together with knowledge of index notation.

KEY TERMS AND DEFINITIONS

- The **product** is the result when two or more values are multiplied together.
- The **quotient** is the result when a value (dividend or numerator) is divided by another value (divisor or denominator).

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: VectorMine/Shutterstock.com

Many companies use viral marketing to spread knowledge about their products or services. This type of advertising targets the whole population and relies on people sharing information with each other at an exponential rate. In other words, getting the word out to any group of people will ultimately lead to more exposure and sales. With the invention of social media this has become a cheap and effective way for companies to advertise.

Key ideas

1. The Third Index Law states that when raising a term in index form to another power the indices are multiplied.

$$(x^p)^q = x^{p \times q} = x^{pq}$$

2. The Fourth Index Law states that a power of a product is equivalent to the product of the powers.

$$(xy)^p = x^p \times y^p = x^p y^p$$

3. The Fifth Index Law states that a power of a quotient is equivalent to the quotient of the powers.

$$\left(\frac{x}{y}\right)^p = x^p \div y^p = \frac{x^p}{y^p}$$

Back to contents

Worked example 1

Using the Third and Fourth index laws

Simplify the following.

a. $(a^3)^6$ WE1a

Working

$(a^3)^6 = a^{3\times 6} = a^{18}$

Thinking

Multiply the index of the pronumeral inside the brackets by the index on the outside of the brackets.

Visual support

$$\begin{aligned}(a^3)^6 &= \overbrace{a^3 \times a^3 \times a^3 \times a^3 \times a^3 \times a^3}^{\text{Multiply the term inside the brackets by itself six times}}\\ &= a^{3+3+3+3+3+3}\\ &= a^{3\times 6}\\ &= a^{18}\end{aligned}$$

b. $(2x^5)^3$ WE1b

Working

$(2x^5)^3 = 2^{1\times 3}x^{5\times 3}$

$2^{1\times 3} = 2^3 = 2 \times 2 \times 2 = 8$

$2^3x^{5\times 3} = 8x^{15}$

Thinking

Step 1: Multiply the indices of the coefficient and pronumeral inside the brackets by the index on the outside of the brackets.

Step 2: Evaluate the coefficient.

Step 3: Fully simplify the expression.

c. $(-5t^4r^9)^2$ WE1c

Working

$(-5t^4r^9)^2 = (-5)^2t^{4\times 2}r^{9\times 2}$

$(-5)^2 = (-5) \times (-5) = 25$

$(-5)^2t^{4\times 2}r^{9\times 2} = 25t^8r^{18}$

Thinking

Step 1: Multiply the indices of the coefficient and pronumerals inside the brackets by the index on the outside of the brackets.

Step 2: Evaluate the coefficient.

Step 3: Fully simplify the expression.

Student practice

Simplify the following.

a. $(y^4)^5$ **b.** $(3r^9)^3$ **c.** $(-4p^3v^8)^2$

Back to contents

Worked example 2

Using the Fifth Index law

Simplify the following.

a. $\left(\frac{2}{t^3}\right)^4$ WE2a

Working	Thinking
$\left(\frac{2}{t^3}\right)^4 = \frac{(2)^4}{(t^3)^4}$	**Step 1:** Raise the numerator and denominator to the power on the outside of the brackets.
$= \frac{2^{1\times4}}{t^{3\times4}}$	**Step 2:** Apply the Third Index Law to the numerator and denominator.
$= \frac{2^4}{t^{12}} = \frac{16}{t^{12}}$	**Step 3:** Evaluate the numerator and fully simplify the expression.

Visual support

$$\left(\frac{2}{t^3}\right)^4 = \overbrace{\frac{2}{t^3} \times \frac{2}{t^3} \times \frac{2}{t^3} \times \frac{2}{t^3}}$$

$$= \frac{2 \times 2 \times 2 \times 2}{t^3 \times t^3 \times t^3 \times t^3} = \frac{2^{1+1+1+1}}{t^{3+3+3+3}}$$

$$= \frac{2^{1\times4}}{t^{3\times4}} = \frac{2^4}{(t^3)^4}$$

b. $\left(\frac{3^2x^6}{4y^7}\right)^3$ WE2b

Working	Thinking
$\left(\frac{3^2x^6}{4y^7}\right)^3 = \frac{(3^2x^6)^3}{(4y^7)^3}$	**Step 1:** Raise the numerator and denominator to the power on the outside of the brackets.
$= \frac{3^{2\times3}x^{6\times3}}{4^3y^{7\times3}}$ $= \frac{3^6x^{18}}{4^3y^{21}}$	**Step 2:** Apply the Fourth Index Law to the numerator and denominator.
$= \frac{729x^{18}}{64y^{21}}$	**Step 3:** Evaluate the coefficients and fully simplify the expression.

Student practice

Simplify the following.

a. $\left(\frac{2}{p^5}\right)^3$ **b.** $\left(\frac{3^2t^6}{5r^7}\right)^2$

Back to contents

Worked example 3

Simplifying expressions using index laws

Simplify the following.

a. $(3^2y^7)^2 \times 2(y^6)^3$

WE3a

Working	Thinking
$(3^2y^7)^2 \times 2(y^6)^3 = 3^{2\times2}y^{7\times2} \times 2 \times y^{6\times3}$ $= 3^4\times 2 \times y^{14} \times y^{18}$	**Step 1:** Apply the Third Index Law to each of the terms in brackets.
$= 81 \times 2 \times y^{14+18}$	**Step 2:** Evaluate the coefficient and apply the First Index Law.
$= 162y^{32}$	**Step 3:** Fully simplify the expression.

b. $\dfrac{(4a^6b^7)^3}{2b^5}$

WE3b

Working	Thinking
$\dfrac{(4a^6b^7)^3}{2b^5} = \dfrac{4^3a^{6\times3}b^{7\times3}}{2b^5}$ $= \dfrac{4^3a^{18}b^{21}}{2b^5}$	**Step 1:** Apply the Fourth Index Law to the term in the numerator.
$= \dfrac{64a^{18}b^{21-5}}{2}$ $= (64 \div 2)a^{18}b^{16}$	**Step 2:** Evaluate the coefficient and apply the Second Index Law.
$= 32a^{18}b^{16}$	**Step 3:** Fully simplify the expression.

c. $\left(\dfrac{7x^9}{3y}\right)^2 \times \left(\dfrac{y}{x^4}\right)^3$

WE3c

Working	Thinking
$\left(\dfrac{7x^9}{3y}\right)^2 \times \left(\dfrac{y}{x^4}\right)^3 = \dfrac{(7x^9)^2}{(3y)^2} \times \dfrac{y^3}{(x^4)^3}$	**Step 1:** Apply the Fifth Index Law to each of the terms in the expression.
$= \dfrac{7^2x^{9\times2}}{3^2y^2} \times \dfrac{y^3}{x^{4\times3}}$ $= \dfrac{49x^{18}}{9y^2} \times \dfrac{y^3}{x^{12}}$	**Step 2:** Evaluate the coefficients and simplify each of the terms.
$= \dfrac{49x^{18}y^3}{9y^2x^{12}}$ $= \dfrac{49x^{18-12}y^{3-2}}{9}$	**Step 3:** Multiply the terms and apply the Second Index Law.
$= \dfrac{49x^6y}{9}$	**Step 4:** Fully simplify the expression.

Student practice

Simplify the following.

a. $(2^3x^6)^2 \times 7(x^4)^8$ **b.** $\dfrac{(3m^4n^7)^5}{9n^3}$ **c.** $\left(\dfrac{5k^8}{4t}\right)^3 \times \left(\dfrac{t^5}{k^2}\right)^6$

Back to contents

2B Questions

Understanding worksheet

1. Determine which index law applies to each expression.

Example

Expression	Third Index Law	Fourth Index Law	Fifth Index Law
$(x^7y^5)^8$	☐	☑	☐

Expression	Third Index Law	Fourth Index Law	Fifth Index Law
I. $(z^2)^{12}$	☐	☐	☐
II. $\left(\frac{3^5}{y^9}\right)^2$	☐	☐	☐
III. $(2p^6)^{10}$	☐	☐	☐
IV. $-7\left(\frac{t^2}{r^{11}}\right)^4$	☐	☐	☐

2. Fill in the blanks.

Example

$$(a^4b^3)^{10} = a^{4\times10}b^{3\times10} = a^{\boxed{40}}b^{30}$$

a. $\left(\frac{5}{c^6}\right)^3 = \frac{5^{\boxed{}}}{c^{6\times3}} = \frac{125}{c^{18}}$

b. $(2^2x^7)^4 = 2^{2\times4}x^{7\times4} = 2^{\boxed{}}x^{28} = 256x^{28}$

c. $\left(\frac{t^4}{p^9}\right)^5 = \frac{t^{4\times5}}{p^{9\times5}} = \frac{t^{20}}{p^{\boxed{}}}$

d. $(-8v^{12}w^{13})^4 = (-8)^4v^{12\times4}w^{13\times4} = 4096v^{48}w^{\boxed{}}$

3. Fill in the blanks by using the words provided.

same	multiplied	expanded	quotients

Powers must be ______ together when applying the Third Index Law. The Fourth and Fifth index laws are used for raising products and ______ to a power respectively. They state that the power of a product or quotient is the ______ as the product or quotient of the powers. The last three index laws can all be derived by writing indices in ______ form and applying the first two index laws for multiplication and division.

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Simplify the following. WE1a,b

a. $(y^3)^4$ **b.** $(t^{10})^5$ **c.** $(3k^4)^2$ **d.** $(2^2x^5)^3$

e. $(4p^{11})^4$ **f.** $(10x^0)^2$ **g.** $(-6n^{13})^2$ **h.** $5(2m^8)^{10}$

---

5. Simplify the following. WE1c

a. $(xy^6)^4$ **b.** $(p^3r^5)^3$ **c.** $(2m^7n^2)^4$ **d.** $(-2a^8b)^2$

e. $(7c^5d^{10})^3$ **f.** $(122^0v^{12}w^9)^8$ **g.** $(-3y^2x^{15})^3$ **h.** $3(-2t^6r)^7$

---

6. Simplify the following. WE2

a. $\left(\frac{3}{x^5}\right)^2$ **b.** $\left(\frac{v^6}{w^9}\right)^3$ **c.** $\left(\frac{2^2}{y^8}\right)^3$ **d.** $\left(\frac{4x}{3y^4}\right)^2$

e. $\left(\frac{5^2k^8}{2^3m^{10}}\right)^2$ **f.** $\left(\frac{10p^6r^3}{3q}\right)^4$ **g.** $\left(\frac{2^2m^{11}n^5}{3t^{12}}\right)^5$ **h.** $\left(\frac{4u^8v^{15}}{3^2x^7y^3}\right)^4$

---

7. Simplify the following. WE3a

a. $(x^2y^4)^3 \times x^7$ **b.** $(m^5n^2)^4 \times (n^4)^2$

c. $(t^9r)^5 \times (r^6t^2)^3$ **d.** $(2a^5b^4)^3 \times (b^7a^2)^5$

e. $4(y^2x^9)^4 \times (2x^4y^6)^5$ **f.** $(3u^9v^6)^4 \times 3(v^8u^5)^7$

g. $(2^3p^4q^8)^3 \times 3^2(q^0p^{10})^5$ **h.** $(7c^3d^6)^2 \times 2(d^7)^6 \times (3c^8)^3$

---

8. Simplify the following. WE3b,c

a. $\frac{(x^4y^6)^3}{y^5}$ **b.** $\frac{(3t^3r^5)^2}{2t}$ **c.** $\frac{(2ba^7)^3}{5a^4}$ **d.** $\frac{(2p^4r^9)^5}{(rp^3)^4}$

e. $\frac{(4c^3)^2}{d^3} \times \left(\frac{d}{c}\right)^3$ **f.** $\frac{(2y^4x^2)^6}{(xy)^3} \times \left(\frac{x^5}{y}\right)^2$ **g.** $\left(\frac{n^3}{m^2}\right)^5 \times \frac{7(m^6n^9)^4}{(6m^4n^5)^2}$ **h.** $\left(\frac{6d^5c^{10}}{2^3c^4}\right)^3 \times \left(\frac{2c^3}{d^2}\right)^4$

---

9. Once the given expression has been fully simplified, what is the final value of the coefficient?

$3(y^6)^2 \times (2^2x^8)^3$

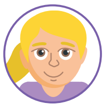
A. 12 **B.** 36 **C.** 96 **D.** 192 **E.** 576

Spot the mistake

10. Select whether Student A or Student B is incorrect.

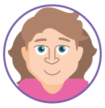
a. Simplify the following.

$(2^5x^7y)^2$



Student A

$$\begin{aligned}(2^5x^7y)^2 &= 2^{5\times2}x^{7\times2}y^2\\ &= 2^{10}x^{14}y^2\\ &= 1024x^{14}y^2\end{aligned}$$



Student B

$$\begin{aligned}(2^5x^7y)^2 &= 2^5x^{7\times2}y^2\\ &= 32x^{14}y^2\end{aligned}$$

b. Simplify the following.

$\left(\frac{5p^{10}}{t^9}\right)^3$

Student A

$$\begin{aligned}\left(\frac{5p^{10}}{t^9}\right)^3 &= \frac{5^3p^{10\times3}}{t^{9\times3}}\\ &= \frac{125p^{30}}{t^{27}}\end{aligned}$$

Student B

$$\begin{aligned}\left(\frac{5p^{10}}{t^9}\right)^3 &= \frac{5^3p^{10+3}}{t^{9+3}}\\ &= \frac{125p^{13}}{t^{12}}\end{aligned}$$

Back to contents

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. At 7 am, the number of people who have read an article online was 32. This number then increased by a factor of $\frac{5}{2}$ every 15 minutes. How many people read the article by 8 am?

12. Dominick's computer had 4^3 GB of available storage space. When he upgraded it, the amount of available storage space doubled a number of times. If the amount of available storage space on Dominick's computer is now 2048 GB, how many times did it double?

13. Each side length of a rectangular digital image, measuring 3600 by 2800 pixels, was resized by a factor of $\frac{1}{3}$. What is the area of the resized image, in pixels?

14. A hole appears in a tank containing 4096 m^3 of water. The amount of water reduces by a factor of four every second. After how many seconds will the volume of water in the tank be less than 1 m^3?

15. A biased coin has been designed so that the probability of landing tails is $\frac{2}{3}$. What is the probability of landing tails on every throw for three throws?

Reasoning

Question working paths

Mild 16 (a,b,d) 🌶	**Medium** 16 (a,b,d), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. All moving particles and objects have kinetic energy. The kinetic energy of any object can be calculated using the given formula.

$$K = \frac{mv^2}{2}$$

where m is mass in kg, v is velocity in m/s and K is kinetic energy in Joules (J).

- **a.** Substitute $m = 24$ into the equation and simplify.
- **b.** Calculate the kinetic energy of a 24 kg object with a velocity of $\frac{7}{2}$ m/s.
- **c.** The object from part **b** speeds up so that its velocity increases by a factor of 3. What is the kinetic energy of the object now?
- **d.** Kinetic energy is always converted from other types of energy. For living things, this is often chemical energy produced by the body after consuming nutrients. Where does the kinetic energy of machines come from?

17. Consider the following equation.

$$y = (-2)^x$$

- **a.** Substitute $x = 4$ and determine the value of y by writing the calculation in expanded form.
- **b.** Substitute $x = 5$ and determine the value of y by writing the calculation in expanded form.
- **c.** Analyse the answers from parts **a** and **b**. What is the pattern followed by negative values raised to a power?

Exam-style

18. Which of the options shows the given expression when fully simplified? (1 MARK)

$$\frac{9(a^3b^7)^5}{(3b^3)^2}$$

A. $a^{15}b^{29}$ **B.** $a^{15}b^{41}$ **C.** $3a^{15}b^{29}$ **D.** $3a^{15}b^{41}$ **E.** $6561a^{15}b^{29}$

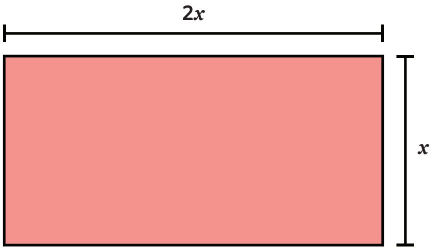
19. Consider the given equation. (3 MARKS)

$16^x = 2^{12}$

a. Write 16 as a power of 2 and substitute it into the equation. 1 MARK

b. Determine the value of x. 2 MARKS

20. Write an expression for the ratio of area (A) to the perimeter (P) of the given rectangle and use it to calculate the value of x when the ratio is equal to 1. (3 MARKS)



21. Fully simplify the given expression, showing all necessary working steps. (3 MARKS)

$$\left(\frac{x^6}{y^4}\right)^3 \times \frac{(2y^2)^7}{x^5}$$

Remember this?

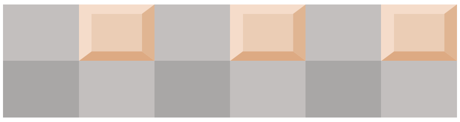
22. Rodney has spent 65% of his money on a new PS4 game.

The game cost $25.

How much money did Rodney have left after he bought the surfboard?

A. $12.25
B. $13.46
C. $16.25
D. $19.46
E. $25

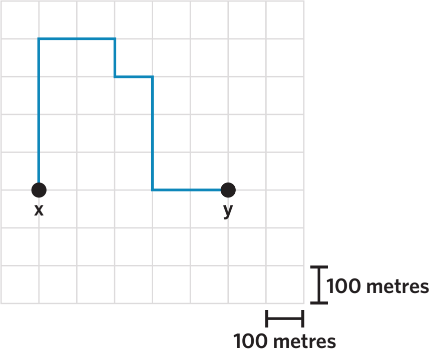
23. A large block of chocolate is split up in 12 equal-sized blocks as shown. The grey shaded blocks have already been eaten.



What percentage of the chocolate still needs to be eaten?

A. 15%
B. 20%
C. 25%
D. 40%
E. 75%

24. Mitchell walks along the path from x to y.



How far does he walk in kilometres?

A. 0.13 km
B. 1.3 km
C. 13 km
D. 130 km
E. 1300 km

Back to contents

2C Negative indices

LEARNING INTENTIONS

Students will be able to:

- evaluate negative indices
- express negative indices as positive
- apply index laws to simplify expressions involving negative indices.

While positive indices allow us to represent repeated multiplication, negative indices do the same for repeated division. Using a combination of index laws while applying conceptual understanding of negative indices allows us to simplify and manipulate algebraic expressions and formulas.

KEY TERMS AND DEFINITIONS

- The **reciprocal of a fraction** can be obtained by swapping the values or expressions in the numerator with those in the denominator.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

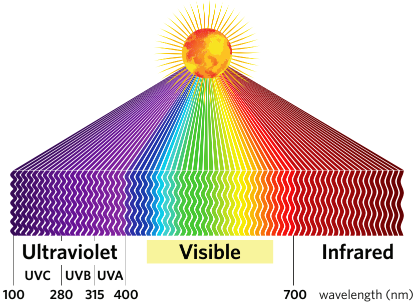


Image: Fouad A. Saad/Shutterstock.com

Negative indices allow us to describe inversely proportional relationships mathematically. For example, the frequency of a light wave is inversely proportional to its wavelength. Longer waves have lower frequencies, whereas shorter waves have higher frequencies. These aspects of light waves specifically determine whether they are detectable by the human eye.

Key ideas

1. Negative indices can be expressed as positive indices using division when $x \neq 0$.

$$x^{-p} = \frac{1}{x^p} \Leftrightarrow \frac{1}{x^{-p}} = x^p$$

2. Index laws apply to negative indices.

Multiplication

$$x^{-p} \times x^{-q} = x^{-p-q} = \frac{1}{x^p} \times \frac{1}{x^q} = \frac{1}{x^{p+q}}$$

Division

$$\frac{x^{-p}}{x^{-q}} = x^{-p-(-q)} = x^{q-p} = \frac{x^q}{x^p}$$

Power of a power

$$(x^p)^{-q} = (x^{-p})^q = x^{-pq} = \frac{1}{x^{pq}}$$

Back to contents

Worked example 1

Evaluating negative indices

Evaluate the following.

a. 3^{-2}

WE1a

Working

$= \frac{1}{3^2}$

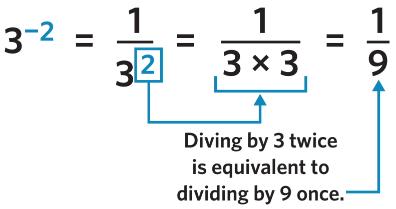
$= \frac{1}{9}$

Thinking

Step 1: Express the negative index as a fraction with a positive index.

Step 2: Evaluate.

Visual support



b. $\left(\frac{2}{5}\right)^{-2}$

WE1b

Working

$= \left(\frac{2^1}{5^1}\right)^{-2} = \left(\frac{2^{1\times(-2)}}{5^{1\times(-2)}}\right) = \frac{2^{-2}}{5^{-2}}$

$= \frac{5^2}{2^2}$

$= \frac{25}{4}$

Thinking

Step 1: Multiply the indices of the terms inside the brackets by the negative index on the outside of the brackets.

Step 2: Write a reciprocal of the fraction with positive indices.

Step 3: Evaluate.

c. $(2^3 \times 4^2)^{-1}$

WE1c

Working

$= 2^{3\times(-1)} \times 4^{2\times(-1)} = 2^{-3} \times 4^{-2}$

$= \frac{1}{2^3} \times \frac{1}{4^2}$

$= \frac{1}{8} \times \frac{1}{16} = \frac{1}{128}$

Thinking

Step 1: Multiply the indices of the terms inside the brackets by the negative index on the outside of the brackets.

Step 2: Express the negative indices as fractions with positive indices.

Step 3: Simplify and evaluate.

Student practice

Evaluate the following.

a. 8^{-2} **b.** $\left(\frac{3}{4}\right)^{-3}$ **c.** $(3^2 \times 2^3)^{-1}$

Back to contents

Worked example 2

Writing negative indices as positive

Write using positive indices only.

a. x^{-5}

WE2a

Working

$= \frac{1}{x^5}$

Thinking

Express the negative index as a fraction with a positive index.

Visual support

$$x^{\boxed{-5}} = \frac{1}{\underbrace{x \times x \times x \times x \times x}} = \frac{1}{x^5}$$

The negative power of 5 shows there is a repeated division by *x* five times.

b. $3y^{-7}$

WE2b

Working

$y^{-7} = \frac{1}{y^7}$

$3y^{-7} = 3 \times \frac{1}{y^7} = \frac{3}{y^7}$

Thinking

Step 1: Express the negative index as a fraction with a positive index.

Step 2: Multiply the fraction by the coefficient and simplify.

c. $\frac{x^{-3}}{y^{-6}}$

WE2c

Working

$= \frac{y^6}{x^3}$

Thinking

Write a reciprocal of the fraction with positive indices.

Student practice

Write using positive indices only.

a. x^{-8} **b.** $5r^{-2}$ **c.** $\frac{w^{-5}}{z^{-9}}$

Worked example 3

Simplifying expressions with negative indices

Simplify and write using positive indices only.

a. $a^{-8} \times a^2$

WE3a

Working

$= a^{-8+2} = a^{-6}$

$= \frac{1}{a^6}$

Thinking

Step 1: Apply the First Index Law.

Step 2: Convert negative indices to positive.

Continues →

Back to contents

Visual support

$$a^{-8} \times a^{2} = \frac{a^{\boxed{2}}}{a^{\boxed{8}}} = \frac{\overbrace{\cancel{a} \times \cancel{a}}}{\underbrace{\cancel{a} \times \cancel{a} \times a \times a \times a \times a \times a \times a}}$$

$$= \frac{1}{\underbrace{a \times a \times a \times a \times a \times a}} = \frac{1}{a^{\boxed{6}}}$$

b. $\dfrac{6u^4}{3u^9}$

WE3b

Working	**Thinking**
$= (6 \div 3)(u^{4-9}) = 2u^{-5}$	**Step 1:** Divide the coefficients and apply the Second Index Law.
$= 2 \times \dfrac{1}{u^5} = \dfrac{2}{u^5}$	**Step 2:** Convert negative indices to positive and simplify the expression.

c. $2t^{-6}r^{-4} \times 5tr^2$

WE3c

Working	**Thinking**
$= (2 \times 5)(t^{-6+1})(r^{-4+2}) = 10t^{-5}r^{-2}$	**Step 1:** Multiply the coefficients and apply the First Index Law to terms with the same bases.
$t^{-5} = \dfrac{1}{t^5}$ and $r^{-2} = \dfrac{1}{r^2}$	**Step 2:** Convert negative indices to positive.
$\therefore 10t^{-5}r^{-2} = 10 \times \dfrac{1}{t^5} \times \dfrac{1}{r^2} = \dfrac{10}{t^5r^2}$	**Step 3:** Simplify the expression.

Student practice

Simplify and write using positive indices only.

a. $c^{-9} \times c^3$ **b.** $\dfrac{12x^3}{3x^7}$ **c.** $3q^{-2}r^{-4} \times 5r^2q$

Back to contents

2C Questions

Understanding worksheet

1. Complete each of the following by filling in the blanks.

Example

$$6^{-3} = \frac{1}{6^{\boxed{3}}}$$

a. $4^{-6} = \dfrac{1}{4^{\boxed{}}}$ **b.** $x^{-10} = \dfrac{1}{x^{\boxed{}}}$ **c.** $7^{\boxed{}} = \dfrac{1}{7^2}$ **d.** $y^{\boxed{}} = \dfrac{1}{y}$

2. Complete each of the following by filling in the blanks.

Example

$$5^{-9} \times 5^6 = 5^{-9+6} = 5^{\boxed{-3}}$$

a. $2^{-3} \times 2^2 = 2^{-3+2} = 2^{\boxed{}}$

b. $\dfrac{4^{-2}}{4} = 4^{-2-1} = 4^{\boxed{}}$

c. $b^6 \times b^{-13} = b^{6+(-13)} = b^{\boxed{}}$

d. $\dfrac{a^3}{a^{11}} = a^{3-11} = a^{\boxed{}}$

3. Fill in the blanks by using the words provided.

reciprocal **proportional** **division** **laws**

Negative indices allow us to represent repeated $\boxed{}$ more efficiently. When working with any indices, including those that are negative, the appropriate index $\boxed{}$ must be followed. When a value or variable has a negative index, it can be expressed with a positive index by using its $\boxed{}$ instead. Negative indices indicate an inversely $\boxed{}$ relationship in equations and formulas.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Evaluate the following. WE1a

a. 4^{-2} **b.** 5^{-3} **c.** 3^{-1} **d.** 10^{-4}

e. 12^{-2} **f.** 223^{-1} **g.** 10^{-8} **h.** 2^{-10}

Back to contents

5. Evaluate the following.

WE1b,c

a. $\left(\frac{1}{2}\right)^{-2}$

b. $(2^4 \times 3^2)^{-1}$

c. $\left(\frac{2}{3}\right)^{-3}$

d. $(4^2 \times 3)^{-2}$

e. $\left(\frac{5}{7}\right)^{-2}$

f. $(6^3 \times 3^0)^{-1}$

g. $\left(\frac{3^2}{10}\right)^{-3}$

h. $\frac{(2^4 \times 7)^{-3}}{2^{-7}}$

6. Write using positive indices only.

WE2a,b

a. x^{-4}

b. y^{-2}

c. r^{-1}

d. $2y^{-3}$

e. $5t^{-7}$

f. $10z^{-12}$

g. $\frac{x^{-3}}{2}$

h. $\frac{3p^{-5}}{5}$

7. Write using positive indices only.

WE2c

a. $\frac{1}{x^{-3}}$

b. $\frac{x^{-4}}{y^{-1}}$

c. $\frac{p^{-3}}{q^{-2}}$

d. $\frac{t^{-1}}{r^{-3}}$

e. $\frac{3u^{-7}}{v^{-4}}$

f. $\frac{a^{-4}}{2b^{-2}}$

g. $\frac{4c^{-1}}{5d^{-6}}$

h. $\frac{2p^{-3}}{7t^{-8}}$

8. Simplify and write using positive indices only.

WE3

a. $x^{-6} \times x$

b. $4t^{-7} \times t^5$

c. $\frac{10p^4}{5p^8}$

d. $3x^{-7} \times 6x^2$

e. $\frac{32t^3}{6t^{11}}$

f. $2a^{-9}b^{-5} \times 3ab$

g. $\frac{42x^{-5}}{3x^5}$

h. $5m^3n^{-4} \times 6m^{-2}n$

9. Simplify and write using positive indices only.

a. $(x^4 \times x^{-1})^{-2}$

b. $\left(\frac{a^{-3}}{a^{-1}}\right)^{-3}$

c. $(p^3q^{-6})^{-1}$

d. $(2x^5y^{-1})^{-2}$

e. $\frac{4r^{-3}p}{16p^{-1}}$

f. $\frac{x^7y^{-5}z^{-3}}{xyz}$

g. $\frac{t^{-1}r^{-8}}{(7t^{-2}r)^2}$

h. $\frac{(v^{-1}u^6)^{-4}}{u^4v^2}$

10. Simplify and write using positive indices only.

$\frac{2x^{-7}y^{-1}}{8xy}$

A. $\frac{4}{x^8y^2}$

B. $-4x^8y^2$

C. $\frac{1}{4x^8y^2}$

D. $\frac{x^8y^2}{4}$

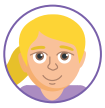
E. $\frac{1}{4x^6}$

Spot the mistake

11. Select whether Student A or Student B is incorrect.

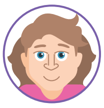
a. Simplify and write using positive indices only.

$\frac{t^4}{t^{-13}}$



Student A

$= t^{4-13} = t^{-9} = \frac{1}{t^9}$



Student B

$= t^{4-(-13)} = t^{4+13} = t^{17}$

b. Simplify and write using positive indices only.

$6x^5y^{-3}$

Student A

$= \frac{6x^5}{y^3}$

Student B

$= \frac{-6x^5}{y^3}$

Back to contents

Problem solving

Question working paths

Mild 12, 13, 14 🌶	**Medium** 13, 14, 15 🌶🌶	**Spicy** 14, 15, 16 🌶🌶🌶

12. One carat of gold is equivalent to 5^{-1} grams. What fraction of one gram is one carat of gold?

13. Sienna is looking at a small insect through a microscope. The length of the insect is 1.5 mm, as seen through the eyepiece of the microscope, which has been set to magnify objects by a factor of 10^2. Calculate the actual length of the insect and give your answer in mm.

14. Frank is uploading a 2^9 GB file to an online shared drive. Every 15 minutes, the amount of data yet to be uploaded decreases by a factor of 2. Evaluate how many GB are yet to be uploaded after three hours, leaving your answer as a fraction.

15. Dougie is practising for a mathematics test by completing past exam questions. He can currently complete 10 multiple choice questions in half an hour. If Dougie wants to be able to finish 20 multiple choice questions in half an hour, then by what factor should he reduce the amount of time he spends on each question?

16. The mass of a newborn joey kangaroo is approximately 2^{-9} kg. What is the mass of a newborn joey, to the nearest gram?

Reasoning

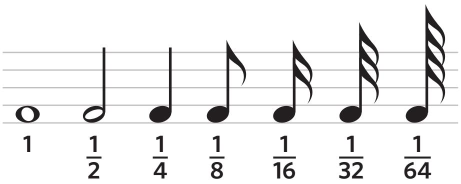
Question working paths

Mild 17 (a,b,c,e) 🌶	**Medium** 17 (a,b,c,e), 18 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

17. The length of time a musical note is played is called note duration. Types of notes show note duration, for example, a whole note has the standardised length of 1, while a half note has half of the duration of the whole note, which is given by $\frac{1}{2}$. This relationship can be shown by the following formula:

$l = 2^{-n}$

where l is note duration and n can be any integer 0 or greater.



- **a.** Write the formula for note duration using positive indices only.
- **b.** What type of note is played when $n = 0$?
- **c.** Determine the value of n when an eighth note (when $l = \frac{1}{8}$) is played.
- **d.** How many sixteenths notes equal the duration of a half note?
- **e.** In theory, any number can be halved any number of times. Give a reason why in practice, it is virtually impossible to play or hear a note with a duration of less than $\frac{1}{64}$.

18. Simplify the following using the First and Second index laws respectively.

- **a.** $x^{-6} \times x^4$
- **b.** $\frac{x^4}{x^6}$
- **c.** Compare your answers to parts **a** and **b**. How can the commutative law for the addition of numbers be used to convert negative indices to positive?

Back to contents

Exam-style

19. Which of the options is equivalent to the given expression? (1 MARK)

$x^{-2}y^5$

A. x^2y^5 **B.** $\frac{x^2}{y^5}$ **C.** $\frac{1}{y^5} \times x^2$ **D.** $\frac{y^5}{x^2}$ **E.** $\frac{1}{x^2} \times \frac{1}{y^5}$

20. Consider the number sequence given below. (2 MARKS)

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

a. Determine and evaluate the next (sixth) value in the sequence. 1 MARK

b. Write the seventh value in the sequence expressed in negative indices. 1 MARK

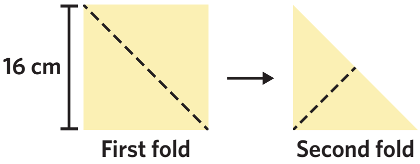
21. When an area of mould is treated, the decrease in its size can be modelled by the equation: (2 MARKS)

$A = 2300 \times 3^{-t}$

where A is the area of mould in cm^2 and t is the number of days since treatment.

Calculate the area of mould on the fifth day of treatment, correct to 2 decimal places.

22. A square piece of paper, pictured, has been folded over diagonally into a right-angled triangle with half of the area of the square. It was then folded over repeatedly into smaller similar triangles, thus halving the area each time. (3 MARKS)



Calculate the area of the paper after five folds.

Remember this?

23. Marcus' water bottle can hold up to 1 L of water. He drinks three-quarters of a full bottle. How much water is left in it?

A. 0.25 L **B.** 0.5 L **C.** 0.7 L **D.** 0.75 L **E.** 1 L

24. Letitia buys a laptop online that costs $525.

With the packaging, it weighs 2.1 kg.

Postage costs are presented in the given table.

Total mass of package	Up to 500 g	500 g up to 1 kg	1 kg up to 1.5 kg	1.5 kg up to 2 kg	2 kg up to 2.5 kg
Postage cost	$12.70	$16.75	$20.65	$27.05	$35.40

In total, how much does Letitia pay for her laptop and postage?

A. $537.70 **B.** $541.75 **C.** $545.65 **D.** $552.05 **E.** $560.40

25. The mean height of students in a classroom is 1.65 m.

The median height of the students in the same class is 1.70 m.

A new student transfers into the class and has a height of 1.92 m.

Which of these is possible?

A. The mean increases and the median decreases.

B. Both the mean and the median decrease.

C. The mean stays the same and the median increases.

D. The median stays the same and the mean increases.

E. Both the mean and the median stay the same.

Back to contents

2D Scientific notation

LEARNING INTENTIONS

Students will be able to:

- write very large numbers using scientific notation
- write very small numbers using scientific notation
- convert numbers written in scientific notation to decimal notation.

Scientific notation uses positive or negative powers of 10 to represent large and small numbers in a readable form. As place values in decimal numbers differ by a factor of 10, this means that any decimal number can be written using powers of 10. It is an application of existing number theory and index laws knowledge.

KEY TERMS AND DEFINITIONS

- A number written in **decimal notation** can be expressed using digits separated by a decimal point.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Sergey Nivens/Shutterstock.com

One area of study where scientific notation is applied regularly is astronomy. For example, 1 astronomical unit (AU) is the distance between the Earth and the Sun. It is a standardised unit, approximately equal to 1.496×10^{11} m, used primarily to express and compare distances within our Solar System.

Key idea

1. Numbers written in **scientific notation** (or standard form) are expressed in the form

$a \times 10^n$

where $1 \le a < 10$ and n can be any integer.

x can be any number

$$\boxed{x} = \boxed{a} \times 10^{\boxed{n}}$$

n is a negative integer when $0 < x < 1$
$n = 0$ when $1 \le x < 10$
n is a positive integer when $x \ge 10$

a can be any number such that $1 \le a < 10$

Back to contents

Worked example 1

Writing numbers in scientific notation

Write the following in scientific notation.

a. 784.1

WE1a

Working

$\underline{7}84.1 \to a = \underline{7}.841$

$n = 2$

$784.1 = 7.841 \times 10^2$

Thinking

Step 1: Determine the value of a by moving the first non-zero digit (or significant figure) to the ones place value position.

Step 2: Determine the value of n by counting the number of places the digit was moved to the right (positive direction).

Step 3: Write the number in scientific notation by substituting the values of a and n into the expression $a \times 10^n$.

Visual support

$$784.1 = 7.841 \times 10 \times 10$$
$$= 7.841 \times 10^2$$

Each time the leading non-zero digit is moved to the right, the number must be multiplied by 10.

b. 0.00293

WE1b

Working

$0.00\underline{2}93 \to a = \underline{2}.93$

$n = -3$

$0.00293 = 2.93 \times 10^{-3}$

Thinking

Step 1: Determine the value of a by moving the first non-zero digit (or significant figure) to the ones place value position.

Step 2: Determine the value of n by counting the number of places the digit was moved to the left (negative direction).

Step 3: Write the number in scientific notation by substituting the values of a and n into the expression $a \times 10^n$.

Visual support

$$0.00293 = 2.93 \div 10 \div 10 \div 10$$
$$= \frac{2.93}{10 \times 10 \times 10}$$
$$= \frac{2.93}{10^3}$$
$$= 2.93 \times 10^{-3}$$

Each time the leading non-zero digit is moved to the left, the number must be divided by 10.

Student practice

Write the following in scientific notation.

a. 627.3 **b.** 0.00062

Back to contents

Worked example 2

Writing numbers in scientific notation using significant figures

Write the following in scientific notation, correct to 3 significant figures.

a. 68 075 000

WE2a

Working	Thinking
$\underline{68\,0}75\,000 \approx 68\,100\,000$	**Step 1:** Round the given number to the specified number of significant figures.
$\underline{6}8\,100\,000 \rightarrow a = \underline{6}.8\,100\,000$	**Step 2:** Determine the value of *a* by moving the first significant figure to the ones place value position.
$n = 7$	**Step 3:** Determine the value of *n* by counting the number of places the digit was moved to the right (positive direction).
$68\,075\,000 \approx 68\,100\,000 = 6.81 \times 10^7$	**Step 4:** Write the number in scientific notation by substituting the values of *a* and *n* into the expression $a \times 10^n$.

b. 0.00030641

WE2b

Working	Thinking
$0.000\underline{306}41 \approx 0.000306$	**Step 1:** Round the given number to the specified number of significant figures.
$0.000\underline{3}06 \rightarrow a = \underline{3}.06$	**Step 2:** Determine the value of *a* by moving the first significant figure to the ones place value position.
$n = -4$	**Step 3:** Determine the value of *n* by counting the number of places the digit was moved to the left (negative direction).
$0.0030641 \approx 0.000306 = 3.06 \times 10^{-4}$	**Step 4:** Write the number in scientific notation by substituting the values of *a* and *n* into the expression $a \times 10^n$.

Student practice

Write the following in scientific notation, correct to 3 significant figures.

a. 78 270 000 000 **b.** 0.0020192

Back to contents

Worked example 3

Converting numbers in scientific notation to decimal notation

Write the following in decimal notation.

a. 3.27×10^{8}

WE3a

Working	Thinking
$a = 3.27$ $n = 8$	**Step 1:** Identify the values of a and n.
$\underline{3}.27 \rightarrow \underline{3}27\ 000\ 000$ $\therefore 3.27 \times 10^{8} = 327\ 000\ 000$	**Step 2:** For a, move the digit in the ones place value position n places to the left. Write the number in decimal notation.

Visual support

$$3.27 \times 10^{8} = 3.27 \overbrace{\times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}^{\text{The number must be multiplied by 10 eight times}}$$

$$= 327\ 000\ 000$$

b. 9.05×10^{-6}

WE3b

Working	Thinking
$a = 9.05$ $n = -6$	**Step 1:** Identify the values of a and n.
$\underline{9}.05 \rightarrow 0.00000\underline{9}05$ $\therefore 9.05 \times 10^{-6} = 0.00000905$	**Step 2:** For a, move the digit in the ones place value position n places to the right. Write the number in decimal notation.

Student practice

Write the following in decimal notation.

a. 8.32×10^{5} **b.** 4.75×10^{-4}

2D Questions

Understanding worksheet

1. Fill in the blanks.

Example

$$350 = 3.5 \times 10 \times 10 = 3.5 \times 10^{\boxed{2}}$$

a. $2800 = 2.8 \times 10 \times 10 \times 10 = 2.8 \times 10^{\boxed{}}$

b. $507.8 = 5.078 \times 10 \times 10 = 5.078 \times 10^{\boxed{}}$

c. $12\,700 = 1.27 \times 10 \times 10 \times 10 \times 10 = 1.27 \times 10^{\boxed{}}$

d. $7\,030\,000 = 7.03 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 7.903 \times 10^{\boxed{}}$

2. Fill in the blanks.

Example

$$0.0005 = \frac{5}{10 \times 10 \times 10 \times 10} = 5 \times 10^{\boxed{-4}}$$

a. $0.3 = \dfrac{3}{10} = 3 \times 10^{\boxed{}}$

b. $0.007 = \dfrac{7}{10 \times 10 \times 10} = 7 \times 10^{\boxed{}}$

c. $0.00052 = \dfrac{5.2}{10 \times 10 \times 10 \times 10} = 5.2 \times 10^{\boxed{}}$

d. $0.000006109 = \dfrac{6.109}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = 6.109 \times 10^{\boxed{}}$

3. Fill in the blanks by using the words provided.

decimal | **negative** | **zero** | **extremely**

Scientific notation is useful for working with $\boxed{}$ small or large values. Any $\boxed{}$ number can be written in scientific notation using positive or negative powers of 10. Small numbers with a value between 0 and 1 are written using $\boxed{}$ powers of 10, while those that are equal to 10 or more are written using positive powers of 10. When the number is between 1 and 10, it is written using the $\boxed{}$ power of 10.

Back to contents

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Write the following in scientific notation. WE1a

a. 500 **b.** 46 000 **c.** 691.2 **d.** 10 400

e. 2703.8 **f.** 96 000 000 **g.** 10 206 000 **h.** 650 050 000 000 000

5. Write the following in scientific notation. WE1b

a. 0.06 **b.** 0.0068 **c.** 0.104 **d.** 0.000745

e. 0.03901 **f.** 0.0004903 **g.** 0.00000001 **h.** 0.00000000000208

6. Write the following in scientific notation, correct to 3 significant figures. WE2

a. 54.21 **b.** 251.3 **c.** 0.06754 **d.** 0.005459

e. 95 213 000 **f.** 0.000035856 **g.** 849 680 000 000 **h.** 0.09996

7. Write the following in decimal notation. WE3a

a. 2×10^4 **b.** 4.2×10^2 **c.** 3.43×10^3 **d.** 1.05×10^2

e. 9.4×10^5 **f.** 1.08×10^5 **g.** 8.06×10^6 **h.** 9.304×10^{11}

8. Write the following in decimal notation. WE3b

a. 1.2×10^{-1} **b.** 3.4×10^{-2} **c.** 4.3×10^{-4} **d.** 5.01×10^{-3}

e. 1.089×10^{-2} **f.** 2.36×10^{-4} **g.** 5.4023×10^{-5} **h.** 6.006×10^{-10}

9. Which of the options shows the given number in scientific notation?

0.0000054

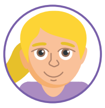
A. 5.4×10^{-7} **B.** 5.4×10^{-6} **C.** 0.54×10^{-5} **D.** 5.4×10^{-5} **E.** 5.4×10^{6}

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Write the following in scientific notation.

0.000078



Student A

$0.000078 \rightarrow a = 78$

$n = -6$

$\therefore 0.000078$

$= 78 \times 10^{-6}$

Student B

$0.000078 \rightarrow a = 7.8$

$n = -5$

$\therefore 0.000078$

$= 7.8 \times 10^{-5}$

b. Write the following in decimal notation.

8.5×10^7

Student A

$a = 8.5$

$n = 7$

$8.5 \rightarrow 85\,000\,000$

$\therefore 8.5 \times 10^7$

$= 85\,000\,000$

Student B

$a = 8.5$

$n = 7$

$8.5 \rightarrow 8\,500\,000$

$\therefore 8.5 \times 10^7$

$= 8\,500\,000$

Back to contents

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. There are 5.078×10^6 people in Melbourne. How many people live in Melbourne, in decimal notation?

12. Determine the number of seconds in one day and write it using scientific notation.

13. The mass of an animal cell is around 3×10^{-9} g. Write the mass of an animal cell in kg using decimal notation.

14. A grain of sand is 0.8 mm in diameter. What is the diameter of a grain of sand in metres using scientific notation?

15. Usain Bolt and Yohan Blake are two of the fastest humans in the world. Bolt can run 100 m in 9.58 seconds, whereas Blake can run it in 9.69 seconds. Using scientific notation, calculate the difference between the two runners' 100 m times in seconds.

Reasoning

Question working paths

Mild 16 (a,b,c,e) 🌶	**Medium** 16 (a,b,c,e), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Prefixes are used in mathematics and science to represent powers of 10. Used together with metric units, they indicate the magnitude of a measurement or value. The given table shows commonly used prefixes for powers of 10.

Prefixes	Value	Scientific notation	Symbol
Tera	1 000 000 000 000	10^{12}	T
Giga	1 000 000 000	10^{9}	G
Mega	1 000 000	10^{6}	M
Kilo	1000	10^{3}	k
Deci	0.1	10^{-1}	d
Centi	0.01	10^{-2}	c
Milli	0.001	10^{-3}	m
Micro	0.000001	10^{-6}	µ
Nano	0.000000001	10^{-9}	n
Pico	0.000000000001	10^{-12}	p

a. A sheet of paper is approximately 1 micrometre (µm) thick. Convert the thickness of a sheet of paper to metres and express it using scientific notation.

b. An average human hair is around 60 000 nanometers (nm) thick. Convert the thickness of human hair to metres and express it using scientific notation.

c. The Great Wall of China is 21 196 km long. Write the length of the Great Wall of China in scientific notation using metres, correct to 4 significant figures.

d. Convert 2 terabytes to kilobytes using the given table and write it using scientific notation.

e. Name a measurement unit that uses a prefix from the table in its name.

17. Convert the numbers in parts **a** and **b** to scientific notation.

a. 15 000 000

b. 0.000000004

c. Using the First Index Law, multiply the values from parts **a** and **b** without a calculator and write the product in decimal notation. Explain how it is possible to multiply or divide numbers written in scientific notation using the index laws.

Exam-style

18. Which of the options shows the given number in scientific notation? (1 MARK)

89 050 000 000

A. 8.905×10^{-10} **B.** 89.05×10^{9} **C.** 8.905×10^{10} **D.** 0.8905×10^{11} **E.** 8.905×10^{11}

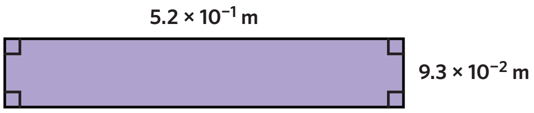
19. Consider the following calculation. (3 MARKS)

$48\,500\,000\,000 \times 4 \times 10^{-11}$

a. Rewrite the multiplication so that both factors are expressed in scientific notation. 1 MARK

b. Simplify the calculation using the index laws and write the product in decimal notation. 2 MARKS

20. Calculate the area of the given rectangle in cm^2 and write the answer using decimal notation. (3 MARKS)



21. 1 astronomical unit (AU) approximately equals 1.496×10^{11} m. The distance from the Sun to Neptune is 30 AU. Determine the distance from the Sun to Neptune in kilometres and write the answer in scientific notation. (3 MARKS)

Remember this?

22. A rectangular prism is made by stacking four identical smaller rectangular prisms, shown in the diagram.

Using the dimensions labelled on the diagram, what is the volume of the larger rectangular prism?

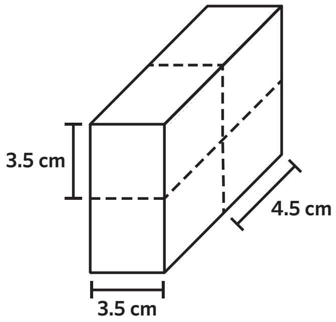
A. 12.25 cm^3

B. 15.75 cm^3

C. 55.125 cm^3

D. 110.25 cm^3

E. 220.5 cm^3



23. Duyen spends 15 minutes on the tram when she travels to work in the morning with an average speed of 30 km/hr. She took another route in the afternoon and spent 12 minutes on a train with an average speed of 50 km/hr. What is the total distance that Duyen travelled?

A. 7.5 km **B.** 10 km **C.** 15 km **D.** 17.5 km **E.** 30 km

24. To solve the linear equation $\frac{5}{3} - 4(x + 3) = 1$, which of the following operations is a correct first step to be applied on both sides of the equation?

A. $-\frac{5}{3}$ **B.** $\times \frac{3}{5}$ **C.** -3 **D.** $\div 4$ **E.** $+4$

Back to contents

2E Fractional indices

LEARNING INTENTIONS

Students will be able to:

- use a root sign to rewrite numbers with a fractional index
- evaluate numbers with a fractional index
- use index laws with fractional indices.

Indices can be any numbers including those that are not integers. Fractional indices are used for roots of numbers, where the denominator represents the order of the root. As index laws allow us to add, subtract, multiply and divide indices in order to simplify expressions, the same can be applied to fractional indices.

KEY TERMS AND DEFINITIONS

- A **radical** symbol $\sqrt{\ }$ is used to denote the root of a number.
- The **radicand** is the number or expression under the radical.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: stockfour/Shutterstock.com

The square root function is used for scaling the uncertainty or variability of data. In finance, returns on investments (ROI), such as stocks, may show a lot of variability day to day but are consistent over longer periods of time. To get a realistic picture of ROI, an economist will use a square root function to scale the variation and work out the volatility of the data. This shows how risky the investment is and can help decide whether it is worth investing in.

Key ideas

1. The n^{th} (order of a) root of a number (radicand) is a value that equals the number when it is multiplied by itself n times.

Note: A **square root** occurs when $n = 2$ and is usually denoted by the radical symbol $\sqrt{\ }$ only.

x equals y multiplied by itself n times

$$\sqrt[\boxed{n}]{x} = y \Longrightarrow x = \overbrace{y \times y \times \dots \times y} = y^{n}$$

2. For a number with a fractional index, the numerator represents the power and the denominator represents the order of the root.

$$x^{\frac{p}{n}} = \sqrt[n]{x^{p}}$$

Back to contents

Worked example 1

Writing fractional indices using a root sign

Write the following using a root sign.

WE1a

a. $4^{\frac{1}{2}}$

Working

$\frac{1}{n} = \frac{1}{2}$

$\therefore n = 2$

$4^{\frac{1}{2}} = \sqrt[2]{4} = \sqrt{4}$

Thinking

Step 1: Identify the order of the root (n) by looking at the denominator of the fractional index.

Step 2: Rewrite the number using an n^{th} root sign.

WE1b

b. $21^{\frac{2}{3}}$

Working

$\frac{p}{n} = \frac{2}{3}$

$\therefore p = 2$ and $n = 3$

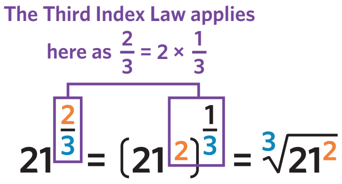
$21^{\frac{2}{3}} = \sqrt[3]{21^2}$

Thinking

Step 1: Identify the power (p) of the number and order of the root (n) by looking at the numerator and denominator of the fractional index.

Step 2: Rewrite the number using the power (p) and an n^{th} root sign.

Visual support



Student practice

Write the following using a root sign.

a. $8^{\frac{1}{3}}$ **b.** $12^{\frac{2}{5}}$

Worked example 2

Evaluating numbers with a fractional index

Evaluate the following.

WE2a

a. $16^{\frac{1}{2}}$

Working

$16^{\frac{1}{2}} = \sqrt[2]{16} = \sqrt{16}$

$\therefore n = 2$

Thinking

Step 1: Rewrite the number using a root sign and determine the order of the root (n).

Continues →

Back to contents

$4 \times 4 = 16$

$\therefore \sqrt{16} = 4$

Step 2: Evaluate by identifying which number, multiplied by itself *n* times, results in the radicand.

Visual support

$$4 \times 4 = 4^2 = 16$$

$$\therefore 16^{\frac{1}{2}} \times \underbrace{(4^2)^{\frac{1}{2}} = 4^{2 \times \frac{1}{2}} = 4^1 = \boxed{4}}_{\text{The Third Index Law can be used here to show that } 16^{\frac{1}{2}} = 4}$$

b. $625^{\frac{1}{4}}$

WE2b

Working

$625^{\frac{1}{4}} = \sqrt[4]{625}$

$\therefore n = 4$

$5 \times 5 \times 5 \times 5 = 625$

$\therefore \sqrt[4]{625} = 5$

Thinking

Step 1: Rewrite the number using a root sign and determine the order of the root (*n*).

Step 2: Evaluate by identifying which number, multiplied by itself *n* times, results in the radicand.

Student practice

Evaluate the following.

a. $9^{\frac{1}{2}}$

b. $216^{\frac{1}{3}}$

Worked example 3

Using index laws with fractional indices

Simplify the following.

a. $x^{\frac{1}{2}} \times x^{\frac{1}{3}}$

WE3a

Working

$x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}}$

$= x^{\frac{3}{6}+\frac{2}{6}}$

$= x^{\frac{5}{6}}$

Thinking

Step 1: Apply the First Index Law to the terms with common bases.

Step 2: Convert the indices to equivalent fractions with equal denominators and add them to simplify the expression.

Visual support

$$x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{3}{6}+\frac{2}{6}} = x^{\frac{5}{6}}$$

Continues →

b. $\dfrac{b^{\frac{1}{2}}}{b^{\frac{1}{4}}}$

WE3b

Working

$$\frac{b^{\frac{1}{2}}}{b^{\frac{1}{4}}} = b^{\frac{1}{2}} \div b^{\frac{1}{4}} = b^{\frac{1}{2}-\frac{1}{4}}$$

$$= b^{\frac{2}{4}-\frac{1}{4}}$$

$$= b^{\frac{1}{4}}$$

Thinking

Step 1: Apply the Second Index Law to the terms with common bases.

Step 2: Convert the indices to equivalent fractions with equal denominators and subtract them to simplify the expression.

c. $\left(t^{\frac{2}{3}}\right)^{\frac{1}{5}}$

WE3c

Working

$$\left(t^{\frac{2}{3}}\right)^{\frac{1}{5}} = t^{\frac{2}{3}\times\frac{1}{5}}$$

$$= t^{\frac{2\times1}{3\times5}}$$

$$= t^{\frac{2}{15}}$$

Thinking

Step 1: Apply the Third Index Law to the expression in brackets.

Step 2: Multiply the fractional indices to simplify the expression.

Student practice

Simplify the following.

a. $y^{\frac{1}{5}} \times y^{\frac{1}{2}}$

b. $\dfrac{a^{\frac{1}{3}}}{a^{\frac{1}{4}}}$

c. $\left(x^{\frac{3}{4}}\right)^{\frac{1}{2}}$

Back to contents

2E Questions

Understanding worksheet

1. Match the following roots to the index form of their radicands.

Example

n^{th} root			Radicand
$\sqrt[4]{16}$	●	●	2^4

n^{th} root			Radicand
$\sqrt{25}$	●	●	6^4
$\sqrt[5]{32}$	●	●	4^5
$\sqrt[4]{1296}$	●	●	5^2
$\sqrt[5]{1024}$	●	●	2^5

2. Fill in the blanks.

Example

$$5^{\frac{3}{4}} = \sqrt[\boxed{4}]{5^3}$$

a. $4^{\frac{1}{7}} = \sqrt[\boxed{}]{4}$

b. $2^{\frac{4}{5}} = \sqrt[\boxed{}]{2^4}$

c. $12^{\frac{9}{8}} = \sqrt[8]{12^{\boxed{}}}$

d. $9^{\frac{3}{2}} = \sqrt{9^{\boxed{}}}$

3. Fill in the blanks by using the words provided.

power **laws** **radical** **order** **cube**

The ________ symbol (√) is used to denote the root of a number. The root can be square, ________ , or of a higher order. Fractional indices are used for roots of numbers, where the numerator represents the ________ and the denominator represents the ________ of the root. Index ________ apply to fractional indices.

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Write the following using a root sign. WE1

a. $5^{\frac{1}{2}}$ **b.** $10^{\frac{1}{4}}$ **c.** $26^{\frac{1}{9}}$ **d.** $4^{\frac{2}{3}}$

e. $7^{\frac{2}{5}}$ **f.** $11^{\frac{3}{8}}$ **g.** $27^{\frac{5}{4}}$ **h.** $13^{1\frac{1}{5}}$

5. Evaluate the following. WE2

a. $4^{\frac{1}{2}}$ **b.** $25^{\frac{1}{2}}$ **c.** $27^{\frac{1}{3}}$ **d.** $16^{\frac{1}{4}}$

e. $64^{\frac{1}{3}}$ **f.** $1\,000\,000^{\frac{1}{6}}$ **g.** $243^{\frac{1}{5}}$ **h.** $2^{\frac{10}{2}}$

6. Simplify the following. WE3a

a. $x^{\frac{1}{3}} \times x^{\frac{1}{3}}$ **b.** $y^{\frac{1}{5}} \times y^{\frac{2}{5}}$ **c.** $x^{\frac{1}{4}} \times x^{\frac{1}{2}}$ **d.** $t^{\frac{2}{3}} \times t^{\frac{1}{6}}$

e. $p^{\frac{1}{2}} \times p^{\frac{1}{7}}$ **f.** $a^{\frac{2}{7}} \times a^{\frac{2}{5}}$ **g.** $b^{\frac{3}{4}} \times b^{1\frac{1}{5}}$ **h.** $x^{\frac{1}{3}} \times x^{\frac{1}{2}} \times x^{\frac{2}{5}}$

7. Simplify the following. WE3b

a. $\dfrac{x^{\frac{5}{7}}}{x^{\frac{1}{7}}}$ **b.** $\dfrac{b^{\frac{3}{5}}}{b^{\frac{2}{5}}}$ **c.** $\dfrac{a^{\frac{1}{2}}}{a^{\frac{1}{3}}}$ **d.** $\dfrac{u^{\frac{4}{3}}}{u^{\frac{1}{9}}}$

e. $\dfrac{y}{y^{\frac{1}{2}}}$ **f.** $\dfrac{p^{\frac{11}{2}}}{p^{\frac{2}{3}}}$ **g.** $\dfrac{r^{1\frac{1}{4}}}{r^{\frac{1}{3}}}$ **h.** $\dfrac{z^{2\frac{2}{3}}}{z^{\frac{7}{4}}}$

8. Simplify the following. WE3c

a. $\left(x^{\frac{1}{2}}\right)^{\frac{1}{3}}$ **b.** $\left(p^{\frac{1}{3}}\right)^{\frac{2}{5}}$ **c.** $\left(y^{\frac{3}{5}}\right)^{\frac{1}{4}}$ **d.** $\left(r^{\frac{1}{2}}\right)^{2}$

e. $(t^3)^{\frac{2}{3}}$ **f.** $\left(k^{\frac{2}{7}}\right)^{\frac{21}{13}}$ **g.** $5\left(x^{\frac{5}{12}}\right)^{\frac{3}{20}}$ **h.** $\left(8^{\frac{2}{3}}b^{\frac{3}{11}}\right)^{\frac{1}{2}}$

9. Write the following in index form.

a. $\sqrt[3]{10}$ **b.** $\sqrt{5}$ **c.** $\sqrt[9]{12}$ **d.** $\sqrt[4]{2^3}$

e. $\sqrt[5]{9^{11}}$ **f.** $\sqrt[2]{6^4}$ **g.** $\sqrt[7]{13^{21}}$ **h.** $\sqrt[6]{10^3}$

10. Which of the options is equivalent to the given expression?

$x^{\frac{7}{3}}$

A. $\sqrt[4]{x}$ **B.** $\sqrt[7]{x^3}$ **C.** $\sqrt[3]{x^7}$ **D.** x^4 **E.** x^{10}

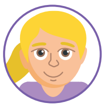
Back to contents

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. Evaluate the following.

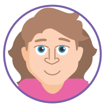
$100\ 000^{\frac{1}{5}}$



Student A

$$100\ 000^{\frac{1}{5}} = \frac{100\ 000}{5}$$

$$= 20\ 000$$



Student B

$$100\ 000^{\frac{1}{5}} = \sqrt[5]{100\ 000}$$

$$= 10$$

b. Simplify and write using positive indices only.

$\sqrt[5]{7^4}$

Student A

$$\sqrt[5]{7^4} = (7^4)^{\frac{1}{5}} = 7^{\frac{4}{5}}$$

Student B

$$\sqrt[5]{7^4} = (7^5)^{\frac{1}{4}} = 7^{\frac{5}{4}}$$

Problem solving

Question working paths

Mild 12, 13, 14	**Medium** 13, 14, 15	**Spicy** 14, 15, 16

12. A square painting has a total area of 6400 cm^2. Express one side length of the painting in centimetres, using a fractional index and evaluate it.

13. Mr Steinberg thinks of a number x. He tells his class that when he multiplies the number by itself five times, the result is 32. Express Mr Steinberg's number using a root sign and evaluate it.

14. Sully is building a fish tank that will be placed inside an aquarium. The client has asked for it to be a perfect cube with a total volume of 216 m^3 to house the animals comfortably. Express the height of the fish tank using a fractional index and evaluate it.

15. Cindy and Emily both walk away from their campsite. Cindy walks north and stops after travelling 6 km and Emily walks east and also stops after travelling some distance. The direct distance between them is now 10 km. How far east did Emily walk from the campsite?

16. Duyen is playing a building video game where her mission is to construct a room with a minimum total volume of 100 blocks. If Duyen wants to ensure that the height of the room is four blocks, and that the floor is a square, how many blocks should she make each side length?

Reasoning

Question working paths

Mild 17 (a,b,d)	**Medium** 17 (a,b,d), 18 (a,b)	**Spicy** All

17. Dr Lovecraft is monitoring the volume of clean water in a lake contaminated with algae. After adding some chemicals to a small sample of distilled water, she pours it into the lake. She models the volume of clean water in the lake by using the given formula.

$$V = 12 \times 9^{\frac{t}{2}}$$

where V is the volume of clean water in cm^3 and t is the number of weeks since treatment started.

a. Calculate the initial volume of clean water when $t = 0$.

b. Calculate the volume of clean water after one week of treatment.

c. If 1 L $= 1000\ \text{cm}^3$, how many weeks will it take for the volume of clean water in the lake to reach 8.748 L?

d. Lakes and rivers can become contaminated or polluted due to human activities. Identify one simple rule to follow in the outdoors to help reduce any negative effects on the environment.

Back to contents

18. Write the expressions in parts **a** and **b** using fractional indices.

a. $\sqrt{4^6}$

b. $\sqrt{5^{10}}$

c. Analyse the simplified indices from parts **a** and **b** and explain the effect of a square root on the power of a number.

Exam-style

19. Which of the options correctly identifies the order of the root given by the fractional index? (1 MARK)

$x^{1\frac{4}{9}}$

A. $\frac{4}{9}$ **B.** 1 **C.** $1\frac{4}{9}$ **D.** 4 **E.** 9

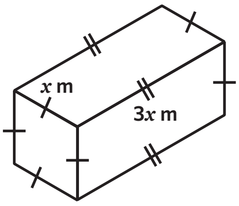
20. Consider the given expression. (3 MARKS)

$\sqrt{2^3} \times \sqrt{2^7}$

a. Rewrite the expression using fractional indices. 1 MARK

b. Fully simplify and evaluate. 2 MARKS

21. The given cuboid has a total volume of 192 m^3. Determine the length of x using the information given in the diagram. (3 MARKS)



22. Fully simplify the given expression, showing all necessary working steps. (3 MARKS)

$3\left(a^{\frac{1}{4}}\right)^{\frac{2}{3}} \times (a^5)^{\frac{1}{6}}$

Remember this?

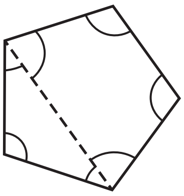
23. Bela is planning to enter a reading competition. She started to read a 240-page book on Monday. On Monday she read 59 pages, on Tuesday she read 43 pages, and on Wednesday she read 78 pages. What percentage of the book has Bela not read yet by the end of Wednesday?

A. 17% **B.** 25% **C.** 32.5% **D.** 45% **E.** 75%

24. Rajani is participating in a push-up challenge to fundraise for a charity. For every 15 push-ups, Rajani will raise $2.50 dollars. Within a week, Rajani has completed 80 push-ups. Rajani's sister has completed 60 push-ups, and Rajani's friends have completed 100 push-ups in total. How much money will Rajani raise in total?

A. $37.5 **B.** $200 **C.** $242.50 **D.** $250 **E.** $600

25. What is the sum of all of the internal angles of this pentagon?



A. 180° **B.** 270° **C.** 360° **D.** 450° **E.** 540°

Back to contents

2F Simple operations with surds

LEARNING INTENTIONS

Students will be able to:

- identify and understand surds
- add and subtract surds
- multiply and divide surds.

Surds are values that cannot be expressed exactly using decimal notation. For this reason, when we perform calculations with surds, the root symbol is used to represent them instead. This is particularly important for calculations that require a number of steps, where rounding can severely affect the accuracy of the outcome.

KEY TERMS AND DEFINITIONS

- **Irrational numbers** cannot be written as a simple fraction and contain an infinite number of non-recurring decimal digits.
- **Surds** are roots that are irrational numbers.
- A **quadratic surd** has the order $n = 2$.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Africa Studio/Shutterstock.com

The golden ratio, also known as the divine proportion, is a ratio between two numbers that equals approximately 1.618. Usually denoted by the Greek letter phi (ϕ), the exact value of the golden ratio is given by $\phi = \frac{1+\sqrt{5}}{2}$. When the proportions of an image follow the golden ratio, it is considered aesthetically pleasing. The golden ratio occurs in nature in the form of a Fibonacci spiral that can be seen in the proportions of plants and seashells.

Key ideas

1. Only like surds of the same order (n) can be added and subtracted.

Like surds	Unlike surds
$-\sqrt{2}, \sqrt{2}, 3\sqrt{2}$, etc.	$-\sqrt{6}, \sqrt{2}, \sqrt{5}$, etc.
$-\sqrt{x}, \sqrt{x}, 4\sqrt{x}$, etc.	$\sqrt{x}, \sqrt{y}, 2\sqrt{z}$, etc.

2. The product of surds equals the root of the product of their radicands.

$$\sqrt{x} \times \sqrt{y} = \sqrt{x \times y} = \sqrt{xy}$$

3. The quotient of surds equals the root of the quotient of their radicands.

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{x \div y} = \sqrt{\frac{x}{y}}$$

Back to contents

Worked example 1

Identifying surds

Determine if the following are surds.

a. $\sqrt[3]{27}$

WE1a

Working	Thinking
$\sqrt[3]{27} = 3$	**Step 1:** Evaluate the root.
3 is not an irrational number $\therefore \sqrt[3]{27}$ is not a surd.	**Step 2:** Determine if the root is irrational and hence decide if it is a surd.

b. $\sqrt{30}$

WE1b

Working	Thinking
$\sqrt{30} = 5.4772255750...$	**Step 1:** Evaluate the root.
5.4772255750... is an irrational number $\therefore \sqrt{30}$ is a surd.	**Step 2:** Determine if the root is irrational and hence decide if it is a surd.

c. $-\sqrt{0.16}$

WE1c

Working	Thinking
$-\sqrt{0.16} = -0.4$	**Step 1:** Evaluate the root.
-0.4 is not an irrational number $\therefore -\sqrt{0.16}$ is not a surd.	**Step 2:** Determine if the root is irrational and hence decide if it is a surd.

Student practice

Determine if the following are surds.

a. $\sqrt[4]{16}$ **b.** $\sqrt{46}$ **c.** $-\sqrt{0.09}$

Worked example 2

Adding and subtracting surds

Simplify the following.

a. $\sqrt{2} + 3\sqrt{5} + 2\sqrt{2}$

WE2a

Working	Thinking
$\sqrt{2} + 3\sqrt{5} + 2\sqrt{2}$	**Step 1:** Identify the like surds in the expression.
$= \sqrt{2} + 2\sqrt{2} + 3\sqrt{5}$ $= (1 + 2)\sqrt{2} + 3\sqrt{5}$ $= 3\sqrt{2} + 3\sqrt{5}$	**Step 2:** Add the coefficients of the like surds to simplify the expression.

Continues →

Back to contents

b. $5\sqrt{7} - \sqrt{6} - 3\sqrt{7}$

WE2b

Working

$5\sqrt{7} - \sqrt{6} - 3\sqrt{7}$

$= 5\sqrt{7} - 3\sqrt{7} - \sqrt{6}$

$= (5 - 3)\sqrt{7} - \sqrt{6}$

$= 2\sqrt{7} - \sqrt{6}$

Thinking

Step 1: Identify the like surds in the expression.

Step 2: Subtract the coefficients of the like surds to simplify the expression.

Student practice

Simplify the following.

a. $3\sqrt{5} + 2\sqrt{3} + \sqrt{5}$

b. $4\sqrt{2} - 2\sqrt{6} - 2\sqrt{2}$

Worked example 3

Multiplying and dividing surds

Simplify the following.

a. $2\sqrt{2} \times 3\sqrt{5}$

WE3a

Working

$$\begin{aligned}2\sqrt{2} \times 3\sqrt{5} &= 2 \times 3 \times \sqrt{2} \times \sqrt{5}\\ &= 6 \times \sqrt{2} \times \sqrt{5}\\ &= 6 \times \sqrt{2 \times 5}\\ &= 6\sqrt{10}\end{aligned}$$

Thinking

Step 1: Multiply the coefficients of the surds.

Step 2: Multiply the values (radicands) under the root signs together to determine the product of the surds.

Step 3: Simplify the expression.

Visual support

$$\begin{aligned}2\sqrt{2} \times 3\sqrt{5} &= 2 \times 3 \times \sqrt{2 \times 5}\\ &= 6\sqrt{10}\end{aligned}$$

b. $\dfrac{9\sqrt{18}}{3\sqrt{3}}$

WE3b

Working

$$\begin{aligned}\frac{9\sqrt{18}}{3\sqrt{3}} &= (9 \div 3)\left(\frac{\sqrt{18}}{\sqrt{3}}\right)\\ &= 3\left(\frac{\sqrt{18}}{\sqrt{3}}\right)\\ &= 3\left(\sqrt{\frac{18}{3}}\right)\\ &= 3\sqrt{18 \div 3}\\ &= 3\sqrt{6}\end{aligned}$$

Thinking

Step 1: Divide the coefficients of the surds.

Step 2: Divide the values (radicands) under the root signs to determine the quotient of the surds.

Step 3: Simplify the expression.

Student practice

Simplify the following.

a. $4\sqrt{3} \times 2\sqrt{7}$

b. $\dfrac{6\sqrt{42}}{2\sqrt{6}}$

Back to contents

2F Questions

Understanding worksheet

1. Identify the like surds in each list.

Example

$\boxed{-2\sqrt{3}}, \sqrt{2}, \boxed{\sqrt{3}}, \boxed{5\sqrt{3}}$

a. $\sqrt{2}, 3\sqrt{2}, \sqrt{3}, \sqrt{5}$

b. $-\sqrt{7}, 5\sqrt{6}, 5\sqrt{7}, 7\sqrt{7}$

c. $-6\sqrt{8}, \sqrt{10}, 6\sqrt{10}, 7\sqrt{10}$

d. $-\sqrt{5}, -2\sqrt{3}, \sqrt{2}, 8\sqrt{3}$

2. Fill in the blanks.

Example

$5\sqrt{3} - \sqrt{2} + \sqrt{3} = \boxed{6}\sqrt{3} - \sqrt{2}$

a. $3\sqrt{5} + 2\sqrt{5} + \sqrt{7} = \boxed{}\sqrt{5} + \sqrt{7}$

b. $\sqrt{3} + 2\sqrt{6} + 3\sqrt{3} = 2\sqrt{6} + \boxed{}\sqrt{3}$

c. $7\sqrt{2} - 4\sqrt{2} - 3\sqrt{5} = \boxed{}\sqrt{2} - 3\sqrt{5}$

d. $9\sqrt{8} + 5\sqrt{7} - 2\sqrt{8} = \boxed{}\sqrt{8} + 5\sqrt{7}$

3. Fill in the blanks by using the words provided.

like	radicands	infinite	irrational

Surds are $\boxed{}$ numbers that cannot be expressed exactly using decimal notation. Surds have an $\boxed{}$ number of non-recurring decimal digits and they cannot be written as a simple fraction. Only $\boxed{}$ surds can be added and subtracted. The product or quotient of two surds equals the surd of the product or quotient of their $\boxed{}$

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Determine if the following are surds. **WE1**

a. $\sqrt{4}$

b. $\sqrt[3]{8}$

c. $\sqrt{5}$

d. $\sqrt{2}$

e. $\sqrt{8}$

f. $\sqrt{27}$

g. $-\sqrt{36}$

h. $-\sqrt{0.64}$

5. Simplify the following. **WE2a**

a. $4\sqrt{3} + 2\sqrt{3}$

b. $\sqrt{6} + 2\sqrt{6}$

c. $3\sqrt{7} + 2\sqrt{7} + \sqrt{3}$

d. $5\sqrt{3} + 4\sqrt{6} + 3\sqrt{3}$

e. $7\sqrt{7} + 6\sqrt{11} + 3\sqrt{7}$

f. $\sqrt{3} + \sqrt{10} + 4\sqrt{3} + \sqrt{3}$

g. $2\sqrt{12} + 3\sqrt{12} + \sqrt{6} + \sqrt{12}$

h. $3\sqrt{13} + 2\sqrt{14} + 2\sqrt{13} + \sqrt{14}$

Back to contents

6. Simplify the following. WE2b

a. $9\sqrt{2} - 3\sqrt{2}$

b. $4\sqrt{5} - \sqrt{5}$

c. $\sqrt{7} - 5\sqrt{3} - 2\sqrt{3}$

d. $10\sqrt{6} - \sqrt{5} - 4\sqrt{6}$

e. $12\sqrt{8} - 4\sqrt{5} - \sqrt{8}$

f. $\sqrt{10} - 11\sqrt{3} + 4\sqrt{10}$

g. $8\sqrt{23} + 7\sqrt{15} - 3\sqrt{23} - \sqrt{15}$

h. $-2\sqrt{5} - 4\sqrt{5} + 7\sqrt{6} + 5\sqrt{5}$

7. Simplify the following. WE3a

a. $\sqrt{4} \times \sqrt{2}$

b. $2\sqrt{3} \times 3\sqrt{2}$

c. $7\sqrt{2} \times \sqrt{7}$

d. $6\sqrt{5} \times 2\sqrt{3}$

e. $5\sqrt{6} \times 4\sqrt{2}$

f. $\sqrt{2} \times \sqrt{5} \times 6\sqrt{7}$

g. $3\sqrt{10} \times \sqrt{5} \times 2\sqrt{3}$

h. $2\sqrt{9} \times 4\sqrt{2} \times 3\sqrt{5}$

8. Simplify the following. WE3b

a. $\frac{\sqrt{21}}{\sqrt{3}}$

b. $\frac{3\sqrt{10}}{\sqrt{2}}$

c. $\frac{8\sqrt{30}}{2\sqrt{5}}$

d. $\frac{2\sqrt{42}}{5\sqrt{7}}$

e. $\frac{3\sqrt{20}}{6\sqrt{10}}$

f. $\frac{6\sqrt{35}}{8\sqrt{5}}$

g. $\frac{24\sqrt{56}}{8\sqrt{7}}$

h. $\frac{28\sqrt{120}}{35\sqrt{15}}$

9. Which of the options shows the given expression once fully simplified?

$10\sqrt{3} - \sqrt{2} + \sqrt{3}$

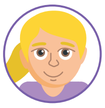
A. $9\sqrt{3} - \sqrt{2}$
B. $10\sqrt{3} - \sqrt{2}$
C. $9\sqrt{3} + \sqrt{2}$
D. $11\sqrt{3} - \sqrt{2}$
E. $10\sqrt{18}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Determine if the following is a surd.

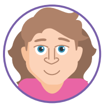
$-\sqrt{0.2916}$



Student A

$-\sqrt{0.2916} = -0.54$ is a decimal

$\therefore -\sqrt{0.2916}$ is a surd.



Student B

$-\sqrt{0.2916} = -0.54$ is not an irrational number

$\therefore -\sqrt{0.2916}$ is not a surd.

b. Simplify the following.

$2\sqrt{6} + 3\sqrt{5} - \sqrt{6}$

Student A

$$\begin{aligned} 2\sqrt{6} + 3\sqrt{5} - \sqrt{6} &= 2\sqrt{6} + \sqrt{6} - 3\sqrt{5} \\ &= 3\sqrt{6} - 3\sqrt{5} \end{aligned}$$

Student B

$$\begin{aligned} 2\sqrt{6} + 3\sqrt{5} - \sqrt{6} &= 2\sqrt{6} - \sqrt{6} + 3\sqrt{5} \\ &= \sqrt{6} + 3\sqrt{5} \end{aligned}$$

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. The ratio of lengths of the shorter to the longer side of an A4 piece of paper is $1 : \sqrt{2}$. If the shorter side is 21 cm long, what is the length of the longer side, to the nearest mm?

12. The volume of a fish tank with a square base is 6000 cm^3. If each side of the base is $\sqrt{120}$ cm long, determine the height of the tank.

13. Sally is choosing an outfit for her computer game character. There are five categories: a hat, shirt, pants, boots, and accessories. Each category contains an equal number of items. If there are a total of 7776 combinations of outfits available, how many items are there in each category?

14. A bathroom wall has been tiled with 216 square tiles. There are 1.5 times as many tiles along the height of the wall as there are along the length of the wall. What is the height of the wall in metres if each side of one tile is 15 cm long?

15. An electrician wants to run an underground cable between opposite corners of a square room. The floor area of the room is 50 m^2. Calculate the length of the cable.

Reasoning

Question working paths

Mild 16 (a,b,d) 🌶	**Medium** 16 (a,b,d), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Louise is a botanist who is closely monitoring the growth of a plant. She has found that under the current conditions the plant's height can be modelled by the given equation.

$$h = 0.2 + \sqrt{t}$$

where h is the height of the plant, in cm, and t is the number of days since sprouting.

a. Determine the initial height of the plant.

b. What was the height of the plant on the fourth day?

c. This type of plant has a maximum height of 5 cm when fully grown. Approximately how many days will it take the plant to fully grow?

d. State a reason for studying the behaviour of plants and animals.

17. Write the expressions in parts **a** and **b** using fractional indices and simplify by applying index laws.

a. $\sqrt{3} \times \sqrt{3}$

b. $\sqrt{x} \times \sqrt{x}$

c. Analyse your answers for parts **a** and **b**. Explain the effect of multiplying a quadratic surd by itself.

Exam-style

18. Which of the options shows the given expression once fully simplified? (1 MARK)

$$\frac{8\sqrt{21}}{4\sqrt{3}}$$

A. $\frac{2}{\sqrt{7}}$ **B.** $2\sqrt{7}$ **C.** $\frac{\sqrt{7}}{2}$ **D.** $2\sqrt{18}$ **E.** $2\sqrt{63}$

Back to contents

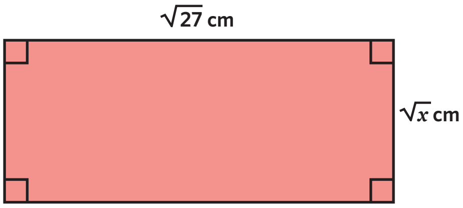
19. Consider the given expression. (2 MARKS)

$$\frac{4\sqrt{8}+6\sqrt{8}}{2\sqrt{2}}$$

a. Simplify the numerator of the expression. 1 MARK

b. Fully simplify and evaluate the expression. 1 MARK

20. The area of the pictured rectangle equals 9 cm^2. Use the given information to determine the value of x. (3 MARKS)



21. Solve for x, showing all necessary working steps. (3 MARKS)

$$2\sqrt{x} \times \sqrt{3} = 12$$

Remember this?

22. Courtney drove to a beach that was 58 km from her house. Her average speed throughout the trip was 64 km/h. How long did the trip take, to the nearest minute?

A. 50 minutes **B.** 54 minutes **C.** 58 minutes **D.** 62 minutes **E.** 66 minutes

23. Farmer Henley records the types and sexes of his animals in the given table.

	Male	**Female**
Cows	16	25
Sheep	8	4
Pigs	23	21

What percentage of the female animals are female pigs?

A. 20% **B.** 21% **C.** 40% **D.** 42% **E.** 50%

24. Poppy and Summer are in a tall building that has multiple levels, some of which go underground. Poppy is on the lowest level which is 14 metres below ground. Summer is on the highest level which is 165 metres above ground. What is the distance between Poppy and Summer, in metres?

A. 14 m **B.** 144 m **C.** 151 m **D.** 165 m **E.** 179 m

Back to contents

Chapter 2 extended application

1. In architectural design and construction, surds find valuable application in representing precise measurements and dimensions of structures. Architects and engineers rely on these mathematical expressions to ensure accuracy and efficiency in creating buildings.

- **a.** The side of a building with a square floor plan is $\sqrt{50}$ m long. Given that $\sqrt{50} = \sqrt{25 \times 2}$, fully simplify the product.
- **b.** The building is a rectangular prism. Architects can calculate the buildings' volume by multiplying the area of its base by its height. If the building's height is represented by $3\sqrt{2}$m, what is the volume of the building?
- **c.** The volume of a different building is given as $5\sqrt{200}$ m^3. Fully simplify the surd as in part **a**.
- **d.** What is the simplified ratio of the volumes of the smaller building to the larger building from parts **b** and **c**?
- **e.** When designing large residential buildings for university students, what factors should an architect consider to enhance the livability of the building.

2. In everyday life, electrical devices consume power that affects energy usage. Prefixes are used in mathematics and science to represent powers of 10. Used together with watts, they indicate the magnitude of the electrical power.

The most common units of power are watt (W), kilowatt (kW) and megawatt (MW). Converting between different units of power, such as watt (W), kilowatt (kW), and megawatt (MW), involves changing the magnitude of the measurement while keeping the same value.

1 MW = 1000 kW = 1 000 000 W

Prefixes	Value (watts)	Scientific notation	Symbol
Milliwatt (mW)	0.001	1×10^{-3} W	mW
Watt (W)	1	1×10^{0} W	W
Kilowatt (kW)	1000	1×10^{3} W	kW
Megawatt (MW)	1 000 000	1×10^{6} W	MW
Gigawatt (GW)	1 000 000 000	1×10^{9} W	GW
Terawatt (TW)	1 000 000 000 000	1×10^{12} W	TW

- **a.** Convert 2.5 MW to watts and express the result in scientific notation.
- **b.** Convert 6.74×10^9 W to gigawatts.
- **c.** Write 4 850 000 000 000 W using scientific notation and state its value in terawatts.
- **d.** Explain the relationship between kilowatts and megawatts.
- **e.** Calculate the total power usage of three devices, each consuming 5200 W, and express it in kilowatts.
- **f.** Provide a real-life example of a device or system where understanding power and energy consumption is important for optimising usage.

3. Biologists use population growth formulas to study the growth of bacterial colonies, and plant populations. They investigate how factors such as birth rates, death rates, migration, and environmental conditions affect population sizes over time.

Different populations can be modelled using the equation:

$P(t) = P_0 \times 2^{kt}$

where:

- $P(t)$ is the population of bacteria at time t
- P_0 is the initial population at $t = 0$
- k is a constant that represents the growth rate of the bacteria.

Use a CAS calculator as required to complete the following. Round the populations to the nearest whole number.

a. If the initial bacterial population is 1000 and the growth rate of the bacteria is 0.05, what is the number of bacteria after 4 hours? Round to the nearest whole number.

b. Using the equation and growth rate from part **a**, copy and complete the table to show the population of bacteria in the first 10 hours.

Time (hours)	1	2	3	4	5	6	7	8	9	10
Bacteria population	1035	1072	1110							

c. When antibiotics are administered the population of bacteria begins to decay and $k = -0.3$. Write the equation modelling the decay rate of bacteria growth using positive indices only.

d. Using the equation from part **c** calculate the difference between the population of bacteria with an initial population of 1000 after 4 hours when the antibiotic is administered compared to when it is not.

e. Scientists have found a super bacteria that grows faster than what they had previously calculated. They represent its growth with the equation $P(t) = P_0 \times (2^{kt})^3$. If this strain of bacteria has a growth rate of 0.08 and an initial population of 1000, what is its population after 4 hours?

f. Other than studying bacteria growth and decay, scientists use formulas to study and predict human populations. List two factors scientists should consider when trying to understand population changes in Australia.

Back to contents

Chapter 2 review

Multiple choice

1. Which of the options correctly fills in the blank? 2A

$$\left(\frac{c^2 \times d^3}{c^0 \times d^2}\right) = c^{2-0}d^{3-\boxed{}} = c^2 \times d$$

A. 0
B. 1
C. 2
D. 3
E. 5

---

2. Which of the options correctly fills in the blank? 2C

$$\frac{x^{-3}}{x^{-3}} = x^{-3-(-3)} = x^{\boxed{}}$$

A. −6
B. −3
C. 0
D. 3
E. 6

---

3. Which of the options correctly fills in the blank? 2D

$$0.00016 = \frac{1.6}{10 \times 10 \times 10 \times 10} = 1.6 \times 10^{\boxed{}}$$

A. −4
B. −3
C. −1
D. 1
E. 4

---

4. Which of the options correctly fills in the blank? 2E

$$4^{\frac{5}{6}} = \sqrt[\boxed{}]{4^5}$$

A. 2
B. 5
C. 6
D. $\frac{5}{6}$
E. $\frac{6}{5}$

---

5. Which of the options correctly fills in the blank? 2F

$$7\sqrt{3} + 3\sqrt{7} + 17\sqrt{3} = \boxed{}\sqrt{3} + 3\sqrt{7}$$

A. 3
B. 7
C. 13
D. 17
E. 24

Fluency

6. Simplify the following. 2A

a. $d^9 \times d^{12}$
b. $7p^2q \times q^2p$
c. $\frac{24x^7y^{10}}{8x^2y^{10}}$
d. $\frac{3f^0t^{11} \times 15f^{14}t^3}{9tf^{13} \times t}$

---

7. Simplify the following. 2B

a. $(3u^4w^6)^5$
b. $\left(\frac{2r^9}{s^2}\right)^6$
c. $(2a^2b^9)^5 \times 2(a^3b^2)^7$
d. $\left(\frac{c^2}{d^4}\right)^5 \times \frac{2(d^5c^2)^4}{(3c^0d^7)^0}$

---

8. Evaluate the following. 2C

a. 3^{-4}
b. $\left(\frac{5}{3}\right)^{-3}$
c. $(2^2 \times 3)^{-2}$
d. $\frac{(3^0 \times 4)^{-2}}{2^{-5}}$

---

9. Simplify and write using positive indices only. 2C

a. $6d^{-5}$
b. $\frac{5h^{-2}}{k^{-4}}$
c. $4e^{-2}f^{-17} \times 11fe$
d. $\frac{x^{-9}y^{-1}}{(3y^{-3}x)^3}$

Back to contents

10. Write the following in scientific notation, correct to 3 significant figures. 2D

a. 8250 **b.** 0.06375 **c.** 411.428 **d.** 0.0999998

11. Simplify the following. 2E

a. $x^{\frac{1}{3}} \times x^{\frac{1}{9}}$ **b.** $\dfrac{a^{\frac{3}{5}}}{a^{\frac{1}{10}}}$ **c.** $2\left(p^{\frac{3}{4}}\right)^{\frac{1}{12}}$ **d.** $\left(16^{\frac{3}{2}}t^{\frac{2}{11}}\right)^{\frac{1}{6}}$

12. Write the following in index form. 2E

a. $\sqrt{21}$ **b.** $\sqrt[6]{7}$ **c.** $\sqrt[11]{2^{12}}$ **d.** $\sqrt[20]{10^{10}}$

13. Simplify the following. 2F

a. $4\sqrt{3} - 3\sqrt{3}$ **b.** $3 - \sqrt{3} - 3\sqrt{3}$

c. $\sqrt{7} - 13\sqrt{7} + \sqrt{10}$ **d.** $-3\sqrt{12} - 2\sqrt{14} - \sqrt{14} + 13\sqrt{12}$

14. Simplify the following. 2F

a. $\sqrt{3} \times \sqrt{5}$ **b.** $\sqrt{7} \times \sqrt{6} \times 2\sqrt{2}$ **c.** $\dfrac{5\sqrt{26}}{\sqrt{2}}$ **d.** $\dfrac{24\sqrt{55}}{8\sqrt{5}}$

Problem solving

15. Chidi is doing an experiment in his chemistry class. There are originally 30 bacteria in a petri dish, and every five seconds the number of bacteria triple. What is the final number of bacteria in the petri dish after 15 seconds? 2A

16. Chi rolls a biased die with seven faces, from one to seven. The probability of landing a two is $\frac{2}{5}$. What's the probability of not landing a two for three consecutive rolls? 2B

17. Rag is studying biology. He discovered that an average ant weighs 2^{-3} g. There are approximately 10^{14} ants in the world. How much do all ants weigh when combined? Express your answer in the simplest index form. 2C

18. Mr Babu is going on a holiday to Iceland. Iceland is approximately 15 186 km from Australia. How far is this distance in metres, in scientific notation, correct to 3 significant figures? 2D

19. Bela has a large cube with identical numbers of small cubes for length, width, and height, where each cube has a volume of 1 cm^3. Given that the total volume of the large cube is 343 cm^3, express one side length of the cube using a fractional index and evaluate. 2E

20. Catherine is looking to buy a frame for a photo. The ratio of the shorter to the longer side of this photo is $1 : \sqrt{3}$. If the shorter side of the photo is 9 cm long, what is the diagonal length of the photo? 2F

Reasoning

21. The International Space Station has discovered a new planet in another solar system and has gathered information about it.

a. The light intensity of the star as observed from Earth is expressed as 5×10^{-4} lumens. Express this light intensity as a fraction.

b. The new planet is 2.062×10^{8} km away from the star that it orbits. What is this distance in decimal notation?

c. The draft reporting information about the new planet states that the radius of its moon is $\frac{6\sqrt{10} + 2\sqrt{10}}{2 \times 4^{\frac{1}{2}}}$ km. When reviewing the report, one astronaut notices that this expression could be simplified. Determine the simplified expression of the radius.

d. A space probe is sent to the new planet to collect samples of its surface. After being launched from Earth at an initial velocity of 2000 m/s, the space probe continuously doubles in velocity every 20 seconds for 1 minute until it reaches its maximum speed. Calculate the maximum speed of the space probe, in km/h.

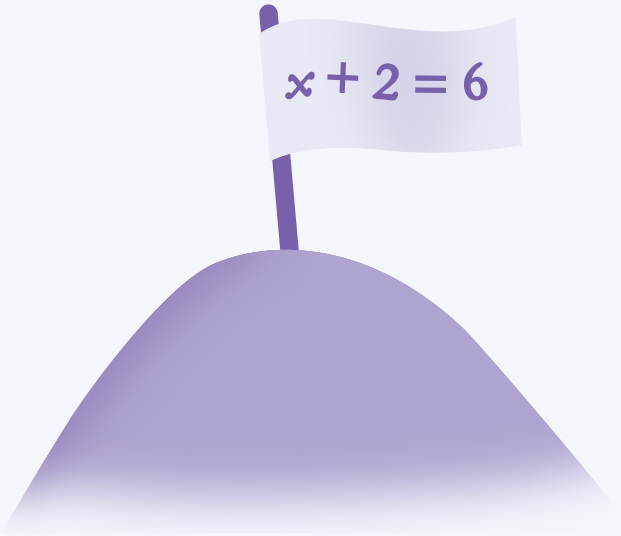
e. Other than discovering new planets, suggest two other possible projects of the International Space Station.

22. Evaluate:

a. $\frac{(3\sqrt{3})^2}{3}$

b. $\left(\frac{3\sqrt{3}}{\sqrt{3}}\right)^2$

c. Compare the expressions in parts **a** and **b**. Comment on how the differences between the expressions influence the answers.



Chapter 3

Equations

Number and algebra

Calculator skills

See online in additional materials for using CAS calculator guides.

3C Equations with pronumerals on both sides
3F Simultaneous equations using substitution and elimination

Back to contents

Chapter 3 research summary

Equations

Big ideas

An equation is a mathematical statement that asserts the equality of two expressions. In simple terms, it means that what is on the left side of the equals sign should be equal to what is on the right side. For example, $2x + 3 = 7$ is an equation. In Year 9, the concept of equations plays a significant role. Students will understand the balance of equations in order to solve for variables represented in coordinate systems. The big ideas of number sense, operations, algebraic thinking, mathematical reasoning, mathematical representation, and proportional reasoning all underpin the concept of equations.

Number sense

Equations often involve numbers and their relationships. A strong sense of numbers helps students to understand and solve equations effectively.

Operations

To solve equations, students need to be proficient in various mathematical operations. They should understand how to manipulate equations using addition, subtraction, multiplication, and division.

Algebraic thinking

This is perhaps the most directly relevant big idea. Algebraic thinking encompasses the use of symbols, variables, and expressions to represent and solve mathematical problems. In Year 9, students delve deeper into algebra, including the use of equations to represent and solve problems.

Mathematical reasoning

Solving equations requires logical thinking and reasoning. Students need to decide on a strategy to solve an equation and then apply that strategy systematically.

Mathematical representation

Equations are a form of mathematical representation. Understanding how equations represent relationships and quantities is crucial to understanding algebra and other Year 9 topics.

Proportional reasoning

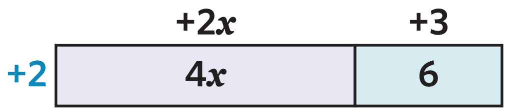
While this might not be directly related to every equation, many real-world problems in Year 9 involve proportions, rates, and ratios, which can be represented and solved using equations.

Visual representations

Area model or open array

Area models, also known as open arrays, can be used to represent equations particularly those involving multiplication or division. For example an equation $2(2x + 3) = 4x + 6$ can be represented as a rectangle with one side length of 2 and an unknown side length of $2x + 3$.

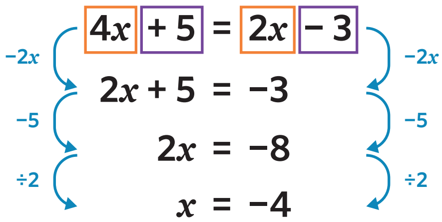
$2(2x + 3)$



$$2(2x + 3) = 2 \times 2x + 2 \times 3$$
$$= 4x + 6$$

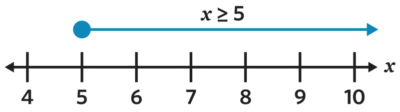
Balance scale

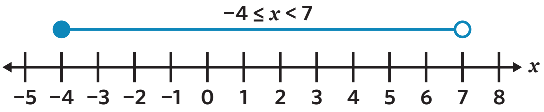
In Year 9, the idea of a balance scale can be represented in a more abstract manner where arrows are used to represent the operations that are applied to balance both sides of the equation.



Number lines

A number line can be used to solve simple equations including inequalities and represent solutions.



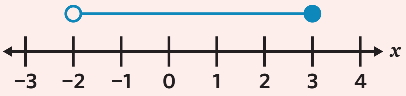
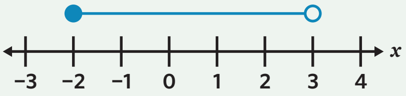


Misconceptions

Misconception	Incorrect ✕	Correct ✓	Lesson
Students multiply the first term inside the brackets, but disregard any further terms in the brackets.	$2(2x + 3) = 4x + 3$	$2(2x + 3) = 4x + 6$	3A
Students add or subtract when expanding, instead of multiplying.	$2(2x + 3) = 4x + 5$	$2(2x + 3) = 4x + 6$	3A
Students ignore negative signs when expanding brackets.	$1.5(4 - 3a) = 1.5 \times 4 + 1.5 \times (3a)$ $= 6 + 4.5a$	$1.5(4 - 3a) = 1.5 \times 4 + 1.5 \times (-3a)$ $= 6 - 4.5a$	3A
Students only apply the inverse operation to one side of the equation.	$5x - 4 = 46$ $5x - 4 + 4 = 46$ $5x = 46$ $x = \frac{46}{5}$	$5x - 4 = 46$ $5x - 4 + 4 = 46 + 4$ $5x = 50$ $x = 10$	3B
Students assume that the unknown value is the next subsequential value followed after itself.	$6x + 4 = 12$ $x = 4$	$6x + 4 - 4 = 12 - 4$ $6x = 8$ $\frac{6x}{6} = \frac{8}{6}$ $x = \frac{4}{3}$	3B
Students don't apply the inverse operation when isolating the unknown variable.	$7x - 2 - 2 = 61 - 2$ $7x = 59$ $7x \times 7 = 59 \times 7$ $x = 413$	$7x - 2 + 2 = 61 + 2$ $7x = 63$ $\frac{7x}{7} = \frac{63}{7}$ $x = 9$	3B, 3C
Students expand brackets by adding the term outside of the brackets with the terms inside of the brackets.	$3(p + 4) = 4p + 7$ $2(3p - 6) = 5p - 4$	$3(p + 4) = 3p + 12$ $2(3p - 6) = 6p - 12$	3C

Continues →

Back to contents

Misconception	Incorrect ✗	Correct ✓	Lesson
Students don't understand the notation using opened and closed circles on a number line.	$-2 \le x < 3$ 	$-2 \le x < 3$ 	3D
Students don't understand the different symbols.	$2x - 3 > 7$ $2x - 3 + 3 \ge 7 + 3$ $2x \ge 10$ $x \ge 5$	$2x - 3 > 7$ $2x - 3 + 3 > 7 + 3$ $2x > 10$ $x > 5$	3D
Students don't know when to reverse the inequality sign.	$-\frac{x}{3} - 7 + 7 \ge -5 + 7$ $-\frac{x}{3} \times (-3) \ge 2 \times (-3)$ $x \ge -6$	$-\frac{x}{3} - 7 + 7 \ge -5 + 7$ $-\frac{x}{3} \times (-3) \le 2 \times (-3)$ $x \le -6$	3D
Students try to solve a formula for a value when it contains more than one variable.	Simplify $V = \frac{a}{b}(12 + a)^2$, when $a = 0.75$. $V = \frac{0.75}{0.75}(12 + 0.75)^2$ $V = \frac{0.75}{0.75}(12.75)^2$ $V = \frac{0.75}{0.75} \times 162.5625$ $V = 162.5625$	Simplify $V = \frac{a}{b}(12 + a)^2$, when $a = 0.75$. $V = \frac{0.75}{b}(12 + 0.75)^2$ $V = \frac{0.75}{b}(12.75)^2$ $V = \frac{0.75}{b} \times 162.5625$	3E
Students use the wrong inverse operations when transposing a formula.	$bx = \frac{v}{12} + c$ $bx + c = \frac{v}{12} + c + c$ $\frac{bx + c}{12} = \frac{v}{12} \div 12$ $\frac{bx + c}{12} = v$	$bx = \frac{v}{12} + c$ $bx - c = \frac{v}{12} + c - c$ $12(bx - c) = \frac{v}{12} \times 12$ $12(bx - c) = v$	3E
Students try to eliminate variables with their coefficients being equal.	$x + 2y = 8 \quad [1]$ $3x - 2y = 4 \quad [2]$ $[2] - [1]$ $\begin{array}{lr} & 3x - 2y = 4 \\ - & x + 2y = 8 \\ \hline & 2x = -4 \end{array}$	$x + 2y = 8 \quad [1]$ $3x - 2y = 4 \quad [2]$ $[1] + [2]$ $\begin{array}{lr} & x + 2y = 8 \\ - & 3x - 2y = 4 \\ \hline & 4x = 12 \end{array}$	3F
Students incorrectly substitute an expression or term when solving an equation.	$y = x + 1 \quad [1]$ $x + 3y = 15 \quad [2]$ $x + 3 + x + 1 = 15$ $2x + 4 = 15$ $2x = 11$ $x = \frac{11}{2}$ $y = \frac{11}{2} + 1$ $y = \frac{13}{2}$ $\therefore x = \frac{11}{2}, y = \frac{13}{2}$	$y = x + 1 \quad [1]$ $x + 3y = 15 \quad [2]$ $x + 3(x + 1) = 15$ $x + 3x + 3 = 15$ $4x + 3 = 15$ $4x = 12$ $x = 3$ $y = 3 + 1$ $y = 4$ $\therefore x = 3, y = 4$	3F

Back to contents

3A Expanding algebraic expressions

LEARNING INTENTIONS

Students will be able to:

- use the distributive law to expand brackets
- simplify expressions by expanding brackets and collecting like terms.

Brackets are used to group expressions that are being multiplied or divided by a common factor. When simplifying expressions that involve multiple terms and expressions inside brackets, it is helpful to expand or remove the brackets first. This process involves applying the distributive law to distribute the value outside of the brackets to each term inside the brackets.

KEY TERMS AND DEFINITIONS

- The **distributive law** for multiplication means that multiplying a number by a group of numbers is the same as multiplying the number by the sum of the other numbers.
- An **expression** is a number of terms grouped together by operations.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

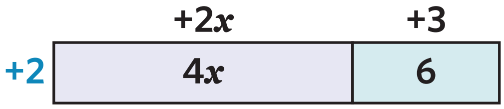
Image: prapann/Shutterstock.com

Algebraic expressions can be used to help calculate scenarios in our everyday life. For example when calculating the total cost of buying a bag of oranges and apples at a grocery store, we can create an algebraic expression to help determine the total cost.

Key idea

1. Expanding brackets involves applying the distributive law to find an equivalent expression without brackets.

$2(2x + 3)$



$$\begin{aligned} 2(2x + 3) &= 2 \times 2x + 2 \times 3 \\ &= 4x + 6 \end{aligned}$$

Back to contents

Worked example 1

Expanding expressions with brackets

Expand the brackets in each expression.

a. $1.5(4 - 3a)$

WE1a

Working

$$1.5(4 - 3a) = 1.5 \times 4 + 1.5 \times (-3a)$$

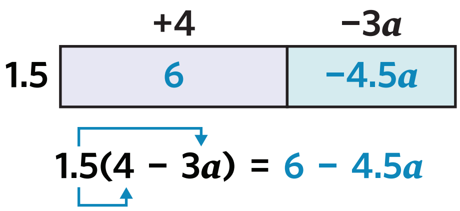
$$= 6 - 4.5a$$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify.

Visual support



b. $2b(-4 + 2b)$

WE1b

Working

$$2b(-4 + 2b) = 2b \times (-4) + 2b \times 2b$$

$$= -8b + 4b^2$$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify.

Student practice

Expand the brackets in each expression.

a. $2.5(6 - 2w)$ **b.** $3v(-5 + 2v)$

Worked example 2

Expanding and simplifying expressions with brackets

Expand the brackets in each expression and simplify.

a. $2(3q + 4) + 5q$

WE2a

Working

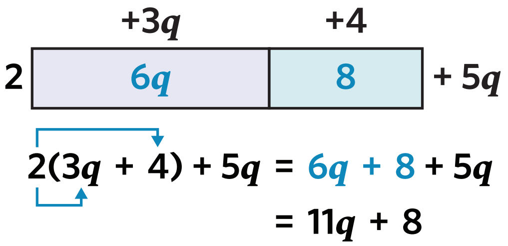
$$\begin{aligned} 2(3q + 4) + 5q &= 2 \times 3q + 2 \times 4 + 5q \\ &= 6q + 8 + 5q \\ &= 6q + 5q + 8 \\ &= 11q + 8 \end{aligned}$$

Thinking

Step 1: Expand the brackets and simplify.

Step 2: Simplify by collecting like terms.

Visual support



b. $4(2x - 3) - 2(x + 5)$

WE2b

Working

$$\begin{aligned} &4(2x - 3) - 2(x + 5) \\ &= 4 \times 2x + 4 \times (-3) + (-2) \times x + (-2) \times 5 \\ &= 8x - 12 - 2x - 10 \\ &= 8x - 2x - 12 - 10 \\ &= 6x - 22 \end{aligned}$$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify by collecting like terms.

Student practice

Expand the brackets in each expression and simplify.

a. $3(2h + 5) + 4h$

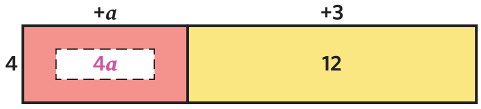
b. $5(3x - 4) - 2(2x + 3)$

3A Questions

Understanding worksheet

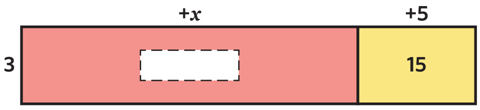
1. Fill in the blanks.

Example



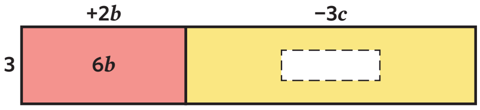
$4(a + 3) = 4a + 12$

a.



$3(x + 5) = 3x + 15$

b.

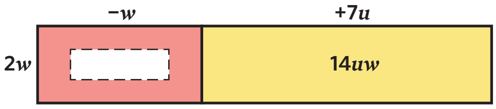


$3(2b - 3c) = 6b - 9c$

c.

$h(h - 6) = h^2 - 6h$

d.



$2w(-w + 7u) = -2w^2 + 14uw$

2. Fill in the blanks to make each equation true.

Example

$4\left(x + \boxed{6}\,y\right) = 4x + 6y$

a. $2\left(x + \boxed{}\right) = 2x + 2$

b. $3\left(\boxed{} - y\right) = 3x - 3y$

c. $\boxed{}(3a + 2b) = -6a - 4b$

d. $\boxed{}(2x + 4y - 3z) = 3x + 6y - 4.5z$

3. Fill in the blanks by using the words provided.

expressions **like** **multiplying** **distributive**

Expanding algebraic $\boxed{}$ is the process of removing brackets from an expression by $\boxed{}$ the terms inside the brackets by a number outside the brackets. This is an application of the $\boxed{}$ law. Simplifying an algebraic expression involves combining $\boxed{}$ terms and reducing the expression to its simplest form.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Expand the brackets in each expression. WE1a

a. $2(x + 3)$

b. $-5(2y - 1)$

c. $3.5(4a + 2)$

d. $6.25(b - 2c)$

e. $-\frac{1}{2}(3x - 4y)$

f. $\frac{1}{4}(2a - 3b + 5c)$

g. $-8(x + y - z)$

h. $3(2x + 4y - 5z)$

5. Expand the brackets in each expression. WE1b

a. $2x(x - 3)$ **b.** $-4y(2y + 5)$ **c.** $6x(x + 2)$ **d.** $-2z(3z - 4)$

e. $6x(2x - 1)$ **f.** $7w(4w^2 - 2)$` **g.** $-2a^2(3a + 5)$ **h.** $2a^2(3a - 24 + 4a)$

6. Expand the brackets in each expression and simplify. WE2a

a. $3(2x - 5) + 4$ **b.** $5(3y + 1) - 2$ **c.** $-5(2k + 3) + 4k$ **d.** $5a + 3(2a + 2)$

e. $4v - 7(v - 8)$ **f.** $9 - (h + 5)$ **g.** $5 + (g^2 - 2)$ **h.** $-3b(2b + 7) + 8b$

7. Expand the brackets in each expression and simplify. WE2b

a. $6(a + 4) + 4(a + 2)$ **b.** $8(x - 1) + 2(x - 2)$

c. $-4(r + 2) + 6(4r - 2)$ **d.** $5(2h + 5) - 3(3h + 2)$

e. $-2(5b - 3) + 4(3b + 7)$ **f.** $y(y + 5) - 4(y - 5)$

g. $3v(2v - 1) - 2v(3v - 1)$ **h.** $s(s + 3t) - t(s - 3t)$

8. Expand and simplify the following expression.

$2(4x - 3) - 3(x + 2)$

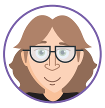
A. $5x + 1$ **B.** $2x - 2$ **C.** $5x - 12$ **D.** $11x + 10$ **E.** $11x + 12$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Expand the following expression.

$-1.5(2 + 3b)$



Student A

$-1.5(2 + 3b) = -1.5 \times 2 + (-1.5) \times 3b$

$= -3 - 4.5b$

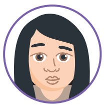
Student B

$-1.5(2 + 3b) = 1.5 \times 2 + 1.5 \times 3b$

$= 3 + 4.5b$

b. Expand and simplify the following expression.

$2(3b - 4) - 1.5(2 - 3b)$



Student A

$2(3b - 4) - 1.5(2 - 3b)$

$= 6b - 8 - 3 + 4.5b$

$= 10.5b - 11$

Student B

$2(3b - 4) - 1.5(2 - 3b)$

$= 6b - 4 - 3 + 3b$

$= 3b - 7$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Xavier has purchased a new iPad. The iPad has a length of $2x - 7$ cm and a width of x cm. Write an expanded and simplified expression for the area of the iPad.

Back to contents

11. Johnny is considering taking a certain number of additional hours of driving lessons from Drive4Life. The total cost of the lessons can be represented by the algebraic expression $C = (2.5 + h)(30)$. Expand and simplify the expression to find the total cost (C) for each additional hour in a lesson.

12. Village Cinemas charges $\$a$ for each adult ticket and $\$b$ for each child ticket. A family package consists of two adult tickets and two children tickets. The cost of the family package is $7 less than if the tickets are purchased individually. Three families purchase the family package and three additional children tickets. Construct an expression that represents the total cost of all the tickets.

13. A tennis court's perimeter has an expanded expression equal to $6x + 18$. It has a length of $x + 3$. Calculate the width of the tennis court.

14. Write an expression for the cost of a fence and gate, in terms of x, around Chirag's pool. The length of the fence is $2x - 3$ metres and the width is $3x + 4$ metres. A contractor charges $12 per metre for the fence, and an additional $100 to install the gate.

Reasoning

Question working paths

Mild 15 (a,b,d) 🌶	**Medium** 15 (a,b,d), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. A school is planning to update the rectangular gardens in the junior school campus and senior school campus. The current gardens have a length of $(4x + 3)$ metres and a width of $(2x + 1)$ metres. The dimensions of the rectangular garden will be doubled in length and tripled in width for the senior school campus. The length of the garden in the junior school will be halved and the width will be divided by four.

a. Use the distributive law to expand the expression to calculate the new length and width of the garden for the senior school campus.

b. Use the distributive law to expand the expression to calculate the new length and width of the garden for the junior school campus.

c. Calculate the difference in the lengths and the difference in the widths between the senior and junior school campus garden.

d. List two factors the school should consider if they were to lay a path around the garden.

16. Expand and simplify the following expressions:

a. $5(x + 3) - 2(x - 5)$

b. $1.25(2.4x + 20)$

c. Comment on a similarity and difference for both expressions in parts **a** and **b**.

Exam-style

17. Expand and simplify $2x(x + 2y) + 3x(2x - 3y)$. (1 MARK)

A. $3y - 2y$ **B.** $8x - 5xy$ **C.** $-3y + 2y$ **D.** $8x^2 - 5xy$ **E.** $8x^2 + 10xy$

18. A triangle has side lengths of $4x$ cm, $2x$ cm and $6x - 3$ cm and a square has four side lengths of $3x$ cm. (2 MARKS)

a. Write an expression each for the perimeter of both shapes. 1 MARK

b. If the side lengths of both shapes were halved, write an expression each for the perimeters of these shapes. 1 MARK

19. For the expression $4(x - 2) - 3(x + 2)$, determine the simplified expression and explain how a student would incorrectly simplify the expression to $7x + 14$. (2 MARKS)

20. Using the following information, construct two expressions in expanded form. Compare the constant and coefficient of t values of both expressions. (3 MARKS)

- Irene receives a \$30 bonus for every TV she sells above her quota of 20 units per week. If she sells t TVs in a week and $t > 20$, write an expression for Irene's bonus in that week.
- Sophie receives a \$20 bonus for every TV she sells above her quota of 30 units per week. If she sells t TVs in a week and $t > 30$, write an expression for Sophie's bonus in that week.

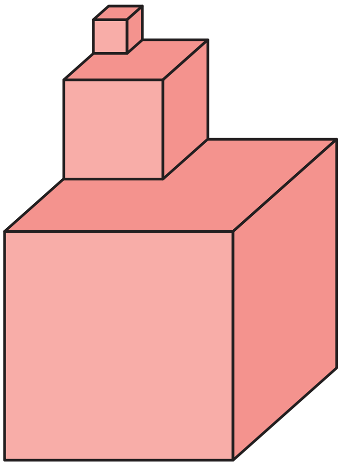
Remember this?

21. The speed of light is 299 792 458 metres per second.

What is the speed of light in metres per minute?

A. 2 997 924.6
B. 4 996 540.9
C. 499 654 096.7
D. 17 987 547 480
E. 29 979 245 800

22. A cube of side length 1 cm is glued onto the top corner of a cube of side length 3 cm, which is then glued onto the top corner of a cube of side length 7 cm.



What is the surface area of the new object?

A. 334 cm^2 **B.** 336 cm^2 **C.** 340 cm^2 **D.** 344 cm^2 **E.** 354 cm^2

23. The bus on route 709 usually runs late.

The following formula can be used to estimate the number of minutes that the bus will be late by:

$t = 2 + 0.7r$

where t is the number of minutes the bus is late and r is the amount of rainfall in millimetres.

Which statement is false?

A. The bus always runs at least 2 minutes late.
B. When there is 3 mm of rainfall, the bus is 4.1 minutes late.
C. When there is 5 mm of rainfall, the bus is 5.5 minutes late.
D. Every extra millimetre of rainfall delays the bus by 0.7 minutes.
E. If the bus was 10 minutes late, there must have been 13 mm of rainfall.

Back to contents

3B Solving linear equations

LEARNING INTENTIONS

Students will be able to:

- solve linear equations using inverse operations
- check solutions using substitution.

To solve an equation, inverse operations are used to find the value of the variable that makes the equation true. Equations may contain fractions and brackets. Fractions in an equation are used to indicate division, while brackets clarify the order of operations.

KEY TERMS AND DEFINITIONS

- **Solving equations** is a process to find the value of the unknown by performing a series of inverse operations.
- A **one-step equation** is an equation that can be solved by applying a single inverse operation.
- A **two-step equation** is an equation that can be solved by applying two inverse operations.
- A **multi-step equation** is an equation that can be solved by applying more than two inverse operations.
- A **variable** is a letter used to represent a value that is unknown or may vary. This is also known as a pronumeral or an unknown.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Standret Natalia/Shutterstock.com

Solving linear equations can be used to understand relationships and make predictions in real-life scenarios. For example a basketball player can analyse the relationship between the number of shots taken in practice to their overall shooting percentage by creating a linear equation. Solving this equation can help them make informed decisions about their practice sessions.

Key ideas

1. To isolate and find the value of the unknown in an equation, use the inverse operation.

Addition + ⟷ Subtraction −

Multiplication × ⟷ Division ÷

Square x^2 ⟷ Square root $\sqrt{x}$

2. When solving equations, it is important to understand the order of operations involved in constructing the equation.

$$\begin{tikzcd}
x = 5 \arrow[r, bend left, "\times 2"] & 2x = 10 \arrow[r, bend left, "+5"] \arrow[l, bend left, "\div 2"] & 2x + 5 = 15 \arrow[r, bend left, "\div 3"] \arrow[l, bend left, "-5"] & \dfrac{2x+5}{3} = 5 \arrow[l, bend left, "\times 3"]
\end{tikzcd}$$

Back to contents

Worked example 1

Solving equations

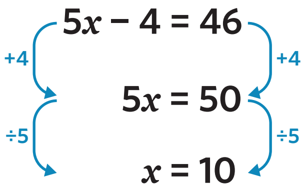
Solve each of the following equations for the unknown values. Provide the answer in exact form as needed.

a. $5x - 4 = 46$

WE1a

Working	Thinking
$\times\ 5$ and $-\ 4$	**Step 1:** Identify the operations being applied to the unknown value.
$+\ 4$ and $\div\ 5$	**Step 2:** The last operation used to construct the equation is the first inverse operation.
$5x - 4 + 4 = 46 + 4$ $5x = 50$ $\frac{5x}{5} = \frac{50}{5}$	**Step 3:** Isolate the unknown variable by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$x = 10$	**Step 4:** Simplify both sides of the equation to find the solution for the unknown variable.
Check: $5 \times 10 - 4 = 46$ ✓	**Step 5:** Substitute the solution in the original equation and check to see if the LHS equals the RHS.

Visual support



b. $-6t + 1 = 55$

WE1b

Working	Thinking
$\times\ -6$ and $+\ 1$	**Step 1:** Identify the operations being applied to the unknown value.
$-\ 1$ and $\div\ -6$	**Step 2:** The last operation used to construct the equation is the first inverse operation.
$-6t + 1 - 1 = 55 - 1$ $-6t = 54$ $\frac{-6t}{-6} = \frac{54}{-6}$	**Step 3:** Isolate the unknown variable by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$t = -9$	**Step 4:** Simplify both sides of the equation to find the solution for the unknown variable.
Check: $-6 \times (-9) + 1 = 55$ ✓	**Step 5:** Substitute the solution in the original equation and check to see if the LHS equals the RHS.

Student practice

Solve each of the following equations for the unknown values. Provide the answer in exact form as needed.

a. $3a + 5 = 44$

b. $-3x + 10 = 22$

Back to contents

Worked example 2

Solving equations with fractions

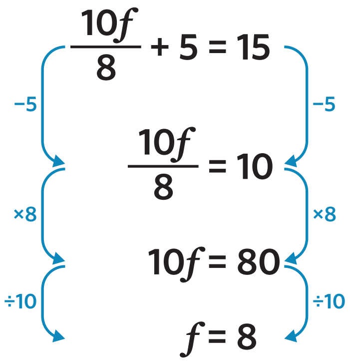
Solve each of the following equations for the unknown values. Provide the answer in exact form as needed.

a. $\frac{10f}{8} + 5 = 15$

WE2a

Working	Thinking
$\times\ 10,\ \div\ 8$ and $+\ 5$	**Step 1:** Identify the operations being applied to the unknown value.
$-\ 5,\ \times\ 8$ and $\div\ 10$	**Step 2:** The last operation used to construct the equation is the first inverse operation.
$\frac{10f}{8} + 5 - 5 = 15 - 5$ $\frac{10f}{8} = 10$ $\frac{10f}{8} \times 8 = 10 \times 8$ $10f = 80$ $\frac{10f}{10} = \frac{80}{10}$	**Step 3:** Isolate the unknown variable by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$f = 8$	**Step 4:** Simplify both sides of the equation to find the solution for the unknown variable.
Check: $\frac{10 \times 8}{8} + 5 = 15$ ✓	**Step 5:** Substitute the solution in the original equation and check to see if the LHS equals the RHS

Visual support



b. $\frac{-21a + 12}{2} = 13$

WE2b

Working	Thinking
$\times\ -21,\ +\ 12$ and $\div\ 2$	**Step 1:** Identify the operations being applied to the unknown value.
$\times\ 2,\ -\ 12$ and $\div\ -21$	**Step 2:** The last operation used to construct the equation is the first inverse operation.

Continues →

Back to contents

$$\frac{-21a+12}{2}\times 2 = 13\times 2$$

$$-21a+12=26$$

$$-21a+12-12=26-12$$

$$-21a=14$$

$$\frac{-21a}{-21}=\frac{14}{-21}$$

Step 3: Isolate the unknown variable by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

$$a=-\frac{2}{3}$$

Step 4: Simplify both sides of the equation to find the solution for the unknown variable.

Check:

$$\frac{-21\times\left(-\frac{2}{3}\right)+12}{2}=13\ ✓$$

Step 5: Substitute the solution in the original equation and check to see if the LHS equals the RHS

Student practice

Solve each of the following equations for the unknown values. Provide the answer in exact form as needed.

a. $\frac{12c}{5}+6=30$

b. $\frac{-15a+10}{6}=5$

Worked example 3

Solving equations with brackets

Solve each of the following equations for the unknown values. Provide the answer in exact form as needed.

a. $5(a-3)=25$

WE3a

Working

Thinking

$-\ 3$ and $\times\ 5$

Step 1: Identify the operations being applied to the unknown value.

$\div\ 5$ and $+\ 3$

Step 2: Isolate the unknown variable by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

$$\frac{5(a-3)}{5}=\frac{25}{5}$$

Step 3: The last operation used to construct the equation will be its inverse.

$$a-3=5$$

$$a-3+3=5+3$$

$$a=8$$

Step 4: Simplify both sides of the equation to find the solution for the unknown variable.

Check:

$$5(8-3)=25\ ✓$$

Step 5: Substitute the solution in the original equation and check to see if the LHS equals the RHS

Visual support

$$\begin{tikzcd}
5(a-3)=25 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
a-3=5 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\
a=8
\end{tikzcd}$$

Continues →

b. $3(2x - 4) = 16$

WE3b

Working	**Thinking**
$3(2x - 4) = 6x - 12$	**Step 1:** Expand the brackets.
× 6 and − 12	**Step 2:** Identify the operations being applied to the unknown value.
+ 12 and ÷ 6	**Step 3:** The last operation used to construct the equation will be its inverse.
$6x - 12 + 12 = 16 + 12$ $6x = 28$ $\frac{6x}{6} = \frac{28}{6}$	**Step 4:** Isolate the unknown variable by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$x = \frac{28}{6} = \frac{14}{3}$	**Step 5:** Simplify both sides of the equation to find the solution for the unknown variable.
Check: $3\left(2 \times \frac{14}{3} + 4\right) = 16$ ✓	**Step 6:** Substitute the solution in the original equation and check to see if the LHS equals the RHS

Student practice

Solve each of the following equations for the unknown values. Provide the answer in exact form as needed.

a. $6(b - 8) = 24$

b. $2(6x + 12) = 27$

Back to contents

3B Questions

Understanding worksheet

1. State the inverse operation required to solve for the unknown value.

Example

$a + 7 = 15$

Inverse operation: $\boxed{-7}$

a. $a + 4 = 17$

Inverse operation: $\boxed{}$

b. $-7 + b = 15$

Inverse operation: $\boxed{}$

c. $5a = 20$

Inverse operation: $\boxed{}$

d. $\frac{a}{-3} = 9$

Inverse operation: $\boxed{}$

2. Complete the missing step to show how each equation is solved.

Example

$$\begin{tikzcd}
4(5b+8)=72 \arrow[d, "\div 4"', "\div 4"] \\
\boxed{5b+8=18} \arrow[d, "-8"', "-8"] \\
5b=10 \arrow[d, "\div 5"', "\div 5"] \\
b=10
\end{tikzcd}$$

a.

$$\begin{tikzcd}
\frac{4m-3}{5}=9 \arrow[d, "\times 5"', "\times 5"] \\
\boxed{} \arrow[d, "+3"', "+3"] \\
4m=48 \arrow[d, "\div 4"', "\div 4"] \\
m=12
\end{tikzcd}$$

b.

$$\begin{tikzcd}
2(3c+4)=20 \arrow[d, "\div 2"', "\div 2"] \\
\boxed{} \arrow[d, "-4"', "-4"] \\
3c=6 \arrow[d, "\div 3"', "\div 3"] \\
c=2
\end{tikzcd}$$

c.

$$\begin{tikzcd}
4(2r-1)=20 \arrow[d, "\div 4"', "\div 4"] \\
2r-1=5 \arrow[d, "+1"', "+1"] \\
\boxed{} \arrow[d, "\div 2"', "\div 2"] \\
r=3
\end{tikzcd}$$

d.

$$\begin{tikzcd}
\frac{5k+7}{3}=4 \arrow[d, "\times 3"', "\times 3"] \\
5k+7=12 \arrow[d, "-7"', "-7"] \\
\boxed{} \arrow[d, "\div 5"', "\div 5"] \\
k=1
\end{tikzcd}$$

3. Fill in the blanks by using the words provided.

solution **isolate** **inverse** **substitution**

One way to solve linear equations is by using $\boxed{}$ operations. This involves manipulating the equation to $\boxed{}$ a variable on one side of the equation. To ensure the $\boxed{}$ is correct, it is important to check the answer using $\boxed{}$.

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (b,c,d), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (c,d,e), 10

4. Solve each of the following equations for the unknown values. Provide the answer in exact form as needed. **WE1**

a. $2s + 3 = 9$ **b.** $4a - 5 = 11$ **c.** $8t - 3 = 62$ **d.** $5 + 6g = 18$

e. $-2h + 5 = 19$ **f.** $-4z + 2 = -10$ **g.** $3 - 2x = 10$ **h.** $-5 - 3y = 17$

---

5. Solve each of the following equations for the unknown values. Provide the answer in exact form as needed. **WE2a**

a. $\frac{4g}{2} - 3 = 11$ **b.** $9 + \frac{7x}{3} = 16$ **c.** $\frac{3v}{2} + 7 = -14$ **d.** $-9 + \frac{7x}{5} = -16$

e. $\frac{-7y}{3} - 2 = 6$ **f.** $2 + \frac{-9z}{4} = 7$ **g.** $\frac{-5y}{6} - 2 = 10$ **h.** $5 + \frac{-9y}{3} = -15$

---

6. Solve each of the following equations for the unknown values. Provide the answer in exact form as needed. **WE2b**

a. $\frac{z + 8}{3} = 12$ **b.** $\frac{16 + 8z}{4} = 18$ **c.** $\frac{-2x + 6}{3} = 8$ **d.** $\frac{18 - 9s}{9} = 21$

e. $\frac{3y + 6}{5} = 8$ **f.** $\frac{10 + 5w}{2} = 9$ **g.** $\frac{-6p + 12}{7} = 10$ **h.** $\frac{14 - 7q}{3} = 3$

---

7. Solve each of the following equations for the unknown values. Provide the answer in exact form as needed. **WE3a**

a. $2(b + 3) = 10$ **b.** $4(c - 3) = 28$ **c.** $-6(p + 8) = 12$ **d.** $-4(x - 4) = 28$

e. $5(-d + 6) = 25$ **f.** $3(-a + 4) = 21$ **g.** $3(x + 4) = 31$ **h.** $4(-k + 5) = 21$

---

8. Solve each of the following equations for the unknown values. Provide the answer in exact form as needed. **WE3b**

a. $5(4x + 3) = 55$ **b.** $-6(2w + 9) = -18$ **c.** $6(3t - 9) = 66$ **d.** $-2(5y + 1) = 22$

e. $2(-3x + 5) = 4$ **f.** $-2(3 - 3f) = -12$ **g.** $8(9 - 5h) = 16$ **h.** $-3(-2 + 4b) = 15$

---

9. Solve for the following unknown values. Where necessary, provide your answer in exact form.

a. $3(d + 2) - 5 = 10$ **b.** $4(2x - 3) + 5 = 33$

c. $6(2a + 3) - 5a = 22$ **d.** $-6h + 4(8 - 4h) = 14$

e. $-5c - 4(-8 - 3c) = 17$

---

10. Solve

$$\frac{6(-y - 20)}{18} = 20$$

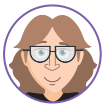
A. $y = -80$ **B.** $y = -20$ **C.** $y = -13.33$ **D.** $y = 80$ **E.** $y = 20$

Back to contents

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. Solve $7x - 2 = 61$



Student A

$7x - 2 + 2 = 61 + 2$

$7x = 63$

$\frac{7x}{7} = \frac{63}{7}$

$x = 9$

Student B

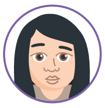
$7x - 2 - 2 = 61 - 2$

$7x = 59$

$7x \times 7 = 59 \times 7$

$x = 413$

b. Solve $5(2x + 7) = -5$



Student A

$5(2x + 7) = -5$

$10x + 35 = -5$

$\frac{10x}{10} + \frac{35}{10} = -5$

$x + 35 = -\frac{1}{2}$

$x + 35 - 35 = -\frac{1}{2} - 35$

$x = -35.5$

Student B

$5(2x + 7) = -5$

$10x + 35 = -5$

$10x + 35 - 35 = -5 - 35$

$10x = -40$

$\frac{10x}{10} = \frac{-40}{10}$

$x = -4$

Problem solving

Question working paths

Mild 12, 13, 14 🌶	**Medium** 13, 14, 15 🌶🌶	**Spicy** 14, 15, 16 🌶🌶🌶

12. Ken buys a new packet of tennis balls with w balls in each packet. On the following day, he purchases another two packets and four tennis balls separately. He now has a total of 13 tennis balls. Ken's number of tennis balls can be represented using the following equation $3w + 4 = 13$. Solve for w to determine the number of tennis balls in each packet.

13. Billo's savings can be represented using the following equation, $8w + 65 = 250$. Where Billo needs \$250 to attend his soccer camp, he already has \$65, and he will earn an additional \$8 allowance each week. Solve the following equations to determine how many full weeks it will take Billo to save up enough money for his soccer camp.

14. Andrew has j number of fidget spinners. He divides the fidget spinners amongst 6 people, including himself. His friend gives him an additional 8. Andrew was left with 10 fidget spinners. Calculate the initial number of fidget spinners Andrew had. The following equation can be used $\frac{j}{6} + 8 = 10$.

15. Maria hires a Neuron scooter. It costs \$1 to get started and then an additional 45 cents per minute. Maria was charged \$11.80. Maria's trip can be represented as as an equation, where m represents minutes. Write an equation and solve for the number of minutes (m) Maria was on the bike.

16. How old is Peter, in years and months, if he is five times as old as Jenny and the sum of their ages is twenty one? Jenny's age is represented as r and Peter's age is represented as $5r$. Determine Peter's age.

Back to contents

Reasoning

Question working paths

Mild 17 (a,b,d) 🌶	**Medium** 17 (a,b,d), 18 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

17. Miss Jeggy is organising a bus for interschool sports day. She requires a bus to travel 50 km and has a total budget of $1300.

Miss Jeggy has found the following companies and the equations used to calculate the hiring cost:

No1. Coaches: $C = 25d + 300$

Coach4you: $C = \dfrac{120d + 3}{5}$

Coach Heroes: $C = 3(9d + 3)$

C represents the cost and d represents distance, in km.

a. When $1300 is substituted for C, the cost of No1. Coaches can be expressed as $1300 = 25d + 300$. Determine whether she will have enough money to hire No1. Coaches.

b. When $1300 is substituted for C, the cost of Coach4you can be expressed as $1300 = \dfrac{120d + 3}{5}$. Determine whether she will have enough money to hire Coach4you.

c. What is the maximum distance Miss Jeggy could use Coach Heroes with her budget?

d. What are two things Miss Jeggy could consider when choosing to hire a coach?

18. For each of the following:

a. Solve $2(3x + 4) = 26$ by expanding the brackets first.

b. Solve $2(3x + 4) = 26$ by dividing by 2 on both sides first.

c. Compare and contrast the steps used to solve for x in both parts **a** and **b**. Suggest when it is easier to use the method in part **a** compared to part **b**.

Exam-style

19. Solve $3(-9d - 3) = 45$ (1 MARK)

A. $d = -1218$ **B.** $d = -3$ **C.** $d = -2$ **D.** $d = -1.3$ **E.** $d = 2$

20. Use $2a - 4 + 3a + 1 = 13$ for parts **a** and **b**. (3 MARKS)

a. Simplify the expression and solve for a. Provide the answer in exact form. 1 MARK

b. Insert brackets into the equation in order to make the value of a equal -16. Prove that it is true by solving the equation for a. 2 MARKS

21. A school class is selling boxes of chocolates to raise money for an end-of-year field trip. Each box of chocolates costs them $5 to purchase from the supplier. They sell each box for $11 to customers. (3 MARKS)

a. Write an expression showing the profit made by the class. Use b to represent the number of boxes sold. 1 MARK

b. How many boxes of chocolates does the class need to sell in order to raise $105 of profit. 2 MARKS

22. The following equation has been incorrectly solved. Identify and explain the error(s) made and show the correct steps to solve for x. (3 MARKS)

$$\frac{4(x + 4)}{12} = 24$$

$$\frac{4(x + 4)}{12} \div 12 = 24 \div 12$$

$$4(x + 4) = 2$$

$$4(x + 4) \times 4 = 2 \times 4$$

$$x + 4 = 8$$

$$x + 4 + 4 = 72 + 4$$

$$x = 76$$

Back to contents

Remember this?

23. Three years ago, Xaden was 150 cm tall.

He is now 16% taller than he was then.

How tall is Xaden now?

A. 126 cm **B.** 152 cm **C.** 166 cm **D.** 174 cm **E.** 222 cm

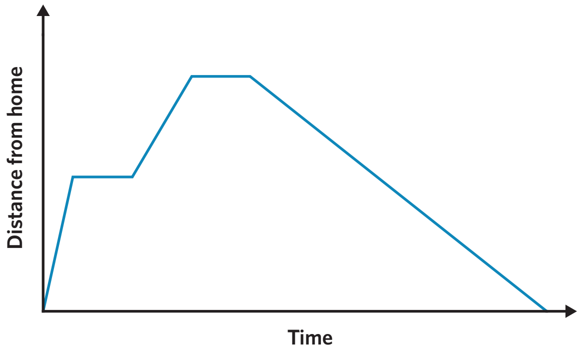
24. The following table shows the number of people who entered the dragon racing tournament at Navarre each day.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Number of entrants	11 400	8600	3400	9300	10 200

To the nearest thousand, what was the total number of entrants for the tournament?

A. 41 000 **B.** 42 000 **C.** 43 000 **D.** 44 000 **E.** 45 000

25. Which of the following stories best fits the graph?



A. Violet sprinted to the park, had a rest, then jogged a bit more, and had another rest before walking home.

B. Violet walked up a steep hill, had a rest before walking up another hill, then rested again and walked downhill.

C. Violet ran to the shops, did some shopping, walked to another shop to do some shopping, and then drove home.

D. Violet walked north-east, turned right, walked further north-east, turned right again, and walked back home.

E. Violet drove north-east, ate lunch, drove at the same speed and did some shopping, and then drove back home.

Back to contents

3C Equations with pronumerals on both sides

LEARNING INTENTIONS

Students will be able to:

- solve equations with pronumerals on both sides by collecting like terms
- expand brackets with pronumerals on both sides to solve equations.

This lesson navigates through solving equations featuring pronumerals on both sides. The emphasis is on the key skills of expanding brackets and collecting like terms on each side of the equation. By applying inverse operations, like terms are grouped and unknown variables are isolated.

KEY TERMS AND DEFINITIONS

- To **equate** expressions is to make them equal to each other.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Julia Tim/Shutterstock.com

Equations with pronumerals on both sides can apply to situations like planning an event. For instance, if an event coordinator needs to determine the price of tickets to a music concert based on the total number of tickets sold and the total money collected, they can use these equations to find the solution.

Key idea

1. Solving equations with pronumerals on both sides involves expanding brackets, grouping like terms, and applying inverse operations to isolate the unknown variable.

$$\begin{alignedat}{3}
& & 3(2a-3) &= 5(a+2) & & \\
&\text{Expand} & & & &\text{Expand} \\
& & \boxed{6a}\,\boxed{-9} &= \boxed{5a}\,\boxed{+10} & & \\
&\text{Group like terms} & & & &\text{Group like terms} \\
&\text{Apply inverse operations} & & & &\text{Apply inverse operations} \\
& & 6a-5a &= 10+9 & & \\
&\text{Solve} & & & &\text{Solve} \\
& & a &= 19 & &
\end{alignedat}$$

Back to contents

Worked example 1

Solving equations with pronumerals on both sides

Solve each equation for the given variable. Provide your answer in exact form where required.

a. $4x = 2x + 10$

WE1a

Working

$4x - 2x = 2x + 10 - 2x$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Check:

$4 \times 5 = 2 \times 5 + 10$

$20 = 20$ ✓

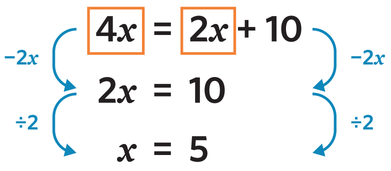
Thinking

Step 1: Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Step 2: Solve and simplify for the unknown value.

Step 3: Check to see if the LHS equals the RHS.

Visual support



b. $-2y + 7 = 5y$

WE1b

Working

$-2y + 2y + 7 = 5y + 2y$

$7 = 7y$

$\frac{7}{7} = \frac{7y}{7}$

$1 = y$

Check:

$-2 \times 1 + 7 = 5 \times 1$

$5 = 5$ ✓

Thinking

Step 1: Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Step 2: Solve for the unknown value.

Step 3: Check to see if the LHS equals the RHS.

Student practice

Solve each equation for the given variable. Provide your answer in exact form where required.

a. $7a = 5a + 8$ **b.** $-2s + 10 = 3s$

Back to contents

Worked example 2

Solving equations with pronumerals and numerals on both sides

Solve each equation for the given variable. Provide your answer in exact form where required.

a. $4x + 5 = 2x - 3$

WE2a

Working

$4x + 5 - 2x = 2x - 3 - 2x$

$2x + 5 = -3$

$2x + 5 - 5 = -3 - 5$

$2x = -8$

$\frac{2x}{2} = \frac{-8}{2}$

$x = -4$

Check:

$4 \times (-4) + 5 = 2 \times (-4) - 3$

$-11 = -11$ ✓

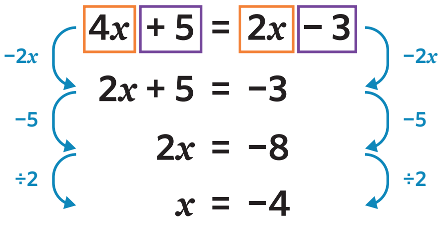
Thinking

Step 1: Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Step 2: Solve and simplify for the unknown value.

Step 3: Check to see if the LHS equals the RHS.

Visual support



b. $7 - 3k = 6k + 4$

WE2b

Working

$7 - 3k + 3k = 6k + 4 + 3k$

$7 = 9k + 4$

$7 - 4 = 9k + 4 - 4$

$3 = 9k$

$\frac{3}{9} = \frac{9k}{9}$

$\frac{1}{3} = k$

Check:

$7 - 3 \times \frac{1}{3} = 6 \times \frac{1}{3} + 4$

$6 = 6$ ✓

Thinking

Step 1: Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Step 2: Solve and simplify for the unknown value.

Step 3: Check to see if the LHS equals the RHS.

Student practice

Solve each equation for the given variable. Provide your answer in exact form where required.

a. $3b + 6 = 2b - 4$ **b.** $6 - 2v = 4v + 4$

Worked example 3

Solving equations with brackets and pronumerals on both sides

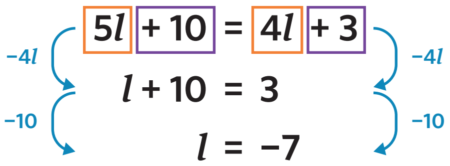
Solve each equation involving brackets for the given variable.

a. $5(l + 2) = 4l + 3$

WE3a

Working	Thinking
$5(l + 2) = 4l + 3$ $\therefore 5l + 10 = 4l + 3$	**Step 1:** Expand the brackets.
$5l + 10 - 4l = 4l + 3 - 4l$ $l + 10 = 3$	**Step 2:** Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.
$l + 10 - 10 = 3 - 10$ $l = -7$	**Step 3:** Solve for the unknown value.
Check: $5((-7) + 2) = 4 \times (-7) + 3$ $-25 = -25$ ✓	**Step 4:** Check to see if the LHS equals the RHS.

Visual support



b. $6(3p + 5) = -\ 3(2p + 6)$

WE3b

Working	Thinking
$6(3p + 5) = 18p + 30$ $-3(2p + 6) = -6p - 18$	**Step 1:** Expand the brackets.
$\therefore 18p + 30 = -6p - 18$ $18p + 30 - 30 = -6p - 18 - 30$ $18p = -6p - 48$ $18p + 6p = -6p - 48 + 6p$ $24p = -48$	**Step 2:** Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.
$\frac{24p}{24} = \frac{48}{24}$ $p = -2$	**Step 3:** Solve for the unknown value.
Check: $-3(2 \times (-2) + 6) = -6 \times (-2) - 18$ $-6 = -6$ ✓	**Step 4:** Check to see if the LHS equals the RHS.

Student practice

Solve each equation involving brackets for the given variable.

a. $4(h + 3) = 5h + 2$

b. $4(5a + 2) = -4(4a + 7)$

Back to contents

3C Questions

Understanding worksheet

1. Circle the like terms for the following equations.

> **Example**
>
> $\boxed{3x} = 12 \boxed{- 2x}$

a. $-5p = 2p + 2$ **b.** $3k - 3 = 15$ **c.** $9g + 3 = 5g - 8$ **d.** $-3x - 1 = 3 + x$

2. Match the given equations with its subsequent step.

Equation	Step
$15 = 7 + 8a$ ●	● Add $3n$ to both sides
$5n = -3n - 16$ ●	● Group like terms, if required apply the inverse operation
$7k - 54 = 4(k - 3)$ ●	● Subtract 7 from both sides
$5n - 9 = 3n + 17$ ●	● Expand brackets

3. Fill in the blanks by using the words provided.

same **substituting** **inverse** **isolate**

To solve equations with unknowns on both sides, it is essential to use __________ operations. These operations involve performing the __________ operation on both sides of the equation to maintain equality. By carefully applying inverse operations, one can simplify the equation and __________ the unknown variable on one side. Finally, verifying the solution by __________ it back into the original equation ensures its accuracy.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Solve each equation for the given variable. Provide your answer in exact form where required. WE1a

a. $2m = m + 6$ **b.** $3n = n + 8$ **c.** $5p = 2p + 15$ **d.** $4q = q - 9$

e. $6r = 2r - 18$ **f.** $5s = 2s - 12$ **g.** $7t = 4t - 22$ **h.** $8u = 5u - 25$

5. Solve each equation for the given variable. Provide your answer in exact form where required. WE1b

a. $2a = a + 3$ **b.** $2b = b - 5$ **c.** $-3c = c - 4$ **d.** $7 - 5g = g$

e. $-6h - 8 = h$ **f.** $-6 - 4f = f$ **g.** $-2d - 4 = d$ **h.** $2 - 3e = e$

6. Solve each equation for the given variable. Provide your answer in exact form where required. WE2

a. $2m - 3 = m + 1$ **b.** $2p + 5 = p - 3$ **c.** $3n - 4 = n + 2$ **d.** $4q - 2 = 2q + 4$

e. $7u - 5 = -4u + 6$ **f.** $-8p + 6 = 3p + 7$ **g.** $5r + 1 = -3r + 3$ **h.** $-6t + 4 = -3t - 5$

7. Solve each equation involving brackets for the given variable. Provide your answer in exact form where required. WE3a

a. $5(z - 1) = z + 7$

b. $2x + 5 = 3(x - 1)$

c. $4(y + 3) = 2y - 2$

d. $a - 6 = 3(a + 2)$

e. $4(p - 3) = 2p + 7$

f. $b + 4 = 2(b + 3)$

g. $-4(c + 2) = c - 5$

h. $-d - 7 = -5(d - 2)$

8. Solve each equation involving brackets for the given variable. Provide your answer in exact form where required. WE3b

a. $3(z - 4) = 2(z + 1)$

b. $4(x + 3) = 5(x - 2)$

c. $2(v + 5) = -3(v - 5)$

d. $-3(2r - 5) = 2(r + 4)$

e. $9(2w + 3) = -6(3w + 2)$

f. $-5(4b + 7) = -8(2b + 5)$

g. $2.5(4y + 6) = -4(5y + 2)$

h. $-1.5(2r + 8) = -3.5(4r + 5)$

9. Solve

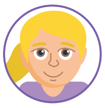
$3(x - 2) = 2(x + 4)$

A. $x = \frac{2}{5}$ **B.** $x = 1$ **C.** $x = 6$ **D.** $x = 11$ **E.** $x = 14$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. $5 - 2k = 6k + 5$



Student A

$5 - 2k + 2k = 6k + 5 + 2k$

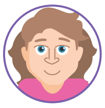
$5 = 8k + 5$

$5 - 5 = 8k + 5 - 5$

$0 = 8k$

$\frac{0}{8} = \frac{8k}{8}$

$0 = k$



Student B

$5 - 2k - 2k = 6k + 5 - 2k$

$5 = 4k + 5$

$5 + 5 = 8k + 5 + 5$

$10 = 8k$

$\frac{10}{8} = \frac{8k}{8}$

$\frac{5}{4} = k$

b. $3(p + 4) = 2(3p - 6)$

Student A

$3(p + 4) = 4p + 7$

$2(3p - 6) = 5p - 4$

$\therefore 4p + 7 = 5p - 4$

$4p + 7 - 7 = 5p - 4 - 7$

$4p = 5p - 11$

$4p - 5p = 5p - 11 - 5p$

$-p = -11$

$p = 11$

Student B

$3(p + 4) = 3p + 12$

$2(3p - 6) = 6p - 12$

$\therefore 3p + 12 = 6p - 12$

$3p + 12 - 12 = 6p - 12 - 12$

$3p = 6p - 24$

$3p - 6p = 6p - 24 - 6p$

$\frac{-3p}{-3} = \frac{-24}{-3}$

$p = 8$

Back to contents

Problem solving

Question working paths

Mild 11, 12, 13	**Medium** 12, 13, 14	**Spicy** 13, 14, 15

11. Alex and Bella are saving money for a summer trip. They both started saving at the same time. Alex saves \$15 every week, while Bella saves \$10 every week. However, Bella had a head start and had already saved \$50 before they started saving together. The following equation $15x = 10x + 50$, represents Alex and Bella's savings after x weeks. After how many weeks (x) will Alex and Bella have saved the same amount of money?

12. The equation $b = 2(b - 6)$ represents the total number of boys at Gary's soccer club. The soccer club originally contained an equal number of boys and girls. Six boys left to play tennis, leaving twice as many girls as boys at the club. Use the equation given to calculate the original total number of people at Gary's soccer club?

13. Calculate the width of the rectangular play area at Kerry's childcare centre. The length is four times the width, and the perimeter is 120 metres.

14. Jenny has a part time job at a rental car company where they charge \$25 per day and 10 cents per kilometre travelled. Rachel has a part time job at another rental car company where they charge \$30 per day and 7 cents per kilometre. Calculate the number of kilometres you would need to travel for the cost of Jenny's company to be equal to Rachel's company, rounded to two decimal places.

15. George Green is planning to buy the newest iPhone, which is priced at \$500. He has already saved up \$80. He can accumulate an additional \$30 every month, but the iPhone's price increases on average by \$2 each month. Can he gather sufficient funds to purchase his iPhone in 16 months?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	**Medium** 16 (a,b,c,e), 17 (a,b)	**Spicy** All

16. The table compares the cost of using two different Internet Service Providers (ISPs) in New York and Los Angeles. The fixed monthly fee is the cost of the subscription and is the same regardless of data used. The cost per gigabyte is the additional cost for each gigabyte of data used.

	Fixed monthly fee (\$)	**Cost per gigabyte (\$)**
New York	30	0.50
Los Angeles	20	0.75

a. Formulate an expression that computes the cost of utilising the internet in New York based on the data usage (d) measured in gigabytes.

b. Formulate an expression that computes the cost of utilising the internet in Los Angeles based on the data usage (d) measured in gigabytes.

c. Set the expressions in part **a** and **b** equal to each other. Determine the solution to this equation in order to identify the data usage where the internet cost is equal in both cities.

d. How much does it cost to use the internet for the data usage found in part **c** in New York and Los Angeles?

e. Some ISPs also charge per hour of internet usage. Discuss the benefits of cost per hour or cost per gigabyte.

17. With reference to $4a + 2 = 9a - 3$.

a. Solve the equation by subtracting $4a$ first from both sides.

b. Solve the equation by subtracting $9a$ first from both sides.

c. Explain two differences in the methods used in parts **a** and **b**.

Exam-style

18. Consider the following equation: $3(x - 2) = 2(x + 3)$. (1 MARK)

Which of the following steps is the best way to start solving this equation using the key skills outlined?

A. Expand brackets on both sides of the equation

B. Group like terms on the same side of the equation

C. Apply inverse operations to isolate the unknown variable

D. Divide the left-hand side by 3 and the right-hand side by 2

E. Apply inverse operations to group like terms on either side of the equation

19. Use the following equation: $4(y + 3) = -5(y - 2)$ (3 MARKS)

a. Perform the first step to solving the equation and write the new form of the equation. 1 MARK

b. Explain and perform the next step to further simplify the equation. 2 MARKS

20. Beanica and Paulie are planning their birthday parties and they need to calculate the number of children that are attending. The following equation shows the total number of people attending their party, $3(6 + b) = 2(3 + 2b)$. Calculate and solve for the number of children attending their party (b). (3 MARKS)

21. Consider the equation: $-2a + 6 = -3(a + 2)$ (3 MARKS)

To solve for a, you will need to:

- Expand the brackets
- Group like terms and apply inverse operations, where necessary to isolate the unknown variable

Using these steps, outline and show each step required to solve for a.

Remember this?

22. Olivia has 480 stickers to distribute equally among her 12 friends. How many stickers will each friend receive?

A. 20 **B.** 30 **C.** 40 **D.** 50 **E.** 60

23. In a population of 250 birds in a sanctuary, different species were observed. The table shows the distribution of the species:

Species	Number of birds
Eagle	60
Parrot	75
Owl	45
Sparrow	45
Others	25

One bird is chosen at random. What is the probability that the bird is an Owl?

A. 0.15 **B.** 0.18 **C.** 0.20 **D.** 0.22 **E.** 0.25

24. John scored 92 in a mathematics test out of 100. Other students, Lisa, Mark, Jenna, and Bill also took the same test. Their scores are:

- Lisa's score: 45
- Mark's score: 46
- Jenna's score: 47
- Bill's score: 50

Whose score is exactly half that of John's?

A. Bill **B.** Jenna **C.** Lisa **D.** Mark **E.** Lisa and Mark

Back to contents

3D Inequalities

LEARNING INTENTIONS

Students will be able to:

- work with the symbols $>, \geq, <, \leq$
- understand that an inequality represents an infinite set of values
- illustrate an inequality on a number line using known conventions
- understand when an inequality sign needs to be reversed
- solve a linear inequality.

Inequalities, represented by the symbols $>, \geq, <, \leq$ denote a range of values rather than a single value. Understanding this concept is crucial in various mathematical and real-life scenarios. It's also important to know when to reverse an inequality sign and how to represent an inequality on a number line using open and closed circles. The ability to solve linear inequalities forms a key part of this skill set.

KEY TERMS AND DEFINITIONS

- **Exclusive** is excluding the value(s).
- **Inclusive** is including the value(s).
- **Inequality** is a statement when one value or algebraic expression is less than or greater than another.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

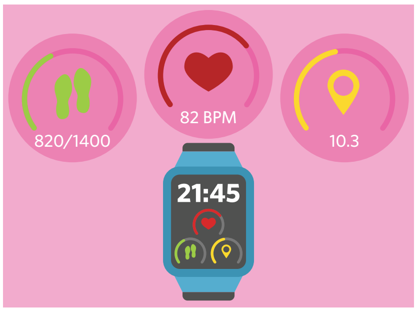


Image: Freepik/freepik.com

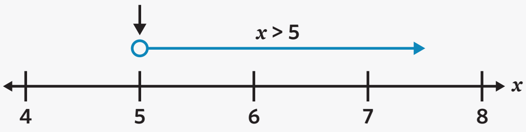
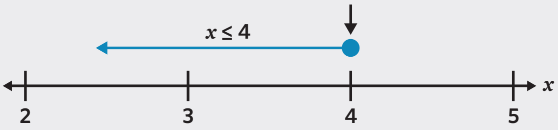
Inequalities are used in health and fitness. For instance, when monitoring calorie intake or the number of steps walked in a day, inequalities can help establish if personal health goals are being met or exceeded.

Key ideas

1. Understanding inequality symbols $(>, \geq, <, \leq)$ is fundamental to interpreting and solving inequalities.

Symbol	Symbol to words	Inequation example	Example in words
$>$	Greater than	$x > 4$	x is greater than four
$\geq$	Greater than or equal to	$x \geq 12$	x is greater than or equal to twelve
$<$	Less than	$x < 5$	x is less than five
$\leq$	Less than or equal to	$x \leq 11$	x is less than or equal to eleven

2. An inequality represents an infinite set of values, which can be illustrated on a number line using known conventions.

Symbol	Symbol to word	Number line diagram
○	Greater than, $>$ Less than, $<$	**Open circle indicates that 5 is not included.** 
●	Greater than or equal to, $\geq$ Less than or equal to, $\leq$	**Closed circle indicates that 4 is included.** 

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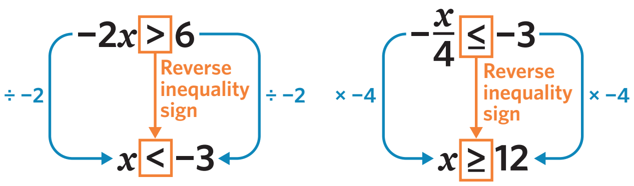
Back to contents

3. Solving a linear inequality involves understanding when to reverse the inequality sign. When we multiply or divide both sides of an inequality by a negative number, the direction of the inequality sign is reversed.



Worked example 1

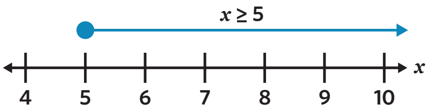
Representing inequalities on a number line

Represent the inequalities on a number line.

a. $x \geq 5$

WE1a

Working



Thinking

Step 1: To show the solution on a number line, use a closed circle to indicate the number given is also included in the set.

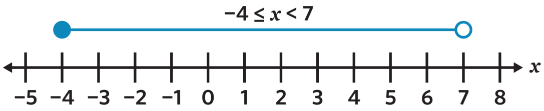
Step 2: Generate a number line that includes the given number.

Step 3: Draw an arrow pointing to the right to all numbers greater than or equal to the given number.

b. $-4 \leq x < 7$

WE1b

Working



Thinking

Step 1: To show the solution on a number line, use an open circle to indicate the number(s) given is not included in the set and use a closed circle to indicate the number(s) given is also included in the set.

Step 2: Generate a number line that includes the given numbers.

Step 3: Draw a line between the two circles.

Student practice

Represent the inequalities on a number line.

a. $x \geq 7$ **b.** $-2 \leq x < 5$

Worked example 2

Solving inequalities

Solve for x.

a. $x - 4 < 6$

WE2a

Working

$-\ 4$

Thinking

Step 1: Identify the operations being applied to the unknown variable.

Continues →

$+ 4$	**Step 2:** The last operation used to construct the inequality is the first inverse operation.
$x - 4 + 4 < 6 + 4$	**Step 3:** Isolate the unknown variable by applying the inverse operations. To keep the inequality true, apply the same operations on both sides of the inequality sign.
$x < 10$	**Step 4:** Simplify both sides of the inequality to find the solution for the unknown variable.

Visual support

$$\begin{tikzcd} x - 4 < 6 \arrow[d, bend right, "+4"'] \arrow[d, bend left, "+4"] \\ x < 10 \end{tikzcd}$$

b. $9 + 3x > -6$

WE2b

Working	Thinking
$\times\ 3$ and $+\ 9$	**Step 1:** Identify the operations being applied to the unknown variable.
$-\ 9$ and $\div\ 3$	**Step 2:** The last operation used to construct the inequality is the first inverse operation.
$9 - 9 + 3x > -6 - 9$ $\frac{3x}{3} > \frac{-15}{3}$	**Step 3:** Isolate the unknown variable by applying the inverse operations. To keep the inequality true, apply the same operations on both sides of the inequality sign.
$x > -5$	**Step 4:** Simplify both sides of the inequality to find the solution for the unknown variable.

Student practice

Solve for x.

a. $x - 3 < 5$ **b.** $8 + 4x > -4$

Worked example 3

Solving inequalities and reversing inequality signs

Solve for x.

a. $-\frac{x}{5} - 2 \geq -10$

WE3a

Working	Thinking
$\div\ -5$ and $-\ 2$	**Step 1:** Identify the operations being applied to the unknown variable.
$+\ 2$ and $\times\ -5$	**Step 2:** The last operation used to construct the inequality is the first inverse operation.

Continues →

$-\frac{x}{5} - 2 + 2 \ge -10 + 2$

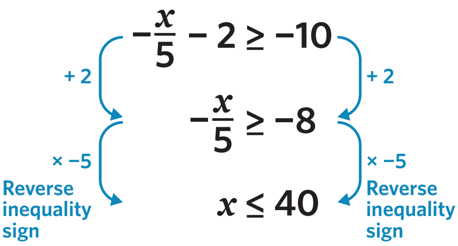
Step 3: Isolate the unknown variable by applying the inverse operations. To keep the inequality true, apply the same operations on both sides of the inequality sign.

$-\frac{x}{5} \times -5 \le -8 \times -5$

$x \le 40$

Step 4: When multiplying or dividing by a negative value, reverse the inequality sign and simplify both sides of the inequality to find the solution for the unknown variable.

Visual support



b. $3x + 4 \le 5x + 1$

WE3b

Working

Thinking

3x and 5x are like terms

4 and 1 are like terms

$3x - 5x + 4 \le 5x - 5x + 1$

$-2x + 4 \le 1$

$-2x + 4 - 4 \le 1 - 4$

$-2x \le -3$

Step 1: Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

$\times -2$

Step 2: Identify the operation being applied to the unknown value.

$\div -2$

Step 3: The last operation used to construct the inequality is the first inverse operation.

$\frac{-2x}{-2} \ge \frac{-3}{-2}$

Step 4: Isolate the unknown variable by applying the inverse operations. To keep the inequality true, apply the same operations on both sides of the inequality sign.

$x \ge \frac{3}{2}$

Step 5: When multiplying or dividing by a negative value, reverse the inequality sign and simplify both sides of the inequality to find the solution for the unknown variable.

Student practice

Solve for x.

a. $-\frac{x}{3} - 2 \ge -8$

b. $3x + 2 \ge 7x + 3$

3D Questions

Understanding worksheet

1. State whether the given inequalities will have an open or closed circle when represented on a number line.

Example

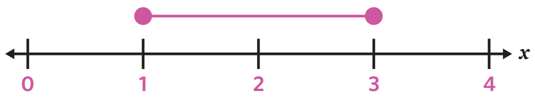
Inequality	Open circle	Closed circle
$x > 2$	☑	☐

Inequality	Open circle	Closed circle
I. $x < 4$	☐	☐
II. $x \geq 3$	☐	☐
III. $x > 5$	☐	☐
IV. $x \leq 1$	☐	☐

2. Represent the inequality on the incomplete number line.

Example

$1 \leq x \leq 3$



a. $2 < x < 5$

b. $3 \leq x \leq 6$

c. $-4 \leq x < 1$

d. $-2 < x \leq 3$

3. Fill in the blanks by using the words provided.

number line **symbols** **inequalities** **linear** **set**

In mathematics, ______ such as >, ≥, <, ≤ are used to represent ______ . Unlike equations, an inequality often represents an infinite ______ of values. This concept can be visualised on a ______ , using conventions such as open and closed circles. When solving a ______ inequality, multiplying or dividing by a negative number reverses the inequality sign.

Back to contents

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Represent the inequalities on a number line. WE1a

a. $x > 7$ **b.** $x \leq 10$ **c.** $x < 8$ **d.** $x \geq 9.5$

e. $11 > x$ **f.** $8.5 \leq x$ **g.** $x < -6.5$ **h.** $x \geq -0.5$

5. Represent the inequalities on a number line. WE1b

a. $5 < x < 7$ **b.** $3 \leq x \leq 8$ **c.** $10 < x < 13$ **d.** $-6 \leq x < 11$

e. $-6 < x \leq 2$ **f.** $-3.5 \leq x \leq 3$ **g.** $-2.5 \leq x \leq 8.5$ **h.** $-3.5 \leq x < -2.5$

6. Solve for x. WE2a

a. $x + 4 < 10$ **b.** $3 + x \geq 7$ **c.** $x - 2 \geq -8$ **d.** $-4 + x < 8$

e. $3x < 21$ **f.** $6x > -18$ **g.** $\frac{x}{5} \leq 8$ **h.** $\frac{x}{4} > -9$

7. Solve for x. WE2b

a. $6x - 3 \geq 21$ **b.** $2x - 3 > -17$ **c.** $\frac{x}{3} + 2 \leq 6$ **d.** $\frac{x}{8} - 7 > 5$

e. $\frac{x+4}{3} < -7$ **f.** $\frac{x-6}{5} > 3$ **g.** $13(x + 5) \leq 39$ **h.** $7(x + 9) < -49$

8. Solve for x. Provide your answer in exact form where required. WE3a

a. $-3x > -9$ **b.** $-\frac{x}{6} \geq -2$ **c.** $-2x - 4 < 8$ **d.** $-\frac{x}{3} - 4 \geq 3$

e. $-\frac{x}{5} + 3 < -2$ **f.** $\frac{-x+5}{6} > -3$ **g.** $-2(x + 6) \geq 3$ **h.** $-3(x + 4) < -5$

9. Solve for x. Provide your answer in exact form where required. WE3b

a. $4x < 6 + 6x$ **b.** $-8x \geq -9 - 5x$ **c.** $3 - 2x \leq 7x$ **d.** $-5 - 4x > 6x$

e. $3x + 10 \leq 7x - 2$ **f.** $3x + 3 > 4x + 6$ **g.** $x + 5 < 8 + 3x$ **h.** $5x - 11 \leq 9x - 3$

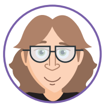
10. Solve the inequality $2x - 3 > 7$

A. $x > 5$ **B.** $x < 5$ **C.** $x < -8$ **D.** $x > -5$ **E.** $x \leq -5$

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. Solve the inequality $-\frac{x}{3} - 7 \geq -5$



Student A

$-\frac{x}{3} - 7 + 7 \geq -5 + 7$

$-\frac{x}{3} \times (-3) \geq 2 \times (-3)$

$x \geq -6$

Student B

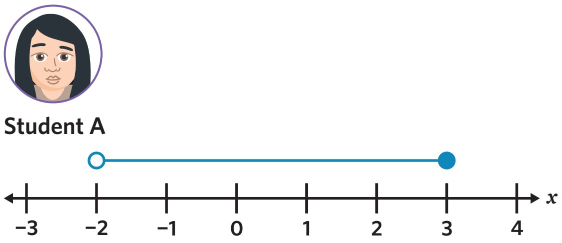
$-\frac{x}{3} - 7 + 7 \geq -5 + 7$

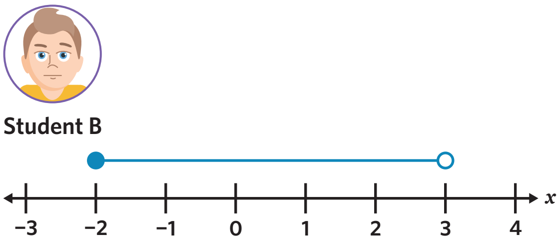
$-\frac{x}{3} \times (-3) \leq 2 \times (-3)$

$x \leq -6$

Back to contents

b. Represent $-2 \leq x < 3$ on a number line.





Problem solving

Question working paths

Mild 12, 13, 14 🌶	**Medium** 13, 14, 15 🌶🌶	**Spicy** 14, 15, 16 🌶🌶🌶

12. Generate an inequality that describes the temperature (t) of boiling water that is greater than or equal to 120°C.

13. Christopher has \$200 in his savings. He visits an online store and buys three video games and a headset. The headset costs \$60. Each video game costs \$$m$. Formulate an inequality in terms of m to represent Christopher's purchase.

14. Generate and solve, how many weeks (w) will Susan need to save in order to have at least \$150? She puts away \$10 each week. She also makes an additional \$20 by babysitting her cousin.

15. Daphne needs to buy some pencils and an eraser. She can spend no more than \$5. The eraser costs \$1 and the pencils cost \$0.25 each. Solve for the number of pencils (p) Daphne can purchase.

16. George has a pool that has a leak and is losing 2 L of water per minute. The pool currently has a volume of 4000 L. Using inequalities, calculate when the pool reaches less than 1500 L, in minutes.

Reasoning

Question working paths

Mild 17 (a,b,c,e) 🌶	**Medium** 17 (a,b,c,e), 18 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

17. Eastdale College is having their annual end of year biathlon where students cycle 10 km for C units of time and run 5 km for R units of time. Peter completes the race in less than 1 hour. Peter has completed the biathlon every year since Year 7. This year in Year 9 he wants to compare his average speed to see his improvements over the last two years.

a. Write an inequality to represent the time it took Peter to finish the race.

b. Speed is calculated by the formula $S = \frac{\text{Distance}}{\text{Time}}$ and Peter's average running speed can be represented by S. Formulate an expression for Peter's running speed using the given information and rearrange it to make R the subject.

c. Peter's average cycling speed is double his running speed. Write an expression for C using the method from part **b** and substitute both expressions (for C and R) in the inequality from part **a**.

d. Solve the inequality to find Peter's average speed s for running and for cycling.

e. When performing any forms of physical activity, it is important to consider some sort of active recovery. Provide an active recovery activity.

18. Solve:

a. $3x + 1 \leq 10$

b. $-3x + 1 \leq 10$

c. Explain how and why the answers differ in parts **a** and **b**.

Back to contents

Exam-style

19. The inequality $50 > 12x + 2$ is equivalent to (1 MARK)

A. $x < 4$

B. $48 > \frac{x}{4}$

C. $x + 12 > 36$

D. $4x + 80 > 96$

E. $-50 > 12x + 2$

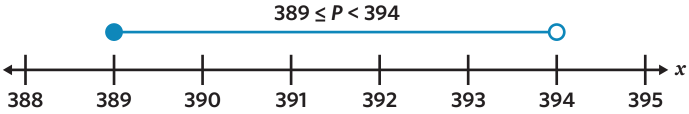
20. With reference to $-4x + 2 \geq 3x - 1$ (2 MARKS)

a. State the like terms for this inequality. 1 MARK

b. With reference to part **a**, solve for x. 1 MARK

21. Plot $-8.25 < x \leq 2.75$ on a number line. (2 MARKS)

22. A soccer field has a perimeter that is greater than 389 metres and less than or equal to 394 metres. This has been displayed on a number line. Explain why this is not a correct representation and then provide the correct number line. (2 MARKS)



Remember this?

23. In a certain town, $\frac{2}{3}$ of the population are registered voters. There are 900 people living in the town. How many people are registered voters?

A. 300 **B.** 400 **C.** 450 **D.** 600 **E.** 750

24. Cassie has $200 saved for her upcoming school trip. She spends $46.50 on a new backpack and buys 3 guidebooks, each costing $18.75. How much money does Cassie have left?

A. $87.25 **B.** $89.25 **C.** $90.75 **D.** $97.25 **E.** $100.50

25. At the start of her journey, Emily's odometer read 45.6 km. During her trip, she checked her odometer 20 times, each time having travelled an additional 260 m. After the 20th check, Emily stopped looking at her odometer for a while. When she reached her destination, the odometer read 52.0 km.

1 km = 1000 m

How far, in kilometres, did Emily travel during the part of the journey when she didn't check the odometer? Give your answer as a decimal, to two decimal places.

A. 1.20 km **B.** 2.20 km **C.** 3.20 km **D.** 4.20 km **E.** 5.20 km

Back to contents

3E Using formulas

LEARNING INTENTIONS

Students will be able to:

- use formulas to represent scenarios and identify the subject of a formula
- find unknowns in a formula by evaluating or solving an equation
- transpose a formula.

Formulas are used to represent scenarios and relate two or more variables. Unknowns in a formula can be found by evaluating or solving an equation. Transposing a formula involves rearranging it to change the subject of the equation. This process requires the ability to construct an equation from a scenario and to substitute and solve for a specified pronumeral.

KEY TERMS AND DEFINITIONS

- A **formula** is a rule written using mathematical symbols and pronumerals that are connected using an equals sign.
- The **subject** of a formula is the variable isolated on one side of the equal sign.
- An equation can be **transposed** to make any variable the subject.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

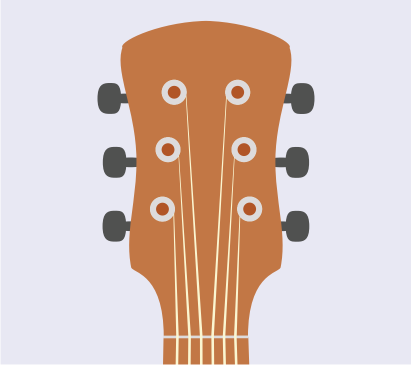


Image: Klaus Kunstler/Shutterstock.com

In music, the relationship between the length of a guitar string and its pitch can be represented with a formula. By identifying the subject, substituting known values, and transposing the formula, one can calculate how changing the string's length will affect the pitch it produces.

Key ideas

1. A formula is an equation that describes the relationship between two or more variables.

Perimeter of a rectangle

$$\underset{\text{perimeter}}{\boxed{P}} = 2\underset{\text{length}}{\boxed{l}} + 2\underset{\text{width}}{\boxed{w}}$$

Converting between Celsius and Fahrenheit

$$\underset{\text{Temperature in Fahrenheit}}{\boxed{F}} = \frac{9}{5}\underset{\text{Temperature in Celsius}}{\boxed{C}} + 32$$

2. Transposing a formula involves rearranging the equation to change the subject, allowing for the calculation of different variables based on the given values.

Perimeter of a rectangle when *w* is the unknown

$$\begin{array}{rcl} & P = 2l + 2w & \\ -2l \downarrow & & \downarrow -2l \\ & P - 2l = 2w & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & \frac{P}{2} - \frac{2}{2l} = w & \\ \text{simplify} \downarrow & & \downarrow \text{simplify} \\ & \frac{P}{2} - l = w & \end{array}$$

Converting between Fahrenheit to Celsius

$$\begin{array}{rcl} & F = \frac{9}{5}C + 32 & \\ -32 \downarrow & & \downarrow -32 \\ & F - 32 = \frac{9}{5}C & \\ \times 5 \downarrow & & \downarrow \times 5 \\ & 5(F - 32) = 9C & \\ \div 9 \downarrow & & \downarrow \div 9 \\ & \frac{5(F - 32)}{9} = C & \end{array}$$

Back to contents

Worked example 1

Substituting values in a formula

Calculate the value of the subject by substituting the given values into the formula.

a. $V = b(3 + a)$, when $b = 2$ and $a = 0.75$

WE1a

Working

$V = 2(3 + 0.75)$

$V = 2(3.75)$

$V = 7.5$

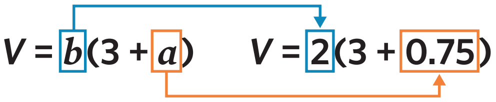
Thinking

Step 1: Substitute the given values into the formula.

Step 2: Simplify the expression inside the brackets.

Step 3: Evaluate the expression to find the value of the subject.

Visual support



b. $R = e(w + c)^2 + c$, when $c = 6$, $e = 2$ and $w = 4$

WE1b

Working

$R = 2(4 + 6)^2 + 6$

$R = 2(10)^2 + 6$

$R = 2(100) + 6$

$R = 200 + 6$

$R = 206$

Thinking

Step 1: Substitute the given values into the formula.

Step 2: Perform and evaluate the operations in the correct order according to the order of operations.

Student practice

Calculate the value of the subject by substituting its given values into the formula.

a. $y = c(2 + d)$, when $c = 3$ and $d = 1.5$

b. $P = a(b + c)^2 + c$, when $c = 4$, $a = 3$, and $b = 5$

Worked example 2

Finding the unknown value in a formula

WE2

Solve for the unknown variable.

The area of a triangle is given by $A = \frac{b \times h}{2}$. Substitute $A = 10$ cm and $b = 4$ cm to solve for the value of h.

Working

$10 = \frac{4 \times h}{2}$

$10 \times 2 = \frac{4 \times h}{2} \times 2$

$\frac{20}{4} = \frac{4 \times h}{4}$

$5 \text{ cm} = h$

Thinking

Step 1: Substitute the given values into the formula.

Step 2: Apply the inverse operation to isolate the unknown variable.

Step 3: Solve for the unknown value.

Continues →

Back to contents

Visual support

$$\begin{tikzcd}
A = \dfrac{b \times h}{2} \arrow[d, bend right, "\text{Substitute}"'] \arrow[d, bend left, "\text{Substitute}"] \\
10 = \dfrac{4 \times h}{2} \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
20 = 4 \times h \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
5 = h
\end{tikzcd}$$

Student practice

Solve for the unknown variable.

The area of a triangle is given by $A = \frac{b \times h}{2}$. Substitute $A = 15$ cm and $b = 3$ cm to solve for the value of h.

Worked example 3

Transposing formulas

Transpose the following equations to make the pronumeral shown in brackets the subject.

a. $y = mx + c \quad (m)$

WE3a

Working

$y - c = mx + c - c$

$y - c = mx$

$\frac{y - c}{x} = \frac{mx}{x}$

$m = \frac{y - c}{x}$

Thinking

Step 1: Apply the inverse operation to both sides of the equation to isolate the variable needed as the subject.

Step 2: Write the equation in the standard form with the new variable as the subject.

Visual support

$$\begin{tikzcd}
y = mx + c \arrow[d, bend right, "-c"'] \arrow[d, bend left, "-c"] \\
y - c = mx \arrow[d, bend right, "\div x"'] \arrow[d, bend left, "\div x"] \\
\dfrac{y - c}{x} = m
\end{tikzcd}$$

Continues →

b. $z = dx^2 + e \quad (x > 0) \quad (x)$

WE3b

Working	**Thinking**
$z - e = dx^2 + e - e$ $z - e = dx^2$ $\frac{z-e}{d} = \frac{dx^2}{d}$ $\frac{z-e}{d} = x^2$	**Step 1:** Apply the inverse operation to both sides of the equation to isolate the variable needed as the subject.
$\sqrt{\frac{z-e}{d}} = \sqrt{x^2}$ $x = \sqrt{\frac{z-e}{d}}$	**Step 2:** Write the equation in the standard form with the new variable as the subject. Take the square root of both sides, as the unknown variable is a positive.

Student practice

Transpose the following equations to make the pronumeral shown in brackets the subject.

a. $y = mx + c \quad (x)$

b. $p = vr^2 + a \quad (r > 0) \quad (r)$

3E Questions

Understanding worksheet

1. Fill in the first inverse operation that needs to be applied on both sides of the equation to make x the subject.

Example

$v = x - p \ \boxed{+p}$

a. $z = y + x \ \boxed{}$ **b.** $\frac{x}{2} = \frac{y}{z} \ \boxed{}$ **c.** $x - y = \frac{1}{z} \ \boxed{}$ **d.** $y = 3(x - z) \ \boxed{}$

2. Match the equation with its correct transposed equation.

Equations			Transposed equations
$2a + b = 5$	●	●	$b = \frac{a-5}{2}$
$a + 2b = 5$	●	●	$b = \frac{a}{2} - \frac{5}{4}$
$a - 2b = 5$	●	●	$b = 5 - 2a$
$2a - 4b = 5$	●	●	$b = \frac{5-a}{2}$

3. Fill in the blanks by using the words provided.

equations **subject** **transposed** **pronumeral** **substituted**

Formulas or ____________ can represent various scenarios. Within these, there is typically a ____________ , or the main variable of interest. When solving for a different variable, the formula can be ____________ , meaning rearranging it to make a new variable the subject. To find an unknown in an equation, known values are ____________ and the equation is solved for the specified ____________ .

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Calculate the value of the subject by substituting the given values into the formula. Round to two decimal places where required. **WE1**

a. $P = 2a + b$, when $a = 2$ and $b = 3$

b. $Q = a^2 + b$, when $a = 1$ and $b = 4$

c. $R = a(b + c)$, when $a = 0.5$, $b = 2$ and $c = 3.5$

d. $S = a^2 + b^2$, when $a = 4.5$ and $b = 3$

e. $T = a(b + c^2)$, when $a = 6.75$, $b = 2.5$ and $c = 4$

f. $U = \sqrt{abc}$, when $a = 2.5$, $b = 3.7$ and $c = 4$

g. $V = \sqrt{a^2} + \left(b + c^2\right)$, when $a = 2.25$, $b = 3.2$ and $c = 4.75$

h. $W = \sqrt{a^2 + d} + b^2 - c$, when $a = 2.3$, $b = 0.3$, $c = 4.2$ and $d = 5.4$

Back to contents

5. Solve for the unknown variable. Round to three decimal places where required. WE2

a. The perimeter of a square is given by $P = 4s$. Substitute $P = 20$ to solve for the value of s.

b. The area if a rectangle is given by $A = lw$. Substitute $A = 24$ and $l = 6$ to solve for the value of w.

c. The circumference of a circle is given by $C = 2\pi r$. Substitute $C = 31.4$ to solve for the value of r.

d. The area of a triangle is given by $A = \frac{bh}{2}$. Substitute $A = 15$ and $b = 5$ to solve for the value of h.

e. The area of a trapezium is given by $A = \frac{(a + b)}{2}h$. Substitute $A = 30$, $a = 3$,$b = 6$ to solve for h.

f. The volume of a cylinder is given by $V = \pi r^2 h$. Substitute $V = 324$ and $r = 5$ to solve for the value of h.

g. The surface area of a sphere is given by $A = 4\pi r^2$. Substitute $A = 251$ to solve for the value of r.

h. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. Substitute $V = 122$ and $r = 7$ to solve for the value of h.

---

6. Transpose the following equations to make the pronumeral shown in brackets the subject. WE3

a. $a = qb + c \quad (c)$

b. $p = qr + s \quad (q)$

c. $y = \frac{x}{z} + w \quad (x)$

d. $v = \frac{wx}{y} + z \quad (x)$

e. $m = n^2 + p \quad (n > 0) \quad (n)$

f. $r = \frac{t^2}{s} + u \quad (t > 0) \quad (t)$

g. $a = \frac{bc^2}{d} + e \quad (c > 0) \quad (c)$

h. $3f = \sqrt{i} + g \quad (i)$

---

7. Transpose $r = 4(s - p)$ to make p the subject.

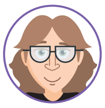
A. $\frac{r}{4} + s = p$ **B.** $\frac{r}{4} = s - p$ **C.** $\frac{r}{4} - s = -p$ **D.** $-\frac{r}{4} + s = p$ **E.** $-\frac{r}{4} + s = -p$

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Simplify the given equation by substituting a with a value of 0.75.

$$V = \frac{a}{b}(12 + a)^2$$



Student A

$$V = \frac{0.75}{b}(12 + 0.75)^2$$

$$V = \frac{0.75}{b}(12.75)^2$$

$$V = \frac{0.75}{b} \times 162.5625$$

Student B

$$V = \frac{0.75}{0.75}(12 + 0.75)^2$$

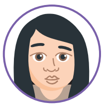
$$V = \frac{0.75}{0.75}(12.75)^2$$

$$V = \frac{0.75}{0.75} \times 162.5625$$

$$V = 162.5625$$

b. Transpose the following equation to make the pronumeral shown in brackets the subject.

$$bx = \frac{v}{12} + c \quad (v)$$



Student A

$$bx + c = \frac{v}{12} + c + c$$

$$\frac{bx + c}{12} = \frac{v}{12} \div 12$$

$$\frac{bx + c}{12} = v$$

Student B

$$bx - c = \frac{v}{12} + c - c$$

$$12(bx - c) = \frac{v}{12} \times 12$$

$$12(bx - c) = v$$

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. Jill is travelling to the United States and is needing to calculate what the weather in degrees Fahrenheit is equal to in degrees Celsius. Transpose the formula $F = \frac{9}{5}C + 32$ to help Jill calculate this conversion.

Back to contents

10. A large helium balloon is losing air over time. The volume of air (V), in litres, remaining in the balloon is given by the equation $V = 100 - 10.5t$, where t represents the time in hours since the balloon was filled. Calculate the volume, in litres, of air remaining in the balloon after an hour and a half.

11. Tori is competing in the Run Melbourne 10 km run. The formula $s = \frac{d}{t}$ represents the speed (s) at which she travels, where d is the distance and t is the time. Calculate the speed in kilometres per minute, to three decimal places, if Tori completed the 10 km run in 53 minutes.

12. Jade wants to rent a car for her weekend getaway. The cost to rent a car is given by the formula $C = 40d + 0.25k + 250$, where d represents the number of days rented and k represents the number of kilometres driven. Rearrange the formula to make k the subject.

13. A bakery produces specialty cakes. The cost of producing each cake is \$15, plus a fixed setup cost of \$500 for each new cake design. The bakery has a budget of \$3000 allocated for three different cake designs. If they want to produce an equal number of cakes for each design, write a formula for this scenario and solve to find what is the maximum number of cakes that can be produced for each design.

Reasoning

Question working paths

Mild 14 (a,b,c,e) 🌶	**Medium** 14 (a,b,c,e), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. A cyclist is participating in a straight line race. The formula $v = \frac{d}{t}$ represents the velocity (v) in metres per second of a cyclist who has travelled a distance of (d) metres in (t) seconds.

a. Calculate the velocity of a cyclist who has travelled 800 metres in 40 seconds.

b. Transpose the formula $v = \frac{d}{t}$ to make d the subject.

c. Determine the distance covered if a cyclist maintains a velocity of 12 metres per second for 25 seconds.

d. Using the rule $v = \frac{d}{t}$, calculate the expected time of a cyclist who has maintained a velocity of 14 metres per second who has travelled a distance of 0.7 km.

e. Provide an item a cyclist would consider taking with them for a long-distance race.

15. Rearrange the following equations to make the pronumeral shown in brackets the subject.

a. $g = ab + \frac{hv}{1}$ (a)

b. $g = ab + \frac{1}{hv}$ (a)

c. Compare the transposed equations generated in parts **a** and **b**.

Exam-style

16. The expected height of a plant, P, in centimetres, n months after it was planted in a garden is modelled by the equation (1 MARK)

$P = 30 + 0.10n$

The expected height of the plant, in centimetres, seven months after it was planted is

A. 0.7 cm **B.** 30.7 cm **C.** 30.8 cm **D.** 36.1 cm **E.** 37 cm

17. In the annual school fair, Samantha runs a baking contest. Nine participants pay an entry fee of \$75 each. Fourteen spectators pay \$10 each to taste the baked goods and vote for their favourites. (3 MARKS)

a. How much money in total is raised from participants and spectators at the baking contest? 1 MARK

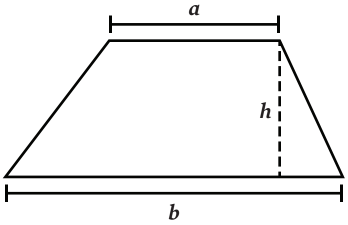
b. The following equation represents the total money raised

$T = 5c + 7a$

Rearrange the equation to make a the subject. 1 MARK

c. Using your answer from part **b**, calculate how many adults (a) attended the fair if there were 75 children (c) and a total of \$1635 was raised. 1 MARK

18. The area of a trapezium is calculated using $A = \frac{(a + b)}{2}h$, where h is the distance between the two parallel sides and a and b are the lengths of the two parallel sides. (3 MARKS)

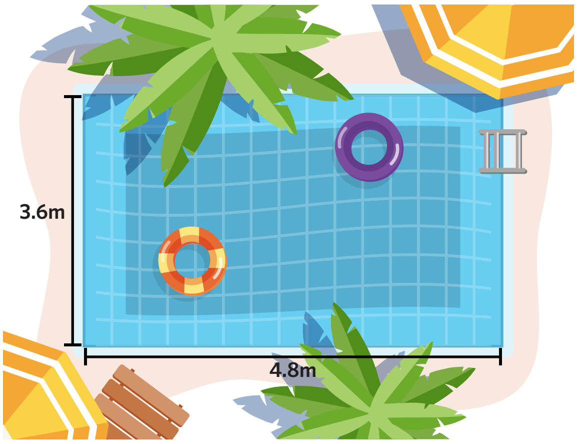


Transpose and calculate the value of b when $A = 16$, $a = 2$ and $h = 4$.

19. The total surface area of a cylinder is calculated using $T = 2\pi r^2 + 2\pi rh$, where $r =$ radius and $h =$ height. A cylinder has a radius of 5 cm and a total surface area of 300 cm^2. Show that the height of the cylinder is approximately 4.549 cm, rounded to three decimal places. (3 MARKS)

Remember this?

20. Daniel is planning to lay a new rectangular pool that measures 4.8 metres long and 3.6 metres wide. He's chosen square paving stones to outline the perimeter of the pool including each corner (shaded in grey). Each stone measures 60 cm × 60 cm. How many paving stones will Daniel need to purchase?



A. 30 **B.** 32 **C.** 34 **D.** 36 **E.** 38

21. Jack has started running and is tracking the total number of kilometres he has completed each week. He has created a table to keep a record.

Number of weeks (x)	1	3	5	7
Total number of kilometres (y)	10	20	30	40

Complete the rule for the linear relationship between total number of kilometres and the number of weeks for his running journey.

$y = \square x + 5$

A. −5 **B.** 0 **C.** 1 **D.** 5 **E.** 10

22. The gourmet chocolate mix is composed of dark chocolate, milk chocolate, and white chocolate in the ratio 5 : 3 : 2. What is the mass, in grams, of dark chocolate in a gourmet chocolate bar that weighs 10 grams?

A. 2.0 **B.** 3.0 **C.** 4.0 **D.** 5.0 **E.** 6.0

Back to contents

3F Simultaneous equations using substitution and elimination

LEARNING INTENTIONS

Students will be able to:

- know when the method of substitution or elimination can be used to solve simultaneous equations
- use substitution techniques to solve simultaneous equations
- use elimination techniques to solve simultaneous equations.

Simultaneous equations are a set of equations with multiple unknown variables. They can be solved using either substitution or elimination methods. The choice of method depends on the specific characteristics of the equations. Substitution involves solving one equation for one variable and then substituting this expression into the other equation. Elimination involves adding or subtracting the equations to eliminate one variable, making it possible to solve for the remaining variable.

KEY TERMS AND DEFINITIONS

- The **elimination method** is the process of eliminating one of the variables by adding the two equations.
- **Simultaneous equations** is a set of two or more equations, each containing two or more variables whose values can simultaneously satisfy both or all of the equations in the set.
- The **substitution method** is the process of adjusting one equation so that the value of one variable is defined in terms of the other.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

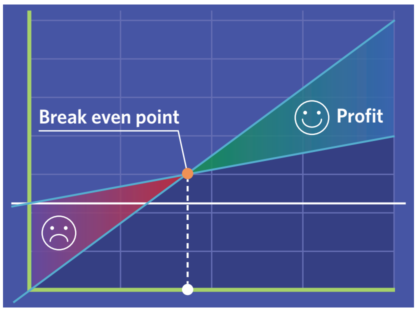
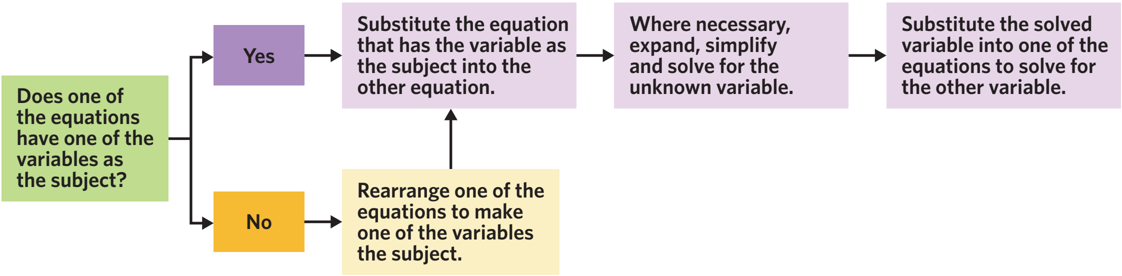


Image: Freepik.com

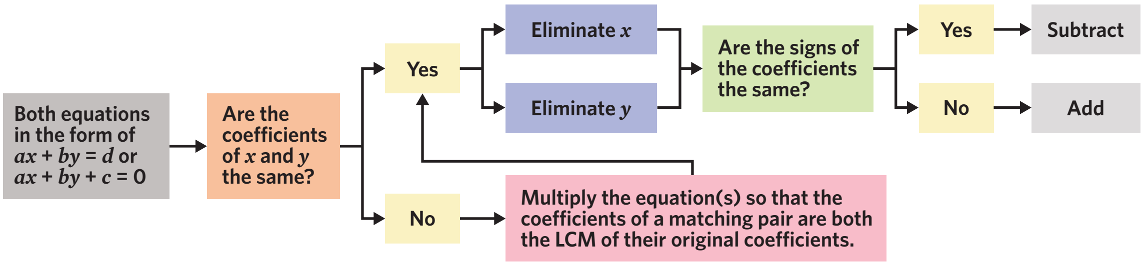
Simultaneous equations are often used in business to determine the optimal production levels that will maximise profit. Both substitution and elimination methods can be applied to these equations to find the most cost-effective production quantities for multiple products.

Key ideas

1. The **substitution** method involves solving one equation for one variable and then substituting that expression into the other equation.



2. The **elimination** method involves adding or subtracting the equations to eliminate one variable, making it possible to solve for the other variable.



Back to contents

Worked example 1

Solving simultaneous equations using substitution

Solve the following pairs of simultaneous equations using substitution.

a. $2x + y = 12$ and $y = 4x$

WE1a

Working

$2x + y = 12 \quad [1]$

$y = 4x \quad [2]$

$2x + 4x = 12$

$6x = 12$

$x = 2$

$y = 4 \times 2$

$y = 8$

$\therefore x = 2, \ y = 8$

Thinking

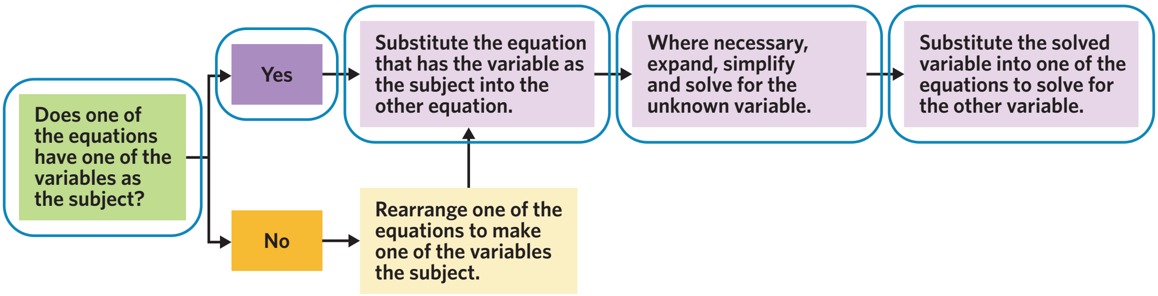
Step 1: Recognise the structure of the equations and identify which equation can be substituted into the other.

Step 2: Substitute the second equation into the first equation.

Step 3: Simplify the equation by collecting like terms and solve.

Step 4: Substitute the value of the known variable into one of the original equations to solve for the value of the unknown variable.

Visual support



b. $y = 3x + 4$ and $4x - 2y = 20$

WE1b

Working

$y = 3x + 4 \quad [1]$

$4x - 2y = 20 \quad [2]$

$4x - 2(3x + 4) = 20$

$4x - 6x - 8 = 20$

$-2x - 8 = 20$

$x = - 14$

$y = 3(-14) + 4$

$y = -38$

$\therefore x = -14, \ y = -38$

Thinking

Step 1: Recognise the structure of the equations and identify which equation can be substituted into the other.

Step 2: Substitute the first equation into the second equation.

Step 3: Expand the brackets.

Step 4: Simplify the equation by collecting like terms and solve.

Step 5: Substitute the value of the known variable into one of the original equations to solve for the value of the unknown variable.

Student practice

Solve the following pairs of simultaneous equations using substitution.

a. $4x + y = 14$ and $y = 3x$

b. $y = 4x + 5$ and $5x - 5y = 20$

Back to contents

Worked example 2

Solving simultaneous equations using elimination

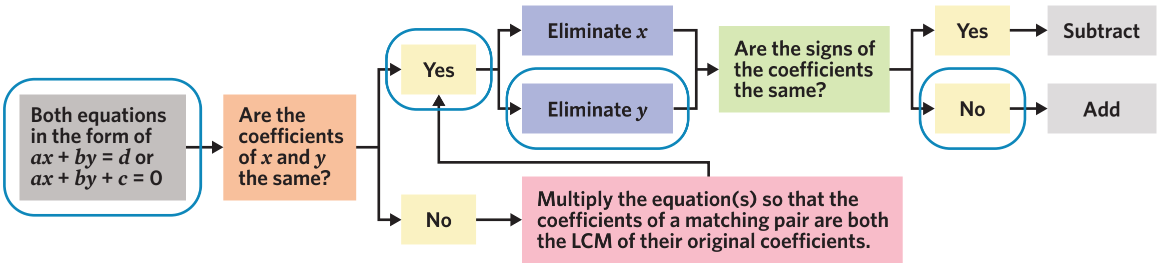
Solve the following pairs of simultaneous equations using elimination.

a. $x + y = 9$ and $x - y = 7$

WE2a

Working	Thinking
$x + y = 9 \quad [1]$ $x - y = 7 \quad [2]$	**Step 1:** Recognise the structure of the equations and use the elimination method by adding.
$[1] + [2]$ $\begin{array}{rr} & x + y = 9 \\ + & x - y = 7 \\ \hline & 2x = 16 \end{array}$	**Step 2:** Add the two equations together to eliminate one variable.
$2x = 16$ $x = 8$	**Step 3:** Solve.
$8 - y = 7$ $y = 1$ $\therefore x = 8,\ y = 1$	**Step 4:** Substitute the value of the known variable into one of the original equations to solve for the value of the unknown variable.

Visual support



b. $5x + y = 9$ and $3x + y = -1$

WE2b

Working	Thinking
$5x + y = 9 \quad [1]$ $3x + y = -1 \quad [2]$	**Step 1:** Recognise the structure of the equations and use the elimination method by subtracting.
$[1] - [2]$ $\begin{array}{rr} & 5x + y = 9 \\ - & 3x + y = -1 \\ \hline & 2x = 10 \end{array}$	**Step 2:** Subtract the second equation from the first equation to eliminate y.
$2x = 10$ $x = 5$	**Step 3:** Solve.
$5(5) + y = 9$ $25 + y = 9$ $y = -16$ $\therefore x = 5,\ y = -16$	**Step 4:** Substitute the value of the known variable into one of the original equations to solve for the value of the unknown variable.

Continues →

Back to contents

Student practice

Solve the following pairs of simultaneous equations using elimination.

a. $x + y = 12$ and $x - y = 4$

b. $7x + y = 11$ and $2x + y = -4$

Worked example 3

Solving simultaneous equations using a matching pair

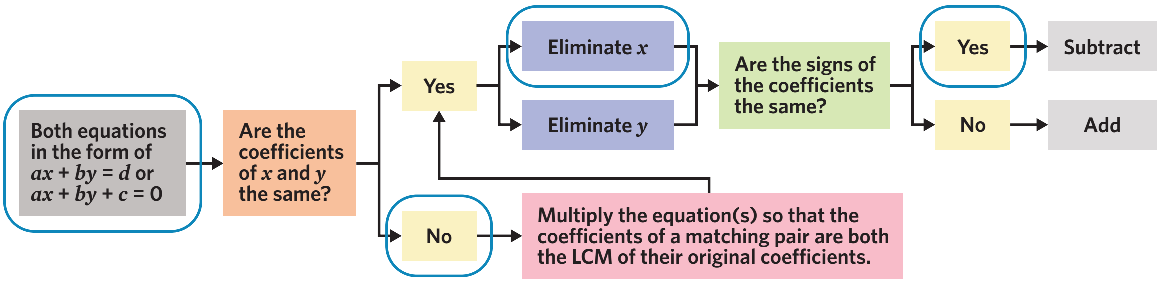
Solve the following pairs of simultaneous equations using elimination.

a. $7y + 3x = 23$ and $y + x = 5$

WE3a

Working	Thinking
$7y + 3x = 23$ [1] $y + x = 5$ [2]	**Step 1:** Recognize the structure of the equations and identify a matching pair.
Multiply [2] by 3 $3y + 3x = 15$ [3]	**Step 2:** Manipulate the equations to create a matching pair.
[1] − [3] $\begin{array}{r} 7y + 3x = 23 \\ -\ \ 3y + 3x = 15 \\ \hline 4y = 8 \end{array}$	**Step 3:** Apply the elimination method by subtracting the third equation from the first equation.
$4y = 8$ $y = 2$	**Step 4:** Solve.
$2 + x = 5$ $x = 3$ $\therefore x = 3,\ y = 2$	**Step 5:** Substitute the value of the known variable into one of the original equations to solve for the value of the unknown variable.

Visual support



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Back to contents

b. $7x - 6y = 61$ and $4x + 9y = 10$

WE3b

Working	Thinking
$7x - 6y = 61 \quad [1]$ $4x + 9y = 10 \quad [2]$	**Step 1:** Recognize the structure of the equations and identify a matching pair.
Multiply [1] by 9 $63x - 54y = 549 \quad [3]$ Multiply [2] by 6 $24x + 54y = 60 \quad [4]$	**Step 2:** Multiply the equation so that the coefficients of a matching pair are both the LCM of their original coefficients
[3] + [4] $\begin{array}{r} 63x - 54y = 549 \\ +\ 24x + 54y = 60 \\ \hline 87x = 609 \end{array}$	**Step 3:** Apply the elimination method by adding the two equations together.
$87x = 609$ $x = 7$	**Step 4:** Solve.
$4\,(7) + 9y = 10$ $28 + 9y = 10$ $9y = -18$ $y = -2$ $\therefore x = 7,\ y = -2$	**Step 5:** Substitute the value of the known variable into one of the original equations to solve for the value of the unknown variable.

Student practice

Solve the following pairs of simultaneous equations using elimination.

a. $6x + 2y = 32$ and $5x + y = 10$

b. $2x - 5y = -2$ and $-5x + 6y = -8$

Back to contents

3F Questions

Understanding worksheet

1. State the value of x or y by substituting the given value.

Example

$y = 3x + 2 \qquad (x = 4) \qquad \therefore y = \boxed{14}$

a. $y = 5x \qquad (x = 6) \qquad \therefore y = \boxed{\quad}$

b. $y = 2x - 4 \qquad (x = 3) \qquad \therefore y = \boxed{\quad}$

c. $x = 2y + 3 \qquad (y = 4) \qquad \therefore x = \boxed{\quad}$

d. $3y + 4x = 26 \qquad (y = 2) \qquad \therefore x = \boxed{\quad}$

2. Place a '+' or '−' to make the equations true.

Example

$4a \boxed{-} 4a = 0$

a. $3p \boxed{\quad} 3p = 0$

b. $-4x \boxed{\quad} 4x = 0$

c. $7k \boxed{\quad} (-7k) = 0$

d. $-5r \boxed{\quad} (-5r) = 0$

3. Fill in the blanks by using the words provided.

elimination **simultaneous** **substitution** **variables** **solve**

In mathematics, $\boxed{\qquad\qquad}$ equations are a set of equations with multiple $\boxed{\qquad\qquad}$, which are to be solved together. To $\boxed{\qquad\qquad}$ these equations, we can use one of two main methods. The first method, $\boxed{\qquad\qquad}$, involves replacing one variable in one equation with the corresponding expression from the other equation. The second method, $\boxed{\qquad\qquad}$, involves adding or subtracting the equations to eliminate one variable, making it possible to solve for the remaining variable.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Solve the following pairs of simultaneous equations using substitution. Provide the answer in exact form. WE1a

a. $x + y = 12$ and $x = 3y$

b. $y = 4x$ and $x + y = 10$

c. $y + 5x = 8$ and $x = 3y$

d. $5x + 2y = 13$ and $y = 4x$

e. $y = 6x$ and $2x - y + 2 = -6$

f. $2x + y - 2 = 7$ and $x = 4y$

g. $y + 2x = 15$ and $x = -3y$

h. $y = 2x$ and $3x + 2y = 20$

5. Solve the following pairs of simultaneous equations using substitution. Provide the answer in exact form. WE1b

a. $y = 4x - 3$ and $y + 4x = 13$

b. $y = 6x + 5$ and $y - 5x = -4$

c. $4x + 2y = 16$ and $y = 2x - 4$

d. $4x - 3y = 10$ and $x = 3y - 2$

e. $2x + 2y = 12$ and $4x - 3y = 10$

f. $2x - 4y = 12$ and $2x - 3y = 8$

g. $7x + 3y - 2 = 0$ and $x - y = 6$

h. $4x + y + 1 = 0$ and $3x - 2y + 20 = 0$

6. Solve the following pairs of simultaneous equations using elimination. Provide the answer in exact form. **WE2**

a. $x + y = 5$ and $x - y = 1$

b. $x - y = 3$ and $x + y = 5$

c. $2x + y = 9$ and $x - y = 3$

d. $3x - 2y = 11$ and $x + 2y = 1$

e. $2x + 2y = 12$ and $-2x - y = 4$

f. $2x + 3y = 10$ and $3x - 3y = -5$

g. $4x + 5y = 20$ and $-5y + 3x = 8$

h. $3x + 4y = 14$ and $-4y - 2x = 7$

---

7. Solve the following pairs of simultaneous equations using elimination. **WE3**

a. $3x + y = 2$ and $5x - 4y = 9$

b. $y + 6x = 25$ and $3y + 2x = 43$

c. $2x + 2y = -1$ and $x + 4y = -5$

d. $8x - 3y = 40$ and $5x - 2y = 26$

e. $4x - 4y = 2$ and $3x + 2y = 10$

f. $4y + 4x = 20$ and $3y - 5x = 3$

g. $7x + 4y = -3$ and $-4x + 3y = 7$

h. $5x - 2y - 1 = 0$ and $3x + 5y - 44 = 0$

---

8. Solve the following pairs of simultaneous equations using substitution.

$x + 4y = 21$ and $x = 12 - y$

A. $x = 12 - y$

B. $x = 9,\ y = 3$

C. $x = 3,\ y = 9$

D. $x = 12,\ y = 9$

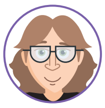
E. $x = 12 - y,\ y = 3$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Solve the following pair of simultaneous equations using substitution.

$y = x + 1$ and $x + 3y = 15$



Student A

$y = x + 1$ [1]

$x + 3y = 15$ [2]

$x + 3(x + 1) = 15$

$x + 3x + 3 = 15$

$4x + 3 = 15$

$4x = 12$

$x = 3$

$y = 3 + 1$

$y = 4$

$\therefore x = 3,\ y = 4$

Student B

$y = x + 1$ [1]

$x + 3y = 15$ [2]

$x + 3 + x + 1 = 15$

$2x + 4 = 15$

$2x = 11$

$x = \frac{11}{2}$

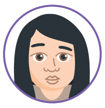
$y = \frac{11}{2} + 1$

$y = \frac{13}{2}$

$\therefore x = \frac{11}{2},\ y = \frac{13}{2}$

b. Solve the following pairs of simultaneous equations using elimination. Provide the answer in exact form.

$x + 2y = 8$ and $3x - 2y = 4$



Student A

$x + 2y = 8$ [1]

$3x - 2y = 4$ [2]

[1] + [2]

$$\begin{array}{rr} & x + 2y = 8 \\ + & 3x - 2y = 4 \\ \hline & 4x = 12 \end{array}$$

$4x = 12$

$x = 3$

$3 + 2y = 8$

$y = \frac{5}{2}$

$\therefore x = 3,\ y = \frac{5}{2}$

Student B

$x + 2y = 8$ [1]

$3x - 2y = 4$ [2]

[2] − [1]

$$\begin{array}{rr} & 3x - 2y = 4 \\ - & x + 2y = 8 \\ \hline & 2x = -4 \end{array}$$

$2x = -4$

$x = -2$

$-2 + 2y = 8$

$y = -5$

$\therefore x = -2,\ y = -5$

Back to contents

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. $g + b = 540$ and $g = b + 80$ represents the number of boys (b) and girls (g) at Kingston College. There are a total of 540 students. If there are 80 more girls than boys, how many boys and girls are there?

11. Heath got 30 more marks in his Science test than in Maths. The total of his marks was 120. Calculate his marks in each test.

12. Mia has purchased a new rectangular dining table and the difference between the length and the width of the table is 1.8 m and the perimeter of the table is 8.4 m. Calculate the length (l) and width (w) of the table in cm.

13. Two complementary angles differ by 18°. Write two equations and calculate the two angles.

14. Jonathan purchased three bags of coffee and five bars of dark chocolate for a total of $30. At the same market, Katherine bought two bags of the same coffee and five bars of the same dark chocolate for $25. Using the elimination method, calculate how much each bag of coffee and each bar of dark chocolate cost.

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Lennox browses the used record stand at the Cherry Lake market and observes that all pop albums are $7 each and jazz albums are $12 each. Lenox purchases a total of 50 albums. Lennox spends a total of $400.

- **a.** Generate two equations to display Lennox's scenario, where x represents the number of pop albums and y represents the number of jazz albums.
- **b.** Using the elimination method, solve for the number of pop and jazz albums that Lennox purchased.
- **c.** Transpose a rule in terms of x.
- **d.** Using the substitution method, solve for the number of pop and jazz albums that Lennox purchased.
- **e.** Outline a reason why someone would rather purchase a physical album instead of streaming music online.

16. Consider the two simultaneous equations.

$$5x + 2y = 4 \quad [1]$$

$$-3x + 2y = 4 \quad [2]$$

- **a.** Solve the equations using the elimination method and subtracting [2] from [1].
- **b.** Solve the equations using the elimination method and subtracting [1] from [2].
- **c.** Comment on the differences and similarities between the steps in part **a** compared to part **b**.

Exam-style

17. At a local amusement park, ticket prices vary for adults and children: (1 MARK)

- The Anderson family bought three adult tickets and two child tickets for $85.00.
- The Lee family bought two adult tickets and four child tickets for $94.00.
- The Gonzalez family bought four adult tickets and three child tickets.

What is the total amount spent by the Gonzalez family?

A. A. $98.00 **B.** $108.00 **C.** $118.00 **D.** $122.00 **E.** $127.00

18. Samuel organises charity football matches at the community centre. (3 MARKS)

Entry fees for the match are \$4 per adult and \$1 per child. \$1278 was raised from the 453 people who attended the most recent match.

a. Generate two equations to represent this situation, where a represents the number of adults and c represents the number of children. 2 MARKS

b. Solve for the number of children who attended this football match. 1 MARK

19. Solve the pair of simultaneous equations using substitution. (3 MARKS)

$2a + 5k = 10$ and $a = 20k$

20. Natalie hosts charity book fairs at her company. The following system of simultaneous linear equations represents two university clubs who have rented projectors and laptops, where x represents the hourly cost, in dollars, of renting one projector and y represents the hourly cost, in dollars, of renting one laptop. (3 MARKS)

Solve the set of simultaneous equations using the process of elimination.

$15x + 25y = 2000$

$35x + 50y = 4200$

Remember this?

21. At Lilli's school, $\frac{6}{8}$ of the students own a pet.

There are 548 students at Lilli's school.

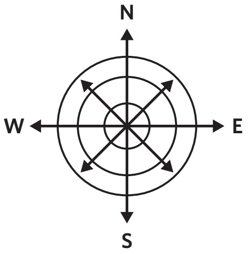
How many students own a pet?

A. 68 **B.** 137 **C.** 274 **D.** 411 **E.** 548

22. Sarah is watching the sunset, the sun sets in the west. She turns 120° clockwise.

Sarah then turns anticlockwise until she faces North.

How many degrees did Sarah turn anticlockwise?



A. 15° **B.** 30° **C.** 80° **D.** 120° **E.** 240°

23. The fraction of butterfly species that are considered to be under the threat of extinction is $\frac{10\,300}{17\,500}$.

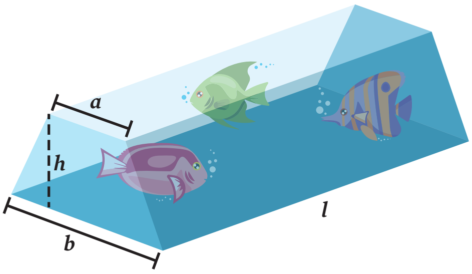
What percentage of butterfly species are not considered to be under the threat of extinction? Rounded to the nearest whole number.

A. 17% **B.** 27% **C.** 41% **D.** 50% **E.** 68%

Back to contents

Chapter 3 extended application

1. A new custom-made aquarium in the shape of the trapezoidal prism shown is being constructed. Currently, the dimensions are set such that $a = 15$ m, $h = 10$ m and $l = 30$ m. The volume of a trapezoidal prism is given by $V = \frac{a + b}{2}hl$.



a. Using CAS or by hand, for what values of b would the aquarium's volume be equal to or less than 13 500 m^3?

b. Using CAS or by hand, rearrange the volume formula to make b the subject.

c. With reference to part **b**, calculate the length of b, in metres, that is required for the aquarium to have a volume of 15 000m^3, correct to 2 decimal places.

d. Rearrange the volume formula to make h the subject.

e. If b is set as 16 m, calculate the height (h) if the aquarium's volume is 9300 m^3?

f. What factors may an aquarium need to consider before installing a new fish tank?

2. Bart (b) is currently eight years older than his sister Lisa (l). In six years time Bart will be twice as old as Lisa.

a. Generate a system of linear equations that represents this situation.

b. Using elimination, calculate the current ages of both Bart and Lisa.

c. $b - l = 8$ and $b = 2l + 6$ are a pair of simultaneous equations that have been transposed. Using CAS or by hand, transpose the equation(s) to make l the subject.

d. Using the equations $b - l = 8$ and $b = 2l - 6$ show and calculate Bart and Lisa's age, using the substitution method.

e. Provide an engaging activity that an older sibling or babysitter could do with a young child.

3. Joe runs a small business where he sells handmade bracelets. Each bracelet costs him m dollars to manufacture and he manufactures b bracelets per month. He also has fixed costs, such as rent and utilities, which is represented as f dollars per month. He sells each bracelet for s dollars.

Joe's total cost can be represented using the following rule $C = bm + f$.

a. A 50% mark down on the manufacture and fixed costs can be represented as $C = 0.5(bm + f)$. Generate an expanded equation in the same form that represents a 75% mark up on the manufacture and fixed costs, then expand the brackets.

b. Joe's brother, Marlon, runs the same business, but the monthly cost to manufacture the bracelets and the fixed costs are double to Joe's monthly and fixed costs. Write a factorised equation to represent the monthly cost of Marlon's business if he also manufactures b bracelets per month.

c. Generate an algebraic equation to represent the monthly profit, P, of Joe's business if he also sells b bracelets in a month.

d. Given the cost of manufacturing each bracelet is \$10, the fixed costs are \$2000 per month, and Joe sells each bracelet for \$25. If Joe wants to make a profit of \$3000 in a month, how many bracelets must he manufacture and sell? Generate and solve the equation, using CAS.

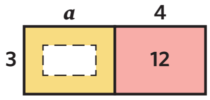
e. In the context of Joe and Marlon's business, outline how the changes in fixed costs and selling price can affect their decision to expand their business.

Back to contents

Chapter 3 review

Multiple choice

1. Identify the missing value. **3A**



A. 3 **B.** 4 **C.** $3a$ **D.** $4a$ **E.** $12a$

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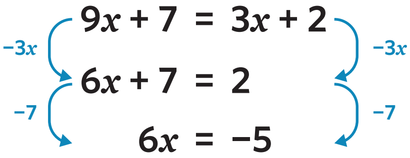
2. Identify the next inverse operation in the diagram. **3B**



A. + 5 **B.** ÷ 5 **C.** × 5 **D.** ÷ 6 **E.** × 6

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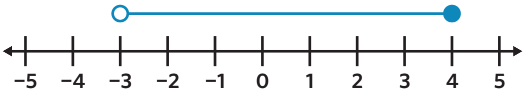
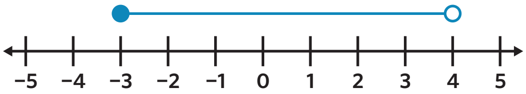
3. Select the correct representation of the inequality $-3 < x \leq 4$ on the number line. **3D**

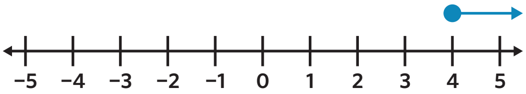
A.



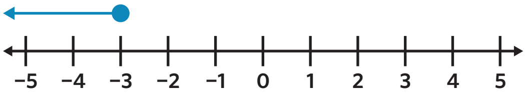
B.



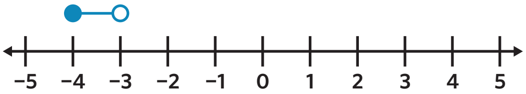
C.



D.



E.



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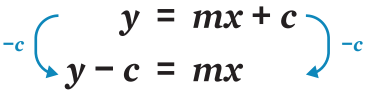
4. What is the next operation required to make x the subject in the equation $y = mx + c$? **3E**



A. $\div x$ **B.** $+ c$ **C.** $\div m$ **D.** $\div mx$ **E.** $- y$

---

5. Substitute the given value into the following equation to solve for the unknown variable. **3F**

$7y + 3x = 33 \quad (y = 3) \quad x = \boxed{}$

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

Fluency

6. Expand the brackets in each expression and simplify. **3A**

a. $3.5(8 - 2w)$ **b.** $7v(- 6 + 3v)$

c. $2(4q + 3) + 10q$ **d.** $10(3x - 4) - 7(x + 7)$

---

7. Solve each of the following equations for the unknown values. Provide the answer in exact form as needed. **3B**

a. $2x - 3 = 7$ **b.** $-4y + 5 = 13$ **c.** $5(2z - 3) = 17$ **d.** $3(4x + 2) - 5 = 11$

Back to contents

8. Solve each of the following equations for the unknown values. Provide the answer in exact form where required. 3B

a. $\frac{-2x}{5} + 3 = 1$ **b.** $\frac{3z}{2} - 1 = 5$ **c.** $\frac{4a + 2}{3} = 6$ **d.** $\frac{2(y - 2)}{3} = 4$

9. Solve each equation for the given variable. Provide your answer in exact form where required. 3C

a. $-5w + 10 = 3w$ **b.** $10 - 6w = 4w + 2$

c. $5(w + 3) = 4w + 2$ **d.** $7(3w + 1) = -2(4w + 3)$

10. Represent the inequalities on a number line. 3D

a. $x > 3$ **b.** $x \leq 7$ **c.** $-2 \leq x < 5$ **d.** $-4 < x \leq 6$

11. Solve for x. 3D

a. $x + 3 > 7$ **b.** $6 - 2x \leq 10$ **c.** $4x - 2 \leq 3x + 1$ **d.** $\frac{-x}{3} + 2 \geq -9$

12. Solve for the unknown variable. Round to three decimal places where required. 3E

a. The perimeter of an equilateral triangle is given by $P = 3s$. Substitute $P = 21$ to solve for the value of s.

b. The perimeter of a rectangle is given by $P = 2l + 2w$. Substitute $l = 5$ and $w = 3$ to solve for the value of P.

c. The area of a circle is given by $A = \pi r^2$. Substitute $r = 3$ to solve for the value of A.

d. The volume of a cylinder is given by $V = \pi r^2 h$. Substitute $V = 100$ and $r = 3$ to solve for the value of h.

13. Transpose the following equations to make the pronumeral shown in brackets the subject. 3E

a. $ax + b = c \quad (a)$ **b.** $y = \frac{p}{x} + z \quad (p)$

c. $ut + 0.5at^2 = x \quad (u)$ **d.** $2m = \sqrt{n} + x \quad (n)$

14. Solve the following pairs of simultaneous equations. Provide the answer in exact form. 3F

a. $2x - y = 9$ and $x = 5y$ **b.** $3x + y = 11$ and $y = 2x + 1$

c. $x + y = 10$ and $x - y = 2$ **d.** $2x + y = 7$ and $3x + 2y = 10$

Problem solving

15. Alex is a party planner and is currently planning 6 different birthday parties for his clients. Each of the clients want to hire a clown for the event. Alex was told that the clown charges a flat fee of \$30 to come to the party and a further \$20 per hour. What is an expanded and simplified expression that can be used to represent the total cost of hiring a clown for all six parties? 3A

16. Sarah is a basketball player who is trying to improve her shot success rate. Sarah knows that the number of shots she can expect to make can be represented by the equation $3(-18 + m) = 9$, where m represents the number of successfully made shots. How many shots should Sarah expect to successfully make that day? 3B

17. John is an event coordinator planning a music concert. He knows that the total money collected from ticket sales is represented by the equation $7t + 150 = 5t + 250$, where t represents the price of each ticket in dollars. Solve the equation to find the price of one ticket. 3C

18. Alice is a fitness enthusiast who tracks her daily steps. She aims to walk at least 13 204 steps a day. One day, she walks x steps in the morning and 7259 steps in the afternoon. If the inequality $x + 7259 \geq 13\,204$ represents her step count for the day, solve for x to find out the minimum number of steps Alice needs to walk in the morning to meet her daily goal. 3D

19. A company uses the formula $C = \frac{1}{5}(P - 3R)$ to calculate the expected per unit profit (C) from selling a certain product. P is the price they sell each unit for and R is the cost of raw materials per unit. If the company sells each unit for \$20 and the cost of raw materials per unit is \$2, what is the expected per unit profit of the product?

3E

---

20. A company is planning to produce two types of products: A and B. The profit they make from each unit of product A is \$7, and from each unit of product B is \$5. The company has a target to make a total profit of \$3500. They also know that the number of units of product A they will sell is 200 more than the number of units of product B. How many units of each product does the company need to sell to meet their profit target?

3F

Reasoning

21. Sam is a young entrepreneur who runs a small lemonade stand. He uses a formula to calculate his daily profit: profit $=$ price per glass $\times$ number of glasses sold $-$ expenses. In order to make his business sustainable, he also has to consider other factors like the weather, competition, and customer demand.

- **a.** Sam sells each glass of lemonade for \$2.50 and he manages to sell 200 glasses. His expenses for that day, including lemons, sugar, and ice, were \$70. Use the formula to calculate Sam's profit for that day.
- **b.** Sam noticed that if he increases the price per glass by \$0.50, his customer demand decreases and he only manages to sell $200 - 20x$ glasses, where x is the number of times he increases the price by \$0.50. His expenses remain constant at \$70. If Sam increases his price on Tuesday to \$3.00 per glass, how many glasses does he sell and what is his profit?
- **c.** On Wednesday, it rained and Sam couldn't sell as many glasses. His sales dropped to $150 - 30y$ glasses, where y is the number of hours it rained. If it rained for 2 hours, Sam sold each glass for \$2.50 and his expenses were still \$70, how much less profit did he make on this day compared to the day before?
- **d.** Sam is looking to expand his business by also selling orange juice. The profit from the orange juice sales is determined by the equation $p = 3.3q - 12$, where q represents the number of orange juice sales and p represents the profit in dollars. Determine the minimum number of orange juices Sam would have to sell in order to make a profit.
- **e.** What other factors should Sam consider before increasing his prices?

22. Consider the following:

- **a.** Solve for x in the equation $4x + 1 \leq 5$.
- **b.** Solve for x in the equation $-4x + 1 \leq 5$.
- **c.** Comparing your answers from parts **a** and **b**, discuss the differences and similarities in the methods you used to solve each equation. How did the presence or absence of the negative sign affect the direction of the inequality?

Chapter 4

Algebraic techniques

Number and algebra

Calculator skills

See online in additional materials for using CAS calculator guides.

4E Factorising by grouping

Back to contents

Chapter 4 research summary

Algebraic techniques

Big ideas

The big ideas that underpin the concept of algebraic techniques in Year 9 Mathematics include the understanding of variables and expressions, equations and inequalities, functions and relationships, patterns and generalisations, symbolic representation, and problem-solving. These big ideas provide a conceptual framework that helps students make sense of the various techniques and concepts they encounter in algebra and are essential for developing a deep understanding of algebra.

Variables and expressions

One of the foundational big ideas in algebra is the concept of variables and expressions. Variables are symbols that represent unknown values, and expressions are combinations of variables, numbers, and operations. Understanding the role of variables and how to manipulate expressions is crucial for solving algebraic problems. This includes simplifying expressions, expanding brackets, and factorising expressions.

Equations and inequalities

Another key big idea is the understanding of equations and inequalities. An equation is a mathematical statement that two expressions are equal, while an inequality is a statement that one expression is greater than or less than another. Solving equations and inequalities involves finding the values of the variables that make the statement true. This is a fundamental skill in algebra and is essential for solving a wide range of problems.

Functions and relationships

Functions describe the relationship between two variables. Understanding the concept of functions and how to represent them graphically, algebraically, and in tables is a crucial big idea in algebra. This includes understanding the concept of linear and non-linear functions, and how to work with them.

Patterns and generalisations

Algebra involves recognising patterns and making generalisations. This is a key big idea that underpins many algebraic techniques. For example, recognising the pattern in a sequence of numbers can lead to a formula that describes the *n*th term of the sequence. Similarly, recognising the pattern in the way that two variables are related can lead to a formula that describes the relationship.

Symbolic representation

Algebra involves working with symbols to represent mathematical ideas. This big idea is fundamental to all of algebra and underpins many algebraic techniques. For example, using symbols to represent unknown values, using letters to represent variables, and using equations to represent relationships.

Problem solving

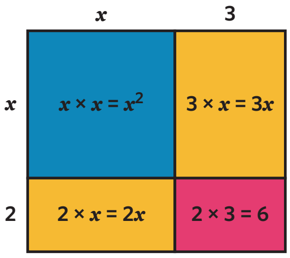
Algebra is a powerful tool for solving problems. This big idea involves using algebraic techniques to model and solve real-world problems. This includes translating word problems into algebraic expressions or equations, solving the equations, and interpreting the results.

Back to contents

Visual representations

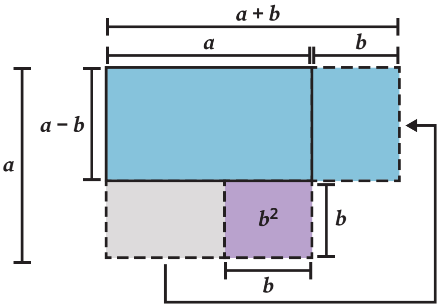
Area models

Area models are useful for visually representing the multiplication of binomials and the factorisation of quadratic expressions. They help students to understand the concept of distributive property and the process of multiplying and factorising expressions.



$$(x + 3)(x + 2) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

$$(a + b)(a - b) = a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$



This portion can be rearranged to show that the area $a^2 - b^2$ is equivalent to the area given by $(a + b)(a - b)$.

Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students disregard the signs of the common terms when expanding and simplifying binomial products.	$(x + 4)(x - 7)$ $= x^2 + 3x - 28$	$(x + 4)(x - 7)$ $= x^2 - 3x - 28$	4A
Students disregard the signs of the constants when multiplying to expand binomial products.	$(y - 2)(y - 7) = y^2 - 7y - 2y - 14$ $= y^2 - 9y - 14$	$(y - 2)(y - 7) = y^2 - 7y - 2y + 14$ $= y^2 - 9y + 14$	4A
Students sum the common terms of expanded perfect squares to zero.	$(y - 8)^2 = (y - 8)(y - 8)$ $= y^2 - 8y - 8y + 64$ $= y^2 + 64$	$(y - 8)^2 = (y - 8)(y - 8)$ $= y^2 - 8y - 8y + 64$ $= y^2 - 16y + 64$	4B
Students make the constant negative when expanding a perfect square involving subtraction.	$(x - 7)^2 = x^2 + 14x - 49$	$(x - 7)^2 = x^2 - 14x + 49$	4B
Students expand to form the sum of two squares instead of difference.	$(x - 10)(x + 10)$ $= x^2 + 10x - 10x + 100$ $= x^2 + 100$	$(x - 10)(x + 10)$ $= x^2 + 10x - 10x - 100$ $= x^2 - 100$	4B
Students find a common factor, but not the highest common factor.	$-9x + 27x^2 = -9(x - 3x)$	$-9x + 27x^2 = -9x(1 - 3x)$	4C
Students do not divide the HCF out of every term in an expression.	$-9x + 27x^2 = -9x(1 - 27x)$	$-9x + 27x^2 = -9x(1 - 3x)$	4C
Students disregard the signs of the terms when factorising.	$-9x + 27x^2 = -9x(1 + 3x)$	$-9x + 27x^2 = -9x(1 - 3x)$	4C
Students factorise a difference of two squares to a perfect square.	$16m^2 - 9 = (4m - 3)(4m - 3)$	$16m^2 - 9 = (4m + 3)(4m - 3)$	4D

Continues →

Misconception	Incorrect ✗	Correct ✓	Lesson
Students halve the coefficients of square terms when taking the square root.	$16m^2 - 9 = (8m + 3)(8m - 3)$	$16m^2 - 9 = (4m + 3)(4m - 3)$	4D
Students only take the square root of pronumerals in square terms.	$16m^2 - 9 = (16m + 3)(16m - 3)$	$16m^2 - 9 = (4m + 3)(4m - 3)$	4D
Students factorise so that the binomial factors are different.	$2x^2 + 20 - 8x - 5x$ $= 2x^2 - 8x - 5x + 20$ $= 2x(x - 4) - 5(x + 4)$ $= (2x - 5)(x - 4)(x + 4)$	$2x^2 + 20 - 8x - 5x$ $= 2x^2 - 8x - 5x + 20$ $= 2x(x - 4) - 5(x - 4)$ $= (2x - 5)(x + 4)$	4E
Students group like terms in a pair when rearranging to factorise.	$2x^2 + 20 - 8x - 5x$ $= 2(x^2 + 10) - 13x$	$2x^2 + 20 - 8x - 5x$ $= (2x - 5)(x + 4)$	4E
Students square the binomial factor in fully factorised expression.	$2x^2 + 20 - 8x - 5x$ $= (2x - 5)(x - 4)^2$	$2x^2 + 20 - 8x - 5x$ $= (2x - 5)(x + 4)$	4E
Students simplify the terms in an algebraic fraction without factorising.	$\frac{4x - 20}{2x - 10} = (4 \div 2)x - (20 \div 10)$ $= 2x - 2$ $= 2(x - 1)$	$\frac{4x - 20}{2x - 10} = \frac{4(x - 5)}{2(x - 5)}$ $= \frac{4\cancel{(x - 5)}}{2\cancel{(x - 5)}}$ $= \frac{4}{2} = 2$	4F
When dividing, students do not reciprocate the divisor before multiplication, i.e. multiplication is performed instead.	$\frac{9(x + 1)}{6(x - 1)} \div \frac{3(x + 1)(x + 3)}{4(x - 1)}$ $= \frac{9(x + 1)^2(x + 3)}{8(x - 1)^2}$	$\frac{9(x + 1)}{6(x - 1)} \div \frac{3(x + 1)(x + 3)}{4(x - 1)}$ $= \frac{2}{x + 3}$	4F
When dividing, students reciprocate the dividend before multiplication.	$\frac{5x^2}{3(x - 1)} \div \frac{15x}{(x - 1)(x + 5)}$ $= \frac{3(x - 1)}{5x^2} \times \frac{15x}{(x - 1)(x + 5)}$ $= \frac{3\cancel{(x - 1)}}{5x^2} \times \frac{15x}{\cancel{(x - 1)}(x + 5)}$ $= \frac{3}{5x^2} \times \frac{15x}{x + 5}$ $= \frac{3}{5x^2 \div 5x} \times \frac{15x \div 5x}{x + 5}$ $= \frac{3}{x} \times \frac{3}{x + 5}$ $= \frac{9}{x(x + 5)}$	$\frac{5x^2}{3(x - 1)} \div \frac{15x}{(x - 1)(x + 5)}$ $= \frac{5x^2}{3(x - 1)} \times \frac{(x - 1)(x + 5)}{15x}$ $= \frac{5x^2 \div 5x}{3(x - 1)} \times \frac{(x - 1)(x + 5)}{15x \div 5x}$ $= \frac{x}{3(x - 1)} \times \frac{(x - 1)(x + 5)}{3}$ $= \frac{x}{3\cancel{(x - 1)}} \times \frac{\cancel{(x - 1)}(x + 5)}{3}$ $= \frac{x}{3} \times \frac{x + 5}{3}$ $= \frac{x(x + 5)}{9}$	4F
Students do not adjust the numerator(s) when converting fractions to equivalent with a common denominator.	$\frac{5}{3x} - \frac{3}{x} = \frac{2}{3x}$	$\frac{5}{3x} - \frac{3}{x} = \frac{5}{3x} - \frac{9}{3x} = -\frac{4}{3x}$	4G
Students incorrectly adjust the value(s) of the numerator(s) when converting fractions to equivalent with a common denominator.	$\frac{5}{3x} - \frac{3}{x} = \frac{5}{3x} - \frac{5 \times 3}{3 \times x}$ $= \frac{5}{3x} - \frac{15}{3x}$ $= -\frac{10}{3x}$	$\frac{5}{3x} - \frac{3}{x} = \frac{5}{3x} - \frac{3 \times 3}{3 \times x}$ $= \frac{5}{3x} - \frac{9}{3x}$ $= -\frac{4}{3x}$	4G
Students combine the denominators of fractions by adding or subtracting them.	$\frac{5}{3x} - \frac{3}{x} = \frac{2}{2x}$	$\frac{5}{3x} - \frac{3}{x} = \frac{5}{3x} - \frac{9}{3x} = -\frac{4}{3x}$	4G

Back to contents

4A Expanding binomial products

LEARNING INTENTIONS

Students will be able to:

- apply the distributive law to expand binomial products
- simplify expressions with binomial products.

Binomial expressions contain two terms, at least one of which must be a variable with a positive integer exponent and the other may be a constant value. A binomial expression represents a sum or difference of two values, and so it is possible to apply the distributive law to simplify the products of binomial expressions.

KEY TERMS AND DEFINITIONS

- A **binomial expression** is a polynomial expression which contains two terms.
- A **polynomial expression** must contain variables with positive integer exponents and may contain a constant.
- The **distributive law** for multiplication states that multiplying a number by a group of numbers is the same as multiplying the number by the sum of the other numbers.

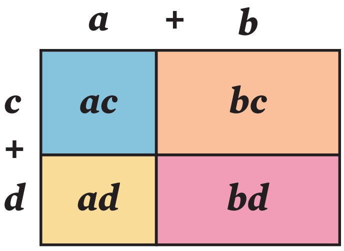
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: VectorMine/Shutterstock.com

Binomial products are often used by economists when predicting the best selling price for an item. Using a combination of research and probability skills, they are able to theorise about the optimal price for a product, so that as many items can be sold as possible. They take into consideration data on past similar product sales, production costs, and the number of items available for sale.

Key idea

1. The distributive law can be applied to expanding products of binomial expressions.



General formula:

$$(a+b)(c+d) = ac + bc + ad + bd$$

Back to contents

Worked example 1

Expanding binomial products

Expand and simplify the following.

a. $(x + 3)(x + 2)$ WE1a

Working

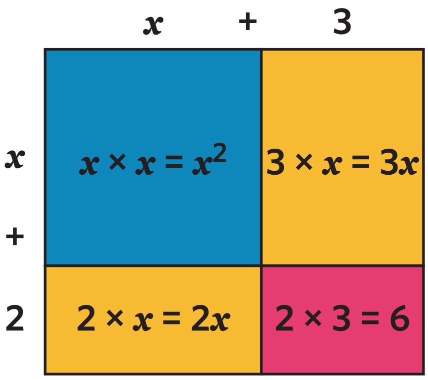
$$\begin{aligned}(x + 3)(x + 2) &= x \times x + 3 \times x + 2 \times x + 2 \times 3 \\ &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

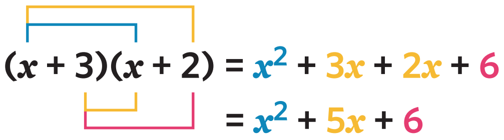
Thinking

Step 1: Expand the brackets by using the distributive law.

Step 2: Simplify the expression by collecting like terms.

Visual support





b. $(t - 2)(t - 5)$ WE1b

Working

$$\begin{aligned}(t - 2)(t - 5) &= t \times t - 5 \times t - 2 \times t - 2 \times (-5) \\ &= t^2 - 5t - 2t + 10 \\ &= t^2 - 7t + 10\end{aligned}$$

Thinking

Step 1: Expand the brackets using the distributive law.

Step 2: Simplify the expression by collecting like terms.

c. $(y - 5)(y + 4)$ WE1c

Working

$$\begin{aligned}(y - 5)(y + 4) &= y \times y + 4 \times y - 5 \times y - 5 \times 4 \\ &= y^2 + 4y - 5y - 20 \\ &= y^2 - y - 20\end{aligned}$$

Thinking

Step 1: Expand the brackets using the distributive law.

Step 2: Simplify the expression by collecting like terms.

d. $(2x + 7)(x - 2)$ WE1d

Working

$$\begin{aligned}(2x + 7)(x - 2) &= 2x \times x - 2 \times 2x + 7 \times x + 7 \times (-2) \\ &= 2x^2 - 4x + 7x - 14 \\ &= 2x^2 + 3x - 14\end{aligned}$$

Thinking

Step 1: Expand the brackets using the distributive law.

Step 2: Simplify the expression by collecting like terms.

Student practice

Expand and simplify the following.

a. $(y + 4)(y + 3)$ **b.** $(p - 3)(p - 2)$ **c.** $(x - 7)(x + 2)$ **d.** $(3t + 5)(t - 1)$

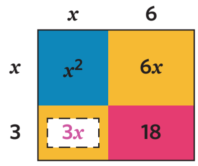
Back to contents

4A Questions

Understanding worksheet

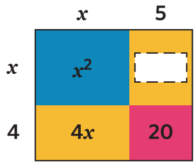
1. Fill in the blanks.

Example



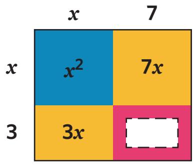
	x	6
x	x^2	$6x$
3	$\boxed{3x}$	18

a.



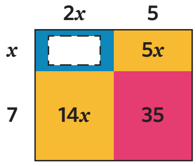
	x	5
x	x^2	$\boxed{}$
4	$4x$	20

b.



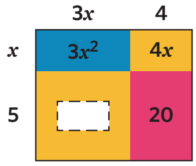
	x	7
x	x^2	$7x$
3	$3x$	$\boxed{}$

c.



	$2x$	5
x	$\boxed{}$	$5x$
7	$14x$	35

d.



	$3x$	4
x	$3x^2$	$4x$
5	$\boxed{}$	20

2. Fill in the blanks.

Example

$(n + 2)(n + 4) = n^2 + 6n + \boxed{8}$

a. $(x + 4)(x + 1) = x^2 + \boxed{}x + 4$

b. $(y - 4)(y + 1) = y^2 - 3y - \boxed{}$

c. $(a + 2)(a - 5) = a^2 - \boxed{}a - 10$

d. $(x - 8)(x - 3) = x^2 \boxed{} 11x + 24$

3. Fill in the blanks by using the words provided.

two **distributive** **sums** **polynomial**

A binomial expression is a ______ expression which contains only ______ terms, including variables and constants. The ______ law applies to the products of binomial expressions because they can be expressed as ______ and differences of two values.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Expand and simplify the following. WE1a

a. $(y + 1)(y + 2)$
b. $(r + 2)(r + 4)$
c. $(x + 5)(x + 3)$
d. $(d + 6)(d + 4)$
e. $(n + 10)(n + 6)$
f. $(a + 11)(a + 3)$
g. $(m + 7)(m + 6)$
h. $(x + 12)(x + 8)$

5. Expand and simplify the following. WE1b

a. $(v - 1)(v - 3)$
b. $(c - 2)(c - 4)$
c. $(x - 5)(x - 2)$
d. $(y - 3)(y - 4)$
e. $(m - 9)(m - 3)$
f. $(w - 7)(w - 6)$
g. $(p - 8)(p - 5)$
h. $(x - 11)(x - 9)$

Back to contents

6. Expand and simplify the following. WE1c

a. $(x + 4)(x - 2)$ **b.** $(p + 5)(p - 3)$ **c.** $(e + 9)(e - 1)$ **d.** $(x - 6)(x + 3)$

e. $(z - 4)(z + 5)$ **f.** $(y - 6)(y + 5)$ **g.** $(x + 7)(x - 9)$ **h.** $(r - 13)(r + 8)$

7. Expand and simplify the following. WE1d

a. $(2x + 3)(x + 2)$ **b.** $(3y + 2)(y + 1)$ **c.** $(2x + 4)(x - 1)$ **d.** $(2t - 5)(t - 1)$

e. $(2z - 1)(z + 3)$ **f.** $(x + 4)(2x - 5)$ **g.** $(2r + 7)(2r - 1)$ **h.** $(3p - 2)(2p - 5)$

8. Which of the options shows the given expression in expanded and simplified form?

$(x + 4)(x - 7)$

A. $x^2 - 3x - 28$
B. $x^2 + 3x - 28$
C. $x^2 - 3x + 28$
D. $x^2 + 11x - 28$
E. $x^2 - 11x - 28$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Expand and simplify the given expression.

$(x + 3)(x - 1)$

Student A

$$\begin{aligned}(x + 3)(x - 1) &= x^2 + x - 3x - 3\\ &= x^2 - 2x - 3\end{aligned}$$

Student B

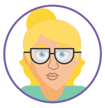
$$\begin{aligned}(x + 3)(x - 1) &= x^2 - x + 3x - 3\\ &= x^2 + 2x - 3\end{aligned}$$

b. Expand and simplify the given expression.

$(y - 2)(y - 7)$

Student A

$$\begin{aligned}(y - 2)(y - 7) &= y^2 - 7y - 2y + 14\\ &= y^2 - 9y + 14\end{aligned}$$



Student B

$$\begin{aligned}(y - 2)(y - 7) &= y^2 - 7y - 2y - 14\\ &= y^2 - 9y - 14\end{aligned}$$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Sam wants to extend her square living room. She will be able to add two and three metres to the length and width respectively. Using x m to represent the current length and width, formulate a binomial product expression for the area of the new extended living room.

Back to contents

11. Duyen is changing the layout of her farm in a computer game. She wants to alter the square cabbage patch so that one side is three blocks longer and the other is one block shorter. Using p to represent the number of blocks along one side of the current cabbage patch, write an expanded expression for the area of the cabbage patch after Duyen changes it.

12. Two high school football teams have an equal number of players (n) each, including substitutes. On the day of a match three people from one team and two from the other were absent. Write an expanded expression for the number of handshakes that occurred at the end of the game if each present member of one team shook hands with each present member of the other team exactly once.

13. The city council is extending the boundaries of a rectangular suburb which is currently x km long and y km wide. An additional 3 km will be added to the length and 4 km will be added to the width of the area. Express the new area of the suburb in expanded form.

14. Salma wants to frame her 25 cm by 20 cm diploma certificate and display it on the wall. The wooden border of the frame she chose is y cm wide. Write an expanded expression showing the total area of the entire picture including the frame.

Reasoning

Question working paths

Mild 15 (a,b,d) 🌶️	**Medium** 15 (a,b,d), 16 (a,b) 🌶️🌶️	**Spicy** All 🌶️🌶️🌶️

15. Graham is an architect who is using a formula to calculate the area of a room on a blueprint for a new house.

$A = (x + a)(x + b)$

where x is the length of one side of the largest square that fits inside the floor plan of the room in mm, and a and b represent the number of mm by which the dimensions of the room exceed this area.

a. Expand the formula for A.

b. Write an expanded and simplified expression for the area of the room (A) if $a = 15$ mm and $b = 20$ mm.

c. Graham realises that the total area of the room will not fit inside the allowable space. He reduces the side lengths of its floor plan from part **b** by 16 mm each. Write an expanded and simplified expression for the area of this room's plan.

d. If you were given the opportunity to design your dream home, describe a feature that you would include.

16. Expand and simplify the given expressions.

a. $(x + 5)(x + 2)$

b. $(2 + x)(5 + x)$

c. Analyse the simplified expressions from parts **a** and **b**. Does the order in which the terms are added inside binomial expressions affect their product?

Exam-style

17. What is the coefficient of x in the expanded and simplified form of the given expression? (1 MARK)

$(x - 7)(x + 3)$

A. -21 **B.** -10 **C.** -4 **D.** 4 **E.** 10

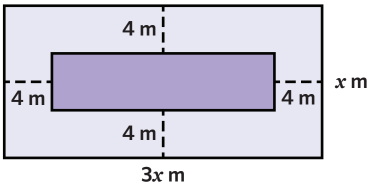
18. Consider the given expression. (3 MARKS)

$(3x - 1)(x + 3)$

a. Expand and simplify. 2 MARKS

b. Evaluate for $x = 1$. 1 MARK

19. Show that the area of the inner rectangle in the diagram is given by $A = 3x^2 - 32x + 64$. (3 MARKS)



20. A square pool with an area of y^2 m^2 is being remodelled. The length of the pool will be increased by 5 m whereas the width will be reduced by 2 m. Write an expanded and simplified expression for the new area of the pool. (3 MARKS)

Remember this?

21. Tom and Jerry both take taxis during their holidays. They travelled with the same taxi company with the same starting rate and rate per kilometre.

Tom travelled 3 km in a taxi and was charged $17. Jerry travelled 12 km in a taxi and was charged $35.

Gianmarco also wants to travel with the same taxi. How much would Gianmarco pay if he travels 6 km?

A. $11
B. $17.5
C. $23
D. $34
E. $45

22. Joanna spent $34.50 during her winter holiday.

During her summer holiday, Joanna spent three times as much as she did in her winter holiday, plus $10 more.

How much did Joanna spend during her summer holiday?

A. $11.50
B. $21.50
C. $102
D. $103.50
E. $113.50

23. Which of the following is the correct solution for a?

$$4\left(\frac{2^a}{5}+1\right)=7.2$$

A. $a = 1$
B. $a = 2$
C. $a = 3$
D. $a = 4$
E. $a = 5$

4B Perfect squares and the difference of two squares

LEARNING INTENTIONS

Students will be able to:

- identify and expand a perfect square
- identify and expand expressions that form a difference of two squares.

The concept of perfect squares can be extended to binomial products because binomial expressions represent sums and differences of values. Binomial products possess the distributive property and further patterns can be established for those that are perfect squares. A difference of two squares is a binomial expression that contains two square values subtracted from one another and is the result of a binomial product of the form $(a + b)(a - b)$.

KEY TERMS AND DEFINITIONS

- A **perfect square**, also known as a square number, in a given number system is the product of a number multiplied by itself.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Marina9/Shutterstock.com

Square numbers are the basis for the game of chess. The chessboard itself is a perfect square with exactly eight alternating dark and light tiles along each side. Each of the pieces has different allowable movements based around the geometric properties of the board. Due to these mathematical consistencies, chess is a game that can be played by anyone in the world, provided they are following the rules.

Key ideas

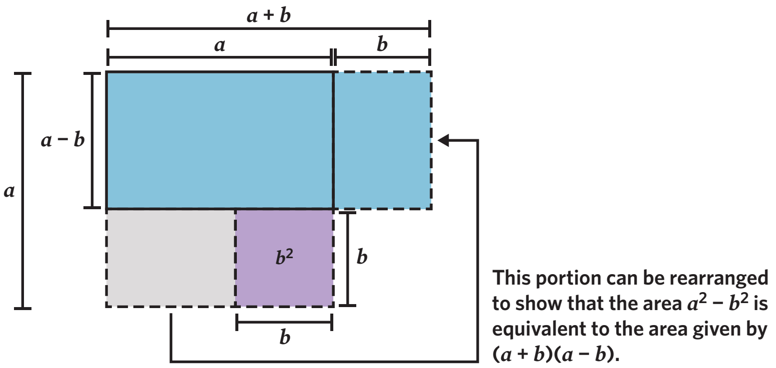
1. A binomial expression multiplied by itself forms a perfect square that can be expanded using the distributive law.

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

2. Expanding a binomial product of the form $(a + b)(a - b)$ results in a difference of two squares.

$$\begin{aligned}(a+b)(a-b) &= a^2 - ab + ab - b^2\\ &= a^2 - b^2\end{aligned}$$



This portion can be rearranged to show that the area $a^2 - b^2$ is equivalent to the area given by $(a + b)(a - b)$.

Back to contents

Worked example 1

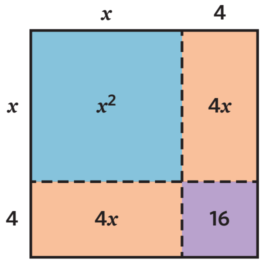
Expanding perfect squares

Expand and simplify the following.

a. $(x+4)^2$ WE1a

Working	Thinking
$(x+4)^2 = (x+4)(x+4)$ $= x^2 + 4x + 4x + 16$	**Step 1:** Expand the binomial product.
$= x^2 + 2 \times 4x + 16$ $= x^2 + 8x + 16$	**Step 2:** Simplify the expression by collecting like terms.

Visual support



$$\begin{aligned}(x+4)^2 &= (x+4)(x+4)\\ &= x^2 + 4x + 4x + 16\\ &= x^2 + 2 \times 4x + 16\\ &= x^2 + 8x + 16\end{aligned}$$

b. $(y-2)^2$ WE1b

Working	Thinking
$(y-2)^2 = (y-2)(y-2)$ $= y^2 - 2y - 2y + 4$	**Step 1:** Expand the binomial product.
$= y^2 - 2 \times 2y + 4$ $= y^2 - 4y + 4$	**Step 2:** Simplify the expression by collecting like terms.

c. $(2x+3)^2$ WE1c

Working	Thinking
$(2x+3)^2 = (2x+3)(2x+3)$ $= 4x^2 + 6x + 6x + 9$	**Step 1:** Expand the binomial product.
$= 4x^2 + 2 \times 6x + 9$ $= 4x^2 + 12x + 9$	**Step 2:** Simplify the expression by collecting like terms.

Student practice

Expand and simplify the following.

a. $(y+2)^2$ **b.** $(p-4)^2$ **c.** $(2r+5)^2$

Worked example 2

Expanding to form a difference of two squares

Expand and simplify the following.

a. $(x + 3)(x - 3)$

WE2a

Working

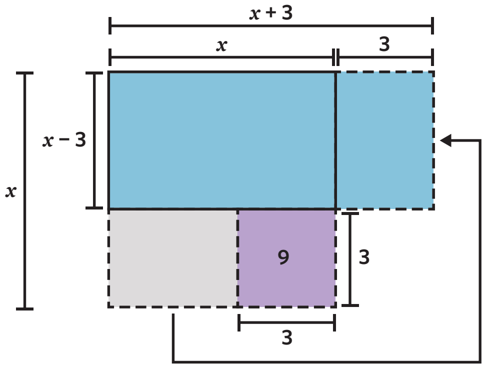
$$\begin{aligned}(x + 3)(x - 3) &= x^2 - 3x + 3x - 9\\ &= x^2 - 9\end{aligned}$$

Thinking

Step 1: Expand the binomial product.

Step 2: Simplify the expression by collecting like terms.

Visual support



$$\begin{aligned}(x + 3)(x - 3) &= x^2 - 3x + 3x - 3^2\\ &= x^2 - 3^2\\ &= x^2 - 9\end{aligned}$$

b. $(2y - 5)(2y + 5)$

WE2b

Working

$$\begin{aligned}(2y - 5)(2y + 5) &= 4y^2 + 10y - 10y - 25\\ &= 4y^2 - 25\end{aligned}$$

Thinking

Step 1: Expand the binomial product.

Step 2: Simplify the expression by collecting like terms.

Student practice

Expand and simplify the following.

a. $(y + 4)(y - 4)$ **b.** $(2t - 3)(2t + 3)$

Back to contents

4B Questions

Understanding worksheet

1. Match the expressions to their expanded simplified forms.

Factorised form		**Expanded form**
$(x + 1)^2$ ●		● $x^2 + 12x + 36$
$(x + 6)^2$ ●		● $x^2 + 2x + 1$
$(x - 3)^2$ ●		● $x^2 - 14x + 49$
$(x - 7)^2$ ●		● $x^2 - 6x + 9$

2. Fill in the blanks.

> **Example**
>
> $(x + 2)(x - 2) = x^2 - \boxed{4}$

a. $(y + 1)(y - 1) = y^2 - \boxed{}$ **b.** $(x + 5)(x - 5) = x^2 - \boxed{}$

c. $(p - 4)(p + 4) = p^2 - \boxed{}$ **d.** $(3x - 7)(3x + 7) = 9x^2 - \boxed{}$

3. Fill in the blanks by using the words provided.

binomial **squares** **difference** **expanded**

Perfect $\boxed{}$ are produced by multiplying $\boxed{}$ expressions by themselves. The distributive law can be used to show that those products follow a general formula when $\boxed{}$ and simplified. A $\boxed{}$ of two squares is produced when binomial products of the form $(a + b)(a - b)$ are expanded and simplified.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Expand and simplify the following. **WE1a,b**

a. $(y + 3)^2$ **b.** $(x + 5)^2$ **c.** $(a + 10)^2$ **d.** $(t - 3)^2$

e. $(x - 6)^2$ **f.** $(b - 7)^2$ **g.** $(11 + m)^2$ **h.** $(8 - n)^2$

5. Expand and simplify the following. **WE1c**

a. $(2c + 1)^2$ **b.** $(3x + 4)^2$ **c.** $(4t + 3)^2$ **d.** $(2y - 7)^2$

e. $(4r - 3)^2$ **f.** $(5d - 8)^2$ **g.** $(10 + 3x)^2$ **h.** $(9 - 7p)^2$

6. Expand and simplify the following. **WE2a**

a. $(y + 5)(y - 5)$ **b.** $(x + 6)(x - 6)$ **c.** $(c + 8)(c - 8)$ **d.** $(d - 3)(d + 3)$

e. $(n - 11)(n + 11)$ **f.** $(m + 14)(m - 14)$ **g.** $(9 - x)(9 + x)$ **h.** $(12 - y)(12 + y)$

Back to contents

7. Expand and simplify the following. WE2b

a. $(2y - 1)(2y + 1)$ **b.** $(2x + 3)(2x - 3)$ **c.** $(3t + 2)(3t - 2)$ **d.** $(4a - 3)(4a + 3)$

e. $(5p + 6)(5p - 6)$ **f.** $(7b - 8)(7b + 8)$ **g.** $(5 + 6x)(5 - 6x)$ **h.** $(15 - 8d)(15 + 8d)$

---

8. Expand and simplify the following.

a. $(p + t)(p - t)$ **b.** $(x + y)^2$ **c.** $(2c - d)(2c + d)$ **d.** $(2a + b)^2$

e. $(3r - q)^2$ **f.** $(2m + 3n)^2$ **g.** $(5x - 9y)(5x + 9y)$ **h.** $(4k - 5m)^2$

---

9. Which of the options shows the given expression in expanded form?

$(x - 7)^2$

A. $x^2 - 49$
B. $x^2 + 49$
C. $x^2 - 14x + 49$
D. $x^2 + 14x - 49$
E. $x^2 + 14x + 49$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Expand and simplify.

$(x - 10)(x + 10)$

Student A

$$\begin{aligned}(x - 10)(x + 10) &= x^2 + 10x - 10x - 100\\ &= x^2 - 100\end{aligned}$$

Student B

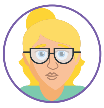
$$\begin{aligned}(x - 10)(x + 10) &= x^2 + 10x - 10x + 100\\ &= x^2 + 100\end{aligned}$$

b. Expand and simplify.

$(y - 8)^2$

Student A

$$\begin{aligned}(y - 8)^2 &= (y - 8)(y - 8)\\ &= y^2 - 8y - 8y + 64\\ &= y^2 + 64\end{aligned}$$



Student B

$$\begin{aligned}(y - 8)^2 &= (y - 8)(y - 8)\\ &= y^2 - 8y - 8y + 64\\ &= y^2 - 16y + 64\end{aligned}$$

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. A square room has been extended, so that exactly 2 m is added to the length and width. Write a binomial product expression for the new area of the room, using x to represent the length and width of the room prior to extension.

12. A large square park is surrounded by a 1 m wide footpath on all sides. Write an expanded simplified expression for the total area of the park and footpath, using p to represent the length and width of the park.

13. Charlie is building a house in a simulation computer game. He uses the square room template to place a room inside the house. Charlie decides to add five units to the length and reduce the width by five units. Write an expanded simplified expression for the area of Charlie's room after alterations.

14. Dawn is putting a completed cross-stitch in a square frame that has a border that is 3 cm wide. Write an expanded simplified expression for the visible part of the picture, using *f* to represent the length and width of the entire frame.

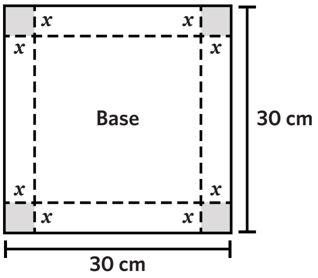
15. An airport in the shape of a square is extending their runway to accommodate larger planes. They are planning on adding to the overall length of the airport in the direction of the runway, but reducing the width by an equal amount due to council regulations. Write an expanded simplified expression for the new area of the airport if they were to add 650 m to the length of the airport while reducing the width by the same amount.

Reasoning

Question working paths

Mild 16 (a,b,c,e) 🌶	**Medium** 16 (a,b,c,e), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Mazzy and Chris are making a box for their mother's birthday gift. They are using a piece of cardboard 30 cm by 30 cm for the bottom part of the box. To fold it up they cut squares out of each corner.



a. Write an expression for one side length of the base.

b. Write an expression for the total area of the base.

c. Expand and simplify the expression from part **b**.

d. Calculate the area of the base when $x = 4$ cm.

e. Identify an advantage or disadvantage of giving people homemade gifts.

17. Expand and simplify the expressions given in parts **a** and **b**.

a. $(x + 5)^2$

b. $(2x + 5)^2$

c. Compare the answers from parts **a** and **b** and comment on whether changing the coefficient of x affects the constant value of an expanded binomial product.

Exam-style

18. Which of the binomial products will result in a difference of two squares? (1 MARK)

A. $(x - 2)^2$

B. $(x + 3)^2$

C. $(x - 2)(x + 3)$

D. $(x - 3)(x + 3)$

E. $(2x - 3)(x + 3)$

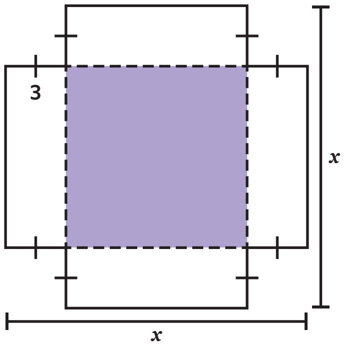
19. Consider the given expression. (3 MARKS)

$(y - 4)^2$

a. Expand and simplify the expression. 2 MARKS

b. Evaluate for $y = 5$. 1 MARK

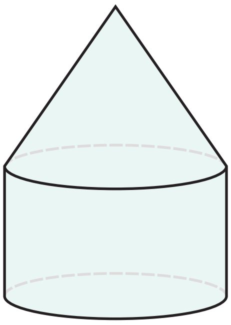
20. Show that the shaded area in the picture is given by $A = x^2 - 12x + 36$. (3 MARKS)



21. A square area with sides 9 m long has been altered so that the length is x m shorter and the width is x m longer. Write an expanded simplified expression for the reduced area. (3 MARKS)

Remember this?

22. Consider the given shape.



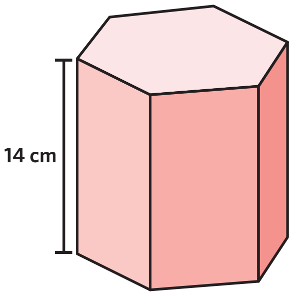
This 3D shape is comprised of

A. a cylinder and cone.
B. a hemisphere and cylinder.
C. a rectangular prism and cone.
D. a rectangular prism and cylinder.
E. a cube and square-based pyramid.

23. How many metres are in 6.2 km?

A. 620 **B.** 602 **C.** 6020 **D.** 6200 **E.** 6220

24. The diagram shows a water bottle as a prism.



The bottom side is a hexagon with an area of 15 cm^2.

The capacity of one cubic centimetre is one millilitre.

What is the capacity of the water bottle in millilitres?

A. 15 mL **B.** 60 mL **C.** 196 mL **D.** 200 mL **E.** 210 mL

4C Factorising algebraic expressions

LEARNING INTENTIONS

Students will be able to:

- determine the highest common factor of a group of terms
- factorise expressions involving a common factor.

Factorising is the reverse process of expanding expressions with brackets. To factorise an expression, the highest common factor (HCF) should be divided out of the terms and the remaining factors written in a pair of brackets. The HCF can be a number only or an entire algebraic term or expression.

KEY TERMS AND DEFINITIONS

- The **highest common factor (HCF)** is the largest number, term or expression that is a common factor of two or more terms or expressions.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: NicoElNino/Shutterstock.com

Data online is encrypted, which means that it is stored as a series of numbers called a cipher. Encryption keys are used to encrypt and decrypt data. A good online security system ensures that if a hacking attack occurs, the numbers making up the encryption keys are very difficult to determine. As most attacks rely on a computer's ability to calculate an encryption key by factorising the number, choosing numbers with large prime factors makes the process significantly more difficult for the attacker.

Key ideas

1. Factorising expressions involves dividing out the highest common factor from its terms.

$$ab + ac = \overset{\text{The HCF has been divided out of each term}}{a}\underbrace{(b + c)}_{\text{Each term has been divided by the HCF}}$$

2. Factorising is the reverse process of expanding.

$$ab + ac \underset{\text{Expanding}}{\overset{\text{Factorising}}{\rightleftarrows}} a(b + c)$$

$$a \times b + a \times c$$

3. When factorising expressions, the leading term inside the brackets needs to be positive.

$$-ab \underbrace{+\ ac}_{} = -a \times b + \underbrace{-a \times -c}_{\text{Multiplying two negatives produce a positive}} = \overset{\text{Negative HCF}}{-a}(\underset{\text{Positive leading term}}{b} - c)$$

Back to contents

Worked example 1

Finding the highest common factor

Determine the highest common factor of the following.

a. $4x$ and $2xy$

WE1a

Working

$4x = 2 \times 2 \times x$

$2xy = 2 \times x \times x \times y$

The HCF of $4x$ and $2xy$ is $2 \times x = 2x$.

Thinking

Step 1: Determine the factors of each term.

Step 2: Determine the HCF by identifying the largest value or term that is in both lists of factors.

Visual support

$$4x = 2 \times 2 \times x$$

$$2xy = 2 \times x \times y$$

$$\therefore \text{HCF is } 2x$$

b. $15x^2y$ and $10xy$

WE1b

Working

$15x^2y = 3 \times 5 \times x \times x \times y$

$10xy = 5 \times 2 \times x \times y$

The HCF of $15x^2y$ and $10xy$ is $5 \times x \times y = 5xy$.

Thinking

Step 1: Determine the factors of each term.

Step 2: Determine the HCF by identifying the largest value or term that is in both lists of factors.

Student practice

Determine the highest common factor of the following.

a. $6a$ and $3ab$ **b.** $12a^2b$ and $8ab$

Worked example 2

Factorising expressions

Factorise the following.

a. $3x + 12$

WE2a

Working

$3x + 12 = 3(\quad)$

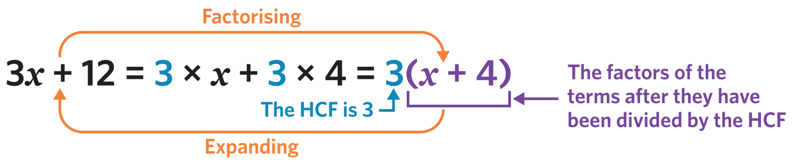
$= 3(x + 4)$

Thinking

Step 1: Identify the HCF of the terms that make up the expression and write it on the outside of a pair of brackets.

Step 2: Divide each term by the HCF and write the remaining factors in the brackets.

Visual support



Continues →

b. $-6x^2 - 15x$

WE2b

Working

$-6x^2 - 15x = -3x(\quad\quad)$

$= -3x(2x + 5)$

Thinking

Step 1: Identify the HCF of the terms that make up the expression and write it on the outside of a pair of brackets, including the negative sign.

Step 2: Divide each term by the HCF and write the remaining factors in the brackets.

Student practice

Factorise the following.

a. $2x + 10$

b. $-8x^2 - 12x$

Worked example 3

Factorising expressions using binomial factors

Factorise the following.

a. $2(a + b) + a(a + b)$

WE3a

Working

$2(a + b) + a(a + b)$

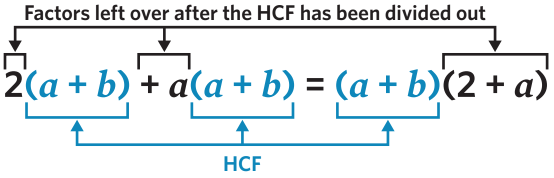
$2(a + b) + a(a + b) = (a + b)(2 + a)$

Thinking

Step 1: Identify the common binomial factor of the parts making up the expression.

Step 2: Divide the binomial HCF out of every part to factorise.

Visual support



b. $2(5x - 3) - x(5x - 3)$

WE3b

Working

$2(5x - 3) - x(5x - 3)$

$2(5x - 3) - x(5x - 3) = (5x - 3)(2 - x)$

Thinking

Step 1: Identify the common binomial factor of the parts making up the expression.

Step 2: Divide the binomial factor out of every part to factorise.

Student practice

Factorise the following.

a. $4(x + y) + x(x + y)$

b. $6(3a - 1) - a(3a - 1)$

Back to contents

4C Questions

Understanding worksheet

1. Fill in the blanks.

Example

$6x + 21 = \boxed{3}(2x + 7)$

a. $4x + 24 = \boxed{}(x + 6)$

b. $2x + 8x^2 = \boxed{}(1 + 4x)$

c. $6c^2 - 10cd = \boxed{}(3c - 5d)$

d. $-12a^2 + 15a = \boxed{}(4a - 5)$

2. Match the factorised expressions to their expanded forms.

Factorised form			**Expanded form**
$7(2x + 3)$	●	●	$-2x - 8x^2$
$4x(3x - 5)$	●	●	$14x + 21$
$-2x(1 + 4x)$	●	●	$-15x^2 + 24x$
$-3x(5x - 8)$	●	●	$12x^2 - 20x$

3. Fill in the blanks by using the words provided.

common **leading** **reverse** **HCF**

Factorising is the __________ process of expanding. When an expression is factorised, it means that the __________ has been divided out of the terms and the remaining factors have been written inside a pair of brackets. The HCF is the largest __________ factor of a group of numbers, terms, or entire expressions. A negative HCF is used for expressions where the __________ term is negative.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Determine the highest common factor of the following. WE1

a. $2a$ and 8
b. $5t^2$ and $15t$
c. $18a^2$ and $12a$
d. $6x$ and $12xy$
e. $3xy^2$ and $12y$
f. $6x^2y$ and $4xy$
g. $10a^2b^2$ and $25ab$
h. $8prq^2$ and $36p^2q$

---

5. Factorise the following. WE2a

a. $6x + 12$
b. $3y + 24y^2$
c. $7a - 21a^2$
d. $4ab - 5b$
e. $10x^2 + 4xy$
f. $12yz + 4y^2$
g. $5tr - 35r^2$
h. $14abc - 28b^2$

6. Factorise the following. WE2b

a. $-5a - 15$
b. $-4x + 16$
c. $-3x^2 - 6x$
d. $-12t + 3t^2$
e. $-7xy^2 - xy$
f. $-13ab + 39a^2$
g. $-12t^2 + 24tr$
h. $-21xy - 3xy^2$

7. Factorise the following. WE3

a. $5(t + r) + t(t + r)$
b. $3(x - 4) + x(x - 4)$
c. $9(8a + 3) - a(8a + 3)$
d. $x(6x + 1) - 2(6x + 1)$
e. $2x(7 - 4x) + 5(7 - 4x)$
f. $4x(x + y) - y(x + y)$
g. $7t(5 - 3t) - 6(5 - 3t)$
h. $(2y + 5) - x(2y + 5)$

8. Which of the options shows the given expression in factorised form?

$-9x + 27x^2$

A. $-9x(1 - 27x)$
B. $-9x(1 - 3x)$
C. $-9(x - 3x)$
D. $9x(1 + 3x)$
E. $9x(1 - 3x)$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the highest common factor of the following.

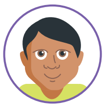
$8x^2$ and $24x$

Student A

$8x^2 = 2 \times 2 \times 2 \times x \times x$

$24x = 2 \times 2 \times 2 \times 3 \times x$

The HCF of $8x^2$ and $24x$ is $2 \times 2 \times 2 \times x = 8x$



Student B

$8x^2 = 2 \times 2 \times 2 \times x^2$

$24x = 2 \times 2 \times 2 \times 3 \times x$

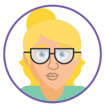
The HCF of $8x^2$ and $24x$ is $2 \times 2 \times 2 = 8$.

b. Factorise the following.

$-4x - 12x^2$

Student A

$-4x - 12x^2 = -4x(1 - 3x)$



Student B

$-4x - 12x^2 = -4x(1 + 3x)$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. A number of adults and children attended a school production, with tickets costing \$5 per person. Using a to represent the number of adults and c to represent the number of children, write a factorised expression showing the total cost for the whole group.

11. Junji is planning a concert for his band and is charging \$7 pre-sale and \$10 on the door for the tickets. Junji's band played two shows and sold out of every ticket on both nights. Write a factorised expression showing the total profits made by Junji's band over the two nights.

12. Tony is a fruit picker at an orchard where apples and pears grow. Every day, he picks exactly the same amount of each type of fruit and is paid \$3 for a kg of apples and \$5 for a kg of pears. Write a factorised expression showing Tony's total pay over a four-day work week.

Back to contents

13. A number of students from five classes and three teachers participated in an origami crane making competition. There is an equal number of students in each class and each person who participated made an average of 10 cranes. Write a factorised expression showing the total number of origami cranes made by the students and teachers.

14. At an all ages music festival there were 50 children under 12 and twice as many teenagers as there were adults. Write a factorised equation showing total ticket sales of \$41 800 if each person paid \$95 to be there.

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. A store is offering discounts on all items, including those that have already been discounted. The price of any item can be modelled by the given equation.

$$p = 0.6x - 0.3y$$

where p is the new price, x is the original price, and y is the existing discount, in dollars.

- **a.** Factorise the formula given for p.
- **b.** Write a factorised expression for the new price of a couch that has already been discounted by \$50.
- **c.** Write a factorised expression for the new price of a computer that has already been discounted by \$129.
- **d.** Amber buys a TV for the new price of \$699. Write a factorised equation showing the new price of the TV in terms of the original price and existing discount of \$99.95.
- **e.** Identify an advantage or disadvantage of waiting for sales in order to purchase an item.

16. Factorise the expressions given in parts **a** and **b**.

- **a.** $-2x + 8$
- **b.** $-2x - 8$
- **c.** Compare your answers for parts **a** and **b** and comment on the effect of a negative HCF on the remaining factors of positive and negative terms.

Exam-style

17. The HCF of the terms in the expression $12x^2 - 3xy$ is (1 MARK)

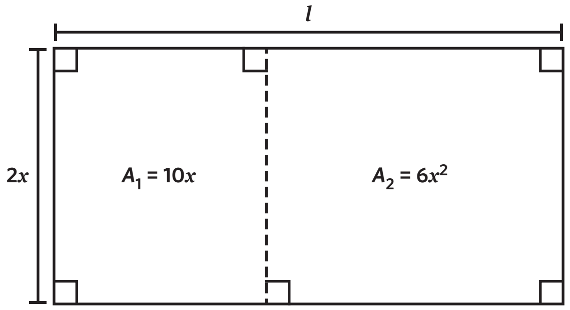
A. 3 **B.** 4 **C.** x **D.** $3x$ **E.** $3x^2$

18. Consider the given expression. (2 MARKS)

$$5(x + 3) - x(x + 3)$$

- **a.** Identify the common binomial factor in the expression. 1 MARK
- **b.** Factorise the expression. 1 MARK

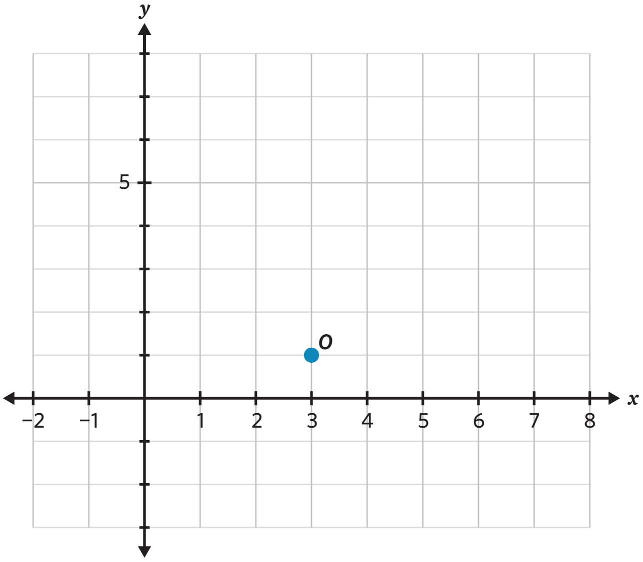
19. Determine the length of l in terms of x using the information given in the diagram. (3 MARKS)



20. Each student in year 9 is asked to take one chair from the hall to the gym, while the teachers are asked to take two chairs each. A total of 432 chairs were moved by the students and teachers over three trips. Write a factorised equation showing the total number of chairs that were moved by the students and teachers. (2 MARKS)

Remember this?

21. Point *O* is translated up 5 units and left 2 units.



What are the coordinates of the new position of point *O*?

A. (1,6) **B.** (3,1) **C.** (6,5) **D.** (6,1) **E.** (8,3)

22. The given table presents data on how long five students spent on homework one evening after school.

Student	Jimmy	Tessa	Seamus	Zoe	Thomas
Time (minutes)	68	71	15	80	84

If Seamus' time is removed from the data, what will happen to the mean time spent on homework that evening?

A. It will increase.
B. It will decrease.
C. It is impossible to tell.
D. It will remain the same.
E. It will increase and then decrease.

23. Joey and Sophie both order small pizzas for dinner. Joey eats $\frac{5}{6}$ of his pizza. Sophie eats less of her pizza than Joey eats of his pizza. What is a fraction of her pizza that Sophie could have eaten?

A. $\frac{6}{7}$ **B.** $\frac{8}{9}$ **C.** $\frac{8}{10}$ **D.** $\frac{7}{8}$ **E.** $\frac{10}{12}$

4D Factorising the difference of two squares

LEARNING INTENTIONS

Students will be able to:

- identify a difference of two squares
- factorise a difference of two squares
- factorise expressions involving a difference of two squares.

A difference of two squares of the form $a^2 - b^2$ can be factorised to a binomial product of the form $(a + b)(a - b)$, where the pronumerals a and b can represent values, terms or whole expressions. This means that any binomial expression that is a difference of two squares can be factorised using a general formula.

KEY TERMS AND DEFINITIONS

- A **general formula** is a rule or mathematical relationship between values that is expressed using symbols or pronumerals.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Pythagorean Theorem

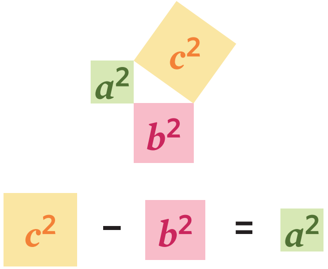


Image: Dream01/Shutterstock.com

The Pythagorean Theorem for calculating the side lengths of a right-angled triangle can be derived by applying an area model. In order to calculate the length of one of the shorter sides of a right-angled triangle, a difference of two squares must be evaluated.

Key idea

1. The difference of two squares can be factorised using a general formula.

$$\underbrace{a^2 - b^2}_{\text{Values, terms or expressions}} = (\overset{\sqrt{a^2}}{a} + \underset{\sqrt{b^2}}{b})(\overset{\sqrt{a^2}}{a} - \underset{\sqrt{b^2}}{b})$$

Back to contents

Worked example 1

Factorising a difference of two squares

Factorise the following.

a. $x^2 - 9$

WE1a

Working

$a^2 = x^2$

$b^2 = 9$

$a = \sqrt{x^2} = x$

$b = \sqrt{9} = 3$

$\therefore x^2 - 9 = (x + 3)(x - 3)$

Thinking

Step 1: Identify a^2 and b^2 in the given expression of the form $a^2 - b^2$.

Step 2: Determine the values of a and b and substitute them into the general formula $(a + b)(a - b)$ to factorise the difference of two squares.

Visual support

General formula: $a^2 - b^2 = (a + b)(a - b)$

$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

b. $4t^2 - 25$

WE1b

Working

$a^2 = 4t^2$

$b^2 = 25$

$a = \sqrt{4t^2} = 2t$

$b = \sqrt{25} = 5$

$\therefore 4t^2 - 25 = (2t + 5)(2t - 5)$

Thinking

Step 1: Identify a^2 and b^2 in the given expression of the form $a^2 - b^2$.

Step 2: Determine the values of a and b and substitute them into the general formula $(a + b)(a - b)$ to factorise the difference of two squares.

c. $36x^2 - 25y^2$

WE1c

Working

$a^2 = 36x^2$

$b^2 = 25y^2$

$a = \sqrt{36x^2} = 6x$

$b = \sqrt{25y^2} = 5y$

$\therefore 36x^2 - 25y^2 = (6x + 5y)(6x - 5y)$

Thinking

Step 1: Identify a^2 and b^2 in the given expression of the form $a^2 - b^2$.

Step 2: Determine the values of a and b and substitute them into the general formula $(a + b)(a - b)$ to factorise the difference of two squares.

Student practice

Factorise the following.

a. $x^2 - 4$ **b.** $9t^2 - 16$ **c.** $16x^2 - 49y^2$

Worked example 2

Simplifying expressions involving a difference of two squares

Factorise and simplify the following.

a. $2x^2 - 8$

WE2a

Working

$2x^2 - 8 = 2(x^2 - 4)$

$a^2 = x^2$

$b^2 = 4$

$a = \sqrt{x^2} = x$

$b = \sqrt{4} = 2$

$\therefore 2x^2 - 8 = 2(x + 2)(x - 2)$

Thinking

Step 1: Factorise the given binomial expression by dividing the HCF out of each term.

Step 2: Identify a^2 and b^2 in the expression $a^2 - b^2$ inside the brackets.

Step 3: Determine the values of a and b and substitute them into the general formula $(a + b)(a - b)$ to factorise the difference of two squares. Include the HCF in the final factorised expression.

Visual support

$$2x^2 - 8 = 2(x^2 - 4) = 2(x^2 - 2^2) = 2(x + 2)(x - 2)$$

General formula: $a^2 - b^2 = (a + b)(a - b)$

b. $(x + 3)^2 - 16$

WE2b

Working

$a^2 = (x + 3)^2$

$b^2 = 16$

$a = \sqrt{(x + 3)^2} = x + 3$

$b = \sqrt{16} = 4$

$$\begin{aligned}\therefore (x + 3)^2 - 16 &= ((x + 3) + 4)((x + 3) - 4)\\ &= (x + 7)(x - 1)\end{aligned}$$

Thinking

Step 1: Identify a^2 and b^2 in the given expression of the form $a^2 - b^2$.

Step 2: Determine the values of a and b, substitute them into the general formula $(a + b)(a - b)$, and collect like terms to factorise the difference of two squares.

Student practice

Factorise and simplify the following.

a. $2x^2 - 18$ **b.** $(x + 2)^2 - 9$

Back to contents

4D Questions

Understanding worksheet

1. Identify *a* or *b* for each difference of two squares.

Example

$a^2 - b^2$	a	b
$16x^2 - 9y^2$	$4x$	$3y$

$a^2 - b^2$	a	b
$x^2 - 25$	x	
$4x^2 - 9$		3
$9t^2 - 4y^2$	$3t$	
$(x - 2)^2 - 16$		4

2. Match each difference of two squares to its factorised form.

$a^2 - b^2$	$(a + b)(a - b)$
$x^2 - 16$ ●	● $(2x + 7)(2x - 7)$
$x^2 - 64$ ●	● $(2x + 9y)(2x - 9y)$
$4x^2 - 49$ ●	● $(x + 4)(x - 4)$
$4x^2 - 81y^2$ ●	● $(x + 8)(x - 8)$

3. Fill in the blanks by using the words provided.

terms **subtracted** **formula** **binomial**

A difference of two squares is a ________ expression where a square value is ________ from another square value. The square values can consist of numbers, ________ or whole expressions. To factorise a difference of two squares, a general ________ can be applied.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Factorise the following. WE1a

a. $x^2 - 1$ **b.** $x^2 - 16$ **c.** $a^2 - 36$ **d.** $y^2 - 81$

e. $p^2 - 144$ **f.** $49 - r^2$ **g.** $100 - c^2$ **h.** $k^2 - 2500$

5. Factorise the following. WE1b

a. $4x^2 - 1$
b. $4t^2 - 9$
c. $9y^2 - 25$
d. $16y^2 - 9$
e. $25x^2 - 81$
f. $144n^2 - 1$
g. $100 - 49m^2$
h. $400 - 81x^2$

6. Factorise the following. WE1c

a. $4a^2 - b^2$
b. $4x^2 - 9y^2$
c. $9x^2 - 16y^2$
d. $m^2 - 4n^2$
e. $49p^2 - 9q^2$
f. $81k^2 - 16m^2$
g. $64t^2 - 81r^2$
h. $9b^2 - 100a^2$

7. Factorise and simplify the following. WE2a

a. $3p^2 - 3$
b. $2y^2 - 32$
c. $3x^2 - 12$
d. $2t^2 - 50$
e. $5x^2 - 180$
f. $147 - 3r^2$
g. $7m^2 - 28n^2$
h. $3x^2y^2 - 27$

8. Factorise and simplify the following. WE2b

a. $(x + 1)^2 - 4$
b. $(y + 4)^2 - 9$
c. $(r + 5)^2 - 16$
d. $(t - 6)^2 - 1$
e. $(p - 2)^2 - 25$
f. $(y + 7)^2 - 25$
g. $81 - (x + 2)^2$
h. $100 - (x - 8)^2$

9. Which of the options shows the given expression in factorised form?

$16m^2 - 9$

A. $(4m - 3)(4m - 3)$
B. $(4m + 3)(4m - 3)$
C. $(4m + 3)(4m + 3)$
D. $(8m + 3)(8m - 3)$
E. $(16m + 3)(16m - 3)$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Factorise $x^2 - 81$

Student A

$a^2 = x^2$ and $a = \sqrt{x^2} = x$
$b^2 = 81$ and $b = \sqrt{81} = 9$
$\therefore x^2 - 81 = (x + 9)(x - 9)$

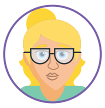
Student B

$a^2 = x^2$ and $a = \sqrt{x^2} = x$
$b^2 = 81$ and $b = \sqrt{81} = 9$
$\therefore x^2 - 81 = (x - 9)(x - 9)$

b. Factorise and simplify $8x^2 - 32y^2$

Student A

$a^2 = 8x^2$ and $a = \sqrt{8x^2} = 4x$
$b^2 = 32y^2$ and $b = \sqrt{32y^2} = 16y$
$\therefore 8x^2 - 32x^2 = (4x + 16y)(4x - 16y)$



Student B

$8x^2 - 32y^2 = 8(x^2 - 4y^2)$
$a^2 = x^2$ and $a = \sqrt{x^2} = x$
$b^2 = 4y^2$ and $b = \sqrt{4y^2} = 2y$
$\therefore 8x^2 - 32x^2 = 8(x + 2y)(x - 2y)$

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. Cindy has added a garage to her square yard. The sides of the yard are y metres long while the length and width of the garage are both 6 metres long. Express the new area of Cindy's yard, excluding the garage, as a difference of two squares.

12. Kevin has a square section in his backyard covered in grass. He redesigns it into a rectangle and now the total area of grass in his yard is given by $g^2 - 16$, where g is one side length of the old square patch of grass in metres. Determine how many metres were taken off the length and added to the width of the grass patch by factorising the given expression.

13. Two table tennis teams, A and B, have exactly t members each. One day, some players from the A team joined the players on the B team for a practice round robin tournament, where each member of one team plays against each member of the other team exactly once. If the total number of games that were played during the tournament is $t^2 - 9$, then how many players from the A team played for the B team?

14. An art gallery in the shape of a square has side lengths $2h$ metres long and contains an open courtyard directly in the centre of the building, accessible only from the gallery. Write a factorised expression for the total interior area of the gallery, excluding the open courtyard, if the area of the courtyard is 49 m^2.

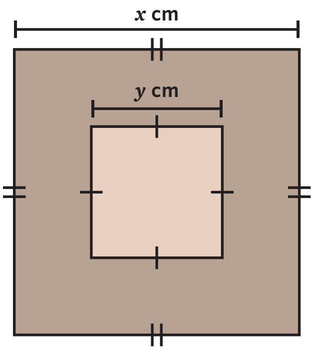
15. Nancy is redeveloping a square area into a playground. She initially adds two metres to both the length and width, then decides to allocate 9 m^2 of this new larger area to a public bathroom. Express the total area of the planned playground, excluding the bathroom, as a fully factorised and simplified binomial product, using x to represent one side length of the original square area.

Reasoning

Question working paths

Mild 16 (a,b,c,e) 🌶	**Medium** 16 (a,b,c,e), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Donnie is programming a machine at his factory so that it can produce similar frames of various sizes. The given picture shows the general dimensions of a wooden frame produced by Donnie's factory.



a. Express the total area of the frame as a difference of two squares.

b. Express the total area of the frame as a difference of two squares when $y = 5$ cm.

c. Factorise the expression from part **b**.

d. What is the length of x if the total area of the frame from part **c** is 96 cm^2?

e. Identify an advantage or disadvantage of having machines produce items that can be made by people.

17. Consider the given expression.

$x(x + 3) - 3(x + 3)$

a. Identify the common binomial factor.

b. Factorise the expression using the common binomial factor.

c. Analyse the answer to part **b** and comment on its nature.

Exam-style

18. In the given expression of the form $a^2 - b^2$, what is the value of b? (1 MARK)

$4x^2 - 49y^2$

A. $2x$ **B.** $2x^2$ **C.** $7y$ **D.** $7y^2$ **E.** $49y$

19. Consider the given expression. (2 MARKS)

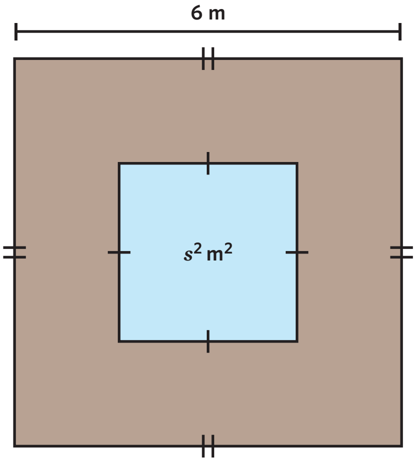
$3x^2 - 108$

a. Factorise the expression by dividing out the HCF. 1 MARK

b. Simplify by factorising the difference of two squares. 1 MARK

20. A wooden deck 6 m long on each side surrounds a square spa pool with an area of s^2 square metres. (2 MARKS)

Express the area of the deck as a binomial product of the form $(a + b)(a - c)$.



21. Factorise and simplify the given expression. (3 MARKS)

$(x + 4)^2 - 81$

Remember this?

22. A particular trail in the Dandenong Ranges is 11.9 kilometres.

If a hiker decides to walk this trail 9 times, how many kilometres would they have travelled?

A. 87 **B.** 99 **C.** 107 **D.** 107.1 **E.** 1071

23. The given table shows the number of students enrolled in all school in Victoria.

	Number of students
Year 10	73 797
Year 11	69 478
Year 12	62 339

What is the total number of students enrolled in Year 10 to 12, to the nearest hundred?

A. 205 000 **B.** 205 600 **C.** 205 614 **D.** 205 700 **E.** 206 000

24. Given that $\frac{1}{5}$ equals to 0.2 as a decimal, what does $\frac{1}{25}$ equal to as a decimal?

A. 0.04 **B.** 0.05 **C.** 0.40 **D.** 0.45 **E.** 1

Back to contents

4E Factorising by grouping

LEARNING INTENTIONS

Students will be able to:

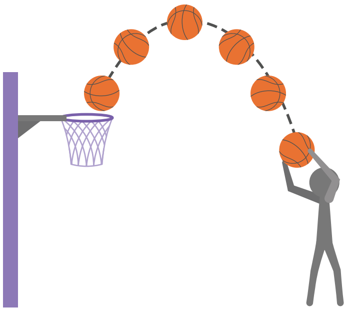
- factorise expressions using the grouping method
- rearrange expressions to factorise by grouping.

Factorising by grouping is used for expressions with four terms and a common binomial factor. When needed, rearranging the expressions first allows for the grouping of pairs of terms that do share a common factor. The entire expression can then be factorised to a binomial product of the form $(a + b)(c + d)$.

KEY TERMS AND DEFINITIONS

- **Grouping** involves pairing a sum or difference of two terms in an expression so that they share a common factor.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Factorising by using binomial factors leads to expressions that provide information about the path of a thrown object. For example, a fully factorised expression of the form $(x + a)(x + b)$ can show the start and end points (a and b) of an object's trajectory, such as the relative positions of a basketball player and the basket into which they are aiming.

Key idea

1. Factorising an expression by grouping involves factorising each group of terms with a common factor.

$$\begin{aligned} ab + ac + bd + cd &= (ab + ac) + (bd + cd) \\ &= a(b + c) + d(b + c) \\ &= (a + d)(b + c) \end{aligned}$$

Back to contents

Worked example 1

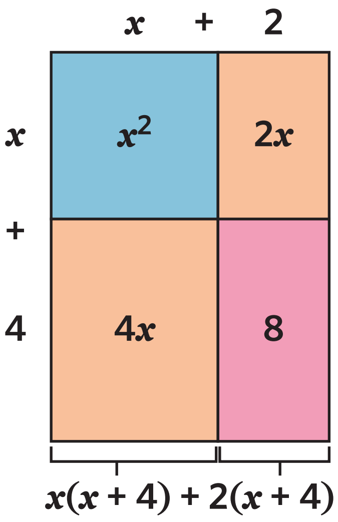
Factorising by grouping

Factorise the following.

a. $x^2 + 4x + 2x + 8$ WE1a

Working	Thinking
$x^2 + 4x + 2x + 8 = (x^2 + 4x) + (2x + 8)$	**Step 1:** Group pairs of terms in order.
$= x(x + 4) + 2(x + 4)$	**Step 2:** Factorise each group of terms by dividing out the HCF.
$= (x + 2)(x + 4)$	**Step 3:** Identify and divide out the common binomial factor to factorise the expression.

Visual support



$$(x^2 + 4x) + (2x + 8) = x(x + 4) + 2(x + 4)$$
$$= (x + 2)(x + 4)$$

b. $x^2 + 5x - 3x - 15$ WE1b

Working	Thinking
$x^2 + 5x - 3x - 15 = (x^2 + 5x) + (-3x - 15)$	**Step 1:** Group pairs of terms in order.
$= x(x + 5) + (-3(x + 5))$ $= x(x + 5) - 3(x + 5)$	**Step 2:** Factorise each group of terms by dividing out the HCF.
$= (x - 3)(x + 5)$	**Step 3:** Identify and divide out the common binomial factor to factorise the expression.

Student practice

Factorise the following.

a. $x^2 + 5x + 2x + 10$

b. $x^2 + 3x - 4x - 12$

Worked example 2

Rearranging expressions to factorise by grouping

Rearrange and factorise the following.

a. $x^2 + 6 + 3x + 2x$

WE2a

Working	Thinking
$x^2 + 6 + 3x + 2x = x^2 + 3x + 2x + 6$	**Step 1:** Rearrange the expression so that each consecutive pair of terms are not like terms and have a common factor.
$= (x^2 + 3x) + (2x + 6)$	**Step 2:** Group pairs of terms in order.
$= x(x + 3) + 2(x + 3)$	**Step 3:** Factorise each group of terms by dividing out the HCF.
$= (x + 2)(x + 3)$	**Step 4:** Identify and divide out the common binomial factor to factorise the expression.

b. $2x^2 - 7 - x + 14x$

WE2b

Working	Thinking
$2x^2 - 7 - x + 14x = 2x^2 + 14x - x - 7$	**Step 1:** Rearrange the expression so that each consecutive pair of terms are not like terms and have a common factor.
$= (2x^2 + 14x) + (-x - 7)$	**Step 2:** Group pairs of terms in order.
$= 2x(x + 7) + (-1(x + 7))$ $= 2x(x + 7) - 1(x + 7)$	**Step 3:** Factorise each group of terms by dividing out the HCF.
$= (2x - 1)(x + 7)$	**Step 4:** Identify and divide out the common binomial factor to factorise the expression.

Student practice

Rearrange and factorise the following.

a. $x^2 + 16 + 2x + 8x$

b. $2x^2 - 3 - x + 6x$

Back to contents

4E Questions

Understanding worksheet

1. Fill in the blanks.

Example

$x^2 + 3x + 2x + 6 = x(x + 3) + \boxed{2}(x + 3)$

a. $x^2 + 4x + 3x + 12 = \boxed{}(x + 4) + 3(x + 4)$

b. $x^2 + 5x + 4x + 20 = x(x + 5) + \boxed{}(x + 5)$

c. $x^2 + 3x - 5x - 15 = x(x + 3)\boxed{}5(x + 3)$

d. $2x^2 - 4x - 3x + 6 = \boxed{}(x - 2) - 3(x - 2)$

2. Match the expressions to their factorised forms.

Expression			**Factorised form**
$x(x + 3) + 5(x + 3)$	●	●	$(x + 7)(x - 4)$
$x(x - 2) + 3(x - 2)$	●	●	$(x + 3)(x + 5)$
$x(x + 7) - 4(x + 7)$	●	●	$(2x + 7)(x - 10)$
$2x(x - 10) + 7(x - 10)$	●	●	$(x + 3)(x - 2)$

3. Fill in the blanks by using the words provided.

grouping **binomial** **common** **rearranged**

Expressions with four terms and a $\boxed{}$ binomial factor can be factorised by $\boxed{}$. When the terms are out of the correct order, an expression can be $\boxed{}$ so that consecutive pairs share a common factor. Factorising by grouping leads to a $\boxed{}$ product of the form $(a + b)(c + d)$.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d) 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Factorise the following. WE1a

a. $x^2 + 5x + 3x + 15$ **b.** $a^2 + 4a + 3a + 12$ **c.** $y^2 + 3y + 6y + 18$ **d.** $p^2 + 4p + 5p + 20$

e. $t^2 + 10t + 3t + 30$ **f.** $2x^2 + 12x + 3x + 18$ **g.** $2r^2 + 14r + 5r + 35$ **h.** $3x^2 + 24x + x + 8$

5. Factorise the following. WE1b

a. $x^2 + 2x - 3x - 6$ **b.** $x^2 + 5x - 3x - 15$ **c.** $y^2 + 6y - 2y - 12$ **d.** $x^2 - 3x + 2x - 6$

e. $r^2 + 7r - 2r - 14$ **f.** $t^2 - 5t - 2t + 10$ **g.** $2x^2 - 6x + 5x - 15$ **h.** $2y^2 - 7y - 2y + 7$

6. Rearrange and factorise the following. WE2a

a. $x^2 + 10 + 5x + 2x$ **b.** $y^2 + 14 + 7y + 2y$ **c.** $p^2 + 12 + 2p + 6p$ **d.** $20 + k^2 + 4k + 5k$

e. $10m + 3m + m^2 + 30$ **f.** $2x^2 + 15 + 6x + 5x$ **g.** $21 + 2n^2 + 14n + 3n$ **h.** $3 + 4x^2 + x + 12x$

Back to contents

7. Rearrange and factorise the following. WE2b

a. $x^2 - 8 + 4x - 2x$ **b.** $y^2 - 10 + 5y - 2y$ **c.** $b^2 - 20 + 4b - 5b$ **d.** $x^2 + 15 - 5x - 3x$

e. $42 + a^2 - 6a - 7a$ **f.** $2t^2 - 15 - 5t + 6t$ **g.** $3x^2 + 4 - 12x - x$ **h.** $24 + 5b^2 - 4b - 30b$

---

8. Which of the options shows the given expression in factorised form?

$2x^2 + 20 - 8x - 5x$

A. $(2x - 5)(x - 4)$

B. $(2x - 5)(x + 4)$

C. $(2x - 5)(x - 4)^2$

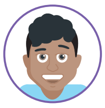
D. $2(x^2 + 10) - 13x$

E. $(2x - 5)(x - 4)(x + 4)$

Spot the mistake

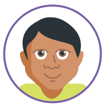
9. Select whether Student A or Student B is incorrect.

a. Factorise $x^2 - 6x - 7x + 42$ by grouping.



Student A

$$\begin{aligned} x^2 - 6x - 7x + 42 &= (x^2 - 6x) + (-7x + 42) \\ &= x(x - 6) + (-7(x - 6)) \\ &= x(x - 6) - 7(x - 6) \\ &= (x - 7)(x - 6) \end{aligned}$$



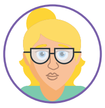
Student B

$$\begin{aligned} x^2 - 6x - 7x + 42 &= (x^2 - 6x) - (7x + 42) \\ &= x(x - 6) - 7(x + 6) \\ &= (x - 7)(x - 6)(x + 6) \end{aligned}$$

b. Rearrange and factorise $18 + 2x^2 + 4x + 9x$ by grouping.

Student A

$$\begin{aligned} 18 + 2x^2 + 4x + 9x &= 2x^2 + 18 + 9x + 4x \\ &= (2x^2 + 18) + (9x + 4x) \\ &= 2(x^2 + 9) + 13x \end{aligned}$$



Student B

$$\begin{aligned} 18 + 2x^2 + 4x + 9x &= 2x^2 + 4x + 9x + 18 \\ &= (2x^2 + 4x) + (9x + 18) \\ &= 2x(x + 2) + 9(x + 2) \\ &= (2x + 9)(x + 2) \end{aligned}$$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. The total area of a rectangular playground can be given by the expression $x(x + 4) + 2(x + 4)$ where x represents an unknown distance in metres. Factorise the expression by grouping in order to determine the dimensions (length and width) of the playground area in terms of x, assuming that length is greater than the width.

11. Oscar is a carpenter who has been cutting a square flat piece of wood with side lengths w cm long. After all the adjustments were made, the area of the wood was given by $w^2 - 4w - 3w + 12$. Factorise the expression to determine how many centimetres were cut from the sides of the piece of wood.

12. Suresh has added an extension to his small cottage with a square floor plan. After the alterations, the total area of the house can be given by $c^2 + 42 + 7c + 6c$ where c represents the original side lengths of the cottage, in metres. Determine how many metres were added to the length and width of the cottage's floor plan by factorising the expression.

13. At the start of the year, Tony had exactly the same number (n) of pairs of shoes and socks. By the end of the year, if he mixed and matched every single pair of shoes he owns with every single pair of socks exactly once, there would be a total of $n^2 - 18 + 6n - 3n$ combinations. How many pairs of shoes did Tony add to his collection over the course of the year?

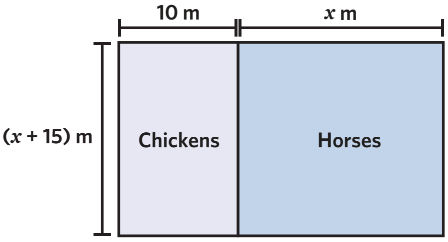
14. A farmhouse with a square floor plan f metres long on each side is located in the north-west corner of a large rectangular property. The total area of the property and farmhouse is given by $f^2 + 300f + 25f + 7500\text{ m}^2$. What is the length of the longest side of the property if the area of the farmhouse is 121 m^2?

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Hannah is developing her land so that she can accommodate some chickens and horses. She has drawn up a diagram and worked out some of the information about the paddock using recommended enclosure area sizes for each type of animal.



a. Formulate an expression for the area designated for chickens.

b. Formulate an expression for the total area of the paddock and factorise it.

c. The chickens will have 1180 m^2 of roaming space in Hannah's paddock. Calculate the total area of the entire paddock.

d. It is recommended that each horse in a paddock has at least 4046 m^2 of roaming space. How many horses can Hannah have in her paddock?

e. Identify a reason for minimum enclosure requirements for domestic animals.

16. Consider the given expression.

$x^2 + 8 - 2x - 4x$

a. The expression can be rearranged to $x^2 - 4x - 2x + 8$. Factorise the rearranged expression by grouping.

b. The expression can also be rearranged to $x^2 - 2x - 4x + 8$. Factorise the rearranged expression by grouping.

c. Compare the results from parts **a** and **b** and comment on the required order of terms when factorising by grouping.

Exam-style

17. Determine which of the options shows a common binomial factor of consecutive pairs of terms in the given expression. (1 MARK)

$3x^2 - 12x - 7x + 28$

A. $-19x$
B. $x - 4$
C. $x + 4$
D. $3x + 7$
E. $x^2 - 4x$

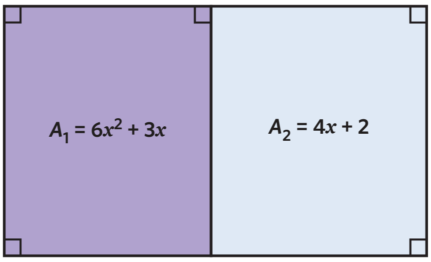
18. Consider the given expression. (3 MARKS)

$p^2 - 4p - 13p + 52$

a. Factorise by grouping. 2 MARKS

b. Evaluate for $p = 15$. 1 MARK

19. Show that the total area of the rectangle can be given by $A = (3x + 2)(2x + 1)$. (3 MARKS)



20. Factorise $4x^2 - 10 + 5x - 8x$ by grouping. (3 MARKS)

Remember this?

21. Tori stacks books. Each book has the same height.

The height of 4 books is 12 cm.

The height of 9 books is 27 cm.

What is the height of 48 books?

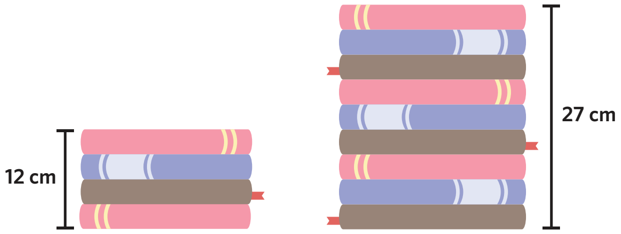
A. 38 cm

B. 54 cm

C. 128 cm

D. 144 cm

E. 330 cm



22. The number of people watching the FIFA world cup has increased by 40% from 2018 to 2023.

Which of the following options represents this increase?

A. 2018 – 47 300 200
2023 – 67 800 000

B. 2018 – 48 500 000
2023 – 67 800 000

C. 2018 – 48 500 000
2023 – 67 900 000

D. 2018 – 48 900 000
2023 – 82 600 000

E. 2018 – 50 000 000
2023 – 100 000 000

23. Layla uses a travel card to pay for train trips around Australia.

At the beginning of the day, she had a balance of \$20 on her travel card.

She makes a total of four trips that day.

Each trip costs \$1.60.

Which sequence shows the balance on her card after she has used the card three consecutive times on that day?

A. \$20.00; \$19.60; \$18.00

B. \$20.00; \$17.40; \$15.80

C. \$18.60; \$17.00; \$15.40

D. \$18.40; \$16.80; \$15.20

E. \$16.80; \$15.20; \$13.00

Back to contents

4F Simplifying algebraic fractions – multiplication and division

LEARNING INTENTIONS

Students will be able to:

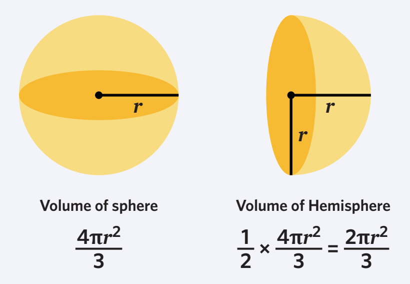
- simplify algebraic fractions using a common factor
- multiply algebraic fractions
- divide algebraic fractions.

Algebraic fractions have numerators and denominators that consist of algebraic terms, expressions, or a combination of both. Simplifying algebraic fractions combines proportional reasoning and algebra skills. The same rules can be applied to the manipulation of algebraic fractions as those used for ordinary fractions, together with the application of algebra skills such as factorising.

KEY TERMS AND DEFINITIONS

- The **reciprocal of a fraction** can be obtained by swapping the values or expressions in the numerator with those in the denominator.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Formulas that describe the features of physical three dimensional objects are often complex and involve algebraic fractions. Manipulating and simplifying algebraic fractions is an integral part of applying these kinds of formulas in branches of mathematics, science, and engineering.

Key ideas

1. Algebraic fractions can be simplified using the highest common factor (HCF) of the numerator and denominator.

The HCF can be a pronumeral or number:

$$\frac{ab + ac}{ad + ae} = \frac{\cancel{a}(b + c)}{\cancel{a}(d + e)} = \frac{b + c}{d + e}$$

The HCF can be an expression:

$$\frac{ab + bc}{ad + dc} = \frac{b\cancel{(a + c)}}{d\cancel{(a + c)}} = \frac{b}{d}$$

2. Algebraic fractions can be **cross-simplified** during multiplication.

$$\frac{a\cancel{(b + c)}}{\cancel{d}} \times \frac{\cancel{d}}{\cancel{b + c}} = a$$

$$\frac{a\cancel{d}\cancel{(b + c)}}{\cancel{d}\cancel{(b + c)}}$$

Worked example 1

Simplifying algebraic fractions

Simplify the following.

a. $\dfrac{12x + 9}{3}$ WE1a

Working

$$\frac{12x + 9}{3} = \frac{3(4x + 3)}{3}$$

$$= \frac{3 \div 3(4x + 3)}{3 \div 3}$$

$$= \frac{4x + 3}{1} = 4x + 3$$

Thinking

Step 1: Factorise the expression in the numerator.

Step 2: Identify and divide out the HCF of the numerator and denominator.

Step 3: Simplify the fraction.

b. $\dfrac{5(x - 3)(x + 2)}{15(x - 3)}$ WE1b

Working

$$\frac{5(x - 3)(x + 2)}{15(x - 3)} = \frac{5\cancel{(x - 3)}(x + 2)}{15\cancel{(x - 3)}}$$

$$= \frac{5(x + 2)}{15}$$

$$= \frac{5 \div 5(x + 2)}{15 \div 5}$$

$$= \frac{x + 2}{3}$$

Thinking

Step 1: Identify and divide out the common binomial factor of the numerator and denominator.

Step 2: Identify and divide out the HCF of the numerator and denominator.

Step 3: Simplify the fraction.

c. $\dfrac{8x - 16}{2x - 4}$ WE1c

Working

$$\frac{8x - 16}{2x - 4} = \frac{8(x - 2)}{2(x - 2)}$$

$$= \frac{8\cancel{(x - 2)}}{2\cancel{(x - 2)}}$$

$$= \frac{8}{2} = 4$$

Thinking

Step 1: Factorise the expressions in the numerator and the denominator.

Step 2: Identify and divide out the common binomial factor of the numerator and denominator.

Step 3: Simplify the fraction.

Student practice

Simplify the following.

a. $\dfrac{15x + 10}{5}$ **b.** $\dfrac{2(x + 1)(x - 6)}{14(x + 1)}$ **c.** $\dfrac{9x - 18}{3x - 6}$

Worked example 2

Multiplying algebraic fractions

Simplify the following.

WE2a

a. $\frac{4x^2}{5} \times \frac{20}{4x}$

Working

$$\frac{4x^2}{5} \times \frac{20}{4x} = \frac{4x^2 \div 4x}{5} \times \frac{20}{4x \div 4x}$$

$$= \frac{x}{5} \times \frac{20}{1}$$

$$= \frac{x}{5 \div 5} \times \frac{20 \div 5}{1}$$

$$= \frac{x}{1} \times \frac{4}{1}$$

$$= \frac{x \times 4}{1 \times 1} = \frac{4x}{1} = 4x$$

Thinking

Step 1: Identify and divide out the HCF of the numerator of the first fraction and the denominator of the second fraction.

Step 2: Identify and divide out the HCF of the denominator of the first fraction and the numerator of the second fraction.

Step 3: Multiply the fractions and simplify.

WE2b

b. $\frac{x - 1}{4(x + 5)} \times \frac{2(x + 5)}{(x - 1)(x + 2)}$

Working

$$\frac{x - 1}{4(x + 5)} \times \frac{2(x + 5)}{(x - 1)(x + 2)}$$

$$= \frac{\cancel{x - 1}}{4(x + 5)} \times \frac{2(x + 5)}{\cancel{(x - 1)}(x + 2)}$$

$$= \frac{1}{4(x + 5)} \times \frac{2(x + 5)}{x + 2}$$

$$= \frac{1}{4 \div 2\cancel{(x + 5)}} \times \frac{2 \div 2\cancel{(x + 5)}}{x + 2}$$

$$= \frac{1}{2} \times \frac{1}{x + 2}$$

$$= \frac{1 \times 1}{2 \times (x + 2)} = \frac{1}{2(x + 2)}$$

Thinking

Step 1: Identify and divide out the HCF of the numerator of the first fraction and the denominator of the second fraction.

Step 2: Identify and divide out the HCF of the denominator of the first fraction and the numerator of the second fraction.

Step 3: Multiply the fractions and simplify.

Student practice

Simplify the following.

a. $\frac{2x^2}{9} \times \frac{45}{2x}$

b. $\frac{x + 3}{9(x - 2)} \times \frac{3(x - 2)}{(x - 7)(x + 3)}$

Back to contents

Worked example 3

Dividing algebraic fractions

Simplify the following.

a. $\frac{9x}{4} \div \frac{27x^2}{16}$

WE3a

Working

$$\frac{9x}{4} \div \frac{27x^2}{16} = \frac{9x}{4} \times \frac{16}{27x^2}$$

$$= \frac{9x \div 9x}{4} \times \frac{16}{27x^2 \div 9x}$$

$$= \frac{1}{4} \times \frac{16}{3x}$$

$$= \frac{1}{4 \div 4} \times \frac{16 \div 4}{3x}$$

$$= \frac{1}{1} \times \frac{4}{3x}$$

$$= \frac{1 \times 4}{1 \times 3x} = \frac{4}{3x}$$

Thinking

Step 1: Convert the division to multiplication by multiplying the first fraction (dividend) by the reciprocal of the second fraction (divisor).

Step 2: Identify and divide out any common factors of the numerator of the first fraction and the denominator of the second fraction.

Step 3: Identify and divide out any common factors of the denominator of the first fraction and the numerator of the second fraction.

Step 4: Multiply the fractions.

b. $\frac{2x}{5(x+7)} \div \frac{6x}{x+7}$

WE3b

Working

$$\frac{2x}{5(x+7)} \div \frac{6x}{x+7} = \frac{2x}{5(x+7)} \times \frac{x+7}{6x}$$

$$= \frac{2x \div 2x}{5(x+7)} \times \frac{x+7}{6x \div 2x}$$

$$= \frac{1}{5(x+7)} \times \frac{x+7}{3}$$

$$= \frac{1}{5\cancel{(x+7)}} \times \frac{\cancel{x+7}}{3}$$

$$= \frac{1}{5} \times \frac{1}{3}$$

$$= \frac{1 \times 1}{5 \times 3} = \frac{1}{15}$$

Thinking

Step 1: Convert the division to multiplication by multiplying the first fraction (dividend) by the reciprocal of the second fraction (divisor).

Step 2: Identify and divide out any common factors of the numerator of the first fraction and the denominator of the second fraction.

Step 3: Identify and divide out any common factors of the denominator of the first fraction and the numerator of the second fraction.

Step 4: Multiply the fractions.

Student practice

Simplify the following.

a. $\frac{5x}{6} \div \frac{35x^2}{18}$

b. $\frac{3x}{7(x-5)} \div \frac{12x}{x-5}$

Back to contents

4F Questions

Understanding worksheet

1. Draw a strike through the common binomial factors that can be cross-simplified.

Example

$$\frac{3(c-5)}{\cancel{c+9}} \times \frac{2\cancel{(c+9)}}{5}$$

a. $\frac{4(x+2)}{7} \times \frac{14}{x+2}$

b. $\frac{9(x+1)}{x-8} \times \frac{3(x-8)}{4}$

c. $\frac{(y+2)(y-7)}{2(y+3)} \times \frac{y+5}{y+2}$

d. $\frac{3(p-5)(p-4)}{2(p+3)} \times \frac{3(p-4)(p+3)}{p-5}$

2. Convert each division to multiplication by filling in the blanks.

Example

$$\frac{2x}{5(x-2)} \div \frac{4}{x-2} = \frac{2x}{5(x-2)} \times \boxed{\frac{x-2}{4}}$$

a. $\frac{3x}{8} \div \frac{15x}{14} = \frac{3x}{8} \times \boxed{}$

b. $\frac{2(x-1)}{3x} \div \frac{x-1}{9x} = \boxed{} \times \frac{9x}{x-1}$

c. $\frac{2(y+3)}{7y} \div \frac{y+3}{3y} = \frac{2(y+3)}{7y} \times \boxed{}$

d. $\frac{7(x+4)(x-1)}{3(x+1)} \div \frac{4(x-1)}{21x} = \frac{7(x+4)(x-1)}{3(x+1)} \times \boxed{}$

3. Fill in the blanks by using the words provided.

factorised **common** **division** **numerator**

The ______ and denominator of an algebraic fraction can consist of pronumerals, algebraic terms or expressions. Rules for the multiplication and ______ of ordinary fractions, consisting of integers only, can be applied to algebraic fractions. Any ______ factors, including those that are algebraic, can be divided out only after all expressions in the fraction have been ______ .

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Simplify the following. **WE1a**

a. $\dfrac{2x+10}{2}$ **b.** $\dfrac{3x+6}{9}$ **c.** $\dfrac{9p-15}{3}$ **d.** $\dfrac{8y-12}{2}$

e. $\dfrac{5t+35}{10}$ **f.** $\dfrac{18x-81}{27}$ **g.** $\dfrac{4x^2-20x}{2x}$ **h.** $\dfrac{18x^2+63x}{3x}$

---

5. Simplify the following. **WE1b,c**

a. $\dfrac{2(x+1)}{3(x+1)}$ **b.** $\dfrac{2y(y-3)}{y(y-3)}$ **c.** $\dfrac{4(x-1)(x+4)}{8(x-1)}$ **d.** $\dfrac{6r+12}{r+2}$

e. $\dfrac{3x+9}{2x+6}$ **f.** $\dfrac{14-7t}{8-4t}$ **g.** $\dfrac{12m-18}{6m-9}$ **h.** $\dfrac{5y^2-25y}{2y-10}$

---

6. Simplify the following. **WE2a**

a. $\dfrac{6x}{5}\times\dfrac{15}{2x}$ **b.** $\dfrac{14r}{9}\times\dfrac{3}{28r}$ **c.** $\dfrac{49}{12t}\times\dfrac{4t}{21}$ **d.** $\dfrac{7y^2}{8}\times\dfrac{16}{21y}$

e. $\dfrac{3p^2}{25}\times\dfrac{10}{27p}$ **f.** $\dfrac{36}{13x^2}\times\dfrac{39x}{4}$ **g.** $\dfrac{63y^2}{6x}\times\dfrac{12x}{7y}$ **h.** $\dfrac{12n^3}{25}\times\dfrac{35}{18n^2}$

---

7. Simplify the following. **WE2b**

a. $\dfrac{x+2}{x+4}\times\dfrac{2(x+4)}{x+2}$ **b.** $\dfrac{(x+7)(x-3)}{x-6}\times\dfrac{x-6}{x-3}$

c. $\dfrac{3(y+1)}{y+5}\times\dfrac{y+5}{(y+1)(y-2)}$ **d.** $\dfrac{6(t+3)}{3(t-4)}\times\dfrac{(t+5)(t-4)}{t+3}$

e. $\dfrac{4(p-1)(p-3)}{p+4}\times\dfrac{3(p+4)}{8(p-1)}$ **f.** $\dfrac{2x(x+1)}{x+4}\times\dfrac{x+4}{x(x+1)}$

g. $\dfrac{10(n-2)(n+5)}{3n(n+1)}\times\dfrac{n(n+1)}{15(n+5)}$ **h.** $\dfrac{6(x+4)}{x(2x-1)}\times\dfrac{5x(2x-1)(x+3)}{12(x+4)}$

---

8. Simplify the following. **WE3a**

a. $\dfrac{2x}{5}\div\dfrac{8x}{25}$ **b.** $\dfrac{21}{4y}\div\dfrac{7}{16y}$ **c.** $\dfrac{2t^2}{3}\div\dfrac{t}{15}$ **d.** $\dfrac{3x^2}{28}\div\dfrac{6x}{7}$

e. $\dfrac{35}{12p}\div\dfrac{5}{3p^2}$ **f.** $\dfrac{10y}{39}\div\dfrac{50y^2}{13}$ **g.** $\dfrac{18n^3}{45}\div\dfrac{32n^2}{15}$ **h.** $\dfrac{81y^2}{6x^3}\div\dfrac{72y}{12x^2}$

---

9. Simplify the following. **WE3b**

a. $\dfrac{2(x+1)}{9}\div\dfrac{x+1}{3}$ **b.** $\dfrac{3(x+5)}{x-2}\div\dfrac{x+5}{x-2}$

c. $\dfrac{4t}{5(x-1)}\div\dfrac{12t}{x-1}$ **d.** $\dfrac{8}{3(y+2)}\div\dfrac{4}{9(y+2)}$

e. $\dfrac{n-7}{(n+4)(n-1)}\div\dfrac{n-7}{2(n-1)}$ **f.** $\dfrac{p+7}{5(p+1)}\div\dfrac{2(p+7)}{15(p+1)}$

g. $\dfrac{6x}{(x-2)(x+3)}\div\dfrac{18x}{3(x-2)}$ **h.** $\dfrac{2(m+4)(m-1)}{20(m-3)}\div\dfrac{6(m-1)}{5(m-3)}$

Back to contents

10. Which of the options shows the given expression once it has been simplified?

$$\frac{9(x+1)}{6(x-1)} \div \frac{3(x+1)(x+3)}{4(x-1)}$$

A. $\frac{2}{x+3}$

B. $\frac{x+3}{2}$

C. $\frac{2(x+1)}{x+3}$

D. $2(x+3)$

E. $\frac{9(x+1)^2(x+3)}{8(x-1)^2}$

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. Simplify $\frac{4x-20}{2x-10}$

Student A

$$\frac{4x-20}{2x-10} = (4 \div 2)x - (20 \div 10)$$
$$= 2x - 2$$
$$= 2(x-1)$$

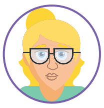
Student B

$$\frac{4x-20}{2x-10} = \frac{4(x-5)}{2(x-5)}$$
$$= \frac{4\cancel{(x-5)}}{2\cancel{(x-5)}}$$
$$= \frac{4}{2} = 2$$

b. Simplify $\frac{5x^2}{3(x-1)} \div \frac{15x}{(x-1)(x+5)}$.

Student A

$$\frac{5x^2}{3(x-1)} \div \frac{15x}{(x-1)(x+5)}$$
$$= \frac{3(x-1)}{5x^2} \times \frac{15x}{(x-1)(x+5)}$$
$$= \frac{3\cancel{(x-1)}}{5x^2} \times \frac{15x}{\cancel{(x-1)}(x+5)}$$
$$= \frac{3}{5x^2} \times \frac{15x}{x+5}$$
$$= \frac{3}{5x^2 \div 5x} \times \frac{15x \div 5x}{x+5}$$
$$= \frac{3}{x} \times \frac{3}{x+5}$$
$$= \frac{3 \times 3}{x \times (x+5)} = \frac{9}{x(x+5)}$$



Student B

$$\frac{5x^2}{3(x-1)} \div \frac{15x}{(x-1)(x+5)}$$
$$= \frac{5x^2}{3(x-1)} \times \frac{(x-1)(x+5)}{15x}$$
$$= \frac{5x^2 \div 5x}{3(x-1)} \times \frac{(x-1)(x+5)}{15x \div 5x}$$
$$= \frac{x}{3(x-1)} \times \frac{(x-1)(x+5)}{3}$$
$$= \frac{x}{3\cancel{(x-1)}} \times \frac{\cancel{(x-1)}(x+5)}{3}$$
$$= \frac{x}{3} \times \frac{x+5}{3}$$
$$= \frac{x \times (x+5)}{3 \times 3} = \frac{x(x+5)}{9}$$

Back to contents

Problem solving

Question working paths

Mild 12, 13, 14 🌶	**Medium** 13, 14, 15 🌶🌶	**Spicy** 14, 15, 16 🌶🌶🌶

12. Joan the electrician works out the total cost of a job by using the equation $C = 90x + 75$ where x is the number of hours spent on the job and \$75 is the callout fee. She has agreed to charge her friend Jilly a third of this amount. Formulate an algebraic fraction and simplify it to show the equation representing the cost of Jilly's job.

13. Dolph's profits for the entire week can be modelled by the expression $280x - 490$, where x represents the number of orders he receives. Formulate a simplified expression for the average profit made in one day of Dolph's 7-day work week.

14. The total floor area of a chicken coop is given by the expression $6x^2 + 27x$ where x is an unknown length in metres. Formulate a simplified expression for the area per chicken, if there are a total of 12 chickens living in the coop altogether.

15. Geoff is working out the best investment plan for his recently acquired inheritance. He comes up with an equation, $G = \frac{900(y-5)}{21y} \times \frac{700y^2}{2(y-2)(y-5)}$, representing his savings after a number of years (y) of investing using a customised plan. Generate a simplified expression for G.

16. Chelsea is analysing the value of an item she sells in her online store. She used the expression $D = \frac{t+3}{6(t-2)}$ to represent the demand and $E = \frac{10}{3(t-2)}$ to represent the total effort put into the production of the item. In both equations, t represents the number of the items currently available for sale. Determine the ratio $\frac{D}{E}$ representing the overall value of the item.

Reasoning

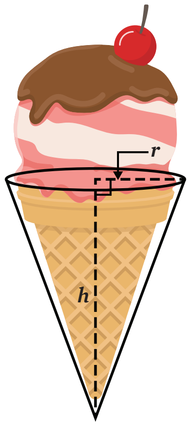
Question working paths

Mild 17 (a,b,d) 🌶	**Medium** 17 (a,b,d), 18 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

17. Sandy owns an ice cream parlour, where she provides waffle cones of three different sizes. The volume of a large cone is given by the formula

$$V_{large} = \frac{\pi r^2 h}{3}$$

where r represents the radius and h represents the height of the cone.



- **a.** A medium cone's volume is $\frac{2}{3}$ of a large cone. Formulate an expression for the volume of a medium cone by simplifying the expression $\frac{2}{3} \times V_{large}$.
- **b.** A small cone's volume is $\frac{3}{5}$ the size of a medium cone. Use the expression from part **a** to formulate an expression for the volume of a small cone.
- **c.** Sandy claims that the large cone's volume is 2.5 times that of a small cone. Determine whether her claim is true by simplifying the ratio of the volume of a large cone to the volume of a small cone.
- **d.** Identify a reason why cafes and restaurants often offer the same meal or product in different portion sizes.

18. Simplify the expressions given in parts **a** and **b**.

- **a.** $\frac{8x}{15} \div \frac{4x}{5}$
- **b.** $\frac{15}{8x} \times \frac{4x}{5}$
- **c.** Compare the answers from parts **a** and **b**. Comment on the importance of the order in which the fractions are reciprocated when converting division to multiplication.

Back to contents

Exam-style

19. The perimeter of an equilateral triangle is given by $12x + 21$. One of its side lengths is given by (1 MARK)

A. $\frac{4x + 7}{3}$ **B.** $4x + 7$ **C.** $3x + 7$ **D.** $3(4x + 7)$ **E.** $3(12x + 21)$

20. Consider the given expression. (3 MARKS)

$$\frac{2x}{(x - 6)(x + 1)} \div \frac{8x}{x - 6}$$

a. Convert the division to multiplication. 1 MARK

b. Simplify the expression by dividing out common factors. 2 MARKS

21. The area of a triangle is given by $A = \frac{8x}{2} \times \frac{20}{2x - 4}$, where x is an unknown length in centimetres. (2 MARKS)

Simplify A by multiplying the fractions.

22. Simplify $\frac{6}{7(x - 1)} \times \frac{(x - 1)(x - 3)}{9}$. (2 MARKS)

Remember this?

23. Mitch and Jess are collecting money for a school fundraiser over 30 days.

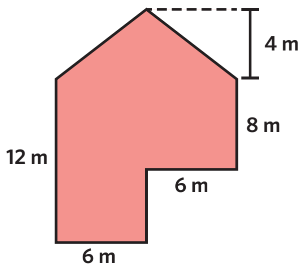
If Jess collected $25 more, she will have collected exactly twice as much as Mitch.

Which of the following options shows how much money they could have earned?

	Mitch	**Jess**
A.	$15	$15
B.	$25	$50
C.	$35	$60
D.	$45	$65
E.	$55	$110

24. James is building a decking for a new house in the backyard.

The area which James will be decking is shown.



What is the total area that James will be decking?

A. 41 m^2 **B.** 82 m^2 **C.** 96 m^2 **D.** 144 m^2 **E.** 164 m^2

25. Miguel buys a value box of chocolates that contains 12 blocks.

She pays with a $10 note.

She gets 30 cents change.

How much does each block of chocolate cost, rounded to the nearest ten cents?

A. $0.40 **B.** $0.80 **C.** $0.90 **D.** $1.20 **E.** $3.30

4G Simplifying algebraic fractions – addition and subtraction

LEARNING INTENTIONS

Students will be able to:

- determine the lowest common denominator of algebraic fractions
- convert algebraic fractions to equivalent fractions with a common denominator before adding or subtracting
- add and subtract numerators of algebraic fractions by simplifying.

Simplifying algebraic fractions using addition and subtraction requires the fractions to have common denominators. The numerators or denominators may consist of algebraic terms or expressions, and so it is important that proportional reasoning is used together with algebra skills in order to manipulate and simplify algebraic fractions.

KEY TERMS AND DEFINITIONS

- The **lowest common multiple (LCM)** is the smallest number that is a multiple of two or more numbers.
- The **lowest common denominator (LCD)** is the lowest common multiple (LCM) of the denominators of two or more fractions.
- **Equivalent fractions** have different numerators and denominators but represent and are equal to the same value.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Dimj/Shutterstock.com

The problem of division of the stakes is a probability scenario where two equally contributing players participate in a game of chance, where after a number of rounds the final winner takes all the money. Algebraic fractions can be used in an attempt to determine the portion of the winnings each player should get, if the game finishes early due to unforeseen circumstances.

Key ideas

1. When converting a fraction to an equivalent fraction, the numerator and denominator must be multiplied or divided by the same value to maintain the proportion represented by the fraction.

$$\frac{a}{b} = \frac{a \times c}{b \times c} = \frac{ac}{bc}$$

The value of the fraction remains the same.

2. Fractions must be converted to equivalent fractions with a common denominator before they can be added or subtracted.

$$\begin{aligned}\frac{a}{b} + \frac{c}{d} &= \frac{d \times a}{d \times b} + \frac{b \times c}{b \times d}\\ &= \frac{ad}{bd} + \frac{bc}{bd}\\ &= \frac{ad + bc}{bd}\end{aligned}$$

Worked example 1

Adding algebraic fractions

Simplify the following.

a. $\frac{3x}{2} + \frac{x}{4}$

WE1a

Working

Multiples of 2: 2, **4**, …

Multiples of 4: **4**, …

$$\frac{3x}{2} + \frac{x}{4} = \frac{2 \times 3x}{2 \times 2} + \frac{x}{4}$$

$$= \frac{6x}{4} + \frac{x}{4}$$

$$= \frac{6x + x}{4} = \frac{7x}{4}$$

Thinking

Step 1: Determine the LCD by finding the LCM of the denominators.

Step 2: Rewrite the addition with equivalent fractions with a common denominator equal to the LCD.

Step 3: Simplify the numerator.

b. $\frac{x + 2}{2} + \frac{x + 4}{3}$

WE1b

Working

Multiples of 2: 2, 4, **6**, …

Multiples of 3: 3, **6**, …

$$\frac{x + 2}{2} + \frac{x + 4}{3} = \frac{3 \times (x + 2)}{3 \times 2} + \frac{2 \times (x + 4)}{2 \times 3}$$

$$= \frac{3(x + 2)}{6} + \frac{2(x + 4)}{6}$$

$$= \frac{3(x + 2) + 2(x + 4)}{6}$$

$$= \frac{3x + 6 + 2x + 8}{6} = \frac{5x + 14}{6}$$

Thinking

Step 1: Determine the LCD by finding the LCM of the denominators.

Step 2: Rewrite the addition with equivalent fractions with a common denominator equal to the LCD.

Step 3: Simplify the numerator.

c. $\frac{3}{2x} + \frac{4}{x}$

WE1c

Working

Multiples of x: x, $\mathbf{2x}$, …

Multiples of $2x$: $\mathbf{2x}$, …

$$\frac{3}{2x} + \frac{4}{x} = \frac{3}{2x} + \frac{2 \times 4}{2 \times x}$$

$$= \frac{3}{2x} + \frac{8}{2x}$$

$$= \frac{3 + 8}{2x} = \frac{11}{2x}$$

Thinking

Step 1: Determine the LCD by finding the LCM of the denominators.

Step 2: Rewrite the addition with equivalent fractions with a common denominator equal to the LCD.

Step 3: Simplify the numerator.

Student practice

Simplify the following.

a. $\frac{2x}{3} + \frac{x}{6}$

b. $\frac{x + 1}{3} + \frac{x + 3}{5}$

c. $\frac{2}{3x} + \frac{5}{x}$

Worked example 2

Subtracting algebraic fractions

Simplify the following.

a. $\frac{x}{4}-\frac{3x}{2}$

WE2a

Working	Thinking
Multiples of 2: 2, **4**, ... Multiples of 4: **4**, ...	**Step 1:** Determine the LCD by finding the LCM of the denominators.
$\frac{x}{4}-\frac{3x}{2}=\frac{x}{4}-\frac{2\times 3x}{2\times 2}$ $=\frac{x}{4}-\frac{6x}{4}$	**Step 2:** Rewrite the subtraction with equivalent fractions with a common denominator equal to the LCD.
$=\frac{x-6x}{4}$ $=-\frac{5x}{4}$	**Step 3:** Simplify the numerator.

b. $\frac{x+4}{2}-\frac{x-1}{5}$

WE2b

Working	Thinking
Multiples of 2: 2, 4, 6, 8, **10**, ... Multiples of 5: 5, **10**, ...	**Step 1:** Determine the LCD by finding the LCM of the denominators.
$\frac{x+4}{2}-\frac{x-1}{5}=\frac{5\times(x+4)}{5\times 2}-\frac{2\times(x-1)}{2\times 5}$ $=\frac{5(x+4)}{10}-\frac{2(x-1)}{10}$	**Step 2:** Rewrite the subtraction with equivalent fractions with a common denominator equal to the LCD.
$=\frac{5(x+4)-2(x-1)}{10}$ $=\frac{5x+20-2x+2}{10}=\frac{3x+22}{10}$	**Step 3:** Simplify the numerator.

c. $\frac{3}{x^2}-\frac{2}{x}$

WE2c

Working	Thinking
Powers of x: x, $\boldsymbol{x^2}$, ... Powers of x^2: $\boldsymbol{x^2}$, ...	**Step 1:** Determine the LCD by finding the LCM of the denominators.
$\frac{3}{x^2}-\frac{2}{x}=\frac{3}{x^2}-\frac{x\times 2}{x\times x}$ $=\frac{3}{x^2}-\frac{2x}{x^2}$	**Step 2:** Rewrite the subtraction with equivalent fractions with a common denominator equal to the LCD.
$=\frac{3-2x}{x^2}$	**Step 3:** Simplify by writing the subtraction as a single fraction.

Student practice

Simplify the following.

a. $\frac{x}{6}-\frac{4x}{3}$ **b.** $\frac{x+3}{2}-\frac{x-2}{3}$ **c.** $\frac{5}{x^2}-\frac{4}{x}$

Back to contents

4G Questions

Understanding worksheet

1. Match each fraction with an equivalent fraction.

Fraction	**Equivalent fraction**
$\frac{x}{4}$ ●	● $\frac{3x}{4}$
$\frac{2x}{7}$ ●	● $\frac{6x}{x^2}$
$\frac{15x}{20}$ ●	● $\frac{6x}{21}$
$\frac{6}{x}$ ●	● $\frac{5x}{20}$

2. Fill in the blanks with the LCD for each calculation.

Example

$$\frac{2x}{3} + \frac{3x}{5} = \frac{10x + 9x}{\boxed{15}}$$

a. $\frac{4x}{3} + \frac{x}{6} = \frac{8x + x}{\boxed{}}$ **b.** $\frac{5x}{2} - \frac{x}{7} = \frac{35x - 2x}{\boxed{}}$ **c.** $\frac{2}{3x} + \frac{5}{x} = \frac{2 + 15}{\boxed{}}$ **d.** $\frac{8}{x} - \frac{6}{x^2} = \frac{8x - 6}{\boxed{}}$

3. Fill in the blanks by using the words provided.

multiplied **common** **LCD** **proportions**

Algebraic fractions can be added or subtracted once they have been converted to equivalent fractions with a $\boxed{}$ denominator. To maintain the $\boxed{}$ of a fraction when converting to an equivalent fraction, the numerator and denominator must be $\boxed{}$ or divided by the same value. The $\boxed{}$ of two or more fractions is the LCM of their denominators.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Simplify the following. WE1a,2a

a. $\frac{3a}{4} + \frac{a}{2}$ **b.** $\frac{x}{2} + \frac{x}{3}$ **c.** $\frac{2y}{3} + \frac{y}{4}$ **d.** $\frac{2t}{3} - \frac{t}{6}$

e. $\frac{2x}{3} + \frac{3x}{2}$ **f.** $\frac{2m}{27} - \frac{5m}{9}$ **g.** $\frac{2x}{9} + \frac{7x}{6}$ **h.** $\frac{4p}{10} - \frac{3p}{15}$

---

5. Simplify the following. WE1b,2b

a. $\frac{t + 2}{6} + \frac{t + 3}{2}$ **b.** $\frac{x + 2}{3} + \frac{x + 5}{2}$ **c.** $\frac{p + 7}{5} + \frac{p - 4}{2}$ **d.** $\frac{y - 1}{7} + \frac{y + 3}{3}$

e. $\frac{x + 3}{2} - \frac{x - 1}{3}$ **f.** $\frac{2r + 5}{2} + \frac{3r + 4}{5}$ **g.** $\frac{2n - 1}{8} - \frac{n - 4}{6}$ **h.** $\frac{3x + 1}{6} - \frac{x - 3}{18}$

6. Simplify the following. WE1c,2

a. $\frac{2}{x} + \frac{3}{2x}$ **b.** $\frac{4}{3x} + \frac{5}{x}$ **c.** $\frac{3}{t} + \frac{7}{5t}$ **d.** $\frac{5}{2y} - \frac{2}{y}$

e. $\frac{5}{6x} + \frac{3}{2x}$ **f.** $\frac{4}{9r} - \frac{2}{3r}$ **g.** $\frac{3}{4n} - \frac{8}{5n}$ **h.** $\frac{-3}{7y} + \frac{5}{3y}$

7. Simplify the following. WE1,2c

a. $\frac{1}{x^2} + \frac{2}{x}$ **b.** $\frac{3}{y} - \frac{2}{y^2}$ **c.** $\frac{4}{t} + \frac{3}{t^2}$ **d.** $\frac{5}{x^2} - \frac{2}{x}$

e. $\frac{3}{a^2} - \frac{7}{a}$ **f.** $\frac{-3}{p} + \frac{5}{p^2}$ **g.** $\frac{7}{2m^2} - \frac{2}{m}$ **h.** $\frac{-6}{n} + \frac{5}{2n^2}$

8. Which of the options is equivalent to $\frac{5}{3x} - \frac{3}{x}$?

A. $\frac{2}{2x}$ **B.** $\frac{2}{3x}$ **C.** $\frac{2}{4x}$ **D.** $-\frac{4}{3x}$ **E.** $-\frac{10}{3x}$

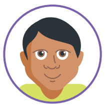
Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Simplify $\frac{3p}{5} + \frac{4p}{7}$

Student A

$$\begin{aligned}\frac{3p}{5} + \frac{4p}{7} &= \frac{7 \times 3p}{7 \times 5} + \frac{5 \times 4p}{5 \times 7}\\ &= \frac{21p}{35} + \frac{20p}{35}\\ &= \frac{21p + 20p}{35}\\ &= \frac{41p}{35}\end{aligned}$$



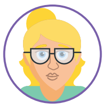
Student B

$$\begin{aligned}\frac{3p}{5} + \frac{4p}{7} &= \frac{5 \times 3p}{5 \times 7} + \frac{7 \times 4p}{7 \times 5}\\ &= \frac{15p}{35} + \frac{28p}{35}\\ &= \frac{15p + 28p}{35}\\ &= \frac{43p}{35}\end{aligned}$$

b. Simplify $\frac{7}{4x} - \frac{3}{2x}$

Student A

$$\begin{aligned}\frac{7}{4x} - \frac{3}{2x} &= \frac{7}{4x} - \frac{3}{4x}\\ &= \frac{7 - 3}{4x}\\ &= \frac{4}{4x}\\ &= \frac{1}{x}\end{aligned}$$



Student B

$$\begin{aligned}\frac{7}{4x} - \frac{3}{2x} &= \frac{7}{4x} - \frac{2 \times 3}{2 \times 2x}\\ &= \frac{7}{4x} - \frac{6}{4x}\\ &= \frac{7 - 6}{4x}\\ &= \frac{1}{4x}\end{aligned}$$

Back to contents

Problem solving

Question working paths

Mild 10, 11, 13 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. On Christmas day Chirag and Cindy ate pavlova together with some friends. Using p to represent the entire pavlova, Chirag ate a portion equivalent to $\frac{p}{4}$ and Cindy ate a portion equivalent to $\frac{p}{5}$. Simplify the expression showing how much Chirag and Cindy ate together in terms of p.

11. Ashley is raising funds to go on a basketball trip to New York City. Using x to represent the total cost of the trip, his mother will contribute $\frac{x}{5}$ towards the trip and his grandfather will contribute $\frac{2x}{3}$. Simplify the expression representing the total amount contributed by Ashley's mother and grandfather in terms of x.

12. After a family dinner, there is a third $\left(\frac{L}{3}\right)$ lasagne left over. Jimmy gets home later and eats a portion equivalent to $\frac{2L}{7}$. Simplify the expression for the remaining portion of lasagne in terms of L.

13. Sylvie withdraws half of her total savings (s) out of her account to take on holiday. During the holiday, Sylvie spends an amount equivalent to $\frac{3s}{8}$. Simplify the expression showing how much money she is bringing back with her from holiday in terms of s.

14. John and Enyd are getting married and their parents have agreed to pay for some of their wedding expenses. John's parents will pay $\frac{w}{4}$ towards the total cost of the wedding (w), while Enyd's parents have agreed to pay $\frac{2w}{5}$. Simplify the expression showing the remaining cost of the wedding in terms of w.

Reasoning

Question working paths

Mild 15 (a,b,d) 🌶	**Medium** 15 (a,b,d), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Spencer receives g dollars from his grandmother for his birthday. He plans on spending some of the money on computer games and a night out with friends. Spencer would like to deposit the rest in his savings account.

a. Spencer spends an amount equivalent to $\frac{g}{3}$ on computer games. Express how much money he has left as a simplified algebraic fraction in terms of g.

b. After purchasing the games, Spencer spends another amount equivalent to $\frac{2g}{5}$ on a night out. Express how much money he has left as a simplified algebraic fraction in terms of g.

c. Spencer deposits \$315 into his savings account. How much money did his grandmother give him, rounded to the nearest dollar?

d. Identify an advantage or disadvantage of spending money instead of saving it.

16. Consider the given expression.

$$\frac{5x}{6} - \frac{2x}{9}$$

a. Simplify by converting both denominators to the LCM of 6 and 9.

b. Simplify by converting both denominators to the product of 6 and 9.

c. Compare the steps taken to complete parts **a** and **b**. Comment on their differences in reaching the final answer.

Back to contents

Exam-style

17. Which of the options shows the value of the LCD of the fractions in the given expression? (1 MARK)

$$\frac{5}{10x} + \frac{7}{15x}$$

A. 30 **B.** 35 **C.** $15x$ **D.** $30x$ **E.** $150x$

18. Consider the given expression. (3 MARKS)

$$\frac{4}{5x^2} + \frac{5}{x}$$

a. Simplify the expression. 2 MARKS

b. Evaluate the expression when $x = 1$. Provide the answer as a mixed number. 1 MARK

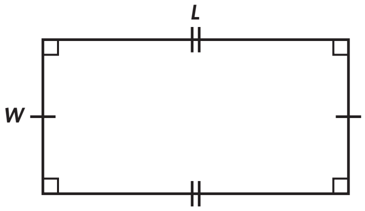
19. Rowan forgets the password to his email account during $\frac{2}{5}$ of his total monthly attempts (a) to log in. During another $\frac{1}{7}$ of his attempts to log in, Rowan incorrectly types in the characters of his password. Simplify the expression showing the number of times Rowan unsuccessfully tries to log in to his email account in terms of a. (3 MARKS)

20. When c is subtracted from $\frac{5x}{12}$, the result is $\frac{3x}{4}$. Show that $c = -\frac{x}{3}$. (3 MARKS)

Remember this?

21. This rectangle has the side lengths L and W.

Which of these expressions could not be used for the perimeter of the rectangle?



A. $2(L + W)$

B. $4(W + L)$

C. $2W + 2L$

D. $W + 2L + W$

E. $L + L + W + W$

22. Miley is facing east towards the ocean.

She makes a quarter turn to the left and continues walking. She then takes another quarter right turn and stops, before taking a quarter left turn.

What direction is she facing now?

A. East

B. West

C. North

D. South

E. None of the above

23. The following table shows the fractions of the common household pets owned in Melbourne.

Which of these common household pets is owned the most?

A. Dog

B. Cat

C. Bird

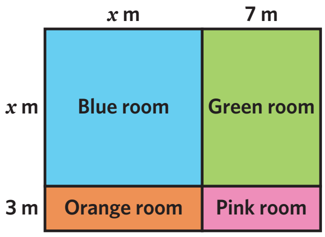
D. Rabbit

E. Guinea pig

Pet	**Fraction of households that own one**
Dog	$\frac{1}{6}$
Cat	$\frac{1}{50}$
Bird	$\frac{1}{8}$
Guinea pig	$\frac{1}{20}$
Rabbit	$\frac{1}{15}$

Back to contents

Chapter 4 extended application



1. At KiddoLand Play Centre, the management is designing distinct playrooms, each characterised by a vibrant colour to stimulate creativity and learning. They are planning to use a combination of square and rectangular rooms on level 1, as shown in the following diagram.

a. Write a factorised quadratic expression to represent the total area of the rooms on level 1.

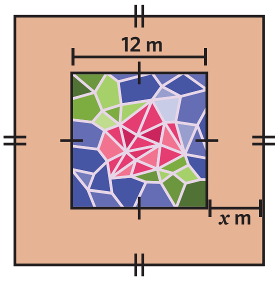
b. Represent the total area of level 1 as an expanded quadratic.

c. Determine the difference in area between the green and orange room in terms of x.

d. KiddoLand is able to double the length and width of their property due to a vacancy next door. Express the new total area of level 1 at KiddoLand in expanded form.

e. On every level at KiddoLand, there is a designated fire safety area. The area of the fire safety area at KiddoLand is given as $(x - 5)^2$ m^2 and it will be built inside the blue room. Determine the expression that represents the total area available for play space in the blue room after the fire safety area is built.

f. Other than safety, what other factors might a parent consider before sending their child to KiddoLand Play Centre?



2. The vast wall in the Royal Gallery is square-shaped. An artist plans to create a square mosaic in the centre of the wall, leaving a painted border that is x m wide all around.

a. Write an expression for the perimeter of the vast wall at the Royal Gallery.

b. Write a factorised expression for the area of the entire wall.

c. Write a factorised expression for the area of the painted border around the mosaic.

d. Using the expression from part **c**, calculate the area of the painted border when $x = 3$.

e. The art gallery director would like the area of the mosaic to be one third of the area of the painted border. Determine the value of x.

f. Suggest something that should be considered by the management to ensure visitors have a positive experience visiting the Royal Gallery.

3. Basketballs are manufactured in various sizes to cater to different age groups and skill levels. Understanding the volume and surface area of these spheres can offer insights into design considerations and material requirements.

The volume of a men's competition basketball is given by the formula

$$V_{men} = \frac{4\pi r^3}{3}$$

a. The volume of the women's competition basketball is $\frac{3}{4}$ of a men's competition basketball. Formulate an expression for the volume of the women's competition basketball.

b. The volume of a youth basketball is $\frac{2}{3}$ the size of the men's competition basketball. Formulate an expression for the volume of a youth basketball.

c. The manufacturer is considering creating a new training basketball for beginners. They want this basketball to have a volume that is the average (mean) of the men's, women's, and youth competition basketballs. Formulate an expression for the volume of this new training basketball using algebraic fractions, and simplify the expression.

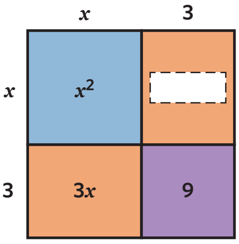
d. If the volume of a men's basketball is 7104 cm^3, determine the radius of the ball. Round the answer to the nearest centimetre.

e. Is it fair that men, women, and youth players use differently sized basketballs? Give a reason for your answer.

Chapter 4 review

Multiple choice

1. Select the option to fill in the blank to expand the product of the binomial expression. **4A,B**



A. x **B.** x^2 **C.** $3x$ **D.** $3x^2$ **E.** $9x$

---

2. Select the option to fill in the blank. **4C**

$7z^2 - 21az = \boxed{}\,(z - 3a)$

A. z **B.** $3a$ **C.** $3z$ **D.** $7a$ **E.** $7z$

---

3. Select the option to fill in the blank. **4D**

$a^2 - b^2$	a	b
$16q^2 - 81$	$\boxed{}$	9

A. $2q$ **B.** $4q$ **C.** $4q^2$ **D.** $16q$ **E.** $16q^2$

---

4. Select the option to fill in the blank. **4E**

$x^2 + 3x - 8x - 24 = x(x + 3) - \boxed{}\,(x + 3)$

A. 6 **B.** 7 **C.** 8 **D.** 9 **E.** 10

---

5. Select the option to fill in the blank. **4F,G**

$$\frac{3x}{4(x - 7)} \div \frac{8}{2(x - 7)} = \frac{3x}{4(x - 7)} \times \boxed{}$$

A. $\frac{3x}{16}$ **B.** $\frac{8}{2(x - 7)}$ **C.** $\frac{3x}{4(x - 7)}$ **D.** $\frac{2(x - 7)}{8}$ **E.** $\frac{4(x - 7)}{3x}$

Fluency

6. Expand and simplify the following. **4A**

a. $(m + 1)(m + 4)$ **b.** $(n + 3)(n - 6)$ **c.** $(p - 4)(p - 1)$ **d.** $(3z + 7)(z - 7)$

---

7. Expand and simplify the following. **4B**

a. $(z + 3)^2$ **b.** $(w - 3)^2$ **c.** $(3x - 2)^2$ **d.** $(4y + 1)^2$

---

8. Expand and simplify the following. **4B**

a. $(z + 4)(z - 4)$ **b.** $(3x - 6)(3x + 6)$ **c.** $(5y - 2)(5y + 2)$ **d.** $(2w + 7)(2w - 7)$

---

9. Factorise the following. **4C**

a. $4y + 20$ **b.** $5z^2 - 15z$ **c.** $-3x + 18$ **d.** $-9w^2 - 27w$

10. Factorise the following. 4C

a. $3(c + d) + c(c + d)$ **b.** $4(2y - 5) - y(2y - 5)$

c. $5f(e + f) + 3(e + f)$ **d.** $6(3z - 4) - 5z(3z - 4)$

11. Factorise and simplify the following. 4D

a. $y^2 - 16$ **b.** $25x^2 - 1$ **c.** $49w^2 - 36z^2$ **d.** $(x - 2)^2 - 9$

12. Factorise the following. 4E

a. $x^2 + 3x + 2x + 6$ **b.** $x^2 + 8x - 4x - 32$ **c.** $2x^2 + 6x + 7x + 21$ **d.** $3x^2 - 12x - 2x + 8$

13. Simplify the following. 4F

a. $\frac{x^2 - 4}{2x + 6} \times \frac{2x + 6}{x^2 - 4}$ **b.** $\frac{3x^2}{12} \times \frac{18}{3x^2}$

c. $\frac{57x^2}{18z} \times \frac{2z}{3x}$ **d.** $\frac{x - 2}{3(x + 4)} \times \frac{6(x + 4)}{(x - 2)(x + 3)}$

14. Simplify the following. 4F

a. $\frac{6y}{5} \div \frac{18y^2}{15}$ **b.** $\frac{3z}{4(z + 6)} \div \frac{9z}{z + 6}$

c. $\frac{7w + 14}{9} \div \frac{14w^2 + 28w}{45}$ **d.** $\frac{a(b + c)^2}{d} \div \frac{d(b + c)}{b + c}$

15. Simplify the following. 4G

a. $\frac{2x}{3} + \frac{x}{6}$ **b.** $\frac{2}{x} + \frac{3}{x^2}$ **c.** $\frac{x + 1}{4} + \frac{x + 3}{5}$ **d.** $\frac{3x + 2}{5} + \frac{2x + 1}{7}$

16. Simplify the following. 4G

a. $\frac{2x}{5} - \frac{4x}{3}$ **b.** $\frac{4}{x^3} - \frac{3}{x^2}$ **c.** $\frac{x + 3}{6} - \frac{x - 2}{4}$ **d.** $\frac{3x + 1}{8} - \frac{2x - 3}{4}$

Problem solving

17. A rectangular garden has a length of x m and a width of y m. The gardener plans to extend the garden by adding 2 m to the length and 4 m to the width. Write an expanded expression for the new area of the garden after the extension. 4A

18. A community garden is in the shape of a square. The local council decides to renovate the garden by extending all sides by 4 m. Write an expanded simplified expression for the new area of the garden, using g to represent the length and width of the garden before the renovation. 4B

19. A local musical band was performing two nights in a row. The profit they make from each ticket is represented by the expression $2x - 7$ where x represents an unknown amount of dollars. On the first night of their performance, they sold y tickets and on the second night of their performance, they sold 28 tickets. Write a factorised expression representing their total profits. 4C

20. A landscape artist isn't satisfied with his square garden design. He adjusts the length by increasing it and adjusts the width by decreasing it, after which the expression for the new area is represented by $p^2 + 3p - 9p - 27$, where p represents the length of one side prior to adjustment. Factorise the expression to determine how many metres he increased the length and decreased the width by. 4E

21. A safety inspector is assessing the dimensions of a stage at a concert. The stage has an area of $4x^2 - 25$ square metres, where x represents an unknown length in metres. The rule is that the longer side of the stage must be at least 8 m more than the shorter side. Factorise the expression to determine whether the stage meets this requirement. 4D

22. Jackson is trying to decide whether he should purchase and renovate a house using all his cash savings (c). Assuming the house alone costs $\frac{2c}{5}$ and the renovations cost $\frac{2c}{7}$, how much money will Jackson have left, in terms of c, after purchasing and renovating the house?

4G

---

23. Sam is a biologist studying two new populations of bacteria. The first population can be represented by the equation $p_1 = \frac{t(t-7)}{3(t+4)}$ and the second population can be represented by the equation $p_2 = \frac{t(t-7)}{t+3}$, where t represents the number of hours after cultivating the populations of bacteria. Write the simplified fraction $\frac{p_1}{p_2}$.

4F

Reasoning

24. In a picturesque region of New South Wales, the Australian Orchard Farm manages vast stretches of land dedicated to growing fruit. Among their diverse range of fruits, apple and orange trees dominate the landscape. Every year, researchers at the facility conduct a thorough survey. They use x to represent the number of apple trees, and y to represent the number of orange trees.

a. The expression representing the total area of soil needed for the fruit trees is $a^2 + 8a$ where a represents the area required for one tree. Factorise this expression.

b. It was found that the expression representing the total number of fruits produced is $(x + 50)(y + 40)$. Expand this expression.

c. Researchers observed that the total number of trees on the property can be expressed as $t^2 + 20t + 100$ where t is the number of weeks since the seeds have been planted. Factorise this expression.

d. The researchers are testing two different fertilisers exclusively on apple trees. Fertiliser A increases the apple yield by an additional $\frac{3x}{5}$ while Fertiliser B increases the apple yield by another $\frac{2x}{7}$. If both fertilisers are used together, determine the new apple yield in terms of x.

e. Why might it be beneficial to use equations to predict fruit production?

25. Consider the given expression.

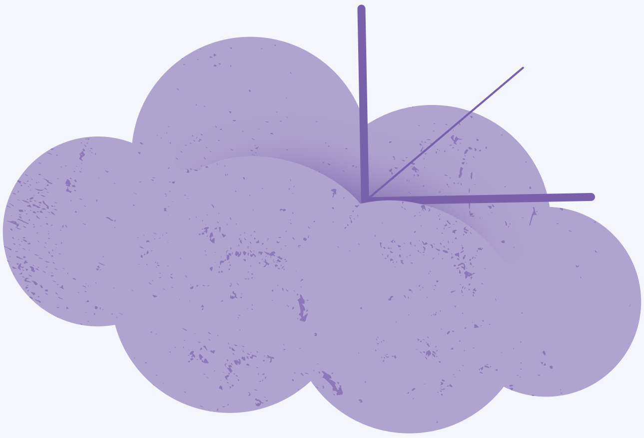
$$\frac{x^2 - 36}{x + 6}$$

a. Factorise $x^2 - 36$.

b. Using your answer from part **a**, simplify the given expression.

c. Using your answers from parts **a** and **b**, comment on the usefulness of factorising expressions when simplifying fractions.

Back to contents



Chapter 5

Linear relations

Number and algebra

Calculator skills

See online in additional materials for using CAS calculator guides.

5A Graphing straight lines using intercept

5G Graphical solutions to simultaneous equations

Chapter 5 research summary

Linear relations

Big ideas

Linear relations is a mathematical concept that involves the representations of relationships between two variables that can be shown in the form of equations, graphs and tables. It includes concepts like gradient, y-intercept, and solving equations to find unknown values. The big ideas of functions and relationships, rate of change, proportional reasoning, rate of change all underpin the concept of linear relations.

Variables and expressions

Variables and expressions are used in representing mathematical relationships. Variables are symbols that represent unknown values, and expressions are combinations of variables, numbers, and operations. In the context of linear relations, variables are used to represent the quantities involved in the relationship, and expressions are used to represent the relationship between the quantities.

Equations and inequalities

Understanding the nature of equations and inequalities and how to solve them underpins linear relations. An equation is a mathematical statement that shows two expressions are equal. An inequality compares the relative size of two expressions. In the context of linear relations, equations and inequalities are used to represent the relationship between two variables and to solve for unknown values.

Functions and relationships

In the context of linear relations, functions are used to represent the relationship between two variables in a way that is easy to understand and work with. A function is a special type of relationship between two variables where each value of the input variable corresponds to exactly one value of the output variable. Understanding the concept of functions and how to represent them in various ways is important to linear relations.

Graphical representation

This big idea involves understanding how to represent mathematical relationships graphically and how to interpret graphs. Graphs are visual representations of mathematical relationships that can provide insights into the nature of the relationship and help to solve problems. In the context of linear relations, graphs are used to represent the relationship between two variables and to understand the characteristics of the relationship, such as its gradient and y-intercept.

Rate of change

The concept of rate of change is significant in linear relations. The rate of change is a measure of how one variable changes as the other variable changes. In the context of linear relations, the rate of change is constant and is represented by the slope of the line.

Proportional reasoning

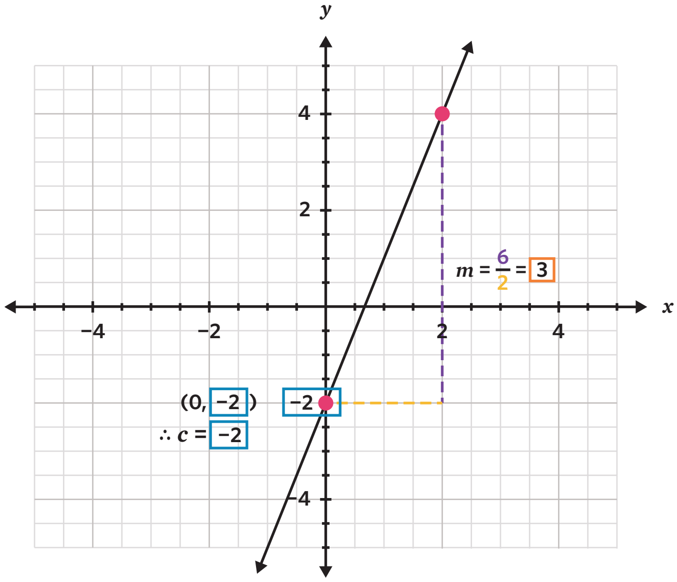
Proportional reasoning is the ability to understand and work with ratios and proportions. In the context of linear relations, proportional reasoning is used to understand the relationship between two variables and to solve problems involving ratios and proportions.

Back to contents

Visual representations

Cartesian plane

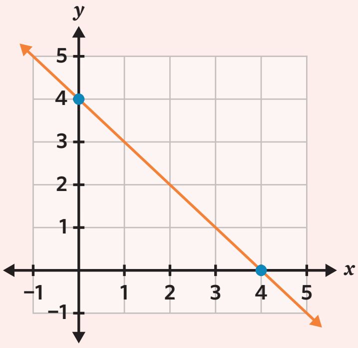
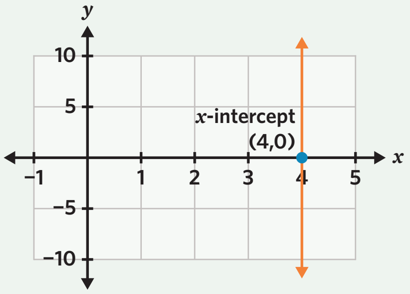
The Cartesian plane or coordinate plane is fundamental for visually representing and working with linear relations. It is used to graph linear equations and inequalities, helping students to understand the relationship between variables, the concept of gradient, y-intercept, and the characteristics of the relationship.



$\therefore y = mx + c$ is:

$y = \boxed{3}x\boxed{-2}$

Misconceptions

Misconception	Incorrect ✕	Correct ✓	Lesson
Students substitute the y-value into the x-value when finding x-intercepts and vice versa.	Calculate the x- and y-intercepts for $y = -2x - 2$. (−2,0) and (0,−1)	Calculate the x- and y-intercepts for $y = -2x - 2$. (−1,0) and (0,−2)	5A
Students think the calculated x- and y-intercepts form one coordinate point.	Calculate the x- and y-intercepts for $y = -2x - 2$. (−1,−1) and (−2,−2)	Calculate the x- and y-intercepts for $y = -2x - 2$. (−1,0) and (0,−2)	5A
Students use the constant in horizontal and vertical lines as both x- and y-intercepts.	$x = 4$ 	$x = 4$ 	5B

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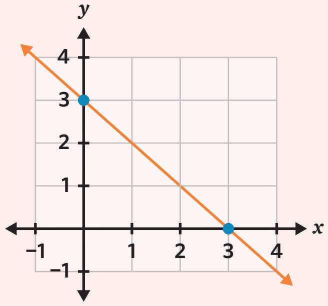
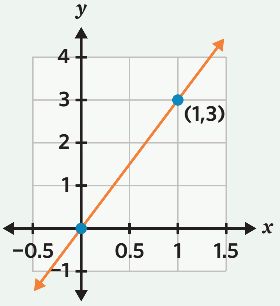
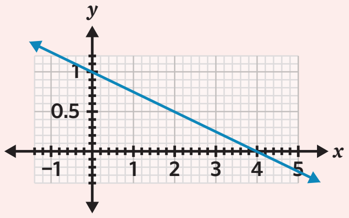
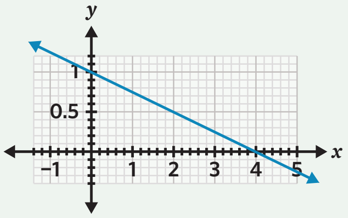
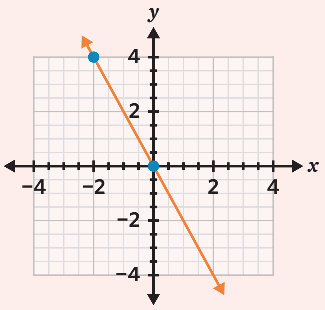
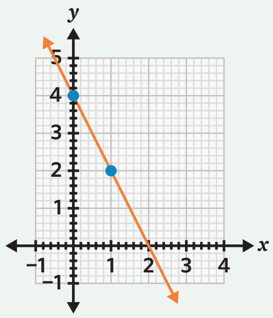
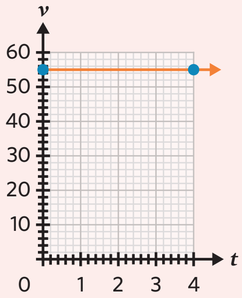
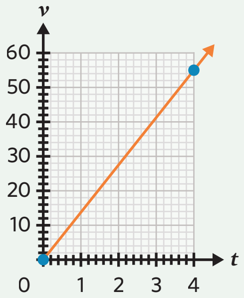
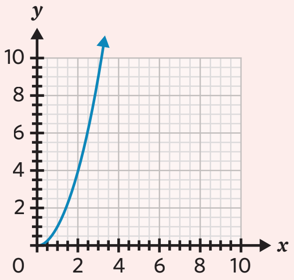
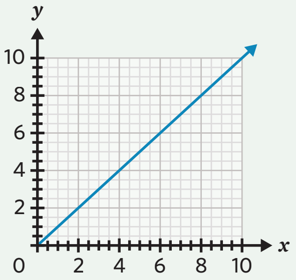
Back to contents

Misconception	Incorrect ✕	Correct ✓	Lesson
Students use m-values as intercepts when given an equation in the form $y = mx$.	$y = 3x$  	$y = 3x$  	5B
Students calculate gradient using $\frac{\text{run}}{\text{rise}}$.	 $m = \frac{\text{run}}{\text{rise}}$ $m = \frac{-4}{1}$ $= -4$	 $m = \frac{\text{run}}{\text{rise}}$ $m = \frac{-1}{4}$ $= \frac{-1}{4}$	5C
Students use the point (m,c) and (0,0) to plot a straight line.	$y = -2x + 4$  	$y = -2x + 4$  	5C
Students believe that any straight line represents direct proportion.	 	 	5D
Students think that direct proportion can be non-linear.	 	 	5D

Continues →

Misconception	Incorrect ✗	Correct ✓	Lesson
Students do not identify a rate to a unit rate of $x : 1$ when performing calculations.	A manufacturer produces 30 items in 15 minutes at a constant rate. The rate is 30 items per hour.	A manufacturer produces 30 items in 15 minutes at a constant rate. Using 30 items in 15 minutes, the rate is 120 items per hour.	5D
When using the distance or midpoint formula students use the coordinates from one point instead of the both x or both y.	$(x_1,y_1) = (1,20)$ and $(x_2,y_2) = (11,2)$ $M(x,y) = \left(\frac{1+20}{2},\frac{11+2}{2}\right)$ $= \left(\frac{21}{2},\frac{13}{2}\right)$ $= \left(10\frac{1}{2},6\frac{1}{2}\right)$	$(x_1,y_1) = (1,20)$ and $(x_2,y_2) = (11,2)$ $M(x,y) = \left(\frac{1+11}{2},\frac{20+2}{2}\right)$ $= \left(\frac{12}{2},\frac{22}{2}\right)$ $= (6,11)$	5E
Students incorrectly use all positive values when a coordinate is negative.	Calculate the length of the line segment joining the points (0,–8) and (−6,2). Vertical distance $y_2 - y_1 = 8 - 2$ $= 6$ Horizontal distance $x_2 - x_1 = 0 - 6$ $= -6$	Calculate the length of the line segment joining the points (0,–8) and (−6,2). Vertical distance $y_2 - y_1 = -8 - 2$ $= -10$ Horizontal distance $x_2 - x_1 = 0 - (-6)$ $= 6$	5E
Students swap the x and y values when substituting into the equation.	$y = -\frac{1}{4}x + c$ $(-1,0)$ $-1 = -\frac{1}{4}(0) + c$ $c = -1$	$y = -\frac{1}{4}x + c$ $(-1,0)$ $0 = -\frac{1}{4}(-1) + c$ $0 = \frac{1}{4} + c$ $c = -\frac{1}{4}$	5A, 5F
Students forget to use the negative reciprocal for perpendicular lines.	$y = 3x - 4$ $m_1 = 3, m_2 = \frac{1}{3}$	$y = 3x - 4$ $m_1 = 3, m_2 = -\frac{1}{3}$	5F
Students think all parallel lines have zero solutions to simultaneous equations.	$y = 3x + 5$ $y = -3x - 2$ Lines have zero solutions.	$y = 3x + 5$ $y = -3x - 2$ Lines have one solution.	5G
Students incorrectly substitute equations into simultaneous equations.	Use substitution to verify whether the (−1,–5) is the solution to the following pairs of simultaneous equations. 1: $y = x - 4$ 2: $y = 2x - 3$ $y = x - 4$, let $x = -1$ $y = -5 - 4$ $y = -9$ ✗ $y = 2x - 3$, let $x = -1$ $y = -5 - 3$ $y = -8$ ✗ ∴ (−1,–5) is not the solution to the simultaneous equations.	Use substitution to verify whether the (−1,–5) is the solution to the following pairs of simultaneous equations. 1: $y = x - 4$ 2: $y = 2x - 3$ $y = x - 4$, let $x = -1$ $y = -1 - 4$ $y = -5$ ✓ $y = 2x - 3$, let $x = -1$ $y = -2 - 3$ $y = -5$ ✓ ∴ (−1,–5) is the solution to the simultaneous equations.	5G

Back to contents

5A Graphing straight lines using intercepts

LEARNING INTENTIONS

Students will be able to:

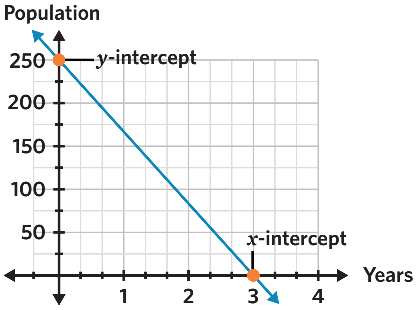
- calculate x and y-intercepts from a rule
- sketch a linear graph using x and y-intercepts.

Sketching a linear graph involves plotting two points and drawing a straight line through them. Calculating x- and y-intercepts from a rule is one method for graphing linear equations. These intercepts are the points where the line crosses the x- and y-axes.

KEY TERMS AND DEFINITIONS

- A **linear rule** is an equation used for a straight line.
- **Substitution** is the process of replacing a variable or an unknown with a given value.
- A **Cartesian plane** is a set of two perpendicular number lines that intersect at the origin.

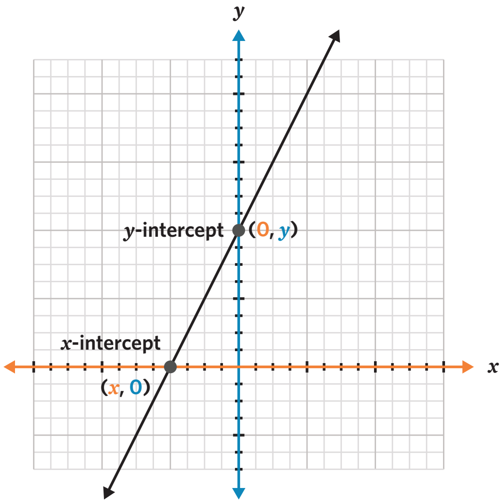
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



In environmental studies, plotting x- and y-intercepts on a linear graph can be used to represent the decline of a plant species in a forest over time. This can help in understanding the rate of growth or decline and predicting future populations.

Key ideas

1. A linear graph can be sketched by drawing a straight line through the plotted x- and y-intercepts.



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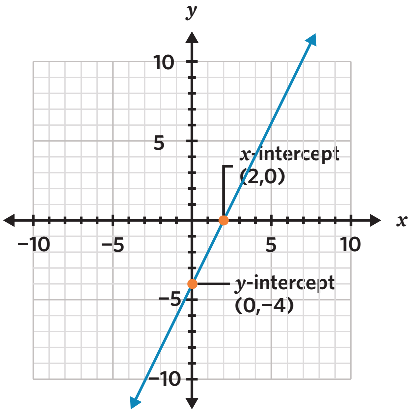
2. The x- and y-intercepts of a linear equation can be calculated from its rule by substituting $x = 0$ to determine the y-intercept and $y = 0$ to determine the x-intercept. These intercepts can be plotted on a graph as the points where the line crosses the x-axis and y-axis.

The graph of $y = 2x - 4$ has:

- a y-intercept at the point on the y-axis when $x = 0$
- a x-intercept at the point on the x-axis when $y = 0$



Worked example 1

Sketching using x- and y-intercepts

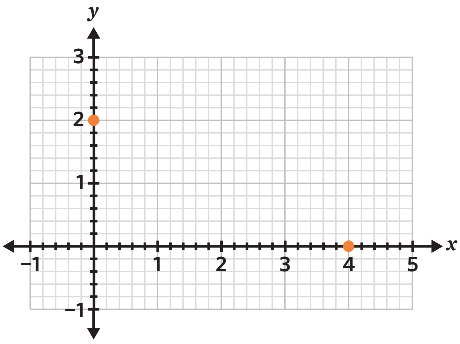
Sketch the straight line that passes through the given x- and y-intercepts.

a. $A(0,2)$ and $B(4,0)$

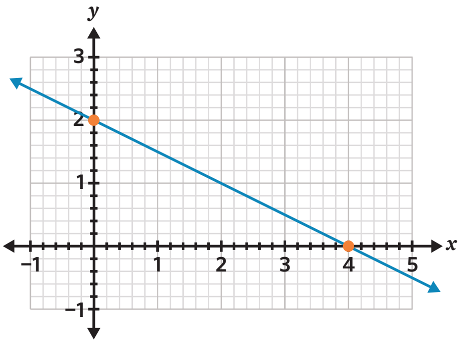
WE1a

Working

Thinking



Step 1: Plot point A and point B on a Cartesian plane.



Step 2: Use a ruler to draw a straight line though and past the two points, adding arrows to each end.

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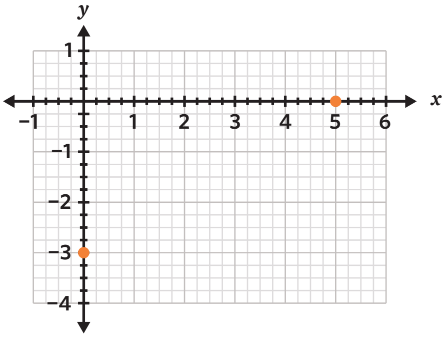
b. $A(0,-3)$ and $B(5,0)$

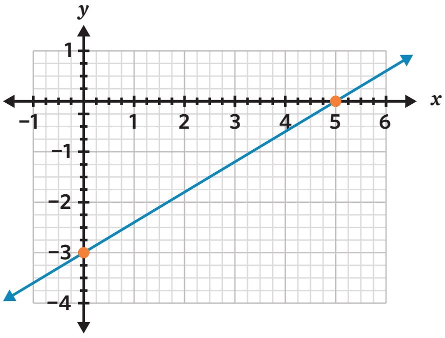
WE1b

Working

Thinking



Step 1: Plot point A and point B on a Cartesian plane.



Step 2: Use a ruler to draw a straight line though and past the two points, adding arrows to each end.

Student practice

Sketch the straight line that passes through the given x- and y-intercepts.

a. $A(0,1)$ and $B(3,0)$

b. $A(0,-2)$ and $B(4,0)$

Worked example 2

Calculating x- and y-intercepts

Calculate the x- and y-intercepts for the following linear equations.

a. $y = 5x - 2$

WE2a

Working

x-intercept:

let $y = 0$

$0 = 5x - 2$

$-5x = -2$

$x = \frac{-2}{-5}$

$x = \frac{2}{5}$

$\therefore$ x-intercept is $\frac{2}{5}$

$(\frac{2}{5},0)$

Thinking

Step 1: To determine the x-intercept, substitute $y = 0$ into the rule and solve for x. State the intercept, writing it as a coordinate pair.

Continues →

Back to contents

y-intercept:

let $x = 0$

$y = 5(0) - 2$

$y = -2$

$\therefore$ y-intercept is -2

$(0,-2)$

Step 2: To determine the y-intercept, substitute $x = 0$ into the rule and solve for y.

Step 3: State the intercept, writing it as a coordinate pair.

b. $3x + y = 6$

WE2b

Working

x-intercept:

let $y = 0$

$3x + 0 = 6$

$3x = 6$

$x = 2$

$\therefore$ x-intercept is 2

$(2,0)$

y-intercept:

let $x = 0$

$3(0) + y = 6$

$y = 6$

$\therefore$ y-intercept is 6

$(0,6)$

Thinking

Step 1: To determine the x-intercept, substitute $y = 0$ into the rule and solve for x. State the intercept, writing it as a coordinate pair.

Step 2: To determine the y-intercept, substitute $x = 0$ into the rule and solve for y.

Step 3: State the intercept, writing it as a coordinate pair.

Student practice

Calculate the x- and y-intercepts for the following linear equations.

a. $y = 3x - 1$

b. $2x + y = 4$

Worked example 3

Sketching by first calculating x- and y-intercepts

Sketch the graph, showing the x- and y-intercepts.

a. $y = -\frac{1}{2}x - 3$

WE3a

Working

x-intercept:

let $y = 0$

$(0) = -\frac{1}{2}x - 3$

$\frac{1}{2}x = -3$

$x = -6$

$\therefore$ x-intercept is -6

$(-6,0)$

Thinking

Step 1: To determine the x-intercept, substitute $y = 0$ into the rule and solve for x. State the intercept, writing it as a coordinate pair.

Continues →

Back to contents

y-intercept:

let $x = 0$

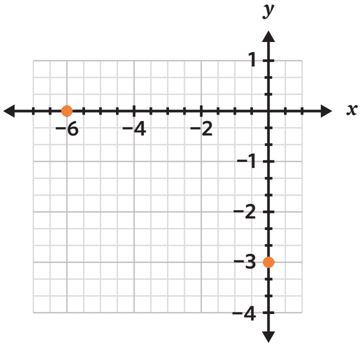
$y = -\frac{1}{2}(0) - 3$

$y = -3$

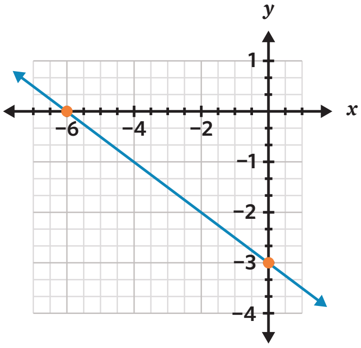
$\therefore$ y-intercept is -3

$(0,-3)$

Step 2: To determine the y-intercept, substitute $x = 0$ into the rule and solve for y. State the intercept, writing it as a coordinate pair.

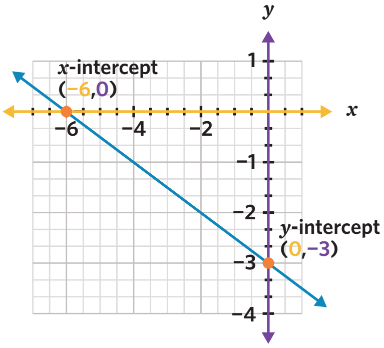


Step 3: Plot the x- and y-intercepts on a Cartesian plane.



Step 4: Use a ruler to draw a straight line though and past the two points, adding arrows to each end.

Visual support



b. $2x + 5y = 30$

WE3b

Working

x-intercept:

let $y = 0$

$2x + 5(0) = 30$

$2x = 30$

$x = 15$

$\therefore$ x-intercept is 15

$(15,0)$

Thinking

Step 1: To determine the x-intercept, substitute $y = 0$ into the rule and solve for x. State the intercept, writing it as a coordinate pair.

Continues →

y-intercept:

let $x = 0$

$2 + 5y = 30$

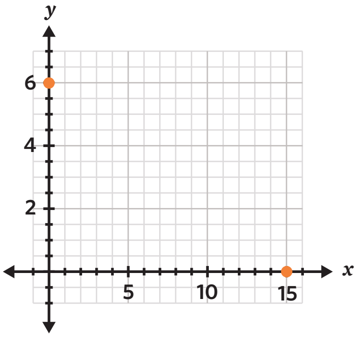
$5y = 30$

$y = 6$

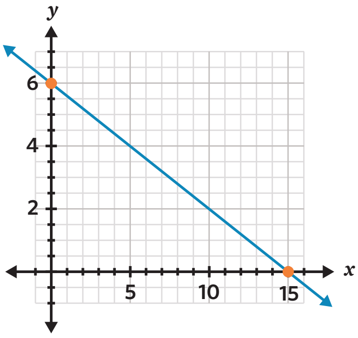
$\therefore$ y-intercept is 6

(0,6)

Step 2: To determine the y-intercept, substitute $x = 0$ into the rule and solve for y. State the intercept, writing it as a coordinate pair.



Step 3: Plot the x- and y-intercepts on a Cartesian plane.



Step 4: Use a ruler to draw a straight line though and past the two points, adding arrows to each end.

Student practice

Sketch the graph, showing the x- and y-intercepts.

a. $y = -\frac{1}{3}x - 8$

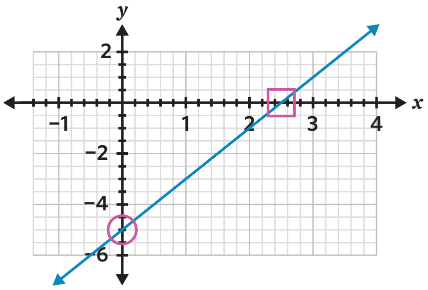
b. $4x + 5y = 20$

5A Questions

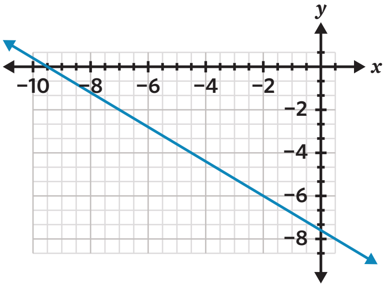
Understanding worksheet

1. Place a square around the x-intercept and a circle around the y-intercept.

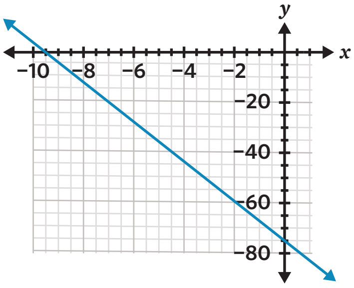
Example



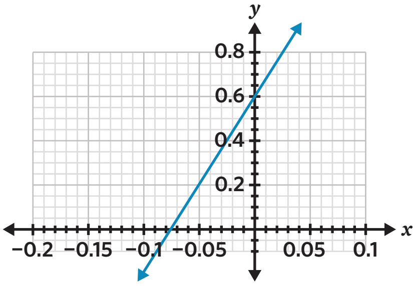
a.



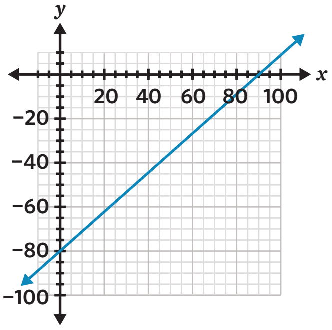
b.



c.



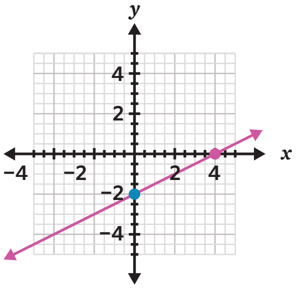
d.



2. Plot the given coordinate and draw the straight line extending through the intercepts.

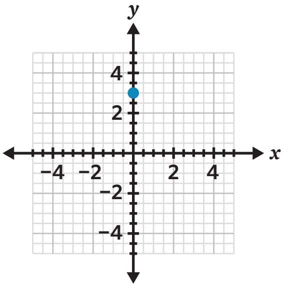
Example

(4,0)

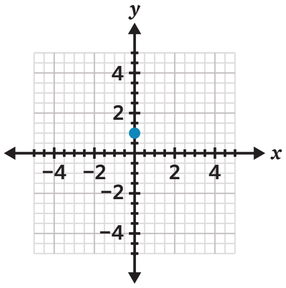


Back to contents

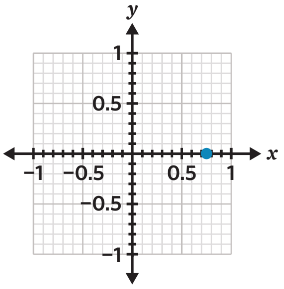
a. (2,0)



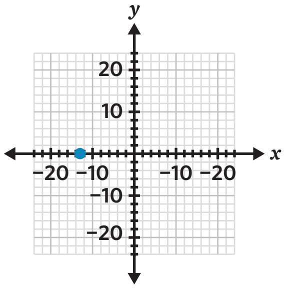
b. (−3,0)



c. (0,0.5)



d. (0,20)



3. Fill in the blanks by using the words provided.

equation	substituting	Cartesian plane	*y*-intercept	*x*-intercept

To graph a linear rule, identify the __________, when $x = 0$, and the __________, when $y = 0$. These intercepts can be found by __________ the respective variable with zero into the __________ of the line. By plotting these intercepts on a __________, a straight line can be represented by connecting the points.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Sketch the straight line that passes through the given *x*- and *y*-intercepts. WE1

a. $A(0,2)$ and $B(4,0)$ **b.** $A(0,-3)$ and $B(7,0)$ **c.** $A(0,6)$ and $B(-1,0)$ **d.** $A(0,5)$ and $B(8,0)$

e. $A(0,1)$ and $B(5,0)$ **f.** $A(0,-4)$ and $B(0.5,0)$ **g.** $A(0,9)$ and $B(-9,0)$ **h.** $A(0,-8)$ and $B(8,0)$

---

5. Calculate the *x*- and *y*-intercepts for the following linear equations. WE2a

a. $y = x + 1$ **b.** $y = 4x - 2$ **c.** $y = \frac{1}{3}x - 1$ **d.** $y = \frac{2}{5}x + 4$

e. $y = \frac{3}{2}x - 6$ **f.** $y = -0.5x + 2$ **g.** $y = 0.4x + 9.5$ **h.** $y = 1.4x - 0.7$

Back to contents

6. Calculate the x- and y-intercepts for the following linear equations. WE2b

a. $2x - y = -4$ **b.** $3x + y = -6$ **c.** $0.5x - y = -2$ **d.** $4x + y = 5$

e. $-x - y = -4$ **f.** $\frac{5}{2}x + y = 1$ **g.** $-\frac{4}{5}x + y = \frac{1}{5}$ **h.** $1.2x + y = -3$

7. Sketch the graph, showing the x- and y-intercepts. WE3a

a. $y = 2x + 4$ **b.** $y = 3x + 6$ **c.** $y = \frac{2}{5}x + 2$ **d.** $y = -\frac{4}{3}x - 4$

e. $y = -3x - 2$ **f.** $y = -5x - 3$ **g.** $y = \frac{1}{4}x + \frac{3}{2}$ **h.** $y = -0.5x - 3$

8. Sketch the graph, showing the x- and y-intercepts. WE3b

a. $x + 4y = 12$ **b.** $4x + y = -8$ **c.** $2x + y = 7$ **d.** $-x + 7y = 14$

e. $-2x - 5y = -8$ **f.** $-4x + 5y = 6$ **g.** $2x - 5y = 10$ **h.** $3x - 2y = -3$

9. Calculate the x- and y-intercepts for $y = -2x - 2$.

A. (2,2) and (−2,−2)
B. (−2,0) and (0,−1)
C. (−1,0) and (0,−2)
D. (−1,−2) and (0,0)
E. (−2,−2) and (0,0)

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Calculate the x- and y-intercepts for $y = \frac{3}{2}x + 2$

Student A

x-intercept:

let $x = 0$

$y = \frac{3}{2}(0) + 2$

$y = 2$

$\therefore$ x-intercept is (0,2)

y-intercept:

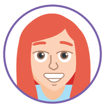
let $y = 0$

$0 = \frac{3}{2}x + 2$

$-\frac{3}{2}x = 2$

$x = -\frac{4}{3}$

$\therefore$ y-intercept is $(-\frac{4}{3}, 0)$



Student B

x-intercept:

let $y = 0$

$0 = \frac{3}{2}x + 2$

$-2 = \frac{3}{2}x$

$-4 = 3x$

$x = -\frac{4}{3}$

$\therefore$ x-intercept is $(-\frac{4}{3}, 0)$

y-intercept:

let $x = 0$

$y = \frac{3}{2}(0) + 2$

$y = 2$

$\therefore$ y-intercept is (0, 2)

Back to contents

b. Sketch the graph of $2x + y = 1$, showing the x- and y-intercepts.

Student A

x-intercept:

let $y = 0$

$2x + 0 = 1$

$2x = 1$

$x = \frac{1}{2}$

$\therefore$ x-intercept is $(0,\frac{1}{2})$

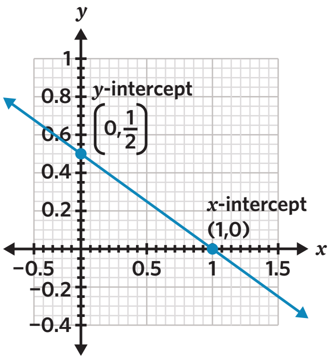
y-intercept:

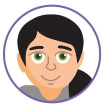
let $x = 0$

$2(0) + y = 1$

$y = 1$

$\therefore$ y-intercept is $(1,0)$





Student B

x-intercept:

let $y = 0$

$2x + 0 = 1$

$2x = 1$

$x = \frac{1}{2}$

$\therefore$ x-intercept is $(\frac{1}{2},0)$

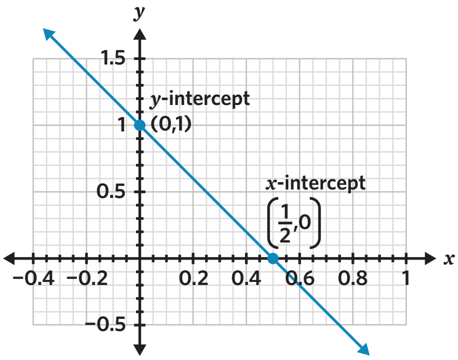
y-intercept:

let $x = 0$

$2(0) + y = 1$

$y = 1$

$\therefore$ y-intercept is $(0,1)$



Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. The declining population of a pest in a local park can be modelled using the equation $y = -20x + 350$. x represents the periods after the initial population and y represents the population. Sketch the graph showing the initial population and the time taken for the population to reach zero.

12. The population (y) of a town can be modelled over time in years (x) using the linear equation $y = 200x + 250$. Sketch a graph showing the population of the town over different years showing the initial population of the town.

13. A plant is known to grow at 3 cm per day after it has germinated and can be modelled using the equation $y = 3x + 1$. Sketch the graph that would represent the height of the plant from the day it germinates.

14. A person has \$300 in a bank account and withdraws the same amount each week for 10 weeks leaving a balance of \$0. Sketch a linear graph to represent this situation, where the x-axis represents the weeks and the y-axis represents the balance in the account.

15. The water level in a tank decreases at a constant rate of 10 L per minute. The initial water level is 500 L. Sketch the graph showing the initial water level and the time taken for the tank to completely drain.

Back to contents

Reasoning

Question working paths

Mild 16 (a,b,c,e)	**Medium** 16 (a,b,c,e), 17 (a,b)	**Spicy** All

16. A car rental company charges a flat fee of \$50 plus \$20 per day for renting a car. The total cost (\$$y$) of renting a car can be modelled using the equation $y = 20x + 50$, where x represents the number of days.

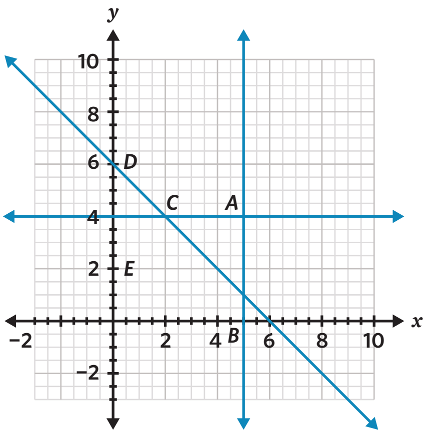
- **a.** Using the equation, calculate the x- and y-intercepts.
- **b.** Sketch the graph, showing the x- and y-intercepts.
- **c.** In the context of the question, explain what the y-intercept represents.
- **d.** Choose a reasonable scale for the x-axis for this situation and explain any changes you would make to part **b**.
- **e.** Provide a reason why someone may hire a car rather than using public transport when travelling.

17. Consider the equation $y = ax + b$.

- **a.** Calculate the x-intercept.
- **b.** Calculate the y-intercept.
- **c.** Use your answers from parts **a** and **b** to write a statement summarising your findings.

Exam-style

18. What term could be used to describe point B? (1 MARK)



- **A.** origin
- **B.** x-axis
- **C.** y-intercept
- **D.** x-intercept
- **E.** point of intersection

19. A marathon has 500 registered participants and the numbers appear to be declining at a constant rate. It is expected that the marathon will not have any participants in it's 10th year. This can be graphically displayed using the coordinates (0,500) and (10,0). (3 MARKS)

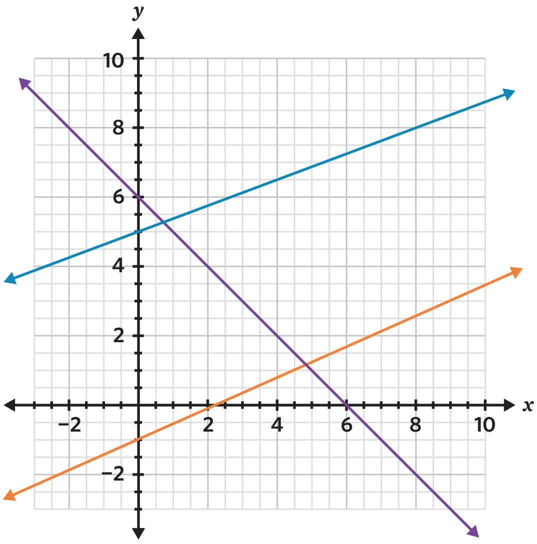
- **a.** Sketch a graph of the number of participants (P) against the number of years (t) including the intercept points. 2 MARKS
- **b.** Use your graph to determine the number of years it will take for the number of participants to decrease to 100. 1 MARK

20. Label each line with the corresponding equation on the given graph. (3 MARKS)

$y = 0.4x + 5$

$y = -x + 6$

$3x - 5y = 7$



21. Sketch the graph of $x + 2y = -4$ and $y = x + 4$ labelling the x- and y-intercepts and any point of intersection. (3 MARKS)

Remember this?

22. The given table shows the number of students at a senior high school and their age.

	Year 11	**Year 12**
16 years old	5	0
17 years old	57	12
18 years old	8	61

What fraction of Year 11 students are 16 years old?

A. $\frac{5}{143}$ **B.** $\frac{5}{109}$ **C.** $\frac{5}{70}$ **D.** $\frac{5}{57}$ **E.** $\frac{5}{13}$

23. A circle has an area of 39 m^2.

Which of these is closest to the diameter of the circle?

A. 3.0 m **B.** 3.5 m **C.** 6.2 m **D.** 7.0 m **E.** 12.4 m

24. In Australia, the currency used is the Australian dollars (AUD).

In France, the currency used is the Euro (EUR).

1 AUD is approximately equal to 0.61 EUR.

How many Euros is 450 Australian dollars?

A. 200.09 **B.** 274.50 **C.** 275.88 **D.** 724.52 **E.** 737.71

Back to contents

5B Lines with one intercept

LEARNING INTENTIONS

Students will be able to:

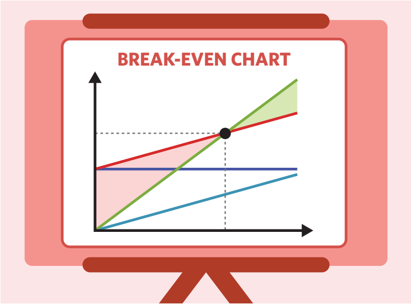
- identify lines that have only one intercept
- identify an equation for horizontal and vertical lines
- sketch vertical and horizontal lines
- sketch lines through the origin.

Certain lines, such as horizontal and vertical lines, have only one intercept. Sketching these lines involves understanding their unique characteristics: horizontal lines have a constant y-value across all x-values, while vertical lines have a constant x-value across all y-values. Straight lines through the origin have the form $y = mx$ and also have only one intercept.

KEY TERMS AND DEFINITIONS

- A line is said to be **horizontal** if two points on the line have the same y-coordinate points.
- The **origin**, (0,0), is the point of intersection of the x and y axes on a Cartesian plane.
- A line is said to be **vertical** if two points on the line have the same x-coordinate points.

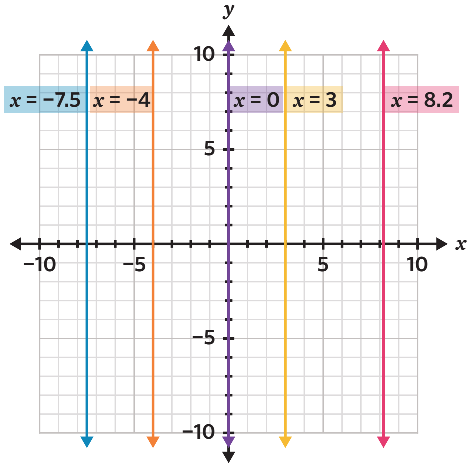
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



In the business world, a company's revenue over a specific period could be represented by a line with only one intercept. This line indicates the start of the measurement period with the slope representing the rate of revenue increase. Other financial measures could be represented by linear equations with one intercept.

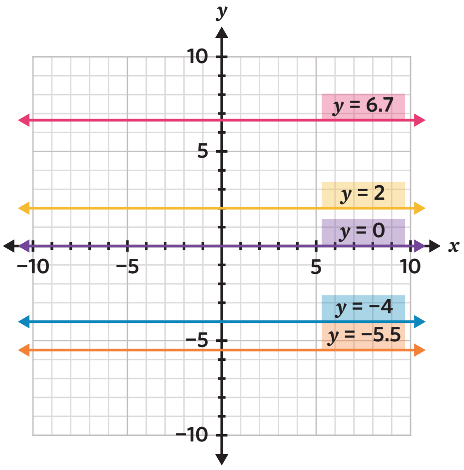
Key ideas

1. Lines that are vertical have only one intercept and their equations can be identified as $x = a$.

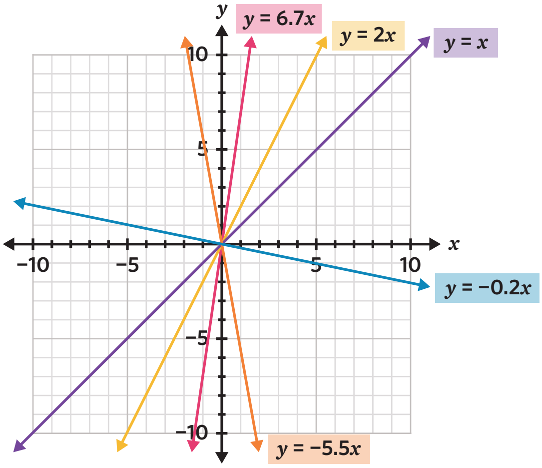


Continues →

2. Lines that are horizontal have only one intercept and their equations can be identified as $y = b$.



3. Lines through the origin have an equation of the form $y = mx$, where m is the gradient (slope) of the line.



Worked example 1

Graphing vertical lines

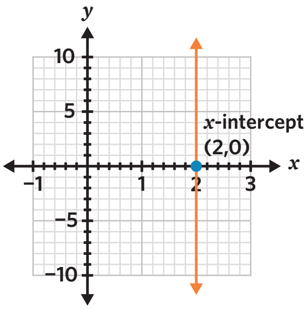
Sketch the graph of the following vertical lines.

a. $x = 2$

WE1a

Working

The x-intercept is at (2,0).



Thinking

Step 1: The equation is in the form $x = a$ and the x-intercept occurs at $(a,0)$.

Step 2: Sketch a vertical line through the x-intercept.

Continues →

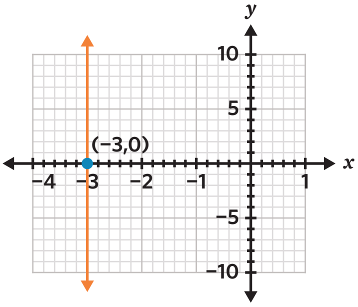
Back to contents

b. $x = -3$

WE1b

Working

The x-intercept is at $(-3,0)$.



Thinking

Step 1: The equation is in the form $x = a$ and the x-intercept occurs at $(a,0)$.

Step 2: Sketch a vertical line through the x-intercept.

Student practice

Sketch the graph of the following vertical lines.

a. $x = 12$ **b.** $x = -9$

Worked example 2

Graphing horizontal lines

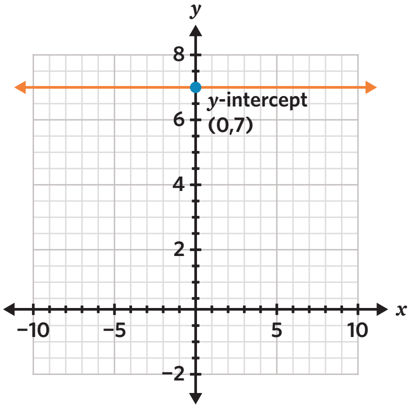
Sketch the graph of the following horizontal lines.

a. $y = 7$

WE2a

Working

The y-intercept is at $(0,7)$.



Thinking

Step 1: The equation is in the form $y = b$ and the y-intercept occurs at $(0,b)$.

Step 2: Sketch a horizontal line through the y-intercept.

Continues →

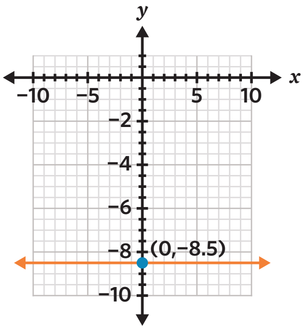
Back to contents

b. $y = -8.5$

WE2b

Working

The y-intercept is at $(0,-8.5)$.



Thinking

Step 1: The equation is in the form $y = b$ and the y-intercept occurs at $(0,b)$

Step 2: Sketch a horizontal line through the y-intercept.

Student practice

Sketch the graph of the following horizontal lines.

a. $y = 1$ **b.** $y = -6$

Worked example 3

Graphing lines through the origin

Sketch the graph of the following lines through the origin.

a. $y = 4x$

WE3a

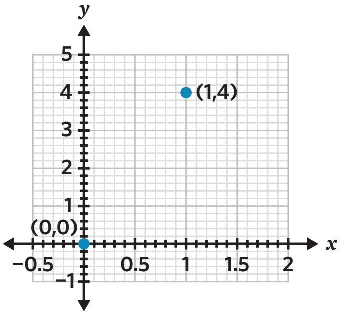
Working

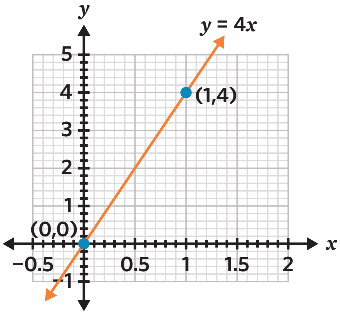
Substitute $x = 1$:

$y = 4(1)$

$y = 4$

$\therefore (x,y) = (1,4)$





Thinking

Step 1: The equation is in the form $y = mx$ so it passes through the origin $(0,0)$. Substitute an x-value to find the corresponding y-value. Identify another point on the graph, (x,y).

Step 2: Plot the origin $(0,0)$ and (x,y) on a Cartesian plane.

Step 3: Sketch the graph by drawing a line through the points.

Continues →

b. $y = -\frac{3}{4}x$

WE3b

Working

Substitute $x = 4$:

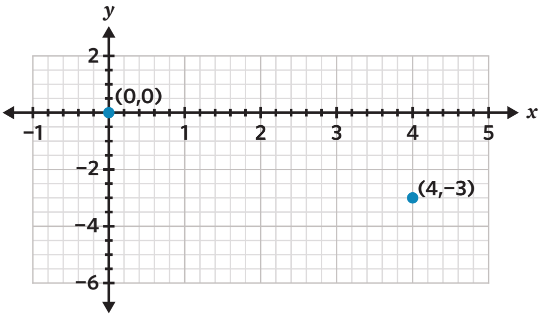
$y = -\frac{3}{4}(4)$

$y = -3$

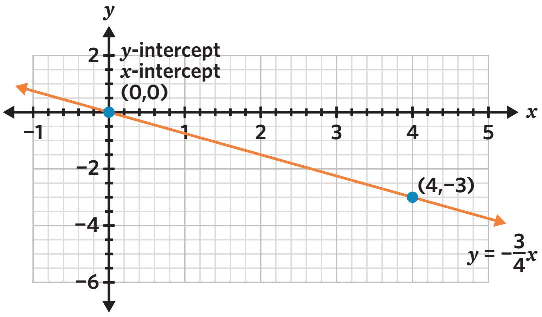
$\therefore (x,y) = (4,-3)$

Thinking

Step 1: The equation is in the form $y = mx$ so it passes through the origin (0,0). Substitute an x-value to find the corresponding y-value. Identify another point on the graph, (x,y).



Step 2: Plot the origin (0,0) and (x,y) on a Cartesian plane.



Step 3: Sketch the graph by drawing a line through the points.

Student practice

Sketch the graph of the following lines through the origin.

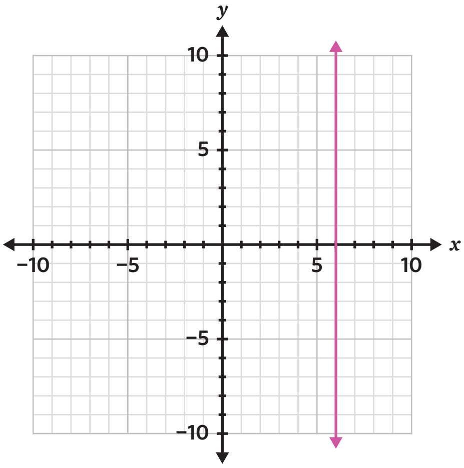
a. $y = 6x$ **b.** $y = -\frac{2}{5}x$

5B Questions

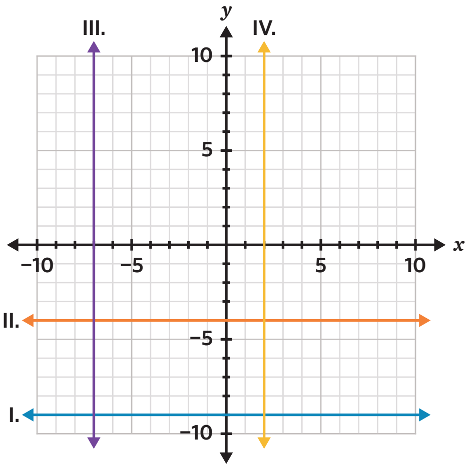
Understanding worksheet

1. Check the box to identify each line as either vertical or horizontal.

Example



Vertical	Horizontal
☑	☐



	Vertical	Horizontal
I.	☐	☐
II.	☐	☐
III.	☐	☐
IV.	☐	☐

2. For each of these lines that pass through the origin, use the given x-coordinate to calculate the y-coordinate for a point on the line.

Example

	x-cordinate	y-cordinate
$y = 4x$	1	4

	x-cordinate	y-cordinate
$y = 10x$	1	
$y = -2x$	1	
$y = \frac{1}{2}x$	2	
$y = -\frac{2}{5}x$	5	

3. Fill in the blanks by using the words provided.

origin	horizontal	vertical	intercept

When sketching straight lines there are some that have only one ______ .

A ______ line is one that goes from left to right and has a single y-intercept, as it remains at a constant y-value. A ______ line goes up and down, and has only an x-intercept, as it remains at a constant x-value. Lines that pass through the ______ are special, as they have both x- and y-intercepts at the same point, (0,0).

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Sketch the graph of the following vertical lines. **WE1**

a. $x = 4$ **b.** $x = -2$ **c.** $x = 0$ **d.** $x = 7$

e. $x = -1.5$ **f.** $x = 9$ **g.** $x = -3.2$ **h.** $x = 2.7$

5. Sketch the graph of the following horizontal lines. **WE2**

a. $y = 3$ **b.** $y = -2$ **c.** $y = 0$ **d.** $y = 5$

e. $y = -1.2$ **f.** $y = 7$ **g.** $y = -4.5$ **h.** $y = \frac{5}{2}$

6. Sketch the graph of the following lines through the origin. **WE3a**

a. $y = x$ **b.** $y = 2x$ **c.** $y = 3x$ **d.** $y = 0.5x$

e. $y = 1.5x$ **f.** $y = 2.5x$ **g.** $y = 0.8x$ **h.** $y = 1.3x$

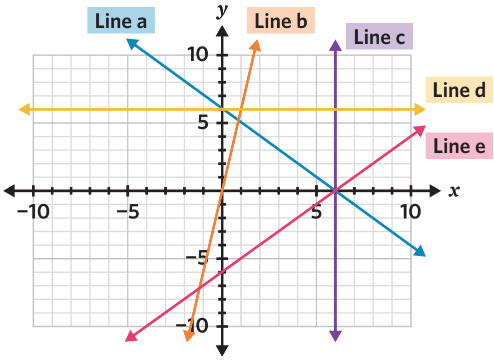
7. Sketch the graph of the following lines through the origin. **WE3b**

a. $y = -0.5x$ **b.** $y = -x$ **c.** $y = -1.5x$ **d.** $y = -2x$

e. $y = -3x$ **f.** $y = -0.4x$ **g.** $y = -\frac{3}{7}x$ **h.** $y = -\frac{5}{3}x$

8. Which of the following lines matches the equation $y = 6$?

A. Line a
B. Line b
C. Line c
D. Line d
E. Line e



Spot the mistake

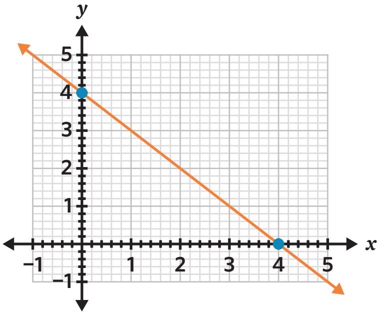
9. Select whether Student A or Student B is incorrect.

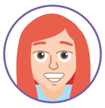
a. Sketch the graph of $x = 4$.

Student A

The x-intercept is at (4,0).

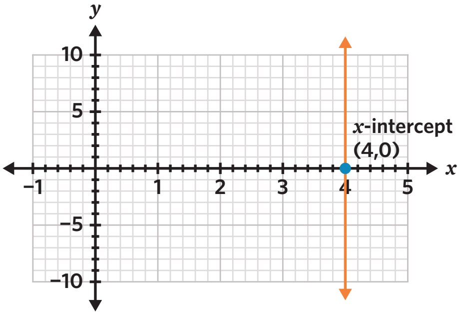
The y-intercept is at (0,4).





Student B

The x-intercept is at (4,0).

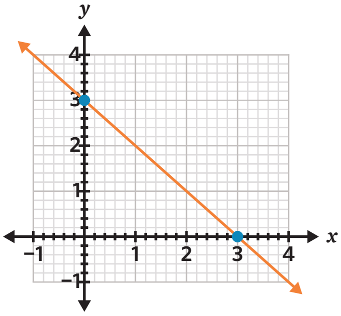


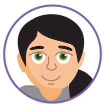
b. Sketch the graph of $y = 3x$.

Student A

The x-intercept is at (3,0).

The y-intercept is at (0,3).





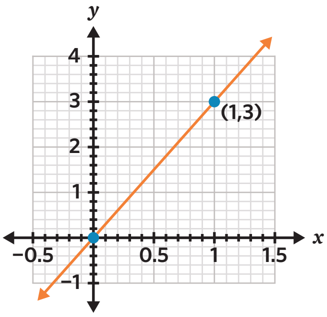
Student B

Substitute $x = 1$:

$y = 3(1)$

$y = 3$

$\therefore (x,y) = (1,3)$



Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. An IT help line is offering a service with a fixed fee (f) of \$5 per month with unlimited questions answered (q). Sketch a horizontal graph to represent this information, $0 \leq q$.

11. An internet provider offers a fixed cost (c) of \$50 per month regardless of the data (d) used. Sketch a horizontal graph to represent this information.

12. A tennis club charges \$25 per lesson. Sketch a graph to represent this information showing the costs (c) for 0 to 10 lessons l.

13. A graphic designer uses a digital program to form a square. The program uses a Cartesian plane and the equations: $y = 0$, $x = 0$, $y = 1$, $x = 1$. What are the coordinates of each of the corners of the square.

14. Three students are planning where they will meet during an orienteering challenge by plotting their paths on a map. They each walk along a different straight line each represented by one of these equations: $y = x$, $y = 7$, $x = 7$. What are the coordinates of the point they will meet?

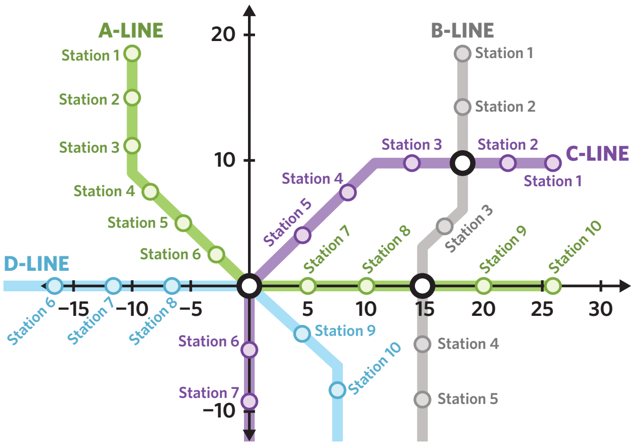
Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. The sections of a train line can be plotted using linear equations. This train network has been placed on a Cartesian plane.

- **a.** Write an equation that could represent A-line and D-lines horizontal sections.
- **b.** Write an equation that, when plotted, would go through A-lines vertical section.
- **c.** Write an equation that, when plotted, would go through C-line stations 1–3.
- **d.** Write an equation that, when plotted, would go through C-line stations 5–4.
- **e.** What other types of transport maps have you seen that use linear equations?



16. Consider the equations $x = a$ and $y = b$.

- **a.** State which equation is horizontal and which is vertical.
- **b.** Plot each equation on a Cartesian plane labelling the point the two lines intersect.
- **c.** Explain how to identify the intersection point of a vertical and horizontal line.

Exam-style

17. What is the equation of line that passes through (1,3) and (20,3)? (1 MARK)

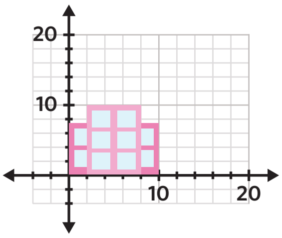
A. $x = 3$ **B.** $x = \frac{3}{20}$ **C.** $y = 3$ **D.** $y = 3x$ **E.** $y = \frac{3}{20}x$

18. A triangle can be drawn using the linear equations $y = -3x$, $y = 3x$ and $y = 3$. (3 MARKS)

a. Sketch the equations and draw the triangle on a Cartesian plane. 2 MARKS

b. What is the distance between the two top corners? 1 MARK

19. An architect is sketching a house and has decided to add an additional three floors to the central part of the design. Draw the extension by sketching and labelling the lines used to extend the walls, as well as the line indicating the new height of the roof. (3 MARKS)



20. A student with a small business walking dogs has predicted that for a year they will have fixed costs (c) of \$500 and they will be charging \$20 per dog walked (d). They will begin making a profit after the break even point which is when revenue = fixed costs. (4 MARKS)

Fixed costs: $c = 500$

Revenue: $c = 20d$

Plot these two equations on the same graph, labelling all key features and use the graph to find the many dogs the student will need to walk before they begin making a profit.

Remember this?

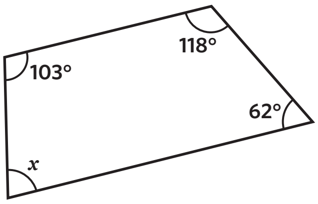
21. The given table shows the approximate population in the sovereign states of the Nordic region in 2022.

The difference between the population in Sweden and Norway is

A. 4 580 000
B. 5 020 000
C. 5 430 000
D. 5 432 000
E. 10 450 000

Country	Population
Denmark	5 870 000
Finland	5 550 000
Iceland	380 000
Norway	5 430 000
Sweden	10 450 000

22. Amy has measured three angles in her garden bed and makes the given diagram.



What is the magnitude of angle x?

A. 62° **B.** 77° **C.** 103° **D.** 118° **E.** 283°

23. There were 15 people who booked a table at an all you can eat Korean BBQ restaurant.

The cost was \$50 per person, plus an extra \$21 for drinks for the whole group.

The total bill can be calculated as

A. $(50 + 21) \times 15$
B. $(15 \times 50) + 21$
C. $15 + 50 + 21$
D. 15×50
E. $(15 \times 21) + 50$

Back to contents

5C Gradient-intercept form

LEARNING INTENTIONS

Students will be able to:

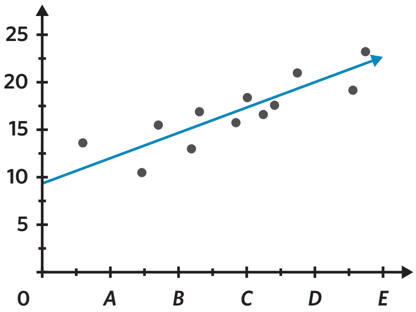
- determine the gradient and y-intercept from a rule or a linear graph
- use the gradient and y-intercept to sketch a linear graph
- determine the equation of a line using the gradient and y-intercept.

The gradient and y-intercept are key features of a linear graph, providing information about the slope and its positioning on a Cartesian plane. Determining these features from a rule allows for the sketching of a line graph. By identifying the gradient and y-intercept of a straight line it is possible to formulate the equation.

KEY TERMS AND DEFINITIONS

- An equation can be **transposed** to make any variable the subject.
- The **gradient** (slope) is the degree of steepness of a line.

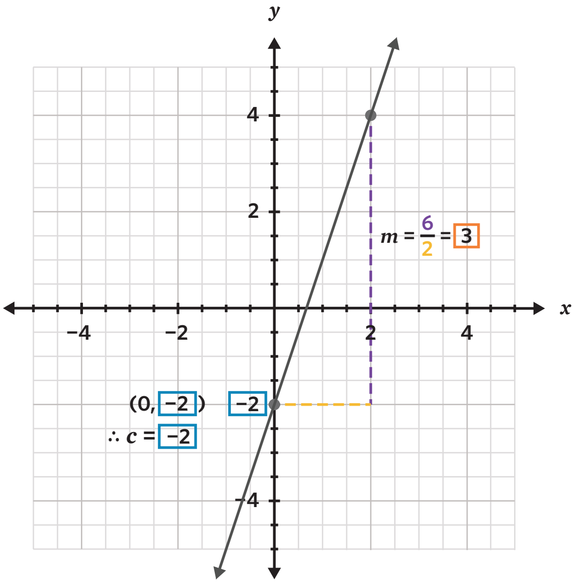
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Knowing how to determine the gradient and y-intercept of a line, and sketching a line graph, can be useful in various fields such as economics. Economists often use these concepts when conducting cost analysis or predicting future trends based on current data.

Key idea

1. The gradient-intercept form is written as $y = mx + c$, where m is the gradient, c is the y-intercept, and y is the subject.



$\therefore y = mx + c$ is:

$$y = \boxed{3}\,x\,\boxed{-2}$$

Back to contents

Worked example 1

Stating the gradient and y-intercept

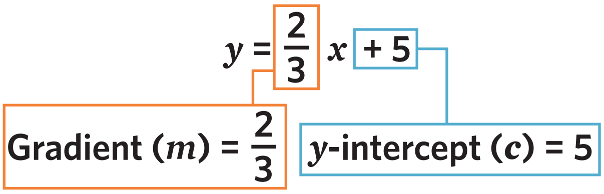
State the gradient and the y-intercept for the following equations.

a. $y = \frac{2}{3}x + 5$

WE1a

Working	Thinking
$y = \frac{2}{3}x + 5$ ✓	**Step 1:** Confirm that the equation is in gradient-intercept form, $y = mx + c$.
y-intercept $(c) = 5$	**Step 2:** Identify the y-intercept (c) by referring to the constant.
Gradient $(m) = \frac{2}{3}$	**Step 3:** Identify the coefficient of x, known as the gradient (m). This is the number that is being multiplied by x.

Visual support



b. $12x + 3y = 3$

WE1b

Working	Thinking
$12x + 3y = 3$ ✗	**Step 1:** Confirm that the equation is in gradient-intercept form, $y = mx + c$.
$12x + 3y = 3$ $3y = 3 - 12x$ $y = \frac{3}{3} - 4x$ $y = -4x + 1$	**Step 2:** Transpose to make y the subject and write the equation in the form of $y = mx + c$.
y-intercept $(c) = 1$	**Step 3:** Identify the y-intercept (c) by referring to the constant.
Gradient $(m) = -4$	**Step 4:** Identify the coefficient of x, known as the gradient (m). This is the number that is being multiplied by x.

Student practice

State the gradient and the y-intercept for the equations.

a. $y = \frac{1}{4}x + 2$ **b.** $20x + 2y = 10$

Worked example 2

Sketching linear graphs using the gradient and y-intercept

Sketch the graphs of the following linear equations.

a. $y = \frac{1}{2}x - 4$

WE2a

Working

$y = \frac{1}{2}x - 4$ ✓

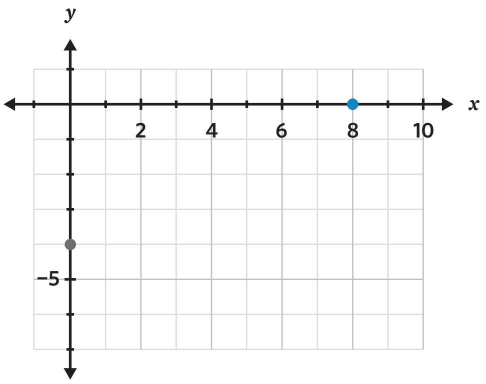
Thinking

Step 1: Confirm that the equation is in gradient-intercept form, $y = mx + c$.

y-intercept $(c) = -4$

$(0,-4)$

Step 2: Identify the coordinate for the y-intercept (c) by referring to the constant.

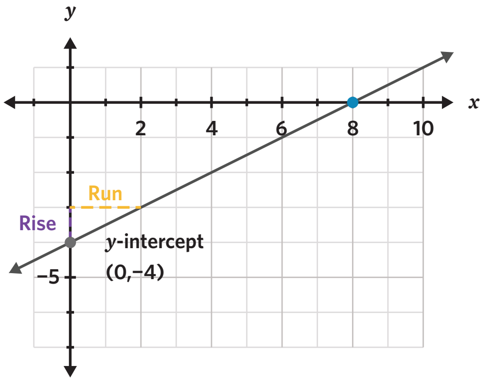


Step 3: Plot the coordinate for the y-intercept.

$m = \frac{1}{2}$

$\frac{\text{rise}}{\text{run}} = \frac{1}{2}$, gradient is positive.

Step 4: Calculate the gradient and determine if the slope is positive or negative.This is determined by the sign in front of the coefficient of x and/or determining if the y-value increases or decreases for each unit increase in x.



Step 5: Using $\frac{\text{rise}}{\text{run}}$ and starting at the y-intercept plot a second point and draw a line through the two points.

b. $3x + y = 8$

WE2b

Working

$3x + y = 8$ ✗

Thinking

Step 1: Confirm that the equation is in gradient-intercept form, $y = mx + c$.

$y = 8 - 3x$

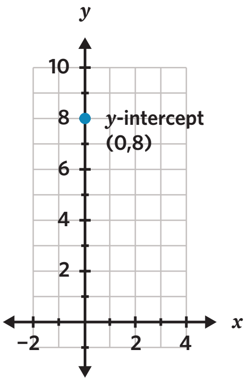
$y = -3x + 8$

Step 2: Transpose to make y the subject and write the equation in the form of $y = mx + c$.

Continues →

y-intercept $(c) = 8$

Step 3: Identify the coordinates for the y-intercept (c) by identifying the constant. State the y-intercept as a coordinate point.

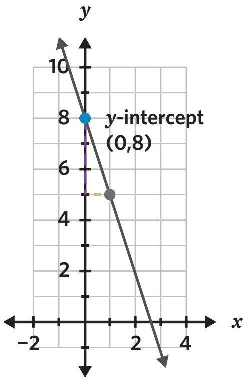


Step 4: Plot the coordinate for the y-intercept.

$m = -3$

$\frac{\text{rise}}{\text{run}} = \frac{-3}{1}$, gradient is negative.

Step 5: Calculate the gradient and determine if the slope is positive or negative.This is determined by the sign in front of the coefficient of x and/or determining if the y-value increases or decreases for each unit increase in x.



Step 6: Using $\frac{\text{rise}}{\text{run}}$ and starting at the y-intercept plot a second point and draw a line through the two points.

Student practice

Sketch the graphs of the following linear equations.

a. $y = \frac{2}{5}x - 1$

b. $4x + y = 5$

Back to contents

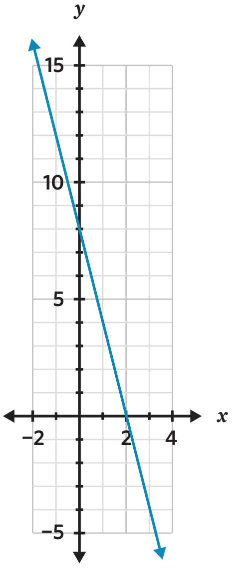
Worked example 3

Determining the linear equation

Determine the linear equation for the following graphs.

a.

WE3a



Working

(0,8)

$c = +8$

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{-8}{2}$

$= -4$

$y = -4x + 8$

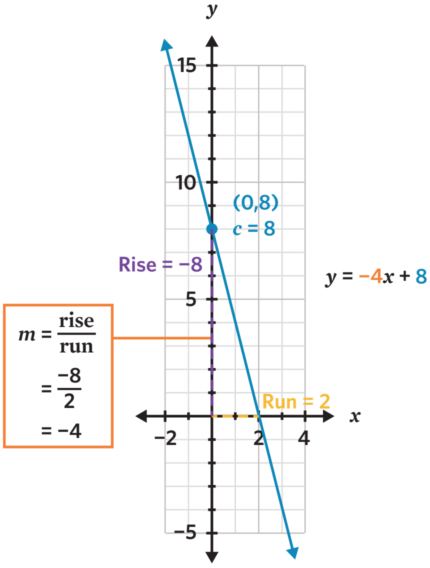
Thinking

Step 1: Identify the y-intercept coordinate from the graph.

Step 2: Calculate the gradient by using $m = \frac{\text{rise}}{\text{run}}$

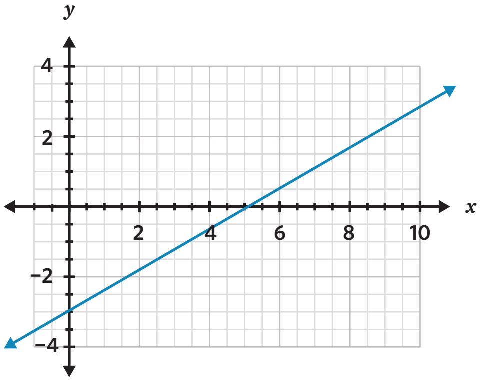
Step 3: Write the equation in the form $y = mx + c$.

Visual support



Continues →

b.

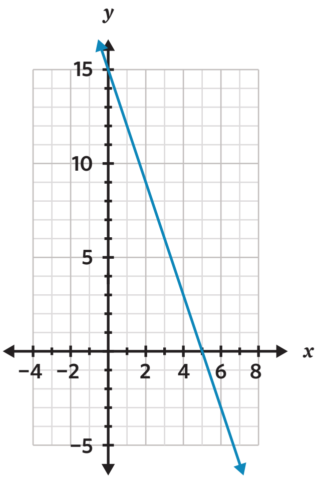


Working	Thinking
(0,−3) $\therefore c = -3$	**Step 1:** Identify the y-intercept coordinate from the graph.
$m = \frac{\text{rise}}{\text{run}}$ $= \frac{3}{5}$	**Step 2:** Calculate the gradient by using $m = \frac{\text{rise}}{\text{run}}$.
$y = \frac{3}{5}x - 3$	**Step 3:** Write the equation in the form $y = mx + c$.

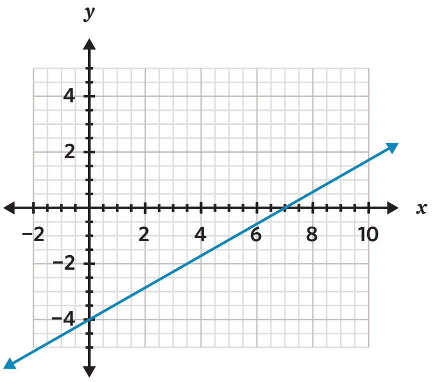
Student practice

Determine the linear equation for the following graphs.

a.



b.



5C Questions

Understanding worksheet

1. State the gradient for each of the linear equations.

Example

$y = 2x + 4 \therefore m = \boxed{2}$

a. $y = 4x + 7 \therefore m = \boxed{}$

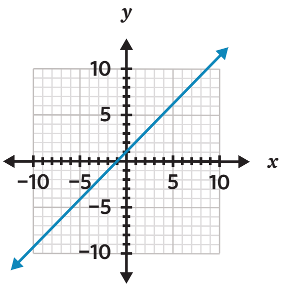
b. $y = -3x - 2.5 \therefore m = \boxed{}$

c. $y = 4 - 5x \therefore m = \boxed{}$

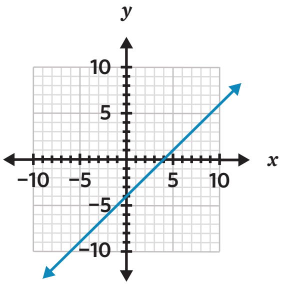
d. $y = -2 + 0.5x \therefore m = \boxed{}$

2. Match the graph with its equation.

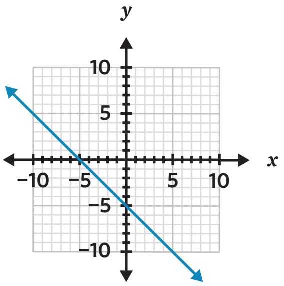
Graph **Equation**



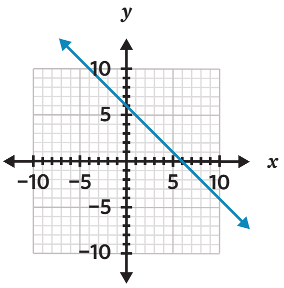
● ● $y = -x + 6$



● ● $y = x - 4$



● ● $y = x + 1$



● ● $y = -x - 5$

Back to contents

3. Fill in the blanks by using the words provided.

equation	gradient	*y*-intercept	sketch

To sketch a linear graph, determine the ______ and *y*-intercept from a given rule.

Use these two values to plot the corresponding points on a Cartesian plane to ______

the graph. The ______ of a line can be found using the gradient and ______ .

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. State the gradient and the *y*-intercept for the equations. **WE1**

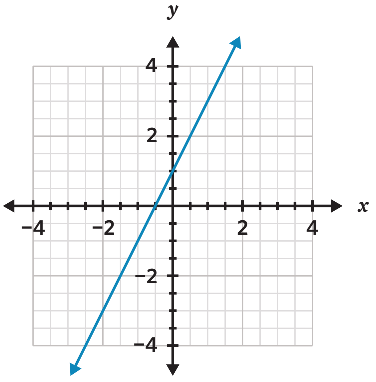
a. $y = 2x + 3$
b. $y = -3x - 1$
c. $y = 0.5x + 2$
d. $2x + y = 9$
e. $y = -1.2x - 3$
f. $2.5x + y = 1$
g. $-0.8x + 2y = 0.2$
h. $1.2x - y = -3$

5. Sketch the graphs of the following linear equations. **WE2**

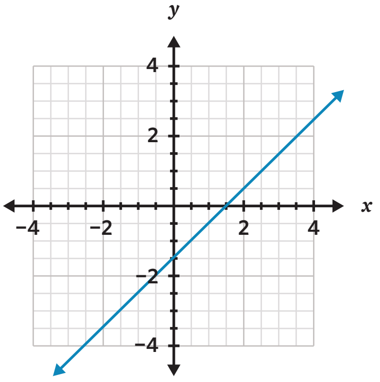
a. $y = x + 5$
b. $y = 2x - 4$
c. $y = -4x + 1$
d. $4x + y = 5$
e. $y = \frac{3}{5}x + 1$
f. $-3x + y = -1$
g. $0.5x - y = 2$
h. $-1.5x - y = -4$

6. Determine the linear equation for the following graphs. **WE3a**

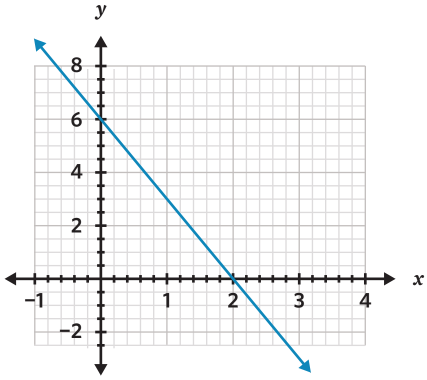
a.



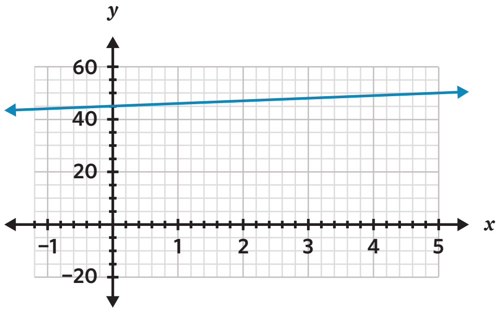
b.



c.

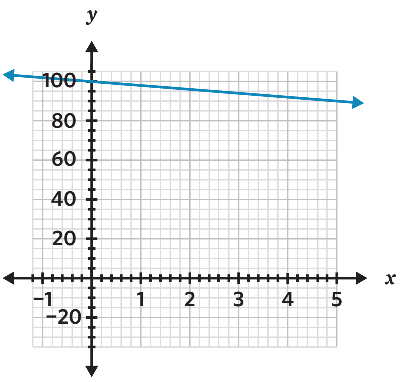


d.

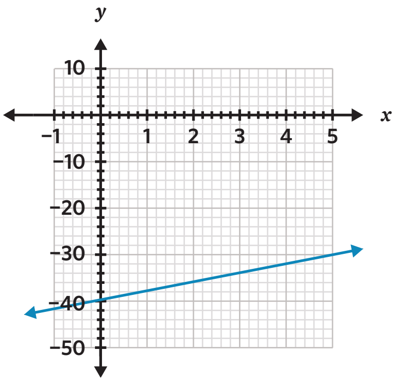


Back to contents

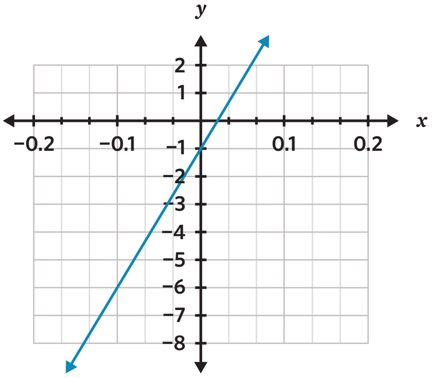
e.



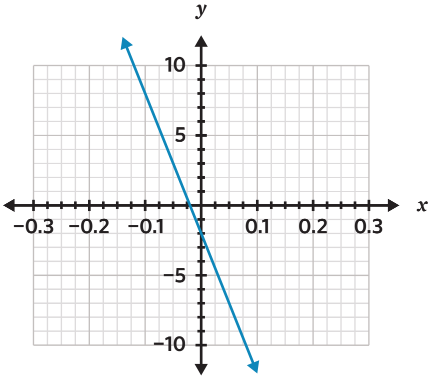
f.



g.



h.



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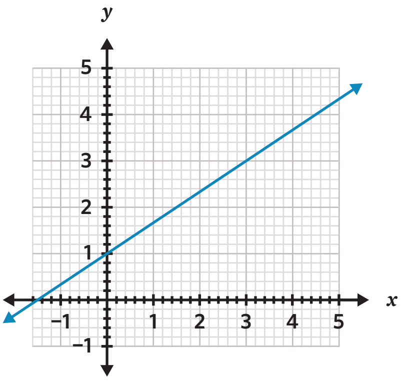
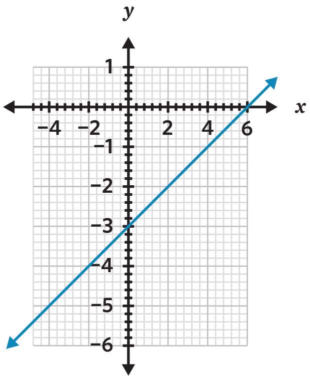
7. Determine the linear equation for the following graphs.

WE3b

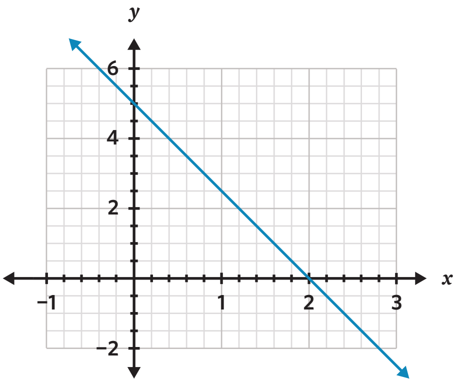
a.



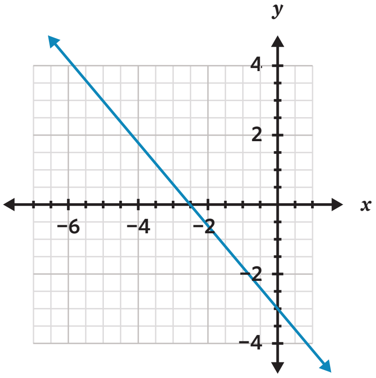
b.



c.

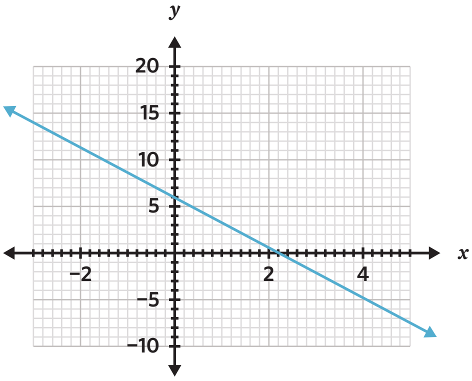


d.

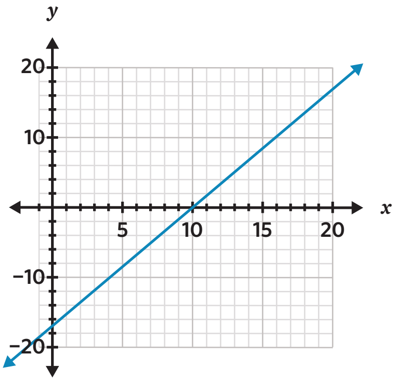


Back to contents

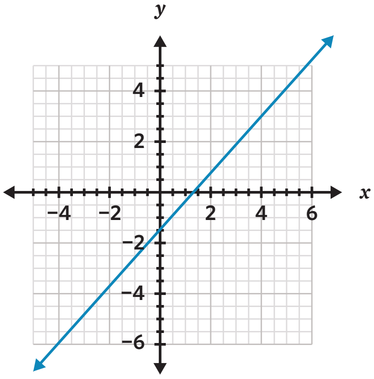
e.



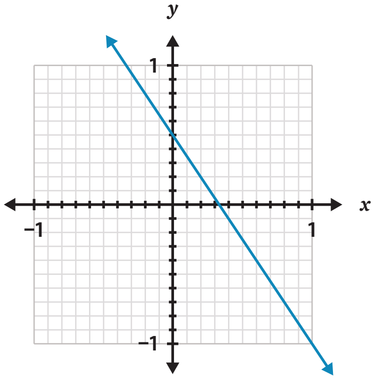
f.



g.



h.



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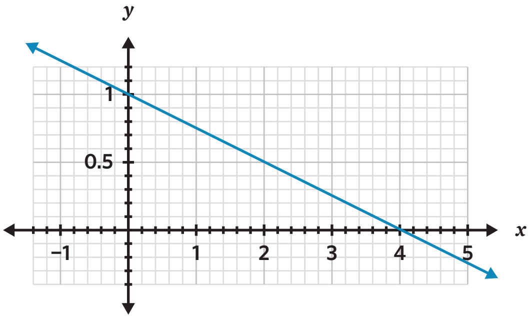
8. State the gradient for the equation $2x + 2y = -3$.

A. -3 **B.** -1 **C.** 0 **D.** 2 **E.** 3

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the equation of the line.



Student A

y-intercept is $(0,1)$

$c = 1$

$m = \frac{-4}{1}$

$= -4$

$\therefore y = -4x + 1$

Student B

y-intercept is $(0,1)$

$c = 1$

$m = \frac{-1}{4}$

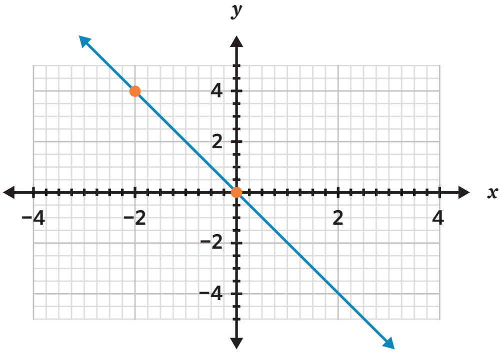
$= \frac{-1}{4}$

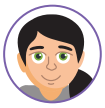
$\therefore y = -\frac{1}{4}x + 1$

Back to contents

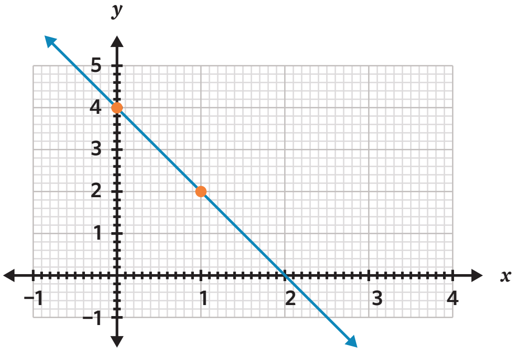
b. Sketch the graph of $y = -2x + 4$.

Student A





Student B



Problem solving

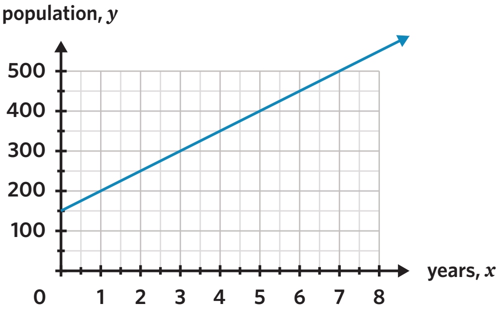
Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. An electricity company charges a monthly fee of \$10 plus \$0.25 per kilowatt hour used. This information can be graphed using the the rule $b = 0.25k + c$, where b is the amount of the monthly electricity bill, k is the number of kilowatt hours used. What is the value of c?

11. A student puts \$30 in a moneybox and then adds \$5 every week. Generate an equation representing the total amount of money in the moneybox (y) after (x) weeks.

12. The graph shows the population of a town over time.



Generate an equation that represents the population over time.

13. A cup of hot chocolate is placed in a freezer and the temperature is measured each minute until it reaches −2°C. Sketch the graph if the initial temperature is 85°C and the temperature is dropping at a constant rate of 3°C each minute.

14. The cost of renting an e-scooter for one ride is advertised as \$1 plus an additional fee of \$1.20 per kilometre travelled. Generate an equation to represent the total cost (c) depending on the distance travelled (d).

Back to contents

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. A postal service offers 2 options for sending a parcel. The total cost (c) for both of these options dependent on the weight (w), in kilograms, is

Option A: total cost $= \$0.50 \times$ weight $+ \$10$

Option B: total cost $= \$3 + \$1 \times$ weight

a. Plot Option A on a Cartesian plane.

b. Plot Option B on the same Cartesian plane.

c. Calculate which option is the cheaper option for a parcel weighing 5 kg.

d. Using the graph from parts **a** and **b**, what does the graph display about the cost of sending a parcel weighing 14 kg for both options?

e. Explain why some postal services offer different rates based on the priority and weight.

16. Linear equations can be writing in the form $ax + by = c$.

a. Transpose the equation to make y the subject, writing in the form $y = mx + c$.

b. With reference to part **a**, state the gradient and y-intercept of $ax + by = c$.

c. Write a statement summarising your findings from part **b** relating how to calculate the gradient and y-intercept from the form $ax + by = c$.

Exam-style

17. The equation of a straight line with a y-intercept of (0,3) is (1 MARK)

A. $x = 3$ **B.** $y = x + 3$ **C.** $y = x + 4$ **D.** $y = 3x + 1$ **E.** $y = 3x + 4$

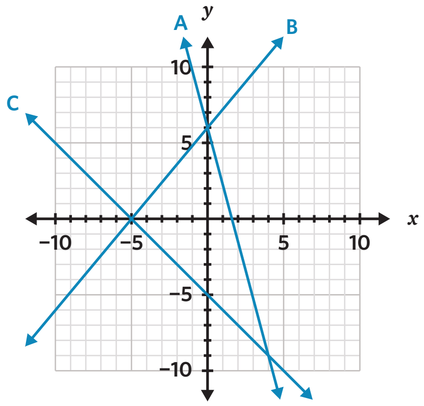
18. A straight line has a y-intercept of (0,2) and passes through the point (3,5). (3 MARKS)

a. Plot the two points and draw the line passing through them. 1 MARK

b. Determine the gradient of the line. 1 MARK

c. Determine the equation of the line. 1 MARK

19. The graph displays three lines, which all intersect. Generate the equation of each of the lines. (3 MARKS)



20. A line passes through the points (0,3) and (7,0). Plot the graph labelling the y- intercept, and calculate the equation of the line. (3 MARKS)

Back to contents

Remember this?

21. At a local cafe, the ratio of lattes to flat whites is always 4 : 3.

On a particular day, there were 20 more lattes than flat whites.

In total, how many lattes and flat whites were sold?

A. 20 **B.** 60 **C.** 80 **D.** 140 **E.** 200

22. Patrick is designing a custom Lego build.

He spent 5 hours designing and 7 hours building.

The percentage of time that was spent building is approximately

A. 35% **B.** 41% **C.** 42% **D.** 58% **E.** 140%

23. John has a pond with three Koi fish.

Blaze is $\frac{5}{6}$ the length of Iris.

Kai is $\frac{4}{5}$ the length of Iris.

Blaze is 2 cm longer than Kai.

What is the total length of Iris, in centimetres?

A. 2 cm **B.** 48 cm **C.** 50 cm **D.** 60 cm **E.** 90

5D Gradient and direct proportion

LEARNING INTENTIONS

Students will be able to:

- understand the concept of direct proportion
- understand direct proportion as gradient
- form an equation that links two variables that are directly proportional
- use the constant rate of change to sketch a graph.

Direct proportionality links two variables in a way that their ratio remains constant. Forming an equation that represents this relationship involves understanding the constant rate of change, also known as the gradient. A relationship that is in direct proportion can be graphed using the linear rule in the form $y = mx$ where the gradient (m) represents the constant rate of change in one variable with respect to the other.

KEY TERMS AND DEFINITIONS

- A **rate** is used to compare quantities in different units.
- A **variable** is a letter used to represent a value that is unknown or may vary. This is also known as a pronumeral or an unknown.

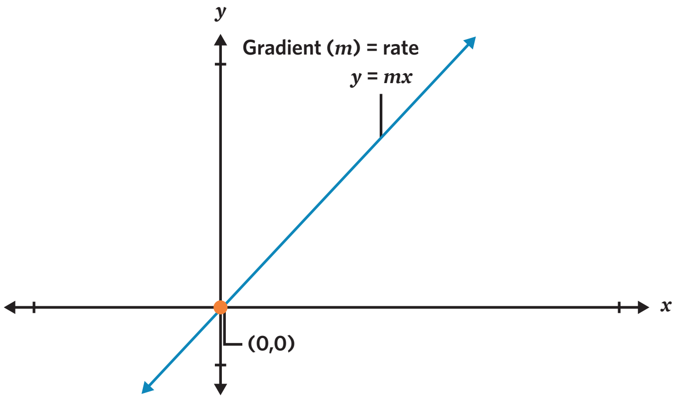
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Robyn Mackenzie/Shutterstock.com

Direct proportionality is a concept that is prevalent in cooking. When a recipe is scaled up or down to serve a different number of people, the ingredients increase or decrease in direct proportion to the original amounts.

Key idea

1. Two variables are directly proportional when the rate of change of one variable with respect to the other is constant. This can be represented by an equation of the form $y = mx$, where the gradient (m) represents the rate of change of y with respect to x.



Worked example 1

Graphing representations of direct proportion

Plot the graph for each directly proportional relationship.

a. A school group plans to walk 12 km per day during a 5-day hiking camp.
Plot a graph of distance (d) vs time (t) for $0 \leq t \leq 5$.

WE1a

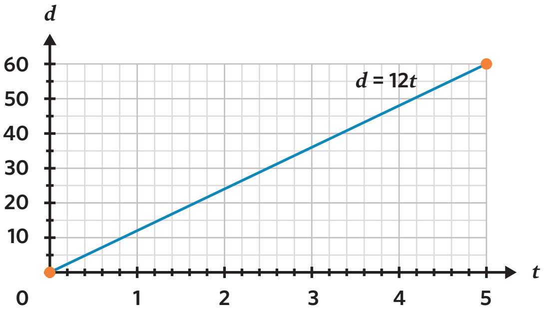
Working

The rate is 12 km per day (12 km/day).

Distance when $t = 5$:

$12 \times 5 = 60$ km

$\therefore (5,60)$



Thinking

Step 1: Identify the described rate.

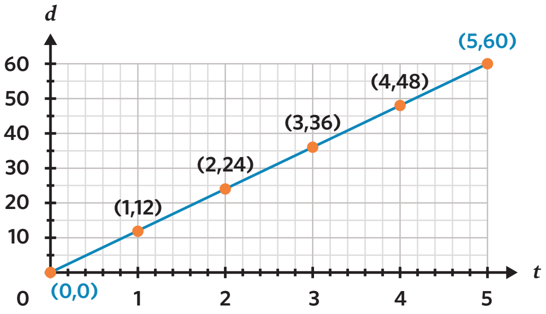
Step 2: Calculate any point using the given information.

Step 3: Choose a suitable scale, and draw a Cartesian plane, using the horizontal axis to represent time.

Step 4: Plot the origin (0,0) together with the calculated point on the graph. Connect both points with a straight line.

Visual support

Time (t)	0	1	2	3	4	5
Distance (d)	0	12	24	36	48	60



b. A tap is leaking at a constant rate and fills a 4.8 L bucket over a 2 day weekend.
Sketch a graph of volume in millilitres (V) vs time in hours (h), $0 \leq h \leq 10$.

WE1b

Working

4.8 L in 2 days = 4800 mL in 48 h.

$$= \frac{4800}{48}$$

= 100 mL per hour

Volume when $t = 2$:

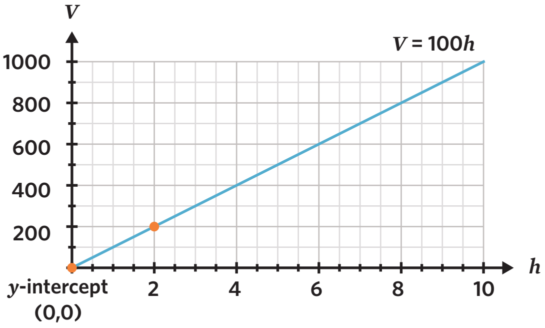
$V = 100 \times 2 = 200$ mL

Thinking

Step 1: Convert values into the described rate.

Step 2: Calculate any point by using the given information.

Continues →



Step 3: Choose a suitable scale, and draw a Cartesian plane, using the horizontal axis to represent time.

Step 4: Plot the origin (0,0) together with the calculated point on the graph. Connect both points with a straight line.

Student practice

Plot the graph for each directly proportional relationship.

a. A market stall advertises \$4 per strawberry punnet. Plot a graph of cost (\$$c$) vs punnets, ($p$) $0 \leq p \leq 10$.

b. On average a 24-hour business generates 0.5 kilograms of waste per hour.
Sketch a graph of waste generated in kilograms(k) vs days (d), $0 \leq d \leq 7$.

Worked example 2

Forming an equation to link two variables in direct proportion

Write the rule connecting the directly proportional variables.

a. Fuel is advertised at \$1.89 per litre. Write an equation connecting the cost (\$$c$) of filling a tank and the litres (l). WE2a

Working	Thinking
The rate is \$1.89 per L $m = 1.89$	**Step 1:** Identify the rate and write it as the gradient.
$c = 1.89l$	**Step 2:** Write an equation including the two variables, and the rate, in the form $y = mx$.

b. A pump running constantly for 3 hours filters 600 L of water. Write an equation connecting the volume of water in litres (V) of filling a tank and the time the pump is running (t). WE2b

Working	Thinking
The rate is 600 L in 3 hours	**Step 1:** Identify the rate.
600L in 3 hours = 200 L per hour. $m = 200$	**Step 2:** Simplify the rate and write it as the gradient.
$V = 200t$	**Step 3:** Write an equation including the two variables, and the rate, in the form $y = mx$.

Student practice

Generate the rule connecting the directly proportional variables.

a. A dance school charges \$22 per lesson. Generate an equation connecting the cost (c) and the number of lessons attended (l).

b. A 10kg bag of rice costs \$38.00. Write an equation connecting the total cost of rice (c) and the number of kilograms (n).

Back to contents

Worked example 3

Solving problems involving direct proportion

Solve the following problems using directly proportional relationships.

a. The final bill ($\$b$) at a cafe with a set lunch menu can be described using the rule $b = 27g$, where g is the number of guests. Calculate the bill amount for 10 guests. **WE3a**

Working	Thinking
$b = 27g$, where $g = 10$: $b = 27 \times 10$ $b = 270$	**Step 1:** Substitute the known value into the given rule and solve.
The bill for 10 guests is $270.	**Step 2:** Write the solution in the context of the question.

b. A train carriage holds 66 passengers. Generate a rule connecting the total number of passengers (p) and number of carriages (c) and use it to calculate the number of carriages required for 198 passengers. **WE3b**

Working	Thinking
The rate (m) is 66 passengers per carriage. $m = 66$ $p = 66c$	**Step 1:** Use the given rate to generate a rule in the form $y = mx$.
$p = 66c$, where $p = 198$: $198 = 66c$ $c = \frac{198}{66}$ $c = 3$	**Step 2:** Substitute the known value into the given rule and solve.
3 carriages are required for 198 passengers.	**Step 3:** Write the solution in the context of the question.

Student practice

Solve the following problems using directly proportional relationships.

a. The cost ($\$c$) for an order of hot drinks can be described using the rule $c = 4.5d$, where d is the number of hot drinks. Calculate the bill amount for 5 hot drinks.

b. A bus holds 52 passengers. Generate a rule connecting the total number of passengers (p) and number of buses (b), and use it to calculate the number of buses required for a school group of 208 students.

Back to contents

5D Questions

Understanding worksheet

1. State the rate for each of these situations using the rate shown in brackets.

Example

A gardener charges a constant rate per hour and receives $135 for 3 hours work.

$ 45 /h

a. A cyclist rides at a constant rate for 2 hours and travels 86 km.

km/h

b. A 40 000 L tank takes 2 hours to fill.

L/h

c. A swimmer takes 20 strokes in a 30 second race.

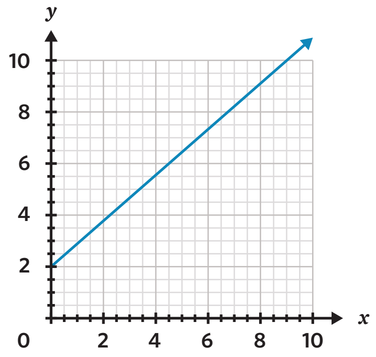
strokes/min

d. A stall selling cupcakes at a set price sold 10 cupcakes for $25.50.

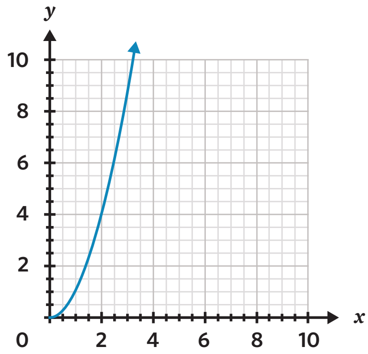
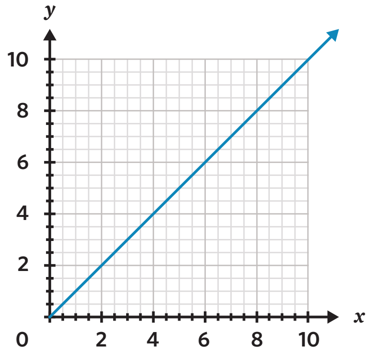
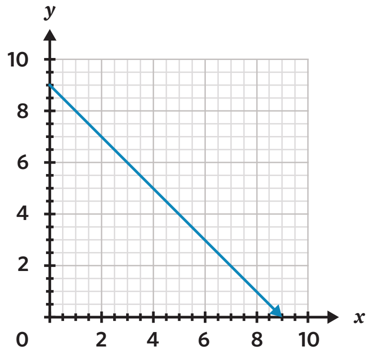
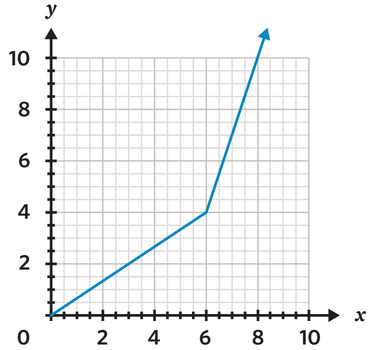
$ /cupcake

2. Identify the graphs as direct proportion or not direct proportion.

Example

	Direct proportion	**Not direct proportion**
	☐	☑

Back to contents

	Direct proportion	Not direct proportion
I. 	☐	☐
II. 	☐	☐
III. 	☐	☐
IV. 	☐	☐

3. Fill in the blanks by using the words provided.

gradient **variables** **rate** **proportional** **constant**

Direct proportion is a concept where two ________ are related such that as one increases, the other increases at a constant ________ . This can be understood as the ________ of a linear graph and represents the ________ rate of change. When two variables are directly ________ , they can be linked using the rule $y = mx$.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Plot the graph for each directly proportional relationship. WE1

- **a.** The distance travelled (d) by a car and the time (t) it takes when travelling at 60 km/h when $0 \leq t \leq 5$.
- **b.** Total rental cost (\$$c$) for a property costing \$420 per week (w) when $0 \leq w \leq 4$.
- **c.** A gym membership costs (\$$c$) \$50 per month (m) when $0 \leq m \leq 12$.
- **d.** A phone plan which costs (\$$c$) \$20 per week (w) when $0 \leq w \leq 4$.
- **e.** A cyclist rides at a constant speed of 45 km/h for 4 hours, $0 \leq t \leq 4$. Use d for distance in kilometres and t for time in hours.
- **f.** A water filter pumps runs constantly at 10 000 m^3/h for 8 hours. Use V for volume in cubic metres and t for time in hours. $0 \leq t \leq 8$.
- **g.** A person types five 500-word documents in 3 hours. Sketch a graph showing words (w) vs minute (m) when $0 \leq m \leq 5$.
- **h.** A pet supplies shop sells 9.8 kg of fish feed every 2 weeks. Sketch a graph showing the amount of fish sold (a) vs day (d) when $0 \leq d \leq 7$.

5. Generate the rule connecting the directly proportional variables. WE2

- **a.** A cyclist travelling at 38 km/h. Use d for distance in km and t for time in hours.
- **b.** An air filter is advertised as filtering 12 000 m^3/h. Use v volume for cubic metres and t for time in hours.
- **c.** I type a 1500 word presentation in 30 minutes. Use w for words typed and t for time in minutes.
- **d.** A bakery sells 120 items of hot food in 4 hours. Use h for the number of items sold and t for time in hours.
- **e.** A car travels 495 kms in 9 hours. Use d for distance in kilometres and t for time in hours.
- **f.** A holiday rental costs \$2500 for 5 nights. Use c for cost in dollars and n for the number of nights.
- **g.** A gym membership can be prepaid at \$1260 for 1 year. Use c for cost in dollars and t for time in months.
- **h.** A phone plan can be prepurchased for \$320 for 365 days. Use c for cost in dollars (to the nearest cent), and t for time in weeks.

6. Solve the following problems using directly proportional relationships. WE3

- **a.** Profit (\$$p$) can be described using the rule $p = 30n$, where n is the number of items sold. Calculator the profit on 20 items.
- **b.** Distance (d) in kilometres can be described using the rule $d = 2t$, where t is time in minutes. Calculate the distance travelled in 30 minutes.
- **c.** Commission paid can be described using the rule $c = 0.02s$, where s is the total sales made in dollars. Calculate the commission paid on \$34 000.
- **d.** A phone bill (\$$b$) can be described using the rule $b = 0.05t$, where t is the number of text messages sent. Calculate the total number of messages that can be sent for \$5.
- **e.** The number of fruit boxes packed (n) at a distribution centre can be described by the rule $n = 30p$, where p is the number of packers working that day. How many fruit boxes can be packed by 15 packers?
- **f.** On average 342 L of water flows into a dam each hour. Generate a rule connecting amount of water (v) and time (t) and use this to calculate the amount of water flowing into the dam in 6 hours.
- **g.** A student typing an assignment, types at 25 words per minute. Write a rule connecting the number of words (w) typed and the time (t), and use it to calculate the time it will take to write a 2500 word essay.
- **h.** One movie ticket costs \$12.50 and the theatre has a sales target of \$6425. Write a rule connecting ticket sales (s) and number of tickets sold (t), and use it to calculate the tickets required to meet the sales target.

Back to contents

7. A heavy truck can travel 6.2 km/L. If the fuel gauge shows 30 L, calculate how many kilometres the truck can travel before the tank is empty.

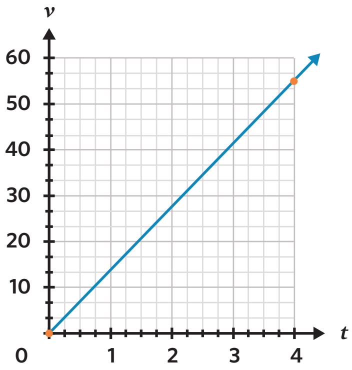
A. 0.2 km **B.** 4.8 km **C.** 6.2 km **D.** 62 km **E.** 186 km

Spot the mistake

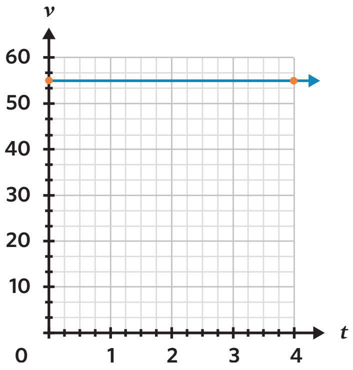
8. Select whether Student A or Student B is incorrect.

a. A 55L fuel tank takes 4 mins to fill from empty. Sketch a graph of volume vs time, $0 \leq t \leq 4$.

Student A



Student B



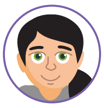
b. A manufacturer produces 30 items in 15 minutes at a constant rate.
How many items are produced in 3 hours?

Student A

The rate is 30 items per hour.

$30 \times 3 = 90$

90 items are produced in 3 hours.



Student B

Using 30 items in 15 minutes, the rate is 120 items per hour.

$120 \times 3 = 360$

360 items are produced in 3 hours.

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. A student deposits \$30 per month into a money box. Calculate the amount in the money box after 5 months.

10. A leaf travels down a river at 2 km per hour. Calculate the distance a leaf would travel in 2 hours.

11. The tide is receding at 2 cm per minute. Write a rule connecting the distance (d) and time (t) and calculate the time, in minutes, it would take for the tide to recede 7 cm.

12. A pond is being filled with water using two garden hoses. One hose has a flow rate of 20 L per minute and the other 30 L per minute. Calculate the time, in minutes, it will take to fill a 1000 L pond.

13. A kayaker is paddling at a rate of 3.5 km per hour. If the kayaker is paddling upstream against a river flowing at 1.5 km per hour, calculate how far they will travel in 3 hours.

Back to contents

Reasoning

Question working paths

Mild 14 (a,b,c,e) 🌶	**Medium** 14 (a,b,c,e), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. Runners completing a 10 km fun run are provided their time to the nearest minute at various markers on the course. Two runners' times are shown in the table.

	Time (minutes)	
Distance (km)	**Tina**	**Jami**
0	0	0
1	6	5
3	18	15
5	30	27
10	60	62

a. Plot the data points for both runners on the same set of axes, with distance on the x-axis and time on the y-axis. Then, draw a straight line passing through the data points that are directly proportional.

b. Write a rule for the runner who is running at a constant rate.

c. Write a rule for a third runner who is constantly running 5 minutes per kilometre.

d. Using your answer from part **c**, plot the directly proportional relationship in the graph generated in part **a**.

e. Outline a running strategy to help with endurance running.

15. The area of a square is $A = l^2$ and the perimeter is $p = 4l$.

a. Complete the table of values using the rule for the area.

Length (m)	0	1	2	3	4
Area (m^2)					

b. Complete the table of values using the rule for perimeter.

Length (m)	0	1	2	3	4
Perimeter (m)					

c. With reference to parts **a** and **b**, explain if the area or perimeter of a square is directly proportional to its length?

Exam-style

16. If variables x and y are directly proportional, which rule is correct? (1 MARK)

A. $x + y =$ constant

B. $x - y =$ constant

C. $\frac{y}{x} =$ constant

D. $x =$ constant

E. $y =$ constant

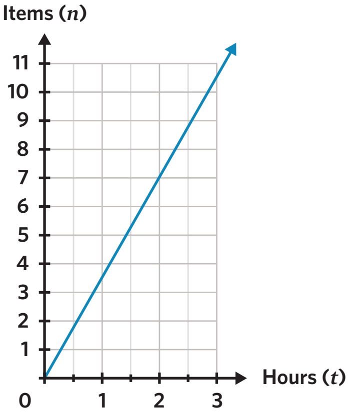
17. For a certain car, the manufacturer states that, on average, the car will travel 825 km on a 55L tank of fuel. (3 MARKS)

a. What is the rate the car uses fuel in kilometres per litre? 1 MARK

b. Write a rule for how far the car can travel, using *d* for distance in kilometres and *l* for litres used. 1 MARK

c. How far can the car travel with 40 L of fuel? 1 MARK

18. This graph shows the number of items produced by a machine. Generate the rule for how many items the machine can produce using *n* for the number of items and *t* for the time taken. (3 MARKS)
Use this rule to predict how many items could be produced in 200 hours.



19. If two swimmers swim at a constant rate, which swimmer has the greatest speed? Explain. (3 MARKS)

- 200m in 4:30 minutes
- 50m in 1:15 minutes

Remember this?

20. An electric bike manufacturer has rolled out two brand new bike models. Their top speeds were recorded.

Bike A can travel 9.65 km in 30 mins.

Bike B can travel 15.3 km in 45 mins.

What is the difference in the top speed, in kilometres per hour, of the two bikes?

A. 0.9 km/hr
B. 1.1 km/hr
C. 19.3 km/hr
D. 20.4 km/hr
E. 66 km/hr

21. A cinema charges $20 per ticket plus an additional $2 booking fee for each order.

The cost, C, per booking can be calculated using the formula $C = 2 + 20n$, where n is the number of tickets purchased.

A group of 7 friends wants to watch the new Disney movie.

How much would they pay?

A. $22
B. $100
C. $140
D. $142
E. $154

Back to contents

22. At a school athletics carnival, scores are given based on the degree of participation.

Each team receives either 1 point for taking part in an event, 2 points for successfully completing it, or 3 points for setting a new record, as shown in the following table.

Participation	Completion	Record-breaker
1 point	2 points	3 points

Laver, the yellow team, received 41 points in total.

What is the largest possible number of events that Laver broke the record for?

A. 11 events
B. 12 events
C. 13 events
D. 14 events
E. 15 events

Back to contents

5E Midpoint and length of a line segment

LEARNING INTENTIONS

Students will be able to:

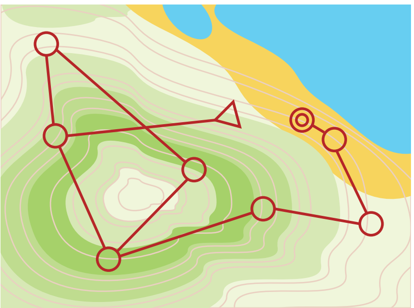
- calculate the midpoint of a line segment between two points
- use Pythagoras' Theorem to calculate the distance between two points.

Determining the midpoint and length of a line segment between two points on a Cartesian plane involves key mathematical concepts. Calculating the midpoint requires an understanding of coordinate geometry, while determining the distance between two points relies on Pythagoras' Theorem. These skills enable the precise determination of the length of a line segment, essential for various applications in geometry.

KEY TERMS AND DEFINITIONS

- A **line segment** is part of a line connecting two points that has definite ends.
- A **line** extends in both directions with no end.

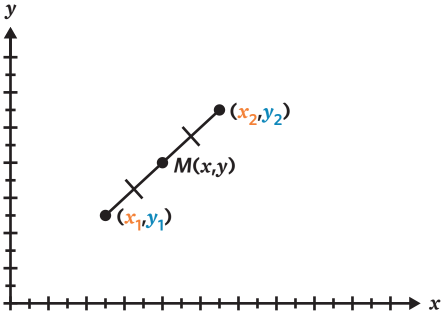
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Navigation systems commonly utilise concepts like calculating the midpoint between two points or determining the distance between them. These principles apply to both 2D maps and 3D models, helping to determine the best route between two locations.

Key ideas

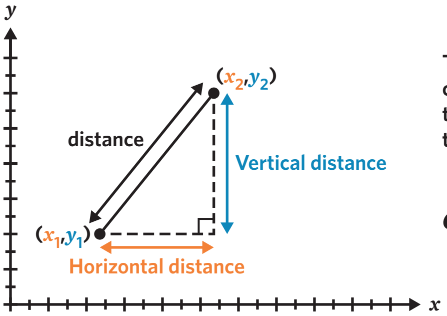
1. The midpoint of a line segment between two points can be found by averaging the x-coordinates and y-coordinates of the endpoints.



The midpoint, $M(x,y)$, of the line segment joining the points (x_1,y_1) and (x_2,y_2) is :

$$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

2. Pythagoras' Theorem can be used to determine the length of a line segment, given two points in a Cartesian plane.



The distance between two points can be calculated using Pythagoras' Theorem where the hypotenuse of the right-angle triangle is the distance between the two points.

$$c^2 = a^2 + b^2$$

Back to contents

Worked example 1

Calculating the midpoint of a line segment between two points

Calculate the midpoint $M(x,y)$ of the line segment between the following pairs of points.

a. (2,3) and (10,7)

WE1a

Working

$(x_1,y_1) = (2,3)$ and $(x_2,y_2) = (10,7)$

$$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2 + 10}{2}, \frac{3 + 7}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{10}{2}\right)$$

$$= (6,5)$$

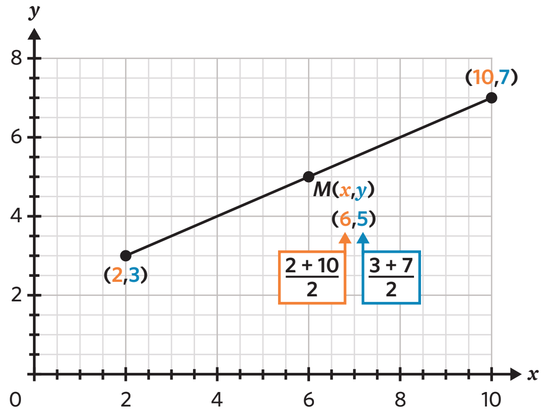
$M(x,y) = (6,5)$

Thinking

Step 1: Nominate (x_1,y_1) and (x_2,y_2).

Step 2: Substitute the x and y values into the formula for the midpoint.

Visual support



b. (−4,6) and (2,5)

WE1b

Working

$(x_1,y_1) = (2,5)$ and $(x_2,y_2) = (-4,6)$

$$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2 - 4}{2}, \frac{5 + 6}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{11}{2}\right)$$

$$= (-1,5.5)$$

$M(x,y) = (-1,5.5)$

Thinking

Step 1: Nominate (x_1,y_1) and (x_2,y_2).

Step 2: Substitute the x and y values into the formula for the midpoint.

Student practice

Calculate the midpoint $M(x,y)$ of the line segment between the following pairs of points.

a. (1,4) and (9,6)   **b.** (−3,5) and (1,4)

Back to contents

Worked example 2

Calculating the endpoint of a line segment given one endpoint and the midpoint

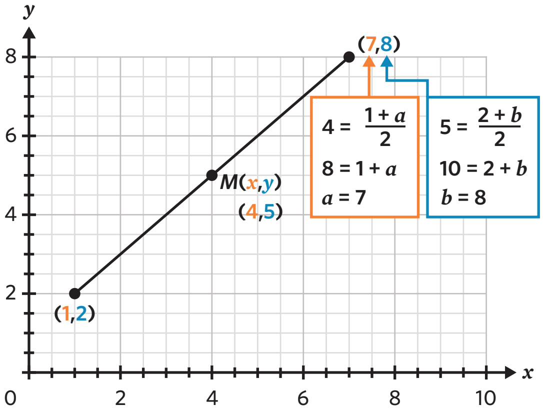
Calculate the endpoint of the line segment given the midpoint $M(x,y)$, and an endpoint.

a. (1,2) and $M(4,5)$

WE2a

Working	Thinking
$(x_1,y_1) = (1,2)$ $(x_2,y_2) = (a,b)$ $M(x,y) = (4,5)$	**Step 1:** Nominate $(x_1,y_1),(x_2,y_2)$ and $M(x,y)$.
$M(x,y) = \left(\frac{x_1 + x_2}{2},\frac{y_1 + y_2}{2}\right)$ $M(4,5) = \left(\frac{1 + a}{2},\frac{2 + b}{2}\right)$	**Step 2:** Substitute the x and y values into the formula for midpoint.
$4 = \frac{1 + a}{2}$ $8 = 1 + a$ $a = 7$	**Step 3:** Calculate the x-coordinate of the endpoint.
$5 = \frac{2 + b}{2}$ $10 = 2 + b$ $b = 8$	**Step 4:** Calculate the y-coordinate of the end point.
$(x_2,y_2) = (a,b)$ $= (7,8)$	**Step 5:** Write the end point as a coordinate pair.

Visual support



b. (−4,2) and $M(-3,-1)$

WE2b

Working	Thinking
$(x_1,y_1) = (-4,2)$ $(x_2,y_2) = (a,b)$ $M(x,y) = (-3,-1)$	**Step 1:** Nominate (x_1,y_1), $(x_2,y_2)$and $M(x,y)$.
$M(x,y) = \left(\frac{x_1 + x_2}{2},\frac{y_1 + y_2}{2}\right)$ $M(-3,-1) = \left(\frac{-4 + a}{2},\frac{2 + b}{2}\right)$	**Step 2:** Substitute the x and y values into the formula for midpoint.

Continues →

Back to contents

$$-3 = \frac{-4 + a}{2}$$

$$-6 = -4 + a$$

$$a = -2$$

Step 3: Calculate the x-coordinate of the endpoint.

$$-1 = \frac{2 + b}{2}$$

$$-2 = 2 + b$$

$$b = -4$$

Step 4: Calculate the y-coordinate of the end point.

$$(x_2, y_2) = (a, b)$$
$$= (-2, -4)$$

Step 5: Write the end point as a coordinate pair.

Student practice

Calculate the endpoint of the line segment given the midpoint $M(x,y)$, and an endpoint.

a. $(4,6)$ and $M(9,11)$

b. $(-10,8)$ and $M(-6,-1)$

Worked example 3

Calculating the length of a line segment

Using Pythagoras' Theorem, calculate the length of the line segment joining the following pairs points. Round to two decimal places, where necessary.

a. $(2,4)$ and $(5,8)$

WE3a

Working

Vertical distance

$$y_2 - y_1 = 8 - 4$$
$$= 4$$

Thinking

Step 1: Calculate the vertical distance by subtracting the y values of each coordinate point.

Horizontal distance

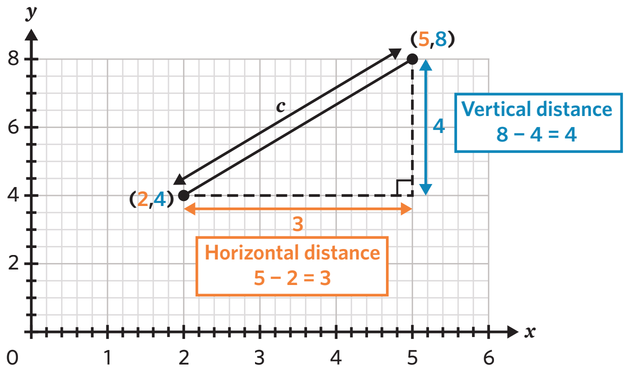
$$x_2 - x_1 = 5 - 2$$
$$= 3$$

Step 2: Calculate the horizontal distance by subtracting the x values of each coordinate point.

$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c^2 = 25$$
$$c = 5$$

Step 3: Use Pythagoras' Theorem to calculate the distance between the points (c) where the horizontal and vertical distances are a and b.

Visual support



Continues →

b. (−10,2) and (4,11)

WE3b

Working	**Thinking**
Vertical distance $y_2 - y_1 = 11 - 2$ $= 9$	**Step 1:** Calculate the vertical distance by subtracting the y values of each coordinate point.
Horizontal distance $x_2 - x_1 = 4 - (-10)$ $= 14$	**Step 2:** Calculate the horizontal distance by subtracting the x values of each coordinate point.
$c^2 = a^2 + b^2$ $c^2 = 9^2 + 14^2$ $c^2 = 81 + 196$ $c^2 = 277$ $c \approx 16.64$	**Step 3:** Use Pythagoras' Theorem to calculate the distance between the points (c) where the horizontal and vertical distances are a and b.

Student practice

Using Pythagoras' Theorem, calculate the length of the line segment joining the following pairs points. Round to two decimal places, where necessary.

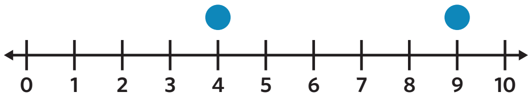
a. (2,2) and (7,14) **b.** (−9,5) and (3,12)

5E Questions

Understanding worksheet

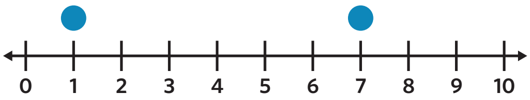
1. State the midpoint between the two numbers.

Example



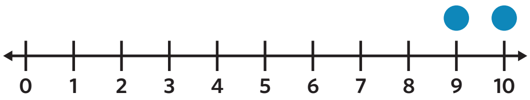
The midpoint between 4 and 9 is $\boxed{6.5}$.

a.



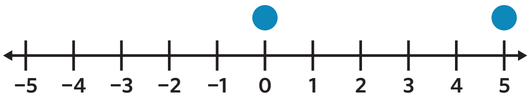
The midpoint between 1 and 7 is $\boxed{}$.

b.



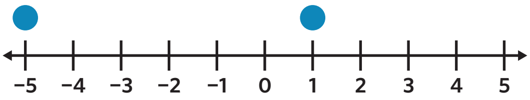
The midpoint between 9 and 10 is $\boxed{}$.

c.



The midpoint between 0 and 5 is $\boxed{}$.

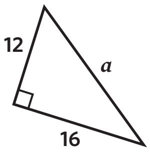
d.



The midpoint between −5 and 1 is $\boxed{}$.

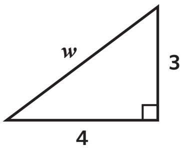
2. Fill in the blanks to use Pythagoras' Theorem to calculate the unknown length.

Example



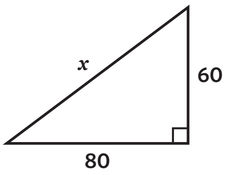
$$\boxed{a}^2 = 16^2 + 12^2$$

a.



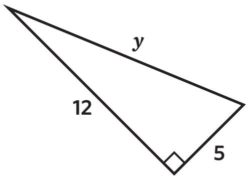
$$\boxed{}^2 = 4^2 + 3^2$$

b.



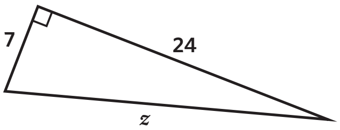
$$x^2 = \boxed{}^2 + 60^2$$

c.



$$y^2 = 144 + \boxed{}$$

d.



$$\boxed{}^2 = 49 + 576$$

3. Fill in the blanks by using the words provided.

midpoint **distance** **average** **line**

On a Cartesian plane, the line segment is a part of a $\boxed{}$ with two endpoints.

To calculate the $\boxed{}$ of a line segment between two points, $\boxed{}$ the x-coordinates and y-coordinates of the endpoints. Use Pythagoras' Theorem to determine the $\boxed{}$ between two points on the Cartesian plane.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Calculate the midpoint $M(x,y)$ of the line segment between the following pairs of points. **WE1**

a. (0,20) and (20,0)
b. (10,5) and (−15,7)
c. (−7,4) and (−1,2)
d. (20,−10) and (−5,−15)
e. (8,13) and (−12,18)
f. (1,3) and (2,15)
g. (−1,−5) and (−4,−9)
h. (−1,−1) and (1,1)

5. Calculate the endpoint of the line segment given the midpoint $M(x,y)$, and an endpoint. **WE2**

a. (2,3) and $M(5,7)$
b. (0,−3) and $M(-5,2)$
c. (−7,−6) and $M(1,5)$
d. (−4,6) and $M(15,-2)$
e. (3,4) and $M(0,0)$
f. (−6,0) and $M(2,7.5)$
g. (−4.5,6) and $M(12,8)$
h. (−4,−9) and $M(0,2)$

6. Using Pythagoras' Theorem, calculate the length of the line segment joining the following pairs of points. **WE3a**

a. (1,3) and (4,7)
b. (1,−3) and (−5,5)
c. (−1,4) and (11,−1)
d. (−3,−4) and (2,8)
e. (0,0) and (8,15)
f. (1.5,10) and (12.5,70)
g. (−6,8) and (2,23)
h. (−14,−2) and (−4,22)

7. Using Pythagoras' Theorem, calculate the length of the line segment joining the following pairs of points. Round to two decimal places, where necessary. **WE3b**

a. (2,3) and (5,4)
b. (0,−3) and (−4,6)
c. (−3,−5) and (1,7)
d. (−1,3) and (12,−2)
e. (1,−3) and (5,11)
f. (−7,9) and (2,15)
g. (3,1.5) and (−4,8)
h. (7,0) and (−9,15)

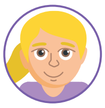
8. Which of the following values is closest to the length of the line segment joining the points (−4,−5) and (−9,2)?

A. 8
B. 9
C. 12
D. 37
E. 74

Spot the mistake

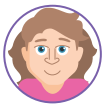
9. Select whether Student A or Student B is incorrect.

a. Calculate the midpoint of the line segment joining the points (1,20) and (11,2).



Student A

$$(x_1,y_1) = (1,20) \text{ and } (x_2,y_2) = (11,2)$$
$$M(x,y) = \left(\frac{1+20}{2},\frac{11+2}{2}\right)$$
$$= \left(\frac{21}{2},\frac{13}{2}\right)$$
$$= \left(10\tfrac{1}{2},6\tfrac{1}{2}\right)$$
$$M(x,y) = \left(10\tfrac{1}{2},6\tfrac{1}{2}\right)$$



Student B

$$(x_1,y_1) = (1,20) \text{ and } (x_2,y_2) = (11,2)$$
$$M(x,y) = \left(\frac{1+11}{2},\frac{20+2}{2}\right)$$
$$= \left(\frac{12}{2},\frac{22}{2}\right)$$
$$= (6,11)$$
$$M(x,y) = (6,11)$$

Back to contents

b. Using Pythagoras' Theorem, calculate the length of the line segment joining the points (0,−8) and (−6,2), rounded to two decimal places.

Student A

Vertical distance

$y_2 - y_1 = 8 - 2$

$= 6$

Horizontal distance

$x_2 - x_1 = 0 - 6$

$= -6$

$c^2 = a^2 + b^2$

$c^2 = 6^2 + (-6)^2$

$c^2 = 36 + 36$

$c^2 = 72$

$c = 8.49$

Student B

Vertical distance

$y_2 - y_1 = -8 - 2$

$= -10$

Horizontal distance

$x_2 - x_1 = 0 - (-6)$

$= 6$

$c^2 = a^2 + b^2$

$c^2 = (-10)^2 + 6^2$

$c^2 = 100 + 36$

$c^2 = 136$

$c = 11.66$

Problem solving

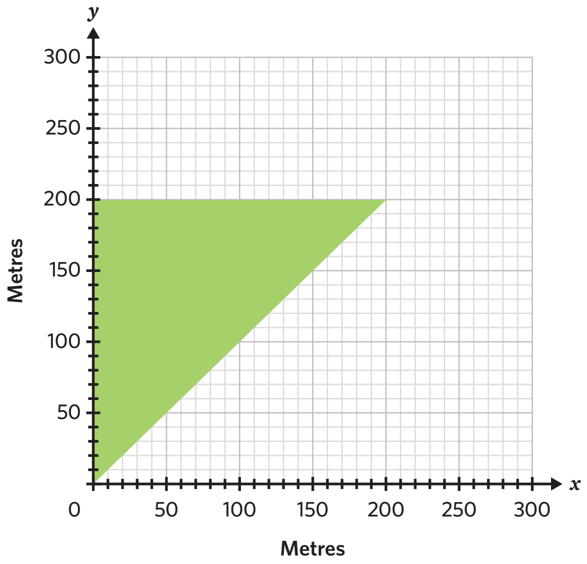
Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

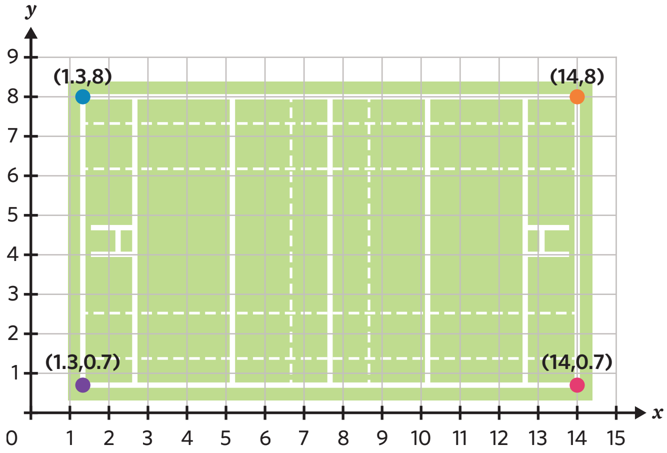
10. Two hikers start walking towards each other from separate locations, $A(150, 30)$ and $B(400, 90)$. Both hikers will cover the same distance and meet in the middle point. Calculate the coordinates of the meeting point, if they were to walk in a straight line.

11. A cricket ball is hit and lands exactly between two players located at (25,30) and (15,27). Calculate the coordinates of the ball's landing point.

12. A dog park is to be fenced so the dogs do not have access to the road. Calculate the exact length, in metres, of fencing required to fence the perimeter of the park shown on the map in green. Round to one decimal place.



13. A rugby game begins with a kickoff from the centre of the field. Using the information provided, calculate the coordinates of the location of the centre of this field. Round to one decimal place.



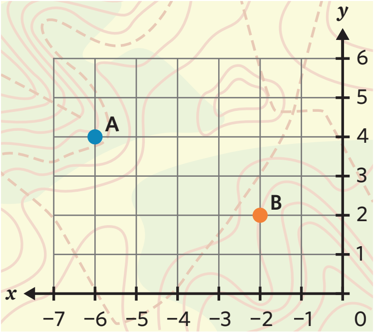
14. A sailing race is set out using markers at locations $A(-10,5)$, $B(-70,-20)$ and $C(30,50)$. Calculate the distance around the course (1 unit = 1 km). Round to one decimal place.

Reasoning

Question working paths

Mild 15 (a,b,d) 🌶	**Medium** 15 (a,b,d), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. There are two drop points located on the following topology map, where each square is 1 km.



a. Calculate the distance between point A and point B in exact form and to one decimal place.

b. If point B is the midpoint of the line segment AC, calculate the coordinates of the end point C.

c. Show that the distance between point A and C is twice the distance calculated in part **a**.

d. Describe a situation where topology maps are used.

16. Consider the line segment with endpoints (x_1,y_1) and (x_2,y_2).

a. Formulate a rule for the vertical distance between the endpoints.

b. Formulate a rule for the horizontal distance between endpoints.

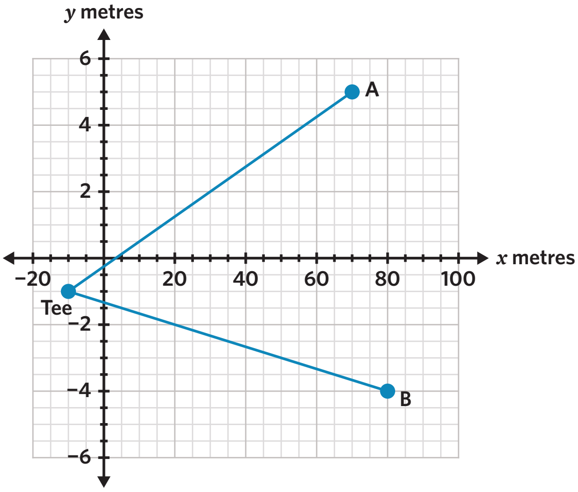
c. Use your answers from part **a** and part **b** to formulate a rule for the length of the line segment.

Exam-style

17. Calculate the exact distance between (−5,0) and (0,5). (1 MARK)

A. $\sqrt{5}$ **B.** 5 **C.** $5\sqrt{2}$ **D.** 25 **E.** 50

18. Two golf balls (A and B) are hit from the tee, landing at point A and B. Round to two decimal places. (3 MARKS)

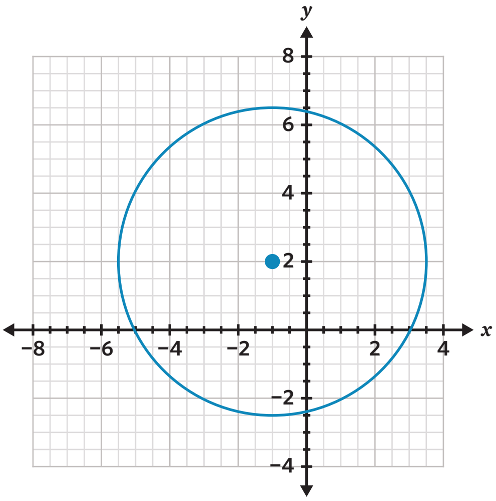


a. Calculate the distance of ball A from the tee. 1 MARK

b. Calculate the distance of ball B from the tee. 1 MARK

c. A third ball (C) lands exactly in the middle of the balls A and B. Calculate the coordinate point of ball C. 1 MARK

19. If the diameter of the circle is a line segment $\overline{AB}$ with an end point $A(-5,4)$ calculate the coordinates of B and the length of $\overline{AB}$. (2 MARKS)



20. A group of friends are taking part in a race where each person completes one section of the course around points marked on a map $A(1,2)$ $B(2,8)$ $C(12,6)$. Show that the distance $\overline{AC}$ is the longest leg. (3 MARKS)

Remember this?

21. Calculate 6×4^3?

A. 72 **B.** 96 **C.** 384 **D.** 648 **E.** 1536

22. Which of the following expressions is equal to 35^2?

A. $5 \times 5 \times 7 \times 7$ **B.** 5×7^2 **C.** $5^2 + 70 + 7^2$ **D.** 7×5^2 **E.** 7×28^2

23. The product of three single digit numbers is 48.

What is the largest possible sum of these three numbers?

A. 10 **B.** 12 **C.** 15 **D.** 45 **E.** 50

Back to contents

5F Equations of lines

LEARNING INTENTIONS

Students will be able to:

- determine the equation of a line
- understand the relationship between the gradients of parallel and perpendicular lines
- determine the equation of a line given a point and a line to which it is parallel
- determine the equation of a line given a point and a line to which it is perpendicular.

The equation of a line can be generated given two points, or a gradient and a single point. Using the gradient relationship between parallel and perpendicular lines it is possible to determine the equation of another line that is either parallel or perpendicular to the reference. This involves finding the gradient and substituting coordinate points appropriately. Parallel lines share the same gradient, while the gradients of perpendicular lines are negative reciprocals of each other.

KEY TERMS AND DEFINITIONS

- The **intersection point** is the point at which two lines cross.
- The **reciprocal of a number** is 1 divided by that number.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

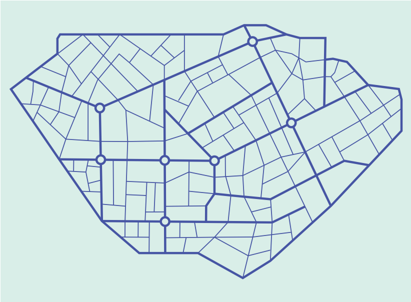
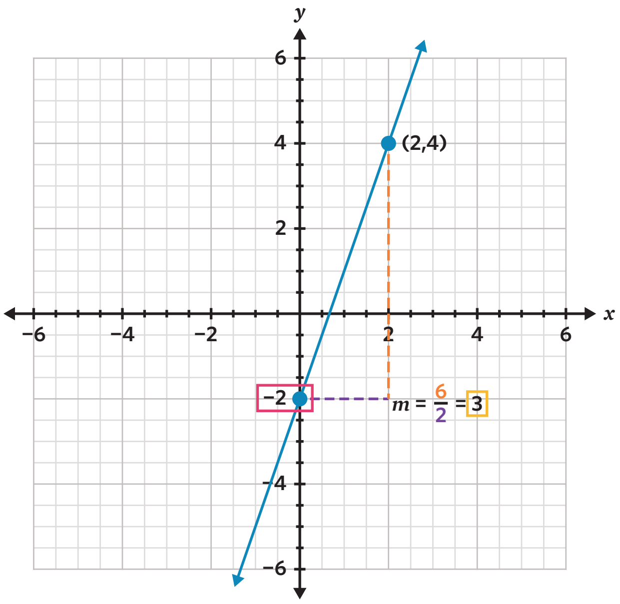


Image: Green angel/Shutterstock.com

City planners use the concept of line equations when designing roads and intersections. For instance, to ensure two roads remain parallel and never intersect, they would ensure both roads have the same gradient. On the other hand, to create a perfect right-angled intersection, they'd design one road to have a gradient perpendicular to the other.

Key ideas

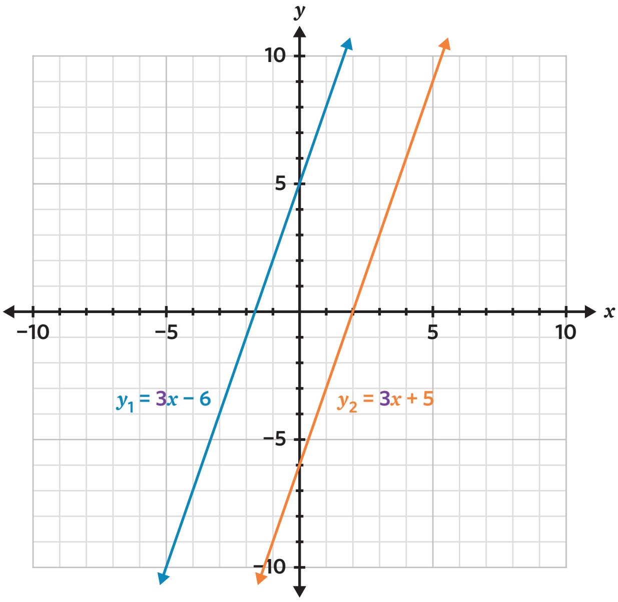
1. The equation of a line can be determined given two points, or a gradient and a single point.



$$\therefore y = mx + c \text{ is:}$$
$$y = \boxed{3}x\boxed{-2}$$

Continues →

2. Parallel lines have the same gradient.



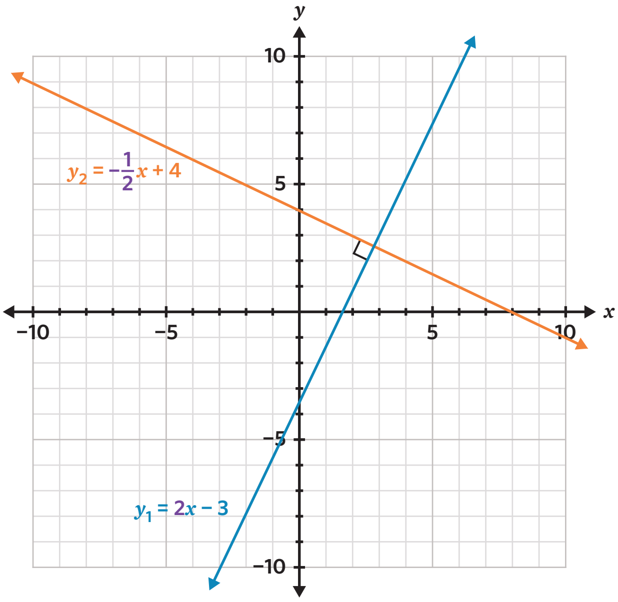
$m_1 = 3$

$m_2 = 3$

$m_1 = m_2$

$\therefore y_1$ and y_2 are parallel

3. Perpendicular lines intersect at a right angle and have negative reciprocal gradients.



$m_1 = 2$

$m_2 = -\frac{1}{2}$

$m_1 m_2 = -1$

$m_2 = -\frac{1}{m_1}$

$\therefore y_1$ and y_2 are perpendicular

Worked example 1

Determining the equation of a line

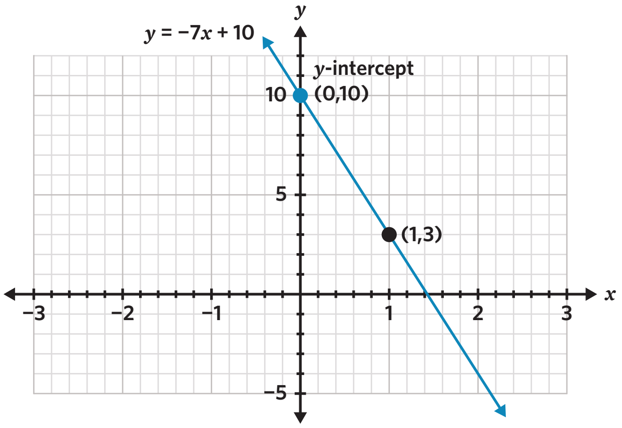
Determine the equation of the line using the given information.

a. A line with a gradient of −7 that passes through the point (1,3).

WE1a

Working	Thinking
Gradient, $m = -7$	**Step 1:** Identify the gradient, m.
$(x,y) = (1,3)$ $3 = -7(1) + c$	**Step 2:** Substitute known x and y values from the given point, and the gradient into $y = mx + c$.
$3 = -7 + c$ $c = 10$	**Step 3:** Simplify and solve to find c.
$y = -7x + 10$	**Step 4:** Write the equation in the form $y = mx + c$.

Visual support



b. A line passes through the points (−4,5) and (0,−3).

WE1b

Working	Thinking
Gradient, $m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$ $(x_1,y_1) = (-4,5)$, and $(x_2,y_2) = (0,-3)$ $m = \frac{-3 - 5}{0 - (-4)}$ $= \frac{-8}{4}$ $= -2$	**Step 1:** Calculate the gradient, m, using the given points.
$(x,y) = (0,-3)$ $\therefore c = -3$	**Step 2:** Identify the value for c. The y-intercept is given.
$y = -2x - 3$	**Step 3:** Write the equation in the form $y = mx + c$.

Student practice

Determine the equation of the line using the given information.

a. A line with a gradient of −3 that passes through the point (2,5).

b. A line that passes through the points (−1,6) and (0,2).

Worked example 2

Determining the equation of a parallel line

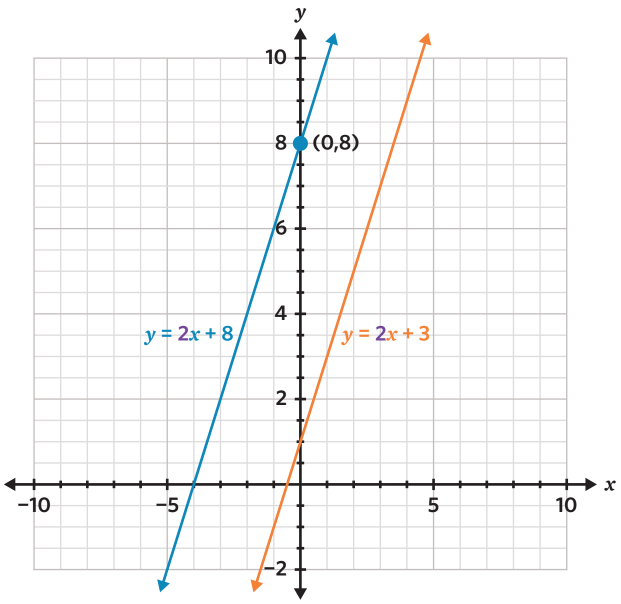
Determine the equations of the lines using the given information.

a. A line which is parallel to $y = 2x + 3$ and passes through (0,8).

WE2a

Working	Thinking
$y = 2x + 3$ $\therefore$ Gradient, $m_1 = m_2 = 2$	**Step 1:** Identify the gradient from the given equation. Parallel lines have the same gradient, $m_1 = m_2$.
$(x,y) = (0,8)$ $\therefore c = 8$	**Step 2:** Identify the value for c. The y-intercept is given.
$y = 2x + 8$	**Step 3:** Write the equation in the form $y = mx + c$.

Visual support



b. A line which is parallel to $y = 5x - 7$ and passes through (2,11).

WE2b

Working	Thinking
$y = 5x - 7$ $\therefore$ Gradient $m_1 = m_2 = 5$	**Step 1:** Identify the gradient from the given equation. Parallel lines have the same gradient, $m_1 = m_2$.
$(x,y) = (2,11)$	**Step 2:** Substitute known x and y values from the given point, and the gradient into the equation $y = mx + c$.
$11 = 5(2) + c$ $11 = 10 + c$ $\therefore c = 1$	**Step 3:** Simplify and solve for c.
$y = 5x + 1$	**Step 4:** Write the equation in the form $y = mx + c$.

Student practice

Determine the equation of the parallel line using the given information.

a. A line which is parallel to the line $y = 3x + 9$ and passes through (0,2).

b. A line which is parallel to the line $y = 2x - 5$ and passes through (1,8).

Back to contents

Worked example 3

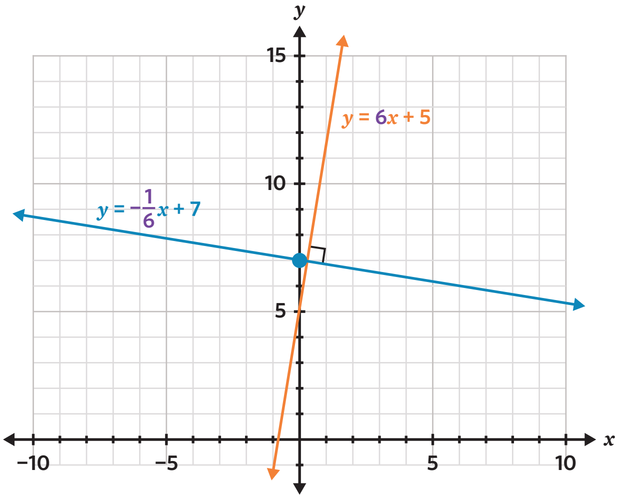
Determining the equation of a perpendicular line

Determine the equations of the lines using the given information.

a. A line which is perpendicular to the line $y = 6x + 5$ and passes through (0,7). WE3a

Working	Thinking
$y = 6x + 5$ $m_1 = 6, m_2 = -\frac{1}{6}$	**Step 1:** Identify the gradient from the given equation. For perpendicular lines $m_2 = -\frac{1}{m_1}$.
$(x,y) = (0,7)$ $c = 7$	**Step 2:** Identify the value for c. The y-intercept is given.
$y = -\frac{1}{6}x + 7$	**Step 3:** Write the equation in the form $y = mx + c$.

Visual support



b. A line which is perpendicular to the line $y = \frac{4}{5}x + 6$ and passes through (8,–1). WE3b

Working	Thinking
$y = \frac{4}{5}x + 6$ $m_1 = \frac{4}{5}, m_2 = -\frac{5}{4}$	**Step 1:** Identify the gradient. For perpendicular lines $m_2 = -\frac{1}{m_1}$.
$(x,y) = (8,-1)$ $-1 = -\frac{5}{4}(8) + c$	**Step 2:** Substitute known x and y values from the given point, and the gradient into the equation $y = mx + c$.
$-1 = -10 + c$ $c = 9$	**Step 3:** Simplify and solve for c.
$y = -\frac{5}{4}x + 9$	**Step 4:** Write the equation in the form $y = mx + c$.

Student practice

Determine the equation of the perpendicular line using the given information.

a. A line which is perpendicular to the line $y = 5x + 1$ and passes through (0,3).

b. A line which is perpendicular to the line $y = \frac{2}{3}x + 8$ and passes through (10,–6).

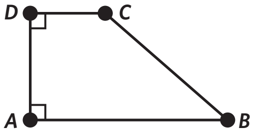
Back to contents

5F Questions

Understanding worksheet

1. For each shape complete the sentence identifying parallel or perpendicular lines.

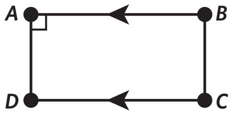
Example



$\overline{AB}$ and $\overline{DC}$ are

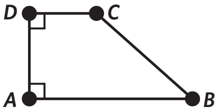
parallel

a.



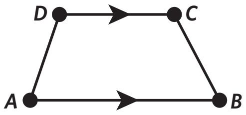
$\overline{AB}$ and $\overline{DC}$ are

b.



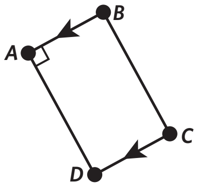
$\overline{AB}$ and $\overline{AD}$ are

c.



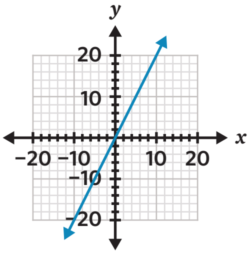
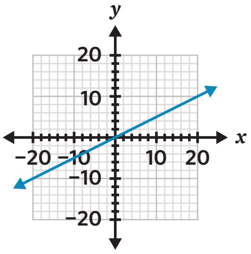
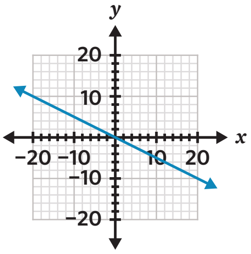
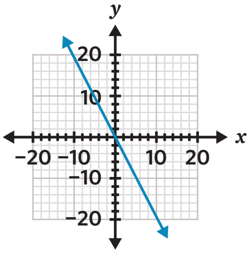
$\overline{AB}$ and $\overline{DC}$ are

d.



$\overline{AB}$ and $\overline{BC}$ are

2. Match each linear graph with its gradient.

Linear graph	Gradient
 ●	● $m = -2$
 ●	● $m = 2$
 ●	● $m = -\frac{1}{2}$
 ●	● $m = \frac{1}{2}$

Back to contents

3. Fill in the blanks by using the words provided.

gradient	**reciprocal**	**equal**	**perpendicular**	**parallel**

When lines are ________ to each other, their gradients are ________ . When two lines are ________ to each other, their gradients have a negative ________ relationship. Determining the equation of a line using $y = mx + c$, given a point and parallel or perpendicular reference lines, involves calculating the ________ and using this and a point on the line to calculate c.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Determine the equation of the straight line using the given information. **WE1a**

a. A line with a gradient of 7 and passes through the point (2,0).

b. A line with a gradient of −6 and passes through the point (4,0).

c. A line with a gradient of 1 and passes through the point (2,5).

d. A line with a gradient of 5 and passes through the point (2,4).

e. A line with a gradient of −3 and passes through the point (0,6).

f. A line with a gradient of 3 and passes through the point (−1,−2).

g. A line with a gradient of 4 and passes through the point (3,5).

h. A line with a gradient of 5 and passes through the point (−3,4).

5. Determine the equation of the straight line using the given information. **WE1b**

a. A line passes through the points (−4,5) and (0,−3)

b. A line passes through the points (7,0) and (0,−4)

c. A line passes through the points (3,3) and (7,−2)

d. A line passes through the points (2,−2) and (6,4)

e. A line passes through the points (5,5) and (0,9)

f. A line passes through the points (4,3) and (2,0)

g. A line passes through the points (8,9) and (7,−3)

h. A line passes through the points (−1,5) and (3,0)

6. Determine the equation of the parallel line using the given information. **WE2**

a. A line which is parallel to the line $y = 4x + 2$ and passes through (0,9).

b. A line which is parallel to the line $y = -2x + 1$ and passes through (1,5).

c. A line which is parallel to the line $y = -6x - 5$ and passes through (3,1).

d. A line which is parallel to the line $y = \frac{1}{2}x + 3$ and passes through (0,4).

e. A line which is parallel to the line $y = \frac{4}{5}x + 9$ and passes through (1,1).

f. A line which is parallel to the line $y = -\frac{1}{4}x + 1$ and passes through (6,−2).

g. A line which is parallel to the line $y = -4.5x + 9$ and passes through (−1,−3).

h. A line which is parallel to the line $y = 0.3x + 4$ and passes through (2,3).

Back to contents

7. Determine the equation of the perpendicular line using the given information. WE3

a. A line which is perpendicular to the line $y = 2x + 1$ and passes through (0,6).

b. A line which is perpendicular to the line $y = -4x + 3$ and passes through (0,–3).

c. A line which is perpendicular to the line $y = \frac{2}{3}x + 4$ and passes through $(-2,4)$.

d. A line which is perpendicular to the line $y = -\frac{1}{7}x - 1$ and passes through (1,8).

e. A line which is perpendicular to the line $y = -x - 4$ and passes through $(-1,0)$.

f. A line which is perpendicular to the line $y = \frac{4}{5}x + 9$ and passes through (10,–6.5).

g. A line which is perpendicular to the line $y = -2.5x + 6$ and passes through (5,–1).

h. A line which is perpendicular to the line $y = 0.3x + 7$ and passes through (6.5,–5).

---

8. Select the linear equation parallel to $8x + 2y = 15$.

A. $y = 4x + 4$ **B.** $y = 8x + 2$ **C.** $y = -4x + 3$ **D.** $y = -8x + 15$ **E.** $y = -\frac{1}{4}x - 15$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the equation of the line that passes through the points (–1,0) and (7,–2).

Student A

Gradient

$m = \frac{\text{rise}}{\text{run}}$ or $\frac{y_2 - y_1}{x_2 - x_1}$

$(-1,0)$, (7,–2)

$m = \frac{0 - (-2)}{-1 - 7}$

$= \frac{2}{-8}$

$= -\frac{1}{4}$

$(-1,0)$

$-1 = -\frac{1}{4}(0) + c$

$c = -1$

$y = -\frac{1}{4}x - 1$

Student B

Gradient

$m = \frac{\text{rise}}{\text{run}}$ or $\frac{y_2 - y_1}{x_2 - x_1}$

$(-1,0)$, (7,–2)

$m = \frac{0 - (-2)}{-1 - 7}$

$= \frac{2}{-8}$

$= -\frac{1}{4}$

$(- 1,0)$

$0 = -\frac{1}{4}(-1) + c$

$0 = \frac{1}{4} + c$

$c = -\frac{1}{4}$

$y = -\frac{1}{4}x - \frac{1}{4}$

b. Determine the equation of the line perpendicular to $y = 3x - 4$ passing through the point (2,–9).

Student A

$y = 3x - 4$

$m_1 = 3, m_2 = -\frac{1}{3}$

(2,–9)

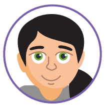
$y = mx + c$

$-9 = -\frac{1}{3}(2) + c$

$-9 = -\frac{2}{3} + c$

$c = 8.3$

$y = -\frac{1}{3}x - 8.3$



Student B

$y = 3x - 4$

$m_1 = 3, m_2 = \frac{1}{3}$

(2,–9)

$y = mx + c$

$-9 = \frac{1}{3}(2) + c$

$-9 = \frac{2}{3} + c$

$c = 9.67$

$y = \frac{1}{3}x - 9.67$

Back to contents

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. The council planted mature trees on the nature strip and conducted annual health checks, measuring their growth. Upon planting, a tree had a height of 1 m. After one year, its height increased to 1.2 m. Determine the equation that represents the tree's growth over time.

11. A fencing design uses parallel lines to represent the bottom, mid and top rails that are 60 cm apart. On sloping ground equations of the bottom and mid lines are $y = 0.5x$ and $y = 0.5x + 60$. State the equation used to represent the top rail.

12. An architect drafted a design for a roof shaped like a right-angled triangle. The design outlines the roof's vertices at points (0,1), (2,3), and (4,1). Determine the equations for each of the sides of the triangle.

13. A zip-line is anchored at the top of a platform and extends to the ground, 17.5 m away from the platform's base. Determine the linear equation for the zip-line, given that it has a gradient of -0.4.

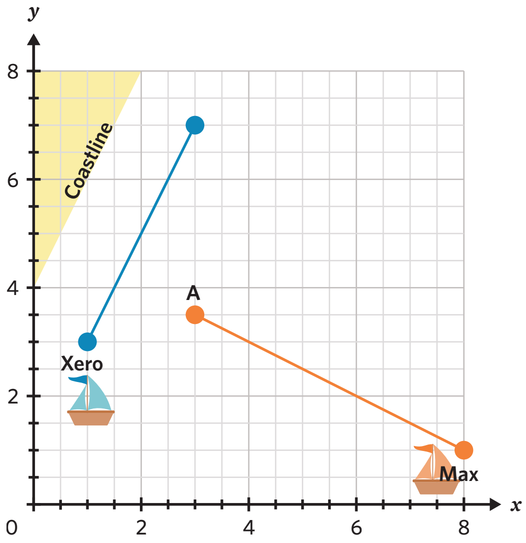
14. The design of a bridge incorporates parallel and perpendicular lines. The segment of the bridge that spans the river begins at (0,70) and ends at (120,70). Determine the equations for the two pillars supporting the bridge, given that they are perpendicular to the span of the bridge.

Reasoning

Question working paths

Mild 15 (a,b,d) 🌶	**Medium** 15 (a,b,d), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Two boats are mapping separate courses to travel along the lines as shown.



a. Show that Xero is travelling parallel to the coastline.

b. Show that the courses the boats are travelling are perpendicular.

c. Max changes course at point A to also travel parallel to the coastline. Determine the equation of the new course taken by Max.

d. Give a reason why it is useful to understand the concepts of parallel and perpendicular paths when sailing.

16. Answer the following using linear relationships.

- **a.** State the gradient of a line that is parallel to $2x + 3y = 5$.
- **b.** State the gradient of a line that is parallel to $ax + by = c$.
- **c.** Given your understanding of the properties of perpendicular lines and the result from part **b**, derive a rule to determine the gradient of a line perpendicular to $ax + by = c$.

Exam-style

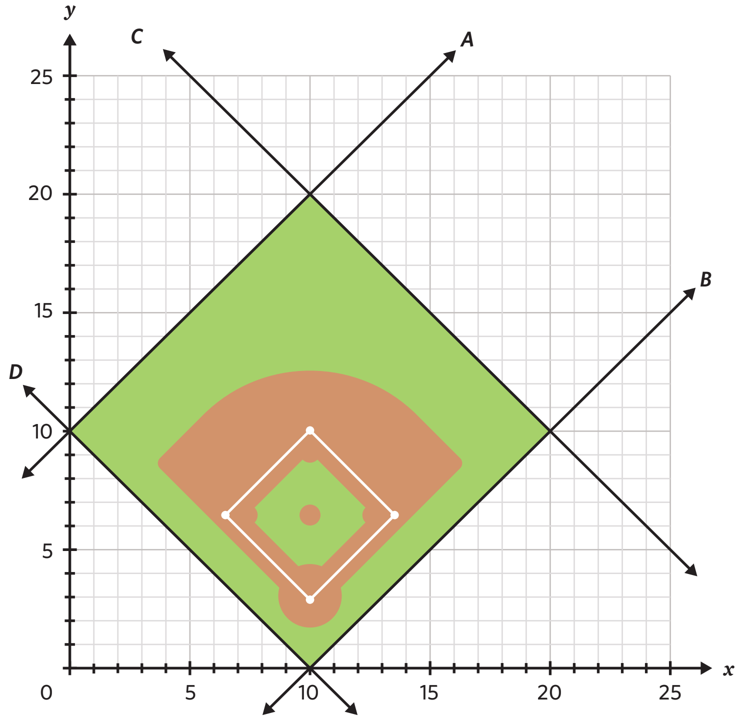
17. Select the linear equation perpendicular to $6x + 2y = 9$ (1 MARK)

- **A.** $y = 6x + 1$
- **B.** $y = -3x + 4$
- **C.** $y = \frac{1}{3}x + 5$
- **D.** $y = -\frac{1}{3}x + 6$
- **E.** $y = 3x + 2$

18. A shape has vertices at $A(0,5)$, $B(2,9)$, $C(6,2)$ and $D(8,6)$. (3 MARKS)

- **a.** Show that $\overline{AB}$ and $\overline{CD}$ are parallel. 1 MARK
- **b.** Show that $\overline{AB}$ and $\overline{BD}$ are perpendicular. 1 MARK
- **c.** What is the relationship between $\overline{BD}$ and $\overline{AC}$? 1 MARK

19. A baseball field is constructed in the shape of a diamond as shown. Determine the four linear equations that make up the sides of the pitch. Use the equations to state any parallel and perpendicular lines, giving reasons. (3 MARKS)



20. A town planner designs a park in the shape of a triangle. The plans show vertices at $(-6.5,2)$, $(-4,12)$ and $(36,2)$. Calculate the equation of the lines that form the triangle and identify any perpendicular lines. (3 MARKS)

Back to contents

Remember this?

21. During an athletics carnival, four teams – red, yellow, green and blue, compete in various events.

At Edward High School, the carnival is conducted over two days.

The following table shows the amount of points each team scored on both days.

	Red	**Yellow**	**Green**	**Blue**
Day 1	17	12	23	8
Day 2	21	10	19	15

Which team won the athletics carnival?

A. Red
B. Blue
C. Green
D. Yellow
E. There was a draw.

22. Michael has 4 dogs.

In total, they eat 7 cups of dry food a day.

How many cups of dry food would each dog eat in a fortnight?

A. 4 cups **B.** 20 cups **C.** 21 cups **D.** 24.5 cups **E.** 25 cups

23. The bullet train from Tokyo to Kyoto takes 122 minutes.

The first train departs from Tokyo at 06:07 am. When will the train reach Kyoto?

A. 06:09 am **B.** 08:07 am **C.** 08:09 am **D.** 12:09 pm **E.** 08:09 pm

Back to contents

5G Graphical solutions to simultaneous equations

LEARNING INTENTIONS

Students will be able to:

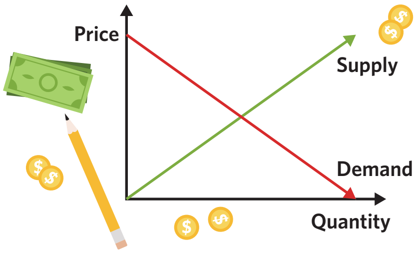
- understand that straight line graphs can have zero, one or infinite solutions
- use a graph to identify the point of intersection of two lines as the solution to simultaneous equations
- use linear rules to check if a point is at the intersection of two lines.

Straight line graphs can intersect in various ways, resulting in zero, one, or an infinite number of solutions. By accurately graphing a system of equations, any point of intersection of the lines can be identified as the solution to simultaneous equations. Additionally, using linear rules allows for verification if a specific point is the intersection of two lines.

KEY TERMS AND DEFINITIONS

- A **system of equations** consists of two or more equations that share the same variables.
- A **solution** is a value, or values, that when substituted into an equation, make that equation true.

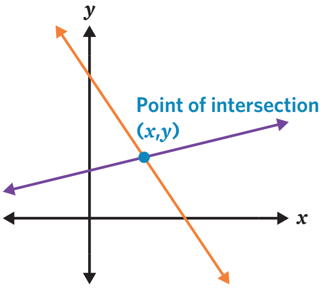
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



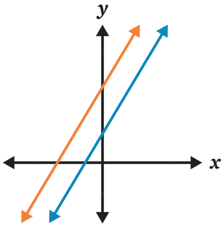
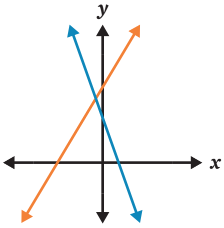
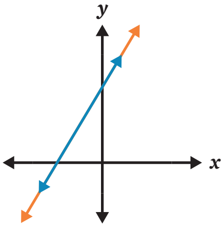
Straight line graphs can represent various factors, such as the relation between supply and demand. The intersection of two graphs might signify a balance between supply and demand. If two straight lines never crossover, it indicates a persistent imbalance, whereas if they coincide, it represents a direct correlation.

Key ideas

1. The point of intersection of two lines on a graph represents the solution to simultaneous equations represented by those lines.



2. Straight line graphs can have zero, one, or infinite solutions.

Parallel lines have the same gradient $(m_1 = m_2)$ and different y-intercepts $(c_1 \neq c_2)$ so there are **zero** solutions.	Straight lines with different gradient $(m_1 \neq m_2)$ have **one** solution.	Straight lines with equal gradient $(m_1 = m_2)$ and equal y-intercept $(c_1 = c_2)$ have **infinite** solutions.
		

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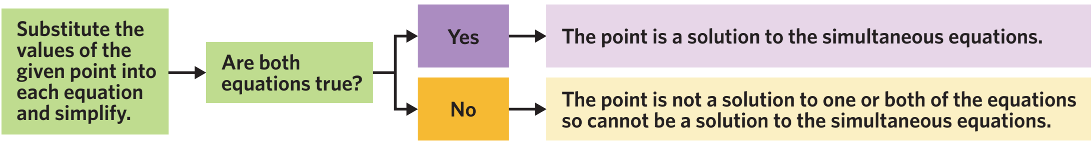
Back to contents

3. Linear equations can be used to check if a point is a solution of two lines by substituting its coordinates into both equations.



Worked example 1

Verifying a solution using substitution

Use substitution to verify whether the given pair of coordinates is the solution to the following pair of simultaneous equations.

a. Verify if (−3,−4) is the solution to the simultaneous equations:
$y = 4x + 8$
$y = -2x - 10$

WE1a

Working

$y = 4x + 8$, let $x = -3$

$y = 4(-3) + 8$

$y = -12 + 8$

$y = -4$ ✓

$y = -2x - 10$, let $x = -3$

$y = -2(-3) - 10$

$y = 6 - 10$

$y = -4$ ✓

∴ (−3,−4) is the solution to the simultaneous equations.

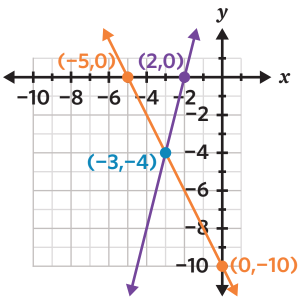
Thinking

Step 1: Substitute the x-coordinate into one equation to verify the y-coordinate.

Step 2: Substitute the x-coordinate into the remaining equation to verify the y-coordinate.

Step 3: State if the point was a solution for both equations.

Visual support



b. Verify if (−2,−3) a solution to the simultaneous equations:
$y = 3x + 9$
$y + 1 = -4x$

WE1b

Working

$y = 3x + 9$, let $x = -2$

$y = 3(-2) + 9$

$y = -6 + 9$

$y = 3$ ✗

Thinking

Step 1: Substitute the x-coordinate into one equation to verify the y-coordinate.

Continues →

Back to contents

$y + 1 = -4x$, let $x = -2$

$y + 1 = -4(-2)$

$y + 1 = 8$

$y = 7$ ✗

Step 2: Substitute the x-coordinate into the remaining equation to verify the y-coordinate.

∴ $(-2,-3)$ is not the solution to the simultaneous equations.

Step 3: State if the point was a solution for both equations.

Student practice

Verify if $(-1,-2)$ a solution to the simultaneous equations.

a. $y = 3x + 1$
$y = -2x - 4$

b. $y = -4x - 6$
$y - 2 = 2x$

Worked example 2

Solving simultaneous equations graphically

Plot the graphs of the given equations to determine the solution of the simultaneous equations.

a. $y = x + 2$
$y = -2x + 8$

WE2a

Working

$y = x + 2$
x-intercept, let $y = 0$
$0 = x + 2$
$x = -2$
∴ x-intercept is -2
$(-2,0)$

y-intercept, let $x = 0$
$y = (0) + 2$
$y = 2$
∴ y-intercept is 2
$(0,2)$

$y = -2x + 8$
x-intercept, let $y = 0$
$0 = -2x + 8$
$2x = 8$
$x = 4$
∴ x-intercept is 4
$(4,0)$

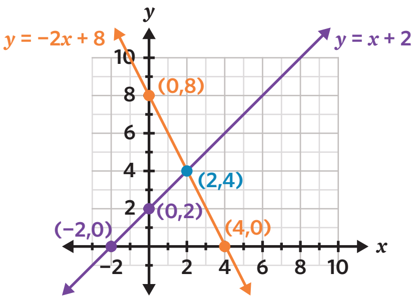
y-intercept, let $x = 0$
$y = -2(0) + 8$
$y = 8$
∴ y-intercept is 8
$(0,8)$

Thinking

Step 1: Accurately plot each graph by first determining the x and y intercepts.

Continues →

Back to contents



Step 2: Accurately plot both lines on the same set of axes.

The intersection point is (2,4).

Step 3: Locate the intersection point and read the coordinates from the graph.

$4 = 2 + 2$ ✓

$4 = -2(2) + 8$ ✓

Step 4: Verify by substitution.

b. $y = -4x + 2$
$7x + 3y = 21$

WE2b

Working

$y = -4x + 2$
x-intercept, let $y = 0$
$0 = -4x + 2$
$4x = 2$
$x = \frac{1}{2}$
$\therefore$ x-intercept is $\frac{1}{2}$
$\left(\frac{1}{2},0\right)$
y-intercept, let $x = 0$
$y = -4(0) + 2$
$y = 2$
$\therefore$ y-intercept is 2
$(0,2)$

$7x + 3y = 21$
x-intercept, let $y = 0$
$7x + 3(0) = 21$
$7x = 21$
$x = 3$
$\therefore$ x-intercept is 3
$(3,0)$

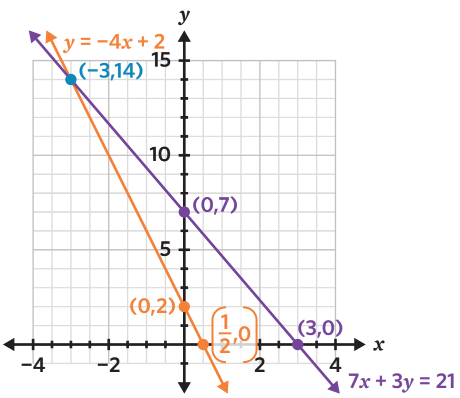
y-intercept, let $x = 0$
$7(0) + 3y = 21$
$3y = 21$
$y = 7$
$\therefore$ y-intercept is 7
$(0,7)$

Thinking

Step 1: Accurately plot each graph by first determining the x and y intercepts.

Continues →

Back to contents



Step 2: Accurately plot both lines on the same set of axes.

The intersection point is $(-3,14)$.

Step 3: Locate the intersection point and read the coordinates from the graph.

$14 = -4 \times (-3) + 2$ ✓

$7 \times (-3) + 3 \times 14 = 21$ ✓

Step 4: Verify by substitution.

Student practice

Plot the graphs of the given equations to determine the solution of the simultaneous equations.

a. $y = x + 15$
$y = -4x + 5$

b. $y = -2x + 3$
$8x + 2y = 2$

Worked example 3

Determining the number of solutions for two lines

Determine the number of solutions for the following pairs of simultaneous equations.

a. $y = 3x + 7$
$y = 3x - 5$

WE3a

Working

$y = 3x + 7, m_1 = 3$

$y = 3x - 5, m_2 = 3$

$m_1 = m_2$

$y = 3x + 7, c_1 = 7$

$y = 3x - 5, c_2 = -5$

$c_1 \neq c_2$

$m_1 = m_2$ and $c_1 \neq c_2$

$\therefore$ This pair of simultaneous equations has zero solutions.

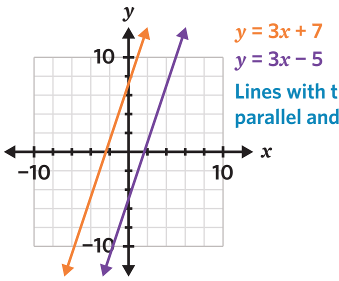
Thinking

Step 1: Identify and compare the gradient of each equation.

Step 2: If $m_1 = m_2$, identify and compare c_1 and c_2 for each equation.

Step 3: State the number of solutions.

Visual support



Lines with the same gradient are parallel and will never intersect.

Continues →

b. $y = 4x + 1$
$y + 2 = 7x$

WE3b

Working	Thinking
$y = 4x + 1, m_1 = 4$ $y + 2 = 7x$ $y = 7x - 2, m_2 = 7$	**Step 1:** Identify and compare the gradient of each equation. Where necessary, rearranging the equation(s) to the form $y = mx + c$.
$m_1 \neq m_2$ $\therefore$ This pair of simultaneous equations has one solution.	**Step 2:** State the number of solutions.

Student practice

Determine the number of solutions for the following pairs of simultaneous equations.

a. $y = 8x + 3$
$y = 8x - 1$

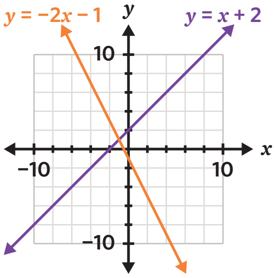
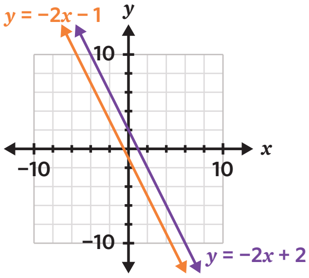
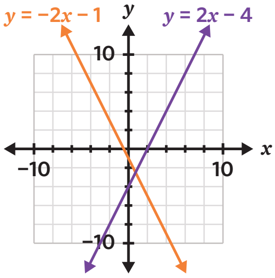
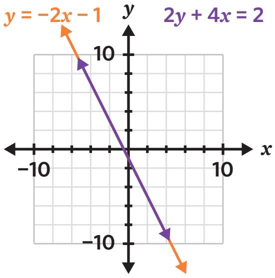
b. $y = 3x + 2$
$y + 9 = 5x$

Back to contents

5G Questions

Understanding worksheet

1. Identify if the pairs of simultaneous equations have zero solutions, one solution, or infinite solutions.

	Zero solutions	One solution	Infinite solutions
I. 	☐	☐	☐
II. 	☐	☐	☐
III. 	☐	☐	☐
IV. 	☐	☐	☐

Back to contents

2. Using substitution, state whether the equation is true or false using the point (1,4).

Example

Equation	True	False
$y = 2x + 2$	☑	☐

Equation	True	False
I. $y = 3x - 4$	☐	☐
II. $2x + 3y = 14$	☐	☐
III. $x + 4y = 10$	☐	☐
IV. $y = -4x + 5$	☐	☐

3. Fill in the blanks by using the words provided.

parallel **solving** **intersection** **infinite** **coordinates**

Simultaneous linear equations can have 0, 1, or __________ solutions. When __________ simultaneous equations graphically the solution can be read from the __________ of the point of intersection. When two lines have the same slope and different y-intercepts, they are __________ and do not have a point of __________ .

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (b,c,d,e), 5 (c,d,e,f), 6 (b,c,d,e), 7 (b,c,d,e), 8	4 (c,d,e,f), 5 (e,f,g,h), 6 (c,d,e,f), 7 (c,d,e,f), 8

4. Use substitution to verify whether the given pair of coordinates is the solution to the following pairs of simultaneous equations. **WE1**

a. $y = x + 4$ and $y = -3x - 4$ for the point (−2,2)

b. $y = 2x + 1$ and $2x + 2y = 8$ for the point (1,3)

c. $y = 0.5x + 4$ and $y = 3x - 5$ for the point (2,5)

d. $y = \frac{1}{2}x - 4$ and $y = -2x + 1$ for the point (2,−3)

e. $y = 4x - 1$ and $15x + y = -30$ for the point (−1,−5)

f. $y = 2x - 3$ and $-5x + y = 2$ for the point (−2,−7)

5. Plot the graphs of the given equations to determine the solution of the simultaneous equations. **WE2**

a. $y = x + 1, y = -2x + 7$

b. $y = 2x - 3, y = -2x + 1$

c. $y = x + 4, y - 8 = -x$

d. $y = 4x + 3, y = -3x + 24$

e. $y = -3x - 4, y - 6 = 2x$

f. $y = 2x + 11, 2y = -1x - 8$

g. $y = x + 4, y = 3x - 2$

h. $y = 6x - 3, y - 11 = 2x$

6. Determine the number of solutions for the following pairs of simultaneous equations. **WE3a**

a. $y = x + 4$ and $y = -x - 4$

b. $y = 2x + 4$ and $y = 2x + 4$

c. $y = 0.5x + 4$ and $y = 0.5x - 5$

d. $y = \frac{1}{2}x + \frac{1}{3}$ and $y = 2x + 1$

e. $y = 4x - 1$ and $y = 4x - 1$

f. $y = 5x - 3$ and $y = 2 + 5x$

7. Determine the number of solutions for the following pairs of simultaneous equations. WE3b

a. $y = -x - 4$ and $2y = -2x - 8$

b. $y = x + 5$ and $2x + 2y = 10$

c. $2y = 3x + 8$ and $y = 3x + 8$

d. $y = \frac{1}{3}x + \frac{2}{3}$ and $3y = x + 12$

e. $2y = 8x - 1$ and $-2x + \frac{1}{2}y = -\frac{1}{2}$

f. $5x - 3 = y$ and $-5x - y = -3$

---

8. Without graphing, determine how many solutions do the following simultaneous equations have.

$y = 3x + 5$

$y = -3x - 2$

A. 0 **B.** 1 **C.** 2 **D.** Infinite **E.** Undefined

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Use substitution to verify whether the $(-1,-5)$ is the solution to the following pairs of simultaneous equations.

1: $y = x - 4$

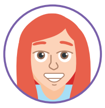
2: $y = 2x - 3$

Student A

$y = x - 4$, let $x = -1$
$y = -1 - 4$
$y = -5$ ✓

$y = 2x - 3$, let $x = -1$
$y = -2 - 3$
$y = -5$ ✓

∴ $(-1,-5)$ is the solution to the simultaneous equations.



Student B

$y = x - 4$, let $x = -1$
$y = -5 - 4$
$y = -9$ ✗

$y = 2x - 3$, let $x = -1$
$y = -5 - 3$
$y = -8$ ✗

∴ $(-1,-5)$ is not the solution to the simultaneous equations.

b. State the number of solutions to this system of equations:

1: $y = 2x + 4$

2: $2y - 8 = 4x$

Student A

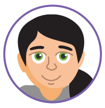
1: $y = 2x + 4$

2: $2y - 8 = 4x$

$2y = 4x + 8$

$y = 2x + 4$

Both equations have the same gradient so they are parallel and have zero solutions.



Student B

1: $y = 2x + 4$

2: $2y - 8 = 4x$

$2y = 4x + 8$

$y = 2x + 4$

Both equations are the same so there are infinite solutions.

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Two stalls are promoting strawberry punnets (p) using distinct pricing approaches. Stall A applies a fixed rate of \$6 per punnet, whereas Stall B employs a pricing structure of \$5 per punnet along with an additional \$2 per order. Using the equations $c = 6p$ and $c = 5p + 2$, verify if purchasing 2 punnets will result in an identical cost of \$12 at both stalls.

11. A company computes its weekly costs using the equation $y = 150x + 300$ for its expenses, and its sales are described by $y = 200x$, where x is the number of weeks. They predict they will break even in less than 7 weeks. Plot the graphs on a single set of axes and state the number of weeks required before they break even.

12. To assess the number of building permits to be granted, a city council employs a simplified model for forecasting housing needs. Supply is represented by $y = 5x + 80$, while $y = x + 100$ predicts demand, with x is the time in years and y indicating the count of houses. Use a graph to identify the point at which supply equals demand and give the corresponding timeframe and house count.

13. Store A sells notebooks for \$3 each, while Store B charges \$2.50 per notebook plus a \$2 fee for each order. This can be written as $c = 3n$ for Store A and $c = 2.50n + 2$ for Store B. Store A says that you need to buy 5 notebooks, costing \$15 in total, before Store B becomes cheaper. Plot both graphs and verify if Store A is correct.

14. An IT business uses the equations $y = 100x + 40$ to anticipate revenue and $y = -20x + 100$ to project costs, where x represents time in years and y represents thousands of dollars. Use a graph to determine the time in years the business will achieve a break-even scenario.

Reasoning

Question working paths

Mild 15 (a,b,c,d) 🌶	**Medium** 15 (a,b,c,d), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. A city planner uses two equations to represent a road and a large river.

Road: $y = -x + 4$

River: $y = 2x + 1$

- **a.** Plot the line that represents the road, $-1 \leq x \leq 6$.
- **b.** Plot the line that represents the river on the same graph.
- **c.** State if a form of river crossing is required for this section of road, giving reasons and any location coordinates.
- **d.** Give the equation for a road that would not cross this river and has a y-intercept at 2.
- **e.** What are some alternatives to bridges that would enable traffic to cross from one side of the river to the other?

16. Use simultaneous equations to answer the following.

- **a.** Explain why $y = 2x + 4$ and $y = 2x - 5$ do not have a point of intersection.
- **b.** Explain the location of the point of intersection of $y = x + 4$ and $y = 2x + 4$.
- **c.** Using the coordinates of the y-intercepts for the equations $y = ax + b$ and $y = dx + b$ write a statement connecting the c value and the solutions to simultaneous equations in the form $y = mx + c$.

Exam-style

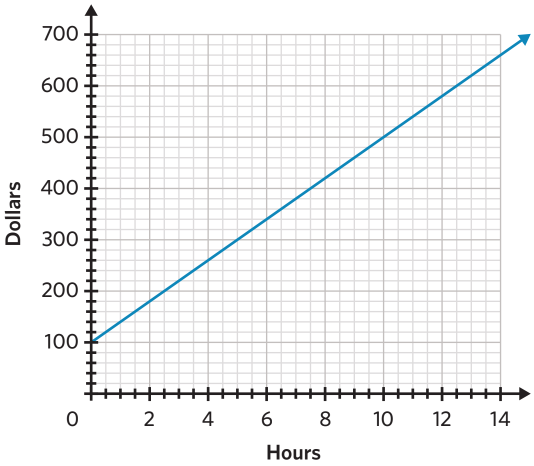
17. How many solutions do the following simultaneous equation have? (1 MARK)

$y = 2x + 3$

$y = 2x - 3$

A. 0 **B.** 1 **C.** 2 **D.** Infinite **E.** Undefined

18. Two industrial cleaners charge differently for their services. Friendly Cleaning charges $100 call out fee plus $40 per hour. This graph shows Friendly Cleaning's fees in dollars. (2 MARKS)

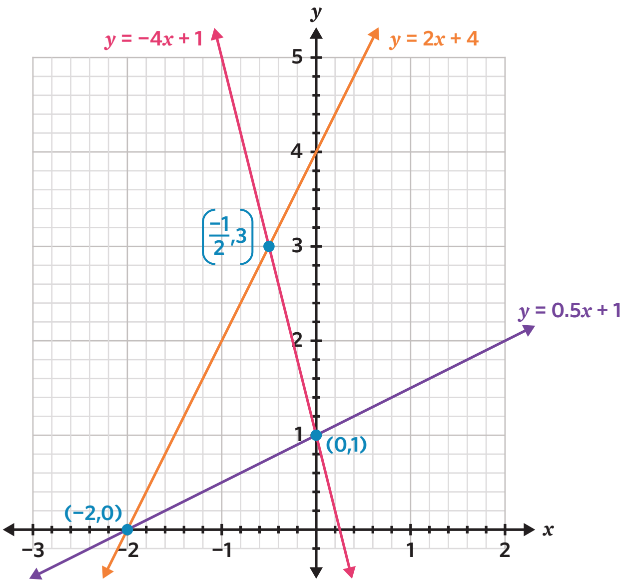


Happy Cleaning charge $50 per hour but does not charge a call out fee.

a. Using the same set of axes, plot the line that represents Happy Cleaning's fees. 1 MARK

b. How many hours of work would both cleaners charge the same amount? 1 MARK

19. A system of equations is shown in the following diagram. Using substitution, verify if the labelled points are solutions or not to each pair of simultaneous equations. (3 MARKS)



20. Plot each line and then state the solution to the simultaneous equations. (3 MARKS)

$y = 2x + 4$

$y = -0.5x - 6$

Remember this?

21. Which number shows 4 thousands, 6 hundreds, 2 tenths and 1 hundredth?

A. 4600.021

B. 4600.21

C. 4610.2

D. 4620.1

E. 4620.01

22. Julian counted the number of dogs he saw on his daily walk.

$\frac{2}{5}$ of the dogs were golden retrievers and $\frac{1}{4}$ were cavoodles.

He saw 3 more golden retrievers than cavoodles.

Calculate the total number of dogs Julian saw on his walk.

A. 10

B. 15

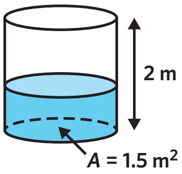
C. 20

D. 40

E. 60

23. A cylindrical water tank has a base with an area of 1.5 m^2.

One cubic metre can hold 1000 L of water.



How many litres of water would fill the tank to a depth of 90 cm?

A. 1.35 L

B. 13.5 L

C. 135 L

D. 1350 L

E. 135 000 L

Back to contents

Chapter 5 extended application

1. An art gallery offers a special subscription model with a sign up fee of $80 for new members and a monthly fee of $30. The total cost, y, for the subscription can be modelled by the equation $y = 30x + 80$, where x is the number of months. The subscription gives a member unlimited access to the gallery.

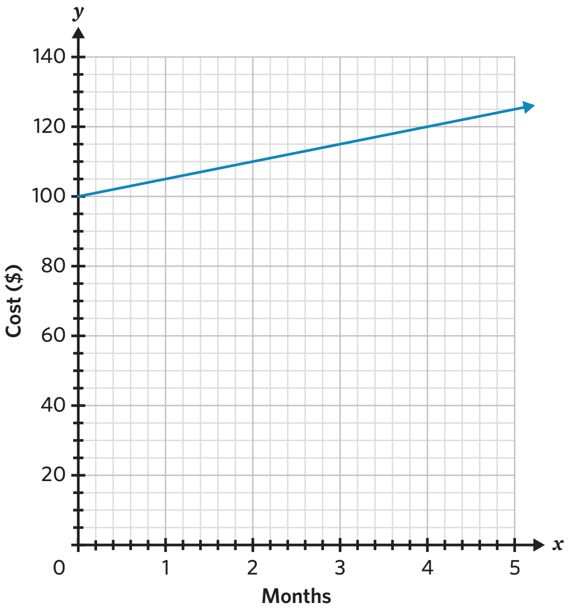
a. Using the given equation, determine both the x- and y-intercepts.

b. Sketch a graph representing the subscription costs for $0 \leq x \leq 5$, and label the end points.

c. A new art gallery is opening in the same city and offering a monthly fee of $50 per month with no sign up fee. Create an equation to model the cost of the new art gallery's subscription.

d. Sketch the graph for the equation in part **c** on the same axes used in part **a.** Label the point at which the lines intersect and explain the significance of this point in the context of the problem.

e. A third gallery is opening in a nearby location. The subscription cost is modelled in the graph below. Determine the equation of the line and explain the subscription model in words.



f. Suggest the benefits for the gallery of offering a subscription model for regular visitors compared to one-time ticket purchases.

2. Two chefs are preparing pasta for a big party and will cook the pasta in batches due to limited cookware. The table illustrates the amount of pasta, in kilograms, and the time taken to cook the pasta, in minutes, for each chef.

	Time (minutes)	
Pasta (kg)	**Chef Marco**	**Chef Maria**
0	0	0
1	6	7
2	12	13
3	18	17
6	36	28

a. Plot the data for both chefs on the same set of axes, with the amount of pasta on the x-axis and time on the y-axis.

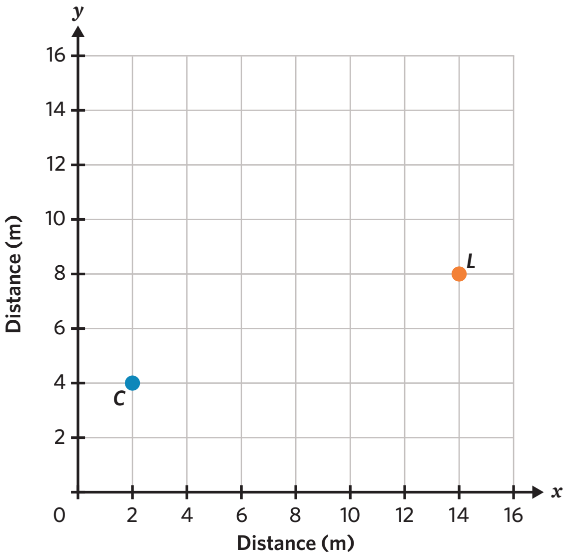
b. State which chef seems to be cooking at a constant rate, and write a rule to model the time it takes them to cook in terms of the amount of pasta there is.

c. A third chef, Chef Brett is brought in who takes 18 minutes to clean the kitchen before he starts cooking. He then cooks at a constant rate of 1 kg of pasta every 6 minutes until he has cooked 6 kg. Plot a line to represent Chef Brett's pasta cooking on the same axes as part **a**.

d. Assuming that Chef Brett and Chef Marco maintain their consistent cooking, explain why the lines that represent their cooking rates will never intercept.

e. Suggest a strategy for the chefs to implement to speed up the cooking process.

3. A Cartesian plane can be used to show the position of two players during indoor soccer practice. During the warm up, two of the players position themselves a distance away from each other to practise their passing skills. Christie is located at point *C*, and Lauren is located at point *L*.



a. The path along which Christie and Lauren pass the ball to one another is a straight line which can be modelled by an equation. Determine the equation of this line.

b. There is a marker along the kicking line between Christie and Lauren and it is located an equal distance from both of them. Determine the coordinates of the marker.

c. Determine the distance, in metres, that Christie needs to pass the ball for it to reach Lauren. Round the answer to 2 decimal places.

d. The coach is observing the warm up from the location represented by the origin. Calculate how many metres, in a straight line, the coach will need to walk from this position to Lauren's position. Round the answer to 2 decimal places.

e. Carlotta is located at (16,4). The path along which Carlotta passes the ball to her partner crosses the midpoint of Christie and Lauren's path and forms a perpendicular line with it. State the equation that models the path of the straight line along which Carlotta and her partner pass a ball.

f. Explain why warming up is an important part of soccer training.

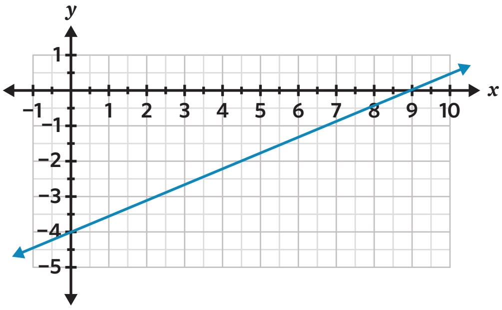
Back to contents

Chapter 5 review

Multiple choice

1. What are the x- and y-intercepts of the following graph? 5A,B

A. (0,9) and (0,−4)
B. (0,9) and (−4,0)
C. (9,0) and (−4,0)
D. (9,0) and (0,−4)
E. (9,9) and (−4,−4)



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2. What is the gradient of the line represented by the following equation? 5C

$y = \frac{2}{5}x - 4 \quad \therefore m =$ ☐

A. −4 **B.** $\frac{2}{5}$ **C.** 2 **D.** 4 **E.** 5

---

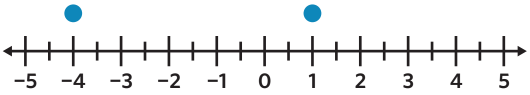
3. A penguin swims at a constant rate for 4 hours and travels 28 km. The rate is: 5D

☐ km/h

A. $\frac{1}{7}$ **B.** $\frac{4}{7}$ **C.** 4 **D.** 7 **E.** 28

---

4. What is the midpoint between −4 and 1? 5E

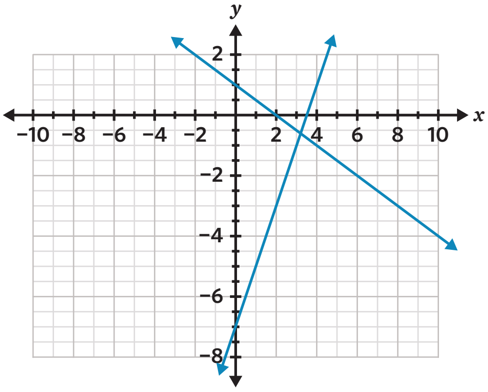


A. −2.5 **B.** −2 **C.** −1.5 **D.** −1 **E.** −0.5

---

5. Identify whether the following lines are classified as parallel, perpendicular or neither, and determine how many solutions the equations have. 5F,G

A. Parallel; 0
B. Parallel; 1
C. Perpendicular; 0
D. Perpendicular; 1
E. Neither; 1



Back to contents

Fluency

6. Calculate the x- and y-intercepts for the following linear equations. 5A

a. $y = -3x + 6$ **b.** $y = \frac{4}{5}x - 8$ **c.** $5x - y = 3$ **d.** $-\frac{2}{3}x + y = -1$

---

7. Sketch the graph of the following vertical and horizontal lines. 5B

a. $x = 8$ **b.** $x = -2$ **c.** $y = 5$ **d.** $y = -\frac{3}{5}$

---

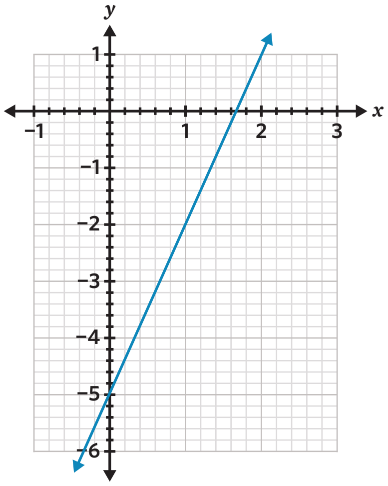
8. State the gradient and the y-intercept for the following equations. 5C

a. $y = -\frac{1}{2}x - 4$ **b.** $y = 2.2x - 10$ **c.** $-1.4x + y = -1$ **d.** $\frac{2}{5}x - y = -3$

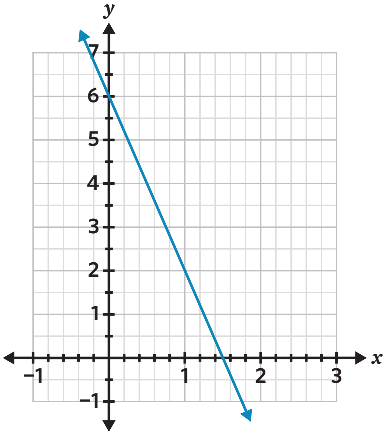
---

9. Determine the linear equation for the following graphs. 5C

a.



b.



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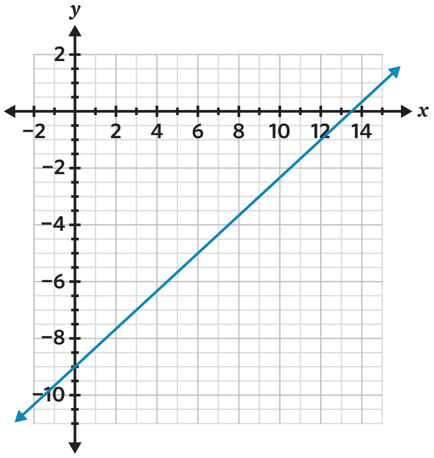

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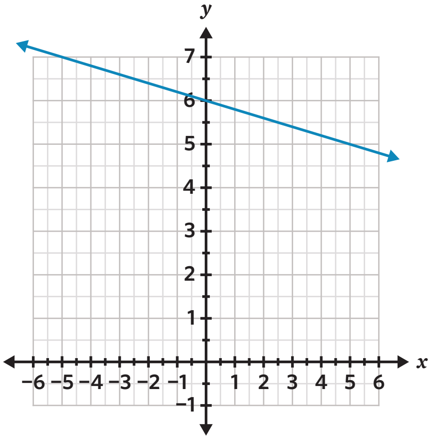

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10. Solve the following problems using directly proportional relationships. 5D

a. The cost of printing in dollars (c) can be described using the rule $c = 0.1p$, where p is the number of pages printed. Calculate the cost of printing 150 pages.

b. A factory produces toys (t) at a rate described by the rule $t = 50h$, where h is the number of hours the factory operates. How many toys can be produced in 8 hours?

c. A plant grows 2.5 cm each day. Write a rule connecting the plant's growth (g) and the number of days since planting (d), and use it to calculate the growth of the plant after 6 days.

d. On average, a car uses 0.07 L of fuel for every kilometre travelled. Generate a rule connecting the car's fuel consumption (f) in litres and distance travelled (d) and use this to calculate the fuel consumption for a 300 km trip.

11. Calculate the midpoint $M(x,y)$ of the line segment between the following pairs of points. 5E

a. (0,6) and (6,0) **b.** (2,–9) and (10,11) **c.** (–1,4) and (4,8) **d.** (–3,–5) and (–8,3)

12. Using Pythagoras' Theorem, calculate the length of the line segment joining the following pairs of points. Round to two decimal places where necessary. 5E

a. (–1,5) and (–5,2) **b.** (4,3) and (2,2) **c.** (–2,7) and (4,–1) **d.** (7,–3) and (–9,14)

13. Determine the equation of the straight line using the given information. 5F

a. A line passes through the points (–2,0) and (0,6).

b. A line passes through the points (–8,4) and (1,–5).

c. A line passes through the points (3,–5) and (4,3).

d. A line passes through the points (10,4) and (–2,10).

14. Determine the equation of the straight line using the given information. 5F

a. A line which is parallel to the line $y = \frac{1}{2}x - 4$ and passes through (0,11).

b. A line which is perpendicular to the line $y = -3x + 8$ and passes through (6,–4).

c. A line which is parallel to the line $y = -4x - 4$ and passes through $\left(-2,\frac{3}{5}\right)$.

d. A line which is perpendicular to the line $y = \frac{3}{8}x - 4$ and passes through (3,–10).

15. Plot the graphs of the given equations to determine the solution of the simultaneous equations. 5G

a. $y = -4x + 12$ and $y = 2x - 6$

b. $y = 0.5x + 3$ and $y = 6x - 8$

c. $6x - y = -1$ and $2x + y = 5$

d. $8x + y = -2$ and $y = -5x + 7$

Problem solving

16. A marine conservation team is tracking the declining population of a particular fish species. The team has modelled the population using the linear equation $y = -4x + 200$, where y represents the number of fish and x represents the number of years since the study began. Sketch the graph showing the fish population and state the number of years until the population reaches zero. 5A

17. A local gym is planning to offer a new membership package that includes a fixed monthly fee of \$45 for unlimited access to the gym. The cost (c) remains the same regardless of how many times a member visits the gym in a month (v). Sketch a graph to represent this information, where $v \geq 0$. 5B

18. A coffee shop wants to explore the connection between coffee pricing and daily sales. They've observed a decrease of 20 cups sold for every \$1 increase in price. At the current rate of \$4.00 per cup, they sell 100 cups per day. Sketch the graph and formulate an equation depicting the total cups sold per day (c) dependent on the price (p). 5C

19. A bakery is planning to bake cupcakes for a charity event. The bakery is able to prepare 72 cupcakes in half an hour. Calculate the number of cupcakes it can bake in 3.5 hours. 5D

20. A city planner is working on a new park design. He wants to connect two points in the park, located at (3,6) and (9,12), with a pathway. Calculate the length of the pathway in metres, where 1 unit equals 10 m. Round to two decimal places. 5E

21. In a coastal city, two lighthouses are being built to guide ships. The first lighthouse's beam follows the equation $y = -3x + 12$. The second lighthouse's beam will be perpendicular to the first one, and the city wants it to reach a specific island located at the coordinates (3,17). Determine the equation of the second lighthouse's beam. 5F

22. Two businesses are planning their production schedules. The first business's production is represented by the equation $y = 0.4x + 6$, where y is the number of products and x is the number of hours. The second business's production is represented by the equation $y = 0.9x + 1$. After how many hours will both businesses have produced the same number of products?

Reasoning

23. The city council is planning to build a new park with walking paths. Magnolia is the lead architect who is responsible for designing the paths.

a. The main path through the park is represented by the equation $y = 6x - 12$. Calculate the x- and y-intercepts and sketch the graph using these intercepts as the path's endpoints.

b. Magnolia wants to add a vertical path extending from the endpoint located at the x-intercept. Identify an equation for this path and sketch this on the same graph.

c. A drinking fountain is to be built exactly halfway along the original path. Determine the coordinates for the location of this fountain.

d. Magnolia decides to add a third path which will be perpendicular to the original path and passes through the point at $\left(\frac{3}{2}, -3\right)$. State the equation that represents this path.

e. The park's design is almost complete. What are some possible challenges that Magnolia might face when translating these mathematical designs into real-world construction?

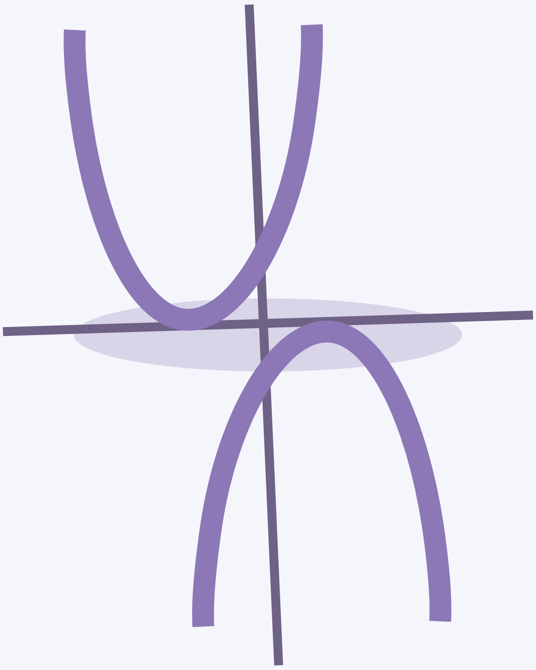
24. Consider the following pair of simultaneous equations.

$$4x + y = 3, \quad y = \frac{1}{4}x - 2$$

a. Show that the two lines are perpendicular.

b. Determine the number of solutions for the simultaneous equations.

c. Using your answers from parts **a** and **b**, comment on what this shows about perpendicular lines.



Chapter 6

Quadratic equations and graphs

Number and algebra

Calculator skills

See online in additional materials for using CAS calculator guides.

6B Factorising and solving monic quadratic equations
6C Graphs of quadratic functions
6F Non-linear graphs

Back to contents

Chapter 6 research summary

Quadratic equations and graphs

Big ideas

Quadratic equations are a fundamental topic for Year 9 mathematics. Quadratic equations are polynomial equations of the second degree usually written in the form $ax^2 + bx + c = 0$. Quadratic equations represent parabolic relationships between variables and their graphs, called parabolas. Number sense, algebraic thinking, mathematical reasoning, and mathematical representation are the main big ideas that underpin the concept of quadratic equations and graphs.

Number sense

Quadratic equations often involve real numbers, and sometimes complex numbers when considering solutions that are not real. A solid understanding of numbers and their properties is essential for comprehending the nature of the solutions.

Operations

Solving quadratic equations especially by factoring requires a grasp of mathematical operations. Students need to understand how to expand and factorise quadratic expressions.

Algebraic thinking

This is central to the concept of quadratic equations. Algebraic thinking involves using symbols, variables, and expressions to represent and solve mathematical problems. Quadratic equations are a progression in algebraic understanding, building on linear equations.

Mathematical reasoning

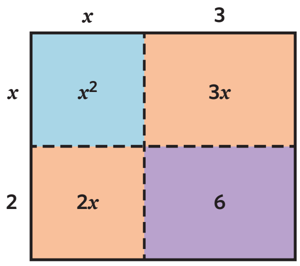
To solve quadratic equations, students need to apply logical thinking. Depending on the method used (factoring, completing the square, quadratic formula), students must reason through the steps to arrive at the solution.

Visual representations

Area models

Represent a quadratic equation like $x^2 + 5x + 6$ as a square or rectangle. The length could be $x + 3$ and the width could be $x + 2$ demonstrating the factorisation of the quadratic equation.

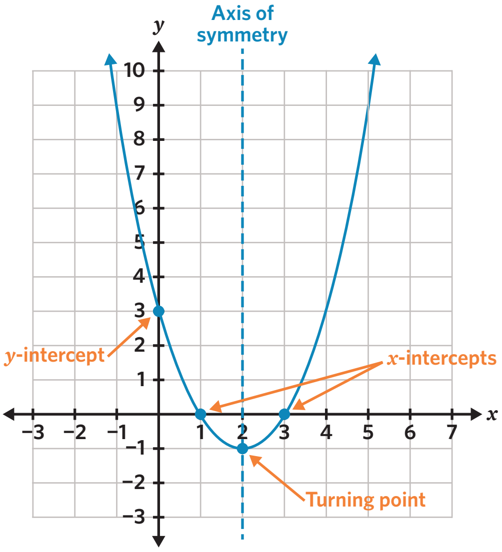
$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 3x + 2x + 3 \times 2 \\ &= x^2 + (3 + 2)x + 3 \times 2 \\ &\therefore p = 3 \text{ and } q = 2 \\ \text{So } x^2 + 5x + 6 &= (x + 3)(x + 2) \end{aligned}$$



Back to contents

Cartesian plane

One of the most profound ideas associated with quadratic equations is their graphical representation as parabolas. The shape, direction (opening upwards or downwards), and position of the parabola provide visual insights into the nature of the quadratic equation. This graphical perspective aids in understanding concepts like the change to turning point, axis of symmetry, and the roots or zeros of the equation.



Interactive software

There are many online tools and software like Desmos or GeoGebra that allow students to manipulate the coefficients of a quadratic equation and see in real-time how the graph changes. This can help them understand the effects of each term in the equation.

Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students assign the correct magnitude but incorrect direction to the solutions.	$(3x + 1)(x - 8) = 0$ $x = \frac{1}{3}$ and $x = -8$	$(3x + 1)(x - 8) = 0$ $x = -\frac{1}{3}$ and $x = 8$	6A
Students do not divide the final value of the solution by the coefficient of x.	$(3x + 1)(x - 8) = 0$ $x = -1$ and $x = 8$	$(3x + 1)(x - 8) = 0$ $x = -\frac{1}{3}$ and $x = 8$	6A
Students substitute negative values incorrectly when testing for a solution.	$3x^2 + 8x - 3 = 3(-3)^2 + 8(-3) - 3$ $= 3 \times (-9) + 8 \times (-3) - 3$ $= -27 - 24 - 3$ $= -54$ $\neq 0$ ✗ $\therefore x = -3$ is not a solution.	$3x^2 + 8x - 3 = 3(-3)^2 + 8(-3) - 3$ $= 3 \times 9 + 8 \times (-3) - 3$ $= 27 - 24 - 3$ $= 0$ ✓ $\therefore x = -3$ is a solution.	6A
Students assign the wrong direction to the factors of c when trying to find a pair that sums to b.	$x^2 + 16x + 64 = 0$ $(x - 8)(x - 8)$ $x = 8$	$x^2 + 16x + 64 = 0$ $(x + 8)(x + 8)$ $x = -8$	6B
Students identify two solutions to a perfect square quadratic equation.	$x^2 + 16x + 64 = 0$ $(x + 8)(x - 8)$ $x = -8, x = 8$	$x^2 + 16x + 64 = 0$ $(x + 8)(x + 8)$ $x = -8$	6B

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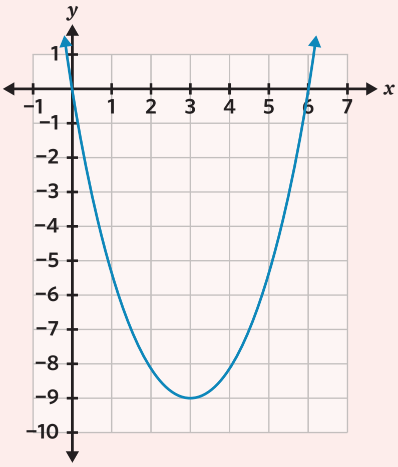
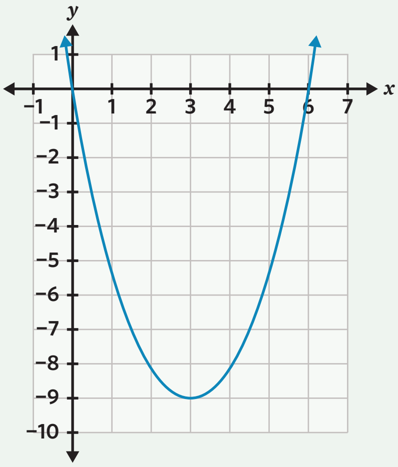
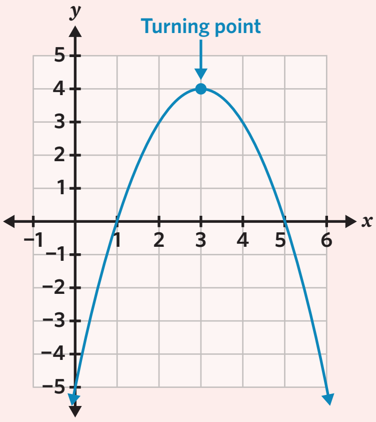
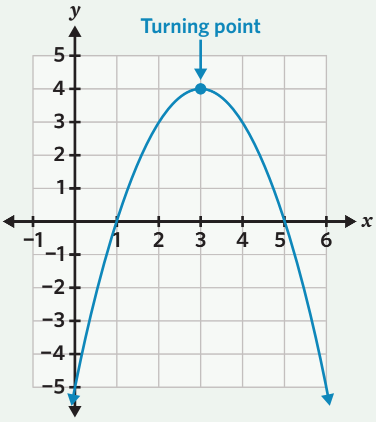
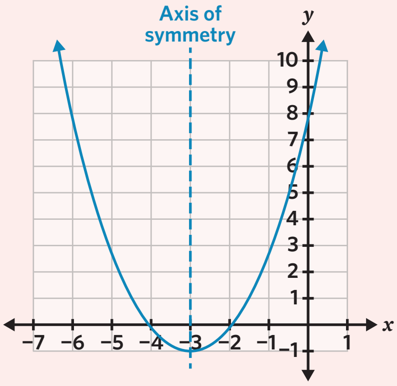
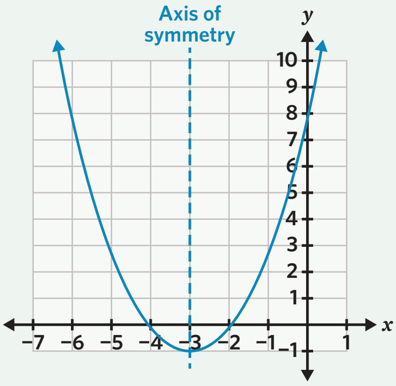
Back to contents

Misconception	Incorrect ✗	Correct ✓	Lesson
Students identify the wrong number of x-intercepts.	 (6,0)	 (0,0), (6,0)	6C
Students identify the turning point by the x- or y-coordinate only.	 Maximum turning point: $x = 3$	 Maximum turning point: (3,4)	6C
Students describe the axis of symmetry as a number or coordinates but not a function.	 Axis of symmetry $= -3$	 Axis of symmetry: $x = -3$	6C

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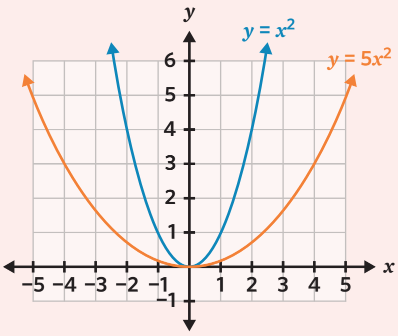
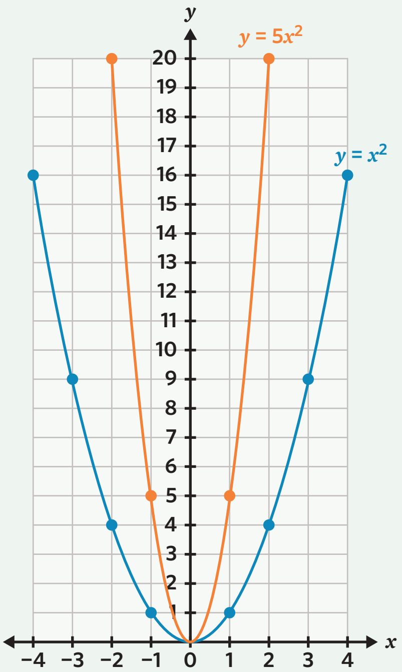
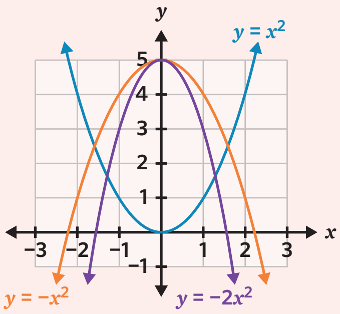
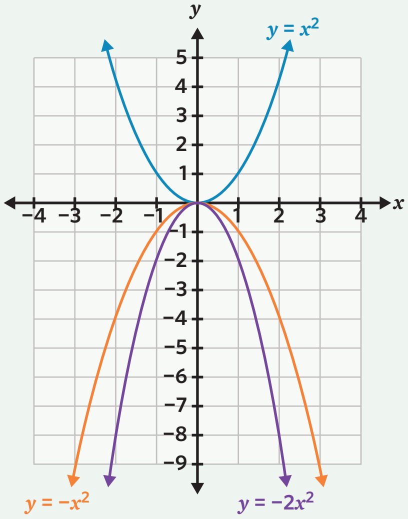
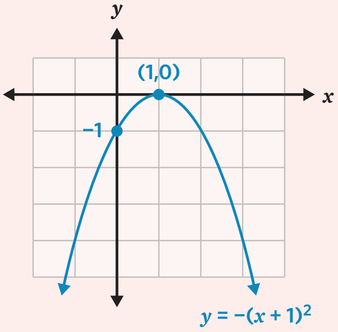
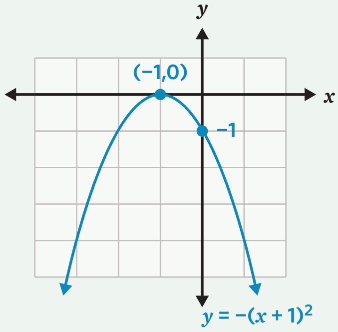
Back to contents

Misconception	**Incorrect ✕**	**Correct ✓**	**Lesson**
Students think that for $y = ax^2, 0 < a < 1$ indicates dilation towards the y-axis and $a > 1$ a dilation away from the axis.			6D
Students do not plot inverted parabolas of the form $y = -ax^2$ with a turning point of (0,0).			6D
Students think that for $y = (x - h)^2 + k, h > 0$ indicates a translation to the left and $h < 0$ a translation to the right.			6E

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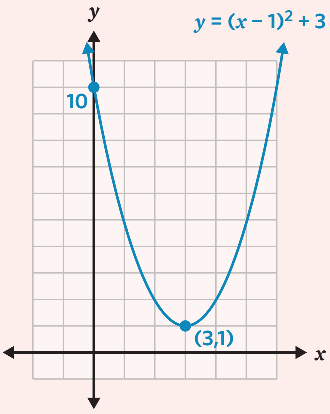
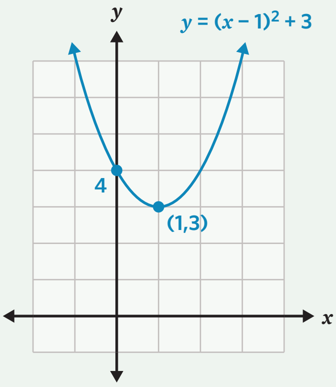
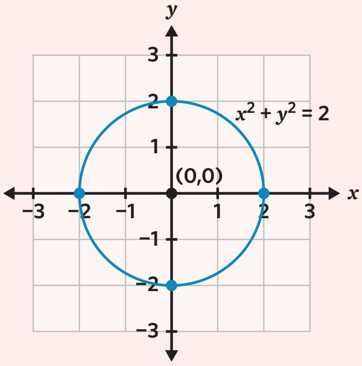
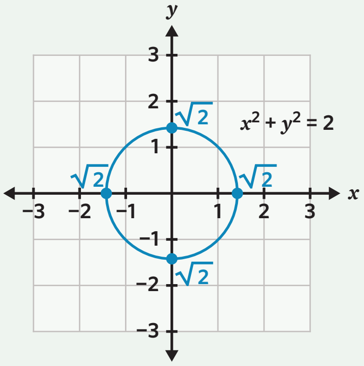
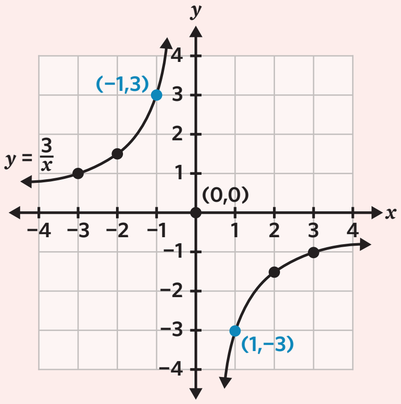
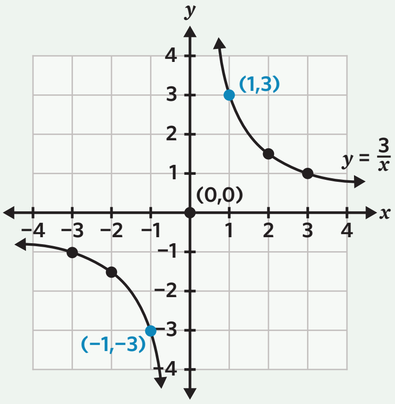
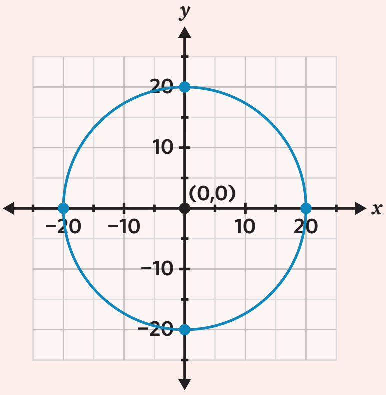
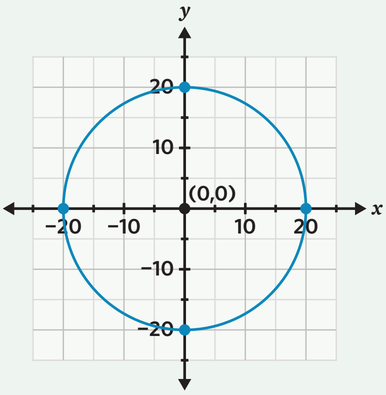
Back to contents

Misconception	Incorrect ✕	Correct ✓	Lesson
For $y = (x - h)^2 + k$, students use h for vertical and k for the horizontal translation.			6E
Students think that for $y = (x - h)^2 + k, k > 0$ indicates a translation down, while $k < 0$ indicates a translation up.	What are the coordinates of the turning point of $y = (x + 5)^2 - 7$? (−5,7)	What are the coordinates of the turning point of $y = (x + 5)^2 - 7$? (−5,−7)	6E
Students do not take the square root of r^2 when determining the radius from $x^2 + y^2 = r^2$.			6F
Students graph a hyperbola with the form $y = \frac{a}{x}$ in the 2nd and/or 4th quadrant.			6F
Students substitute the diameter of a circle for r.	 $x^2 + y^2 = 1600$	 $x^2 + y^2 = 400$	6F

6A Quadratic equations

LEARNING INTENTIONS

Students will be able to:

- identify quadratic equations
- write quadratic equations in standard form
- understand and apply the Null Factor Law to solve factorised quadratics.

A quadratic equation, also called a second degree equation, can be identified by its form. It has to contain a second degree variable with an index equal to two, and may contain a linear term (or first degree variable) as well as a constant value. It cannot contain variables of any other degree.

KEY TERMS AND DEFINITIONS

- The **Null Factor Law** states that if the product of two values is zero then either or both of the two values is zero.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Thaiview/Shutterstock.com

The shape of a satellite dish follows a parabolic pattern which can be modelled by a quadratic equation. When the dimensions of a dish fit perfectly within the parameters of a quadratic, the signal can be focused at one precise point, which in turn allows for information to be transmitted across vast distances.

Key ideas

1. Any **quadratic equation** can be written in the form $ax^2 + bx + c = 0$.

Standard form of a quadratic equation:

$$ax^2 + bx + c = 0 \text{ where } a \neq 0$$

a, b, c are known values

2. The Null Factor Law can be applied to solve quadratic equations in factorised form.

Null Factor Law:

If $a \times b = 0$, then $a = 0$ and/or $b = 0$

For factorised quadratics of the form $(x + p)(x + q)$

If $(x + p)(x + q) = 0$, then $x + p = 0$ and/or $x + q = 0$

Back to contents

Worked example 1

Writing quadratic equations in standard form

Rearrange the following quadratic equations to standard form $ax^2 + bx + c = 0$ where $a > 0$.

a. $x^2 = 3x + 5$

WE1a

Working

$$x^2 = 3x + 5$$
$$x^2 - 5 = 3x + 5 - 5$$
$$x^2 - 5 = 3x$$
$$x^2 - 5 - 3x = 3x - 3x$$
$$x^2 - 5 - 3x = 0$$
$$x^2 - 3x - 5 = 0 \checkmark$$

Thinking

Step 1: Apply inverse operations on either side of the equation until only zero remains on one side.

Step 2: Arrange the terms in the equation to match the form $ax^2 + bx + c = 0$ where $a > 0$.

b. $4x = -2x^2 + 5$

WE1b

Working

$$4x = -2x^2 + 5$$
$$4x - 5 = -2x^2 + 5 - 5$$
$$4x - 5 = -2x^2$$
$$4x - 5 + 2x^2 = -2x^2 + 2x^2$$
$$4x - 5 + 2x^2 = 0$$
$$2x^2 + 4x - 5 = 0 \checkmark$$

Thinking

Step 1: Apply inverse operations on either side of the equation until only zero remains on one side.

Step 2: Arrange the terms in the equation to match the form $ax^2 + bx + c = 0$ where $a > 0$.

c. $2(x^2 - 5x) = 3$

WE1c

Working

$$2(x^2 - 5x) = 3$$
$$2 \times x^2 + 2 \times (-5x) = 3$$
$$2x^2 - 10x = 3$$
$$2x^2 - 10x - 3 = 3 - 3$$
$$2x^2 - 10x - 3 = 0$$
$$2x^2 - 10x - 3 = 0 \checkmark$$

Thinking

Step 1: Expand the expression.

Step 2: Apply inverse operations on either side of the equation until only zero remains on one side.

Step 3: Check if the terms in the equation match the form $ax^2 + bx + c = 0$ where $a > 0$.

Student practice

Rearrange the following quadratic equations to standard form $ax^2 + bx + c = 0$ where $a > 0$.

a. $x^2 = 6x + 7$ **b.** $3x = -5x^2 - 3$ **c.** $3(x^2 - 4x) = 5$

Back to contents

Worked example 2

Checking the solutions of a quadratic equation

Substitute the given values into the quadratic equations to check whether they are solutions.

a. $x^2 + 2x - 3 = 0 \quad (x = 1)$ WE2a

Working	Thinking
$x^2 + 2x - 3 = (1)^2 + 2(1) - 3$	**Step 1:** Substitute the given value in the quadratic equation.
$= 1 + 2 - 3$ $= 0$ ✓	**Step 2:** Evaluate the equation and check if it is equal to zero.
$\therefore x = 1$ is a solution.	**Step 3:** Identify the given value as a solution to the quadratic equation if the equation is equal to zero when the given value has been substituted.

b. $2x^2 - 5x + 2 = 0 \quad (x = -2)$ WE2b

Working	Thinking
$2x^2 - 5x + 2 = 2(-2)^2 - 5(-2) + 2$	**Step 1:** Substitute the given value in the quadratic equation.
$= 2 \times 4 + 5 \times 2 + 2$ $= 8 + 10 + 2$ $= 20$ $\neq 0$ ✗	**Step 2:** Evaluate the equation and check if it is equal to zero.
$\therefore x = -2$ is not a solution.	**Step 3:** Identify the given value as a solution to the quadratic equation if the equation is equal to zero when the given value has been substituted.

Student practice

Substitute the given values into the quadratic equations to check whether they are solutions.

a. $x^2 + 3x - 10 \quad (x = 2)$

b. $2x^2 + 3x - 5 \quad (x = -1)$

Worked example 3

Using the Null Factor Law

Solve the following equations.

a. $x(x + 4) = 0$ WE3a

Working	Thinking
$x(x + 4) = x \times (x + 4) = 0$ The factors are x and $x + 4$.	**Step 1:** Identify the factors of the factorised quadratic equation.

Continues →

Step 2: Make each factor equal to zero and solve.

$x = 0$

and

$$x + 4 = 0$$
$$x + 4 - 4 = 0 - 4$$
$$x = -4$$

The solutions are $x = -4, x = 0$.

Step 3: State the solutions of the quadratic.

b. $(x - 1)(x + 3) = 0$

WE3b

Working

Thinking

Step 1: Identify the factors of the factorised quadratic equation.

$(x - 1)(x + 3) = (x - 1) \times (x + 3) = 0$

The factors are $x - 1$ and $x + 3$.

Step 2: Make each factor equal to zero and solve.

$$x - 1 = 0$$
$$x - 1 + 1 = 0 + 1$$
$$x = 1$$

and

$$x + 3 = 0$$
$$x + 3 - 3 = 0 - 3$$
$$x = -3$$

Step 3: State the solutions of the quadratic.

The solutions are $x = -3, x = 1$.

c. $(2x + 1)(3x - 2) = 0$

WE3c

Working

Thinking

Step 1: Identify the factors of the factorised quadratic equation.

$(2x + 1)(3x - 2) = (2x + 1) \times (3x - 2) = 0$

The factors are $2x + 1$ and $3x - 2$.

Step 2: Make each factor equal to zero and solve.

$$2x + 1 = 0$$
$$2x + 1 - 1 = 0 - 1$$
$$2x = -1$$
$$\frac{2x}{2} = \frac{-1}{2}$$
$$x = -\frac{1}{2}$$

and

$$3x - 2 = 0$$
$$3x - 2 + 2 = 0 + 2$$
$$3x = 2$$
$$\frac{3x}{3} = \frac{2}{3}$$
$$x = \frac{2}{3}$$

Step 3: State the solutions of the quadratic.

The solutions are $x = -\frac{1}{2}, x = \frac{2}{3}$.

Student practice

Solve the following equations.

a. $x(x + 7) = 0$

b. $(x - 2)(x + 6) = 0$

c. $(3x + 2)(2x - 5) = 0$

Back to contents

6A Questions

Understanding worksheet

1. Identify the values of a, b or c in the following quadratic equations in the form $ax^2 + bx + c = 0$.

Example

$x^2 - 7x + 12 = 0 \quad b = \boxed{-7}$

a. $3x^2 - 5x + 2 = 0 \quad a = \boxed{}$

b. $x^2 + 2x + 3 = 0 \quad a = \boxed{}$

c. $2x^2 - 11x + 5 = 0 \quad b = \boxed{}$

d. $4x^2 + 3x = 0 \quad c = \boxed{}$

2. Identify if the quadratic equations are written in standard form $ax^2 + bx + c = 0$.

Example

Equation	Standard form	Not in standard form
$5x^2 - 2 = 0$	☑	☐

Equation	Standard form	Not in standard form
I. $x^2 + 3x + 5 = 0$	☐	☐
II. $x^2 - 4x + 6 = 1$	☐	☐
III. $2x^2 - 5x = 0$	☐	☐
IV. $4x^2 = 0$	☐	☐

3. Fill in the blanks by using the words provided.

solutions **Null Factor Law** **quadratic** **zero**

All $\boxed{}$ equations can be rearranged to the standard form $ax^2 + bx + c = 0$ where a, b, and c are constants and a cannot be $\boxed{}$. The x-values for which the quadratic equation is equal to zero are the $\boxed{}$ of the quadratic. The $\boxed{}$ is applied when solving quadratic equations in factorised form.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Rearrange the following quadratic equations to standard form $ax^2 + bx + c = 0$ where $a > 0$. WE1a,b

a. $x^2 = 2x + 4$

b. $2x^2 = 1 + 3x$

c. $x^2 = 5 - x$

d. $4x = 3 - x^2$

e. $x^2 = -8x - 7$

f. $2x = -5x^2 + 9$

g. $5 = 10x - 3x^2$

h. $-4x + 9 = x^2$

5. Rearrange the following quadratic equations to standard form $ax^2 + bx + c = 0$ where $a > 0$. WE1c

a. $2(x^2 + 3x) = 4$

b. $3(x^2 + x) = -5$

c. $2(x^2 - 5x) = 11$

d. $5(x^2 - 3) = 7x$

Back to contents

e. $3(x^2 - 4) = -2x$ **f.** $-(2x^2 + 3x) = 8$

g. $2(4x^2 - 3) = -3x$ **h.** $-3(2x^2 + 5x) = -10$

---

6. Substitute the given values into the quadratic equations to check whether they are solutions. WE2

a. $x^2 - 4 = 0 \quad (x = 2)$ **b.** $x^2 - 2x = 0 \quad (x = 1)$

c. $2x^2 - 3 = 0 \quad (x = -1)$ **d.** $x^2 + x - 2 = 0 \quad (x = 1)$

e. $6 - 7x + x^2 = 0 \quad (x = 2)$ **f.** $x^2 - 5x - 14 = 0 \quad (x = -2)$

g. $2x^2 - 5x - 3 = 0 \quad (x = 4)$ **h.** $15 + 8x + x^2 = 0 \quad (x = -3)$

---

7. Solve the following equations. WE3a,b

a. $x(x + 3) = 0$ **b.** $2x(x - 5) = 0$

c. $(x + 2)(x - 2) = 0$ **d.** $(x - 3)(x + 5) = 0$

e. $(x - 7)(x - 3) = 0$ **f.** $(x + 11)(x - 10) = 0$

g. $\left(x + \frac{2}{3}\right)(x - 8) = 0$ **h.** $\left(x - \frac{1}{2}\right)\left(x + \frac{1}{4}\right) = 0$

---

8. Solve the following equations. WE3c

a. $x(2x + 1) = 0$ **b.** $2x(3x - 1) = 0$

c. $(2x - 1)(x + 3) = 0$ **d.** $(2x + 3)(x - 1) = 0$

e. $(3x + 1)(2x + 5) = 0$ **f.** $(4x + 3)(3x - 5) = 0$

g. $(5x - 3)(6x - 11) = 0$ **h.** $(4x - 9)(7x + 13) = 0$

---

9. Which option shows the solutions to $(3x + 1)(x - 8) = 0$?

A. $x = \frac{1}{3}$ and $x = 8$ **B.** $x = -1$ and $x = 8$

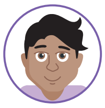
C. $x = 3$ and $x = -8$ **D.** $x = -\frac{1}{3}$ and $x = 8$

E. $x = \frac{1}{3}$ and $x = -8$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Solve $(x - 7)(2x + 9) = 0$.



Student A

$(x - 7)(2x + 9) = 0$

The factors are $x - 7$ and $2x + 9$

$x - 7 = 0$

$x = 0 - 7$

$x = -7$

and

$2x + 9 = 0$

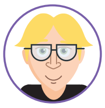
$2x = 0 + 9$

$2x = 9$

$\frac{2x}{2} = \frac{9}{2}$

$x = \frac{9}{2}$

The solutions are $x = -7$ and $x = \frac{9}{2}$.



Student B

$(x - 7)(2x + 9) = 0$

The factors are $x - 7$ and $2x + 9$

$x - 7 = 0$

$x - 7 + 7 = 0 + 7$

$x = 7$

and

$2x + 9 = 0$

$2x + 9 - 9 = 0 - 9$

$2x = -9$

$\frac{2x}{2} = -\frac{9}{2}$

$x = -\frac{9}{2}$

The solutions are $x = 7$ and $x = -\frac{9}{2}$.

Back to contents

b. Substitute $x = -3$ into the quadratic equation $3x^2 + 8x - 3 = 0$ to check whether it is a solution.

Student A

$$\begin{aligned} 3x^2 + 8x - 3 &= 3(-3)^2 + 8(-3) - 3 \\ &= 3 \times (-9) + 8 \times (-3) - 3 \\ &= -27 - 24 - 3 \\ &= -54 \\ &\neq 0 \ \times \end{aligned}$$

$\therefore x = -3$ is not a solution.

Student B

$$\begin{aligned} 3x^2 + 8x - 3 &= 3(-3)^2 + 8(-3) - 3 \\ &= 3 \times 9 + 8 \times (-3) - 3 \\ &= 27 - 24 - 3 \\ &= 0 \ \checkmark \end{aligned}$$

$\therefore x = -3$ is a solution.

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. The shape of a canal running through a city can be modelled by the equation $x^2 - 3x = d$ where x is the width of the canal and d is its depth, relative to the ground. Show that when the canal is 3 m wide, its depth is 0 m.

12. A hammock has been tied between two trees. Its shape can be modelled by the equation $x(x - 4) = y$ where x is the distance from one of the trees and y is the depth of the hammock, relative to the points where it was tied. Calculate the depth of the hammock when the distance from one of the trees is 4 m.

13. Colin writes a cooking book. His profit per sold book can be modelled by the equation $10n - n^2 - 21 = s$, where n represents the retail price of one book and s is the profit from its sale, in dollars. Show that when the retail price of one book is \$7, the profit per book is equal to zero.

14. The shape of a half-pipe at the skate park can be modelled by the equation $w(w - 6) = y$, where w is the width of the half-pipe and y is its depth, relative to the edges. Assuming that the half-pipe's edges are at the points where depth $y = 0$ m, how wide is the half-pipe?

15. To calculate the best price for her homemade soaps, Jenny uses the equation $(2 - x)(x - 5) = p$, where x is the price per bar of soap in dollars and p is the potential profit from its sale. Solve the equation when $p = 0$ and identify the two retail prices per bar of soap that will result in zero profit according to Jenny's model.

Reasoning

Question working paths

Mild 16 (a,b,d) 🌶	**Medium** 16 (a,b,d), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Kenny is analysing the views on his streaming channel. He finds that the time people spend watching one of his videos is related to the length of the video. Kenny can model this by the equation

$v = 5m - 4 - m^2$

where v is the length of time people spend watching a video and m is the length of the video, in minutes.

a. Substitute $m = 1$ and calculate the value of v.

b. Substitute $m = 4$ and calculate the value of v.

c. What happens to the views on Kenny's videos if he does not ensure that they are between one and four minutes long? Explain in context.

d. Identify an aspect of a presentation or performance that helps keep an audience interested.

Back to contents

17. Solve the equations given in parts **a** and **b**.

a. $x(x + 9) = 0$

b. $5x(2x - 5) = 0$

c. Analyse the answers from parts **a** and **b**. Make a generalisation about the solutions to quadratic equations of the form $ax(bx + c) = 0$ where $a \neq 0$ and $b \neq 0$.

Exam-style

18. One of the solutions to the quadratic equation $x^2 - 7x + 12 = 0$ is (1 MARK)

A. $x = -7$
B. $x = -4$
C. $x = -3$
D. $x = 3$
E. $x = 12$

19. Consider the given quadratic equation. (3 MARKS)

$1 = 5x - 4x^2$

a. Show that the equation is equivalent to $4x^2 - 5x + 1 = 0$. 2 MARKS

b. Show that $x = 1$ is a solution to the equation. 1 MARK

20. The area of a paddock can be modelled by the quadratic equation $(40 - x)(x - 50) = a$ where x is an unknown length in metres. Show that the area cannot exist when $x = 40$ m or $x = 50$ m. (3 MARKS)

21. Rearrange the quadratic equation $-5x = 3 - 2x^2$ to standard form $ax^2 + bx + c = 0$ where $a > 0$ and show that $x = 3$ is a solution to the equation. (3 MARKS)

Remember this?

22. In the city of Metropolis, there are 75 000 cars. Each day, the average car travels 20 km. Which of these gives the best estimate for the total number of kilometres travelled by cars in Metropolis each day?

A. 75 000 × 25
B. 75 000 × 20
C. 75 000 ÷ 25
D. 75 000 ÷ 20
E. 75 000 × 15

23. A timetable for the train from Central Station to Blacktown is given.

Departure times				
Central Station	10:15	11:15	12:15	13:15
Redfern	10:20	11:20	12:20	13:20
Strathfield	10:45	11:45	12:45	13:45
Blacktown	11:30	12:30	13:30	14:30

Emma is catching a train from Central Station to Blacktown.

She catches the first train that leaves Central Station after 12 pm.

At what time will Emma arrive at Blacktown?

A. 12:30 pm
B. 1:00 pm
C. 1:15 pm
D. 1:30 pm
E. 2:30 pm

24. $8 \times (3 + 2)$ ★ $5 \times 6 + 4$

Which symbol (<, =, >, ≤, ≥) should replace ★ to make the number sentence correct?

A. <
B. =
C. >
D. ≤
E. ≥

Back to contents

6B Factorising and solving monic quadratic equations

LEARNING INTENTIONS

Students will be able to:

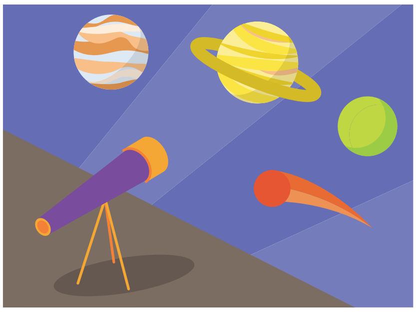
- factorise monic quadratic equations
- solve monic quadratic equations.

Factorisation of quadratic equations is an essential step in the process of solving them. Under certain conditions, such as in the case of monic quadratic equations of the form $ax^2 + bx + c = 0$ where $a = 1$, the factorisation process can be streamlined. The Null Factor Law can then be applied to determine the solutions of the quadratic equation.

KEY TERMS AND DEFINITIONS

- A **monic quadratic trinomial** is a quadratic expression of the form $ax^2 + bx + c$ where $a = 1$.
- A **trinomial** is a polynomial expression containing three different terms, one of which may be a constant.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Optics is the study of lenses and mirrors. The curvature of a lens or mirror can be modelled by a quadratic equation. Studying the way light rays interact with these surfaces led to the invention of the telescope. A telescope is able to magnify faraway objects due to the precise arrangement of the lenses and mirrors inside of it.

Key ideas

1. Monic quadratic expressions of the form $ax^2 + bx + c$ where $a = 1$ can be factorised to $(x + p)(x + q)$ where $b = p + q$ and $c = pq$.

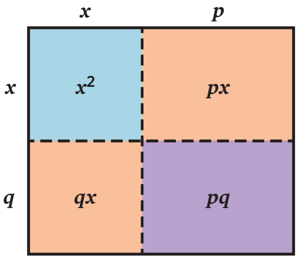
Expanding

$$(x + p)(x + q) = x^2 + px + qx + pq$$

$$= x^2 + (p + q)x + pq$$

Factorising

An expression of the form $ax^2 + bx + c$ where $a = 1$, $b = p + q$ and $c = pq$



2. Monic quadratic equations in standard form must be factorised so that the Null Factor Law can be applied to solve them.

When $(x + p)(x + q) = 0$

$$x + p = 0$$
$$x = -p$$

and

$$x + q = 0$$
$$x = -q$$

Worked example 1

Solving monic quadratic equations

Factorise and solve the following.

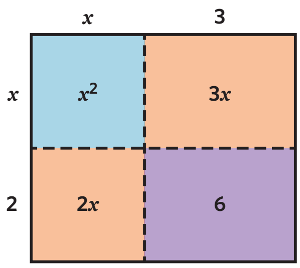
a. $x^2 + 5x + 6 = 0$

WE1a

Working	Thinking
$b = 5$ $c = 6$	**Step 1:** Identify the values of b and c for the equation of the form $x^2 + bx + c = 0$.
$c = 2 \times 3 = 6$ ✓ $b = 2 + 3 = 5$ ✓ $\therefore p = 2$ and $q = 3$	**Step 2:** Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.
$x^2 + 5x + 6 = (x + 2)(x + 3) = 0$	**Step 3:** Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.
$x + 2 = 0$ $x = -2$ and $x + 3 = 0$ $x = -3$	**Step 4:** Solve the equation by applying the Null Factor Law.

Visual support

$$x^2 + 5x + 6 = x^2 + 3x + 2x + 3 \times 2$$
$$= x^2 + (3 + 2)x + 3 \times 2$$
$$\therefore p = 3 \text{ and } q = 2$$
$$\text{So } x^2 + 5x + 6 = (x + 3)(x + 2)$$



b. $x^2 - 3x - 10 = 0$

WE1b

Working	Thinking
$b = -3$ $c = -10$	**Step 1:** Identify the values of b and c for the equation of the form $x^2 + bx + c = 0$.
$c = -5 \times 2 = -10$ ✓ $b = -5 + 2 = -3$ ✓ $\therefore p = -5$ and $q = 2$	**Step 2:** Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.
$x^2 - 3x - 10 = (x - 5)(x + 2) = 0$	**Step 3:** Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.
$x - 5 = 0$ $x = 5$ and $x + 2 = 0$ $x = -2$	**Step 4:** Solve the equation by applying the Null Factor Law.

Continues →

c. $x^2 = 12 - 7x$

WE1c

Working	Thinking
$x^2 = 7x - 12$ $x^2 - 7x = 12$ $x^2 - 7x + 12 = 0$ $b = -7$ $c = 12$	**Step 1:** Rearrange the equation to the form $x^2 + bx + c = 0$ and identify the values of b and c.
$c = -4 \times (-3) = 12$ ✓ $b = -4 + (-3) = -7$ ✓ $\therefore p = -4$ and $q = -3$	**Step 2:** Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.
$x^2 - 7x + 12 = (x - 4)(x - 3) = 0$	**Step 3:** Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.
$x - 4 = 0$ $x = 4$ and $x - 3 = 0$ $x = 3$	**Step 4:** Solve the equation by applying the Null Factor Law.

Student practice

Factorise and solve the following.

a. $x^2 + 3x + 2 = 0$ **b.** $x^2 - 2x - 8 = 0$ **c.** $x^2 = 6 - 5x$

Worked example 2

Solving with perfect squares and differences of two squares

Factorise and solve the following.

a. $x^2 - 6x + 9 = 0$

WE2a

Working	Thinking
$b = -6$ $c = 9$	**Step 1:** Identify the values of b and c for the equation of the form $x^2 + bx + c = 0$.
$c = -3 \times (-3) = 9$ $b = -3 + (-3) = -6$ $\therefore p = -3$ and $q = -3$	**Step 2:** Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.
$x^2 - 6x + 9 = (x - 3)(x - 3)$ $= (x - 3)^2 = 0$	**Step 3:** Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.
$x - 3 = 0$ $x = 3$	**Step 4:** Solve the equation by applying the Null Factor Law. Note that there is only one solution as the quadratic is a perfect square.

Continues →

b. $x^2 - 16 = 0$

WE2b

Working	**Thinking**
$c = -16$ $b = 0$	**Step 1:** Identify the value of c in the equation of the form $x^2 + c = 0$.
$c = -4 \times 4$ ✓ $b = -4 + 4 = 0$ ✓ $\therefore p = -4$ and $q = 4$	**Step 2:** Determine a pair of factors of c that sum to zero. Let this pair of factors be p and q.
$x^2 - 16 = (x - 4)(x + 4)$	**Step 3:** Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.
$x - 4 = 0$ $x = 4$ and $x + 4 = 0$ $x = -4$	**Step 4:** Solve the equation by applying the Null Factor Law.

Student practice

Factorise and solve the following.

a. $x^2 - 8x + 16 = 0$ **b.** $x^2 - 25 = 0$

Back to contents

6B Questions

Understanding worksheet

1. Fill in the blanks.

Example

$x^2 - 2x - 15 = x^2 + 3x + \boxed{-5}\,x - 15$

a. $x^2 + 7x + 12 = x^2 + 3x + \boxed{}\,x + 12$

b. $x^2 + 3x - 10 = x^2 + 5x + \boxed{}\,x - 10$

c. $x^2 - 4x + 4 = x^2 - 2x + \boxed{}\,x + 4$

d. $x^2 - 9 = x^2 + \boxed{}\,x + 3x - 9$

2. Match the factorised quadratic equations to their solutions.

Equation			Solutions
$(x - 2)(x - 3) = 0$	●	●	$x = -2, x = 7$
$(x - 7)(x + 2) = 0$	●	●	$x = 2, x = 3$
$(x + 2)(x + 3) = 0$	●	●	$x = 2, x = -7$
$(x - 2)(x + 7)$	●	●	$x = -3, x = -2$

3. Fill in the blanks by using the words provided.

one **trinomial** **single** **factorised**

Monic quadratic $\boxed{}$ equations have the form $ax^2 + bx + c = 0$ where the coefficient of x^2, a is equal to $\boxed{}$. Quadratic equations in this form can be $\boxed{}$ and solved using the Null Factor Law. These equations usually have two solutions, except for perfect squares that have a $\boxed{}$ solution.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Factorise and solve the following. WE1a

a. $x^2 + 4x + 3 = 0$
b. $x^2 + 5x + 4 = 0$
c. $x^2 + 7x + 10 = 0$
d. $x^2 + 8x + 12 = 0$
e. $x^2 + 9x + 14 = 0$
f. $x^2 + 13x + 36 = 0$
g. $x^2 + 17x + 42 = 0$
h. $2x^2 + 14x + 24 = 0$

5. Factorise and solve the following. WE1b

a. $x^2 - 2x - 3 = 0$
b. $x^2 - 5x + 4 = 0$
c. $x^2 - 4x - 12 = 0$
d. $x^2 + 9x - 10 = 0$
e. $x^2 + 3x - 10 = 0$
f. $x^2 - 7x - 18 = 0$
g. $x^2 - x - 56 = 0$
h. $x^2 - 16x + 63 = 0$

6. Factorise and solve the following. **WE1c**

a. $x^2 + 3x = 4$
b. $x^2 - 8 = 2x$
c. $x^2 = 3 - 2x$
d. $x^2 = 6 - x$
e. $32 + 4x = x^2$
f. $18 - x^2 = 7x$
g. $-16 = 6x - x^2$
h. $-9x = -21 - x^2 + x$

7. Factorise and solve the following. **WE2a**

a. $x^2 + 4x + 4 = 0$
b. $x^2 + 6x + 9 = 0$
c. $x^2 - 2x + 1 = 0$
d. $x^2 - 10x + 25 = 0$
e. $x^2 - 12x + 36 = 0$
f. $x^2 - 14x + 49 = 0$
g. $x^2 - 18x + 81 = 0$
h. $2x^2 - 32x + 128 = 0$

8. Factorise and solve the following. **WE2b**

a. $x^2 - 4 = 0$
b. $x^2 - 36 = 0$
c. $x^2 - 49 = 0$
d. $x^2 - 100 = 0$
e. $x^2 - 196 = 0$
f. $x^2 - y^2 = 0$
g. $x^2 - 4y^2 = 0$
h. $-9p^2 + x^2 = 0$

9. Which of the options shows the solutions to the given expression?

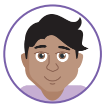
$x^2 + 16x + 64 = 0$

A. $x = 8$
B. $x = 64$
C. $x = -8$
D. $x = -8, x = 8$
E. $x = -32, x = 32$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Factorise and solve $x^2 - 10x + 25 = 0$.



Student A

$b = -10$

$c = 25$

$c = -5 \times 5 = 25$

$b = -5 + 5 = -10$

$\therefore p = -5$ and $q = 5$

$x^2 - 10x + 25 = (x - 5)(x + 5) = 0$

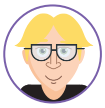
$x - 5 = 0$

$x = 5$

and

$x + 5 = 0$

$x = -5$



Student B

$b = -10$

$c = 25$

$c = -5 \times (-5) = 25$

$b = -5 + (-5) = -10$

$\therefore p = -5$ and $q = -5$

$x^2 - 10x + 25 = (x - 5)(x - 5)$

$= (x - 5)^2 = 0$

$x - 5 = 0$

$x = 5$

Back to contents

b. Factorise and solve $x^2 + 2x - 15 = 0$.

Student A

$b = 2$

$c = -15$

$c = 5 \times (-3) = -15$

$b = 5 - 3 = 2$

$\therefore p = 5$ and $q = -3$

$x^2 + 2x - 15 = (x + 5)(x - 3) = 0$

$x + 5 = 0$

$x = -5$

and

$x - 3 = 0$

$x = 3$

Student B

$b = 2$

$c = -15$

$c = -5 \times 3 = -15$

$b = -5 + 3 = 2$

$\therefore p = -5$ and $q = 3$

$x^2 + 2x - 15 = (x - 5)(x + 3) = 0$

$x - 5 = 0$

$x = 5$

and

$x + 3 = 0$

$x = -3$

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. Over a number of weeks (w), Jenny's savings account balance (a) could be modelled by the equation $a = w^2 - 8w + 12$. Factorise and solve the equation to find the weeks during which Jenny's account balance was zero (when $a = 0$).

12. During a cold winter night, outside temperature can be modelled by the equation $t = h^2 - 6h + 8$ where h is the hours since midnight and t is the temperature in degrees Celsius. Identify the times during which the temperature was 0°C.

13. During a dive session, a diver's depth, d is measured relative to the surface, in metres. It can be modelled by the equation $d = m^2 - 6m$ where m is their horizontal distance from the edge of the pool. Factorise and solve the equation when $d = -9$ m to determine how far the diver was from the edge of the pool when they were at a depth of 9 m.

14. The number of people (p) who visited a website within t minutes of it being live is given by the equation $t^2 - 5t = p$. Assuming that t may only be positive, how many minutes did it take for 24 people to visit the website?

15. A rectangular playground's area is given by the equation $x(x + 4) = 45$ where x is an unknown length in metres. Assuming that x must be a positive value, solve the equation to determine the length of x.

Reasoning

Question working paths

Mild 16 (a,b,d) 🌶	**Medium** 16 (a,b,d), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. The number of people visiting a market can be modelled by the formula

$t^2 + 6t = p$

where t is the number of hours since opening and p is the number of people at the market.

a. Substitute $p = 40$ and rearrange the equation to standard form.

b. Solve the equation from part **a**.

c. Assuming that t may only be positive and that the market opens at 6:30 am, when does the market have 40 visitors?

d. Why would a market or shop have more visitors during certain times of the day?

17. Solve the equations given in parts **a** and **b**.

a. $x^2 - 64 = 0$

b. $x^2 - 81 = 0$

c. Comment on the nature of solutions to equations of the form $x^2 = n$, where $n > 0$.

Exam-style

18. Which of the given options is the factorised form of $x^2 + 10x - 24$? (1 MARK)

A. $(x + 4)(x + 6)$
B. $(x + 2)(x + 12)$
C. $(x + 2)(x - 12)$
D. $(x - 2)(x + 12)$
E. $(x - 2)(x - 12)$

19. Consider the given equation. (3 MARKS)

$x^2 - 9x = -20$

a. Rearrange the equation to standard form $ax^2 + bx + c = 0$ where $a = 1$. 1 MARK

b. Factorise the equation. 1 MARK

c. Determine the solutions. 1 MARK

20. Jane's profits from running a small business can be modelled by the equation $p = x^2 - 2x - 35$ where p is the total profits in dollars and x is the number of weeks since opening. Assuming that x may only be a positive number, calculate how many weeks it took for the business to start making a profit. (3 MARKS)

21. The solutions to the equation $x^2 + bx + c = 0$ are $x = 5$ and $x = 6$. Determine the values of b and c. (2 MARKS)

Remember this?

22. The Summer Olympics occur every leap year. The first Summer Olympics of the 21st century took place in 2000. When will the 15th Summer Olympics of the 21st century take place?

A. 2054
B. 2056
C. 2060
D. 2064
E. 2068

23. Liam uses blocks to build a pyramid.

Number of layers	1	2	3	4	5
Number of blocks	1	4	9	16	25

When a pyramid consists of 15 layers, how many blocks will the 15th layer in that pyramid have?

A. 200
B. 225
C. 230
D. 235
E. 240

24. Friends Ava and Adam complete a training session. Ava swims $\frac{1}{4}$ of the total distance and cycles $\frac{1}{5}$ of the distance, and runs the rest of the way. Adam swims $\frac{1}{3}$ of the total distance, cycles $\frac{1}{4}$ of the distance, and runs the rest of the way. What fraction of the total distance did each friend run?

A. Ava and Adam both run $\frac{3}{5}$ of the total distance.

B. Ava runs $\frac{9}{20}$ and Adam runs $\frac{5}{12}$ of the total distance.

C. Ava runs $\frac{11}{20}$ and Adam runs $\frac{5}{12}$ of the total distance.

D. Ava runs $\frac{9}{20}$ and Adam runs $\frac{7}{12}$ of the total distance.

E. Ava runs $\frac{11}{20}$ and Adam runs $\frac{7}{12}$ of the total distance.

Back to contents

6C Graphs of quadratic functions

LEARNING INTENTIONS

Students will be able to:

- identify the key features of a parabola
- plot a parabola by creating and using a table of values
- determine values using a parabola.

Parabolas are the visual representations of quadratic functions. The key features of a parabola include an axis of symmetry, a vertex or turning point, a y-intercept, and any x-intercepts. All parabolas have the same basic shape, although each individual parabola's key features define its equation and make it different from the others.

KEY TERMS AND DEFINITIONS

- A **parabola** is the graph of a quadratic function with the general equation $y = ax^2 + bx + c$ where a, b, c are known values and $a \neq 0$.
- The **vertex** or turning point of a parabola is the point at which the axis of symmetry intersects with the parabola.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

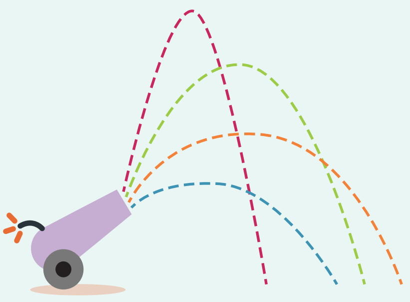
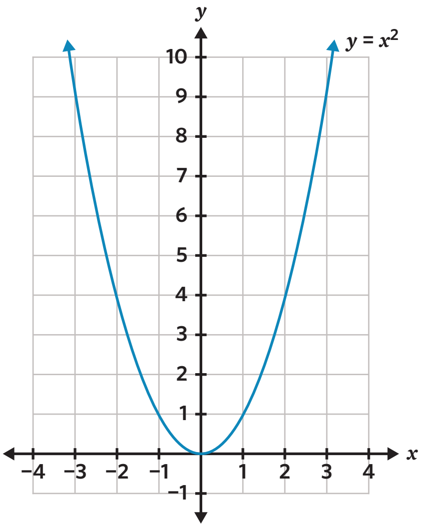


Image: Oleksandr Panasovskyi/Shutterstock.com

Any thrown object follows a flight path called a trajectory. Trajectories are often modelled by quadratic functions as parabolas can accurately represent the relationship between time and height. A typical trajectory is symmetrical about a maximum, which represents the highest point of the thrown object.

Key ideas

1. The basic parabola is given by the rule $y = x^2$.



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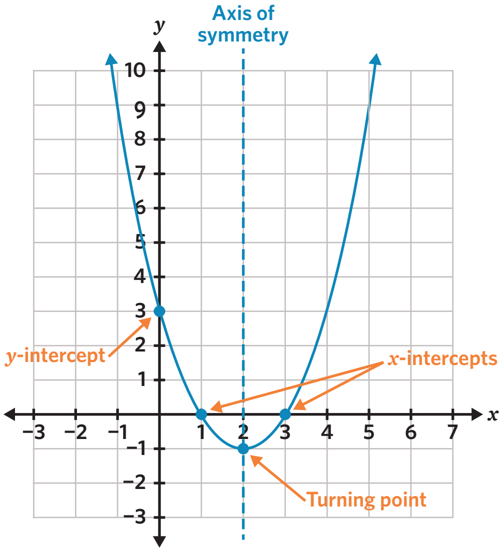
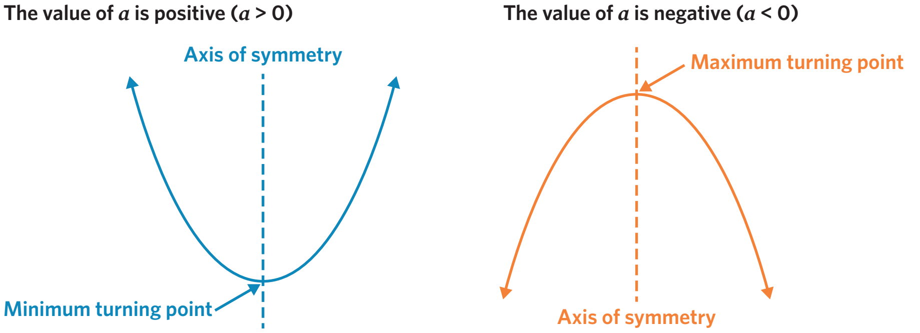
2. The **key features** in parabolas are an **axis of symmetry**, one **y-intercept**, and one **turning point** (or vertex), which may be a maximum or a minimum. If the parabola makes contact with the x-axis, it also has one or two **x-intercept(s)**.



3. For a function with the general equation $y = ax^2 + bx + c$, the parabola is concave up with a **minimum** turning point if $a > 0$ and concave down with a **maximum** turning point if $a < 0$.



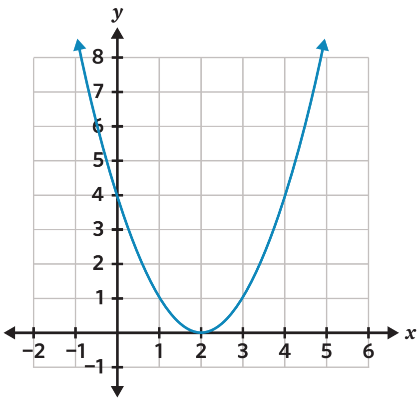
Worked example 1

Identifying features of a parabola

Use the graphs to determine the equation of the axis of symmetry, coordinates and type of turning point, and any x- and y-intercepts of the parabolas.

a.

WE1a



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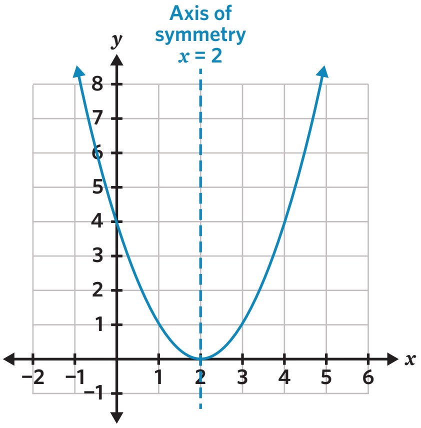
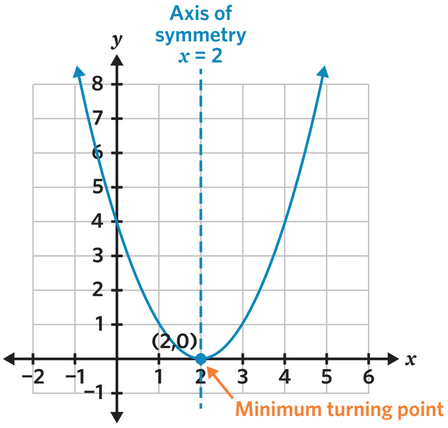
Back to contents

Working

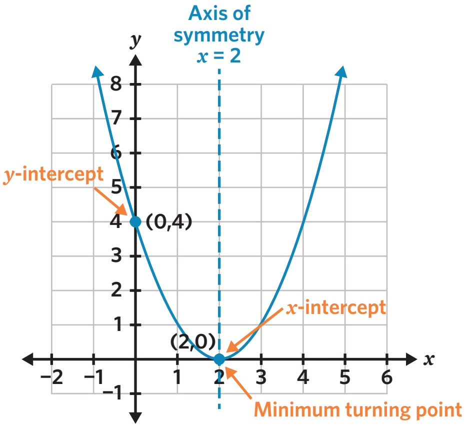
Thinking



Step 1: Identify the vertical line about which the graph is symmetrical and determine its equation.



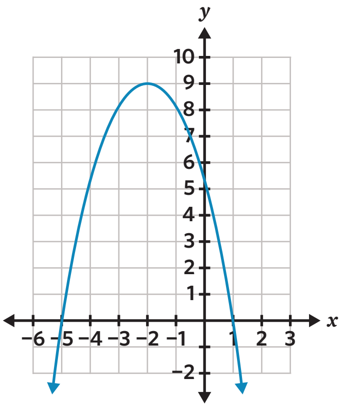
Step 2: Identify the turning point by the point where the axis of symmetry intersects with the parabola and determine its coordinates. Decide if the turning point is a minimum or maximum by identifying whether it is the lowest or highest point on the parabola, respectively.



Step 3: Identify the x- and y-intercepts by the points where the parabola makes contact with the x- and y-axes, respectively. Determine their coordinates.

b.

WE1b



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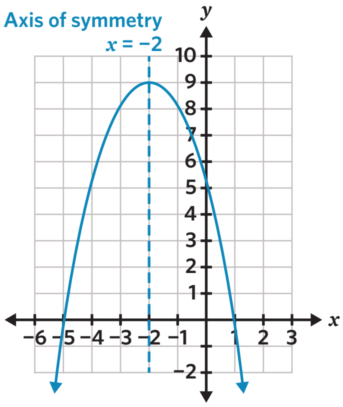
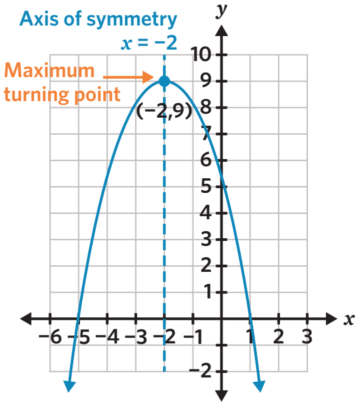
Working

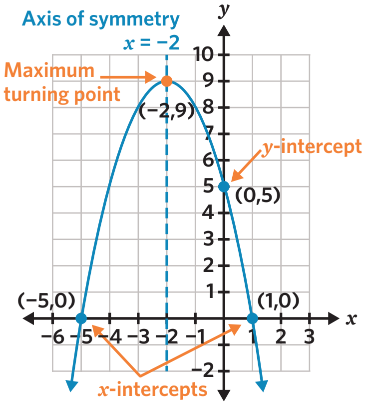
Thinking



Step 1: Identify the vertical line about which the graph is symmetrical and determine its equation.



Step 2: Identify the point where the axis of symmetry intersects with the parabola and determine its coordinates. Decide if the turning point is a minimum or maximum by identifying whether it is the lowest or highest point on the parabola, respectively.

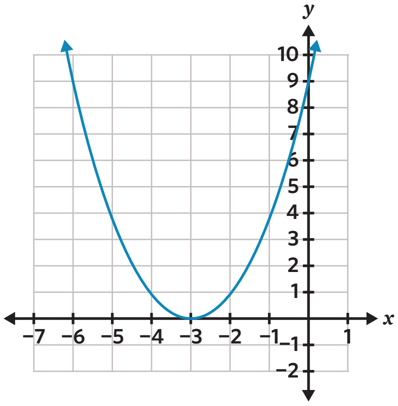


Step 3: Identify the x- and y-intercepts by the points where the parabola makes contact with the x- and y-axes, respectively. Determine their coordinates.

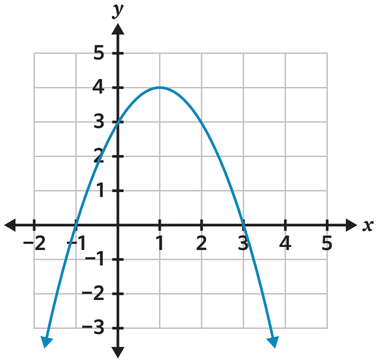
Student practice

Use the graphs to determine the equation of the axis of symmetry, coordinates and type of turning point, and any x- and y-intercepts of the parabolas.

a.



b.



Worked example 2

Plotting a quadratic function

Plot the graphs of the quadratic functions and identify their key features.

a. $y = x^2 - 9$ for $-3 \leq x \leq 3$

WE2a

Working

$(-3)^2 - 9 = 9 - 9 = 0$

$(-2)^2 - 9 = 4 - 9 = -5$

$(-1)^2 - 9 = 1 - 9 = -8$

$(0)^2 - 9 = 0 - 9 = -9$

$(1)^2 - 9 = 1 - 9 = -8$

$(2)^2 - 9 = 4 - 9 = -5$

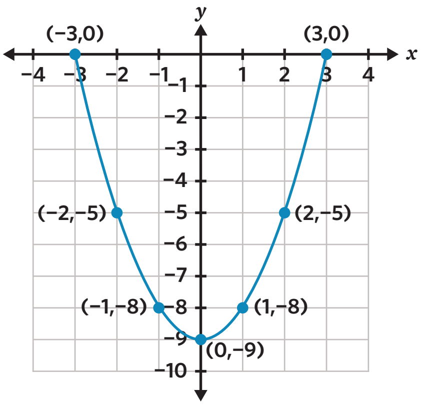
$(3)^2 - 9 = 9 - 9 = 0$

Thinking

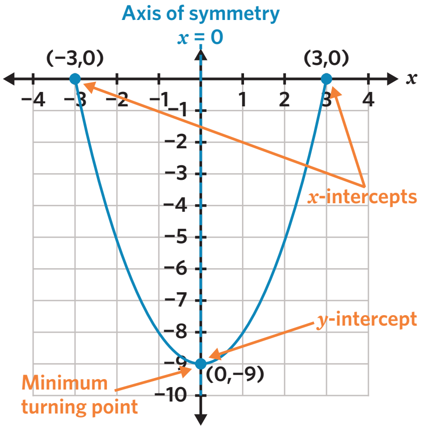
Step 1: Substitute each of the required x-values in the given equation to determine the y-values.

x	y	(x,y)
−3	0	(−3,0)
−2	−5	(−2,−5)
−1	−8	(−1,−8)
0	−9	(0,−9)
1	−8	(1,−8)
2	−5	(2,−5)
3	0	(3,0)

Step 2: Draw a table, including all the required x-values and enter the y-values from Step 1. Determine the coordinates of each point.



Step 3: Plot the coordinates (x,y) from the table on a Cartesian plane and join them with a smooth curve.



Step 4: Identify the axis of symmetry equation, coordinates and type of turning point, and any x- and y-intercepts of the parabola.

Continues →

b. $y = x^2 + 2x - 3$ for $-4 \leq x \leq 2$

WE2b

Working

$(-4)^2 + 2(-4) - 3 = 16 - 8 - 3 = 5$

$(-3)^2 + 2(-3) - 3 = 9 - 6 - 3 = 0$

$(-2)^2 + 2(-2) - 3 = 4 - 4 - 3 = -3$

$(-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$

$(0)^2 + 2(0) - 3 = 0 + 0 - 3 = -3$

$(1)^2 + 2(1) - 3 = 1 + 2 - 3 = 0$

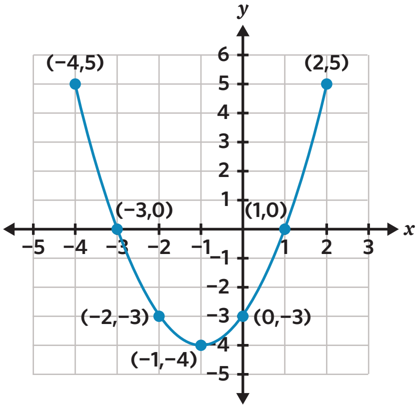
$(2)^2 + 2(2) - 3 = 4 + 4 - 3 = 5$

Thinking

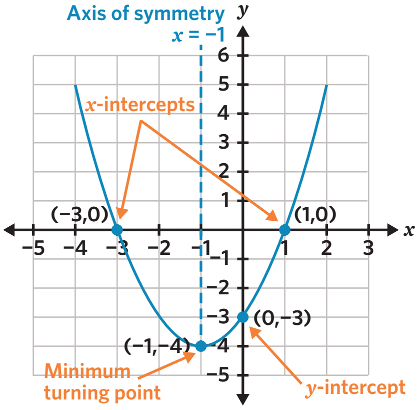
Step 1: Substitute each of the required x-values in the given equation to determine the y-values.

x	y	(x,y)
-4	5	$(-4,5)$
-3	0	$(-3,0)$
-2	-3	$(-2,-3)$
-1	-4	$(-1,-4)$
0	-3	$(0,-3)$
1	0	$(1,0)$
2	5	$(2,5)$

Step 2: Draw a table, including all the required x-values and enter the y-values from Step 1. Determine the coordinates of each point.



Step 3: Plot the coordinates (x,y) from the table on a Cartesian plane and join them with a smooth curve.



Step 4: Identify the axis of symmetry equation, coordinates and type of turning point, and any x- and y-intercepts of the parabola.

Student practice

Plot the graphs of the quadratic functions and identify their key features.

a. $y = x^2 - 4$ for $-3 \leq x \leq 3$

b. $y = x^2 + 2x - 8$ for $-4 \leq x \leq 2$

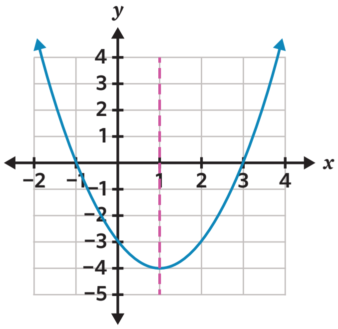
Back to contents

6C Questions

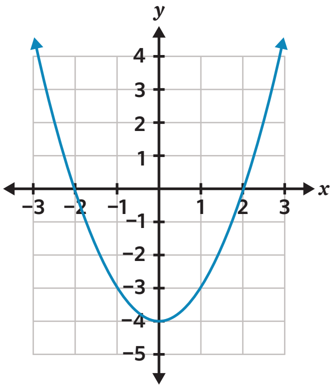
Understanding worksheet

1. Draw the axes of symmetry on the parabolas.

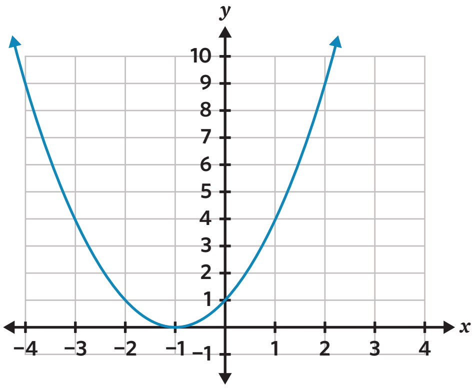
Example



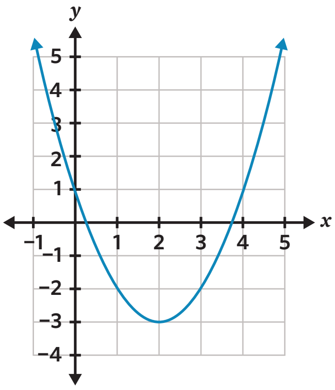
a.



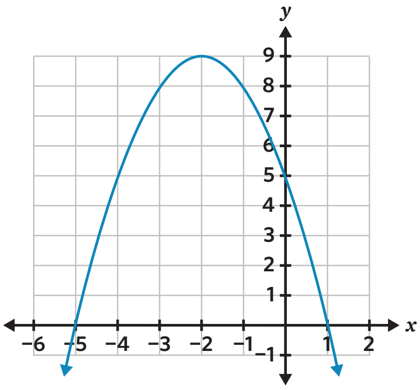
b.



c.



d.



2. Fill in the blanks in the tables of values using the given equations.

Example

$y = x^2 - 1$

x	-2	-1	0	1	2
y	3	0	-1	0	3

a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	

b. $y = x^2 + 4$

x	-2	-1	0	1	2
y	8	5		5	8

c. $y = x^2 - 2x$

x	-1	0	1	2	3
y	3	0		0	3

d. $y = x^2 + x - 2$

x	-3	-2	-1	0	1	2
y	4		-2	-2	0	4

3. Fill in the blanks by using the words provided.

two	parabola	maximum	symmetry	minimum

Quadratic functions of the form $y = ax^2 + bx + c$ where a, b, c are known values, producing a ________ when graphed. Each parabola has a unique set of key features, including an axis of ________, a turning point, a y-intercept, and it may also have none, one or ________ x-intercept(s). A turning point represents a ________ y-value if the parabola is concave up while a concave down parabola contains a ________ y-value at the turning point.

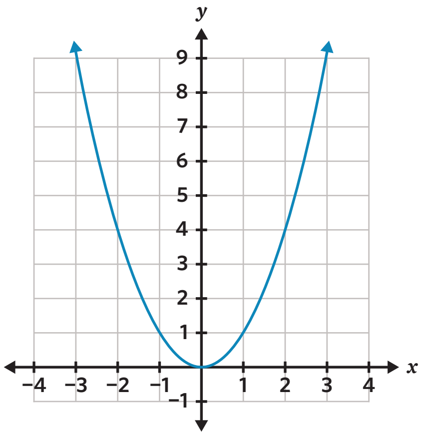
Fluency

Question working paths

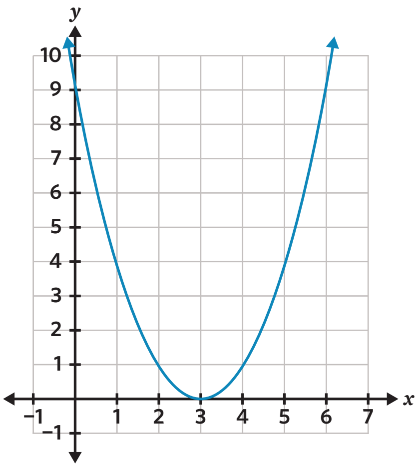
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Use the graphs to determine the equation of the axis of symmetry, coordinates and type of turning point, and any x- and y-intercepts of the parabolas. WE1a

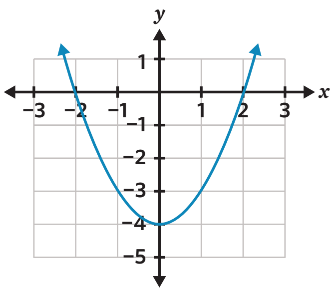
a.



b.



c.



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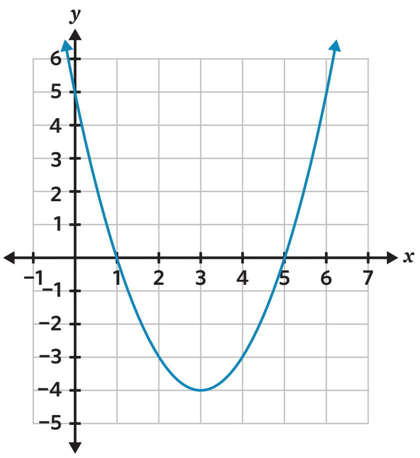
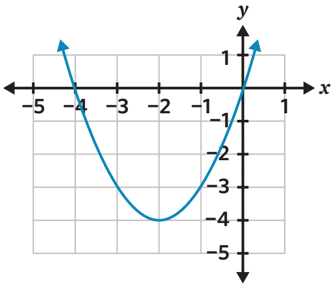


Back to contents

e.



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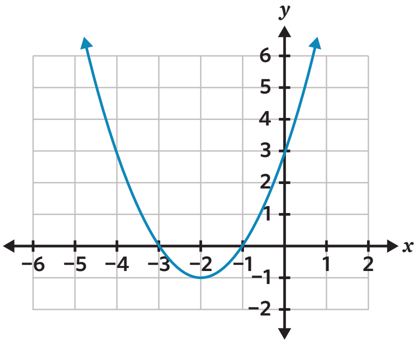

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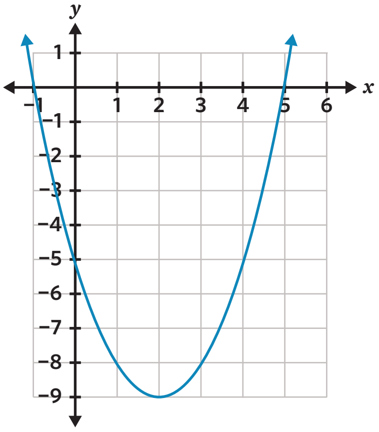

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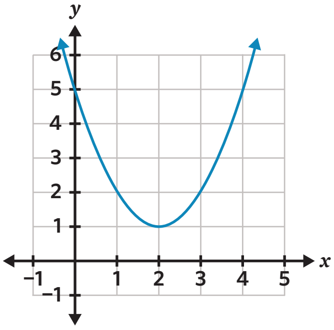

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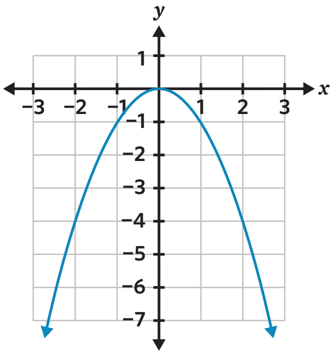
5. Use the graphs to determine the equation of the axis of symmetry, coordinates and type of turning point, and any x- and y-intercepts of the parabolas.

WE1b

a.



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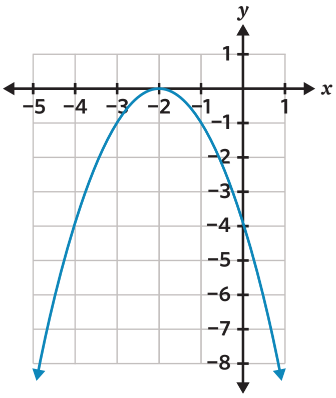

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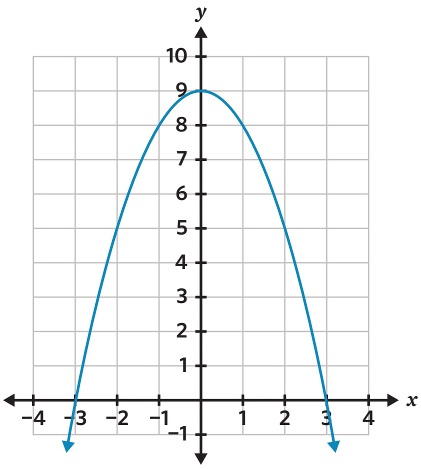
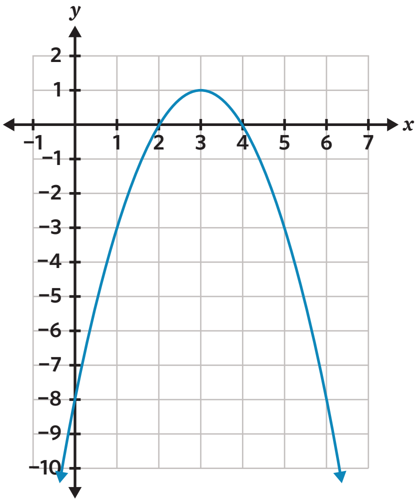
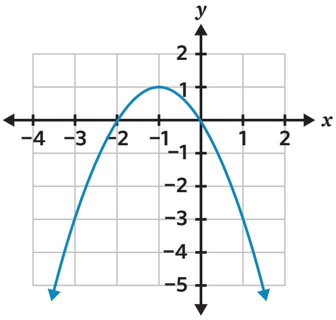


d.



Back to contents

e.



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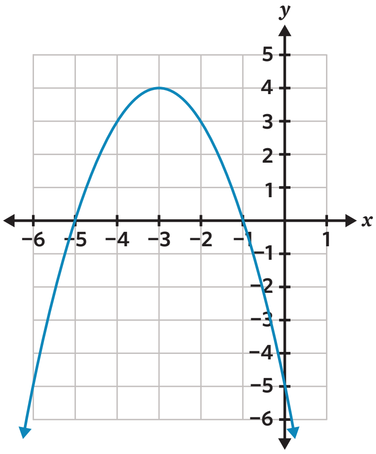

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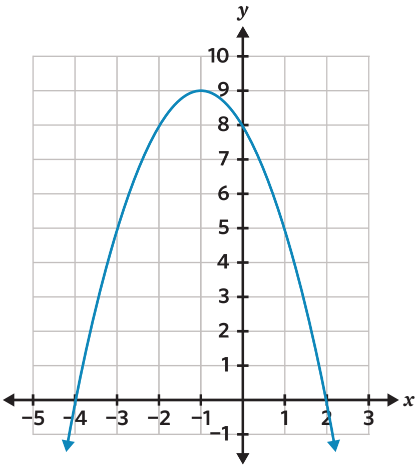
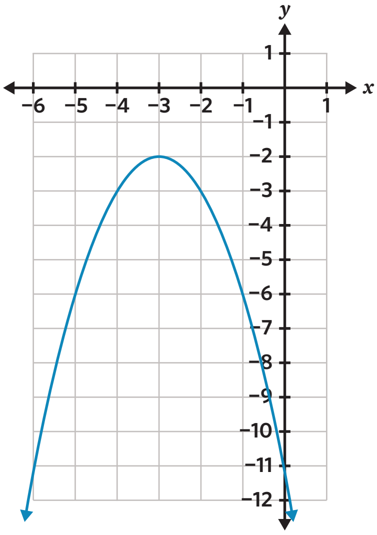


h.



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6. Plot the graphs of the quadratic functions and identify their key features.

WE2

a. $y = x^2$ for $-3 \le x \le 3$

b. $y = x^2 - 1$ for $-3 \le x \le 3$

c. $y = x^2 + 2x$ for $-3 \le x \le 1$

d. $y = x^2 - 4x$ for $-1 \le x \le 5$

e. $y = x^2 - 2x - 3$ for $-2 \le x \le 4$

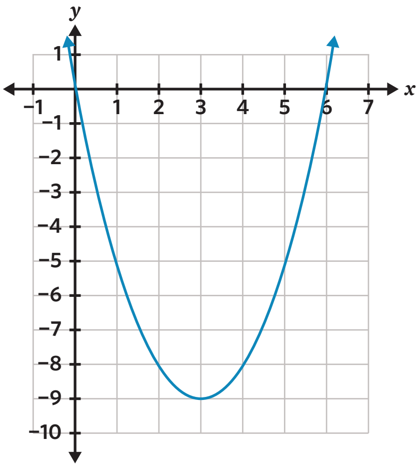
f. $y = x^2 + 4x + 4$ for $-5 \le x \le 1$

g. $y = -x^2 + 2x + 8$ for $-2 \le x \le 4$

h. $y = -x^2 + 4x - 5$ for $-1 \le x \le 5$

---

7. What are the coordinates of the x-intercept(s) in the given parabola?



A. (0,0)

B. (6,0)

C. (0,0), (3,0)

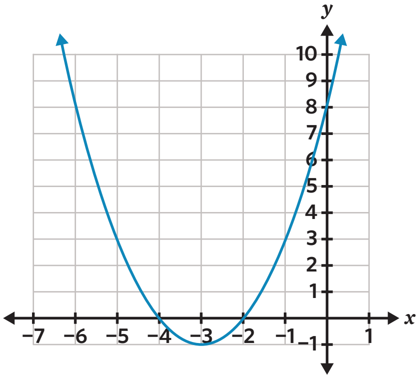
D. (0,0), (6,0)

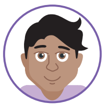
E. (6,0), (3,–9)

Spot the mistake

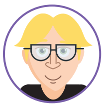
8. Select whether Student A or Student B is incorrect.

a. Determine the equation of the axis of symmetry of the given parabola.

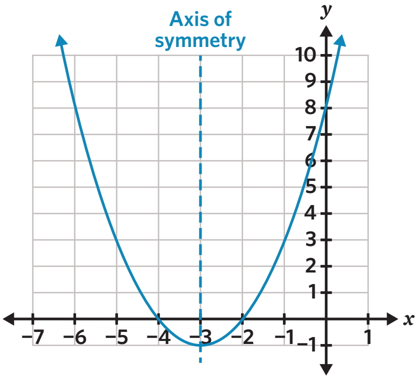




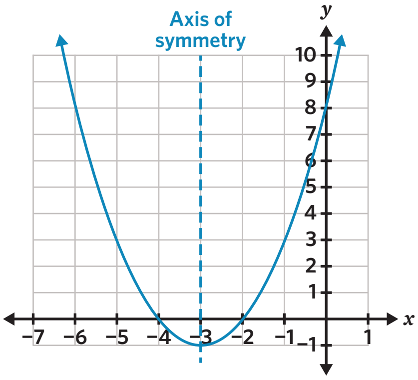
Student A



Student B

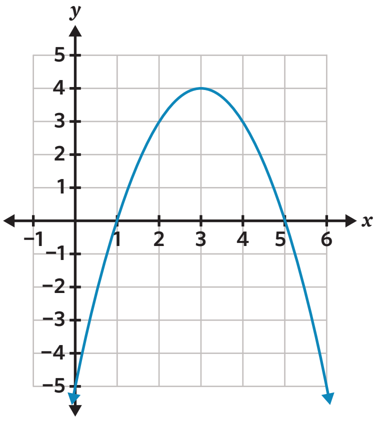


Axis of symmetry $= -3$

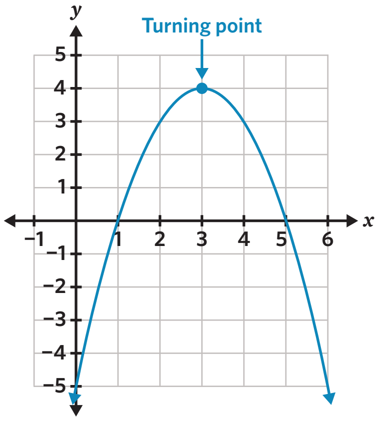


Axis of symmetry: $x = -3$

b. Determine the turning point of the given parabola.

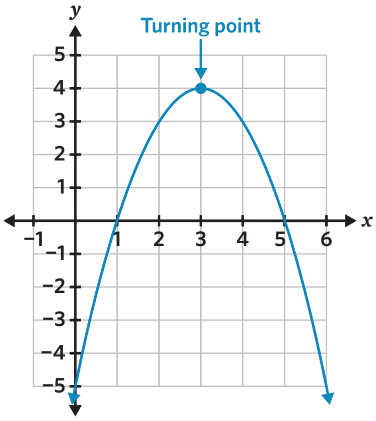


Student A



Maximum turning point (3,4)

Student B



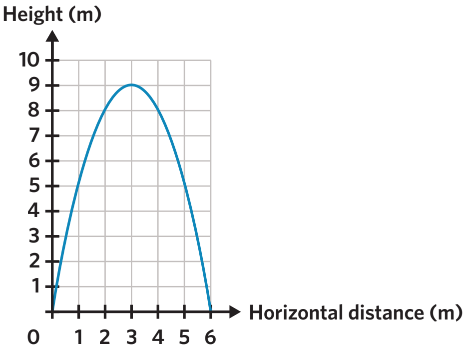
Maximum turning point: $x = 3$

Problem solving

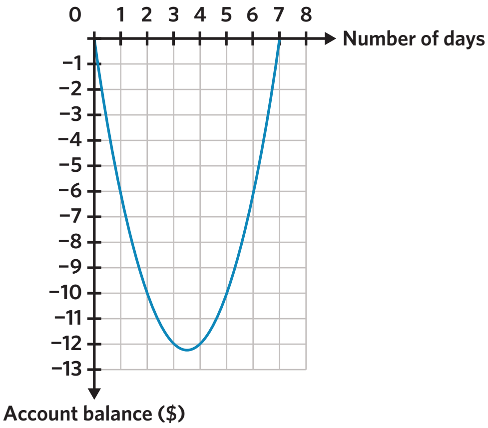
Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. A ball was thrown into the air and followed the path modelled by the given parabola. The horizontal and vertical axes respectively represent the ball's horizontal distance and height in metres, relative to its starting point at (0,0). Determine the ball's maximum height and its horizontal distance from the starting point when it occurred.



10. The balance of an account over a week can be modelled by the given parabola. Assuming that day number one was a Monday, on which days of the week was the account balance equal to −$10?



Back to contents

11. Jolene aims and throws a ball of paper in the rubbish bin from her desk in the classroom. The trajectory of the ball is given by the equation $y = -x^2 + 2x$ where x is the horizontal distance of the ball from Jolene's desk and y is the ball's height relative to the point from which it was thrown, both in metres. Plot the graph of the given quadratic function for $0 \leq x \leq 2$ and determine the horizontal distance from Jolene's desk to the bin.

12. Sally is building an enclosure for her rabbits. The possible area of the enclosure can be modelled by the equation $a = -m^2 + 6m$, where m is the length of one side in metres. Plot the quadratic function for $0 \leq m \leq 6$ and determine the maximum possible area of the rabbit enclosure.

13. Over three weeks, the average weekly temperature of a town could be modelled by the equation $t = w^2 - 2w - 3$ for $0 \leq w \leq 3$, where t is temperature in degrees Celsius and w is the number of weeks. Plot the graph of the function for the given number of weeks to determine the minimum average temperature over the three weeks.

Reasoning

Question working paths

Mild 14 (a,b,d)	**Medium** 14 (a,b,d), 15 (a,b)	**Spicy** All

14. On school sports day, Jackie participated in high jump. His height during the jump can be modelled by the equation $y = -x^2 + 3x$ for $0 \leq x \leq 3$ where y is Jackie's height from the ground in metres and x is the time in seconds.

a. Complete a table of values using the given equation and time interval.

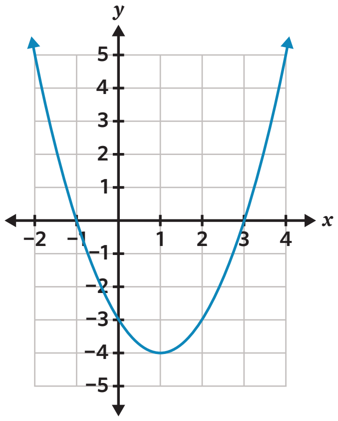
b. Plot the quadratic function and identify the x- and y-intercepts.

c. Jackie beat the school's high jump record by 0.32 m. What was the school's previous high jump record?

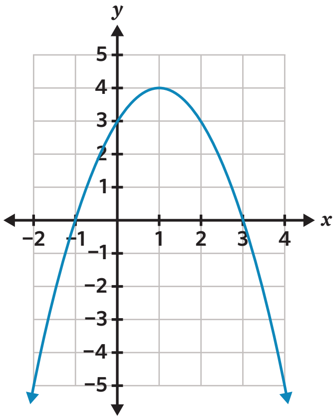
d. Identify another type of school event during which students can engage in activities outside of the classroom.

15. Identify the key features of the parabolas in parts **a** and **b**.

a.



b.



c. Identify the difference between the key features of the two parabolas. How does this difference affect the appearance of the parabola?

Exam-style

16. What is the y-coordinate of the point where $x = -1$ on the parabola given by $y = x^2 + 2x - 15$? (1 MARK)

A. $y = -18$ **B.** $y = -16$ **C.** $y = -15$ **D.** $y = -12$ **E.** $y = 16$

17. Consider the given quadratic equation. (3 MARKS)

$y = x^2 + 2x + 3$

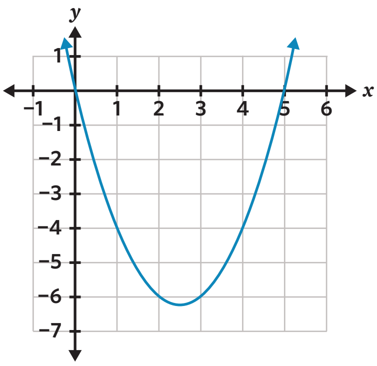
a. Fill in a table of values for $-3 \leq x \leq 1$. 1 MARK

b. Plot the parabola. 2 MARKS

18. Fill in the given table of values for $y = x^2 + 2x + 1$ and plot the parabola. (3 MARKS)

x	-3	-2	-1	0	1
y					

19. The graph of $y = x^2 - 5x$ is given. Determine the coordinates of its minimum turning point. (3 MARKS)



Remember this?

20. In a recent drought, a local reservoir lost $\frac{7}{8}$ of its water capacity. What is $\frac{7}{8}$ as a percentage?

A. 75% **B.** 77.5% **C.** 80% **D.** 87.5% **E.** 90%

21. John has a digital music player. Each song plays for 4 minutes. John has 60 songs on his playlist. How many hours will it take to play all the songs?

A. 3 hours **B.** 4 hours **C.** 4.5 hours **D.** 6 hours **E.** 6.5 hours

22. This table shows some information about the five largest planets in the fictional Zeta Quadrant.

Planet	**Diameter (km)**
Zeta Prime	139 820
Gigantus	116 460
Titanus	50 724
Neptor	49 244
Terranis	12 742

How much larger in diameter is Zeta Prime than Terranis?

A. 120 000 km **B.** 127 078 km **C.** 130 000 km **D.** 140 000 km **E.** 150 000 km

Back to contents

6D Sketching parabolas with dilations and reflections

LEARNING INTENTIONS

Students will be able to:

- sketch parabolas of the form $y = ax^2$
- describe the features of parabolas of the form $y = ax^2$
- describe how the value of a affects parabolas of the form $y = ax^2$.

All parabolas with an equation of the form $y = ax^2$ share common key features, such as an axis of symmetry, turning point, and x and y-intercepts. The value of a defines the proportions of the parabola as compared to the basic $y = x^2$ and may indicate a dilation away or towards the y-axis, a reflection in the x-axis, or both.

KEY TERMS AND DEFINITIONS

- An **inverted parabola** (concave down) with an equation of the form $y = ax^2$ is produced when $a < 0$.
- An **upright parabola** (concave up) with an equation of the form $y = ax^2$ is produced when $a > 0$.

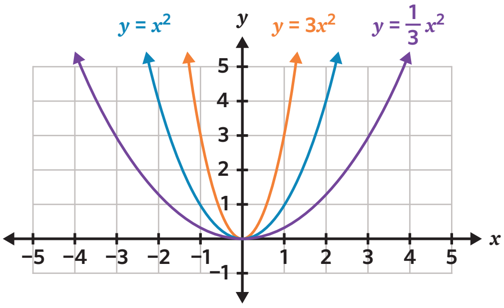
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Aviavlad/Shutterstock.com

The parabolic shape of the reflector inside car headlights allows for the light beams to be projected using a single light source, such as a small light bulb. Projecting the light beams symmetrically produces high beam lights, useful for driving in the countryside. When the light beams are projected at an angle, pointing towards the ground, low beam lights are produced. Low beam lights are required for driving in populated areas, where the driver is likely to encounter another motorist.

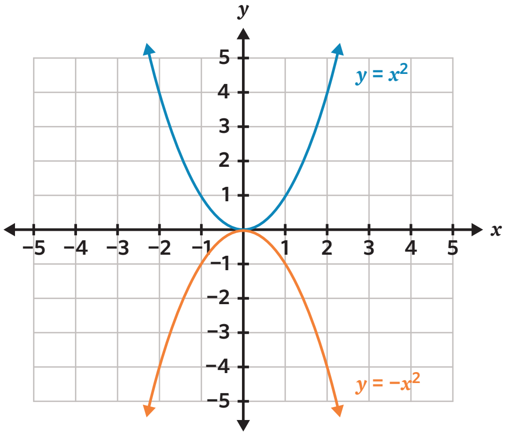
Key ideas

1. For the equation $y = ax^2$, the parabola **dilates away** from the y-axis and is wider than the basic parabola $y = x^2$ when $0 < a < 1$. It **dilates towards** the y-axis and is narrower than the basic parabola $y = x^2$ when $a > 1$.

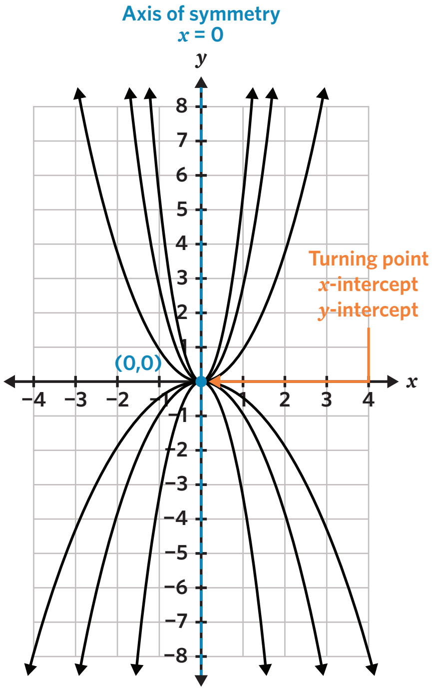


Continues →

2. The graph of a parabola with an equation of the form $y = -ax^2$ is a **reflection** in the x-axis of a parabola with an equation of the form $y = ax^2$.



3. All parabolas with equations of the form $y = ax^2$ share the same axis of symmetry at $x = 0$, turning point coordinates (0,0), and x and y-intercept (0,0).



Worked example 1

Comparing graphs of $y = ax^2$ when $a > 0$

WE1

Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of a affects each parabola.

$y = x^2$ and $y = 2x^2$

Working

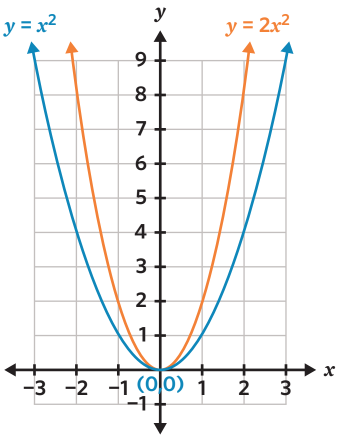
x	−2	−1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 2x^2$	8	2	0	2	8

Thinking

Step 1: Complete a table of values for $-2 \leq x \leq 2$ for both equations.

Continues →

Back to contents



Step 2: Plot the sets of coordinates given by the table of values on the same set of axes. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.

$y = 2x^2$ is narrower than $y = x^2$.

Step 3: Comment on the widths of the parabolas as compared to each other.

Student practice

Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of *a* affects each parabola.

$y = x^2$ and $y = 3x^2$

Worked example 2

Comparing graphs of $y = ax^2$ when $a < 0$

WE2

Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of *a* affects the parabolas.

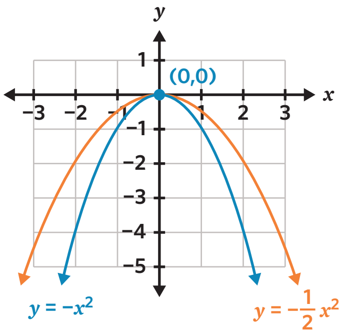
$y = -\frac{1}{2}x^2$ and $y = -x^2$

Working

Thinking

x	-2	-1	0	1	2
$y = -\frac{1}{2}x^2$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2
$y = -x^2$	-4	-1	0	-1	-4

Step 1: Complete a table of values for each quadratic equation for $-2 \le x \le 2$.



Step 2: Plot the sets of coordinates given by the table of values on the same set of axes. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.

$y = -\frac{1}{2}x^2$ is wider than $y = -x^2$

Step 3: Comment on the widths of the parabolas as compared to each other.

Continues →

Back to contents

Student practice

Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of a affects each parabola.

$y = -\frac{1}{4}x^2$ and $y = -x^2$

Worked example 3

Comparing graphs of $y = ax^2$ when $a \neq 0$

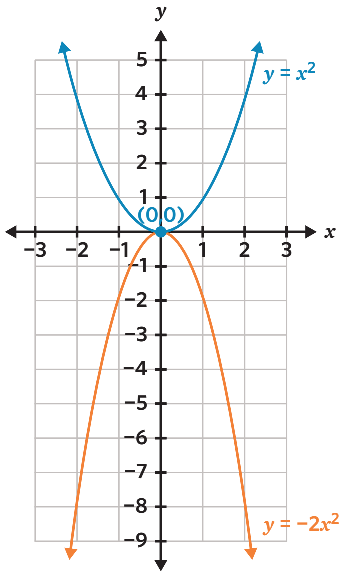
WE3

Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of a affects the parabolas.

$y = -2x^2$ and $y = x^2$

Working

x	-2	-1	0	1	2
$y = -2x^2$	-8	-2	0	-2	-8
$y = x^2$	4	1	0	1	4



$y = -2x^2$ is inverted and is narrower than $y = x^2$.

Thinking

Step 1: Complete a table of values for each quadratic equation for $-2 \leq x \leq 2$.

Step 2: Plot the sets of coordinates given by the table of values on the same set of axes. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.

Step 3: Comment on the widths and orientations of the parabolas as compared to each other.

Student practice

Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of a affects each parabola.

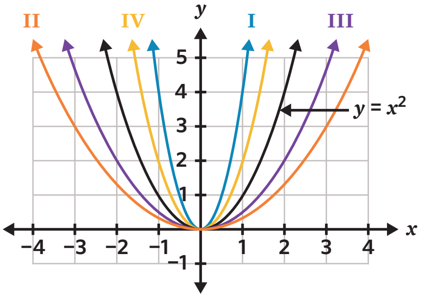
$y = -3x^2$ and $y = x^2$

Back to contents

6D Questions

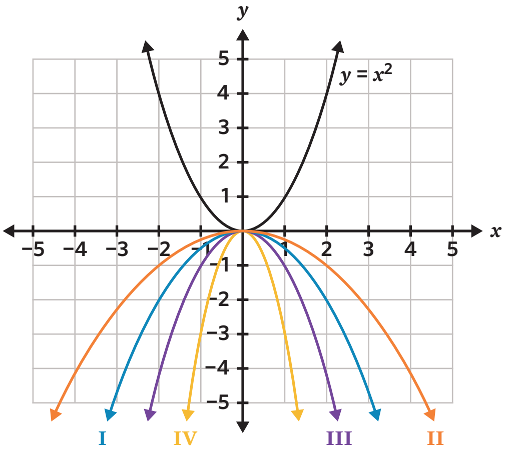
Understanding worksheet

1. Match the parabolas to the given equations.



a. $y = \frac{1}{3}x^2$ **b.** $y = \frac{1}{2}x^2$ **c.** $y = 2x^2$ **d.** $y = 4x^2$

2. Match the parabolas to the given equations.



a. $y = -\frac{1}{4}x^2$ **b.** $y = -\frac{1}{2}x^2$ **c.** $y = -x^2$ **d.** $y = -3x^2$

3. Fill in the blanks by using the words provided.

wider	basic	turning	narrower

Parabolas with equations of the form $y = ax^2$ all share the same key features, such as a common axis of symmetry at $x = 0$, a [ ] point at (0,0), and a shared x and y-intercept at (0,0). The value of a defines the proportions and orientation of the parabola, as compared to the [ ] parabola $y = x^2$. When $0 < a < 1$, the parabola is [ ] than $y = x^2$ and [ ] when $a > 1$, while a negative a inverts it.

Back to contents

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Plot the parabolas of the form $y = ax^2$ on the same set of axes and identify their key features. Determine how the value of a affects each parabola. **WE1**

a. $y = x^2$ and $y = 4x^2$

b. $y = \frac{1}{2}x^2$ and $y = x^2$

c. $y = \frac{1}{2}x^2, y = x^2$ and $y = 2x^2$

d. $y = x^2, y = 2x^2$ and $y = 3x^2$

e. $y = \frac{1}{4}x^2, y = x^2$ and $y = 4x^2$

f. $y = \frac{1}{4}x^2, y = \frac{1}{2}x^2$ and $y = x^2$

g. $y = \frac{1}{4}x^2, y = x^2$ and $y = 3x^2$

h. $y = \frac{1}{5}x^2, y = x^2$ and $y = 2x^2$

---

5. Plot the parabolas of the form $y = ax^2$ on the same set of axes and identify their key features. Determine how the value of a affects each parabola. **WE2**

a. $y = -2x^2$ and $y = -x^2$

b. $y = -3x^2$ and $y = -x^2$

c. $y = -2x^2, y = -x^2$ and $y = -\frac{1}{2}x^2$

d. $y = -4x^2, y = -2x^2$ and $y = -x^2$

e. $y = -3x^2, y = -x^2$ and $y = -\frac{1}{2}x^2$

f. $y = -2x^2, y = -x^2$ and $y = -\frac{1}{4}x^2$

g. $y = -x^2, y = -\frac{1}{2}x^2$ and $y = -\frac{1}{4}x^2$

h. $y = -3x^2, y = -x^2$ and $y = -\frac{1}{5}x^2$

---

6. Plot the parabolas of the form $y = ax^2$ on the same set of axes and identify their key features. Determine how the value of a affects each parabola. **WE3**

a. $y = -x^2$ and $y = 2x^2$

b. $y = -x^2$ and $y = 3x^2$

c. $y = -\frac{1}{2}x^2$ and $y = x^2$

d. $y = -x^2$ and $y = \frac{1}{4}x^2$

e. $y = -4x^2$ and $y = x^2$

f. $y = -x^2, y = -\frac{1}{2}x^2$ and $y = 3x^2$

g. $y = -2x^2, y = -\frac{1}{4}x^2$ and $y = x^2$

h. $y = -x^2, y = -\frac{1}{2}x^2$ and $y = \frac{1}{4}x^2$

---

7. Which of the given equations results in an upright parabola that is wider than $y = -x^2$?

A. $y = -8x^2$

B. $y = -\frac{1}{8}x^2$

C. $y = \frac{1}{8}x^2$

D. $y = x^2$

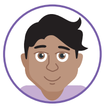
E. $y = 8x^2$

Back to contents

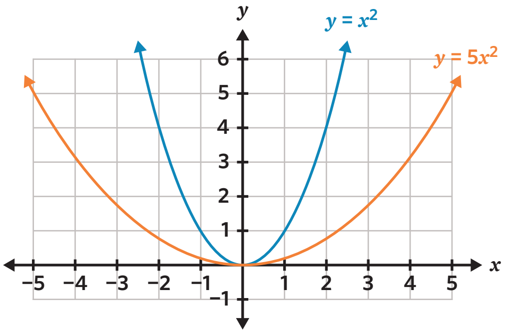
Spot the mistake

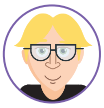
8. Select whether Student A or Student B is incorrect.

a. Sketch the graphs of $y = x^2$ and $y = 5x^2$ on the same set of axes.

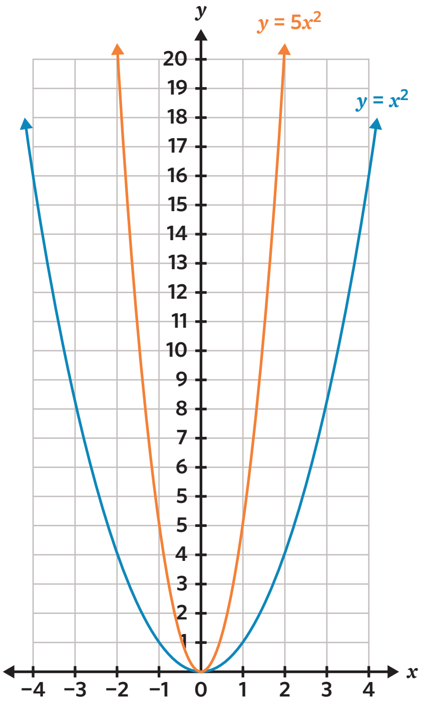


Student A



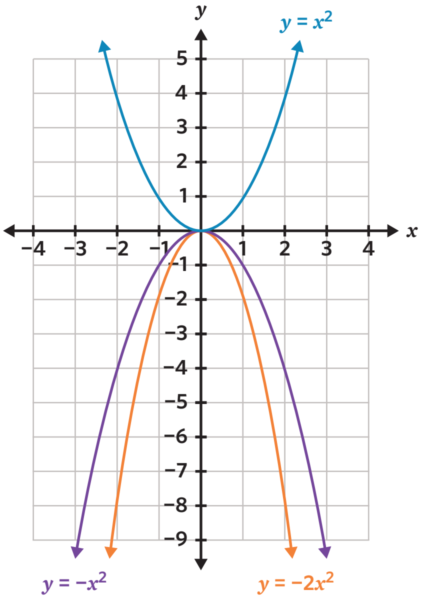


Student B

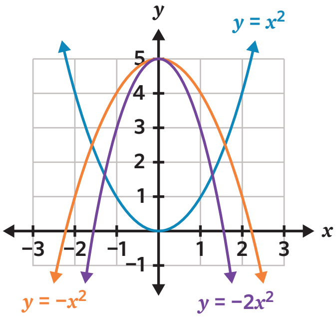


b. Sketch the graphs of $y = x^2$ and $y = -x^2$ and $y = -2x^2$ on the same set of axes.

Student A



Student B



Back to contents

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. The shape of an archway can be modelled by the equation $y = -x^2$ where it is symmetrical about the line $x = 0$. During renovation, the archway was made wider and its equation was scaled by a factor of a third. Determine an equation which could be used to model the shape of the archway after renovation.

10. A water bird dives into the sea to catch a fish. The bird's path can be modelled by the equation $y = ax^2$ where it is symmetrical about the point at which the bird reaches its greatest depth and starts coming back up again immediately after. Determine the value of a if the point $(1,3)$ lies on the graph.

11. The shape of a bridge over a river can be modelled by the equation $y = -\frac{1}{4}x^2$. Determine an equation that could be used to model the reflection of the bridge in the river and sketch the parabola.

12. The shape of the cross-section of a small bowl in a dinner set is given by $y = \frac{1}{2}x^2$ where the axis of symmetry splits the bowl in two identical halves. In the same set, a large bowl has the same shape, but the equation modelling it has half the scale factor of the small bowl's. Determine an equation that could be used to describe the shape of a large bowl and sketch both equations on the same set of axes.

13. Three different parts of a roller coaster can be modelled by the quadratic equations $y = \frac{1}{3}x^2$, $y = -x^2$, and $y = 2x^2$. Sketch all three parabolas on the same set of axes and determine which of the three equations modelling parts of the roller coaster describe the steepest incline.

Reasoning

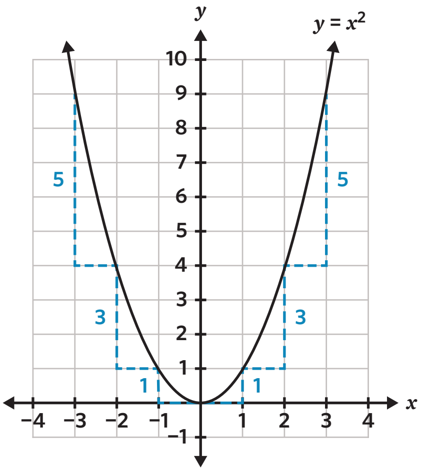
Question working paths

Mild 14 (a,b,c,e) 🌶	**Medium** 14 (a,b,c,e), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. Giovanni is using his 3D printer to sketch patterns on a piece of metal. He decides to use a basic parabola with the equation $y = x^2$ as the basis for all his designs.

- **a.** Determine the key features of Giovanni's basic parabola so he can enter the values accurately when prompted by his printer.
- **b.** Giovanni decides to narrow the shape of his basic parabola by using a factor of three. What is the equation of this new parabola?
- **c.** Giovanni decides to widen the shape of the parabola from part **b** by using a factor of two. What is the equation of this new parabola?
- **d.** Giovanni wants to reflect the parabola from part **c** in the x-axis. Determine the equation of this parabola and sketch it on the same axes as the basic parabola $y = x^2$.
- **e.** Identify an advantage or disadvantage of using a machine to create art.

15. Consider the changes in y-values with each change in x-values shown on the basic parabola $y = x^2$.



a. Plot the parabola $y = 2x^2$ and show the change in y-values with change in x-values on the graph for $-3 \leq x \leq 3$.

b. Plot the parabola $y = \frac{1}{2}x^2$ and show the change in y-values with change in x-values on the graph for $-3 \leq x \leq 3$.

c. Compare the numbers shown on each graph and identify a pattern in the change in y-values with change in x-values on either side of the y-axis for a parabola of the form $y = ax^2$.

Exam-style

16. A parabola has an equation of the form $y = ax^2$. If the point (1,5) lies on the graph, the equation of the parabola is (1 MARK)

A. $y = -5x^2$

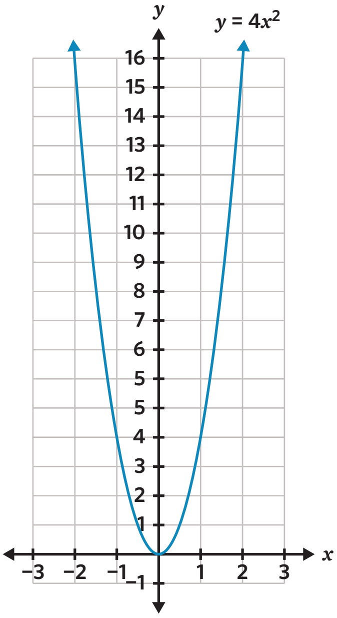
B. $y = -\frac{1}{5}x^2$

C. $y = x^2$

D. $y = \frac{1}{5}x^2$

E. $y = 5x^2$

17. Consider the given parabola. (3 MARKS)

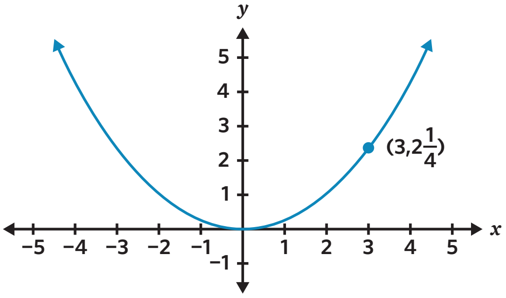


a. Sketch the reflection of the graph in the x-axis. 2 MARKS

b. Determine the equation of a parabola which is twice as wide as the reflection of $y = 4x^2$ in the x-axis. 1 MARK

Back to contents

18. The graph of a parabola of the form $y = ax^2$ is shown. Determine the value of a. (3 MARKS)



19. Sketch the graph of $y = -2x^2$. Label the x and y-intercepts with their coordinate(s). (3 MARKS)

Remember this?

20. Molly subscribes to a streaming service that has a monthly fee of \$15. Subscribers pay an extra \$2 per movie if they want to rent a new release. The expression that represents Molly's monthly streaming bill if she rents y new releases during the month is

A. $2y$ **B.** $15y$ **C.** $15y + 2$ **D.** $15 + 2y$ **E.** $2 + 15y$

21. The battery level on Sam's electric scooter showed that he had used two-thirds of the battery life. He then spent \$36 charging it until full at a cost of \$0.90 per kilowatt-hour (kWh). Approximately how much energy in kWH can Sam's scooter battery hold when it is full?

A. 30 kWh **B.** 40 kWh **C.** 60 kWh **D.** 90 kWh **E.** 120 kWh

22. Sally makes and sells handmade candles. As the number of candles she makes increases, the cost to make each candle decreases.

Number of candles	Cost to make each candle	Total cost
10	\$3.50	\$35.00
20	\$3.20	\$64.00
30	\$2.90	\$87.00

Using the pattern, determine the total cost of making 50 candles.

A. \$87.00 **B.** \$104.00 **C.** \$115.00 **D.** \$120.00 **E.** \$145.00

Back to contents

6E Sketching translations of parabolas

LEARNING INTENTIONS

Students will be able to:

- understand the turning point form of a quadratic equation
- identify the turning point from a quadratic equation of the form $y = a(x - h)^2 + k$
- sketch graphs of quadratic equations in turning point form.

Parabolas are often used to model curved surfaces or trajectories of thrown objects. Translating a parabola, together with dilations and reflections on a Cartesian plane allows for more accurate modelling of real-life situations and objects. The turning point form of a quadratic equation $y = a(x - h)^2 + k$ is written using the turning point (h,k) of the parabola.

KEY TERMS AND DEFINITIONS

- A **translation** of a graph requires all points to move the same distance and in the same direction horizontally and/or vertically.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

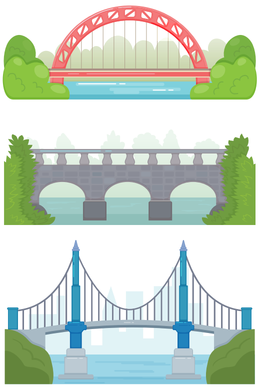
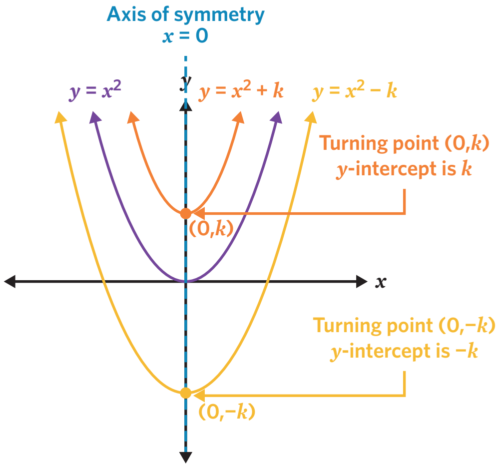


Image: Anny Chaban/Shutterstock.com

Many bridge designs are based on parabolas due to their symmetrical nature. Uniform and symmetrical distribution of weight is pivotal for a successful and safe bridge design, and so parabolas often form the perfect mathematical models. Using cables and special materials for different parts of the bridge allows for a variety of parabola inspired bridge designs seen in many countries around the world.

Key idea 1

1. Quadratic equations of the form $y = ax^2 + k$ show a vertical translation. If $k > 0$, the graph is translated k units up and if $k < 0$, the graph is translated k units down.



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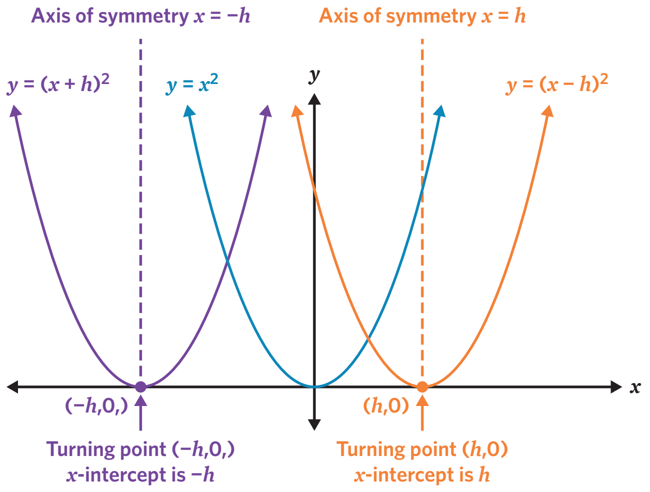
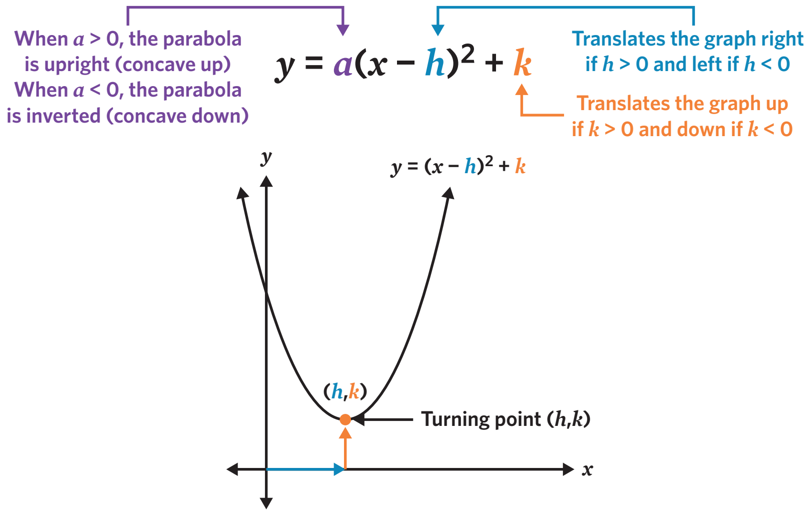
Back to contents

2. Quadratic equations of the form $y = a(x - h)^2$ show a horizontal translation. If $h > 0$, the graph is translated h units to the right and if $h < 0$, the graph is translated h units to the left.



3. The turning point form of a quadratic equation is given by $y = a(x - h)^2 + k$, where (h,k) are the coordinates of the turning point.



Worked example 1

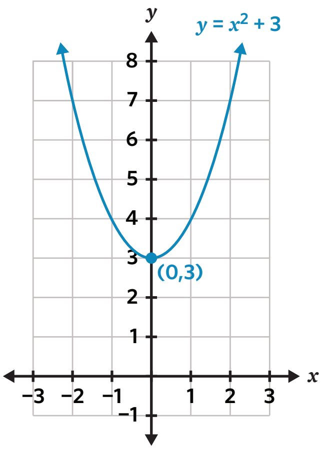
Sketching parabolas with vertical translations

Sketch the parabolas showing the y-intercept and coordinates of the turning point. **WE1a**

a. $y = x^2 + 3$

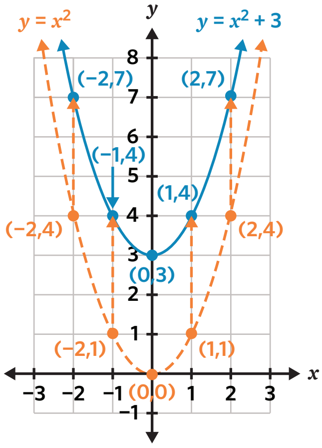
Working	Thinking
For $y = x^2 + 3$ $k = 3$ $\therefore y = x^2$ is translated 3 units up	**Step 1:** In the equation of the form $y = ax^2 + k$, determine the value of k. This shows by how many units the graph of $y = ax^2$ must be translated vertically.
Turning point $(0,3)$	**Step 2:** Determine the coordinates of the turning point $(0,k)$.

Continues →



Step 3: Sketch an upright parabola with an axis of symmetry at $x = 0$ and a turning point at $(0,k)$.

Visual support



WE1b

b. $y = -x^2 - 2$

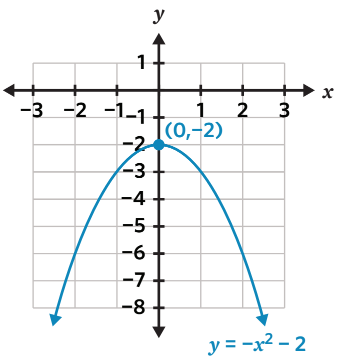
Working

For $y = -x^2 - 2 = -x^2 + (-2)$

$k = -2$

$\therefore$ $y = -x^2$ is translated 2 units down.

Turning point $(0,-2)$



Thinking

Step 1: In the equation of the form $y = ax^2 + k$, determine the value of k. This shows by how many units the graph of $y = ax^2$ must be translated vertically.

Step 2: Determine the coordinates of the turning point $(0,k)$.

Step 3: Sketch an inverted parabola with an axis of symmetry at $x = 0$ and a turning point at $(0,k)$.

Student practice

Sketch the parabolas showing the y-intercept and coordinates of the turning point.

a. $y = x^2 + 1$

b. $y = -x^2 - 3$

Back to contents

Worked example 2

Sketching parabolas with horizontal translations

Sketch the parabolas showing the y-intercept and coordinates of the turning point.

a. $y = (x - 2)^2$

WE2a

Working

For $y = (x - 2)^2$

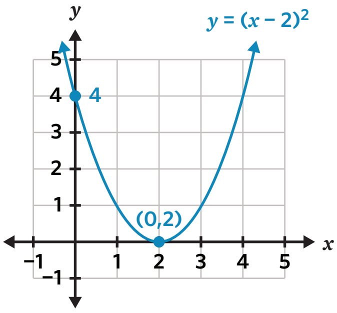
$h = 2$

$\therefore\ y = x^2$ is translated 2 units right

Turning point (2,0)

$y = (0 - 2)^2 = (-2)^2 = 4$

y-intercept is 4



Thinking

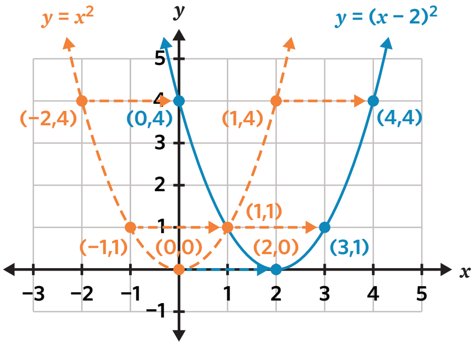
Step 1: In the equation of the form $y = a(x - h)^2$, determine the value of h. This shows by how many units the graph of $y = ax^2$ must be translated horizontally.

Step 2: Determine the coordinates of the turning point $(h,0)$.

Step 3: Substitute $x = 0$ into the equation of the parabola to determine the y-intercept.

Step 4: Sketch an upright parabola with an axis of symmetry at $x = h$, a turning point at $(h,0)$ and a y-intercept from the previous step.

Visual support



b. $y = -(x + 1)^2$

WE2b

Working

For $y = -(x + 1)^2 = -(x - (-1))^2$

$h = -1$

$\therefore\ y = -x^2$ is translated 1 unit left

Turning point (−1,0)

$y = -(0 + 1)^2 = -(1)^2 = -1$

y-intercept is −1

Thinking

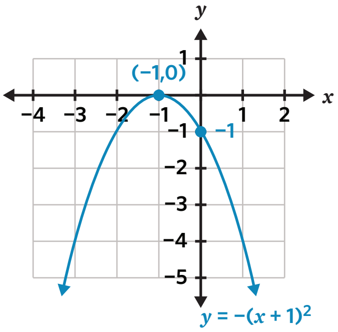
Step 1: In the equation of the form $y = a(x - h)^2$, determine the value of h. This shows by how many units the graph of $y = ax^2$ must be translated horizontally.

Step 2: Determine the coordinates of the turning point $(h,0)$.

Step 3: Substitute $x = 0$ into the equation of the parabola to determine the y-intercept.

Continues →

Back to contents



Step 4: Sketch an inverted parabola with an axis of symmetry at $x = h$, a turning point at $(h,0)$ and a y-intercept from the previous step.

Student practice

Sketch the parabolas showing the y-intercept and coordinates of the turning point.

a. $y = (x - 1)^2$

b. $y = -(x + 2)^2$

Worked example 3

Sketching parabolas with combined translations

Sketch the parabolas showing the y-intercept and coordinates of the turning point.

a. $y = (x - 1)^2 + 2$

WE3a

Working

For $y = (x - 1)^2 + 2$

$h = 1$ and $k = 2$

$\therefore y = x^2$ is translated 1 unit right and 2 units up

Thinking

Step 1: In the equation of the form $y = a(x - h)^2 + k$, determine the values of h and k. This shows by how many units the graph of $y = ax^2$ must be translated horizontally and vertically.

Turning point (1,2)

Step 2: Determine the coordinates of the turning point (h,k).

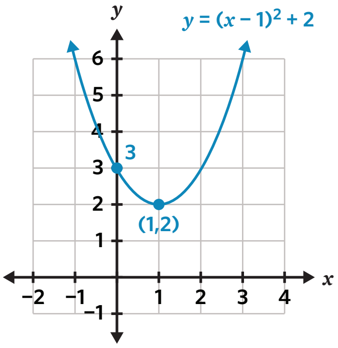
$y = (0 - 1)^2 + 2 = (-1)^2 + 2$

$= 1 + 2$

$= 3$

y-intercept is 3

Step 3: Substitute $x = 0$ into the equation of the parabola to determine the y-intercept.

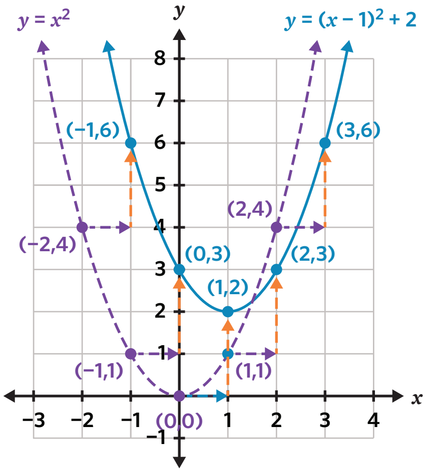


Step 4: Sketch an upright parabola with an axis of symmetry at $x = h$, a turning point at (h,k) and a y-intercept from the previous step.

Continues →

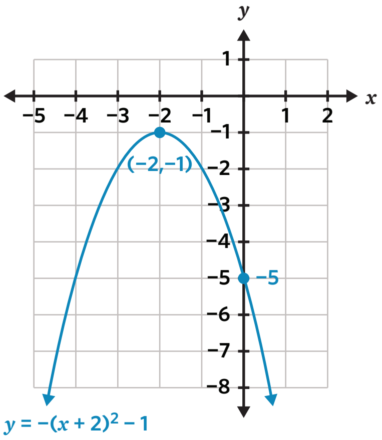
Back to contents

Visual support



b. $y = -(x + 2)^2 - 1$

WE3b

Working	Thinking
For $y = -(x + 2)^2 - 1$ $= -(x - (-2))^2 + (-1)$ $h = -2$ and $k = -1$ ∴ $y = -x^2$ is translated 2 units left and 1 unit down	**Step 1:** In the equation of the form $y = a(x - h)^2 + k$, determine the values of h and k. This shows by how many units the graph of $y = ax^2$ must be translated horizontally and vertically.
Turning point $(-2,-1)$	**Step 2:** Determine the coordinates of the turning point (h,k).
$y = -(0 + 2)^2 - 1 = -(2)^2 - 1$ $= -4 - 1$ $= -5$ y-intercept is -5	**Step 3:** Substitute $x = 0$ into the equation of the parabola to determine the y-intercept.
	**Step 4:** Sketch an inverted parabola with an axis of symmetry at $x = h$, a turning point at (h,k) and a y-intercept from the previous step.

Student practice

Sketch the parabolas showing the y-intercept and coordinates of the turning point.

a. $y = (x - 2)^2 + 1$

b. $y = -(x + 1)^2 - 3$

Back to contents

6E Questions

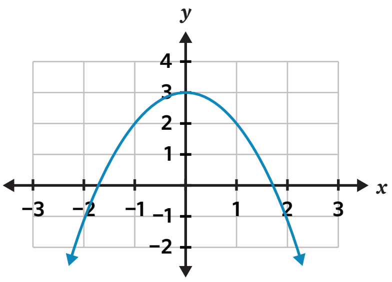
Understanding worksheet

1. Match each quadratic equation to a translation it describes.

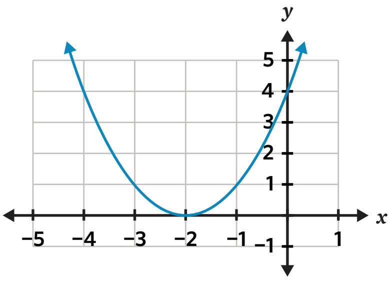
Equation			Translation
$y = x^2 + 6$	●	●	$y = x^2$ translated 6 units right
$y = -x^2 - 6$	●	●	$y = -x^2$ translated 6 units left
$y = (x - 6)^2$	●	●	$y = x^2$ translated 6 units up
$y = -(x + 6)^2$	●	●	$y = -x^2$ translated 6 units down

2. Match each graph to the coordinates of its turning point.

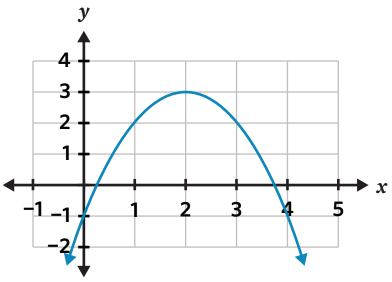
Graph **Turning point**



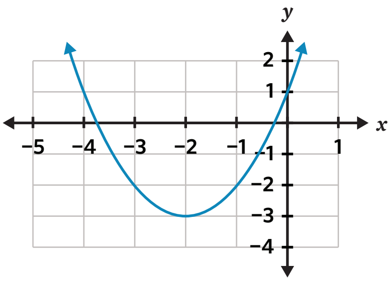
● ● (−2,−3)



● ● (2,3)



● ● (−2,0)



● ● (0,3)

Back to contents

3. Fill in the blanks by using the words provided.

coordinates	point	translated	units

A parabola with an equation of the form $y = ax^2$ can be __________ horizontally and/or vertically. It then has the turning __________ $y = a(x - h)^2 + k$ where the values and directions of h and k represent the number of __________ by which the parabola was moved left, right, up or down. The __________ of the turning point of the parabola are also given by (h,k).

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Sketch the parabolas showing the y-intercept and coordinates of the turning point. **WE1**

a. $y = x^2 + 2$ **b.** $y = x^2 + 4$ **c.** $y = x^2 - 3$ **d.** $y = -x^2 + 1$

e. $y = -x^2 + 2$ **f.** $y = -x^2 - 4$ **g.** $y = 5 - x^2$ **h.** $y = -3 - x^2$

5. Sketch the parabolas showing the y-intercept and coordinates of the turning point. **WE2**

a. $y = (x - 3)^2$ **b.** $y = (x + 1)^2$ **c.** $y = (x + 2)^2$ **d.** $y = -(x - 1)^2$

e. $y = -(x + 3)^2$ **f.** $y = -(x + 4)^2$ **g.** $y = -(x - 4)^2$ **h.** $y = -(x + 5)^2$

6. Sketch the parabolas showing the y-intercept and coordinates of the turning point. **WE3**

a. $y = (x - 1)^2 + 3$ **b.** $y = (x + 1)^2 + 2$ **c.** $y = (x - 2)^2 - 2$ **d.** $y = (x + 3)^2 - 2$

e. $y = -(x - 2)^2 + 3$ **f.** $y = -(x + 3)^2 - 2$ **g.** $y = -(x - 4)^2 - 3$ **h.** $y = 4 - (x - 1)^2$

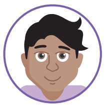
7. What are the coordinates of the turning point of $y = (x + 5)^2 - 7$?

A. (−7,−5) **B.** (−5,−7) **C.** (−5,7) **D.** (5,−7) **E.** (5,7)

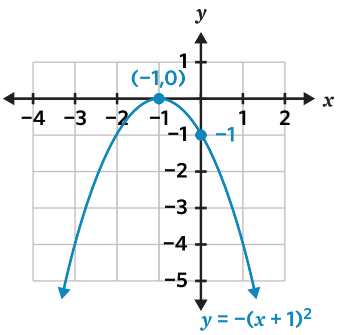
Spot the mistake

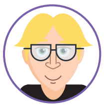
8. Select whether Student A or Student B is incorrect.

a. Sketch the graph of $y = -(x + 1)^2$ showing the y-intercept and coordinates of the turning point.

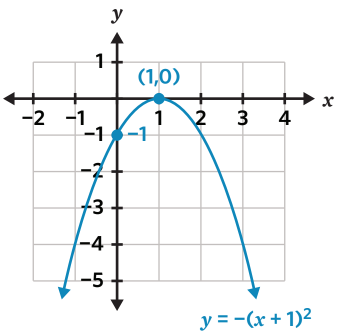


Student A



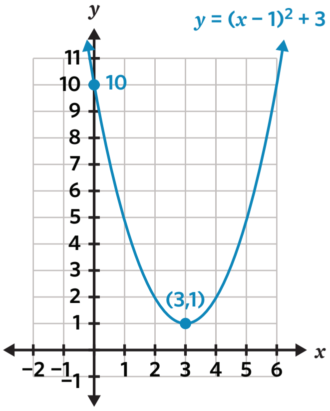


Student B

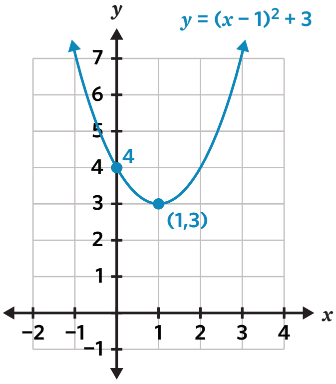


b. Sketch the graph of $y = (x - 1)^2 + 3$ showing the y-intercept and coordinates of the turning point.

Student A



Student B



Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. The height above sea level of a mountain can be modelled by the equation $y = -(x - 4)^2 + 10$, where the average sea level is represented by the line $y = 0$ (x-axis) and the values of x represent horizontal distance. Determine the coordinates of the mountain's summit.

10. Frances wants to install a pool in her backyard. The area of the pool can be modelled by the equation $y = -(x - 6)^2 + 36$ where x represents the length of one of the sides in metres. Determine how long Frances should make the side length of the pool in order to achieve the maximum possible area.

11. A flare is released from the top of a hill. The height of the flare following its release can be modelled by the equation $h = -(x - 5)^2 + 45$, where x is the horizontal distance in metres. Determine the height of the hill from which the flare was released.

12. The number of students enrolled in a course over the first four weeks of the semester can be modelled by the equation $s = (w - 1)^2 + 20$ for $0 \leq w \leq 4$. Sketch the parabola and determine how many students were enrolled in the course at the beginning of the semester.

13. The parabolic cable of a suspension bridge can be modelled by the equation $y = \frac{1}{4}(x - 4)^2$ for $0 \leq x \leq 8$, where the y-axis represents a vertical metal beam to which the cable is attached. Sketch this part of the bridge and determine at what height the cable attaches to the vertical beam, given that the units are metres.

Back to contents

Reasoning

Question working paths

Mild 14 (a,b,c,e)	**Medium** 14 (a,b,c,e), 15 (a,b)	**Spicy** All

14. The cross section of a skating bowl can be modelled by the equations $y = (x - 1)^2 + 1$ for $0 \leq x \leq 2$ and $y = -(x - 3)^2 + 3$ for $2 \leq x \leq 4$, where y is the height of the skating bowl and x is the horizontal distance from the start of the skating bowl.

a. Identify the coordinates of the turning point of the skating bowl for $0 \leq x \leq 2$.

b. Identify the coordinates of the turning point of the skating bowl for $2 \leq x \leq 4$.

c. Sketch the graph of the cross section for $0 \leq x \leq 2$, showing the y-intercept and coordinates of the turning point.

d. On the same set of axes, sketch the graph of the cross section for $2 \leq x \leq 4$, showing the coordinates of the turning point of the skating bowl and the points where $x = 2$ and $x = 4$.

e. Skating bowls are often owned by the city council and are public areas that can be used by everyone. Name another type of public facility provided for citizens and maintained by the council.

15. Sketch the parabolas given in parts **a** and **b**, showing the y-intercept and coordinates of the turning point.

a. $y = (x - 2)^2 - 4$

b. $y = (x - 3)^2 - 9$

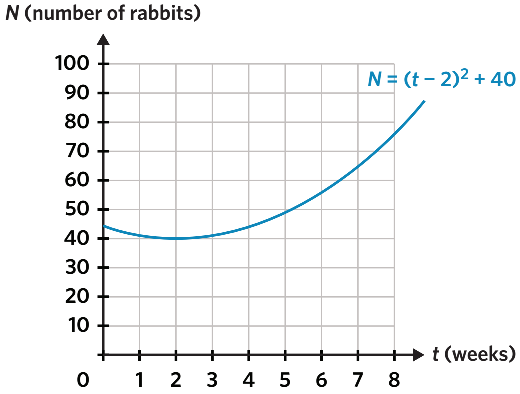
c. What is the y-intercept of a parabola with the equation of the form $y = (x - h)^2 - h^2$?

Exam-style

16. The coordinates of the point where $y = -(x + 6)^2 - 5$ crosses the y-axis are (1 MARK)

A. (−6,−5) **B.** (0,−41) **C.** (0,−6) **D.** (0,41) **E.** (6,0)

17. The given graph shows the population of rabbits on an island over an eight week period. It can be modelled by the equation $N = (t - 2)^2 + 40$, where N is the numbers of rabbits and t is the time in weeks. (3 MARKS)

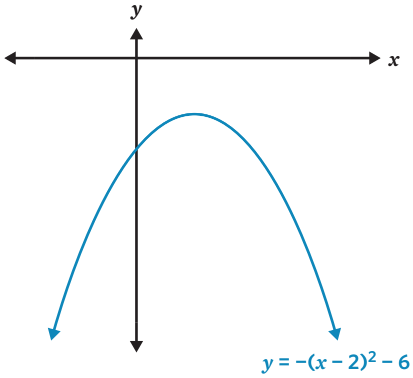


a. Determine the initial number of rabbits. 1 MARK

b. Determine the minimum population of rabbits and the week during which it occurred. 2 MARKS

Back to contents

18. Show the y-intercept and the coordinates of the turning point on the given graph of $y = -(x - 2)^2 - 6$. (2 MARKS)



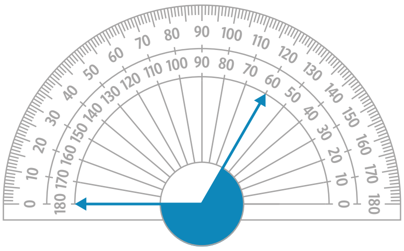
19. A parabola has the turning point form equation $y = (x + h)^2 - 5$ and its y-intercept $= 11$. Show that the turning point of the parabola has the coordinates (−4,−5). (3 MARKS)

Remember this?

20. A 100 g bar of alloy is composed of silver and copper in the ratio of 4 : 1. How many grams of silver are in the 100 g bar?

A. 20 g **B.** 25 g **C.** 50 g **D.** 75 g **E.** 80 g

21. What is the size of the shaded angle?



A. 60° **B.** 120° **C.** 240° **D.** 300° **E.** 360°

22. There are approximately 20 000 trees in a forest. The number of trees is increasing by about 1000 each year due to a reforestation project. Which of these will give the best estimate of the number of trees in this forest in 5 years' time?

A. $20000 + 1000 + 5$

B. $(20000 + 1000) \times 5$

C. $(20000 + 5) \times 1000$

D. $20000 + 5 \times 1000$

E. $20000 \times 1000 \times 5$

6F Non-linear graphs

LEARNING INTENTIONS

Students will be able to:

- identify and sketch a circle centred at the origin from its equation
- identify and sketch a rectangular hyperbola from its equation.

Together with parabolas, circles and hyperbolas are part of a group of curves called conic sections. Circles form a continuous curve, able to be drawn in one step without lifting pen from paper. Hyperbolas are examples of discontinuous curves, where certain values of x and y are undefined and form asymptotes.

KEY TERMS AND DEFINITIONS

- An **asymptote** is a line that the graph of a function approaches, but does not touch at any point.
- A **continuous** relation has a graph without any breaks or abrupt changes in values.
- A **discontinuous** relation has a graph which is not connected or contains abrupt changes in values.
- An **undefined** value or expression does not have mathematical meaning, such as division by zero.

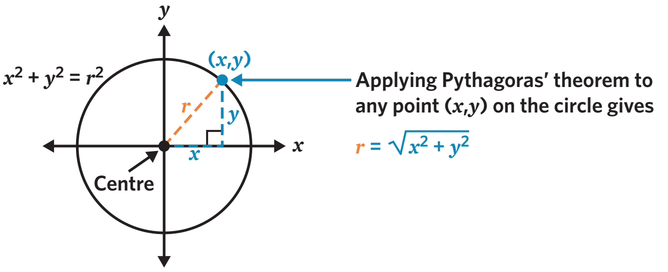
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: inter reality/Shutterstock.com

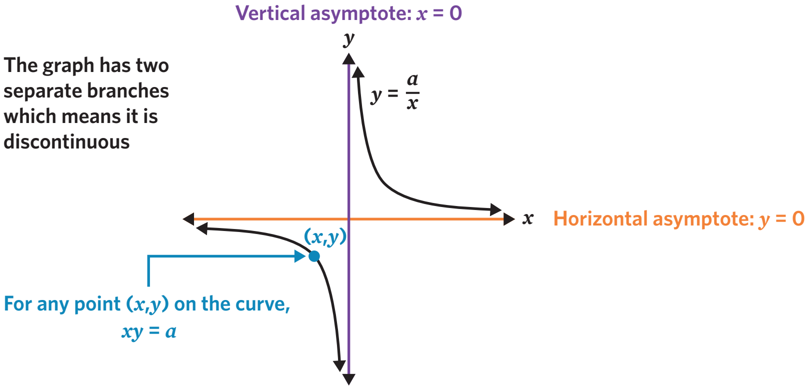
Circles and hyperbolas are types of conic sections, which are curves formed by intersecting a cone's surface with a plane. This can be visualised by looking at the shape of the light cast by a bulb on a surface, such as a wall. The light source is designed to cast two identical cones of light, one from the top and another from the bottom, while the wall acts as the intersecting plane, producing a hyperbola.

Key ideas

1. A **circle** with a centre at (0,0) has the equation $x^2 + y^2 = r^2$ where r is the radius.



2. A **rectangular hyperbola** with asymptotes at $y = 0$ and $x = 0$ has the equation $y = \frac{a}{x}$ where $a \neq 0$.



Worked example 1

Graphing circles

Sketch the circle showing the intercepts and coordinates of the centre. **WE1**

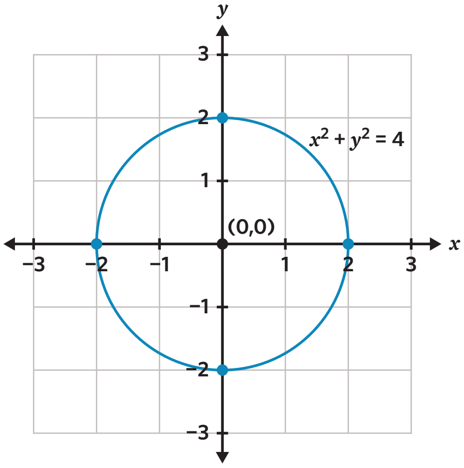
$x^2 + y^2 = 4$

Working

$$x^2 + y^2 = 4$$
$$= 2^2$$
$$= r^2$$
$$\therefore r = 2$$

Thinking

Step 1: Determine the length of the radius, r.



Step 2: Draw a circle with centre (0,0) and radius r on a set of axes. Mark the coordinates of the centre and x- and y-intercepts.

Student practice

Sketch the circle showing the intercepts and coordinates of the centre.

$x^2 + y^2 = 9$

Worked example 2

Graphing hyperbolas

Fill in the tables of values for the given hyperbolas and graph them showing the points where $x = -1$ and $x = 1$.

a. $y = \frac{1}{x}$ **WE2a**

x	y
-2	
-1	
$-\frac{1}{2}$	
0	
$\frac{1}{2}$	
1	
2	

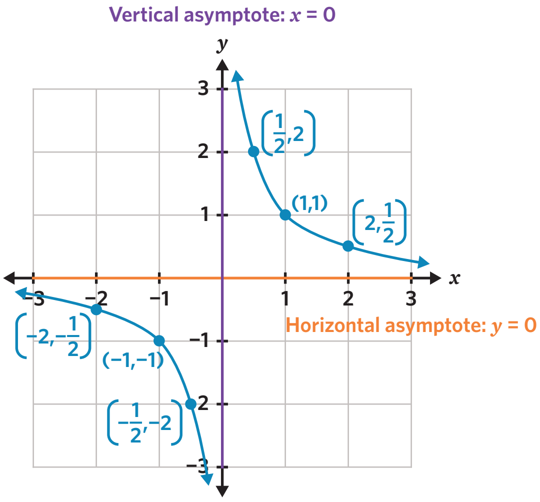
Continues →

Working

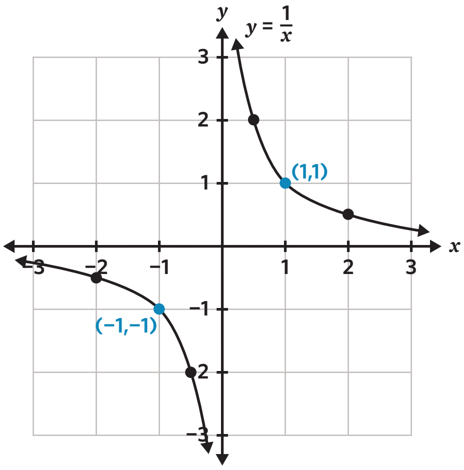
x	$y = \frac{1}{x}$	(x,y)
-2	$-\frac{1}{2}$	$\left(-2,-\frac{1}{2}\right)$
-1	-1	$(-1,-1)$
$-\frac{1}{2}$	-2	$\left(-\frac{1}{2},-2\right)$
0	Undefined	Asymptote
$\frac{1}{2}$	2	$\left(\frac{1}{2},2\right)$
1	1	$(1,1)$
2	$\frac{1}{2}$	$\left(2,\frac{1}{2}\right)$

Thinking

Step 1: Complete the given table of values using the equation of the hyperbola and determine the coordinates of the points.



Step 2: Plot the coordinates (x,y) from the table on a Cartesian plane and join each separate branch with a smooth curve, adding arrows at the ends. Ensure that the curves do not touch the x- or y-axis.



Step 3: Mark the coordinates of the points where $x = 1$ and $x = -1$ and label the graph with the given equation.

Continues →

Back to contents

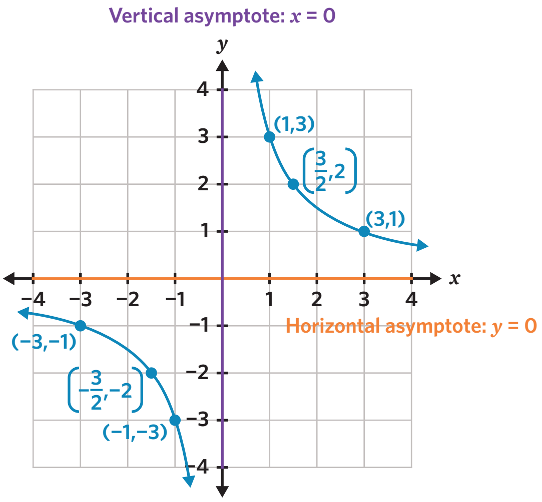
WE2b

b. $y = \frac{3}{x}$

x	y
-3	
$-\frac{3}{2}$	
-1	
0	
1	
$\frac{3}{2}$	
3	

Working

x	$y = \frac{3}{x}$	(x,y)
-3	-1	$(-3,-1)$
$-\frac{3}{2}$	-2	$\left(-\frac{3}{2},-2\right)$
-1	-3	$(-1,-3)$
0	Undefined	Asymptote
1	3	$(1,3)$
$\frac{3}{2}$	2	$\left(\frac{3}{2},2\right)$
3	1	$(3,1)$

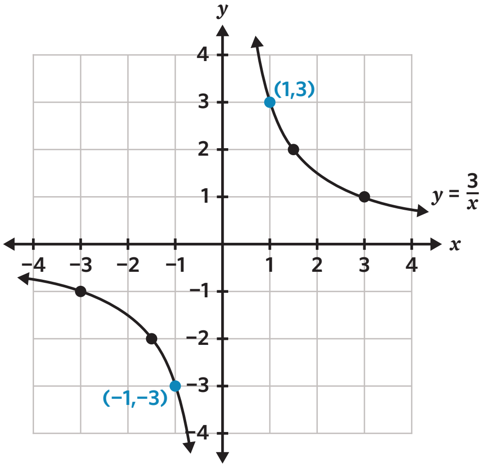


Thinking

Step 1: Complete the given table of values using the equation of the hyperbola and determine the coordinates of the points.

Step 2: Plot the coordinates (x,y) from the table on a Cartesian plane and join each separate branch with a smooth curve, adding arrows at the ends. Ensure that the curves do not touch the x- or y-axis.

Continues →



Step 3: Mark the coordinates of the points where $x = 1$ and $x = -1$ and label the graph with the given equation.

Student practice

Fill in the tables of values for the given hyperbolas and graph them showing the points where $x = -1$ and $x = 1$.

a. $y = \frac{1}{x}$

x	y
-3	
-1	
$-\frac{1}{3}$	
0	
$\frac{1}{3}$	
1	
3	

b. $y = \frac{5}{x}$

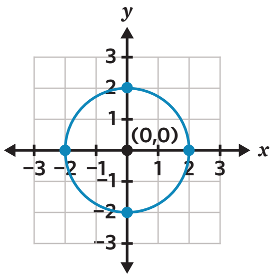
x	y
-5	
$-\frac{5}{2}$	
-1	
0	
1	
$\frac{5}{2}$	
5	

6F Questions

Understanding worksheet

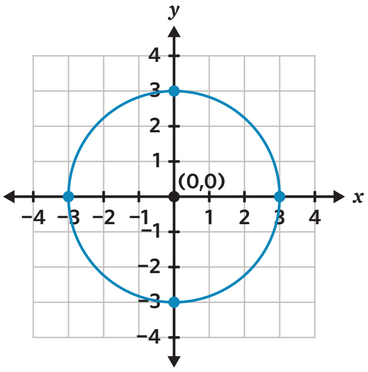
1. Determine the length of the radius for each circle.

Example



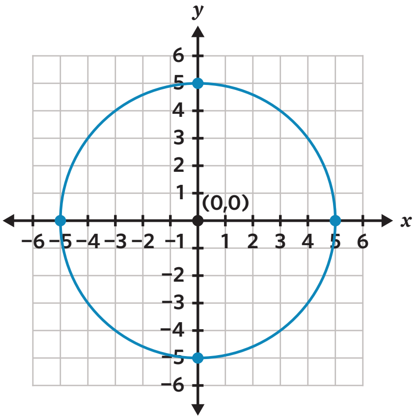
$r =$ 2

a.



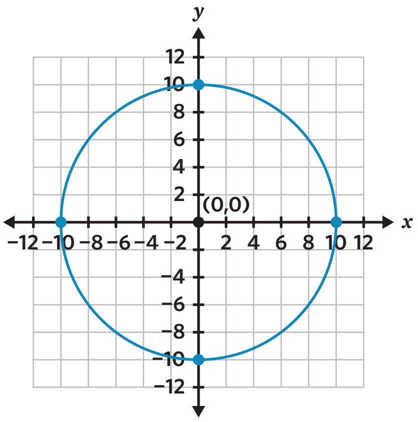
$r =$

b.



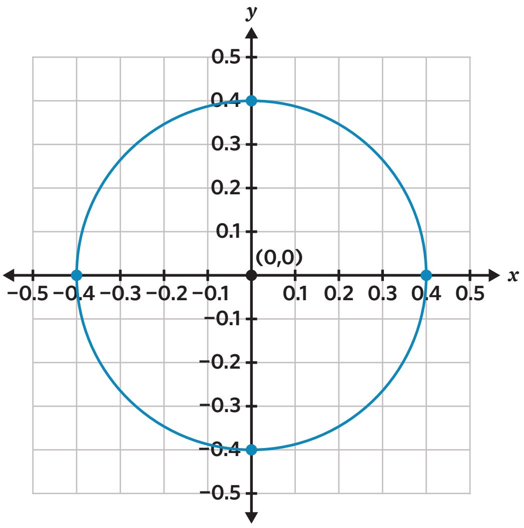
$r =$

c.



$r =$

d.



$r =$

Back to contents

2. Fill in the blanks.

Example

x	-5	-2	-1	0	1	2	5
$y = \frac{20}{x}$	-4	-10	-20	Undefined	20	10	4

x	-5	-2	-1	0	1	2	5
$y = \frac{10}{x}$		-5	-10			5	

3. Fill in the blanks by using the words provided.

asymptotes **conic** **continuous** **discontinuous**

Circles and hyperbolas are types of ______ sections. A circle is a ______ curve that forms a graph without any breaks, while the hyperbola is an example of a ______ graph. The hyperbola is not connected and contains ______ , which are lines that the graph approaches, but does not touch.

Fluency

Question working paths

Mild 🌶️	Medium 🌶️🌶️	Spicy 🌶️🌶️🌶️
4 (a,b,c,d), 5 (a,b,c,d), 6	4 (c,d,e,f), 5 (c,d,e,f), 6	4 (e,f,g,h), 5 (e,f,g,h), 6

4. Sketch the circles showing the intercepts and coordinates of the centre. **WE1**

a. $x^2 + y^2 = 1$ **b.** $x^2 + y^2 = 16$ **c.** $x^2 + y^2 = 25$ **d.** $x^2 + y^2 = 49$

e. $x^2 + y^2 = 64$ **f.** $x^2 + y^2 = 100$ **g.** $x^2 + y^2 = 0.09$ **h.** $x^2 + y^2 = 1.44$

---

5. Fill in the tables of values for the given hyperbolas and graph them showing the points where $x = -1$ and $x = 1$. **WE2**

a. $y = \frac{2}{x}$

x	y
-2	
-1	
0	
1	
2	

b. $y = \frac{4}{x}$

x	y
-4	
-2	
-1	
0	
1	
2	
4	

c. $y = \frac{6}{x}$

x	y
-6	
-2	
-1	
0	
1	
2	
6	

d. $y = \frac{8}{x}$

x	y
-8	
-4	
-1	
0	
1	
4	
8	

Back to contents

e. $y = \frac{9}{x}$

x	y
-9	
-3	
-1	
0	
1	
3	
9	

f. $y = \frac{12}{x}$

x	y
-12	
-4	
-1	
0	
1	
4	
12	

g. $y = \frac{20}{x}$

x	y
-20	
-5	
-4	
-1	
0	
1	
4	
5	
20	

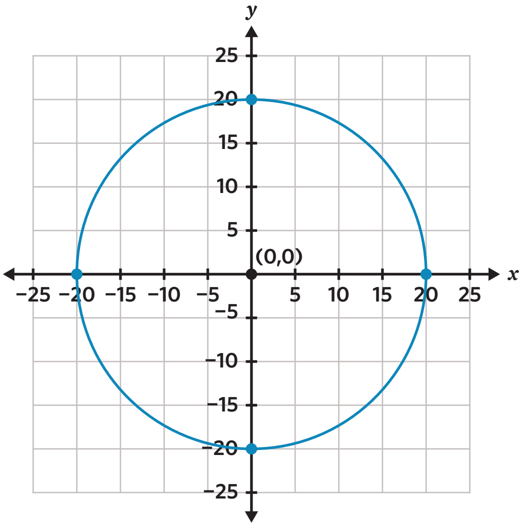
h. $y = \frac{15}{x}$

x	y
-15	
-5	
$-\frac{5}{3}$	
-1	
0	
1	
$\frac{5}{3}$	
5	
15	

---

6. What is the equation of the given circle?

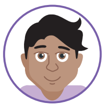
A. $x^2 + y^2 = 10$
B. $x^2 + y^2 = 20$
C. $x^2 + y^2 = 40$
D. $x^2 + y^2 = 400$
E. $x^2 + y^2 = 1600$



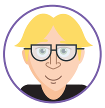
Spot the mistake

7. Select whether Student A or Student B is incorrect.

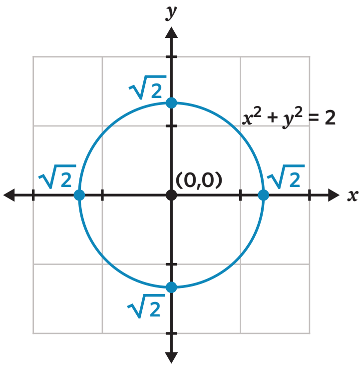
a. Sketch the circle with the equation $x^2 + y^2 = 2$.

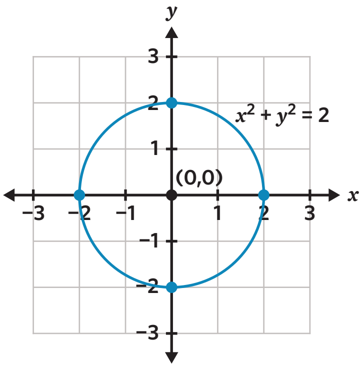


Student A



Student B



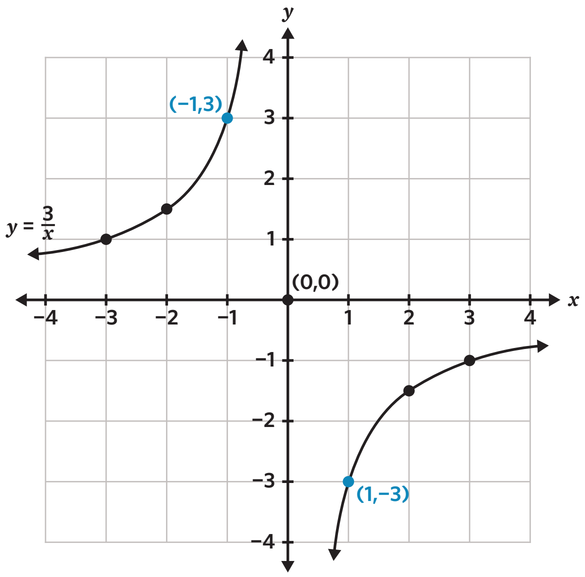


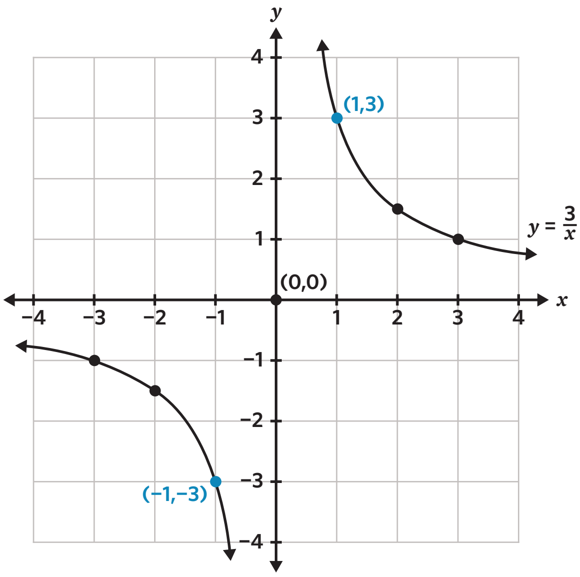
Back to contents

b. Graph the hyperbola $y = \frac{3}{x}$.

Student A

Student B





Problem solving

Question working paths

Mild 8, 9, 10 🌶	**Medium** 9, 10, 11 🌶🌶	**Spicy** 10, 11, 12 🌶🌶🌶

8. The shape of a roundabout in bird's eye view can be modelled by the equation $x^2 + y^2 = 16$. Determine the radius of the roundabout, in metres.

9. At the outermost lane of a circular cycling track, the diameter of the track is equal to 100 m. Write an equation of the form $x^2 + y^2 = r^2$ modelling the complete circular path of a cyclist at the outermost lane of the track.

10. The shape of a bicycle wheel can be modelled by the equation $x^2 + y^2 = 1225$, where all sprockets extend directly from the centre of the wheel to a point on its rim. Determine the length of each sprocket, given that the units are centimetres.

11. A statue at the centre of a circular garden bed is surrounded by a path. The path's outer edge can be modelled by the equation $x^2 + y^2 = 36$, relative to the statue at the centre. Determine the width of the path if the shape of the circular garden bed can be modelled by the equation $x^2 + y^2 = 9$, relative to the statue at the centre.

12. A space station observed by astronomers from Earth follows a path modelled by $y = \frac{810\,000}{x}$ for $405 \le x \le 2000$ where x and y respectively represent the horizontal and vertical distance of the space station from the point of observation on Earth. Plot the hyperbolic path of the station using the given table of values and determine the coordinates of the space station when it is at its closest point to Earth.

x	405	900	1000	2000
$y = \frac{810\,000}{x}$				

Reasoning

Question working paths

Mild 13 (a,b,d) 🌶	**Medium** 13 (a,b,d), 14 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

13. The High Roller, located in Las Vegas, USA, is the second tallest ferris wheel in the world with a radius of 80 m.

Image: Aneese/Shutterstock.com

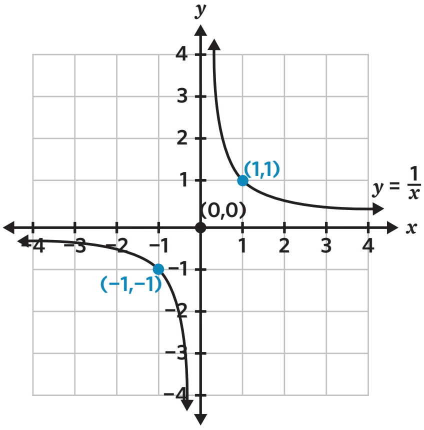
a. Write an equation of the form $x^2 + y^2 = r^2$ to model the shape of The High Roller, using (0,0) to represent the coordinates of its centre.

b. Sketch a graph of the equation from part **a**.

c. The lowest point of The High Roller is 8 m from the ground. Determine the real-life height of a cabin that is in the upper half of the ferris wheel and 40 m horizontally from the centre. Round the answer to the nearest metre.

d. Name a famous Australian tourist attraction or destination .

14. Sketch the hyperbolas in parts **a** and **b** on the same axes as the given graph of $y = \frac{1}{x}$ and mark the coordinates where $x = -1$ and $x = 1$ for each curve.



a. $y = \frac{2}{x}$

b. $y = \frac{3}{x}$

c. Describe the effect of increasing the value of a on the graph of a hyperbola with the equation $y = \frac{a}{x}$.

Exam-style

15. A circular plate has a diameter of 24 cm. An equation that could be used to model its shape is (1 MARK)

A. $x^2 + y^2 = 12$

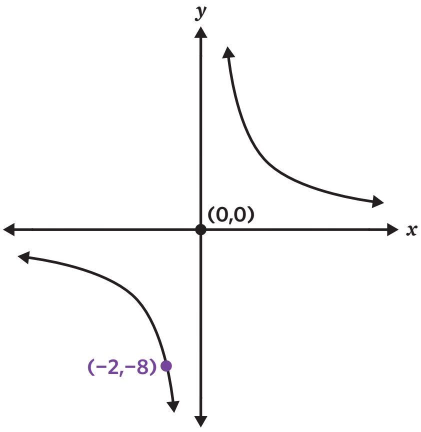
B. $x^2 + y^2 = 24$

C. $x^2 + y^2 = 48$

D. $x^2 + y^2 = 144$

E. $x^2 + y^2 = 576$

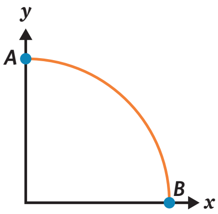
16. Consider the given graph of a hyperbola with an equation of the form $y = \frac{a}{x}$. (2 MARKS)



a. Determine the value of a. 1 MARK

b. Determine the value of y when $x = 64$. 1 MARK

17. The given diagram shows part of the graph of $x^2 + y^2 = 121$. Determine the coordinates of points A and B. (2 MARKS)



18. Sketch the graph of $x^2 + y^2 = 81$ showing the intercepts and coordinates of the centre. (3 MARKS)

Remember this?

19. Sara measures the volume of a liquid to be 3 litres and 18 millilitres. Which of these shows how Sara can write this measurement in litres?

A. 3.018 L **B.** 3.18 L **C.** 3.2 L **D.** 4.18 L **E.** 4.2 L

20. Emily and Jack are saving money for a trip. If Jack had saved $15 more, he would have saved exactly twice as much as Emily. Which row of the table shows how much money they could have saved?

	Emily	Jack
1	$12	$21
2	$15	$35
3	$18	$36
4	$30	$45
5	$35	$40

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

21. Which statement is always true?

A. The diagonals of a square are unequal in length.
B. The opposite angles of a rectangle are supplementary.
C. The opposite sides of a rectangle are unequal in length.
D. The angles in an isosceles trapezium are all right angles.
E. The opposite sides of a parallelogram are unequal in length.

Chapter 6 extended application

1. Tyrone is a graphic designer and is using quadratic equations to create different visual designs. The three equations he is using are as follows:

Equation 1: $y = 2x^2 + 1$

Equation 2: $y = 2x^2 + x + 1$

Equation 3: $y = 2x^2 - x + 1$

a. Fill in this table of values for $-3 \le x \le 3$ for all three equations.

x	-3	-2	-1	0	1	2	3
$y = 2x^2 + 1$							
$y = 2x^2 + x + 1$							
$y = 2x^2 - x + 1$							

b. Plot all three quadratic equations on the same set of axes for $-3 \le x \le 3$.

c. Identify the dilation(s) and/or translation(s) that must occur in order to transform the basic parabola, $y = x^2$ to the graph given by Equation 1.

d. Tyrone would like to create another visual design that mirrors the current design over the x-axis. Write the three equations of the reflected parabolas.

e. Sketch the new reflected design for $-3 \le x \le 3$.

f. Identify a reason why graphic designers may use parabolas to create visual designs.

2. At a local golf tournament, Owen has to hit a bunker shot, which occurs when the ball is stuck inside a large sandpit. These types of shots are typically a short distance. Part of the golf ball's trajectory can be modelled by the equation $y = \frac{1}{3}x^2$, $0 \le x \le 5$, where y is the height of the golf ball above the ground in metres, and x is the horizontal distance the ball travels in metres.

a. Complete a table of values using the given equation and the horizontal distance interval of 0 m to 5 m. Use exact values.

b. Plot the quadratic function on a graph.

c. A pitch is another short golf shot that typically has a higher trajectory. Owen models part of the path of a pitch by the equation $y = \frac{1}{2}x^2$, $0 \le x \le 5$. Determine the difference in height of the bunker shot and pitch after traversing a horizontal distance of 5 m. Round to two decimal places.

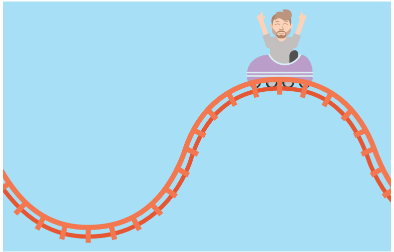
Owen is not sure whether to model the entire trajectory of a golf ball for a longer shot using the equation $y = -\frac{1}{100}x^2 + x$ or $y = \frac{1}{100}x^2 + x$ for $0 \le x \le 100$.

d. Which long distance equation is more suitable to model the entire trajectory of a golf ball for a longer shot? Justify your answer.

e. Some people may think that golf is boring or only a suitable sport for retirees. Give a reason why golf can be enjoyed by anyone.

3. The first section of a roller coaster track can be modelled by the equations $y = (x - 2)^2 - 3$ for $0 \le x \le 4$ and $y = -(x - 6)^2 + 5$ for $4 \le x \le 8$ where y is the height of the roller coaster above or below ground level and x is the horizontal distance from the start in tens of metres.

a. Sketch the graph of the first section of the roller coaster track for $0 \le x \le 8$, showing the y-intercept, the point when $x = 4$ and $x = 8$, and the coordinates of all turning points.



Back to contents

The second section of the roller coaster is called The Monster and can be modelled by the equation $y = (x - 10)^2 - 3, 8 \leq x \leq 22$. The Monster is well known for being extremely fast, steep, and high.

b. Determine the lowest point of this section.

c. Show that when the Monster section reaches 141 m above ground level the roller coaster has travelled a horizontal distance of 220 m from the start of the first section.

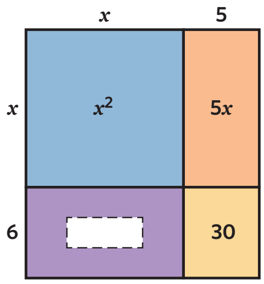
d. Describe what happens when $x = 2$ and when $x = 6$ in the model for the first section of the roller coaster and explain why this is important for the safety of any roller coaster that is modelled using parabolas.

e. What factors should visitors consider when planning a trip to a theme park during peak seasons?

Chapter 6 review

Multiple choice

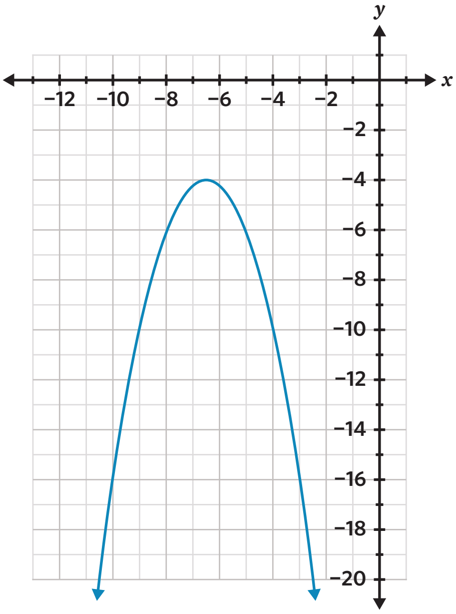
1. Identify the missing value to expand the quadratic expression $(x + 6)(x + 5)$. 6A,B



A. x
B. 6
C. $6x$
D. 11
E. $6 + x$

---

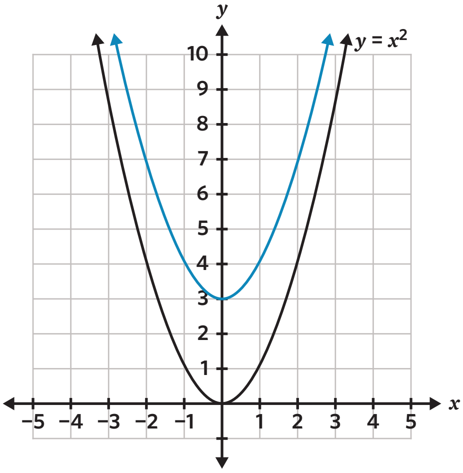
2. Determine the equation for the axis of symmetry in the following graph. 6C



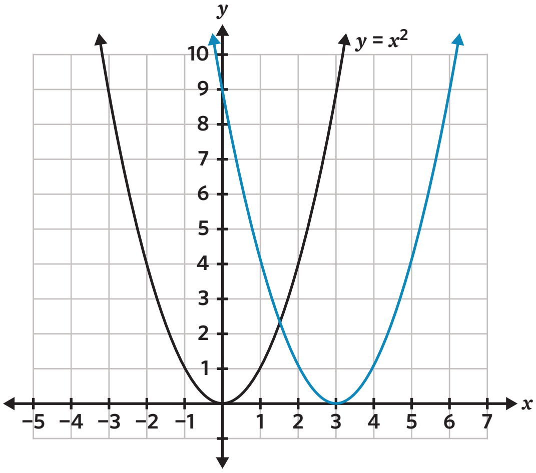
A. $x = -7$
B. $x = -6.5$
C. $y = -4$
D. $x = 6.5$
E. $x = 7$

3. Which of the following graphs accurately illustrates the dilation from x^2 to $3x^2$?

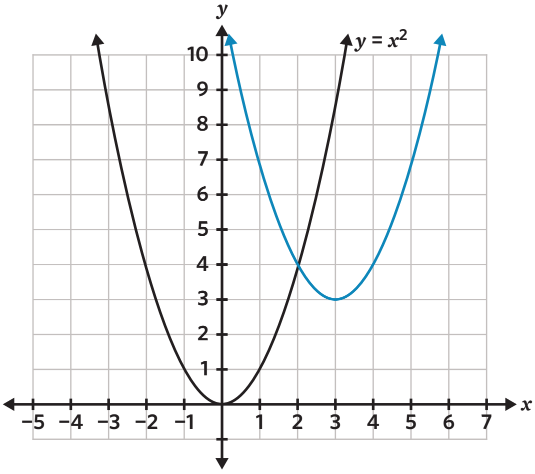
A.



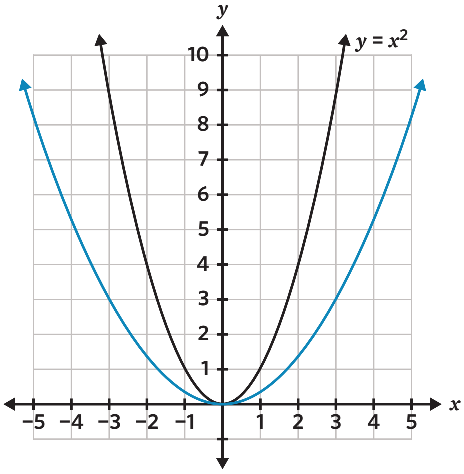
B.



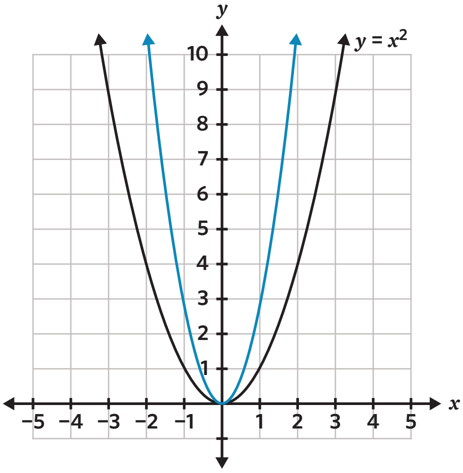
C.



D.

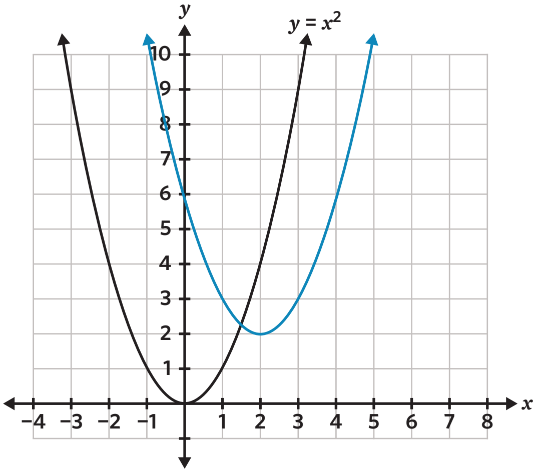


E.

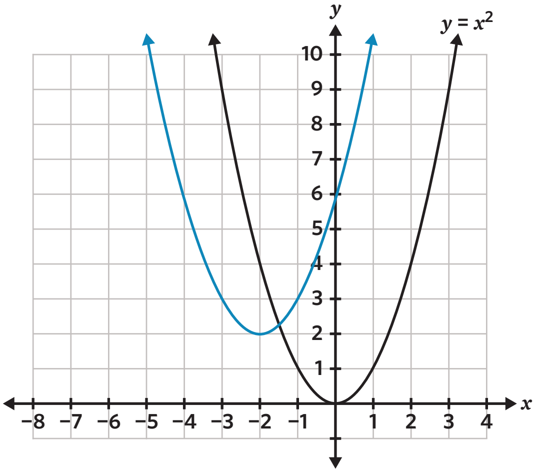


4. Which of the following graphs accurately illustrates the translation from x^2 to $(x - 4)^2 + 2$?

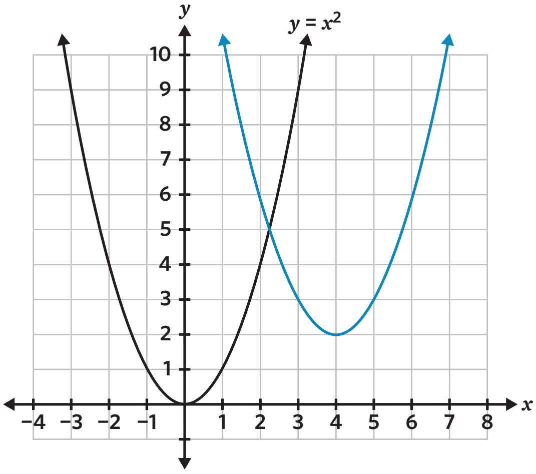
A.



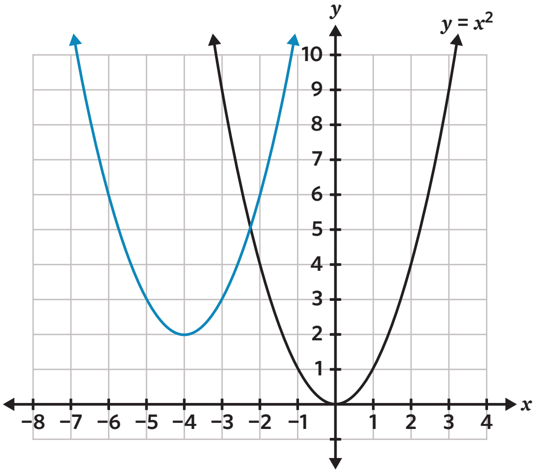
B.



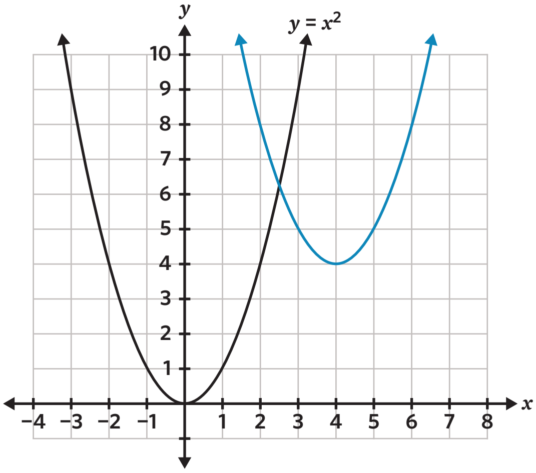
C.



D.

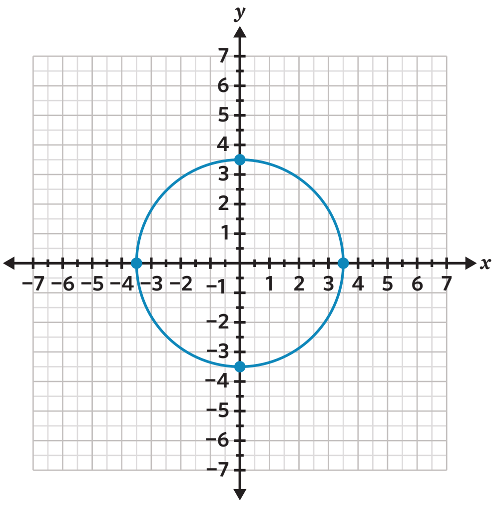


E.



Back to contents

5. Determine the length of the radius for the following circle. **6F**



A. 0 **B.** 3 **C.** 3.5 **D.** 6 **E.** 7

Fluency

6. Substitute the given values into the quadratic equations to check whether they are solutions. **6A**

a. $3x^2 - 4x + 1 = 0 \quad (x = 1)$ **b.** $x^2 + 5x - 6 = 0 \quad (x = -6)$

c. $4x^2 - 8x + 2 = 0 \quad (x = 0)$ **d.** $2x^2 + 3x - 4 = 0 \quad (x = -1)$

---

7. Solve the following equations. **6A**

a. $(x + 2)(x - 3) = 0$ **b.** $(3x - 4)(x + 1) = 0$

c. $(x - 5)(2x + 3) = 0$ **d.** $(4x + 1)(x - 2) = 0$

---

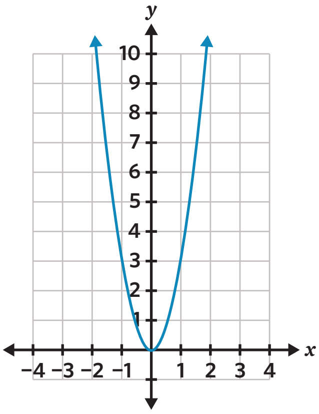
8. Factorise and solve the following. **6B**

a. $x^2 + 4x - 5 = 0$ **b.** $x^2 - 5x + 6 = 0$ **c.** $x^2 = 8x - 15$ **d.** $x^2 - 9 = 0$

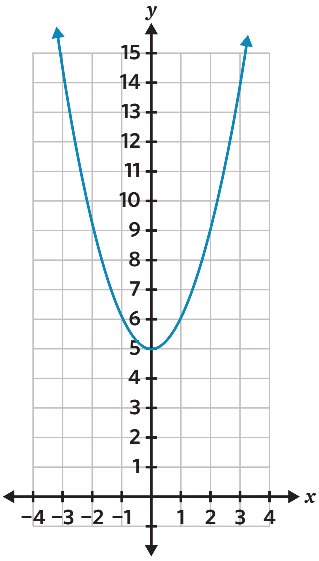
---

9. Use the graphs to determine the equation of the axis of symmetry, coordinates and type of turning point, and any x- and y-intercepts of the parabolas. **6C**

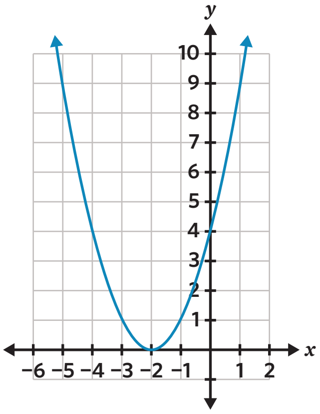
a.



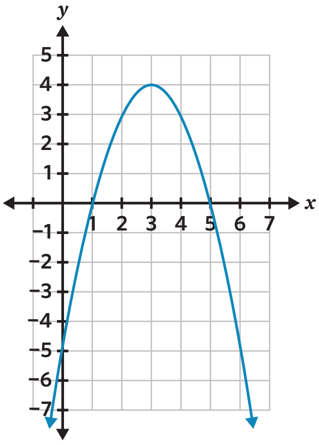
b.



c.



d.



10. Plot the graphs of the quadratic functions and identify their key features. 6C

a. $y = x^2 - 4x + 3$ for $-1 \leq x \leq 5$

b. $y = x^2 - 2x + 3$ for $-2 \leq x \leq 4$

c. $y = -x^2 - 2x + 3$ for $-4 \leq x \leq 2$

d. $y = x^2 + 3x - 4$ for $-5 \leq x \leq 2$

11. Plot the parabolas of the form $y = ax^2$ on the same set of axes and mark the coordinates of the turning point and intercept. Determine how the value of a affects each parabola. 6D

a. $y = x^2$ and $y = \frac{1}{2}x^2$

b. $y = -x^2$ and $y = -2x^2$

c. $y = 3x^2$ and $y = -4x^2$

d. $y = 5x^2$ and $y = -\frac{3}{4}x^2$

12. Sketch the parabolas showing the y-intercept and coordinates of the turning point. 6E

a. $y = (x + 3)^2 - 4$

b. $y = -(x - 4)^2 + 3$

c. $y = (x - 2)^2 + 1$

d. $y = (x + 5)^2 - 6$

13. Sketch the circles showing the intercepts and coordinates of the centre. 6F

a. $x^2 + y^2 = 1$

b. $x^2 + y^2 = 9$

c. $x^2 + y^2 = 20.25$

d. $x^2 + y^2 = 3.61$

14. Fill in the tables of values for the given hyperbolas and graph them showing the points where $x = -1$ and $x = 1$. 6F

a. $y = \frac{2}{x}$

x	y
−2	
−1	
0	
1	
2	

b. $y = \frac{3}{x}$

x	y
−3	
−2	
−1	
0	
1	
2	
3	

c. $y = \frac{4}{x}$

x	y
−4	
−2	
−1	
0	
1	
2	
4	

d. $y = \frac{6}{x}$

x	y
−6	
−2	
−1	
0	
1	
2	
6	

Problem solving

15. Sophie is designing a garden bridge that has a parabolic shape given by $y = -x(x - 2)$, where y is the height of the bridge in metres, and x is the horizontal distance from the start of the bridge, in metres. Assuming that the bridge starts at ground level, find the points where the bridge meets the ground. 6A

16. In the design of a new microscope, an engineer is working on the curvature of a lens that can be modelled by the equation $y = -x^2 + 3x$, where y is the height of the lens in millimetres, and x is the horizontal distance from the edge of the lens in millimetres. Factorise and solve the equation when $y = 2$ mm to determine the horizontal distance from the edge of the lens when the height is 2 mm. 6B

17. A local park is designing a new water fountain. The height of the water stream, h, in metres, can be modelled by the equation $h = -x^2 + 4x$, where x is the time in seconds since the water was released. Plot the quadratic function for $0 \leq x \leq 4$ and determine the maximum height of the water stream. 6C

18. In a skatepark, there's a half-pipe ramp used for skateboarding. The shape of the cross-section of the half-pipe is given by the equation $y = \frac{1}{2}x^2$ where y is the height in metres and x is the horizontal distance in metres. The axis of symmetry splits the half-pipe into two identical halves. The skatepark is planning to build a new half-pipe with a similar shape. It can be described by an equation with a scaling factor that is a third of the original half-pipe. Determine an equation that could be used to describe the shape of the new half-pipe. Sketch the equations of both the existing and the new half-pipe on the same set of axes. 6D

19. In an amusement park, a section of a roller coaster track is being designed. The shape of the section can be modelled by the equation $y = \frac{1}{3}(x - 3)^2$ for $0 \leq x \leq 6$, where y is the height in metres and x is the horizontal distance in metres. Sketch the section of the roller coaster track and determine the distance at which the track reaches its lowest elevation. 6E

20. In a wildlife reserve, the number of kangaroos expected to be observed by researchers is modelled by $y = \frac{5000}{x}$, where y represents the number of kangaroos and x is the distance in kilometres from the research centre for $1 \leq x \leq 5$. Sketch the graph and determine the number of kangaroos that will be observed when the research team is 4 km away from the research centre. 6F

Reasoning

21. In a small town, the local council decides to build a new park called 'The Parabolic Park'. The park features various parabolic-shaped objects and activities, including a parabolic fountain, a parabolic slide, and a parabolic archway.

a. The parabolic archway at the centre of the park can be modelled by the equation $y - 4x + x^2 = 0$, where y is the height of the archway and x is the horizontal distance. Write the equation in the form $y = ax^2 + bx + c$.

b. The slide, which is partly underground, is modelled by the quadratic equation $y = x^2 - 4x + 3$ for $0 \leq x \leq 2$, where y represents the height and x represents its horizontal distance from the start of the slide. Identify where the slide intersects the ground by solving for $y = 0$.

c. The shape of the pond in the park can be modelled by the equation $x^2 + y^2 = 30.25$. Sketch the graph given by the equation showing the intercepts and coordinates of the centre.

d. Another archway in the park can be modelled by the equation $y = -2x^2 + 4$. Sketch the graph of the archway and identify its key features.

e. What are some activities that could be added to the Parabolic Park to make it more engaging for visitors?

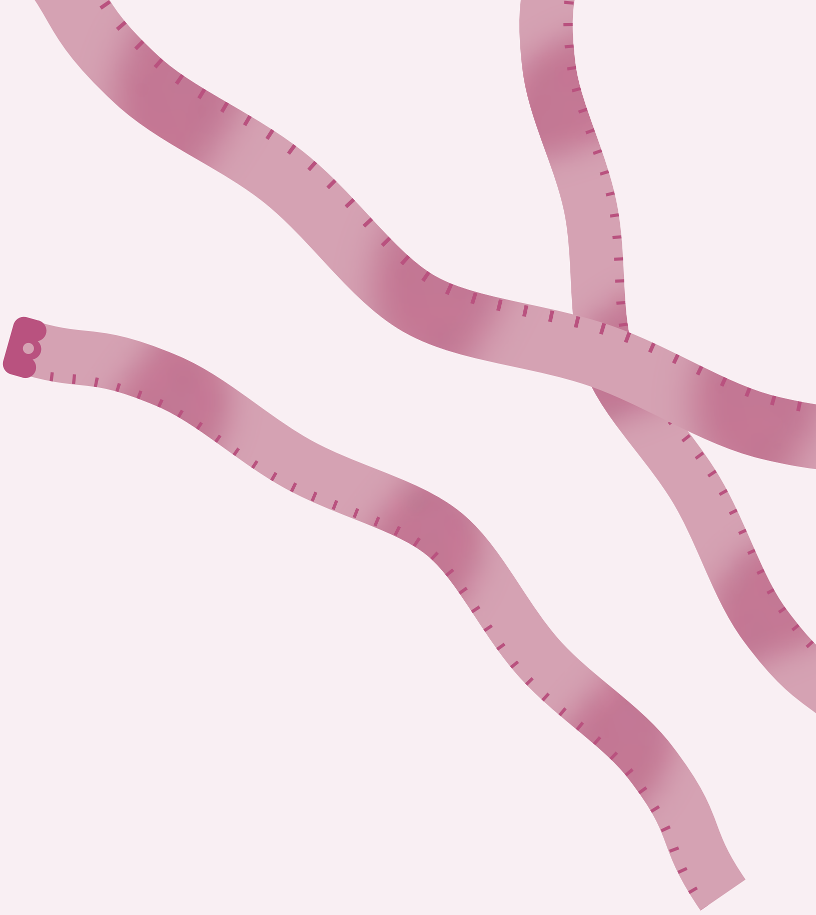
22. Graph the following equations on the same axis.

a. $y = 3x^2$

b. $y = \frac{1}{3}x^2$

c. Comparing your answers from part **a** and part **b**, describe how the magnitude of a in $y = ax^2$ affects the 'width' of the parabola.

Back to contents



Chapter 7

Measurement

Measurement and geometry

Back to contents

Chapter 7 research summary

Measurement

Big ideas

Measurement focuses on quantifying the attributes of objects and spaces, including length, area, volume and capacity. In Year 9, students delve deeper in the subject and apply their knowledge from earlier years to expand the concepts. The big ideas that underpin the concept of measurement at this level encompass several foundational mathematical principles including geometry, proportionality, and algebraic relationships. These foundational ideas provide students with a comprehensive understanding of how and why we measure in mathematics.

Number sense

Measurement is intrinsically tied to the idea of numbers and quantities. Whether students are measuring lengths, areas, volumes, or other quantities, they are assigning numbers to represent these quantities.

Geometry and spatial relationships

Understanding geometric properties of shapes and solids is important in order to determine area and volume.

Proportionality and scale

As students measure, they often encounter situations where they need to scale up or down. This could be in the context of maps models, or any scenario where different scales are used to represent the same item or object.

Algebraic thinking

In Year 9 students start to see how measurements can be used in algebraic formulas. Students can also use algebra to find unknown lengths or angles.

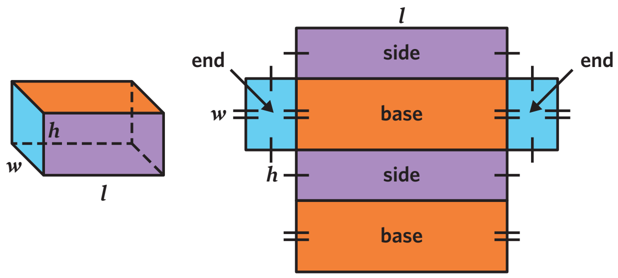
Patterns and regularity

Students can identify patterns through repeated measurements and observations. For example, as the radius of a circle increases, its area increases in a specific pattern.

Visual representations

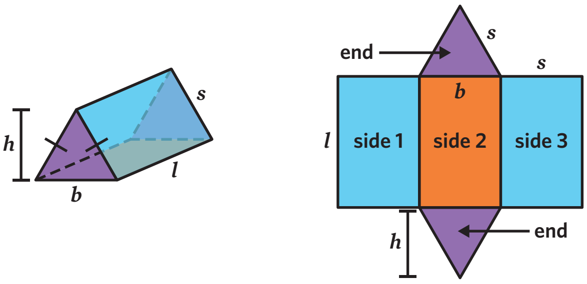
Nets

Using nets to represent 3D objects can allow students to connect the concept of area and surface area. Nets help students to visualise all the surfaces of a 3D object and to understand why we use different total surface area formulas for specific objects.



Total surface area = $2 \times \text{base} + 2 \times \text{side} + 2 \times \text{end}$

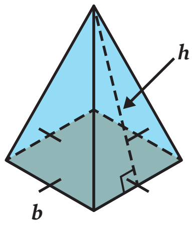
$= 2lw + 2lh + 2wh$

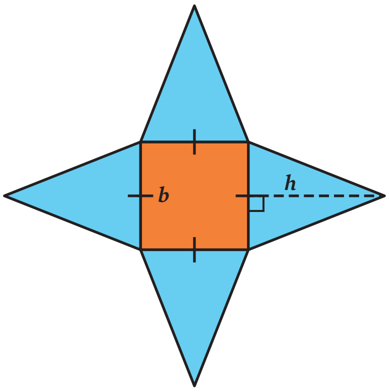


Total surface area = $2 \times \text{end} + \text{side 2} + \text{side 1} + \text{side 3}$

$= 2\left(\frac{b \times h}{2}\right) + l \times b + l \times s + l \times s$

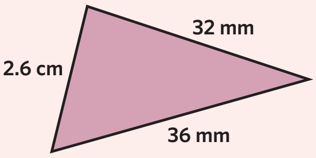
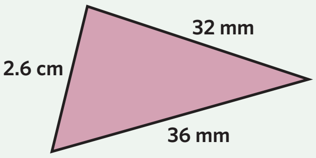
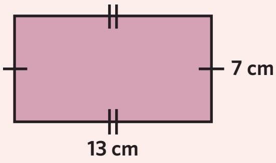
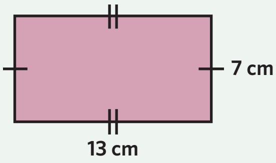
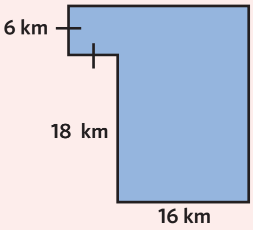
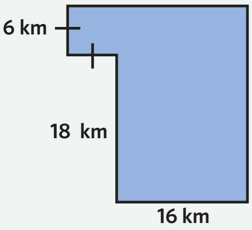
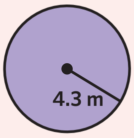
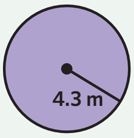
Back to contents





Total surface area = $b^2 + 4\left(\frac{b \times h}{2}\right)$

Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students do not convert all units of length to the same unit.	 $P = 32 + 36 + 2.6$ mm $P = 70.6$ mm	 2.6 cm = (2.6 × 10) mm = 26 mm $P = 32 + 36 + 26$ mm $P = 94$ mm	7A
Students calculate area instead of perimeter.	 $P = 13 \times 7$ cm $P = 91$ cm	 $P = 13 + 13 + 7 + 7$ cm $P = 40$ cm	7A, 7B, 7C, 7D
Students do not add all the missing side lengths when calculating perimeter of shapes.	 $P = 86$ km	 $P = 92$ km	7A, 7B, 7D
Students confuse diameter and radius.	 $C = 4.3 \times \pi$ ≈ 13.5 m	 $C = 8.3 \times \pi$ ≈ 27.02 m	7B, 7C, 7H

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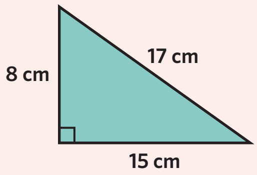
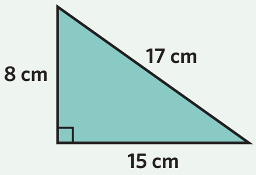
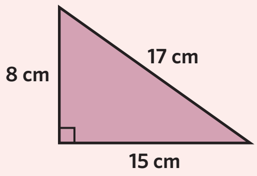
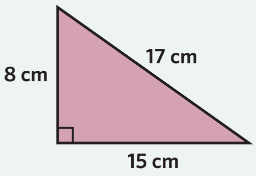
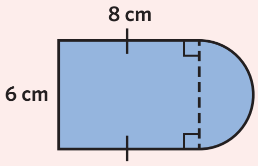
Back to contents

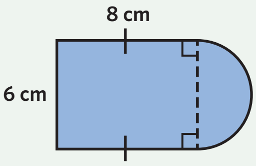
<table>
<tr><th>Misconception</th><th>Incorrect ✗</th><th>Correct ✓</th><th>Lesson</th></tr>
<tr><td>Students use the height and the side that is not the base when calculating the area of a triangle.</td><td>

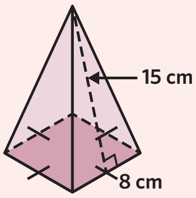
$A = \frac{17 \times 8}{2}$
$= 68\text{ cm}^2$</td><td>

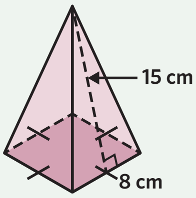
$A = \frac{15 \times 8}{2}$
$= 60\text{ cm}^2$</td><td>7C</td></tr>
<tr><td>Students do not divide by 2 when calculating the area of a triangle.</td><td>

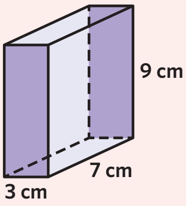
$A = 15 \times 8$
$= 120\text{ cm}^2$</td><td>

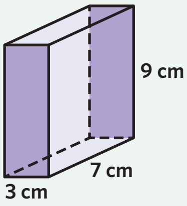
$A = \frac{15 \times 8}{2}$
$= 60\text{ cm}^2$</td><td>7C</td></tr>
<tr><td>Students add the length of adjoining edges when calculating perimeter.</td><td>

$P = \frac{6\pi}{2} + 6 + 6 + 8 + 8$
$\approx 37.42\text{ cm}$</td><td>

$P = \frac{6\pi}{2} + 6 + 8 + 8$
$\approx 31.42\text{ cm}$</td><td>7D</td></tr>
<tr><td>Students only add the visible faces when calculating TSA.</td><td>

$\text{TSA} = 8^2 + 3\left(\frac{8 \times 15}{2}\right)$
$= 64 + 180$
$= 244\text{ cm}^2$</td><td>

$\text{TSA} = 8^2 + 4\left(\frac{8 \times 15}{2}\right)$
$= 64 + 240$
$= 304\text{ cm}^2$</td><td>7E</td></tr>
<tr><td>Students confuse volume with surface area.</td><td>

$\text{TSA} = lwh$
$= 3 \times 7 \times 9$
$= 189\text{ cm}^2$</td><td>

$\text{TSA} = 2lw + 2wh + 2lh$
$= 2(3 \times 7) + 2(7 \times 9) + 2(3 \times 9)$
$= 42 + 126 + 54$
$= 222\text{ cm}^2$</td><td>7E, 7F, 7G, 7H</td></tr>
</table>

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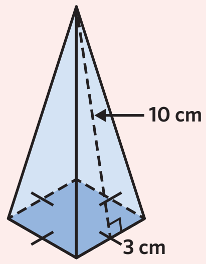
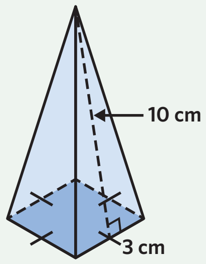
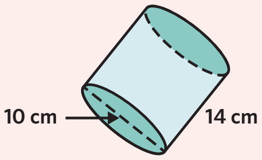
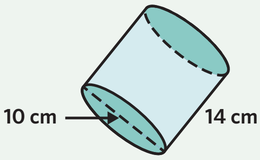
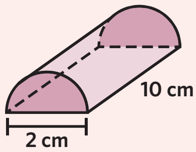
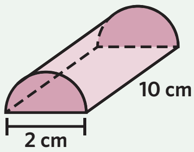
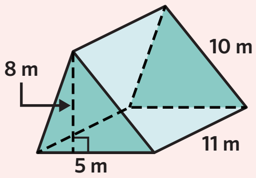
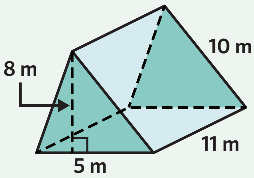
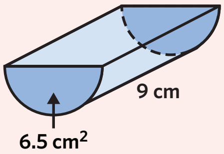
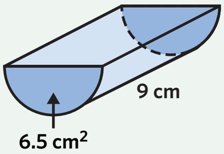
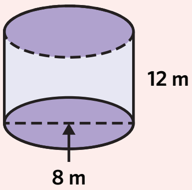
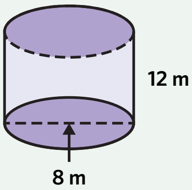
Back to contents

Misconception	Incorrect ✗	Correct ✓	Lesson
Students do not halve the area of triangular faces when calculating the total surface area of a square-based pyramid.	 $\text{TSA} = 3^2 + 4(3 \times 10)$ $= 129\text{ cm}^2$	 $\text{TSA} = 3^2 + 4\left(\frac{3 \times 10}{2}\right)$ $= 69\text{ cm}^2$	7E
Students do not add both circular bases when calculating the total surface area of a cylinder.	 $\text{TSA} = 2 \times \pi \times 5^2 + \pi \times 5 \times 14$ $= 50\pi + 70\pi$ $\approx 376.99\text{ cm}^2$	 $\text{TSA} = 2 \times \pi \times 5^2 + 2 \times \pi \times 5 \times 14$ $= 50\pi \times 140\pi$ $\approx 125.66\text{ cm}^2$	7F
Students do not add the rectangular flat face of a semi-cylinder when calculating its surface area.	 $\text{TSA} = 2 \times \frac{1}{2} \times \pi \times 1^2 + \frac{1}{2} \times 2 \times \pi \times 1 \times 10$ $= \pi + 10\pi$ $\approx 34.56\text{ cm}^2$	 $\text{TSA} = 2 \times \frac{1}{2} \times \pi \times 1^2 + \frac{1}{2} \times 2 \times \pi \times 1 \times 10 + 2 \times 10$ $= \pi + 10\pi + 20$ $\approx 54.56\text{ cm}^2$	7F
Students square the area of the cross section rather than multiplying by *h* or *w* or *l* when calculating volume.	 $\text{Volume} = \left(\frac{5 \times 8}{2}\right)^2$ $= 4400\text{ m}^3$	 $\text{Volume} = \left(\frac{5 \times 8}{2}\right) \times 11$ $= 220\text{ m}^3$	7G
Students use the area of a full circle instead of the area of a semicircle when calculating the volume of a portion of a cylinder	 $\text{Volume} = 6.5 \times 2 \times 9$ $= 117\text{ cm}^3$	 $\text{Volume} = 6.5 \times 9$ $= 58.5\text{ cm}^3$	7H
Students multiply the height of a cylinder by the length of the circular face's circumference instead of using the area of the cross-section when calculating volumer.	 $\text{Volume} = 8 \times \pi \times 12$ $= 96\pi$ $\approx 301.59\text{ m}^3$	 $\text{Volume} = \pi \times 4^2 \times 12$ $= 192\pi$ $\approx 603.19\text{ m}^3$	7H

Back to contents

7A Length and perimeter

LEARNING INTENTIONS

Students will be able to:

- convert between units of length
- determine the perimeter of simple shapes
- determine the unknown side of a shape given the perimeter.

Different units of measurement are used to measure the distance between two or more points or the length of the sides of a shape. When calculating the perimeter of shapes that involve different units of measurement, it is helpful to convert all of the shapes' side lengths to the same unit. The perimeter of the shape is often referred to as the distance around a shape.

KEY TERMS AND DEFINITIONS

- A **centimetre (cm)** is one-hundredth of a metre.
- A **kilometre (km)** is one thousand metres.
- A **metre (m)** is a standardised unit measuring length.
- A **millimetre (mm)** is one-thousandth of a metre.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

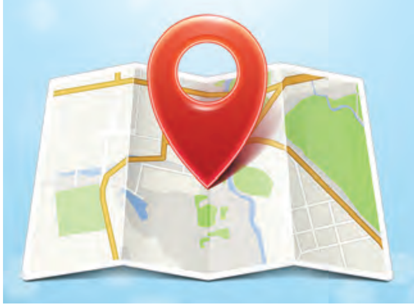
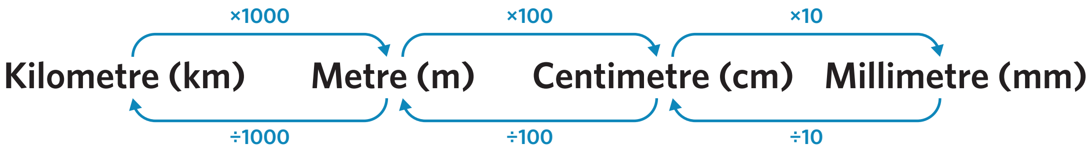


Image: Perfect Vectors/Shutterstock.com

When planning a road trip, understanding units of length and how to calculate the length of different routes can help estimate the time required for driving. This aids in efficient route planning and fuel requirement estimations.

Key idea

1. We can convert different units of length by multiplying or dividing by powers of 10.



Worked example 1

Converting unit lengths

Convert the following.

a. 19.6 cm to mm

WE1a

Working

19.6 cm = (19.6 × 10) mm

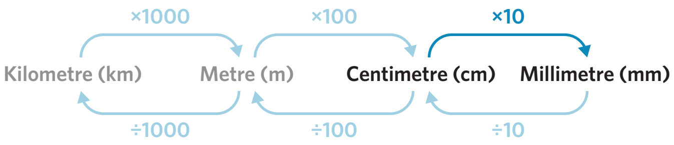
= 196 mm

Thinking

Determine the multiplication or division required.

1 cm = 10 mm

Visual support



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Back to contents

b. 276 500 mm to m

WE1b

Working

$276\,500 \text{ mm} = (276\,500 \div 10) \text{ cm}$

$= 27\,650 \text{ cm}$

$27\,650 \text{ cm} = (27\,650 \div 100) \text{ m}$

$= 276.5 \text{ m}$

Thinking

Step 1: Determine the multiplication or division required.

10 mm = 1 cm

Step 2: Determine the multiplication or division required.

100 cm = 1 m

Student practice

Convert the following.

a. 32.9 cm to mm

b. 679 200 mm to m

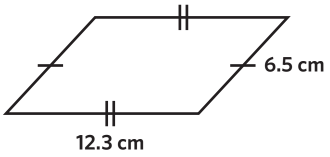
Worked example 2

Calculating the perimeter

Determine the perimeters of the following shapes.

a.

WE2a



Working

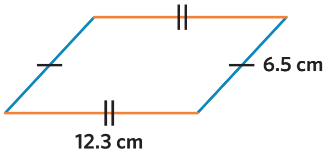
$P = (12.3 + 12.3 + 6.5 + 6.5) \text{ cm}$

$= 37.6 \text{ cm}$

Thinking

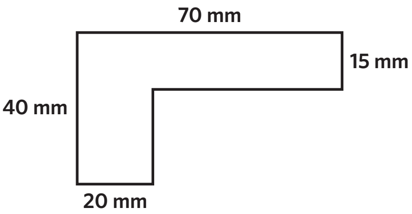
Add the lengths if they are all the same unit of measurement. Take note of equal side lengths based on the markings.

Visual support



b.

WE2b



Working

$70 - 20 = 50 \text{ mm}$

$40 - 15 = 25 \text{ mm}$

$P = (40 + 70 + 15 + 50 + 25 + 20) \text{ mm}$

$= 220 \text{ mm}$

Thinking

Step 1: Use subtraction to determine the length of the two missing sides.

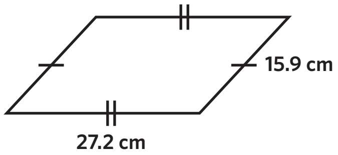
Step 2: Add the lengths if they are all the same unit of measurement.

Continues →

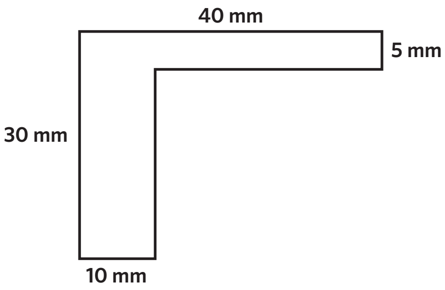
Student practice

Determine the perimeters of the following shapes.

a.



b.

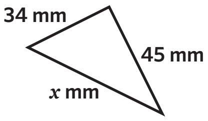


Worked example 3

Calculating the missing value given a perimeter

Determine the length of the unknown side.

a. WE3a



Perimeter = 134 mm

Working

$P = 134$ mm

$$134 = 34 + 45 + x$$

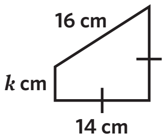
$$134 - 34 - 45 = x$$

$$x = 55 \text{ mm}$$

Thinking

Subtract known side lengths from the perimeter.

b. WE3b



Perimeter = 48 cm

Working

$P = 48$ cm

$$48 = 16 + 14 + 14 + k$$

$$48 - 16 - 14 - 14 = k$$

$$k = 4 \text{ cm}$$

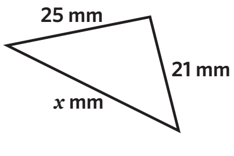
Thinking

Subtract known side lengths from the perimeter. Take note of equal side lengths based on the markings.

Student practice

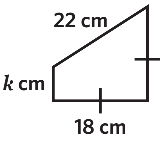
Determine the length of the unknown side.

a.



Perimeter = 79 mm

b.



Perimeter = 64 cm

Back to contents

7A Questions

Understanding worksheet

1. Fill in the missing information to convert the units.

Example

10 m to cm

10 $\boxed{\times}$ 100 = 10 000

a. 150 mm to cm

150 ☐ 10 = 15 cm

b. 7250 m to km

7250 ÷ ☐ = 7.25 km

c. 268 m to cm

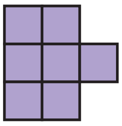
268 × 100 = ☐

d. 8657 mm to m

8657 ÷ ☐ = 8.657 m

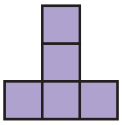
2. Calculate the perimeter of each shape. Each unit block has a side length of 1 cm.

Example



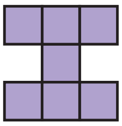
Perimeter = $\boxed{12}$ **cm**

a.



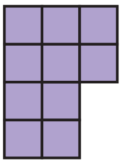
Perimeter = ☐ **cm**

b.



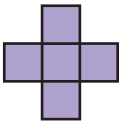
Perimeter = ☐ **cm**

c.



Perimeter = ☐ **cm**

d.



Perimeter = ☐ **cm**

3. Fill in the blanks by using the words provided.

convert **perimeter** **length** **measurement**

The ☐ is the distance around the outside of a shape. Metric units of ☐ are all related by powers of ten. Sides that have the same markings have the same ☐ . When calculating the perimeter of a shape that has different units of measurement for different sides it is important to ☐ all measurements to the same unit of length.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

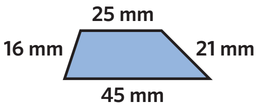
4. Convert the following. WE1

a. 300 cm to m
b. 15.3 cm to mm
c. 7850 m to km
d. 127.5 cm to m
e. 7.5 m to mm
f. 8459 cm to km
g. 0.0263 m to mm
h. 8 560 000 mm to km

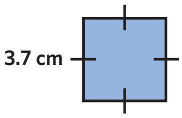
Back to contents

5. Determine the perimeters of the following shapes. WE2a

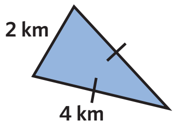
a.



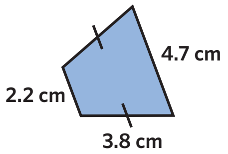
b.



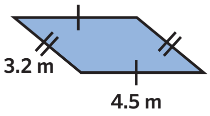
c.



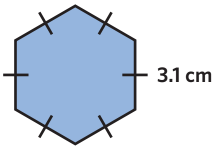
d.



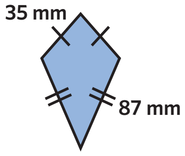
e.



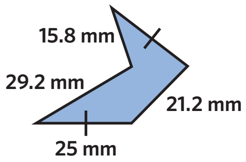
f.



g.

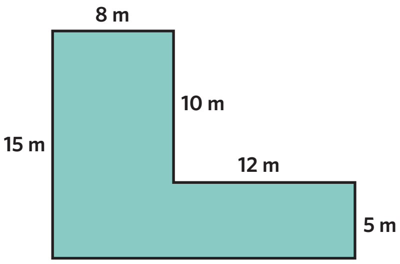


h.

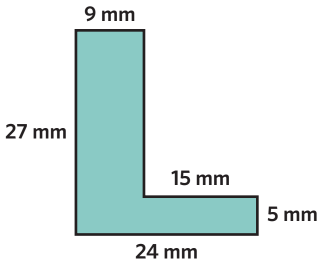


6. Determine the perimeters of the following shapes. WE2b

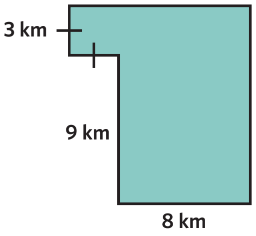
a.



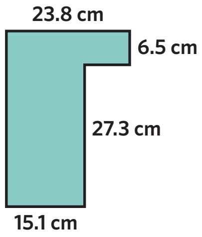
b.



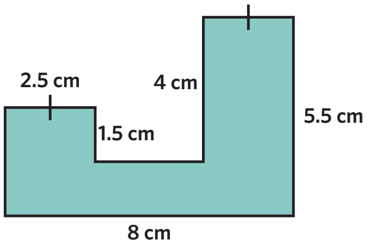
c.



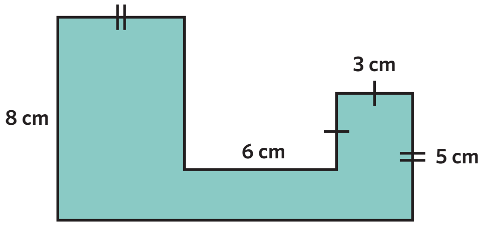
d.



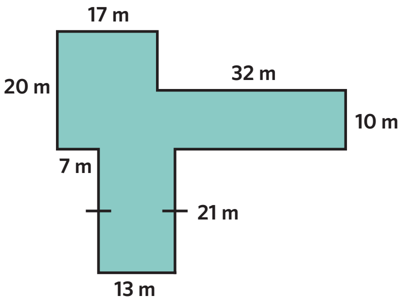
e.



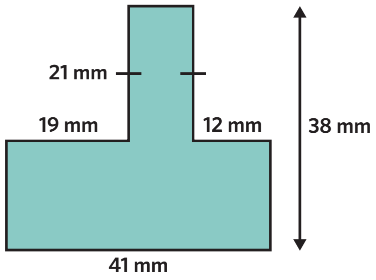
f.



g.

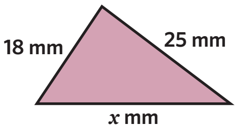


h.



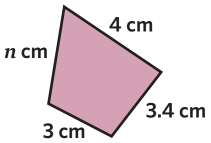
7. Determine the length of the unknown side. WE3

a.



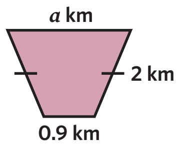
Perimeter = 73 mm

b.



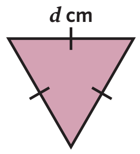
Perimeter = 15 cm

c.



Perimeter = 7.6 km

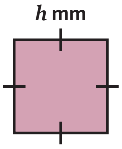
d.



Perimeter = 183 cm

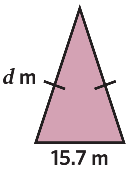
Back to contents

e.



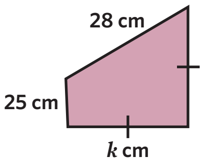
Perimeter = 160 mm

f.



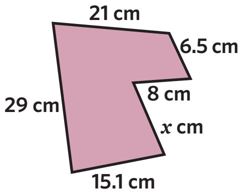
Perimeter = 57.9 m

g.



Perimeter = 96 cm

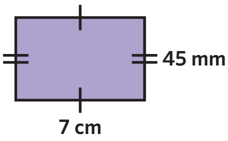
h.



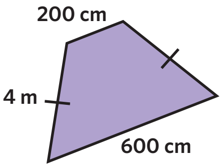
Perimeter = 99.6 cm

8. Calculate the perimeter of the following shapes in the specified unit.

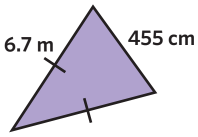
a. (cm)



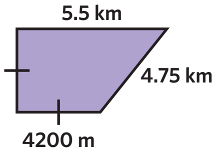
b. (m)



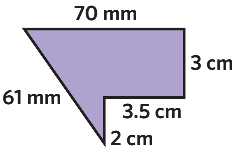
c. (m)



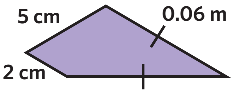
d. (km)



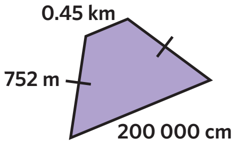
e. (mm)



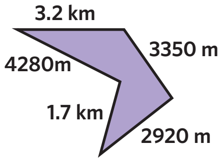
f. (mm)



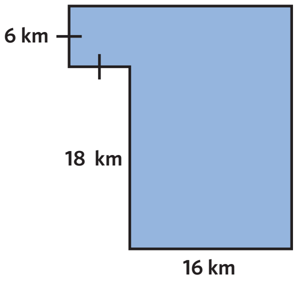
g. (km)



h. (cm)



9. Determine the perimeter of the shape.

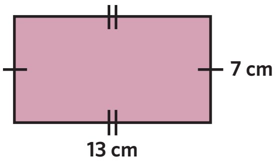


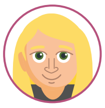
A. 40 km **B.** 46 km **C.** 68 km **D.** 86 km **E.** 92 km

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Determine the perimeter of the shape.

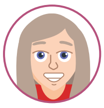




Student A

$P = 13 + 13 + 7 + 7$ cm

$P = 40$ cm

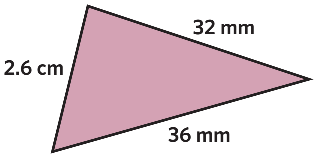


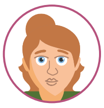
Student B

$P = 13 \times 7$ cm

$P = 91$ cm

b. Determine the perimeter of the shape.





Student A

$P = 32 + 36 + 2.6$ mm

$P = 70.6$ mm

Student B

2.6 cm $= (2.6 \times 10)$ mm

$= 26$ mm

$P = 32 + 36 + 26$ mm

$P = 94$ mm

Problem solving

Question working paths

Mild 11, 12, 13	**Medium** 12, 13, 14	**Spicy** 13, 14, 15

11. How many centimetres of purple ribbon does Jemma have if she has 4 different pieces that measure 21 cm, 1.7 m, 3520 mm and 98 cm?

12. The perimeter of Ralph's rectangular dining table is 2.6 m. Each of the long sides are 95 cm. What is the length of the short sides in metres?

13. Cassandra charges her customers $21.50 for each kilometre they travel in her horse and carriage. How much will a customer pay for a 5600 metre journey?

14. Peter's Pool Yard Company builds a pool that is 15 m by 11 m. The pool comes with a standard decking that can be assembled around the edges of the pool. The width of the decking is 55 cm. In metres, what is the length of fencing if it is built around the outer side of the decking?

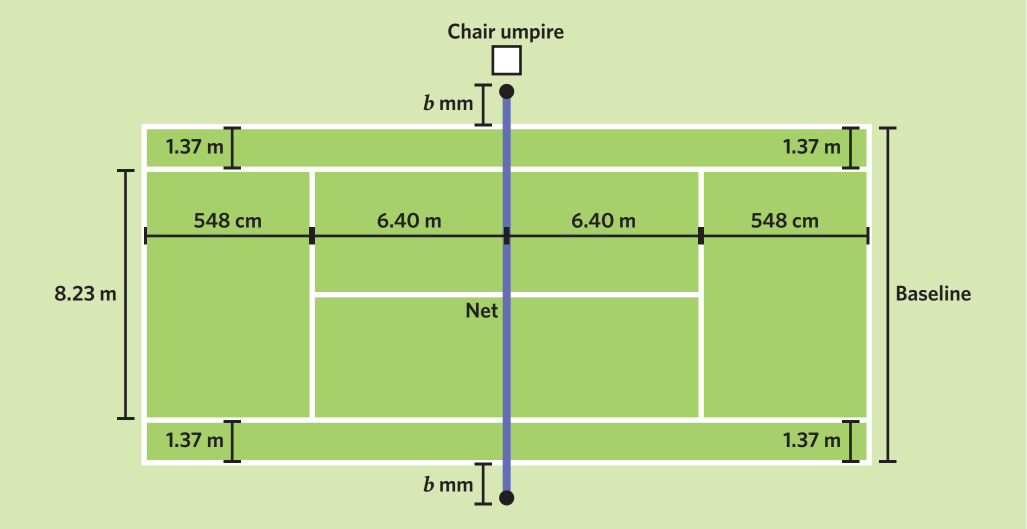
15. Phil made a mosaic around a 2.25 m perimeter of a kite shaped mirror. Two of the equal sides sum to 128 cm. Using *a* to represent one of the unknown sides, write an equation to represent the situation and calculate the missing side lengths in metres.

Reasoning

Question working paths

Mild 16 (a,b,c,e)	**Medium** 16 (a,b,c,e), 17 (a,b)	**Spicy** All

16. Helen lives on a farm in the countryside and wants to learn how to play tennis. She has decided to build a tennis court on her property. She uses the following design to understand the dimensions of the space required to construct a court.



a. What is the length of the baseline in centimetres?

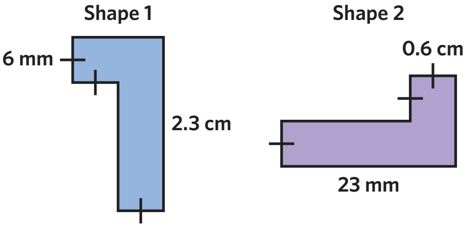
b. The chair umpire sits in the middle of the court. What is the length of the entire side of the court from the right-hand side baseline to the left-hand side base line? Give your answer in metres.

c. The total length of the net is 12.79 m. Calculate the value of *b*.

d. Helen needs 25 ml of white paint for every 1 m of each line. How much paint, in ml, does she need to paint all the lines of the court?

e. Other than the size of the court, what other factors might Helen need to consider before deciding to build the court?

17. Consider the following shapes.



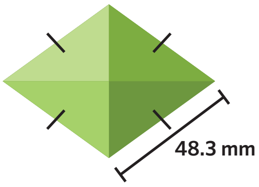
a. Calculate the perimeter of shape 1. Give your answer in centimetres.

b. Calculate the perimeter of shape 2. Give your answer in millimetres.

c. Comment on the similarities and differences between shape 1 and shape 2 and explain why the answers in parts **a** and **b** are equal.

Exam-style

18. The four sides of a rhombus shaped diamond jewel have a side length of 48.3 mm, as shown in the following diagram. (1 MARK)

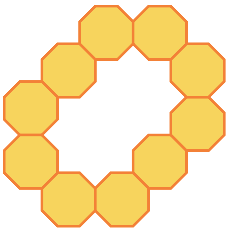


A jeweller outlined the outer side of the diamond with platinum gold. What is the minimum distance that the gold needs to cover so that the full diamond is encircled by gold?

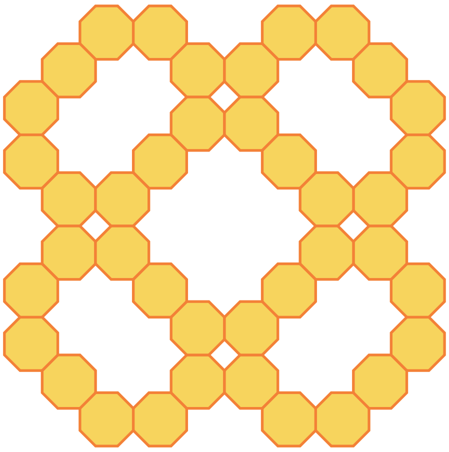
A. 48.3 mm **B.** 96.6 mm **C.** 144.9 mm **D.** 193.2 mm **E.** 241.5 mm

19. A standard octagon coin has a side length of 1.7 cm. (3 MARKS)

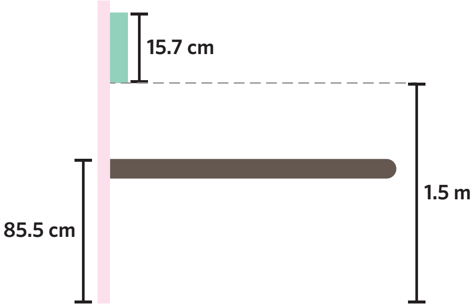
a. What is the perimeter of the outside of the following layout of standard octagon coins? Give your answer in millimetres. 1 MARK



b. Four identical layouts are put side by side, as shown in the diagram. What is the outer perimeter adjoining layout? Give your answer in millimetres. 2 MARKS



20. A table folds out of a wall unit. The length of the wall from the floor to the top of the table is 85.5 cm. A picture frame hangs on the wall of the wall unit. The frame is 15.7 cm long. The bottom of the frame is 1.5 m from the floor. (2 MARKS)

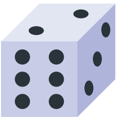


The current picture frame is replaced by a frame that is 25% shorter in length. The top of the new frame is hung in the same position as the original, so that its top reaches the same highest point of the original. What is the length from the top of the table to the bottom of the new hanging picture frame in metres?

21. Freddie uses 150 cm^2 of wrapping paper to wrap a cube shaped present. He then uses a special diamond encrusted ribbon to cover the perimeter of the top face of the gift. If the cost of the special ribbon is $123.25 per centimetre, what is the cost of the ribbon rounded to the nearest 50 dollars? (2 MARKS)

Remember this?

22. Jenny rolls a six sided die 300 times.



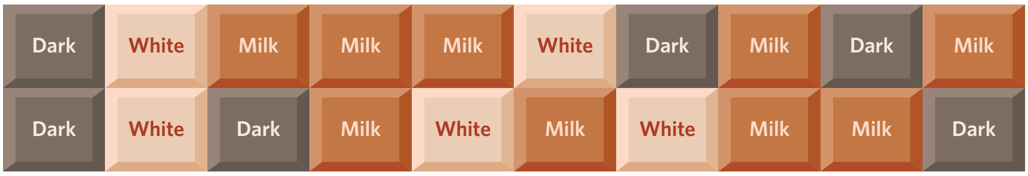
Which of the following results is most likely to occur?

A. 43 ones **B.** 52 twos **C.** 57 threes **D.** 104 fours **E.** 295 fives

23. Which of the following probabilities best describes the chances of an event happening as very unlikely?

A. $\frac{1}{4}$ **B.** $\frac{3}{10}$ **C.** $\frac{4}{25}$ **D.** $\frac{6}{12}$ **E.** $\frac{8}{20}$

24. A block of chocolate has three different types of chocolate; milk chocolate, dark chocolate and white chocolate. Each block is equally sized.



Dark	White	Milk	Milk	Milk	White	Dark	Milk	Dark	Milk
Dark	White	Dark	Milk	White	Milk	White	Milk	Milk	Dark

What proportion of the bar consists of dark chocolate?

A. 0.03 **B.** 0.06 **C.** 0.3 **D.** 0.6 **E.** 3.0

Back to contents

7B Circumference and perimeter of a sector

LEARNING INTENTIONS

Students will be able to:

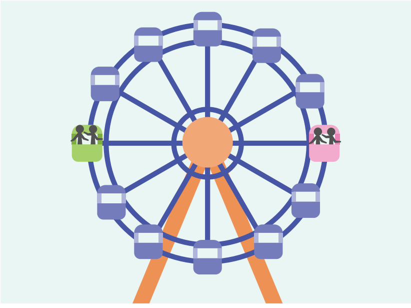
- understand and use the formula for the circumference of a circle
- calculate circumference to an exact number and with a calculator
- understand sectors as a fraction of a circle
- calculate the perimeter of a sector.

The perimeter of a circle is referred to as circumference. Formulas can be used to calculate the perimeter of a circle or sector. When calculating the circumference of a circle or the perimeter of a sector it is helpful to be able to identify the diameter or radius of the shapes. A sector is made up of three sides: two radii and an arc. An arc is the circular part of a sector.

KEY TERMS AND DEFINITIONS

- An **arc** is a part of the circumference of a circle.
- The **circumference** is the perimeter of a circle.
- The **diameter** is a straight line between two points on the circumference of a circle that passes through the centre.
- The **radius** is the direct distance from the centre of a circle to any point on the circumference.
- A **sector** is part of a circle that is enclosed by two radii and an arc.

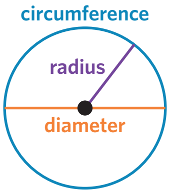
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



When creating a ferris wheel, calculating the circumference of a circle and perimeter of a sector can help with the accurate and safe design of the ride. This aids the efficient placement of seats around the ferris wheel so that the seats are evenly distributed and the ride remains balanced.

Key ideas

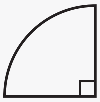
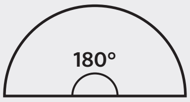
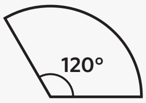
1. Formulas are used to calculate the **circumference** of a circle.



$$\frac{C}{d} = \pi \qquad \frac{C}{2r} = \pi$$

$$C = \pi d \qquad C = 2\pi r$$

2. A **sector** is a fraction of a circle, and its **arc** length represents a fraction of the **circumference**.

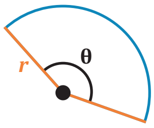
Sector	Fraction of circle	Arc length
	$\frac{90}{360} = \frac{1}{4}$	$\frac{1}{4} \times 2\pi r$
 180°	$\frac{180}{360} = \frac{1}{2}$	$\frac{1}{2} \times 2\pi r$
 120°	$\frac{120}{360} = \frac{1}{3}$	$\frac{1}{3} \times 2\pi r$

Continues →

3. The perimeter of a **sector** is calculated by adding the length of the **arc** and the two radii that enclose the sector.

Perimeter of a **sector** = Arc length + radius + radius

Perimeter of a **sector** $= \frac{\theta}{360} \times 2\pi r + 2r$

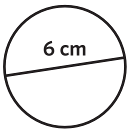


Worked example 1

Calculating the circumference of a circle in terms of π

Calculate the circumference in terms of π and correct to two decimal places.

a. WE1a



Working

$C = \pi d$

$= 6\pi$ cm

$= 6 \times \pi$

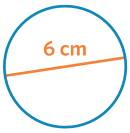
≈ 18.85 cm

Thinking

Step 1: Apply the formula and express the circumference in terms of π.

Step 2: Calculate the circumference and round the value to two decimal places.

Visual support

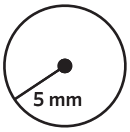


$C = \pi 6$

$C = 6\pi$

$C \approx 18.85$ cm

b. WE1b



Working

$C = 2\pi r$

$= 2 \times \pi \times 5$

$= 10\pi$ mm

$= 10 \times \pi$

≈ 31.42 mm

Thinking

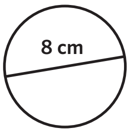
Step 1: Apply the formula and express the circumference in terms of π.

Step 2: Calculate the circumference and round the value to two decimal places.

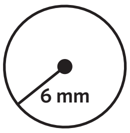
Student practice

Calculate the circumference in terms of π and correct to two decimal places.

a.



b.



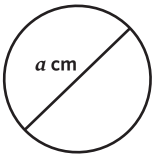
Worked example 2

Calculating the radius and diameter given the circumference

Determine the length of the unknown in terms of π and correct to two decimal places.

a.

WE2a



Circumference = 22 cm

Working

$C = \pi d$

$22 = a\pi$

$a = \frac{22}{\pi}$ cm

≈ 7.00 cm

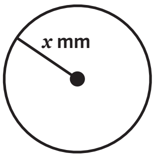
Thinking

Step 1: Use the formula and express the circumference in terms of π.

Step 2: Divide the circumference by π and round to two decimal places.

b.

WE2b



Circumference = 356 mm

Working

$C = 2\pi r$

$356 = 2x\pi$

$x = \frac{356}{2\pi}$

$x = \frac{178}{\pi}$ mm

$x \approx 56.66$ mm

Thinking

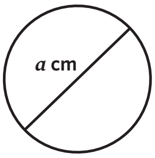
Step 1: Use the formula and express the circumference in terms of π.

Step 2: Divide the circumference by 2π and round to two decimal places.

Student practice

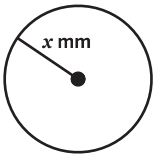
Determine the length of the unknown in terms of π and correct to two decimal places.

a.



Circumference = 55 cm

b.



Circumference = 246 mm

Back to contents

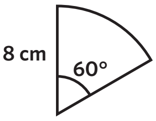
Worked example 3

Calculating the perimeter of a sector

Calculate the perimeter of the sector in terms of π and correct to two decimal places.

a.

WE3a



Working

$P = \frac{\theta}{60} \times 2\pi r + 2r$

$P = \frac{60}{360} \times 2\pi \times 8 + 2 \times 8$

$P = \frac{1}{6} \times 16\pi + 16$

$P = \frac{8}{3}\pi + 16$ cm

$P \approx 8.38 + 16$

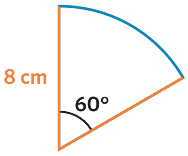
$P \approx 24.38$ cm

Thinking

Step 1: Apply the formula and express the perimeter in terms of π.

Step 2: Calculate the perimeter and round the value to two decimal places.

Visual support



$P = \frac{\theta}{360} + 2\pi r + 2r$

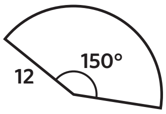
$P = \frac{60}{360} + 2\pi \times 8 + 2 \times 8$

$P \approx 8.38 + 16$

$P \approx 24.38$ cm

b.

WE3b



Working

$P = \frac{\theta}{360} \times 2\pi r + 2r$

$P = \frac{150}{360} \times 2\pi \times 12 + 2 \times 12$

$P = \frac{5}{12} \times 24\pi + 24$ units

$P = 10\pi + 24$

$P \approx 55.42$ units

Thinking

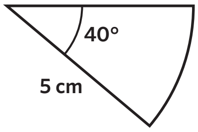
Step 1: Apply the formula and express the perimeter in terms of π.

Step 2: Calculate the perimeter and round the value to two decimal places.

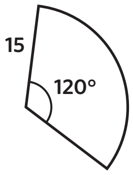
Student practice

Calculate the perimeter of the sector in terms of π and correct to two decimal places.

a.



b.

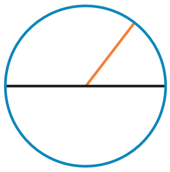


7B Questions

Understanding worksheet

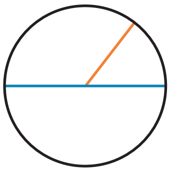
1. State what part of each circle or sector is coloured in blue.

Example

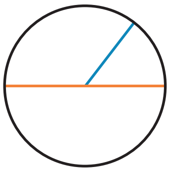


Circumference

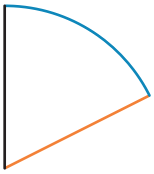
a.



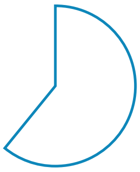
b.



c.

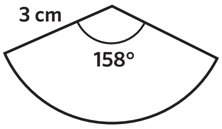


d.



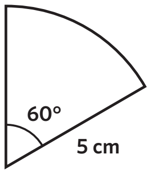
2. Complete the missing information.

Example



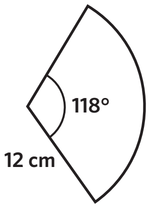
$$\frac{\boxed{158}}{360} \times 2\pi \times 3 + 2 \times 3$$

a.



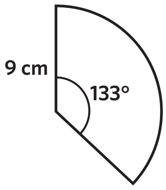
$$\frac{60}{360} \times 2\pi \times 5 + 2 \times \boxed{}$$

b.



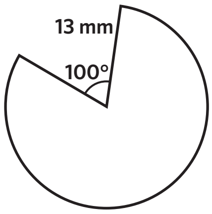
$$\frac{\boxed{}}{360} \times 2\pi \times 12 + 2 \times 12$$

c.



$$\frac{133}{360} \times \boxed{} \times 9 + 2 \times 9$$

d.



$$\frac{\boxed{}}{360} \times 2\pi \times 13 + 2 \times 13$$

3. Fill in the blanks by using the words provided.

arc **circumference** **radius** **sector**

The perimeter of a circle is referred to as the $\boxed{}$. The $\boxed{}$ of a circle is the straight line joining its centre to any point on the circumference. The curved edge edge of a $\boxed{}$ is an $\boxed{}$.

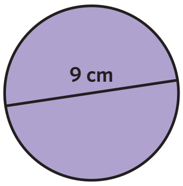
Fluency

Question working paths

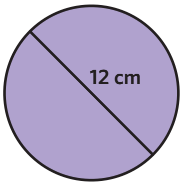
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Calculate the circumference in terms of π and correct to two decimal places. **WE1**

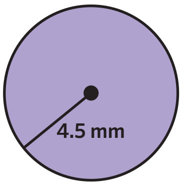
a.



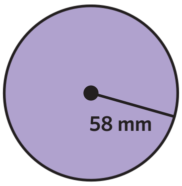
b.



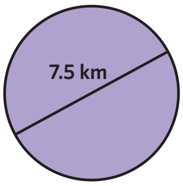
c.



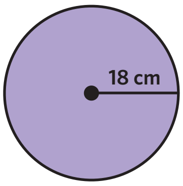
d.



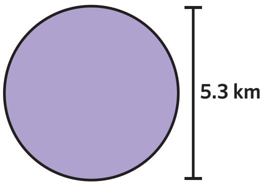
e.



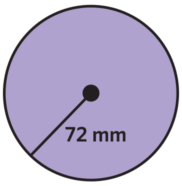
f.



g.



h.



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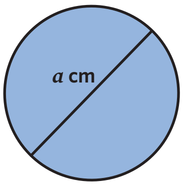
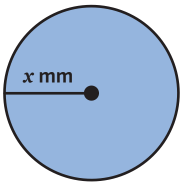
5. Determine the length of the unknown in terms of π and correct to two decimal places. **WE2**

a.



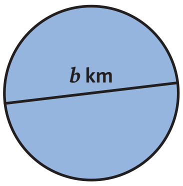
Circumference = 34 cm

b.



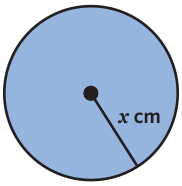
Circumference = 60 mm

c.



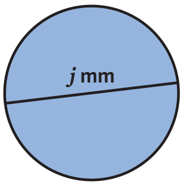
Circumference = 121 km

d.



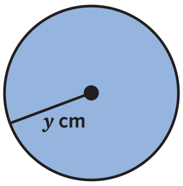
Circumference = 78.5 cm

e.



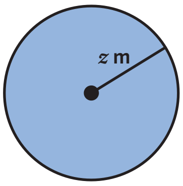
Circumference = 88 mm

f.



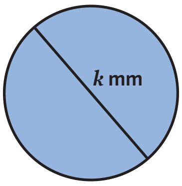
Circumference = 173 cm

g.



Circumference = 237.6 m

h.



Circumference = 169.64 cm

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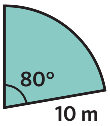
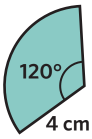
6. Calculate the perimeter of the sector in terms of π and correct to two decimal places. **WE3**

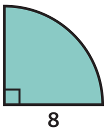
a.



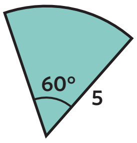
b.



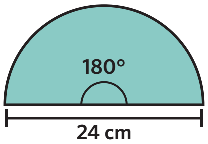
c.



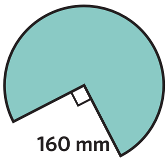
d.



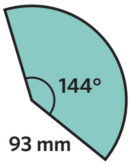
e.



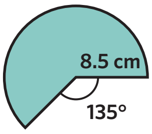
f.



g.



h.



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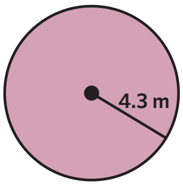
7. Determine the circumference, correct to two decimal places.



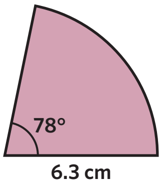
A. 4.3 m
B. 13.5 m
C. 8.6π m
D. 27.02 m
E. 58.09 m

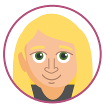
Back to contents

Spot the mistake

8. Select whether Student A or Student B is incorrect.

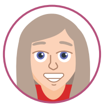
a. Calculate the perimeter of the sector.





Student A

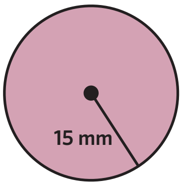
$$P = \frac{78}{360} \times 2\pi \times 6.3 + 6.3$$
$$\approx 8.58 + 6.3$$
$$\approx 14.88 \text{ cm}$$

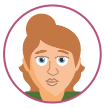


Student B

$$P = \frac{78}{360} \times 2\pi \times 6.3 + 6.3 + 6.3$$
$$\approx 8.58 + 6.3 + 6.3$$
$$\approx 21.18 \text{ cm}$$

b. Determine the perimeter of the circle.





Student A

$$A = 2\pi \times 15$$
$$= 30\pi$$
$$\approx 94.25 \text{ mm}$$

Student B

$$A = \pi \times 15^2$$
$$= 225\pi$$
$$\approx 706.86 \text{ mm}$$

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. A circular playpen for toddlers has a radius of 5.8 m. What is the circumference of the playpen in metres? Round your answer to two decimal places.

10. The diameter of a bike wheel is 66 cm. How many rotations does the wheel need to complete to travel 5 km?

11. Joe's Pizza sells pizza by the slice. Joe makes large circular pizzas with a diameter of 18 cm and cuts them into 6 slices. What is the perimeter of one slice in centimetres? Round your answer to two decimal places.

12. The large hand on a clock measures 16 cm and reaches from the centre of a clock to its outer edge. How many centimetres, rounded to two decimal places, will the tip traverse in 25 minutes?

13. A roundabout in the centre of Paris has a diameter of 187 m. One sector of the roundabout is lined with hedges to create a dog play area. Two of the hedges form an angle of 129° at the centre of the roundabout. What is the perimeter of the dog play area rounded to two decimal places?

Back to contents

Reasoning

Question working paths

Mild 14 (a,b,d) 🌶	**Medium** 14 (a,b,d), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. Millie is designing a logo for a new active wear brand. She is experimenting by adjoining a circle. The original design contains an orange sector with a radius of 10 cm and a blue sector with an internal angle of 145° and a radius of 8 cm.

Figure 1

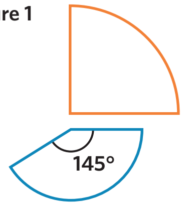
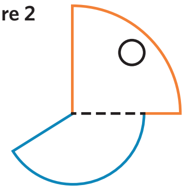


Figure 2



a. The orange part of the logo was created by cutting a full circle into four equally sized sectors. What is the internal angle of the orange sector?

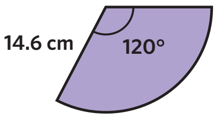
b. If Millie changes the internal angle of the blue sector in Figure 1 to 150° and maintains the same radius, what is the difference between the perimeter of the adjusted blue sector and the original? Give your answer in millimetres rounded to two decimal places.

c. Millie maintains the dimensions of the original design to create the logo shown in Figure 2. What is the perimeter of the logo rounded to two decimal places?

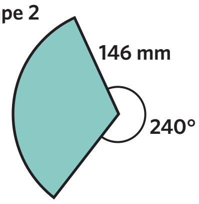
d. Millie's boss tells her that the logo needs to be more eye-catching. Propose a constructive way that Millie could respond to her boss.

15. Consider the following shapes.

Shape 1



Shape 2



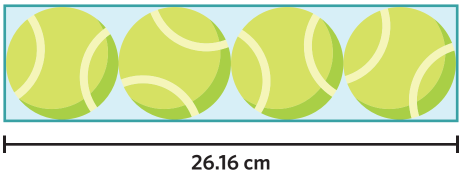
a. Calculate the perimeter of shape 1. Give your answer in cm, rounded to two decimal places.

b. Calculate the perimeter of shape 2. Give your answer in mm, rounded to two decimal places.

c. Comment on the similarities and differences between shape 1 and shape 2 and explain why the answers in parts **a** and **b** are equal.

Exam-style

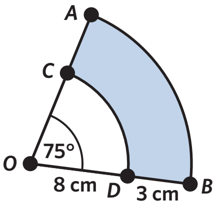
16. A tennis ball is spherical in shape. A standard tennis ball container holds four balls. (1 MARK)



What is the radius of a tennis ball, in millimetres?

A. 3.27 mm **B.** 6.54 mm **C.** 32.7 mm **D.** 654 mm **E.** 2616 mm

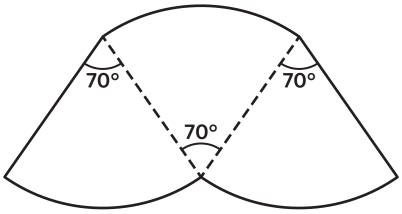
17. *OAB* is a sector of a circle. *OCD* is a sector of a circle. Both sectors have the same centre point (O). (3 MARKS)



a. Calculate the perimeter of the white sector. Give the exact answer. 1 MARK

b. Calculate the perimeter of the shaded region. Give the exact answer. 2 MARKS

18. The following diagram shows a shape that consists of three identical sectors. The perimeter of the shape is 51 cm. (2 MARKS)



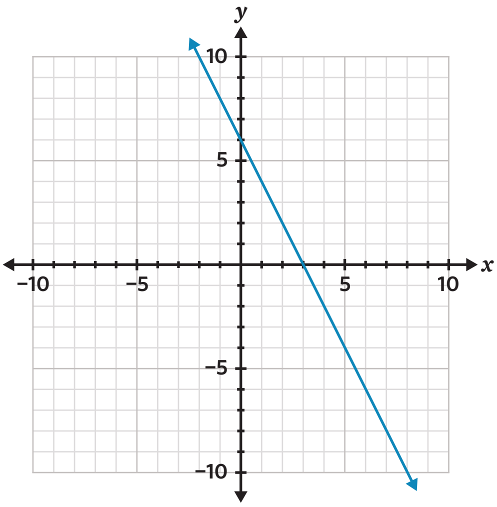
Calculate the radius of the sector. Round to the nearest whole centimetre.

19. Sector A and Sector B have the same perimeter. Sector A has an internal angle of 197° and a radius of 8.75 cm. Sector B has a radius of 17.5 cm. (3 MARKS)

State the internal angle of Sector B. Round your answer to one decimal place.

Remember this?

20. Consider the given line.



The equation that best describes this line is

A. $y = 2x + 6$
B. $y = 2x - 6$
C. $y = -2x + 6$
D. $y = -2x - 6$
E. $x = -2y + 6$

21. Edrolo High School conducts an end of year ceremony to celebrate their students' achievements in the past year.

In 2020, $\frac{4}{7}$ of the attendees were parents and guardians.

If there were 602 attendees, there were

A. 86 parents and guardians.
B. 157 parents and guardians.
C. 258 parents and guardians.
D. 344 parents and guardians.
E. 528 parents and guardians.

22. A Biology class is conducting an experiment on the rate of reaction of certain enzymes.

Which of the following beakers has the greatest amount of hydrogen peroxide?

A. Beaker A with 0.5 mL of hydrogen peroxide.
B. Beaker B with 0.24 L of hydrogen peroxide.
C. Beaker C with 500 mL of hydrogen peroxide.
D. Beaker D with 250 mL of hydrogen peroxide.
E. Beaker E with 0.05 L of hydrogen peroxide.

Back to contents

7C Area

LEARNING INTENTIONS

Students will be able to:

- understand the meaning of a square unit
- convert between units of area
- calculate the area of two-dimensional shapes using formulas.

Different units of measurement are used to measure the distance between two or more points or the length of the sides of a shape. When calculating the area of shapes that involve different units of measurement, it is helpful to convert all of the shapes' side lengths to the same unit. The area of a 2D shape is often referred to as the amount of two-dimensional space taken up by the shape. Area is always given in square units.

KEY TERMS AND DEFINITIONS

- The **area** is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.
- A **hectare** is a metric unit of area equal to 10 000 square metres.

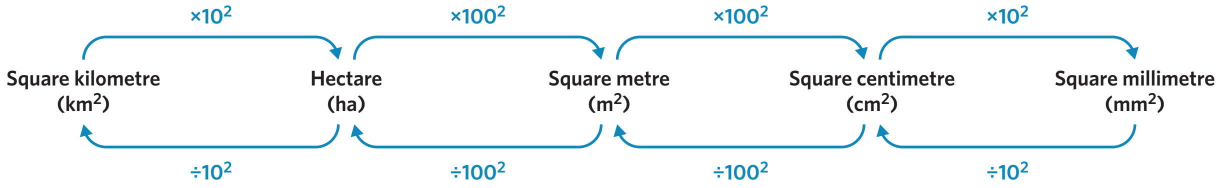
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: ymgerman/Shutterstock.com

Real estate agents and prospective buyers use the location and area of properties. Knowing the area of a property can help agents to estimate the value of a house or block of flats. This helps agents to guide prospective buyers to look at properties that are suitable for their budgets.

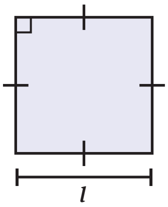
Key ideas

1. We can convert between different units of area measurement by multiplying or dividing by powers of 10.

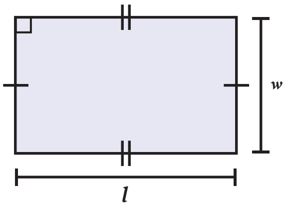


2. Formulas are used to calculate the **area** of 2D shapes.

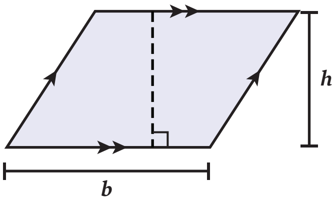
Square $A = l^2$



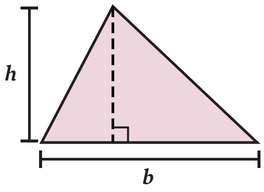
Rectangle $A = lw$



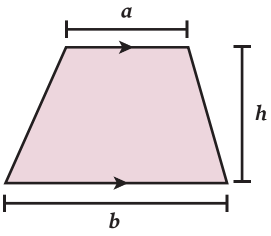
Parallelogram $A = bh$



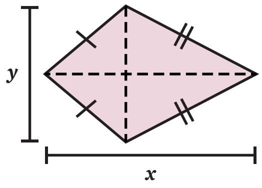
Triangle $A = \frac{b \times h}{2}$



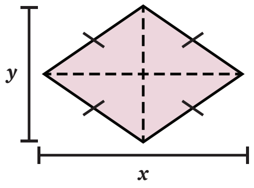
Trapezium $A = \frac{1}{2}(a + b)h$



Kite $A = \frac{1}{2}xy$



Rhombus $A = \frac{1}{2}xy$



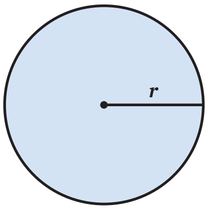
Continues →

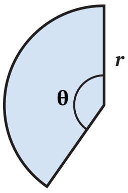
Back to contents

3. The **area** of a sector is calculated by multiplying $\frac{\theta}{360}$ by the area of a full circle with the same radius.

Circle $A = \pi r^2$

Sector $A = \frac{\theta}{360} \times \pi r^2$





Worked example 1

Converting between units of area measurement

Convert.

a. 560 mm^2 to cm^2

WE1a

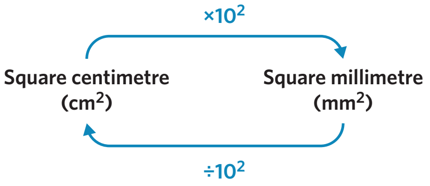
Working

$560 \div 10^2 = 5.6 \text{ cm}^2$

Thinking

Complete the necessary multiplication or division and state the value, with units.

Visual support



b. 0.75 km^2 to m^2

WE1b

Working

$0.75 \times 10^2 \times 100^2 = 750\ 000 \text{ m}^2$

Thinking

Complete the necessary multiplication or division and state the value, with units.

Student practice

Convert.

a. 380 mm^2 to cm^2 **b.** 0.89 km^2 to m^2

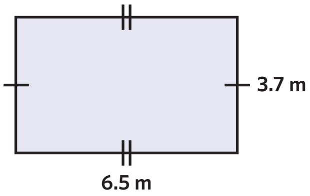
Worked example 2

Calculating the area of rectangles, triangles and parallelograms

Determine the areas of the following shapes.

a.

WE2a



Working

$A = l \times w$

$A = 6.5 \times 3.7$

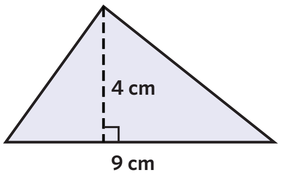
$= 24.05\ \text{m}^2$

Thinking

Write the formula for the area of a rectangle and substitute the side lengths to calculate the area, with units.

b.

WE2b



Working

$A = \frac{b \times h}{2}$

$A = \frac{9 \times 4}{2}$

$= \frac{36}{2}$

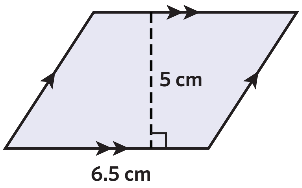
$= 18\ \text{cm}^2$

Thinking

Write the formula for the area of a triangle and substitute the side lengths to calculate the area, with units.

c.

WE2c



Working

$A = b \times h$

$A = 6.5 \times 5$

$= 32.5\ \text{cm}^2$

Thinking

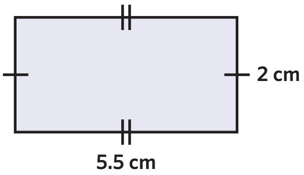
Write the formula for the area of a parallelogram and substitute the side lengths to calculate the area, with units.

Continues →

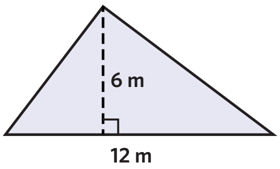
Student practice

Determine the areas of the following shapes.

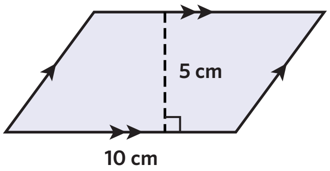
a.



b.



c.



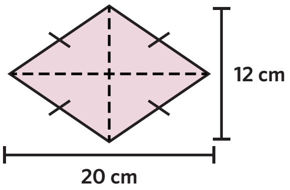
Worked example 3

Calculating the area of rhombuses and trapeziums

Determine the areas of the following shapes.

a.

WE3a



Working

$A = \frac{1}{2}xy$

$A = \frac{1}{2} \times 20 \times 12$

$= 10 \times 12$

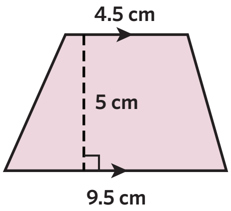
$= 120 \text{ cm}^2$

Thinking

Write the formula for the area of a rhombus and substitute the lengths to calculate the area, with units.

b.

WE3b



Working

$A = \frac{1}{2}(4.5 + 9.5) \times 5$

$= 7 \times 5$

$= 35 \text{ cm}^2$

Thinking

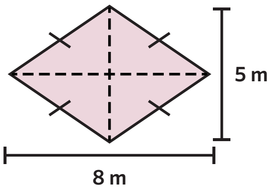
Write the formula for the area of a trapezium and substitute the lengths to calculate the area, with units.

Continues →

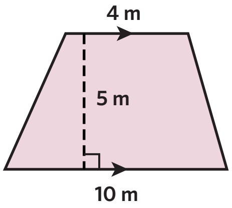
Student practice

Determine the areas of the following shapes.

a.



b.

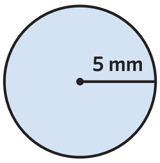


Worked example 4

Calculating the area of circles and sectors

Calculate the areas of the following shapes, correct to 2 decimal places.

a. WE4a



Working

$A = \pi r^2$

$A = \pi \times 5^2$

$= 78.539...$

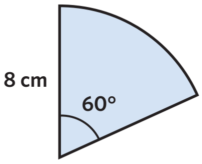
$\approx 78.54 \text{ mm}^2$

Thinking

Step 1: Write the formula for the area of a circle and substitute the radius to calculate the area.

Step 2: Round the answer to the specified number of decimal places and include units.

b. WE4b



Working

$A = \frac{\theta}{360} \pi r^2$

$A = \frac{60}{360} \times \pi \times 8^2$

$= 33.510...$

$\approx 33.51 \text{ cm}^2$

Thinking

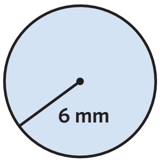
Step 1: Write the formula for the area of a sector and substitute the radius to calculate the area.

Step 2: Round the answer to the specified number of decimal places and include units.

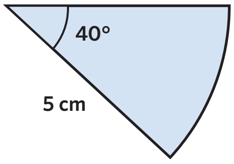
Student practice

Calculate the areas of the following shapes, correct to 2 decimal places.

a.



b.



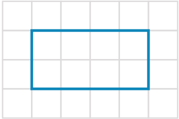
Back to contents

7C Questions

Understanding worksheet

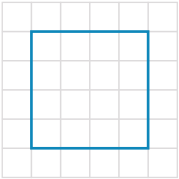
1. Calculate the area of each shape. Each unit block has an area of 1 cm^2.

Example



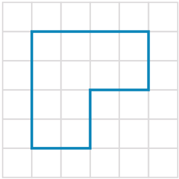
Area = $\boxed{8}$ cm^2

a.



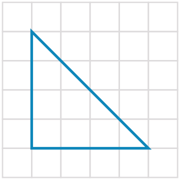
Area = $\boxed{}$ cm^2

b.



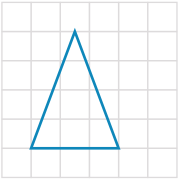
Area = $\boxed{}$ cm^2

c.



Area = $\boxed{}$ cm^2

d.



Area = $\boxed{}$ cm^2

2. Fill in the missing unit of measurement.

Example

328.9864 m^2 = 32 898 640 $\boxed{\text{mm}^2}$

a. 21 cm^2 = 2100 $\boxed{}$

b. 15 ha = 150 000 $\boxed{}$

c. 98 m^2 = 0.0098 $\boxed{}$

d. 320 415 000 m^2 = 320.415 $\boxed{}$

3. Fill in the blanks by using the words provided.

area | **circle** | **radius** | **sector**

The $\boxed{}$ of triangles, squares, rectangles or any other 2D shapes can be found by counting the number of square units a shape covers on a grid. The area of a $\boxed{}$ is calculated by multiplying $\frac{\theta}{360}$ by the area of a full $\boxed{}$ with the same $\boxed{}$, where θ represent the internal angle of the sector.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Convert. WE1

a. 12 cm^2 to mm^2

b. 3000 cm^2 to m^2

c. 37 m^2 to cm^2

d. 2 ha to km^2

e. 845 000 mm^2 to m^2

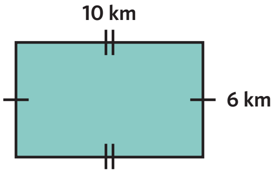
f. 1.8 ha to cm^2

g. 0.089 km^2 to m^2

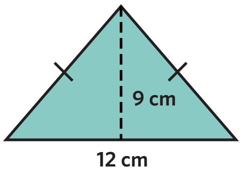
h. 92 050 000 mm^2 to km^2

5. Determine the areas of the following shapes. WE2

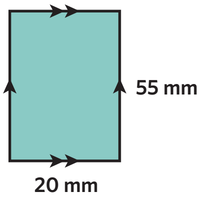
a.



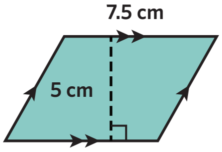
b.



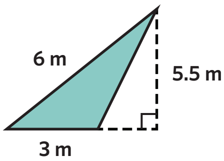
c.



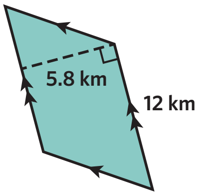
d.



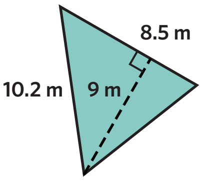
e.



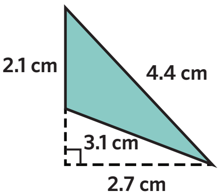
f.



g.

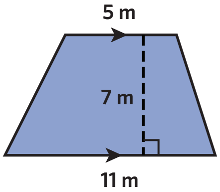


h.

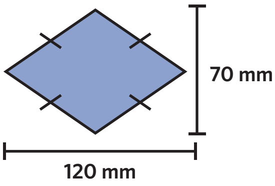


6. Determine the areas of the following shapes. WE3

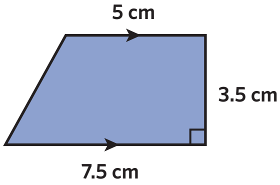
a.



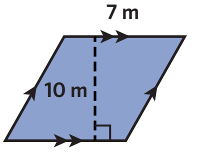
b.



c.



d.



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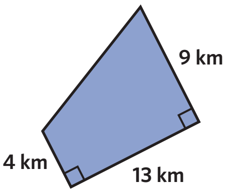

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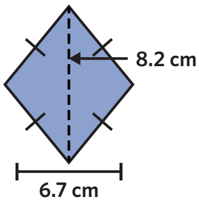
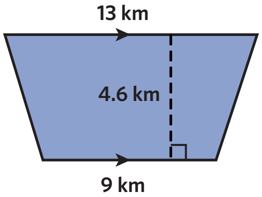
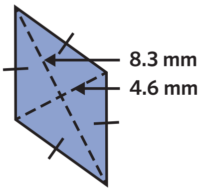


g.

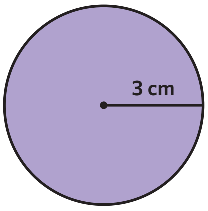


h.

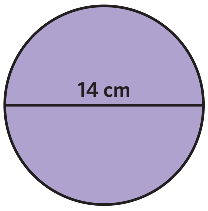


7. Calculate the areas of the following shapes, correct to 2 decimal places. WE4

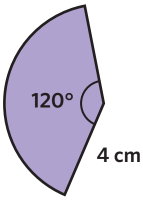
a.



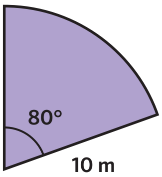
b.



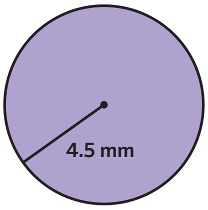
c.



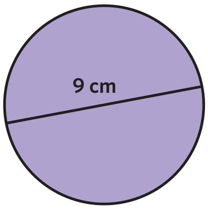
d.



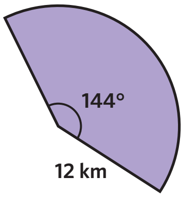
e.



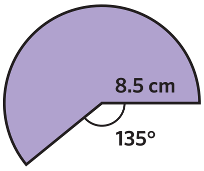
f.



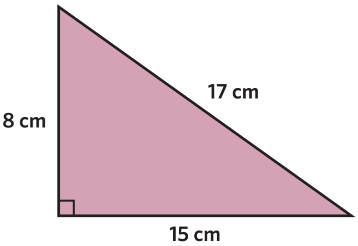
g.



h.



8. Calculate the area of this shape.



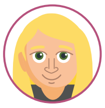
A. 40 cm^2 **B.** 60 cm^2 **C.** 68 cm^2 **D.** 120 cm^2 **E.** 127.5 cm^2

Back to contents

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the area of the circle, correct to 2 decimal places.



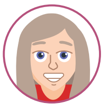
Student A

$A = \pi r^2$

$A = \pi \times 7.5^2$

$= 176.714...$

$\approx 176.71\ \text{km}^2$



Student B

$d = 2r = 7.5$

$r = 3.75$

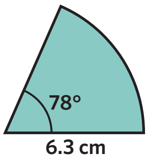
$A = \pi r^2$

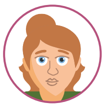
$A = \pi \times 3.75^2$

$= 44.178...$

$\approx 44.18\ \text{km}^2$

b. Determine the area of the sector, correct to 2 decimal places.





Student A

$P = \frac{\theta}{360} \times 2\pi r + 2r$

$P = \frac{78}{360} \times 2\pi \times 6.3 + 6.3 + 6.3$

$\approx 8.58 + 6.3 + 6.3$

$\approx 21.18\ \text{cm}^2$

Student B

$A = \frac{\theta}{360} \times \pi r^2$

$A = \frac{78}{360} \times \pi \times 6.3^2$

$= 27.016...$

$\approx 27.02\ \text{cm}^2$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Kim has combined three adjoining properties she owns. If the areas of the properties are 3250 m^2, 2.5 hectares and 5 km^2, how many hectares of land does Kim own altogether?

11. Jim is putting down parquetry flooring in a rectangular room. What is the cost of the flooring if the room measures 7.5 m by 11 m and the cost of each square metre is $220.60?

12. A clown wears two different large circular buttons. One with a radius of 150 mm and another with a diameter of 16 cm. What is the difference in area between the two buttons? Give the answer in cm^2, correct to 2 decimal places.

Back to contents

13. Steven's glass shop is replacing four identical triangular windows in a church. Each window has a base length of 3.4 m and a height of 6.3 m. How many square metres of glass does Steven need to replace all of the windows?

14. A school has increased the diameter of their circular play area from 50 m to 62 m. What is the percentage increase in the play area?

Reasoning

Question working paths

Mild 15 (a,b,d)	**Medium** 15 (a,b,d), 16 (a,b)	**Spicy** All

15. Rishmi is an origami enthusiast because she finds doing origami very relaxing. Origami is the art of folding paper. The aim of origami is to fold paper to create paper sculptures. Rishmi is working with circular pieces of paper.

Figure 1

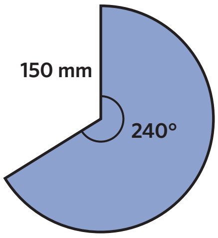
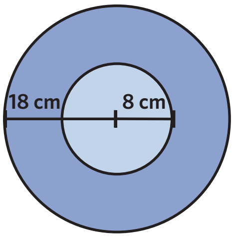


Figure 2



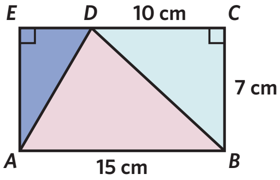
a. What is the area of Rishmi's original piece of paper (Figure 1) in cm^2, rounded to 1 significant figure?

b. Rishmi cuts a sector with a radius of 3 cm from the centre of the original piece of paper. Using the exact value of π, what is the area of the remaining shape? Recalculate the area of Figure 1 using the exact value of π.

c. For her next design Rishmi uses another circular piece of paper and cuts a circular hole in the middle of the paper (Figure 2). What is the percentage decrease in the area of the paper after Rishmi cuts the white hole? Use a calculator and round the answer to 2 decimal places.

d. Suggest two other similar hobbies that Rishmi could take up that she may find relaxing.

16.



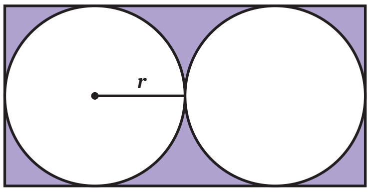
a. Calculate the area of $\triangle ABD$.

b. Calculate the areas of $\triangle BCD$ and $\triangle ADE$.

c. Use the answers to parts **a** and **b** to explain how the area of the quadrilateral *ABCE* is related to the sum of the areas of $\triangle BCD$, $\triangle ADE$ and $\triangle ABD$

Exam-style

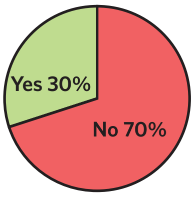
17. Two identical circles of radius r are drawn inside a rectangle, as shown in the given diagram. (1 MARK)



The shaded area can be found using

A. $2\pi r^2$ **B.** $2r - 2\pi r^2$ **C.** $2\pi r^2 - 8r^2$ **D.** $6r^2 - 2\pi r^2$ **E.** $8r^2 - 2\pi r^2$

18. The given pie chart shows the results of a survey. (3 MARKS)

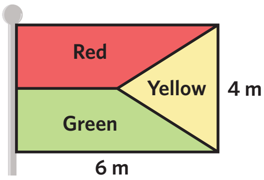


The radius of the pie chart is 21 mm.

a. What is the length of the arc of the sector representing 'yes'? Give the answer as an exact value in mm. 1 MARK

b. What is the area of the sector representing 'no'? Give the answer as an exact value in cm^2. 2 MARKS

19. A flag is made up of three colour sections: yellow, red and green. The flag is 6 m long and 4 m wide, as shown in the diagram. (2 MARKS)



The tip of the yellow section extends to the centre of the flag.

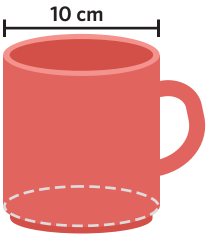
What percentage of the flags' area is green?

20. A rectangular prism box has a height of 15 cm and a volume of 1920 cm^3. The length of the lid is twice as long as the width. (2 MARKS)

What are the measurements of the length and width of the lid of the prism?

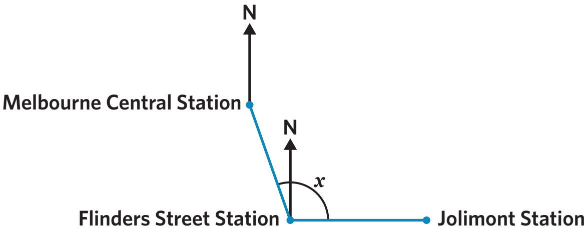
Remember this?

21. A coffee mug has a diameter of 10 centimetres. The circumference of the rim of the mug is closest to



A. 15.7 cm
B. 15.7 cm^2
C. 16.0 cm
D. 31.4 cm
E. 31.4 cm^2

22. A straight line is drawn on a map from Melbourne Central station to Flinders Station. Another line segment is drawn from Flinders Street Station to Jolimont Station.



The angle marked x in the diagram is likely to be:

A. 19° **B.** 91° **C.** 109° **D.** 180° **E.** 251°

23. The given data set shows the number of German shepherds at a dog park. This data was collected every day at 12 pm for 3 weeks.

5, 4, 1, 2, 8, 7, 1, 9, 1, 2, 5, 1, 8, 4, 2, 7, 10, 8, 4, 1, 10

The number of days where there were more than 5 German shepherds at the dog park is

A. 7 **B.** 8 **C.** 9 **D.** 10 **E.** 11

Back to contents

7D Composite shapes

LEARNING INTENTIONS

Students will be able to:

- identify the shapes that make up a composite shape
- calculate the perimeter and area of composite shapes.

When calculating the area of a composite shape that is made up of different regular shapes, it is helpful to be able to identify the shapes that make up the entire composite shape. Knowing the formulas for the area of the regular shapes makes calculating the area of composite shapes more efficient. Depending on the composite shape, both addition and subtraction can be used to find its area.

KEY TERMS AND DEFINITIONS

- A **composite shape** consists of two or more regular shapes.

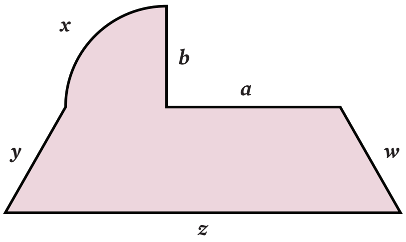
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: andysavchenko/Shutterstock.com

Dog kennel designers use the perimeter and area of composite shapes to design dog houses. Using the dimensions of different regular shapes they can create a layout that is suitable for the size of a dog, ensuring that the area of door and window openings provides good ventilation.

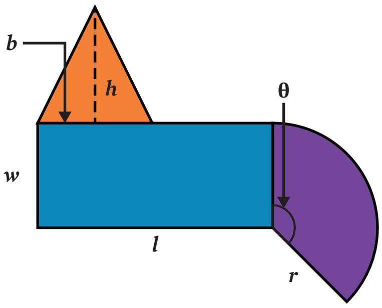
Key ideas

1. The perimeter of a composite shape is the sum of all its edges.



$P = a + b + x + y + z + w$

2. The area of a composite shape is the sum of the areas of the regular shapes that it is made up of.



$\text{Area} = l \times w + \frac{bh}{2} + \frac{\theta}{360} \times \pi r^2$

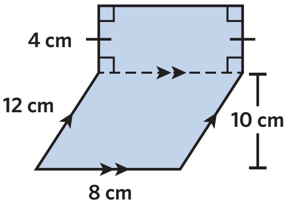
Worked example 1

Calculating the perimeter and area of composite shapes

Calculate the perimeter and the area for the following composite shapes. Where necessary, round to two decimal places.

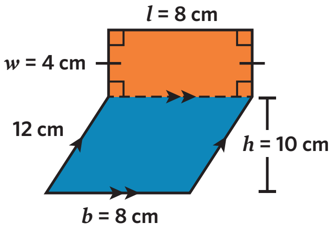
a.

WE1a



Working	Thinking
$P = 8 + 12 + 12 + 4 + 4 + 8$ $= 48$ cm	**Step 1:** Add the lengths if they are all the same unit of measurement. Take note of equal side lengths based on the markings.
Area = parallelogram area + rectangle area	**Step 2:** Break the composite shape up into regular shapes.
$A = b \times h + l \times w$ $= 8 \times 10 + 4 \times 8$ $= 80 + 32$ $= 112 \text{ cm}^2$	**Step 3:** Write the formulas for the area of the regular shapes and substitute the side lengths to calculate the area, with units.

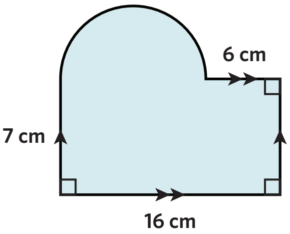
Visual support



$$A = b \times h + l \times w$$
$$= 8 \times 10 + 4 \times 8$$
$$= 80 + 32$$
$$= 112 \text{ cm}^2$$

b.

WE1b



Working	Thinking
Arc length $= \frac{\theta}{360} \times 2\pi r$ $= \frac{180}{360} \times 2 \times \pi \times 5$ $= 5\pi$ cm	**Step 1:** Calculate the length of the missing side.
$P = 16 + 7 + 7 + 6 + 5\pi$ ≈ 51.71 cm	**Step 2:** Add the lengths if they are all the same unit of measurement. Take note of equal side lengths based on the markings.

Continues →

Back to contents

Area = rectangle area + sector area

$$A = l \times w + \frac{\theta}{360} \times \pi r^2$$

$$= 16 \times 7 + \frac{180}{360} \times \pi \times 5^2$$

$$\approx 151.27 \text{ cm}^2$$

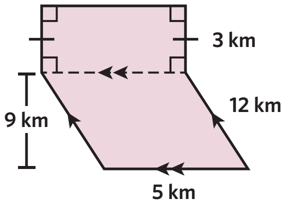
Step 3: Divide the composite shape up into regular shapes.

Step 4: Write the formulas for the area of the regular shapes and substitute the side lengths to calculate the area, with units.

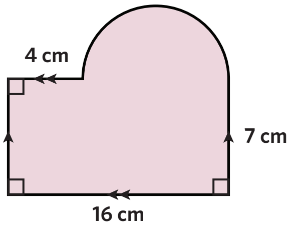
Student practice

Calculate the perimeter and the area for the following composite shapes. Where necessary, round to two decimal places.

a.



b.

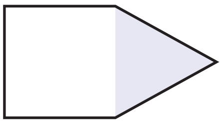


7D Questions

Understanding worksheet

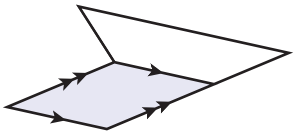
1. Identify the shaded regular shape that is shaded in each composite shape.

Example

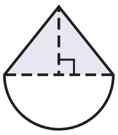


Triangle

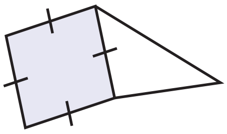
a.



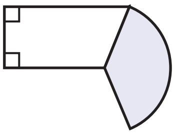
b.



c.



d.

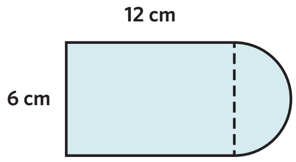


Back to contents

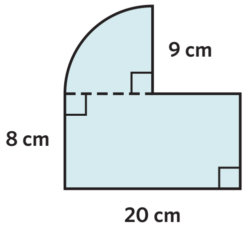
2. Fill in the missing information.

Example

Area= $12 \times \boxed{6} + \frac{1}{2} \times \pi \times 3^2$

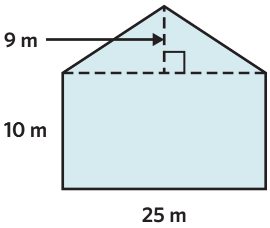


a.



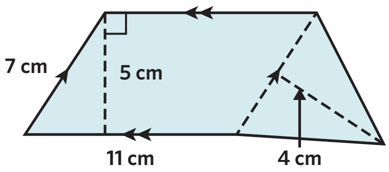
Area $= 20 \times 8 + \frac{1}{4} \times \pi \times \boxed{}^2$

b.



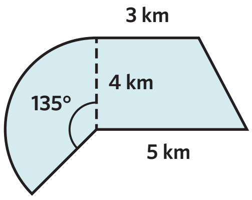
Area $=10 \times 25 + \frac{\boxed{} \times 9}{2}$

c.



Area $= 11 \times 5 + \frac{\boxed{} \times 4}{2}$

d.



Area $= \frac{1}{2}(5 + 3) \times 4 + \frac{\boxed{}}{360} \times \pi \times 4^2$

3. Fill in the blanks by using the words provided.

area	composite	perimeter	regular

A $\boxed{}$ shape is made up of two or more $\boxed{}$ shapes.

The $\boxed{}$ of a composite shape is the sum of the area of the regular shapes that adjoin to form the entire composite shape. The $\boxed{}$ of a composite shape is the sum of the external edges of the shape.

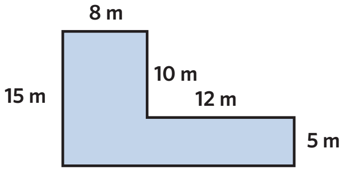
Fluency

Question working paths

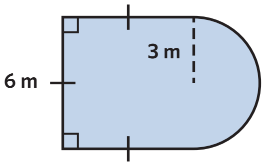
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6	4 (c,d,e,f), 5 (c,d,e,f), 6	4 (e,f,g,h), 5 (e,f,g,h), 6

4. Calculate the perimeter and the area for the following composite shapes. Where necessary, round to two decimal places. WE1

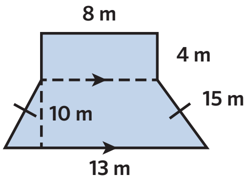
a.



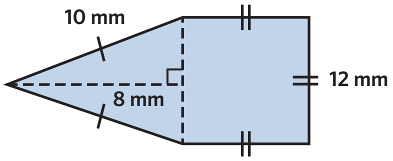
b.



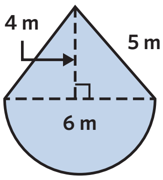
c.



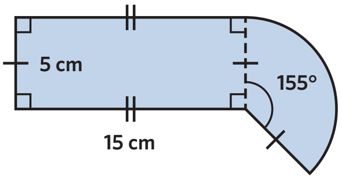
d.



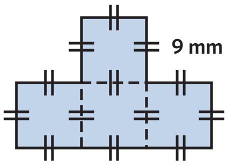
e.



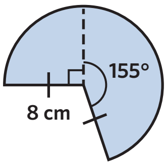
f.



g.



h.



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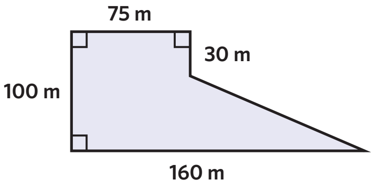
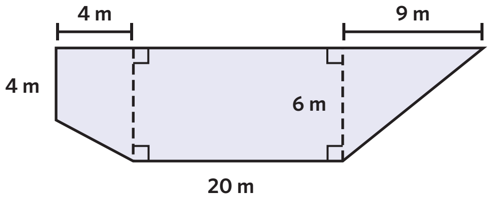
5. Calculate the areas of the following shapes, correct to two decimal places.

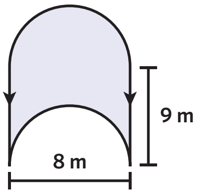
a.



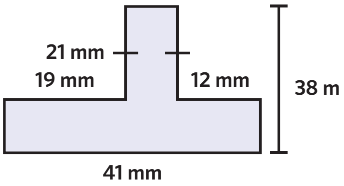
b.



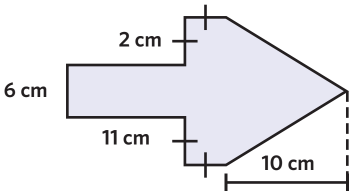
c.



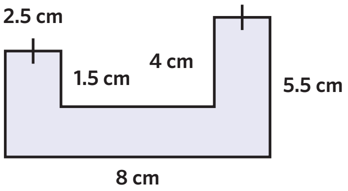
d.



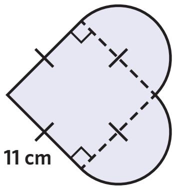
e.



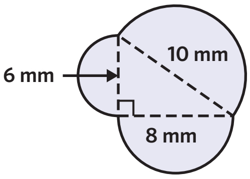
f.



g.



h.



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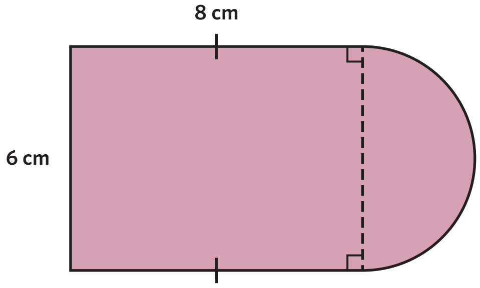
6. Determine the perimeter of the shape, correct to two decimal places.

A. 23.42 cm
B. 28 cm
C. 31.42 cm
D. 37.42 cm
E. 62.14 cm

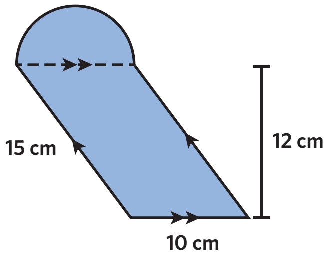


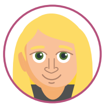
Back to contents

Spot the mistake

7. Select whether Student A or Student B is incorrect.

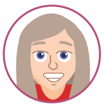
a. Determine the area of the composite shape, correct to two decimal places.





Student A

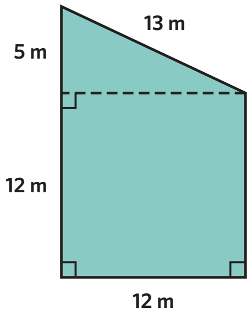
$$\text{Area} = 10 + 15 + 15 + \frac{180}{360} \times \pi \times 10$$
$$= 40 + 15.71$$
$$= 57.1\ \text{cm}^2$$

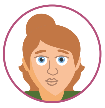


Student B

$$\text{Area} = 10 \times 12 + \frac{180}{360} \times \pi \times 5^2$$
$$=120 + 39.30$$
$$= 159.27\ \text{cm}^2$$

b. Determine the perimeter of the shape.





Student A

$P = 12 + 12 + 12 + 5 + 13$

$= 54$ cm

Student B

$P = 12 + 12 + 12 + 12 + 5 + 13$

$= 66$ cm

Problem solving

Question working paths

Mild 8, 9, 10 🌶	**Medium** 9, 10, 11 🌶🌶	**Spicy** 10, 11, 12 🌶🌶🌶

8. A rectangular tile with dimensions of 12 cm by 10 cm is glued together with a triangular tile with a base length of 100 mm and a height of 55 mm. Calculate the area of the adjoined tile in centimetres.

9. A carpenter is making a wooden tabletop by joining a large rectangle with a rhombus on one side. The rectangle measures 1.2 m by 0.8 m. The length between opposite vertices of the rhombus are 0.9 m and 0.7 m. Calculate the total area of the tabletop, in square metres.

10. A school running track is made up of a rectangle that is 50 m long and 25 m wide. Adjoined to each of the 25 m sides is a semicircle with a diameter of 25 m. Calculate the perimeter, in exact form, of the running track, in metres.

Back to contents

11. Samantha is redecorating her room and uses an old rectangular rug and cuts a circular hole in its centre. The rectangular rug has dimensions of 2.5 m by 3 m, and the circle cut out has a radius of 150 cm. Calculate the area, in exact form, of the redecorated rug in square centimetres.

12. A swimming pool has a rectangular main section that measures 12 m by 8 m. On one side, there is an adjoining large shallow pool that is two thirds of a full circle with a radius of 8 m. Calculate the perimeter and area, in exact form, of the entire pool.

Reasoning

Question working paths

Mild 13 (a,b,d) 🌶	**Medium** 13 (a,b,d), 14 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

13. Penny Lane Secondary school has a new running track. It is made up of two parallel sides that continue and then curve to form two semi-circles at each end of the track. The track is 12 m wide.

Note: the images are not drawn to scale.

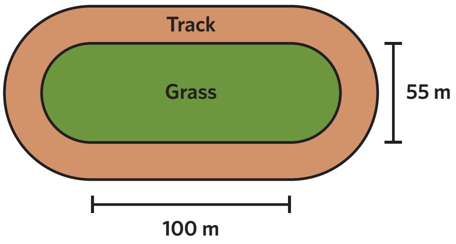
a. Anika's first lap is around the inside of the track, how far will she need to run? Round the answer to two decimal places.

b. How much further will she run on her second lap if she runs on the outside of the track? Round the answer to two decimal places.

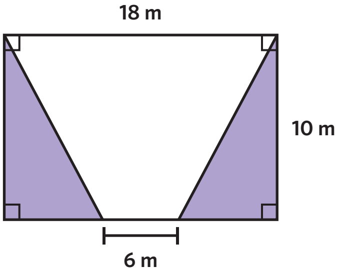
c. The material used for the track is polyurethane bound rubber. How many square metres of the material is required to complete the entire track? Round the answer to two decimal places.

d. The school has a limited construction budget. Suggest a way the school management can ensure that the construction budget is not exceeded.

Figure 1



14. Consider the diagram.



a. Calculate the shaded area by only using the area formula for a triangle. Provide the answer in exact form.

b. Calculate the shaded area by using the area formula for a rectangle and trapezium. Provide the answer in exact form.

c. Compare and contrast the steps and answer used to calculate the areas in parts **a** and **b**. Explain why the answers are the same.

Exam-style

15. Two identical right angled triangles are drawn inside a circle with a radius, as shown in the following diagram. (1 MARK)

The shaded area can be found using

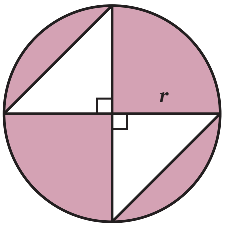
A. $\pi r^2 - \frac{2\pi}{2}$

B. $r^2 - \pi r^2$

C. $\pi r^2 - r^2$

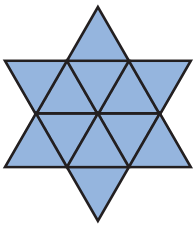
D. $2\pi r - r^2$

E. $r^2 - 2\pi r$



Back to contents

16. Multiple equilateral triangles are joined together to form a star, as shown in the diagram. (3 MARKS)

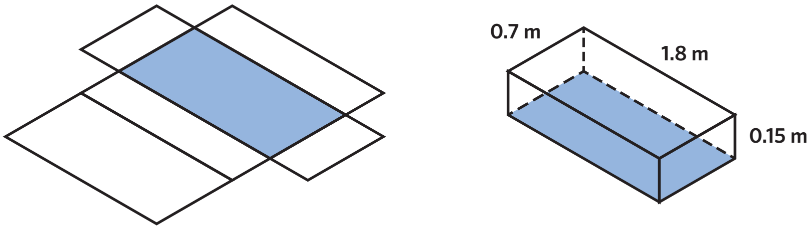


The side length of each equilateral triangle is 2 mm.

a. Calculate the perimeter of the star. Provide the answer in exact form, in millimetres. 1 MARK

b. Calculate the area of the star. Provide the answer in exact form, in square millimetres. 2 MARKS

17. A storage cardboard box can open and fold flat. (3 MARKS)



When folded flat, what is the total area that the box occupies? Provide the answer in exact form, in square metres.

18. A can of baked beans is in the shape of a cylinder. The height of the can is 12 cm. The label wraps around the curved side. The area of the label that covers the entire curved side of the cylinder is 226.19 cm^2. (2 MARKS)

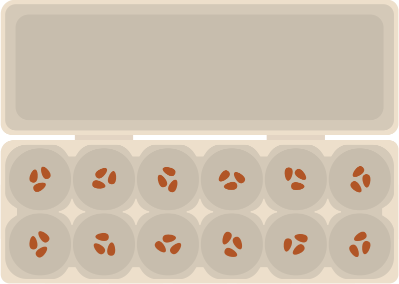
What is the radius of the can? Round the answer to one significant figure.

Remember this?

19. Yosemite has a bag of 50 tomato seeds.

He adds 3 seeds into each well in an egg carton.

The calculation that shows the number of tomato seeds left is



A. $50 - 3$
B. $50 - 12$
C. $50 - 12 \times 3$
D. $50 - 3 + 3 + 3$
E. $50 - 12 + 12 + 12$

20. Simplify the expression $-5x + 12x + 4 + 2$.

A. $6x + 7$ **B.** $7x + 6$ **C.** $12x + 6$ **D.** $12x + 1$ **E.** $13x$

21. The equation $y = \frac{1}{4}x - 16$ is rearranged so that x is the subject.

Which of the following correctly gives x as the subject?

A. $x = 4y - 16$ **B.** $x = 4y + 16$ **C.** $x = 4y + 64$ **D.** $y = 4x + 64$ **E.** $y = 4x + 16$

Back to contents

7E Surface area of prisms and pyramids

LEARNING INTENTIONS

Students will be able to:

- understand that surface area is the total area of a net
- identify the two dimensional surfaces in a prism or pyramid
- calculate the surface area of a prism or pyramid using 2D area formulas.

Prisms and pyramids are 3D objects that can be represented as 2D objects (nets). These nets are made up of individual faces in the shape of rectangles and triangles. Knowing the formulas for the surface area of the rectangular and triangular faces makes calculating the total surface area of prisms and pyramids more efficient.

KEY TERMS AND DEFINITIONS

- A **face** is a flat surface of a solid object.
- A **net** is a 2-dimensional representation of a solid object.
- A **prism** is a 3-dimensional object which has two identical polygon faces on either end, connected by rectangular faces. A prism has the same cross-section when cut anywhere along its length.
- **Total surface area (TSA)** is the sum of the area of all faces of a solid object.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

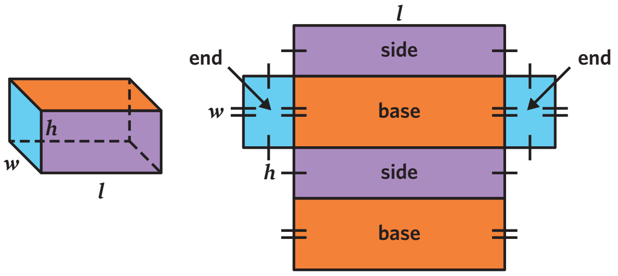
Image: Ground Picture/Shutterstock.com

For shop owners who offer gift wrapping services, it can be helpful to use total surface area calculations. Using the dimensions of differently shaped gifts and the number of gifts that require wrapping it is possible to estimate the total amount of wrapping paper required.

Key idea

1. Total surface area is the sum of the area of the faces of a solid object.

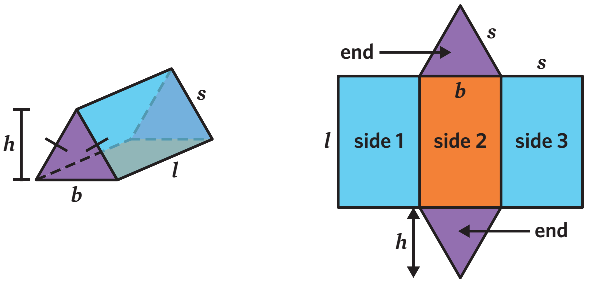
Rectangular prism



Total surface area = 2 × base + 2 × side + 2 × end

$= 2lw + 2lh + 2wh$

Triangular prism



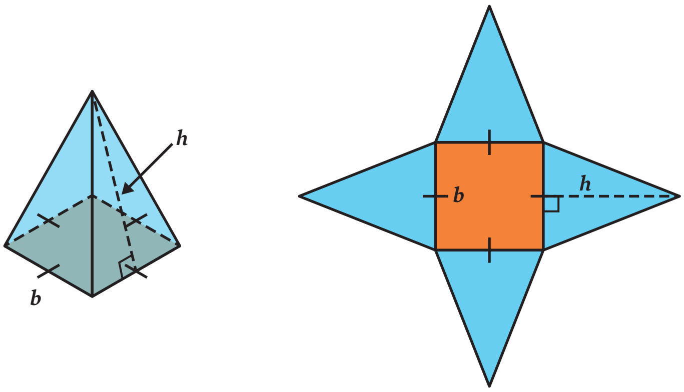
Total surface area = 2 × end + side 2 + side 1 + side 3

$= 2\left(\frac{b \times h}{2}\right) + l \times b + l \times s + l \times s$

Continues →

Back to contents

Square-based pyramid



Total surface area = $b^2 + 4\left(\frac{b \times h}{2}\right)$

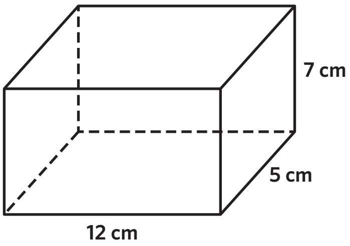
Worked example 1

Calculating the surface area of prisms

Calculate the total surface area of the prisms.

a.

WE1a



Working

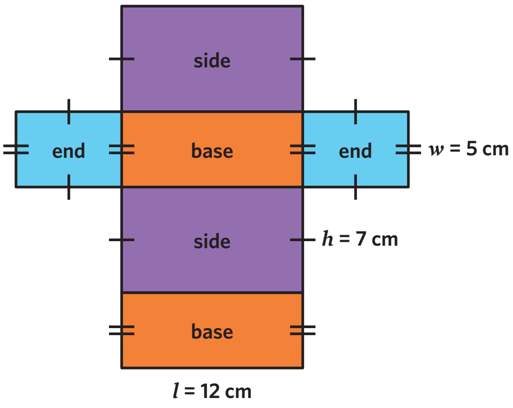
$$\begin{aligned} \text{TSA} &= 2 \times \text{base} + 2 \times \text{side} + 2 \times \text{end} \\ &= 2lw + 2lh + 2wh \\ &= 2(12 \times 5) + 2(12 \times 7) + 2(5 \times 7) \\ &= 120 + 168 + 70 \\ &= 358 \text{ cm}^2 \end{aligned}$$

Thinking

Step 1: Write the formula for the total surface area of a rectangular prism.

Step 2: Substitute the side lengths to calculate the total surface area, with units.

Visual support

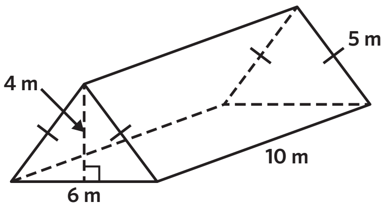


Total surface area = 2(12 × 5) + 2(12 × 7) + 2(5 × 7)

Continues →

b.

WE1b



Working

TSA = 2 × base + side 2 + side 1 + side 3

$$= 2\left(\frac{6 \times 4}{2}\right) + (6 \times 10) + (10 \times 5) + (10 \times 5)$$

$$= 24 + 60 + 50 + 50$$

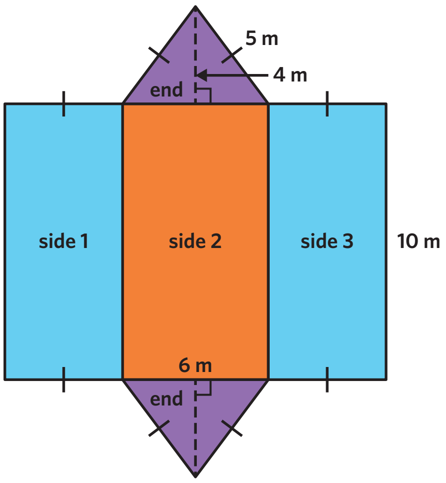
$$= 184 \text{ m}^2$$

Thinking

Step 1: Write the formula for the total surface area of a triangular prism.

Step 2: Substitute the side lengths to calculate the total surface area, with units.

Visual support

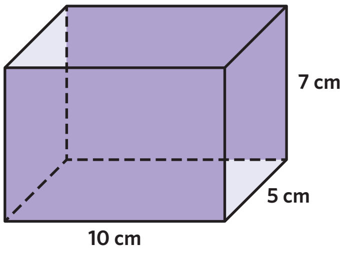


Total surface area = $2\left(6 \times \frac{4}{2}\right) + (6 \times 10) + (10 \times 5) + (10 \times 5)$

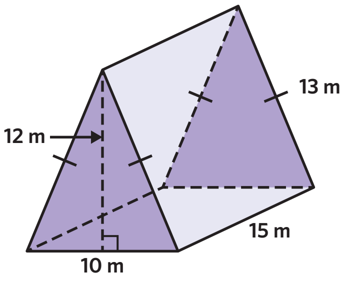
Student practice

Calculate the total surface area of the prisms.

a.



b.

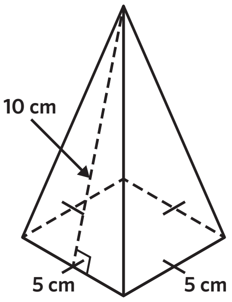


Worked example 2

Calculating the surface area of pyramids

WE2

Calculate the total surface area of the pyramid.



Working

$$\text{TSA} = b^2 + 4\left(\frac{b \times h}{2}\right)$$

$$= 5^2 + 4\left(\frac{5 \times 10}{2}\right)$$

$$= 25 + 100$$

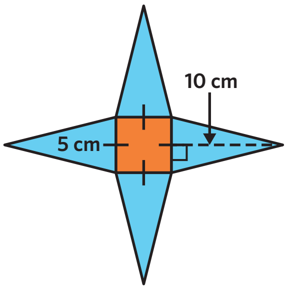
$$= 125 \text{ cm}^2$$

Thinking

Step 1: Write the formula for the total surface area of a square-based pyramid.

Step 2: Substitute the side lengths to calculate the total surface area, with units.

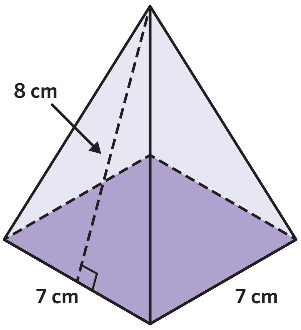
Visual support



Total surface area = $5^2 + 4\left(\frac{5 \times 10}{2}\right)$

Student practice

Calculate the total surface area of the pyramid.



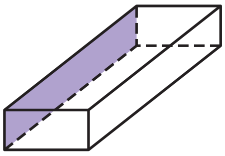
Back to contents

7E Questions

Understanding worksheet

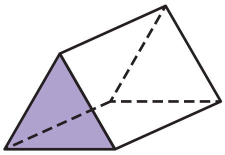
1. Identify the shape of the shaded face.

Example

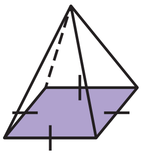


Rectangle

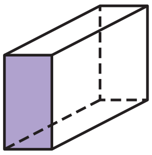
a.



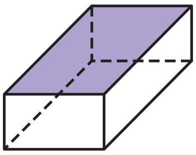
b.



c.

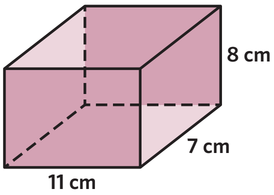


d.



2. Fill in the missing information.

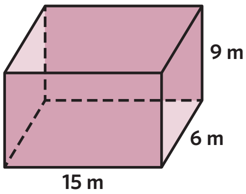
Example



Total surface area

$= 2(11 \times \boxed{7}) + 2(11 \times 8) + 2(7 \times 8)$

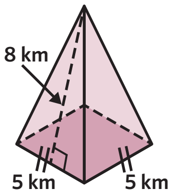
a.



Total surface area

$= 2(15 \times 6) + 2(15 \times 9) + \boxed{}(9 \times 6)$

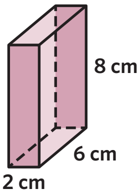
b.



Total surface area

$= 5^2 + 4\left(\frac{5 \times \boxed{}}{2}\right)$

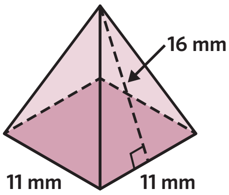
c.



Total surface area

$= 2(2 \times 6) + 2(2 \times \boxed{}) + 2(8 \times 6)$

d.



Total surface area

$= 11^2 + \boxed{}\left(\frac{11 \times 16}{2}\right)$

Back to contents

3. Fill in the blanks by using the words provided.

faces **net** **prism** **surface**

A ______ is a 2D representation of a solid object which shows all of the solid object's ______ . The total ______ area of a ______ is calculated by adding together the area of each of the prism's faces.

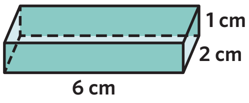
Fluency

Question working paths

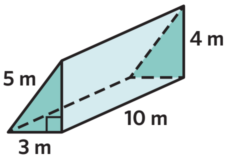
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Calculate the total surface area of the following prisms. WE1

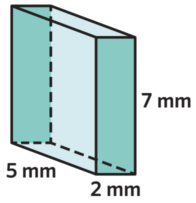
a.



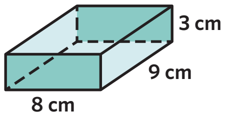
b.



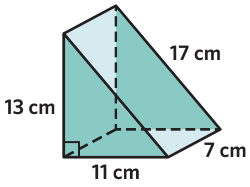
c.



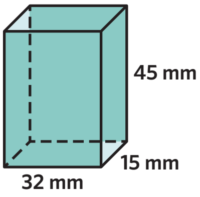
d.



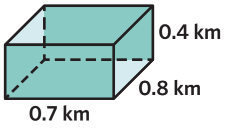
e.



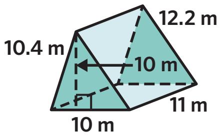
f.



g.

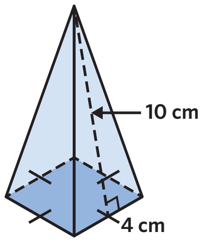


h.

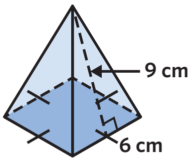


5. Calculate the total surface area of the following pyramids. WE2

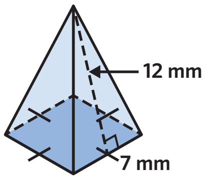
a.



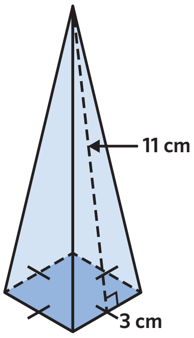
b.



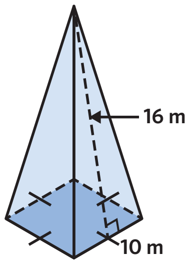
c.



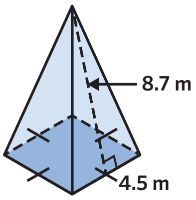
d.



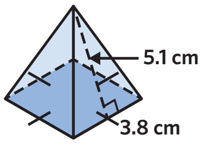
e.



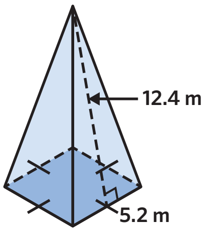
f.



g.

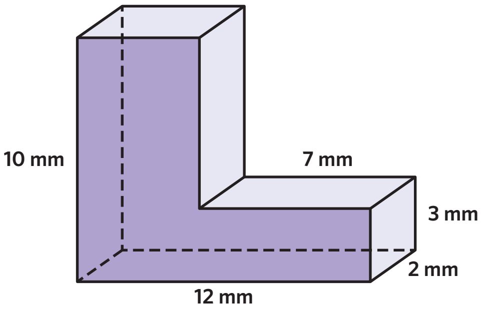


h.

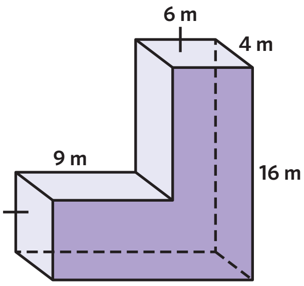


6. Calculate the total surface area of the following solid objects.

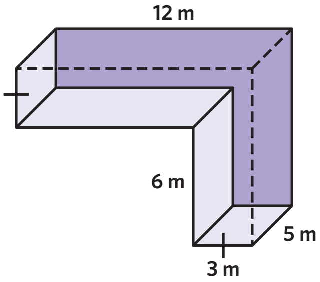
a.



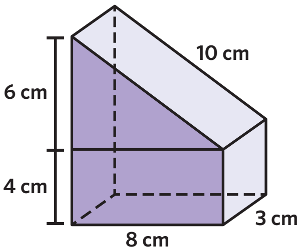
b.



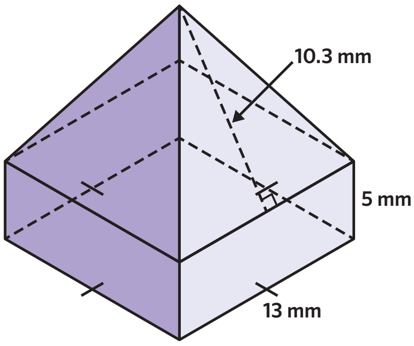
c.



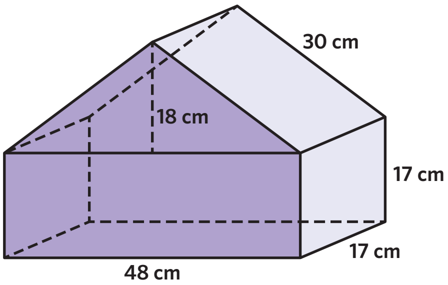
d.



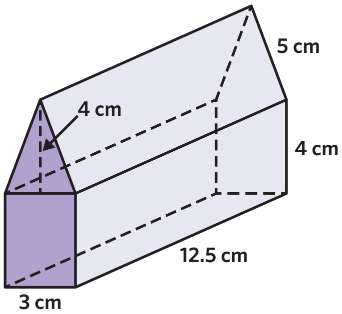
e.



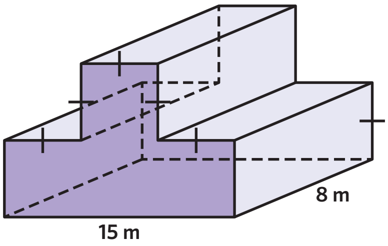
f.



g.



h.



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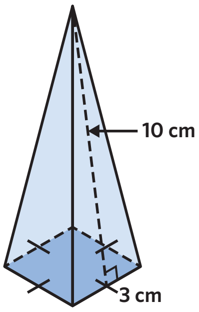
7. Calculate the total surface area of this pyramid.

A. 13 cm^2
B. 30 cm^2
C. 60 cm^2
D. 69 cm^2
E. 129 cm^2

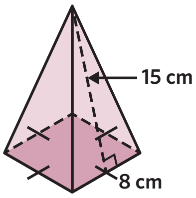


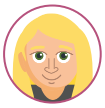
Back to contents

Spot the mistake

8. Select whether Student A or Student B is incorrect.

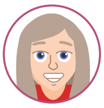
a. Calculate the total surface area of the pyramid.





Student A

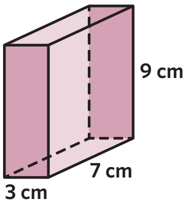
$$\begin{aligned} \text{TSA} &= 8^2 + 3\left(\frac{8 \times 15}{2}\right) \\ &= 64 + 180 \\ &= 244 \text{ cm}^2 \end{aligned}$$

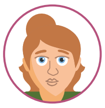


Student B

$$\begin{aligned} \text{TSA} &= 8^2 + 4\left(\frac{8 \times 15}{2}\right) \\ &= 64 + 240 \\ &= 304 \text{ cm}^2 \end{aligned}$$

b. Calculate the total surface area of the prism.





Student A

$$\begin{aligned} \text{TSA} &= 2lw + 2wh + 2lh \\ &= 2(3 \times 7) + 2(7 \times 9) + 2(3 \times 9) \\ &= 42 + 126 + 54 \\ &= 222 \text{ cm}^2 \end{aligned}$$

Student B

$$\begin{aligned} \text{TSA} &= lwh \\ &= 3 \times 7 \times 9 \\ &= 189 \text{ cm}^2 \end{aligned}$$

Problem solving

Question working paths

Mild 9, 10, 11	**Medium** 10, 11, 12	**Spicy** 11, 12, 13

9. A large inflatable cube die has edge lengths of 15 cm. What is the total surface area of the die?

10. What is the total surface area of a standard rectangular tissue box with the dimensions 22.5 cm, 12.2 cm, and 8.5 cm?

11. Abigail's camping tent has two triangular entries. Both entries have a base of 2.1 m, a height of 1.4 m, and slanted sides of 1.75 m. If the tent is 3 m long, what is its total surface area?

12. A rectangular water storage tank is 5.8 m long, 2.5 m wide, and 10 m deep. What is the total cost of the plastic for the tank if the plastic costs $5.80 per square metre?

13. A green room is built for an immersive movie. The room is 8 m long, 6 m wide and 2.8 m tall. How much green material is required to only cover the walls of the room?

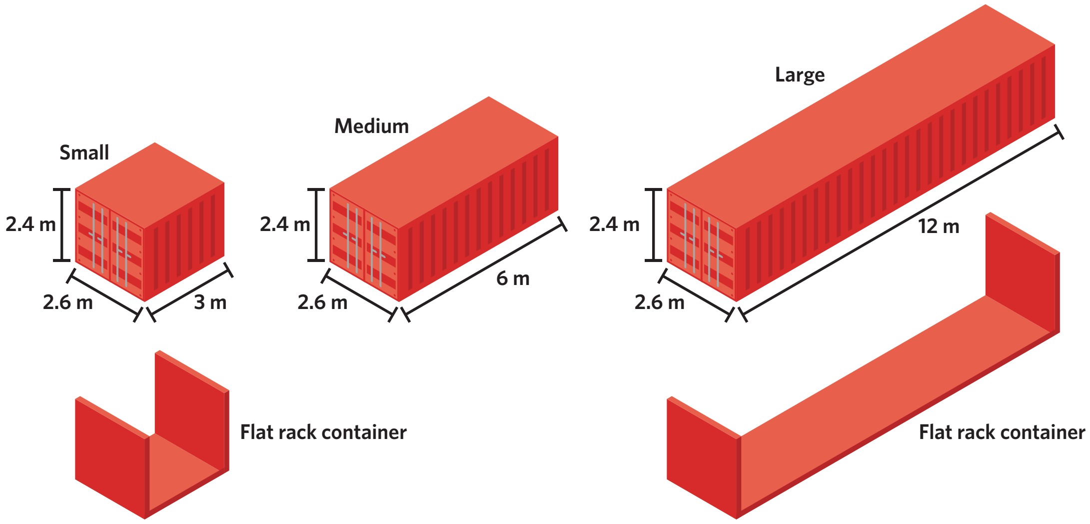
Back to contents

Reasoning

Question working paths

Mild 14 (a,b,c,e)	**Medium** 14 (a,b,c,e), 15 (a,b)	**Spicy** All

14. Shipping containers are large, standardised steel boxes that are specifically designed for transporting goods across various modes of transport. The images show the dimensions of three standard containers and the design of the non-standard 'flat rack container'.



a. A special hardened steel is used for the top and bottom of each container. How many square metres of hardened steel are required for the medium container?

b. What is the total surface area of the small container?

c. Calculate the combined total surface area of a small and large container.

d. How many more squared metres are required to construct a large flat rack container compared to a small flat rack container?

e. When determining the price to charge a customer for sending goods with a shipping container, other than total surface area, what factors could be considered when setting the price?

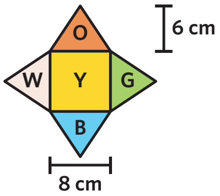
15. The net of a square based pyramid can be drawn multiple ways.

a. Using the same colours, draw two more nets with different layouts that represent the same pyramid as Figure 1.

b. Calculate the area of the two nets that were drawn in part **a**.

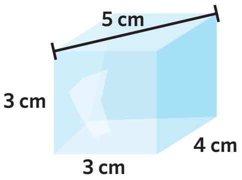
c. Compare and contrast the similarities and differences between the nets drawn in part **a** and justify why both nets have an equal total surface area.

Figure 1



Exam-style

16. An ice block is in the shape of a rectangular prism, as shown in the diagram. (1 MARK)



The block is cut in half to create two equally sized blocks of ice.

The cut is made diagonally, as represented by the line.

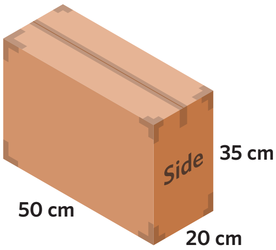
What is the total surface area of one of the equally sized blocks of ice in square centimetres?

A. 15 cm^2 **B.** 30 cm^2 **C.** 48 cm^2 **D.** 60 cm^2 **E.** 120 cm^2

Back to contents

17. Lin is planning to put some boxes into storage. He plans on using a standard small box size. (3 MARKS)

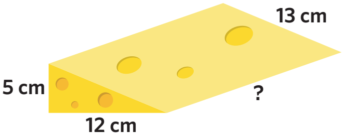
The diagram shows the dimensions of the box for storage.



a. One side of the storage box is labelled. What is the surface area of this side of the box? 1 MARK

b. What is the total surface area of Lin's storage box? 2 MARKS

18. A block of cheese is made up of five flat faces. The diagram shows the dimensions of the block of cheese. (3 MARKS)



The surface area of the block of cheese is 660 cm^2.

What is the missing length of the block of cheese?

19. A standard container for pet snakes is in the shape of a rectangular prism with the dimensions 1 m by 0.5 m by 0.9 m. For safety purposes, all of the sides of the container need to be covered with a protective screen. (2 MARKS)

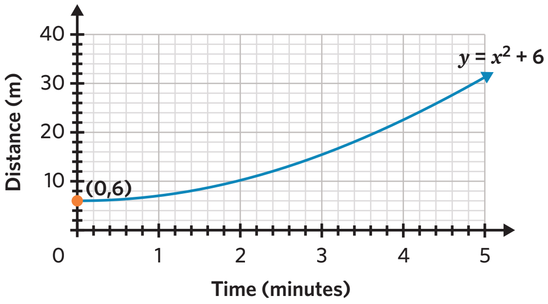
How many square centimetres of protective screen are required for one container?

Remember this?

20. There are 26 268 379 Australians as of 31 December 2022. If the total number of hours volunteered by Australians in 2022 is 499 099 201, how many hours on average does an Australian spend volunteering in 2022?

A. 0.0526 **B.** 1.311 **C.** 16 **D.** 19 **E.** 20

21. A balloon floats in the air. The below graph shows the distance of the balloon above ground over 5 minutes.

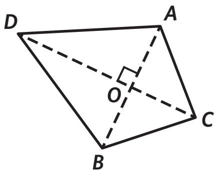


For how many minutes was the balloon within 10 m of the ground?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

22. Given that $AO = 3$ mm and $CD = 10$ mm, calculate the area of the kite $ABCD$ in cm^2.

A. 0.3
B. 0.6
C. 13
D. 30
E. 60



Back to contents

7F Surface area of a cylinder

LEARNING INTENTIONS

Students will be able to:

- identify the shapes that form the net of a cylinder
- calculate the surface area of a cylinder
- calculate the surface area of cylindrical portions.

Cylinders are 3D solid objects that can be represented as 2D objects (nets). These nets are made up of individual faces in the shape of two circles and a rectangle. Knowing the formulas for the surface area of the circular and rectangular faces makes calculating the total surface area of cylinders more efficient.

KEY TERMS AND DEFINITIONS

- A **cylinder** is a 3-dimensional object that has 2 flat, circular bases that are connected by a curved surface.

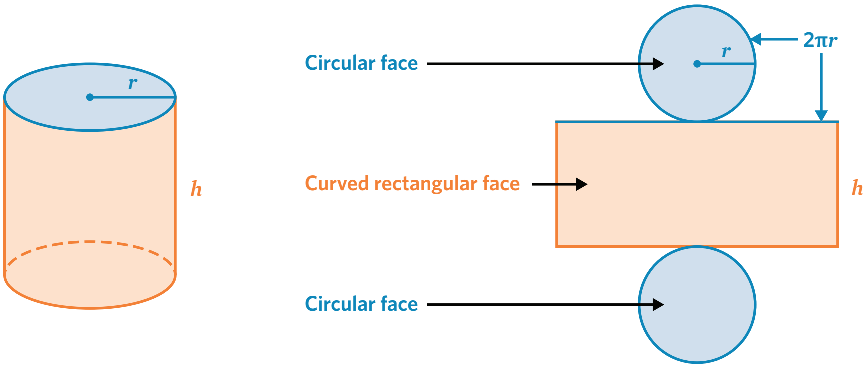
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: indra-east/Shutterstock.com

For manufacturers of food labels, it can be helpful to use total surface area calculations for cylinders. Using the dimensions of differently shaped food bottles or cans, it is possible to estimate the total amount of material and adhesive required for batches of label production.

Key idea

1. The total surface area (TSA) of a cylinder is the sum of the area of its two circular faces and one rectangular face.



Total surface area $= 2\pi r^2 + 2\pi rh$

Back to contents

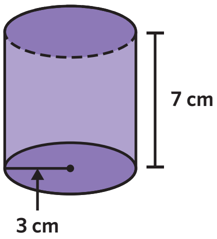
Worked example 1

Calculating the total surface area of a cylinder

Calculate the total surface area of the cylinder, correct to two decimal places.

a.

WE1a



Working

$h = 7 \text{ cm}, r = 3 \text{ cm}$

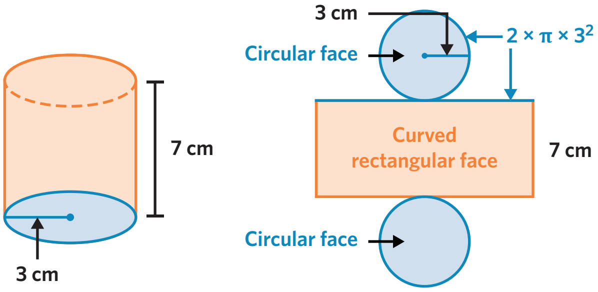
$$\begin{aligned} \text{TSA} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 7 \\ &= 18\pi + 42\pi \\ &\approx 188.50 \text{ cm}^2 \end{aligned}$$

Thinking

Step 1: Identify the side lengths of the cylinder.

Step 2: Break the composite shape up into regular shapes.

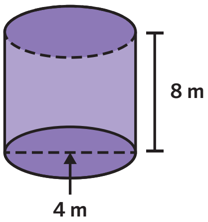
Visual support



$$\begin{aligned} \text{Total surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 7 \end{aligned}$$

b.

WE1b



Working

$h = 8 \text{ m}$

$d = 4 \text{ m}$

$r = \frac{d}{2} = \frac{4}{2} = 2 \text{ m}$

$$\begin{aligned} \text{TSA} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 2^2 + 2 \times \pi \times 2 \times 8 \\ &= 8\pi + 32\pi \\ &\approx 125.66 \text{ m}^2 \end{aligned}$$

Thinking

Step 1: Identify the height and the radius of the cylinder.

Step 2: Write the formula for the total surface area of a cylinder and substitute the side lengths to calculate the total surface area, with units.

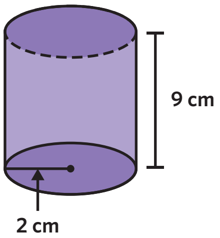
Continues →

Back to contents

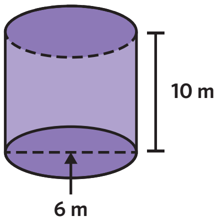
Student practice

Calculate the total surface area of the cylinder, correct to two decimal places.

a.



b.



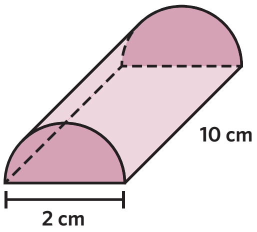
Worked example 2

Calculating the total surface area of a cylindrical portion

Calculate the total surface area of the cylindrical portions, correct to two decimal places.

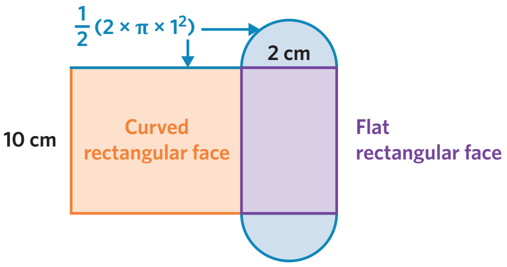
a.

WE2a



Working	Thinking
$h = 10$ cm $d = 2$ cm and $r = \frac{d}{2} = \frac{2}{2} = 1$ cm	**Step 1:** Identify the height and radius of the cylinder.
$\text{SA} = \frac{1}{2}(2\pi r^2) + \frac{1}{2}(2\pi rh)$ $= \frac{1}{2}(2 \times \pi \times 1^2) + \frac{1}{2}(2 \times \pi \times 1 \times 10)$ $= \pi + 10\pi$ $= 11\pi \text{ cm}^2$	**Step 2:** Write the formula for the surface area of a curved face and semi-circle faces and substitute the side lengths to calculate the surface area, with units.
$\text{SA} = 2 \times 10$ $\text{SA} = 20 \text{ cm}^2$ $\text{TSA} = 11\pi + 20$ $\approx 54.56 \text{ cm}^2$	**Step 3:** Calculate the surface area of the flat rectangular face then sum all the faces to calculate the total surface area.

Visual support



$= \frac{1}{2}(2\pi r^2) + \frac{1}{2}(2\pi rh) + lw$

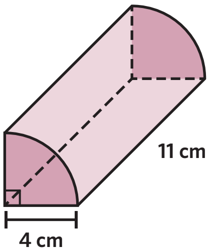
$= \frac{1}{2}(2 \times \pi \times 1^2) + \frac{1}{2}(2\pi \times 1 \times 10) + 2 \times 10$

Continues →

Back to contents

b.

WE2b



Working

$h = 11$ cm, $r = 4$ cm

$$\begin{aligned} \text{SA} &= \frac{1}{4}(2\pi r^2) + \frac{1}{4}(2\pi rh) \\ &= \frac{1}{4}(2 \times \pi \times 4^2) + \frac{1}{4}(2 \times \pi \times 4 \times 11) \\ &= 8\pi + 22\pi \\ &= 30\pi \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{SA} &= 2(l \times w) \\ &= 2(4 \times 11) \\ &= 88 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{TSA} &= 30\pi + 88 \\ &\approx 182.25 \text{ cm}^2 \end{aligned}$$

Thinking

Step 1: Identify the height and radius of the cylinder.

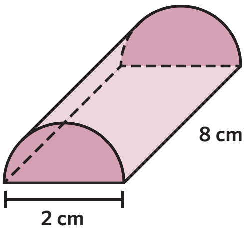
Step 2: Write the formula for the surface area of the quadrant faces and curved face and substitute the side lengths to calculate the surface area, with units.

Step 3: Write the formula for the surface area of the flat rectangular faces and substitute the side lengths to calculate the surface area. Sum all the faces to calculate the total surface area.

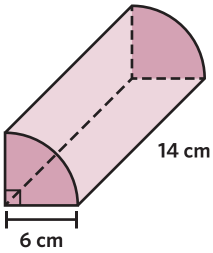
Student practice

Calculate the total surface area of the cylindrical portions, correct to two decimal places.

a.



b.



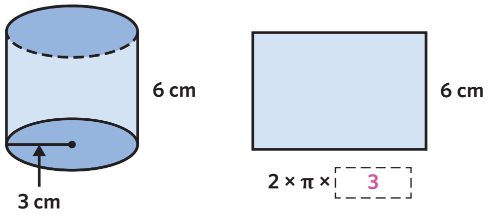
Back to contents

7F Questions

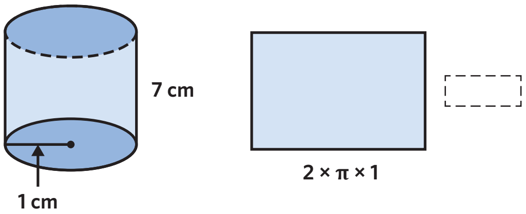
Understanding worksheet

1. Complete the missing information.

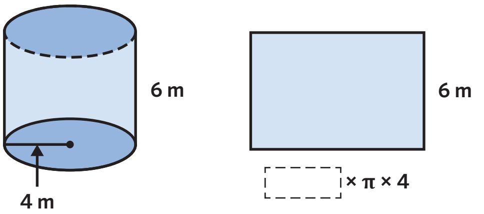
Example



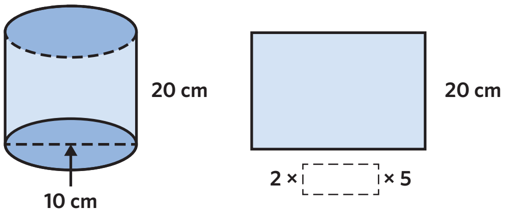
a.



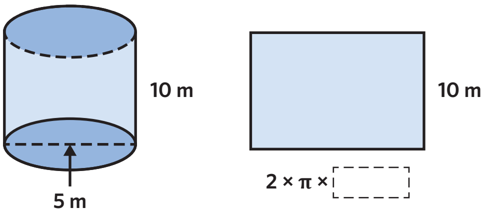
b.



c.



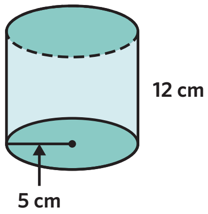
d.



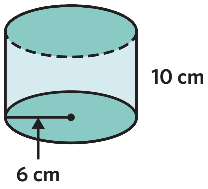
2. Complete the missing information.

Example

TSA $= 2 \times \pi \boxed{5}^2 + 2 \times \pi \times 5 \times 12$

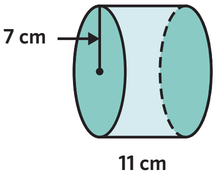


a.



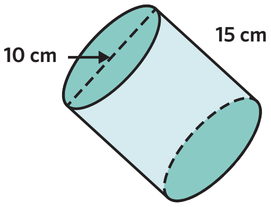
TSA $= 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times \boxed{}$

b.



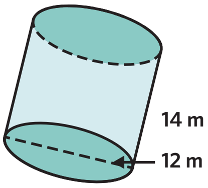
TSA $= \boxed{} \times \pi \times 7^2 + 2 \times \pi \times 7 \times 11$

c.



TSA $= 2 \times \pi \times 5^2 + 2 \times \pi \times \boxed{} \times 15$

d.



TSA $= 2 \times \pi \times \boxed{}^2 + 2 \times \pi \times 6 \times 14$

Back to contents

3. Fill in the blanks by using the words provided.

circumference **cylinder** **face** **net**

The ______ of a cylinder is made up of two circles and a rectangle. The length of one of the sides of the rectangular ______ is the ______ of the circular face.

The length of the other side of the rectangular face is the height of the ______ .

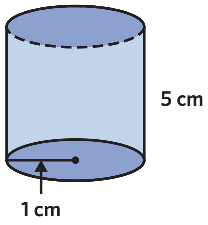
Fluency

Question working paths

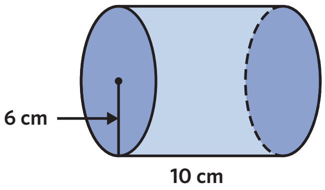
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Calculate the total surface area of the cylinder, correct to two decimal places. Where necessary, give the answer in the units in the bracket. WE1a

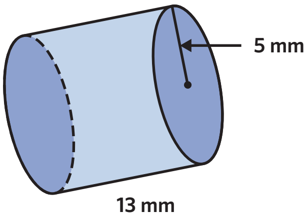
a.



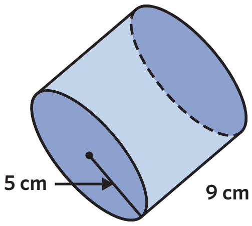
b.



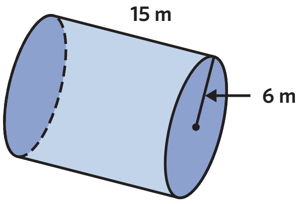
c.



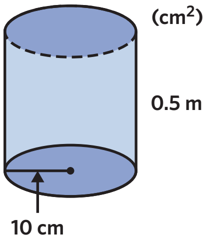
d.



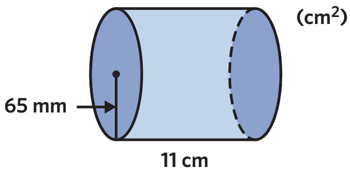
e.



f.



g.



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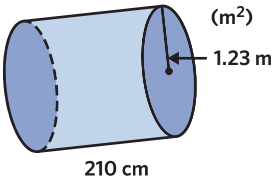
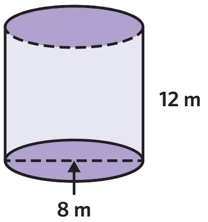


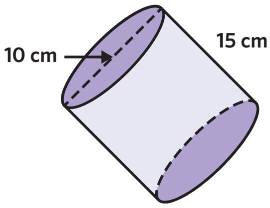
5. Calculate the total surface area of the cylinder, correct to two decimal places. Where necessary, give the answer in the units in the bracket.

WE1b

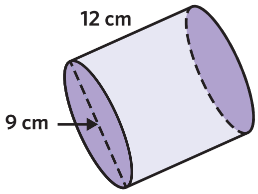
a.



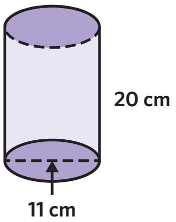
b.



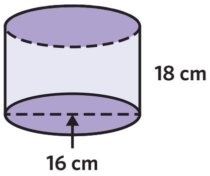
c.



d.



e.



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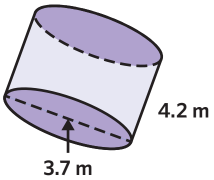
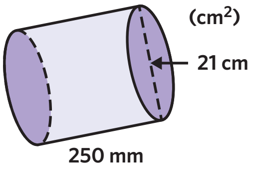


g.



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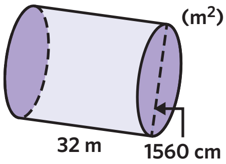
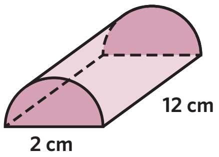


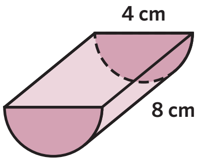
6. Calculate the total surface area of the cylindrical portions, correct to two decimal places. Where necessary, give the answer in the units in the bracket.

WE2a

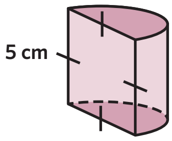
a.



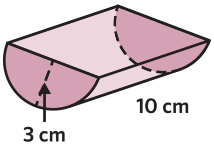
b.



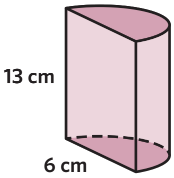
c.



d.



e.



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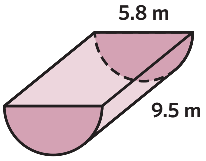

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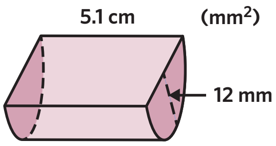

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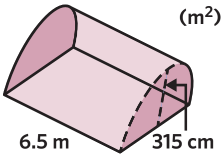
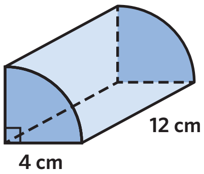


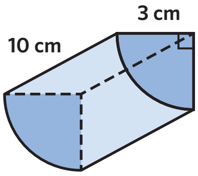
7. Calculate the total surface area of the cylindrical portions, correct to two decimal places. Where necessary, give the answer in the units in the bracket.

WE2b

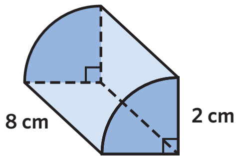
a.



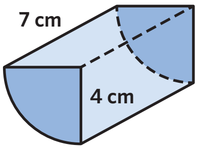
b.



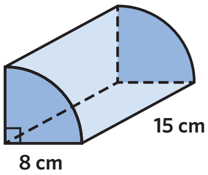
c.



d.



e.



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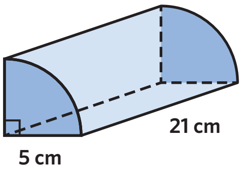
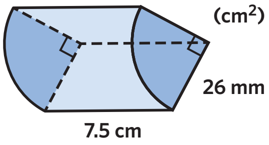
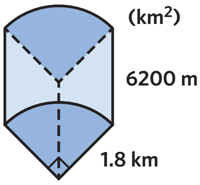


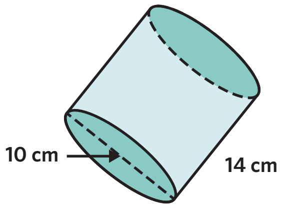
g.



h.



8. Calculate the total surface area of the cylinder, correct to two decimal places.



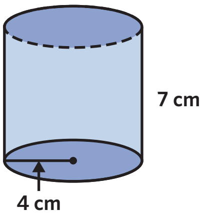
A. 140 cm^2
B. 439.82 cm^2
C. 518.36 cm^2
D. 596.90 cm^2
E. 1507.96 cm^2

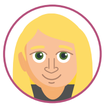
Back to contents

Spot the mistake

9. Select whether Student A or Student B is incorrect.

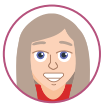
a. Calculate the total surface area of the cylinder, correct to two decimal places.





Student A

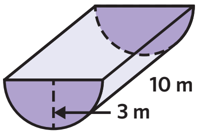
$$\begin{aligned} \text{TSA} &= 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 7 \\ &= 32\pi + 56\pi \\ &\approx 276.46 \text{ cm}^2 \end{aligned}$$

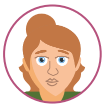


Student B

$$\begin{aligned} \text{TSA} &= \pi \times 4^2 \times 7 \\ &= 112\pi \\ &\approx 351.86 \text{ cm}^2 \end{aligned}$$

b. Calculate the total surface area of the cylindrical portion, correct to two decimal places.





Student A

$$\begin{aligned} \text{TSA} &= 2 \times \frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times 2 \times \pi \times 3 \times 10 \\ &\quad + 6 \times 10 \\ &= 9\pi + 30\pi + 60 \\ &\approx 182.52 \text{ cm}^2 \end{aligned}$$

Student B

$$\begin{aligned} \text{TSA} &= 2 \times \frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times 2 \times \pi \times 3 \times 10 \\ &= 9\pi + 30\pi \\ &\approx 122.52 \text{ cm}^2 \end{aligned}$$

Problem solving

Question working paths

Mild 10, 11, 12	**Medium** 11, 12, 13	**Spicy** 12, 13, 14

10. A standard kitchen roller has a radius of 40 mm and a length of 360 mm. How many square centimetres can a chef roll with one full roll, correct to two decimal places?

11. Multiple 10 cent pieces are wrapped in cardboard and form a cylinder that is 200 mm tall. A 10 cent piece has a diameter of 23.6 mm. Assuming that none of the cardboard overlaps, how many square millimetres of cardboard are used for the roll, correct to two decimal places?

12. The concrete road roller is a cylindrical shape and has a radius of 0.7 m and is 160 cm in length. How many square metres of the road does it level if it travels straight and rotates 30 times, correct to two decimal places?

13. Kamal charges \$0.01 per square centimetre that he has to paint. How much does Kamal charge a customer for painting the external surfaces of 15 cylinder containers, each with a radius of 60 cm and a height of 1 m, rounded to the nearest cent?

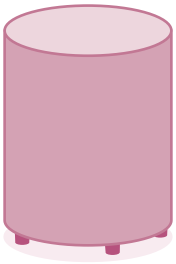
14. The butterfly greenhouse is shaped like a half-cylinder. It has a diameter of 5 m and is 16 m long. Not including the flooring, how many square metres of plastic is required for its cover, correct to two decimal places?

Reasoning

Question working paths

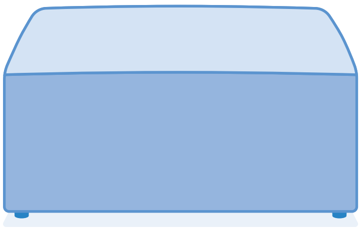
Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Omari has just moved into his own apartment and wants to reupholster some of the ottomans that he is using to furnish some of the rooms in his apartment.



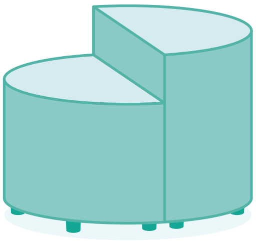
Ottoman 1

Radius: 30 cm
Height: 0.8 m



Ottoman 2

Length: 1 m
Width: 0.6 m
Height: 0.55 m



Ottoman 3

a. If Omari does not reupholster the bottom of Ottoman 1, how many square metres of material does he need, rounded to two decimal places?

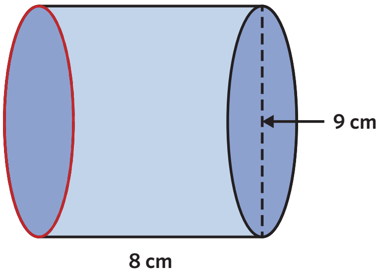
b. For Ottoman 2, Omari intends to put material on all of its faces. How much more material will he need to upholster Ottoman 2 than Ottoman 1? Give the answer in square metres and round to two decimal places.

c. Ottoman 3 is modular and is made up of two half cylinders. Both parts have the same diameter of 80 cm. The shorter half cylinder has a height of 60 cm. How many square metres of material does Omari need to cover all the faces of the shorter half cylinder?

d. The taller part of Ottoman 3 is 25% taller than the shorter part. How many square metres of material is required to cover the non-circular sides?

e. Omari is overwhelmed by the cost of new furniture. Propose a strategy Omari could implement to keep the cost of furnishing his apartment low.

16.



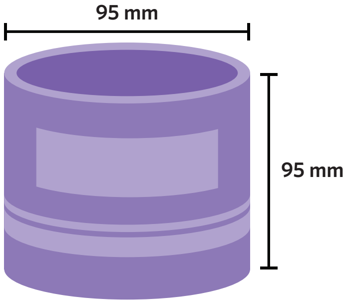
a. Calculate the circumference of the cylinder. Give the answer in centimetres and rounded to two decimal places.

b. Calculate the total surface area of the rectangular curved face. Give the answer in square centimetres and rounded to two decimal places.

c. Compare and contrast the answers from part **a** and the surface area calculated in part **b** to justify why the formula for the surface area of a cylinder includes $2\pi r$.

Exam-style

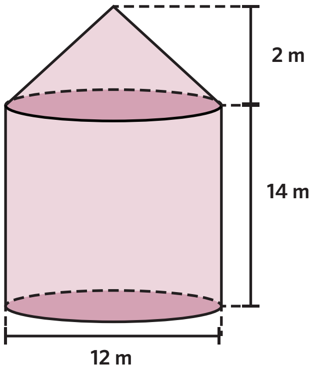
17. A standard foam sleeve is designed to insulate a drink to keep it cold. It is in the shape of a cylinder and the top is open. (1 MARK)



Calculate the total outer surface area in square millimetres, rounded to the nearest whole number.

A. 190 mm^2 **B.** 4513 mm^2 **C.** 35 441 mm^2 **D.** 42 529 mm^2 **E.** 113 412 mm^2

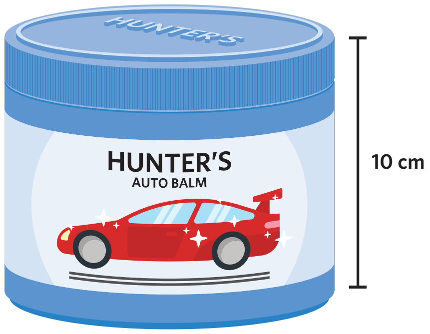
18. A large water storage tank is in the shape of a cylinder. It has a top that is a cone and is shown below. (2 MARKS)



a. Calculate the area of the base of the tank in exact form. 1 MARK

b. Calculate the total surface area of the tank, excluding its top, in exact form. 1 MARK

19. Hunter manufactures a balm wax to polish motor vehicles. The balm comes in a cylindrical container. The height of the container is 10 cm. The area of the curved face is 80π cm^2. (2 MARKS)



Calculate the surface area of the base of the balm container in exact form.

20. A large cylindrical tube is used to collect tennis balls. It has no base or top. (3 MARKS)

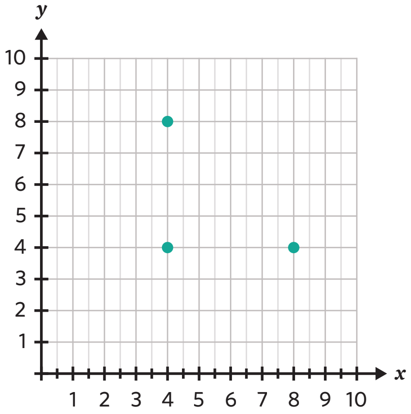
The radius of the circular hole is 3.25 cm. The surface area of the outside of the tube is 1846 cm^2.

Calculate the height of the tube in centimetres, rounded to two decimal places.

Back to contents

Remember this?

21. This graph shows three vertices of a square.



Which of these points is the fourth vertex?

A. (8,6) **B.** (8,8) **C.** (6,8) **D.** (6,6) **E.** (8,10)

22. Alex earns \$60 for mowing 5 lawns. Which expression shows how many dollars he will earn for mowing p lawns?

A. $(60 \times 5) \times p$ **B.** $(60 \div 5) \div p$ **C.** $(60 \times 5) \div p$ **D.** $(60 \div 5) \times p$ **E.** $(60 + 5) \times p$

23. A coach wants to carry out a survey about the new training program using a sample of athletes from the track team. He wants his survey to be representative of the events in the team. In the team, there are three times as many sprinters as long-distance runners. Which of these samples should the coach use to get the most representative results?

A. All the sprinters
B. All the long-distance runners
C. 5 sprinters and 15 long-distance runners
D. 15 sprinters and 15 long-distance runners
E. 15 sprinters and 5 long-distance runners

7G Volume of a prism

LEARNING INTENTIONS

Students will be able to:

- convert between units of volume
- understand the relationship between volume and capacity
- calculate the volume of right prisms and solids with consistent cross-sections.

Volume is a measure of the space occupied by a three-dimensional object. Converting between units of volume ensures consistent measurements, while understanding the relationship between volume and capacity provides insights into the space an object can hold. The volume of right prisms and solids with consistent cross-sections can be determined by finding the area of their cross-section and extending it through its height.

KEY TERMS AND DEFINITIONS

- **Capacity** is the maximum amount of fluid or substance a container can hold.
- A **cross-section** is a surface that is created when making a straight cut through a 3D shape.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Fikret Eskisarli/Shutterstock.com

Architects and builders frequently convert between units of volume when designing buildings and structures. Understanding how to calculate the volume of right prisms, for example, can be crucial when designing a swimming pool or a multi-storey parking garage.

Key ideas

1. Converting between units of measurement involves multiplying or dividing by powers of 10.

$$\begin{tikzcd} \text{Cubic kilometre (km}^3\text{)} \arrow[r, bend left, "\times 1000^3"] & \text{Cubic metre (m}^3\text{)} \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{Cubic centimetre (cm}^3\text{)} \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{Cubic millimetre (mm}^3\text{)} \arrow[l, bend left, "\div 10^3"] \end{tikzcd}$$

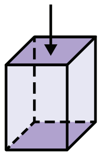
2. Converting between measures of capacity involves multiplying or dividing by 1000.

$$\begin{tikzcd} \text{Megalitre (ML)} \arrow[r, bend left, "\times 1000"] & \text{Kilolitre (kL)} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{Litre (L)} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{Millilitre (mL)} \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$

3. The volume of a shape with a uniform cross-section is equal to the area of the cross-section multiplied by the perpendicular height

Rectangular Prism

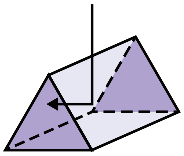
area of cross-section = $w \times l$



Volume = $l \times w \times h$

Triangular Prism

area of cross-section = $\dfrac{b \times h}{2}$



Volume = $\dfrac{b \times h}{2} \times l$

Back to contents

Worked example 1

Converting volumes and capacities

Convert the following.

a. 15 000 mm^3 to cm^3

WE1a

Working

$1000 \text{ mm}^3 = 1 \text{ cm}^3$

$15\,000 \text{ mm}^3 = (15\,000 \div 10^3) \text{ cm}^3$

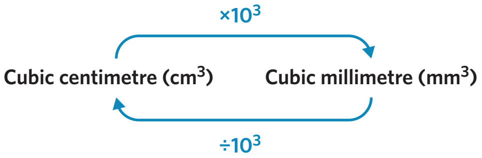
$= 15 \text{ cm}^3$

Thinking

Step 1: Determine the multiplication or division required.

Step 2: Apply the conversion.

Visual support



b. 1675 kL to L

WE1b

Working

$1 \text{ kL} = 1000 \text{ L}$

$1675 \text{ kL} = (1675 \times 1000) \text{ L}$

$= 1\,675\,000 \text{ L}$

Thinking

Step 1: Determine the multiplication or division required.

Step 2: Apply the conversion.

Student practice

Convert the following.

a. 21 000 mm^3 to cm^3

b. 2935 kL to L

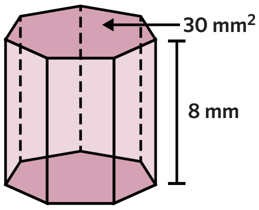
Worked example 2

Calculating volumes when given the area of the cross-section

Calculate the volume of the solid objects.

a.

WE2a



Working

Volume = area of cross-section × height

$= 30 \times 8$

$= 240 \text{ mm}^3$

Thinking

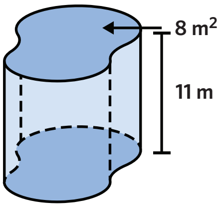
Step 1: Write the formula for the volume of the solid object.

Step 2: Substitute the values to calculate the volume, with units.

Continues →

Back to contents

b.



WE2b

Working

Volume $=$ area of cross-section $\times$ height

$= 8 \times 11$

$= 88 \text{ mm}^3$

Thinking

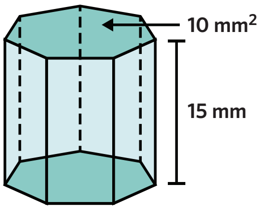
Step 1: Write the formula for the volume of the solid object.

Step 2: Substitute the values to calculate the volume, with units.

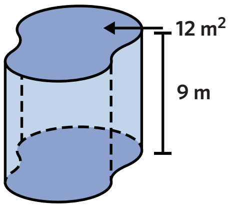
Student practice

Calculate the volume of the solid objects.

a.



b.

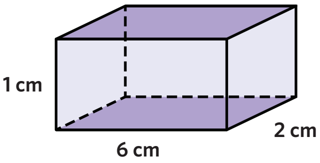


Worked example 3

Calculating the volume of prisms

Calculate the total volume of the prisms.

a.



WE3a

Working

Volume $= l \times w \times h$

$= 6 \times 2 \times 1$

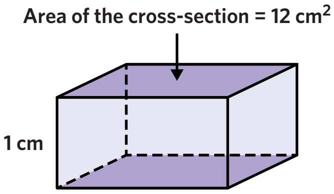
$= 12 \text{ cm}^3$

Thinking

Step 1: Write the formula for the volume of a rectangular prism.

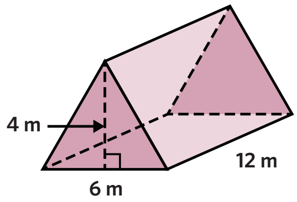
Step 2: Substitute the side lengths to calculate the volume, with units.

Visual support



Continues →

b.



Working

$$\text{Volume} = \frac{b \times h}{2} \times l$$

$$= \frac{6 \times 4}{2} \times 12$$

$$= 144 \text{ m}^3$$

Thinking

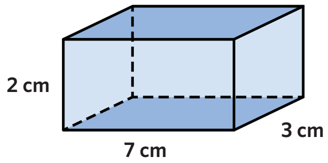
Step 1: Write the formula for the volume of a triangular prism.

Step 2: Substitute the side lengths to calculate the volume, with units.

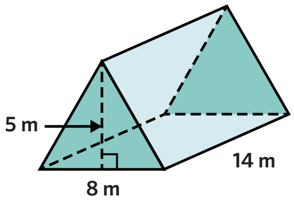
Student practice

Calculate the total volume of the prisms.

a.



b.



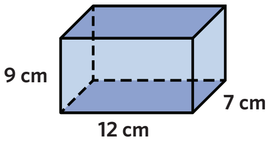
Back to contents

7G Questions

Understanding worksheet

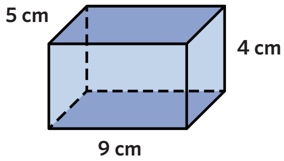
1. State the perpendicular height of the following solid objects.

Example



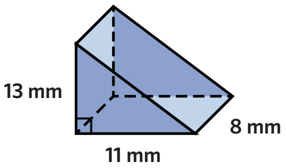
Perpendicular height: $\boxed{9}$

a.



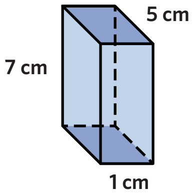
Perpendicular height: $\boxed{}$

b.



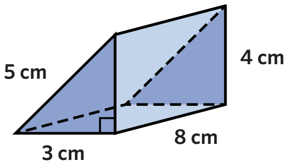
Perpendicular height: $\boxed{}$

c.



Perpendicular height: $\boxed{}$

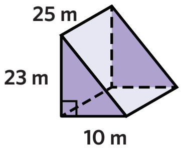
d.



Perpendicular height: $\boxed{}$

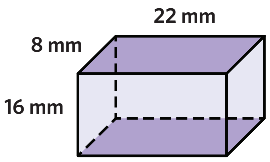
2. Complete the missing information.

Example



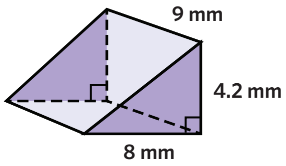
Volume $= \frac{23 \times 10}{2} \times \boxed{25}$

a.



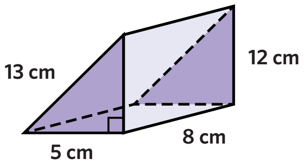
Volume $= 16 \times \boxed{} \times 8$

b.



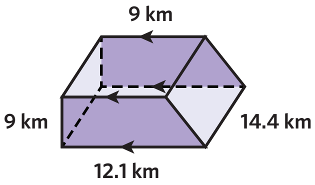
Volume $= \frac{4.2 \times \boxed{}}{2} \times 9$

c.



Volume $= \frac{\boxed{} \times 5}{2} \times 8$

d.



Volume $= \frac{1}{2}(\boxed{} + 12.1) \times 9 \times 14.4$

Back to contents

3. Fill in the blanks by using the words provided.

capacity **cubic** **cross-section** **volume**

The amount of space an object occupies is referred to as its ________ . Volume is measured with ________ units. The ________ of a container refers to the amount of liquid a container can hold. The area of the ________ of prisms and cylinders is used to determine the volume of these solid objects.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

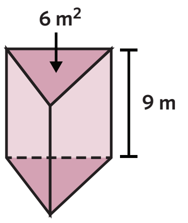
4. Convert the following. **WE1a**

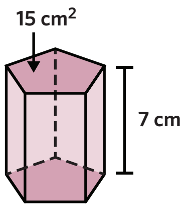
a. 10 000 mm^3 to cm^3
b. 5 cm^3 to mm^3
c. 2 500 000 mm^3 to m^3
d. 8 m^3 to mm^3
e. 15 000 cm^3 to m^3
f. 0.25 m^3 to cm^3
g. 50 000 mm^3 to m^3
h. 0.001 km^3 to mm^3

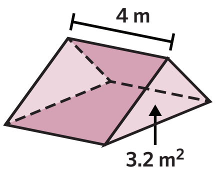
5. Convert the following. **WE1b**

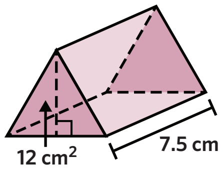
a. 5 ML to kL
b. 5 000 mL to L
c. 1 500 L to kL
d. 2 kL to ML
e. 2 ML to L
f. 750 L to ML
g. 50 kL to mL
h. 7 000 000 mL to kL

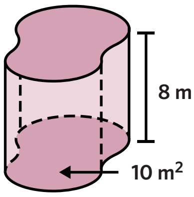
6. Calculate the volume of the solid objects. **WE2**

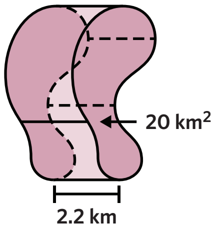
a.


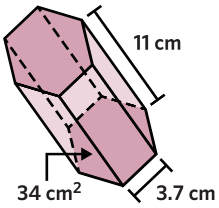
b.


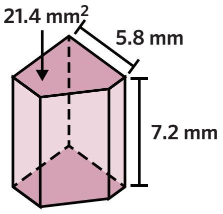
c.


d.


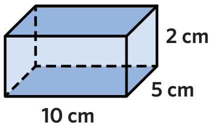
e.


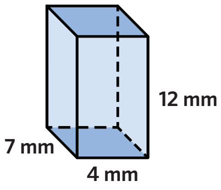
f.


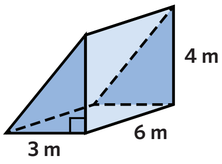
g.


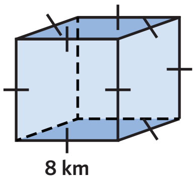
h.


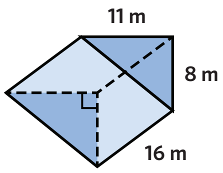
7. Calculate the volume of the following prisms. **WE3**

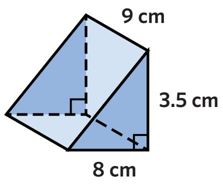
a.


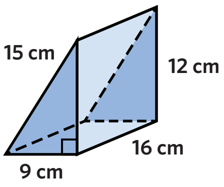
b.


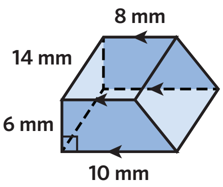
c.


d.


e.


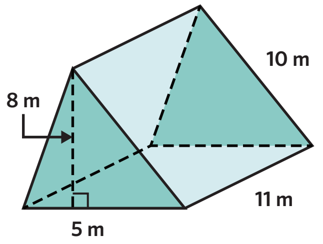
f.


g.


h.


Back to contents

8. Calculate the volume of the following prism.

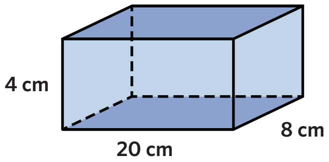


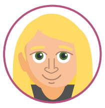
A. 34 m^3 **B.** 200 m^3 **C.** 220 m^3 **D.** 440 m^3 **E.** 4400 m^3

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the volume of the rectangular prism.



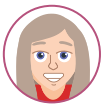


Student A

Volume $= l \times w \times h$

$= 20 \times 8 \times 4$

$= 640 \text{ cm}^3$



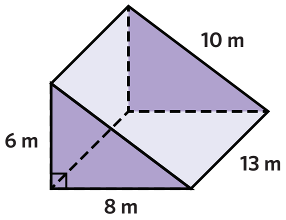
Student B

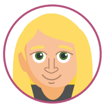
Volume $= (l \times w)^2$

$= (20 \times 8)^2$

$= 25\,600 \text{ cm}^3$

b. Calculate the volume of the triangular prism.





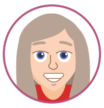
Student A

Volume $= 2 \times \text{base} + \text{side } 2 + \text{side } 1 + \text{side } 3$

$= 2\left(\frac{8 \times 6}{3}\right) + (6 \times 13) + 2(8 \times 13)$

$= 48 + 78 + 208$

$= 334 \text{ m}^3$



Student B

Volume $= \frac{b \times h}{2} \times l$

$= \frac{8 \times 6}{2} \times 13$

$= 312 \text{ m}^3$

Problem solving

Question working paths

Mild 10, 11, 12	**Medium** 11, 12, 13	**Spicy** 12, 13, 14

10. How much space does a rectangular shoebox take up in cubic centimetres if its dimensions are 30 cm by 15 cm by 10 cm?

11. A chef bakes a cake that is 11 cm tall. The top of the cake has an area of 65 cm^2. What is the volume of the cake in cubic centimetres?

12. A tent shaped as a triangular prism measures 2.8 m in length, 1.6 m in width, and 1.5 m in height. What is the volume of the tent in cubic metres?

13. A creative ice sculpture is in the shape of a cube. How much liquid in millilitres is used to create the sculpture if all of its edges are 50 cm long? 1 cubic centimetre is equal to 1 millilitre.

14. Tracy's garden pond is empty and has a uniform cross-section from its lowest to highest point. The surface has an area of 35 m^2. If 60 mm depth of rain fell in a day, what is the total volume of rain contained in the garden pond in litres? 1 cubic metre is equal to 100 litres.

Reasoning

Question working paths

Mild 15(a,b,c,e) 🌶	**Medium** 15(a,b,c,e), 16(a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. The Boeing 747-400 is one of the world's largest and most iconic aeroplanes. The cargo hold of a Boeing 747-400 can vary depending on the specific model and configuration. Below are the details of one of the most common models which has two rectangular prism shaped cargo decks.

Main deck cargo volume

- Length: 29.7 m
- Width: 6.1 m
- Height: 2.7 m

Lower deck cargo volume

- Length: 6.3 m
- Width: 3.7 m
- Height: 1.73 m

a. Calculate the main deck cargo volume of a Boeing 747-400. Round to two decimal places.

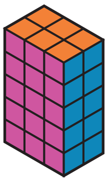
b. The main deck is used to transport large equipment with an average density of 200 kg/m^3. Calculate the maximum weight of the cargo that can be loaded onto the main deck. Rounded to the nearest whole number

c. Due to some large cargo pieces, the available height in the main deck is reduced by 0.5 m. Recalculate the new main deck cargo volume. Round to two decimal places.

d. The lower deck cargo is completely filled and then all of its contents are removed and packed into the main deck. What percentage of the main deck's volume is now full, rounded to two decimal places? Use the volume of the main deck with its reduced height.

e. How should aid organisations organise the Boeing 747-400 cargo hold to transport emergency supplies to regions affected by disasters?

16. Each unit block in the diagram is equal to 1 cm^3.



a. Using the pink face as the cross-section, calculate the volume of the solid object.

b. Using the orange face as the cross-section, calculate the volume of the solid object.

c. Compare and contrast the volumes calculated in part **a** and **b**, and explain why different cross-sections can be used to calculate the volume of some prisms.

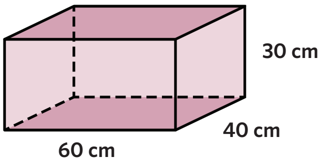
Exam-style

17. A Jenga tower is a square-based prism. The tower has a length and width of 19.05 cm and a volume of 25 348.74 cm^3. (1 MARK)

The height of the Jenga tower, in centimetres, is closest to

A. 19 cm **B.** 25 cm **C.** 65 cm **D.** 70 cm **E.** 1330 cm

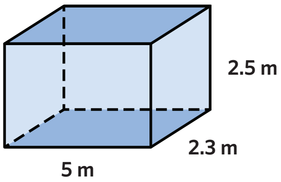
18. Mimi is planning to decorate an empty fish tank. The fish tank is a rectangular prism with a length of 60 cm, a width of 40 cm, and a height of 30 cm. (3 MARKS)



a. Calculate the volume of the fish tank, in cubic centimetres. 1 MARK

b. Mimi has colourful pebbles to decorate the fish tank. Each pebble is a rectangular prism and has a length of 2 cm, width of 2 cm, and a height of 1 cm. Calculate the maximum number of small pebbles Mimi can fit inside the tank. 2 MARKS

19. A portable storage container is in the shape of a rectangular prism. It has a height of 2.5 m, a width of 2.3 m, and a length of 5 m, as shown in the diagram. (2 MARKS)



Calculate the volume of four portable storage containers in cubic metres.

20. A standard 3 × 3 × 3 Rubik's cube is made up of individual coloured cubelets. Each cubelet has a length, width and height of 1.57 cm. (3 MARKS)

Calculate the volume, in cubic centimetres, of one Rubik's cube. Round to two decimal places.

Remember this?

21. Carlos has completed $\frac{3}{4}$ of a 400 km bike race. Which decimal shows how much of the race Carlos has completed?

A. 0.250 **B.** 0.500 **C.** 0.625 **D.** 0.750 **E.** 0.875

22. John makes three necklaces for a total cost of $120. He sells one necklace for $50 and another necklace for $45. For what price must he sell the third necklace to achieve a profit?

A. More than $25
B. Less than $25
C. $25
D. The profit is not possible
E. The price doesn't matter

23. The table shows the fractions of the population in a city that use different modes of transportation.

Mode of transportation	Bus	Bicycle	Car	Walking
Fraction of the population	$\frac{1}{5}$	$\frac{1}{60}$	$\frac{1}{3}$	$\frac{1}{20}$

Which form of transport is the most uncommon?

A. Bus
B. Bicycle
C. Car
D. Walking
E. All modes of transportation are used equally

Back to contents

7H Volume of a cylinder

LEARNING INTENTIONS

Students will be able to:

- convert between volume and capacity
- understand cylinders as solids with a constant circular cross-section
- calculate the volume of a cylinder.

Volume and capacity are closely related concepts, often requiring conversion between the two. Cylinders, as three-dimensional shapes, have a constant circular cross-section throughout their height. To calculate the volume of a cylinder, understanding of the area of circles, semi-circles, and quadrants is essential. This involves skills such as multiplying and dividing by 1000 and accurately identifying the cylinder's cross-section.

KEY TERMS AND DEFINITIONS

- A **cylinder** is a 3-dimensional object that has 2 flat, circular bases that are connected by a curved surface.

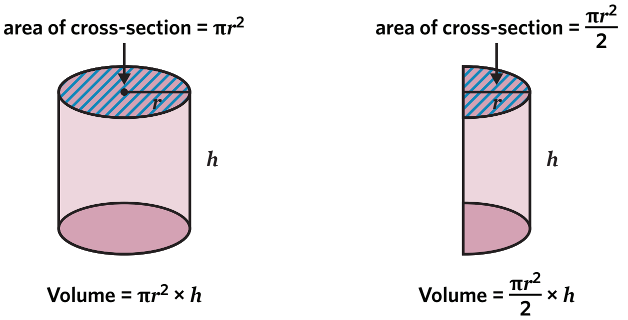
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Vera Larina/Shutterstock.com

In the world of beverages, understanding the concept of volume and capacity is essential. For instance, drink manufacturers need to determine how much liquid a cylindrical can or bottle can hold, which requires knowledge about the volume of a cylinder.

Key ideas

1. A cylinder has a uniform circular cross-section; therefore, the volume of a cylinder is calculated by multiplying the area of its base (a circle) by its height.



2. Volume and capacity are related measures, with volume often expressed in cubic units and capacity in mL, L, or kL.

Volume (square units)	Capacity (how much liquid?)
1 cm^3	1 mL
1000 cm^3	1 L
1 m^3	1000 L or 1 kL

Back to contents

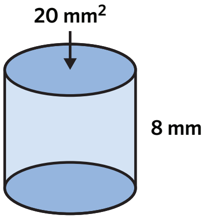
Worked example 1

Calculating the volume of a cylinder given the area of the cross-section

Calculate the volume of the cylinders.

a.

WE1a



Working

Volume = area of cross-section × height

$= 20 \times 8$

$= 160 \text{ mm}^3$

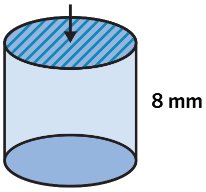
Thinking

Step 1: Write the formula for the volume of the solid object.

Step 2: Substitute the values to calculate the volume, with units.

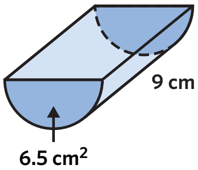
Visual support

area of cross-section = 20 mm^2



b.

WE1b



Working

Volume = area of cross-section × height

$= 6.5 \times 9$

$= 58.5 \text{ cm}^3$

Thinking

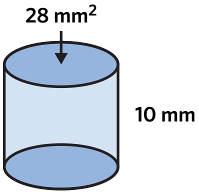
Step 1: Write the formula for the volume of the solid object.

Step 2: Substitute the values to calculate the volume, with units.

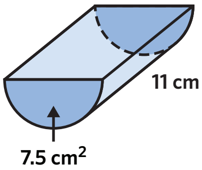
Student practice

Calculate the volume of the solid objects.

a.



b.



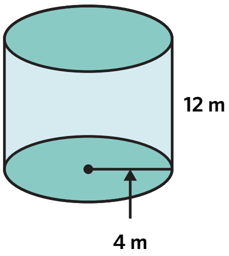
Worked example 2

Calculating the volume of a cylinder

Calculate the volume of the cylinders, correct to two decimal places.

a.

WE2a



Working

Volume $= \text{area of cross-section} \times \text{height}$

$= \pi r^2 \times h$

$= \pi \times 4^2 \times 12$

$= 192\pi$

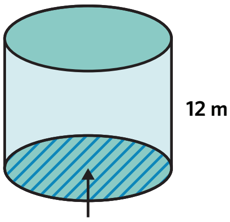
$\approx 603.19 \text{ m}^3$

Thinking

Step 1: Write the formula for the volume of a cylinder.

Step 2: Substitute the side lengths to calculate the volume, with units.

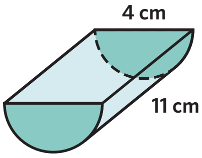
Visual support



Area of cross-section = $\pi r^2 = 16\pi$

b.

WE2b



Working

Volume $= \text{area of cross-section} \times \text{height}$

$= \frac{1}{2}(\pi r^2) \times h$

$h = 11 \text{ cm}$

$d = 4 \text{ cm}$ and $r = \frac{4}{2} = 2 \text{ cm}$

$= \frac{1}{2}(\pi \times 2^2) \times 11$

$= 22\pi$

$\approx 69.12 \text{ cm}^3$

Thinking

Step 1: Write the formula for the volume of a semi-cylinder.

Step 2: Identify the height and radius of the cylinder.

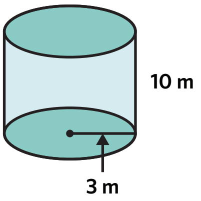
Step 3: Substitute the side lengths to calculate the volume, with units.

Back to contents

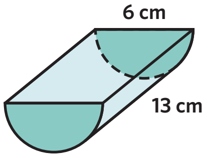
Student practice

Calculate the volume of the cylinders, correct to two decimal places.

a.



b.

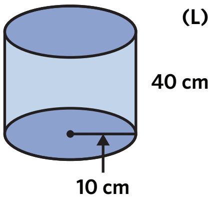


Worked example 3

Calculating the capacity of a cylinder

Calculate the capacity for the given metric, of the cylinders. Round to two decimal places.

a.



WE3a

Working

$$\text{Volume} = \text{area of cross-section} \times \text{height}$$
$$= \pi r^2 \times h$$
$$= \pi \times 10^2 \times 40$$
$$= 4000\pi \text{ cm}^3$$

$1000 \text{ cm}^3 = 1 \text{ L}$

$4000\pi \div 1000$

$\approx 12.57 \text{ L}$

Thinking

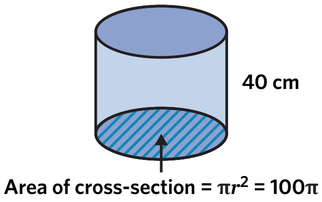
Step 1: Write the formula for the volume of a cylinder.

Step 2: Substitute the side lengths to calculate the volume, with units.

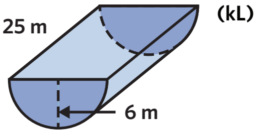
Step 3: Determine the multiplication or division required.

Step 4: Apply the conversion.

Visual support



b.



WE3b

Working

$$\text{Volume} = \text{area of cross-section} \times \text{height}$$
$$= \frac{1}{2}(\pi r^2) \times h$$

Thinking

Step 1: Write the formula for the volume of a semi-cylinder.

Continues →

$= \frac{1}{2}(\pi \times 6^2) \times 25$

$= 450\pi \text{ m}^3$

Step 2: Substitute the side lengths to calculate the volume, with units.

$1 \text{ m}^3 = 1000 \text{ L}$

Step 3: Determine the multiplication or division required.

$450\pi \times 1000$

$= 450\ 000\pi \text{ L}$

$1 \text{ L} = 1 \text{ kL}$

$450\ 000\pi \div 1000$

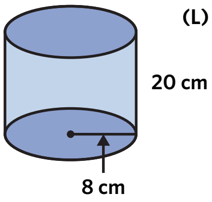
$\approx 1413.72 \text{ kL}$

Step 4: Apply the conversions.

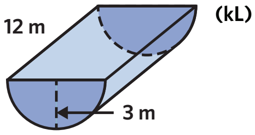
Student practice

Calculate the volume of the cylinders, correct to two decimal places.

a.



b.



Back to contents

7H Questions

Understanding worksheet

1. Fill in the missing information.

Example

$4\text{ cm}^3 = \boxed{4}\text{ ml}$

a. $86\text{ cm}^3 = \boxed{}\text{ mL}$

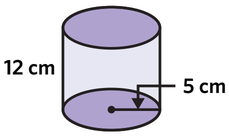
b. $\boxed{}\text{ cm}^3 = 734\text{ mL}$

c. $9\text{ m}^3 = 9000\boxed{}$

d. $\boxed{}\text{ m}^3 = 63\text{ L}$

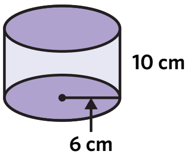
2. Complete the missing information.

Example



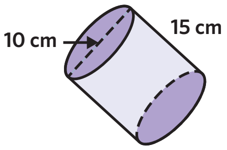
$\text{Volume} = \pi \times \boxed{5}^2 \times 12$

a.



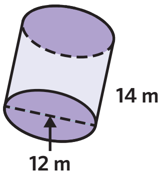
$\text{Volume} = \boxed{} \times 6^2 \times 10$

b.



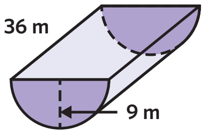
$\text{Volume} = \pi \times \boxed{}^2 \times 15$

c.



$\text{Volume} = \pi \times 6^2 \times \boxed{}$

d.



$\text{Volume} = \boxed{}(\pi \times 9^2) \times 36$

3. Fill in the blanks by using the words provided.

capacity **cross-section** **cylinder** **volume**

How much space an object occupies is referred to as its $\boxed{}$. When this is specifically about liquids, it's called $\boxed{}$. A $\boxed{}$ has a unique property: it has a circular $\boxed{}$, regardless of where it's cut parallel to its base. To calculate how much space is inside it, we have to use formulas related to circles and its height.

Back to contents

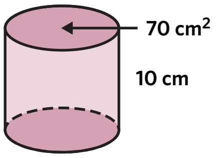
Fluency

Question working paths

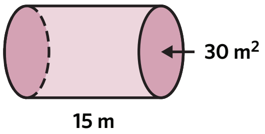
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f), 8

4. Calculate the volume of the following cylinders. **WE1**

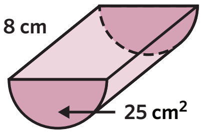
a.



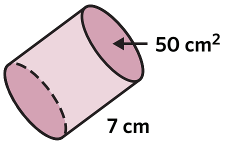
b.



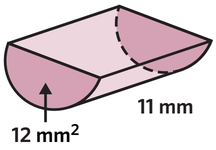
c.



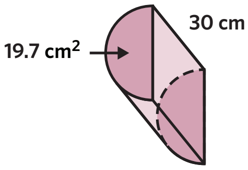
d.



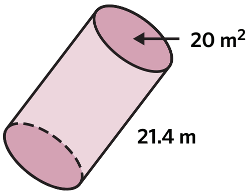
e.



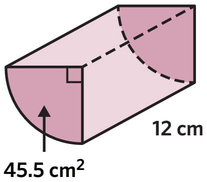
f.



g.

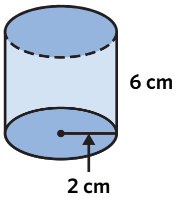


h.

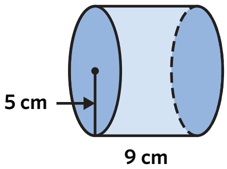


5. Calculate the volume of the cylinders, correct to two decimal places. **WE2a**

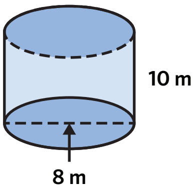
a.



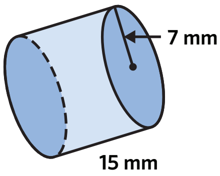
b.



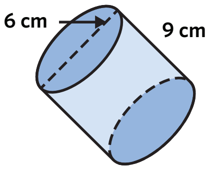
c.



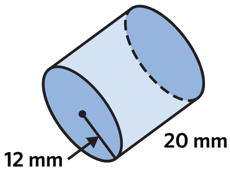
d.



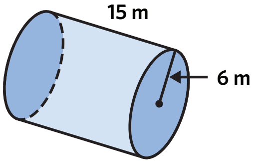
e.



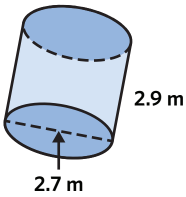
f.



g.

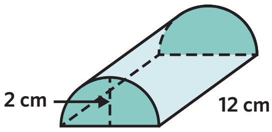


h.

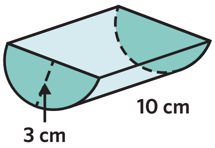


6. Calculate the volume of the cylinders, correct to two decimal places. **WE2b**

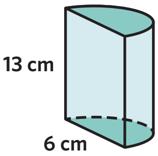
a.



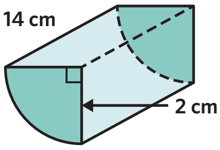
b.



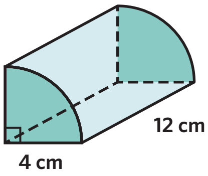
c.



d.



e.



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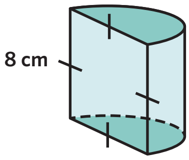
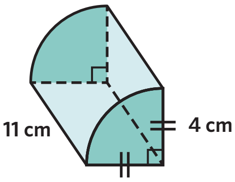
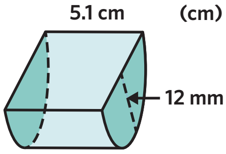


g.



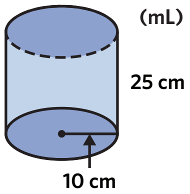
h.



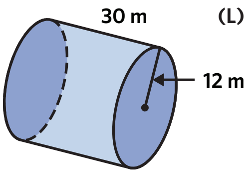
7. Calculate the capacity for the given metric, of the cylinders. Round to two decimal places.

WE3

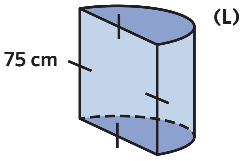
a.



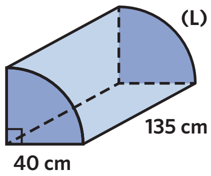
b.



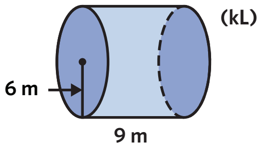
c.



d.



e.



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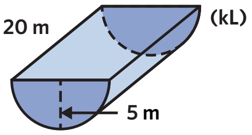

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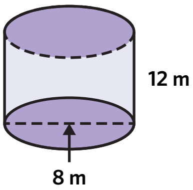
8. Calculate the volume of the cylinder, correct to two decimal places.

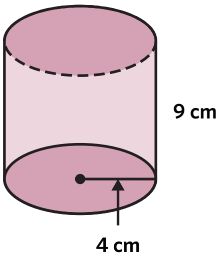


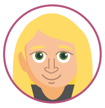
A. 96 m^3 **B.** 301.59 m^3 **C.** 384.00 m^3 **D.** 603.19 m^3 **E.** 2412.74 m^3

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the volume of the cylinder, correct to two decimal places.



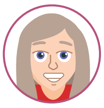


Student A

Volume $= 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 9$

$= 32\pi + 72\pi$

$= 326.73 \text{ cm}^3$



Student B

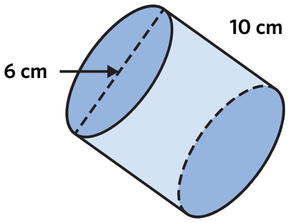
Volume $= \pi \times 4^2 \times 9$

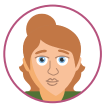
$= 144\pi$

$= 452.39 \text{ cm}^3$

Back to contents

b. Calculate the volume of the cylinder, correct to two decimal places.





Student A

$d = 6$

$r = 3$

$\text{Volume} = \pi \times 3^2 \times 10$

$= 90\pi$

$= 282.74 \text{ cm}^3$

Student B

$\text{Volume} = \pi \times 6^2 \times 10$

$= 360\pi$

$= 1130.97 \text{ cm}^3$

Problem solving

Question working paths

Mild 10, 11, 12	**Medium** 11, 12, 13	**Spicy** 12, 13, 14

10. A cylindrical container has a radius of 10 cm and height of 15 cm. Calculate the volume in cubic centimetres. Round the answer to two decimal places.

11. A gas cylinder has a diameter of 20 cm and is 75 cm tall. Calculate its volume in cubic centimetres. Round the answer to two decimal places.

12. The Lismore Water Tower is located in rural Victoria and has a height of 31 m and a diameter of 14 m. How much water can it store, in litres, when it is three quarters full? Round the answer to two decimal places.

13. The Shark Dive Xtreme experience at the Melbourne Aquarium includes a 15 minute scuba dive with real sharks. The cylindrical tank for the dive has a depth of 8 m and a radius of 4 m. What is the capacity of the tank in litres, to the nearest whole number?

14. 20 g of a sports electrolyte powder is required to be mixed with 500 mL of water to formulate a sport rehydration supplement. Approximately, how many grams of the electrolyte powder are required to be mixed with water to fill a cylindrical cooler with a diameter of 50 cm and a height of 25.56 cm?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	**Medium** 15 (a,b,c,e), 16 (a,b)	**Spicy** All

15. Cylinder chemical storage tanks are used for fluid containment. Efficient cylindrical storage distributes internal pressure more evenly, reducing stress concentration points and enhancing the tank's structural integrity.

a. A cylindrical storage tank for storing chemicals has a radius of 2 m and a height of 5 m. Calculate the capacity of the tank in litres. Round the answer to the nearest litre.

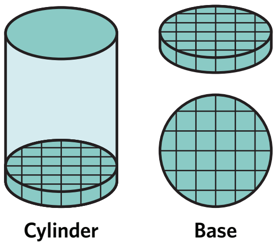
b. The tank can hold liquids with a density of 1000 kg/m^3. Determine the maximum weight of the liquid the tank can hold when completely full. Round the answer to two decimal places.

c. A cylindrical rod is placed vertically inside the tank, taking up 20% of the tank's radius and extending from the bottom to the top. Calculate the tank's capacity with the rod inside. Round the answer to two decimal places.

d. A different cylindrical storage container has a radius of 1.8 m. Calculate the height of the tank if its capacity is the same as the tank with the rod in it from part **c**. Round the answer to the nearest metre.

e. Space in some storage facilities is very limited. Propose a way to optimise cylindrical tank placement and organisation while maintaining safety.

16. The following diagram attempts to show why the area of a base of a cylinder multiplied by its height is equal to the volume of the cylinder. The units shown are 1 cm × 1 cm.



a. Count the area of the base and state its approximate area. Give an approximate whole number answer in square centimetres.

b. The cylinder has a height of 8 units. State the approximate volume of the cylinder. Give an approximate whole number answer in cubic centimetres.

c. Compare and contrast the image of the base and cylinder, and justify why πr^2 is used in the formula for the volume of a cylinder.

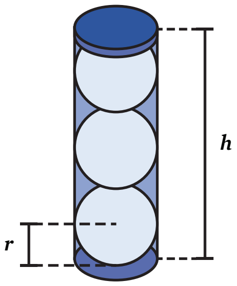
Exam-style

17. A cylinder has a radius of r centimetres. The height of the cylinder is 14 cm. (1 MARK)

If the volume of the cylinder is approximately 1580 cm^3, then the radius of the cylinder, in centimetres, is closest to.

A. 4 **B.** 5 **C.** 6 **D.** 8 **E.** 10

18. A cylindrical squash ball container can hold a maximum of three balls stacked on top of each other. The top ball touches the lid and the bottom ball touches the base of the container, as shown in the diagram. The radius of a squash ball is 20 mm. (3 MARKS)



a. Calculate the height of the cylindrical squash ball container. Give the answer in millimetres. 1 MARKS

b. Calculate the volume of a cylindrical squash ball container in cubic centimetres. Round the answer to two decimal places. 2 MARKS

19. A toilet roll is in the shape of a cylinder with a cylindrical hole in its centre as shown in the diagram. The diameter of the entire roll is 12.75 cm and its height is 11 cm. The hole in its centre has a radius of 2.5 cm. (3 MARKS)

Calculate the volume of a toilet roll in cubic centimetres. Round the answer to two decimal places.

20. A jar of marbles is designed as a cylinder. The jar has a circumference of 25.12 cm and a volume of 628 cm^3. Determine the height of the jar in centimetres. Round the answer to the nearest whole number. (3 MARKS)

Remember this?

21. The table shows a pattern. The left and right numbers are connected by a rule. Select the number to replace the star (★) to complete the table.

Left number	Right number
2	5
4	10
6	15
8	20
★	35

A. 10 **B.** 14 **C.** 17 **D.** 20 **E.** 25

22. A total of 500 000 books were sold in a bookstore in 2023. Of these books, 275 000 were fiction books. How many non-fiction books were sold in the bookstore in 2023?

A. 125 000 **B.** 175 000 **C.** 225 000 **D.** 235 000 **E.** 275 000

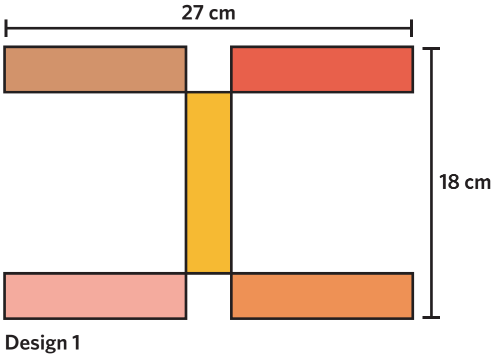
23. Sara saved \$210 in 4 weeks. After the first week, she saved \$15 more each week than the week before. How much did Sara save in the first week?

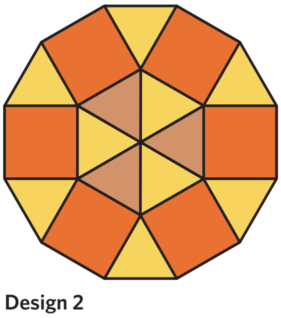
A. \$20 **B.** \$30 **C.** \$40 **D.** \$50 **E.** \$60

Back to contents

Chapter 7 extended application

1. Michelle is a mosaic artist who is currently working on multiple designs that include triangles and different quadrilaterals. Designing and creating mosaics is time intensive. On average, it takes Michelle 55 minutes to complete 10 square centimetres of her designs.





Michelle is yet to create Design 3, but she would like to create a trapezium that has an area of 34 cm^2, to celebrate her 34th birthday.

a. Calculate the length of the unknown parallel side of Design 3, if its perpendicular height is 4 cm and the known parallel side is 6 cm.

b. Design 1 is made up of 5 congruent rectangles. Create two equations to determine the length and width of a rectangle. Then, calculate the perimeter and area of the coloured parts for design 1.

c. Design 2 is a dodecagon that is made up of squares and equilateral triangles. If the side length of each square is 4 cm, determine the perimeter and area of Design 2, rounded to 2 decimal places where needed.

d. Michelle prices her mosaics at $10.55 per square centimetre. List the price of each design for her customers, rounded to the nearest cent.

e. One of Michelle's customers has requested that she triple the size of all three of her designs. Michelle is concerned that this will require too much time and effort. If Michelle works 6 hours per day, estimate how many work days it will take Michelle to complete this work for her customer?

f. How is designing mosaics different from other forms of art?

2. At standard dairy farms, optimal storage capacity and efficiency are paramount. Meticulous planning and precise calculations by engineers ensure that milk is stored in a safe and sterile environment, maximising space without compromising on storage quality.

Image: Ken Felepchuk/Shutterstock.com

a. The cylindrical part of the silo has a circumference of 28.26 metres. Calculate the diameter of the silo's base rounded to the nearest whole number.

b. If a quarter of the circular top of the silo (with the diameter found in part **a**) is to be used as a special ventilation section, determine the perimeter and area of this sector, rounded to the nearest whole number.

c. A rectangular storage vessel with a length of 7 metres, width of 5 metres and a height of 4.5 metres is built in the same storage facility as the cylindrical silo. Calculate the surface area of the rectangular storage vessel, excluding its floor base.

d. Using the diameter from part **a**, determine the capacity, in litres, of the cylindrical silo rounded to the nearest whole number. Assume the silo has the same height as the rectangular storage extension.

e. During the storage process it is optimal that the silo is only full to 80% of its capacity. Using the capacity from part **d**, determine how many litres of milk the silo can store when 80% full. Note that 1 cubic metre holds 1000 litres of milk.

f. Sustainability has become an essential factor in modern dairy farming. Suggest a sustainable practice dairy farmers might adopt to be more environmentally friendly.

3. A city is planning to construct an underground transportation tunnel. The proposed tunnel is to have a straight run of 5 km with a consistent diameter of 26 m. The primary machine used for boring this tunnel is a cylindrical drill with conical tips. The engineering team has two drill options:

- Drill A: Tip cone with a base diameter of 2 metres and a height of 3 metres. The cylindrical body has a diameter of 2 metres and a height of 10 metres.
- Drill B: Tip cone with a base diameter of 2.5 metres and a height of 3.5 metres. The cylindrical body has a diameter of 2.5 metres and a height of 12 metres.

The costs of a full insertion is:

- Drill A: $15.20
- Drill B: $29.00

Note: A full insertion of a drill, will result in the displacement of earth. The volume of earth displaced will be equivalent to the drill's volume.

a. Calculate the total volume, in cubic kilometres, of the tunnel. Round the answer to 3 decimal places

b. A metallic material is painted on each drill to make its surface even stronger. Calculate the combined surface area, in square metres, of Drill A and Drill B, excluding the base of the conical parts and excluding the bases of the cylindrical parts. Round the answer to 2 decimal places.

c. For Drill A and Drill B, determine the total volume, in cubic metres, of earth displaced by each of the drills during one full insertion. Round the answer to 2 decimal places.

d. The entire tunnel is excavated and cement is poured along the entire length of the 5 km tunnel. To build the road, 15% of the tunnel's volume is filled with cement. How many litres of cement are required for the road, rounded to the nearest whole number?

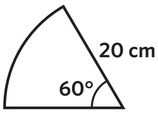
e. The city would like to ensure that the construction project is within the budget. If the builders use one drill to excavate the entire tunnel, which drill will be cheaper to use and how much cheaper will it be than using the alternative option, rounded to the nearest whole dollar?

f. Suggest which drill should be used to construct the tunnel. Give a reason for your suggestion.

Chapter 7 review

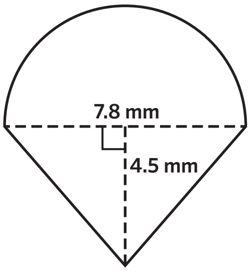
Multiple choice

1. What is the perimeter of this sector in millimetres, rounded to one decimal place? **7A,B**



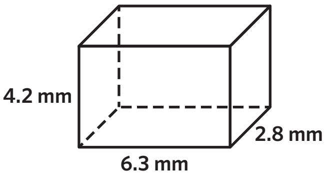
A. 209.4 mm **B.** 569.4 mm **C.** 600.2 mm **D.** 609.4 mm **E.** 1656.6 mm

2. What is the area of this composite shape in centimetres squared, rounded to two decimal places? **7C,D**



A. 0.29 cm^2 **B.** 0.41 cm^2 **C.** 0.83 cm^2 **D.** 2.09 cm^2 **E.** 41.44 cm^2

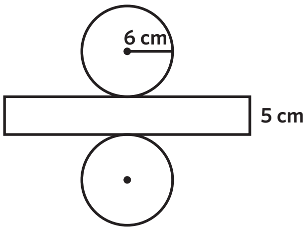
3. What is the missing information for calculating the total surface area of this prism? **7E**



Total surface area = 2(4.2 × 6.3) + 2(6.3 × ☐) + 2(2.8 × 4.2)

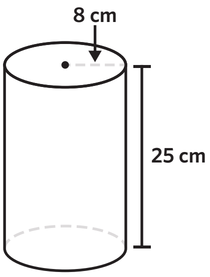
A. 2.8 **B.** 4.2 **C.** 6.3 **D.** 7.0 **E.** 13.3

4. What is the total surface area of the cylinder represented by the following net, rounded to two decimal places? **7F**



A. 30.00 cm^2 **B.** 256.19 cm^2 **C.** 309.59 cm^2 **D.** 414.69 cm^2 **E.** 603.19 cm^2

5. What is the capacity of this cylinder, in litres, rounded to two decimal places. **7H**



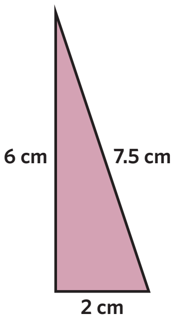
A. 0.063 L **B.** 1.97 L **C.** 5.00 L **D.** 5.03 L **E.** 20.11 L

Back to contents

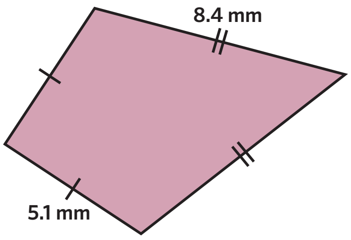
Fluency

6. Determine the perimeters of the following shapes. **7A**

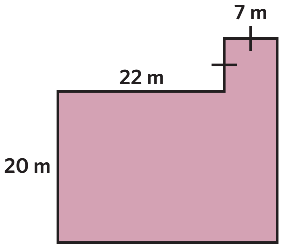
a.



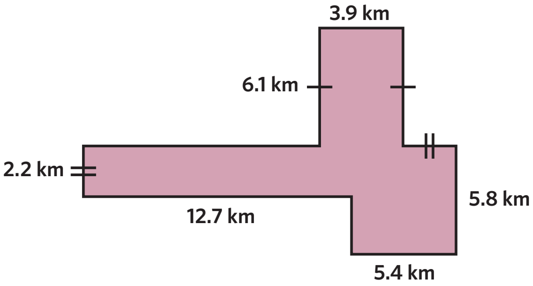
b.



c.

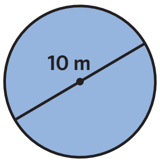


d.

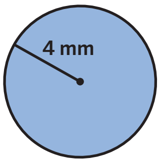


7. Calculate the circumference, correct to two decimal places. **7B**

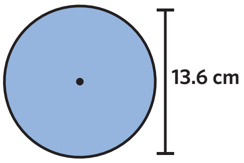
a.



b.



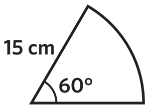
c.



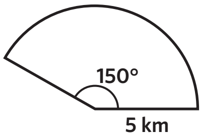
d.

8. Calculate the perimeter of the sectors, correct to two decimal places. **7B**

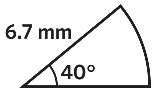
a.



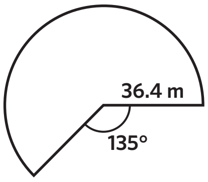
b.



c.

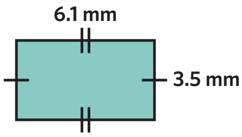


d.

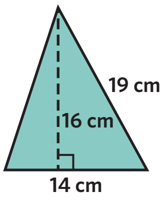


9. Determine the areas of the following shapes. **7C**

a.



b.



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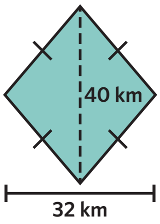
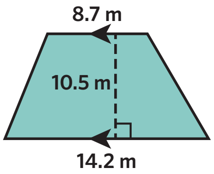
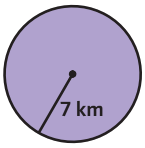


d.

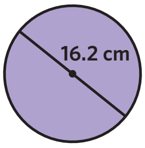


10. Calculate the areas of the following shapes, correct to two decimal places. **7C**

a.



b.



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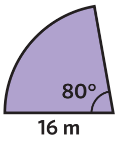

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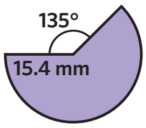
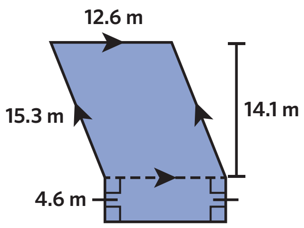


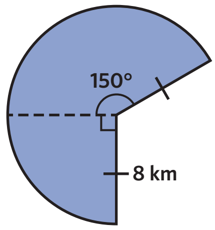
Back to contents

11. Calculate the perimeter and the area for the following composite shapes, rounded to two decimal places. 7D

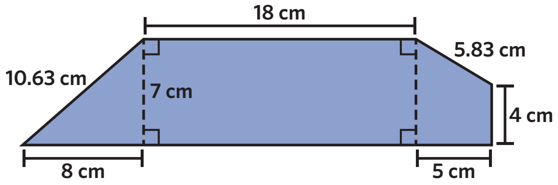
a.



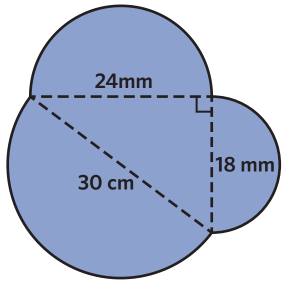
b.



c.

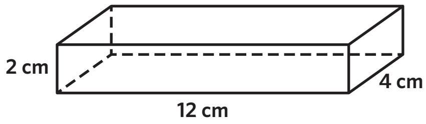


d.

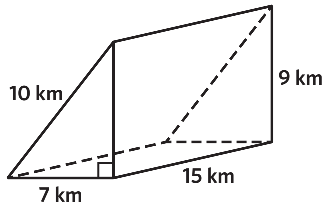


12. Calculate the total surface area of the following prisms. 7E

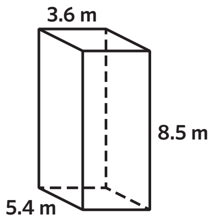
a.



b.



c.



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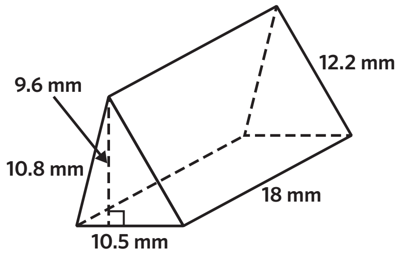
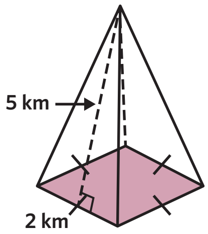
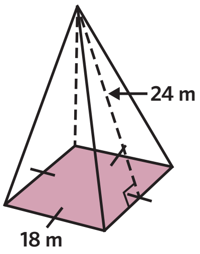


13. Calculate the total surface area of the following pyramids. 7E

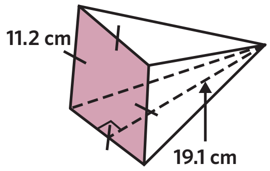
a.



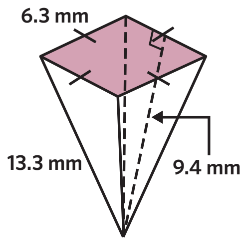
b.



c.

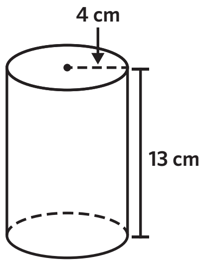


d.

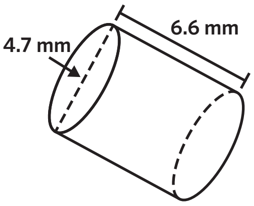


14. Calculate the total surface area of the objects, correct to two decimal places. 7F

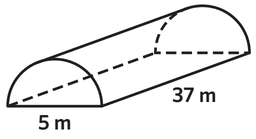
a.



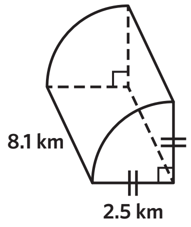
b.



c.

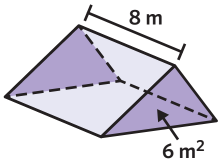


d.

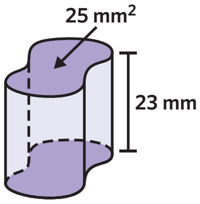


15. Calculate the volume of the solid objects. 7G

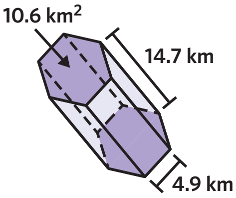
a.



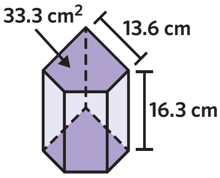
b.



c.

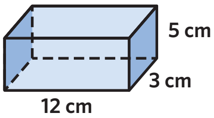


d.

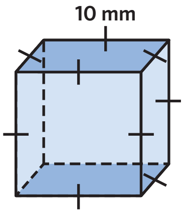


16. Calculate the total volume of the prisms. 7G

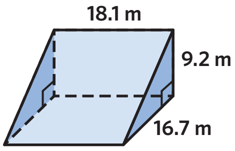
a.



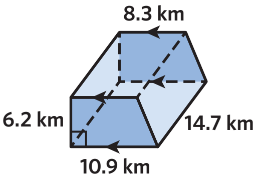
b.



c.

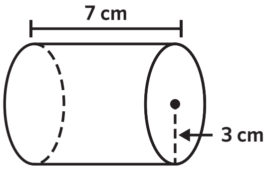


d.

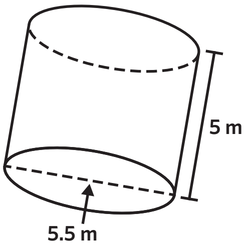


17. Calculate the volume of the cylinders, correct to two decimal places. 7H

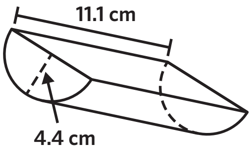
a.



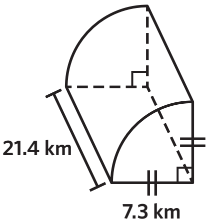
b.



c.



d.



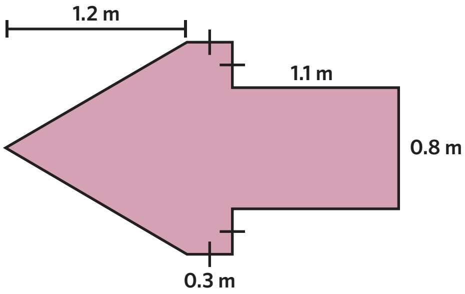
Problem solving

18. Sophie measures the height of three different plants in her garden. What is the total height of the three plants, in centimetres, if the three individual measurements are 16 cm, 1441 mm, and 2.87 m? 7A

19. Seamus bakes a circular cake that has a radius of 12.3 cm to celebrate his birthday. He shares the cake equally between six people. Each piece is the shape of a sector. What is the perimeter of the base of one piece of cake, rounded to the nearest centimetre? 7B

20. A standard bicycle wheel has a diameter of 622 mm. Tessa measures the diameter of her bicycle wheel as 58 cm. What is the difference in the area between a standard bicycle wheel and Tessa's bicycle wheel in squared centimetres? Round to two decimal places. 7C

21. An arrow is to be painted on a road to indicate the direction of traffic. 7D

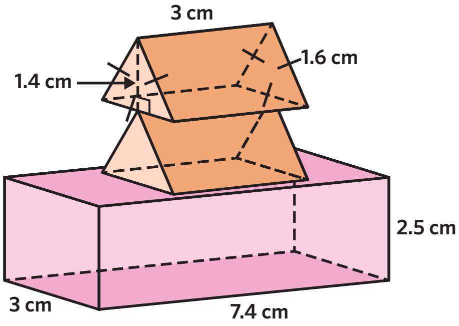


What is the total area to be painted in square metres?

22. Jimmy wants to construct his own camping tent rather than purchasing one. He builds a metal frame shaped as a triangular prism with a base of 1.9 m, a height of 1.3 m, a slant height of 2.3 m, and a length of 2.8 m. How much tent fabric, in squared metres, is needed to completely cover the metal frame? 7E

23. Zoe is using stickers to decorate her cylindrical drink bottle that is 22.5 cm high and has a diameter of 9.4 cm. If it costs Zoe $0.02 to cover one square centimetre of the bottle in stickers, how much will it cost to cover the entire drink bottle? Round to two decimal places. 7F

24. Tom is an artist who is creating a sculpture consisting of a rectangular prism with two identical triangular prisms stacked on top. 7G

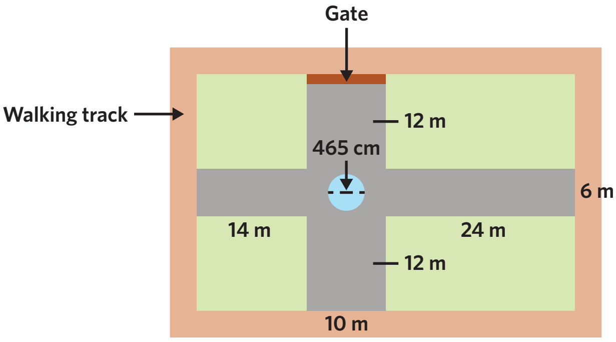


To enter a particular art competition, the volume of the sculpture must not exceed 60 000 mm^3. By how much does the volume of Tom's sculpture exceed the competition's limit in cubed millimetres?

25. Annabel wants to install a water trough shaped as a half cylinder in a paddock. Annabel needs the trough to hold 650 L of water when completely full so her horses have enough to drink. If the water trough has a radius of 65 cm, what is the minimum length it must be to hold this amount of water, rounded to two decimal places? 7H

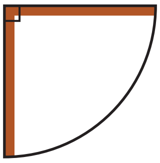
Reasoning

26. A park has a fountain located at the intersection of two paths, as well as grass areas. The local council decides to visit the park and measures various dimensions. They want to use these measurements to understand more about the park and its features. The council's measurements are shown in the diagram.



a. The entire park is surrounded by a walking track. Calculate the total length of the inner edges of the walking track, in metres.

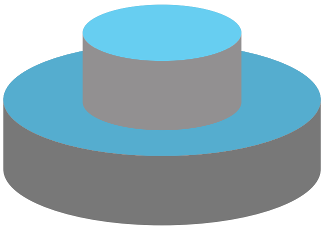
b. There is a gate at the main entrance. It covers the area of a sector when completely opened and closed.



What is the area that the gate covers as it is opened and closed, rounded to two decimal places?

c. The paths inside the park have weathered over time and need to be repaved. If the pavement material costs \$8.50 per square metre, how much will it cost the council to repave the paths? Assume that the area under the fountain does not need to be repaved.

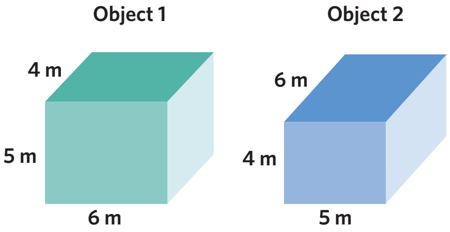
d. The council decides to remodel the design of the fountain to **two** cylindrical vessels that are filled with water.



From smallest to largest, the diameters of the vessels are 210 cm and 420 cm. Each vessel is 60 cm high. What is the total capacity of the fountain in litres, rounded to two decimal places?

e. What other features could the local council add to this park?

27. Consider the objects.

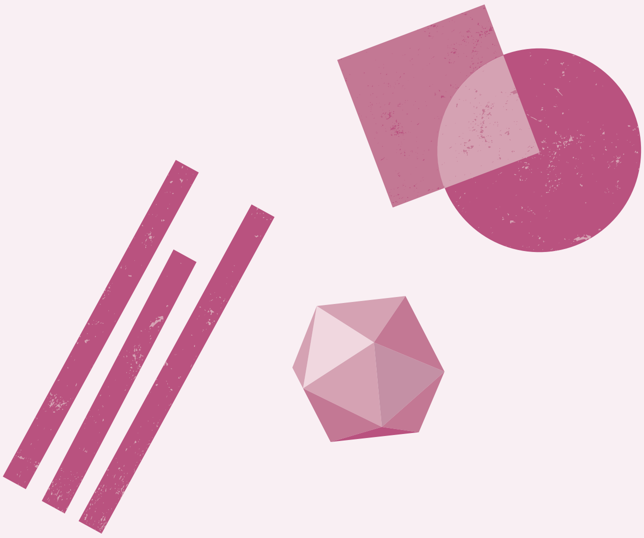


a. Calculate the volume of object 1.

b. Calculate the volume of object 2.

c. Compare parts **a** and **b** to explain how the commutative law relates to the calculation of the volume of rectangular prisms.

Back to contents



Chapter 8

Geometry

Measurement and geometry

Back to contents

Chapter 8 research summary

Geometry

Big ideas

Geometry is a branch of mathematics that deals with the properties, measurement, and relationships of points, lines, angles, surfaces and solids. The big ideas that underpin the study of geometry in Year 9 mathematics are crucial for students to develop a deep understanding of the subject and its applications. Understanding geometry lays the foundation for more advanced studies in Mathematics and equips students with the skills and knowledge they need to navigate the world around them.

Spatial reasoning

One of the primary big ideas in geometry is spatial reasoning. This involves the ability to visualise and understand the position, direction and movement of objects in space. By Year 9, students should be able to mentally manipulate two-dimensional shapes and three-dimensional objects, predict the results of transformations and understand the properties that remain invariant under such transformations.

Congruence and similarity

These are two fundamental ideas in geometry that deal with the comparison of shapes. While congruent shapes are exact matches in size and shape, similar shapes maintain the same shape but can have different sizes. Understanding the criteria for congruence and similarity, especially for triangles is a key big idea for geometry.

Proportional reasoning

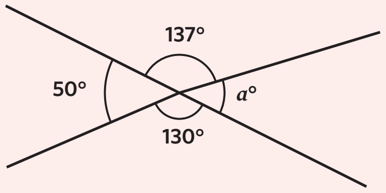
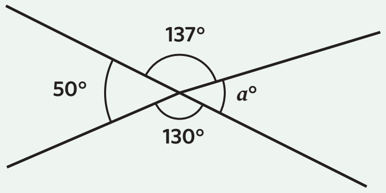
Proportional reasoning enables students to understand and describe the relationship between geometric figures and their dimensions, ensuring a deeper understanding of geometric principles and their applications. Proportional reasoning serves as a bridge between algebraic and geometric thinking, helping students to see and work with relationships, patterns, and structures within and across shapes.

Visual representations

Interactive software

There are many online tools and software like Desmos or GeoGebra that allow students to create, modify, and explore geometric constructions and figures. With friendly simple interfaces, users can create and manipulate geometric shapes, lines and points. When choosing the best software, consider the curriculum, the specific learning objectives, the technological capabilities of the devices being used, and student preferences.

Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students think co-interior angles are equal.	Angles are equal.	Angles sum to 180°.	8A
Students apply vertically opposite properties to any four lines with a common vertex.	 $a° = 50°$	 $a° = 43°$	8A

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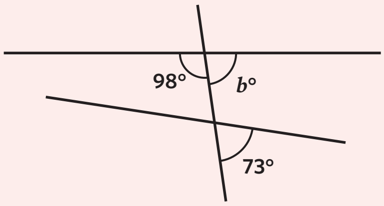
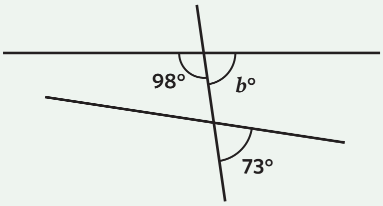
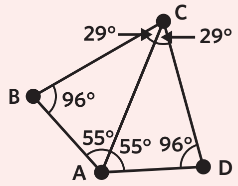
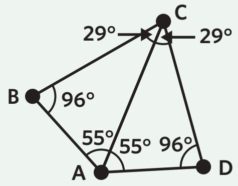
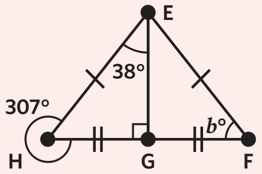
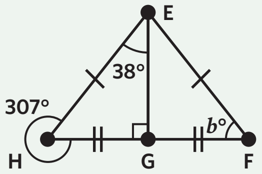
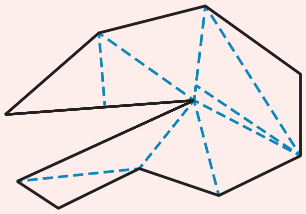
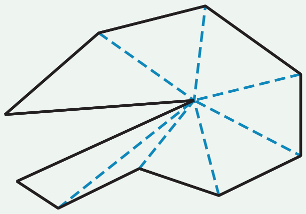
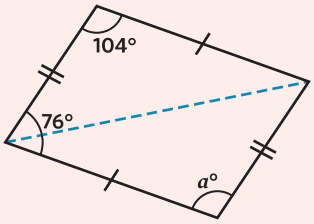
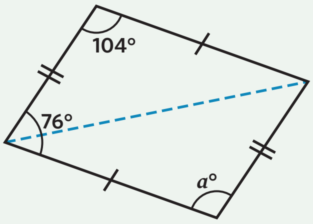
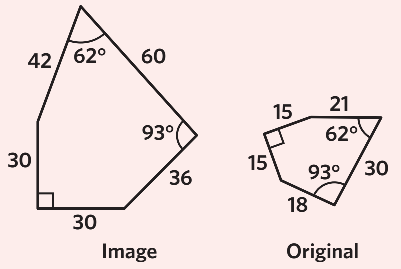
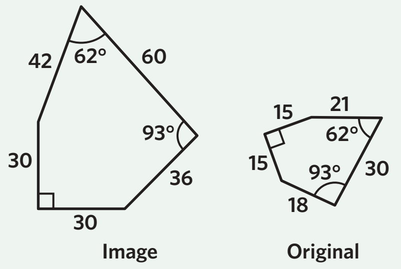
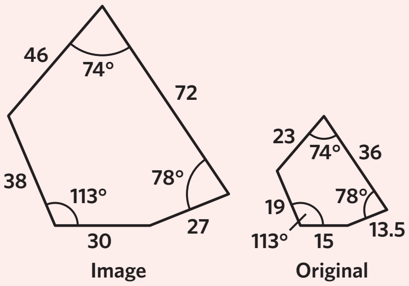
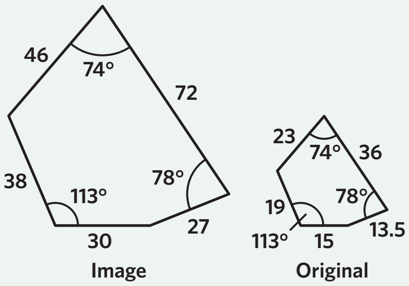
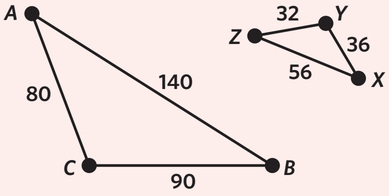
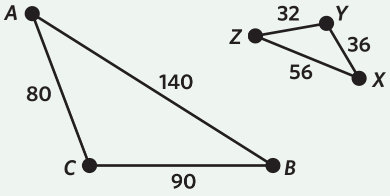
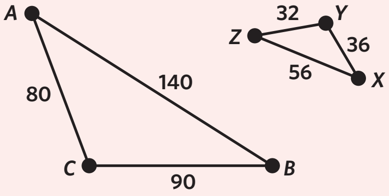
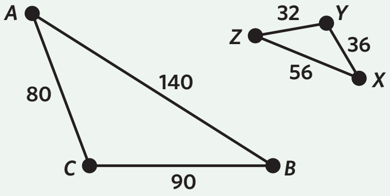
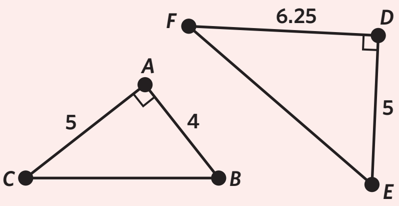
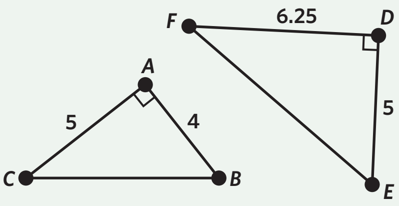
Back to contents

Misconception	Incorrect ✗	Correct ✓	Lesson
Students apply angle properties of parallel lines when the lines are not parallel.	 $a° = 73°$	 $a° = 82°$	8A
Students use AAA as a congruence test.	 $\angle BAC = 55°$ $\angle ACB = 29°$ $\angle ABC = 96°$ ∴ $\triangle ABC$ and $\triangle ADC$ are congruent due to AAA.	 $\angle BAC = 55°$ $\angle ACB = 29°$ $\angle ABC = 96°$ AC is a common side length. ∴ $\triangle ABC$ and $\triangle ADC$ are congruent due to ASA.	8B
Students locate corresponding angles or sides by translating the image.	 $\angle HEG$ is corresponding to $\angle F = b°$. $\angle HEG = 38°$ $b° = 38°$	 $\angle H$ is corresponding to $\angle F = b°$. $\angle H = 360° - 307°$ $= 53$ $b° = 53°$	8B
Students use any arrangement of triangles.	 10 internal triangles, 10 sides $S = 10 \times 180°$ $= 1800°$	 8 internal triangles, 10 sides $S = (n - 2) \times 180°$ $= (10 - 2) \times 180°$ $= 8 \times 180°$ $= 1440°$	8C
Students incorrectly name quadrilaterals therefore apply the incorrect rules for finding missing angle measure or side lengths.	 In a kite opposite angles sum to 180°. $104° + a° = 180°$ $a° = 76°$	 In a parallelogram opposite angles are equal so $a° = 104°$.	8C

Continues →

Back to contents

Misconception	Incorrect ✗	Correct ✓	Lesson
Students believe reduction in size must be a negative number.	 Scale factor $= -0.5$	 Scale factor $= 0.5$	8D
Students believe a scale factor must always be greater than 1.	 Scale factor $= \frac{\text{image length}}{\text{original length}}$ $= \frac{30}{15}$ $= 2$	 Scale factor $= \frac{\text{image length}}{\text{original length}}$ $= \frac{15}{30}$ $= \frac{1}{2}$	8D
Students compare corresponding features in the incorrect order.	 Scale factor of $\triangle ABC$ to $\triangle XYZ = \frac{9}{35}$.	 Scale factor of $\triangle ABC$ to $\triangle ZXY = \frac{2}{5}$.	8E
Students write the scale factor in the incorrect order using the reciprocal.	 Scale factor of $\triangle ABC$ to $\triangle ZYX = \frac{5}{2}$.	 Scale factor of $\triangle ABC$ to $\triangle ZYX = \frac{2}{5}$.	8E
Students believe any similarity test used on a right-angled triangle must be RHS.	 $\triangle ABC \sim \triangle DEF$ by RHS.	 $\triangle ABC \sim \triangle DEF$ by SAS.	8E

Back to contents

8A Angles and parallel lines

LEARNING INTENTIONS

Students will be able to:

- work with complementary and supplementary angles, vertically opposite angles and angles in a full circle
- identify the types of angles formed by parallel lines and a transversal
- use angle properties to determine if two lines are parallel
- calculate unknown angles in parallel lines.

Angles play a fundamental role in geometry, with various types and properties that dictate their relationships. Complementary and supplementary angles, vertically opposite angles, and angles in a full circle each have distinct characteristics. Parallel lines intersected by a transversal create specific angles, and understanding these angle properties can determine if two lines are parallel. Mastery of these concepts allows for the determination of unknown angles, especially in contexts involving parallel lines.

KEY TERMS AND DEFINITIONS

- **Perpendicular** lines form a right angle (90°).
- **Parallel** lines never touch and are always the same distance apart.
- **Adjacent angles** are angles that share a vertex and a common side.
- The **angle** between line segments AB and BC is denoted $\angle ABC$.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

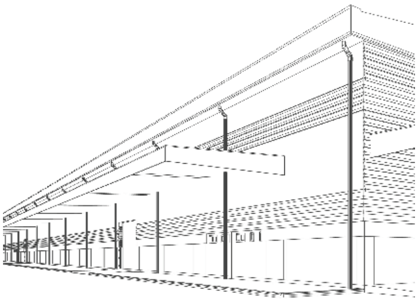


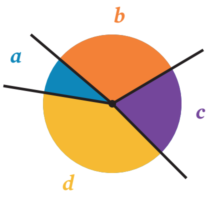
Image: Jinda.design/Shutterstock.com

Drawing in perspective is foundational in art. When artists want to depict a three-dimensional space on a two-dimensional canvas, they often use parallel lines and transversals to create vanishing points. This technique gives depth to the artwork, with angles formed by parallel lines and a transversal playing a key role in achieving the desired effect.

Key ideas

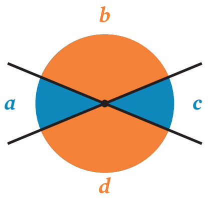
1. Angles meeting at a vertex can share special relationships.

A full **revolution** is when all angles at a point sum to 360°.



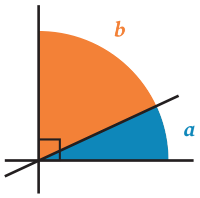
$$a + b + c + d = 360°$$

Vertically opposite angles are equal to each other.



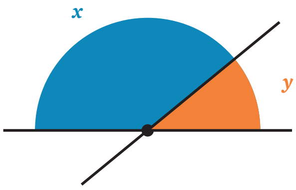
$$b = d \text{ and } a = c$$

Complementary angles sum to 90°.



$$a + b = 90°$$

Supplementary angles sum to 180°.



$$x + y = 180°$$

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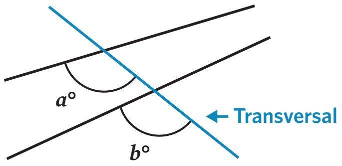
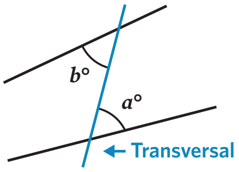
Back to contents

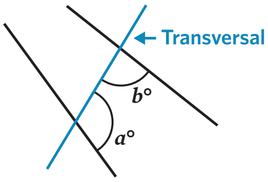
2. Angles formed by two lines and a transversal form corresponding, alternate and co-interior angle pairs.



Corresponding angles are in matching positions. They are on the same side on the transversal.

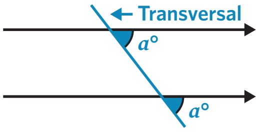


Alternate angles are on either side of the transversal. They are either both in between two straight lines, or both outside of two straight lines.

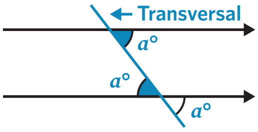


Co-interior angles are in between two straight lines and on the same side of the transversal.

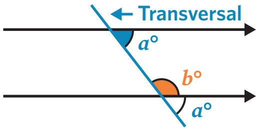
3. Angle properties can be used to determine if two lines are parallel.



Corresponding angles must be equal.



Alternate angles are equal.



$$a° + b° = 180°$$

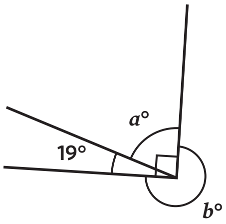
Co-interior angles sum to 180° (supplementary).

Worked example 1

Determining angles at a point

Determine the unknown angles, giving reasons, for each diagram.

a. WE1a



Working

$a°$ and 19° form a complementary angle.

$a°$, 19°, and $b°$ form a full revolution.

$a + 19° = 90°$

$a = 71°$

$a° + b° + 19° = 360°$

$71° + b° + 19° = 360°$

$90° + b° = 360°$

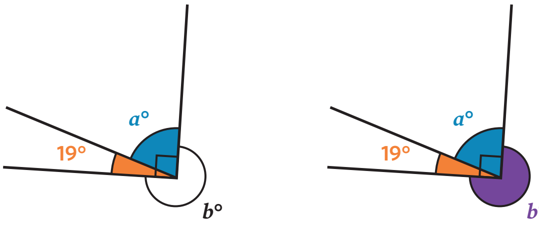
$b° = 270°$

Thinking

Step 1: Establish the connection between the known and unknown angles.

Step 2: State the relationship using an equation and solve.

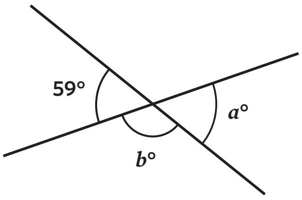
Visual support



$a° + 19° = 90°$ $a° + b° + 19° = 360°$

Continues →

b.



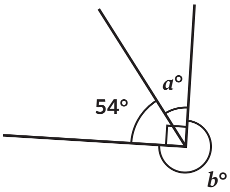
WE1b

Working	Thinking
$a°$ and 59° are vertically opposite angles. $b°$ and 59° form a supplementary angle.	**Step 1:** Establish the connection between the known and unknown angles.
$a° = 59°$ $b° + 59° = 180°$ $b° = 121°$	**Step 2:** State the relationship using an equation and solve.

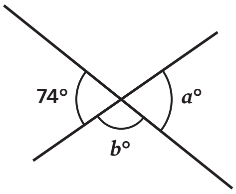
Student practice

Determine the unknown angles, giving reasons, for each diagram.

a.



b.

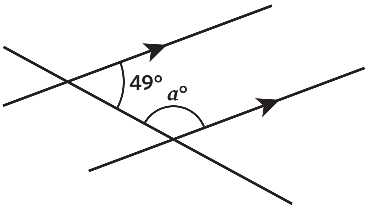


Worked example 2

Determining angles involving parallel lines and a transversal

Determine the unknown angles, giving reasons, for each diagram.

a.



WE2a

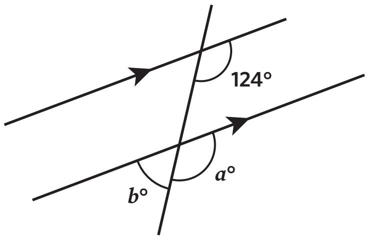
Working	Thinking
49° and $a°$ are co-interior angles on parallel lines.	**Step 1:** Establish the connection between the known and unknown angles.
$a° + 49° = 180°$ $a° = 131°$	**Step 2:** State the relationship using an equation and solve.

Continues →

Back to contents

b.

WE2b



Working

124° and $a°$ are corresponding angles on parallel lines.

$a°$ and $b°$ are supplementary angles.

$a° = 124°$

$a° + b° = 180°$

$124° + b° = 180°$

$b° = 56°$

Thinking

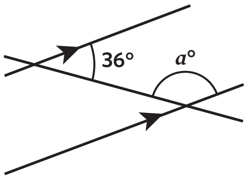
Step 1: Establish the connection between the known and unknown angles.

Step 2: State the relationship using an equation and solve.

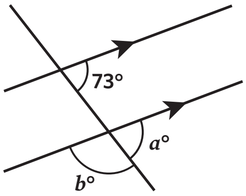
Student practice

Determine the unknown angles, giving reasons, for each diagram.

a.



b.



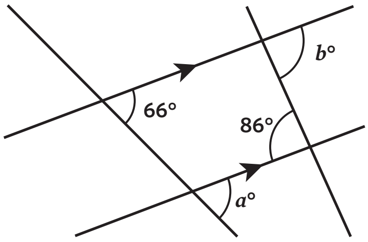
Worked example 3

Determining angles involving parallel lines and multiple transversals

Determine the unknown angles, giving reasons, for each diagram.

a.

WE3a



Working

66° and $a°$ are corresponding angles on parallel lines.

86° and $b°$ are alternate angles on parallel lines.

$a° = 66°$

$b° = 86°$

Thinking

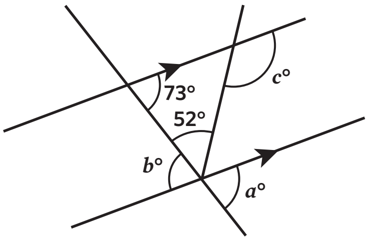
Step 1: Establish the connection between the known and unknown angles.

Step 2: State the relationship using an equation and solve.

Continues →

b.

WE3b



Working

73° and $a°$ are corresponding angles on parallel lines.

73° and $b°$ are alternate angles on parallel lines.

$(52° + b°)$ and $c°$ are alternate angles on parallel lines.

$a° = 73°$

$b° = 73°$

$c° = 52° + b°$

$= 52° + 73°$

$= 125°$

Thinking

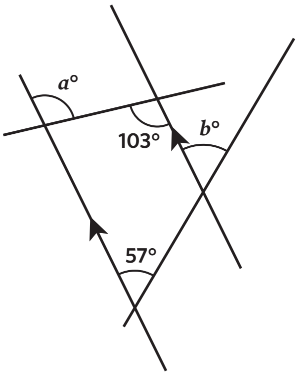
Step 1: Establish the connection between the known and unknown angles.

Step 2: State the relationship using an equation and solve.

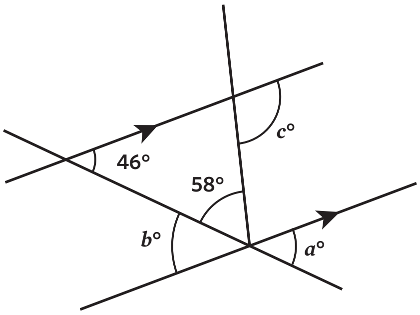
Student practice

Determine the unknown angles, giving reasons, for each diagram.

a.



b.

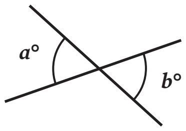


8A Questions

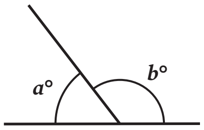
Understanding worksheet

1. Match the relationship between the angles in these diagrams.

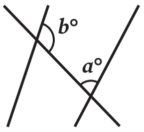
Angle



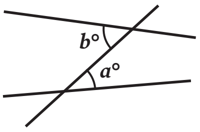
●



●



●



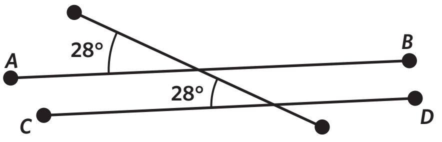
●

Relationship

- ● Alternate
- ● Vertical opposite
- ● Supplementary
- ● Co-interior

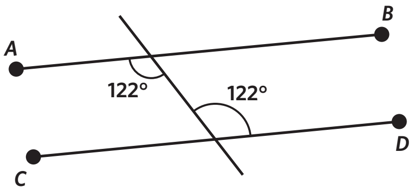
2. For each diagram, identify the angle relationship that indicates that lines *AB* and *CD* are parallel.

Example



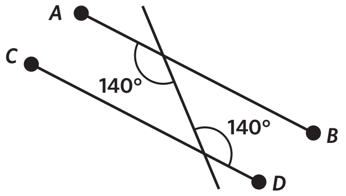
Corresponding angles are equal.

a.



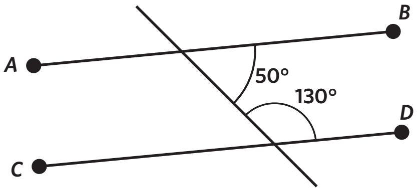
_________ angles are equal.

b.



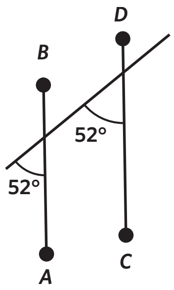
_________ angles are equal.

c.



_________ angles sum to 180°.

d.



_________ angles are equal.

Back to contents

3. Fill in the blanks by using the words provided.

equal **supplementary** **complementary** **transversal** **co-interior**

Angles are ______ if they have a shared vertex and sum to 90°, while those that add up to 180° are ______ angles. When a ______ crosses two parallel lines the angles on the same side of the transversal and inside the parallel lines are called ______ angles. If a line crosses two parallel lines, the alternate interior angles are ______ .

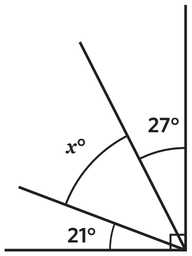
Fluency

Question working paths

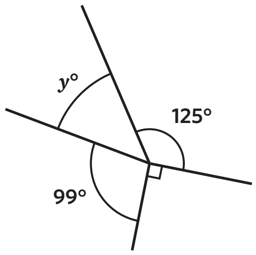
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Determine the unknown angles, giving reasons, for each diagram. **WE1**

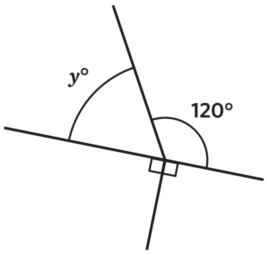
a.



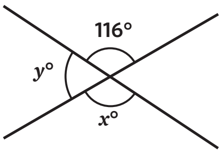
b.



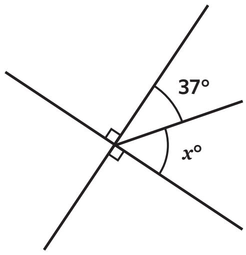
c.



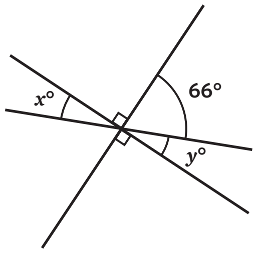
d.



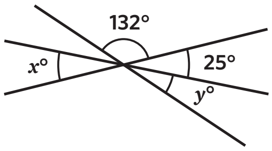
e.



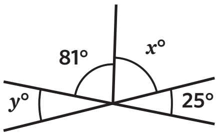
f.



g.



h.



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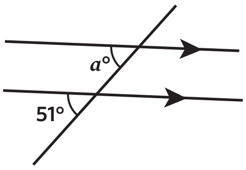
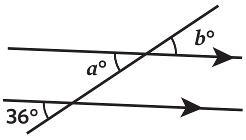
5. Determine the unknown angles, giving reasons, for each diagram. **WE2**

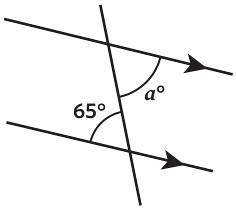
a.



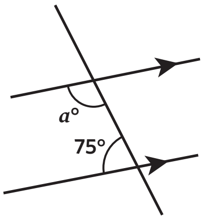
b.



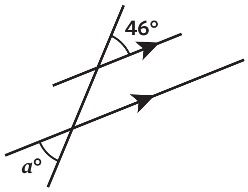
c.



d.



e.



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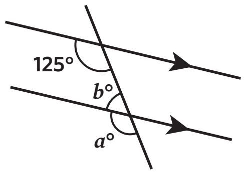

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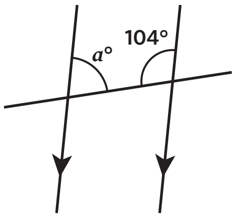
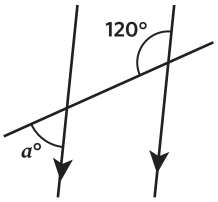


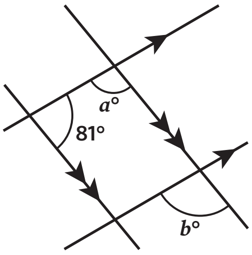
h.



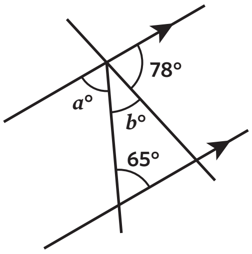
Back to contents

6. Determine the unknown angles, giving reasons, for each diagram. WE3

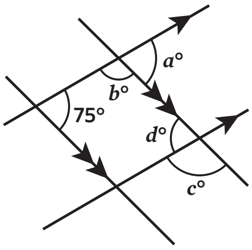
a.



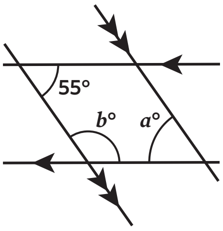
b.



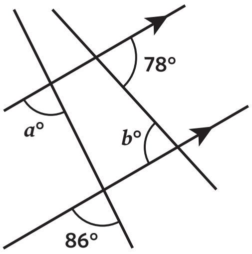
c.



d.



e.



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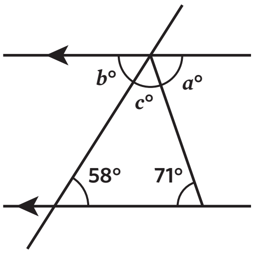
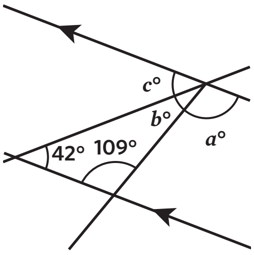
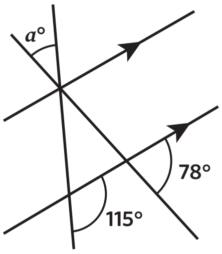


g.



h.



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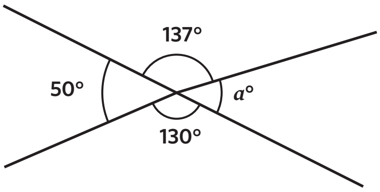
7. Co-interior angles between two parallel lines have which relationship?

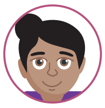
A. Angles are equal
B. Angles sum to 90°
C. Angles sum to 180°
D. Angles sum to 360°
E. Angles have no relationship

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Determine the value of the unknown angle.

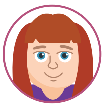




Student A

$a°$ is vertically opposite 50°.

$a° = 50°$



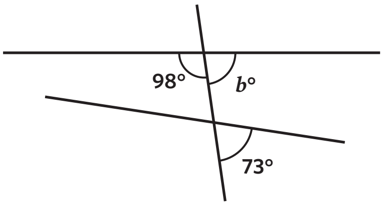
Student B

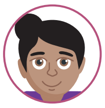
$a°$ and 137° are supplementary.

$a° + 137° = 180°$

$a° = 43°$

b. Determine the value of the unknown angle.

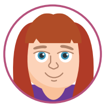




Student A

$b°$ is corresponding to 73°.

$b° = 73°$



Student B

$b°$ and 98° are supplementary.

$b° + 98° = 180°$

$b° = 82°$

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. A botanic garden designer intends to incorporate a central fountain surrounded by 5 straight paths radiating outward. To achieve a harmonious design, the paths will be uniformly spaced. Calculate the angle formed between each adjacent pair of paths.

10. A bridge spans across a straight section of a double lane highway diagonally. The bridge forms an angle of 68° with one road and an angle of 112° with the other road. Illustrate a diagram that demonstrates how, in this section, the two roads are parallel to each other.

11. An engineer is examining the alignment of two parallel lines of wiring. Instead of measuring the constant distance between the two wires, they use a ruler as a transversal and measure an interior angle as 53°. Provide a visual representation illustrating the measurements of all angles formed when the wires are parallel.

12. At a roundabout, where five roads (*A*–*E*) intersect, the angles between the roads are as follows: Road *A* and *B* form a right angle (90°), Road *B* and *C* form a 65° angle, Road *C* and *D* form a 25° angle, Road *D* and *E* form a 155° angle, and Road *E* and *A* form a 25° angle. Determine which roads, if any, appear to continue straight through the roundabout.

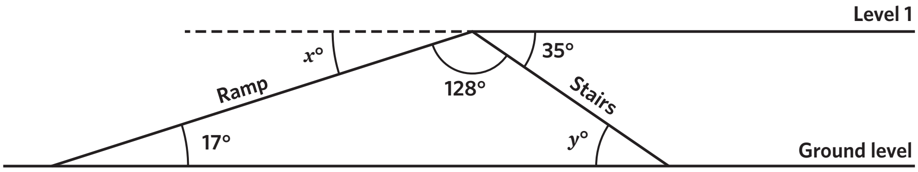
13. A city park is enclosed by four roads. The internal angles within the park are given as 110°, $a°$, $b°$ and 65°. Given that two of the roads forming the park are parallel, determine the values of $a°$ and $b°$.

Reasoning

Question working paths

Mild 14 (a,b,d) 🌶	**Medium** 14 (a,b,d), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. The engineering manager of a new construction project is verifying the angles of the ramps to ensure that Level 1 and the Ground level are parallel.



a. Determine the value of the angle $x°$, explain your reasoning.

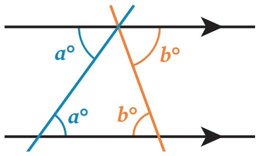
b. Determine the value of the angle $y°$, explain your reasoning.

c. Using your answer from part **a** and **b**, decide if Level 1 and the ground level are parallel, explain your reasoning.

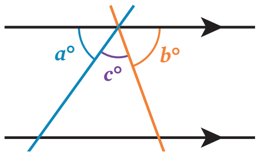
d. Explain the significance of ensuring that ramps and stairs adhere to regulations.

15. Consider the following angles.

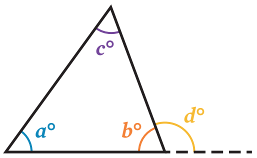
a. State the relationship that can be used to explain the marked angles.



b. State the relationship that can be used to calculate angles $c°$ using angle $a°$ and $b°$.



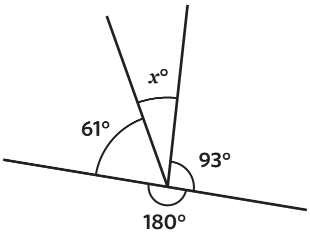
c. Using the answers from part **a** and **b**, write a mathematical statement to calculate the external angle ($d°$) of a triangle using $a°$ and $c°$.



Back to contents

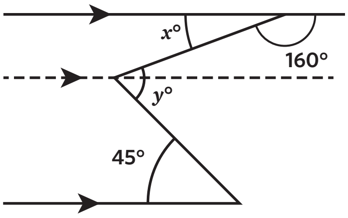
Exam-style

16. What is the value of $x°$ in the following diagram? (1 MARK)



A. 26° **B.** 29° **C.** 32° **D.** 36° **E.** 39°

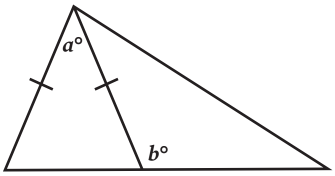
17. Consider the following diagram. (2 MARKS)



a. Determine the value of $x°$. 1 MARK

b. Calculate the value of $y°$. 1 MARK

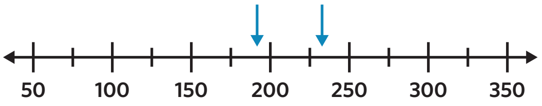
18. If $a° = 40°$, calculate the value of $b°$. (2 MARKS)



19. A regular octagon is used inside a bike wheel to support the eight spokes that radiate from the centre of the wheel. Draw a diagram labelling the internal angles of the triangles formed by the spokes. (2 MARKS)

Remember this?

20.



Which one of these numbers lie between the two arrows?

A. 120 **B.** 160 **C.** 210 **D.** 270 **E.** 300

21. Alex earned $500 in the past 5 days.

After the first day, he earned $25 more each day than the day before.

How much did Alex earn on the first day?

A. $50 **B.** $75 **C.** $100 **D.** $125 **E.** $150

22. Lisa works in a toy factory assembling toy sets. She assembles one set at a time. Each set contains 3 cars, 4 planes, and 2 boats. Every hour, Lisa assembles a total of 180 toys. How many planes does she assemble every hour?

A. 36 **B.** 45 **C.** 60 **D.** 72 **E.** 80

Back to contents

8B Congruent triangles

LEARNING INTENTIONS

Students will be able to:

- understand the meaning of congruence
- identify the minimal conditions required to prove triangles are congruent
- identify corresponding pairs of sides and angles in triangles
- choose which congruence test shows that a pair of triangles are congruent
- find unknown angles or side lengths using congruence of triangles.

Congruence in geometry refers to the exact match in shape and size between two figures. In the context of triangles, certain conditions, when met, can prove their congruence. It is essential to identify corresponding pairs of sides and angles in these triangles. Determining the right congruence test for a pair of triangles allows for the deduction of unknown angles or side lengths within the triangles.

KEY TERMS AND DEFINITIONS

- **Corresponding** sides or angles appear in the same position in similar or congruent shapes.

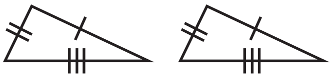
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: doomu/Shutterstock.com

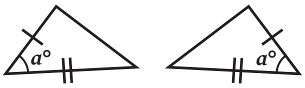
In nature, congruence is often observed in organisms that exhibit bilateral symmetry, like butterflies. Both wings of a butterfly are congruent in shape and size, allowing them to fly effectively. Understanding congruence in this context can help in studying and appreciating the beauty and functionality of nature.

Key ideas

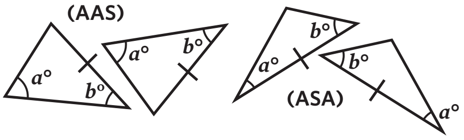
1. Congruence means two geometric figures have the same shape and size, with minimal conditions being SSS, SAS, and ASA congruence tests for triangles.



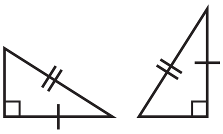
All three sides in each triangle have a corresponding side length **(SSS)**.



Two pairs of corresponding sides and a corresponding angle (the angle between them) **(SAS)**.



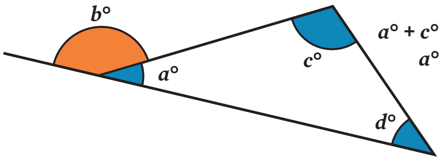
Two corresponding pairs of angles and one corresponding pair of side lengths.



Two right-angled triangles have a corresponding hypotenuse and another pair of corresponding side lenghts **(RHS)**.

2. Congruence of triangles, combined with interior angle sum and exterior angle theorem, helps determine unknown values.

- The interior angle sum of any triangle is equal to 180°.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



$$a^\circ + c^\circ + d^\circ = 180^\circ$$
$$a^\circ + b^\circ = 180^\circ$$
$$b^\circ = c^\circ + d^\circ$$

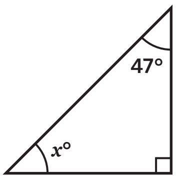
Worked example 1

Using triangle properties to determine unknowns

Given the triangle, find the value of the unknown angle.

a.

WE1a



Working

$a° + c° + d° = 180°$

$x° + 47° + 90° = 180°$

$x° + 137° = 180°$

$x° = 180° - 137°$

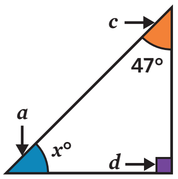
$x° = 43°$

Thinking

Step 1: Using the interior angle sum of a triangle, write an equation using the known and unknown information.

Step 2: Solve.

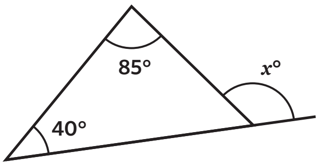
Visual support



$a + c + d = 180°$

b.

WE1b



Working

$c° + d° = b°$

$40° + 85° = x°$

$x° = 125°$

Thinking

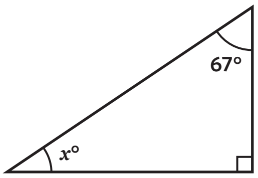
Step 1: Using the exterior angle theorem, write an equation using the known and unknown information.

Step 2: Solve.

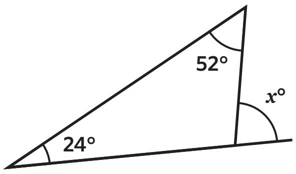
Student practice

Given the triangle, find the value of the unknown angle.

a.



b.



Back to contents

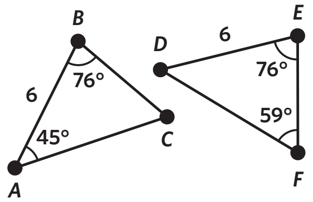
Worked example 2

Testing triangles for congruency

Determine if the following triangles are congruent. Explain using the congruence test.

a.

WE2a



Working	Thinking
For $\triangle ABC$ $76° + 45° + \angle C = 180°$ $\angle C = 59°$	**Step 1:** Calculate any unknown angles for each triangle using the interior angle sum.
For $\triangle DEF$ $76° + 59° + \angle D = 180°$ $\angle D = 45°$ $\angle A = \angle D$ $\angle B = \angle E$ $AB = DE$	**Step 2:** Identify the corresponding features.
$\therefore \triangle ABC \equiv \triangle DEF$ (**ASA**)	**Step 3:** Indicate whether the triangles are congruent or not, and refer to an appropriate congruence test to support your statement.

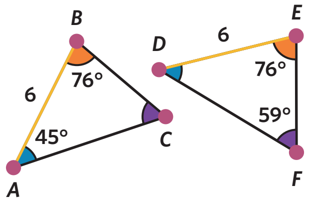
Visual support

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

$AB = DE$



$76° + 45° + \angle C° = 180°$

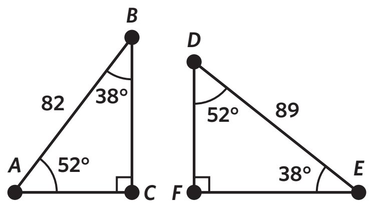
$\angle C° = 59°$

$76° + 59° + \angle D° = 180°$

$\angle D° = 45°$

b.

WE2b



Working	Thinking
$\angle A = \angle D$ $\angle B = \angle E$ $AB \neq DE$	**Step 1:** Identify the corresponding features.
$\therefore \triangle ABC$ and $\triangle DEF$ are not congruent (**ASA**)	**Step 2:** Indicate whether the triangles are congruent or not, and refer to an appropriate congruence test to support your statement.

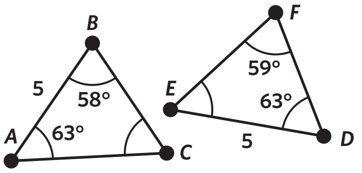
Continues →

Back to contents

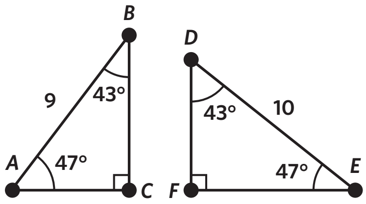
Student practice

Determine if the following triangles are congruent. Explain using the congruence test.

a.



b.



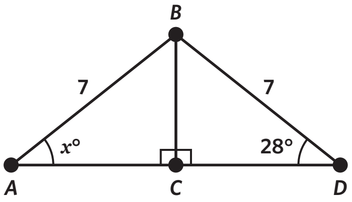
Worked example 3

Using congruent triangles to determine unknowns

Given two congruent triangles, find the value of the unknown.

a.

WE3a



Working

∠BCA = ∠BCD = 90° (R)

△ABC and △DBC are both right-angled triangles

$AB = BD$ (H)

BC is common to both △ABC and △DBC (S)

∠BAC = ∠BDC

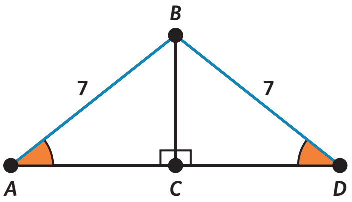
∴ $x° = 28°$

Thinking

Step 1: Identify the corresponding features that indicate the two triangles are congruent.

Step 2: Identify the side or angle that corresponds with the unknown.

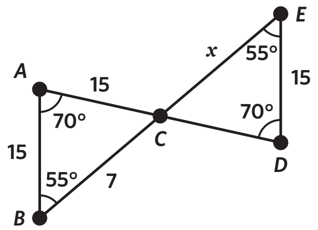
Visual support



Continues →

WE3b

b.



Working

$\angle BAC = \angle EDC$ (A)

$\angle ABC = \angle DEC$ (A)

$AB = BD$ (S)

$BC = EC$

$\therefore x = 17$

Thinking

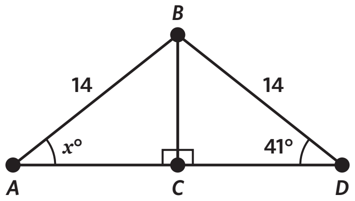
Step 1: Identify the corresponding features that indicate the two triangles are congruent.

Step 2: Identify the side or angle that corresponds with the unknown.

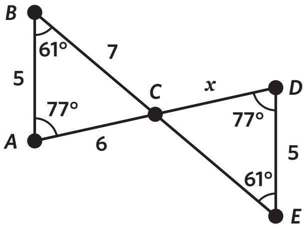
Student practice

Given two congruent triangles, find the value of the unknown

a.



b.

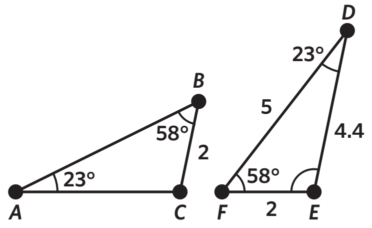


8B Questions

Understanding worksheet

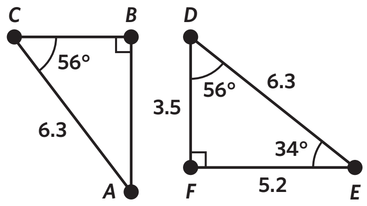
1. Name the corresponding side or angle for each pair of congruent triangles.

Example



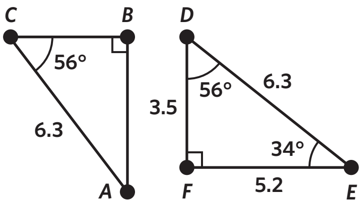
$\angle ABC = \angle\boxed{DFE}$

a.



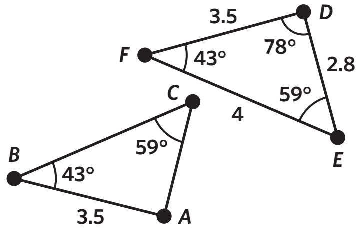
$AB = \boxed{}$

b.



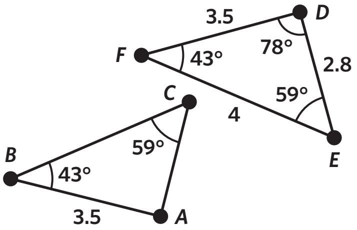
$\angle ABC = \angle\boxed{}$

c.



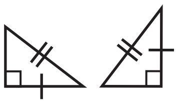
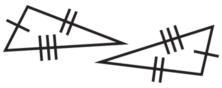
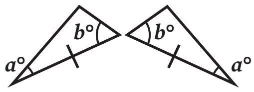
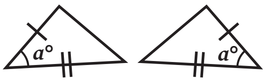
$BC = \boxed{}$

d.



$\angle CBA = \angle\boxed{}$

2. Match the triangle pairs to the appropriate congruence test.

Triangle pairs			Congruence test
	●	●	SSS
	●	●	ASA
	●	●	SAS
	●	●	RHS

Back to contents

3. Fill in the blanks by using the words provided.

corresponding | **equal** | **unknown** | **congruence**

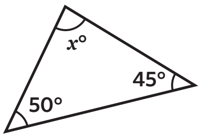
Congruence in geometry means two objects have ________ shape and size. For triangles, identifying ________ sides and angles is vital to prove their congruence. Minimal conditions required for triangle congruence must also be met. Choosing the appropriate ________ test depends on available information, and congruence can help find ________ angles or side lenghts.

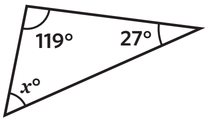
Fluency

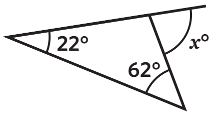
Question working paths

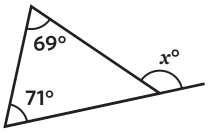
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

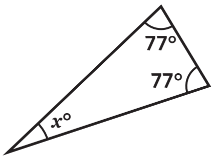
4. Given the triangle, find the value of the unknown angle. WE1

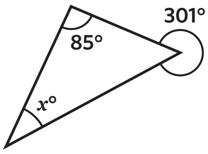
a.


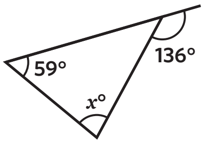
b.


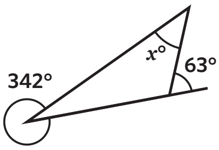
c.


d.


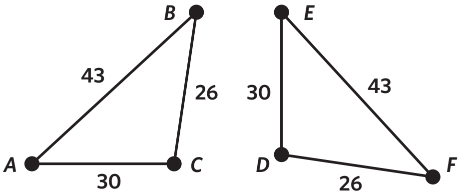
e.


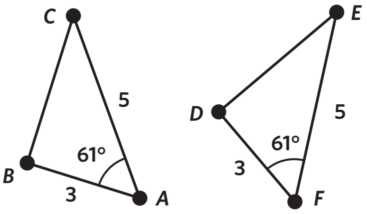
f.


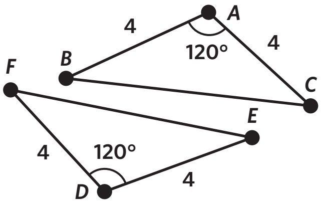
g.


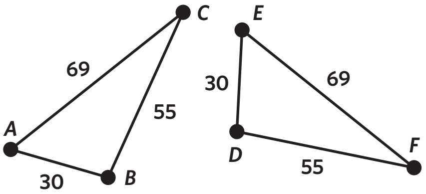
h.


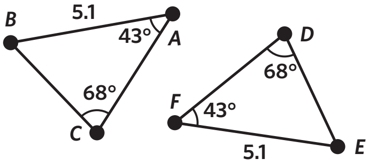
5. Determine if the following triangles are congruent. Explain using the congruence test. WE2a

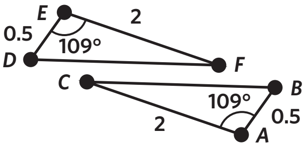
a.


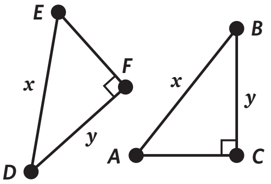
b.


c.


d.


e.


f.


g.


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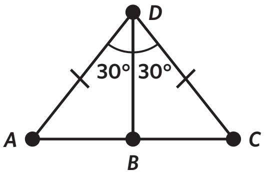
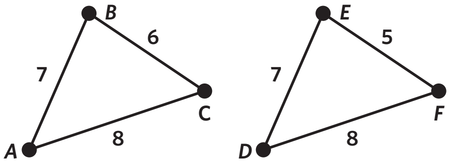
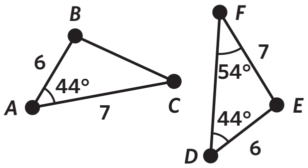


6. Determine if the following triangles are congruent. Explain using the congruence test. WE2b

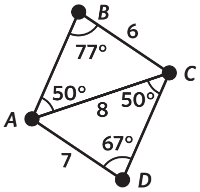
a.



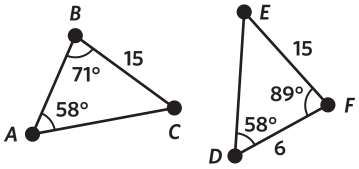
b.



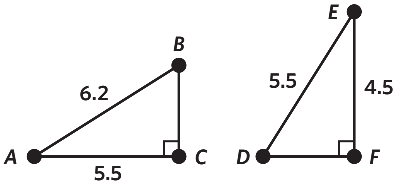
c.



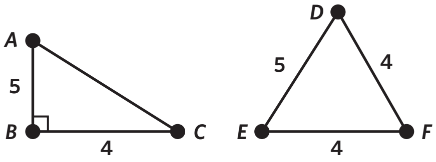
d.



e.



f.



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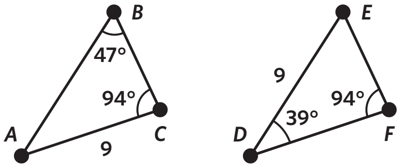

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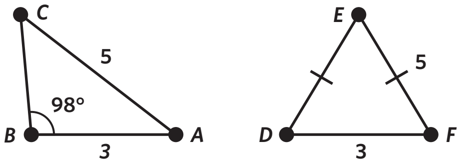
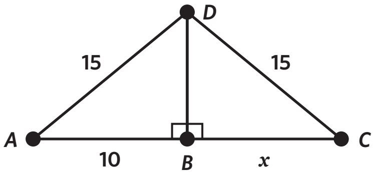
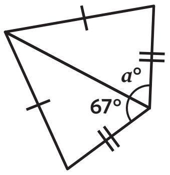


7. Given two congruent triangles, find the value of the unknown. WE3

a.



b.



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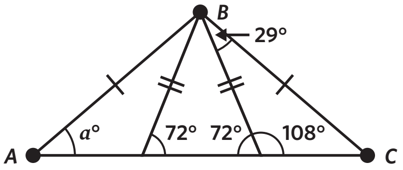

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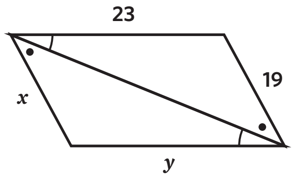

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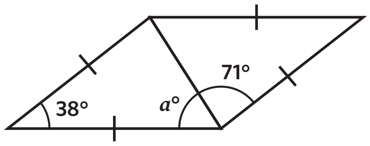

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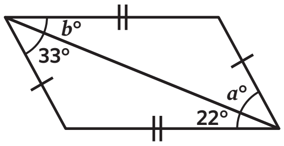

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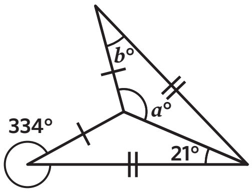

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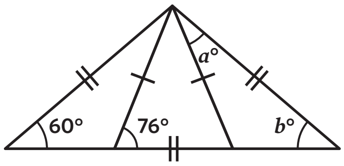
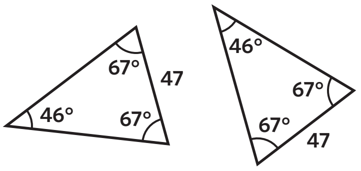


8. Which congruence test shows these two triangles are congruent?

A. AAA
B. SSS
C. SAS
D. ASA
E. SSA

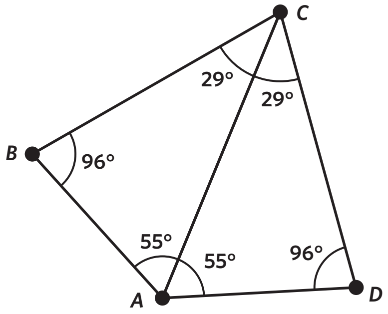


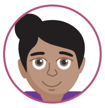
Back to contents

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Using a congruence test, explain if $\triangle ABC \equiv \triangle ADC$.





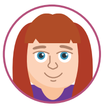
Student A

$\angle BAC = 55°$

$\angle ACB = 29°$

$\angle ABC = 96°$

$\therefore$ $\triangle ABC$ and $\triangle ADC$ are congruent due to AAA



Student B

$\angle BAC = 55°$

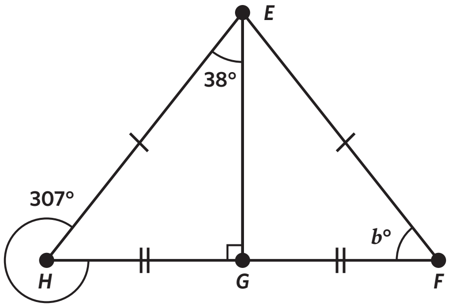
$\angle ACB = 29°$

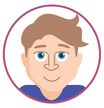
$\angle ABC = 96°$

AC is a common side length

$\therefore$ $\triangle ABC$ and $\triangle ADC$ are congruent due to ASA

b. Determine the value of the unknown angle.





Student A

$\angle H$ is corresponding to $\angle F = b°$.

$\angle H = 360° - 307°$

$= 53$

$b° = 53°$

Student B

$\angle HEG$ is corresponding to $\angle F = b°$

$\angle HEG = 38°$

$b° = 38°$

Problem solving

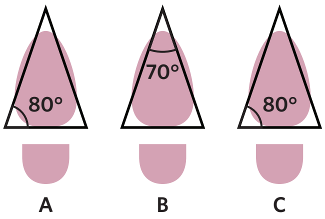
Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. A landscape gardener is in the process of devising a layout for a garden bed by utilising three rope lengths: 10 m, 8 m, and 6 m. Using these ropes, the gardener generates two triangular garden beds for the client. Use a congruence test to determine if these two designs are different shapes.

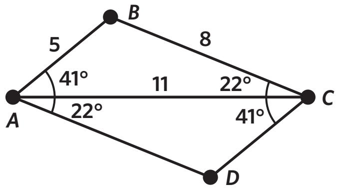
11. Alec and Sasha are making origami using identical square pieces of paper. Alec folds the paper in half once on the diagonal forming a right-angled triangle. Sasha also forms a right-angled triangle by folding the paper twice on the diagonal. Sasha says the two triangles are congruent because all the angles are equal in both triangles. Explain using a congruence test if Sasha is correct.

12. A detective is studying footprints found at three different scenes. The detective suspects they could be made by the same person since the isosceles triangles formed by the footprints have the same height when compared. Using a congruence test, explain which pairs of shoes, if any, could be worn by the same person.



13. In a four-way tug of war, four teams compete against each other. Each team is equipped with an identical length of rope, and all four ropes converge at the centre. At the game's initiation, the angles formed between the ropes are all right angles. Utilise congruent triangles to demonstrate that for each team, the distances to the teams on the left and right of them are equivalent.

14. A long distance running course is being pegged out and is shown below. The instructions state that participants will need to go around the markings in the following order: A–B–C–A–D–C–D. What is the distance of the course?

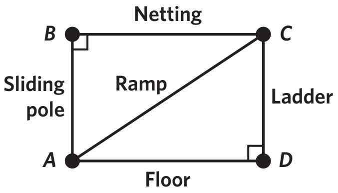


Reasoning

Question working paths

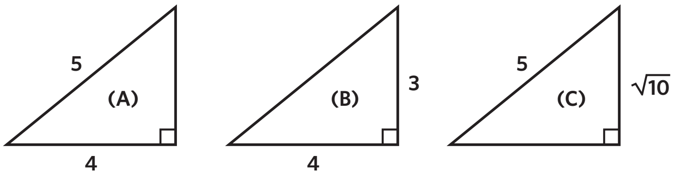
Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Play centres often use rectangular scaffolding to construct large play areas. The following diagram shows the side view of a plan for a climbable section.



- **a.** Formulate an equation to show the relationship between the lengths of the sliding pole (AB) and the ladder (CD).
- **b.** Formulate an equation to show the relationship between the length of the netting (BC) and the floor section (AD).
- **c.** Formulate an equation to show the relationship between $\angle ACD$ and $\angle CAB$.
- **d.** Explain how the congruence test SSS can be used to show that $\triangle ABC \equiv \triangle CDA$.
- **e.** Give a reason why scaffolding is used in construction.

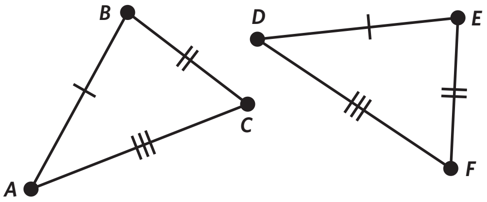
16. Three right-angled triangles are shown.



- **a.** Identify which two triangles are congruent. Explain using a congruence test.
- **b.** Explain why the third triangle is not congruent to the other two, using congruence criteria.
- **c.** Based on your answer in part **a**, explain two congruence tests that could be used for any right-angled triangles when given two side lengths.

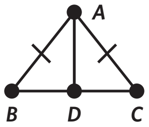
Exam-style

17. For these congruent triangles what is the corresponding side to EF? (1 MARK)



A. DE **B.** DF **C.** BC **D.** CA **E.** BA

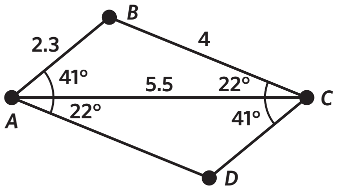
18. Use congruence to determine the following. (2 MARKS)



a. If $BD = DC$ explain how $\triangle ABD \equiv \triangle ACD$. 1 MARK

b. Determine the relationship between $\angle BAD$ and $\angle DAC$. 1 MARK

19. An orienteering course is illustrated below, comprising a series of markers. According to the guidelines, participants must navigate the markers following either the sequence: A–B–C–A or A–C–D–A. Explain why, for both of these course variations, participants will cover the same distance. (2 MARKS)



20. SSA is not a test for congruence. Use a diagram to explain why. (2 MARKS)

Remember this?

21. Emily has been awarded a rating of 4.8 out of 5 in an online restaurant review.

Oliver, Mia, Ava, and Noah have also had ratings in the same online review.

- Oliver's restaurant rating: 2.2
- Mia's restaurant rating: 2.3
- Ava's restaurant rating: 2.4
- Noah's restaurant rating: 2.5

Whose restaurant has a rating that is exactly half that of Emily's restaurant?

A. Mia's
B. Ava's
C. Noah's
D. Oliver's
E. None of the above

22. Sophia lives in Sydney. One day the temperature was 28°C.

The same day she phoned her friend Ethan, who lives in New York.

Ethan said that the temperature there was −8°C.

What was the temperature difference between the two cities on this day?

A. 16°C **B.** 20°C **C.** 36°C **D.** 40°C **E.** 48°C

23. This data shows how long some employees spent in a training session:

Employee	Amy	Bob	Cal	Deb	Eva
Time (in hours)	3	2	Over 4	4	1

If Eva's time is removed from the data, what happens to the mean time spent in the training session?

A. It increases
B. It decreases
C. It stays the same
D. It increases by exactly 1
E. It is not possible to tell

Back to contents

8C Quadrilaterals and other polygons

LEARNING INTENTIONS

Students will be able to:

- identify the different properties of quadrilaterals using congruent triangles
- determine if polygons are regular, irregular, convex or non convex based on properties
- find interior angle sums of a polygon
- find unknown angles using the angle sum and polygon properties.

Polygons, with their varying sides and angles, can be categorised as regular, irregular, convex, or non-convex based on specific properties. Understanding these properties often involves the use of congruent triangles, especially when dealing with quadrilaterals. Additionally, the ability to calculate unknown angles in polygons is crucial, relying on the angle sum property and knowledge of the interior angle sum of a triangle. The exterior angle theorem further aids in determining the relationships between the angles of a polygon.

KEY TERMS AND DEFINITIONS

- A **convex polygon** has all internal angles less than 180°.
- A **concave polygon** has at least one internal angle greater than 180°.
- Two triangles are **congruent** if each has the same three corresponding side lengths and the same three corresponding angles.

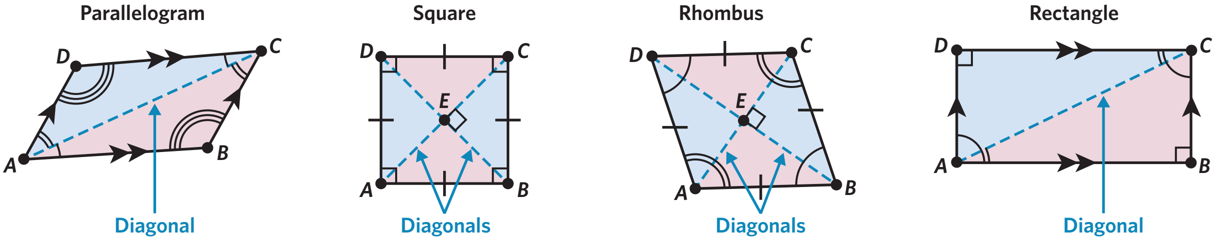
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Francesco Scatena/Shutterstock.com

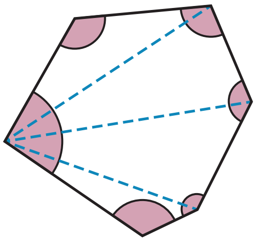
Land surveying is vital in determining property boundaries, infrastructure planning, and more. Surveyors utilise knowledge of polygon properties, such as identifying angles and determining the nature of the polygon, to produce accurate and legally binding plots.

Key ideas

1. Different properties of quadrilaterals can be determined using congruent triangles and their corresponding sides and angles.



2. The interior angle sum of a polygon can be found dividing a polygon into triangles and by using the formula $(n - 2) \times 180°$, where n is the number of sides of the polygon.



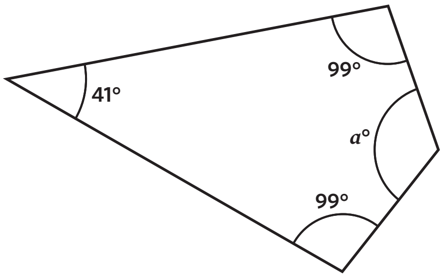
Worked example 1

Finding angles in a quadrilateral

Find the value(s) of the pronumeral(s) in the diagrams using properties of quadrilaterals.

a.

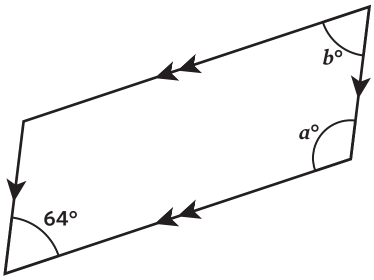
WE1a



Working	Thinking
$41° + 99° + 99° + a° = 360°$	**Step 1:** The internal angles of a quadrilateral add to 360°.
$a° + 239° = 360°$	**Step 2:** Solve for the unknown.
$a° = 360° - 239°$	
$a° = 121°$	

b.

WE1b

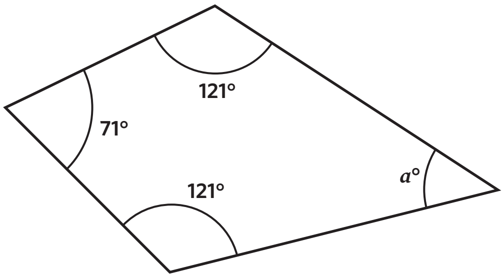


Working	Thinking
$a° + 64 = 180°$ $a° = 180° - 64°$ $a° = 116°$	**Step 1:** Internal angles that share an edge in a parallelogram are co interior angles on parallel lines.
$b° = 64°$	**Step 2:** Opposite angles in a parallelogram are equal.

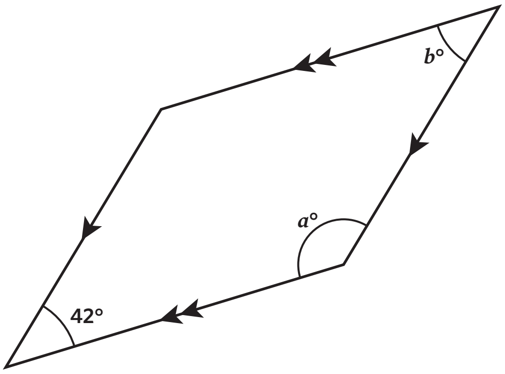
Student practice

Find the value(s) of the pronumeral(s) in the diagrams.

a.



b.



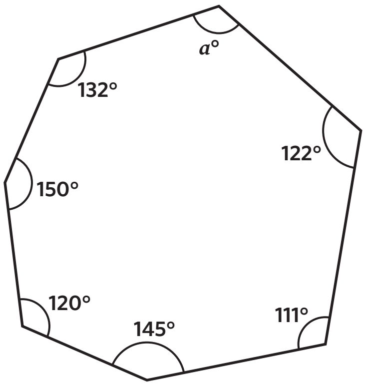
Worked example 2

Finding angles in polygons

For each polygon, find the angle sum and then find the value of any pronumerals.

a.

WE2a



Working

$S = (n - 2) \times 180°$ when $n = 7$

$S = (7 - 2) \times 180°$

$S = 900°$

$120° + 145° + 111° + 122° + 132° + 150° + a° = 900°$

$780° + a° = 900°$

$a° = 900° - 780°$

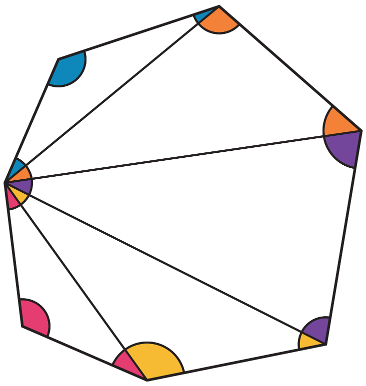
$a° = 120°$

Thinking

Step 1: Calculate the internal angle sum of the polygon.

Step 2: Use the internal angle sum of the polygon to calculate any unknowns.

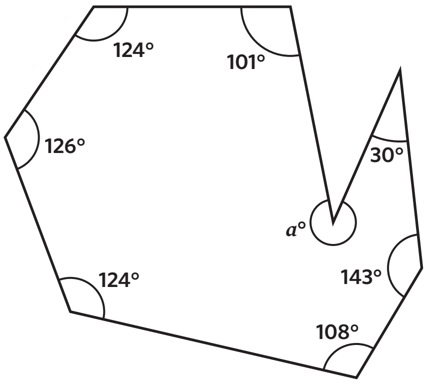
Visual support



$S = (n - 2) \times 180°$
$S = 5 \times 180°$
$S = 900°$

b.

WE2b



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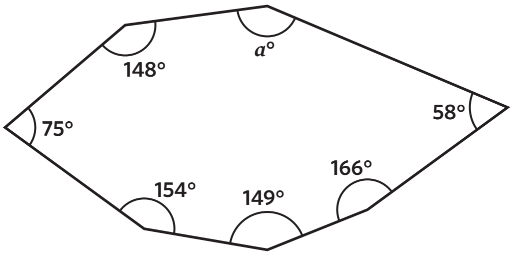
Back to contents

Working	Thinking
$S = (n - 2) \times 180°$ when $n = 8$ $S = (8 - 2) \times 180°$ $S = 1080°$	**Step 1:** Calculate the internal angle sum of the polygon.
$124° + 126° + 124° + 108° +$ $143° + 30° + a° + 101 = 1080°$ $756° + a° = 1080°$ $a° = 1080° - 756°$ $a° = 324°$	**Step 2:** Use the internal angle sum of the polygon to calculate any unknowns.

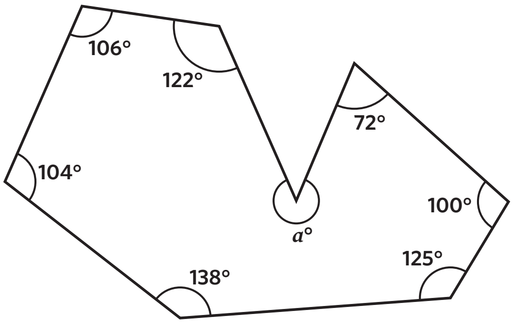
Student practice

For each polygon, find the angle sum and then find the value of any pronumerals.

a.



b.



Back to contents

8C Questions

Understanding worksheet

1. Match the characteristics with the special quadrilaterals.

Characteristics	Special quadrilaterals
A parallelogram with 4 sides of equal length ●	● Kite
2 pairs of adjacent sides, equal in length ●	● Rhombus
2 pairs of sides equal in length and 4 right angles ●	● Square
4 sides of equal length and 4 right angles ●	● Rectangle

2. Determine the number of sides for each of the polygons.

Example

A quadrilateral has 4 sides.

a. An octagon has ☐ sides.

b. A hexagon has ☐ sides.

c. A decagon has ☐ sides.

d. A pentagon has ☐ sides.

3. Fill in the blanks by using the words provided.

interior **concave** **quadrilaterals** **sum** **angles**

Understanding the properties of polygons is essential to determine whether a shape is regular or irregular, and whether it's convex or ☐ . One effective method of exploring the properties of ☐ is by utilising congruent triangles. The angle ☐ and polygon properties can be harnessed to solve for unknown ☐ within polygons. By delving into these concepts, we can accurately calculate the ☐ angle sums of various polygons.

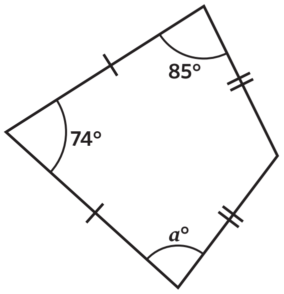
Fluency

Question working paths

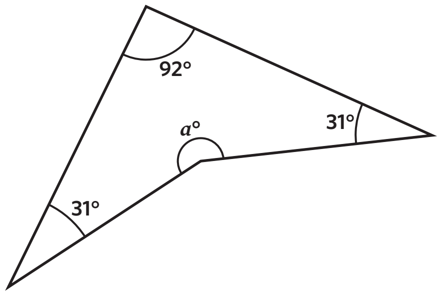
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6	4 (c,d,e,f), 5 (c,d,e,f), 6	4 (e,f,g,h), 5 (e,f,g,h), 6

4. Find the value(s) of the unknown(s) in the diagrams using properties of quadrilaterals. WE1

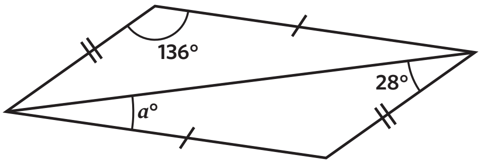
a.



b.



c.



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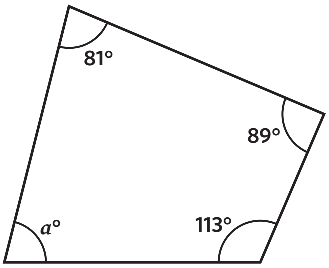

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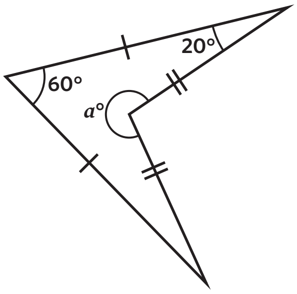

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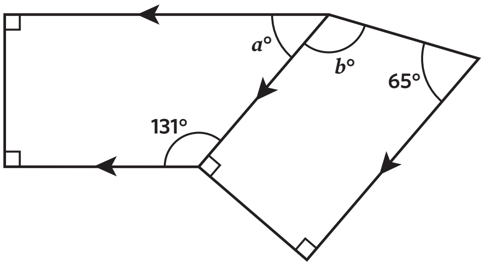
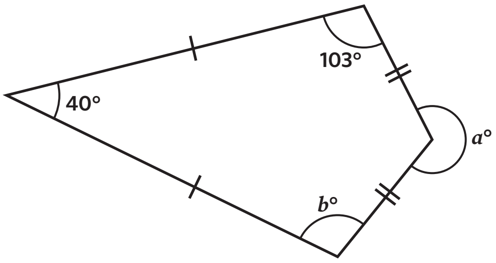
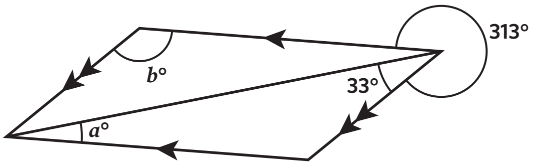


g.



h.



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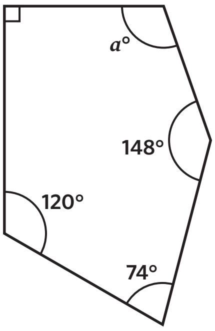
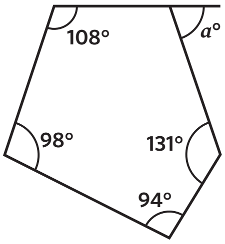
5. For each polygon, find the angle sum and then find the value of the unknown(s). WE2

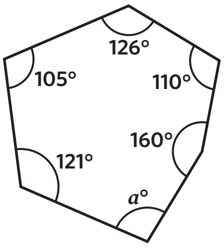
a.



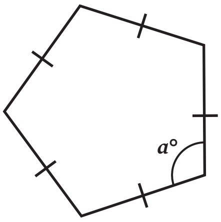
b.



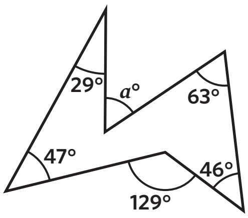
c.



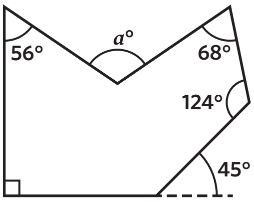
d.



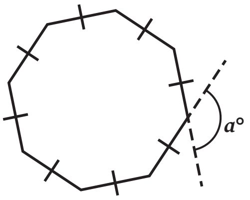
e.



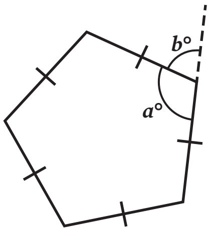
f.



g.

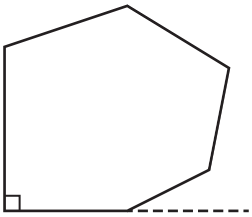


h.



---

6. What is the internal angle sum of this polygon?

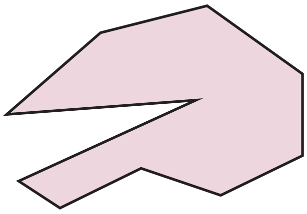


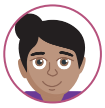
A. 90° **B.** 180° **C.** 360° **D.** 720° **E.** 1080°

Spot the mistake

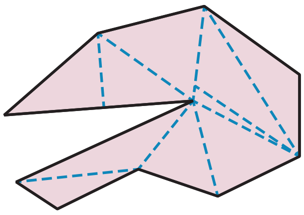
7. Select whether Student A or Student B is incorrect.

a. Determine the internal angle sum of this irregular, concave polygon.





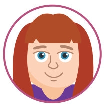
Student A



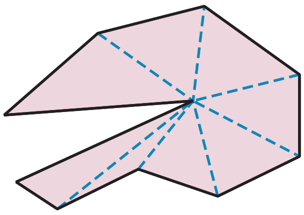
10 internal triangles, 10 sides

$S = 10 \times 180°$

$= 1800°$



Student B



8 internal triangles, 10 sides

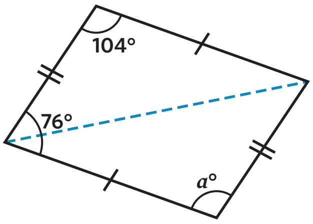
$S = (n - 2) \times 180°$

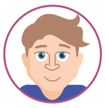
$= (10 - 2) \times 180°$

$= 8 \times 180°$

$= 1440°$

b. Use properties of special quadrilaterals to calculate the unknown.





Student A

In a parallelogram opposite angles are equal so $a° = 104°$

Student B

In a kite opposite angles sum 180°

$104° + a° = 180°$

$a° = 76°$

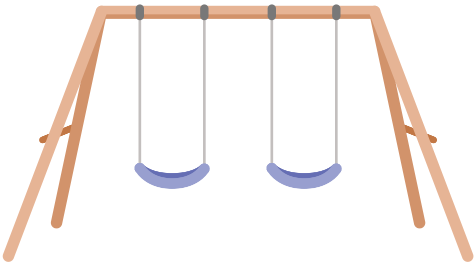
Problem solving

Question working paths

Mild 8, 9, 10 🌶	**Medium** 9, 10, 11 🌶🌶	**Spicy** 10, 11, 12 🌶🌶🌶

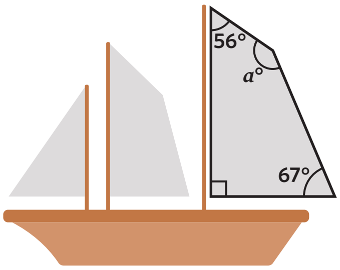
8. A builder is assessing whether a finished room can be considered a rectangle. After measuring all the dimensions of the room, the builder asserts that the room is rectangular due to two walls measuring 10 m and the other two measuring 5 m long. Explain why this information does not confirm that the room is rectangular and provide visual representations of two potential room designs.

9. A swingset is built by connecting two isosceles triangles at the top with a single crossbar. To enhance stability, the triangles are inclined inward at the top, creating an angle of 68° with the ground. Given that the resulting shape is a trapezium, what is the angle formed between the crossbar and one of the triangles?



10. An extendable table takes the form of a regular hexagon designed to seat 6 people when fully extended. By removing the central rectangular section and sliding the remaining triangles a smaller table is formed, capable of seating four people. What is the name of the shape of the smaller table, and what are the measurements of the angles formed at each vertex?

11. Sail boats often have triangular sails. Schooners are one of the various types of boats that use differently shaped sails. Calculate the internal angles of the sail shown.



12. A tri-blade wind-turbine is formed by connecting congruent rectangles to the edges of an equilateral triangle. Determine the angle made between the blades.

Reasoning

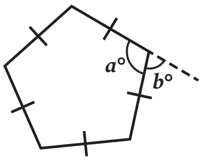
Question working paths

Mild 13 (a,b,c,e) 🌶	**Medium** 13 (a,b,c,e), 14 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

13. A signage craftsman uses an irregular polygon to fashion the shape of a letter 'P.' This polygon has interior angles measuring 100°, 90°, 93°, 87°, 80° and a°.

- **a.** Calculate the total sum of the interior angles in the polygon.
- **b.** Calculate the angle a° that completes the polygon.
- **c.** Classify whether this polygon is convex or concave.
- **d.** Determine the total sum of the interior angles for the polygon representing the letter 'F'.
- **e.** Why would it be useful to know the dimensions and angles of quadrilaterals when designing signs?

14. Using this regular polygon answer the following questions.

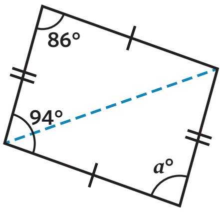


- **a.** Determine the interior angle a° for this polygon.
- **b.** Determine the exterior angle b° for this polygon.
- **c.** Using part **b**, formulate a rule for the exterior angle for any n-sided regular polygon.

Back to contents

Exam-style

15. Determine the size of the unknown angle. (1 MARK)



A. 86°
B. 94°
C. 104°
D. 106°
E. 180°

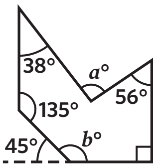
16. A robot follows this sequence of instructions: (2 MARKS)

- Start
- Move forward 2 units
- Turn right 72 degrees
- Repeat until returning to the starting point.

a. Draw the shape formed by the bot's movement, labelling both the starting point and each internal angle. 1 MARK

b. If the bot were to trace a regular hexagon, how many degrees to the right would the new set of instructions guide the bot? 1 MARK

17. For the following polygon calculate the unknown values. (2 MARKS)



18. In a geometry assignment, students are asked to design a symmetrical roof for a building using various quadrilaterals. One student suggests using a rhombus, asserting that its opposite sides are of equal length and it could create a symmetric roof. Determine if the roof can indeed be designed using a rhombus. If a rhombus isn't suitable, propose alternative quadrilaterals that would create a symmetrical roof design. (2 MARKS)

Remember this?

19. There are 73 guests at a gathering.

There are 11 more adults than children.

How many children are at the gathering?

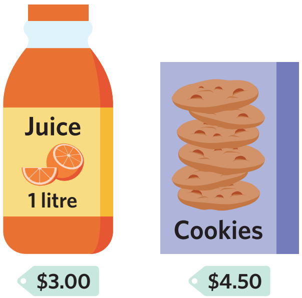
A. 31 **B.** 32 **C.** 33 **D.** 41 **E.** 42

20. John buys 2 litres of juice and a pack of cookies.

He pays with a $50 note.

How much change should John receive?

A. $38.50
B. $39.50
C. $40.50
D. $41.50
E. $42.00



21. Which statement is always true?

A. The adjacent sides of a rectangle are equal in length.
B. The diagonals of a square are perpendicular to each other.
C. The opposite angles of a trapezium are equal in size.
D. The angles in an isosceles triangle are all equal in size.
E. The diagonals of a parallelogram are perpendicular to each other.

Back to contents

8D Enlargement and similar figures

LEARNING INTENTIONS

Students will be able to:

- enlarge a figure using a scale factor and centre of enlargement
- understand the meaning of the term similar figures
- determine the scale factor between two similar figures
- determine the value of corresponding side lengths in similar figures using the scale factor.

Enlarging geometric figures involves the use of a scale factor and a specific point called the centre of enlargement. This process leads to the creation of similar figures, which maintain the same shape but differ in size. Determining the scale factor between two similar figures allows for the calculation of corresponding side lengths. Ensuring accuracy in enlargement and consistency in the application of the scale factor is essential for maintaining the integrity of the geometric relationships.

KEY TERMS AND DEFINITIONS

- A **ratio** is a proportional relationship between two or more quantities with the same unit.
- If two quantities have a **proportional relationship** they can be written as a ratio.

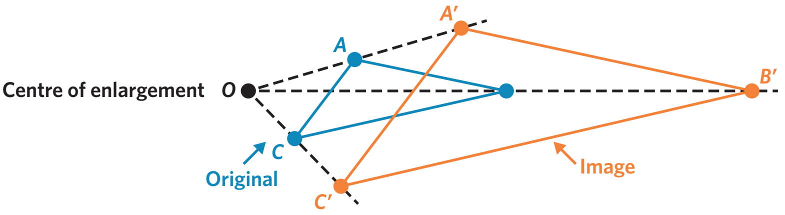
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Net Vector/Shutterstock.com

In cartography, maps are often created by shrinking the vastness of real-world locations into a manageable size. By understanding scale factors, one can determine the actual distance between two points on a map and how it corresponds to the real world.

Key ideas

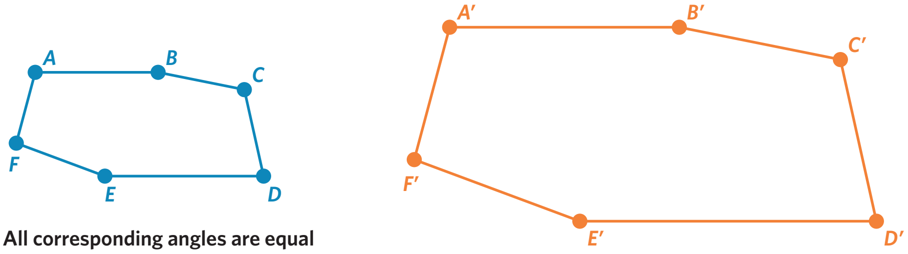
1. Enlarging a figure using a scale factor and centre of enlargement involves stretching or shrinking the figure while keeping all corresponding points in a proportional relationship from the centre.



Enlargement or Scale factor

$$\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

2. Similar figures are geometric figures that have the same shape but may have different sizes.



All corresponding angles are equal

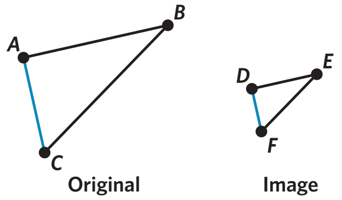
$\angle A = \angle A'$
$\angle B = \angle B'$
$\angle C = \angle C'$
$\angle D = \angle D'$
$\angle E = \angle E'$
$\angle F = \angle F'$

Ratio between pairs of corresponding sides is equal

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{D'E'}{DE} = \frac{E'F'}{EF} = \frac{F'A'}{FA}$$

Continues →

3. The scale factor between two similar figures is the ratio of corresponding side lengths.



Scale factor = $\frac{\text{image length}}{\text{original length}}$

$\Delta ABC \sim \Delta DEF$

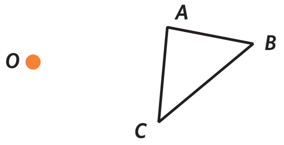
Scale factor = $\frac{DF}{AC}$

$0 <$ Scale factor < 1	The image will be smaller than the original
Scale factor $= 1$	The image and the original will be congruent
Scale factor > 1	The image will be larger than the original

Worked example 1

Enlarging figures

Copy the given diagram and use the given centre of enlargement (O) and the scale factors to enlarge the shape.

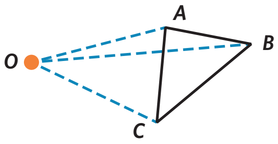


a. Scale factor $= \frac{1}{2}$

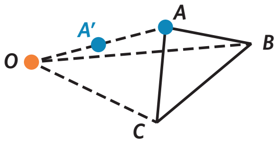
WE1a

Working

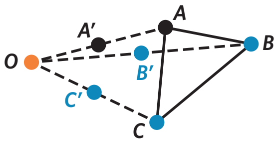
Thinking

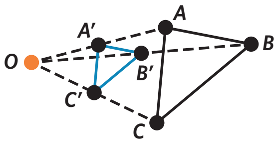


Step 1: The scale factor is less than 1 so the image will be smaller than the object. Draw three lines beginning at O and extending to each point on the object.



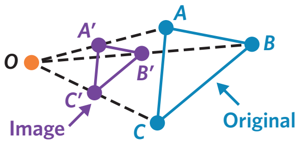
Step 2: Given the scale factor is $\frac{1}{2}$, place A' so that OA' is half of OA. Repeat for any remaining points.





Step 3: Join the points to create a smaller similar shape.

Visual support



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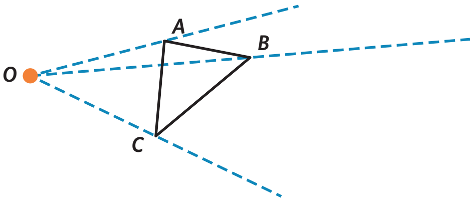
Back to contents

b. Scale factor $= 2$

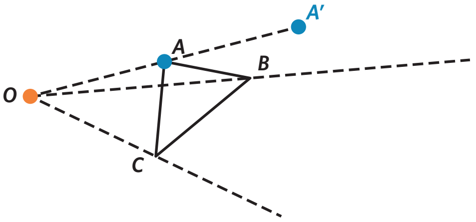
WE1b

Working

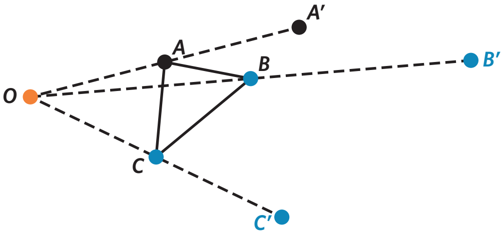
Thinking

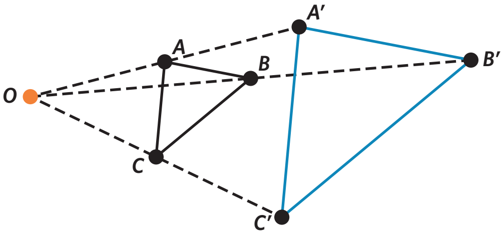


Step 1: The scale factor is greater than 1 so the image will be larger than the object. Draw three lines beginning at *O* and extending through each point on the object.



Step 2: Given the scale factor is 2, extend the line *OA* by a factor of 2 to calculate the distance of *OA*′ and place *A*′. Repeat for any remaining points.

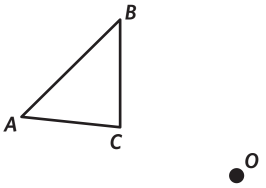




Step 3: Join the points to create a larger similar shape.

Student practice

Copy the given diagram and use the given centre of enlargement (*O*) and the scale factors to enlarge the shape.



a. Scale factor $= \frac{1}{2}$

b. Scale factor $= 2$

Back to contents

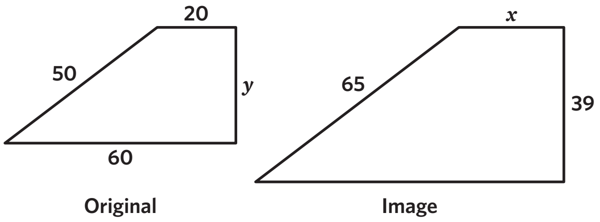
Worked example 2

Using scale factor

Using the given pairs of similar figures, use the scale factors to determine the values of the pronumerals.

a.

WE2a



Working

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$= \frac{65}{50}$$

$$= \frac{13}{10}$$

$$x = 20 \times \frac{13}{10}$$

$$= 26$$

$$y = 39 \div \frac{13}{10}$$

$$= 39 \times \frac{10}{13}$$

$$= 3 \times 10$$

$$= 30$$

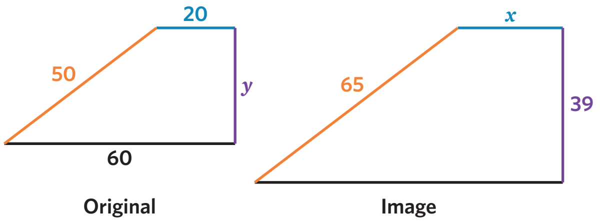
Thinking

Step 1: Calculate the scale factor.

Step 2: Calculate the unknown side length of the image by multiplying the known, corresponding side length of the original by the scale factor.

Step 3: Calculate the unknown side length of the original by dividing the known, corresponding side length of the image by the scale factor.

Visual support



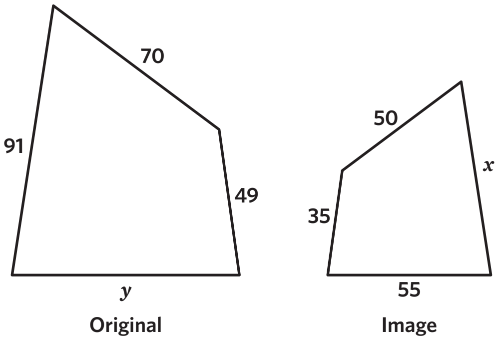
Scale factor = $\frac{65}{50} = \frac{13}{10}$

$x = 20 \times$ scale factor

$y = 39 \div$ scale factor

b.

WE2b



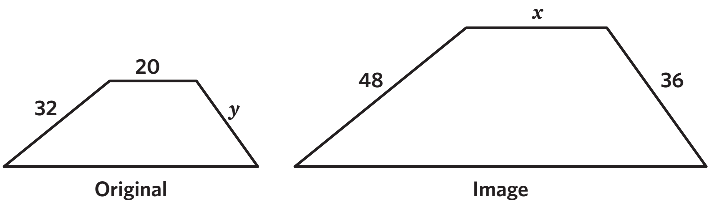
Continues →

Working	Thinking
$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$ $= \frac{50}{70}$ $= \frac{5}{7}$	**Step 1:** Calculate the scale factor.
$x = 91 \times \frac{5}{7}$ $= 65$	**Step 2:** Calculate the unknown side length of the image by multiplying the known, corresponding side length of the original by the scale factor.
$y = 55 \div \frac{5}{7}$ $= 55 \times \frac{7}{5}$ $= 11 \times 7$ $= 77$	**Step 3:** Calculate the unknown side length of the original by dividing the known, corresponding side length of the image by the scale factor.

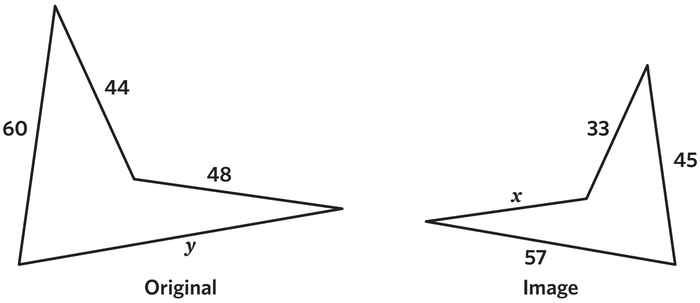
Student practice

Using the given pairs of similar figures, use the scale factors to determine the values of the pronumerals.

a.



b.



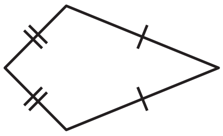
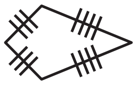
Back to contents

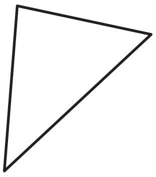
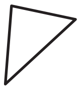
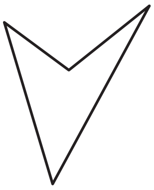
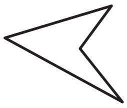
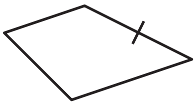
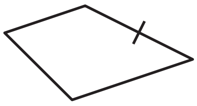
8D Questions

Understanding worksheet

1. Determine whether the scale factor of these pairs of similar figures is less than 1, equal to 1, or greater than 1.

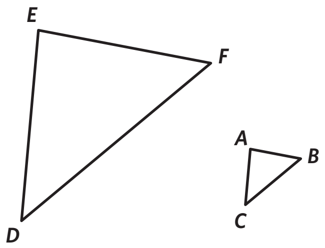
Example

Original	Image	Less than 1	Equal to 1	Greater than 1
		☑	☐	☐

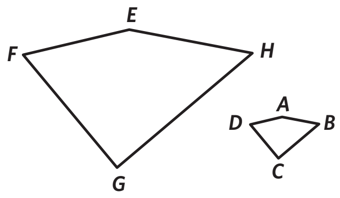
Original	Image	Less than 1	Equal to 1	Greater than 1
I. 		☐	☐	☐
II. 		☐	☐	☐
III. 		☐	☐	☐
IV. 		☐	☐	☐

2. Identify the side corresponding to AB in each of these pairs of similar figures.

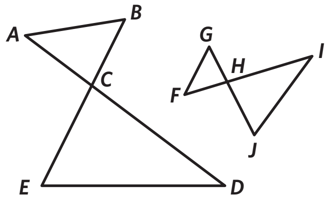
a. $AB \equiv$ ☐



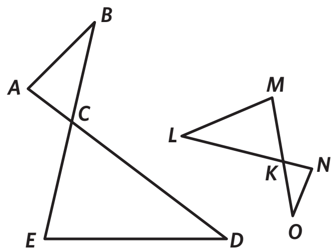
b. $AB \equiv$ ☐



c. $AB \equiv$ ☐



d. $AB \equiv$ ☐



Back to contents

3. Fill in the blanks by using the words provided.

corresponding **enlargement** **scale** **similar** **multiplied**

In geometry, when enlarging a figure, a ________ factor and a centre of ________ must be selected. Enlargement is particularly relevant when dealing with ________ figures, which share the same shape but may differ in size. To find the scale factor between two similar figures, ________ side lengths are compared. To find the value of corresponding side lengths in similar figures, each side length in the original figure is ________ by the scale factor.

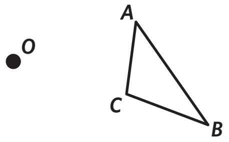
Fluency

Question working paths

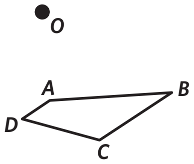
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b), 5 (a,b), 6 (a,b,c), 7	4 (b,c), 5 (b,c), 6 (b,c,d), 7	4 (c,d), 5 (c,d), 6 (c,d,e), 7

4. Copy the given diagrams and use the given centres of enlargement (O) and the scale factors to enlarge the shapes. **WE1a**

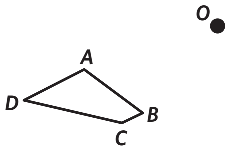
a. Scale factor = 0.5



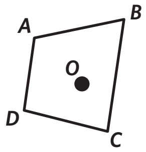
b. Scale factor = 0.25



c. Scale factor = 0.5

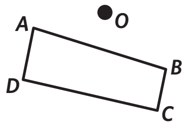


d. Scale factor = 0.5

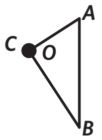


5. Copy the given diagrams and use the given centres of enlargement (O) and the scale factors to enlarge the shapes. **WE1b**

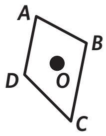
a. Scale factor = 2



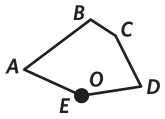
b. Scale factor = 3



c. Scale factor = 2

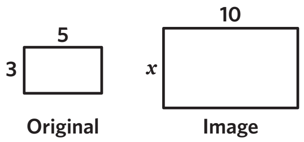


d. Scale factor = 2.5

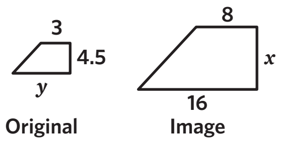


6. Using the given pairs of similar figures, use the scale factors to determine the values of the pronumerals. **WE2**

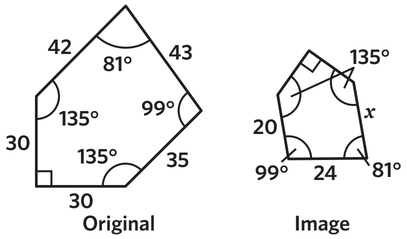
a.



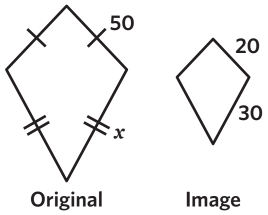
b.



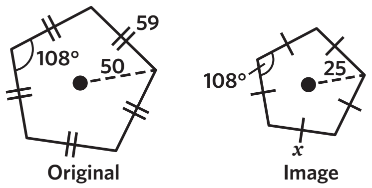
c.



d.

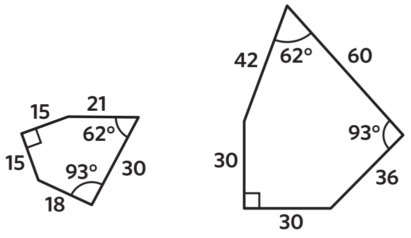


e.



Back to contents

7. Determine the scale factor for these two similar figures.



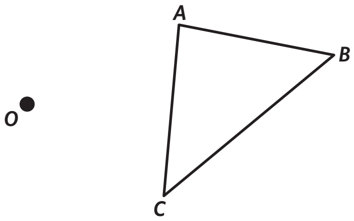
Original   Image

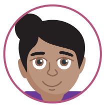
A. −2   **B.** −0.5   **C.** 0.5   **D.** 1   **E.** 2

Spot the mistake

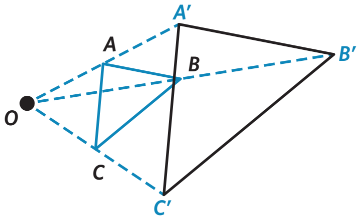
8. Select whether Student A or Student B is incorrect.

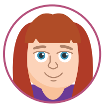
a. Use the given centre of enlargement (O) and a scale factor of 0.5 to enlarge the shape.



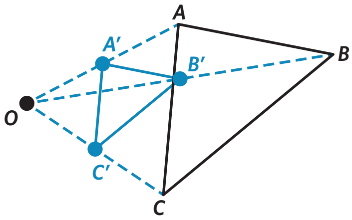


Student A





Student B



b. Calculate the scale factor for these two similar figures.

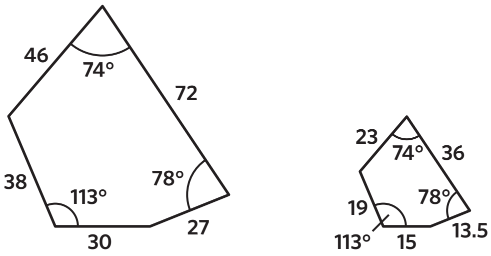
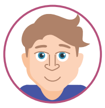


Image   Original



Student A

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$= \frac{30}{15}$$

$$= 2$$

Student B

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$= \frac{15}{30}$$

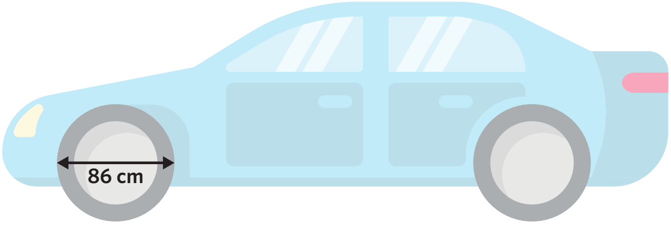
$$= \frac{1}{2}$$

Problem solving

Question working paths

Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. Max is creating a scale drawing of a car. The original wheels have a diameter of 86 cm. If Max is using a scale factor of $\frac{1}{10}$, determine the diameter of the wheels in the drawing.



10. An adventure park is planning to place some model dinosaurs around the park for decoration. They have a model Tyrannosaurus Rex that is 2 m tall. If an average adult Tyrannosaurus Rex is 12.4 m tall, calculate the scale factor if the actual dinosaur was used as the original, rounded to 1 decimal place.

11. A team of architects are creating a model of an upcoming project. They have been provided with a rectangular plot of land measuring 4 km in length and 2 km in width. The model will be constructed on a platform that is 3 m long. Determine the scale factor they will be using for the model.

12. The Burj Khalifa in Dubai stands at a height of 828 m, while the Merdeka 118 in Kuala Lumpur reaches a height of 677 m. If Merdeka 118 was built first, calculate the scale factor of Merdeka 118 in comparison to the Burj Khalifa, rounded to 2 decimal places.

13. In a photograph of an old house, the windows appear to be 3 cm tall, and the first floor is 5 cm above the ground. If the original windows were 2.5 m tall, calculate the height, in metres, of the first floor from the ground, rounded to 2 decimal places

Reasoning

Question working paths

Mild 14 (a,b,d) 🌶	**Medium** 14 (a,b,d), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. A graphic designer is using Photoshop to resize a photo. The original is a rectangle measuring 8 cm in length and 4 cm in width. The graphic designer needs to create a larger version of the photo while maintaining the same shape (aspect ratio).

- **a.** Determine the scale factor that will double the dimensions of the photo, while maintaining the same aspect ratio.
- **b.** After enlarging the photo, the width of the rectangle now measures 16 cm. Calculate the length of the rectangle in the enlarged image, in centimetres.
- **c.** To print the photo on a poster that measures 40 cm wide, determine the scale factor required to enlarge the photo while maintaining the aspect ratio.
- **d.** Describe what would happen to a photo if it is enlarged and the aspect ratio is not maintained.

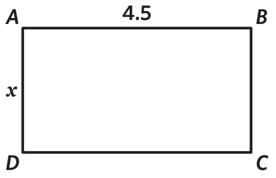
15. Determine whether the following shapes are similar.

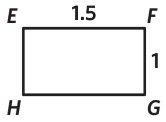
- **a.** Two circles; the first circle has a radius of 3 and the second circle has a radius of 5.
- **b.** Two rectangles; the first rectangle has a length of 3 and a width of 2, and the second rectangle has a length of 6 and a width of 3.
- **c.** Using your answers from parts **a** and **b**, explain what makes shapes similar.

Back to contents

Exam-style

16. Rectangle *ABCD* is similar to rectangle *EFGH*. (1 MARK)





The length of *AD* is:

A. 1 **B.** 1.5 **C.** 2 **D.** 2.5 **E.** 3

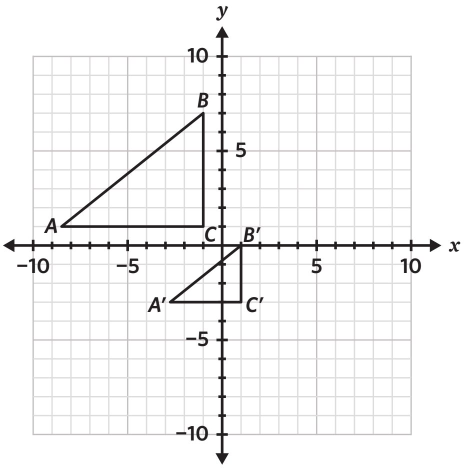
17. A hobby farmer is planning to create a scaled model of their farm layout for a presentation. The original paddock layout is represented as a rectangle measuring 1 km in length and 0.5 km in width. The goal is to create a scaled model of the paddock while maintaining the same shape. (3 MARKS)

a. The farmer wants to print a poster of the paddock layout for an agricultural fair. The poster is 2 m wide. Determine the scale factor needed to scale the farm layout image for the poster. 1 MARK

b. Determine the scale factor required to create a smaller model of the paddock, where the length is 2.5 m. 1 MARK

c. Calculate the width, in metres, of the rectangle on the scaled-down model from part **b**. 1 MARK

18. Two similar triangles are shown on the Cartesian plane. Determine the scale factor and locate the coordinates of the centre of enlargement. (3 MARKS)



19. In toy manufacturing, a toy car model is created as a miniature replica of a real car. The real car is 4 m long with a 2.2 m door, and the toy car is designed to be 10 cm long. Determine the scale factor used to create the miniature toy car from the real car and calculate the length of the door on the toy car, in centimetres. (2 MARKS)

Remember this?

20. Mike buys 4 boxes of pencils.

He pays with a $10 note.

He gets 40 cents change.

How much does each box of pencils cost?

A. 220 cents **B.** 240 cents **C.** 260 cents **D.** 280 cents **E.** 300 cents

21. James, Sarah, William, Alec, and Ali each measure the height of their plant on their desk.

- James's plant is 240 mm
- Sarah's plant is 0.16 m
- William's plant is 15 cm
- Alec's plant is 150 mm
- Ali's plant is 0.26 cm

Who has the tallest plant?

A. Alec **B.** Ali **C.** James **D.** Sarah **E.** William

22. A hospital cafeteria provides 7 meal services a day, and each meal service uses 5 apples in the fruit salad.

The table shows some information about two different apple suppliers.

Apple supplier	Amount of water used per apple (litres)
Local Orchard	2.5
Big Farm	1.25

How many litres of water would be saved in a day by using apples from Big Farm instead of Local Orchard?

A. 25 L **B.** 37.5 L **C.** 43.75 L **D.** 50 L **E.** 75 L

Back to contents

8E Similar triangles

LEARNING INTENTIONS

Students will be able to:

- identify similar triangles
- determine the scale factor between similar triangles
- calculate an unknown side length of a triangle using the scale factor.

When it comes to triangles, their unique properties allow us to test them for similarity using specific features. In general, a triangle's shape can be defined by knowing only three of its unique properties, such as all angle sizes and side lengths, or certain combinations of both.

KEY TERMS AND DEFINITIONS

- An **included angle** is an angle between two sides of a triangle.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

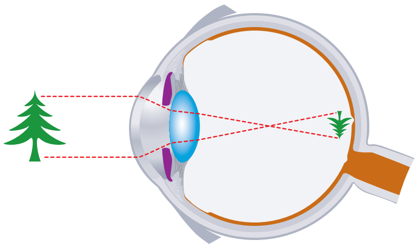


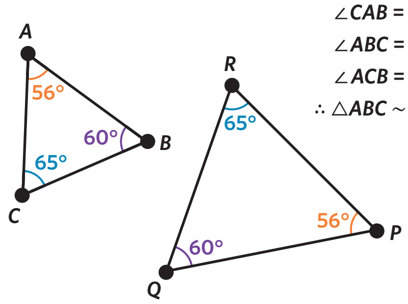
Image: Peter Hermes Furian/Shutterstock.com

Our eyes act as a lens that projects an image of what we see to the back of the retina. This image is always upside down, but our brains can process it and allow us to perceive it as normal. Similar triangles are used to visualise how light rays interact with the eye, as well as help understand certain eye conditions, such as near - or long-sightedness.

Key idea

1. A pair of triangles are similar if they satisfy the conditions of at least one of the similarity tests below.

AAA (angle-angle-angle)
If all three corresponding angles are equal, the triangles are similar.



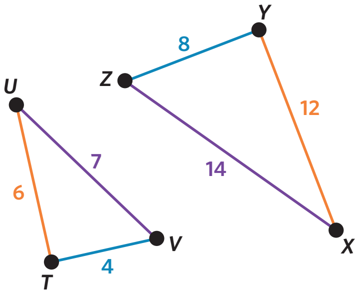
$\angle CAB = \angle RPQ = 56°$

$\angle ABC = \angle PQR = 60°$

$\angle ACB = \angle PRQ = 65°$

$\therefore \triangle ABC \sim \triangle PQR$ (AAA)

SSS (side-side-side)
If all three pairs of corresponding side lengths are in the same ratio, the triangles are similar.

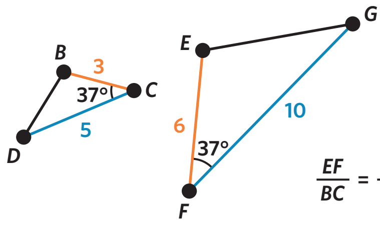


$$\frac{YZ}{TV} = \frac{YX}{TU} = \frac{XZ}{UV}$$

$$= \frac{8}{4} = \frac{12}{6} = \frac{14}{7} = 2$$

$\therefore \triangle TUV \sim \triangle YXZ$ (SSS)

SAS (side-angle-side)
If two pairs of corresponding side lengths are in the same ratio and the included angles are equal, the triangles are similar.

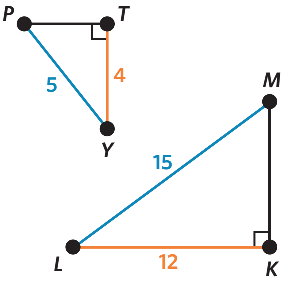


$$\frac{EF}{BC} = \frac{FG}{CD} = \frac{6}{3} = \frac{10}{5} = 2$$

$\angle BCD = \angle EFG = 37°$

$\therefore \triangle BCD \sim \triangle EFG$ (SAS)

RHS (right angle-hypotenuse-side)
If the hypotenuse and another pair of corresponding side lengths are in the same ratio, the right-angled triangles are similar.



$\angle MKL = \angle PTY = 90°$

$$\frac{ML}{PY} = \frac{KL}{TY} = \frac{15}{5} = \frac{12}{4} = 3$$

$\therefore \triangle KLM \sim \triangle TYP$ (RHS)

Back to contents

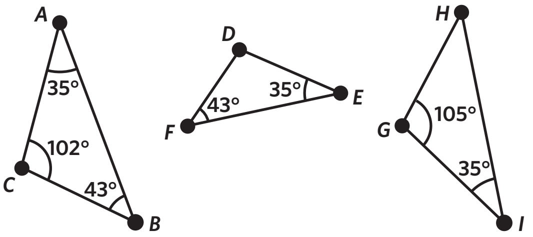
Worked example 1

Identifying similar triangles

Determine which pair of triangles are similar for each of the following, with reasons.

a.

WE1a



Working

Use AAA (angle-angle-angle) as all three internal angles are known or can be calculated.

$\angle BAC = 35° = \angle FED = \angle HIG$

$\angle ABC = 43° = \angle EFD$

$\neq \angle IHG = 180° - 105° - 35° = 40°$

$\angle ACB = 102°$

$= \angle EDF = 180° - 43° - 35° = 102°$

$\neq \angle IGH = 105°$

$\therefore \triangle ABC \sim \triangle EFD$ (AAA)

Thinking

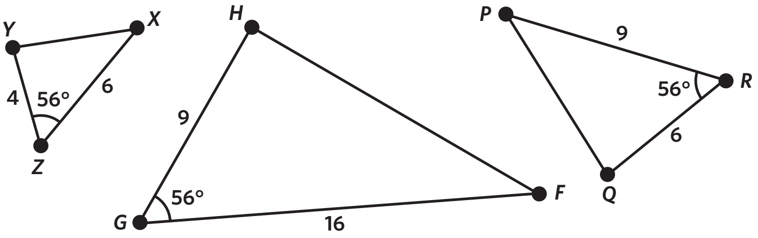
Step 1: Determine which of the similarity tests applies based on the given properties.

Step 2: Compare the corresponding features and calculate any unknown angles.

Step 3: Identify the pair of similar triangles.

b.

WE1b



Working

Use SAS (side-angle-side) as two side lengths and the included angle are known.

$\angle YZX = \angle HGF = \angle QRP = 56°$

For $\triangle YZX$ and $\triangle HGF$:

$$\frac{YZ}{HG} = \frac{4}{9}$$

$$\frac{ZX}{GF} = \frac{6}{16} = \frac{3}{8}$$

$$\frac{YZ}{HG} \neq \frac{ZX}{GF}$$

$\therefore \triangle YZX$ is not similar to $\triangle HGF$.

For $\triangle YZX$ and $\triangle QRP$:

$$\frac{YZ}{QR} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{ZX}{RP} = \frac{6}{9} = \frac{2}{3}$$

$\therefore \triangle YZX \sim \triangle QRP$ (SAS).

Thinking

Step 1: Determine which of the similarity tests applies based on the given properties.

Step 2: Compare the ratios of two pairs of corresponding features.

Step 3: Identify the pair of similar triangles.

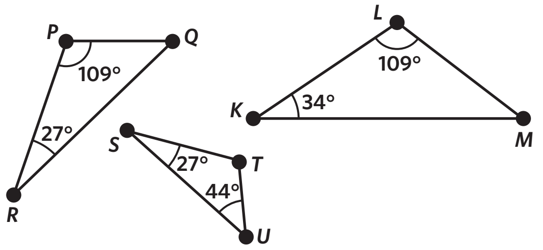
Continues →

Back to contents

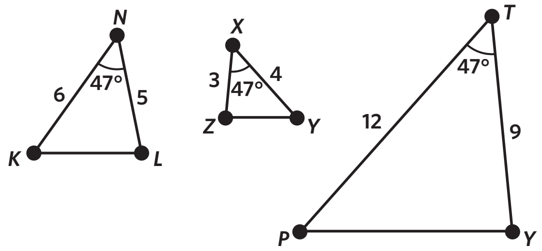
Student practice

Determine which pair of triangles are similar for each of the following, with reasons.

a.



b.



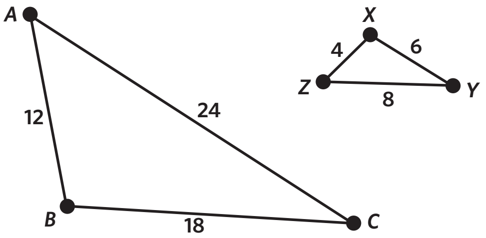
Worked example 2

Determining the scale factor

Calculate the scale factors for the following pairs of similar triangles.

a. $\triangle ZXY$ to $\triangle ABC$

WE2a



Working

$\triangle ZXY \sim \triangle ABC$ by SSS.

$\therefore$ AB and ZX, BC and XY, CA and YZ are corresponding side lengths.

$\frac{AB}{ZX} = \frac{12}{4} = 3$

The scale factor of $\triangle ZXY$ to $\triangle ABC$ is 3.

Thinking

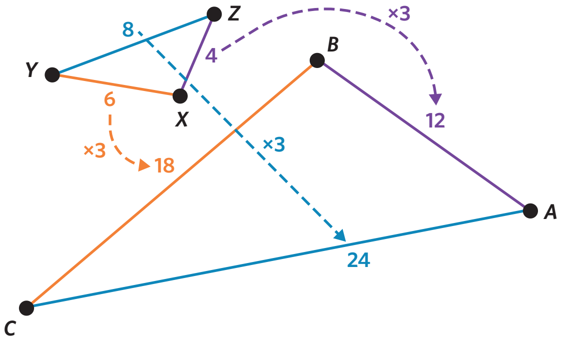
Step 1: Identify the corresponding side lengths.

Step 2: Calculate the ratio of a pair of corresponding side lengths.

Step 3: Determine the scale factor.

Visual support

Each side length in $\triangle ABC$ is exactly 3 times as long as its corresponding side length in $\triangle XYZ$.



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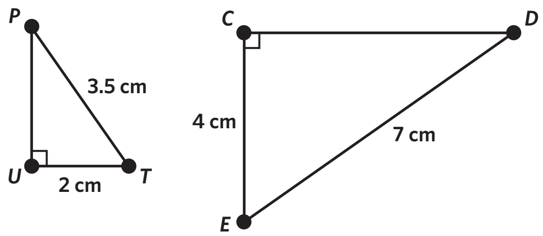
Back to contents

b. $\triangle DCE$ to $\triangle PUT$

WE2b



Working

$\triangle DCE \sim \triangle PUT$ by RHS.

$\therefore$ PT and DE, and TU and EC are corresponding side lengths.

$$\frac{PT}{DE} = \frac{3.5}{7} = \frac{1}{2}$$

$$\frac{TU}{EC} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{PT}{DE} = \frac{TU}{EC}$$

The scale factor of $\triangle DCE$ to $\triangle PUT$ is $\frac{1}{2}$.

Thinking

Step 1: Identify the corresponding side lengths.

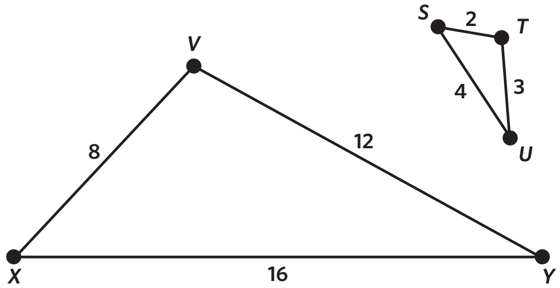
Step 2: Calculate the ratio of corresponding side lengths.

Step 3: Determine the scale factor.

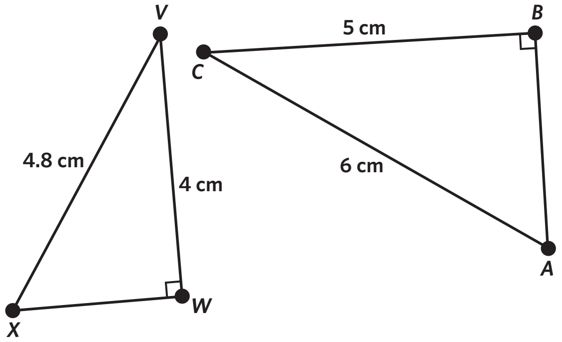
Student practice

Calculate the scale factors for the following pairs of similar triangles.

a. $\triangle STU$ to $\triangle XVY$



b. $\triangle ABC$ to $\triangle WXV$

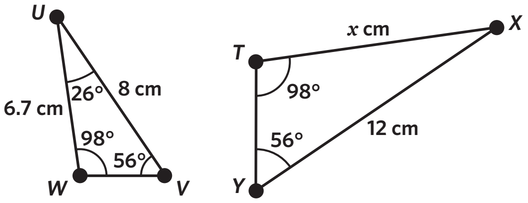


Worked example 3

Using the scale factor to calculate unknown side lengths

Calculate the values of the pronumerals for the following pairs of similar triangles.
Round to one decimal place as needed.

a.



WE3a

Working

$$\frac{UV}{XY} = \frac{UW}{XT}$$

$$\frac{8}{12} = \frac{6.7}{x}$$

$$8x = 6.7 \times 12$$

$$x = 10.05$$

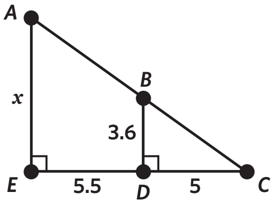
$$\approx 10.1 \text{ cm}$$

Thinking

Step 1: Form an equation for the side length ratio using a pair of corresponding side lengths.

Step 2: Substitute the given values into the side length ratio equation and solve for the unknown.

b.



WE3b

Working

$$\frac{AE}{BD} = \frac{EC}{DC}$$

$$\frac{x}{3.6} = \frac{5 + 5.5}{5}$$

$$\frac{x}{3.6} = \frac{10.5}{5}$$

$$5x = 3.6 \times 10.5$$

$$x = \frac{3.6 \times 10.5}{5}$$

$$= 7.56$$

$$\approx 7.6$$

Thinking

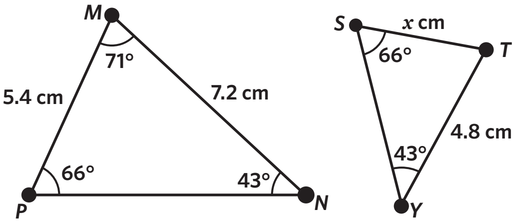
Step 1: Form an equation for the side length ratio using a pair of corresponding side lengths.

Step 2: Substitute the given values into the side length ratio equation and solve for the unknown.

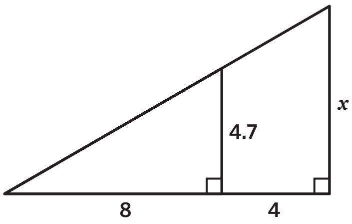
Student practice

Calculate the values of the pronumerals for the following pairs of similar triangles.
Round to one decimal place as needed.

a.



b.



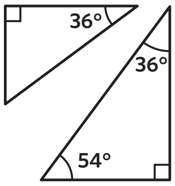
Back to contents

8E Questions

Understanding worksheet

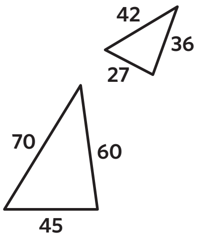
1. Circle the most suitable similarity test for each of the following pairs of triangles.

Example



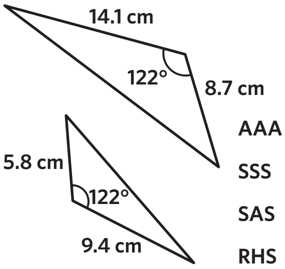
[✓ AAA (angle-angle-angle)]
SSS (side-side-side)
SAS (side-angle-side)
RHS (right angle-hypotenuse-side)

a.



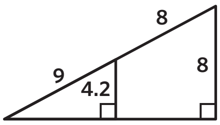
AAA (angle-angle-angle)
SSS (side-side-side)
SAS (side-angle-side)
RHS (right angle-hypotenuse-side)

b.



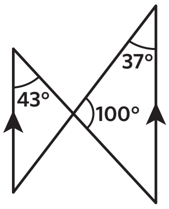
AAA (angle-angle-angle)
SSS (side-side-side)
SAS (side-angle-side)
RHS (right angle-hypotenuse-side)

c.



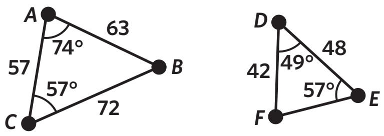
AAA (angle-angle-angle)
SSS (side-side-side)
SAS (side-angle-side)
RHS (right angle-hypotenuse-side)

d.



AAA (angle-angle-angle)
SSS (side-side-side)
SAS (side-angle-side)
RHS (right angle-hypotenuse-side)

2. Complete the statements about the following pair of similar triangles by filling in the blanks.



Example

$\triangle ABC \sim \triangle\boxed{FDE}$ by AAA.

a. $\angle ABC = \boxed{}°$

b. $\angle DFE = \boxed{}°$

c. $\dfrac{AB}{FD} = \dfrac{\boxed{}}{42}$

d. $\dfrac{AB}{FD} = \dfrac{BC}{DE} = \dfrac{CA}{EF} = \dfrac{3}{\boxed{}}$

3. Fill in the blanks by using the words provided.

angles	equal	vertices	shape

When a pair of triangles are similar, their $\boxed{}$ is exactly the same. This means that the length ratio is $\boxed{}$ for all corresponding sides. Triangles can be tested for similarity by comparing exactly three specific corresponding features, including either all three $\boxed{}$ or sides, or a combination of two sides and an angle. When stating that two triangles are similar, the order of their $\boxed{}$ must be listed consistently, where they correspond to each other.

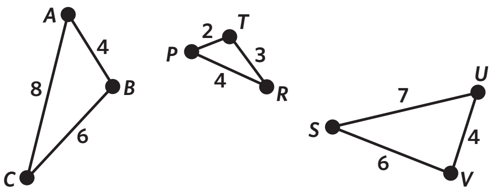
Fluency

Question working paths

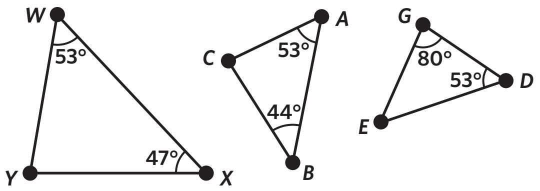
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7	4 (c,d,e), 5 (c,d,e), 6 (c,d,e), 7

4. Determine which pair of triangles are similar for each of the following, with reasons. WE1

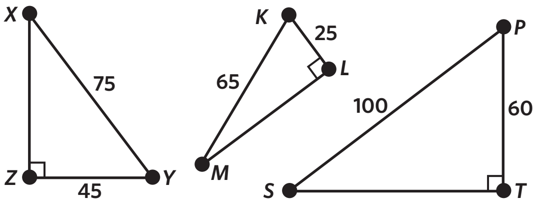
a.



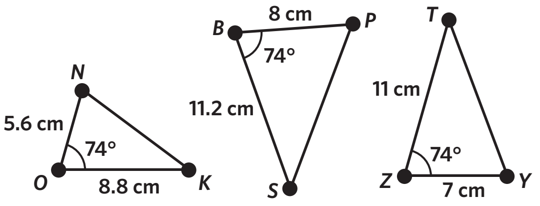
b.



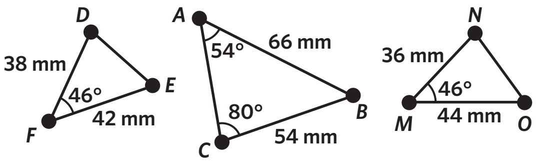
c.



d.

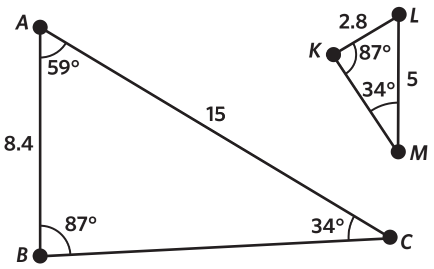


e.

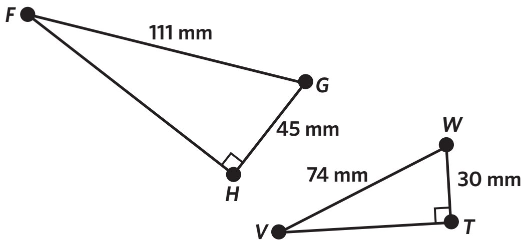


5. Calculate the scale factors for the following pairs of similar triangles. WE2

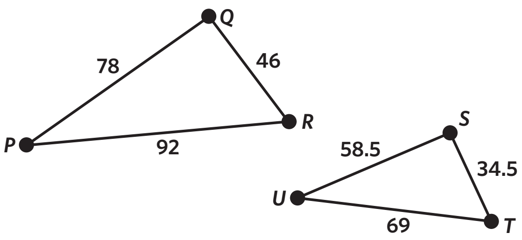
a. $\triangle KLM$ to $\triangle BAC$



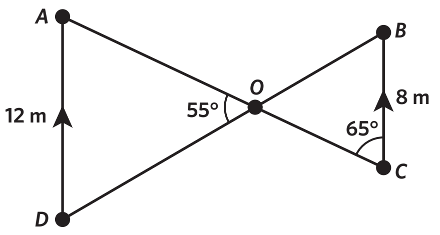
b. $\triangle TVW$ to $\triangle HFG$



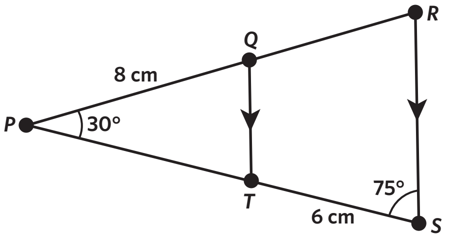
c. $\triangle PQR$ to $\triangle UST$



d. $\triangle AOD$ to $\triangle COB$



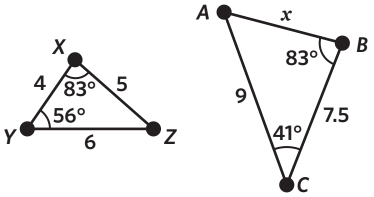
e. $\triangle PQT$ to $\triangle PRS$



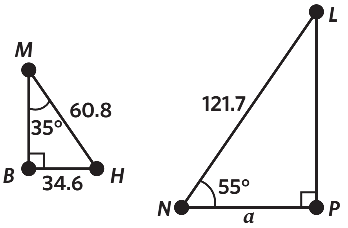
Back to contents

6. Calculate the values of the pronumerals for the following pairs of similar triangles. Round to one decimal place as needed. WE3

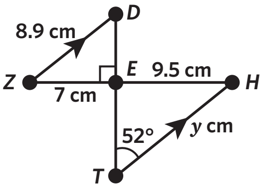
a.



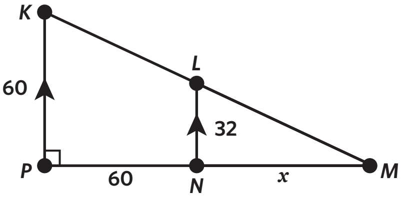
b.



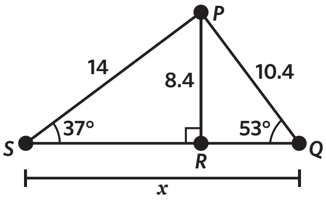
c.



d.



e.



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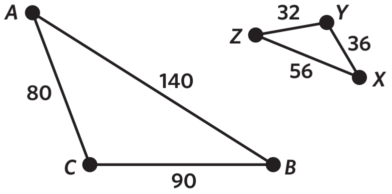
7. What is the scale factor of △*ABC* to △*ZXY*?

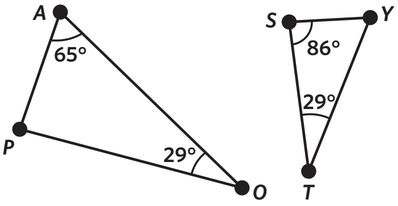


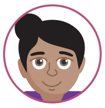
A. $\frac{2}{5}$ **B.** $\frac{10}{7}$ **C.** $\frac{5}{2}$ **D.** $\frac{45}{16}$ **E.** $\frac{35}{9}$

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Determine if the following pair of triangles are similar, with reasons.



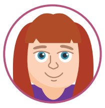


Student A

$\angle AOP = 29° = \angle STY$

$\angle PAO = 65° \neq \angle YST = 86°$

∴ △*APO* is not similar to △*YST*.



Student B

$\angle AOP = 29° = \angle STY$

$\angle APO = 180° - 65° - 29° = 86°$

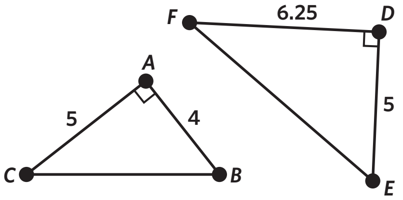
$= \angle YST$

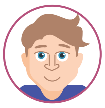
$\angle PAO = 65°$

$= \angle SYT = 180° - 86° - 29° = 65°$

∴ △*APO* ~ △*YTS* by AAA.

b. Determine if the following pair of triangles are similar, with reasons.





Student A

$\angle CAB = 90° = \angle FDE$

$\frac{CA}{FD} = \frac{5}{6.25} = \frac{AB}{DE} = \frac{4}{5}$

$\therefore \triangle ABC \sim \triangle DEF$ by RHS.

Student B

$\angle CAB = 90° = \angle FDE$

$\frac{CA}{FD} = \frac{5}{6.25} = \frac{AB}{DE} = \frac{4}{5}$

$\therefore \triangle ABC \sim \triangle DEF$ by SAS.

Problem solving

Question working paths

Mild 9, 10, 11	**Medium** 10, 11, 12	**Spicy** 11, 12, 13

9. A triangular area has been traced on a map. The dimensions of the drawn triangle are 2.4 cm, 3.5 cm, and 5.1 cm. The map has a scale of 150 km to 1 cm. What are the real-life dimensions of the triangular area, in km?

10. Two triangles are similar and their longest sides are 81 and 54 units. What is the scale factor of the larger triangle to the smaller triangle?

11. A ski lift cable extends from the top of the mountain to the restaurant. The top of the mountain is 300 m above the ground where the restaurant is located 790 m away, and the cable is approximately 950 m long. Calculate how high up from the ground a skier is halfway down the line, to the nearest metre.

12. An equilateral triangle has been split into similar triangles by drawing lines between the midpoints of each side. What is the scale factor of the smaller triangles to the larger triangle?

13. The sun is shining down on Bernie and Chucky at the same angle. They cast shadows on the ground that are perpendicular to each person's body. Bernie is 80 cm tall and her shadow is 3.5 m long. If Chucky's shadow is 7.5 m long, then how tall is Chucky, to the nearest centimetre?

Reasoning

Question working paths

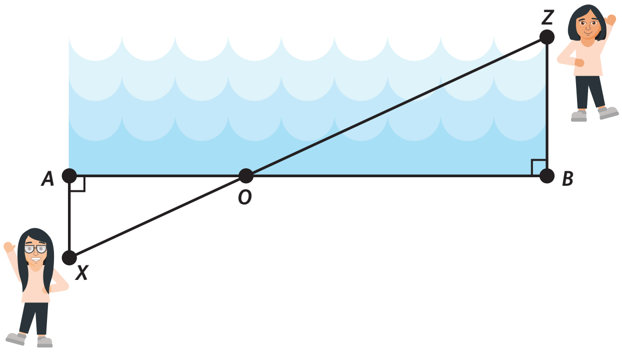
Mild 14 (a,b,c,e)	**Medium** 14 (a,b,c,e), 15 (a,b)	**Spicy** All

14. Two guides are setting up a zipline across the river. They find an efficient way to approximate the width of any river using similar triangles, pictured. Use the diagram to answer the following questions.

a. Name the length which can be used to approximate the width of the river, using appropriate notation.

b. Name the similar triangles in the diagram.

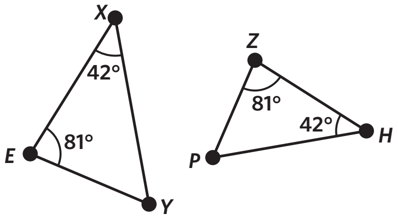
c. Which test can be used to prove that the triangles in part **b** are similar?



d. The guide at point X is 3.8 m away from the bank at point A, while the lengths of AO and BO are 15 m and 50 m, respectively. Calculate the width of the river, in metres, rounded to one decimal place.

e. Find one other possible real-life application for the measuring method shown in the diagram.

15. Consider the following diagram.



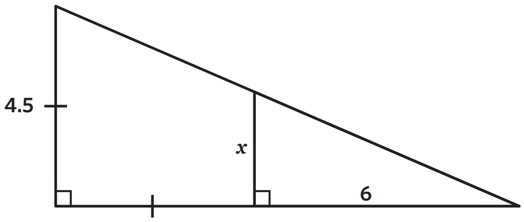
a. Determine the size of $\angle XYE$.

b. Which test proves that $\triangle XYE$ is similar to $\triangle HPZ$?

c. Using your answers for parts **a** and **b**, explain whether it is necessary to know all three internal angles of a pair of triangles in order to test them for similarity.

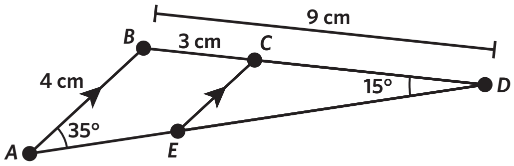
Exam-style

16. What is the value of x, rounded to one decimal place? (1 MARK)



A. 1.5 **B.** 1.8 **C.** 1.9 **D.** 2.6 **E.** 3.4

17. Consider the following diagram. (3 MARKS)

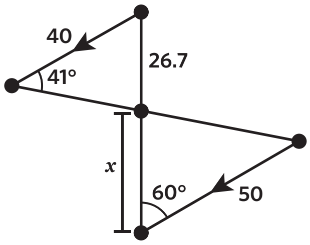


a. Identify the pair of similar triangles in the diagram, using appropriate notation. 1 MARK

b. Calculate the scale factor of the larger triangle to the smaller triangle from part **a**. 1 MARK

c. Calculate the length of CE, rounded to one decimal place. 1 MARK

18. Determine the value of x in the following diagram, rounded to one decimal place. (3 MARKS)



19. A triangle is similar to a smaller triangle with an area of 108 cm^2 and a base length of 12 cm. If the scale factor between the triangles is 4, then what is the height of the larger triangle? (3 MARKS)

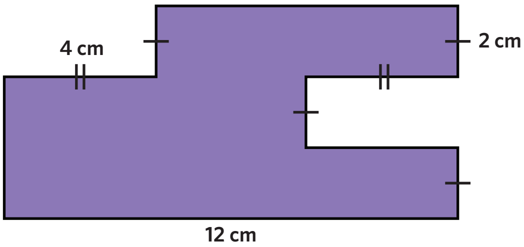
Remember this?

20. The fraction $\frac{6}{16}$ equals 0.375 as a decimal.

What does $\frac{6}{4}$ equal as a decimal?

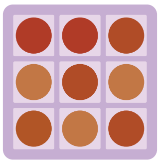
A. 0.09375 **B.** 0.25 **C.** 0.75 **D.** 1.5 **E.** 2.7

21. Find the area of the composite shape.

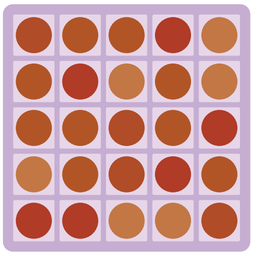


A. 8 cm^2 **B.** 16 cm^2 **C.** 56 cm^2 **D.** 72 cm^2 **E.** 144 cm^2

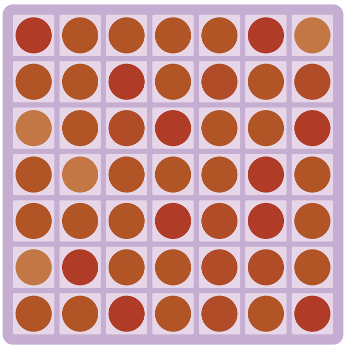
22. Bharath is buying chocolate for his friends. He buys four small packages, one medium package and two large packages.



Small package
$7.50 each



Medium package
$10.80 each



Large package
$13.50 each

Bharath pays with a $100 note.

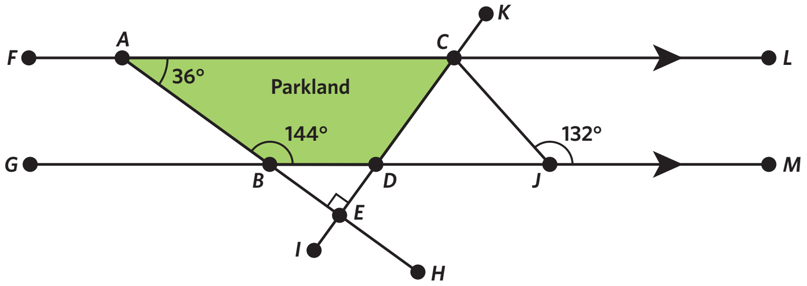
How much change should Bharath receive?

A. $31.80 **B.** $32.20 **C.** $52.50 **D.** $67.80 **E.** $68.20

Back to contents

Chapter 8 extended application

1. As part of city planning, engineers are designing a complex system of roads. They need to ensure that certain roads remain parallel to each other and that specific intersections are designed with precise angles. This diagram shows five roads and six intersections.



a. Identify and describe the types of angles that are formed when two parallel roads are intersected by a transversal road.

b. The engineers must determine if Road *FL* and Road *GM* are parallel to each other. Using angle properties, explain how they can use the angles formed by Road *FL* and Road *GM* with a transversal road to decide if the two roads are parallel.

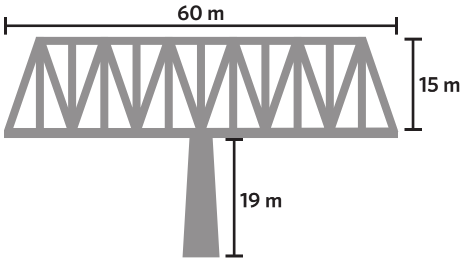
c. At intersection *E*, it is critical to have a right angle for safety reasons. If one of the roads is known to form a 90° angle with the transversal road, explain how to determine if any other road also forms a right angle in this system of roads.

d. The city has safeguarded a section of parkland, ensuring no roads traverse through it. Calculate the interior angles within the parkland area identified by the shape *CDBA*.

e. The intersection at *C* is a roundabout. Calculate the angles between the roads that converge at the roundabout.

f. Explain some considerations that city planners should take into account when planning the locations of new infrastructure or upgrades to existing infrastructure.

2. A construction project has a team of architects designing a pedestrian bridge to span across a river. The bridge is a replica of an existing bridge in a different city that is shown in the following diagram, with the architects ensuring that the bridges are similar.



The pedestrian bridge will cross a smaller section of the river and will have a length of 40 m. The bridge's steel supports are designed as congruent triangles that run along both sides of the bridge.

a. Identify the scale factor of the existing bridge to the new pedestrian bridge.

b. Calculate the exact dimensions of the new pedestrian bridge that is to be constructed, maintaining similarity with the original bridge. Round to the nearest metre where necessary.

c. Describe the minimal conditions required to prove that two triangles are congruent in shape and size for the pedestrian bridge.

d. Given that the diagonal segments of the steel supports on the existing bridge each measure 16 m, calculate the total length of steel necessary for the construction of these diagonal segments for both sides of the pedestrian bridge.

e. Discuss the safety considerations architects should take into account when constructing a pedestrian bridge, including factors such as weight distribution, materials used, and the bridge's intended use.

Back to contents

3. In a biology research project, scientists are studying a population of migratory birds. They have captured two birds, Bird A and Bird B, and are using their data to make inferences about the entire population. Bird A has a wingspan of 20 cm, while Bird B has a wingspan of 25 cm. The scientists notice that the skeletal structure of Bird A is similar in shape to Bird B.

a. Identify the scale factor that relates the wingspan of Bird A (20 cm) to the wingspan of Bird B (25 cm).

b. Given that the length of Bird A's small wing bone is 2 cm, calculate the length of the corresponding wing bone in Bird B, ensuring it is proportionate to Bird A's bone length, correct to one decimal place.

A scan of the wing of Bird A indicates that it has a surface area of 22.5 cm^2 and the shape can be generalised as a kite as shown.

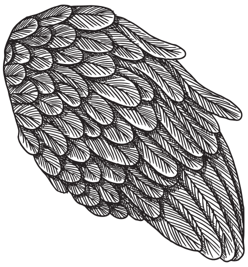
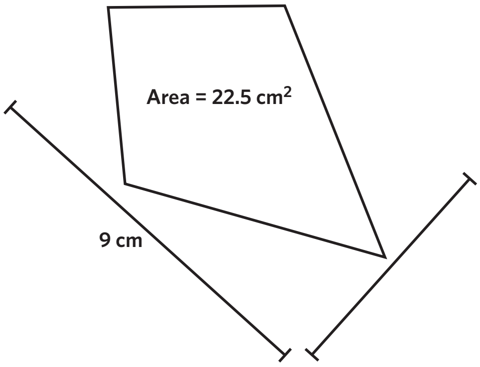


Image: Golden ShrimpShutterstock.com



c. Describe the minimal conditions required to prove that the skeletal structures of Bird A and Bird B are similar.

d. Using the scan details from Bird A, sketch the generalisation of the wing of Bird B and predict the surface area of the wing of Bird B, rounded to three decimal places.

e. Discuss how changes in the birds' habitat, such as food availability, may affect the average wingspan of the population.

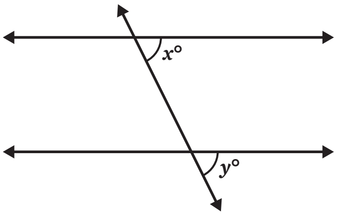
Back to contents

Chapter 8 review

Multiple choice

8A

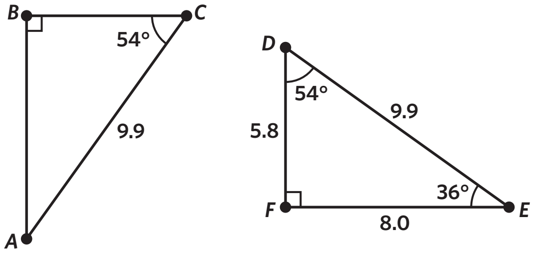
1. What is the relationship between the two angles $x°$ and $y°$?



A. Alternate
B. Co-interior
C. Corresponding
D. Supplementary
E. Complementary

8B

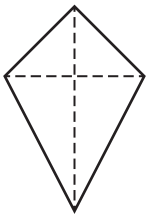
2. Name the corresponding angle to $\angle ACB$ for this pair of congruent triangles.



A. $\angle ABC$
B. $\angle BAC$
C. $\angle EFD$
D. $\angle FDE$
E. $\angle FED$

8C

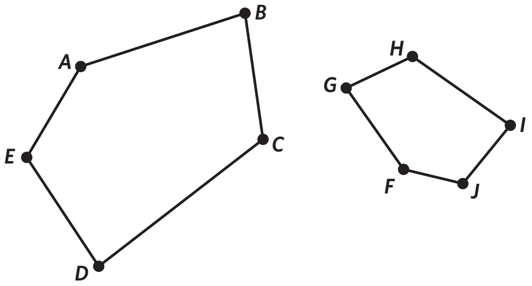
3. Which of the following best describes the properties of a kite?



A. 2 pairs of adjacent sides, equal in length.
B. 4 sides of equal length and equal angles.
C. 4 sides of equal length and 4 right angles.
D. A parallelogram with 4 sides of equal length.
E. 2 pairs of sides equal in length and 4 right angles.

8D

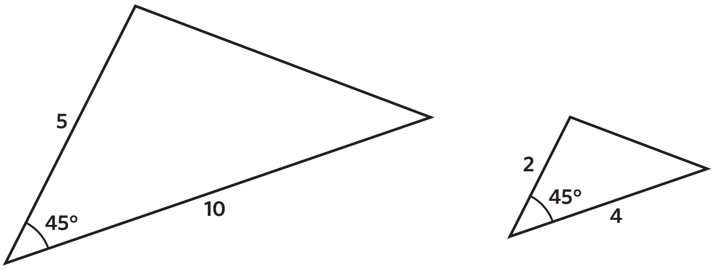
4. Identify the side corresponding to AB for this pair of similar shapes.



A. BA
B. CD
C. FG
D. GH
E. IJ

8E

5. Under what condition are the following pair of triangles similar?



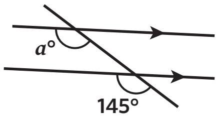
A. AAA
B. SAS
C. SSS
D. RHS
E. Not similar

Back to contents

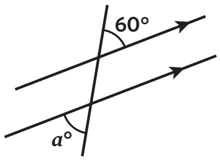
Fluency

6. Determine the unknown angles, giving reasons, for each diagram. 8A

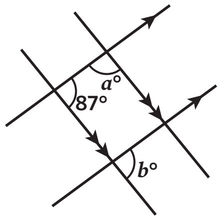
a.



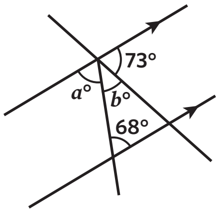
b.



c.

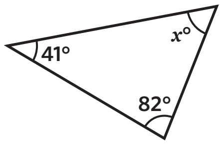


d.

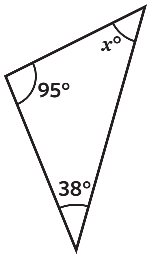


7. Given the triangle, find the value of the unknown angle. 8B

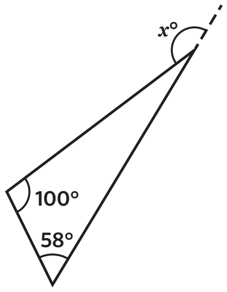
a.



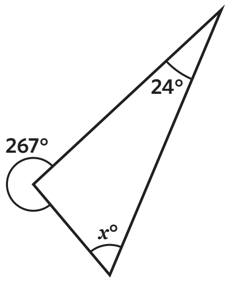
b.



c.

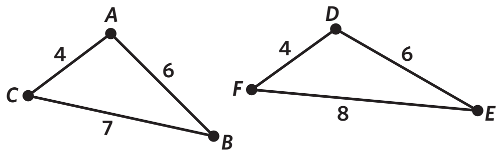


d.

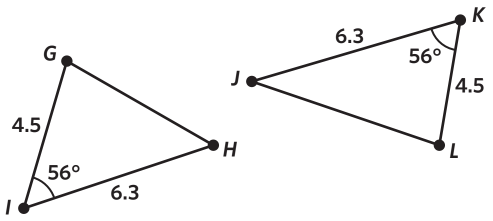


8. Determine if the following triangles are congruent. Explain using the congruence test. 8B

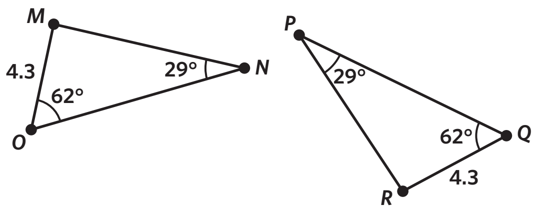
a.



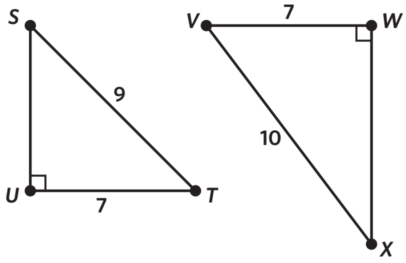
b.



c.

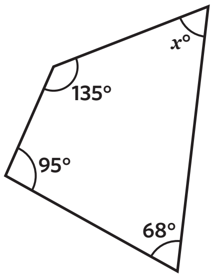


d.

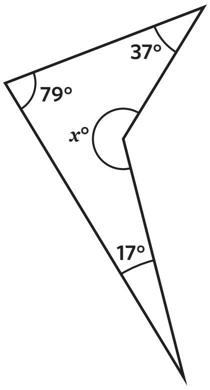


9. Find the value of the unknown for each diagram using properties of quadrilaterals. 8C

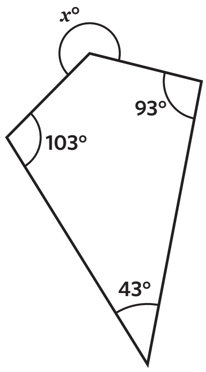
a.



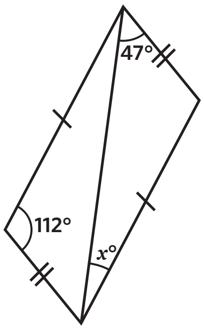
b.



c.

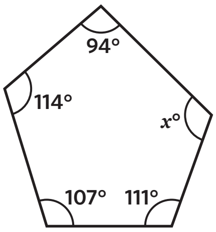


d.

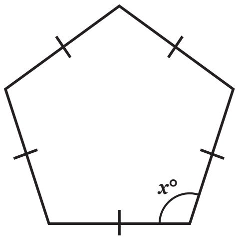


10. For each polygon, find the angle sum and then find the value of the unknown. 8C

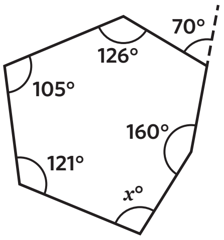
a.



b.



c.



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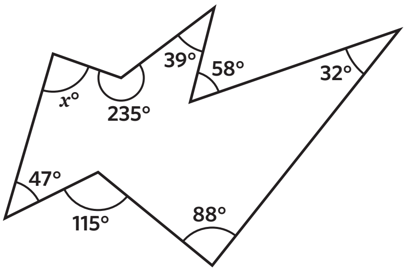

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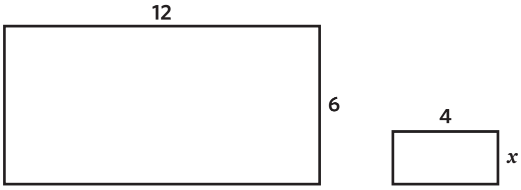
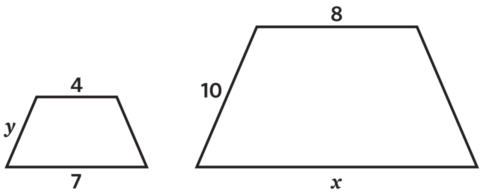
11. For the given pairs of similar figures, use the scale factors to determine the values of the pronumerals. Round to two decimal places where necessary. 8D

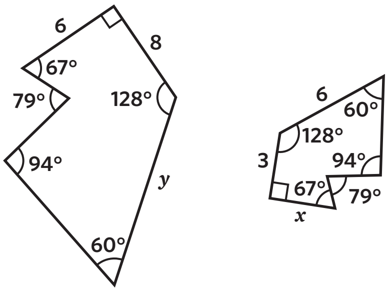
a.



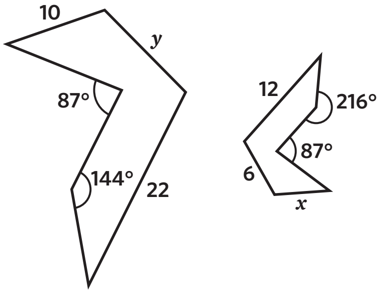
b.



c.



d.



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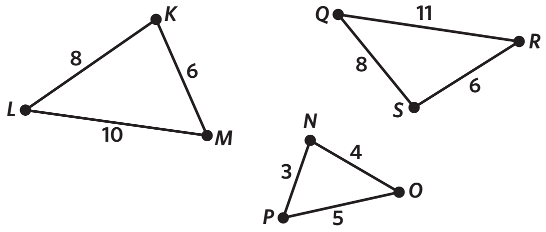
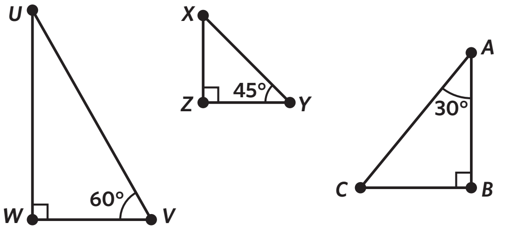
12. Determine which pair of triangles are similar for each of the following, with reasons. 8E

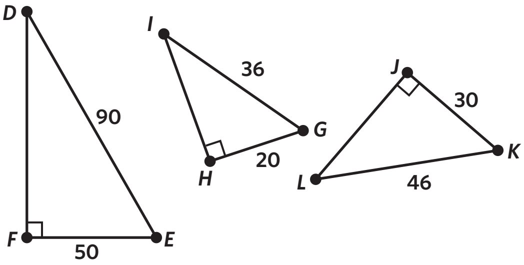
a.



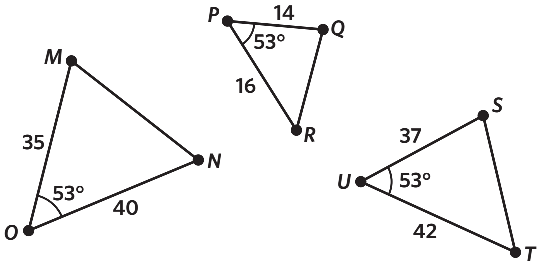
b.



c.

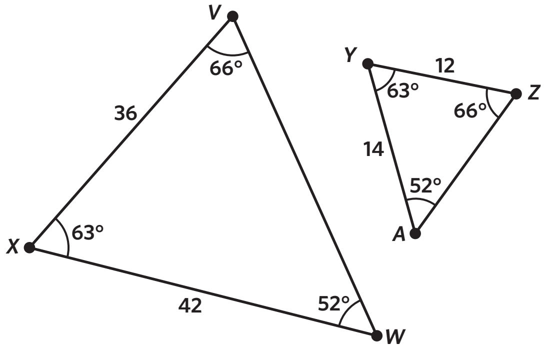


d.

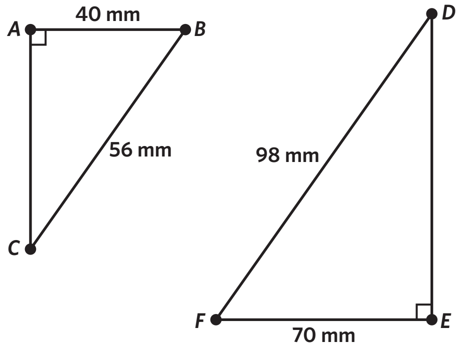


13. Calculate the scale factors for the following pairs of similar triangles. 8E

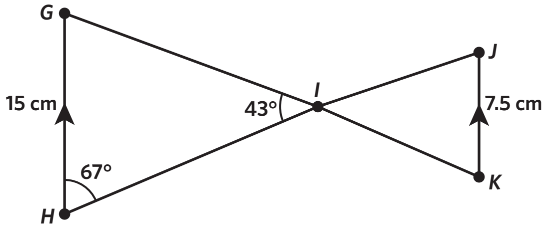
a. △*XVW* to △*YZA*



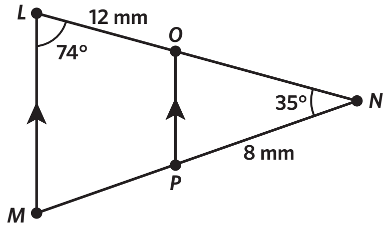
b. △*ABC* to △*EFD*



c. △*GHI* to △*KJI*



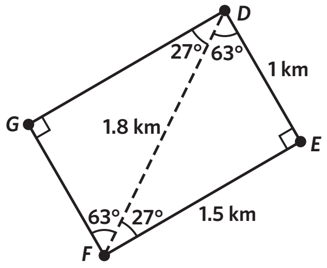
d. △*LMN* to △*OPN*



Problem solving

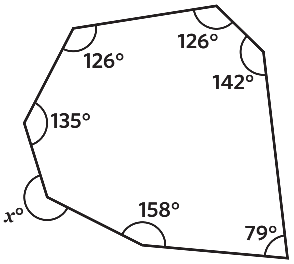
14. A courtyard is enclosed by four walls. The internal angles within the courtyard are given as 95°, *x*°, *y*°, and 103°. Given that two of the walls forming the courtyard are parallel, determine the values of *x*° and *y*°. 8A

15. The run and swim legs of a triathlon course are mapped out as shown below, with the line DF representing the shoreline. 8B



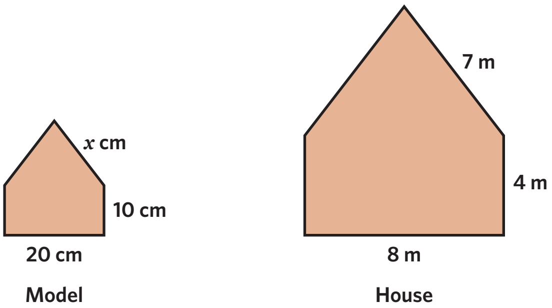
The athletes are to first complete the swim leg, going from D-E-F followed by running back to the starting point D and completing the D-G-F-D run leg. They then complete their bike leg, which is 10 km. Determine the total distance of the triathlon.

16. A greenhouse roof is built in an irregular shape to maximise the sunlight exposure. Calculate the missing angle *x*° from the greenhouse. 8C



Back to contents

17. Tom is an architect who builds models of houses. The house he is currently working on has a front facade shaped as a pentagon. The dimensions of the facades of both the model and house are shown in the diagram. 8D



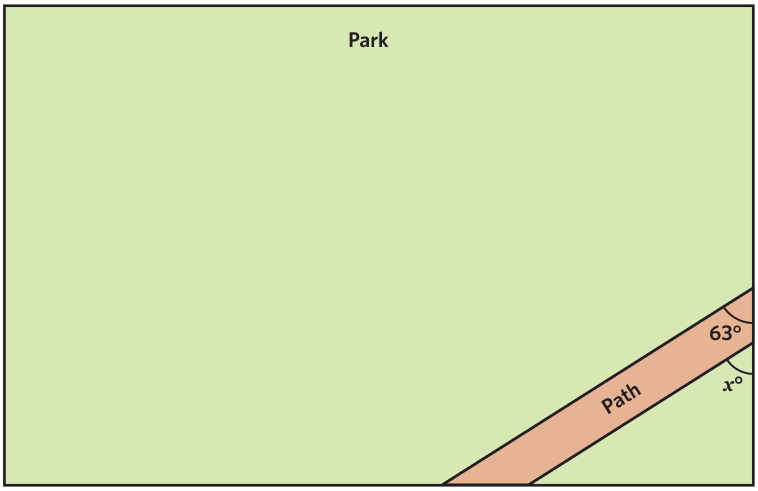
Determine the scale factor from the model to the house, and determine the value of x in centimetres.

18. Emily wants to hang a banner on a wall that is 4 m high. She uses a pole that is 5 m long to reach the top of the wall. Later, she needs to hang a smaller banner on a different wall that is 2.5 m high. She uses a shorter pole that is 3 m long for this task. Determine if two similar triangles are formed from the poles. 8E

Reasoning

19. Sarah is an architect who is designing a new park. The park will feature two triangular playgrounds, a hexagonal fountain, and a rectangular observation deck. Sarah uses a scale model to plan the layout.

a. There is a path that goes through the bottom right corner of the park as displayed in the diagram. Solve for the unknown angle $x°$.



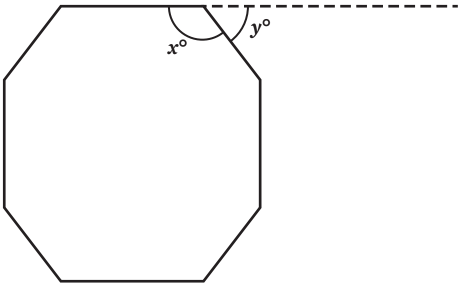
b. In her design, Sarah wants to place two triangular playgrounds at opposite ends of the park. Each playground has sides of 30 m, 40 m, and 50 m. Explain if the playgrounds are congruent using the congruence test.

c. Sarah wants to ensure that the hexagonal fountain has equal angles at each vertex. What should the value of each internal angle of the hexagonal fountain be?

d. Sarah's scale model uses a scale factor of $\frac{1}{50}$. If the actual length of one side of the observation deck is 100 m, what is the length of that side in the scale model?

e. What are some possible challenges Sarah might face when transitioning from the scale model to the actual construction of the park?

20. Consider the following regular polygon.

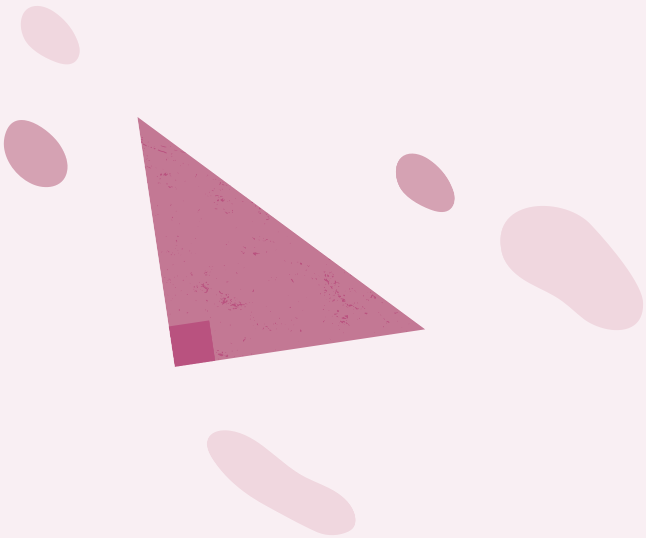


a. Determine the interior angle $x°$ for this polygon.

b. Determine the exterior angle $y°$ for this polygon.

c. Is the sum $x° + y°$ specific to this polygon? What can you conclude about the relationship of interior and exterior angles of regular polygons?

Back to contents



Chapter 9

Pythagoras' theorem and trigonometry

Measurement and geometry

Calculator skills

See online in additional materials for using CAS calculator guides.

9C Calculating unknown side lengths
9D Calculating unknown angles

Back to contents

Chapter 9 research summary

Pythagoras' theorem and trigonometry

Big ideas

Pythagoras' theorem and trigonometry are significant topics in Year 9. These concepts are built upon foundational mathematical ideas. Understanding these concepts in Year 9 will be integral in helping students pursue senior mathematical studies in Year 10 and beyond.

Proportional reasoning

Proportional reasoning is crucial in trigonometry, where ratios of side lengths in right-angled triangles remain constant (sine, cosine, and tangent). Proportional reasoning allows students to scale triangles up or down and still apply the same trigonometric relationships.

Spatial reasoning

Understanding the relationships between the sides and angles of triangles requires students to visualise shapes, rotate them mentally, and see the connections between different parts of a figure.

Algebraic reasoning

Using letters and symbols to represent quantities allows for generalisations. For example in Pythagoras' theorem, $a^2 + b^2 = c^2$, variables can represent any real number side lengths that fit the criteria of a right-angled triangle.

Geometric properties

Concepts such as congruence and similarities are fundamental in understanding why the Pythagorean theorem and trigonometric ratios work. Similar triangles have the same shape but not necessarily the same size and this property is foundational in trigonometry.

Number sense

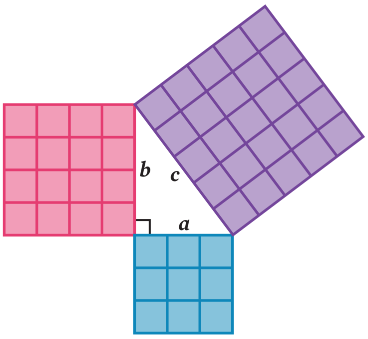
Intuitive understanding of numbers, their magnitudes, and relationships are important. For example, recognizing that the hypotenuse is always the longest side in a right-angled triangle can guide a student's thinking.

Together these big ideas form a foundation upon which students can build a deeper understanding of more advanced mathematical topics. As they progress, these foundational concepts will continually reappear, reinforcing their importance in the broader landscape of mathematics.

Visual representations

Square areas

Square areas can be used to show that the square area of the hypotenuse is equal to the sum of the square areas of the sides.

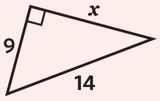
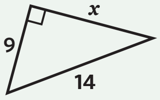
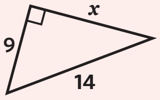
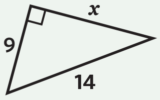
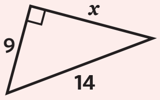
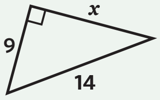
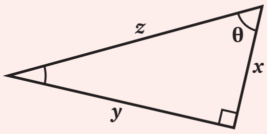
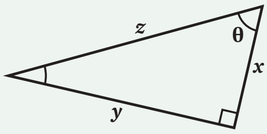
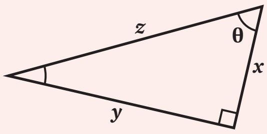
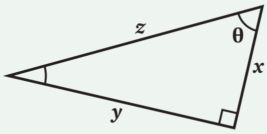
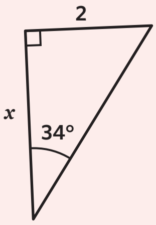
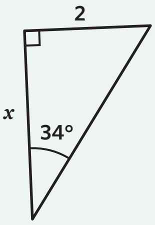


Interactive software

Software tools like Geogebra or Desmos can allow students to manipulate triangle dimensions and angles, observing how trigonometry ratios change.

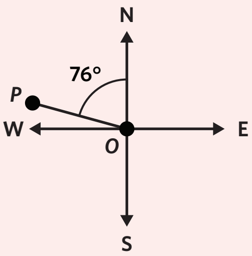
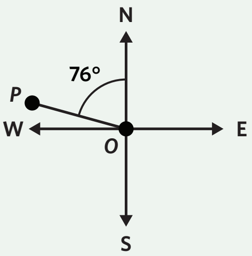
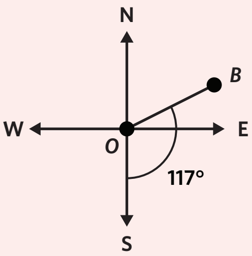
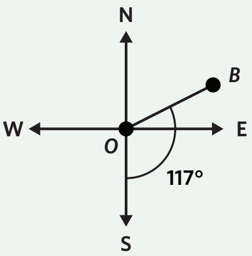
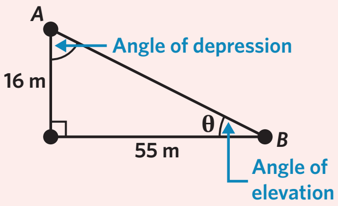
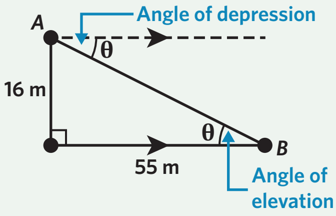
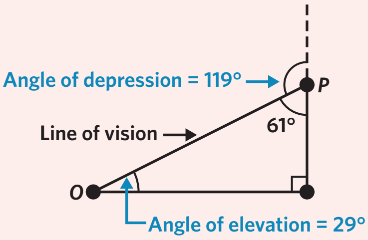
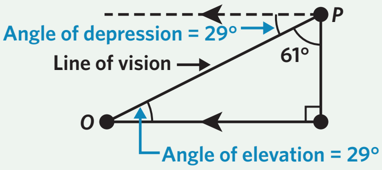
Back to contents

Misconceptions

Misconception	Incorrect ✕	Correct ✓	Lesson
Students sum or subtract side lengths without squaring them first.	 $x^2 = 14 - 9$ $= 5$ $x = \sqrt{5}$ ≈ 2.24	 $x^2 = 14^2 - 9^2$ $= 115$ $x = \sqrt{115}$ ≈ 10.72	9A
Students add squares of side lengths to find the length of a shorter side or subtract to find hypotenuse.	 $x^2 = 14^2 + 9^2$ $= 277$ $x = \sqrt{277}$ ≈ 16.64	 $x^2 = 14^2 - 9^2$ $= 115$ $x = \sqrt{115}$ ≈ 10.72	9A
Students assign a squared side length to a side.	 $x = 14^2 - 9^2$ $= 115$	 $x^2 = 14^2 - 9^2$ $= 115$ $x = \sqrt{115}$ ≈ 10.72	9A
Students use the right angle as the reference angle.	 $\cos\theta = \frac{y}{z}$	 $\cos\theta = \frac{x}{z}$	9B
Students do not apply the correct trigonometric ratio when calculating an unknown side length.	 $\sin\theta = \frac{x}{z}$	 $\cos\theta = \frac{x}{z}$	9B, 9C
Students always write the unknown side length as the numerator value.	 $\tan 34° = \frac{x}{2}$	 $\tan 34° = \frac{2}{x}$	9C
Students incorrectly transpose the equation.	$\cos 18° = \frac{6}{x}$ $x = 6\cos 18°$ $x = 5.706339...$ ≈ 5.71	$\cos 18° = \frac{6}{x}$ $x = \frac{6}{\cos 18°}$ $x = 6.3087733...$ ≈ 6.31	9C

Continues →

Back to contents

Misconception	Incorrect ✗	Correct ✓	Lesson
Students think the inverse relates to a reciprocal.	$\sin\theta = 0.5$ $\theta = \sin^{-1}(0.5)$ $= \frac{1}{\sin 0.5°}$ $\approx 115°$	$\sin\theta = 0.5$ $\theta = \sin^{-1}(0.5)$ $= 30°$	9D
Students use a trigonometric ratio instead of an inverse trigonometric ratio to solve.	$\tan\theta = \frac{O}{A}$ $\tan\theta = \frac{4}{5}$ $\theta = \tan\left(\frac{4}{5}\right)$ $\theta = 0.01398...$ $\approx 0.01°$	$\tan\theta = \frac{O}{A}$ $\tan\theta = \frac{4}{5}$ $\theta = \tan^{-1}\left(\frac{4}{5}\right)$ $\theta = 38.65980...$ $\approx 39.66°$	9D
Students measure a true bearing from north in the anti-clockwise direction.	 076° T	 284° T	9E
Students misinterpret the direction of the bearing.	 The angle from north to *B*: $180° - 117° = 63°$ ∴ The bearing of *O* from *B* is 063° T	 The angle from north to *B*: $180° - 117° = 63°$ ∴ The bearing of *B* from *O* is 063° T. $63° < 180°$ $63° + 180° = 243°$ ∴ The bearing of *O* from *B* is 243° T.	9E
Students think the angles of elevation and depression between two points are complementary.			9F
Students measure the angle of depression from the vertical.			9F

Back to contents

9A Pythagoras' theorem

LEARNING INTENTIONS

Students will be able to:

- identify the hypotenuse of a right-angled triangle
- apply Pythagoras' theorem to calculate the length of the hypotenuse
- apply Pythagoras' theorem to calculate the length of a shorter side of a right-angled triangle.

Pythagoras' theorem is a relation in geometry that links the side lengths of a right-angled triangle. This allows for calculation of an unknown side length, given the lengths of the other two sides. Pythagoras' theorem is the basis for trigonometry, which involves the study of angles in two and three dimensions.

KEY TERMS AND DEFINITIONS

- A value in **exact form** is a number that is not expressed as a decimal approximation, rather it is left in fraction or surd form.

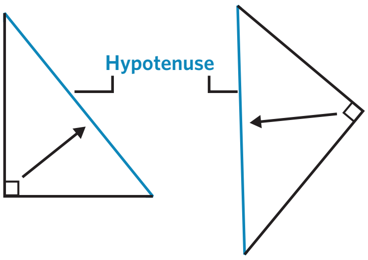
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Guenter Albers/Shutterstock.com

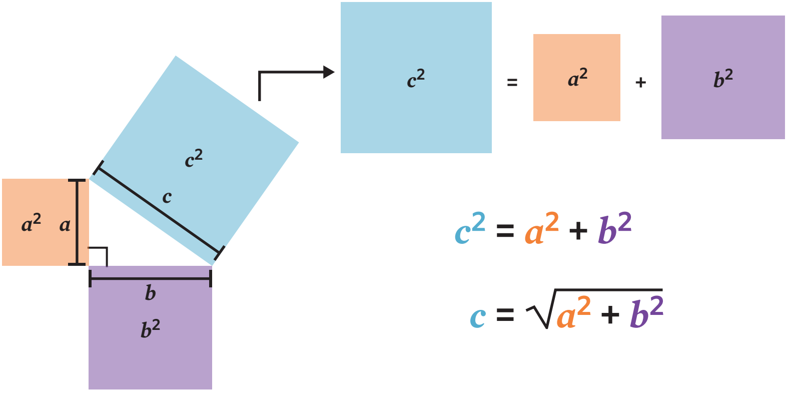
The Great Pyramids at Giza in Egypt pre-dated the formulation of Pythagoras' theorem in Greece. It is known that the builders of the pyramids used knotted ropes to approximate right angles by applying their knowledge of triples of whole numbers which always produce side lengths of a right-angled triangle. These collections of numbers will later be known as Pythagorean triples.

Key ideas

1. The **hypotenuse** is the longest side of a right-angled triangle, and it is always opposite the right angle.



2. **Pythagoras' theorem** establishes that in a right-angled triangle, the square of the hypotenuse (c) equals the sum of the squares of the two shorter sides (a and b), expressed as $c^2 = a^2 + b^2$.



$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

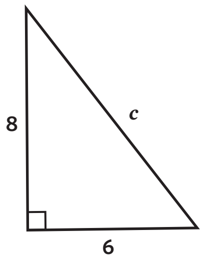
Worked example 1

Calculating the length of the hypotenuse

Calculate the lengths of the hypotenuse, correct to 2 decimal places as required.

a.

WE1a



Working

$c = c$

$a = 8$

$b = 6$

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 6^2$

$c = \sqrt{8^2 + 6^2}$

$= \sqrt{100}$

$= 10$

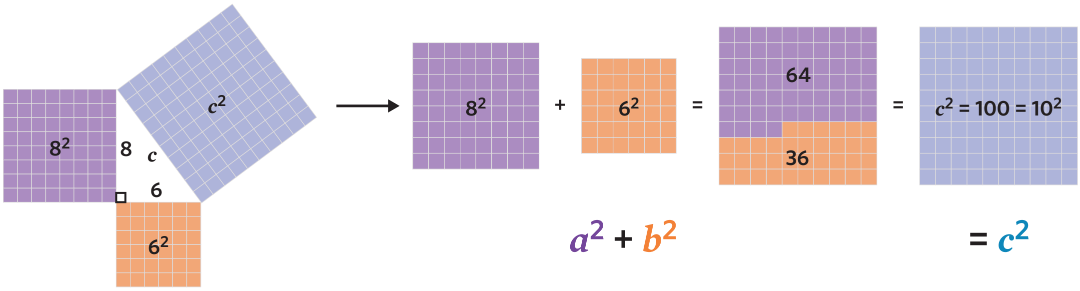
Thinking

Step 1: Identify the lengths of the hypotenuse (c), and the two shorter sides of the right-angled triangle (a and b).

Step 2: Substitute the values of a and b in Pythagoras' theorem formula.

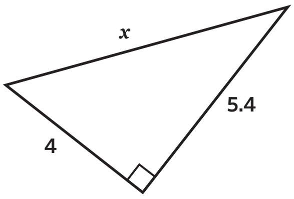
Step 3: Solve for the length of the hypotenuse (c).

Visual support



b.

WE1b



Working

$c = x$

$a = 4$

$b = 5.4$

$c^2 = a^2 + b^2$

$x^2 = 4^2 + 5.4^2$

Thinking

Step 1: Identify the lengths of the hypotenuse (c), and the two shorter sides of the right-angled triangle (a and b).

Step 2: Substitute the values of a and b in Pythagoras' theorem formula.

Continues →

Back to contents

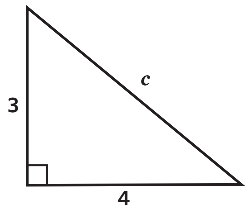
$$x = \sqrt{4^2 + 5.4^2}$$
$$= \sqrt{45.16}$$
$$= 6.720...$$
$$\approx 6.72$$

Step 3: Solve for the length of the hypotenuse (c), and round the value to the required number of decimal places.

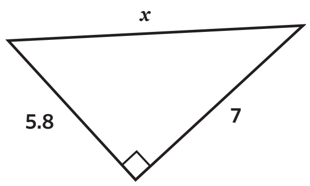
Student practice

Evaluate the following.

a.



b.



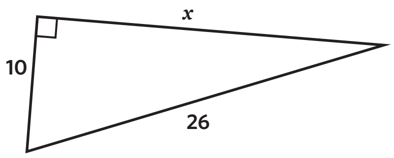
Worked example 2

Calculating the length of a shorter side

Calculate the values of the pronumerals, correct to 2 decimal places as required.

a.

WE2a



Working

$c = 26$

$a = 10$

$b = x$

$c^2 = a^2 + b^2$

$26^2 = 10^2 + x^2$

$26^2 - 10^2 = x^2$

$\sqrt{26^2 - 10^2} = x$

$\sqrt{576} = x$

$\therefore x = 24$

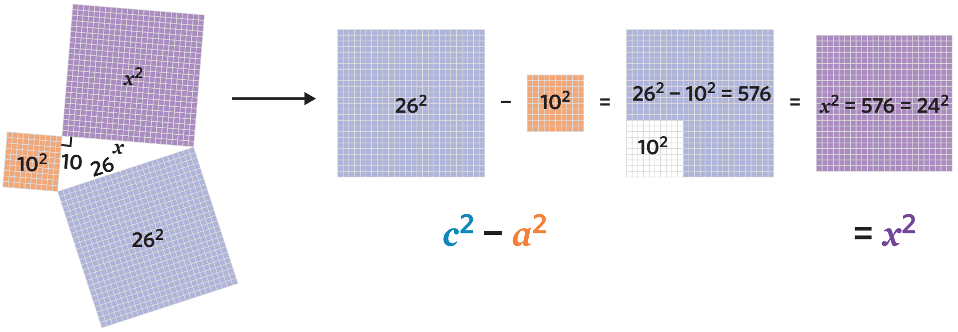
Thinking

Step 1: Identify the lengths of the hypotenuse (c), and the two shorter sides of the right-angled triangle (a and b).

Step 2: Substitute the values of a, b, and c in Pythagoras' theorem formula.

Step 3: Solve for the unknown length.

Visual support

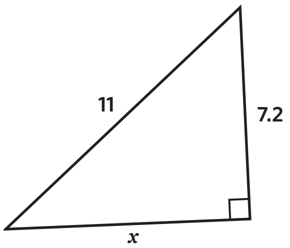


Continues →

Back to contents

b.

WE2b



Working

$c = 11$

$a = x$

$b = 7.2$

$c^2 = a^2 + b^2$

$11^2 = x^2 + 7.2^2$

$11^2 - 7.2^2 = x^2$

$\sqrt{11^2 - 7.2^2} = x$

$\sqrt{69.16} = x$

$x = 8.316...$

≈ 8.32

Thinking

Step 1: Identify the lengths of the hypotenuse (c), and the two shorter sides of the right-angled triangle (a and b).

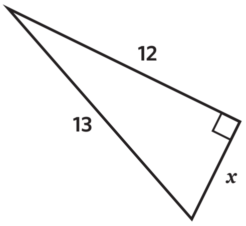
Step 2: Substitute the values of a, b, and c in Pythagoras' theorem formula.

Step 3: Solve for the unknown length and round the value to the required number of decimal places.

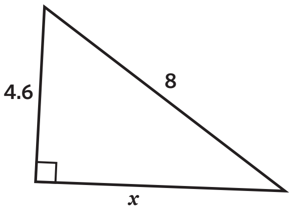
Student practice

Calculate the values of the pronumerals, correct to 2 decimal places as required.

a.



b.



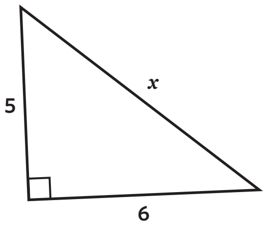
Worked example 3

Calculating lengths using exact values

Calculate the values of the pronumerals in exact form.

a.

WE3a



Working

$c = x$

$a = 5$

$b = 6$

Thinking

Step 1: Identify the lengths of the hypotenuse (c), and the two shorter sides of the right-angled triangle (a and b).

Continues →

Back to contents

$c^2 = a^2 + b^2$

$x^2 = 5^2 + 6^2$

Step 2: Substitute the values of *a*, *b*, and *c* in Pythagoras' theorem formula.

$x^2 = 25 + 36$

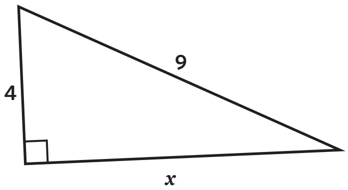
$= 61$

$\therefore x = \sqrt{61}$

Step 3: Solve for the unknown length and express the answer in surd form.

b.

WE3b



Working

$c = 9$

$a = x$

$b = 4$

Thinking

Step 1: Identify the lengths of the hypotenuse (*c*), and the two shorter sides of the right-angled triangle (*a* and *b*).

$c^2 = a^2 + b^2$

$9^2 = x^2 + 4^2$

Step 2: Substitute the values of *a*, *b*, and *c* in Pythagoras' theorem formula.

$9^2 - 4^2 = x^2$

$81 - 16 = x^2$

$65 = x^2$

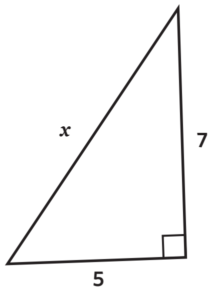
$\therefore x = \sqrt{65}$

Step 3: Solve for the unknown length and express the answer in surd form.

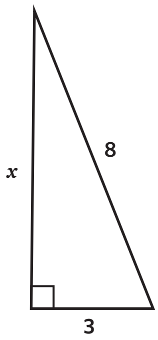
Student practice

Calculate the values of the pronumerals in exact form.

a.



b.

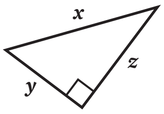


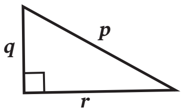
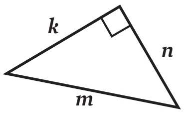
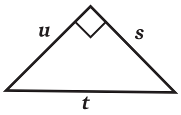
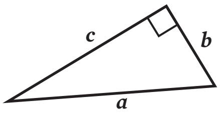
9A Questions

Understanding worksheet

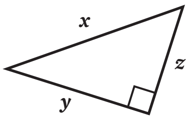
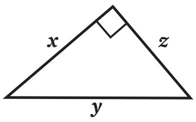
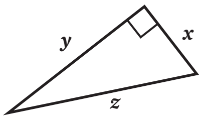
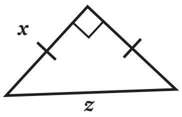
1. Determine which side is the hypotenuse for each of the given right-angled triangles.

Example

Triangle	Hypotenuse
	$\boxed{x}$

Triangle	Hypotenuse
	☐
	☐
	☐
	☐

2. Match the right-angled triangles to equations that accurately show the relationships between their side lengths.

Triangle	Side length relationship
 ●	● $z^2 = y^2 - x^2$
 ●	● $z^2 = 2x^2$
 ●	● $x^2 = y^2 + z^2$
 ●	● $y^2 = z^2 - x^2$

Back to contents

3. Fill in the blanks by using the words provided.

sum **lengths** **two** **square** **hypotenuse**

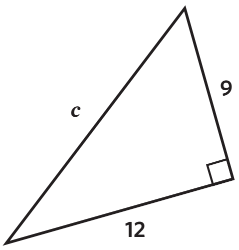
In the right-angled triangle, the longest side is called the __________ . Pythagoras' theorem shows the relationship between the side __________ of a right-angled triangle, where the __________ of the hypotenuse is equal to the __________ of the squares of the other two sides. This formula can be used to determine any unknown side length of a right-angled triangle, given the lengths of the other __________ sides.

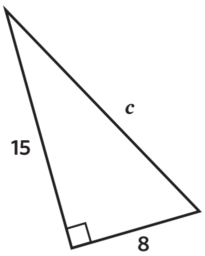
Fluency

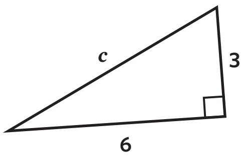
Question working paths

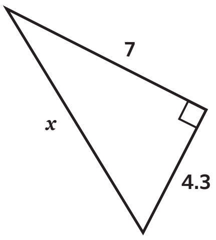
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

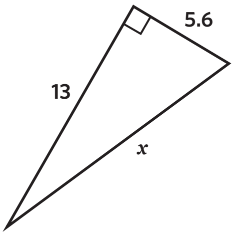
4. Calculate the lengths of the hypotenuse, correct to 2 decimal places as required. WE1

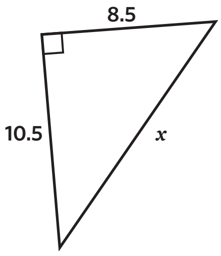
a.


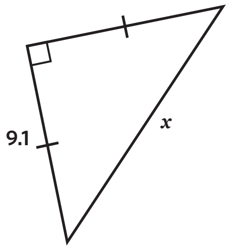
b.


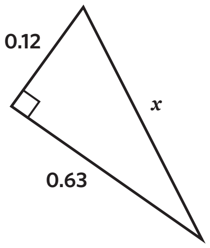
c.


d.


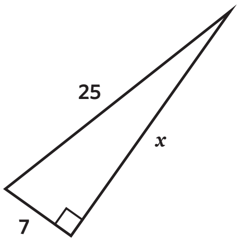
e.


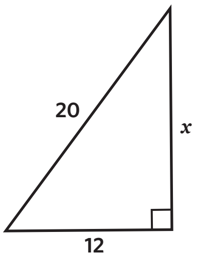
f.


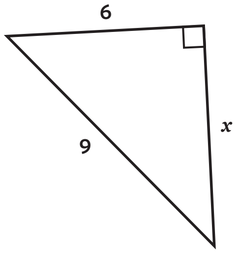
g.


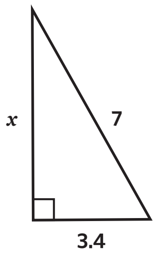
h.


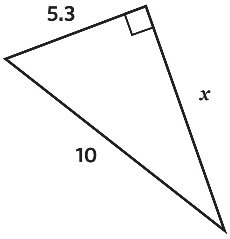
5. Calculate the values of the pronumerals, correct to 2 decimal places as required. WE2

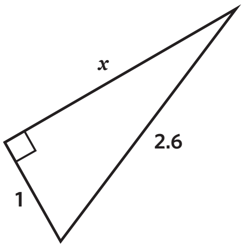
a.


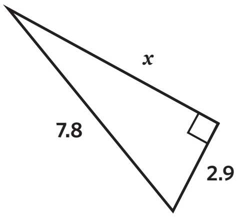
b.


c.


d.


e.


f.


g.


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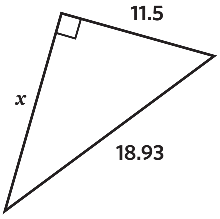
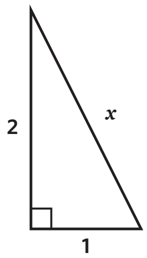


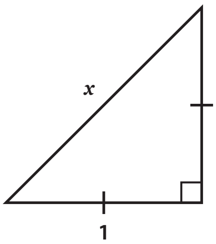
Back to contents

6. Calculate the values of the pronumerals in exact form. WE3

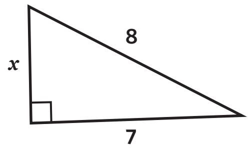
a.



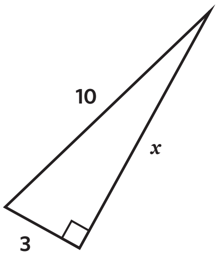
b.



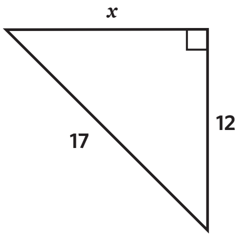
c.



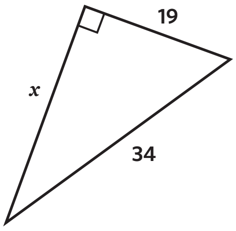
d.



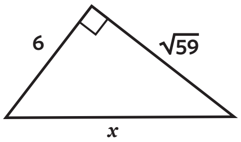
e.



f.



g.



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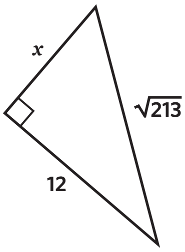
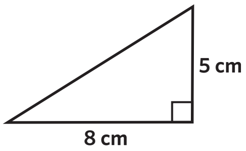
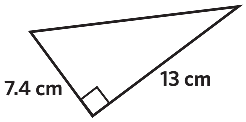


7. Calculate the unknown lengths. Provide the answers in centimetres, rounded to 2 decimal places.

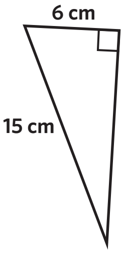
a.



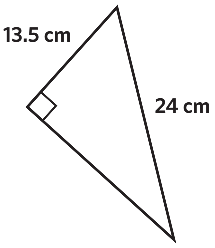
b.



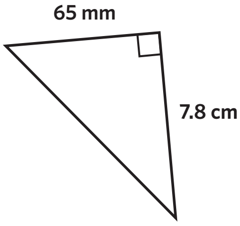
c.



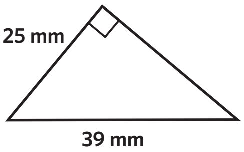
d.



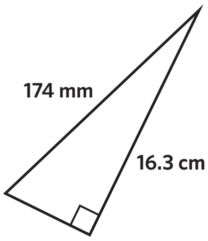
e.



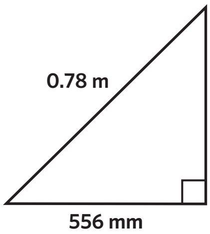
f.



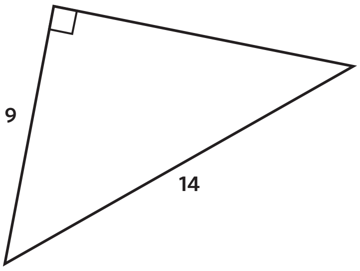
g.



h.



8. Calculate the length of the unmarked side in the given right-angled triangle, correct to 2 decimal places.

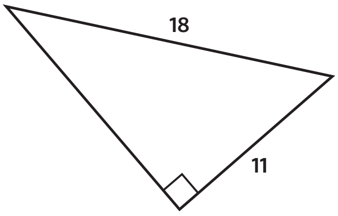


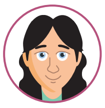
A. 2.24
B. 5
C. 10.72
D. 16.64
E. 115.00

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the unknown side length in exact form.





Student A

$a = 18$

$b = 11$

$c =$ unknown side length

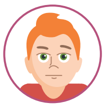
$c^2 = a^2 + b^2$

$= 18^2 + 11^2$

$= 324 + 121$

$= 445$

$\therefore c = \sqrt{445}$



Student B

$a = 11$

$b =$ unknown side length

$c = 18$

$c^2 = a^2 + b^2$

$18^2 = 11^2 + b^2$

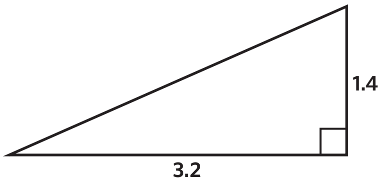
$18^2 - 11^2 = b^2$

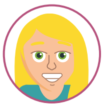
$324 - 121 = b^2$

$203 = b^2$

$\therefore b = \sqrt{203}$

b. Calculate the unknown side length, correct to 1 decimal place.





Student A

$a = 1.4$

$b = 3.2$

$c^2 = a^2 + b^2$

$= 1.4^2 + 3.2^2$

$c = \sqrt{1.4^2 + 3.2^2}$

$= \sqrt{12.2}$

$= 3.49...$

≈ 3.5

Student B

$a = 1.4$

$b = 3.2$

$c^2 = a^2 + b^2$

$= 1.4 + 3.2$

$c = \sqrt{1.4 + 3.2}$

$= \sqrt{4.6}$

$= 2.14...$

≈ 2.1

Problem solving

Question working paths

Mild 10, 11, 12	**Medium** 11, 12, 13	**Spicy** 12, 13, 14

10. Jenny's backyard is rectangular with sides 13 m and 5 m long. She wants to install a fence running diagonally from one corner to the opposite corner of the yard. Calculate the length of the fence to the nearest metre.

11. A swimming pool, 24 m long, has a sloped floor so that the increase in depth is gradual from 0 m on one side to its maximum depth of 7 m on the other side. Calculate the length of the sloped floor of the pool, in metres.

12. A wheelchair ramp is shaped like a right-angled triangle, where the hypotenuse represents the ramp's length and the side which makes a right angle with the height represents the ramp's run. To meet requirements, a 1 m high wheelchair ramp's length must be at least 14 m. Determine the minimum ramp run, in metres, of a 1 m high wheelchair ramp, rounded to 2 decimal places.

13. Rory is standing on a river bank when he sees his friend Gino on the other side at the end of a bridge. The direct distance between the two friends is 14.8 m at this point. Rory walks a total of 8 m to the end of the bridge along his side of the river and walks across it to meet Gino. Determine the distance, in metres, across the river, rounded to 1 decimal place.

14. Sam is casting a shadow on the ground. The direct distance between the top of Sam's head and the top of his shadow's head is 2.3 m. Given that Sam's shadow is 1.7 m long, calculate Sam's height, in metres, correct to 2 decimal places.

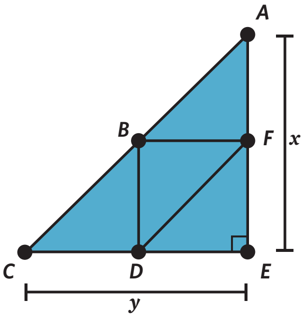
Back to contents

Reasoning

Question working paths

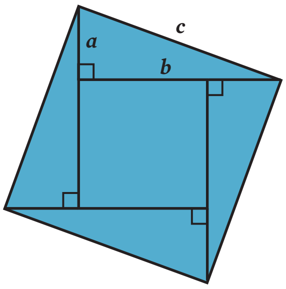
Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. Adrianna is building her house and has designed one half of the frame for the roof. The given diagram shows her design, not to scale.



- **a.** Identify the hypotenuse in $\triangle ACE$.
- **b.** Using c to represent the length of the hypotenuse, write an equation showing the relationship between the side lengths of $\triangle ACE$.
- **c.** $\triangle ABF$, $\triangle BCD$, $\triangle FDE$, and $\triangle DFB$ are congruent. Side AF is 3 m and $y = 5$ m. Determine the length of the hypotenuse of $\triangle ACE$, rounded to 2 decimal places.
- **d.** Adrianna will complete her design by reflecting the frame across side AE, letting this central beam be shared by both halves. Calculate the total length of timber needed for the entire frame, rounded up to the nearest metre.
- **e.** Identify an advantage or disadvantage of building your own home.

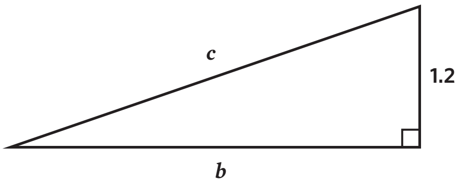
16. The given diagram shows four identical right-angled triangles arranged in a pattern.



- **a.** Write an expression for the area of one of the pictured triangles using $A = \frac{bh}{2}$, where b is the length of the base and h is the height of the triangle.
- **b.** Write an expression for the side length of the smaller square in the middle of the diagram in terms of a and b.
- **c.** Show that the area of the square with sides c is equivalent to $a^2 + b^2$ in the given picture.

Exam-style

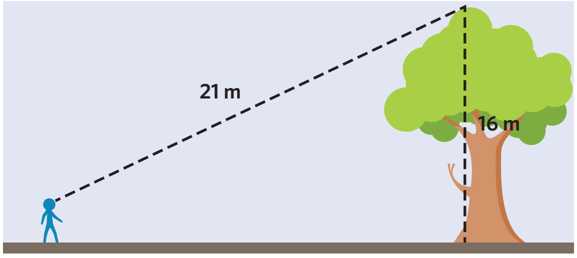
17. Which of the options correctly shows the values of b and c for the given triangle? (1 MARK)



A. $b = 2.4, c = 2.5$ **B.** $b = 3.5, c = 3.7$ **C.** $b = 4, c = 5$ **D.** $b = 5, c = 13$ **E.** $b = 8, c = 17$

Back to contents

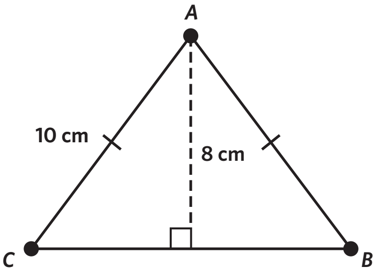
18. Sylvia is standing some distance away from a tree. The tree is 16 m high and the direct distance between the top of Sylvia's head and the top of the tree is 21 m. This is shown in the diagram, not to scale. (3 MARKS)



a. Sylvia is 1.7 m tall. Determine the vertical distance between the top of Sylvia's head and the top of the tree. 1 MARK

b. Calculate the distance Sylvia will need to walk to get to the base of the tree, rounded to 1 decimal place. 2 MARKS

19. Calculate the area, in centimetres, of $\triangle ABC$. (3 MARKS)



20. A cable runs under the floor of a rectangular room from one corner to the opposite corner. The cable is 13.5 m long and the room's length is 11 m. Show that the width of the room is 7.83 m, rounded to 2 decimal places. (2 MARKS)

Remember this?

21. Which of these is the closest to 1?

A. 0.98 **B.** 0.999 **C.** 1.002 **D.** 1.1 **E.** 1.5

22. How many more books were sold in the East Coast and West Coast than in the North and South regions?

Region	Number of books sold
East Coast	120 456
West Coast	98 321
Central	75 890
North	45 678
South	56 789

A. 100 310 **B.** 114 310 **C.** 116 310 **D.** 126 310 **E.** 130 310

23. A factory produces three types of widgets: small, medium, and large. A small widget weighs 2 kg, a medium widget weighs 4 kg, and a large widget weighs 6 kg. If a shipment contains 32 kg of widgets, what is the smallest number of widgets that could be in the shipment?

A. 4 **B.** 5 **C.** 6 **D.** 7 **E.** 8

Back to contents

9B Trigonometric ratios

LEARNING INTENTIONS

Students will be able to:

- label the sides of a right-angled triangle with reference to an angle
- understand the three trigonometric ratios for right-angled triangles
- understand that for any right-angled triangles with the same angles, the trigonometric ratios are the same
- use the ratios of a right-angled triangle's sides to calculate the sine, cosine and tangent of an angle.

Trigonometry explores the core principles governing right-angled triangles and their associated trigonometric ratios. These ratios offer valuable insights into the relationships between the sides of such triangles. Importantly, they remain constant when dealing with right-angled triangles that share the same angles. These ratios, including sine, cosine, and tangent, provide a framework for calculating angles within triangles.

KEY TERMS AND DEFINITIONS

- When two objects are **adjacent**, they are positioned directly next to each other or share a common boundary or side.
- When two objects are **opposite**, they are situated on opposite sides of a specified or implied boundary or facing in contrasting directions.

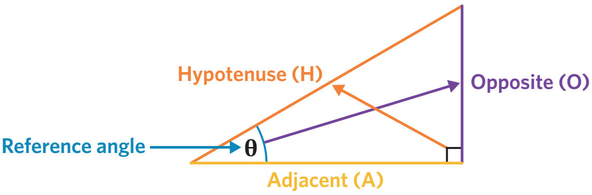
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Imageman/Shutterstock.com

In the realm of computer graphics, the use of right-angled triangles and understanding trigonometric ratios are essential. These triangles assist in rendering realistic images and animations, allowing for the precise calculation of angles and dimensions in creating visual elements.

Key ideas

1. In a right-angled triangle, sides can be labelled as adjacent, opposite and hypotenuse with reference to a specific angle.



2. There are three trigonometric ratios for right-angled triangles: sine, cosine, and tangent, which relate the angles to the sides of the triangle.

$$\sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{O}{H}$$

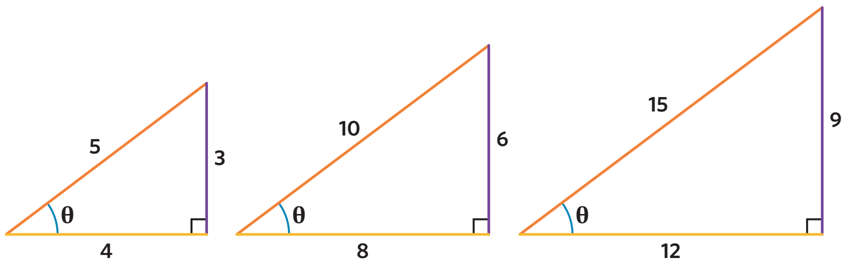
$$\cos\theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{A}{H}$$

$$\tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}} = \frac{O}{A}$$

Continues →

Back to contents

3. For similar right-angled triangles, the trigonometric ratios are the same.



$$\sin\theta = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

$$\cos\theta = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\tan\theta = \frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

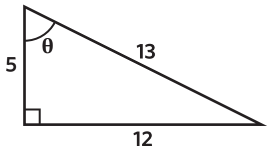
Worked example 1

Identifying trigonometric ratios

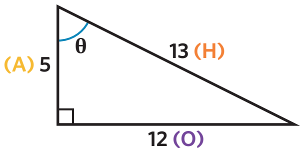
For the triangles shown, write the equations for the sine, cosine and tangent ratios of the reference angle (θ).

a.

WE1a



Working



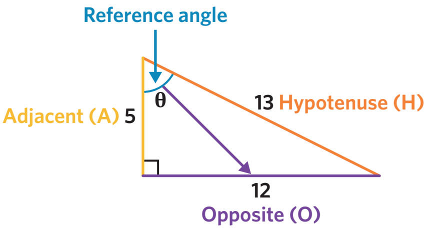
$$\sin\theta = \frac{\text{O}}{\text{H}}$$
$$= \frac{12}{13}$$
$$\cos\theta = \frac{\text{A}}{\text{H}}$$
$$= \frac{5}{13}$$
$$\tan\theta = \frac{\text{O}}{\text{A}}$$
$$= \frac{12}{5}$$

Thinking

Step 1: Label the sides of the right-angled triangle in relation to the reference angle.

Step 2: Write each trigonometric ratio by substituting the values of the labelled sides.

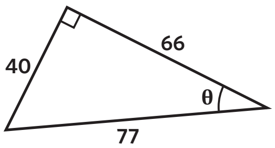
Visual support



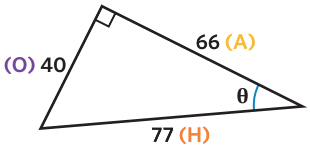
Continues →

b.

WE1b



Working



$$\begin{aligned}\sin\theta &= \frac{\text{O}}{\text{H}}\\ &= \frac{40}{77}\\ \cos\theta &= \frac{\text{A}}{\text{H}}\\ &= \frac{66}{77}\\ &= \frac{6}{7}\\ \tan\theta &= \frac{\text{O}}{\text{A}}\\ &= \frac{40}{66}\\ &= \frac{20}{33}\end{aligned}$$

Thinking

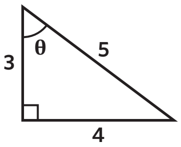
Step 1: Label the sides of the right-angled triangle in relation to the reference angle.

Step 2: Write each trigonometric ratio by substituting the values of the labelled sides and simplify where possible.

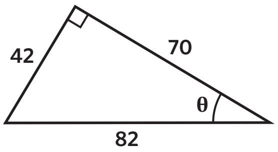
Student practice

For the triangles shown, write the equations for the sine, cosine and tangent ratios of the reference angle (θ).

a.



b.



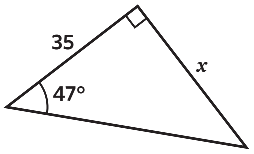
Worked example 2

Relating sides to trigonometric ratios

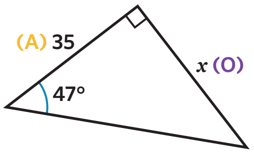
For the following triangles, write the trigonometric ratio that relates the two given sides and the reference angle.

a.

WE2a



Working



Thinking

Step 1: Label the sides of the right-angled triangle in relation to the reference angle.

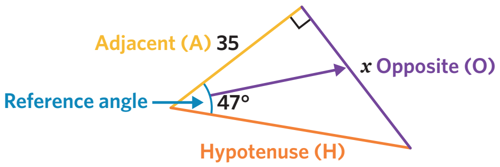
Continues →

$$\tan\theta = \frac{O}{A}$$

$$\tan 47° = \frac{x}{35}$$

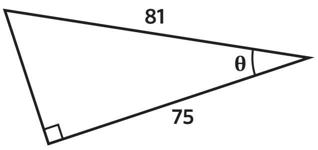
Step 2: Write the trigonometric ratio by substituting the given values and pronumerals representing the side lengths and/or angles of the triangle.

Visual support

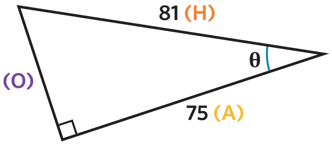


b.

WE2b



Working



Thinking

Step 1: Label the sides of the right-angled triangle in relation to the reference angle.

$$\cos\theta = \frac{A}{H}$$

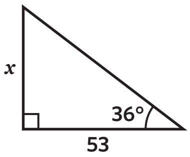
$$\cos\theta = \frac{75}{81}$$

Step 2: Write the trigonometric ratio by substituting the given values and pronumerals representing the side lengths and/or angles of the triangle.

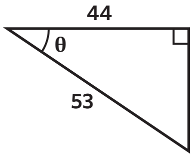
Student practice

For each of the following triangles, write the trigonometric ratio that relates the two given sides and the reference angle.

a.



b.

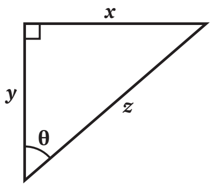


9B Questions

Understanding worksheet

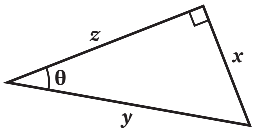
1. Identify the adjacent (A) side using the given reference angle for each triangle.

Example



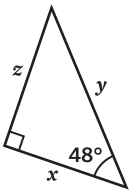
A = y

a.



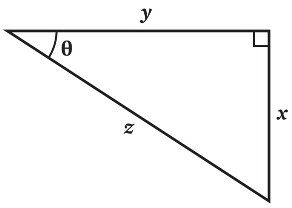
A = ☐

b.



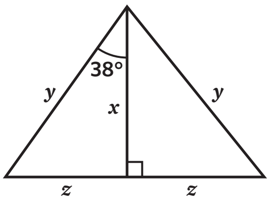
A = ☐

c.



A = ☐

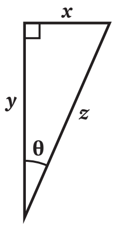
d.



A = ☐

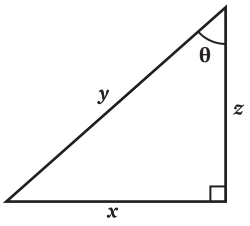
2. Identify the opposite (O) side using the given reference angle for each triangle.

Example



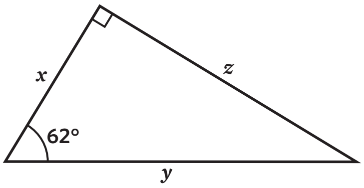
O = x

a.



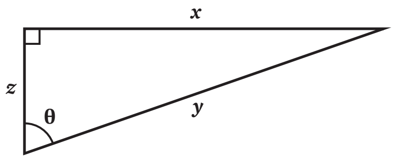
O = ☐

b.



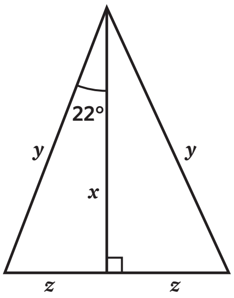
O = ☐

c.



O = ☐

d.



O = ☐

Back to contents

3. Fill in the blanks by using the words provided.

adjacent	tangent	hypotenuse	side	angles

In a right-angled triangle, the side opposite the right angle is called the ______ . The side that forms an arm of the reference angle is the ______ , and the opposite ______ is opposite the reference angle. The three trigonometric ratios: sine, cosine, and ______ , relate side lengths of right-angled triangles to their internal angles. These ratios remain the same for triangles with the same ______ .

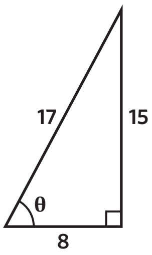
Fluency

Question working paths

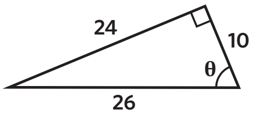
Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (b,c), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (c,d), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. For the triangles shown, write the equations for the sine, cosine and tangent ratios of the reference angle (θ). WE1

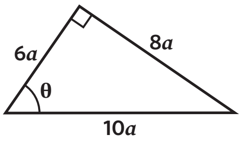
a.



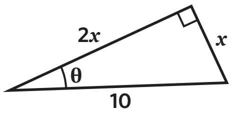
b.



c.

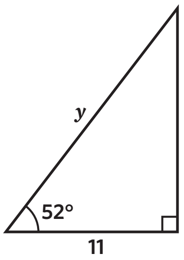


d.

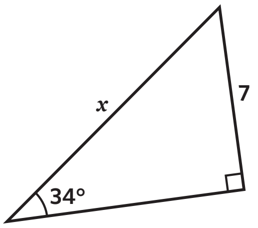


5. For the following triangles, write the trigonometric ratio that relates the two given sides and the reference angle. WE2a

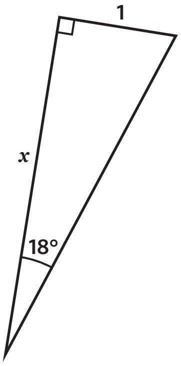
a.



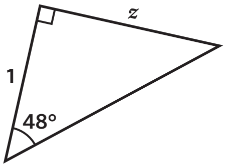
b.



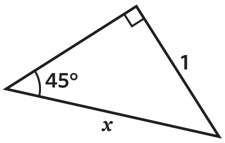
c.



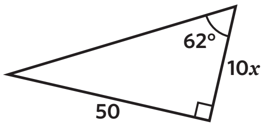
d.



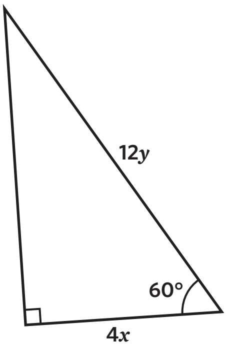
e.



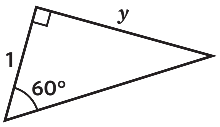
f.



g.



h.

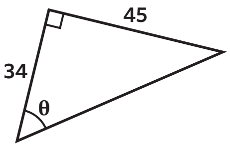


Back to contents

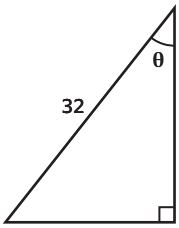
6. For the following triangles, write the trigonometric ratio that relates the two given sides and the reference angle.

WE2b

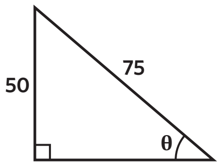
a.



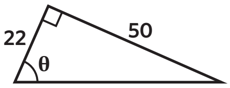
b.



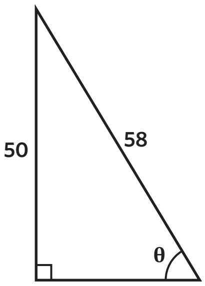
c.



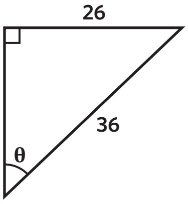
d.



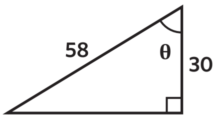
e.



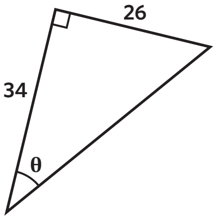
f.



g.



h.



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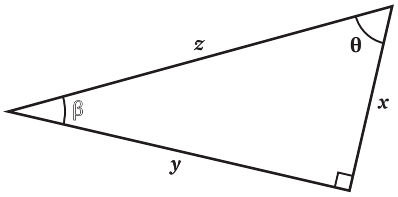
7. The cosine trigonometric ratio for this triangle using θ as the reference angle is:

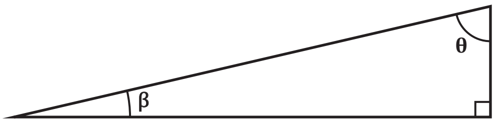


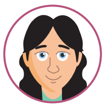
A. $\cos\theta = \frac{x}{y}$ **B.** $\cos\theta = \frac{x}{z}$ **C.** $\cos\theta = \frac{y}{z}$ **D.** $\cos\theta = x$ **E.** $\cos\theta = y$

Spot the mistake

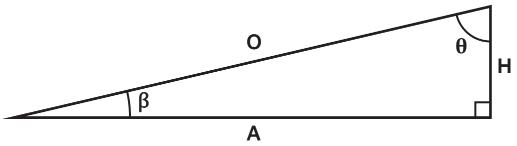
8. Select whether Student A or Student B is incorrect.

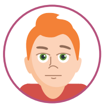
a. Label the triangle's sides using β as the reference angle.



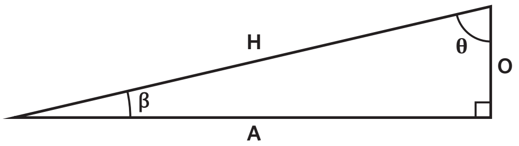


Student A

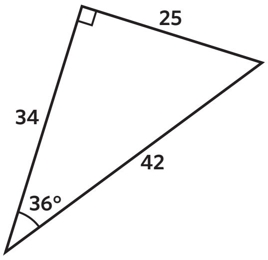


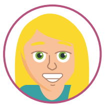


Student B

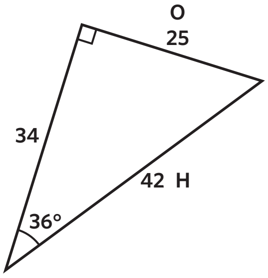


b. Write the sine ratio for the following triangle.



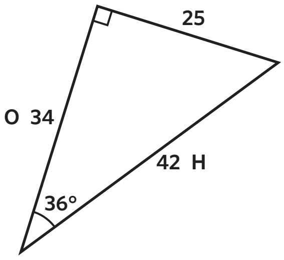


Student A



$$\sin 36° = \frac{25}{42}$$

Student B



$$\sin 36° = \frac{34}{42}$$

Problem solving

Question working paths

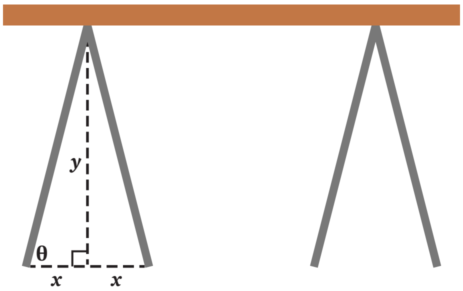
Mild 9, 10, 11 🌶	**Medium** 10, 11, 12 🌶🌶	**Spicy** 11, 12, 13 🌶🌶🌶

9. Council stipulates that the tangent ratio of an access ramp cannot differ from $\frac{1}{14}$. Create a triangle to illustrate the steepest ramp, and label the reference angle and any known side lengths.

10. A clothesline is attached to a wall, making a 90° angle. The clothesline is 2 m high off the ground, protrudes 0.75 m from the wall and has a diagonal strut that attaches to the ground at the base of the wall. Write a trigonometric ratio to represent the situation if the wall is opposite the reference angle.

11. In a triangular race course, participants first run 300 m in a straight line, make a 90° turn to the left, then continue for 400 m before returning directly to the starting point, 500 m away. Illustrate the course, and with the starting point as the reference angle, write the three trigonometric ratios for the formed triangle.

12. A trestle table is held up by A-frames as shown below. Each A-frame has 1 m length arms. Write the cosine ratio using θ as the reference angle.



13. A frame for training a plant up a wall is constructed using a right-angled isosceles triangle, with two side lengths of 3 m. Express the tangent ratio for this specific triangle.

Reasoning

Question working paths

Mild 14 (a,b,d) 🌶	**Medium** 14 (a,b,d), 15 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

14. A city council has commissioned an artist to create a rectangular mural that is 1 m by $\sqrt{3}$ m with a diagonal of 2 m. The internal angles of the two triangles that form the rectangular mural are 30°, 60°, and 90°.

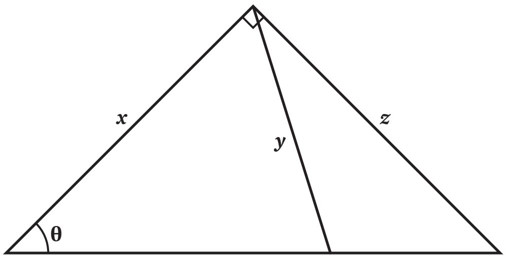
- **a.** Draw the rectangle showing all given measurements.
- **b.** Write the three trigonometric ratios for 60°.
- **c.** Another part of the mural uses a square with side lengths of 2 m. Use your knowledge of trigonometric ratios to write the tangent ratio of 45°.
- **d.** Provide a location where city councils might display community art projects.

15. Using knowledge of trigonometric ratios:

- **a.** Draw a triangle that represents $\tan\theta = \frac{1}{2}$.
- **b.** Draw a triangle that represents $\cos\theta = \frac{1}{2}$.
- **c.** With reference to parts **a** and **b**, and a diagram, explain whether it's possible to create a triangle representing $\cos\theta = \frac{2}{1}$.

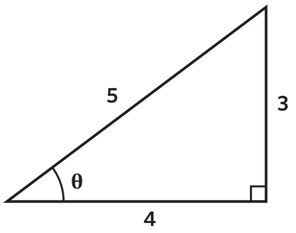
Exam-style

16. The tangent trigonometric ratio using θ as the reference angle is: (1 MARK)

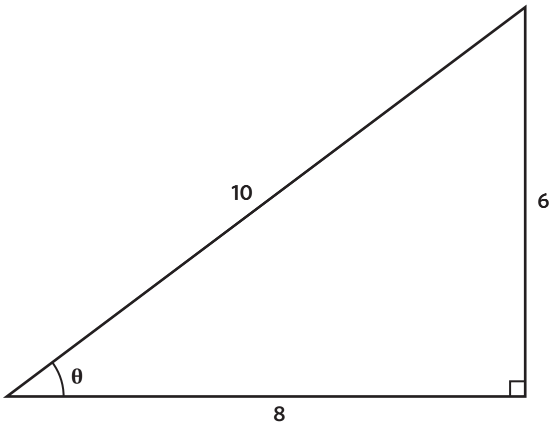


A. $\tan\theta = \frac{x}{y}$ **B.** $\tan\theta = \frac{x}{z}$ **C.** $\tan\theta = \frac{z}{y}$ **D.** $\tan\theta = \frac{z}{x}$ **E.** $\tan\theta = \frac{y}{z}$

17. These are two similar triangles. (3 MARKS)



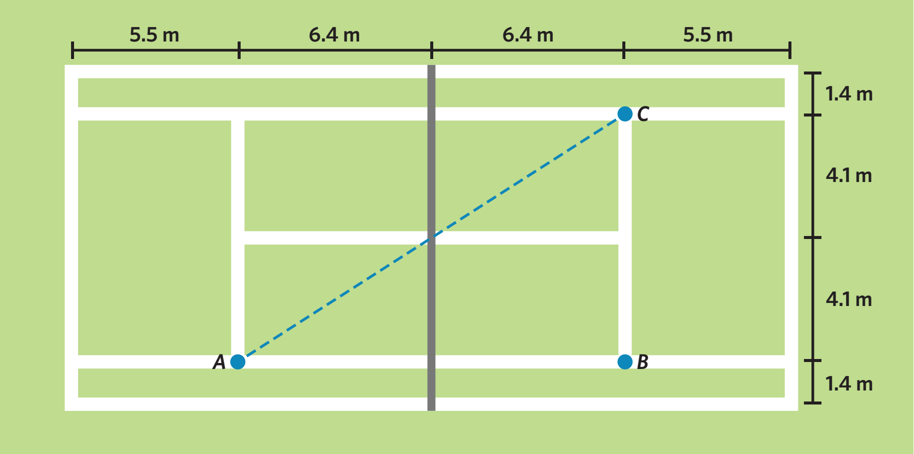
Triangle A



Triangle B

- **a.** Write the ratio cosθ for triangle A. 1 MARK
- **b.** Write the ratio cosθ for triangle B. 1 MARK
- **c.** Using your answers from parts **a** and **b**, write a statement about the cosine ratio for similar triangles. 1 MARK

18. Alex (A) hits the ball past Cate (C) on a tennis court with the following dimensions. Assuming the court lines form right angles, write the three trigonometric ratio using $\angle CAB$ as the reference angle. (3 MARKS)



19. Max is flying a kite with Sal. The kite is flying directly above Sal's head while attached to a string 8.5 m long. Sal is positioned 3 m away from Max. The string forms an angle (θ) with the horizontal. Create a diagram to depict this scenario and express the trigonometric ratio using θ as the reference angle along with the provided distances. (3 MARKS)

Remember this?

20. This table below compares the approximate population of two countries.

Country	**Approximate population (millions)**
Portugal	10.196
Greece	10.423

Approximately how many more people live in Greece than in Portugal?

A. 22 700 **B.** 227 000 **C.** 2 270 000 **D.** 22 700 000 **E.** 227 000 000

21. Two fair six-sided dice are rolled at the same time. What is the probability both dice will show a 3?

A. $\frac{1}{36}$ **B.** $\frac{1}{18}$ **C.** $\frac{1}{12}$ **D.** $\frac{1}{6}$ **E.** $\frac{2}{3}$

22. John has a breakfast consisting of a bowl of cereal and a glass of orange juice.

- The bowl of cereal contains $\frac{3}{8}$ of John's recommended daily vitamin C intake.
- The glass of orange juice contains $\frac{1}{3}$ of his recommended daily vitamin C intake.

In total, what percentage of his recommended daily vitamin C intake does John consume at breakfast?

A. 68.27% **B.** 69.58% **C.** 70.83% **D.** 71.32% **E.** 72.15%

Back to contents

9C Calculating unknown side lengths

LEARNING INTENTIONS

Students will be able to:

- evaluate a trigonometric value on a calculator
- set up and solve trigonometric equations to calculate an unknown side length in the numerator
- set up and solve trigonometric equations to calculate an unknown side length in the denominator.

Trigonometric ratios relate the angles of a triangle to the lengths of its sides. Evaluating a trigonometric value can provide precise measures related to these angles. When an unknown side length is in the numerator, multiplication or division can be used to isolate and solve for this value. If the unknown side length is in the denominator, rearranging the trigonometric equation allows for the solution of the unknown.

KEY TERMS AND DEFINITIONS

- The **subject** of a formula is the variable isolated on one side of the equal sign.
- **Substitution** is the process of replacing a variable or an unknown with a given value.

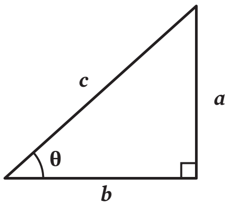
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: N_Sakarin/Shutterstock.com

In the realm of photography, trigonometry assists in setting up the right angles for capturing tall structures, like skyscrapers or monuments. When a photographer knows the height of the structure and the distance they are from its base, they can use trigonometry to ensure they position the camera at the correct angle to capture the entire structure in their frame.

Key ideas

1. Trigonometric equations with the unknown side lengths in the numerator can be transposed using the inverse operation multiplication.

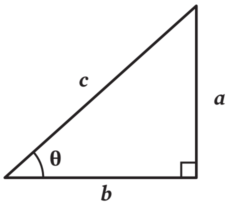


$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

$a = c \times \sin\theta$ $b = c \times \cos\theta$ $a = b \times \tan\theta$

2. Rearranging equations, including those that are trigonometric, to make a pronumeral in the denominator the subject involves applying multiple inverse operations.



$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

$c = \frac{a}{\sin\theta}$ $c = \frac{b}{\cos\theta}$ $b = \frac{a}{\tan\theta}$

Worked example 1

Evaluating trigonometric ratios using a calculator

Evaluate each of the following, correct to four decimal places.

a. sin35°

WE1a

Working	Thinking
$\sin 35° = 0.5735764...$	**Step 1:** Using a calculator with trigonometric functions and ensuring the calculator is in degree mode, input the required calculation.
≈ 0.5736	**Step 2:** Round to the required number of decimal places.

b. cos40°

WE1b

Working	Thinking
$\cos 40° = 0.7660444...$	**Step 1:** Using a calculator with trigonometric functions and ensuring the calculator is in degree mode, input the required calculation.
≈ 0.7660	**Step 2:** Round to the required number of decimal places.

c. tan23°

WE1c

Working	Thinking
$\tan 23° = 0.4244748...$	**Step 1:** Using a calculator with trigonometric functions and ensuring the calculator is in degree mode, input the required calculation.
≈ 0.4245	**Step 2:** Round to the required number of decimal places.

Student practice

Evaluate each of the following, correct to four decimal places.

a. sin15° **b.** cos23° **c.** tan12°

Worked example 2

Solving for x involving trigonometric ratios

Calculate the value of x in each equation, correct to two decimal places.

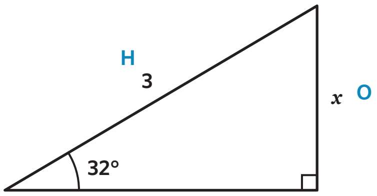
a. $\sin 32° = \frac{x}{3}$

WE2a

Working	Thinking
$\sin 32° = \frac{x}{3}$ $\sin 32° \times 3 = \frac{x}{3} \times 3$ $3\sin 32° = x$ $x = 3\sin 32°$	**Step 1:** Transpose the equation to make x the subject.
$x = 1.58975...$ ≈ 1.59	**Step 2:** Evaluate using a calculator, rounding to the required number of decimal places.

Continues →

Visual support



b. $\cos 36° = \frac{4}{x}$

WE2b

Working

$$\cos 36° = \frac{4}{x}$$

$$\cos 36° \times x = \frac{4}{x} \times x$$

$$x\cos 36° = 4$$

$$\frac{x\cos 36°}{\cos 36°} = \frac{4}{\cos 36°}$$

$$x = \frac{4}{\cos 36°}$$

$$x = 4.94427...$$

$$\approx 4.94$$

Thinking

Step 1: Transpose the equation to make x the subject.

Step 2: Evaluate using a calculator, rounding to the required number of decimal places.

Student practice

Calculate the value of x in each equation, correct to two decimal places.

a. $\sin 15° = \frac{x}{2}$

b. $\cos 24° = \frac{3}{x}$

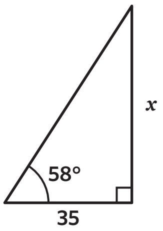
Worked example 3

Calculating side lengths

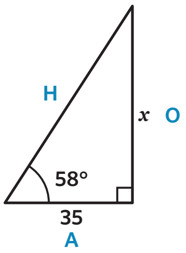
For each triangle, calculate the value of x correct to two decimal places.

a.

WE3a



Working



Thinking

Step 1: Label the sides of the right-angled triangle in relation to the reference angle.

Continues →

$\tan\theta = \frac{O}{A}$

$\tan 58° = \frac{x}{35}$

Step 2: Identify the trigonometric ratio and substitute the relevant values into the equation.

$x = 35\tan 58°$

Step 3: Transpose the equation to make x the subject.

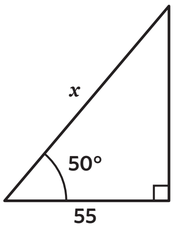
$x = 56.01170...$

≈ 56.01

Step 4: Evaluate using a calculator, rounding to the required number of decimal places.

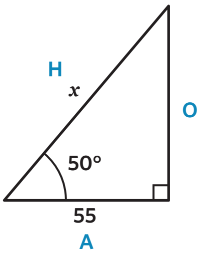
b.

WE3b



Working

Thinking



Step 1: Label the sides of the right-angled triangle in relation to the reference angle.

$\cos\theta = \frac{A}{H}$

$\cos 50° = \frac{55}{x}$

Step 2: Identify the trigonometric ratio and substitute the relevant values into the equation.

$x = \frac{55}{\cos 50°}$

Step 3: Transpose the equation to make x the subject.

$x = 85.56481\ldots$

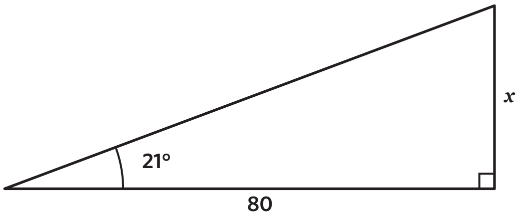
≈ 85.56

Step 4: Evaluate using a calculator, rounding to the required number of decimal places.

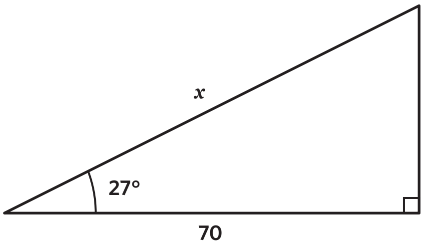
Student practice

For each triangle, calculate the value of x correct to two decimal places.

a.



b.



Back to contents

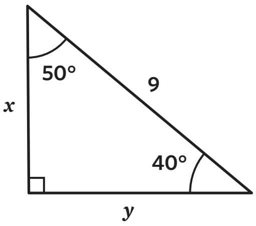
9C Questions

Understanding worksheet

1. Match the equation to the one that has been transposed to make x the subject.

Equation			**Transposed equation**
$a = bx$	●	●	$x = ab$
$a = x + b$	●	●	$x = a - b$
$a = \frac{b}{x}$	●	●	$x = \frac{a}{b}$
$a = \frac{x}{b}$	●	●	$x = \frac{b}{a}$

2. Using the following triangle, fill in the equations to find the required side length.



Example

$\sin 50° = \frac{y}{\boxed{9}}$

a. $\cos 40° = \frac{\boxed{\quad}}{9}$ **b.** $\tan 40° = \frac{x}{\boxed{\quad}}$ **c.** $\tan 50° = \frac{y}{\boxed{\quad}}$ **d.** $\sin 40° = \frac{x}{\boxed{\quad}}$

3. Fill in the blanks by using the words provided.

reference **trigonometric** **subject** **denominator** **right**

When working with $\boxed{\qquad}$ -angled triangles, it is often necessary to evaluate a $\boxed{\qquad}$ value using a calculator. By labelling the sides of the triangle using a $\boxed{\qquad}$ angle, it is then possible to set up and solve trigonometric equations to find an unknown side length in the numerator or $\boxed{\qquad}$ of the fraction. This involves rearranging equations to make the pronumeral the $\boxed{\qquad}$.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d,e), 8 (b,c,d,e), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (c,d,e,f), 8 (c,d,e,f), 9

4. Evaluate each of the following, correct to four decimal places. WE1

a. $\sin 40°$ **b.** $\cos 9°$ **c.** $\tan 41°$ **d.** $\sin 10.5°$

e. $\cos 32°$ **f.** $\tan 70°$ **g.** $\sin 25.7°$ **h.** $\cos 13.4°$

5. Calculate the value of x in each equation, correct to two decimal places. WE2a

a. $\sin 10° = \frac{x}{5}$ **b.** $\cos 70° = \frac{x}{13}$ **c.** $\tan 15° = \frac{x}{20}$ **d.** $\sin 32.4° = \frac{x}{56}$

e. $\cos 22.5° = \frac{x}{0.3}$ **f.** $\tan 30.4° = \frac{x}{100}$ **g.** $\sin 72° = \frac{x}{8}$ **h.** $\cos 9° = \frac{x}{1}$

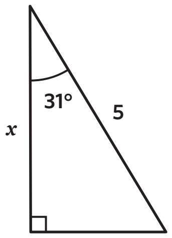
6. Calculate the value of x in each equation, correct to two decimal places. WE2b

a. $\cos 21° = \frac{2}{x}$ **b.** $\tan 45° = \frac{10}{x}$ **c.** $\sin 67° = \frac{5}{x}$ **d.** $\sin 70.5° = \frac{28}{x}$

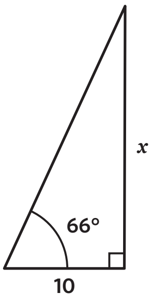
e. $\tan 51° = \frac{1.5}{x}$ **f.** $\cos 28.1° = \frac{0.9}{x}$ **g.** $\sin 4.6° = \frac{7}{x}$ **h.** $\tan 80° = \frac{3}{x}$

7. For each triangle, calculate the value of x correct to two decimal places. WE3a

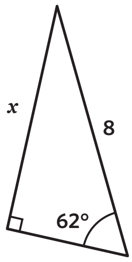
a.



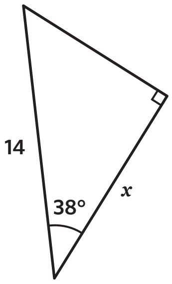
b.



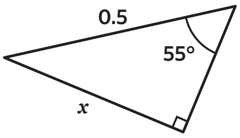
c.



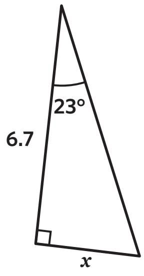
d.



e.

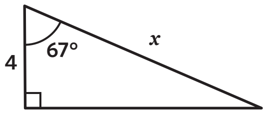


f.

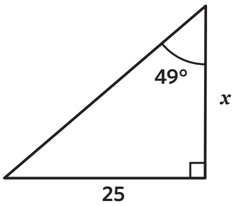


8. For each triangle, calculate the value of x correct to two decimal places. WE3b

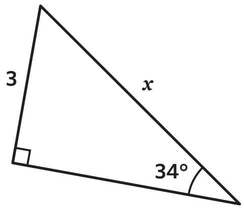
a.



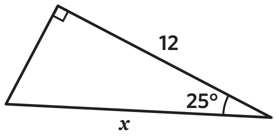
b.



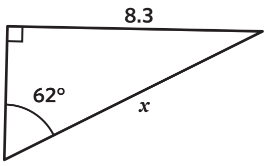
c.



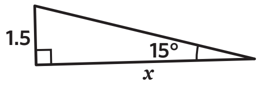
d.



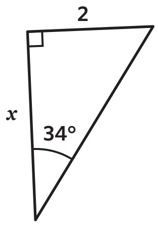
e.



f.



9. Which trigonometric ratio matches the information given in the following triangle?



A. $\cos 34° = \frac{2}{x}$ **B.** $\sin 34° = \frac{x}{2}$ **C.** $\tan 34° = \frac{2}{x}$ **D.** $\tan 34° = \frac{x}{2}$ **E.** $\tan 34° = 2x$

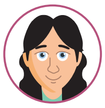
Back to contents

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Calculate the value of x in the equation, correct to two decimal places.

$\cos18° = \frac{6}{x}$



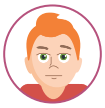
Student A

$\cos18° = \frac{6}{x}$

$x = 6\cos18°$

$x = 5.70633...$

≈ 5.71



Student B

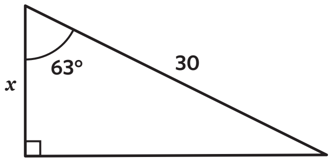
$\cos18° = \frac{6}{x}$

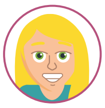
$x = \frac{6}{\cos18°}$

$x = 6.30877...$

≈ 6.31

b. Calculate the value of x, correct to two decimal places.





Student A

$\cos\theta = \frac{A}{H}$

$\cos63° = \frac{x}{30}$

$x = 30\cos63°$

$x = 13.61971...$

≈ 13.62

Student B

$\sin\theta = \frac{O}{H}$

$\sin63° = \frac{x}{30}$

$x = 30\sin63°$

$x = 26.73019...$

≈ 26.73

Problem solving

Question working paths

Mild 11, 12, 13	**Medium** 12, 13, 14	**Spicy** 13, 14, 15

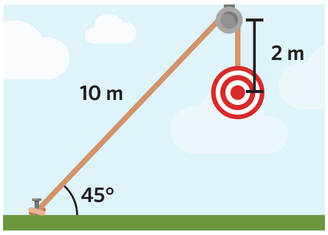
11. A person fishing off the beach uses a rod-stand that holds the rod at an angle of 65° with the ground. If the rod is 2 m long, what is the vertical height of the tip of the rod? Round to two decimal places.

12. A rectangle of origami paper is folded on the diagonal, then opened back out forming two triangles. The longest side of the rectangle paper is 10 cm and makes an angle of 40° with the diagonal. Calculate the length of the diagonal to the nearest centimetre.

13. On a construction site, a ramp is assembled to provide wheelbarrows access to an area normally accessed via stairs only. The ramp, made from a spare 4.2 m long plank of wood, forms a 30° angle with the ground. Calculate the exact vertical height of the stairs.

14. A sound engineer is determining the optimal speaker placement for even sound distribution. The speaker will be wall-mounted, angled downward, on a bracket that makes a 70° angle with the wall, and should deliver the best sound quality on the ground at a horizontal distance of 10 m from the wall. How high up the wall should the speaker be positioned? Round to one decimal place.

15. In an archery competition, an elevated target is suspended above the ground via a pulley system. The angle made with the rope and the ground is 45° and the distance from the anchor point on the ground to the top of the pulley is 10 m. Calculate the height of the bullseye in metres if it is hanging 2 m from the pulley, correct to two decimal places.

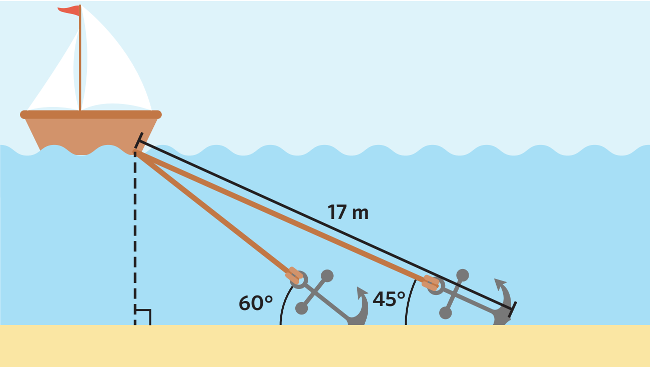


Reasoning

Question working paths

Mild 16 (a,b,d)	**Medium** 16 (a,b,d), 17 (a,b)	**Spicy** All

16. A boat is anchored offshore using two anchors, as shown in the diagram. Use the given information to answer the questions, correct to two decimal places.



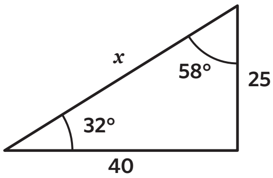
a. What is the depth of the ocean at the boat's anchor point?

b. What length of chain has been released for the second anchor?

c. How far apart are the two anchors?

d. What might be the benefit of using multiple anchors when stationary?

17. Consider the right-angled triangle shown.



a. Calculate x using cosine, rounding to one decimal place.

b. Calculate x using sine, rounding to one decimal place.

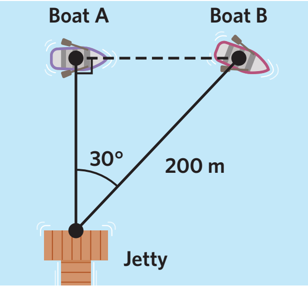
c. Using your answers from part **a** and **b** write a statement explaining multiple methods to find a side length.

Exam-style

18. Select the value equivalent to sin20°. (1 MARK)

A. sin70° **B.** cos20° **C.** tan20° **D.** cos70° **E.** tan70°

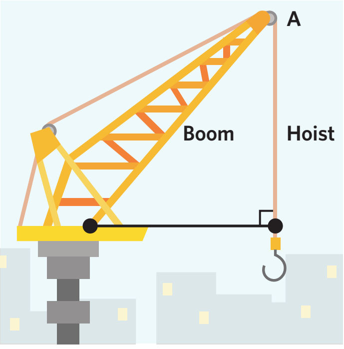
19. Two fishing boats can be seen from a jetty. (2 MARKS)



a. How far apart are the two boats? 1 MARK

b. What is Boat A's distance from the jetty, rounded to two decimal places? 1 MARK

20. A crane has a boom that is 9 m long. With a load attached, the hoist makes an angle with the boom of 40°. The load is lifted higher and the boom makes a 20° angle with the hoist. How high has the load been lifted in metres, rounded to two decimal places? (3 MARKS)



21. Using the right-angled triangle $\triangle ABC$, where: (3 MARKS)

- AB $=$ 2 and the longest side
- AC $=$ 1 and is the shortest side and
- $\angle B = 30°$

state the exact values of sin30° and cos60°.

Remember this?

22. This list shows the weights of ten packages, in kilograms.
45, 52, 60, 40, 55, 50, 48, 53, 46, 58

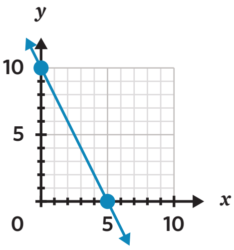
What is the range of the weights?

A. 18 kg **B.** 20 kg **C.** 22 kg **D.** 24 kg **E.** 26 kg

23. One-fourth of Tom's books is equal to one-fifth of Jerry's books. Together they have 45 books. How many books does Jerry have?

A. 20 **B.** 25 **C.** 30 **D.** 35 **E.** 40

24.



Which of these equations represents the line in the graph?

A. $y = 10 - 2x$ **B.** $y = 10 + 5x$ **C.** $y = 10 - 5x$ **D.** $y = 10 + 2x$ **E.** $y = 10 - x$

Back to contents

9D Calculating unknown angles

LEARNING INTENTIONS

Students will be able to:

- use inverse trigonometric ratios to calculate angles in right-angled triangles
- use a calculator to calculate a value using an inverse trigonometric ratio
- calculate the value of an angle in a right-angled triangle given two side lengths.

Inverse trigonometric ratios provide a method to determine angles in right-angled triangles using known side lengths. Using a calculator to evaluate a value with an inverse trigonometric ratio ensures accurate angle measurement. To calculate the value of an angle in a right-angled triangle, it's essential to first identify the two relevant side lengths, then apply the appropriate inverse ratio. This process ensures a comprehensive understanding of the relationship between side lengths and angles in right-angled triangles.

KEY TERMS AND DEFINITIONS

- **Inverse operations** are mathematical operations that are the reverse of each other. The inverse of multiplication is division and the inverse of division is multiplication; the inverse of addition is subtraction and the inverse of subtraction is addition.

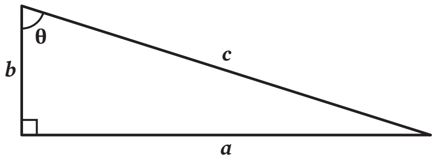
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Terelyuk/Shutterstock.com

When installing solar panels, technicians must angle them optimally towards the sun to maximise efficiency. Knowing the height of a nearby obstacle, like a tree or building, and its distance from the panel, technicians can use inverse trigonometric ratios to adjust the panels ensuring minimal shade during the day.

Key idea

1. Inverse trigonometric ratios can be used to determine angles in right-angled triangles when given two side lengths.



$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{H}$

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

$\theta = \sin^{-1}\left(\frac{a}{c}\right)$ $\theta = \cos^{-1}\left(\frac{b}{c}\right)$ $\theta = \tan^{-1}\left(\frac{a}{b}\right)$

Worked example 1

Determining angles using inverse trigonometric ratios

Calculate the value of θ in each equation, correct to 2 decimal places.

a. $\sin\theta = 0.1$

WE1a

Working	Thinking
$\sin\theta = 0.1$ $\theta = \sin^{-1}(0.1)$	**Step 1:** Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.
$\theta = 5.739...$ $\approx 5.74°$	**Step 2:** Evaluate using a calculator, rounding to the required number of decimal places.

b. $\tan\theta = \frac{12}{5}$

WE1b

Working	Thinking
$\tan\theta = \frac{12}{5}$ $\theta = \tan^{-1}\left(\frac{12}{5}\right)$	**Step 1:** Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.
$\theta = 67.380...$ $\approx 67.38°$	**Step 2:** Evaluate using a calculator, rounding to the required number of decimal places.

Student practice

Calculate the value of θ in each equation, correct to 2 decimal places.

a. $\sin\theta = 0.6$ **b.** $\tan\theta = \frac{14}{9}$

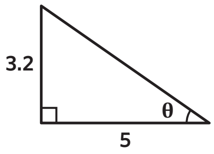
Worked example 2

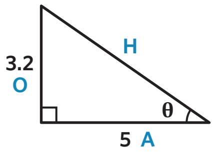
Determining an unknown angle

For each triangle, calculate the value of θ to the nearest degree.

a.

WE2a



Working	Thinking
	**Step 1:** Label the sides of the right-angled triangle using the reference angle.
$\tan\theta = \frac{O}{A}$ $\tan\theta = \frac{3.2}{5}$	**Step 2:** Identify the relevant trigonometric ratio and substitute the values into the equation.

Continues →

Back to contents

$$\theta = \tan^{-1}\left(\frac{3.2}{5}\right)$$

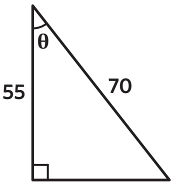
Step 3: Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

$$\theta = 32.619...$$
$$\approx 33°$$

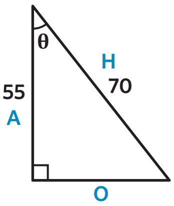
Step 4: Evaluate using a calculator, rounding to the required number of decimal places.

b.

WE2b



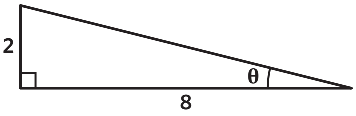
Working	Thinking
	Step 1: Label the sides of the right-angled triangle using the reference angle.
$\cos\theta = \frac{A}{H}$ $\cos\theta = \frac{55}{70}$	**Step 2:** Identify the relevant trigonometric ratio and substitute the values into the equation.
$\theta = \cos^{-1}\left(\frac{55}{70}\right)$	**Step 3:** Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.
$\theta = 38.213...$ $\approx 38°$	**Step 4:** Evaluate using a calculator, rounding to the required number of decimal places.



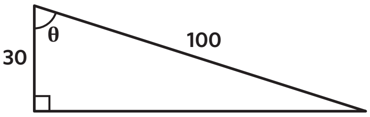
Student practice

For each triangle, calculate the value of θ to the nearest degree.

a.



b.



Back to contents

9D Questions

Understanding worksheet

1. Complete each of the following by filling in the blanks.

Example

$\cos\theta = 0.2$

$\theta = \cos^{-1}\left(\boxed{0.2}\right)$

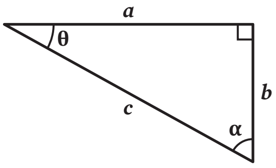
a. $\cos\theta = 0.5$
$\theta = \cos^{-1}\left(\boxed{}\right)$

b. $\tan\theta = \frac{2}{5}$
$\theta = \tan^{-1}\left(\frac{\boxed{}}{5}\right)$

c. $\sin\theta = \boxed{}$
$\theta = \sin^{-1}(0.9)$

d. $\tan\theta = \boxed{}$
$\theta = \tan^{-1}(1.2)$

2. Fill in the equation used to determine the required angle for the following triangle.



Example

$\theta = \cos^{-1}\left(\frac{a}{\boxed{c}}\right)$

a. $\theta = \sin^{-1}\left(\frac{b}{\boxed{}}\right)$

b. $\theta = \tan^{-1}\left(\frac{\boxed{}}{a}\right)$

c. $\alpha = \sin^{-1}\left(\frac{\boxed{}}{c}\right)$

d. $\alpha = \tan^{-1}\left(\frac{a}{\boxed{}}\right)$

3. Fill in the blanks by using the words provided.

angle **trigonometric** **inverse** **lengths**

When working with right-angled triangles, $\boxed{}$ trigonometric ratios can be used to calculate angles. To do this, the $\boxed{}$ ratio required to determine the angle must be identified first. Once this is known, a calculator can be used to evaluate the $\boxed{}$ value. This skill is useful when calculating the value of an angle in a right-angled triangle, given the $\boxed{}$ of two sides.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6	4 (c,d,e,f), 5 (c,d,e,f), 6	4 (e,f,g,h), 5 (e,f,g,h), 6

4. Calculate the value of θ in each equation, correct to 2 decimal places. WE1

a. $\sin\theta = 0.4$

b. $\cos\theta = \frac{4}{13}$

c. $\tan\theta = \frac{1}{20}$

d. $\sin\theta = 0.34$

e. $\cos\theta = \frac{0.3}{1.2}$

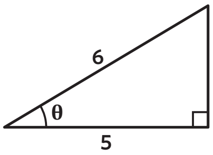
f. $\tan\theta = 2.5$

g. $\cos\theta = 0.5$

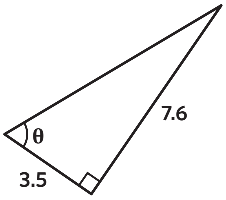
h. $\sin\theta = \frac{7}{10}$

5. For each triangle, calculate the value of θ to the nearest degree. WE2

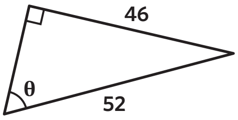
a.



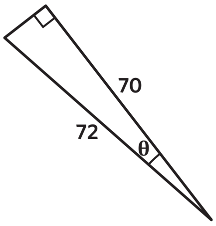
b.



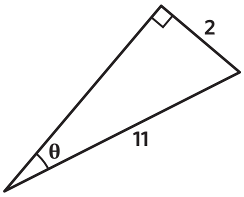
c.



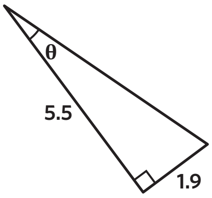
d.



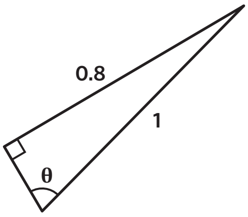
e.



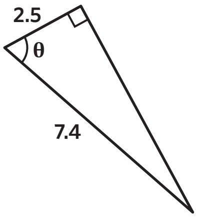
f.



g.



h.



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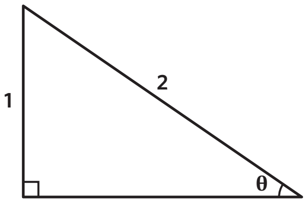
6. Which inverse trigonometric ratio would be used to calculate θ in the following triangle?



A. $\theta = \sin^{-1}(2)$

B. $\theta = \tan\left(\frac{1}{2}\right)$

C. $\theta = \cos^{-1}\left(\frac{1}{2}\right)$

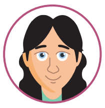
D. $\theta = \sin^{-1}\left(\frac{1}{2}\right)$

E. $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

Spot the mistake

7. Select whether Student A or Student B is incorrect.

a. Evaluate $\sin\theta = 0.5$ to solve for θ, correct to the nearest degree.



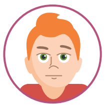
Student A

$\sin\theta = 0.5$

$\theta = \sin^{-1}(0.5)$

$= \frac{1}{\sin 0.5}$

$\approx 115°$



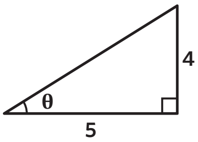
Student B

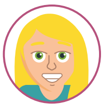
$\sin\theta = 0.5$

$\theta = \sin^{-1}(0.5)$

$= 30°$

b. Calculate the value of θ correct to 2 decimal places.





Student A

$\tan\theta = \frac{\text{O}}{\text{A}}$

$\tan\theta = \frac{4}{5}$

$\theta = \tan^{-1}\left(\frac{4}{5}\right)$

$\theta = 38.65980...$

$\approx 39.66°$

Student B

$\tan\theta = \frac{\text{O}}{\text{A}}$

$\tan\theta = \frac{4}{5}$

$\theta = \tan\left(\frac{4}{5}\right)$

$\theta = 0.01398...$

$\approx 0.01°$

Back to contents

Problem solving

Question working paths

Mild 8, 9, 10 🌶	**Medium** 9, 10, 11 🌶🌶	**Spicy** 10, 11, 12 🌶🌶🌶

8. In a playground, there is a slide attached to a platform that is 1.5 m above the ground. The slide itself is 3 m long. What angle does the slide make with the ground?

9. A sound engineer is looking at where to place a speaker for optimal sound distribution. They know the speaker will be placed on a wall bracket at a height of 2.5 m and angled down so that the best sound will be heard 3 m from the base of the wall. Calculate the angle the bracket should make with the wall, correct to the nearest degree.

10. In a theatre production, a spotlight is mounted on a scaffold 6 m above the stage floor. The spotlight can be rotated to focus on different areas of the stage. What angle does the spotlight's beam make with the stage floor when it is aimed at a point 8 m away from the base of the scaffold, correct to 1 decimal place?

11. A seesaw, supported by a stand 40 cm tall, has a total length of 6 m. When one side is on the ground, determine the angle formed between the seesaw and the ground, correct to 2 decimal places.

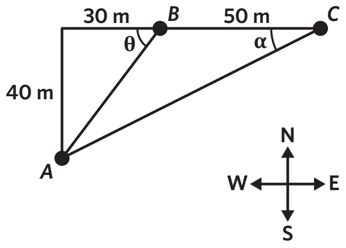
12. In a science experiment, a laser beam on a table is directed at a prism placed 2 m away. The prism is then raised 1 m above the lab table. Determine the angle the laser beam makes with the lab table when it enters the prism, correct to 2 decimal places.

Reasoning

Question working paths

Mild 13 (a,b,d) 🌶	**Medium** 13 (a,b,d), 14 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

13. In a navigation class, students are learning to use a compass. A student stands at point A and walks 40 m north. From there, they walk 30 m east to point B. They then continue east another 50 m to point C.



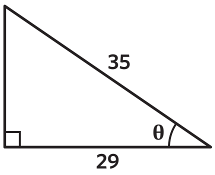
- **a.** Determine the angle from point B to point A (θ), correct to 2 decimal places.
- **b.** Determine the angle from point C to point A (α), correct to 2 decimal places.
- **c.** If another point D is placed 10 m directly south of C, determine the angle from point D to point A, correct to 2 decimal places.
- **d.** In what situation might it be useful to have an understanding of how to use a compass?

14. Use trigonometric ratios to answer the following.

- **a.** Calculate the angles in a right-angled isosceles triangle with two side lengths of 1.
- **b.** Write expressions for the hypotenuse of the triangle in part **a**, using sine and cosine.
- **c.** Using your answers from part **b**, write a general statement to explain why $\sin 45° = \cos 45°$.

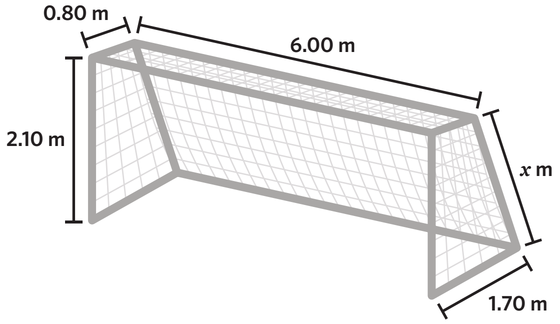
Exam-style

15. Select the value closest to θ in the given triangle.



A. 20° **B.** 34° **C.** 39° **D.** 50° **E.** 56°

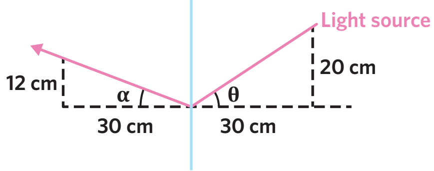
16. The dimensions of a soccer goal are shown in the diagram. (3 MARKS)



a. Calculate the angle the back of the net makes with the ground correct to the nearest degree. 2 MARKS

b. Use trigonometry to calculate the sloping distance (x) of the net, correct to 2 decimal places. 1 MARK

17. In a science experiment, a beam of light is directed through a glass pane. Inside the pane, the light ray refracts, bending towards a line perpendicular to the pane of glass. The observers measure the distance from the perpendicular line to the light beam at a point 30 cm on each side of the glass as shown in the diagram. (3 MARKS)



Determine the difference in the angle made by the beam of light after it exits the pane of glass compared to the angle it enters the pane of glass, correct to 2 decimal places.

18. A floating shelf forms a 90° angle with the wall and is supported by two different struts, both attached to its underside 30 cm away from the wall. The struts are attached to the wall by brackets and form right-angled triangles. The installation instructions specify the following: (3 MARKS)

Bracket A should be attached to the wall 11 cm below the shelf.

Bracket B should be attached to the wall 25 cm below the shelf.

Calculate the angles that each strut makes with the wall correct to the nearest degree.

Remember this?

19. This table shows the storage capacity of two large data centres.

Name of data centre	Storage capacity (terabytes)
Alpha	500
Beta	2500

What is the ratio of the storage capacity of the Beta data centre to the storage capacity of the Alpha data centre?

A. 1 : 2 **B.** 2 : 1 **C.** 1 : 5 **D.** 5 : 1 **E.** 5 : 2

20. Milk costs 125.5 cents per litre. How much does 8 L of milk cost?

A. 804 cents **B.** 904 cents **C.** 1004 cents **D.** 1104 cents **E.** 1204 cents

21. There are 72 people who chose to participate in either Basketball, Soccer, or Volleyball.

- Eight more people chose Basketball than Soccer.
- Twice as many people chose Volleyball as Soccer.

How many people chose to participate in Basketball?

A. 16 **B.** 24 **C.** 28 **D.** 32 **E.** 36

Back to contents

9E Bearings

LEARNING INTENTIONS

Students will be able to:

- understand how to measure and write true bearings and compass bearings
- state the true bearing from one point to another
- identify that opposite directions differ by 180°
- use trigonometry to solve problems involving bearings.

Bearings are descriptive angles, often used together with maps to show the relative positions of two points. Trigonometry can be applied to work out distances and lengths, based on the given directions about travel from one point to another.

KEY TERMS AND DEFINITIONS

- An **acute angle** is greater than 0° but less than 90° in size.
- A **due** bearing refers to an exact direction, such as directly north, east, west, or south.

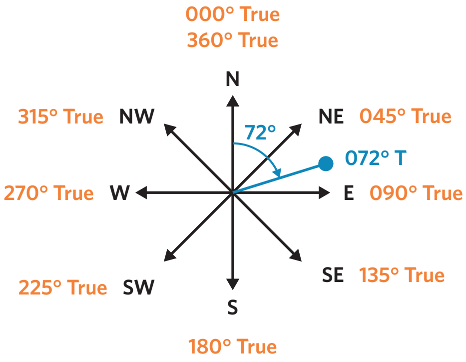
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Aastels/Shutterstock.com

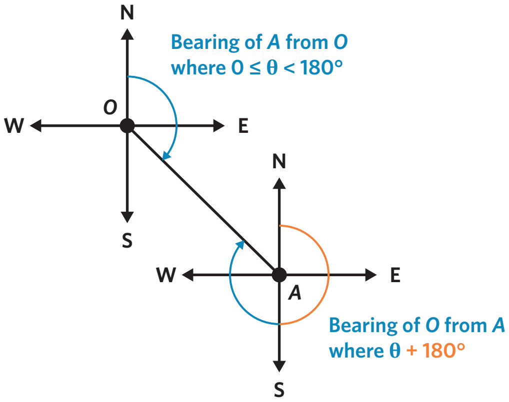
During a long journey across the seas, sailors use true bearings to navigate the waters. It can be difficult to establish exactly where a ship is located when landmarks are too far to use as a visual reference. The consistency of true bearings allows for ships to accurately map and plan their journeys, as well as communicate their exact location to other vessels.

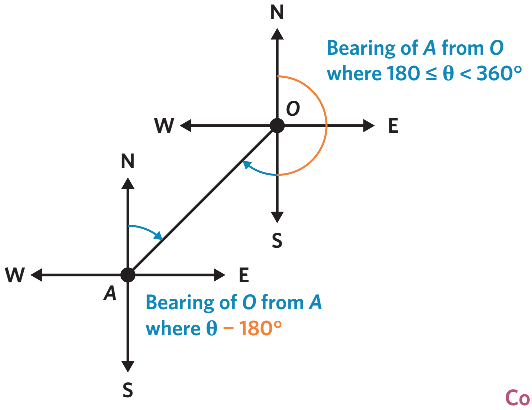
Key ideas

1. **True bearings** are angles that are measured clockwise from north, written using three figures and can also include T at the end.



2. Opposite directions differ by 180°. For a bearing of size θ, the opposite bearing is given by:
   - $\theta + 180°$ if $0° \leq \theta < 180°$
   - $\theta - 180°$ if $180° \leq \theta < 360°$





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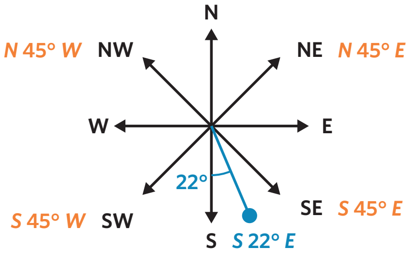
Back to contents

3. **Compass bearings** are acute angles measured from north or south in the direction of east or west. They are written using a starting direction (*N* or *S*), followed by an acute angle of rotation, and the direction of rotation (*E* or *W*).



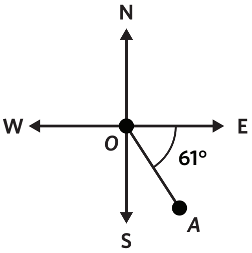
Worked example 1

Writing true bearings

Determine the true bearings of *A* from *O* and *O* from *A* in the given diagrams.

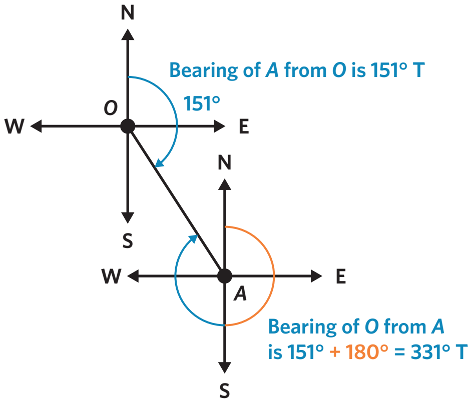
a.

WE1a



Working	Thinking
The angle from north to *A*: $90° + 61° = 151°$ ∴ The bearing of *A* from *O* is 151° T.	**Step 1:** Determine the size of the entire angle measured clockwise from north to point *A*. Write it using three digits and T at the end.
$151° < 180°$	**Step 2:** Identify if the true bearing of *A* from *O* is greater than or less than 180°.
∴ The bearing of *O* from *A*: $151° + 180° = 331°$ T	**Step 3:** If the bearing of *A* from *O* is less than 180°, add 180° to get the opposite direction. If the bearing of *A* from *O* is greater than 180°, subtract 180° to get the opposite direction.

Visual support



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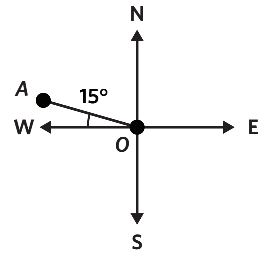
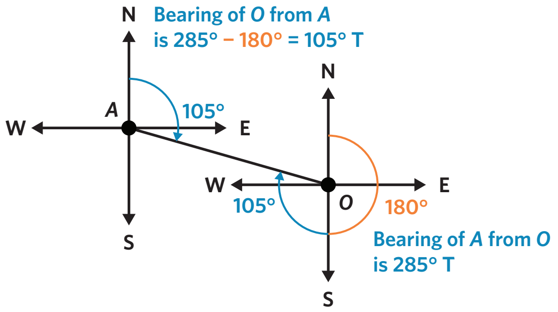
b.

WE1b



Working	Thinking
The angle from north to A: $270° + 15° = 285°$ ∴ The bearing of A from O is 285° T.	**Step 1:** Determine the size of the entire angle measured clockwise from north to point A. Write it using three digits and T at the end.
$285° > 180°$	**Step 2:** Identify if the true bearing of A from O is greater than or less than 180°.
∴ The bearing of O from A: $285° - 180° = 105°$ T	**Step 3:** If the bearing of A from O is less than 180°, add 180° to get the opposite direction. If the bearing of A from O is greater than 180°, subtract 180° to get the opposite direction.

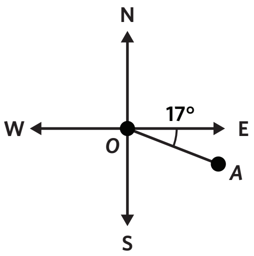
Visual support



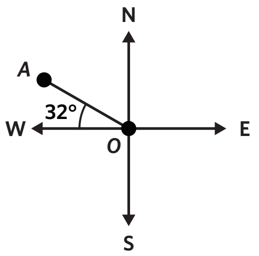
Student practice

Determine the true bearings of A from O and O from A in the given diagrams.

a.



b.



Back to contents

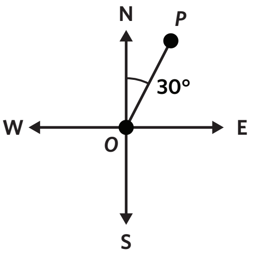
Worked example 2

Writing compass bearings

Determine the compass bearings of P from O in the given diagrams.

a.

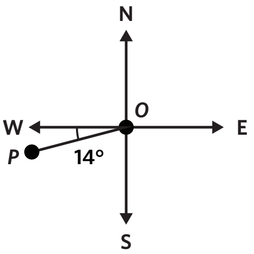
WE2a



Working	Thinking
Acute angle from N to $P = 30°$	**Step 1:** Identify the acute angle of rotation from north (N) to point P.
Direction of rotation is east (E).	**Step 2:** Identify the direction of rotation from north to point P.
$N\ 30°\ E$	**Step 3:** Write the compass bearing using the starting direction, acute angle of rotation, and the direction of rotation.

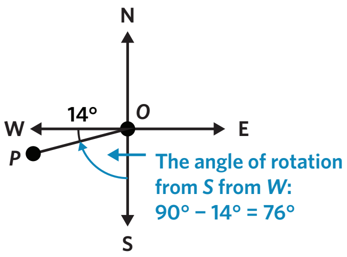
b.

WE2b



Working	Thinking
Acute angle from S to P: $90° - 14° = 76°$	**Step 1:** Calculate the acute angle of rotation from south (S) to point P.
Direction of rotation is west (W).	**Step 2:** Identify the direction of rotation from south to point P.
$S\ 76°\ W$	**Step 3:** Write the compass bearing using the starting direction, acute angle of rotation, and the direction of rotation.

Visual support



The angle of rotation from S from W: $90° - 14° = 76°$

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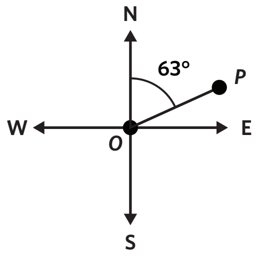
Back to contents

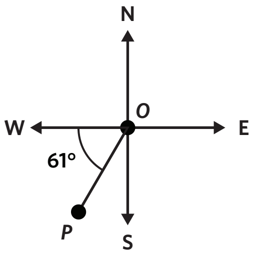
Student practice

Determine the compass bearings of *P* from *O* in the given diagrams.

a.



b.



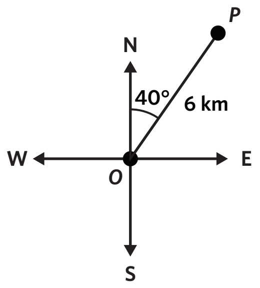
Worked example 3

Using bearings with trigonometry

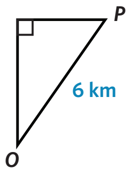
Use the given diagrams to calculate the required distances (km), correct to 2 decimal places.

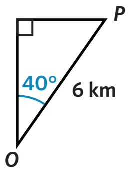
a. Determine how far east point *P* is from point *O*.

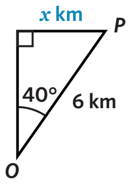
WE3a



Working







$$\sin 40° = \frac{x}{6}$$

$$x = 6\sin 40°$$

$$= 3.856...$$

$$\approx 3.86 \text{ km}$$

Thinking

Step 1: Form a right-angled triangle and label the hypotenuse with the distance between points *O* and *P*.

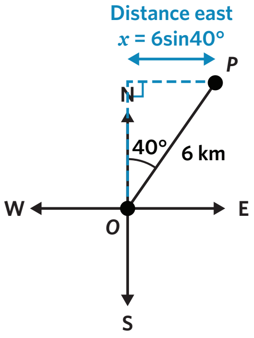
Step 2: Use the given bearing to label a reference angle in the triangle.

Step 3: Label the required side with a pronumeral and solve using trigonometric ratios, rounding to the required number of decimal places.

Continues →

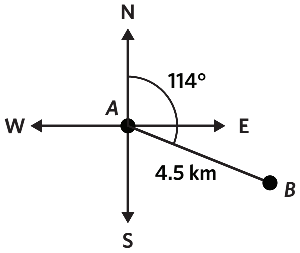
Back to contents

Visual support

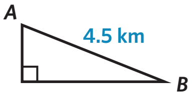


b. Determine how far east point B is from point A.

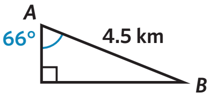
WE3b

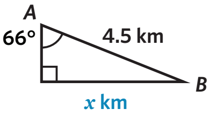


Working



$180° - 114° = 66°$





$$\sin 66° = \frac{x}{4.5}$$

$$x = 4.5\sin 66°$$

$$= 4.110...$$

$$\approx 4.11 \text{ km}$$

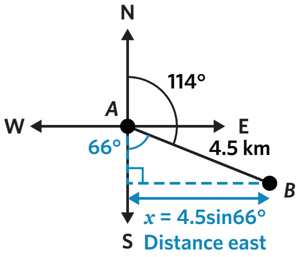
Thinking

Step 1: Form a right-angled triangle and label the hypotenuse with the distance between points A and B.

Step 2: Calculate the internal angle which is supplementary with the given bearing and mark it on the right-angled triangle.

Step 3: Label the required side with a pronumeral and solve using trigonometric ratios, rounding to the required number of decimal places.

Visual support



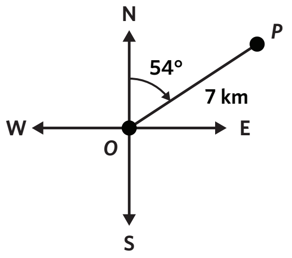
Continues →

Back to contents

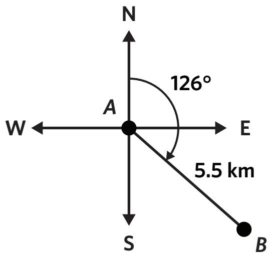
Student practice

Use the given diagrams to calculate the required distances (km), correct to 2 decimal places.

a. Determine how far east point *P* is from point *O*.



b. Determine how far east point *B* is from point *A*.



9E Questions

Understanding worksheet

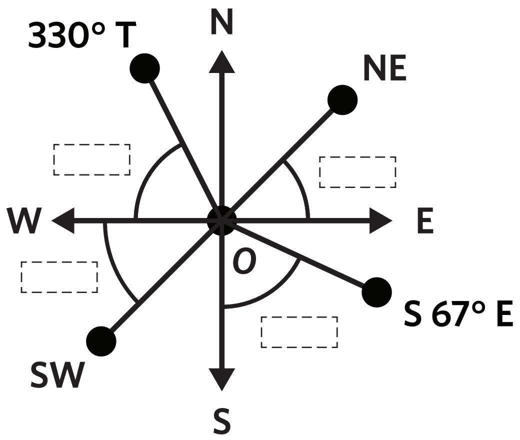
1. Determine the true bearings of opposite directions to the ones given in the table.

Example

A from *O* (θ)	*O* from *A* (θ ± 180°)
030° T	210° T

A from *O* (θ)	*O* from *A* (θ ± 180°)
060° T	° T
090° T	° T
170° T	° T
283° T	° T

2. Fill in the blanks with the sizes of the indicated angles.



3. Fill in the blanks by using the words provided.

clockwise **compass** **internal** **opposite**

True bearings are measured from north in the ______ direction, while ______ bearings are measured from the north or south in the direction of east or west. The ______ direction of a bearing can be obtained by adding or subtracting 180°. Trigonometry can be applied to calculate distances, using bearings to determine an ______ angle of a formed right-angled triangle, together with a given length.

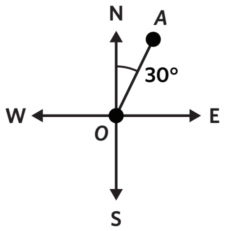
Fluency

Question working paths

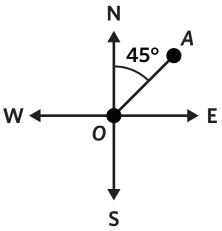
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Determine the true bearings of *A* from *O* and *O* from *A* in the given diagrams. **WE1a**

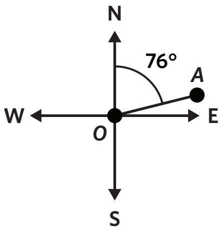
a.



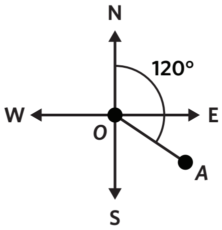
b.



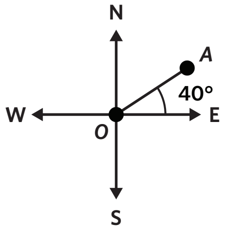
c.



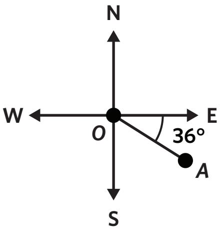
d.



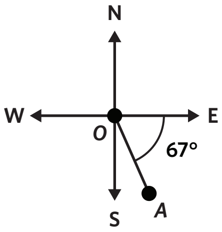
e.



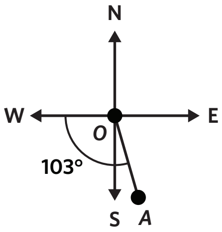
f.



g.

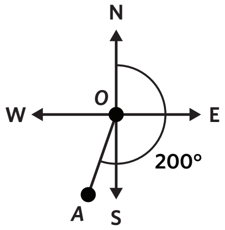


h.

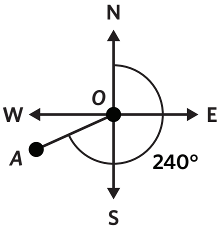


5. Determine the true bearings of *A* from *O* and *O* from *A* in the given diagrams. **WE1b**

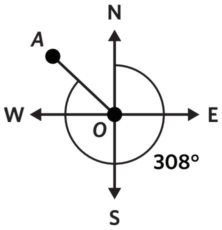
a.



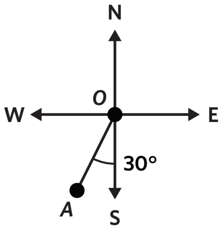
b.



c.



d.



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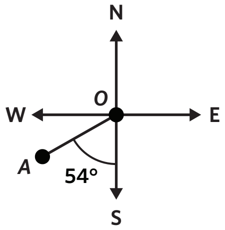

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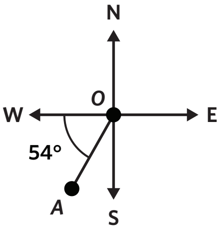

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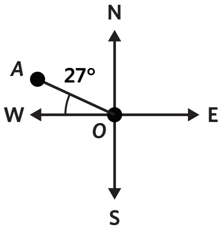

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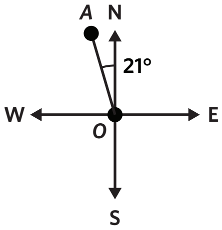
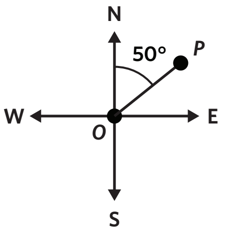


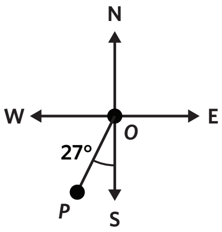
Back to contents

6. Determine the compass bearings of *P* from *O* in the given diagrams. WE2

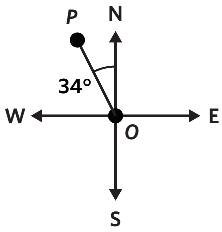
a.



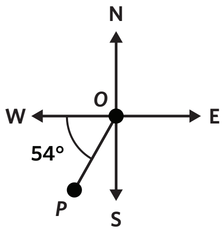
b.



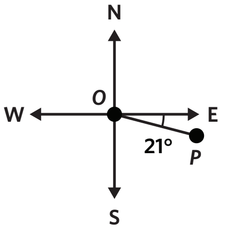
c.



d.



e.



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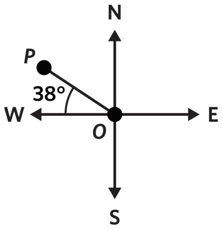
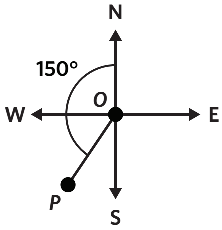
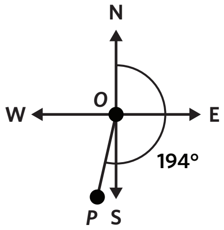


g.

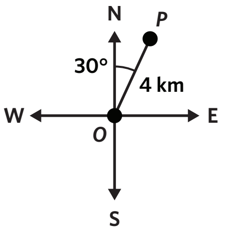


h.

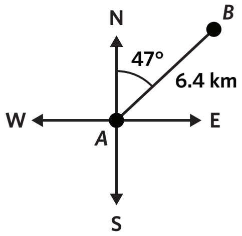


7. Use the given diagrams to calculate the required distances (km), correct to 2 decimal places. WE3

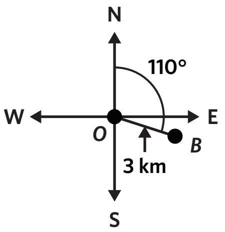
a. Determine how far east point *P* is from point *O*.



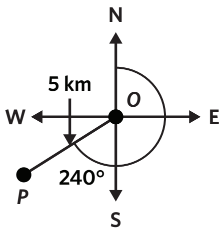
b. Determine how far east point *B* is from point *A*.



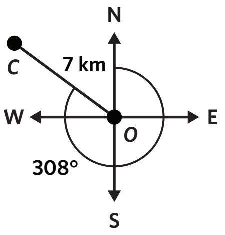
c. Determine how far east point *B* is from point *O*.



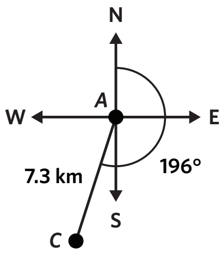
d. Determine how far west point *P* is from point *O*.



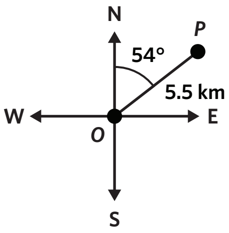
e. Determine how far west point *C* is from point *O*.



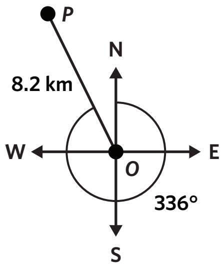
f. Determine how far south point *C* is from point *A*.



g. Determine how far north point *P* is from point *O*.

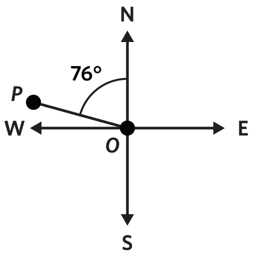


h. Determine how far north point *P* is from *O*.



Back to contents

8. Determine the true bearing of P from O in the given diagram.

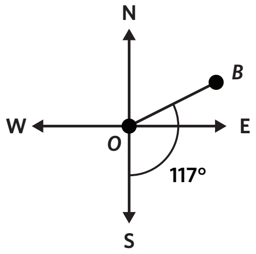


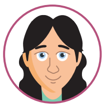
A. 014° T **B.** 076° T **C.** 104° T **D.** 284° T **E.** 436° T

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the true bearing of O from B.



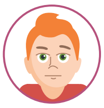


Student A

The angle from north to B:

180° − 117° = 63°

∴ The bearing of O from B is 063° T



Student B

The angle from north to B:

180° − 117° = 63°

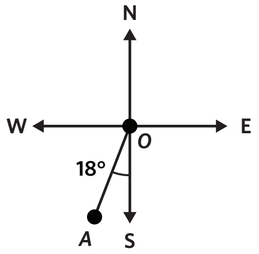
∴ The bearing of B from O is 063° T.

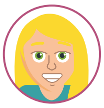
63° < 180°

63° + 180° = 243°

∴ The bearing of O from B is 243° T.

b. Determine the true bearing of A from O.





Student A

The angle from north to A:

180° − 18° = 162°

∴ The bearing of A from O is 162° T.

Student B

The angle from north to A:

180° + 18° = 198°

∴ The bearing of A from O is 198° T.

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. A ranger walks on a bearing of 058° T from a forest hut to his car. Determine the true bearing of the direction the ranger should take in order to return back to the hut from the car.

11. The bearing of a ship at sea from its docking point on the shore is 290° T. Determine the true bearing of the direction the ship must sail in order to reach the dock from its position at sea at the time of observation.

12. A hiker walks 5 km on a bearing of $N\ 55°\ E$ from point O to point A. Determine how far east point A is from point O, correct to 1 decimal place.

Back to contents

13. A plane flies on a bearing of 189° T for 450 km. Determine how far south, to the nearest kilometre, the plane flew from its starting point.

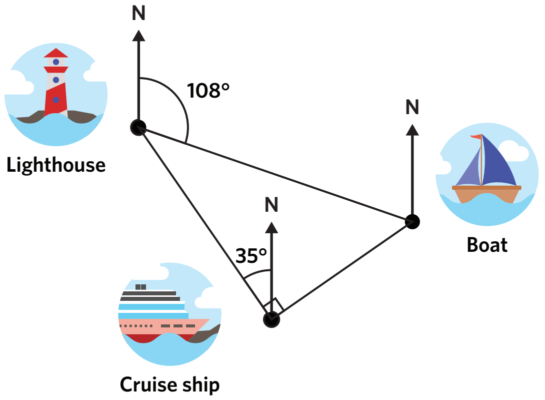
14. Rowan leaves his house and walks due west, then turns and walks due north until he gets to the grocery store. The store is on a true bearing of 305° and a direct distance of 5.6 km from Rowan's house. Determine how far north Rowan walked on his way to the store from his house. Round the answer to 1 decimal place.

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. The given diagram shows the relative positions of a lighthouse, a boat, and a cruise ship. Use the information to answer the questions and round all distances to the nearest kilometre.



a. Determine the true bearing of the boat from the cruise ship.

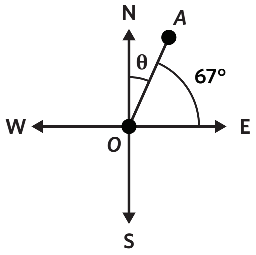
b. Determine the true bearing of the lighthouse from the boat.

c. The direct distance from the cruise ship to the boat is 34 km. Determine how far east the boat is from the cruise ship.

d. Determine the direct distance between the cruise ship and the lighthouse.

e. Identify a reason why ships and boats at sea must be able to communicate with each other at all times.

16. Use the given diagram to answer parts **a** and **b**. Move anti-clockwise from north when given a negative bearing.



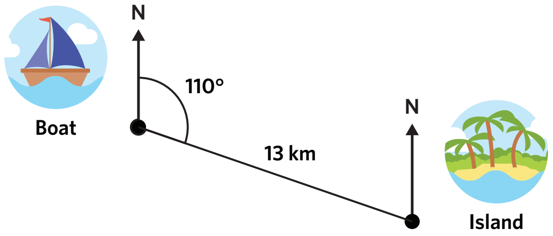
a. Calculate $\theta + 180°$ and mark point B in the given direction.

b. Calculate $\theta - 180°$ and mark point C in the given direction.

c. Comment on the relationship between the directions given by adding 180° to and subtracting 180° from a bearing.

Exam-style

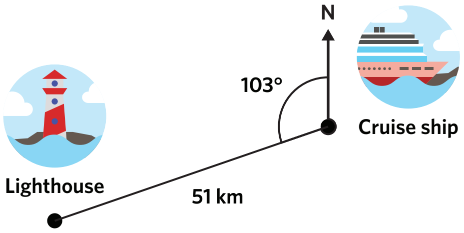
17. An island is at a true bearing of 110° from a boat 13 km away. (1 MARK)



The distance due east from the boat to the island is closest to

A. 4 km **B.** 5 km **C.** 11 km **D.** 12 km **E.** 13 km

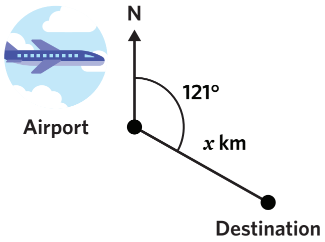
18. Use the given diagram to answer the questions. (3 MARKS)



a. Determine the true bearing of the ship from the lighthouse. 1 MARK

b. Determine how far east the ship is from the lighthouse, to the nearest kilometre. 2 MARKS

19. In the given diagram, a passenger plane leaves the airport and flies on a true bearing of 121° T towards its destination, which is 789 km due south from the airport. Determine the direct distance (x) between the airport and the plane's destination, to the nearest kilometre. (3 MARKS)



20. In a national park, a cabin is located at a bearing of 056° T from the information centre 4.7 km away. Determine how far north the cabin is from the information centre, correct to 1 decimal place. (3 MARKS)

Remember this?

21. $2.5 \times [?] = 1.25$

What value of [?] makes this number sentence correct?

A. 0.25

B. 0.5

C. 0.75

D. 1

E. 1.5

Back to contents

22. Four teammates share a prize.

They share their total prize based on the number of games each played.

	Games played
Alice	3
Bob	5
Carol	2
David	4

This month their total prize is $560.

What is Carol's share?

A. $80 **B.** $100 **C.** $120 **D.** $160 **E.** $200

23. A truck driver in New South Wales plans to travel from Sydney to Melbourne. The travel time for the journey is 15 hours. The driver must stop for at least 5 hours of rest after every 8 hours of travel.

If the driver must arrive at 6 pm on Tuesday, what is the latest time the driver can depart Sydney on Monday?

A. 6 am **B.** 10 am **C.** 6 pm **D.** 10 pm **E.** 12 am

Back to contents

9F Elevation and depression

LEARNING INTENTIONS

Students will be able to:

- understand the meaning of angles of elevation and depression
- use angles of elevation and depression to calculate heights and distances
- apply trigonometry to determine angles of elevation and depression.

Angles of elevation and depression are the angles made between an observer's line of vision and the horizontal. Angles of elevation occur when an observer is looking up at an object, whereas angles of depression occur when an observer is looking down.

KEY TERMS AND DEFINITIONS

- **Line of vision** is the implied straight line along which an observer looks at an object.
- A **horizontal line** is parallel with the horizon.
- A **vertical line** is perpendicular (at a 90° angle) with the horizon.

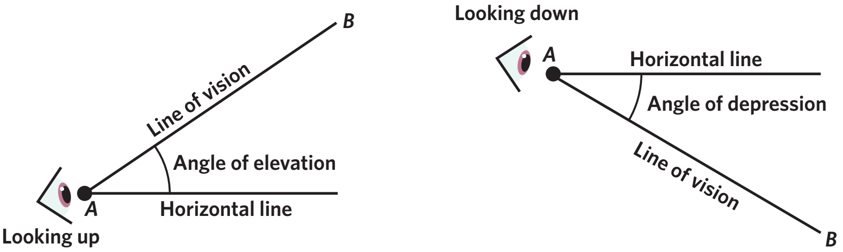
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: SPK Studio Images/Shutterstock.com

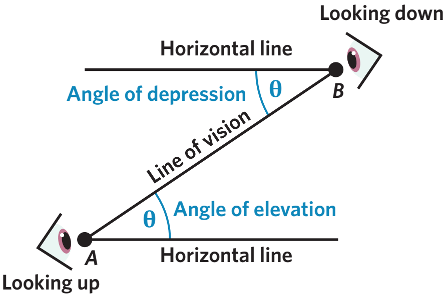
Land surveying involves taking precise measurements and accurately mapping a piece of land in two and three dimensions. A surveyor uses an instrument called a clinometer to measure the angles between objects, as well as any angles of elevation or depression. This allows for the heights of inclines and depths of bodies of water to be calculated by applying trigonometry.

Key ideas

1. The **angle of elevation** or **depression** of a point *B*, from another point *A*, is the angle between the horizontal line and the line of vision, which can also be referred to as the direct distance, *AB*.



2. The angle of elevation of point *B* from point *A* is equal to the angle of depression of point *A* from point *B* because they are alternate angles in parallel lines.



∴ Angle of elevation of *B* from *A* = Angle of depression of *A* from *B*

Back to contents

Worked example 1

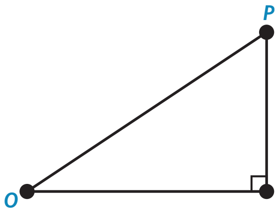
Using angles of elevation and depression

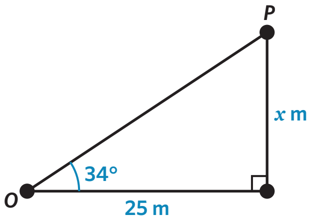
Determine the distances to the nearest metre.

a. Point P is 25 m horizontally from point O, at an angle of elevation of 34°. Determine the vertical distance of P from O.

WE1a

Working





$$\tan 34° = \frac{x}{25}$$
$$x = 25\tan 34°$$
$$= 16.86...$$
$$\approx 17 \text{ m}$$

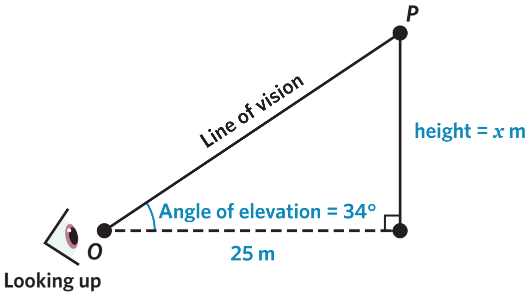
Thinking

Step 1: Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the given points.

Step 2: Label two sides of the triangle using the given information. Mark the angle of elevation opposite the side representing vertical distance.

Step 3: Solve for the unknown length by using a trigonometric ratio. Round the answer to the nearest metre.

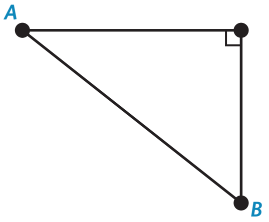
Visual support



b. Point B is 7 m vertically from point A, at an angle of depression of 39°. Determine the horizontal distance of B from A.

WE1b

Working

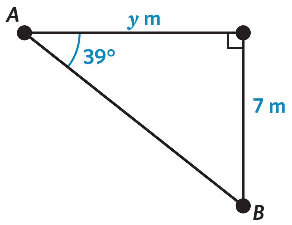


Thinking

Step 1: Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the given points.

Continues →

Back to contents



Step 2: Label two sides of the triangle using the given information. Mark the angle of depression opposite the side representing vertical distance.

$$\tan 39^\circ = \frac{7}{y}$$

$$y = \frac{7}{\tan 39^\circ}$$

$$= 8.64...$$

$$\approx 9 \text{ m}$$

Step 3: Solve for the unknown length by using a trigonometric ratio. Round the answer to the nearest metre.

Student practice

Determine the distances to the nearest metre.

a. Point *P* is 15 m horizontally from point *O*, at an angle of elevation of 25°. Determine the vertical distance of *P* from *O*.

b. Point *B* is 10 m vertically from point *A*, at an angle of depression of 24°. Determine the horizontal distance of *B* from *A*.

Worked example 2

Determining angles of elevation and depression

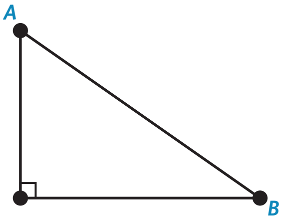
Determine the angles to the nearest degree.

WE2

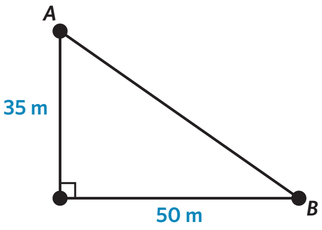
Point *A* is 35 m above and a horizontal distance of 50 m from point *B*. Determine the angles of elevation and depression between *A* and *B*.

Working

Thinking

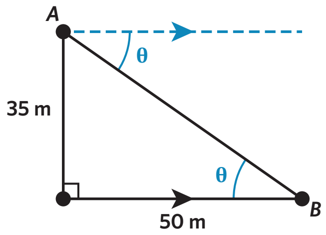


Step 1: Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the given points.



Step 2: Label the shorter sides of the triangle with the given vertical and horizontal distances.

Continues →



Step 3: Mark the angles of elevation and depression between the given points by using alternate angles on parallel lines.

$$\tan\theta = \frac{35}{50}$$

$$\theta = \tan^{-1}\left(\frac{35}{50}\right)$$

$$= 34.99...$$

$$\approx 35°$$

Step 4: Solve for the unknown angle by using a trigonometric ratio. Round the answer to the nearest degree.

Student practice

Determine the angles to the nearest degree.

Point *A* is 18 m above and a horizontal distance of 40 m from point *B*. Determine the angles of elevation and depression between *A* and *B*.

Back to contents

9F Questions

Understanding worksheet

1. Match the descriptions with the right-angled triangles that show the given information.

Description

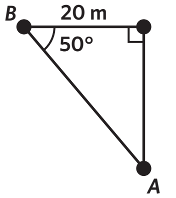
Point *B* is 15 m horizontally from point *A*, at an angle of elevation of 30°. ●

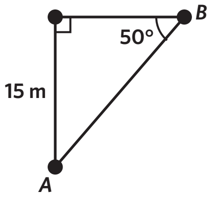
Point *B* is 20 m vertically from point *A*, at an angle of elevation of 30°. ●

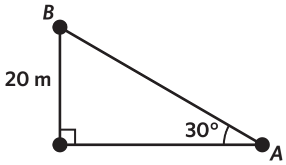
Point *A* is 20 m horizontally from point *B*, at an angle of depression of 50°. ●

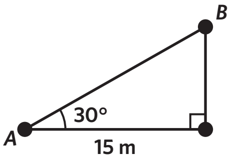
Point *A* is 15 m vertically from point *B*, at an angle of depression of 50°. ●

Right-angled triangle

● 

● 

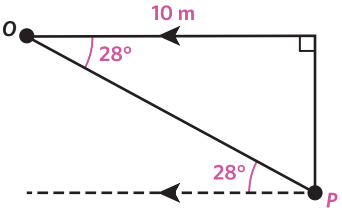
● 

● 

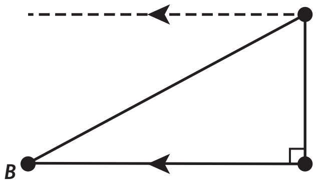
2. Label the known points, angles and distances using the given information.

Example

Point *P* is 10 m horizontally from point *O*, at an angle of depression of 28°.



Point *A* is 16 m vertically from *B*, at an angle of elevation of 34°.



Back to contents

3. Fill in the blanks by using the words provided.

alternate	depression	elevation	equal

When an observer is looking up at an object, the angle between the line of vision and the horizontal is called the angle of ________ . When an observer is looking down at an object, an angle of ________ is formed instead. The angle of elevation of the object from the observer is ________ to the angle of depression of the observer from the object as they are ________ angles in parallel lines.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6	4 (c,d,e,f), 5 (c,d,e,f), 6	4 (e,f,g,h), 5 (e,f,g,h), 6

4. Determine the distances to the nearest metre. **WE1**

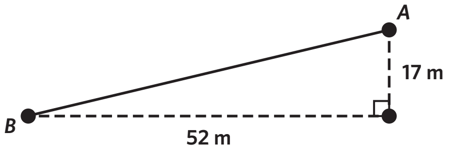
- **a.** Point B is 10 m horizontally from point A, at an angle of elevation of 30°. Determine the vertical distance of B from A.
- **b.** Point B is 23 m vertically from point A at an angle of depression of 20°. Determine the horizontal distance of B from A.
- **c.** Point P is 28 m horizontally from point O at an angle of depression of 40°. Determine the vertical distance of P from O.
- **d.** Point P is 31 m vertically from point O at an angle of elevation of 35°. Determine the horizontal distance of P from O.
- **e.** Point C is 21.5 m horizontally from point O at an angle of depression of 47°. Determine the vertical distance of C from O.
- **f.** Point Q is 19.3 m vertically from point A at an angle of elevation of 18°. Determine the horizontal distance of Q from A.
- **g.** Point X is 57.8 m horizontally from point Y at an angle of depression of 21°. Determine the direct distance of X from Y.
- **h.** Point P is 19.56 m vertically from point Q at an angle of elevation of 32°. Determine the direct distance of P from Q.

5. Determine the angles to the nearest degree. **WE2**

- **a.** Point A is 10 m above and a horizontal distance of 30 m from point B. Determine the angles of elevation and depression between A and B.
- **b.** Point A is 15 m below and a horizontal distance of 35 m from point B. Determine the angles of elevation and depression between A and B.
- **c.** Point O is 12 m above and a horizontal distance of 20 m from point P. Determine the angles of elevation and depression between O and P.
- **d.** Point O is 9 m below and a horizontal distance of 16 m from point P. Determine the angles of elevation and depression between O and P.
- **e.** Point A is 21 m above and a horizontal distance of 19 m from point C. Determine the angles of elevation and depression between A and C.
- **f.** Point P is 50.5 m below and a horizontal distance of 64.5 m from point Q. Determine the angles of elevation and depression between P and Q.
- **g.** Point S is 29.5 m above and a direct distance of 85.3 m from point T. Determine the angles of elevation and depression between S and T.
- **h.** Point X is 39.5 m below and a direct distance of 145.3 m from point Y. Determine the angles of elevation and depression between X and Y.

Back to contents

6. Determine the angle of depression of B from A in the given diagram (not drawn to scale), to the nearest degree.

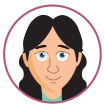


A. 18° **B.** 19° **C.** 71° **D.** 72° **E.** 162°

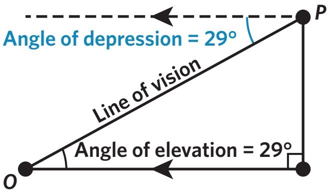
Spot the mistake

7. Select whether Student A or Student B is incorrect.

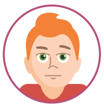
a. The angle of elevation of P from O is 29°. Draw a diagram and determine the angle of depression of O from P.



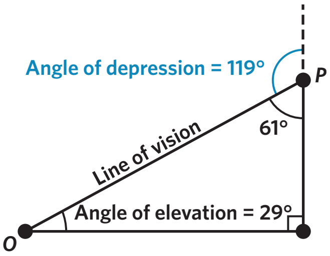
Student A



Angle of depression of O from $P = 29°$

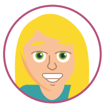


Student B

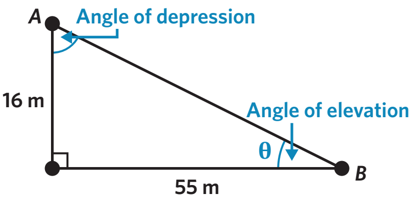


Angle of depression of O from $P = 119°$

b. Point A is 16 m above and a horizontal distance of 55 m from point B. Determine the angles of elevation and depression between A and B, to the nearest degree.



Student A



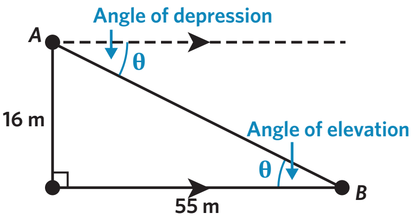
$$\tan\theta = \frac{16}{55}$$

$$\theta = \tan^{-1}\left(\frac{16}{55}\right) = 16.22... \approx 16°$$

Angle of elevation $\approx 16°$

Angle of depression $\approx 90° - 16° \approx 74°$

Student B



$$\tan\theta = \frac{16}{55}$$

$$\theta = \tan^{-1}\left(\frac{16}{55}\right) = 16.22... \approx 16°$$

Angle of elevation = Angle of depression $\approx 16°$

Problem solving

Question working paths

Mild 8, 9, 10 🌶	**Medium** 9, 10, 11 🌶🌶	**Spicy** 10, 11, 12 🌶🌶🌶

8. From a point on the ground 50 m away, the angle of elevation to the top of a building is 30°. Determine the height of the building to the nearest metre.

9. From a boat out at sea, the angle of elevation to the top of a lighthouse is 10°. Determine the distance from the boat to the lighthouse if the lighthouse and the small island it is positioned on have a combined height of 30 m. Round the answer to the nearest metre.

10. A helicopter is at a height of 60 m and a horizontal distance of 100 m from a landing pad. Determine the angle of elevation of the helicopter from the landing pad to the nearest degree.

11. A sea bird flying over the water spots a fish at a horizontal distance of 28 m at an angle of depression of 37°. Determine the height of the bird above the surface of the water if the fish was at a depth of 2 m when it was seen by the bird. Round the answer to the nearest metre.

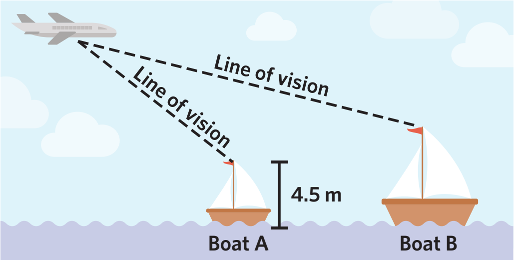
12. Ishmael observes a sky train carriage directly above him at a height of 50 m. A few seconds later, Ishmael observes the same carriage again, this time at an angle of elevation of 22°. Determine the distance travelled by the sky train between the two observations, to the nearest metre.

Reasoning

Question working paths

Mild 13 (a,b,c,e) 🌶	**Medium** 13 (a,b,c,e), 14 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

13. A crow's nest on a boat is an accessible area close to the highest point of the boat. The given diagram shows the position of a plane, as observed from the crow's nests of boat A and boat B, not to scale.



a. The angle of elevation of the plane from boat A is 53° and it is 500 m horizontally from the point of observation. Draw a right-angled triangle showing this information.

b. Determine the height of the plane to the nearest metre.

c. The crow's nest of boat A is at an angle of depression of 11° from the crow's nest of boat B and they are 12 m apart horizontally. Determine the height of the crow's nest of boat B, correct to 1 decimal place.

d. Determine the angle of depression of boat B from the plane to the nearest degree.

e. Identify a reason for the placement of a boat's crow's nest at or very close to the highest point of the vessel.

14. Determine the angles of elevation and depression between points X and Y for parts **a** and **b** to the nearest degree.

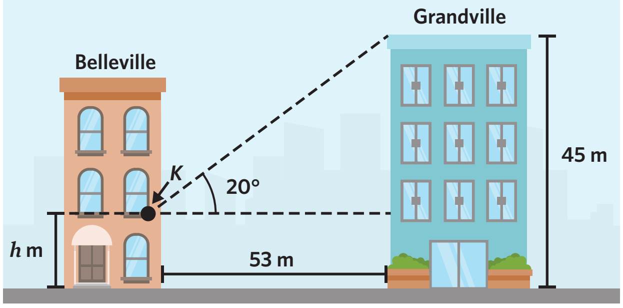
a. Point X is 10 m vertically and 20 m horizontally from point Y.

b. Point X is 20 m vertically and 10 m horizontally from point Y.

c. Make generalisations about the size of the angles of elevation and depression between two points when the horizontal distance between them is greater than, equal to, or less than the vertical distance.

Exam-style

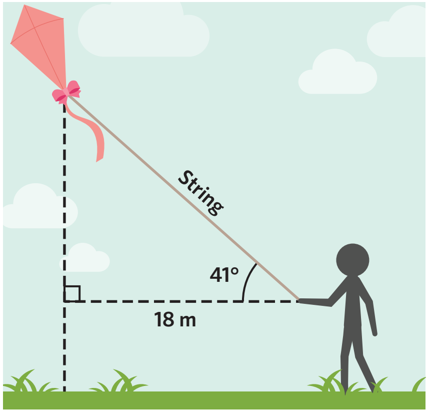
15. Two residential buildings, Grandville and Belleville, are 53 m apart. From Kathy's window in Belleville the top of Grandville is at an angle of elevation of 20°. The height of Grandville is 45 m. This information is shown in the diagram, not to scale. (1 MARK)



The height (h) from the ground to Kathy's window, in metres, is closest to

A. 8 **B.** 19 **C.** 20 **D.** 25 **E.** 26

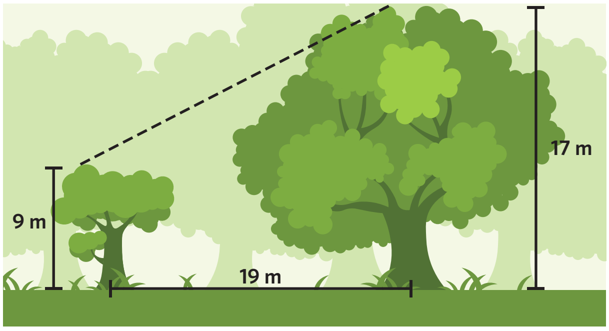
16. Ronnie is holding the string of a kite in her hand. The string is taut and the angle of elevation of the kite, from Ronnie's hand, is 41°. Ronnie and the kite are 18 m apart horizontally. This is shown in the diagram, not to scale. (3 MARKS)



a. Determine the angle of depression of Ronnie's hand from the kite. 1 MARK

b. Determine the length of the string, correct to 2 decimal places. 2 MARKS

17. Two trees in a forest, 9 m and 17 m in height are positioned 19 m horizontally from each other. This is shown in the diagram, not to scale. (2 MARKS)



Determine the angle of elevation of the top of the taller tree from the top of the shorter tree to the nearest degree.

18. A spotlight at a height of 0.8 m from the ground projects a beam towards a possum in a tree. The angle of elevation of the light beam towards the possum is 41° and there is a distance of 31 m between the spotlight and the possum. Determine the height of the possum from the ground, correct to 1 decimal place. (3 MARKS)

Back to contents

Remember this?

19. This table shows the relationship between the number of pages in a book and the thickness of the book in millimetres.

Pages	100	200	300	400	500	600	700
Thickness	5	10	15	20	25	30	35

What is the thickness of a book with 800 pages?

A. 35 mm
B. 40 mm
C. 45 mm
D. 50 mm
E. 55 mm

20. $50 \times \frac{\sqrt{75^2 \times 120^2}}{2} = \boxed{}$

A. 225 000
B. 375 500
C. 450 000
D. 562 500
E. 675 000

21. Emily is facing north. She turns 45 degrees clockwise, then 90 degrees anticlockwise, then 180 degrees clockwise, at which point she stops. By how many degrees clockwise does Emily's final position differ from her starting position?

A. 135°
B. 225°
C. 270°
D. 315°
E. 360°

Back to contents

Chapter 9 extended application

1. In a surveying project, an engineer is determining the height of a distant building using a measuring device. The building is situated a considerable distance away from the engineer's observation point.

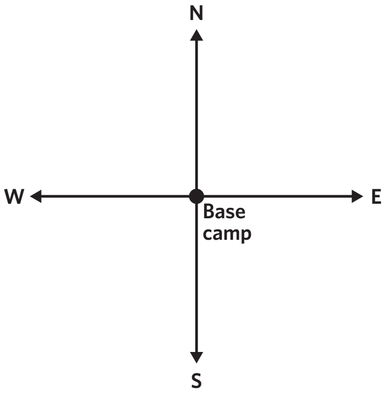
They have set up a measuring device on a tripod, which stands 1 m off the ground. When the device is level, it indicates that the distance from the observation point to the tower is 100 m, with an angle of elevation measuring 30° when directed towards the top of the building. Round all answers to the nearest unit.

- **a.** Complete the diagram labelling the given distances and angles.
- **b.** Calculate the height of the tower.
- **c.** The laser device also has the capability to measure straight-line distances. What will the reading indicate for the distance to the top of the tower?

At the engineers' current position, they can also see the top of a second building. The device reads a distance of 195 m to the top of this building at an angle of elevation of 40°.

- **d.** Determine the vertical height of the second tower.
- **e.** Calculate the horizontal distance between the two towers.
- **f.** Describe any methods that would have been used to obtain these measurements before the advent of laser measuring devices.

2. A wilderness search and rescue operation has a team coordinating with two hikers who have become lost in the mountains. Hiker Max is known to be 10 km away from the team's base camp, positioned at a bearing of $N\ 45°\ E$. Hiker Sara is located 8 km away, on a bearing of $S\ 30°\ W$ from the base camp. Round all answers to 2 decimal places.



- **a.** Complete an accurate diagram that illustrates the locations of the hikers in reference to the base camp.
- **b.** Calculate how far north of the base camp Max is located.
- **c.** Calculate how far south of the base camp Sara is located.
- **d.** Determine the east-west distance between Max and Sara.

Sara is instructed to hike due east for 5 km to an extraction point.

- **e.** Determine the distance and compass bearing of her new location from the base camp.
- **f.** What safety precautions could be taken to prevent becoming lost or to assist in the event of becoming lost.

3. A flight controller at a regional airport is managing two aircraft for separate landings. Aircraft Charlie is positioned 35 km away and on a bearing of 120° T from the control room. Aircraft Delta is 50 km away and positioned on a bearing of 300° T from the control room. Round all answers to the nearest unit.

- **a.** Calculate how far east of the control room Aircraft Charlie is located.
- **b.** Calculate how far west of the control room Aircraft Delta is located.
- **c.** Determine the north-south distance between Aircraft Charlie and Aircraft Delta.
- **d.** Charlie changes course and flies due north for 20 km. Calculate Aircraft Charlie's new true bearing from the control room.

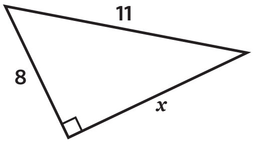
Given that Delta is flying at an altitude of 5000 m and is 100 m horizontally away from the control room.

- **e.** Determine the angle of elevation from the control room to Aircraft Delta's position.
- **f.** What are some of the tasks an air traffic controller may be responsible for?

Chapter 9 review

Multiple choice

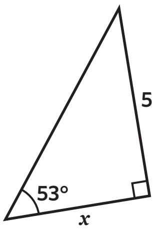
1. Calculate the value of x in exact form. 9A



A. $\sqrt{3}$ **B.** $\sqrt{19}$ **C.** $\sqrt{57}$ **D.** $\sqrt{58}$ **E.** $\sqrt{185}$

---

2. Determine the trigonometric ratio that relates the two given sides and the reference angle. 9B,C



A. $\cos 53° = \frac{5}{x}$

B. $\sin 53° = \frac{5}{x}$

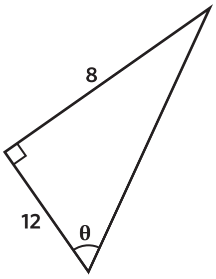
C. $\sin 53° = \frac{x}{5}$

D. $\tan 53° = \frac{5}{x}$

E. $\tan 53° = \frac{x}{5}$

---

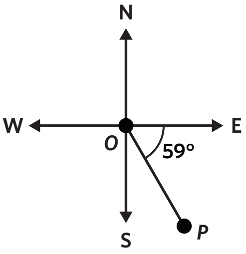
3. Calculate the value of θ to the nearest degree. 9D



A. 34° **B.** 42° **C.** 48° **D.** 56° **E.** 60°

---

4. Determine the compass bearings of P from O in the given diagram. 9E

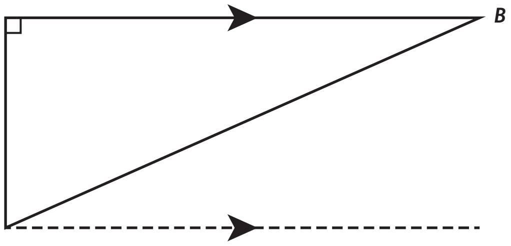


A. $E\,31°\,S$ **B.** $S\,31°\,E$ **C.** $E\,59°\,S$ **D.** $S\,59°\,E$ **E.** $N\,59°\,W$

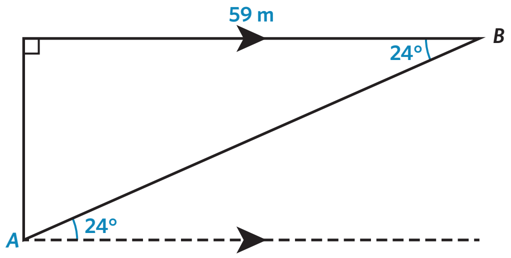
Back to contents

5. Label the known points, angles and distances using the given information. 9F

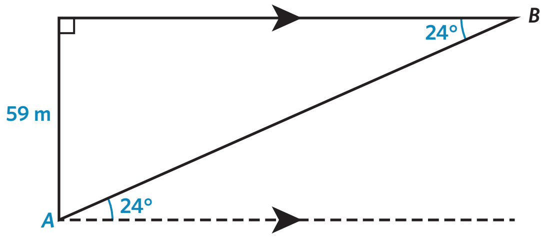
Point A is 59 m horizontally from B, at an angle of depression of 24°.



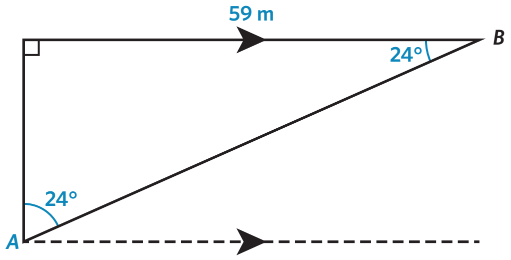
A.



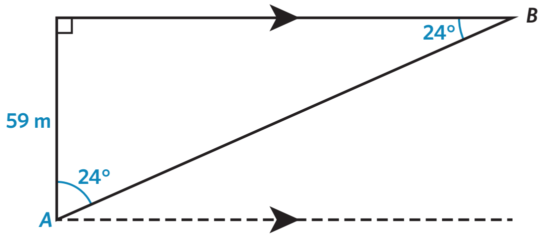
B.



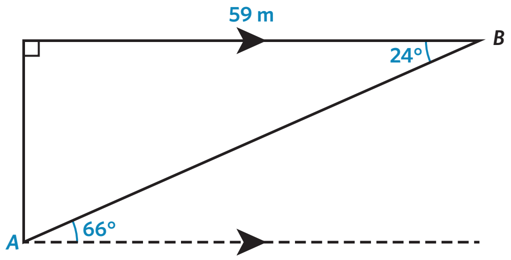
C.



D.



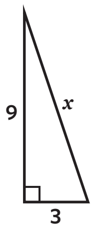
E.



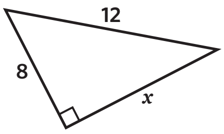
Fluency

6. Calculate the values of the pronumerals, correct to 2 decimal places. 9A

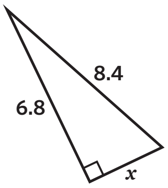
a.



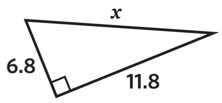
b.



c.

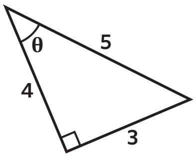


d.

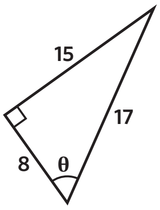


7. For the triangles shown, write the equations for the sine, cosine and tangent ratios of the reference angle (θ). 9B

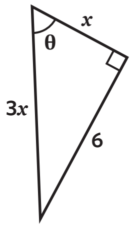
a.



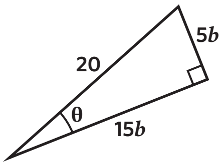
b.



c.



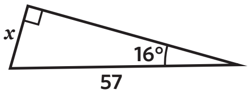
d.



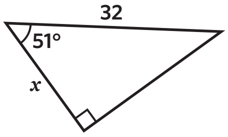
Back to contents

8. For each triangle, calculate the value of x correct to 2 decimal places. 9C

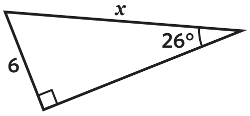
a.



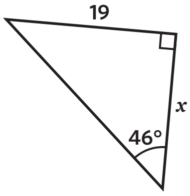
b.



c.



d.

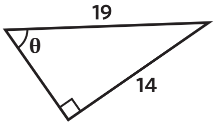


9. Calculate the value of θ in each equation, correct to 2 decimal places. 9D

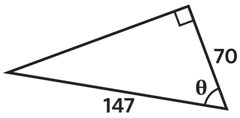
a. $\sin\theta = \frac{3}{5}$ **b.** $\tan\theta = 5.5$ **c.** $\cos\theta = 0.74$ **d.** $\tan\theta = \frac{2}{15}$

10. For each triangle, calculate the value of θ to the nearest degree. 9D

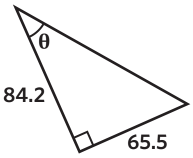
a.



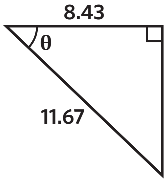
b.



c.

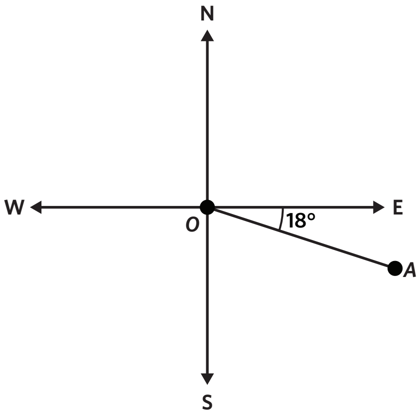


d.

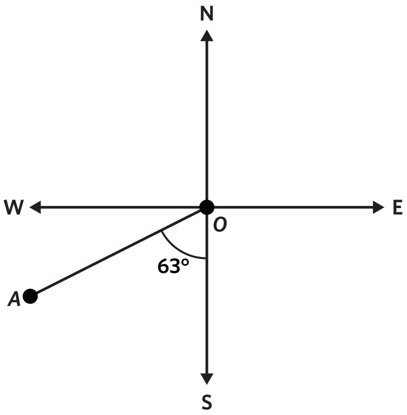


11. Determine the true bearings of A from O and O from A in the given diagrams. 9E

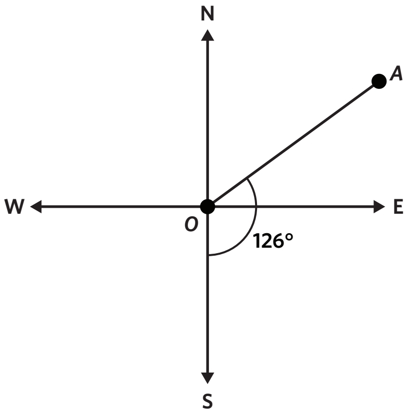
a.



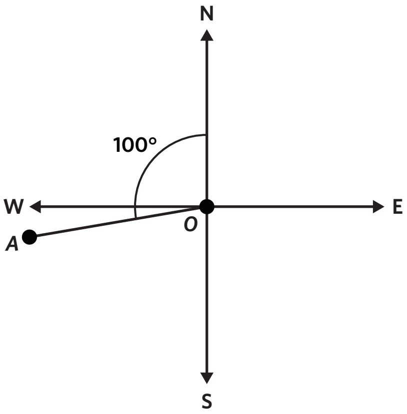
b.



c.

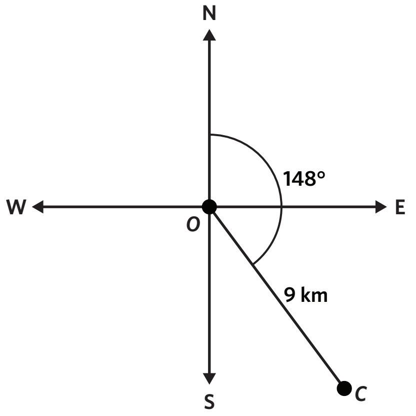


d.

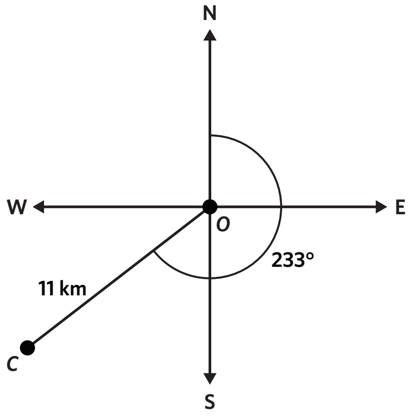


12. Use the given diagrams to calculate the required distances, correct to 2 decimal places. 9E

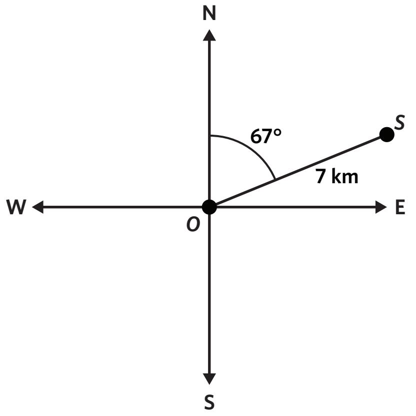
a. Determine how far east point *C* is from point *O*.



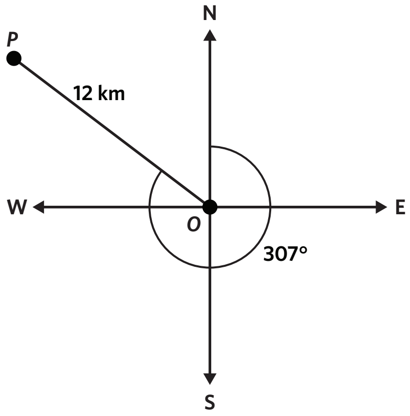
b. Determine how far south point *C* is from point *O*.



c. Determine how far north point *S* is from point *O*.



d. Determine how far west point *P* is from point *O*.



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13. Determine the distances to the nearest metre. 9F

a. Point *X* is 13 m horizontally from point *Y*, at an angle of elevation of 60°. Determine the vertical distance of *X* from *Y*.

b. Point *A* is 12 m vertically from point *B* at an angle of depression of 30°. Determine the horizontal distance of *A* from *B*.

c. Point *P* is 13.4 m horizontally from point *O* at an angle of elevation of 16°. Determine the vertical distance of *P* from *O*.

d. Point *C* is 110 m horizontally from point *O* at an angle of depression of 52°. Determine the direct distance of *C* from *O*.

---

14. Determine the angles to the nearest degree. 9F

a. Point *P* is 15 m above and a horizontal distance of 32 m from point *O*. Determine the angles of elevation and depression between *P* and *O*.

b. Point *A* is 6 m below and a horizontal distance of 40 m from point *B*. Determine the angles of elevation and depression between *A* and *B*.

c. Point *S* is 52.1 m below and a horizontal distance of 102.3 m from point *T*. Determine the angles of elevation and depression between *S* and *T*.

d. Point *X* is 12.4 m above and a direct distance of 37.9 m from point *Y*. Determine the angles of elevation and depression between *X* and *Y*.

Back to contents

Problem solving

15. Lani is sailing her boat, and decides to anchor it 4.6 m above the sea floor. The boat can drift a maximum distance of 3.6 m horizontally from the anchor. Calculate the length of the anchor's rope, correct to two decimal places. 9A

16. In a professional baking contest, participants are required to design a triangular cake with a tangent ratio $\frac{2}{9}$. Create a right-angled triangle to illustrate the cake, and label the reference angle and any known side lengths. 9B

17. After a storm, the top part of a tree has snapped at the trunk, fallen over, and now makes a 40° angle with the horizontal ground. The lower part of the tree's trunk is still standing at a right angle with the ground and is 1.5 m long. Calculate the total height of the tree prior to the storm, correct to 2 decimal places. 9C

18. Mason is flying his remote controlled plane. He starts to descend for landing when the plane is 5 m above the ground. The runway is 12 m away from the point directly below the plane. Form a right-angled triangle showing this scenario and calculate the angle with opposite and adjacent side lengths of 12 m and 5 m, respectively, rounded to the nearest degree. 9D

19. Patrick is an astronaut piloting his spacecraft on a bearing of $S\,40°\,W$ for 700 km to research an orbiting space station, Station Joy. Calculate how far west Station Joy is from the spacecraft's starting point, rounded to 2 decimal places. 9E

20. Timmy spots a plane directly above him in the sky at a height of 9.5 km. After a few seconds, Timmy notices that the plane is now at an angle of elevation of 82°. Determine the distance travelled by the plane between the two observations, in kilometres, rounded to 2 decimal places. 9F

Reasoning

21. Alex is a marine biologist who is studying the migration pattern of whales. He is standing on a boat and his eye level is at 3 m above the surface of the ocean. Alex spots an emerging whale at the surface 10 m horizontally from him.

a. Draw a right-angle triangle that represents this scenario, and calculate the length of the hypotenuse, correct to 2 decimal places.

b. The whale is at an angle of θ below Alex's eye level. Write the equations for sine, cosine and tangent ratios of the reference angle θ.

c. Alex spots another whale at the surface of the water 20 m from him horizontally. Determine the angles of elevation and depression between Alex and the whale, to the nearest degree.

d. Alex receives information that both whales are at a bearing of 145° T from his current location. If his boat is currently aligned with a bearing of $N\,10°\,W$, calculate the angle clockwise Alex needs to turn to align his boat towards the whales.

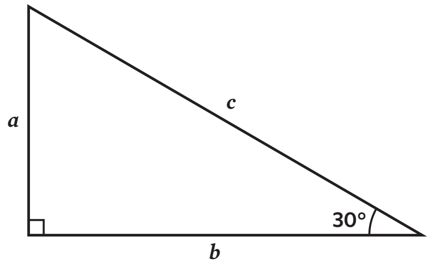
e. What are some ethical considerations Alex should keep in mind while studying marine life in their natural habitat?

22. Consider the following triangle.

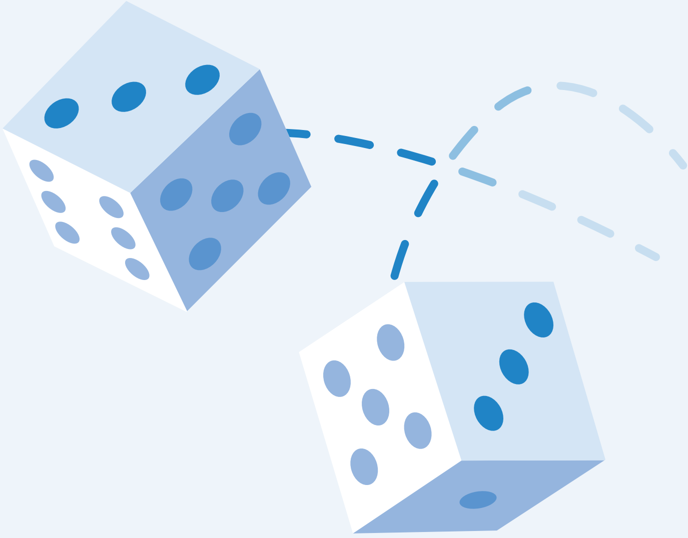
a. If $c = 1$, calculate the value of a, rounding to 2 decimal places.

b. If $c = 1$, calculate the value of b, rounding to 2 decimal places.

c. Using your answers from parts **a** and **b**, calculate the values of a and b, rounding to 1 decimal place, when $c = 10$. What can you conclude about the relationship between angles and side lengths of a right angle triangle?



Back to contents



Chapter 10

Probability

Statistics and probability

Back to contents

Chapter 10 research summary

Probability

Big ideas

In Year 9 Mathematics students begin to advance their study and dive deeper into the concept of probability. Students will progress from their learning of chance and single events to explore exclusivity and independence of multiple events.

Proportional reasoning

Proportional reasoning is the fundamental skill that underpins probability. Proportional reasoning is used in understanding fractions and ratios as probability is often expressed as a fraction. The ability to interpret, compare, and use fractions or ratios correctly requires proportional reasoning skills.

Complementary events

For any event A its complement is the event that A does not occur. The probability of an event and its complement always adds up to 1.

Randomness and variation

Recognising that individual outcomes are random, but there are patterns or regularities that emerge in the long run.

Visual representations

Venn diagrams

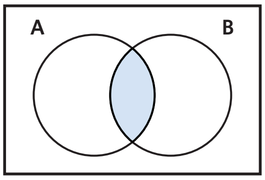
Venn diagrams are used to represent sets and subsets. They are particularly effective for visualising the union, intersection and complements of events.

Two-way tables

Two-way tables display frequencies or probability for combinations of two categorical variables.

And

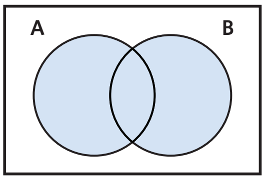
Elements that belong to both A **and** B



	A	Not A
B	▨	
Not B		

Or

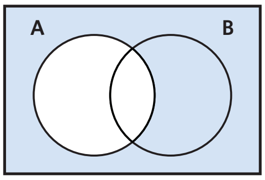
Elements that belong to either A **or** B **or** both



	A	Not A
B	▨	▨
Not B	▨	▨

Not

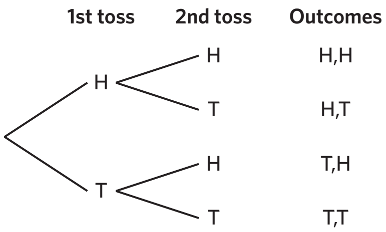
Elements that do **not** belong to A



	A	Not A
B		▨
Not B		▨

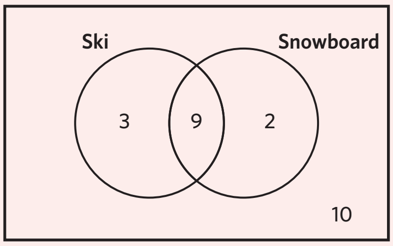
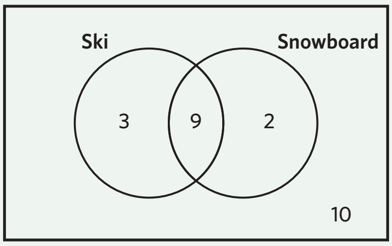
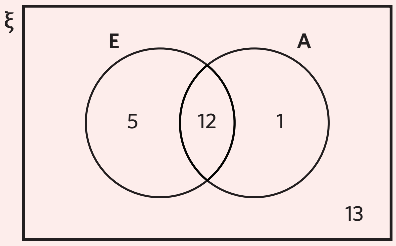
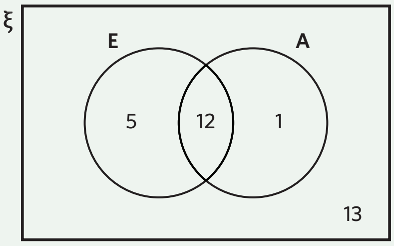
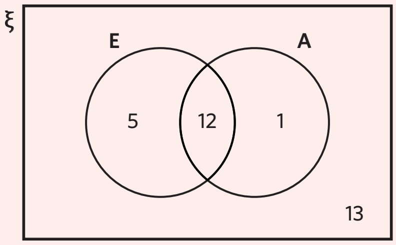
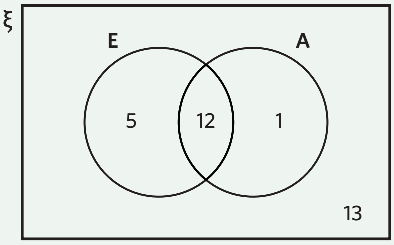
Tree diagrams

Tree diagrams are branching diagrams that show all possible outcomes or combinations of a sequence of events. They are particularly helpful for visualising compound events.



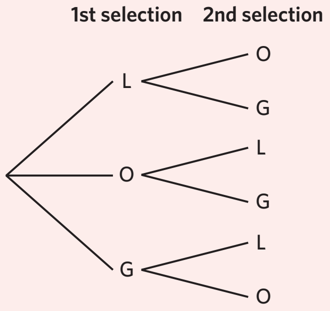
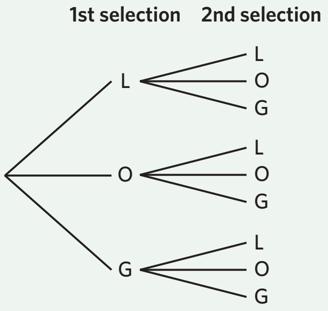
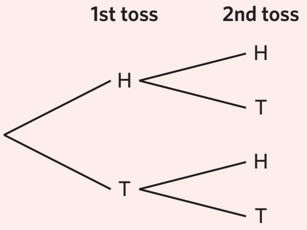
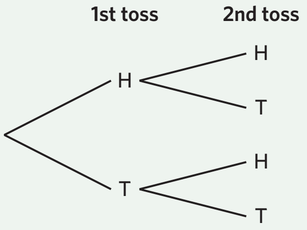
Back to contents

Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students believe that the word ‘and’ requires that they add up all parts of set A and set B rather than the intersection.	 Ski and snowboard $= 3 + 2$ $= 5$ people	 Ski and snowboard $= 9$ people	10A
Students give the numerator of a probability, rather than providing the fraction with a denominator.	What is the probability of randomly selecting a student who can snowboard only? People who can snowboard only $= 2$ Pr(snowboard only) $= 2$	What is the probability of randomly selecting a student who can snowboard only? People who can snowboard only $= 2$ Total number of students $= 24$ Pr(snowboard only) $= \frac{2}{24}$ $= \frac{1}{12}$	10A
Students confuse the symbols of union and intersection.	 $n(E \cap A) = 5 + 12 + 1$ $= 18$	 $n(E \cap A) = 12$	10B
Students do not consider the full sample space.	 $n(E') = 1$	 $n(E') = 1 + 13$ $= 14$	10B
Students think the complement of a set means ‘not that set’.	$\xi = \{2, 3, 4, 5, 6, 7, 8\}$ $E = \{2, 3, 8\}$ $F = \{2, 4, 5, 7\}$ The complement of F is equal to: $\{2, 3, 8\}$	$\xi = \{2, 3, 4, 5, 6, 7, 8\}$ $E = \{2, 3, 8\}$ $F = \{2, 4, 5, 7\}$ The complement of F is equal to: $\{3, 6, 8\}$	10B

Continues →

Back to contents

<table>
<tr><th>Misconception</th><th>Incorrect ✕</th><th>Correct ✓</th><th>Lesson</th></tr>
<tr>
<td>Students do not consider the effect on the number of outcomes when events do not include replacement.</td>
<td>Two letters are selected from the word TRAY without replacement.
<table>
<tr><td></td><td></td><td colspan="4">2nd</td></tr>
<tr><td></td><td></td><td>T</td><td>R</td><td>A</td><td>Y</td></tr>
<tr><td rowspan="4">1st</td><td>T</td><td>(T,T)</td><td>(T,R)</td><td>(T,A)</td><td>(T,Y)</td></tr>
<tr><td>R</td><td>(R,T)</td><td>(R,R)</td><td>(R,A)</td><td>(R,Y)</td></tr>
<tr><td>A</td><td>(A,T)</td><td>(A,R)</td><td>(A,A)</td><td>(A,Y)</td></tr>
<tr><td>Y</td><td>(Y,T)</td><td>(Y,R)</td><td>(Y,A)</td><td>(Y,Y)</td></tr>
</table>
ξ = {(T,T), (T,R), (T,A), (T,Y), (R,T), (R,R), (R,A), (R,Y), (A,T), (A,R), (A,A), (A,Y), (Y,T), (Y,R), (Y,A), (Y,Y)}</td>
<td>Two letters are selected from the word TRAY without replacement.
<table>
<tr><td></td><td></td><td colspan="4">2nd</td></tr>
<tr><td></td><td></td><td>T</td><td>R</td><td>A</td><td>Y</td></tr>
<tr><td rowspan="4">1st</td><td>T</td><td>X</td><td>(T,R)</td><td>(T,A)</td><td>(T,Y)</td></tr>
<tr><td>R</td><td>(R,T)</td><td>X</td><td>(R,A)</td><td>(R,Y)</td></tr>
<tr><td>A</td><td>(A,T)</td><td>(A,R)</td><td>X</td><td>(A,Y)</td></tr>
<tr><td>Y</td><td>(Y,T)</td><td>(Y,R)</td><td>(Y,A)</td><td>X</td></tr>
</table>
ξ = {(T,R), (T,A), (T,Y), (R,T), (R,A), (R,Y),(A,T), (A,R), (A,Y), (Y,T), (Y,R), (Y,A)}</td>
<td>10C</td>
</tr>
<tr>
<td>Students confuse the number of outcomes with the probability and vice versa.</td>
<td>Two fair coins are thrown. What is the probability of landing at least one head?
$\text{Pr(at least one head)} = 4$</td>
<td>Two fair coins are thrown. What is the probability of landing at least one head?
$\text{Pr(at least one head)} = \frac{3}{4}$</td>
<td>10C</td>
</tr>
<tr>
<td>Students draw tree diagrams without replacement.</td>
<td>Two letters are selected from the word LOG with replacement.
Draw a tree diagram.
 </td>
<td>Two letters are selected from the word LOG with replacement.
Draw a tree diagram.
 </td>
<td>10D</td>
</tr>
<tr>
<td>Students use the events instead of the number of outcomes when determining the probability of an event.</td>
<td>Calculate the probability of selecting the letter G twice.
Number of events = 2
$n(G,G) = 1$
$\Pr(G,G) = \frac{1}{2}$</td>
<td>Calculate the probability of selecting the letter G twice.
Possible outcomes = 9
$n(G,G) = 1$
$\Pr(G,G) = \frac{1}{9}$</td>
<td>10D</td>
</tr>
<tr>
<td>Students do not see the order of events as important to a tree diagram.</td>
<td>A fair sided coin is tossed twice.

What is the probability of landing a head (H) then a tail (T)?
$\Pr(H,T) = \frac{1}{2}$</td>
<td>A fair sided coin is tossed twice.

What is the probability of landing a head (H) then a tail (T)?
$\Pr(H,T) = \frac{1}{4}$</td>
<td>10D</td>
</tr>
</table>

Continues →

Back to contents

<table>
<tr><th>Misconception</th><th>Incorrect ✗</th><th>Correct ✓</th><th>Lesson</th></tr>
<tr>
<td>Students confuse experimental probability with theoretical probability.</td>
<td>Calculate the experimental probability that a person is born during the summer.
<table>
<tr><th>Outcome</th><th>Frequency</th></tr>
<tr><td>Summer</td><td>12</td></tr>
<tr><td>Autumn</td><td>10</td></tr>
<tr><td>Winter</td><td>5</td></tr>
<tr><td>Spring</td><td>13</td></tr>
</table>
$\begin{aligned}\text{Pr(summer)} &= \frac{\text{number of summers}}{\text{number of seasons}} \\ &= \frac{1}{4}\end{aligned}$</td>
<td>Calculate the experimental probability that a person is born during the summer.
<table>
<tr><th>Outcome</th><th>Frequency</th></tr>
<tr><td>Summer</td><td>12</td></tr>
<tr><td>Autumn</td><td>10</td></tr>
<tr><td>Winter</td><td>5</td></tr>
<tr><td>Spring</td><td>13</td></tr>
</table>
Frequency of summer: 12
Number of trials: 40
$\begin{aligned}\text{Pr(summer)} &= \frac{\text{frequency of summer}}{\text{total number of trials}} \\ &= \frac{12}{40} \\ &= \frac{3}{10}\end{aligned}$</td>
<td>10E</td>
</tr>
<tr>
<td>Students think the expected occurrence equals the frequency of an experiment.</td>
<td>If $\text{Pr(summer)} = \frac{3}{10}$, calculate the expected number of people that are born in summer if the birth season of 1000 people is recorded.
Expected number of occurrences
$\begin{aligned}&= \frac{3}{10} \times 40 \\ &= \frac{120}{10} \\ &= 12\end{aligned}$</td>
<td>If $\text{Pr(summer)} = \frac{3}{10}$, calculate the expected number of people that are born in summer if the birth season of 1000 people is recorded.
Expected number of occurrences
$\begin{aligned}&= \frac{3}{10} \times 1000 \\ &= \frac{3000}{10} \\ &= 300\end{aligned}$</td>
<td>10E</td>
</tr>
</table>

Back to contents

10A Venn diagrams and two-way tables

LEARNING INTENTIONS

Students will be able to:

- interpret information represented in Venn diagrams and two-way tables
- calculate probabilities represented in Venn diagrams and two-way tables
- construct Venn-diagrams and two-way tables.

Venn diagrams and two-way tables are powerful tools for representing and analysing data, especially when calculating probabilities. These visual aids allow for a clear understanding of events, whether they satisfy 'and', 'or', or 'not' conditions. Constructing and interpreting these diagrams and tables not only aids in data representation but also streamlines the process of probability calculation. Utilising these tools effectively can provide insights into collected data and answer specific questions related to probabilities.

KEY TERMS AND DEFINITIONS

- **Exclusive 'or'** suggests an element belongs to two separate categories but not both.
- **Inclusive 'or'** suggests an element that belongs to two separate categories as well as both.
- A **two-way table** displays the frequency (count) of two categories, written in a table format with rows and columns.
- A **Venn diagram** provides a visual representation of similarities and differences between two or more sets of information.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: 4 PM production/Shutterstock.com

Two-way tables are useful in healthcare when studying how two conditions might be related. Suppose there's a need to understand how many patients exercise regularly and how many of them also follow a specific diet. A two-way table can clarify this relationship, informing decisions in public health campaigns.

Key ideas

1. Representing data in Venn diagrams or two-way tables simplifies the process of interpreting data.

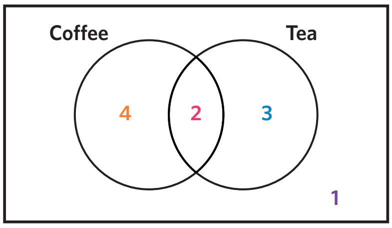
Raw Data:

Person 1: **Drinks tea only**
Person 2: **Drinks coffee only**
Person 3: **Drinks tea only**
Person 4: **Drinks neither tea or coffee**
Person 5: **Drinks coffee only**
Person 6: **Drinks both tea and coffee**
Person 7: **Drinks both tea and coffee**
Person 8: **Drinks coffee only**
Person 9: **Drinks tea only**
Person 10: **Drinks coffee only**

Two-way table:

	Drinks tea	Does not drink tea	Total
Drinks coffee	2	4	6
Does not drink coffee	3	1	4
Total	5	5	10

Venn diagram:



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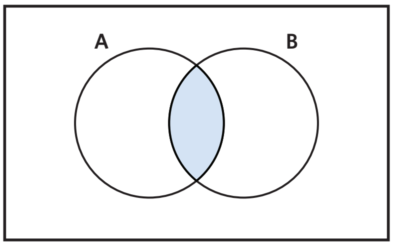
Back to contents

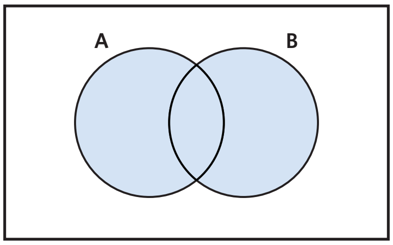
2. Events satisfying ‘and’, ‘or’, and ‘not’ conditions are represented in Venn diagrams and two-way tables.



	A	Not A
B	▨	
Not B		

And

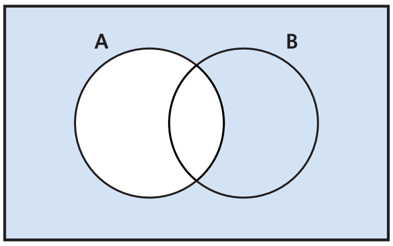
Elements that belong to both A **and** B



	A	Not A
B	▨	▨
Not B	▨	▨

Or

Elements that belong to either A **or** B **or** both



	A	Not A
B		▨
Not B		▨

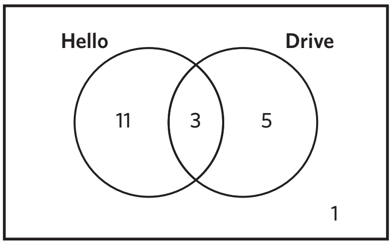
Not

Elements that do **not** belong to A

Worked example 1

Interpreting information from Venn diagrams

Consider the following Venn diagram showing people’s responses to a survey relating to which magazines they read.



a. How many people read Hello?

WE1a

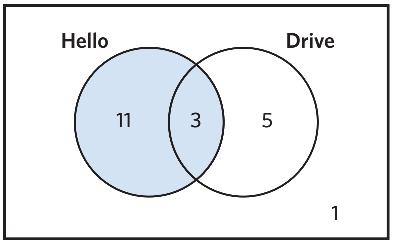
Working

$11 + 3 = 14$ people

Thinking

Sum the numbers within the relevant circle. Note: Without the word ‘only’, both numbers are counted.

Visual support



Continues →

Back to contents

b. How many people read Hello or Drive, but not both?

WE1b

Working

$11 + 5 = 16$ people

Thinking

Sum the numbers in the two circles, not including the overlap.

c. Determine the probability of randomly selecting a person from the sample who reads Drive.

WE1c

Working

$5 + 3 = 8$ people

$$\begin{aligned}\Pr(\text{reads Drive}) &= \frac{8}{20} \\ &= \frac{2}{5}\end{aligned}$$

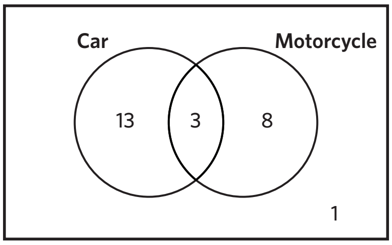
Thinking

Step 1: Determine the number of people who fit the description.

Step 2: Determine the probability and express it as a fraction in the simplest form.

Student practice

Consider the following Venn diagram showing people's responses relating to the vehicle they own.



a. How many people own a car?

b. How many people own a car or a motorcycle, but not both?

c. Determine the probability of randomly selecting a person from the sample who owns both types of vehicles.

Worked example 2

Interpreting information from two-way tables

Consider the following two-way table representing people's responses relating to their sushi eating habits.

	Eats raw fish	Does not eat raw fish	Total
Eats wasabi	10	4	**14**
Does not eat wasabi	9	2	**11**
Total	**19**	**6**	**25**

a. How many people eat raw fish?

WE2a

Working

19 people

Thinking

Find the information in the 'total' column or row for the relevant category.

Visual support

	Eats raw fish	Does not eat raw fish	Total
Eats wasabi	10	4	**14**
Does not eat wasabi	9	2	**11**
Total	**19**	**6**	**25**

Continues →

Back to contents

b. How many people eat raw fish or wasabi or both?

WE2b

Working

10 + 9 + 4 = 23 people

Thinking

Sum the totals for each category, as well as the number for both.

Visual support

	Eats raw fish	Does not eat raw fish	Total
Eats wasabi	10	4	**14**
Does not eat wasabi	9	2	**11**
Total	**19**	**6**	**25**

c. Determine the probability of randomly selecting a person from the sample who does not eat raw fish but eats wasabi.

WE2c

Working

4 people

Pr(does not eat raw fish but does eat wasabi) $= \frac{4}{25}$

Thinking

Step 1: Determine the number of people who fit the description.

Step 2: Determine the probability and express it as a fraction in the simplest form.

Student practice

Consider the following two-way table representing people's responses relating to their burger eating preferences.

	Eats bacon	Does not eat bacon	Total
Eats cheese	10	7	**17**
Does not eat cheese	4	5	**9**
Total	**14**	**12**	**26**

a. How many people eat bacon?

b. How many people eat bacon or cheese or both?

c. Determine the probability of randomly selecting a person from the sample who does not eat bacon or cheese.

Worked example 3

Constructing Venn diagrams and two-way tables

20 people were asked whether they make phone calls and send emails daily. 11 people said they make phone calls daily, 12 people said they email daily and 1 person said they do not do either daily.

a. Use the given information to construct a two-way table.

WE3a

Working

	Sends emails	Does not send emails	Total
Makes phone calls			
Does not make phone calls			
Total			

Thinking

Step 1: Draw the grid to make a two-way table and fill in the headings.

Continues →

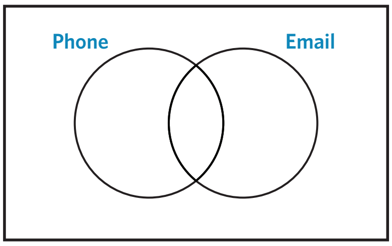
	Sends emails	Does not send emails	Total
Makes phone calls	4	7	**11**
Does not make phone calls	8	1	**9**
Total	**12**	**8**	**20**

Step 2: Fill in the numbers using the information provided.

b. Use the given information to construct a Venn diagram

WE3b

Working



Thinking

Step 1: Draw the rectangle and circles, and write the labels on the Venn diagram.

Overlap of Phone and Email $= 11 + 12 - (20 - 1)$

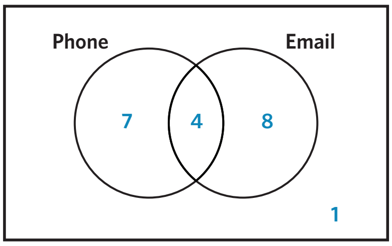
$= 4$

Step 2: Fill in the information that is known.

Phone only $= 11 - 4 = 7$

Email only $= 12 - 4 = 8$

Step 3: Determine what numbers are written in the blank parts of the Venn diagram.



Step 4: Fill in the remaining information.

Student practice

20 people were surveyed about the type of phone they own. 9 people said they own an Android, 8 people said they own an iPhone and 7 people said they own both.

a. Use the given information to construct a two-way table.

b. Use the given information to construct a Venn diagram.

Back to contents

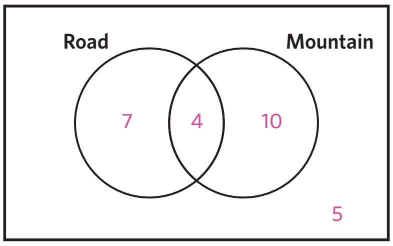
10A Questions

Understanding worksheet

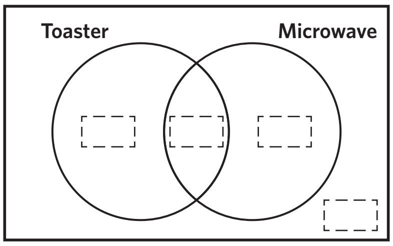
1. Using the two-way table, complete the missing information in the Venn diagram.

Example

	Road bike	**No road bike**	**Total**
Mountain bike	4	10	**14**
No mountain bike	7	5	**12**
Total	**11**	**15**	**26**



	Toaster	**No toaster**	**Total**
Microwave	15	5	**20**
No microwave	6	1	**7**
Total	**21**	**6**	**27**



2. Complete the missing information.

Example

	Likes dogs	**Dislikes dogs**	**Total**
Likes cats	9	4	**13**
Dislikes cats	8	3	**11**
Total	**17**	**7**	**24**

	Watch	**No watch**	**Total**
Ring			**21**
No ring		7	**9**
Total	**19**		**30**

3. Fill in the blanks by using the words provided.

exclusive **inclusive** **Venn diagram** **two-way table**

A ______ displays the frequency of two categories, written in a table format with rows and columns. A ______ provides a visual representation of similarities and differences between two or more sets of information. When interpreting two-way tables and Venn diagrams, the word ‘or’ can include the overlap of both categories, which makes it ______ , or it might be ______ , which does not include the overlap between the categories.

Back to contents

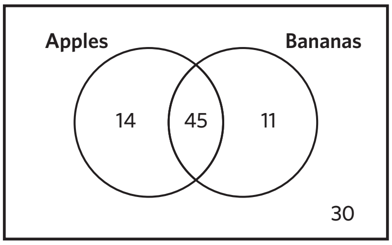
Fluency

Question working paths

Mild	**Medium**	**Spicy**
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b), 9	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c), 9	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (c,d), 9

4. Consider the following Venn diagram showing people's responses to a survey relating to which fruits they purchased at a supermarket.

WE1a,b



a. How many people were surveyed in total?

b. How many people purchased apples?

c. How many people purchased bananas only?

d. How many people did not purchase either fruit?

e. How many people purchased both apples and bananas?

f. How many people purchased apples or bananas, but not both?

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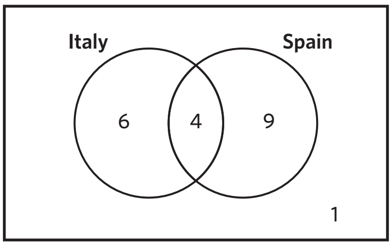
5. Consider the following Venn diagram showing people's responses to a survey relating to which countries they have visited. Determine the probability of randomly selecting a person from the sample who:

WE1c



a. has visited Spain only.

b. has visited Italy.

c. has visited both Italy and Spain.

d. has not visited Italy or Spain.

e. has visited Italy or Spain but not both.

f. has visited Italy only or neither Italy or Spain.

---

6. Consider the following two-way table representing people's responses to a survey relating to the languages they can speak.

WE2a,b

	Chinese	**Not Chinese**	**Total**
French	1	9	**10**
Not French	8	12	**20**
Total	**9**	**21**	**30**

a. How many people were surveyed in total?

b. How many people can speak neither Chinese or French?

c. How many people can speak Chinese only?

d. How many people can speak French but not Chinese?

e. How many people can speak both Chinese and French?

f. How many people can speak Chinese or French, but not both?

Back to contents

7. Consider the following two-way table representing people's responses to a survey relating to the video game consoles they own. Determine the probability of randomly selecting a person from the sample who:

WE2c

	Xbox	**Not an Xbox**	**Total**
PS5	8	21	**29**
Not a PS5	16	5	**21**
Total	24	26	**50**

a. owns an Xbox only.

b. does not own a PS5.

c. owns an Xbox.

d. owns both an Xbox and PS5.

e. owns neither an Xbox or PS5.

f. owns an Xbox or PS5.

8. Construct a two-way table and a Venn diagram given the following information.

WE3

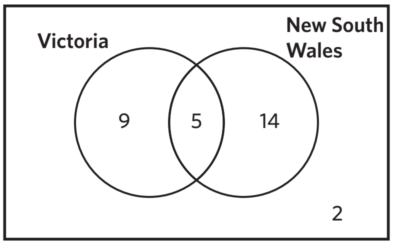
a. 18 participants were surveyed about their favourite type of desserts. Among them, 5 stated they liked cake, 8 like ice cream, and 10 like neither.

b. Among 28 surveyed participants about the type of mustard they have at home: 10 have dijon, 14 have wholegrain, and 8 have both.

c. 250 beachgoers were asked about the types of surfboards they own. Of these, 58 own a longboard, 120 own a shortboard, and 85 did not own a board.

d. In a survey of 180 musicians, 32 can play the trumpet, 90 can play the drums, and 75 can play the drums only.

9. The following Venn diagram shows the number of international travellers who visited different states in Australia. Which statement is correct?



A. There are 28 travellers in the data set.

B. There are 14 travellers who visit Victoria.

C. There are 23 travellers who visit both states.

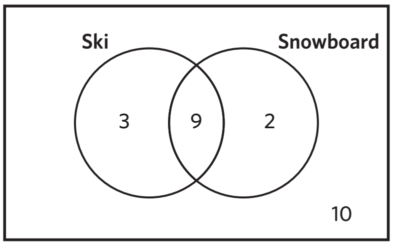
D. There are 14 travellers who visit New South Wales.

E. There are 2 travellers who do not visit Queensland.

Spot the mistake

10. Select whether Student A or Student B is incorrect.

The following Venn diagram shows the number of students that can ski and snowboard.



a. How many students can both ski and snowboard?

Student A

$3 + 2 = 5$ people

Student B

9 people

Back to contents

b. What is the probability of randomly selecting a student who can snowboard only?

Student A

People who can snowboard only $= 2$

$\Pr(\text{snowboard only}) = 2$

Student B

People who can snowboard only $= 2$

Total number of students $= 24$

$$\begin{aligned}\Pr(\text{snowboard only}) &= \frac{2}{24}\\ &= \frac{1}{12}\end{aligned}$$

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. In a book club of 12 friends, 8 members read books on a Kindle, 7 read a physical book, and 3 read both from a Kindle and physical books. Construct a Venn diagram to represent the situation.

12. Eighty Minecraft players were surveyed regarding their in-game building material usage. Of that group, 28 said that they used wood, 42 said that they used stones, and 10 said that they did not use either store or wood. Construct a two-way table to represent the scenario.

13. Among a group of 50 students, 25 like maths, 30 like science, and 10 like both. What's the probability of selecting a student from the group who likes maths only?

14. At a young artists studio, 17 artists focus solely on sculpture, 14 focus only on painting, 5 focus on neither and 9 embrace both forms of artistic expression. If a new artist joins the club, what is the likelihood they will focus on sculpture or painting?

15. Of the 70 puzzle enthusiasts completing a puzzle making course, 40 are interested in Sudoku, 25 are interested in crosswords, and 10 are puzzle enthusiasts who are interested in both. Determine the probability that a randomly selected puzzle lover only favours crosswords.

Reasoning

Question working paths

Mild 16 (a,b,c,e) 🌶	**Medium** 16 (a,b,c,e), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Music streaming services like Apple Music and Spotify have transformed how people access and enjoy music. In a survey of a group of 200 adults, 120 participants use Spotify, 90 use Apple Music, and 40 use both.

a. Create a Venn diagram illustrating this data.

b. Calculate the number of participants who use either Spotify, Apple Music, or both.

c. If a participant is chosen at random, what's the probability they use Spotify only?

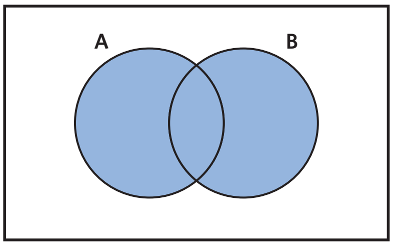
d. What percentage of participants use only Apple Music?

e. What factors should a subscriber consider before selecting which streaming service to subscribe to?

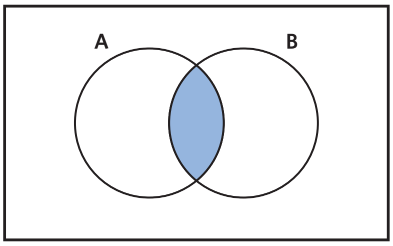
Back to contents

17. Consider the two Venn diagrams.

Venn diagram 1



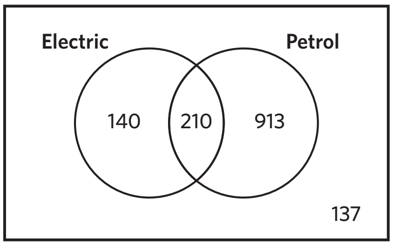
Venn diagram 2



- **a.** What does the shaded area in Venn diagram 1 represent?
- **b.** What does the shaded area in Venn diagram 2 represent?
- **c.** Compare your answers for part **a** and **b**, and explain which words are used to specify which part of the Venn diagram is represented.

Exam-style

18. A climate scientist is researching people's car preferences by recording the types of cars parked in a large city carpark. He categorises each car as either petrol, electric, or hybrid and shows his results in a Venn diagram. (1 MARK)



If the overlapping area represents Hybrid cars, the probability of randomly selecting a hybrid or non-electric car is

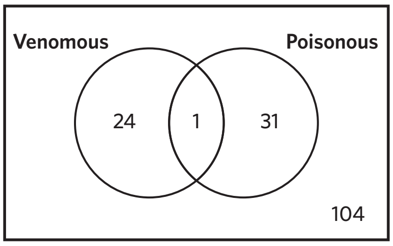
A. 0.1 **B.** 0.3 **C.** 0.5 **D.** 0.7 **E.** 0.9

19. A group of hotel guests were observed while selecting juices from the hotel buffet breakfast. Part of the results were recorded in a two-way table. (3 MARKS)

	Orange juice	**Not orange juice**	**Total**
Apple juice	6		**10**
No apple juice		7	
Total			**25**

- **a.** What is the total number of people who did not select orange juice? 1 MARK
- **b.** What is the probability of a person selecting both orange and apple juice? Give the answer as a percentage. 2 MARKS

20. A reptile veterinarian records the toxicity of 160 different species of wildlife that are housed in a reptile sanctuary. The veterinarian has recorded the results into the Venn diagram. (3 MARKS)



A reptile escapes from the sanctuary. What is the percentage probability that the escaped reptile is poisonous, or neither venomous or poisonous? Round the answer to two decimal places.

Back to contents

21. 75 participants were surveyed about the mode of transportation they use to commute to work. Out of those surveyed, 18 mentioned using bicycles, 39 used cars, and 32 did not use either. (3 MARKS)

What is the probability of selecting someone who uses both modes of transportation? Give the answer as a percentage, rounded to two decimal places.

Remember this?

22. James was saving money for a new bike.

By Friday evening, he had saved 40% of his target amount.

On Saturday, he received a gift of $50, which meant he had now saved 60% of his target amount.

What was James' target amount?

A. $200 **B.** $250 **C.** $300 **D.** $350 **E.** $400

23. John lives in Pineville. He is travelling by train to the city centre to attend a 16:00 meeting. John needs to arrive at the city centre 45 minutes before his meeting starts.

City train timetable			
Maple Grove	12:00 pm	1:00 pm	2:00 pm
Pineville	12:30 pm	1:30 pm	2:30 pm
Oak Street	12:45 pm	1:45 pm	2:45 pm
Birch Square	1:00 pm	2:00 pm	3:00 pm
City Centre	1:15 pm	2:15 pm	3:15 pm

What is the latest time John can catch a train from Pineville?

A. 12:30 pm **B.** 1:30 pm **C.** 2:15 pm **D.** 2:30 pm **E.** 3:15 pm

24. It takes 6 hours for 3 chefs to prepare meals for 300 guests at a banquet.

At the same rate, how many hours should it take 4 chefs to prepare meals for 300 guests?

A. 4 hours **B.** 4.5 hours **C.** 5.5 hours **D.** 6 hours **E.** 6.5 hours

Back to contents

10B Using set notation

LEARNING INTENTIONS

Students will be able to:

- describe different data sets using symbols and notations
- list data sets from descriptions and diagrams
- calculate probabilities for data sets that are represented with symbols.

In set theory, symbols and notations provide a streamlined method to represent and describe data sets. When decoded from descriptions and diagrams such as Venn diagrams, set notation allows for precise data extraction and organisation. Furthermore, this data can be used for calculating probabilities, offering a comprehensive approach to analyse, interpret, and predict outcomes based on given data sets.

KEY TERMS AND DEFINITIONS

- A **sample space** is a set of all possible outcomes or results of an experiment. This is also referred to as the universal set.
- A **subset** is a set that contains only elements that are also in the sample space, without any additional elements.
- A **complement** is a set of all elements in the universal set that are not in a given set.
- The **intersection** is the set of elements that two or more sets have in common.
- The **union** is the set of all elements that are in either of two or more sets.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Drazen Zigic/Shutterstock.com

In sports tournaments, understanding how to use symbols and notations helps in analysing team and player performance data. Set notations can be used to describe a set of players who scored goals in multiple matches, or players who participated in both soccer and basketball games in a school sports event.

Key ideas

1. Symbols and notations in set theory are used to describe different data sets.

$\xi = \{1, 2, 3, 4, 5\}$

$C = \{2, 3, 4, 5\}$

$D = \{3, 4, 5\}$

$E = \{\ \}$

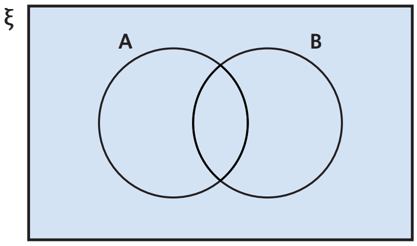
Symbol	Meaning	Example
U or ξ	The universal set, or sample space, is the set of all possible outcomes	$\xi = \{1, 2, 3, 4, 5\}$
$\cup$	Union of sets is the combination of elements in sets (or/both)	$C \cup D = \{2, 3, 4, 5\}$
$\cap$	Intersection of sets is the elements contained in both sets	$C \cap D = \{3, 4, 5\}$
$'$	Complement is the set of all elements in the universal set that are not in a given set	$D' = \{1, 2\}$
$\emptyset$	Empty set is a set that contains no elements	$E = \{\ \} = \emptyset$
n	Cardinal number is the number of elements in a set	$n(C) = 4$ $n(D) = 3$

Continues →

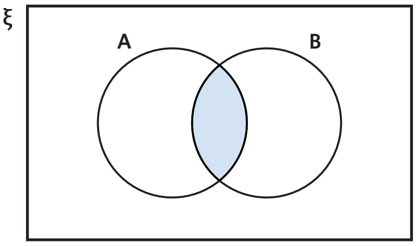
Back to contents

2. Venn diagrams show how data sets in a sample space are grouped.

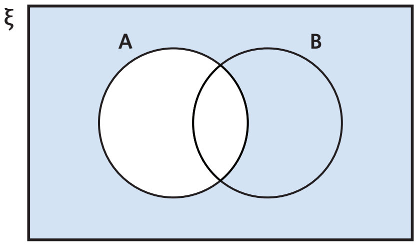
ξ: the universal set



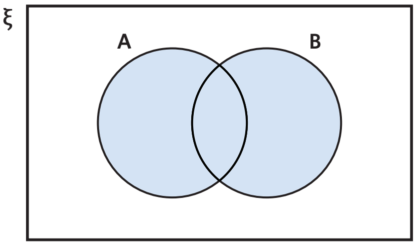
A ∩ B: elements in both A and B



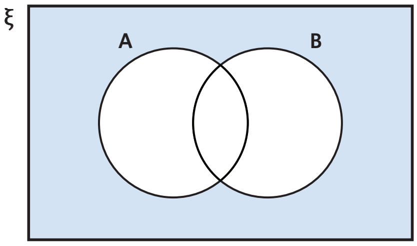
A′: elements not in A



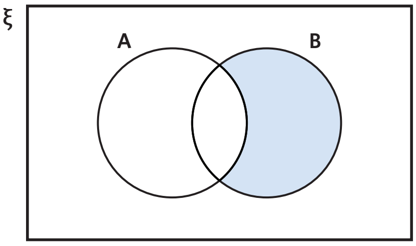
A ∪ B: elements in A or B or both



(A ∪ B)′: elements not in A or not in B



A′ ∩ B: elements not in A but in B



Worked example 1

Using set notation

A number is chosen from the set of positive integers between 11 and 20 inclusive. E is the set of odd numbers between 11 and 20 inclusive. P is the set of multiples of 3 between 11 and 20 inclusive.

a. List the sample space. **WE1a**

Working	Thinking
$\xi = \{\text{integers between 11 and 20, inclusive}\}$ $= \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$	List the sample space based on the described data sets.

b. List the set E. **WE1b**

Working	Thinking
$E = \{\text{odd numbers between 11 and 20, inclusive}\}$ $= \{11, 13, 15, 17, 19\}$	List the set E using set notation based on the given information.

c. List the set P. **WE1c**

Working	Thinking
$P = \{\text{multiples of 3 between 11 and 20, inclusive}\}$ $= \{12, 15, 18\}$	List the set P using set notation based on the given information.

Continues →

d. Draw a Venn diagram.

WE1d

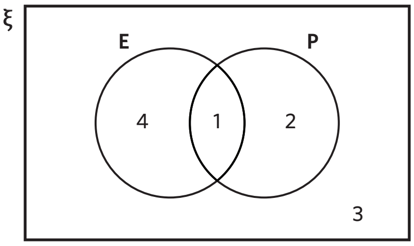
Working

E only = {11, 13, 17, 19}

P only = {12, 18}

Both E and P = {15}

Not E or P = {14, 16, 20}

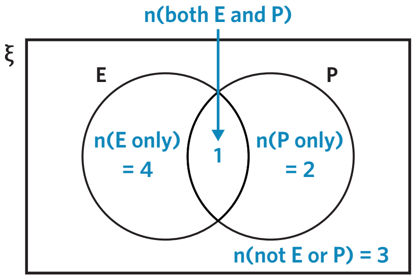


Thinking

Step 1: List the elements that belong to each region of the Venn diagram.

Step 2: Draw a Venn diagram and write the cardinal number of each corresponding region.

Visual support



Student practice

A number is chosen from the set of positive integers between 5 and 15 inclusive. M is the set of even numbers between 5 and 15 inclusive. N is the set of multiples of 4 between 5 and 15 inclusive.

a. List the sample space.

b. List the set M.

c. List the set N.

d. Draw a Venn diagram.

Worked example 2

Interpreting data sets using set notation

Consider the following sample space and subsets.

ξ = {11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

D = {14, 15, 17}

E = {11, 12, 15, 17, 18, 19}

a. List the set D ∪ E.

WE2a

Working

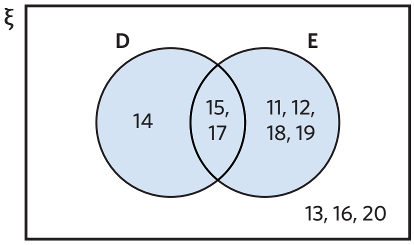
D ∪ E = {11, 12, 14, 15, 17, 18, 19}

Thinking

Use the union symbol (∪) to combine the elements of sets D and E without repetition.

Continues →

Visual support



b. List the set D ∩ E.

WE2b

Working

D ∩ E = {15, 17}

Thinking

Use the intersection symbol (∩) to represent the common elements between sets D and E.

c. List the set E'.

WE2c

Working

E' = {13, 14, 16, 20}

Thinking

Use the complement notation (') to list the elements in the universal set ξ that are not in set E.

Student practice

Consider the following sample space and subsets.

ξ = {1, 2, 3, 4, 5, 6, 7, 8}

A = {1, 3, 5, 7}

B = {1, 2, 5}

a. List the set A ∪ B. **b.** List the set A ∩ B. **c.** List the set B'.

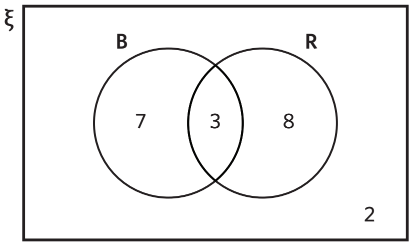
Worked example 3

Calculating probabilities using set notation

Calculate the probabilities.

a.

WE3a



Pr(B ∪ R)

Working

n(ξ) = 7 + 3 + 2 + 8 = 20

n(B ∪ R) = 7 + 3 + 8

= 18

Thinking

Step 1: Determine the number of elements in the union of B and R, and the sample space (ξ).

Continues →

Back to contents

$$\Pr(B \cup R) = \frac{18}{20}$$
$$= \frac{9}{10}$$

Step 2: Calculate the probability of B ∪ R by dividing the number of elements in B ∪ R by the number of elements in the sample space.

b. WE3b

	B	B’	Total
R	3	8	**11**
R’	7	2	**9**
Total	**10**	**10**	**20**

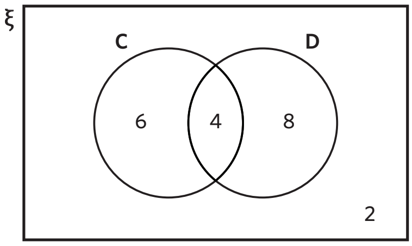
Pr(B ∩ R)

Working	Thinking
$n(B \cap R) = 3$ $n(\xi) = 20$	**Step 1:** Determine the number of elements in the intersection of B and R, and the sample space (ξ).
$\Pr(B \cap R) = \frac{3}{20}$	**Step 2:** Calculate the probability of B ∩ R by dividing the number of elements in B ∩ R by the number of elements in the sample space.

Student practice

Calculate the probabilities.

a.



Pr(C ∪ D)

b.

	C	C’	Total
D	4	8	**12**
D’	6	2	**8**
Total	**10**	**10**	**20**

Pr(C ∩ D)

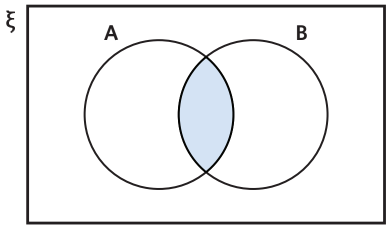
Back to contents

10B Questions

Understanding worksheet

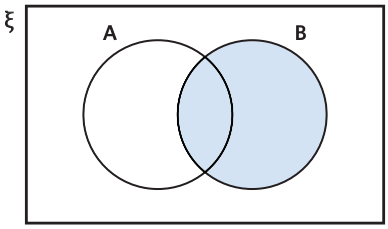
1. Complete the missing information to represent the shaded region.

Example



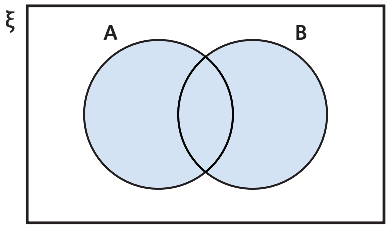
A $\boxed{\cap}$ B

a.



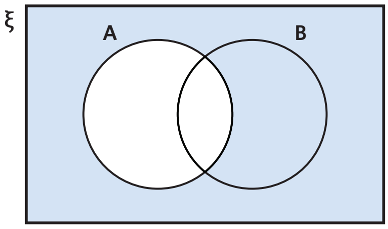
$\boxed{}$

b.



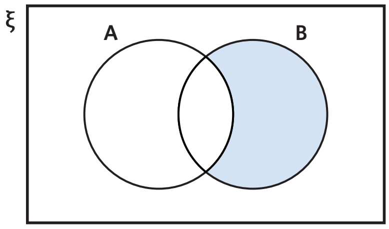
A $\boxed{}$ B

c.



$\boxed{}$

d.



A′ $\boxed{}$ B

2. Complete each of the following by filling in the blanks.

ξ = {1, 3, 5, 7, 8, 9, 10}
C = {1, 9, 10}
D = {1, 3, 5, 10}
E = { }

Example

E = { } = $\boxed{\emptyset}$

a. C $\boxed{}$ D = {1, 3, 5, 9, 10} **b.** C $\boxed{}$ D = {1, 10} **c.** $\boxed{}$ = {3, 5, 7, 8} **d.** n(C) = $\boxed{}$

3. Fill in the blanks by using the words provided.

complement **intersection** **sample space** **subset** **union**

Data sets often receive representation through specific symbols and notations. The entire $\boxed{}$ of possible outcomes in an experiment can be shown using various symbols. For instance, if one data set is entirely contained within another, it's referred to as a $\boxed{}$. Considering everything except a specific set refers to the set's $\boxed{}$. Representing the overlap or common elements between two sets uses the term $\boxed{}$. Conversely, to denote all elements in either of the sets or in both, the term $\boxed{}$ is used.

Back to contents

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c), 9	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d), 9	4 (e,f,g,h), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f), 9

4. Consider a sample space U of integers from 1 to 20 inclusive. C is a set that contains all even numbers from U, and set D contains all prime numbers from U. WE1,2

a. List the sample space.
b. List the set C.
c. List the set D.
d. Draw a Venn diagram.
e. List the set C ∪ D.
f. List the set C ∩ D.
g. List the set C′.
h. List the set D′.

5. Consider the following sample space and subsets. WE2

ξ = {31, 32, 33, 34, 35, 36, 37, 38, 39, 40}
F = {32, 34, 36}
G = {31, 32, 34, 37, 39}

a. List the set F ∪ G.
b. List the set F ∩ G.
c. Calculate n(G).
d. List the set G′.
e. List the set F′.
f. List the set F′ ∪ G.

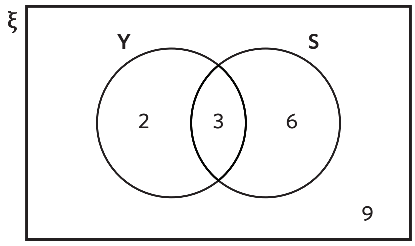
6. Consider the following sample space and subsets.

ξ = {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}
D = {22, 25, 27}
E = {21, 23, 26, 28, 29, 30}

a. Calculate Pr(D).
b. Calculate Pr(E).
c. Calculate Pr(D ∪ E).
d. Calculate Pr(D ∩ E).
e. Calculate Pr(D′).
f. Calculate Pr(E′).

7. Calculate the probabilities. WE3a

a. Pr(Y).
b. Pr(S).
c. Pr(Y ∪ S).
d. Pr(Y ∩ S).
e. Pr(Y′).
f. Pr(S′).



8. Calculate the probabilities. WE3b

a. Pr(F ∪ P).
b. Pr(F ∩ P).
c. Pr(F′).
d. Pr(F).
e. Pr(P).
f. Pr(P′).

	P	**P′**	**Total**
F	5	4	**9**
F′	10	6	**16**
Total	**15**	**10**	**25**

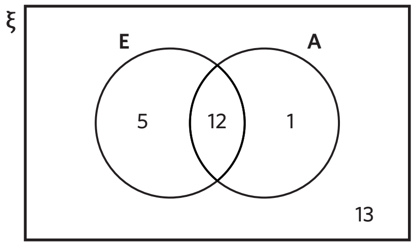
9. ξ = {2, 3, 4, 5, 6, 7, 8}
E = {2, 3, 8}
F = {2, 4, 5, 7}
The complement of F is equal to:

A. 4
B. {2}
C. {2, 3, 8}
D. {3, 6, 8}
E. {2, 4, 5, 7}

Back to contents

Spot the mistake

10. Select whether Student A or Student B is incorrect.



a. Find n(E ∩ A).

Student A

$n(E \cap A) = 5 + 12 + 1$
$= 18$

Student B

$n(E \cap A) = 12$

b. Find n(E’).

Student A

$n(E') = 1$

Student B

$n(E') = 1 + 13$
$= 14$

Problem solving

Question working paths

Mild 11, 12, 13 🌶️	**Medium** 12, 13, 14 🌶️🌶️	**Spicy** 13, 14, 15 🌶️🌶️🌶️

11. A school holiday program consists of 12 students who were all born in different months of the year. The program is split into Group 1 and Group 2. Group 1 consists of all the students that were born in the first half of the year and Group 2 consists of all the students that were born in the second half of the year. List the sets of Group 1 and Group 2.

12. A kennel has a group of different breeds of dogs. The group includes a Bulldog, Poodle, Boxer and a Beagle. List the set of A, if it is equal to {dog breeds with the letter o}.

13. A group of surgeons in a hospital are surveyed about their country of birth. The universal set is represented as ξ = {names of all surgeons}. The subsets of the universal set are represented as A = {names of surgeons born in Australia} and N = {names of surgeons not born in Australia}. Explain in words why $A \cap N = \emptyset$.

14. A group of 150 people attended a seminar on programming languages. Among them, 100 were proficient in Python (P), 75 were proficient in Java (J), and 40 were proficient in both languages. What is the probability of randomly selecting an individual from the group who is not proficient in Python?

15. Anita’s Gelato had a group of customers complete a poll on their favourite flavour. 103 customers said they like pistachio (P), 98 customers said they like stracciatella (S), while 27 customers said they liked both pistachio and stracciatella. Six customers liked neither flavour. What is the probability of randomly selecting a customer that completed the poll who satisfies the condition $P \cup S$?

Reasoning

Question working paths

Mild 16 (a,b,d) 🌶️	**Medium** 16 (a,b,d), 17 (a,b) 🌶️🌶️	**Spicy** All 🌶️🌶️🌶️

16. The Australian Football League (AFL) is made up of teams from around Australia. A total of 8 teams play the finals at the end of a season. B is equal to the teams that played finals in 2020 and C is equal to the teams that played finals in 2021.

All the teams	Teams that played finals in 2020 (B)	Teams that played finals in 2021 (C)
Adelaide Crows	Richmond Tigers	Melbourne Demons
Brisbane Lions	Geelong Cats	Western Bulldogs
Carlton Blues	Brisbane Lions	Geelong Cats
Collingwood Magpies	Port Adelaide Power	Port Adelaide Power
Essendon Bombers	St Kilda Saints	Brisbane Lions
Fremantle Dockers	Collingwood Magpies	Sydney Swans
Geelong Cats	West Coast Eagles	Essendon Bombers
Gold Coast Suns	Western Bulldogs	GWS Giants
Greater Western Sydney Giants		
Hawthorn Hawks		
Melbourne Demons		
North Melbourne Kangaroos		
Port Adelaide Power		
Richmond Tigers		
St Kilda Saints		
Sydney Swans		
West Coast Eagles		
Western Bulldogs		

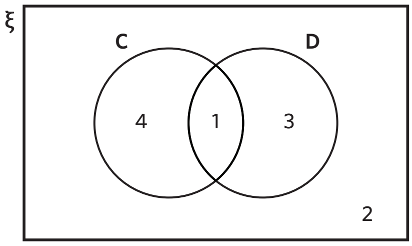
a. How many teams are there in the sample space?

b. List the set B ∩ C.

c. A Carlton Blues fan wants to change the team they support by selecting a team at random from the entire AFL. What is the probability that the fan selects a new team that played in both the 2020 and 2021 finals?

d. From 2017 to 2021, five teams did not make the finals once. Suggest one possible way the AFL Commission could help lower-performing teams improve.

17. Consider the diagram.



a. Calculate n(C) + n(D).

b. Calculate n(C ∪ D).

c. Compare and contrast the answers to parts **a** and **b** and explain when n(C) + n(D) may equal n(C ∪ D).

Exam-style

18. Consider a sample space S of integers from 1 to 10. Let set Y contain all even numbers from S and set Z contain all prime numbers from S. Which of the following represents Y ∩ Z? (1 MARK)

A. {2} **B.** {2, 4} **C.** {3, 5, 7} **D.** {2, 3, 5, 7} **E.** {2, 4, 6, 8}

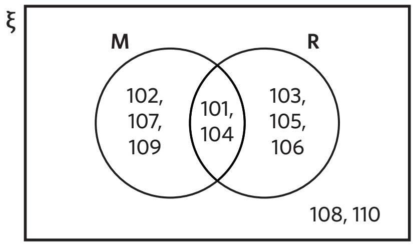
Back to contents

19. Consider a sample space S containing integers from 1 to 20. Let set A contain all multiples of 3 from S and set B contain all prime numbers from S. (3 MARKS)

a. What is the union of sets A and B? 1 MARK

b. If a number is randomly selected from the sample space S, what is the probability that the number belongs to the intersection of A and B? 2 MARKS

20. In a certain school, students were surveyed about their participation in the Mathematics Club (M) and the Robotics Club (R). Their responses were presented in the following Venn diagram with their student numbers placed in the relevant region. (3 MARKS)



Students who participated in both clubs will be considered for an advanced interdisciplinary workshop.

If a student is randomly selected from the entire sample space, what is the probability that they are being considered for the advanced workshop? Give the answer as a percentage.

21. The list of members of a book club is given by the sample space = {FRED, JASON, JACKIE, LUCIA, PETER, MOIRA, ELLEN, HARRY}. Set M consists of members who have read at least one mystery novel this month and is given by {FRED, MOIRA, ELLEN}. Set R represents members who have written a book review this month and is given by {JASON, LUCIA, ELLEN, HARRY}. (3 MARKS)

If a club member is picked at random from the complement of the union of M and R (denoted by (M∪R)’), which member(s) could it possibly be?

Remember this?

22. At a local bakery, a muffin and a cup of coffee are both priced at $4.

The bakery manager uses a formula to determine the amount of money collected from the sale of m muffins and c cups of coffee daily.

Which formula can the manager use to determine the amount of money collected from the sale of muffins and coffee in dollars?

A. $4mc$ **B.** $8mc$ **C.** $4m \times 4c$ **D.** $4(m + c)$ **E.** $48(m + c)$

23. Tornadoes are measured using the Fujita scale.

The scale number of a tornado indicates the wind speed the tornado generates in km/h.

A tornado measuring 2 on the Fujita scale generates 200 km/h of wind speed.

A tornado measuring 4 on the Fujita scale generates 300 km/h of wind speed.

A tornado measuring 4 on the Fujita scale generates how many times as much wind speed as a tornado measuring 2?

A. 0.5 **B.** 1 **C.** 1.5 **D.** 2 **E.** 2.5

24. This is part of a timetable for the train from City Center to Green Valley.

Sophia travels on the first train that leaves City Center after 11:00 am.

At what time will Sophia arrive at Green Valley?

A. 11:00 am

B. 12:00 pm

C. 12:15 pm

D. 1:00 pm

E. 2:00 pm

Departure times				
City Center	09:15	10:15	11:15	12:15
Midtown Station	09:45	10:45	11:45	12:45
Hillside	10:30	11:30	12:30	13:30
Green Valley	11:00	12:00	13:00	14:00

Back to contents

10C Using arrays for two-step experiments

LEARNING INTENTIONS

Students will be able to:

- interpret an array and list the sample space for experiments with two steps
- draw arrays for two-step experiments that are carried out with or without replacement
- calculate probabilities from arrays for two-step experiments that are carried out with or without replacement.

Interpreting arrays is crucial for understanding two-step chance experiments. This involves listing the sample space, which includes all possible outcomes. Arrays can be used when these experiments are carried out with or without replacement. From these arrays, probabilities can be calculated to determine the likelihood of specific outcomes.

KEY TERMS AND DEFINITIONS

- **Two-step experiments** describe a probability event that is made up of two actions.

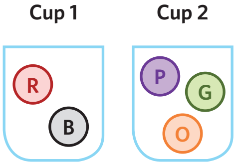
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: txking/Shutterstock.com

Understanding two-step chance experiments can be useful in predicting outcomes in board games, like rolling dice and moving game pieces. For instance, in a game of Monopoly, knowing the possible outcomes of rolling two dice helps players strategise their moves on the game board.

Key ideas

1. An **array** can be used to interpret two-step experiments and list the sample space of outcomes.



		Cup 2		
		Purple	Green	Orange
Cup 1	Red	(R,P)	(R,G)	(R,O)
	Black	(B,P)	(B,G)	(B,O)

Sample space = ξ = {(RP), (RG), (RO), (BP), (BG), (BO)}

2. In experiments with **replacement** or **without replacement**, items or outcomes are returned or not returned to the population or sample after each selection.

A card was selected from a hat:

With replacement

		Second selection		
		Jack	Queen	King
First selection	Jack	(J,J)	(J,Q)	(J,K)
	Queen	(Q,J)	(Q,Q)	(Q,K)
	King	(K,J)	(K,Q)	(K,K)

Sample space = ξ = {(J,J), (J,Q), (J,K), (Q,J), (Q,Q), (Q,K), (K,J), (K,Q), (K,K)}

There are 9 possible outcomes.

Without replacement

		Second selection		
		Jack	Queen	King
First selection	Jack	X	(J,Q)	(J,K)
	Queen	(Q,J)	X	(Q,K)
	King	(K,J)	(K,Q)	X

Sample space = ξ = {(J,Q), (J,K), (Q,J), (Q,K), (K,J), (K,Q)}

There are 6 possible outcomes.

Back to contents

Worked example 1

Interpreting two-step experiments with replacement

A bag contains one green (G) counter, one white (W) counter, one blue (B) counter and one purple (P) counter. Two counters are selected at random with replacement.

a. Draw an array and list the sample space. WE1a

Working

		Second selection			
		G	W	B	P
First selection	G				
	W				
	B				
	P				

		Second selection			
		G	W	B	P
First selection	G	(G,G)	(G,W)	(G,B)	(G,P)
	W	(W,G)	(W,W)	(W,B)	(W,P)
	B	(B,G)	(B,W)	(B,B)	(B,P)
	P	(P,G)	(P,W)	(P,B)	(P,P)

ξ = {(G,G), (G,W), (G,B), (G,P), (W,G), (W,W), (W,B), (W,P), (B,G), (B,W), (B,B), (B,P), (P,G), (P,W), (P,B), (P,P)}

Thinking

Step 1: Draw the array. Label the first event and all possible outcomes in the left column and the second event with all possible outcomes in the top row.

Step 2: Write all pairs of possible outcomes for the two events in the table.

Step 3: List the sample space.

b. Calculate Pr(B, B). WE1b

Working

Possible outcomes $= n(\xi) = 16$

$n\{(B,B)\} = 1$

$\text{Pr}(B,B) = \frac{1}{16}$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

c. Calculate probability that the first selected counter is blue. WE1c

Working

Possible outcomes $= n(\xi) = 16$

$n(\text{B first}) = 4$

$$\text{Pr}(\text{B first}) = \frac{4}{16}$$
$$= \frac{1}{4}$$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

Continues →

Student practice

A bag contains two green (G) counters, one silver (S) counter, and one yellow (Y) counter. Two counters are chosen at random with replacement.

a. Draw an array and list the sample space.

b. Calculate Pr(S,G)

c. Calculate probability that the first selected counter is green.

Worked example 2

Interpreting two-step experiments without replacement

A bag contains two green (G) counters, one yellow (Y) counter and one blue (B) counter. Two counters are chosen at random without replacement.

a. Draw an array and list the sample space.

WE2a

Working

		Second selection			
		G	G	Y	B
First selection	G				
	G				
	Y				
	B				

Thinking

Step 1: Draw the array. Label all possible outcomes, including identical ones, of the first event in the left column and the possible outcomes, including identical ones, of the second event in the top row.

		Second selection			
		G	G	Y	B
First selection	G	X	(G,G)	(G,Y)	(G,B)
	G	(G,G)	X	(G,Y)	(G,B)
	Y	(Y,G)	(Y,G)	X	(Y,B)
	B	(B,G)	(B,G)	(B,Y)	X

Step 2: Write all pairs of possible outcomes for the two events in the table. Without replacement means that the same counter cannot be chosen twice.

ξ = {(G,G), (G,Y), (G,B), (G,G), (G,Y), (G,B), (Y,G), (Y,G), (Y,B), (B,G), (B,G), (B,Y)}

Step 3: List the sample space.

b. Calculate the probability that both counters are green.

WE2b

Working

Possible outcomes $= n(\xi) = 12$

Both G = {(G,G),(G,G)}

n(Both G) = 2

$$\Pr(G,G) = \frac{2}{12} = \frac{1}{6}$$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

Continues →

c. Calculate the probability that one counter is green and one is blue.

WE2c

Working	Thinking
Possible outcomes $= n(\xi) = 12$	**Step 1:** Determine the number of possible outcomes.
G and B $= \{(B,G),(B,G),(G,B),(G,B)\}$ $n(\text{G and B}) = 4$	**Step 2:** Determine the number of desired outcomes.
$\Pr(\text{G and B}) = \frac{4}{12}$ $= \frac{1}{3}$	**Step 3:** Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

Student practice

A bag contains one green (G) counter, one purple (P) counter, one blue (B) counter and one white (W) counter. Two counters are chosen at random without replacement.

a. Draw an array and list the sample space.

b. Calculate the probability that one counter is purple.

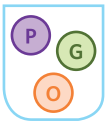
c. Calculate the probability that one counter is blue and one counter is white.

10C Questions

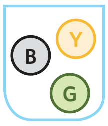
Understanding worksheet

1. Fill in the blanks in the array for an experiment where two coloured counters are selected from a cup with replacement.

Example



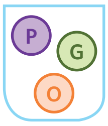
		Second selection		
		P	**G**	**O**
	P	(P,P)	(P,G)	(B,O)
First selection	**G**	(G,P)	(G,G)	(G,O)
	O	(O,P)	(O,G)	(O,O)



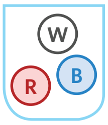
		B	**G**	**Y**
		(B,B)	(B,G)	(B,Y)
First selection	**G**	(G,B)	(G,G)	
	Y		(Y,G)	(Y,Y)

2. Fill in the missing outcomes in the array for an experiment where two coloured counters are selected from a cup without replacement.

Example



		Second selection		
		P	**G**	**O**
First selection	**P**	X	(P,G)	(P,O)
	G	(G,P)	X	(G,O)
	O	(O,P)	(O,G)	X



		Second Selection		
		R	**B**	**W**
		X	(R,B)	
	B	(B,R)		(B,W)
	W	(W,R)	(W,B)	X

3. Fill in the blanks by using the words provided.

array	probability	replacement	two-step

When conducting ______ experiments, it's crucial to understand how outcomes combine. One way to represent outcomes is through an ______ , which lists all possible combinations. This is particularly helpful when dealing with chance experiments carried out with or without ______ . When comparing experiments that only differ by being conducted with or without replacement, we notice that the order of outcomes matters and the ______ of specific outcomes change.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c), 9	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d), 9	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f), 9

4. A bag contains two yellow (Y) counters, one white (W) counter and one blue (B) counter. Two counters are chosen at random with replacement. **WE1**

a. Draw an array and list the sample space.

b. Calculate the probability that both counters are yellow.

c. Calculate the probability that one counter is blue and one counter is white.

d. Calculate Pr(Y,W).

e. Calculate the probability that at least one counter is blue.

f. Calculate the probability that the counters are the same colour.

5. A coin is tossed then a fair six-sided die is rolled. WE1

- **a.** Draw an array and list the sample space.
- **b.** Calculate the probability of obtaining (H,4).
- **c.** Calculate probability of tossing a head and an even number.
- **d.** Calculate Pr((H,2) or (T,3)).
- **e.** Calculate Pr(H, positive integer).
- **f.** Calculate Pr(T, prime number).

6. Two six-sided fair dice are thrown. WE1

- **a.** Draw an array and list the sample space.
- **b.** Calculate Pr(6,5).
- **c.** Calculate the probability of rolling the same number twice.
- **d.** Calculate the probability of rolling exactly one 3.
- **e.** Calculate the probability that the sum of the rolls is greater than 7.
- **f.** Calculate the probability that the sum of the rolls is a prime number.

7. A bag contains one green (G) counter, one purple (P) counter, and one white (W) counter. Two counters are chosen at random without replacement. WE2

- **a.** Draw an array and list the sample space.
- **b.** Calculate Pr(W,G).
- **c.** Calculate the probability that both counters are white.
- **d.** Calculate the probability that one counter is purple.
- **e.** Calculate the probability that the second counter selected is green.
- **f.** Calculate the probability that at least one of the counters is purple or green.

8. Two letters are selected from the word MEET without replacement. WE2

- **a.** Draw an array and list the sample space.
- **b.** Calculate Pr(E,M).
- **c.** Calculate the probability that the first letter is E.
- **d.** Calculate the probability that exactly one letter is E.
- **e.** Calculate probability of selecting M second.
- **f.** Calculate the probability that both letters are E.

9. Two fair coins are thrown. What is the probability of landing at least one head?

- **A.** $\frac{1}{2}$
- **B.** $\frac{3}{4}$
- **C.** 2
- **D.** 3
- **E.** 4

Back to contents

Spot the mistake

10. Select whether Student A or Student B is incorrect.

Two letters are selected from the word TRAY without replacement

a. Draw an array and list the sample space.

Student A

		2nd			
		T	**R**	**A**	**Y**
1st	**T**	(T,T)	(T,R)	(T,A)	(T,Y)
	R	(R,T)	(R,R)	(R,A)	(R,Y)
	A	(A,T)	(A,R)	(A,A)	(A,Y)
	Y	(Y,T)	(Y,R)	(Y,A)	(Y,Y)

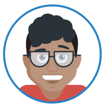
ξ = {(T,T), (T,R), (T,A), (T,Y), (R,T), (R,R), (R,A), (R,Y), (A,T), (A,R), (A,A), (A,Y), (Y,T), (Y,R), (Y,A), (Y,Y)}

Student B

		2nd			
		T	**R**	**A**	**Y**
1st	**T**	X	(T,R)	(T,A)	(T,Y)
	R	(R,T)	X	(R,A)	(R,Y)
	A	(A,T)	(A,R)	X	(A,Y)
	Y	(Y,T)	(Y,R)	(Y,A)	X

ξ = {(T,R), (T,A), (T,Y), (R,T), (R,A), (R,Y),(A,T), (A,R), (A,Y), (Y,T), (Y,R), (Y,A)}

b. Calculate the probability of first selecting the letter A.



Student A

Possible outcomes = n(ξ) = 12

A first = {(A,T), (A,R), (A,Y)}

n(A first) = 3

$\text{Pr(A first)} = \frac{3}{12}$

$= \frac{1}{4}$

Student B

Possible outcomes = n(ξ) = 12

Letter A = {(T,A), (R,A), (A,R), (A,Y), (Y,A)}

n(letter A) = 6

$\text{Pr(A)} = \frac{6}{12}$

$= \frac{1}{2}$

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. Phil is trying to predict the winner of the Australian Open men's and women's competitions by placing the female players' names Keys (K) and Swiatek (S) in one hat and male players' names Djokovic (D) and Sinner (S) in another hat. Create an array to show all possible outcomes of selecting from the women's names first followed by the men.

12. Frederick has two boxes of coloured spray paint. One box has black and purple spray, and the second box has black, green and yellow spray. If he randomly chooses one spray from each box, what is the probability he selected at least one black spray?

13. The names Frank, Gigi, Ibrahim, Davis, Micahel and Carlotta are placed in a hat so a teacher can select pairs randomly for a group project. The teacher takes a name, does not replace it and then takes another name to form a pair. What is the probability that the first pair selected is Davis and Carlotta?

14. Each turn in a game of backgammon a player rolls two fair six-sided dice. What is the probability that the sum of the two dice is greater than 10?

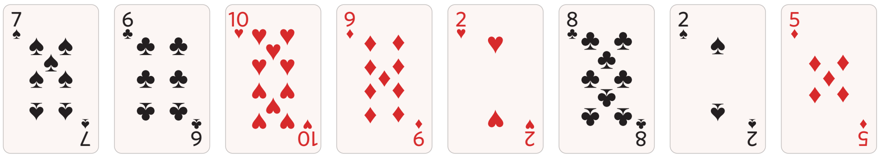
15. Zac has a king of hearts (red), diamonds (red), clubs (black) and spades (black) in his pocket. If he picks two cards from his pocket at random and without replacing the first card selected, what is the probability both cards are red?

Reasoning

Question working paths

Mild 16 (a,b,d)	**Medium** 16 (a,b,d), 17 (a,b)	**Spicy** All

16. Felix and Rosanna have a selection of random cards that they found on the train. They are playing a game by selecting cards at random with their eyes closed. Shown below are the cards that they found.



a. Felix selects a card and does not replace it. He then selects another card. Construct an array to represent the scenario. Represent each outcome in the array by writing the card's number and its suit. For example, if he selected the 7 of spades and then 10 of hearts this would be shown as (7S,10H).

b. What is the probability that Felix selected two cards that are both 5 or below?

c. Rosanna selects a card and replaces it. She then selects another card. Calculate the probability that the cards she selected are the same.

d. Suggest a game that Felix and Rosanna could play with the selection of cards.

17. Two letters are selected from the word RAT. Array 1 shows the experiment with replacement. Array 2 shows the experiment without replacement.

Array 1

		2nd		
		R	**A**	**T**
1st	**R**	(R,R)	(R,A)	(R,T)
	A	(A,R)	(A,A)	(A,T)
	T	(T,R)	(T,A)	(T,T)

Array 2

		2nd		
		R	**A**	**T**
1st	**R**	X	(R,A)	(R,T)
	A	(A,R)	X	(A,T)
	T	(T,R)	(T,A)	X

a. Calculate Pr(R,A) for the experiment with replacement.

b. Calculate Pr(R,A) for the experiment without replacement.

c. Compare and contrast the answers from part **a** and **b**. Explain how replacement affects the probability of some outcomes.

Exam-style

18. Maya is conducting a two-step experiment. In the first step, she randomly selects a card from a deck of 52 playing cards and records the suit. In the second step, she rolls a fair six-sided die. How many possible outcomes are there for the experiment? (1 MARK)

A. 24
B. 36
C. 48
D. 72
E. 96

19. In a two-step experiment a spinner is spun and an eight-sided fair die is rolled. The spinner contains 4 equally sized sectors labelled A, B, C, and D. (3 MARKS)

a. Calculate the number of possible outcomes for this two-step experiment. 1 MARK

b. Calculate the probability of outcome containing both a vowel and an odd number. 2 MARKS

20. Two six-sided fair dice are thrown in an experiment. The diagram below shows an array to represent the two-way experiment. (2 MARKS)

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

Calculate the probability of both rolls summing to less than 9.

21. A drawer in a fridge contains two red capsicums, two green capsicums and one yellow capsicum. Two capsicums are selected at random without replacement. (3 MARKS)

Calculate the probability that the pair of selected capsicums are different in colour.

Remember this?

22. Which of these can be measured in square metres?

A. Time **B.** Area **C.** Weight **D.** Volume **E.** Temperature

23. Which of these expressions is equivalent to n^3?

A. $3n^2$ **B.** $n^2 \times n^2$ **C.** $n \times 3$ **D.** $n \times n \times n$ **E.** $n + n + n$

24. A group of 300 employees in a company were asked which type of hot beverage they prefer.

The table shows the results.

One of these employees is chosen at random.

What is the probability that the employee prefers a latte?

A. 0.1

B. 0.2

C. 0.3

D. 0.4

E. 0.5

Hot Beverage Type	Number of Employees
Espresso	60
Latte	90
Cappuccino	75
Black coffee	45
Tea	30

Back to contents

10D Tree diagrams

LEARNING INTENTIONS

Students will be able to:

- interpret a tree diagram and list the sample space for experiments with two steps
- draw tree diagrams for two step experiments that are carried out with or without replacement
- calculate probabilities from tree diagrams for two-step experiments that are carried out with or without replacement.

Tree diagrams map out sequences of events, with each branch representing a potential outcome after each event. Tree diagrams can be used to calculate the number of possible outcomes from a two-step experiment and to calculate event probabilities by tracing their way through the branches of the tree. Tree diagrams are useful in interpreting results within the context of a problem, which is vital for real-world applications of probability.

KEY TERMS AND DEFINITIONS

- A **tree diagram** is a visual representation used to display all possible outcomes for a series of sequential events or decisions.
- A **node** represents a point where a decision or event occurs, branching out into different possible outcomes or paths.

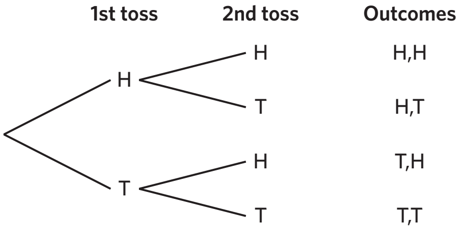
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: nadia_if/Shutterstock.com

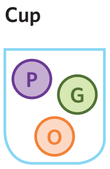
Tree diagrams can be used to help understand and predict the outcomes of cross-breeding plant species. Plant genetics often involves the study of flower colours. For instance, a plant might have dominant genes for purple flowers (P) and recessive genes for white flowers (W). When cross-breeding two plants that both have a mix of these genes (PW), a tree diagram can help illustrate the possible colour outcomes for their offspring.

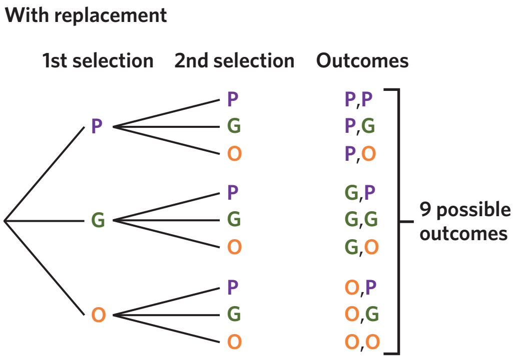
Key ideas

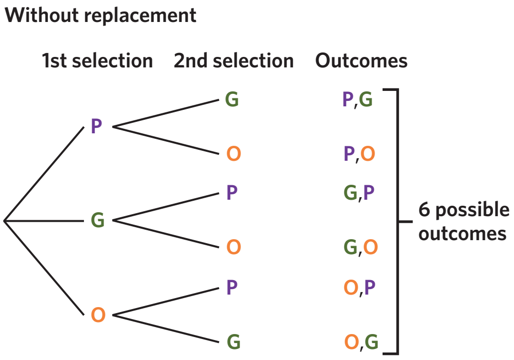
1. A tree diagram can be used to list the sample space and outcomes of experiments with two or more steps.



2. Tree diagrams show that experiments with or without replacement have a different number of possible outcomes. This affects the probability of specific events







Worked example 1

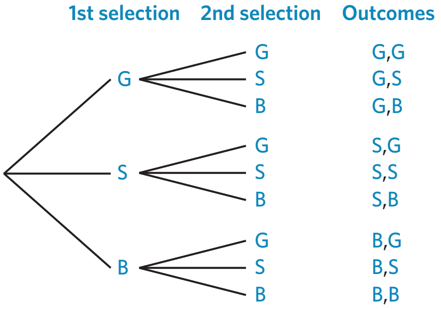
Interpreting two-step experiments with replacement

A bag contains one gold (G) coin, one silver (S) coin, and one bronze (B) coin. Two coins are selected at random with replacement.

a. Draw a tree diagram and list the sample space.

WE1a

Working



ξ = {(G,G), (G,S), (G,B), (S,G), (S,S), (S,B), (B,G), (B,S), (B,B)}

Thinking

Step 1: Label the first and second event and draw a branch for each possible outcome for each event.

Step 2: Write all pairs of possible outcomes for the two events.

Step 3: List the sample space.

b. Calculate Pr(S,S).

WE1b

Working

Possible outcomes = n(ξ) = 9

n{(S,S)} = 1

Pr(S,S) = $\frac{1}{9}$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

c. Calculate the probability that at least one counter is gold.

WE1c

Working

Possible outcomes = n(ξ) = 9

At least one G = {(G,G), (G,S), (G,B), (S,G), (B,G)}

n(at least one G) = 5

Pr(at least one G) = $\frac{5}{9}$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

Student practice

A bag contains one black (B) counter, one white (W) counter, and one red (R) counter. Two counters are selected at random with replacement.

a. Draw a tree diagram and list the sample space.

b. Calculate Pr(B,W).

c. Calculate the probability that the counters are the same colour.

Back to contents

Worked example 2

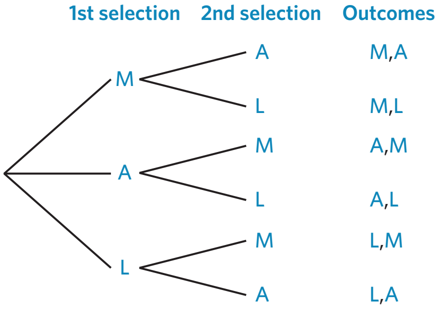
Interpreting two-step experiments without replacement

A hat contains the names Massie (M), Arjun (A), and Li (L). Two names are chosen at random without replacement.

a. Draw a tree diagram and list the sample space.

WE2a

Working



ξ = {(M,A), (M,L), (A,M), (A,L), (L,M), (L,A)}

Thinking

Step 1: Label the first and second event and draw a branch for each possible outcome for each event.

Step 2: Write all the possible outcomes.

Step 3: List the sample space.

b. Calculate Pr(A,L).

WE2b

Working

Possible outcomes = n(ξ) = 6

n{(A,L)} = 1

$\text{Pr}(A,L) = \frac{1}{6}$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

c. Calculate the probability of selecting Li (L).

WE2c

Working

Possible outcomes = n(ξ) = 6

Contains L = {(M,L), (A,L), (L,M), (L,A)}

n(contains L) = 4

$$\text{Pr(contains L)} = \frac{4}{6}$$
$$= \frac{2}{3}$$

Thinking

Step 1: Determine the number of possible outcomes.

Step 2: Determine the number of desired outcomes.

Step 3: Write the probability as a fraction of the number of desired outcomes out of the total number of possible outcomes, in the simplest form.

Student practice

A hat contains the names Timothy (T), Brigette (B), and Sunita (S). Two names are chosen at random without replacement.

a. Draw a tree diagram and list the sample space.

b. Calculate Pr(T,S).

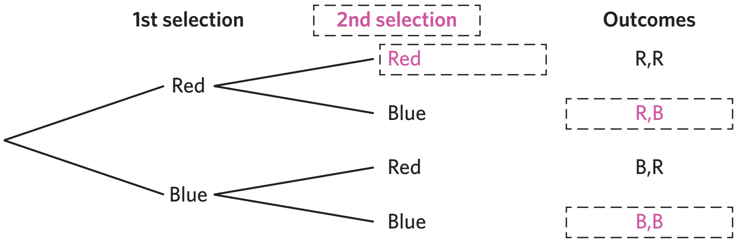
c. Calculate the probability of selecting Timothy (T).

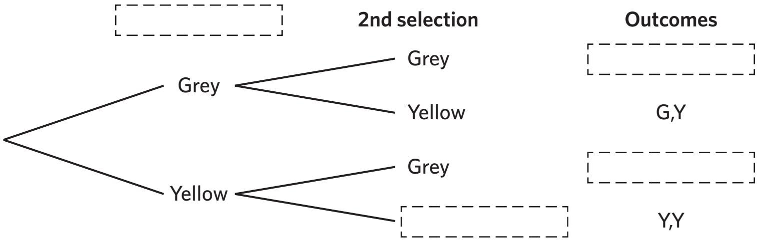
10D Questions

Understanding worksheet

1. Fill in the blanks in the tree diagram for an experiment where two coloured counters are selected from a cup with replacement.

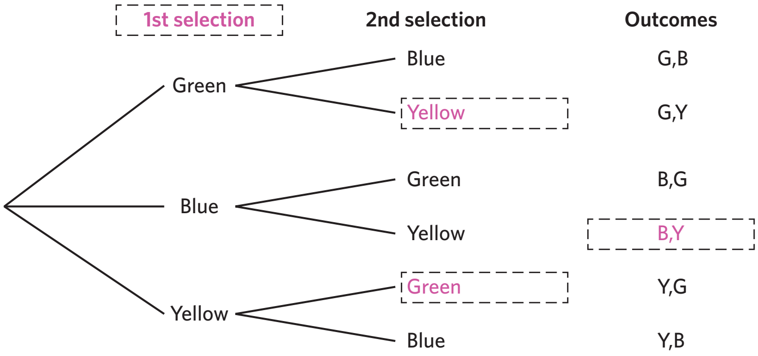
Example

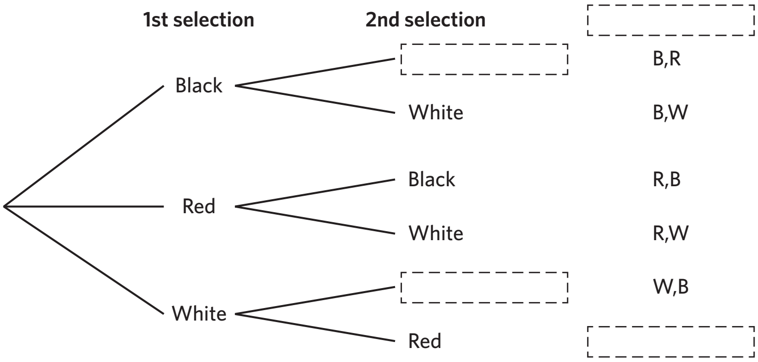




2. Fill in the blanks in the tree diagram for an experiment where three coloured counters are selected from a cup without replacement.

Example





3. Fill in the blanks by using the words provided.

events | **replacement** | **tree diagram** | **two-step**

One way to represent outcomes of a ________ experiment is through a ________ , which lists all possible combinations. When an experiment with two ________ is conducted with or without ________ , the number of outcomes will vary.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c), 9	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d), 9	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f), 9

4. A fridge contains one can of Coca-Cola (C), and one can of Fanta (F). Two cans are selected at random with replacement. **WE1**

a. Draw a tree diagram and list the sample space.
b. Calculate Pr(C,C).
c. Calculate the probability that at least one can is Fanta.
d. Calculate the probability of the first can being a Coca-Cola.
e. State the number of potential outcomes after the first event.
f. Calculate the probability of not selecting a Fanta second.

---

5. A bag contains one green (G) counter, one purple (P) counter, and one turquoise (T) counter. Two counters are selected at random with replacement. **WE1**

a. Draw a tree diagram and list the sample space.
b. Calculate Pr(T,G).
c. Calculate the probability that both counters are purple.
d. Calculate the probability that at least one counter is turquoise.
e. Calculate the probability that the counters are the same colour.
f. Calculate the probability that the second selected counter is green or purple.

---

6. A drawer contains a fork (F), knife (K) and spoon (S). Two utensils are selected at random without replacement. **WE2**

a. Draw a tree diagram and list the sample space.
b. Calculate Pr(K,S).
c. Calculate the probability that at least one utensil is a fork.
d. Calculate the probability of selecting a spoon first.
e. Calculate the probability of selecting a knife first or second.
f. Calculate the probability of selecting a fork or a spoon.

---

7. A bookshelf contains two autobiographies (A), one fiction (F) book, and one picture (P) book. Two books are selected at random without replacement. **WE2**

a. Draw a tree diagram and list the sample space.
b. Calculate Pr(F,P).
c. Calculate the probability that both books are an autobiography.
d. Calculate the probability of selecting a fiction book first.
e. Calculate the probability that at least one book is an autobiography.
f. Calculate the probability that the second book selected is a fiction or picture book.

Back to contents

8. A bag contains coins with the numbers 1, 4, and 7 on them. Two coins are chosen at random without replacement.

- **a.** Draw a tree diagram and list the sample space.
- **b.** Calculate Pr(4,7).
- **c.** Calculate the probability of selecting a 7.
- **d.** Calculate the probability that the sum of the numbers selected is greater than 5.
- **e.** Calculate the probability that the sum of the numbers is not a prime number.
- **f.** Calculate the probability that the product of the numbers is an even number.

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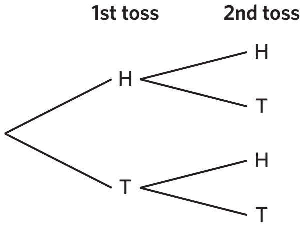
9. A fair sided coin is tossed twice.



What is the probability of landing a head (H) then a tail (T)?

A. $\frac{1}{6}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{2}$ **D.** 1 **E.** 2

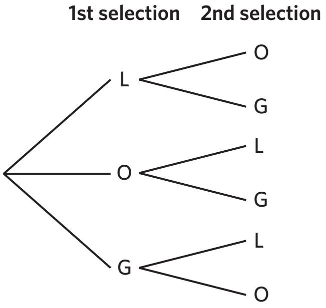
Spot the mistake

10. Select whether Student A or Student B is incorrect.

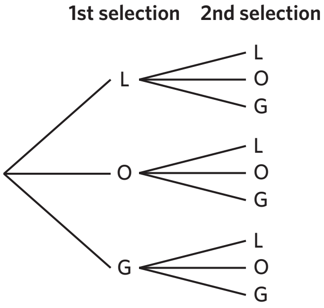
a. Two letters are selected from the word LOG with replacement.

Draw a tree diagram.

Student A



Student B



b. Two letters are selected from the word LOG with replacement.

Calculate the probability of selecting the letter G twice.

Student A

Number of events = 2

n(G,G) = 1

$\Pr(G,G) = \frac{1}{2}$

Student B

Possible outcomes = 9

n(G,G) = 1

$\Pr(G,G) = \frac{1}{9}$

Back to contents

Problem solving

Question working paths

Mild 11, 12, 13 🌶	**Medium** 12, 13, 14 🌶🌶	**Spicy** 13, 14, 15 🌶🌶🌶

11. There are two egg (E) sandwiches and one cheese (C) sandwich left on a shelf at a canteen. Sashin randomly selects two sandwiches. Construct a tree diagram and list the sample space for this scenario.

12. Ben takes the train to and from work. There is an equal chance that the trains will be on time (T) or late (L). Calculate the probability that the two trains he rides on the same day will both be late.

13. Each question on the learner's permit test contains answer options A, B, C or D. Mitchel randomly selects the answers for the last two questions. What is the probability that he selected C then D?

14. Louisa is picking names out of a hat to create pairs for a badminton competition. If there are 6 different names in the hat, what is the probability that the first pair she picks is Darren and Jessica?

15. Rishmi plays a card game in which there is an equal chance of a win, loss or draw. What is the probability that she draws three consecutive matches?

Reasoning

Question working paths

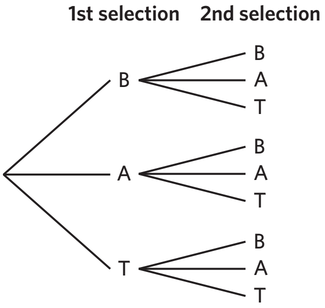
Mild 16 (a,b,d) 🌶	**Medium** 16 (a,b,d), 17 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

16. Erica is a corporate lawyer and has to dress in a suit and shirt when visiting clients. In her wardrobe she has one black suit (B), one navy (N) suit and one maroon (M) suit. She also has one white (W) shirt and one grey (G) shirt. Erica washes her suit and shirt daily, so she never wears the same suit or shirt on consecutive days.

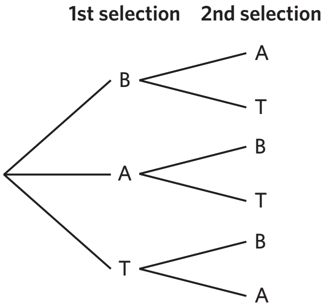
- **a.** On Monday, Erica selects a suit and shirt. Draw a tree diagram to represent the scenario and list the sample space.
- **b.** Calculate the probability that Erica randomly selects the maroon suit and grey shirt combination.
- **c.** Erica is travelling for work and randomly selects the two suits she will pack. Calculate the probability that she packs the navy suit for her work trip.
- **d.** Erica is considering asking her boss to dress more informally for client meetings. Suggest a reason why dressing professionally is important for a corporate lawyer.

17. Two letters are selected from the word BAT. The two tree diagrams show the experiment with or without replacement.

With replacement



Without replacement



- **a.** Calculate Pr(A,A) for the experiment with replacement.
- **b.** Calculate Pr(A,A) for the experiment without replacement.
- **c.** Compare and contrast the answers from parts **a** and **b**. Explain how replacement affects the probability of some outcomes.

Exam-style

18. Sarah is planning her weekend activities. On Saturday, she can either go for a hike, go to the beach, or stay home. On Sunday, she can choose to either read a book, visit a friend, or go shopping. If Sarah wants to plan her weekend by randomly selecting one activity for each day, how many different combinations of weekend plans are possible? (1 MARK)

A. 3 **B.** 5 **C.** 6 **D.** 9 **E.** 12

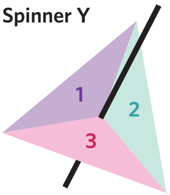
19. Sarah has two packs of lentil (L) pasta, one bag of gluten free (G) pasta in her backpack for a camping trip as she cannot restock her bag during the trip. She randomly selects one pack of pasta for Wednesday and another for Thursday. (3 MARKS)

a. Draw a tree diagram and list all possible outcomes. 1 MARK

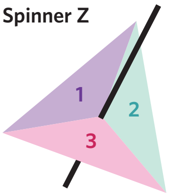
b. Calculate the probability that Sarah selects at least one pack of gluten free pasta. 2 MARKS

20. Two fair 3-sided spinners are used in a two-step experiment. The spinners are shown below. (3 MARKS)

Spinner Y



Spinner Z



Calculate the probability that spinner Y lands on 3 and spinner Z does not land on 1.

21. Two fair 6-sided dice are rolled. (3 MARKS)

Calculate the probability that one die lands on 3 and the other lands on a number equal to or greater than 4.

Remember this?

22. 25 students take a maths test.

A teacher records the scores of each student.

The results are shown in the stem-and-plot.

Stem	Leaf
6	0 2 3
7	1 2 3 4 5 5 6
8	0 1 2 3 4 4 5 6 7
9	0 2 3 3 5 7

Key

7 | 1 = 71

What is the median score?

A. 72 points **B.** 76 points **C.** 80 points **D.** 82 points **E.** 84 points

23. A school is purchasing new library books worth $8 500. They were given a 15% discount by the supplier. What was the price paid by the school after the discount?

A. $7125 **B.** $7225 **C.** $7325 **D.** $7525 **E.** $8000

24. A bakery has 750 cookies to pack into tins. When full, each tin holds 5 layers of cookies with 18 cookies in each layer. The bakery packs as many full tins as possible. How many cookies are left over?

A. 10 **B.** 12 **C.** 15 **D.** 20 **E.** 30

Back to contents

10E Experimental probability

LEARNING INTENTIONS

Students will be able to:

- calculate the experimental probability of an event given the results of the experiment
- calculate the expected number of occurrences given a probability and a number of trials
- understand that running more trials generally gives a better estimate of the true probability of an experiment.

Experimental probability provides insights into the likelihood of an event based on actual outcomes from experiments. By analysing these outcomes, it's possible to determine the expected number of occurrences for a given number of trials. Furthermore, increasing the number of trials typically offers a more accurate estimate of an experiment's true probability, enhancing the reliability of predictions.

KEY TERMS AND DEFINITIONS

- An **experiment** is a series of trials conducted to examine the results of chance activities.
- **Frequency** refers to how many times an event occurs.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Gorodenkoff/Shutterstock.com

Manufacturing industries frequently test products for defects. Determining the experimental probability of a defect in a batch of products can be helpful for production. For example, if a phone factory has a historical probability for defects, they can predict the expected number of faulty phones in the entire production.

Key ideas

1. Experimental probability, also referred to as relative frequency, is determined by conducting multiple trials in an experiment.

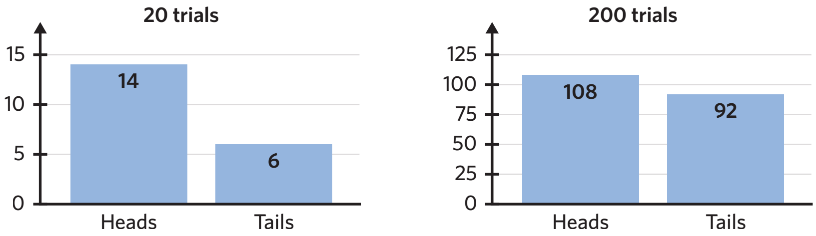
$$\text{Pr(event)} = \frac{\text{frequency of event}}{\text{total number of trials}}$$

2. The accuracy of estimating the true probability of an experiment generally improves with more trials, as larger sample sizes tend to approach the theoretical probability.

Theoretical probability:

$$\text{Pr(heads)} = \frac{1}{2} = 50\%$$

Experimental probability:



$\text{Pr(heads)} = \frac{14}{20} = 70\%$

$\text{Pr(heads)} = \frac{108}{200} = 54\%$

3. The expected number of occurrences of an event, states how many times an event is expected to occur. It is calculated using probability of the event and the total number of trials conducted.

Expected number of occurrences = Pr(event) × number of trials

Worked example 1

Calculating experimental probability and expected occurrences

A spinner has the numbers 1 to 6 inclusive. The spinner is spun 100 times and lands on the number 5 forty times.

a. Calculate the experimental probability of landing on the number 5. WE1a

Working	Thinking
Trials: 100 Frequency: 40	**Step 1:** Determine the number of trials and the frequency of the event.
$\Pr(5) = \frac{40}{100}$ $= \frac{4}{10}$ $= \frac{2}{5}$	**Step 2:** Write the experimental probability as a fraction of the frequency of the event out of the total number of trials, in the simplest form.

b. Calculate the expected number of occurrences of landing on the number 5 if the spinner is spun 250 times. WE1b

Working	Thinking
Expected number of occurrences $= \Pr(5) \times$ number of trials $= \frac{4}{10} \times 250$ $= 100$	Multiply the experimental probability of the event by the number of trials.

Student practice

A spinner has the numbers 1 to 4 inclusive. The spinner is spun 50 times and lands on the number 3 fifteen times.

a. Calculate the experimental probability of landing on the number 3.

b. Calculate the expected number of occurrences of landing on the number 3 if the spinner is spun 120 times.

Worked example 2

Calculating experimental probability and expected occurrences from frequency tables

A bag contains an unspecified number of coloured counters. A counter is selected at random and replaced. The following table contains the results of the experiment.

Outcome	Red (R)	Black (B)	Green (G)	White (W)
Frequency	42	12	40	6

a. Calculate the number of trials. WE2a

Working

$n(R) = 42, n(B) = 12, n(G) = 40, n(W) = 6$

$$\begin{aligned}\text{Total number of trials} &= \text{Sum of all frequencies}\\ &= n(R) + n(B) + n(G) + n(W)\\ &= 42 + 12 + 40 + 6\\ &= 100\end{aligned}$$

Thinking

Step 1: Identify the frequency of each outcome in the table.

Step 2: Sum the frequencies of all the outcomes.

b. Calculate the experimental probability of selecting a black (B) counter. WE2b

Working

Trials: 100

Frequency: 12

$\Pr(B) = \frac{3}{25}$

Thinking

Step 1: Determine the number of trials and the frequency of the event.

Step 2: Write the experimental probability as a fraction of the frequency of the event out of the total number of trials, in the simplest form.

c. Calculate the expected number of occurrences of selecting a black (B) counter if the spinner is spun 500 times. WE2c

Working

$$\begin{aligned}\text{Expected number of occurrences} &= \Pr(B) \times \text{number of trials}\\ &= \frac{3}{25} \times 500\\ &= 75\end{aligned}$$

Thinking

Multiply the experimental probability of the event by the number of trials.

Student practice

A bag contains an unspecified number of coloured counters. A counter is selected at random and replaced. The following table contains the results of the experiment.

Outcome	Purple (P)	Blue (B)	Silver (S)	Gold (G)
Frequency	16	13	10	11

a. Calculate the number of trials.

b. Calculate the experimental probability of selecting a silver (S) counter.

c. Calculate the expected number of occurrences of selecting a silver (S) counter if the spinner is spun 180 times.

Back to contents

10E Questions

Understanding worksheet

1. For each description, indicate whether it is more likely to describe theoretical or experimental probability.

Example

Description	Theoretical probability	Experimental probability
The chance of tossing a head on a fair coin is $\frac{1}{2}$	☑	☐

Description	Theoretical probability	Experimental probability
I. The probability of selecting a teacher at an Australian school that speaks Finnish is $\frac{2}{3}$.	☐	☐
II. The probability of rolling a six on a fair six-sided die is $\frac{1}{6}$.	☐	☐
III. The probability of selecting a person from a sports stadium crowd that can pilot an aeroplane is $\frac{1}{3}$.	☐	☐
IV. The probability of selecting of black counter from a bag with an equal number of black and white counters is $\frac{1}{2}$.	☐	☐

2. Complete the missing information in the table.

Example

A standard 6 sided die is rolled 19 times: 3, 4, 4, 4, 4, 6, 1, 2, 3, 3, 1, 1, 5, 5, 1, 5, 4, 3

Number on die	1	2	3	4	5	6
Relative frequency	4	1	4	5	3	1

A non-standard 6 sided die is rolled 19 times: 7, 7, 9, 10, 11, 12, 7, 7, 9, 8, 8, 10, 10, 8, 9, 11, 12, 12, 10

Number on die	7		9	10	11	12
Relative frequency			3		2	3

3. Fill in the blanks by using the words provided.

experimental **frequency** **theoretical** **trials**

Probability that is based on logic is called __________ probability. Probability that is determined through actual tests or experiments, is known as __________ probability. When conducting a set number of __________ , the experimental probability is calculated by comparing the __________ of the desired outcome to the total number of attempts.

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b), 5 (a,b), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d), 5 (c,d), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f), 5 (e,f), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. A spinner contains black (B), white (W) and green (G) portions. The spinner is spun 100 times and lands on the green 45 times, white 30 times and black 25 times. WE1

a. Calculate the experimental probability of landing on black.

b. Calculate the expected number of occurrences of landing on the black if the spinner is spun 220 times.

c. Calculate the experimental probability of landing on green.

d. Calculate the expected number of occurrences of landing on green if the spinner is spun 420 times.

e. Calculate the experimental probability of landing on black or white.

f. Calculate the expected number of occurrences of landing on black or white if the spinner is spun 260 times.

---

5. In an experiment with 500 people selected at random, 350 people speak one language, 100 speak two languages and 50 speak three or more languages. WE1

a. Calculate the experimental probability of choosing a person who speaks only two languages.

b. Calculate the expected number of people who speak only two languages from a group of 600 people.

c. Calculate the experimental probability of choosing a person who speaks only one language.

d. Calculate the expected number of people who speak only one language from a group of 200 people.

e. Calculate the experimental probability of choosing a person who speaks three or more languages.

f. Calculate the expected number of people who speak three or more languages from a group of 40 people.

---

6. A bag contains an unspecified number of coloured counters. A counter is selected at random and replaced. The following table contains the results of the experiment. WE2

Outcome	Turquoise (T)	Gold (G)	Brown(B)	Cream(C)
Frequency	12	25	6	7

a. Calculate the number of trials.

b. Calculate the experimental probability of selecting a brown (B) counter.

c. Calculate the expected number of occurrences of selecting a brown (B) counter if the experiment is repeated 225 times.

d. Calculate the experimental probability of selecting a gold (G) counter.

e. Calculate the expected number of occurrences of selecting a gold (G) counter if the experiment is repeated 1000 times.

f. Calculate the experimental probability of selecting a cream (C) or gold (G) counter.

g. Calculate the expected number of occurrences of selecting a cream (C) or gold (G) counter if the experiment is repeated 75 times.

h. Calculate the experimental probability of not selecting a cream (C) counter.

7. The age of a group of music festival attendees is recorded in the following frequency table. **WE2**

Outcome	18 years-old	19 years-old	20 years-old	21 years-old
Frequency	40	68	75	17

a. Calculate the number of music festival attendees is the group.

b. Calculate the experimental probability that a festival attendee is exactly 18 years-old.

c. Calculate the expected number of 18 year-old attendees if the age of 500 people is recorded.

d. Calculate the experimental probability that a festival attendee is exactly 20 years-old.

e. Calculate the expected number of 20 year-old attendees if the age of 800 people is recorded.

f. Calculate the experimental probability that a festival attendee is older than 19 years-old.

g. Calculate the expected number of attendees that are older than 19 years-old if the age of 1000 people is recorded.

h. Calculate the experimental probability that a festival attendee is 19 years-old or younger.

---

8. A deck of cards contains 52 cards, and a card is selected from the deck and then replaced. The results are recorded in the following table.

Outcome	Club	Diamond	Heart	Spade
Frequency	10	2	5	7

What is the experimental probability of selecting a club?

A. $\frac{1}{4}$ **B.** $\frac{5}{12}$ **C.** 10 **D.** 24 **E.** 52

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the experimental probability that a person is born during the summer.

Outcome	Summer	Autumn	Winter	Spring
Frequency	12	10	5	13

Student A

Frequency of summer: 12

Number of trials: 40

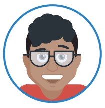
$$\Pr(\text{summer}) = \frac{\text{frequency of summer}}{\text{total number of trials}} = \frac{12}{40} = \frac{3}{10}$$

Student B

$$\Pr(\text{summer}) = \frac{\text{number of summers}}{\text{number of seasons}} = \frac{1}{4}$$

Back to contents

b. If $\Pr(\text{summer}) = \frac{3}{10}$, calculate the expected number of people that are born in summer if the birth season of 1000 people is recorded.



Student A

$$\begin{aligned} \text{Expected number of occurrences} &= \frac{3}{10} \times 40 \\ &= \frac{120}{10} \\ &= 12 \end{aligned}$$

Student B

$$\begin{aligned} \text{Expected number of occurrences} &= \frac{3}{10} \times 1000 \\ &= \frac{3000}{10} \\ &= 300 \end{aligned}$$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. James' hockey team has won three games and lost nine games when playing at an opponents' home venue. What is the relative frequency of losing games played at the opponents' home venue for James' hockey team? Give the answer as a decimal.

11. Derek is a chess enthusiast and has been tracking his wins, draws, and losses over the past month. He has won 6 games, drawn 8 and lost 11. What is the experimental probability of Derek winning his next game? Give the answer as a decimal.

12. A jeweller noticed that 2 out of 40 diamond rings had a minor flaw. If they are planning to showcase 500 rings in a jewellery exhibition, how many might be expected to have a flaw based on the experimental probability?

13. A cinema showcases three types of movies: action, drama, and animation. The experimental probability of a moviegoer watching an action movie is $\frac{4}{7}$. If the cinema has 3500 viewers in a day, how many are predicted to watch an action movie?

14. After assessing traffic flow, a traffic analyst stated that the experimental probability of heavy traffic congestion at morning time in New York was 70%. If the analyst observed heavy traffic congestion on 7 mornings, over how many mornings in total was the traffic flow assessed?

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. A sport drink was introduced claiming to improve hydration after physical activity. Two separate studies were executed to ascertain its effectiveness. After consuming the drink for three days, the athletes were surveyed on whether they felt an increase in hydration, no change, or a decrease in hydration. Distinct athletes were engaged for each study. The feedback is shown in the table below.

Survey response	Increased hydration	No change	Decreased hydration
Study 1	55	15	10
Study 2	300	50	150

a. Determine the total number of athletes who took part in each study.

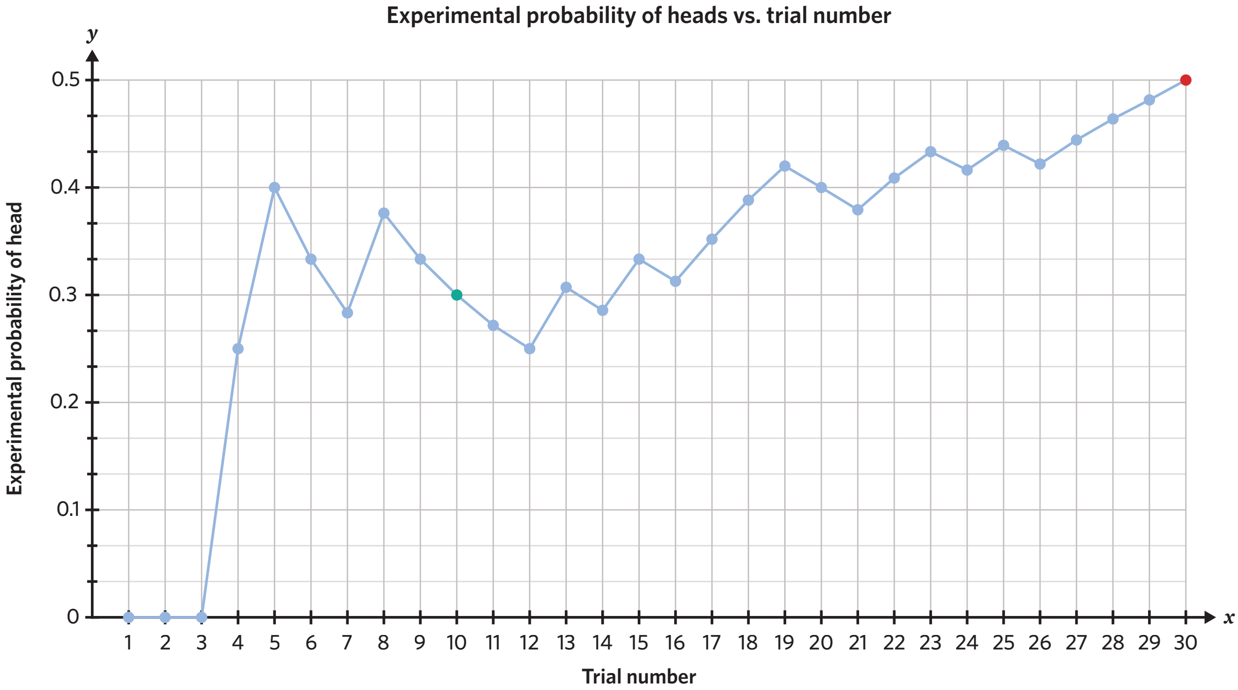
b. For each study, calculate the experimental probabilities of the sport drink increasing hydration. Give the exact answers as decimals.

c. If 1000 athletes consume this drink, how many among them are predicted to feel a decrease in their hydration based on each study's findings?

d. Which of the studies is more likely to be a more reliable reflection of the effectiveness of the sport drink? Justify your answer.

e. Other than changes in hydration, state other factors that should be considered when developing a sport drink for physical activity.

16. The below graph shows the changes in relative frequency of landing on a Head during an unbiased coin toss experiment of 30 tosses. The green point shows the relative frequency after 10 tosses. The red point shows the relative frequency after 30 tosses.



a. State the relative frequency after 10 tosses as a decimal.

b. State the relative frequency after 30 tosses as a decimal.

c. Compare and contrast the answers from part **a** and **b**. Explain how the number of trials affects the experimental probability in an unbiased coin toss experiment.

Exam-style

17. A toy manufacturing company is testing a new toy car design for defects. During the trial phase, out of a sample of 100 cars, 7 were found to have defects. The company now plans to produce a batch of 500 cars. (1 MARK)

Which of the following statements is true?

A. If 200 cars are produced, the expected number of defects is 7.

B. The expected number of defective cars in the batch of 500 is 35.

C. The theoretical probability is always equal to the experimental probability.

D. The experimental probability of a car being defective from the trial phase is 0.7.

E. A larger sample size in the trial phase would definitely change the experimental probability.

18. A tech company is running tests on its latest batch of smartphones for screen sensitivity issues. Out of a sample of 80 phones, 4 were found to have sensitivity problems. (3 MARKS)

a. Calculate the experimental probability of a phone having a screen sensitivity issue from this sample. 1 MARK

b. If the company now plans to sell a shipment of 500 phones, how many phones are expected to have screen sensitivity issues based on the experimental probability? 2 MARKS

19. During a field study, biologists observed a particular bird species. Their observations are shown in the table below. (3 MARKS)

Bird pattern	Number observed
Spotted	20
Plain	100

Based on the sample, if the biologists observe a separate group and notice 50 spotted birds, how many birds were they likely to have observed in total in this other group?

20. In a board game, there's a $\frac{3}{5}$ chance that a player will draw a 'bonus' card. (3 MARKS)

Based on this probability, calculate how many cards a player would need to draw to expect to receive 24 'bonus' cards.

Remember this?

21. Emily has 3 more pencils than Tom. Emily correctly writes this fact as an equation using e for the number of pencils she has and t for the number of pencils Tom has. Which of these could be Emily's equation?

A. $e = 3t$ **B.** $e = t \div 3$ **C.** $e = 3 - t$ **D.** $e = t - 3$ **E.** $e = t + 3$

22. This table shows how the volume of oil is related to its mass.

Volume (litres)	Mass (kilograms)
40	20
80	40
120	60
160	80

A tank contains 90 litres of oil. How many kilograms of oil are in the tank?

A. 45 kg **B.** 50 kg **C.** 55 kg **D.** 60 kg **E.** 65 kg

23. Sara and Tim biked a trail together. Sara finished the trail in one hour and fifteen minutes. Tim took 25% longer than Sara. Approximately how long did Tim take to finish the trail?

A. 90 minutes **B.** 95 minutes **C.** 100 minutes **D.** 105 minutes **E.** 110 minutes

Back to contents

Chapter 10 extended application

1. The rise of online purchasing has significantly altered the landscape of consumer behaviour over the last decade, steering numerous shoppers away from physical retail stores towards digital platforms for their convenience and variety.

In a recent survey conducted in Brisbane, residents were asked about their shopping habits to understand the popularity of online purchasing compared to buying from physical retail stores. The survey investigated whether people purchase household items and clothes online or from physical retail stores.

- 1200 people participated in the survey.
- 1000 people said they purchase household items and clothes online.
- 450 people said they purchase household items and clothes both online and in physical stores.
- Everyone surveyed purchased items online or from a physical store or both.

a. Determine n(Online stores ∩ Physical stores).

b. Construct a Venn diagram that represents the data provided, labelling one circle as 'Online stores' and the other as 'Physical stores'.

c. Create a two-way table that represents the data provided.

d. Determine the probability that a randomly selected person purchases physical retail stores only and the probability that a randomly selected person shops online.

e. Based on the answers in part **d**, comment on the shopping behaviour of residents.

f. What is one factor that may influence a shopper to purchase items in physical stores despite the prevalent online purchasing trend?

2. Dylan, an art teacher, has to select two different art supplies from the store for his next class. The store has a promotion on sketchbooks (SB), coloured pencils (CP), and paint sets (PS). Dylan ensures he doesn't select the same type of item to provide a range of materials for his students.

a. Construct a tree diagram that showcases all the possible combinations of two different art supplies Dylan can select.

b. Draw an array and list the sample space.

c. Determine the probability that Dylan selects a sketchbook and a paint set.

d. Dylan changes his mind and now may select the same types of material for his students. Determine how much less likely it is that he will select a sketchbook and a paint set.

e. Discuss one benefit of providing students with different types of art supplies.

3. In the competitive world of film production, two companies, Stellar Films and Galaxy Studios, decided to test different strategies for predicting the success of movie releases. Stellar Films focused on employing famous actors, while Galaxy Studios focused on utilising innovative, high-budget special effects.

The companies then classified the success of some of their recently released movies into three categories (High, Moderate, or Low) based on box office revenue. Different sets of movies were studied in each case. The results of the companies' studies on the success of their movies are represented in the following table.

	Success		
	High	**Moderate**	**Low**
Stellar Films	50	80	20
Galaxy Studios	160	85	50

Back to contents

a. Determine the total number of movies studied by each production company.

b. For each company, calculate the experimental probabilities of a movie achieving high, moderate, or low success. Provide your answers in decimal form and round to two decimal places.

c. If each production company were to produce 1000 movies, using the answer in part **b**, predict the number of movies that would achieve low success.

d. Considering both strategies, which production company appears to be more successful in avoiding low-revenue movie releases? Justify your answer based on calculations.

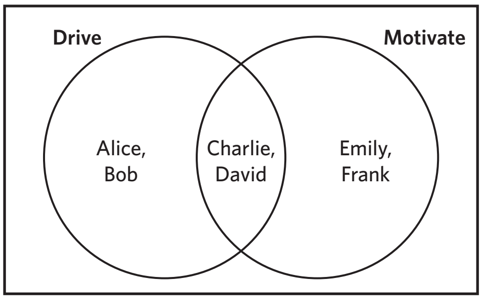
e. Other than the variables studied (renowned actors and high-tech special effects), suggest another factor that may influence a movie's box office success.

Back to contents

Chapter 10 review

Multiple choice

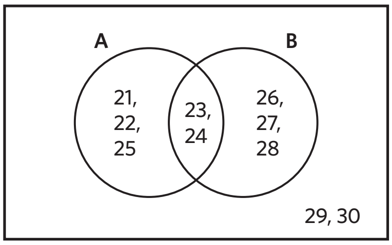
1. Given the following Venn diagram, determine the probability of randomly selecting an individual from the survey who has read ‘Drive’. 10A



A. $\frac{1}{3}$ **B.** $\frac{1}{2}$ **C.** $\frac{2}{3}$ **D.** $\frac{3}{4}$ **E.** $\frac{4}{5}$

---

2. Given the following Venn diagram, list the set B’. 10B



A. {21, 22, 25, 29, 30}
B. {21, 22, 23, 24, 25}
C. {23, 24, 26, 29, 30}
D. {21, 22, 26, 27, 28}
E. {26, 27, 28, 29, 30}

---

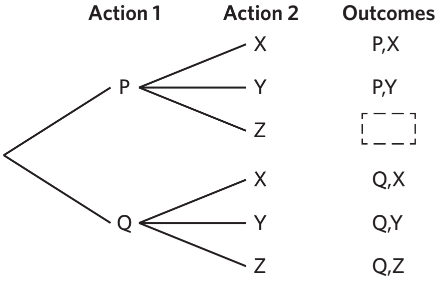
3. The following array shows a two-step experiment with replacement. Calculate Pr(Z,A). 10C

	X	**Y**	**Z**	**A**
X	(X,X)	(X,Y)	(X,Z)	(X,A)
Y	(Y,X)	(Y,Y)	(Y,Z)	(Y,A)
Z	(Z,X)	(Z,Y)	(Z,Z)	(Z,A)

A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{12}$ **E.** $\frac{1}{16}$

---

4. Identify the missing outcome in the following tree diagram. 10D



A. (Q,Y) **B.** (Z,P) **C.** (P,Z) **D.** (Q,Z) **E.** (Z,Q)

5. A box contains an unspecified number of different biscuit types. A biscuit is selected at random and replaced. The following table represents the results from the experiment.

10E

Biscuits	Anzac Biscuit (A)	Shortbread (S)	Chocolate Chip (Ch)	Macaron (M)
Frequency	11	3	5	8

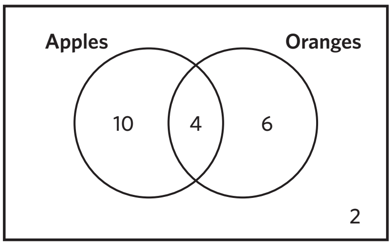
Calculate the experimental probability of selecting a Shortbread (S) biscuit.

A. $\frac{1}{9}$ **B.** $\frac{5}{27}$ **C.** $\frac{8}{27}$ **D.** $\frac{11}{27}$ **E.** $\frac{8}{9}$

Fluency

6. Consider the following Venn diagram showing people's purchasing preferences for two types of fruits: apples and oranges.

10A



a. How many people purchase only apples?

b. How many people purchase either apples or oranges, but not both?

c. Determine the probability of randomly selecting a person from the sample who purchases oranges.

d. How many people do not purchase either apples or oranges?

---

7. Consider the following two-way table representing people's responses relating to their exercise and sleeping habits.

10A

	Exercises	**Does not exercise**	**Total**
Tracks sleep	12	5	17
Does not track sleep	8	3	11
Total	20	8	28

a. How many people exercise?

b. How many people either exercise or track their sleep or both?

c. Determine the probability of randomly selecting a person from the sample who does not exercise but tracks their sleep.

d. How many people neither exercise nor track their sleep?

---

8. Consider the following sample space and subsets.

10B

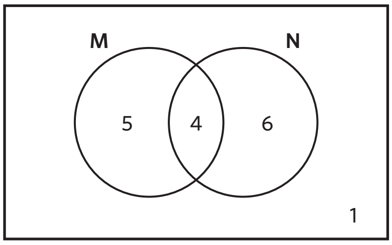
$\xi = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{23, 24, 26\}$

$B = \{21, 23, 25, 27, 29\}$

a. List the set $A \cup B$. b. List the set $A \cap B$. c. List the set B'. d. List the set $A \cup B'$.

9. Calculate the probabilities.



a. Pr(M ∪ N) **b.** Pr(M ∩ N) **c.** Pr(M’) **d.** Pr(N’ ∩ M’)

---

10. A box contains two red (R) balls, two white (W) balls, and one black (B) ball. Two balls are chosen at random without replacement.

a. Draw an array and list the sample space for this experiment.

b. Calculate the probability that both balls are red.

c. Calculate the probability that neither ball is black.

d. Calculate the probability that one ball is red and the other is white.

---

11. A box contains name tags of different employees: Emily (E), David (D), Sarah (S), Xavier (X). Two name tags are chosen at random without replacement.

a. Draw a tree diagram and list the sample space for this experiment.

b. Calculate Pr(X,E).

c. Calculate the probability of selecting Sarah (S).

d. Calculate the probability of not selecting David (D).

---

12. A dice with the numbers 1 to 6 inclusive is rolled 120 times and lands on the number 3 thirty times.

a. Calculate the experimental probability of landing on the number 3.

b. Calculate the expected number of occurrences of landing on the number 3 if the dice is rolled 300 times.

c. If the dice is rolled another 120 times and lands on the number 3 an additional seventy times, calculate the new experimental probability of landing on the number 3.

d. Using the new experimental probability from part c, calculate the expected number of occurrences of landing on the number 3 if the dice is rolled 540 times, rounded to the nearest whole number.

---

13. A box contains an unspecified number of coloured balls. A ball is selected at random and replaced. The following table contains the results of the experiment:

Outcome	Blue (B)	Yellow (Y)	Pink (P)	Orange (O)
Frequency	48	15	35	2

a. Calculate the number of trials.

b. Calculate the experimental probability of selecting a Yellow (Y) ball.

c. Calculate the expected number of occurrences of selecting a Yellow (Y) ball if the ball is selected 280 times.

d. Calculate the experimental probability of not selecting an Orange (O) ball.

Problem solving

14. In a school sports club consisting of 20 students, 12 students play basketball, 10 students play soccer, and 5 students play both basketball and soccer. Construct a Venn diagram to represent the situation.

15. The music club at a school has a group of different musical instruments. The group includes a Violin, Trumpet, Flute, and Guitar. List the set M if it is equal to {musical instruments that contain the letter 'u'}. **10B**

---

16. In a community gardening club, there are six members: Anna, Bruce, Clara, Derek, Emily, and Frank. The club coordinator randomly selects pairs to work on different sections of the garden. The coordinator picks a name, does not replace it, and then picks another name to form a pair. What is the probability that the first pair selected is Anna and Derek? **10C**

---

17. In a local chess tournament, there are 7 players: Alice, Bob, Charlie, David, Emily, Fiona, and Greg. The tournament organiser is randomly selecting two players to face off in the first match. What is the probability that the first match is between Alice and Fiona? **10D**

---

18. On a popular music streaming platform, there are three main genres that users listen to: Pop, Jazz, and Classical. The experimental probability of a user listening to a Jazz track is $\frac{3}{8}$. If the platform has 480 000 users on a particular day, how many are predicted to listen to Jazz? **10E**

Reasoning

19. A primary school is trying to determine a homework policy, so that students are more likely to hand homework in on time. 400 students were asked about their homework preferences. There were 280 students who preferred homework during the school week and 120 students who preferred homework on weekends. 30 students preferred to receive homework during the week and weekend.

a. Draw a Venn diagram to represent the situation and determine how many students preferred only one of the options.

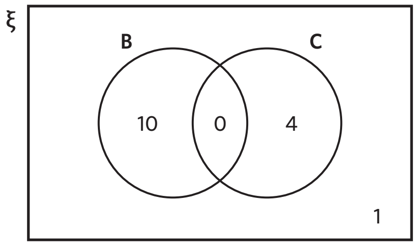
b. Determine the probability of randomly selecting a person from the sample who preferred completing homework during the weekend.

c. Let *D* represent the set of students who prefer homework during the school week and *W* represent the set of students who prefer homework during the weekend. Represent the intersection of *D* and *W* using set notation.

d. A teacher has assigned two separate homework tasks. Students can complete the tasks whenever they like. Construct an array to represent the various combinations of choices for completing the two homework tasks. For example, a student may complete one task during the week and the other during the weekend.

e. What are some possible reasons a student might choose to do the homework tasks during the school week?

20. Consider the following Venn diagram.

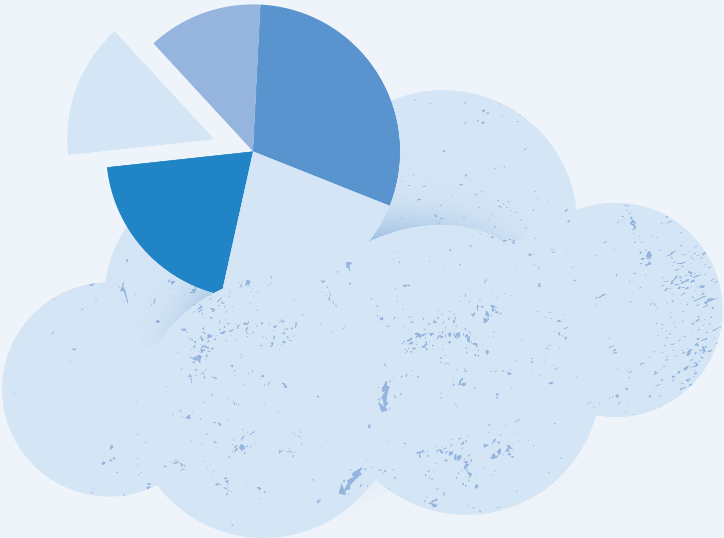


a. Calculate n(B) + n(C).

b. Calculate n(B ∪ C).

c. Compare and contrast the answers to parts **a** and **b** and explain when n(B) + n(C) may equal n(B ∪ C).

Back to contents



Chapter 11

Statistics

Statistics and probability

Calculator skills

See online in additional materials for using CAS calculator guides.

11A Measures of centre and spread

Back to contents

Chapter 11 research summary

Statistics

Big ideas

Summary statistics are a foundational element and they play a crucial role in helping students understand and interpret data in real-world contexts. Statistics is a field of mathematics that deals with collecting, analysing, interpreting, presenting, and organising data. There are several fundamental mathematical ideas and concepts that underpin the field of statistics.

Estimation

Estimation is using data from a sample to estimate parameters of the population.

Discrete and continuous data

The distinction between data that can only take specific values, also known as discrete, and data that can take any value in a given range, also known as continuous, is crucial in determining which statistical methods to use.

Central tendency

Central tendency is about understanding where the centre of a data set is. It can help to summarise the centre of a data set with a single value.

Spread

While the central tendency explains where the data is centred, the spread explains how spread out or variable the data is.

Shape

The shape is about understanding the distribution of data. Symmetry and peaks may indicate that the data is symmetric or bimodal. Asymmetry may indicate the data is positively or negatively skewed.

Outliers

Identifying values that stand out from the rest of the data. They can significantly impact the mean and can provide insights into anomalies or unique characteristics in the data.

Visual representations

Stem-and-leaf plots

A stem-and-leaf plot is used in a similar way to dot plots and column graphs. A stem-and-leaf plot allows visualisation of data in horizontal stacks, but also retains the integrity of each individual data value for future analysis. A key is always included with a stem-and-leaf plot.

Set 1	Stem	Set 2
6	5	8
8 6 4 0	6	2 6 7
8 6 2 0	7	0 0 4 6 8
8 6 4 2 0	8	0 2 4 6 8
0	9	0 2

Key

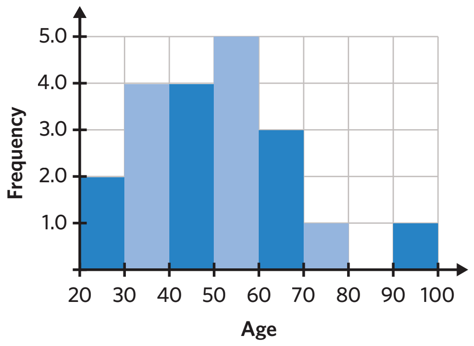
3 | 7 = 37

Back to contents

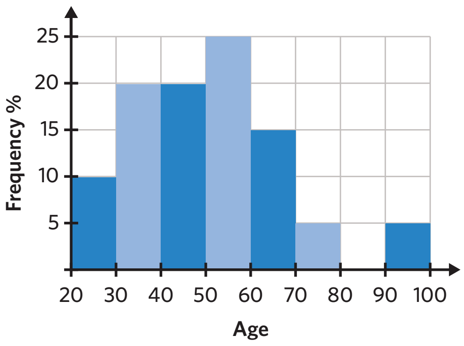
Histograms

Histograms contain data grouped in exclusive intervals, and have no gaps between the bars. Histograms can be symmetric, skewed, or bimodal.

Frequency histogram

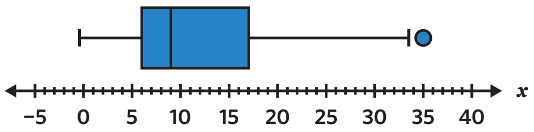


Percentage frequency histogram



Box plot

A box and whisker plot – also called a box plot – displays the five-number summary of a set of data. The five-number summary is the minimum, first quartile, median, third quartile, and maximum. In a box plot, the box is drawn from the first quartile to the third quartile, with a vertical line at the median.



Misconceptions

Misconception	Incorrect ✗	Correct ✓	Lesson
Students determine the interquartile range without ordering the data set.	Calculate the interquartile range of the following data set: 7, 23, 10, 4, 17, 2, 11, 19, 5, 8 First half: 7, 23, 10, 4, 17 $Q_1 = 10$ Second half: 2, 11, 19, 5, 8 $Q_3 = 19$ $IQR = Q_3 - Q_1$ $= 19 - 10$ $= 9$	Calculate the interquartile range of the following data set: 7, 23, 10, 4, 17, 2, 11, 19, 5, 8 Ordered: 2, 4, 5, 7, 8, 10, 11, 17, 19, 23 First half: 2, 4, 5, 7, 8 $Q_1 = 5$ Second half: 10, 11, 17, 19, 23 $Q_3 = 17$ $IQR = Q_3 - Q_1$ $= 17 - 5$ $= 12$	11A
Students exclude outliers when calculating the mean or range.	Calculate the mean and range of the following data set: 9, 15, 47, 6, 8, 11 47 is an outlier and should be removed. Mean $= \frac{9 + 15 + 6 + 8 + 11}{5}$ $= \frac{49}{5}$ $= 9.8$ Range $= 15 - 6$ $= 9$	Calculate the mean and range of the following data set: 9, 15, 47, 6, 8, 11 Mean $= \frac{9 + 15 + 47 + 6 + 8 + 11}{6}$ $= \frac{96}{6}$ $= 16$ Range $= 47 - 6$ $= 41$	11A

Continues →

<table>
<tr><th>Misconception</th><th>Incorrect ✗</th><th>Correct ✓</th><th>Lesson</th></tr>
<tr>
<td>Students disregard referring to the key when extracting data from a stem-and-leaf plot.</td>
<td>The median is 28.5.
<table>
<tr><th>Stem</th><th>Leaf</th><th>Key</th></tr>
<tr><td>0</td><td></td><td>3 | 4 = 3.4</td></tr>
<tr><td>1</td><td>4 8</td><td></td></tr>
<tr><td>2</td><td>1 6 8 | 9</td><td></td></tr>
<tr><td>3</td><td>2 4 7 9</td><td></td></tr>
</table></td>
<td>The median is 2.85.
<table>
<tr><th>Stem</th><th>Leaf</th><th>Key</th></tr>
<tr><td>0</td><td></td><td>3 | 4 = 3.4</td></tr>
<tr><td>1</td><td>4 8</td><td></td></tr>
<tr><td>2</td><td>1 6 8 | 9</td><td></td></tr>
<tr><td>3</td><td>2 4 7 9</td><td></td></tr>
</table></td>
<td>11B</td>
</tr>
<tr>
<td>Students believe that the choice of what constitutes the ‘stem’ and the ‘leaf’ in a stem-and-leaf plot is arbitrary.</td>
<td>Using the following data sets generate an ordered back-to-back stem-and-leaf plot.
Group A
3.4, 2.9, 4.7, 5.6, 3.2, 4.1, 1.4, 5.3, 2.6, 5.9, 3.7, 3.9, 4.8, 2.1, 1.8, 6.7, 4.3, 6.1, 5.4
Group B
4.2, 3.6, 2.4, 5.1, 3.7, 4.6, 0.5, 5.0, 1.5, 5.8, 2.7, 3.1, 4.7, 2.2, 1.0, 6.4, 4.5, 6.2, 5.5
<table>
<tr><th>Group B</th><th>Stem</th><th>Group A</th></tr>
<tr><td>5 1</td><td>0</td><td></td></tr>
<tr><td>3 5</td><td>1</td><td>4 8</td></tr>
<tr><td>6 4 2</td><td>2</td><td>1 6 9</td></tr>
<tr><td></td><td>3</td><td>2 4 7 9</td></tr>
<tr><td>6 2</td><td>4</td><td>1 3 7 8</td></tr>
<tr><td>5 4 1 0</td><td>5</td><td>3 4 6 9</td></tr>
<tr><td>3 4</td><td>6</td><td>1 7</td></tr>
<tr><td>3 4 2</td><td>7</td><td></td></tr>
<tr><td>5</td><td>8</td><td></td></tr>
</table>
Key
2 | 7 = 2.7</td>
<td>Using the following data sets generate an ordered back-to-back stem-and-leaf plot.
Group A
3.4, 2.9, 4.7, 5.6, 3.2, 4.1, 1.4, 5.3, 2.6, 5.9, 3.7, 3.9, 4.8, 2.1, 1.8, 6.7, 4.3, 6.1, 5.4
Group B
4.2, 3.6, 2.4, 5.1, 3.7, 4.6, 0.5, 5.0, 1.5, 5.8, 2.7, 3.1, 4.7, 2.2, 1.0, 6.4, 4.5, 6.2, 5.5
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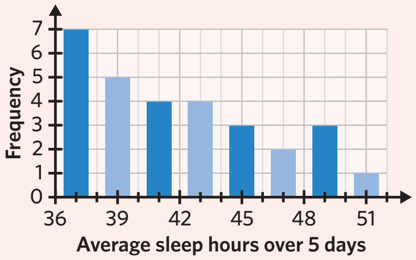
<tr><td>5 0</td><td>1</td><td>4 8</td></tr>
<tr><td>7 4 2</td><td>2</td><td>1 6 9</td></tr>
<tr><td>7 6 1</td><td>3</td><td>2 4 7 9</td></tr>
<tr><td>7 6 5 2</td><td>4</td><td>1 3 7 8</td></tr>
<tr><td>8 5 1 0</td><td>5</td><td>3 4 6 9</td></tr>
<tr><td>4 2</td><td>6</td><td>1 7</td></tr>
</table>
Key
2 | 7 = 2.7</td>
<td>11B</td>
</tr>
</table>

Continues →

<table>
<tr><th>Misconception</th><th>Incorrect ✕</th><th>Correct ✓</th><th>Lesson</th></tr>
<tr>
<td>Students group data in overlapping or incomplete intervals.</td>
<td>
<table>
<tr><th>Class interval</th><th>Frequency</th><th>Percentage frequency</th></tr>
<tr><td>22–27</td><td>5</td><td>$\frac{5}{20} \times 100$
$= 25\%$</td></tr>
<tr><td>28–33</td><td>3</td><td>$\frac{3}{20} \times 100$
$= 15\%$</td></tr>
<tr><td>34–39</td><td>2</td><td>$\frac{2}{20} \times 100$
$= 10\%$</td></tr>
<tr><td>40–45</td><td>4</td><td>$\frac{4}{20} \times 100$
$= 20\%$</td></tr>
<tr><td>46–51</td><td>4</td><td>$\frac{4}{20} \times 100$
$= 20\%$</td></tr>
<tr><td>52–57</td><td>0</td><td>$\frac{0}{20} \times 100$
$= 0\%$</td></tr>
<tr><td>Total</td><td>20</td><td>100%</td></tr>
</table>
</td>
<td>
<table>
<tr><th>Class interval</th><th>Frequency</th><th>Percentage frequency</th></tr>
<tr><td>22–<27</td><td>5</td><td>$\frac{5}{20} \times 100$
$= 25\%$</td></tr>
<tr><td>27–<32</td><td>2</td><td>$\frac{2}{20} \times 100$
$= 10\%$</td></tr>
<tr><td>32–<37</td><td>2</td><td>$\frac{2}{20} \times 100$
$= 10\%$</td></tr>
<tr><td>37–<42</td><td>2</td><td>$\frac{2}{20} \times 100$
$= 10\%$</td></tr>
<tr><td>42–<47</td><td>3</td><td>$\frac{3}{20} \times 100$
$= 15\%$</td></tr>
<tr><td>47–<52</td><td>4</td><td>$\frac{4}{20} \times 100$
$= 20\%$</td></tr>
<tr><td>Total</td><td>20</td><td>100%</td></tr>
</table>
</td>
<td>11C</td>
</tr>
<tr>
<td>Students confuse a histogram with a bar graph.</td>
<td>


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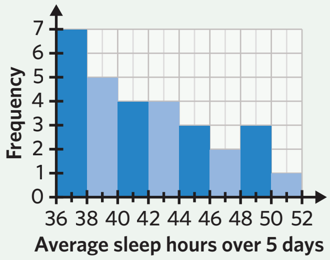
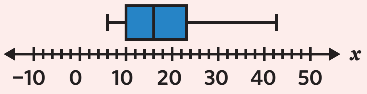
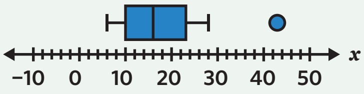
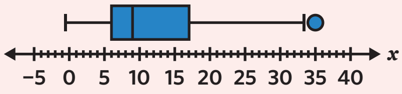
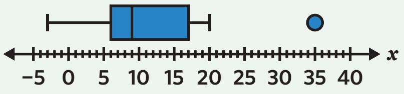


</td>
<td>11C</td>
</tr>
<tr>
<td>Students represent an outlier as part of the whisker.</td>
<td>Using the given data, generate a boxplot.
12, 6, 7, 8, 14, 10, 17, 20, 15, 22, 23, 25, 28, 43


</td>
<td>Using the given data, generate a boxplot.
12, 6, 7, 8, 14, 10, 17, 20, 15, 22, 23, 25, 28, 43


</td>
<td>11D</td>
</tr>
<tr>
<td>Students assume that the upper and lower fence values are noted as the end of the whiskers.</td>
<td>Using the given data, calculate the upper and lower fence and generate a boxplot.
5, 3, 6, 11, 9, 13, 7, 17, 9, 6, 14, 2, 18, 20, 35
Ordered: 2, 3, 5, 6, 6, 7, 9, 9, 11, 13, 14, 17, 18, 20, 35
Minimum $= 2$, $Q_1 = 6$, median $= 9$, $Q_3 = 17$, maximum $= 35$
IQR $= 17 - 6$
$= 11$
Lower fence $= 16 - 1.5 \times 11$
$= -0.5$
Upper fence $= 17 + 1.5 \times 11$
$= 33.5$


</td>
<td>Using the given data, calculate the upper and lower fence and generate a boxplot.
5, 3, 6, 11, 9, 13, 7, 17, 9, 6, 14, 2, 18, 20, 35
Ordered: 2, 3, 5, 6, 6, 7, 9, 9, 11, 13, 14, 17, 18, 20, 35
Minimum $= 2$, $Q_1 = 6$, median $= 9$, $Q_3 = 17$, maximum $= 35$
IQR $= 17 - 6$
$= 11$
Lower fence $= 16 - 1.5 \times 11$
$= -0.5$
Upper fence $= 17 + 1.5 \times 11$
$= 33.5$


</td>
<td>11D</td>
</tr>
</table>

Back to contents

11A Measures of centre and spread

LEARNING INTENTIONS

Students will be able to:

- find the mean, median, and mode of a data set
- find the range and interquartile range of a data set
- identify outliers in a data set.

Summary statistics include measures of centre and measures of spread that are used to summarise data sets. Measures of centre and spread can also be used to compare data sets so that conclusions can be drawn. There are different measures for each that can be selected depending on a number of factors, including the presence of outliers.

KEY TERMS AND DEFINITIONS

- A **quartile** is a statistical measure that divides a data set into four equal parts.
- The **maximum** is the largest value in a data set.
- The **minimum** is the smallest value in a data set.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Henk Vrieselaar/Shutterstock.com

The performance of a public transport service can be summarised using measures of centre and spread. A measure of centre could indicate how many minutes a train is early or late at certain times. A measure of spread could describe how consistently the train arrives early, on time or late.

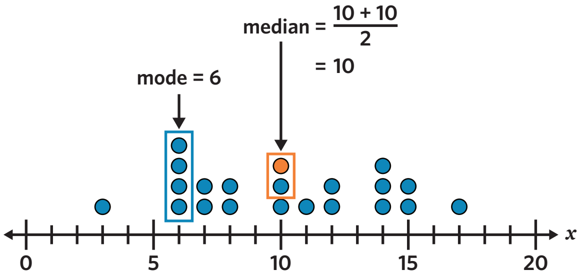
Key ideas

1. Measures of centre include the mean, median, and mode. They indicate the typical value of a data set.

Mean $= \dfrac{\text{sum of all values}}{\text{number of values}}$

Median is the middle value of a data set.

Mode is the most commonly occurring value(s) of a data set.



$$\begin{aligned}\text{mean} &= \frac{\text{sum of all values}}{\text{number of values}}\\ &= \frac{211}{21}\\ &= 10.05\end{aligned}$$

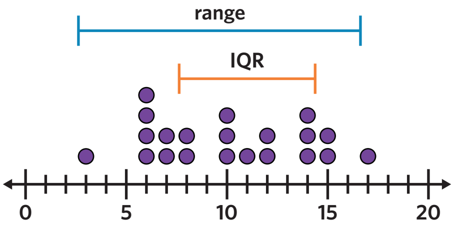
2. Measures of spread include the range and interquartile range. They indicate how spread out the values in a data set are.

Range is the difference between the minimum and maximum.

Range = maximum − minimum

Interquartile range (IQR) is the range of the middle 50% of a data set.

IQR $= Q_3 - Q_1$, where Q_1 is the first quartile and Q_3 is the third quartile.



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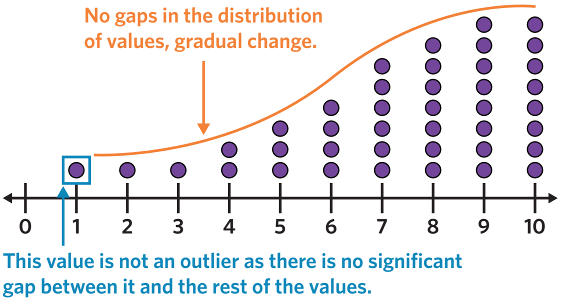
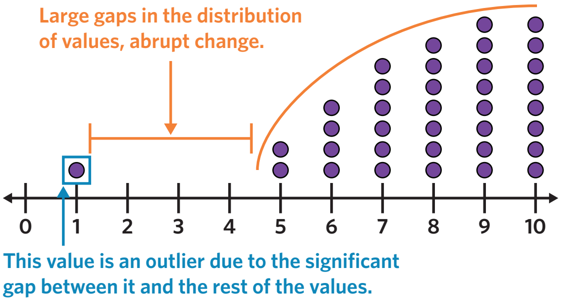
3. To be considered an **outlier**, a numerical value must differ significantly from others in the distribution. Outliers affect the range and mean of numerical data and have little to no effect on the median, mode and interquartile range.


No gaps in the distribution of values, gradual change.

This value is not an outlier as there is no significant gap between it and the rest of the values.


Large gaps in the distribution of values, abrupt change.

This value is an outlier due to the significant gap between it and the rest of the values.

	Best measure of centre	Best measure of spread
Outliers	Median or mode	Interquartile range
No outliers	Mean	Range

Worked example 1

Determining measures of centre

Determine the mean, median and mode(s) of the following data sets. Round to one decimal place where necessary.

a. 2, 2, 12, 1, 3, 17, 12, 0, 6

WE1a

Working	Thinking
0, 1, 2, 2, 3, 6, 12, 12, 17	**Step 1:** Organise the data set in ascending order.
Mean $= \dfrac{\text{sum of all values}}{\text{number of values}} = \dfrac{55}{9} = 6.11 \approx 6.1$	**Step 2:** Calculate the mean by adding all the values together and dividing by the total number of values.
Median $= 3$	**Step 3:** Identify the median by referring to the 'middle' value in the ordered data set.
Mode $= 2, 12$	**Step 4:** Identify the mode. It is the value(s) with the highest frequency in the data set.

Visual support

mode = 2, 12

$$0, 1, \boxed{2, 2}, \boxed{3}, 6, \boxed{12, 12}, 17$$

median = 3

b. 4, 17, 19, 46, 45, 15, 23, 33, 1, 18, 50, 16

WE1b

Working	Thinking
1, 4, 15, 16, 17, 18, 19, 23, 33, 45, 46, 50	**Step 1:** Organise the data set in ascending order.
Mean $= \dfrac{\text{sum of all values}}{\text{number of values}} = \dfrac{287}{12} = 23.92 \approx 23.9$	**Step 2:** Calculate the mean by adding all the values together and dividing by the total number of values.
Median $= \dfrac{18 + 19}{2} = 18.5$	**Step 3:** Identify the median by referring to the 'middle' value in the ordered set. If there is an even number of data values, add the two middle numbers together and divide the result by two.

Continues →

Back to contents

There is no mode.	**Step 4:** Identify the mode. It is the value(s) with the highest frequency in the data set. If all values have the same frequency, there is no mode.

Student practice

Determine the mean, median and mode(s) of the following data sets. Round to one decimal place where necessary.

a. 8, 6, 12, 2, 17, 6, 8, 2, 18

b. 49, 31, 4, 36, 24, 5, 16, 32, 45, 11, 8, 34, 18, 27, 16

Worked example 2

Determining quartiles

Determine the first and third quartiles (Q_1 and Q_3) of the following data sets.

a. 13, 2, 2, 6, 14, 19, 2, 14, 17, 15

WE2a

Working	Thinking
2, 2, 2, 6, 13, 14, 14, 15, 17, 19	**Step 1:** Organise the data set in ascending order.
2, 2, 2, 6, 13, | 14, 14, 15, 17, 19	**Step 2:** Split the data set into two equal groups.
2, 2, $\boxed{2,}$ 6, 13, | 14, 14, 15, 17, 19 $Q_1 = 2$	**Step 3:** Identify the middle value of the first half of the ordered set. This is the first quartile.
2, 2, 2, 6, 13, | 14, 14, $\boxed{15,}$ 17, 19 $Q_3 = 15$	**Step 4:** Identify the middle value of the second half of the data set. This is the third quartile.

Visual support

2, 2, $\boxed{2,}$ 6, 13, | 14, 14, $\boxed{15,}$ 17, 19

$Q_1 = 2$ $Q_3 = 15$

b. 13, 2, 2, 6, 14, 20, 2, 14, 17, 15, 9, 0, 12

WE2b

Working	Thinking
0, 2, 2, 2, 6, 9, 12, 13, 14, 14, 15, 17, 20	**Step 1:** Organise the data set in ascending order.
0, 2, 2, 2, 6, 9, | 12, | 13, 14, 14, 15, 17, 20	**Step 2:** Split the data set into two equal groups. If there is an odd number of data values, exclude the middle value and create two groups using the remaining values.
0, 2, $\boxed{2, 2,}$ 6, 9, | 12, | 13, 14, 14, 15, 17, 20 $Q_1 = \dfrac{2 + 2}{2} = 2$	**Step 3:** Identify the middle value of the first half of the ordered set. If there is an even number of data values, add the two middle numbers together and divide the result by two. This is the first quartile.
0, 2, 2, 2, 6, 9, | 12, | 13, 14, $\boxed{14, 15,}$ 17, 20 $Q_3 = \dfrac{14 + 15}{2} = 14.5$	**Step 4:** Identify the middle value of the second half of the data set. If there is an even number of data values, add the two middle numbers together and divide the result by two. This is the third quartile.

Continues →

Back to contents

Student practice

Determine the first and third quartiles (Q_1 and Q_3) of the following data sets.

a. 7, 13, 2, 16, 9, 0, 18, 5, 20, 11

b. 16, 5, 2, 12, 19, 7, 0, 20, 11, 14, 8, 3, 18

Worked example 3

Determining measures of spread

Calculate the range and interquartile range (IQR) of the following data sets.

a. 18, 7, 12, 3, 10, 19, 2, 14, 16, 5

WE3a

Working

2, 3, 5, 7, 10, 12, 14, 16, 18, 19

$$\begin{aligned}\text{Range} &= \text{maximum} - \text{minimum}\\ &= 19 - 2\\ &= 17\end{aligned}$$

$Q_1 = 5$

$Q_3 = 16$

$$\begin{aligned}\text{IQR} &= 16 - 5\\ &= 11\end{aligned}$$

Thinking

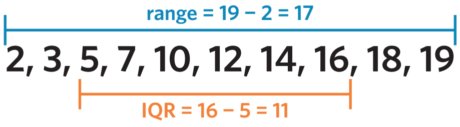
Step 1: Organise the data set in ascending order.

Step 2: Calculate the range.

Step 3: Determine the first and third quartiles.

Step 4: Calculate the IQR.

Visual support



b. 10, 6, 15, 3, 20, 0, 12, 8, 1, 18, 7, 13, 2

WE3b

Working

0, 1, 2, 3, 6, 7, 8, 10, 12, 13, 15, 18, 20

$$\begin{aligned}\text{Range} &= \text{maximum} - \text{minimum}\\ &= 20 - 0\\ &= 20\end{aligned}$$

$$Q_1 = \frac{2 + 3}{2} = 2.5$$

$$Q_3 = \frac{13 + 15}{2} = 14$$

$$\begin{aligned}\text{IQR} &= 14 - 2.5\\ &= 11.5\end{aligned}$$

Thinking

Step 1: Organise the data set in ascending order.

Step 2: Calculate the range.

Step 3: Determine the first and third quartiles.

Step 4: Calculate the IQR.

Student practice

Calculate the range and interquartile range (IQR) of the following data sets.

a. 5, 10, 18, 2, 20, 9, 14, 1, 16, 7

b. 7, 16, 0, 20, 14, 2, 18, 12, 9, 5, 11, 4, 19

11A Questions

Understanding worksheet

1. Circle the middle value(s) in the following ordered data sets.

> **Example**
>
> 3, 4, 5, 6, 7, 8, 9, 10

a. 2, 3, 5, 7, 11

b. 4, 7, 9, 12, 15, 19

c. 1, 5, 9, 13, 15, 20, 25

d. 1, 4, 6, 8, 11, 13, 16, 19

2. Circle any potential outliers in the following data sets.

> **Example**
>
> 9, 4, 6, 5, 26, 7, 2

a. 15, 25, 10, 1, 20

b. 7, 95, 15, 22, 25, 35

c. 23, 7, 19, 43, 13, 41, 31, 29, 70

d. 3, 50, 38, 45, 41, 110, 52, 48, 39

3. Fill in the blanks by using the words provided.

median **spread** **summary** **interquartile range** **typical**

Measures of centre and spread are known as ______ statistics. Examples of measures of centre include the mode, ______ and mean, and they usually indicate the ______ value of a data set. Examples of measures of spread include the range and ______, and they usually indicate the ______ of a data set.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (c,d,e), 8

4. Determine the mean, median and mode(s) of the following data sets. Round to one decimal place where necessary. WE1

a. 4, 7, 11, 13, 14, 14, 15, 17, 19

b. 12, 13, 14, 14, 15, 16, 16, 17, 18, 19

c. 5, 2, 14, 18, 10, 3, 7, 40, 11

d. 2, 3.4, 0.1, 0.5, 1, 1, 8, 3.2, 5, 3, 0.8, 3.8

e. 23, 19, 27, 28, 35, 19, 32, 18, 23, 29, 37, 24, 28, 30, 16

f. 170, 145, 231, 160, 145, 157, 159, 164, 143, 179, 155, 167, 141, 137, 158, 135

g. 6, −9, 0, −5, −1, −3, 8, 10, 0, 3, 2, −7

h. 12.3, 15.6, 16.7, 19.8, 14.2, 17.1, 13.8, 20.1, 11.2, 14.9, 18.5, 16.4, 19.6

Back to contents

5. Determine the first and third quartiles (Q_1 and Q_3) of the following data sets. **WE2**

a. 3, 4, 5, 7, 11, 14, 16, 18, 19

b. 2, 3, 6, 8, 10, 12, 14, 17, 18, 19

c. 12, 7, 20, 18, 10, 5, 9, 19, 11

d. 0.7, 3.5, 1.1, 0.9, 2, 2, 7.2, 4.1, 4.8, 3.6, 0.2, 2.9

e. 18, 32, 24, 43, 11, 27, 6, 38, 22, 49, 21, 29, 14

f. 14, 23, 16, 27, 12, 32, 19, 21, 24, 31, 18, 28, 20, 25, 17

g. −3, −8, 1, 6, −7, 2, 0, 10, 4, −6, −2, 9

h. 17.5, 13.9, 20.2, 15.1, 18.6, 14.8, 12.7, 16.9, 19.3, 16.3, 13.5, 18.2, 14.6

---

6. Calculate the range and interquartile range (IQR) of the data sets from **question 5**. **WE3**

a. 3, 4, 5, 7, 11, 14, 16, 18, 19

b. 2, 3, 6, 8, 10, 12, 14, 17, 18, 19

c. 12, 7, 20, 18, 10, 5, 9, 19, 11

d. 0.7, 3.5, 1.1, 0.9, 2, 2, 7.2, 4.1, 4.8, 3.6, 0.2, 2.9

e. 18, 32, 24, 43, 11, 27, 6, 38, 22, 49, 21, 29, 14

f. 14, 23, 16, 27, 12, 32, 19, 21, 24, 31, 18, 28, 20, 25, 17

g. −3, −8, 1, 6, −7, 2, 0, 10, 4, −6, −2, 9

h. 17.5, 13.9, 20.2, 15.1, 18.6, 14.8, 12.7, 16.9, 19.3, 16.3, 13.5, 18.2, 14.6

---

7. By first identifying any potential outliers, calculate the best measure of centre and spread for the following data sets. Round to one decimal place where necessary.

a. 3, 6, 10, 13, 16, 19, 21, 23, 26, 29

b. 7, 9, 10, 12, 14, 15, 16, 17, 18, 40

c. 14, 9, 22, 20, 12, 7, −10, 11, 23, 15, 2

d. 1.2, 8.7, 6.3, 2.9, 9.8, 5.1, 0.4, 3.6, 7.9, 0.8, 9.0

e. −20, 5, −1, 3, −8, 7, 1, −5, −9, 0, 6, 2, 8, −6, 4

---

8. Consider the following data set:

19, 22, 23, 25, 28, 28, 29, 33, 35, 38, 39, 48, 50, 59

What is the median of this data set?

A. 28 **B.** 29 **C.** 31 **D.** 33 **E.** 34

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the interquartile range of the following data set:

7, 23, 10, 4, 17, 2, 11, 19, 5, 8

Student A

First half: 7, 23, 10, 4, 17

$Q_1 = 10$

Second half: 2, 11, 19, 5, 8

$Q_3 = 19$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 19 - 10 \\ &= 9\end{aligned}$$

Student B

2, 4, 5, 7, 8, 10, 11, 17, 19, 23

First half: 2, 4, 5, 7, 8

$Q_1 = 5$

Second half: 10, 11, 17, 19, 23

$Q_3 = 17$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 17 - 5 \\ &= 12\end{aligned}$$

b. Calculate the mean and range of the following data set:

9, 15, 47, 6, 8, 11, 5, 12, 7, 14

Round any values to two decimal places where necessary.

Student A

47 is an outlier and should be removed.

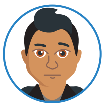
$$\text{Mean} = \frac{9 + 15 + 6 + 8 + 11 + 5 + 12 + 7 + 14}{9}$$

$$= \frac{87}{9}$$

$$= 9.7$$

$$\text{Range} = 15 - 6$$

$$= 9$$



Student B

$$\text{Mean} = \frac{9 + 15 + 47 + 6 + 8 + 11 + 5 + 12 + 7 + 14}{10}$$

$$= \frac{134}{10}$$

$$= 13.4$$

$$\text{Range} = 47 - 5$$

$$= 42$$

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Dylan records the times (in seconds) of nine athletes in a 100 m sprint. The times are 10.8, 11.2, 10.5, 11.1, 10.8, 10.9, 10.6, 11.3 and 10.7. What are the mean and median of the athletes' times in this sprint? Round to one decimal place.

11. A meteorologist records the daily temperatures (in degrees Celsius) for a week. The temperatures are displayed in the following table.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temperature	22	24	23	19	25	21	20

What are the range and interquartile range of temperatures recorded for the week?

12. Karen is a researcher and wants to compare the median length of stay for two different wards in a hospital. The wards treat the same types of patients. Ward A has 10 patients and Ward B has 13 patients. The length of stay (in days) for both wards are:

Ward A: 4, 5, 6, 2, 3, 3, 3, 5, 8, 45

Ward B: 3, 4, 4, 5, 6, 7, 9, 10, 12, 15, 20, 25, 80

Which ward has a shorter median length of stay?

13. A business owner wants to know the number of items sold in a given week. The owner knows that the total revenue generated from the sales is $12 000 and the average selling price of each item is $25. How many items were sold during that week?

14. A car dealer wants to analyse the prices of the used cars they sold on a particular day. The data is shown below.

$16 800, $19 400, $16 200, $18 200, $15 750, $14 500, $66 000, $17 100 and $21 500

They calculate some summary statistics of the data:

Summary statistic	Mean	Median	Range	IQR
Value	$22 827.78	$17 100	$51 500	$4475

They realise that the value $66 000 was actually the price of a new car sold on the same day. How much would each of the mean, median, range, and interquartile range change after this outlier is removed from the data set?

Back to contents

Reasoning

Question working paths

Mild 15 (a,b,c,e)	**Medium** 15 (a,b,c,e), 16 (a,b)	**Spicy** All

15. Yolanda can catch the train or bus to travel to school. She wants to collect data on the travel times of both forms of transport to decide which option is best. She travels on the train and bus for 10 days each and collects the following data.

Train travel times (minutes)	Bus travel times (minutes)
65	55
67	60
68	65
72	70
75	75
70	80
71	85
73	90
74	95
69	100

- **a.** Calculate the best measure of centre for both the train and bus travel times.
- **b.** Calculate the best measure of spread for both the train and bus travel times.
- **c.** Which public transport option is typically quicker? Explain your answer.
- **d.** Which public transport option is more consistent? Explain your answer.
- **e.** List two other factors that Yolanda should consider when deciding which mode of transport to select.

16. Consider the data set 24, 30, 44, 35, 42, 34, 22, 49.

- **a.** The values 30 and 40 are added. Calculate the value of the mean before and after the new values are added.
- **b.** The values 15 and 55 are added. Calculate the value of the mean after the new values are added.
- **c.** Using your answers from parts **a** and **b**, explain why the means remain unchanged when the new values were added.

Exam-style

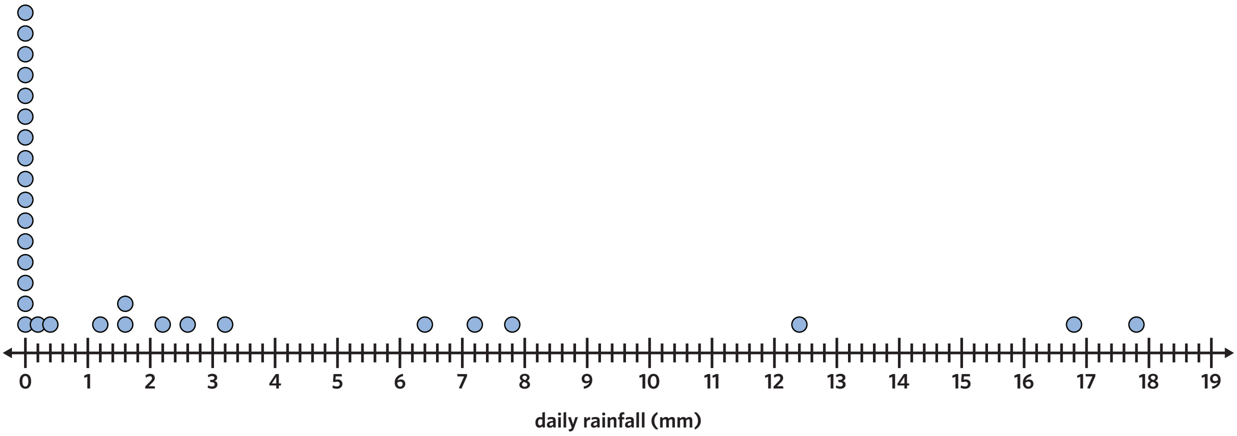
17. The following data set shows the distribution of mathematics test scores for a class of 23 students. (1 MARK)

40, 41, 44, 44, 52, 57, 59, 59, 59, 65, 66, 68, 68, 69, 69, 70, 70, 75, 76, 77, 78, 85, 89

For this class, the interquartile range (IQR) of test scores is

A. 14.5 **B.** 17.5 **C.** 18 **D.** 24 **E.** 49

18. The dot plot below shows the distribution of daily rainfall, in millimetres, at a weather station for 30 days in September. (2 MARKS)



- **a.** Write down the range. 1 MARK
- **b.** Write down the median. 1 MARK

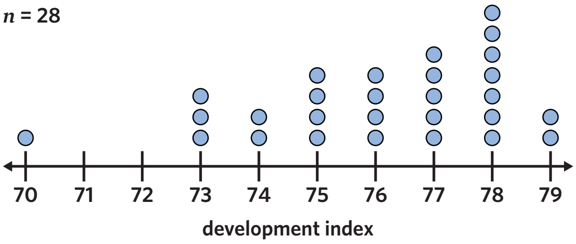
19. The following data sets represent the wingspan, in millimetres, of 32 moths in two locations: forest and grassland. (3 MARKS)

Forest: 12, 14, 16, 18, 24, 28, 32, 36, 40, 44, 46, 50, 52

Grassland: 12, 15, 16, 18, 20, 23, 27, 32, 35, 38, 40, 42, 44, 45, 45, 46, 48, 49, 49

Calculate the difference in means between the moths caught in the forest and the moths caught in the grassland to one decimal place.

20. The development index for each country is a whole number between 0 and 100. The following dot plot displays the values of the development index for each of the 28 countries that have a high development index. (3 MARKS)

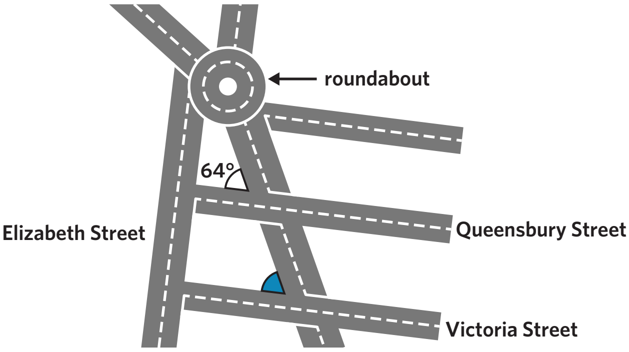


Identify any outliers, and calculate the appropriate measures of centre and spread for this data to one decimal place, where applicable.

Remember this?

21. A city planner is modifying the roundabout at Elizabeth Street.

She needs to know the existing positions and angles of the streets near the roundabout.



She knows that Queensberry Street and Victoria Street are parallel.

What is the size of the shaded angle on the map?

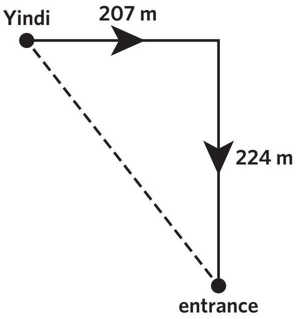
A. 26° **B.** 32° **C.** 45° **D.** 64° **E.** 116°

22. A fair \$1 coin and a fair \$2 coin are tossed at the same time.

What is the probability that one coin lands on heads and the other coin lands on tails?

A. 0 **B.** $\frac{1}{4}$ **C.** $\frac{1}{3}$ **D.** $\frac{1}{2}$ **E.** 1

23. Yindi is walking in a park and realises she needs to rush home. She can get back to the entrance in two ways: walking around the perimeter or cutting straight across the park. This is shown in the following diagram.



How many metres less does she walk by cutting straight across the park?

A. 126 m

B. 288 m

C. 305 m

D. 345 m

E. 431 m

Back to contents

11B Stem-and-leaf plots

LEARNING INTENTIONS

Students will be able to:

- understand the purpose and structure of a stem-and-leaf plot, and how it can be used to represent values in a data set
- display data in a stem-and-leaf plot, use it to calculate summary statistics, and describe the characteristics of the data set
- construct and interpret back-to-back stem-and-leaf plots, and use them as a tool to compare and contrast different data sets.

A stem-and-leaf plot is a tool used to represent values in a data set. Its structure allows for easy calculation of summary statistics and provides a clear visual representation of the data set's characteristics. Back-to-back stem-and-leaf plots can be constructed to compare and contrast different data sets. The ability to create these plots, extract and calculate summary statistics, and use them for comparison is essential for comprehensive data analysis.

KEY TERMS AND DEFINITIONS

- **Frequency** refers to how many times something occurs.
- **Symmetrical data** refers to data sets where the values are evenly distributed around the mean, such that the left and right halves of the data distribution are mirror images of each other.
- **Skewed data** refers to data sets where the values are not symmetrically distributed around the mean.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: MITstudio/Shutterstock.com

Stem-and-leaf plots are commonly used in meteorology to analyse and represent weather data such as temperatures or rainfall over a period. This visual tool assists in calculating summary statistics and describing characteristics of the data set, leading to more accurate weather predictions.

Key ideas

1. A stem-and-leaf plot is a tool for representing values in a data set, with its structure allowing for easy calculation of summary statistics and description of data set characteristics.

Stem	Class A				
0	1				
1	2	4			
2	0	2	2		
3	8	9	9	9	
4	5	5	7	8	9

Negatively skewed

Data in each row is ordered from smallest to largest

Key

3 | 0 = 30 marks

A key is used to show the place value and how the data in the stem-and-leaf plot is read

2. Back-to-back stem-and-leaf plots can be constructed and interpreted to compare and contrast different data sets.

					Class A	Stem	Class B				
	9	6	5	4	3	0	1	1			
7	5	5	4	1	0	1	2	4	7	8	
			9	9	8	2	0	2	2	5	5
				2	3	3	8	9	9	9	
					0	4	0	0			

Positively skewed

40 represents the highest mark

1 represents the lowest mark

Symmetrical

Back to contents

Worked example 1

Constructing and using a stem-and-leaf plot

Using the following data set:
3.0, 4.2, 2.8, 3.5, 5.0, 4.4, 0.2, 4.6, 7.8, 1.8, 5.3, 2.4, 3.6, 3.4, 2.1, 1.3, 6.1, 4.7, 6.7, 5.6

a. Arrange the data set in an ordered stem-and-leaf plot. **WE1a**

Working

Stem	Leaf			
0	2			
1	3	8		
2	1	4	8	
3	0	4	5	6
4	2	4	6	7
5	0	3	6	
6	1	7		
7	8			

Key

2 | 4 = 2.4

Thinking

Step 1: Identify the stem and leaves in the data set. The stem represents the unit values, while the leaves represent the tenths values of each number in the data set. Arrange the data in ascending order and ensure that the stem is long enough to include all values in the set. Enter each value in order from smallest to largest.

Step 2: Include a key.

b. Use the stem-and-leaf plot generated in worked example **1a** to calculate the median and mode, and comment on the shape of the data. **WE1b**

Working

Stem	Leaf			
0	2			
1	3	8		
2	1	4	8	
3	0	4	5	6
4	2	4	6	7
5	0	3	6	
6	1	7		
7	8			

Key

2 | 4 = 2.4

Median = $\frac{3.6 + 4.2}{2} = 3.9$

There is no mode.

The data is symmetrical.

Thinking

Step 1: Split the data set into two equal groups.

Step 2: Identify the median by referring to the 'middle' value in the ordered set. If there is an even number of data values, add the two middle numbers together and divide the result by two.

Step 3: Identify the mode. It is the value(s) with the highest frequency in the data set. If all values have the same frequency, there is no mode.

Step 4: Comment on the data by referring to whether the data displayed is symmetrical or skewed.

Continues →

Back to contents

Student practice

Using the following data set:
4.1, 3.7, 2.5, 5.2, 3.8, 4.9, 0.4, 5.2, 7.4, 1.6, 5.7, 2.6, 3.3, 4.5, 2.3, 1.1, 6.5, 4.4, 6.3, 5.8

a. Arrange the data set in an ordered stem-and-leaf plot.

b. Use the stem-and-leaf plot generated in student practice **1a** to calculate the median and mode, and then comment on the data.

Worked example 2

Constructing back-to-back stem-and-leaf plots

A number of participants were asked to report the number of hours their phone lasted before reaching zero battery.
Brand A: 14, 7, 17, 8, 19, 11, 5, 17, 8, 18, 10
Brand B: 17, 20, 29, 15, 15, 7, 19, 10, 22, 26, 11

a. Generate a back-to-back stem-and-leaf plot with intervals of 10. WE2a

Working

Brand A	Stem	Brand B
8 8 7 5	0	7
9 8 7 7 4 1 0	1	0 1 5 5 7 9
	2	0 2 6 9

Key

1 | 6 = 16 hours

Thinking

Step 1: Identify the stem and leaves in the data set. The stem represents the tens values, while the leaves represent the unit values of each number in the data sets. Arrange the data in each set in ascending order and ensure that the stem is long enough to include all values in both data sets. Enter the values from each data set on either side of the stem, in order from smallest to largest.

Step 2: Include a key.

b. Use the back-to-back stem-and-leaf plot generated in worked example **2a** to compare and contrast the difference between the two phones. WE2b

Working

Brand B has the highest number of hours before the phone has zero battery. Brand A has the lowest number of hours before the phone has zero battery. Brand B has a median battery time of 17 hours, compared to Brand A which has a median battery time of 11 hours. Brand A phone batteries generally last between 5 and 19 hours, and Brand B between 7 and 29 hours.

Thinking

Refer to both sets of data and comment on their similarities and differences.

Student practice

Two classes were asked to report their favourite temperature in degrees Celsius.
Class A: 18, 22, 28, 20, 10, 19, 15, 21, 12, 23, 12
Class B: 5, 21, 20, 11, 12, 18, 15, 15, 13, 10, 6

a. Generate a back-to-back stem-and-leaf plot with intervals of 10.

b. Use the back-to-back stem-and-leaf plot generated in student practice **2a** to compare and contrast the difference between the two classes.

Back to contents

11B Questions

Understanding worksheet

1. Consider the following data:

70, 84, 66, 80, 100, 60, 76, 90, 56, 72, 86, 78, 80, 62, 68, 74, 64, 88, 67, 58.

Fill in the blanks to complete the stem-and-leaf plot for this data set.

Stem	Leaf					
5	6	☐				
6	0	2	4	6	7	8
7	☐	2	4	6	8	
8	0	☐	4	6	8	
9	0					
10	☐					

Key

2 | 4 = 24

2. Fill in the blanks for the following with reference to the back-to-back stem-and-leaf plot.

				Set 1	Stem	Set 2				
				6	5	8				
	8	6	4	0	6	2	6	7		
	8	6	2	0	7	0	0	4	6	8
8	6	4	2	0	8	0	2	4	6	8
				0	9	0	2			

Key

3 | 7 = 37

a. The minimum value for set 2 is ☐.

b. The maximum value for set 1 is ☐.

c. The mode for set 2 is ☐.

d. The median for set 1 is ☐.

3. Fill in the blanks by using the words provided.

median **ordered** **key** **stem** **leaf**

A ☐ -and-leaf plot is a graphic tool where each number is split into two parts. The ☐ is used to give a scale to the data and ensure the values are correctly interpreted. The ☐ part of the number, which is usually the final digit, is then represented graphically. It is essential that the values are ☐ to accurately depict the data set. This type of plot allows easy calculation of the ☐ , among other summary statistics, and facilitates effective comparison of different data sets.

Back to contents

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c), 5 (a,b,c), 6 (a,b), 7 (a,b), 8	4 (b,c,d), 5 (b,c,d), 6 (b,c), 7 (b,c), 8	4 (c,d,e), 5 (c,d,e), 6 (c,d), 7 (c,d), 8

4. For the following data sets arrange the data set in an ordered stem-and-leaf plot. WE1a

a. 38, 27, 30, 21, 7, 17, 34, 52, 33, 24, 27, 18, 22, 30, 25, 27, 39

b. 30, 32, 22, 34, 14, 22, 47, 49, 34, 41, 14, 20, 50, 30, 33, 22, 41, 49, 25, 29, 44, 36, 38, 44

c. 3.4, 2.2, 4.9, 3.2, 5.2, 2.8, 4.6, 3.5, 3.9, 5.0, 5.7, 2.1, 4.2, 4.1

d. 33.9, 34.3, 33.1, 34.0, 35.3, 33.3, 33.8, 34.7, 36.5, 34.7, 35.0, 33.6, 34.9, 36.5, 35.7, 33.0, 34.8, 31.5, 35.6, 34.2

e. 164, 156, 156, 158, 161, 164, 154, 155, 172, 164, 182, 164, 199, 182, 156, 174, 156, 170

5. Use the stem-and-leaf plots generated in **question 4** to calculate the median and mode, and then comment on the data. WE1b

a. 38, 27, 30, 21, 7, 17, 34, 52, 33, 24, 27, 18, 22, 30, 25, 27, 39

b. 30, 32, 22, 34, 14, 22, 47, 49, 34, 41, 14, 20, 50, 30, 33, 22, 41, 49, 25, 29, 44, 36, 38, 44

c. 3.4, 2.2, 4.9, 3.2, 5.2, 2.8, 4.6, 3.5, 3.9, 5.0, 5.7, 2.1, 4.2, 4.1

d. 33.9, 34.3, 33.1, 34.0, 35.3, 33.3, 33.8, 34.7, 36.5, 34.7, 35.0, 33.6, 34.9, 36.5, 35.7, 33.0, 34.8, 31.5, 35.6, 34.2

e. 164, 156, 156, 158, 161, 164, 154, 155, 172, 164, 182, 164, 199, 182, 156, 174, 156, 170

6. For the following data sets generate a back-to-back stem-and-leaf plot using the specified intervals. WE2a

a. Intervals of 10
Set A: 29, 21, 37, 42, 47, 53, 24, 29, 32, 41, 54, 46, 34, 29
Set B: 42, 49, 23, 37, 45, 43, 37, 31, 50, 46, 41, 53, 34, 45

b. Intervals of 1.0
Set X: 0.9, 1.6, 2.2, 7.9, 0.9, 3.1, 4.2, 0.5, 2.1, 1.5, 2.3, 5.7, 6.1, 3.2, 5.6, 1.9, 3.8, 4.2, 1.8, 2.4, 0.6
Set Y: 0.3, 0.7, 0.8, 1.7, 1.3,0.4, 0.6, 2.9, 1.2, 4.1, 5.7, 2.4, 3.1, 6.2, 3.5, 1.5, 4.8, 2.4, 2.9, 3.7, 7.5

c. Intervals of 10
Lakers score over 24 games: 136, 78, 88, 100, 97, 88, 88, 135, 121, 87, 72, 72, 70, 76, 94, 148, 119, 123, 132, 70, 104, 93, 116, 78
Celtics score over 24 games: 114, 90, 137, 98, 113, 105, 108, 128, 99, 92, 113, 125, 108, 87, 129, 101, 81, 80, 73, 99, 113, 107, 82, 78

d. Intervals of 1
Two models of routers were evaluated to measure their transmission speeds in Mbps.
The data below shows the transmission speeds of 15 routers from each model.
Model I: 14.8, 13.0, 13.2, 16.3, 16.4, 15.6, 13.5, 13.7, 13.9, 15.7, 15.9, 14.0, 14.2, 17.1, 17.3
Model II: 10.2, 10.5, 11.1, 12.2, 12.4, 12.6, 10.7, 11.0, 11.2, 13.1, 13.3, 14.5, 10.8, 11.0, 14.3

7. Use the back-to-back stem-and-leaf plots generated in **question 6** to compare and contrast the difference between the two sets of data. WE2b

a. Intervals of 10
Set A: 29, 21, 37, 42, 47, 53, 24, 29, 32, 41, 54, 46, 34, 29
Set B: 42, 49, 23, 37, 45, 43, 37, 31, 50, 46, 41, 53, 34, 45

b. Intervals of 1.0
Set X: 0.9, 1.6, 2.2, 7.9, 0.9, 3.1, 4.2, 0.5, 2.1, 1.5, 2.3, 5.7, 6.1, 3.2, 5.6, 1.9, 3.8, 4.2, 1.8, 2.4, 0.6
Set Y: 0.3, 0.7, 0.8, 1.7, 1.3, 0.4, 0.6, 2.9, 1.2, 4.1, 5.7, 2.4, 3.1, 6.2, 3.5, 1.5, 4.8, 2.4, 2.9, 3.7, 7.5

c. Intervals of 10
Lakers score over 24 games: 136, 78, 88, 100, 97, 88, 88, 135, 121, 87, 72, 72, 70, 76, 94, 148, 119, 123, 132, 70, 104, 93, 116, 78
Celtics score over 24 games: 114, 90, 137, 98, 113, 105, 108, 128, 99, 92, 113, 125, 108, 87, 129, 101, 81, 80, 73, 99, 113, 107, 82, 78

Back to contents

d. Intervals of 1
Two models of routers were evaluated to measure their transmission speeds in Mbps.
The data below shows the transmission speeds of 15 routers from each model.
Model I: 14.8, 13.0, 13.2, 16.3, 16.4, 15.6, 13.5, 13.7, 13.9, 15.7, 15.9, 14.0, 14.2, 17.1, 17.3
Model II: 10.2, 10.5, 11.1, 12.2, 12.4, 12.6 10.7, 11.0, 11.2, 13.1, 13.3, 14.5 10.8, 11.0, 14.3

---

8. Consider the following data displayed in the stem-and-leaf plot. Determine the median.

Stem	Leaf
0	
1	1 3 5
2	5 9
3	1 3 4
4	0 2

Key
1 | 2 = 1.2

A. 2.9 **B.** 3.0 **C.** 3.1 **D.** 30 **E.** 31

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Using the following data sets to generate an ordered back-to-back stem-and-leaf plot
Group A: 3.4, 2.9, 4.7, 5.6, 3.2, 4.1, 1.4, 5.3, 2.6, 5.9, 3.7, 3.9, 4.8, 2.1, 1.8, 6.7, 4.3, 6.1, 5.4
Group B: 4.2, 3.6, 2.4, 5.1, 3.7, 4.6, 0.5, 5.0, 1.5, 5.8, 2.7, 3.1, 4.7, 2.2, 1.0, 6.4, 4.5, 6.2, 5.5

Student A

Group B	Stem	Group A
5	0	
5 0	1	4 8
7 4 2	2	1 6 9
7 6 1	3	2 4 7 9
7 6 5 2	4	1 3 7 8
8 5 1 0	5	3 4 6 9
4 2	6	1 7

Key
2 | 7 = 2.7

Student B

Group B	Stem	Group A
5 1	0	
3 5	1	4 8
6 4 2	2	1 6 9
	3	2 4 7 9
6 2	4	1 3 7 8
5 4 1 0	5	3 4 6 9
3 4	6	1 7
3 4 2	7	
5	8	

Key
2 | 7 = 2.7

b. State the median from the following stem-and-leaf plot.

Stem	Leaf			
0				
1	4	8		
2	1	6	8	9
3	2	4	7	9

Key
3 | 4 = 3.4

Student A

The median is 28.5.

Stem	Leaf			
0				
1	4	8		
2	1	6	8 │	9
3	2	4	7	9

Key
3 | 4 = 3.4

Student B

The median is 2.85.

Stem	Leaf			
0				
1	4	8		
2	1	6	8 │	9
3	2	4	7	9

Key
3 | 4 = 3.4

Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. John has recently taken up running and he has been recording the number of kilometres he runs every day for over a fortnight. Here is his data: 8, 12, 15, 9, 25, 10, 9, 5, 35, 14, 38, 42, 9, 26, 27. Construct a stem-and-leaf plot to display John's running data.

11. Mary is a teacher who has recorded the marks of a recent maths test out of 50. The marks are: 39, 17, 25, 46, 38, 19, 46, 50, 6, 18, 29, 20, 47. Construct a stem-and-leaf plot using Mary's data, and then determine the median score.

12. At a local play centre, the number of cars that enter the play centre car park in one day were recorded for two weeks: 120, 121, 150, 128, 135, 148, 140, 154, 142, 130, 136, 145, 138, 154. Construct a stem-and-leaf plot of the data, determine the median and mode, and comment on whether the data is skewed or symmetrical.

13. A small company recently conducted a survey asking their employees how many hours they worked last Monday.

The results were as follows: 8.5, 9.0, 6.5, 8.0, 9.5, 8.0, 8.0, 7.5, 8.5, 9.0, 7.5, 8.5, 9.0, 9.5, 9.5.

Construct a stem-and-leaf plot of the data, determine the median and mode, and comment on whether the data is skewed or symmetrical.

14. A sports event was conducted where two teams competed. The team scores (out of 50) were recorded as follows:

Team A: 24, 38, 10, 40, 45, 27, 35, 38, 17, 44, 40, 45, 43, 38
Team B: 33, 45, 47, 50, 46, 48, 47, 45, 50, 37, 48, 39, 41, 50

Construct back-to-back stem-and-leaf plots for the two teams. Calculate the mode and median for each team, and compare and contrast the performance of the two teams based on your analysis of the plots.

Back to contents

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶	**Medium** 15 (a,b,c,e), 16 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

15. A local fast-food chain has collected data on the number of customers served per day at two of their busiest locations for a portion of July: Location A and Location B. The data is presented below:

Location A: 85, 98, 90, 85, 130, 120, 95, 82, 110, 88, 120, 95, 100, 85, 88, 100, 110, 90, 85
Location B: 105, 80, 100, 95, 110, 115, 108, 112, 95, 100, 92, 112, 95, 105, 110, 118, 80, 110, 108

a. Arrange the data for Location A and Location B in a back-to-back stem-and-leaf plot.

b. Calculate the median number of customers served per day for both Location A and Location B using the back-to-back stem-and-leaf plot generated in part **a**.

c. From the back-to-back stem-and-leaf plots, identify the mode for both Location A and Location B. Which location has a higher frequency for the mode?

d. With reference to the back-to-back stem-and-leaf plot and your calculations in parts **b** and **c**, compare and contrast the data between Location A and Location B.

e. If the fast-food chain was to run a promotion, what kind of marketing techniques could they use?

16. Consider two data sets:

Set A: 3.1, 2.6, 2.9, 4.3, 5.2, 3.7, 5.5
Set B: 5.0, 4.0, 2.5, 3.0, 4.5, 5.0, 5.3, 3.2, 5.7, 5.5

a. Construct a stem-and-leaf plot for Set A and determine the median.

b. Construct a stem-and-leaf plot for Set B and determine the median.

c. Comment on the difference between calculating the median for Set A compared to Set B.

Exam-style

17. The stem-and-leaf plot displays 16 temperatures recorded in the month of February. (1 MARK)

Stem	Leaf					
1	9	9				
2	0	3	4	5	6	8
3	2	5	5	5	6	7
4	0	1				

Key

3 | 4 = 34°C

The median temperature is

A. 28°C
B. 29°C
C. 30°C
D. 32°C
E. 35°C

18. The back-to-back stem-and-leaf plot displays the heights, in centimetres, of 23 dogs and their breed. (3 MARKS)

Cavoodle				Stem	Poochon					
8	3	1	0	2	2	4				
9	7	3	2	3	0	1	6	6	8	9
		7	6	4	3	4	5	6	7	

Key

2 | 7 = 27 cm

a. State the modal height, in centimetres, for poochon dogs. 1 MARK

b. Show that the median height, in centimetres, for cavoodle dogs is 32.5. 2 MARKS

Back to contents

19. The stem-and-leaf plot shows the distance covered, in kilometres, by Timothy during his last 29 bike rides. (3 MARKS)

Calculate the median and mode and comment on the shape of the data.

Stem	Leaf													
20	5	5												
21	1	1	2	2	3	3	5	5	6	6	7	7		
22	0	0	2	2	3	4	4	5	5	6	7	8	8	8
23	3													

Key

21 | 5 = 21.5 kilometres

20. A stem-and-leaf plot has been generated for 41, 35, 10, 43, 29, 12, 39, 30, 14, 23, 27, 38, 42, 28. An incorrect median value of 31.5 has been calculated. Calculate the median and explain why the median value of 31.5 is incorrect. (2 MARKS)

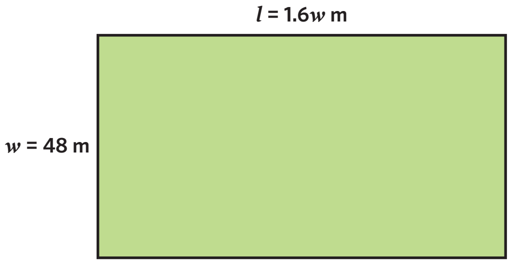
Stem	Leaf			
1	0	2	4	
2	9	3	7	8
3	5	9	0	8
4	1	3	2	

Key

1 | 5 = 15

Remember this?

21. A rectangular playground has two shorter sides each measuring 48 m in length. The longer sides are 1.6 times the length of the shorter sides. Calculate the perimeter of the playground?



A. 96 m
B. 192 m
C. 230.4 m
D. 249.6 m
E. 384 m

22. In a local soccer league, a team played 27 matches in a season. Out of these, they won 11 matches. Which of these fractions is the closest to the proportion of matches they won?

A. $\frac{1}{3}$ **B.** $\frac{2}{5}$ **C.** $\frac{1}{2}$ **D.** $\frac{4}{7}$ **E.** $\frac{3}{5}$

23. For a school assignment, Sarah measures the lengths of leaves in her garden. She records these measurements to the nearest hundredth of a centimetre. Sarah records the length of one of the leaves as 6.32 cm. What is the smallest measurement the actual length of that leaf could be?

A. 6.305 cm **B.** 6.310 cm **C.** 6.315 cm **D.** 6.320 cm **E.** 6.325 cm

Back to contents

11C Grouped data

LEARNING INTENTIONS

Students will be able to:

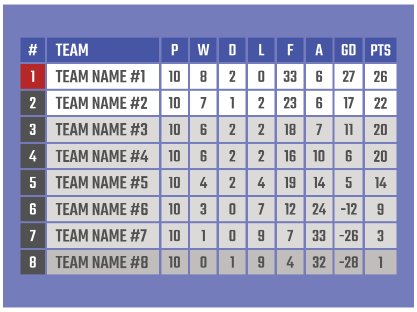
- understand the use of a frequency table for recording data with class intervals
- recognise the different types of data that can be represented in a histogram
- present grouped numerical data in a histogram and interpret the resulting graph.

Frequency tables with class intervals are a useful tool for organising and displaying raw data. They form the basis for creating histograms, which are graphical representations of numerical data. The construction of a histogram involves selecting appropriate class intervals and calculating the percentage frequency for each interval. The resulting graph, with its specific shape and key features, provides a visual display of the data that can be interpreted to gain insights into the underlying distribution.

KEY TERMS AND DEFINITIONS

- A **class interval** defines the data range of the groups into which a set of data has been organised.
- The **range** is a statistical measure of the spread of data. It is the difference between the maximum and minimum values.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



#	TEAM	P	W	D	L	F	A	GD	PTS
1	TEAM NAME #1	10	8	2	0	33	6	27	26
2	TEAM NAME #2	10	7	1	2	23	6	17	22
3	TEAM NAME #3	10	6	2	2	18	7	11	20
4	TEAM NAME #4	10	6	2	2	16	10	6	20
5	TEAM NAME #5	10	4	2	4	19	14	5	14
6	TEAM NAME #6	10	3	0	7	12	24	-12	9
7	TEAM NAME #7	10	1	0	9	7	33	-26	3
8	TEAM NAME #8	10	0	1	9	4	32	-28	1

Image: Anton Prohorov/Shutterstock.com

In sports analytics, coaches often use frequency tables with class intervals to record athletes' performance data, such as run times or goals scored. These can be visualised in a histogram to help analyse performance trends and make decisions about training programs.

Key ideas

1. A frequency table with class intervals is a useful tool for recording and displaying raw data, and the class intervals should be appropriate for the given data.

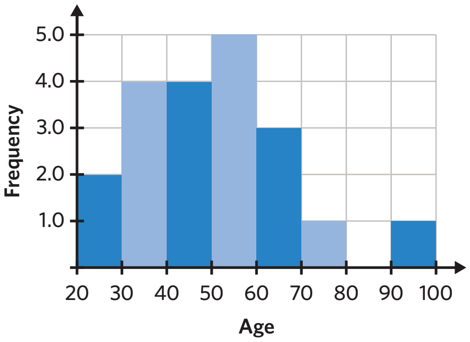
Class interval	Frequency	Percentage frequency
a–$<b$	The number of data values in this class interval	$\frac{\text{frequency}}{\text{total}} \times 100$
b–$<c$	The number of data values in this class interval	$\frac{\text{frequency}}{\text{total}} \times 100$
c–$<d$	The number of data values in this class interval	$\frac{\text{frequency}}{\text{total}} \times 100$
…	…	…
Total	Total number of data values	100%

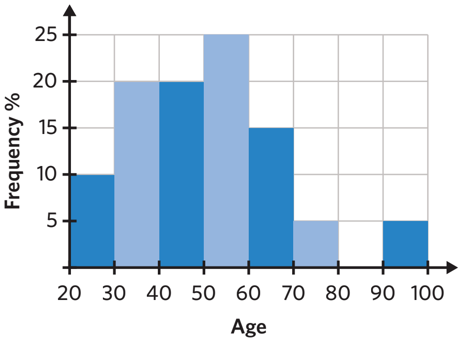
Where a, b, c, and d are dependent on the data.

Continues →

Back to contents

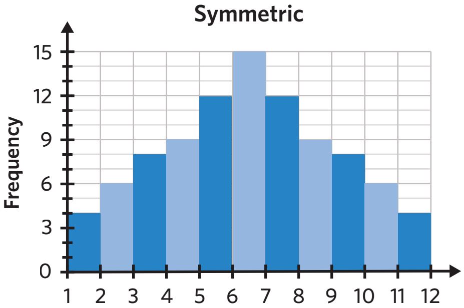
2. A histogram is a statistical graph for displaying the distribution of continuous data.

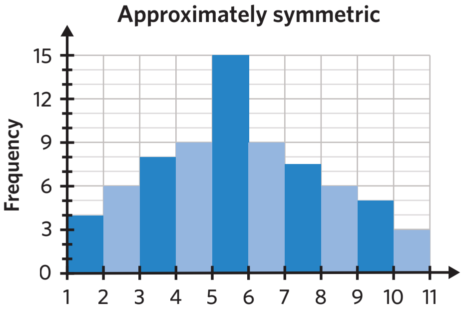

Frequency histogram


Percentage frequency histogram

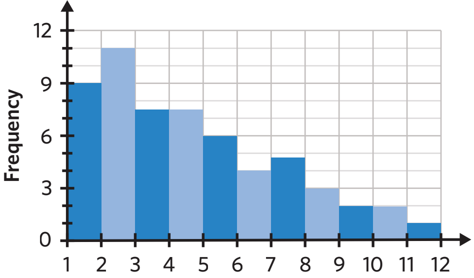
3. Histograms must contain data grouped in exclusive intervals only as there must not be any gaps between the bars. A histogram shape or distribution can be described as symmetric, skewed, or bimodal.

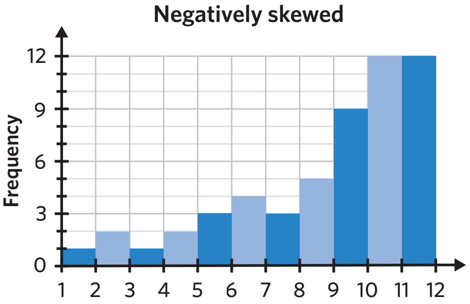
A **symmetric** distribution is the same on both sides of the centre. If the distribution isn't exactly symmetric, it is important to describe the shape as approximately symmetric.


Symmetric

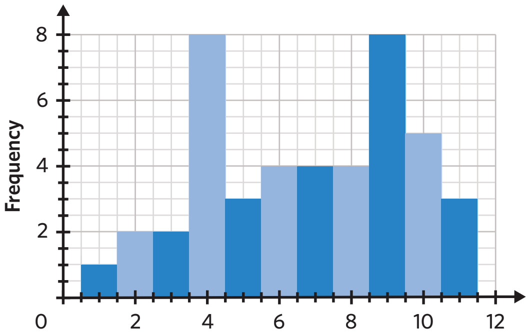

Approximately symmetric

A **positively skewed** distribution trails off in a positive direction on the horizontal axis.
A **negatively skewed** distribution trails off in a negative direction on the horizontal axis.


Positively skewed


Negatively skewed

A **bimodal** distribution has two peaks (two modes). The distribution will have a distinct gap between two cohesive groups of bars.



Worked example 1

Constructing frequency tables and frequency histograms

The numbers of hours individuals in a class have worked in a fortnight are provided.
23, 5, 7, 11, 1, 13, 25, 18, 8, 19, 28, 2, 6, 10, 14, 9, 29, 3, 12, 21, 28, 16, 4, 4, 2

a. Complete the grouped frequency table. **WE1a**

Number of hours worked per fortnight	Frequency	Percentage frequency
0–<5		
5–<10		
10–<15		
15–<20		
20–<25		
25–<30		
Total		

Working

Number of hours worked per fortnight	Frequency	Percentage frequency
0–<5	6	$\frac{6}{25} \times 100 = 24\%$
5–<10	5	$\frac{5}{25} \times 100 = 20\%$
10–<15	5	$\frac{5}{25} \times 100 = 20\%$
15–<20	3	$\frac{3}{25} \times 100 = 12\%$
20–<25	2	$\frac{2}{25} \times 100 = 8\%$
25–<30	4	$\frac{4}{25} \times 100 = 16\%$
Total	**25**	**100%**

Thinking

Step 1: Determine the frequency for each class interval occurrence in the 'frequency' column.

Step 2: Determine the 'percentage frequency' by dividing the frequency by the total and multiplying by 100.

b. Using worked example **1a**, construct a frequency histogram and comment on the distribution. **WE1b**

Working

Minimum value = 0

Maximum value = 30

Minimum frequency = 2

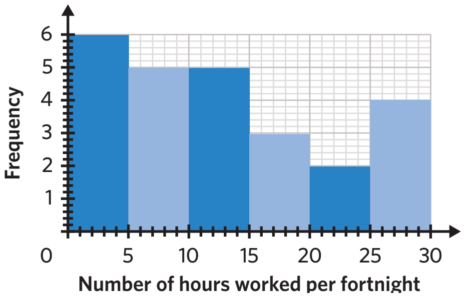
Maximum frequency = 6

Thinking

Step 1: Use the class intervals from the frequency table to determine how to scale the horizontal (x) axis.

Step 2: Identify the smallest and largest frequency value from the frequency table to determine how to scale the vertical (y) axis.

Continues →



Step 3: Draw the bars of the histogram using the frequencies and class intervals given in the table. Label the histogram's axes.

The distribution for the number of hours worked per fortnight is positively skewed.

Step 4: Use the shape of the histogram to comment on the distribution of the data.

Student practice

The distances students travel to school, in kilometres, are provided.
5, 15, 30, 22, 17, 35, 12, 25, 42, 33, 21, 29, 24, 18, 27, 8, 19, 20, 11, 8, 28, 22, 26, 21, 26.

a. Complete the grouped frequency table.

Distance travelled (km)	Frequency	Percentage frequency
5–<10		
10–<15		
15–<20		
20–<25		
25–<30		
30–<35		
35–<40		
40–<45		
Total		

b. Using student practice **1a**, construct a frequency histogram and comment on the distribution.

Worked example 2

Constructing frequency tables and percentage frequency histograms

The times (seconds) recorded for 20 year 9 students to complete a wall sit in Ms Jenny's class are recorded.
34.5, 43.5, 46.6, 48.8,31.7, 32.3, 38.1, 25.6, 27.2, 31.4, 41.1, 41.5, 42.4, 51.9, 32.4, 37.5, 37.8, 35.6, 36.5, 23.4

a. Complete the grouped frequency table.

WE2a

Wall sit (seconds)	Frequency	Percentage frequency
22–<27		
27–<32		
32–<37		
37–<42		
42–<47		
47–<52		
Total		

Continues →

Back to contents

Working

Wall sit (seconds)	Frequency	Percentage frequency
22–<27	2	$\frac{2}{20} \times 100 = 10\%$
27–<32	3	$\frac{3}{20} \times 100 = 15\%$
32–<37	5	$\frac{5}{20} \times 100 = 25\%$
37–<42	5	$\frac{5}{20} \times 100 = 25\%$
42–<47	3	$\frac{3}{20} \times 100 = 15\%$
47–<52	2	$\frac{2}{20} \times 100 = 10\%$
Total	**20**	**100%**

Thinking

Step 1: Determine the frequency for each class interval occurrence in the 'frequency' column.

Step 2: Determine the 'percentage frequency' by dividing the frequency by the total and multiplying by 100.

b. Using worked example **2a**, construct a percentage frequency histogram and comment on the distribution. **WE2b**

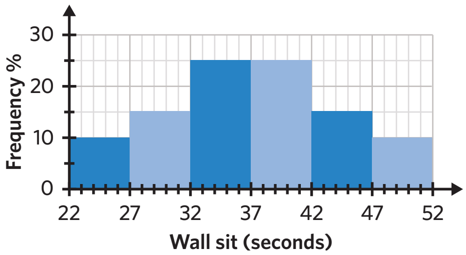
Working

Minimum value $= 22$

Maximum value $= 52$

Minimum percentage frequency $= 10\%$

Maximum percentage frequency $= 25\%$



The distribution for the wall sit times, in seconds, is symmetric.

Thinking

Step 1: Use the class intervals from the frequency table to determine how to scale the horizontal (x) axis.

Step 2: Identify the smallest and largest percentage frequency value from the frequency table to determine how to scale the vertical (y) axis.

Step 3: Draw the bars of the histogram using the percentage frequencies and class intervals given in the table. Label the histogram's axes.

Step 4: Use the shape of the histogram to comment on the distribution of the data.

Student practice

The weekly savings ($) for 15 year 11 students in Mr. Thompson's economics class are provided.
58.5, 62.0, 70.5, 65.9, 58.7, 52.3, 76.4, 80.2, 67.5, 55.4, 52.6, 73.0, 72.1, 59.8, 68.7

a. Complete the grouped frequency table.

Weekly savings ($)	Frequency	Percentage frequency
52–<57		
57–<62		
62–<67		
67–<72		
72–<77		
77–<82		
Total		

b. Using student practice **2a**, construct a percentage frequency histogram and comment on the distribution.

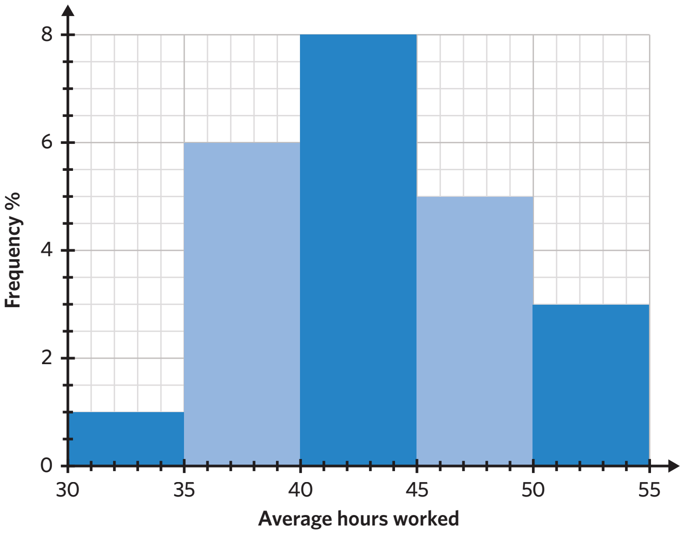
11C Questions

Understanding worksheet

1. Calculate the missing values in the grouped frequency table.

Hours of study	Frequency	Percentage frequency
0–<3		10%
3–<6	4	%
6–<9		30%
9–<12	2	20%
Total	**10**	%

2. With reference to the histogram, complete the sentences.



a. There were a total of student(s) who worked 30–<35 hours over a fortnight.

b. There were a total of student(s) who participated in this data collection.

c. The highest frequency (modal class) was the class interval.

d. The total number of students who worked an average of 35–<50 hours in a fortnight is .

3. Fill in the blanks by using the words provided.

shape	**histogram**	**percentage**	**frequency**	**axis**

A is a tool used to present grouped numerical data. This graph uses the of data within a given class interval as the height of the bars, with the class intervals themselves being represented along the horizontal . It is essential to understand the of a histogram as it can provide insights into the distribution of the data. The frequency for each bar, represents the proportion of the total data that falls within each class interval.

Back to contents

Fluency

Question working paths

Mild 4, 6, 8	Medium 5, 7, 8	Spicy All

4. The number of books each student in a class reads each month is provided. WE1

15, 12, 10, 14, 17, 3, 20, 29, 22, 9, 16, 30, 3, 5, 13, 17, 8, 33, 2, 18, 24, 27, 19, 6, 5, 1, 15

a. Using the given frequency table, complete the frequency column.

b. Using the given frequency table, complete the frequency percentage column. Round to two decimal places where necessary.

c. Using the completed frequency table, construct a frequency histogram.

d. Use the shape of the histogram to comment on the distribution of the data.

e. How many students read less than 10 books in a month?

f. What percentage of students read 20 or more books in a month? Round to two decimal places.

Number of books a student reads	Frequency	Percentage frequency
0–<5		
5–<10		
10–<15		
15–<20		
20–<25		
25–<30		
30–<35		
Total		

---

5. The number of movies each person in a film club watches each month is provided. WE1

5, 0, 24, 8, 1, 2, 12, 4, 20, 16, 9, 25, 7, 0, 3, 2, 11, 21, 6, 27, 14, 13, 4, 10, 17, 5, 1, 2

a. Generate a grouped frequency table with class intervals of 4 units, starting with 0–<4. Round to two decimal places where necessary.

b. Using the completed frequency table, construct a frequency histogram.

c. Use the shape of the histogram to comment on the distribution of the data.

d. How many people watch less than 8 movies each month?

e. What percentage of people watch at least 4 but less than 16 movies each month? Round to two decimal places.

---

6. The electricity consumption (kWh) for 22 houses on Maple Street during the week (excluding the weekend) is provided. WE2

90.2, 75.5, 83.4, 95.7, 87.6, 92.5, 100.0, 78.0, 89.3, 96.5, 70.5, 85.3, 93.4, 81.2, 99.0, 77.1, 91.4, 95.2, 85.1, 99.9, 92.2, 98.9

a. Using the given frequency table, complete the frequency column.

b. Using the given frequency table, complete the percentage frequency column. Round to two decimal places where necessary.

c. Using the completed frequency table, construct a percentage frequency histogram.

d. Use the shape of the histogram to comment on the distribution of the data.

e. How many houses use less than 80 kWh of electricity?

f. What percentage of houses use greater than 90 but less than 100 kWh of electricity?

Electricity consumption (kWh)	Frequency	Percentage frequency
70–<75		
75–<80		
80–<85		
85–<90		
90–<95		
95–<100		
100–<105		
Total		

7. The following data set displays the length, in centimetres, of 20 snakes observed by Dr. Martinez in the desert: **WE2**

29.5, 40.8, 35.4, 46.6, 39.3, 50.2, 52.0, 31.7, 48.8, 41.5, 36.6, 37.0, 44.2, 47.5, 49.9, 38.1, 42.0, 40.0, 34.2, 43.9

a. Generate a grouped frequency table with class intervals of 4 units, starting with 28–<32.

b. Using the completed frequency table, construct a percentage frequency histogram.

c. Use the shape of the histogram to comment on the distribution of the data.

d. How many snakes have a length greater than 36 cm?

e. What percentage of snakes have a length greater than 44 cm but less than 56 cm?

---

8. Which grouped frequency table correctly displays the height, in centimetres, of 12 plants in a biology experiment: 15.5, 22.3, 20.1, 25.8, 18.6, 28.7, 21.9, 27.4, 23.2, 29.5, 19.2, 24.6?

A.

Height of plant (cm)	**Frequency**	**Percentage frequency**
15.5–<21	3	$\frac{3}{12} \times 100 = 25\%$
20–<25	4	$\frac{4}{12} \times 100 = 33.33\%$
25–<30	5	$\frac{5}{12} \times 100 = 41.67\%$
Total	**12**	**100%**

B.

Height of plant (cm)	**Frequency**	**Percentage frequency**
15.5–<21	4	$\frac{4}{12} \times 100 = 33.33\%$
21–<26	5	$\frac{5}{12} \times 100 = 41.67\%$
26–<31	3	$\frac{3}{12} \times 100 = 25\%$
Total	**12**	**100%**

C.

Height of plant (cm)	**Frequency**	**Percentage frequency**
15–21	4	$\frac{4}{12} \times 100 = 33.33\%$
21–26	5	$\frac{5}{12} \times 100 = 41.67\%$
26–31	3	$\frac{3}{12} \times 100 = 25\%$
Total	**12**	**100%**

D.

Height of plant (cm)	**Frequency**	**Percentage frequency**
15.5–<20.5	4	$\frac{4}{12} \times 100 = 33.33\%$
20.5–<25.5	4	$\frac{4}{12} \times 100 = 33.33\%$
25.5–<30.5	4	$\frac{4}{12} \times 100 = 33.33\%$
Total	**12**	**100%**

E.

Height of plant (cm)	**Frequency**	**Percentage frequency**
15–20	4	$\frac{4}{12} \times 100 = 33.33\%$
20–25	5	$\frac{5}{12} \times 100 = 41.67\%$
25–30	3	$\frac{3}{12} \times 100 = 25\%$
Total	**12**	**100%**

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Generate a grouped frequency table with class intervals of size 5 for the given data.

41.7, 34.8, 33.0, 31.5, 42.6, 24.8, 47.8, 50.4, 47.2, 23.8, 29.4, 26.1, 44.5, 48.2, 37.5, 25.9, 45.3, 23.5

Student A

Class interval	Frequency	Percentage frequency
22–<27	5	$\frac{5}{20} \times 100 = 25\%$
27–<32	2	$\frac{2}{20} \times 100 = 10\%$
32–<37	2	$\frac{2}{20} \times 100 = 10\%$
37–<42	2	$\frac{2}{20} \times 100 = 10\%$
42–<47	3	$\frac{3}{20} \times 100 = 15\%$
47–<52	4	$\frac{4}{20} \times 100 = 20\%$
Total	**20**	**100%**

Student B

Class interval	Frequency	Percentage frequency
22–27	5	$\frac{5}{20} \times 100 = 25\%$
28–33	3	$\frac{3}{20} \times 100 = 15\%$
34–39	2	$\frac{2}{20} \times 100 = 10\%$
40–45	4	$\frac{4}{20} \times 100 = 20\%$
46–51	4	$\frac{4}{20} \times 100 = 20\%$
52–57	0	$\frac{0}{20} \times 100 = 0\%$
Total	**20**	**100%**

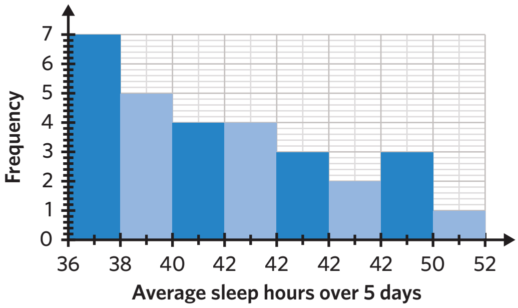
b. Construct a histogram to display the grouped frequency table.

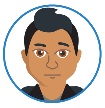
Class interval	Frequency	Percentage frequency
36–<38	7	$\frac{7}{29} \times 100 = 24.14\%$
38–<40	5	$\frac{5}{29} \times 100 = 17.24\%$
40–<42	4	$\frac{4}{29} \times 100 = 13.79\%$
42–<44	4	$\frac{4}{29} \times 100 = 13.79\%$
44–<46	3	$\frac{3}{29} \times 100 = 10.34\%$
46–<48	2	$\frac{2}{29} \times 100 = 6.90\%$
48–<50	3	$\frac{3}{29} \times 100 = 10.34\%$
50–<52	1	$\frac{1}{29} \times 100 = 3.45\%$
Total	**29**	**99.99%***

*Due to rounding.

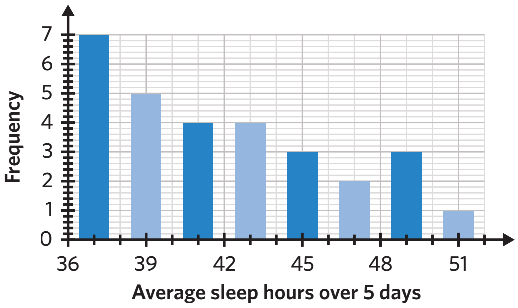
Back to contents

Student A





Student B



Problem solving

Question working paths

Mild 10, 11, 12 🌶	**Medium** 11, 12, 13 🌶🌶	**Spicy** 12, 13, 14 🌶🌶🌶

10. Sarah conducted a survey at school to find out the number of books students read in a month. She collected the following data: 8, 5, 12, 9, 7, 4, 10, 6, 8, 11, 5, 9, 10, 7, 6

Complete the grouped frequency table for the number of books read. Round to two decimal places where necessary.

Number of books read	Frequency	Percentage frequency
0–<5		
5–<10		
10–<15		
Total		

11. During the school's sports day, the PE teacher recorded the time taken (in seconds) by students to complete a 100 m sprint. The data is as follows: 15.2, 14.8, 16.5, 15.0, 15.4, 16.2, 14.9, 15.1, 16.0, 15.3. Using the data, construct a grouped frequency table with class intervals of 0.5, starting with interval 14.5–<15.

12. A local gym recorded the number of hours members exercised in a month. The data is as follows:

15, 20, 18, 22, 17, 19, 21, 15, 20, 18, 22. Using the data, construct a frequency histogram with class intervals of 3 starting with interval 15–<18, and then use the shape of the histogram to comment on the distribution of the data.

13. A basketball coach recorded the number of points scored by each player in a season. The data is displayed in the grouped frequency table.

Points scored	Frequency	Percentage frequency
0–<10	5	27.78%
10–<20	8	44.44%
20–<30	3	16.67%
30–<40	2	11.11%
Total	**18**	**100%**

Construct a percentage frequency histogram to represent the data and analyse the performance of the players. Is the data symmetric, skewed, or bimodal?

Back to contents

14. A local farmer recorded the number of apples picked from his orchard over 16 days. The data is as follows: 52, 50, 55, 60, 45, 52, 50, 55, 53, 60, 45, 53, 50, 55, 60, 51. Construct a frequency table with class intervals of 3 units, starting with the interval 45–<48, and then create a frequency histogram using the frequency table. Comment on the distribution.

Reasoning

Question working paths

Mild 15 (a,b,c,e) 🌶

Medium 15 (a,b,c,e), 16 (a,b) 🌶🌶

Spicy All 🌶🌶🌶

15. The grouped frequency table displays the total minutes of incidental exercise a 15 year old from Ms Holly's Humanities class completes in a day.

Total exercise minutes	Frequency	Frequency percentage
30–<35	6	18.75%
35–<40	7	21.88%
40–<45	10	31.25%
45–<50	5	
50–<55	4	12.5%
Total	**32**	**100%**

- **a.** Using the frequency value, calculate the frequency percentage for a student who completes 45–<50 minutes of exercise each day. Round the answer to two decimal places.
- **b.** Using the frequency table, construct a frequency histogram to display the data.
- **c.** With reference to the answer for part **b**, comment on the distribution of the data.
- **d.** Using the frequency values, what is the percentage of students who complete 35–<50 minutes of exercise each day?
- **e.** Suggest an activity that results in incidental exercise.

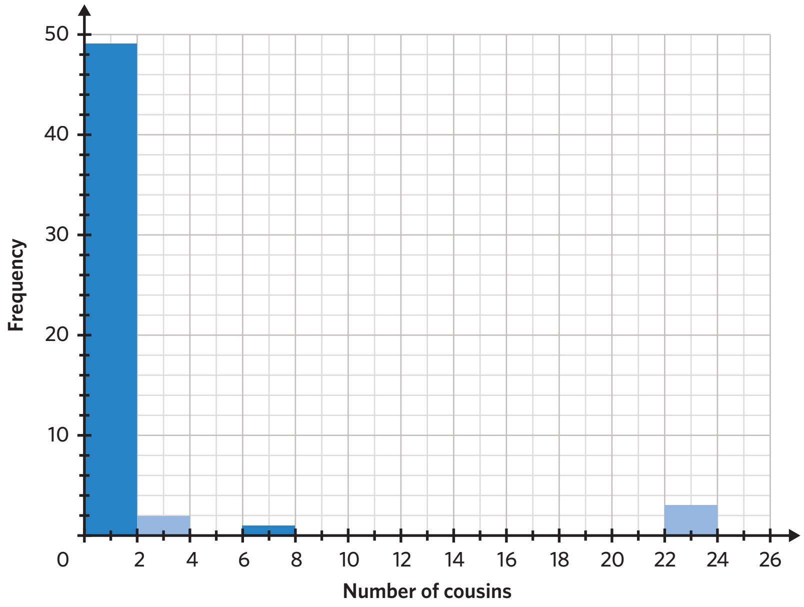
16. With reference to the table

Class interval	Frequency	Frequency percentage
10–<15	3	
15–<20		40%
Total		**100%**

- **a.** Calculate the frequency percentage for the class interval 10–<15.
- **b.** Calculate the frequency value for the class interval 15–<20.
- **c.** In part **a** the frequency percentage total is given, in part **b** the frequency total is not given. Explain whether or not part **b** could have been calculated before part **a**.

Exam-style

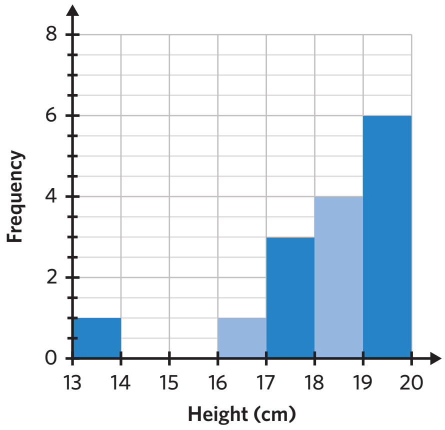
17. The histogram shows the distribution of the number of cousins for 55 students. (1 MARK)



Using the histogram, the percentage of these 55 students with less than 2 cousins is closest to

A. 3% **B.** 49% **C.** 53% **D.** 89% **E.** 93%

18. The heights, in centimetres, of 15 chihuahuas are displayed in the histogram. (3 MARKS)



a. Use the shape of the histogram to comment on the distribution of the data. 1 MARK

b. Calculate the percentage of chihuahuas surveyed that have a height greater than or equal to 18 cm. Round to two decimal places. 2 MARKS

19. The weights of a litter of cats are displayed in the grouped frequency table. (2 MARKS)

Weight (kg)	Frequency	Frequency percentage
0.8–<0.9	3	30%
0.9–<1.0	2	
1.0–<1.1	4	
1.1–<1.2	1	10%
Total	**10**	**100%**

Calculate the percentage of cats that had a weight of 0.9–<1.1 kg.

20. The maximum heights, in metres, of 16 plants were recorded. (3 MARKS)

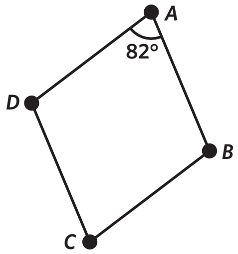
15.6, 14.5, 15.9, 15.2, 16.1, 15.0, 14.8, 16.2, 15.8, 14.9, 15.3, 14.9, 16.2, 15.3, 15.1, 16.2

Construct a grouped frequency table with an interval of size 0.5 to display the data.

Remember this?

21. The diagram shows a rhombus.

What is the size of the angle *ADC* in the rhombus?



A. 39°
B. 49°
C. 54°
D. 82°
E. 98°

22. A soccer match kicks off at 3:15 pm and lasts for 105 minutes.

What time does it end?

A. 4:00 pm **B.** 4:20 pm **C.** 4:40 pm **D.** 4:50 pm **E.** 5:00 pm

23. Sophie has 3.98 kg of sand in bags.

The bags come in three sizes: 1.5 kg, 650 g, and 330 g. All the bags are full.

How many bags does Sophie have?

A. 4 **B.** 6 **C.** 7 **D.** 10 **E.** 12

Back to contents

11D Boxplots

LEARNING INTENTIONS

Students will be able to:

- understand boxplot conventions for displaying data set values
- read and interpret five-number summary statistics from a boxplot
- construct a boxplot for a data set.

Boxplots are graphical representations that provide a snapshot of a data set's distribution. They display key statistics such as the median, interquartile range (IQR), and potential outliers. The median represents the central value, while the IQR shows the spread of the middle 50% of the data. Outliers, which are data points that fall outside the typical range, can significantly influence the overall understanding of the data. By constructing and interpreting boxplots, insights about the data's distribution, skewness, and general trends can be gleaned, offering a comprehensive view of the data set's characteristics.

KEY TERMS AND DEFINITIONS

- **Numerical data** is quantitative data that can either be discrete or continuous.

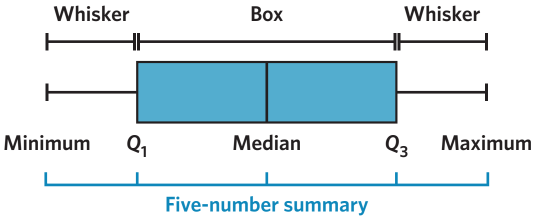
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Bro Crock/Shutterstock.com

In the world of finance, boxplots can be instrumental when analysing the annual performance of stocks. Through a single boxplot, investors can quickly identify the median stock price, the consistency in its performance, and any outliers indicating sudden hikes or drops.

Key ideas

1. A boxplot requires the five-number summary – minimum, Q_1, median, Q_3, and the maximum.

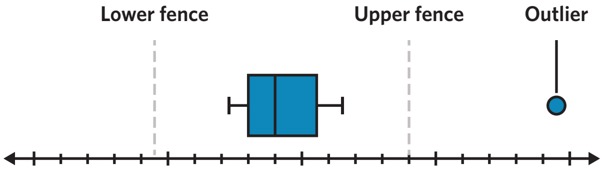


2. A boxplot may contain extreme values, known as outliers.

Less than $Q_1 - 1.5 \times \text{IQR}$ (lower fence)

Greater than $Q_3 + 1.5 \times \text{IQR}$ (upper fence)

Any data values that are less than or greater than the lower and/or upper fence are considered outliers.



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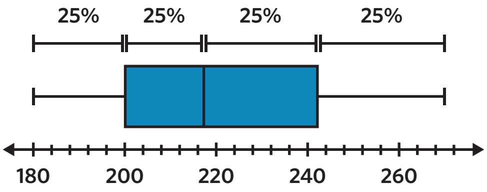
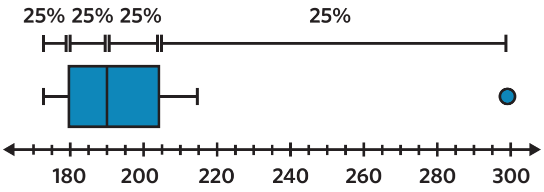
Back to contents

3. A boxplot is made up of four sections, each part consisting of 25% of the data.

No outliers



With outliers



Worked example 1

Constructing boxplots with no outliers

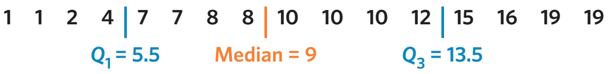
Determine the five-number summary and the upper and lower fences for the given data.

a. 12, 2, 4, 10, 7, 1 10, 8, 16, 8, 7, 19, 10, 1, 15, 19

WE1a

Working	Thinking
1, 1, 2, 4, 7, 7, 8, 8, 10, 10, 10, 12, 15, 16, 19, 19 Smallest value = 1 Largest value = 19	**Step 1:** Organise the data set in ascending order. Identify the smallest and largest values in the data set. These values represent the minimum and maximum, respectively.
Minimum = 1 $Q_1 = 5.5$ Median = 9 $Q_3 = 13.5$ Maximum = 19	**Step 2:** State the five-number summary.
IQR = 13.5 − 5.5 = 8	**Step 3:** Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3.
Lower fence = 5.5 − 1.5 × 8 = −6.5 Upper fence = 13.5 + 1.5 × 8 = 25.5	**Step 4:** Determine the lower and upper fences.

Visual support



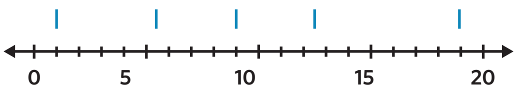
b. Using worked example **1a**, generate a boxplot.

WE1b

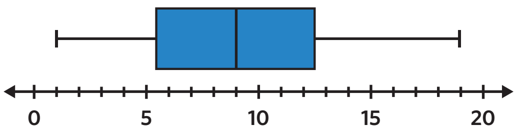
Working	Thinking
There are no data values that are less than −6.5 and/or greater than 25.5. ∴ There are no outliers.	**Step 1:** Use the lower and upper fences to identify if there are any outliers.

Continues →

Back to contents



Step 2: Plot the five-number summary on a number line and construct the boxplot.



Step 3: The box is formed using the Q_1, median, and Q_3 values. The whiskers extend from the box to the minimum and maximum values.

Student practice

Determine the five-number summary and the upper and lower fences for the given data.

a. 3, 5, 9, 6, 11, 14, 2, 17, 7, 13, 4, 18, 10, 15, 11

b. Using student practice **1a**, generate a boxplot.

Worked example 2

Constructing boxplots with outliers

Determine the five-number summary and the upper and lower fences for the given data.

a. 5, 3, 6, 11, 9, 13, 7, 17, 9, 6, 14, 2, 18, 20, 35

WE2a

Working

2, 3, 5, 6, 6, 7, 9, 9, 11, 13, 14, 17, 18, 20, 35

Smallest value = 2

Largest value = 35

Minimum = 2

$Q_1 = 6$

Median = 9

$Q_3 = 17$

Maximum = 35

$\text{IQR} = 17 - 6$

$= 11$

$\text{Lower fence} = 6 - 1.5 \times 11$

$= -10.5$

$\text{Upper fence} = 17 + 1.5 \times 11$

$= 33.5$

Thinking

Step 1: Organise the data set in ascending order. Identify the smallest and largest values in the data set. These values represent the minimum and maximum, respectively.

Step 2: State the five-number summary.

Step 3: Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3.

Step 4: Determine the lower and upper fences.

Visual support

2 3 5 $\boxed{6}$ 6 7 9 $\boxed{9}$ 11 13 14 $\boxed{17}$ 18 20 35

$Q_1 = 6$ Median = 9 $Q_3 = 17$

Continues →

Back to contents

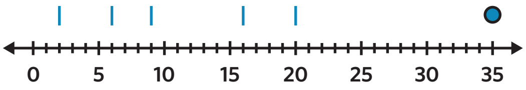
b. Using worked example **2a**, generate a boxplot.

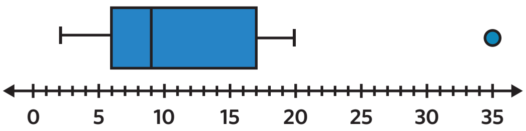
WE2b

Working

There are no data values that are less than −10.5, however there is one data value greater than 33.5.

∴ There is one outlier (35).





Thinking

Step 1: Use the lower and upper fences to identify if there are any outliers in the data.

Step 2: Determine the next maximum data value that is not an outlier. Plot the five-number summary and the next maximum data value on a number line and construct the boxplot. An outlier is displayed by using a '●' or a '✕'.

Step 3: The box is formed using the Q_1, median, and Q_3 values. The whiskers extend from the box to the minimum and maximum values. Outliers are not included in the whiskers.

Student practice

Determine the five-number summary and the upper and lower fences for the given data.

a. 5, 3, 6, 11, 9, 13, 7, 17, 9, 6, 14, 2, 18, 20, 38

b. Using student practice **2a**, generate a boxplot.

Worked example 3

Constructing parallel boxplots

Determine the five-number summary and the upper and lower fences for both sets of data.

a. **Set A:** 19, 17, 30, 12, 9, 20, 15, 22, 11, 4, 23, 20, 8
Set B: 14, 18, 28, 6, 4, 3, 7, 5, 9, 17, 38, 15, 2

WE3a

Working

Set A: 4, 8, 9, 11, 12, 15, 17, 19, 20, 20, 22, 23, 30

Set B: 2, 3, 4, 5, 6, 7, 9, 14, 15, 17, 18, 28, 38

Set A:	**Set B:**
Smallest value = 4	Smallest value = 2
Largest value = 30	Largest value = 38

Set A:	**Set B:**
Minimum = 4	Minimum = 2
$Q_1 = 10$	$Q_1 = 4.5$
Median = 17	Median = 9
$Q_3 = 21$	$Q_3 = 17.5$
Maximum = 30	Maximum = 38
IQR = 21 − 10	IQR = 17.5 − 4.5
= 11	= 11

Thinking

Step 1: Organise the data set in ascending order. Identify the smallest and largest values in the data set. These values represent the minimum and maximum, respectively.

Step 2: State the five-number summary for both sets of data.

Step 3: Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3.

Continues →

Back to contents

Set A:

$$\text{Lower fence} = 10 - 1.5 \times 11$$
$$= -6.5$$
$$\text{Upper fence} = 4.5 + 1.5 \times 11$$
$$= 37.5$$

Set B:

$$\text{Lower fence} = 4.5 - 1.5 \times 13$$
$$= -15$$
$$\text{Upper fence} = 17.5 + 1.5 \times 13$$
$$= 37$$

Step 4: Determine the lower and upper fences.

b. Using worked example **3a**, generate parallel boxplots.

WE3b

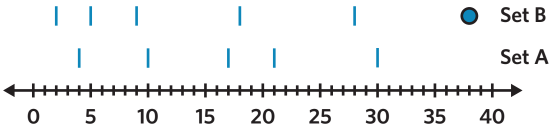
Working

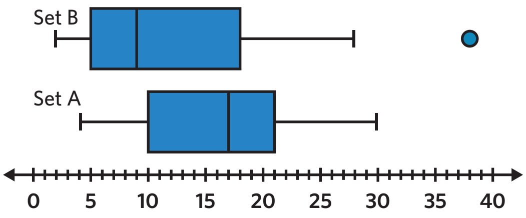
Set A: There are no data values less than −6.5 and/or greater than 37.5.

∴ There are no outliers.

Set B: There are no data values that are less than −15, however there is one data value greater than 37.

∴ There is one outlier (38).





Thinking

Step 1: Use the lower and upper fences to identify if there are any outliers in the data.

Step 2: Determine the next maximum data value for the data set(s) that have outliers. Plot the five-number summary and the next maximum data value(s) on the one number line and construct the boxplot. An outlier is displayed by using a '●' or a '✕'.

Step 3: The box is formed using the Q_1, median, and Q_3 values. The whiskers extend from the box to the minimum and maximum values. Outliers are not included in the whiskers.

Student practice

Determine the five-number summary and the upper and lower fences for both sets of data.

a. Set A: 25, 13, 29, 9, 7, 21, 16, 25, 14, 5, 26, 18, 9
Set B: 12, 19, 27, 8, 3, 2, 6, 4, 11, 20, 44, 17, 1

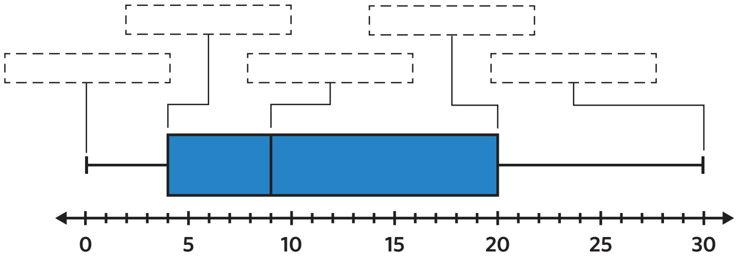
b. Using student practice **3a**, generate parallel boxplots.

Back to contents

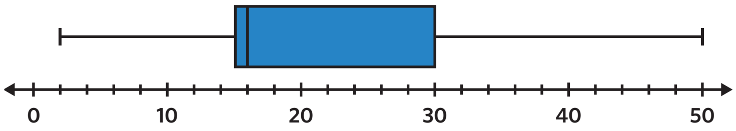
11D Questions

Understanding worksheet

1. Fill in the boxes with the correct mathematical terms.



2. With reference to the boxplot, complete the sentences.



a. The minimum data value is ☐.

b. The third quartile value is ☐.

c. The ‘box’ represents 50% of the data, the box lies between ☐.

d. There is ☐ % of the data that is greater than 30.

3. Fill in the blanks by using the words provided.

whiskers **outlier** **50%** **median**

A boxplot is an effective way to visualise the distribution of a data set. The ☐ represents the middle value, splitting the data in half. This type of graphical representation also contains ☐ , which extend to show the range of the majority of the data. The interquartile range encapsulating the central ☐ of data values is especially important for understanding variability. Any data point falling outside the upper and/or lower fence is considered an ☐ , which may indicate an extreme or unique observation in the data set.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b), 9 (a,b), 10	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c), 9 (b,c), 10	4 (d,e,f), 5 (d,e,f), 6 (d, e,f), 7 (d,e,f), 8 (c,d), 9 (c,d), 10

4. Determine the five-number summary and the upper and lower fences for the given data. WE1a

a. 25, 5, 30, 15, 10, 20, 15

b. 9, 15, 1, 19, 13, 3, 21, 7, 17, 11, 5

c. 13, 7, 19, 3, 11, 17, 9, 15, 5

d. 16, 8, 20, 2, 12, 18, 10, 4, 6

e. 14, 22, 2, 15, 8, 24, 21, 4, 18, 13, 6, 10

f. 19, 24, 16, 4, 9, 27, 2, 27

Back to contents

5. Using **question 4**, generate a boxplot. WE1b

a. 25, 5, 30, 15, 10, 20, 15

b. 9, 15, 1, 19, 13, 3, 21, 7, 17, 11, 5

c. 13, 7, 19, 3, 11, 17, 9, 15, 5

d. 16, 8, 20, 2, 12, 18, 10, 4, 6

e. 14, 22, 2, 15, 8, 24, 21, 4, 18, 13, 6, 10

f. 19, 24, 16, 4, 9, 27, 2, 27

6. Determine the five-number summary and the upper and lower fences for the given data. WE2a

a. 9, 10, 21, 23, 10, 17, 9, 0, 45, 29, 9

b. 29, 21, 27, 19, 18, 28, 17, 19, 1, 18, 30

c. 30, 24, 18, 27, 28, 6, 27, 23, 33

d. 46, 12, 15, 22, 27, 30, 28, 21, 22

e. 4, 14, 15, 16, 15, 18, 25, 20

f. 22, 15, 17, 30, 33, 47, 15, 16, 25, 26, 26, 27

7. Using **question 6**, generate a boxplot. WE2b

a. 9, 10, 21, 23, 10, 17, 9, 0, 45, 29, 9

b. 29, 21, 27, 19, 18, 28, 17, 19, 1, 18, 30

c. 30, 24, 18, 27, 28, 6, 27, 23, 33

d. 46, 12, 15, 22, 27, 30, 28, 21, 22

e. 4, 14, 15, 16, 15, 18, 25, 20

f. 22, 15, 17, 30, 33, 47, 15, 16, 25, 26, 26, 27

8. Determine the five-number summary and the upper and lower fences for both sets of data. WE3a

a. Set A: 8, 19, 21, 33, 28, 5, 14
Set B: 3, 1, 25, 39, 27, 15, 1, 23

b. Set A: 31, 15, 27, 8, 6, 23, 19, 27, 13, 4, 29, 17, 8
Set B: 10, 20, 26, 17, 22, 11, 15, 13, 12, 22, 36, 18, 0

c. Set A: 10, 14, 16, 38, 22, 5, 9, 19
Set B: 18, 8, 15, 1, 4, 30, 22, 20

d. Set A: 7, 10, 24, 26, 50, 5, 6, 13, 19, 16
Set B: 18, 0, 18, 11, 25, 20, 22, 38, 29, 26

9. Using **question 8**, generate parallel boxplots. WE3b

a. Set A: 8, 19, 21, 33, 28, 5, 14
Set B: 3, 1, 25, 39, 27, 15, 1, 23

b. Set A: 31, 15, 27, 8, 6, 23, 19, 27, 13, 4, 29, 17, 8
Set B: 10, 20, 26, 17, 22, 11, 15, 13, 12, 22, 36, 18, 0

c. Set A: 10, 14, 16, 38, 22, 5, 9, 19
Set B: 18, 8, 15, 1, 4, 30, 22, 20

d. Set A: 7, 10, 24, 26, 50, 5, 6, 13, 19, 16
Set B: 18, 0, 18, 11, 25, 20, 22, 38, 29, 26

10. Using the given data set, determine which of the values is an outlier.

13, 5, 19, 9, 3, 16, 11, 26, 10, 4, 35, 21, 15, 18

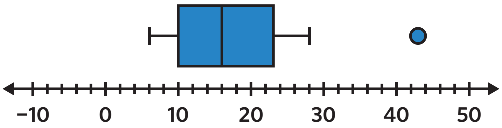
A. −6 **B.** 14 **C.** 19 **D.** 34 **E.** 35

Spot the mistake

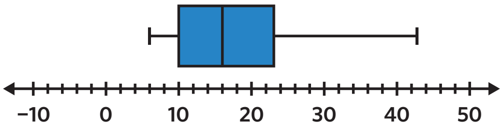
11. Select whether Student A or Student B is incorrect.

a. Using the given data, generate a boxplot.
12, 6, 7, 8, 14, 10, 17, 20, 15, 22, 23, 25, 28, 43

Student A



Student B



b. Using the given data, calculate the upper and lower fence and generate a boxplot.
5, 3, 6, 11, 9, 13, 7, 17, 9, 6, 14, 2, 18, 20, 35

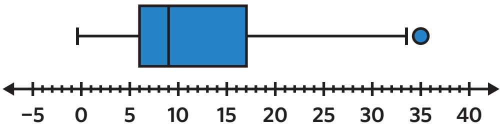
Student A

Minimum $= 2$, $Q_1 = 6$, median $= 9$, $Q_3 = 17$, maximum $= 35$

$$\begin{aligned} \text{IQR} &= 17 - 6 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{Lower fence} &= 16 - 1.5 \times 11 \\ &= -0.5 \end{aligned}$$

$$\begin{aligned} \text{Upper fence} &= 17 + 1.5 \times 11 \\ &= 33.5 \end{aligned}$$



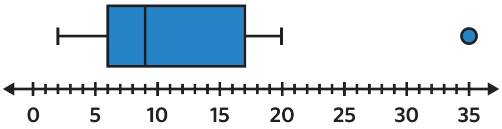
Student B

Minimum $= 2$, $Q_1 = 6$, median $= 9$, $Q_3 = 17$, maximum $= 35$

$$\begin{aligned} \text{IQR} &= 17 - 6 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{Lower fence} &= 16 - 1.5 \times 11 \\ &= -0.5 \end{aligned}$$

$$\begin{aligned} \text{Upper fence} &= 17 + 1.5 \times 11 \\ &= 33.5 \end{aligned}$$



Problem solving

Question working paths

Mild 12, 13, 14	**Medium** 13, 14, 15	**Spicy** 14, 15, 16

12. A tech enthusiast is studying the price range of different iPhone models released over the past few years in AUD. The prices she noted down are: \$799, \$899, \$999, \$1099, \$1599, \$749, \$1199, \$849, \$949, \$1099, \$1199.

Prove that \$1599 is not considered an outlier.

13. A local marathon recorded the finish times (in hours) of its participant:
3.5, 2.4, 4, 3.8, 5.2, 3.9, 4.5, 4.2, 4.7, 4.1, 5, 5.1, 3.7, 4.2.

Determine if there are any outliers in this data.

14. A local library has conducted a survey to determine how many books each visitor borrows every week. Generate a boxplot for 1, 2, 1, 3, 2, 1, 3, 4, 2, 1, 2, 3, 3. The data has a lower fence value of −2 and an upper fence value of 6.

15. During a cricket tournament, a team's individual scores for eleven matches were:
280, 290, 315, 270, 330, 310, 295, 275, 340, 320, 205.

The coach believes that the lowest score (205) is a statistical outlier and wants to verify this. Determine the five-number summary, and then create a boxplot to display and confirm this.

16. For a geography project, John collected data on the daily temperatures (in degrees Celsius) of his town for the month of January and March.

January: 25, 27, 24, 29, 31, 38, 26, 30, 24, 25, 32, 28, 31, 24, 33, 29, 30, 31, 26, 27, 28, 29, 33, 25, 27, 28, 30, 30, 24, 26, 27

March: 23, 21, 20, 32, 34, 25, 23, 21, 24, 25, 27, 22, 25, 32, 27, 31, 26, 38, 19, 21, 20, 24, 23, 20, 20, 19, 19, 18, 23, 20, 20

Determine the five-number summaries and if there are any outliers, then display the data using parallel boxplots.

Back to contents

Reasoning

Question working paths

Mild 17 (a,b,c,e) 🌶	**Medium** 17 (a,b,c,e), 18 (a,b) 🌶🌶	**Spicy** All 🌶🌶🌶

17. Beanica has been analysing the monthly sales of four different products (A, B, C, and D) over the past year. She wants to understand the distribution, consistency, and any unusual trends in the sales data. The data for each product has been summarised into boxplots, and the manager is using them to make decisions.

a. Product A's sales data is given as follows: minimum $= 100$, $Q_1 = 200$, median $= 300$, $Q_3 = 400$, maximum $= 500$. Construct a boxplot for Product A's sales data.

b. Product B's sales data is given, and it includes an outlier: minimum $= 50$, $Q_1 = 150$, median $= 250$, $Q_3 = 350$, maximum $= 450$, outlier $= 600$. Display Product B's sales data together with Product A's using parallel boxplots.

c. Using the parallel boxplots constructed in parts **a** and **b**, calculate the IQR and range.

d. Product C's sales data is given, but the maximum value is missing: minimum $= 80$, $Q_1 = 180$, median $= 280$, $Q_3 = 380$, outlier $= 700$. Determine the upper fence for Product C's sales data to confirm that a data value of 700 is an outlier.

e. Beanica is planning a marketing campaign to boost the sales of Product D, which has been underperforming. Suggest an approach she could take to help with the sales.

18. Two sets of data are given:

Set A: 15, 18, 20, 22, 24, 25, 25, 27, 30, 32, 35, 40

Set B: 5, 15, 18, 20, 22, 24, 25, 25, 27, 30, 32, 35, 40

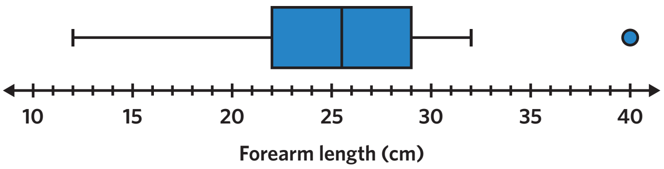
a. Determine the five-number summary and lower fence for Set A.

b. Determine the five-number summary and the lower fence for Set B.

c. Compare the data sets and the answers generated in parts **a** and **b**. Comment on how the inclusion of an additional smaller data value affects the five-number summary and the lower fence.

Exam-style

19. The boxplot shows the distribution of forearm lengths, in centimetres, of 150 people. (1 MARK)
The percentage of these 150 people with a forearm length of greater than 29 cm is



A. 2%
B. 6.45%
C. 25%
D. 75%
E. 94%

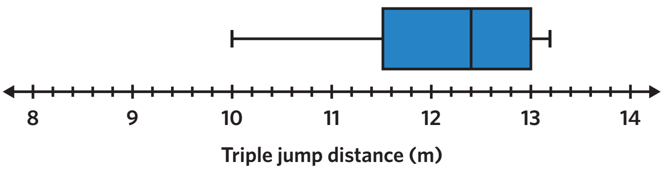
20. The five-number summary for the distribution of daily ice cream sales for the month of February is displayed in the table. (3 MARKS)

	Ice cream sales ($)
Minimum	10
Q_1	22
Median	25
Q_3	27.5
Maximum	30

a. There are no values greater than the upper fence value, calculate and explain if there are any values less than the lower fence value. 2 MARKS

b. The second lowest ice cream sales in February were $15. Using the table and your answer from part **a** generate a boxplot to display the number of ice cream sales. 1 MARK

21. The boxplot displays the distribution of triple jump length for 15 athletes in a competition. (2 MARKS)



Calculate the lower and upper fence using the boxplot to explain that the minimum and maximum values are not outliers.

22. The data below represents the number of books sold at a local bookstore over a period of 18 consecutive days: (3 MARKS)

14, 26, 39, 46, 13, 30, 5, 46, 37, 26, 39, 8, 8, 9, 17, 48, 29, 27

Generate a boxplot to represent the distribution of the number of books sold.

Remember this?

23. Sara solved the equation $4x + 2 = 6$ correctly.

Which of the following could be two lines of her solution?

A. $4x = 2$, $x = \frac{1}{2}$ **B.** $4x = 4$, $x = 0$ **C.** $4x = 4$, $x = 1$ **D.** $4x = 6$, $x = \frac{3}{2}$ **E.** $4x = 8$, $x = 2$

24. The table shows the number of pages Chelsea reads on four different days.

Day	Monday	Tuesday	Wednesday	Thursday
Pages read	64	77	90	103

Every day, Chelsea increases the number of pages she reads by 13 pages.

What is the number of pages Chelsea will read on Saturday?

A. 116 **B.** 129 **C.** 142 **D.** 155 **E.** 168

25. What is the value of $\frac{47.5 + 32.75}{8.5 \times 2.5}$ rounded to one decimal place?

A. 3.8 **B.** 4.7 **C.** 7.9 **D.** 9.4 **E.** 10.2

Chapter 11 extended application

1. The 100 m sprint at the Olympic Games, often labelled the pinnacle of athletic achievement, encapsulates a breathtaking fusion of speed, skill, and spectacle. With winning sprinters hailing from various countries, data on winning times represents not only the triumph of individual athletes but also provides insight into patterns, anomalies, and the evolution of speed across different Olympic years.

Consider the following table, which presents the winning times in seconds of the 100 m sprint at previous Olympic games.

Year	Winning time (seconds)	Winner's name	Winner's country
1904	11.00	Archie Hahn	USA
1972	10.14	Valeriy Borzov	USSR
1976	10.06	Hasely Crawford	Trinidad and Tobago
1980	10.25	Allan Wells	Great Britain
1984	9.99	Carl Lewis	USA
1988	9.92	Carl Lewis	USA
1992	9.96	Linford Christie	Great Britain
1996	9.84	Donovan Bailey	Canada
2000	9.87	Maurice Greene	USA
2004	9.85	Justin Gatlin	USA
2008	9.69	Usain Bolt	Jamaica
2012	9.63	Usain Bolt	Jamaica
2016	9.81	Usain Bolt	Jamaica
2020	9.80	Lamont Marcell Jacobs	Italy

a. Determine the mean, median, and mode of the winning times, rounded to two decimal places where necessary.

b. Determine the range and interquartile range (IQR) of the winning times.

c. Round the winning times to one decimal place and then construct a stem-and-leaf plot to visually display this data.

d. Determine if there are any outliers in this data set. Explain why or why not.

e. Identify any potential factors that may have contributed to different winning times in different Olympic years.

2. The real estate market can be highly variable, influenced by various factors such as location. Analysing house price data within a specific location or during a particular time period can provide valuable insights for buyers, sellers, and investors.

Consider the following data, which represents apartment prices in thousands, rounded to the nearest $10 000, in a Melbourne suburb over the last year:

250, 320, 290, 450, 510, 360, 390, 220, 320, 430, 380, 460, 290, 310, 400, 420, 450, 370, 300, 410, 550, 320, 590, 450, 410, 360, 390, 220, 320, 430, 380, 560, 290, 310, 400, 420, 450, 370, 300, 410

House prices (000s)	Frequency	Percentage frequency
\$200–<\$300		
\$300–<\$400		
\$400–<\$500		
\$500–<\$600		
Total		

a. Explain why apartment prices are considered continuous data.

b. Complete the grouped frequency table for the data.

c. Identify the class intervals with the least and most frequent apartment prices.

d. Using the frequency table completed in part **b**, construct a frequency histogram to display the data.

e. Using the frequency histogram constructed in part **d**, comment on the distribution of the data.

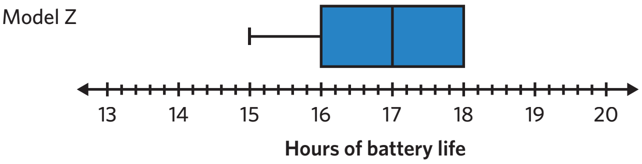
f. What factors may influence the prices of properties within the same neighbourhood?

3. Battery life is an important factor in determining whether a consumer decides to purchase a smartphone. Three different smartphone models (Model X, Model Y, and Model Z) were tested for their battery life in hours under similar conditions. Each number represents the battery life of a different smartphone of that particular model.

Battery life (hours):

- Model X: 22, 22, 23, 23, 23, 24, 24, 24, 24, 24, 25, 25, 25, 25, 25
- Model Y: 16, 19, 20, 20, 21, 21, 22, 22, 22, 22, 22, 23, 24, 25, 28

The following boxplot shows Model Z.



a. Determine the five-number summary for both Model X and Y.

b. Determine and explain if there are any outliers in the data sets.

c. Generate parallel boxplots for Model X and Y.

d. Comment on the spread of the data for all three smartphone models.

e. With reference to the given data, suggest one reason why a consumer may prefer to use Model Z rather than Model Y.

f. Other than battery life, what features may consumers consider when purchasing a new smartphone?

Chapter 11 review

Multiple choice

1. Identify any potential outliers in the following data set. **11A**

1, 4, 2, 9, 22, 9, 5, 11

A. 1 **B.** 2 **C.** 9 **D.** 22 **E.** No outliers

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2. Consider the following data set: 14, 12, 21, 18, 36, 41, 22, 25, 31, 33, 40, 15, 16. **11B**

What is the missing number needed to complete the stem-and-leaf plot?

Stem	Leaf				
1	2	4	5	6	8
2	1	2	5		
3	1	3	⬚		
4	0	1			

Key

2 | 1 = 21

A. 5 **B.** 6 **C.** 7 **D.** 8 **E.** 9

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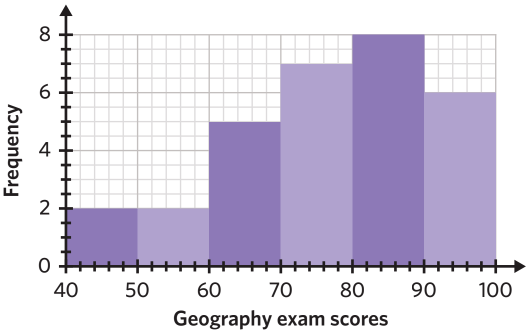
3. The histogram shows the distribution of exam scores in a geography class. **11C**



What is the total number of students who got a score of 70–<90 on the geography exam?

A. 7 **B.** 8 **C.** 12 **D.** 14 **E.** 15

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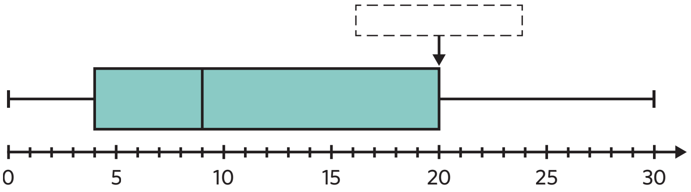
4. With reference to the boxplot, which of the options correctly labels the outlined feature? **11D**



A. Minimum **B.** Q_1 **C.** Median **D.** Q_3 **E.** Maximum

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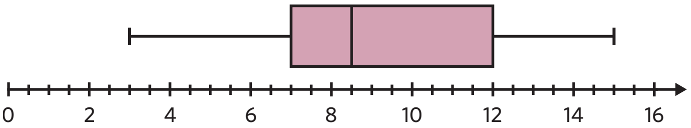
5. With reference to the boxplot, complete the sentence. **11D**



50% of the data lies between ⬚ and ⬚.

A. 3, 12 **B.** 3, 15 **C.** 7, 12 **D.** 8.5, 12 **E.** 8.5, 16

Back to contents

Fluency

6. Determine the mean, median and mode(s) of the following data sets. Round to one decimal place where necessary. **11A**

a. 3, 5, 5, 6, 8, 11, 11, 11, 12, 12, 14, 16, 17

b. 22, 22, 25, 26, 31, 32, 32, 36, 37, 41, 45, 46, 47, 49

c. 5, 16, 3, 29, 31, 12, 90, 12, 88, 2, 10

d. 0.8, −1.2, 4.4, 5.9, 2.7, −13.1, −8.0, 3.6, 9.1, 11.2, −1.5, 4.8

7. Calculate the range and interquartile range (IQR) for the following data sets. Round to one decimal place where necessary. **11A**

a. 2, 2, 4, 5, 7, 9, 12, 15, 18

b. −9, −7, −7, −7, −5, −5, −4, −3, −3, −1

c. 14, 23, 19, 8, 31, 32, 26, 21, 10

d. 8.4, 7.2, 5.1, 9.3, 4.2, 6.6, 7.4, 3.3, 6.9, 5.0

8. Arrange each of the following data sets in an ordered stem-and-leaf plot. **11B**

a. 2, 4, 6, 10, 12, 18, 20, 22, 27, 28, 30, 32, 44, 49, 49, 51

b. 22, 23, 28, 29, 35, 36, 42, 43, 49, 50, 56, 57, 63, 64, 70, 71

c. 9, 23, 14, 28, 35, 12, 16, 37, 11, 46, 32, 10, 13, 31, 15, 34

d. 6.3, 3.1, 7.2, 1.8, 4.6, 5.5, 2.7, 4.0, 7.9, 3.9, 2.4, 1.4, 5.7, 6.7, 5.0, 3.0, 3.8

9. The data provided represents the distances, in kilometres, travelled by 27 participants in a charity walkathon: **11C**

10.2, 8.5, 12.7, 5.3, 2.2, 14.6, 9.8, 11.4, 7.1, 6.9, 16.1, 4.6, 15.5, 8.3, 10.7, 3.9, 12.0, 7.8, 9.1, 11.8, 15.3, 6.5, 13.9, 4.2, 15.2, 13.1, 2.9

a. Generate a grouped frequency table with class intervals of 3, starting with 0–<3. Round to two decimal places where necessary.

b. Using the frequency table in part **a**, generate a frequency histogram.

c. Comment on the shape of the histogram.

d. What percentage of participants walked 12 km or more? Round to two decimal places.

10. Determine the five-number summary and the upper and lower fence for the following data sets. Round to one decimal place where necessary. **11D**

a. 12, 15, 22, 25, 28, 31, 34, 40, 45, 50, 55, 58, 63, 66, 70, 75, 80, 85

b. 19, 23, 16, 30, 28, 14, 27, 21, 26, 12, 22, 25, 17, 29, 18, 24, 20, 15

c. 3.2, 4.6, 2.1, 5.7, 6.3, 7.8, 1.9, 3.5, 2.8, 4.2, 5.9, 6.7, 1.5, 4.8, 3.9, 2.6

d. −8, 10, −5, 12, 7, −4, 14, 2, −9, 15, 6, −2, 13, −7, 9, 3, −11, 11

11. Generate a box plot for the following data sets. **11D**

a. 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53

b. 5, 17, 3, 28, 9, 36, 12, 25, 7, 31, 14, 20, 2, 23, 16, 34, 11, 19

c. 15, 18, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 99

d. 0.2, 3.8, 3.9, 4.2, 4.6, 5.0, 5.3, 5.4, 5.5, 5.6, 5.7, 5.9, 6.5, 6.8

Problem solving

12. A bakery owner is analysing the weights of two types of bread loaves: whole wheat and multigrain. The weights, in grams, for both types of loaves are as follows: **11A**

Whole wheat: 300, 320, 310, 330, 290, 280, 315

Multigrain: 350, 365, 360, 380, 370, 355, 385, 395

Calculate the range and interquartile range of weights for both types of bread loaves.

13. Taura is a cyclist who is tracking the distances, in kilometres, that she covers during her rides over a period of two weeks. The data she collects is: 18, 22, 15, 28, 20, 30, 23, 25, 35, 14, 40, 42, 19, 26. Construct a stem-and-leaf plot to display Taura's cycling distances and comment on whether the data is skewed or symmetrical. 11B

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14. A music teacher recorded the number of practice hours completed by her students in a month. The data is displayed in the grouped frequency table, rounded two decimal places where necessary. 11C

Number of practice hours	Frequency	Percentage frequency
0–<3	10	27.78%
3–<6	15	41.67%
6–<9	7	19.44%
9–<12	3	8.33%
12–<15	1	2.78%
Total	**36**	**100%**

Construct a percentage frequency histogram to represent the data. Is the data symmetric, skewed, or bi-modal?

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15. For a weather study, Daisy collected data on the daily humidity levels (%) of her town for the month of July. The humidity levels were: 60, 65, 70, 55, 75, 80, 62, 68, 72, 58, 64, 69, 73, 57, 81, 76, 74, 59, 66, 67, 63, 71, 79, 11, 78, 61, 70, 65, 62, 52, 55. Determine if there are any outliers and then create a boxplot to display the data. 11D

Reasoning

16. A pet store has gathered data on the weights, in kilograms, of different breeds of dogs owned by their customers at two of their most popular branches: Branch Pawsome and Branch Petopia. The collected data is as follows:

Branch Pawsome: 28, 12, 40, 22, 5, 45, 32, 18, 20, 42, 34, 26, 24, 27, 15, 8, 36, 30, 38

Branch Petopia: 5, 45, 25, 13, 41, 17, 33, 21, 29, 9, 39, 15, 37, 19, 35, 43, 23, 27, 31, 26

a. Construct a back-to-back stem-and-leaf plot to organise the data for both Branch Pawsome and Branch Petopia.

b. Which branch has a higher median dog weight?

c. Determine the five-number summary and use this to construct parallel boxplots.

d. Using the boxplot constructed in part **c**, calculate the interquartile range for each branch.

e. What are some factors that pet owners should consider when choosing a pet store to purchase items for their dog?

17. Consider the following data set:

2, 3, 3, 4, 5, 5, 6, 6, 6, 6, 7, 7, 8, 9, 11, 11, 12, 13, 14, 15, 15, 18, 20, 21, 25

a. Calculate the mean to one decimal place.

b. Construct a frequency histogram, using class intervals of 0–<5, to display the data.

c. Using your answers from parts **a** and **b**, comment on whether the mean can be calculated from the histogram?

Back to contents

Answers

Back to contents

1A Decimals and significant figures

Student practice

Worked example 1

a. 70.4 **b.** 3882.88

Worked example 2

a. 5 **b.** 2 **c.** 68

Worked example 3

a. 0.85 **b.** 360

Understanding worksheet

1. **a.** 1 **b.** 4 **c.** 2 **d.** 2

2. Significant: I; II; IV
Not significant: III

3. truncating; significant; not significant; leading

Fluency

4. **a.** 3.14 **b.** 158.4
c. 10.5044 **d.** 35.744
e. 0.0 **f.** 423.67
g. 1308.44643 **h.** 990.0

5. **a.** 4 **b.** 6 **c.** 2 **d.** 3
e. 5 **f.** 5 **g.** 3 **h.** 6

6. **a.** 85.2 **b.** 159.9
c. 6.01 **d.** 80 000
e. 0.0048 **f.** 78 500
g. 10 **h.** 524 010

7. **a.** 140 **b.** 3.9 **c.** 21 **d.** 2.5
e. 4270 **f.** 317 **g.** 17 000 **h.** 23.4

8. C

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. On a particular day, an exchange rate agency lists one pound sterling (the currency of the United Kingdom) as equal to \$1.8849926 in Australian dollars. How many Australian dollars is equivalent to 3 pound sterling, rounded to the nearest cent?

Key points

- 1 pound sterling equals 1.8849926 Australian dollars.
- How many Australian dollars is 3 pound sterling, rounded to the nearest cent?

Explanation

1 pound sterling = 1.8849926 Australian dollars

3 pound sterling = 3 × 1.8849926
= 5.6549778 Australian dollars

Round \$5.6549778 to the nearest cent, which is 2 decimal places.

\$5.6549778 ≈ \$5.65

Critical digit is 4

Therefore this digit does not change

Answer

3 pound sterling is equivalent to \$5.65 in Australian dollars.

11. The distance between the Earth and the Sun is called 1 AU (astronomical unit). 1 AU is equal to 150 780 000 km. How many significant figures are there in this value?

Key points

- 1 AU = 150 780 000 km.
- How many significant figures are in 150 780 000?

Explanation

All non-zero digits are significant. Therefore, 1, 5, 7, and 8 are significant in **15**0 **78**0 000.

Zeros between non-zero digits are significant. Therefore, the first zero in 15**0** 780 000 is significant.

Zeros to the right of the last non-zero digit are significant only if the number is a decimal. 150 780 000 is not a decimal. Therefore, the last four zeros in 150 780 000 are not significant.

150 780 000

5 significant figures

Answer

There are 5 significant figures in 150 780 000.

12. Cecilia measured the length of a table using a metre ruler with centimetre markings. She measured it to be 1.5 m. She then used a centimetre ruler with millimetre markings to measure the same table. She measured it to be 154.3 cm. Which ruler measures the length of the table to the greatest number of significant figures?

Key points

- A one metre ruler with centimetre markings measures the length of the table as 1.5 metres.
- A centimetre ruler with millimetre markings measures the length of the table as 154.3 cm.
- Which ruler measures length to the greatest number of significant figures?

Explanation

1.5 m has 2 significant figures. 154.3 cm has 4 significant figures.

4 significant figures > 2 significant figures

154.3 cm, which has the greater number of significant figures, was measured by the centimetre ruler.

Answer

The centimetre ruler measures the length of the table to the greatest number of significant figures.

13. Jerone wants to figure out how much faster his new bike will be. He measured the diameter of one wheel of his new bike as 41.2 cm. What is the circumference of the wheels, given the circumference can be calculated using $C = \pi d$, rounded to an appropriate number of significant figures?

Back to contents

Key points

- The diameter of one wheel of the bike is 41.2 cm.
- The circumference can be calculated using $C = \pi d$.
- What is the circumference of the wheels, rounded to an appropriate number of significant figures?

Explanation

Substitute $d = 41.2$ cm into the circumference formula, $C = \pi d$.

$C = \pi \times 41.2$

$= 129.4336173...$ cm

Determine an appropriate number of significant figures. The only given number is the diameter of the bike wheel, which is 41.2. This number has 3 significant figures, therefore, this is the appropriate number of significant figures to round the circumference to.

Round 129.4336173... to 3 significant figures.

129.4336173 = 129 cm

Critical digit is 4

Therefore this digit does not change

Answer

The circumference of the wheels is 129 cm.

14. The time (t, in hours) a car travels given particular speed (s, in km/h) and distance (d, in km) can be calculated using $t = \frac{d}{s}$. The speed of a car was recorded to be $50.\dot{3}$ km/h and the total distance travelled was 206 km. What is the difference between the times calculated using speed rounded to two and three significant figures? Round the answer to the nearest minute.

Key points

- $t = \frac{d}{s}$, where t is time in hours, s is speed in km/h, and d is distance in km.
- $s = 50.\dot{3}$ km/h
- $d = 206$ km
- What is the difference between the times calculated using speed rounded to two or three significant figures, to the nearest minute?

Explanation

Calculate the time when the speed is rounded to three significant figures.

$t = \frac{d}{s}$

$= \frac{206}{50.3}$

$= 4.0954274...$ hours

Calculate the time when the speed is rounded to two significant figures.

$t = \frac{d}{s}$

$= \frac{206}{50}$

$= 4.12...$ hours

Calculate the difference between the times.

$4.12... - 4.0954274... = 0.0245726...$ hours

Convert 0.0245726... hours into minutes.

$0.0245726... \times 60 = 1.474356...$ minutes

Round 1.474356... to the nearest minute.

1.474356... ≈ 1 minute

Critical digit is 4

Therefore this digit does not change

Answer

The difference between the calculated times is 1 minute.

Reasoning

15. **a.** 5 significant figures and 3 decimal places are used to record the finish times after the times have been converted to seconds.

b. Max Verstappen's finish time rounded to 1 decimal place is 76.7 seconds.

c. Nico Hulkenberg's finish time rounded to 2 significant figures is 78 seconds.

d. Lewis Hamilton and Fernando Alonso's race times can be rounded to a minimum of 4 significant figures so that their finish order can be determined.

e. Suggested option 1: Rowing has much slower times than car racing, and so less significant figures are used to record them.

Suggested option 2: Swimming has much slower times than car racing, and so less significant figures are used to record them.

Note: There are other possible options.

16. **a.** The length, width, and area of the wall to 2 significant figures are 3.6 m, 1.3 m, and 4.7 m^2 respectively.

b. The length, width, and area of the wall to 3 significant figures are 3.55 m, 1.34 m, and 4.76 m^2 respectively.

c. The area of the wall is 4.766784 m^2. This is a more accurate measurement for the area than 4.7 m^2 or 4.76 m^2 . This is because the number has not been rounded and therefore has more significant figures. The more significant figures in a number, the more accurate it is.

Exam-style

17. E

18. **a.** 0.04

b. 5.3

19. 0.83 m^2

20. Grant rounded both values incorrectly.

When rounding 9.235946..., he did not increase the digit being rounded. The correct answer is 9.236.

When rounding 1.002493, he rounded to four decimal places, not four significant figures. The correct answer is 1.002.

Remember this?

21. C **22.** C **23.** D

1B Profits and discounts

Student practice

Worked example 1

a. \$148 **b.** \$70

Worked example 2

a. 27% **b.** \$19.58 **c.** \$661.76

Worked example 3

a. 220% profit **b.** 43% loss

Understanding worksheet

1. **a.** \$21 **b.** \$71 **c.** \$1100 **d.** \$452

2. **a.** 1.2 **b.** 0.7 **c.** 1.02 **d.** 0.97

3. mark-up; sale price; loss; cost price

Back to contents

Fluency

4. **a.** $115 **b.** $200
c. $83.20 **d.** $322
e. $633 **f.** $38.39
g. $2712 **h.** $5173.20

5. **a.** 15% **b.** 40% **c.** 18% **d.** 30%
e. 100% **f.** 14% **g.** 300% **h.** 46%

6. **a.** $1000 **b.** $12
c. $238.09 **d.** $24
e. $36.87 **f.** $1107.25
g. $361.98 **h.** $2373.89

7. **a.** 25% profit **b.** 50% loss
c. 40% profit **d.** 33% loss
e. 12% profit **f.** 15% loss
g. 9% profit **h.** 55% loss

8. C

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. M&H, a clothing store, offers a 15% birthday discount on the price of any single item. It is Quinn's birthday and she is considering buying a shirt that costs $35.45. After the discount, how much would Quinn pay?

Key points

- Quinn has a 15% birthday discount.
- She buys a shirt that costs $35.45.
- How much would Quinn pay after the discount?

Explanation

Quinn has a 15% discount.

$100\% - 15\% = 85\%$

∴ Quinn pays 85% of the original price.

Calculate how much Quinn pays for the shirt.

$$\begin{aligned}\text{Discounted price} &= 85\% \text{ of original price}\\ &= 0.85 \times 35.45\\ &= 30.132...\\ &\approx \$30.13\end{aligned}$$

Answer

Quinn pays $30.13 for the shirt after the discount.

11. Patrick is shopping for dress shoes and sees a pair of Julius Marlow shoes on sale. He looks at the sticker and sees that the shoes have been reduced from $159.99 to $100. Calculate the percentage discount on the shoes, rounded to the nearest percent.

Key points

- A pair of shoes have been reduced from $159.99 to $100.
- Calculate the percentage discount on the shoes, rounded to the nearest percent.

Explanation

Calculate the percentage discount on the shoes.

$$\begin{aligned}\text{Percentage discount} &= \frac{\text{Change in price}}{\text{Original price}} \times 100\\ &= \frac{159.99 - 100}{159.99} \times 100\\ &= \frac{59.99}{159.99} \times 100\\ &= 0.374... \times 100\\ &\approx 37\%\end{aligned}$$

Answer

There is a 37% discount on the shoes.

12. Sam and Alex bought a house in 2018 for $650 000 and sold it three years later for $796 278. What was the percentage profit made on the sale of the house, rounded to the nearest percent?

Key points

- A house was bought for $650 000 in 2018.
- The house sold for $796 278 in 2021.
- What is the percentage profit made on the sale of the house, rounded to the nearest percent?

Explanation

Calculate the percentage profit made on the sale of the house.

$$\begin{aligned}\text{Percentage profit} &= \frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100\\ &= \frac{796\,278 - 650\,000}{650\,000} \times 100\\ &= \frac{146\,278}{650\,000} \times 100\\ &= 0.225... \times 100\\ &\approx 23\%\end{aligned}$$

Answer

There is a 23% profit made on the sale of the house.

13. Olivia saw on social media that her favourite bubble tea shop is increasing the prices of their bubble teas by 8%. When she next visits the shop, the new price is $5.20. Determine the original price of the bubble tea before the increase in price, to the nearest cent.

Key points

- The price of bubble tea is increasing by 8%.
- The new price of bubble tea is $5.20.
- What is the original price of the bubble tea before the increase in price, to the nearest cent?

Explanation

The percentage increase is 8%, therefore, the new price is 108% of the original price.

Calculate the original selling price of the bubble tea.

$$\begin{aligned}\text{Original selling price} &= \frac{\text{new price}}{\text{percentage increase}} \times 100\\ &= \frac{5.20}{108} \times 100\\ &= 0.0481... \times 100\\ &\approx \$4.81\end{aligned}$$

Answer

The original price of the bubble tea before the price increase is $4.81.

Back to contents

14. Gareth is analysing the profits and costs of his video game business over the last three weeks. The following table shows the amounts, in dollars, of sales and costs in each week.

Week	**1**	**2**	**3**
Sales	$13 032	$8571	$20 102
Costs	$10 023	$7053	$18 281

In which week did Gareth achieve the highest percentage profit?

Key points

- The table shows the amounts, in dollars, of sales and costs in each week.
- In which week did Gareth achieve the highest percentage profit?

Explanation

$$\text{Percentage profit} = \frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100$$

Calculate the percentage profit for week 1.

$$\begin{aligned}\text{Percentage profit} &= \frac{13\,032 - 10\,023}{10\,023} \times 100\\ &= 30.020...\\ &\approx 30\%\end{aligned}$$

Calculate the percentage profit for week 2.

$$\begin{aligned}\text{Percentage profit} &= \frac{8571 - 7053}{7053} \times 100\\ &= 21.522...\\ &\approx 22\%\end{aligned}$$

Calculate the percentage profit for week 3.

$$\begin{aligned}\text{Percentage profit} &= \frac{20\,102 - 18\,281}{18\,281} \times 100\\ &= 9.961...\\ &\approx 10\%\end{aligned}$$

$30\% > 22\% > 10\%$

Week 1 > week 2 > week 3

Answer

Gareth achieved the highest percentage profit in week 1.

Reasoning

15. **a.** The gluten-free croissant will cost $8.40 after the mark-up.

b. The original price of the vegan pain-au-chocolates was $5.80.

c. Salima expects a 73% profit on the gluten-free croissants after the price increase.

d. The difference in profit is $63.50.

e. Suggested option 1: Use less expensive ingredients.

Suggested option 2: Reduce the opening hours of the speciality bakery.

Note: There are other possible options.

16. **a.** The new price is $9.60.

b. The result of this calculation is $9.60.

c. Percentage increases and decreases can be combined by multiplying them together.

Exam-style

17. A

18. **a.** 67%

b. $54

19. 71 heaters

20. $1.60 per litre

Remember this?

21. D   **22.** B   **23.** D

1C Income

Student practice

Worked example 1

a. $229.04   **b.** $187.55   **c.** $315.14

Worked example 2

a. $2   **b.** $20   **c.** $25

Worked example 3

a. $34 892 per year   **b.** $1467.77 per week

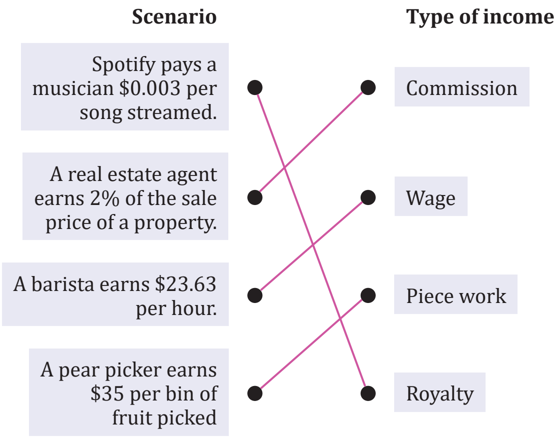
c. $32.03 per hour

Understanding worksheet

1.

Rate	Calculation	New rate
$1000 per week	× 52	Annual or yearly
$3000 per month	÷ 2	Fortnightly
$4500 per month	× 12	Annual or yearly
$45 000 per year	÷ 26	Fortnightly

2.



3. salary; royalty; percentage; piece work

Fluency

4. **a.** $86   **b.** $56.88   **c.** $154.68   **d.** $283.01
e. $230.52   **f.** $270.17   **g.** $183.94   **h.** $283.08

5. **a.** $294   **b.** $183.09   **c.** $302.45   **d.** $244.02
e. $213.41   **f.** $336.66   **g.** $248.57   **h.** $595.25

6. **a.** $45   **b.** $25   **c.** $0.80   **d.** $3500
e. $80   **f.** $52.20

7. **a.** $840 **b.** $4166.67

c. $39 000 **d.** $38.24

e. $96 360 **f.** $35.84

g. $26 832 **h.** $845

8. E

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. Maddie picked up a Saturday shift at the cafe she works at. She will work from 10 am until 4 pm. Her base rate is $26.72 per hour, but she gets time-and-a-half pay on Saturdays. Determine the amount of money she expects to earn for this shift.

Key points

- Maddie works a Saturday shift between 10 am and 4 pm.
- Her base rate is $26.72 per hour.
- She gets time-and-a-half pay on Saturdays.
- How much money can she expect to earn for this shift?

Explanation

Determine the number of hours worked during the shift.

There are 6 hours between 10 am and 4 pm.

Multiply the number of hours worked by the base rate and the Saturday rate.

$6 \times 26.72 \times 1.5 = \$240.48$

∴ Maddie can expect to earn $240.48 for this shift.

Answer

Maddie can expect to earn $240.48 for this shift.

11. Cal is a retail worker who earns $27.91 per hour, as well as 5% commission on any sales made. On a particular day, he worked 6.54 hours and sold $680 worth of goods. How much did he make on this day?

Key points

- Cal earns $27.91 per hour.
- He also earns 5% commission on any sales made.
- He worked 6.54 hours and sold $680 worth of goods one day.
- How much did he make on this day?

Explanation

Calculate Cal's wage by multiplying the number of hours worked by the hourly rate.

$$\begin{aligned} 6.54 \times 27.91 &= \$182.531... \\ &\approx \$182.53 \end{aligned}$$

Calculate the dollar amount of Cal's commission.

$$\begin{aligned} 5\% \text{ of } \$680 &= \frac{5}{100} \times 680 \\ &= \$34 \end{aligned}$$

Calculate the total amount Cal made on this day.

$$\begin{aligned} \text{Income} &= \text{wages} + \text{commission} \\ &= \$182.53 + \$34 \\ &= \$216.53 \end{aligned}$$

∴ Cal made $216.53 on this day.

Answer

Cal made $216.53 on this day.

12. Jane has a side-gig at an e-commerce warehouse as a picker and packer. She earns a packing rate for each item she picks and packs for delivery. The packing rate is $1.25 per item. She usually packs 105 items per day. If she works 3 times a week, calculate how much she expects to earn in one week.

Key points

- The packing rate is $1.25 per item.
- Jane usually packs 105 items per day.
- She works 3 times a week.
- Calculate how much she expects to earn in one week.

Explanation

Calculate how much Jane earns per day.

$$\begin{aligned} \text{Daily pay} &= \text{packing rate} \times \text{number of items packed per day} \\ &= 1.25 \times 105 \\ &= \$131.25 \end{aligned}$$

Calculate how much Jane earns per week.

$$\begin{aligned} \text{Weekly pay} &= \text{daily pay} \times \text{number of times worked per week} \\ &= 131.25 \times 3 \\ &= \$393.75 \end{aligned}$$

∴ Jane can expect to earn $393.75 in one week.

Answer

Jane can expect to earn $393.75 in one week.

13. Salima is an author of two best selling books. She receives royalties of 7.5% for her first book, and 10% for her second book. Both books retail at $19.95. In one year, 1500 copies of her first book and 3400 copies of her second book were sold. How much income from royalties will she earn?

Key points

- Salima receives royalties of 7.5% for her first book.
- Salima receives royalties of 10% for her second book.
- Both books have a price of $19.95.
- 1500 copies of her first book and 3400 copies of her second book were sold.
- How much income from royalties can she expect to earn?

Explanation

Calculate the income from royalties for Salima's first book.

$$\begin{aligned} \text{Income} &= \text{royalties rate} \times \text{price of book} \times \text{number of books sold} \\ &= \frac{7.5}{100} \times 19.95 \times 1500 \\ &= \$2244.375 \\ &\approx \$2244.38 \end{aligned}$$

Calculate the income from royalties for Salima's second book.

$$\begin{aligned} \text{Income} &= \text{royalties rate} \times \text{price of book} \times \text{number of books sold} \\ &= \frac{10}{100} \times 19.95 \times 3400 \\ &= \$6783 \end{aligned}$$

Calculate the total income from royalties.

$$\begin{aligned} \text{Total income} &= \text{income from first book} + \text{income from second book} \\ &= 2244.38 + 6783 \\ &= \$9027.38 \end{aligned}$$

∴ Salima can expect to earn $9027.38 from royalties.

Answer

Salima can expect to earn $9027.38 from royalties.

Back to contents

14. Alex works as a hotel receptionist. In one week, they worked 20 hours, which included 7 hours on a public holiday and 5 hours on Sunday. They received $1263.52 in their bank account.

What is their hourly wage? Assume they earn 1.75 times their base rate for Sunday work, and double-pay for public holiday work.

Key points

- Alex worked 20 hours in one week.
- This included 7 hours on a public holiday, which is double-pay.
- This included 5 hours on Sunday, which is 1.75 times their base rate.
- They were paid $1263.52.
- What is their hourly wage?

Explanation

Determine the number of hours worked at base rate.

$20 - 7 - 5 = 8$ hours

Let Alex's base rate be x.

Form an equation that represents Alex's weekly earnings.

Weekly earnings = base rate × (normal hours + public holiday rate × public holiday hours + Sunday rate × Sunday hours)

$1263.52 = (8 + 2 \times 7 + 1.75 \times 5)x$

$1263.52 = 30.75x$

Solve for x.

$x = \frac{1263.52}{30.75}$

$= \$41.090...$

$\approx \$41.09$

∴ Alex's hourly wage is $41.09.

Answer

Alex's hourly wage is $41.09.

Reasoning

15. **a.** Tobi earned $103.74 working on Wednesday.

b. Tobi earned $272 commission from selling hardware for the week.

c. Tobi earned a total of $726.18 from his wages from the week.

d. Tobi will be able to afford his holiday after 31 weeks.

e. Suggested option 1: Tobi could put a greater percentage of his earnings per week into savings.

Suggested option 2: Tobi could spend less money.

Note: There are other possible options.

16. **a.** The worker earns $322.47.

b. The worker earns $250.81.

c. The worker earns better pay by working 6 hours at 1.5 times their base hourly rate than by working 4 hours at 1.75 times their base hourly rate. This can also be determined by only comparing the reduction in hours and increase in pay rates. An increase in pay by 0.25 times the base hourly rate is not sufficient to compensate for a third of the hours lost.

Exam-style

17. C

18. **a.** $210 **b.** $420 **c.** $550

19. $892.39

20. $47.57

Remember this?

21. C **22.** D **23.** E

1D Taxation

Student practice

Worked example 1

a. $336 **b.** $1234

Worked example 2

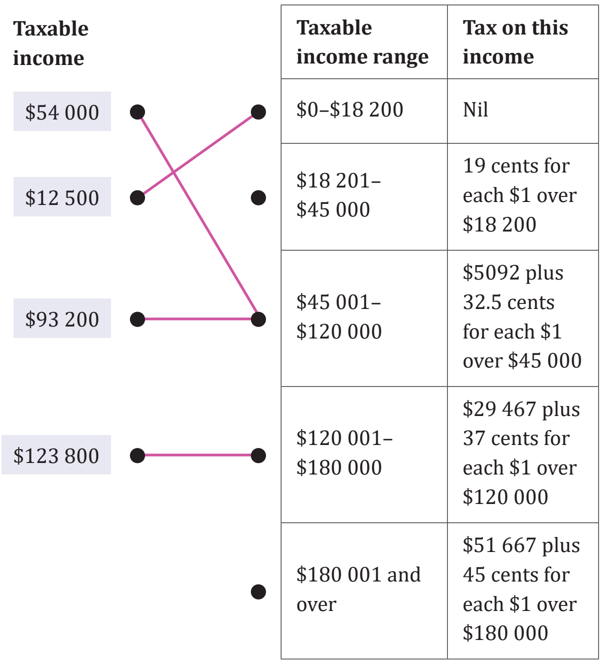
a. $3567.06 **b.** $28 719.18

Understanding worksheet

1.

Gross earnings	Withheld tax	Net income
$850	$112	$738
$1300	$136	$1164
$3750	$498	$3252
$4482	$1182	$3300

2.



Taxable income range	Tax on this income
$0–$18 200	Nil
$18 201–$45 000	19 cents for each $1 over $18 200
$45 001–$120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001–$180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

3. income tax; taxable income; withheld tax; deductions

Fluency

4. **a.** $75 **b.** $168 **c.** $1452 **d.** $120
e. $112 **f.** $2357 **g.** $252 **h.** $212

5. **a.** $4142 **b.** $8179.50
c. $34 018 **d.** $0
e. $18 716.98 **f.** $20 914.63
g. $0 **h.** $77 236.45

6. **a.** \$3724 **b.** \$12 290.75
c. \$30 931.46 **d.** \$5067.30
e. \$20.33 **f.** \$75 346.90

7. B

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

9. Bonnie checks her fortnightly pay slip and sees that \$398 in tax is withheld from her gross pay of \$2352. Determine her net income over a month.

Key points

- Bonnie receives a gross pay of \$2352 in her fortnightly pay slip.
- \$398 in tax is withheld.
- Determine Bonnie's net income over a month.

Explanation

Net income = gross earnings − withheld tax

Substitute the appropriate values into this equation to calculate Bonnie's net income over the fortnight.

Net income = 2352 − 398
= \$1954

There are two fortnights in a month. Therefore, multiply Bonnie's fortnightly net income by 2 to determine her monthly net income.

1954 × 2 = \$3908

Answer

Bonnie's net income over a month is \$3908.

10. Lincoln earns a \$5000 pay rise after a performance review. Before the review, his gross salary was \$43 550. Calculate the difference in income tax he owes after the pay rise.

Key points

- Lincoln earns a \$5000 pay rise.
- Before the pay rise, his gross salary was \$43 550.
- Calculate the difference in income tax Lincoln owes after the pay rise.

Explanation

Calculate the tax Lincoln owed before the pay rise when his gross salary was \$43 550.

The tax rate for this income is 19 cents for each \$1 over \$18 200.

Portion of the taxable income above the threshold
= 43 550 − 18 200
= \$25 350

Income tax before pay rise = 25 350 × 0.19
= \$4816.50

Calculate the tax Lincoln owes after the pay rise when his gross salary increased by \$5000.

Gross salary after pay rise = 43 550 + 5000
= \$48 550

The tax rate for this income is \$5092 plus 32.5 cents for each \$1 over \$45 000.

Portion of the taxable income above the threshold
= 48 550 − 45 000
= \$3 550

Income tax after pay rise = 5092 + 3550 × 0.325
= \$6245.75

Calculate the difference in income tax Lincoln owes after the pay rise.

Difference = income tax after pay rise − income tax before pay rise
= 6245.75 − 4816.50
= \$1429.25

Answer

Lincoln owes \$1429.25 more in income tax after the pay rise.

11. Surae's net income each fortnightly pay cycle is \$1840. Her gross income is \$58 500 per year. Calculate how much tax she owes or is owed at the end of the year.

Key points

- Surae's net income is \$1840 each fortnight.
- Her gross income is \$58 500 per year.
- Calculate how much tax Surae owes or is owed at the end of the year.

Explanation

There are 26 fortnights in a year.

Surae's yearly net income = Surae's fortnightly net income × 26
= 1840 × 26
= \$47 840

Calculate Surae's total withheld tax.

Total withheld tax = gross earnings − net income
= 58 500 − 47 840
= \$10 660

Calculate Surae's total income tax.

The tax rate for this income is \$5092 plus 32.5 cents for each \$1 over \$45 000.

Portion of the taxable income above the threshold
= 58 500 − 45 000
= \$13 500

Income tax = 5092 + 13 500 × 0.325
= \$9479.50

Calculate how much tax Surae owes or is owed.

Tax refund or tax owed = income tax − total withheld tax
= 9479.50 − 10 660
= −\$1180.50

Answer

Surae is owed \$1180.50 tax at the end of the year.

12. Mako's annual gross income is \$56 505. After she submitted her tax return, she discovered she could claim \$600 worth of deductions. How much less income tax would she pay if she claimed those deductions?

Key points

- Mako's annual gross income is \$56 505.
- She can claim \$600 worth of deductions.
- How much less income tax would she pay if she claims the deductions?

Explanation

Mako's taxable income is reduced by \$600 but this does not change the tax range

The rate at which the \$600 would have been taxed is 32.5 cents per \$1.

600 × 0.325 = \$195 less tax paid.

Answer

Mako pays \$195 less tax if she claims the deductions.

Back to contents

13. The total amount of income tax Makhim owes is $11 982. What is his gross income? Assume he has no deductions.

Key points

- Makhim owes $11 982 in income tax.
- He has no deductions.
- What is Makhim's gross income?

Explanation

Let x be Makhim's gross income. Determine the value of x.

The tax rate is $5092 plus 32.5 cents for each $1 over $45 000.

Portion of the taxable income above the threshold $= x - 45\,000$

$11\,982 = 5092 + (x - 45\,000) \times 0.325$

$6890 = (x - 45\,000) \times 0.325$

$21\,200 = x - 45\,000$

$x = \$66\,200$

Answer

Makhim's gross income is $66 200.

Reasoning

14. **a.** Nick's annual gross income is $75 998.

b. Nick's withheld tax is $644. His annual net income is $59 254.

c. Nick's taxable income after deductions is $75 578.

d. Nick's income tax is $15 029.85. He will be owed $1714.15 by the tax office at the end of the year.

e. Suggested option 1: An advantage is that Nick can save time not doing his taxes. A disadvantage is that Nick has to pay the accountant.

Suggested option 2: An advantage is that Nick can receive help when doing his taxes from a professional. A disadvantage is that Nick may not understand his taxes if he does not do them himself.

Note: There are other possible options.

15. **a.** The income tax on a gross income of $54 000 using the table in Key Idea 2 is $8017.

b. The income tax on a gross income of $54 000 using the table from 2019 is $9097.

c. Changing the income tax ranges resulted in less income tax being paid by a person earning a gross income of $54 000 currently than in 2019, as indicated by the higher income tax amount in 2019 compared to Key Idea 2.

Exam-style

16. C

17. **a.** $26 038
b. $6216

18. $127.68

19. $23 425.80

Remember this?

20. B

21. C

22. D

1E Simple interest

Student practice

Worked example 1

a. $45 **b.** $21 **c.** $82.48

Worked example 2

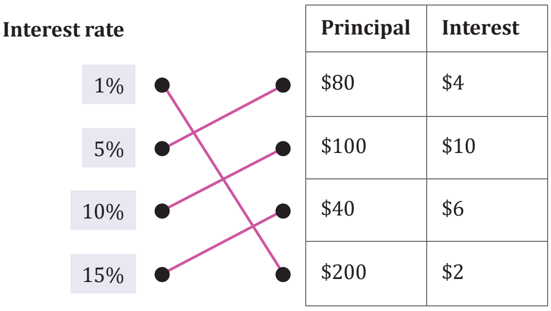
a. $P = \$1875$

b. $t = 6$ years

c. $P \approx \$2666.67$

Understanding worksheet

1.



Principal	Interest
$80	$4
$100	$10
$40	$6
$200	$2

2.

Principal	Interest	Amount
$500	$15	$515
$300	$6	$306
$120	$4.80	$124.80
$790	$20	$810

3. loan; investment; principal; interest rate

Fluency

4. **a.** $30 **b.** $675
c. $750 **d.** $412.50
e. $2297.09 **f.** $354.54
g. $805.87 **h.** $4048.37

5. **a.** $45 **b.** $45
c. $0.21 **d.** $10.54
e. $123.50 **f.** $17.68
g. $384.88 **h.** $7796.42

6. **a.** $P \approx \$2083.33$ **b.** $r = 2.40\%$ p.a.
c. $t = 2$ years **d.** $P = \$800$
e. $t = 4$ years **f.** $r \approx 6.40\%$ p.a.
g. $P \approx \$3410.40$ **h.** $r \approx 2.66\%$ p.a.

7. D

Spot the mistake

8. **a.** Student B is incorrect. **b.** Student B is incorrect.

Back to contents

Problem solving

9. Ryan receives $200 for his birthday. He decides to invest it in a bond that offers an annual interest rate of 4%. Determine the amount of interest he will earn after 2 years.

Key points

- Ryan receives $200.
- He invests it in a bond with an annual interest rate of 4%.
- Determine the amount of interest he will earn after 2 years.

Explanation

Identify the principal, interest rate, and time period.

$P = 200$

$r = 4$

$t = 2$

Substitute the values into the simple interest formula.

$$I = \frac{200 \times 4 \times 2}{100}$$

$$= \$16$$

Answer

Ryan earns $16 interest on his investment.

10. Alex is looking at buying a $1200 electric guitar. He currently has $1000 and he decides to put it in a savings account that will accrue simple interest. He sets a goal of buying the guitar in 16 months. What is the minimum annual interest rate required to afford the guitar in this time?

Key points

- Alex wants to buy a $1200 guitar in 16 months time.
- He puts $1000 in a savings account that accrues simple interest.
- What is the minimum annual interest rate required to afford the guitar in this time?

Explanation

Identify the known information.

$P = 1000$

$I = 200$

$t = 16$

Substitute the known values into the simple interest formula.

$$200 = \frac{1000 \times r \times 16}{100}$$

Solve for the missing value.

$$200 \times 100 = 1000 \times r \times 16$$

$$20\,000 = 16\,000r$$

$$\frac{20\,000}{16\,000} = r$$

$$r = 1.25\% \text{ per month}$$

Convert this monthly interest rate into an annual interest rate.

$$\text{annual interest rate} = \text{monthly interest rate} \times 12$$

$$= 1.25 \times 12$$

$$= 15\%$$

Answer

15% is the minimum annual interest rate required for Alex to afford the guitar in 16 months time.

11. Emily wants to start her own small business in 6 months time. She decides to open a simple interest savings account that offers an annual interest rate of 6%. If she needs $600 more, calculate the amount she will need to invest initially.

Key points

- Emily needs $600 more in 6 months time.
- She puts an amount of money into a simple interest savings account with an annual interest rate of 6%.
- Calculate this amount that Emily invests initially.

Explanation

Convert the annual interest rate into a monthly interest rate.

$$\frac{6}{12} = 0.5\% \text{ per month}$$

Identify the known information.

$r = 0.5$

$I = 600$

$t = 6$

Substitute the known values into the simple interest formula.

$$600 = \frac{P \times 0.5 \times 6}{100}$$

Solve for the missing value.

$$600 \times 100 = P \times 0.5 \times 6$$

$$60\,000 = 3P$$

$$\frac{60\,000}{3} = P$$

$$P = 20\,000$$

Answer

Emily needs to initially invest $20 000.

12. Maki is planning to buy his first car. The car he has set his sights on costs $10 000, but he only has $3000 saved up. He decides to borrow the remaining amount as a personal loan, and pay it off when its value reaches $9000. If the bank charges simple interest at 7.16% p.a., how long, to the nearest year, will he wait to pay off the loan?

Key points

- The car that Maki wants to buy is $10 000.
- He has saved $3000.
- He borrows the remaining amount and pays it off when its value reaches $9000.
- The bank charges simple interest at 7.16% p.a.
- How long, to the nearest year, will Maki wait to pay off the loan?

Explanation

Identify the known information.

$$\text{Amount Maki borrows from the bank} = 10\,000 - 3000$$

$$P = 7000$$

$$\text{Interest accrued when Maki pays off the loan} = 9000 - 7000$$

$$I = 2000$$

$r = 7.16$

Substitute the known values into the simple interest formula.

$$2000 = \frac{7000 \times 7.16 \times t}{100}$$

Solve for the missing value.

$$2000 \times 100 = 7000 \times 7.16 \times t$$

$$200\,000 = 50\,120t$$

$$\frac{200\,000}{50\,120} = t$$

$$t = 3.9904...$$

$$t \approx 4 \text{ years}$$

Answer

Maki will wait 4 years to pay off the loan.

Back to contents

13. Shari is a student who wants to travel abroad for her studies. She borrows $2500 from a lender. The lender states that the amount will attract interest for 16 months at a rate of 8.33% p.a. in which time no repayments can be made by the borrower. After this, the loan stops attracting interest and she'll need to start paying off the value of the loan, including interest. If she wants to pay off her loan in 3 years, how much will her monthly repayments be?

Key points

- Shari borrows $2500.
- The loan attracts interest for 16 months at a rate of 8.33% p.a.
- Then the loan stops attracting interest and Shari must start paying it off.
- She wants to pay off her loan in 3 years.
- How much will Shari's monthly repayments be?

Explanation

Convert 16 months into years.

$\frac{16}{12} = 1.33\ldots$ years

Identify the known information.

$P = 2500$

$r = 8.33$

$t = 1.33\ldots$

Substitute the known values into the simple interest formula.

$$I = \frac{2500 \times 8.33 \times 1.33\ldots}{100}$$
$$= 277.66\ldots$$

∴ The loan attracts $277.66... in interest.

Calculate the value of the loan that Shari must pay off.

$$A = P + I$$
$$= 2500 + 277.66\ldots$$
$$= 2777.66\ldots$$

Shari wants to pay off $2777.66... in 3 years. There are $3 \times 12 = 36$ months in 3 years.

Calculate how much Shari must pay each month to pay off $2777.66... in 36 months.

$$\frac{2777.66\ldots}{36} \approx \$77.16$$

Answer

Shari's monthly repayments will be $77.16.

Reasoning

14. **a.** $250 interest is earned.

b. The value of his investment is $2160.

c. It will take 16 years for account 2 to become the best value savings account.

d. Suggested option 1: Paulo should consider which banks have positive reviews from customers.

Suggested option 2: Paulo should consider which banks have better interest rates.

Note: There are other possible options.

15. **a.** The value of the loan is $525.

b. The value of the loan is $551.25.

c. $25 interest is earned in part **a** and $26.25 interest is earned in part **b**. Therefore, using a value that has already accrued interest as the principal earns the greater amount of interest.

Exam-style

16. C

17. **a.** $203 **b.** 4 months

18. 65 months

19. $780

Remember this?

20. E **21.** C **22.** E

1F Compound interest and depreciation

Student practice

Worked example 1

a. $1166.40 **b.** $855.65

Worked example 2

a. $6635.52 **b.** $672.53

Understanding worksheet

1.

Total amount (A)	Principal (P)	Interest = (Total amount − principal)
$5100	$5000	$100
$5202	$5100	$102
$5306.04	$5202	$104.04
$5412.16	$5306.04	$106.12

2. **a.** 1000 **b.** 15 **c.** 12 **d.** 12

3. simple; compound; sum; percentage

Fluency

4. **a.** $10 500 **b.** $173.03
c. $2730.06 **d.** $21 282.77
e. $6070.20 **f.** $865.41
g. $10 341.99 **h.** $2119.89

5. **a.** $4050 **b.** $4088.83
c. $382.91 **d.** $940.90
e. $1064.09 **f.** $784.12
g. $2030.31 **h.** $1290.49

6. **a.** $12 562.41 **b.** $5456.21
c. $2244.35 **d.** $5150.38
e. $4883.36 **f.** $38 044.71
g. $498.57 **h.** $3548.55

7. B

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student B is incorrect.

Back to contents

Problem solving

9. John has purchased a new photocopier for $3500, which depreciates by 25% p.a. Calculate the value of the photocopier after 5 years. Round to the nearest cent.

Key points

- John purchased a new photocopier for $3500.
- The photocopier depreciates by 25% p.a.
- Calculate the value of the photocopier after 5 years.

Explanation

Identify the principal, interest rate, and number of compounding periods.

$P = 3500$

$r = 25$

$n = 5$

Substitute the values into the compound depreciation formula.

$$A = 3500 \times \left(1 - \frac{25}{100}\right)^5$$

$$\approx \$830.57$$

$$\overset{\text{Total amount}}{\boxed{A}} = \overset{\text{Principal}}{\boxed{3500}} \times \left(\underbrace{1 - \frac{\overset{\text{Interest rate (\%)}}{\boxed{25}}}{100}}_{\text{Percentage decrease}}\right)^{\boxed{5}\ \text{Number of compounding periods}}$$

Answer

The value of the photocopier after 5 years will be $830.57.

10. Jenny invested some money in a savings account that offers a yearly interest rate of 2.7%. After 4 years, the balance in her account has grown to $12 300. If the interest is compounded annually, what was the initial amount of money she invested in the account?

Key points

- The balance in the account compounds annually at 2.7% p.a.
- The investment was made 4 years ago.
- Jenny's account balance is currently $12 300.
- Find the initial amount of money she invested in the account.

Explanation

Identify the final amount, interest rate, and number of compounding periods.

$A = 12\ 300$

$r = 2.7$

$t = 4$

Substitute the values into the compound interest formula and solve for P.

$$12\ 300 = P \times \left(1 + \frac{2.7}{100}\right)^4$$

Solve for the principal.

$$P = \frac{12\ 300}{\left(1 + \frac{2.7}{100}\right)^4}$$

$$\approx \$11\ 056.64$$

Answer

Jenny's initial investment was $11 056.64.

11. Elon's electric car has a value of $50 607.68, rounded to the nearest cent, after 5 years. The car's value depreciated by 3.4% p.a. during that time. Calculate the initial value (P) of Elon's car.

Key points

- The car has a current value of $50 607.68.
- The car was purchased 5 years ago.
- The car's value depreciated by 3.4% p.a.
- Calculate the initial value of Elon's car.

Explanation

Identify the value, interest rate, and number of compounding periods.

$A = 50\ 607.68$

$r = 3.4\%$

$n = 5$

Substitute the values into the compound depreciation formula.

$$50\ 607.68 = P \times \left(1 - \frac{3.4}{100}\right)^5$$

Solve for the principal.

$$P = \frac{50\ 607.68}{\left(1 - \frac{3.4}{100}\right)^5}$$

$$\approx \$60\ 163.18$$

Answer

The initial value of Elon's electric car was $60 163.18.

12. The interest in Liam's savings account compounds monthly. He receives a notification that the interest rate on his savings account will rise from 3.45% to 3.70% p.a. If his current balance is $4560, calculate the difference in interest accrued between the two interest rates over three months.

Key points

- The interest compounds monthly.
- The interest rate will rise from 3.45% to 3.70% p.a.
- The current balance is $4560.
- Calculate the difference in interest accrued between the two interest rates over three months.

Explanation

Identify the principal and number of compounding periods.

$P = 4560$

$n = 3$

Convert the original interest rate to match the compounding period.

$$r = \frac{3.45}{12}\% \text{ per month}$$

Substitute the values into the compound interest formula.

$$A = 4560 \times \left(1 + \frac{\frac{3.45}{12}}{100}\right)^3$$

$$= \$4599.4431...$$

Convert the new interest rate to match the compounding period.

$$r = \frac{3.70}{12} \text{ per month}$$

Substitute the values into the compound interest formula.

$$A = 4560 \times \left(1 + \frac{\frac{3.70}{12}}{100}\right)^3$$

$$= \$4602.3101...$$

$$\begin{aligned}\text{Difference in interest} &= \$4602.3101... - \$4599.4431...\\ &= \$2.867...\\ &\approx \$2.87\end{aligned}$$

Answer

Liam will receive $2.87 more interest over three months.

Back to contents

13. The balance on credit cards typically attracts interest that compounds daily after a certain number of days, known as the interest-free period. Shak's credit card has an interest-free period of 25 days and the interest rate is 9.95% p.a. If the balance of Shak's card is \$583, how much more does he pay if he pays the balance after 40 days?

Key points

- During the interest-free period, the balance does not attract interest.
- The interest-free period is 25 days.
- The interest rate is 9.95% p.a.
- The balance of Shak's card is \$583.
- How much more does he pay if he pays the balance after 40 days?

Explanation

The interest is compounded over $40 - 25 = 15$ days.

Identify the principal and number of compounding periods.

$P = 583$

$n = 15$

Convert the interest rate to match the compounding period.

$r = \frac{9.95}{365}\%$ per day

Substitute the values into the compound interest formula.

$$A = 583 \times \left(1 + \frac{\frac{9.95}{365}}{100}\right)^{15}$$

$$\approx \$585.39$$

Shak will pay $\$585.39 - \$583 = \$2.39$ more after 40 days.

Answer

Shak pays \$2.39 more after 40 days.

Reasoning

14. **a.** The value of option 1 at the end of the term is \$55 230.05.

b. Option 1 is better value than option 2 by \$2517.88.

c. Kahreem would need to invest \$85 795.64 more initially.

d. It will take 14 years for option 1 to double in value.

e. Suggested option 1: An advantage is that term deposits guarantee a specific interest rate.

Suggested option 2: A disadvantage is that the money in a term deposit cannot be withdrawn.

Note: There are other possible options.

15. **a.** The new amount is \$8226.54.

b. The new amount is \$8052.55.

c. The amount increased by \$3226.54 in part **a** whereas the amount increased by \$3052.55 in part **b**. The amount of interest accrued is greater when compounding occurs monthly compared to annually. This is because with monthly compounding, the interest is calculated and added to the balance more frequently, allowing the balance to increase more rapidly.

Exam-style

16. C

17. **a.** \$12 518.75 **b.** \$12 575.17

18. Loan 1 will cost Daniel the least amount of money because less interest accumulates due to the less frequent compounding. Loan 2, despite having the same annual interest rate, ends up costing more because the interest is compounded more frequently, leading to more interest accruing over the course of the year.

19. \$503.50

Remember this?

20. E **21.** D **22.** C

Chapter 1 extended application

1. **a.** Billy's gross profit was \$1333.50.

b. The employee's net pay each week is \$372.

c. Billy's employee's total net pay for the summer is \$4464.

d. Billy's total net profit was \$46 489.

e. Suggested option 1: Having a summer job allows you to have money and independence.

Suggested option 2: Having a summer job means having to miss out on time with friends and family.

Note: There are other possible options.

2. **a.** The total cost of all parts is \$157.80.

b. The discounted cost of the service is \$161.50.

c. The regular hourly rate of the mechanic is \$38.37.

d. The total cost payable by the customer is \$731.06.

e. Suggested option 1: Working overtime brings in additional pay at a higher rate.

Suggested option 2: Working overtime can be exhausting as these hours may be additional to a full workload.

Note: There are other possible options.

3. **a.** The maximum amount the school can withdraw is \$2000.

b. The value of the investment after 10 years is \$1 150 000.

c. The value of the investment after 10 years is \$1 158 220.

d. The investment option described in part **c** shows a 16% return on the principal investment.

e. Suggested option 1: The school could invest this money to build a swimming pool.

Suggested option 2: The school could buy 3D printers.

Note: There are other possible options.

Chapter 1 review

Multiple choice

1. B **2.** C **3.** C **4.** D **5.** D

Fluency

6. **a.** 84 **b.** 49 **c.** 3.3 **d.** 8.6913

7. **a.** \$1700 **b.** \$1682.23
c. \$4.18 **d.** \$1568.05

8. **a.** \$264.24 **b.** \$118.11 **c.** \$421.88 **d.** \$355.78

9. **a.** \$120 **b.** \$1070 **c.** \$26 **d.** \$736

10. **a.** \$0 **b.** \$3942.50
c. \$18 030.25 **d.** \$28 299.60

Back to contents

11. **a.** \$250 **b.** \$8.14
c. \$58.46 **d.** \$30 549.77

---

12. **a.** $p = \$1857.14$ **b.** $p = \$6473.33$
c. $r = 5.11\%$ p.a. **d.** $t = 1.09$ years

---

13. **a.** \$43 600 **b.** \$7916.54
c. \$116 480.36 **d.** \$16.09

---

14. **a.** \$4374 **b.** \$419.79
c. \$892.71 **d.** \$2015.31

Problem solving

15. Beccy is running a marathon in the United States, where they express distance using miles. Given that one mile is 1.60934 km, how far did Beccy run in kilometres if the marathon was 26.2188 miles? Round the answer to three decimal places.

Key points

- 1 mile equals 1.60934 km.
- Convert 26.2188 miles to kilometres, correct to three decimal places.

Explanation

$$\begin{aligned} 1 \text{ mile} &= 1.60934 \text{ km} \\ \therefore 26.2188 \text{ miles} &= 1.60934 \times 26.2188 \text{ km} \\ &= 42.1949... \\ &\approx 42.195 \text{ km} \end{aligned}$$

$$\begin{tikzcd} \textbf{1 mile} \arrow[r, bend left, "\times 1.60934"] & \textbf{1.60934 km} \end{tikzcd}$$

$$\begin{tikzcd} \textbf{26.2188 miles} \arrow[r, bend left, "\times 1.60934"] & \textbf{42.1949 km} \arrow[r, "\text{Round to three decimal places}"] & \approx \textbf{42.195 km} \end{tikzcd}$$

Answer

Beccy ran 42.195 km.

16. OSPM is a retailer for eyewear. The cost price is \$102 for a pair of prescription glasses and \$85 for a pair of sunglasses. OSPM sells a pair of prescription glasses and a pair of sunglasses for \$495, making a 105% profit on the pair of prescription glasses. What is the percentage profit for the pair of sunglasses, correct to two decimal places?

Key points

- The cost price of a pair of prescription glasses is \$102.
- The cost price of a pair of sunglasses is \$85.
- The selling price of a pair of glasses and a pair of sunglasses is \$495.
- A 105% profit was made from the pair of prescription glasses.
- What is the percentage profit for the pair of sunglasses, correct to two decimal places?

Explanation

$$\begin{aligned} \text{Selling price for prescription glasses} &= \text{cost price} \times \\ &\quad (1 + \text{ percentage profit}) \\ &= 102 \times (1 + 105\%) \\ &= 102 \times 2.05 \\ &= \$209.1 \end{aligned}$$

$$\begin{aligned} \text{Selling price for sunglasses} &= \text{selling price for both} \\ &\quad - \text{prescription glasses} \\ &= 495 - 209.1 \\ &= \$285.9 \end{aligned}$$

$$\begin{aligned} \text{Profit for sunglasses} &= \text{selling price} - \text{cost price} \\ &= 285.9 - 85 \\ &= \$200.9 \end{aligned}$$

$$\begin{aligned} \%\text{ profit} &= \frac{\text{profit}}{\text{cost price}} \times 100\% \\ &= \frac{200.9}{85} \times 100\% \\ &= 236.352...\% \\ &\approx 236.35\% \end{aligned}$$

Answer

The percentage profit for the pair of sunglasses is 236.35%.

17. Alexis has been paid \$975.95 for her work during the past two weeks. She knows that she has worked 21.25 regular hours and 4.35 hours overtime. Given that her overtime pay is 150% of her normal pay, calculate Alexis' regular pay rate per hour, correct to two decimal places.

Key points

- Alexis earned \$975.95 over two weeks.
- She worked 21.25 regular hours.
- She worked an additional 4.35 hours overtime.
- Her overtime pay is 150% of her normal pay.
- Calculate Alexis' regular pay rate per hour, correct to two decimal places.

Explanation

Let Alexis' regular pay rate be \$$x$ per hour. Hence, her overtime pay rate is \$$1.5x$ per hour.

$$\begin{aligned} \text{Total earnings} &= \text{regular pay earnings} + \text{overtime} \\ &\quad \text{pay earnings} \\ 975.95 &= (\text{regular pay hours} \times \text{pay rate}) + (\text{overtime} \\ &\quad \text{pay hours} \times \text{pay rate}) \\ 975.95 &= 21.25x + 4.35 \times 1.5x \\ 975.95 &= 21.25x + 6.525x \\ 975.95 &= 27.775x \\ 975.95 \div 27.775 &= 27.775x \div 27.775 \\ x &= 35.138... \\ x &\approx \$35.14 \end{aligned}$$

Answer

Alexis' regular pay rate is \$35.14 per hour.

18. Victoria works for a charity where she is offered a \$15 590 tax deduction per year. She also purchases a private insurance policy which is equivalent to an additional \$1218.40 tax deduction per year. In the end, Victoria's income is within the range of \$45 001–\$120 000. Calculate the possible minimum and maximum of Victoria's gross income prior to tax deductions.

Key points

- Victoria has a \$15 590 tax deduction per year.
- She has an additional \$1218.40 tax deduction per year.
- Victoria's income range after all deductions is \$45 001–\$120 000.
- Calculate the possible minimum and maximum of Victoria's gross income before deductions.

Back to contents

Explanation

Total tax deductions $= 15\,590 + 1218.40 = \$16\,808.40$

Victoria's minimum gross income $=$ minimum in tax bracket + tax deductions

$= 45\,001 + 16\,808.4$

$= \$61\,809.40$

Victoria's maximum gross income $=$ maximum in tax bracket + tax deductions

$= 120\,000 + 16\,808.4$

$= \$136\,808.40$

Answer

Victoria's minimum income is $61 809.40, and her maximum income is $136 808.40.

19. Dhanya opened a Goalsaver account at the start of April. The Goalsaver account pays Dhanya 4.65% simple interest monthly given that the account balance at the end of month is higher than at the beginning of the month; otherwise the interest paid monthly will only be 0.02%. The below table shows Dhanya's account balance over three months. The simple interest is paid on the first day of each month.

Month	Balance on first day of the month	Balance on the last day of month	$I = \dfrac{Prt}{100}$
April	$1278.90	$1525.15	$I = \dfrac{1525.15 \times \frac{4.65}{12} \times 1}{100}$ ≈ 5.91
May	1525.15 + 5.91 = $1531.06	$1448.03	
June		$1918.28	

Using the filled-in example from the table, complete the rest of the table. Calculate and round to the nearest cent.

Key points

- 4.65% simple interest monthly given that the account balance at the end of month is higher than at the beginning of the month.
- 0.02% simple interest monthly given that the account balance at the end of month is lower than at the beginning of the month.
- The simple interest is paid on the first day of each month, shown in the table. The table shows a balance of three months.
- Using the filled-in example from the table, complete the rest of the table. Calculate and round to the nearest cent.

Explanation

Month	Balance on first day of the month	Balance on the last day of month	$I = \dfrac{Prt}{100}$
April	$1278.90	$1525.15	$I = \dfrac{1525.15 \times \frac{4.65}{12} \times 1}{100}$ ≈ $5.91
May	1525.15 + 5.91 = $1531.06	$1448.03	$I = \dfrac{1448.03 \times \frac{0.02}{12} \times 1}{100}$ ≈ $0.02
June	1448.03 + 0.02 = $1448.05	$1918.28	$I = \dfrac{1918.28 \times \frac{4.65}{12} \times 1}{100}$ ≈ $7.43

Answer

Month	Balance on first day of the month	Balance on the last day of month	$I = \dfrac{Prt}{100}$
April	$1278.90	$1525.15	$5.91
May	$1531.06	$1448.03	$0.02
June	$1448.05	$1918.28	$7.43

20. Kashifa wants to invest $49 750.15 for one year, where she has a choice of whether interest is compounded weekly, monthly or yearly. If the interest compounds weekly, the interest rate offered will be 2.4935% p.a. If the interest compounds monthly, the interest rate offered will be 2.5635% p.a. If the interest compounds yearly, the interest rate offered will be 2.7901% p.a. Which product should Kashifa choose to maximise her earnings?

Key points

- Kashifa invests $49 750.15 for one year.
- She can choose whether her investment is compounded weekly, monthly or yearly.
- If her investment is compounded weekly, the interest rate would be 2.4935% p.a.
- If her investment is compounded monthly, the interest rate would be 2.5635% p.a.
- If her investment is compounded yearly, the interest rate would be 2.7901% p.a.
- Which product should Kashifa choose to maximise her earnings?

Explanation

Calculate the amount after one year if it's compounded weekly.

$P = 49\,750.15$

$n = 52$

$r = \dfrac{2.4935}{52}$

$$A = 49\,750.15 \times \left(1 + \frac{\frac{2.4935}{52}}{100}\right)^{52}$$

$\approx \$51\,005.96$

Calculate the amount after one year if it's compounded monthly.

$P = 49\,750.15$

$n = 12$

$r = \dfrac{2.5635}{12}$

$$A = 49\,750.15 \times \left(1 + \frac{\frac{2.5635}{12}}{100}\right)^{12}$$

$\approx \$51\,040.59$

Calculate the amount after one year if it's compounded yearly.

$P = 49\,750.15$

$n = 1$

$r = 2.7901$

$$A = 49\,750.15 \times \left(1 + \frac{2.7901}{100}\right)$$

$\approx \$51\,138.23$

$\$51\,138.23 > \$51\,040.59 > \$51\,005.96$

Yearly > Monthly > Weekly

The yearly compounding interest rate maximises the earnings.

Answer

Kashifa should choose the yearly compounding interest rate.

Reasoning

21. a. The new price of the bundles is $45.50.

b. Jess' gross income from doing these accounts is $25 125.

c. The income tax on Jess' taxable annual income is $56 295.25.

d. The value of the loan when repayments began is $7493.50.

e. Suggested option 1: Candle & Co can expand the range of their products.

Suggested option 2: Candle & Co can promote their products online to attract more customers.

Note: There are other possible options.

22. a. The new amount if $11 000 accrues annual compound interest at 5% p.a. for 3 years is approximately $12 733.88.

b. The new amount if $11 000 accrues monthly compound interest at 5% p.a. for 3 years is approximately $12 776.19.

c. The new amount when compounding monthly ($12 776.19) is greater than when compounding yearly ($12 733.88). Monthly compounding earns more money compared to annual compounding because the interest is calculated and added to the balance more frequently, allowing the balance to increase more rapidly.

Back to contents

2A The First and Second index laws

Student practice

Worked example 1

a. 2^{11} **b.** 3^5

Worked example 2

a. t^{12} **b.** p^6

Worked example 3

a. $6r^3p^{13}$ **b.** $3x^2y^6$ **c.** $4t^5$

Understanding worksheet

1. **a.** 9 **b.** 8 **c.** 5 **d.** 14

2. **a.** 2 **b.** 6 **c.** 5 **d.** 4

3. form; identical; laws; one

Fluency

4. **a.** 5^{10} **b.** 7^{10} **c.** 3^{16} **d.** 8^8
e. 2^9 **f.** 6^9 **g.** 4^7 **h.** 1

5. **a.** y^8 **b.** x^2 **c.** c^9 **d.** 1
e. t^{14} **f.** p^7 **g.** x^3 **h.** y^{10}

6. **a.** x^2y^4 **b.** $3p^7t^7$
c. $6v^7w^7$ **d.** $8a^{12}b^{13}$
e. $14c^6d^{13}$ **f.** $56p^6q^4$
g. $6x^4y^{10}$ **h.** $8r^{18}t^{12}$

7. **a.** x^3y^6 **b.** $2ab^6$
c. $4q^3$ **d.** $3x^3y$
e. $3v^{11}w^5$ **f.** $2n^5$
g. $\frac{6t^2y^{13}}{5}$ **h.** $\frac{4p^{11}r^7}{3}$

8. C

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. In a racing computer game, a player can double their score every five seconds if they continuously avoid any obstacles. What is the final score of a player with 7200 points who continuously avoids obstacles for the last 35 seconds of a race?

Key points

- If a player continuously avoids any obstacles, their score doubles every five seconds.
- A player has 7200 points.
- They continuously avoid obstacles for the last 35 seconds.
- What is their final score?

Explanation

If the score doubles every five seconds for 35 seconds the score will double seven times because $\frac{35}{5} = 7$.

The initial score of 7200 points doubles seven times.

$$\begin{aligned}\text{Final score} &= 7200 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\\ &= 7200 \times 2^7\\ &= 921\,600\end{aligned}$$

Answer

The player's final score is 921 600 points.

11. Every day for a week, the number of people who have seen a video online increases by a factor of four. If initially only 300 people saw the video, then how many have seen it by the end of that week?

Key points

- The number of people who have seen the video increases by a factor of four every day.
- This occurs every day for a week.
- 300 people initially saw the video.
- How many people have seen the video by the end of that week?

Explanation

300 people initially saw the video. This number increases by a factor of four every day for seven days. Calculate the number of viewers after one week.

$$\begin{aligned}\text{Number of viewers} &= 300 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4\\ &= 300 \times 4^7\\ &= 4\,915\,200\end{aligned}$$

Answer

4 915 200 people saw the video by the end of the week.

12. The height of a 2 m tall bamboo plant increases by a factor of four every week. How many days will it take the bamboo to grow to 32 m?

Key points

- A bamboo plant is 2 m tall.
- It increases by a factor of 4 every week.
- How many days will it take the bamboo to grow to 32 m?

Explanation

The bamboo plant is initially 2 m tall. The height increases by a factor of four every week until it reaches 32 m.

After 1 week, the height is $2 \times 4 = 8$ m.

$$\begin{aligned}\text{After 2 weeks, the height is } 2 \times 4 \times 4 &= 2 \times 4^{1+1}\\ &= 2 \times 4^2\\ &= 32 \text{ m}\end{aligned}$$

∴ The plant reaches a height of 32 m after 2 weeks.

Convert 2 weeks into days.

$2 \times 7 = 14$ days

Answer

The plant reaches a height of 32 m after 14 days.

13. Each person at a wedding is able to choose a combination of dishes from a set menu to make up their own five-course meal. A guest can choose one of three types of dishes for each course. How many different combinations of five-course meals are available?

Key points

- There are five courses.
- Guests select one of three types of dishes for each course.
- How many different combinations are available?

Back to contents

Explanation

The total number of combinations can be calculated by multiplying the number of dishes in each course together. There are three different dish options for each of the five courses.

Course 1	Course 2	Course 3	Course 4	Course 5
Options	Options	Options	Options	Options
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3

3 options for course 1 × 3 options for course 2 × 3 options for course 3 × 3 options for course 4 × 3 options for course 5

$$\begin{aligned}\text{Total number of combinations} &= 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^5 \\ &= 243\end{aligned}$$

Answer

There are 243 different combinations of five-course meals.

14. Garth wants to crochet a quilt made of identical squares with a total area of 1.44 m^2. If Garth wants the quilt to consist of exactly 6^2 squares, then how long should he make one side of each individual square, in cm?

Key points

- A quilt has a total area of 1.44 m^2.
- The quilt should consist of exactly 6^2 identical squares.
- How long should Garth make one side of each individual square, in cm?

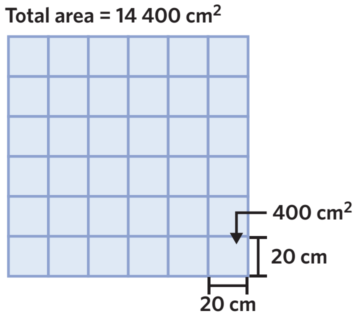
Explanation

The quilt should consist of $6^2 = 36$ identical squares.

Total area of each square $= \frac{1.44}{36} = 0.04$ m^2

$$\begin{aligned}\text{Area of one square} &= \text{length}^2 \\ 0.04 &= \text{length}^2\end{aligned}$$

$$\begin{aligned}\text{length} &= \sqrt{0.04}\text{ m} \\ &= 0.2\text{ m} \\ &= 20\text{ cm}\end{aligned}$$



Answer

Garth should make one side of an individual square 20 cm.

Reasoning

15. **a.** 2 MB of RAM equals 2^{11} KB of RAM.

b. 1 GB equals 2^{20} KB.

c. The memory capacity has doubled 19 times since the 1990s.

d. Suggested option 1: Computers.

Suggested option 2: Playstations.

Note: There are other possible options.

16. **a.** When $n = 3$, the expression equals 8.

b. When $n = 4$, the expression equals 16.

c.
$$\begin{aligned}2^7 &= 2^3 \times 2^4 \\ &= 8 \times 16 \\ &= 128\end{aligned}$$

We can multiply the values of smaller indices together to calculate the values of larger indices.

Exam-style

17. B

18. **a.** $2^x = 32$

b. $x = 5$

19. 268 096 cm^3

20. 9:37 pm on Thursday

Remember this?

21. C

22. D

23. D

2B The Third, Fourth and Fifth index laws

Student practice

Worked example 1

a. y^{20} **b.** $27r^{27}$ **c.** $16p^6v^{16}$

Worked example 2

a. $\frac{8}{p^{15}}$ **b.** $\frac{81t^{12}}{25r^{14}}$

Worked example 3

a. $448x^{44}$ **b.** $27m^{20}n^{32}$ **c.** $\frac{125k^{12}t^{27}}{64}$

Understanding worksheet

1. Third index law: I

Fourth index law: III

Fifth index law: II; IV

2. **a.** 3 **b.** 8 **c.** 45 **d.** 52

3. multiplied; quotients; same; expanded

Fluency

4. **a.** y^{12} **b.** t^{50}

c. $9k^8$ **d.** $64x^{15}$

e. $256p^{44}$ **f.** 100

g. $36n^{26}$ **h.** $5120m^{80}$

5. **a.** x^4y^{24} **b.** p^9r^{15}

c. $16m^{28}n^8$ **d.** $4a^{16}b^2$

e. $343c^{15}d^{30}$ **f.** $v^{96}w^{72}$

g. $-27x^{45}y^6$ **h.** $-384r^7t^{42}$

6. **a.** $\frac{9}{x^{10}}$ **b.** $\frac{v^{18}}{w^{27}}$

c. $\frac{64}{y^{24}}$ **d.** $\frac{16x^2}{9y^8}$

Back to contents

e. $\frac{625k^{16}}{64m^{20}}$ **f.** $\frac{10\,000p^{24}r^{12}}{81q^{4}}$

g. $\frac{1024m^{55}n^{25}}{243t^{60}}$ **h.** $\frac{256u^{32}v^{60}}{6561x^{28}y^{12}}$

7. **a.** $x^{13}y^{12}$ **b.** $m^{20}n^{16}$

c. $r^{23}t^{51}$ **d.** $8a^{25}b^{47}$

e. $128x^{56}y^{38}$ **f.** $243u^{71}v^{80}$

g. $4608p^{62}q^{24}$ **h.** $2646c^{30}d^{54}$

8. **a.** $x^{12}y^{13}$ **b.** $\frac{9t^{5}r^{10}}{2}$

c. $\frac{8a^{17}b^{3}}{5}$ **d.** $32p^{8}r^{41}$

e. $16c^{3}$ **f.** $64x^{19}y^{19}$

g. $\frac{7m^{6}n^{41}}{36}$ **h.** $\frac{27c^{30}d^{7}}{4}$

9. D

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

11. At 7 am, the number of people who have read an article online was 32. This number then increased by a factor of $\frac{5}{2}$ every 15 minutes. How many people read the article by 8 am?

Key points

- At 7 am, 32 people read an article online.
- This number increased by a factor of $\frac{5}{2}$ every 15 minutes.
- How many people read the article by 8 am?

Explanation

32 people had read the article at 7 am. This number increases by a factor of $\frac{5}{2}$ every 15 minutes until 8 am.

There are 60 minutes between 7 am and 8 am. $\frac{60}{15} = 4$.

Therefore, 32 people increases by a factor of $\frac{5}{2}$ four times.

$$\begin{aligned}32 \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} &= 32 \times \left(\frac{5}{2}\right)^4\\ &= 32 \times \frac{5^4}{2^4}\\ &= 32 \times \frac{625}{16}\\ &= 1250\end{aligned}$$

Answer

1250 people have read the article by 8 am.

12. Dominick’s computer had 4^3 GB of available storage space. When he upgraded it, the amount of available storage space doubled a number of times. If the amount of available storage space on Dominick’s computer is now 2048 GB, then how many times did it double?

Key points

- Dominick’s computer had 4^3 GB of available storage space.
- The amount of available storage space doubled a number of times.
- Now the available storage space is 2048 GB.
- How many times did the available storage space double?

Explanation

$$\begin{aligned}\text{Available data space before upgrade} &= 4^3\\ &= 4 \times 4 \times 4\\ &= 64 \text{ GB}\end{aligned}$$

64 GB doubles a number of times until it reaches 2048 GB. Let this number be x.

$64 \times 2^x = 2048$

Solve for x.

$2^x = 32$

$x = 5$ because $2^5 = 32$.

Answer

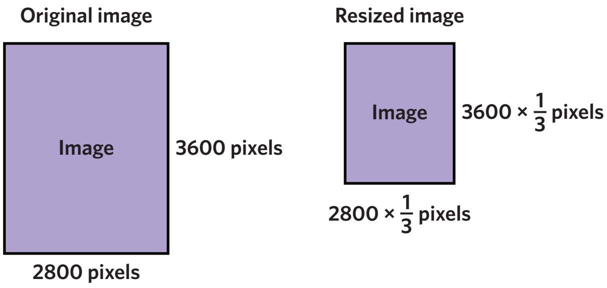
The available data space was doubled 5 times.

13. Each side length of a rectangular digital image, measuring 3600 by 2800 pixels, was resized by a factor of $\frac{1}{3}$. What is the area of the resized image, in pixels?

Key points

- A rectangular digital image measures 3600 by 2800 pixels.
- Each side length is resized by a factor of $\frac{1}{3}$.
- What is the area of the resized image, in pixels?

Explanation



As each side length was resized by $\frac{1}{3}$, we can write the following formula for the new area of the image:

$$\begin{aligned}\text{New area} &= 3600 \times \frac{1}{3} \times 2800 \times \frac{1}{3}\\ &= 3600 \times 2800 \times \frac{1}{3^2}\\ &= 10\,080\,000 \times \frac{1}{9}\\ &= 1\,120\,000\end{aligned}$$

Answer

The area of the resized image is 1 120 000 pixels.

14. A hole appears in a tank containing 4096 m^3 of water. The amount of water reduces by a factor of four every second. After how many seconds will the volume of water in the tank be less than 1 m^3?

Key points

- A tank contains 4096 m^3 of water.
- The volume of water reduces by a factor of 4 every second.
- After how many seconds will there be less than 1 m^3 of water in the tank?

Explanation

4096 reduces by a factor of four for a number of seconds until it reaches 1 m^3. Let x be the number of seconds.

$\frac{4096}{4^x} = 1$

Write 4096 as a power of four.

$4096 = 4^6$

$\frac{4^6}{4^x} = 1$

When $x = 6$, the fraction equals one.

Answer

There will be less than 1 m^3 of water left in the tank after 6 seconds.

15. A biased coin has been designed so that the probability of landing tails is $\frac{2}{3}$. What is the probability of landing tails on every throw for three throws?

Key points

- The probability of landing tails on a biased coin is $\frac{2}{3}$.
- What is the probability of landing tails on every throw for three throws?

Explanation

The probability of landing tails is $\frac{2}{3}$.

Therefore, when tossing the coin n times, the probability of landing tails on every toss is $\left(\frac{2}{3}\right)^n$.

After three tosses, $n = 3$. Substitute $n = 3$ into $\left(\frac{2}{3}\right)^n$ and evaluate:

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

Answer

The probability of landing tails on every one of three throws is $\frac{8}{27}$.

Reasoning

16. **a.** The kinetic energy when $m = 24$ is $K = 12v^2$.

b. The kinetic energy of a 24 kg object with a velocity of $\frac{7}{2}$ m/s is 147 Joules.

c. The kinetic energy of the object is now 1323 Joules after speeding up.

d. Suggested option 1: The kinetic energy of planes comes from jet fuel.

Suggested option 2: The kinetic energy of cars comes from petrol.

Note: There are other possible options.

17. **a.** $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

b. $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$

c. The even power produced a positive answer whereas the odd power produced a negative answer. For negative bases, an even power always produces a positive answer and an odd power always produces a negative answer.

Exam-style

18. A

19. **a.** $(2^4)^x = 2^{12}$

b. $2^{4x} = 2^{12}$

$4x = 12$

$x = 3$

20. $x = 3$

21. $128x^{13}y^2$

Remember this?

22. B **23.** C **24.** B

2C Negative indices

Student practice

Worked example 1

a. $\frac{1}{64}$ **b.** $\frac{64}{27}$ **c.** $\frac{1}{72}$

Worked example 2

a. $\frac{1}{x^8}$ **b.** $\frac{5}{r^2}$ **c.** $\frac{z^9}{w^5}$

Worked example 3

a. $\frac{1}{c^6}$ **b.** $\frac{4}{x^4}$ **c.** $\frac{15}{qr^2}$

Understanding worksheet

1. **a.** 6 **b.** 10 **c.** −2 **d.** −1

2. **a.** −1 **b.** −3 **c.** −7 **d.** −8

3. division; laws; reciprocal; proportional

Fluency

4. **a.** $\frac{1}{16}$ **b.** $\frac{1}{125}$

c. $\frac{1}{3}$ **d.** $\frac{1}{10\,000}$

e. $\frac{1}{144}$ **f.** $\frac{1}{223}$

g. $\frac{1}{100\,000\,000}$ **h.** $\frac{1}{1024}$

5. **a.** 4 **b.** $\frac{1}{144}$ **c.** $\frac{27}{8}$ **d.** $\frac{1}{2304}$

e. $\frac{49}{25}$ **f.** $\frac{1}{216}$ **g.** $\frac{1000}{729}$ **h.** $\frac{1}{10\,976}$

6. **a.** $\frac{1}{x^4}$ **b.** $\frac{1}{y^2}$ **c.** $\frac{1}{r}$ **d.** $\frac{2}{y^3}$

e. $\frac{5}{t^7}$ **f.** $\frac{10}{z^{12}}$ **g.** $\frac{1}{2x^3}$ **h.** $\frac{3}{5p^5}$

7. **a.** x^3 **b.** $\frac{y}{x^4}$ **c.** $\frac{q^2}{p^3}$ **d.** $\frac{r^3}{t}$

e. $\frac{3v^4}{u^7}$ **f.** $\frac{b^2}{2a^4}$ **g.** $\frac{4d^6}{5c}$ **h.** $\frac{2t^8}{7p^3}$

8. **a.** $\frac{1}{x^5}$ **b.** $\frac{4}{t^2}$ **c.** $\frac{2}{p^4}$ **d.** $\frac{18}{x^5}$

e. $\frac{16}{3t^8}$ **f.** $\frac{6}{a^8b^4}$ **g.** $\frac{14}{x^{10}}$ **h.** $\frac{30m}{n^3}$

9. **a.** $\frac{1}{x^6}$ **b.** a^6 **c.** $\frac{q^6}{p^3}$ **d.** $\frac{y^2}{4x^{10}}$

e. $\frac{p^2}{4r^3}$ **f.** $\frac{x^6}{y^6z^4}$ **g.** $\frac{t^3}{49r^{10}}$ **h.** $\frac{v^2}{u^{28}}$

10. C

Spot the mistake

11. **a.** Student A is incorrect. **b.** Student B is incorrect.

Back to contents

Problem solving

12. One carat of gold is equivalent to 5^{-1} grams. What fraction of one gram is one carat of gold?

Key points

- One carat of gold weighs 5^{-1} g.
- What fraction of one gram is one carat of gold?

Explanation

Express the mass of one carat of gold using positive indices only.

$$\text{One carat of gold} = 5^{-1}\text{ g}$$
$$= \tfrac{1}{5}\text{ g}$$

$$5^{\boxed{-1}} = \frac{1}{5^{\boxed{1}}} = \frac{1}{5}$$

The negative power of 1 shows there is a division by 5 once.

Answer

One carat of gold weighs $\frac{1}{5}$ of one gram.

13. Sienna is looking at a small insect through a microscope. The length of the insect is 1.5 mm, as seen through the eyepiece of the microscope, which has been set to magnify objects by a factor of 10^2. Calculate the actual length of the insect and give your answer in mm.

Key points

- The length of an insect through a microscope is 1.5 mm.
- The insect has been magnified 10^2 times.
- What is the actual length of the insect in mm?

Explanation

Write an equation showing the relationship between the actual and magnified length of the insect, where x represents the actual length.

$$1.5 = x \times 10^2$$

Solve the equation for x.

$$\begin{aligned} x &= \frac{1.5}{10^2} \\ &= 1.5 \div 10^2 \\ &= 1.5 \div 100 \\ &= 0.015 \end{aligned}$$

Answer

The actual length of the insect is 0.015 mm.

14. Frank is uploading a 2^9 GB file to an online shared drive. Every 15 minutes, the amount of data yet to be uploaded decreases by a factor of 2. Evaluate how many GB are yet to be uploaded after three hours, leaving your answer as a fraction.

Key points

- Frank is uploading a 2^9 GB file.
- Every 15 minutes, the amount of data to upload reduces by a factor of 2.
- How many GB are yet to be uploaded after 3 hours?

Explanation

Convert the number of hours the file has been uploading to minutes and calculate how many 15 minute intervals make up 3 hours.

$$3\text{ hours} = 3 \times 60 = 180\text{ minutes}$$

$$\frac{180}{15} = 12$$

The size of the file has halved (reduced by a factor of 2) 12 times.

We can use indices to evaluate the size of the 2^9GB file after it was divided by two 12 times.

$$\frac{2^9}{2^{12}} = 2^{9-12} = 2^{-3}$$

Evaluate the size of the remaining file.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}\text{ GB}$$

Answer

$\frac{1}{8}$ GB of the file is yet to be uploaded.

15. Dougie is practising for a mathematics test by completing past exam questions. He can currently complete 10 multiple choice questions in half an hour. If Dougie wants to be able to finish 20 multiple choice questions in half an hour, then by what factor should he reduce the amount of time he spends on each question?

Key points

- Dougie can complete 10 questions in 30 minutes.
- He wants to improve so that he can complete 20 questions in 30 minutes.
- By what factor should Dougie reduce the time he spends on a question?

Explanation

The time spent on one question is given by the formula below.

$$\text{Time per question} = \frac{\text{total time}}{\text{number of questions}}$$

The relationship between the time spent per question and the number of questions completed is inversely proportional. This means that if Dougie wants to double the questions he can complete in a set period of time, he should halve the amount of time spent on each individual question.

Answer

Dougie should reduce the amount of time he spends on each question by a factor of 2.

16. The mass of a newborn joey kangaroo is approximately 2^{-9} kg. What is the mass of a newborn joey, to the nearest gram?

Key points

- The mass of a newborn joey $\approx 2^{-9}$ kg.
- What is the mass of a newborn joey to the nearest gram?

Explanation

Express the mass of a joey using positive indices only.

$$2^{-9} = \frac{1}{2^9}\text{ kg}$$

Multiply this value by 1000 to convert to grams from kilograms, and evaluate.

$$\begin{aligned} \frac{1}{2^9}\text{ kg} &= \frac{1}{2^9} \times 1000\text{ g} \\ &= \frac{1000}{2^9} \\ &= \frac{1000}{512} = 1000 \div 512 = 1.953... \\ &\approx 2\text{ g} \end{aligned}$$

Answer

The mass of a newborn joey is approximately 2 grams.

Reasoning

17. **a.** $l = \frac{1}{2^n}$

b. A whole note is played when $n = 0$.

c. $n = 3$ when $l = \frac{1}{8}$.

d. Eight sixteenths notes equal the duration of one half note.

e. Suggested option 1: It is physically impossible for a human to play a note that lasts for such a short period of time.

Suggested option 2: It is physically impossible for a human to hear a note that lasts for such a short period of time.

Note: There are other possible options.

18. **a.** $x^{-6} \times x^4 = x^{-6+4} = x^{-2} = \frac{1}{x^2}$

b. $\frac{x^4}{x^6} = x^{4-6} = x^{-2} = \frac{1}{x^2}$

c. Parts **a** and **b** produce the same answer. The commutative law states that numbers can be added in any order. This implies that it is possible to express any negative index positively by applying the First or Second index laws.

Exam-style

19. D **20.** **a.** $\frac{1}{243}$ **21.** 9.47 cm^2 **22.** 8 cm^2

b. 3^{-6}

Remember this?

23. A **24.** E **25.** D

2D Scientific notation

Student practice

Worked example 1

a. 6.273×10^2 **b.** 6.2×10^{-4}

Worked example 2

a. 7.83×10^{10} **b.** 2.02×10^{-3}

Worked example 3

a. 832 000 **b.** 0.000475

Understanding worksheet

1. **a.** 3 **b.** 2 **c.** 4 **d.** 6

2. **a.** −1 **b.** −3 **c.** −4 **d.** −6

3. extremely; decimal; negative; zero

Fluency

4. **a.** 5×10^2 **b.** 4.6×10^4
c. 6.912×10^2 **d.** 1.04×10^4
e. 2.7038×10^3 **f.** 9.6×10^7
g. 1.0206×10^7 **h.** 6.5005×10^{14}

5. **a.** 6×10^{-2} **b.** 6.8×10^{-3}
c. 1.04×10^{-1} **d.** 7.45×10^{-4}
e. 3.901×10^{-2} **f.** 4.903×10^{-4}
g. 1×10^{-8} **h.** 2.08×10^{-12}

6. **a.** 5.42×10^1 **b.** 2.51×10^2
c. 6.75×10^{-2} **d.** 5.46×10^{-3}
e. 9.52×10^7 **f.** 3.59×10^{-5}
g. 8.50×10^{11} **h.** 1.00×10^{-1}

7. **a.** 20 000 **b.** 420
c. 3430 **d.** 105
e. 940 000 **f.** 108 000
g. 8 060 000 **h.** 930 400 000 000

8. **a.** 0.12 **b.** 0.034
c. 0.00043 **d.** 0.00501
e. 0.01089 **f.** 0.000236
g. 0.000054023 **h.** 0.0000000006006

9. B

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. There are 5.078×10^6 people in Melbourne. How many people live in Melbourne, in decimal notation?

Key points

- There are 5.078×10^6 people in Melbourne.
- How many people live in Melbourne, in decimal notation?

Explanation

Numbers written in scientific notation are expressed in the form $a \times 10^n$.

Identify the values of a and n.

$a = 5.078$

$n = 6$

For a, move the digit in the ones place value position n places to the left and thus write the number in decimal notation.

$\underline{5}.078 \rightarrow \underline{5}\ 078\ 000$

$\therefore 5.078 \times 10^6 = 5\ 078\ 000$

Answer

There are 5 078 000 people in Melbourne.

12. Determine the number of seconds in one day and write it using scientific notation.

Key points

- Determine the number of seconds in one day and write it using scientific notation.

Explanation

$$\begin{aligned}\text{Seconds in one day} &= \text{hours in one day} \times \text{minutes in one hour} \\ &\quad \times \text{seconds in one minute} \\ &= 24 \times 60 \times 60 \\ &= 86\ 400\end{aligned}$$

Write 86 400 in scientific notation.

Numbers written in scientific notation are expressed in the form $a \times 10^n$.

Determine the value of a by moving the first non-zero digit (or significant figure) to the ones place value position.

$\underline{8}6\ 400 \rightarrow a = \underline{8}.64$

Determine the value of n by counting the number of places the digit was moved to the right (positive direction).

$n = 4$

Write the number in scientific notation by substituting the values of a and n into the expression $a \times 10^n$.

$86\ 400 = 8.64 \times 10^4$

Back to contents

$$86\,400 = 8.64 \times 10 \times 10 \times 10 \times 10$$

$$= 8.64 \times 10^4$$

Each time the leading non-zero digit is moved to the right, the number must be multiplied by 10.

Answer

There are 8.64×10^4 seconds in one day.

13. The mass of an animal cell is around 3×10^{-9} g. Write the mass of an animal cell in kg using decimal notation.

Key points

- The mass of an animal cell is around 3×10^{-9} g.
- Write the mass of an animal cell in kg using decimal notation.

Explanation

Numbers written in scientific notation are expressed in the form $a \times 10^n$.

Identify the values of a and n.

$a = 3$

$n = -9$

In a, move the digit in the ones place value position n places to the right and thus write the number in decimal notation.

$\underline{3} \rightarrow 0.00000000\underline{3}$

$\therefore 3 \times 10^{-9} = 0.000000003$

Convert 0.000000003 g to kg.

$1 \text{ kg} = 1000 \text{ g}$

$0.000000003 \text{ g} = 0.000000003 \div 1000$

$= 0.000000000003 \text{ kg}$

Answer

The mass of an animal cell is 0.00000000003 kg.

14. A grain of sand is 0.8 mm in diameter. Write the diameter of a grain of sand in metres, using scientific notation.

Key points

- A grain of sand is 0.8 mm in diameter.
- Write the diameter of a grain of sand in metres, using scientific notation.

Explanation

Convert 0.8 mm to metres.

$1 \text{ m} = 1000 \text{ mm}$

$0.8 \text{ mm} = 0.8 \div 1000 = 0.0008 \text{ m}$

Convert 0.0008 m to scientific notation.

Numbers written in scientific notation are expressed in the form $a \times 10^n$.

Determine the value of a by moving the first non-zero digit (or significant figure) to the ones place value position.

$0.000\underline{8} \rightarrow a = \underline{8}$

Determine the value of n by counting the number of places the digit was moved to the left (negative direction).

$n = -4$

Write the number in scientific notation by substituting the values of a and n into the expression $a \times 10^n$.

$0.0008 = 8 \times 10^{-4}$

Answer

The diameter of a grain of sand is 8×10^{-4} m.

15. Usain Bolt and Yohan Blake are two of the fastest humans in the world. Bolt can run 100 m in 9.58 seconds, whereas Blake can run it in 9.69 seconds. Using scientific notation, calculate the difference between the two runners' 100 m in seconds.

Key points

- Bolt can run 100 m in 9.58 seconds.
- Blake can run 100 m in 9.69 seconds.
- Using scientific notation, calculate the difference between the two runners' 100 m in seconds.

Explanation

Calculate the difference between the two runners' times:

Difference between times = Blake's time − Bolt's time

$= 9.69 - 9.58$

$= 0.11$ seconds

Write 0.11 in scientific notation.

Numbers written in scientific notation are expressed in the form $a \times 10^n$.

Determine the value of a by moving the first non-zero digit (or significant figure) to the ones place value position.

$0.\underline{1}1 \rightarrow a = \underline{1}.1$

Determine the value of n by counting the number of places the digit was moved to the left (negative direction).

$n = -1$

Write the number in scientific notation by substituting the values of a and n into the expression $a \times 10^n$.

$0.11 = 1.1 \times 10^{-1}$

Answer

The difference between Usain Bolt's and Yohan Blake's 100 m times is 1.1×10^{-1} seconds.

Reasoning

16. **a.** The thickness of a sheet of paper is 1×10^{-6} m.

b. The thickness of the average human hair is 6×10^{-5} m.

c. The length of the Great Wall of China is 2.120×10^7 m.

d. 2 terabytes is equal to 2×10^9 kilobytes.

e. Suggested option 1: Centimetre.

Suggested option 2: Kilowatt.

Note: There are other possible options.

17. **a.** 1.5×10^7

b. 4×10^{-9}

c. $1.5 \times 10^7 \times 4 \times 10^{-9} = 6 \times 10^{7-9} = 6 \times 10^{-2} = 0.06$

As all numbers written in scientific notation use powers of 10, their powers can be combined using the index laws for multiplication and division when the numbers are multiplied or divided.

Exam-style

18. C

19. **a.** $4.85 \times 10^{10} \times 4 \times 10^{-11}$

b. $4.85 \times 10^{10} \times 4 \times 10^{-11} = 19.4 \times 10^{-1} = 1.94$

20. 483.6 cm^2

21. 4.488×10^9 km

Remember this?

22. E **23.** D **24.** A

Back to contents

2E Fractional indices

Student practice

Worked example 1

a. $\sqrt[3]{8}$ **b.** $\sqrt[5]{12^2}$

Worked example 2

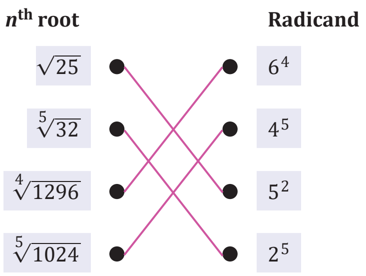
a. 3 **b.** 6

Worked example 3

a. $y^{\frac{7}{10}}$ **b.** $a^{\frac{1}{12}}$ **c.** $x^{\frac{3}{8}}$

Understanding worksheet

1.



2. **a.** 7 **b.** 5 **c.** 9 **d.** 3

3. radical; cube; power; order; laws

Fluency

4. **a.** $\sqrt{5}$ **b.** $\sqrt[4]{10}$ **c.** $\sqrt[9]{26}$ **d.** $\sqrt[3]{4^2}$
e. $\sqrt[5]{7^2}$ **f.** $\sqrt[8]{11^3}$ **g.** $\sqrt[4]{27^5}$ **h.** $\sqrt[5]{13^6}$

5. **a.** 2 **b.** 5 **c.** 3 **d.** 2
e. 4 **f.** 10 **g.** 3 **h.** 32

6. **a.** $x^{\frac{2}{3}}$ **b.** $y^{\frac{3}{5}}$ **c.** $x^{\frac{3}{4}}$ **d.** $t^{\frac{5}{6}}$
e. $p^{\frac{9}{14}}$ **f.** $a^{\frac{24}{35}}$ **g.** $b^{\frac{39}{20}}$ **h.** $x^{\frac{37}{30}}$

7. **a.** $x^{\frac{4}{7}}$ **b.** $b^{\frac{1}{5}}$ **c.** $a^{\frac{1}{6}}$ **d.** $u^{\frac{11}{9}}$
e. $y^{\frac{1}{2}}$ **f.** $p^{\frac{29}{6}}$ **g.** $r^{\frac{11}{12}}$ **h.** $z^{\frac{11}{12}}$

8. **a.** $x^{\frac{1}{6}}$ **b.** $p^{\frac{2}{15}}$ **c.** $y^{\frac{3}{20}}$ **d.** r
e. t^2 **f.** $k^{\frac{6}{13}}$ **g.** $5x^{\frac{1}{16}}$ **h.** $2b^{\frac{3}{22}}$

9. **a.** $10^{\frac{1}{3}}$ **b.** $5^{\frac{1}{2}}$ **c.** $12^{\frac{1}{9}}$ **d.** $2^{\frac{3}{4}}$
e. $9^{\frac{11}{5}}$ **f.** 6^2 **g.** 13^3 **h.** $10^{\frac{1}{2}}$

10. C

Spot the mistake

11. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

12. A square painting has a total area of 6400 cm^2. Express one side length of the painting in centimetres, using a fractional index and evaluate it.

Key points

- The area of a square is 6400 cm^2.
- Express one side length of the painting in centimetres, using a fractional index and evaluate it.

Explanation

$$\text{Area} = \text{side length} \times \text{side length}$$
$$6400 = \text{side length}^2$$
$$\sqrt{6400} = \sqrt{\text{side length}^2}$$

Side length $= \sqrt{6400} = 6400^{\frac{1}{2}} = 80$ cm.

Answer

The side length of the painting is given by $6400^{\frac{1}{2}} = 80$ cm.

13. Mr Steinberg thinks of a number x. He tells his class that when he multiplies the number by itself five times, the result is 32. Express Mr Steinberg's number using a root sign and evaluate it.

Key points

- x multiplied by itself five times gives 32.
- Express Mr Steinberg's number using a root sign and evaluate it.

Explanation

$$x \times x \times x \times x \times x = 32$$
$$x^5 = 32$$
$$\sqrt[5]{x^5} = \sqrt[5]{32}$$
$$x = \sqrt[5]{32} = 2$$

Answer

Mr Steinberg's number is $x = \sqrt[5]{32} = 2$.

14. Sully is building a fish tank that will be placed inside an aquarium. The client has asked for it to be a perfect cube with a total volume of 216 m^3 to house the animals comfortably. Express the height of the fish tank using a fractional index and evaluate it.

Key points

- The tank is a perfect cube with a volume of 216 m^3.
- Express the height of the fish tank using a fractional index and evaluate it.

Explanation

Volume $= l \times w \times h$

Let the height of the fish tank be x m. As it is a perfect cube, the length and width are both also x m.

Volume $= x \times x \times x$

$$216 = x^3$$
$$\sqrt[3]{216} = \sqrt[3]{x^3}$$
$$216^{\frac{1}{3}} = x = 6 \text{ m}$$

Answer

The height of the fish tank is given by $216^{\frac{1}{3}} = 6$ m.

Back to contents

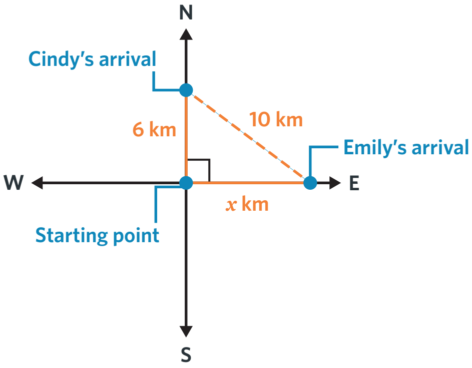
15. Cindy and Emily both walk away from their campsite. Cindy walks north and stops after travelling 6 km and Emily walks east and also stops after travelling some distance. The direct distance between them is now 10 km. How far east did Emily walk from the campsite?

Key points

- Cindy walks north and stops after travelling 6 km.
- Emily walks east and stops after travelling after some distance.
- The direct distance between them is now 10 km.
- How far east did Emily walk from the campsite?

Explanation

Let the distance Emily travelled be x km.



According to Pythagoras' theorem, $6^2 + x^2 = 10^2$.

$6^2 + x^2 = 10^2$

$36 + x^2 = 100$

$x^2 = 100 - 36$

$x^2 = 64$

$\sqrt{x^2} = \sqrt{64}$

$x = 8$

Answer

Emily walked 8 km east from the campsite.

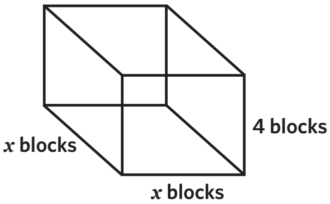
16. Duyen is playing a building video game where her mission is to construct a room with a minimum total volume of 100 blocks. If Duyen wants to ensure that the height of the room is four blocks, and that the floor is a square, how many blocks should she make each side length?

Key points

- The minimum total volume is 100 blocks.
- The height of the room is four blocks.
- The floor is a square.
- How many blocks should she make each side length?

Explanation

Let the side length be x blocks.



Volume $= l \times w \times h$

$100 = x \times x \times 4$

$100 = 4x^2$

$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = 5$

Answer

Duyen should make each side of the square room 5 blocks long.

Reasoning

17. a. The initial volume of clean water is 12 cm^3.

b. The volume of clean water after one week of treatment is 36 cm^3.

c. It will take 12 weeks for the volume of clean water to reach 8.748 L.

d. Suggested option 1: Always pick up after yourself and dispose of waste sensibly.

Suggested option 2: Make sure to read any signs that are encountered in parks and nature reserves.

Note: There are other possible options.

18. a. $4^{\frac{6}{2}} = 4^3$

b. $5^{\frac{10}{2}} = 5^5$

c. Taking the square root, i.e. when the order of the root is 2, halves the power of a number.

Exam-style

19. E

20. a. $2^{\frac{3}{2}} \times 2^{\frac{7}{2}}$

b. $2^{\frac{3}{2}} \times 2^{\frac{7}{2}} = 2^{\frac{3}{2}+\frac{7}{2}} = 2^{\frac{10}{2}} = 2^5 = 32$

21. $x = 4$ m

22. $3a$

Remember this?

23. B **24.** E **25.** E

2F Simple operations with surds

Student practice

Worked example 1

a. Not a surd **b.** Surd **c.** Not a surd

Worked example 2

a. $4\sqrt{5} + 2\sqrt{3}$ **b.** $2\sqrt{2} - 2\sqrt{6}$

Worked example 3

a. $8\sqrt{21}$ **b.** $3\sqrt{7}$

Understanding worksheet

1. a. $\sqrt{2}, 3\sqrt{2}$ **b.** $-\sqrt{7}, 5\sqrt{7}, 7\sqrt{7}$

c. $\sqrt{10}, 6\sqrt{10}, 7\sqrt{10}$ **d.** $-2\sqrt{3}, 8\sqrt{3}$

2. a. 5 **b.** 4 **c.** 3 **d.** 7

3. irrational; infinite; like; radicands

Back to contents

Fluency

4. **a.** Not a surd **b.** Not a surd
c. Surd **d.** Surd
e. Surd **f.** Surd
g. Not a surd **h.** Not a surd

5. **a.** $6\sqrt{3}$ **b.** $3\sqrt{6}$
c. $5\sqrt{7} + \sqrt{3}$ **d.** $8\sqrt{3} + 4\sqrt{6}$
e. $10\sqrt{7} + 6\sqrt{11}$ **f.** $6\sqrt{3} + \sqrt{10}$
g. $6\sqrt{12} + \sqrt{6}$ **h.** $5\sqrt{13} + 3\sqrt{14}$

6. **a.** $6\sqrt{2}$ **b.** $3\sqrt{5}$
c. $\sqrt{7} - 7\sqrt{3}$ **d.** $6\sqrt{6} - \sqrt{5}$
e. $11\sqrt{8} - 4\sqrt{5}$ **f.** $5\sqrt{10} - 11\sqrt{3}$
g. $5\sqrt{23} + 6\sqrt{15}$ **h.** $-\sqrt{5} + 7\sqrt{6}$

7. **a.** $\sqrt{8}$ **b.** $6\sqrt{6}$ **c.** $7\sqrt{14}$ **d.** $12\sqrt{15}$
e. $20\sqrt{12}$ **f.** $6\sqrt{70}$ **g.** $6\sqrt{150}$ **h.** $24\sqrt{90}$

8. **a.** $\sqrt{7}$ **b.** $3\sqrt{5}$ **c.** $4\sqrt{6}$ **d.** $\frac{2\sqrt{6}}{5}$
e. $\frac{\sqrt{2}}{2}$ **f.** $\frac{3\sqrt{7}}{4}$ **g.** $3\sqrt{8}$ **h.** $\frac{4\sqrt{8}}{5}$

9. D

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. The ratio of lengths of the shorter to the longer side of an A4 piece of paper is $1 : \sqrt{2}$. If the shorter side is 21 cm long, what is the length of the longer side, to the nearest mm?

Key points

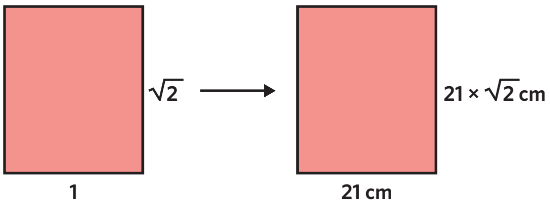
- $1 : \sqrt{2}$ is the ratio of lengths of the shorter to the longer side of an A4 piece of paper.
- The shorter side is 21 cm long.
- What is the length of the longer side, to the nearest mm?

Explanation

Length of shorter side = 21 cm

Multiply this length by $\sqrt{2}$ to determine the length of the longer side in cm.

Shorter side : longer side = 1 : $\sqrt{2}$



$$21 \times \sqrt{2} = 29.6984...$$
$$\approx 29.7 \text{ cm}$$

Convert this length into mm.

$$29.7 \text{ cm} \times 10 = 297 \text{ mm}$$

Answer

The length of the longer side of an A4 piece of paper is 297 mm.

12. The volume of a fish tank with a square base is 6000 cm^3. If each side of the base is $\sqrt{120}$ cm long, determine the height of the tank.

Key points

- A fish tank with a square base has a volume of 6000 cm^3.
- Each side of the square base is $\sqrt{120}$ cm long.
- Determine the height of the tank.

Explanation

Calculate the area of the square base.

$$\begin{aligned}\text{Area of square base} &= \text{length of base} \times \text{length of base}\\ &= \sqrt{120} \times \sqrt{120}\\ &= \sqrt{120 \times 120}\\ &= \sqrt{14\,400} \text{ cm}^2\end{aligned}$$

Volume of fish tank = area of square base × height of fish tank

Substitute known values into this equation.

$$6000 = \sqrt{14\,400} \times \text{height of fish tank}$$

Solve for the height of the fish tank.

$$\begin{aligned}\text{Height of fish tank} &= \frac{6000}{\sqrt{14\,400}}\\ &= 50 \text{ cm}\end{aligned}$$

Answer

The height of the fish tank is 50 cm.

13. Sally is choosing an outfit for her computer game character. There are five categories: a hat, shirt, pants, boots, and accessories. Each category contains an equal number of items. If there are a total of 7776 combinations of outfits available, how many items are there in each category?

Key points

- There are 5 categories.
- Each category contains an equal number of items.
- There are 7776 combinations in total.
- How many items are in each category?

Explanation

Let the number of items in each category be n.

There are 5 categories each with n items. The total number of combinations is 7776. This is represented by the equation $n^5 = 7776$.

Solve for n.

$$\begin{aligned}n &= \sqrt[5]{7776}\\ &= 6\end{aligned}$$

Answer

There are 6 items in each category.

14. A bathroom wall has been tiled with 216 square tiles. There are 1.5 times as many tiles along the height of the wall as there are along the length of the wall. What is the height of the wall in metres if each side of one tile is 15 cm long?

Key points

- A bathroom wall has been tiled with 216 square tiles.
- There are 1.5 times as many tiles along the height of the wall as there are along the length of the wall.
- The side length of each tile is 15 cm.
- What is the height of the wall in metres?

Explanation

Let the number of tiles along the length of the wall be x.

There are 1.5 times as many tiles along the height of the wall as there are along the length of the wall.

$\therefore$ Number of tiles along height of wall $= 1.5x$

The number of tiles on the wall can be represented by the following equation.

$216 = x \times 1.5x$

$216 = 1.5x^2$

Solve for x.

$x^2 = \frac{216}{1.5}$

$x^2 = 144$

$x = \sqrt{144}$

$x = 12$

Determine the number of tiles along the height of the wall.

$$\begin{aligned}\text{Number of tiles along height of wall} &= 1.5x\\ &= 1.5 \times 12\\ &= 18 \text{ tiles}\end{aligned}$$

Determine the height of the wall that has 18 tiles of side length 15 cm, in metres.

$$\begin{aligned}18 \times 15 &= 270 \text{ cm}\\ &= 2.7 \text{ m}\end{aligned}$$

Answer

The height of the wall is 2.7 m.

15. An electrician wants to run an underground cable between opposite corners of a square room. The floor area of the room is 50 m^2. Calculate the length of the cable.

Key points

- An underground cable is run between opposite corners of a square room.
- The floor area of the room is 50 m^2.
- Calculate the length of the cable.

Explanation

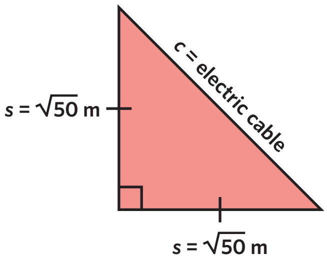
Let the side length of the square room be s.

$$\begin{aligned}\text{Area of room} &= s^2\\ 50 &= s^2\end{aligned}$$

Solve for s.

$s = \sqrt{50}$ m

Pythagoras' theorem states that $a^2 + b^2 = c^2$, where c is the hypotenuse of a right-angled triangle, and a and b are the two shorter lengths. Use this to determine the length of the electric cable.



$$\begin{aligned}c &= \sqrt{s^2 + s^2}\\ &= \sqrt{(\sqrt{50})^2 + (\sqrt{50})^2}\\ &= \sqrt{50 + 50}\\ &= \sqrt{100}\\ &= 10 \text{ m}\end{aligned}$$

Answer

The length of the electric cable is 10 m.

Reasoning

16. a. The initial height of the plant is 0.2 cm.

b. The height of the plant is 2.2 cm on the fourth day.

c. It will take approximately 23 days for the plant to fully grow.

d. Suggested option 1: Learning about plants and animals helps with understanding human behaviour.

Suggested option 2: Learning about plants and animals allows us to understand how to take care of the environment.

Note: There are other possible options.

17. a. $3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^{\frac{1}{2}+\frac{1}{2}} = 3^1 = 3$

b. $x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x$

c. Multiplying a quadratic surd by itself always results in the radicand because when the powers are added, they simplify to 1.

Exam-style

18. B

19. a. $\frac{10\sqrt{8}}{2\sqrt{2}}$

b. 10

20. $x = 3$

21. $x = 12$

Remember this?

22. B

23. D

24. E

Chapter 2 extended application

1. a. $5\sqrt{2}$ m

b. $150\sqrt{2}$ m^3

c. $50\sqrt{2}$ m^3

d. 1 : 3

e. Suggested option 1: Colour scheme.

Suggested option 2: Natural light.

Note: There are other possible options.

2. a. 2.5×10^6 W

b. 6.74 GW

c. 4.85×10^{12} W = 4.85 TW

d. 1 kilowatt (kW) is equal to 1000 megawatts (MW).

e. 15.6 kW

f. Suggested option 1: Engineers planning and working with the energy grid of a city.

Suggested option 2: Companies monitoring electricity usage during production processes.

Note: There are other possible options.

Back to contents

3. **a.** 1149

b.

Time (hours)	Bacteria population
1	1035
2	1072
3	1110
4	1149
5	1189
6	1231
7	1275
8	1320
9	1366
10	1414

c. $P(t) = \frac{P_0}{2^{0.3t}}$

d. 713

e. 1945

f. Suggested option 1: Birth rates.

Suggested option 2: Immigration trends.

Note: There are other possible options.

Chapter 2 review

Multiple choice

1. C **2.** C **3.** A **4.** C **5.** E

Fluency

6. **a.** d^{21} **b.** $7p^3q^3$ **c.** $3x^5$ **d.** $5ft^{12}$

7. **a.** $243u^{20}w^{30}$ **b.** $\frac{64r^{54}}{s^{12}}$

c. $64a^{31}b^{59}$ **d.** $2c^{18}$

8. **a.** $\frac{1}{81}$ **b.** $\frac{27}{125}$ **c.** $\frac{1}{144}$ **d.** 2

9. **a.** $\frac{6}{d^5}$ **b.** $\frac{5k^4}{h^2}$ **c.** $\frac{44}{ef^{16}}$ **d.** $\frac{y^8}{27x^{12}}$

10. **a.** 8.25×10^3 **b.** 6.38×10^{-2}

c. 4.11×10^2 **d.** 1.00×10^{-1}

11. **a.** $x^{\frac{4}{9}}$ **b.** $a^{\frac{1}{2}}$ **c.** $2p^{\frac{1}{16}}$ **d.** $2t^{\frac{1}{33}}$

12. **a.** $21^{\frac{1}{2}}$ **b.** $7^{\frac{1}{6}}$ **c.** $2^{\frac{12}{11}}$ **d.** $10^{\frac{1}{2}}$

13. **a.** $\sqrt{3}$ **b.** $3 - 4\sqrt{3}$

c. $\sqrt{10} - 12\sqrt{7}$ **d.** $10\sqrt{12} - 3\sqrt{14}$

14. **a.** $\sqrt{15}$ **b.** $2\sqrt{84}$ **c.** $5\sqrt{13}$ **d.** $3\sqrt{11}$

Problem solving

15. Chidi is doing an experiment in his chemistry class. There are originally 30 bacteria in a petri dish, and every five seconds the number of bacteria triple. What is the final number of bacteria in the petri dish after 15 seconds?

Key points

- Chidi is doing an experiment in his chemistry class.
- There are originally 30 bacteria in a petri dish.
- Every five seconds the number of bacteria triples.
- Calculate the final number of bacteria in the petri dish after 15 seconds.

Explanation

Determine how many 5 second intervals are in 15 seconds.

$$\begin{aligned}\text{Number of intervals} &= \text{total time} \div \text{time per interval}\\ &= 15 \div 5\\ &= 3 \text{ intervals}\end{aligned}$$

The bacteria's population can be represented by the equation 30×3^t, where t represents the number of times 5 seconds has passed.

$$\begin{aligned}\text{After 15 seconds, } t &= \frac{15}{5}\\ &= 3\end{aligned}$$

$30 \times 3^3 = 810$

Answer

The final number of bacteria in the petri dish after 15 seconds is 810.

16. Chi rolls a biased die with seven faces, from one to seven. The probability of landing a two is $\frac{2}{5}$. What's the probability of not landing a two for three consecutive rolls?

Key points

- A biased dice labelled from one to seven is rolled by Chi.
- The probability of rolling a two is $\frac{2}{5}$.
- Calculate the probability of not rolling a two for three consecutive rolls.

Explanation

Calculate the probability of not rolling a two for one roll.

$1 - \frac{2}{5} = \frac{3}{5}$

Calculate the probability of not rolling a two, in three consecutive rolls.

$$\begin{aligned}\left(\frac{3}{5}\right)^3 &= \frac{3^3}{5^3}\\ &= \frac{27}{125}\end{aligned}$$

Answer

The probability of not landing a two in three consecutive rolls is $\frac{27}{125}$.

17. Rag is studying biology. He discovered that an average ant weighs 2^{-3} g. There are approximately 10^{14} ants in the world. How much do all ants weigh when combined? Express your answer in the simplest index form.

Key points

- Rag discovered that an average ant weighs 2^{-3} g.
- In the world, there are approximately 10^{14} ants in the world.
- Calculate how much all ants weigh when combined? Express your answer in the simplest index form.

Explanation

Calculate the total weight of ants by multiplying the number of ants by the weight of an average ant.

Total weight = individual weight × number of ants.

$$= 2^{-3} \times 10^{14}$$

$$= 2^{-3} \times (2 \times 5)^{14}$$

Express your answer in the simplest index form, using the index laws.

$$= 2^{-3} \times 2^{14} \times 5^{14}$$

$$= 2^{-3+14} \times 5^{14}$$

$$= 2^{11} \times 5^{14}$$

Answer

The weight of all ants combined in simplest index form is $2^{11} \times 5^{14}$ g.

18. Mr Babu is going on a holiday to Iceland. Iceland is approximately 15 186 km from Australia. How far is this distance in metres, in scientific notation, correct to 3 significant figures?

Key points

- Mr Babu is going on a holiday to Iceland, approximately 15 186 km from Australia.
- Represent the distance using scientific notation correct to 3 significant figures.

Explanation

Convert to metres.

15 186 × 1000 = 15 186 000 m

Move the first significant figure to the ones place value position.

15 186 000 → 1.5186000

Count the number of place values that the digit has moved. This will be the power of 10.

Number of place values moved = 7

Represent the distance using scientific notation.

$15\ 186\ 000 = 1.5186 \times 10^7$

Round to 3 significant figures.

$1.5186 \approx 1.52$

$1.5186 \times 10^7 \approx 1.52 \times 10^7$

Answer

Iceland is approximately 1.52×10^7 m from Australia.

19. Bela has a large cube with identical numbers of small cubes for length, width, and height, where each cube has a volume of 1 cm^3. Given that the total volume of the large cube is 343 cm^3, express one side length of the cube using a fractional index and evaluate.

Key points

- Each small cube in the large cube has a volume of 1 cm^3.
- The large cube's total volume is 343 cm^3.
- Express one side length of the large cube using a fractional index and evaluate.

Explanation

Volume of a cube is the side length cubed. Let the side length equal *s*.

$s^3 = 343$

Solve for *s*.

$(s^3)^{\frac{1}{3}} = 343^{\frac{1}{3}}$

$s = 343^{\frac{1}{3}}$

$s = 7$ cm

Answer

The side length of the large cube is 7 cm.

20. Catherine is looking to buy a frame for a photo. The ratio of the shorter to the longer side of this photo is $1 : \sqrt{3}$. If the shorter side of the photo is 9 cm long, what is the diagonal length of the photo?

Key points

- Short to long side ratio of a photo is $1 : \sqrt{3}$.
- The shorter side is 9 cm long.
- Determine the diagonal length of the photo.

Explanation

Pythagoras' theorem can be applied to write an equation for the diagonal length of the photo.

$(\text{diagonal length})^2 = (\text{long side})^2 + (\text{short side})^2$

Using the ratio $1 : \sqrt{3}$, the short side and long side are 9 cm and $\sqrt{3} \times 9 = 9\sqrt{3}$ cm, respectively.

Let the diagonal length be *d*. Substitute the values into the equation for the diagonal length of the photo.

$$d^2 = (9\sqrt{3})^2 + 9^2$$

$$= 81 \times 3 + 81$$

$$= 243 + 81$$

$$= 324$$

$$\therefore d = \sqrt{324} = 18$$

Answer

The diagonal length of the photo is 18 cm.

Reasoning

21. a. The light intensity of the star as observed from Earth as a fraction is $\frac{1}{2000}$ lumens.

b. The new planet is 206 200 000 km away from the star it orbits.

c. The simplified expression of the radius is $2\sqrt{10}$ km.

d. The space probe's maximum speed is 57 600 km/h.

e. Suggested option 1: The International Space Station could work to develop new satellite technology.

Suggested option 2: The International Space Station could work to test new space equipment.

Note: There are other possible options.

22. a. $\frac{(3\sqrt{3})^2}{3}$ is equal to 9.

b. $\left(\frac{3\sqrt{3}}{\sqrt{3}}\right)^2$ is equal to 9.

c. In part **a**, the numerator includes a surd inside of brackets, raised to the power of 2. In part **b**, the numerator is identical to part **a** and the entire fraction has been raised to the power of 2. However, squaring the denominator in part **b** produces the same denominator as part **a**. Therefore, the expressions are equivalent and have the same value.

Back to contents

3A Expanding algebraic expressions

Student practice

Worked example 1

a. $15 - 5w$ **b.** $-15v + 6v^2$

Worked example 2

a. $10h + 15$ **b.** $11x - 26$

Understanding worksheet

1. **a.** $3x$ **b.** $-9c$ **c.** h^2 **d.** $-2w^2$

2. **a.** 1 **b.** x **c.** -2 **d.** 1.5

3. expressions; multiplying; distributive; like

Fluency

4. **a.** $2x + 6$ **b.** $-10y + 5$

c. $14a + 7$ **d.** $6.25b - 12.5c$

e. $-\frac{3}{2}x + 2y$ **f.** $\frac{1}{2}a - \frac{3}{4}b + \frac{5}{4}c$

g. $-8x - 8y + 8z$ **h.** $6x + 12y - 15z$

5. **a.** $2x^2 - 6x$ **b.** $-8y^2 - 20y$

c. $6x^2 + 12x$ **d.** $-6z^2 + 8z$

e. $12x^2 - 6x$ **f.** $28w^3 - 14w$

g. $-6a^3 - 10a^2$ **h.** $14a^3 - 48a^2$

6. **a.** $6x - 11$ **b.** $15y + 3$

c. $-6k - 15$ **d.** $11a + 6$

e. $-3v + 56$ **f.** $4 - h$

g. $3 + g^2$ **h.** $-6b^2 - 13b$

7. **a.** $10a + 32$ **b.** $10x - 12$

c. $20r - 20$ **d.** $h + 19$

e. $2b + 34$ **f.** $y^2 + y + 20$

g. $-v$ **h.** $s^2 + 2st + 3t^2$

8. C

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. Xavier has purchased a new iPad. The iPad has a length of $2x - 7$ cm and a width of x cm. Write an expanded and simplified expression for the area of the iPad.

Key points

- The iPad has a length of $2x - 7$ cm and a width of x cm.
- Write an expanded and simplified expression for the area.

Explanation

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= (2x - 7) \times x \\ &= 2x \times x - 7 \times x \\ &= 2x^2 - 7x \text{ cm}^2 \end{aligned}$$

Answer

The area is $2x^2 - 7x$ cm^2.

11. Johnny is considering taking a certain number of additional hours of driving lessons from Drive4Life. The total cost of the lessons can be represented by the algebraic expression $C = (2.5 + h)(30)$. Expand and simplify the expression to find the total cost (C) for each additional hour in a lesson.

Key points

- The expression for the total cost of the lessons is $C = (2.5 + h)(30)$.
- Expand and simplify the expression $C = (2.5 + h)(30)$.

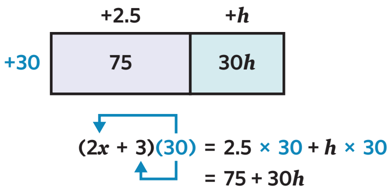
Explanation

$C = (2.5 + h)(30)$

$C = 2.5 \times 30 + h \times 30$

$C = 75 + 30h$

(2.5 + *h*)(30)



Answer

$C = 75 + 30h$

12. Village Cinemas charges \$$a$ for each adult ticket and \$$b$ for each child ticket. A family package consists of two adult tickets and two children tickets. The cost of the family package is \$7 less than if the tickets are purchased individually. Three families purchase the family package and three additional children tickets. Construct an expression that represents the total cost of all the tickets.

Key points

- \$$a$ for each adult ticket and \$$b$ for each child ticket.
- A family package is equal to two adult tickets and two children tickets.
- The cost of the family package is \$7 less than if the tickets are purchased individually.
- Three families purchase the family package plus three additional children tickets.
- Construct an expression that represents the total cost of all the tickets.

Explanation

Total cost = cost of three family packages + cost of three children tickets

Cost of a family package = 2 × cost of adult tickets + 2 × cost of children tickets − 7

$= 2a + 2b - 7$

$$\begin{aligned} \therefore \text{Total cost} &= 3 \times (2a + 2b - 7) + 3b \\ &= 3 \times 2a + 3 \times 2b - 3 \times 7 + 3b \\ &= 6a + 6b - 21 + 3b \\ &= 6a + 9b - 21 \end{aligned}$$

Answer

The total cost of the tickets is \$$(6a + 9b - 21)$.

Back to contents

13. A tennis court's perimeter has an expanded expression equal to $6x + 18$. It has a length of $x + 3$. Calculate the width of the tennis court.

Key points

- A tennis court's perimeter has an expanded expression equal to $6x + 18$.
- It has a length of $x + 3$.
- Calculate the width of the tennis court.

Explanation

$$\begin{aligned}\text{Perimeter} &= 2 \times \text{length} + 2 \times \text{width}\\ 6x + 18 &= 2 \times (x + 3) + 2 \times \text{width}\\ 6x + 18 &= 2 \times x + 2 \times 3 + 2 \times \text{width}\\ 6x + 18 &= 2x + 6 + 2 \times \text{width}\\ 6x + 18 - 2x - 6 &= 2x + 6 + 2 \times \text{width} - 2x - 6\\ \frac{4x + 12}{2} &= \frac{2 \times \text{width}}{2}\\ \text{width} &= 2x + 6\end{aligned}$$

Answer

The width of the tennis court is $2x + 6$.

14. Write an expression for the cost of a fence and gate, in terms of x, around Chirag's pool. The length of the fence is $2x - 3$ metres and the width is $3x + 4$ metres. A contractor charges $12 per metre for the fence, and an additional $100 to install the gate.

Key points

- The length of the fence is $2x - 3$ metres and the width is $3x + 4$ metres.
- A contractor charges $12 per metre for the fence, and the gate costs $100.
- Write an expression for the cost of a fence and gate, in terms of x, around Chirag's pool.

Explanation

Total cost = cost of fence + cost of gate

Cost of fence = perimeter of fence in metres × cost of each metre

$$\begin{aligned}\text{Perimeter of fence in metres} &= 2 \times \text{length} + 2 \times \text{width}\\ &= 2 \times (2x - 3) + 2 \times (3x + 4)\\ &= 2 \times 2x - 2 \times 3 + 2 \times 3x + 2 \times 4\\ &= 4x - 6 + 6x + 8\\ &= 10x + 2 \text{ metres}\end{aligned}$$

$$\begin{aligned}\therefore \text{Cost of fence} &= (10x + 2) \times 12\\ &= 10x \times 12 + 2 \times 12\\ &= 120x + 24\end{aligned}$$

$$\begin{aligned}\therefore \text{Total cost} &= 120x + 24 + 100\\ &= \$(120x + 124)\end{aligned}$$

Answer

The cost of the fence including the gate is $(120x + 124)$.

Reasoning

15. **a.** The new senior school campus garden will have a length of $(8x + 6)$ metres and a width of $(6x + 3)$ metres.

b. The new junior school campus garden will have a length of $(2x + 1.5)$ metres and a width of $(0.5x + 0.25)$ metres.

c. The difference in the lengths between the two gardens is $(6x + 4.5)$ metres and the difference in the widths between the two gardens is $(5.5x + 2.75)$ metres.

d. Suggested option 1: The school should consider the width of the path to ensure that it is not too narrow.

Suggested option 2: The school should consider the material used to lay the path.

Note: There are other possible options.

16. **a.** $3x + 25$

b. $3x + 25$

c. A similarity is that with both expressions in parts **a** and **b**, we expand through multiplying each term inside the brackets by the coefficient of the bracket and then simplify.

A difference is that the expression in part **a** we need to simplify the expressions by expanding brackets and collecting like terms, whereas for the expression in part **b** we only need to use the distributive law to expand brackets.

Exam-style

17. D

18. **a.** The perimeter of the triangle is $12x - 3$ cm and the perimeter of the square is $12x$ cm.

b. The perimeter of the new triangle is $6x - 1.5$ cm and the perimeter of the new square is $6x$ cm.

19. The correct simplified expression is $x - 14$. A student would incorrectly simplify the expression to $7x + 14$ by ignoring negative signs when expanding brackets.

20. An expression for Irene's bonus that week in expanded form is $\$(30t - 600)$.

An expression for Sophie's bonus that week in expanded form is $\$(20t - 600)$.

The constant for both expressions is −600. The coefficient for Irene's expression is 30 while the coefficient for Sophie's expression is 20.

Remember this?

21. D **22.** A **23.** E

3B Solving linear equations

Student practice

Worked example 1

a. $a = 13$ **b.** $x = -4$

Worked example 2

a. $c = 10$ **b.** $a = -\frac{4}{3}$

Worked example 3

a. $b = 12$ **b.** $x = \frac{1}{4}$

Understanding worksheet

1. **a.** -4 **b.** $+7$ **c.** $\div 5$ **d.** $\times -3$

2. **a.** $4m - 3 = 45$ **b.** $3c + 4 = 10$
c. $2r = 6$ **d.** $5k = 5$

3. inverse; isolate; solution; substitution

Back to contents

Fluency

4. **a.** 3 **b.** 4 **c.** $\frac{65}{8}$ **d.** $\frac{13}{6}$

e. -7 **f.** 3 **g.** $-\frac{7}{2}$ **h.** $-\frac{22}{3}$

5. **a.** 7 **b.** 3 **c.** -14 **d.** -5

e. $-\frac{24}{7}$ **f.** $-\frac{20}{9}$ **g.** $-\frac{72}{5}$ **h.** $\frac{20}{3}$

6. **a.** 28 **b.** 7 **c.** -9 **d.** -19

e. $\frac{34}{3}$ **f.** $\frac{8}{5}$ **g.** $-\frac{29}{3}$ **h.** $\frac{5}{7}$

7. **a.** 2 **b.** 10 **c.** -10 **d.** -3

e. 1 **f.** -3 **g.** $\frac{19}{3}$ **h.** $-\frac{1}{4}$

8. **a.** 2 **b.** -3 **c.** $\frac{20}{3}$ **d.** $-\frac{12}{5}$

e. 1 **f.** -1 **g.** $\frac{7}{5}$ **h.** $-\frac{3}{4}$

9. **a.** 3 **b.** 5 **c.** $\frac{4}{7}$ **d.** $\frac{9}{11}$

e. $-\frac{15}{7}$

10. A

Spot the mistake

11. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

12. Ken buys a new packet of tennis balls with w balls in each packet. On the following day, he purchases another two packets and four tennis balls separately. He now has a total of 13 tennis balls. Ken's number of tennis balls can be represented using the following equation $3w + 4 = 13$. Solve for w to determine the number of tennis balls in each packet.

Key points

- There are w balls in each packet of tennis balls.
- He purchases another two packets and four tennis balls, and has a total of 13 tennis balls.
- The number of tennis balls can be represented using: $3w + 4 = 13$.
- Solve for w to find the number of tennis balls in a packet.

Explanation

The operations applied are × 3 and + 4.

The inverse operations are ÷ 3 and − 4, which are applied in reverse order.

$$
\begin{aligned}
3w + 4 &= 13\\
3w + 4 - 4 &= 13 - 4\\
3w &= 9\\
3w \div 3 &= 9 \div 3\\
w &= 3
\end{aligned}
$$

Answer

There are three tennis balls in each packet.

13. Billo's savings can be represented using the following equation, $8w + 65 = 250$. Where Billo needs \$250 to attend his soccer camp, he already has \$65, and he will earn an additional \$8 allowance each week. Solve the following equations to determine how many full weeks it will take Billo to save up enough money for his soccer camp.

Key points

- Billo's savings can be represented using the equation: $8w + 65 = 250$
- Solve the equation to determine how many weeks it will take him to save up enough money for his soccer camp.

Explanation

The operations applied are × 8 and + 65.

The inverse operations are ÷ 8 and − 65, which are applied in reverse order.

$$
\begin{aligned}
8w + 65 &= 250\\
8w + 65 - 65 &= 250 - 65\\
8w &= 185\\
8w \div 8 &= 185 \div 8\\
w &= 23.125
\end{aligned}
$$

Answer

It will take Billo 24 weeks to save up enough money for soccer camp.

14. Andrew has j number of fidget spinners. He divides the fidget spinners amongst 6 people, including himself. His friend gives him an additional 8. Andrew was left with 10 fidget spinners. Calculate the initial number of fidget spinners Andrew had. The following equation can be used $\frac{j}{6} + 8 = 10$.

Key points

- Andrew has j number of fidget spinners.
- He divides the fidget spinners amongst 6 people.
- His friend gives him an additional 8 fidget spinners.
- He was left with 10 fidget spinners.
- Calculate the initial number of fidget spinners, using the equation $\frac{j}{6} + 8 = 10$.

Explanation

The operations applied are ÷ 6 and + 8.

The inverse operations are × 6 and − 8, which are applied in reverse order.

$$
\begin{aligned}
\frac{j}{6} + 8 &= 10\\
\frac{j}{6} + 8 - 8 &= 10 - 8\\
\frac{j}{6} &= 2\\
\frac{j}{6} \times 6 &= 2 \times 6\\
j &= 12
\end{aligned}
$$

Answer

Andrew initially had 12 fidget spinners.

15. Maria hires a Neuron scooter. It costs \$1 to get started and then an additional 45 cents per minute. Maria was charged \$11.80. Maria's trip can be represented as as an equation, where m represents minutes. Write an expression and solve for the number of minutes (m) Maria was on the bike.

Key points

- It costs \$1 to get started and then an additional 45 cents per minute to hire a Neuron scooter.
- Maria was charged \$11.80.
- Write an expression and solve for the number of minutes (m) Maria was on the bike.

Explanation

The fixed cost of hiring the Neuron scooter is \$1.

There is an additional cost of 45 cents per minute, which can be represented as $0.45m$.

The total cost of hiring the Neuron scooter can therefore be expressed as $1 + 0.45m$.

Maria's total cost was \$11.80.

$1 + 0.45m = 11.80$

Solve the equation using inverse operations.

$$\begin{aligned} 1 + 0.45m &= 11.80 \\ 1 + 0.45m - 1 &= 11.80 - 1 \\ 0.45m &= 10.80 \\ 0.45m \div 0.45 &= 10.80 \div 0.45 \\ m &= 24 \end{aligned}$$

Answer

Maria was on the scooter for 24 minutes.

16. How old is Peter, in years and months, if he is five times as old as Jenny and the sum of their ages is twenty one? Jenny's age is represented as r and Peter's age is represented as $5r$. Determine Peter's age.

Key points

- Peter is five times as old as Jenny.
- The sum of their ages is twenty one.
- Jenny's age is represented as r and Peter's as $5r$.
- Determine Peter's age.

Explanation

Jenny's age: r

Peter's age: $5r$

The sum of their ages is 21. Therefore, $r + 5r = 21$

Solve for r.

$$\begin{aligned} r + 5r &= 21 \\ 6r &= 21 \\ 6r \div 6 &= 21 \div 6 \\ r &= 3.5 \end{aligned}$$

Calculate Peter's age.

$$\begin{aligned} 5r &= 5 \times 3.5 \\ &= 17.5 \\ &= 17 \text{ years and 6 months} \end{aligned}$$

Answer

Peter is 17 years and 6 months old.

Reasoning

17. **a.** Miss Jeggy will not have enough money to hire No1. Coaches.

b. Miss Jeggy will have enough money to hire Coach4you.

c. The maximum distance Miss Jeggy could use Coach Heroes with her budget is 47.81 km.

d. Suggested option 1: Miss Jeggy should consider the fuel efficiency of each coach, since petrol costs will also need to be included when budgeting.

Suggested option 2: Miss Jeggy could also consider the environmental impact of each of the coach options.

Note: There are other possible options.

18. **a.** $x = 3$ **b.** $x = 3$

c. In both parts **a** and **b**, we arrived at the same solution for x, which is $x = 3$. This means that both methods, either expanding the brackets first or dividing both sides by 2, yield the same result. The method in part **a** can be easier to use when dealing with more complex expressions inside the brackets since it can help simplify the equation and make it easier to solve.

Exam-style

19. C

20. **a.** $a = \frac{16}{5}$ **b.** $2a - (4 + 3a) + 1 = 13$

21. **a.** $6b$ **b.** 18 boxes

22. The inverse operation of ÷ 12 should be × 12. The correct answer is $x = 68$.

Remember this?

23. D **24.** C **25.** A

3C Equations with pronumerals on both sides

Student practice

Worked example 1

a. $a = 4$ **b.** $s = 2$

Worked example 2

a. $b = -10$ **b.** $v = \frac{1}{3}$

Worked example 3

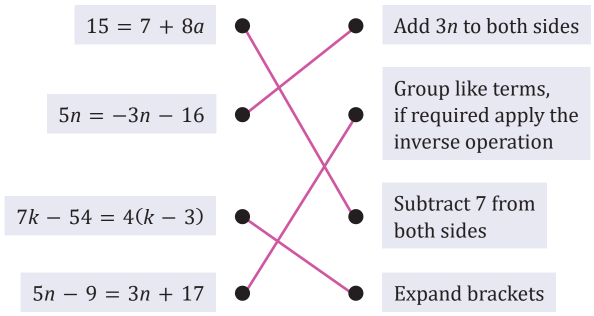
a. $h = 10$ **b.** $a = -1$

Understanding worksheet

1. **a.** $-5p, 2p$ **b.** $-3, 15$

c. $9g, 5g$ and $3, -8$ **d.** $-3x, x$ and $-1, 3$

2.



3. inverse; same; isolate; substituting

Back to contents

Fluency

4. **a.** $m = 6$ **b.** $n = 4$
c. $p = 5$ **d.** $q = -3$
e. $r = -\frac{9}{2}$ **f.** $s = -4$
g. $t = -\frac{22}{3}$ **h.** $u = -\frac{25}{3}$

5. **a.** $a = 3$ **b.** $b = -5$ **c.** $c = 1$ **d.** $g = \frac{7}{6}$
e. $h = -\frac{8}{7}$ **f.** $f = -\frac{6}{5}$ **g.** $d = -\frac{4}{3}$ **h.** $e = \frac{1}{2}$

6. **a.** $m = 4$ **b.** $p = -8$
c. $n = 3$ **d.** $q = 3$
e. $u = 1$ **f.** $p = -\frac{1}{11}$
g. $r = \frac{1}{4}$ **h.** $t = 3$

7. **a.** $z = 3$ **b.** $x = 8$ **c.** $y = -7$ **d.** $a = -6$
e. $p = \frac{19}{2}$ **f.** $b = -2$ **g.** $c = -\frac{3}{5}$ **h.** $d = \frac{17}{4}$

8. **a.** $z = 14$ **b.** $x = 22$
c. $v = 1$ **d.** $r = \frac{7}{8}$
e. $w = -\frac{13}{12}$ **f.** $b = \frac{5}{4}$
g. $y = -\frac{23}{30}$ **h.** $r = -\frac{1}{2}$

9. E

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. Alex and Bella are saving money for a summer trip. They both started saving at the same time. Alex saves $15 every week, while Bella saves $10 every week. However, Bella had a head start and had already saved $50 before they started saving together. The following equation $15x = 10x + 50$, represents Alex and Bella's savings after x weeks. After how many weeks (x) will Alex and Bella have saved the same amount of money?

Key points

- Alex saves $15 every week.
- Bella saves $10 every week.
- Bella had a head start of $50.
- The equation $15x = 10x + 50$ represents Alex and Bella's savings after x weeks.
- How many weeks (x) will Alex and Bella have saved the same amount of money?

Explanation

The given equation $15x = 10x + 50$ where x represents the number of weeks. On the left side, $15x$ represents the amount of money Alex saves every week. On the right side, $10x + 50$ represents the money Bella saves every week, including her initial savings of $50.

Solve for the number of weeks by using the inverse operation and isolating x.

$$15x - 10x = 10x + 50 - 10x$$
$$5x = 50$$
$$5x \div 5 = 50 \div 5$$
$$x = 10$$

Answer

After 10 weeks, Bella and Alex will have saved the same amount of money.

12. The equation $b = 2(b - 6)$ represents the total number of boys at Gary's soccer club. The soccer club originally contained an equal number of boys and girls. Six boys left to play tennis, leaving twice as many girls as boys at the club. Use the equation given to calculate the original total number of people at Gary's soccer club.

Key points

- The equation $b = 2(b - 6)$ represents the total number of boys at Gary's soccer club.
- The soccer club originally had an equal number of boys and girls.
- Six boys left the club to play tennis.
- After the boys left, there were twice as many girls as boys.
- Calculate the original total number of people at Gary's soccer club.

Explanation

The given equation is $b = 2(b - 6)$ where, b represents the initial number of boys at the club.

Solve for the total number of people by using the inverse operation and isolating b.

$$b = 2b - 2 \times 6$$
$$b = 2b - 12$$
$$b - 2b = 2b - 12 - 2b$$
$$-b = -12$$
$$-b \times (-1) = (-12) \times (-1)$$
$$b = 12$$

Before the six boys left, there were 12 boys left at the club.

The soccer club originally had an equal number of boys and girls, so the total number of people at the club was:

12 boys + 12 girls

= 24 people

Answer

The original total number of people at Gary's soccer club was 24.

13. Calculate the width of the rectangular play area at Kerry's childcare centre. The length is four times the width, and the perimeter is 120 metres.

Key points

- The length of the play area is four times the width.
- The perimeter of the play area is 120 metres.
- Calculate the width of the play area.

Explanation

The given problem describes a rectangle, and the perimeter of a rectangle is given by the formula 2 × (length + width). Here, the length is four times the width, which can be expressed as $4w$. The width is the unknown, which can be represented as w. The perimeter can be expressed as $2 \times (4w + w)$.

Given that the perimeter is 120 metres:

$$120 = 2 \times (4w + w)$$

$$120 = 2 \times (5w)$$

Isolate w.

Divide both sides of the equation by 2×5:

$$120 \div (2 \times 5) = 2(5w) \div (2 \times 5)$$
$$12 = w$$

Back to contents

Answer

The width of the play area for room 1 at Kerry's childcare centre is 12 metres.

14. Jenny has a part time job at a rental car company where they charge \$25 per day and 10 cents per kilometre travelled. Rachel has a part time job at another rental car company where they charge \$30 per day and 7 cents per kilometre. Calculate the number of kilometres you would need to travel for the cost of Jenny's company to be equal to Rachel's company, rounded to two decimal places.

Key points

- Jenny's rental company charges \$25 per day and 10 cents per kilometre.
- Rachel's rental company charges \$30 per day and 7 cents per kilometre.
- Calculate the number of kilometres for the cost of hiring from Jenny's company to be equal to the cost of hiring from Rachel's company, rounded to two decimal places.

Explanation

Let k be the number of kilometres.

The cost of hiring from Jenny's company can be expressed as $\$25 + \$0.10 \times k$, and the cost of hiring from Rachel's company can be expressed as $\$30 + \$0.07 \times k$.

Calculate the point where these two costs are equal:

$$\$25 + \$0.10 \times k = \$30 + \$0.07 \times k$$

Group like terms by applying the inverse operation.

$$\$0.10 \times k - \$0.07 \times k = \$30 + \$0.07 \times k - \$0.07 \times k$$

$$\$25 + \$0.10 \times k - \$0.07 \times k - \$25 = \$30 - \$25$$

$$\$0.03 \times k = \$5$$

$$\$0.03 \times k \div \$0.03 = \$5 \div \$0.03$$

$$k = 166.67$$

Answer

Need to travel approximately 166.67 kilometres for the cost of hiring from Jenny's company to be equal to the cost of hiring from Rachel's company.

15. George Green is planning to buy the newest iPhone, which is priced at \$500. He has already saved up \$80. He can accumulate an additional \$30 every month, but the iPhone's price increases on average by \$2 each month. Can he gather sufficient funds to purchase his iPhone in 16 months?

Key points

- The newest iPhone is priced at \$500.
- George Green has already saved up \$80.
- He can save an additional \$30 every month.
- The iPhone's price increases on average by \$2 each month.
- Determine if George can gather sufficient funds to purchase his iPhone in 16 months.

Explanation

To solve this, an equation can be formed to represent George's savings and the cost of the iPhone over time. The savings for George can be expressed as $80 + 30m$, where (m) represents the number of months. The price of the iPhone can be expressed as $500 + 2m$.

The question is asking whether George's savings will be at least equal to iPhone's price after 16 months. So, the equation to solve is:

$$80 + 30m = 500 + 2m$$

Simplify the equation:

$$\begin{array}{rrcll} & 80\boxed{+\,30m} & = & 500\boxed{+\,2m} & \\ -2m & 80 + 28m & = & 500 & -2m \\ -80 & 28m & = & 420 & -80 \\ \div 28 & m & = & 15 & \div 28 \end{array}$$

$$80 + 30m - 2m = 500 + 2m - 2m$$

$$80 + 28m = 500$$

$$80 + 28m - 80 = 500 - 80$$

$$28m = 420$$

$$28m \div 28 = 420 \div 28$$

$$m = 15$$

Answer

George will have saved enough money to buy the phone in 15 months meaning that after 16 months, George will be able to buy the iPhone.

Reasoning

16. a. The cost of using the internet in New York for a data usage of d gigabytes can be calculated using the expression $30 + 0.5d$

b. The cost of using the internet in Los Angeles for a data usage of d gigabytes can be calculated using the expression $20 + 0.75d$.

c. By equating the two expressions and solving for d, we find that the data usage where the cost of using the internet is the same in both cities is equal to 40 GB.

$d = 40$ gigabytes.

d. The cost of using the internet for a data usage of 40 gigabytes in both New York and Los Angeles is \$50.

e. Suggested option 1: The cost per hour or cost per gigabyte would depend on the specific usage patterns of the individual. If a person tends to use a lot of data but not spend much time online (for example, by downloading large files), then a lower cost per gigabyte would be more beneficial.

Suggested option 2: If a person spends a lot of time online but doesn't use much data (for example, by browsing text-heavy websites or sending emails), then a lower cost per hour would be more beneficial.

Note: There are other possible options.

17. a. When we subtract $4a$ from both sides of the equation, we get $a = 1$.

b. When we subtract $9a$ from both sides of the equation, we get $a = 1$.

c. The order in which we perform the operations changes the intermediate steps of the equation, but the final solution remains the same. In part **a**, the intermediate equation simplifies to $2 = 5a - 3$, while in part **b**, it simplifies to $-5a + 2 = -3$. The coefficients of a in the simplified equations differ ($5a$ in part **a** and $-5a$ in part **b**), however, the final solution of $a = 1$ is the same.

Exam-style

18. A

19. a. **Step 1:** Expand brackets

$$4y + 12 = -5y + 10$$

Back to contents

b. **Step 2:** Group like terms, if required apply the inverse operation

$4y$ and $-5y$ are like terms

12 and 10 are like terms

For the next step, isolate the variable y on one side of the equation. Add $5y$ to both sides of the equation and subtract 12 on both sides of the equation.

$4y + 5y = 10 - 12$

$9y = -2$

20. $b = 12$

21.
- Expand brackets
  $-2a + 6 = -3a - 6$
- Group like terms and apply inverse operations, where necessary to isolate the unknown variable
  $-2a + 3a = -6 - 6$
  $a = -12$

Remember this?

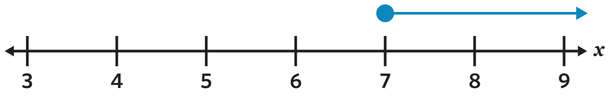
22. C **23.** B **24.** D

3D Inequalities

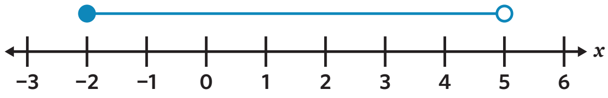
Student practice

Worked example 1

a.



b.



Worked example 2

a. $x < 8$ **b.** $x > -3$

Worked example 3

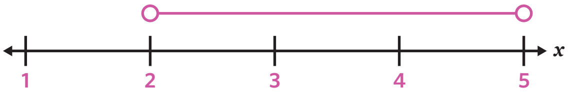
a. $x \leq 18$ **b.** $x \leq -\frac{1}{4}$

Understanding worksheet

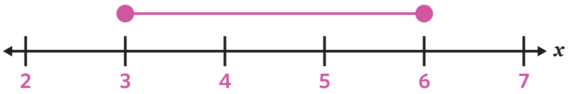
1. Open circle: I, III

Closed circle: II, IV

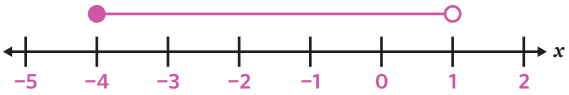
2. a.



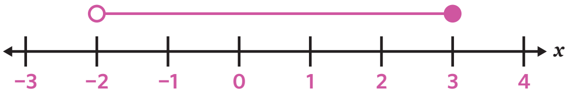
b.



c.



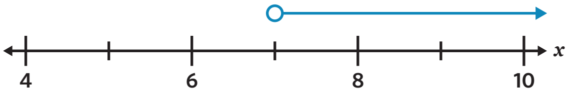
d.



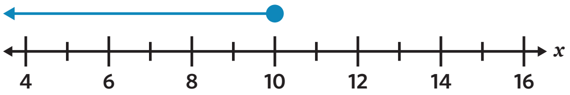
3. symbols; inequalities; set; number line; linear

Fluency

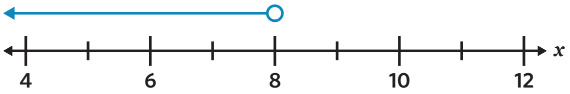
4. a.



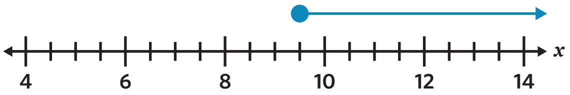
b.



c.



d.



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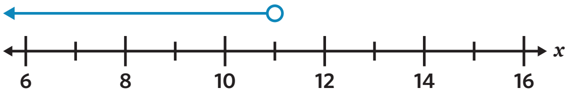
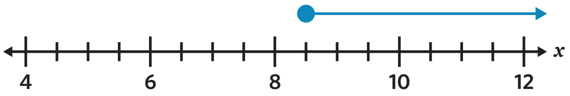
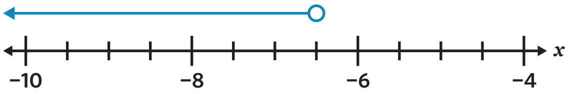


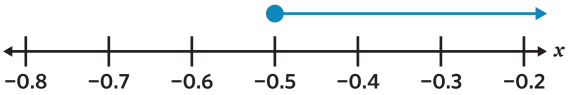
f.



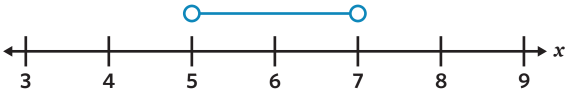
g.



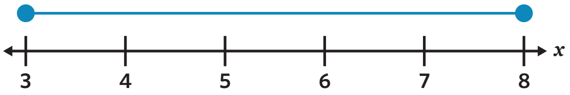
h.



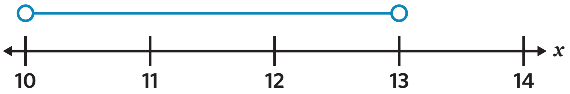
5. a.



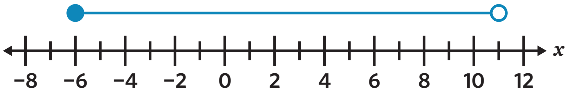
b.



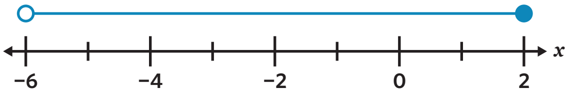
c.



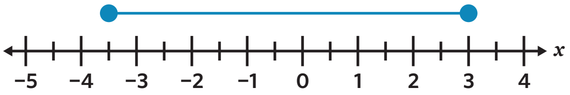
d.



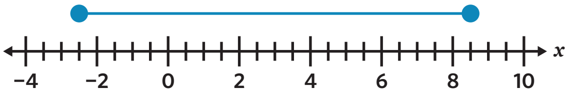
e.



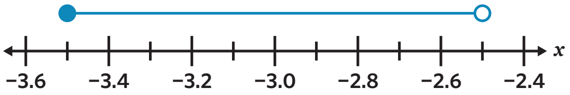
f.



g.



h.



Back to contents

6. **a.** $x < 6$ **b.** $x \geq 4$

c. $x \geq -6$ **d.** $x < 12$

e. $x < 7$ **f.** $x > -3$

g. $x \leq 40$ **h.** $x > -36$

7. **a.** $x \geq 4$ **b.** $x > -7$

c. $x \leq 12$ **d.** $x > 96$

e. $x < -25$ **f.** $x > 21$

g. $x \leq -2$ **h.** $x < -16$

8. **a.** $x < 3$ **b.** $x \leq 12$

c. $x > -6$ **d.** $x \leq -21$

e. $x > 25$ **f.** $x < 23$

g. $x \leq -\frac{15}{2}$ **h.** $x > -\frac{7}{3}$

9. **a.** $x > -3$ **b.** $x \leq 3$ **c.** $x \geq \frac{1}{3}$ **d.** $x < -\frac{1}{2}$

e. $x \geq 3$ **f.** $x < -3$ **g.** $x > -\frac{3}{2}$ **h.** $x \geq -2$

10. A

Spot the mistake

11. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

12. Generate an inequality that describes the temperature (t) of boiling water that is greater than or equal to 120°C.

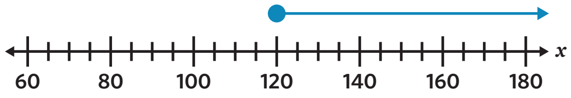
Key points

- The temperature (t) of boiling water is greater than or equal to 120°C.
- Write an inequality in terms of t that can be used to describe the temperature of boiling water.

Explanation

A closed circle is used to indicate that 120°C is included.

The arrow points to the right as the temperature is of the boiling water is greater than 120°C.



Answer

An inequality that can be used to describe the temperature of boiling water is $t \geq 120$°C.

13. Christopher has \$200 in his savings. He visits an online store and buys three video games and a headset. The headset costs \$60. Each video game costs \$$m$. Formulate an inequality in terms of m to represent Christopher's purchase.

Key points

- Christopher has \$200 in savings.
- He purchases three video games, which cost \$$m$ each.
- He purchases a headset, which costs \$60
- Write an inequality in terms of m that can be used to represent Christopher's purchase.

Explanation

$$\begin{aligned}\text{Cost of video games} &= \text{number of games purchased} \times \text{cost of one game}\\ &= 3 \times m\\ &= \$3m\end{aligned}$$

Cost of headset = \$60

Total cost $= 3m + 60$. The total cost must be smaller than or equal to the amount of money that Christoper has saved.

$3m + 60 \leq 200$

Answer

An inequality that can be used to represent Christopher's purchase is $3m + 60 \leq 200$.

14. Generate and solve, how many weeks (w) will Susan need to save in order to have at least \$150? She puts away \$10 each week. She also makes an additional \$20 by babysitting her cousin.

Key points

- Susan wants to have at least \$150 saved up.
- She puts away \$10 each week.
- She makes an additional \$20 by babysitting her cousin.
- Generate an inequality and calculate how many weeks (w) Susan will need to save in order to have at least \$150.

Explanation

Write an expression to show Susan's \$10 savings every week in addition to the \$20 she earned by babysitting.

$$\begin{aligned}\text{Total savings} &= 10 \times \text{number of weeks} + 20\\ &= 10w + 20\end{aligned}$$

Make the expression greater than or equal to \$150.

$$20 + 10w \geq 150$$

Solve for w.

$$\begin{aligned}20 + 10w - 20 &\geq 150 - 20\\ 10w &\geq 130\\ 10w \div 10 &\geq 130 \div 10\\ w &\geq 13\end{aligned}$$

Answer

Susan will need to save for at least 13 weeks in order to reach her goal.

15. Daphne needs to buy some pencils and an eraser. She can spend no more than \$5. The eraser costs \$1 and the pencils cost \$0.25 each. Solve for the number of pencils (p) Daphne can purchase.

Key points

- Daphne needs to buy some pencils, which cost \$0.25 each.
- Daphne needs to buy one eraser, which costs \$1 each.
- She can spend no more than \$5.
- Calculate how many pencils (p) she can buy.

Explanation

$$\begin{aligned}\text{Cost of pencils} &= \text{number of pencils purchased} \times \text{cost of one pencil}\\ &= p \times 0.25\\ &= \$0.25p\end{aligned}$$

Cost of eraser = \$1

Total cost $= 0.25p + 1$. The total cost must be less than or equal to \$5.

$0.25p + 1 \leq 5$

Solve for p.

$$0.25p + 1 \leq 5$$
$$0.25p + 1 - 1 \leq 5 - 1$$
$$0.25p \leq 4$$
$$0.25p \div 0.25 \leq 4 \div 0.25$$
$$p \leq 16$$

Answer

Daphne can buy no more than 16 pencils.

16. George has a pool that has a leak and is losing 2 L of water per minute. The pool currently has a volume of 4000 L. Using inequalities, calculate when the pool reaches less than 1500 L, in minutes.

Key points

- George has a pool that has a leak.
- The pool loses 2 L of water per minute.
- The pool currently has 4000 L of water.
- Using inequalities, calculate when the pool reaches less than 1500 L, in minutes.

Explanation

Let t be the amount of time that has passed since the leak began, in minutes. Write an expression for the volume of water in the pool.

Volume of water = 4000 − 2 × number of minutes since leak began

Make the expression less than or equal to 1500 litres.

$$4000 - 2t < 1500$$

Solve for t.

$$4000 - 2t - 4000 < 1500 - 4000$$
$$-2t < -2500$$
$$-2t \div -2 > -2500 \div -2$$
$$t > 1250$$

Answer

The volume of the pool will be less than 1500 L after 1250 minutes.

Reasoning

17. a. The time it took Peter to complete the race can be represented by the inequality $C + R < 1$.

b. Peter's average running speed is given by $S = \frac{5}{R}$ and can be rearranged to give $R = \frac{5}{S}$.

c. $C + R < 1$ and $\frac{10}{2s} + \frac{5}{s} < 1$.

d. $s > 10$. If $s > 10$, then his running speed is greater than 10 km/h and his cycling speed is greater than 20 km/h.

e. Suggested option 1: Light jogging is a form of active recovery.

Suggested option 2: Stretching is a form of active recovery.

Note: There are other possible options.

18. a. $x \leq 3$

b. $x \geq -3$

c. The solutions to the two inequalities differ due to the signs of the coefficients. In part **a**, dividing by a positive coefficient preserves the inequality direction, while in part **b**, dividing by a negative coefficient reverses the inequality direction. This is because dividing by a negative number flips the direction of the inequality.

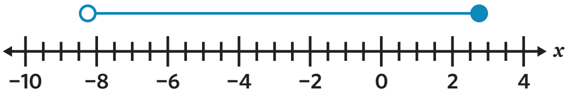
Exam-style

19. A

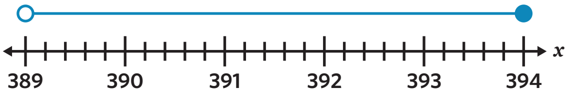
20. a. $-4x$ and $3x$; 2 and -1

b. $x \leq \frac{3}{7}$

21.



22. The number line and given inequality do not match the statement that the perimeter is greater than 389 metres and less than or equal to 394 metres. The correct inequality is $389 < P \leq 394$, subsequently the correct number line is displayed.



Remember this?

23. D **24.** D **25.** A

3E Using formulas

Student practice

Worked example 1

a. 10.5 **b.** 247

Worked example 2

$h = 10$ cm

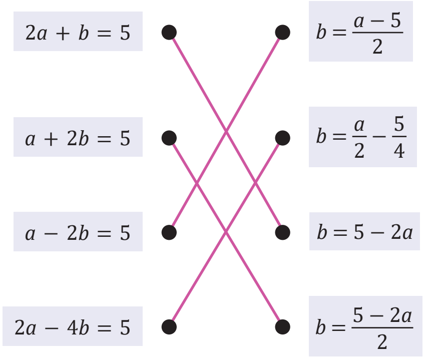
Worked example 3

a. $x = \frac{y - c}{m}$ **b.** $r = \sqrt{\frac{p - a}{v}}$

Understanding worksheet

1. a. $-\,y$ **b.** $\times\, 2$ **c.** $+\,y$ **d.** $\div\, 3$

2. Equations Transposed equations



3. equations; subject; transposed; substituted; pronumeral

Back to contents

Fluency

4. **a.** $P = 7$ **b.** $Q = 5$

c. $R = 2.75$ **d.** $S = 29.25$

e. $T \approx 124.88$ **f.** $U \approx 6.08$

g. $V \approx 28.01$ **h.** $W \approx -0.84$

5. **a.** $s = 5$ **b.** $w = 4$

c. $r \approx 4.997$ **d.** $h = 6$

e. $h \approx 6.667$ **f.** $h \approx 4.125$

g. $r \approx 4.469$ **h.** $h \approx 2.378$

6. **a.** $c = a - qb$ **b.** $q = \frac{p - s}{r}$

c. $x = z(y - w)$ **d.** $x = \frac{y(v - z)}{w}$

e. $n = \sqrt{m - p}$ **f.** $t = \sqrt{s(r - u)}$

g. $c = \sqrt{\frac{d(a - e)}{b}}$ **h.** $i = (3f - g)^2$

7. D

Spot the mistake

8. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. Jill is travelling to the United States and is needing to calculate what the weather in degrees Fahrenheit is equal to in degrees Celsius. Transpose the formula $F = \frac{9}{5}C + 32$ to help Jill calculate this conversion.

Key points

- Jill needs to convert the temperature in degrees Fahrenheit to degrees Celsius.
- Transpose the formula $F = \frac{9}{5}C + 32$ to solve for C.

Explanation

Apply the inverse operation to both sides of the equation to isolate the variable C.

$$F - 32 = \frac{9}{5}C + 32 - 32$$

$$F - 32 = \frac{9}{5}C$$

$$(F - 32) \times 5 = \frac{9}{5}C \times 5$$

$$5(F - 32) = 9C$$

$$5(F - 32) \div 9 = 9C \div 9$$

$$\frac{5(F - 32)}{9} = C$$

Answer

Jill can use the formula $C = \frac{5(F - 32)}{9}$ to convert temperatures in degrees Fahrenheit into degrees Celsius.

10. A large helium balloon is losing air over time. The volume of air(V), in litres, remaining in the balloon is given by the equation $V = 100 - 10.5t$, where t represents the time in hours since the balloon was filled. Calculate the volume, in litres, of air remaining in the balloon after an hour and a half.

Key points

- A helium balloon is losing air over time.
- The formula $V = 100 - 10.5t$ can be used to calculate the remaining volume of air in the balloon, where t represents the time in hours since the balloon was filled.
- An hour and a half has elapsed since the balloon was filled.
- Calculate the volume, in litres, of air remaining in the balloon.

Explanation

Substitute the given value into the formula.

It has been one and a half hours since the balloon was filled, so $t = 1.5$.

$V = 100 - 10.5(1.5)$

Perform and evaluate the operations in the correct order according to the order of operations.

$V = 100 - 10.5 \times 1.5$

$V = 100 - 15.75$

$V = 84.25$

Answer

The balloon will have 84.25 L of air after an hour and a half.

11. Tori is competing in the Run Melbourne 10 km run. The formula $s = \frac{d}{t}$ represents the speed (s) at which she travels, where d is the distance and t is the time. Calculate the speed in kilometres per minute, to three decimal places, if Tori completed the 10 km run in 53 minutes.

Key points

- The formula $s = \frac{d}{t}$ can be used to calculate the speed, where d is the distance travelled and t is the time elapsed.
- The distance that Tori travelled is 10 kilometres.
- The time that Tori takes to travel that distance is 53 minutes.
- Calculate Tori's speed for the race, in kilometres per minute, rounded to three decimal places.

Explanation

Substitute the given values into the formula.

The distance travelled is 10 kilometres so $d = 10$ km.

The time elapsed is 53 minutes so $t = 53$ min.

$s = \frac{10}{53}$

Perform and evaluate the operations in the correct order according to the order of operations.

$s = 10 \div 53$

$s = 0.1886...$

$s \approx 0.189$ km/min

Answer

Tori ran at a speed of 0.189 kilometres per minute in the Run Melbourne 10 km run.

12. Jade wants to rent a car for her weekend getaway. The cost to rent a car is given by the formula $C = 40d + 0.25k + 250$, where d represents the number of days rented and k represents the number of kilometres driven. Rearrange the formula to make k the subject.

Key points

- The formula $C = 40d + 0.25k + 250$ can be used to calculate the cost of renting a car, where d represents the number of days rented and k represents the number of kilometres driven.
- Transpose the formula to make k the subject.

Back to contents

Explanation

Apply the inverse operation to both sides of the equation to isolate the variable k.

$$\begin{tikzcd}
C = 40d + 0.25k + 250 \arrow[d, bend right, "-250"'] \arrow[d, bend left, "-250"] \\
C - 250 = 40d + 0.25k \arrow[d, bend right, "-40d"'] \arrow[d, bend left, "-40d"] \\
C - 250 - 40d = 0.25k \arrow[d, bend right, "\div 0.25"'] \arrow[d, bend left, "\div 0.25"] \\
\frac{C - 250 - 40d}{0.25} = k
\end{tikzcd}$$

$$C - 250 = 40d + 0.25k + 250 - 250$$
$$C - 250 = 40d + 0.25k$$
$$C - 250 - 40d = 40d + 0.25k - 40d$$
$$C - 250 - 40d = 0.25k$$
$$(C - 250 - 40d) \div 0.25 = 0.25k \div 0.25$$
$$\frac{C - 250 - 40d}{0.25} = k$$

Answer

The rearranged formula used to calculate the number of kilometres driven is represented as $k = \frac{C - 250 - 40d}{0.25}$.

13. A bakery produces specialty cakes. The cost of producing each cake is $15, plus a fixed setup cost of $500 for each new cake design. The bakery has a budget of $3000 allocated for three different cake designs. If they want to produce an equal number of cakes for each design, write a formula for this scenario and solve to find what is the maximum number of cakes that can be produced for each design.

Key points

- Each cake costs $15 to make.
- They have a fixed setup cost of $500 for each new cake design and they want to make three different cake designs.
- They have allocated a budget of $3000 allocated.
- They want to produce an equal number of cakes for each design.
- Write a formula for this scenario and calculate the maximum number of cakes that can be produced for each design.

Explanation

Let n be the total number of cakes made.

Calculate the cost of the new cake designs.

The bakery wants to produce 3 different cake designs.

Total cost of cake designs = cost of each cake design × number of cake designs

= 500 × 3

= 1500

Create an equation for the cost.

The bakery has allocated a budget of $3000.

They want to produce an equal number of cakes for each design.

Total cost = total cost of cake designs + cost of each cake × number of cakes

Substitute the given values into the formula.

$$3000 = 1500 + 15n$$

Apply the inverse operation to both sides of the equation to isolate the variable n.

$$3000 - 1500 = 1500 + 15n - 1500$$
$$1500 = 15n$$
$$1500 \div 15 = 15n \div 15$$
$$n = 100$$

In total, 100 cakes can be baked.

Calculate the number of cakes that can be made for each design.

Number of cakes per design = 100 ÷ 3

= 33.333....

= 33 cakes

Answer

The bakery can make a maximum of 33 cakes for each design.

Reasoning

14. a. The velocity of the cyclist is 20 metres per second.

b. The velocity formula transposed with d as the subject is $d = vt$.

c. The distance covered by a cyclist who maintains a velocity of 12 metres per second for 25 seconds is 300 m.

d. The expected time of a cyclist travelling 0.7 km with a velocity of 14 metres per second is 50 seconds.

e. Suggested option 1: Cyclists should consider taking water bottles with them.

Suggested option 2: Cyclists should consider taking energy bars with them.

Note: There are other possible options.

15. a. $a = \frac{g - hv}{b}$ **b.** $a = \frac{g - \frac{1}{hv}}{b}$

c. In part **a**, $\frac{hv}{1}$ is used, which can also be expressed as hv.

In part **b**, $\frac{1}{hv}$ is used, which is the reciprocal of hv.

The difference in the two transposed equations is the expression of the term involving hv, which changes the transposed equation.

Exam-style

16. B

17. a. $815 **b.** $a = \frac{T - 5c}{7}$

c. 180 adults

18. $b = \frac{2A}{h} - a$; $b = 6$

19.

$$300 = 2\pi(5)^2 + 2\pi(5)h$$
$$300 = 2\pi(25) + 2\pi(5)h$$
$$300 = 50\pi + 10\pi h$$
$$300 - 50\pi = 10\pi h$$
$$\frac{300 - 50\pi}{10\pi} = h$$
$$h = 4.5493...$$
$$h \approx 4.549 \text{ cm}$$

Remember this?

20. B **21.** D **22.** D

3F Simultaneous equations using substitution and elimination

Student practice

Worked example 1

a. $x = 2,\ y = 6$ **b.** $x = -3,\ y = -7$

Worked example 2

a. $x = 8,\ y = 4$ **b.** $x = 3,\ y = -10$

Worked example 3

a. $x = -3,\ y = 25$ **b.** $x = 4,\ y = 2$

Understanding worksheet

1. **a.** 30 **b.** 2 **c.** 11 **d.** 5

2. **a.** – **b.** + **c.** + **d.** –

3. simultaneous; variables; solve; substitution; elimination

Fluency

4. **a.** $x = 9,\ y = 3$ **b.** $x = 2,\ y = 8$
c. $x = \frac{3}{2},\ y = \frac{1}{2}$ **d.** $x = 1,\ y = 4$
e. $x = 2,\ y = 12$ **f.** $x = 4,\ y = 1$
g. $x = 9,\ y = -3$ **h.** $x = \frac{20}{7},\ y = \frac{40}{7}$

5. **a.** $x = 2,\ y = 5$ **b.** $x = -9,\ y = -49$
c. $x = 3,\ y = 2$ **d.** $x = 4,\ y = 2$
e. $x = 4,\ y = 2$ **f.** $x = -2,\ y = -4$
g. $x = 2,\ y = -4$ **h.** $x = -2,\ y = 7$

6. **a.** $x = 3,\ y = 2$ **b.** $x = 4,\ y = 1$
c. $x = 4,\ y = 1$ **d.** $x = 3,\ y = -1$
e. $x = -10,\ y = 16$ **f.** $x = 1,\ y = \frac{8}{3}$
g. $x = 4,\ y = \frac{4}{5}$ **h.** $x = 21,\ y = -\frac{49}{4}$

7. **a.** $x = 1,\ y = -1$ **b.** $x = 2,\ y = 13$
c. $x = 1,\ y = -\frac{3}{2}$ **d.** $x = 2,\ y = -8$
e. $x = \frac{11}{5},\ y = \frac{17}{10}$ **f.** $x = \frac{3}{2},\ y = \frac{7}{2}$
g. $x = -1,\ y = 1$ **h.** $x = 3,\ y = 7$

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. $g + b = 540$ and $g + b = 540$ represents the number of boys (b) and girls (g) at Kingston College. There are a total of 540 students. If there are 80 more girls than boys, how many boys and girls are there?

Key points

- The boys and girls and Kingston can be represented with the equations $g + b = 540$ and $g + b = 540$.
- 540 students in total.
- 80 more girls than boys.
- Calculate the number of boys (b) and girls (g).

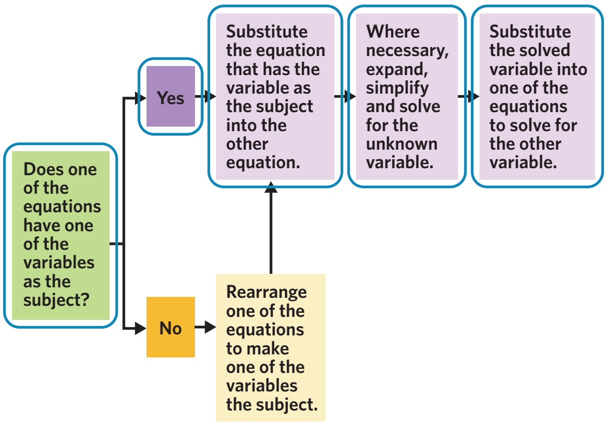
Explanation

Two equations:

$g + b = 540$ [1]

$g = b + 80$ [2]

Where g represents the number of girls and b represents the number of boys.



Substitute equation [2] into equation [1].

$(b + 80) + b = 540$

$2b + 80 = 540$

Solve for b.

$2b + 80 - 80 = 540 - 80$

$2b = 460$

$2b \div 2 = 460 \div 2$

$b = 230$

Substitute the value of b into equation [2] to find the number of girls (g).

$g = 230 + 80$

$g = 310$

Answer

At Kingston College, there are 230 boys and 310 girls.

11. Heath got 30 more marks in her Science test than in Maths. The total of his marks was 120. Calculate her marks in each test.

Key points

- Heath's marks in Science are 30 more than his marks in Maths.
- The total of Heath's marks in both subjects is 120.
- We need to calculate the marks Heath scored in each test.

Explanation

Represent this situation with two equations:

$S = M + 30$ [1]

$S + M = 120$ [2]

Where S represents the marks in Science and M represents the marks in Maths.

Substitute equation [1] into equation [2].

$(M + 30) + M = 120$

$2M + 30 = 120$

Solve for M.

$2M + 30 - 30 = 120 - 30$

$2M = 90$

$2M \div 2 = 90 \div 2$

$M = 45$

Substitute the value of M into equation [1] to find the marks in Science (S).

$S = 45 + 30$

$S = 75$

Answer

Heath scored 45 marks in Maths and 75 marks in Science.

12. Mia has purchased a rectangular new dining table and the difference between the length and the width of the table is 1.8 m and the perimeter of the table is 8.4 m. Calculate the length (l) and width (w) of the table in cm.

Key points

- Mia has purchased a new rectangular dining table.
- The difference between the length and the width of the table is 1.8 m.
- The perimeter of the table is 8.4 m.
- Calculate the length (l) and width (w) of the table in cm.

Explanation

Represent this situation with two equations:

$l - w = 1.8 \quad [1]$

$2l + 2w = 8.4 \quad [2]$

Where l represents the length of the table and w represents the width of the table.

Transpose [1] to make l the subject.

$l - w = 1.8$

$l - w + w = 1.8 + w$

$l = w + 1.8$

Substitute equation [1] into equation [2].

$2(w + 1.8) + 2w = 8.4$

$2w + 3.6 + 2w = 8.4$

$4w + 3.6 = 8.4$

Solve for w.

$4w + 3.6 - 3.6 = 8.4 - 3.6$

$4w = 4.8$

$4w \div 4 = 4.8 \div 4$

$w = 1.2 \text{ m}$

Substitute the value of w into equation [1] to find the length (l).

$l = 1.2 + 1.8$

$l = 3 \text{ m}$

Answer

The length of the table is 300 cm and the width of the table is 120 cm.

13. Two complementary angles differ by 18°. Write two equations and calculate the two angles.

Key points

- Two complementary angles differ by 18°.
- Calculate the two angles.

Explanation

Represent this situation with two equations:

$x + y = 90 \quad [1]$

$x - y = 18 \quad [2]$

Where x and y represent the two angles.

Add equation [1] and equation [2].

$$\begin{array}{lr} & x + y = 90 \\ + & x - y = 18 \\ \hline & 2x = 108 \end{array}$$

Solve for x.

$2x \div 2 = 108 \div 2$

$x = 54°$

Substitute the value of x into equation [1] to calculate the second angle y.

$54 + y = 90$

$y = 90 - 54$

$y = 36°$

Answer

The two angles are 54° and 36°.

14. Jonathan purchased three bags of coffee and five bars of dark chocolate for a total of $30. At the same market, Katherine bought two bags of the same coffee and five bars of the same dark chocolate for $25. Using the elimination method, calculate how much does each bag of coffee and each bar of dark chocolate cost?

Key points

- Jonathan purchased 3 coffee bags and 5 chocolate bars for $30.
- Katherine purchased 2 coffee bags and 5 chocolate bars for $25.
- Calculate the cost of each coffee bag and each chocolate bar.

Explanation

Represent this situation with two equations:

$3c + 5d = 30 \quad [1]$

$2c + 5d = 25 \quad [2]$

Where c represents the price for each coffee bag and d represents the price for each chocolate bar.

Subtract equation [2] from equation [1].

$$\begin{array}{lr} & 3c + 5d = 30 \\ - & 2c + 5d = 25 \\ \hline & c = \$5 \end{array}$$

Substitute the value of c into equation [2] to calculate the cost of dark chocolate (d).

$2(5) + 5d = 25$

$10 + 5d = 25$

$10 + 5d - 10 = 25 - 10$

$5d = 15$

$5d \div 5 = 15 \div 5$

$d = \$3$

Answer

Each bag of coffee costs $5 and each bar of dark chocolate costs $3.

Reasoning

15. a. Two equations to display Lennox's scenario, where x represents the number of pop albums and y represents the number of jazz albums is $7x + 12y = 400$ and $x + y = 50$.

b. Lennox purchased 40 pop albums and 10 jazz albums.

c. A transposed rule in terms of x is $y = 50 - x$.

Back to contents

d. Lennox purchased 40 pop albums and 10 jazz albums.

e. Suggested option 1: Some value physical albums for nostalgia and collectible items like album covers, creating a more memorable experience than digital streaming.

Suggested option 2: Audiophiles prefer physical albums for higher audio quality, such as vinyl records or CDs, offering a richer sound compared to compressed streaming files.

Note: There are other possible options.

16. a. Subtracting [2] from [1] results in $x = 0$ and $y = 2$

b. Subtracting [1] from [2] results in $x = 0$ and $y = 2$

c. This results in the coefficient of x in the resulting equation being positive in part **a** and negative in part **b**. However, this does not affect the solution to the system of equations.

Exam-style

17. C

18. a. $4a + c = 1278; a + c = 453$

b. $c = 178$

19. $a = \frac{40}{9},\ k = \frac{2}{9}$

20. $x = 40,\ y = 56$

Remember this?

21. D **22.** B **23.** C

Chapter 3 extended application

1. a. $15 < b \le 75$ m

b. $b = \frac{2V}{hl} - a$

c. $b = 85$ m

d. $h = \frac{2V}{l(a + b)}$

e. $h = 20$ m

f. Suggested option 1: The compatibility of animals in a fish tank.

Suggested option 2: Methods of access to the inside of a fish tank.

Note: There are other possible options.

2. a. $b = l + 8$

$b + 6 = 2(l + 6)$

b. Bart is 10 years old, Lisa is 2 years old.

c. $l = b - 8, l = \frac{b - 6}{2}$

d. Bart is 10 years old, Lisa is 2 years old.

e. Suggested option 1: Playing with action figures.

Suggested option 2: Drawing.

Note: There are other possible options.

3. a. $1.75bm + 1.75f$

b. $C = 2(bm + f)$

c. $P = bs - bm - f$

d. 334 bracelets

e. Suggested option 1: Increased costs can reduce the profit and impact their ability to expand their bracelet business.

Suggested option 2: Decreased costs can increase profit, improving their ability to expand their bracelet business.

Note: There are other possible options.

Chapter 3 review

Multiple choice

1. C **2.** D **3.** A **4.** C **5.** B

Fluency

6. a. $28 - 7w$ **b.** $-42v + 21v^2$

c. $18q + 6$ **d.** $23x - 89$

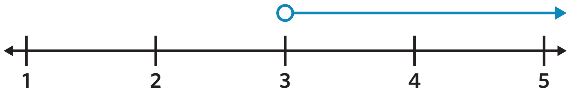
7. a. $x = 5$ **b.** $y = -2$ **c.** $z = \frac{16}{5}$ **d.** $x = \frac{5}{6}$

8. a. $x = 5$ **b.** $z = 4$ **c.** $a = 4$ **d.** $y = 8$

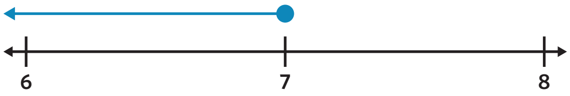
9. a. $w = \frac{5}{4}$ **b.** $w = \frac{4}{5}$

c. $w = -13$ **d.** $w = -\frac{13}{29}$

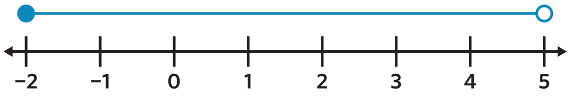
10. a.



b.



c.



d.

11. a. $x > 4$ **b.** $x \ge -2$ **c.** $x \le 3$ **d.** $x \le 33$

12. a. $s = 7$ **b.** $P = 16$

c. $A \approx 28.274$ **d.** $h \approx 3.537$

13. a. $a = \frac{c - b}{x}$ **b.** $p = x(y - z)$

c. $u = \frac{x - 0.5at^2}{t}$ **d.** $n = (2m - x)^2$

14. a. $x = 5, y = 1$ **b.** $x = 2, y = 5$

c. $x = 6, y = 4$ **d.** $x = 4, y = -1$

Problem solving

15. Alex is a party planner and is currently planning 6 different birthday parties for his clients. Each of the clients want to hire a clown for the event. Alex was told that the clown charges a flat fee of \$30 to come to the party and a further \$20 per hour. What is an expanded and simplified expression that can be used to represent the total cost of hiring a clown for all six parties?

Key points

- Alex is planning 6 different birthday parties for his clients.
- The clown charges a flat fee of \$30.
- The clown charges a further \$20 per hour.
- What is an expanded and simplified expression that can be used to represent the total cost of hiring a clown for all six parties?

Explanation

Find an expression for the cost of hiring a clown for 1 party.

Let h be the number of hours that the clown is hired for.

$$\begin{aligned}\text{Cost of hiring clown} &= \text{charge per hour} \times \text{number of hours} \\ &\quad + \text{flat fee} \\ &= 20h + 30\end{aligned}$$

Find an expression for the cost of hiring a clown for 6 parties.

$$\text{Total cost} = 6(20h + 30)$$

Multiply each term inside the brackets by the coefficient of the bracket.

$$\begin{aligned}6(20h + 30) &= 6 \times 20h + 6 \times 30 \\ &= 120h + 180\end{aligned}$$

Answer

The expanded expression that can be used to represent the total cost of hiring a clown for Alex's six clients is $120h + 180$.

16. Sarah is a basketball player who is trying to improve her shot success rate. Sarah knows that the number of shots she can expect to make can be represented by the equation $3(-18 + m) = 9$, where m represents the number of successfully made shots. How many shots should Sarah expect to successfully make that day?

Key points

- Sarah is trying to improve her shot success rate as a basketball player.
- The number of shots can be represented by the equation $3(-18 + m) = 9$ where m represents the number of successful shots.
- How many successful shots should Sarah expect that day?

Explanation

Solve the equation for m.

Distribute the 3 on the left side of the equation.

$$\begin{aligned}3 \times (-18) + 3 \times m &= 9 \\ -54 + 3m &= 9\end{aligned}$$

Add 54 to both sides of the equation to isolate the term with m.

$$\begin{aligned}-54 + 3m + 54 &= 9 + 54 \\ 3m &= 63\end{aligned}$$

Divide both sides by 3 to solve for m.

$$\begin{aligned}\frac{3m}{3} &= \frac{63}{3} \\ m &= 21\end{aligned}$$

Answer

Sarah should expect to make 21 successful shots that day.

17. John is an event coordinator planning a music concert. He knows that the total money collected from ticket sales is represented by the equation $7t + 150 = 5t + 250$, where t represents the price of each ticket in dollars. Solve the equation to find the price of one ticket.

Key points

- The equation $7t + 150 = 5t + 250$, where t represents the price of each ticket in dollars, can be used to represent the total amount of money collected from ticket sales.
- Solve the equation for t.

Explanation

Simplify the equation by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

$$\begin{aligned}7t + 150 - 5t &= 5t + 250 - 5t \\ 2t + 150 &= 250 \\ 2t + 150 - 150 &= 250 - 150 \\ 2t &= 100\end{aligned}$$

Solve and simplify for the unknown value.

$$\begin{aligned}2t \div 2 &= 100 \div 2 \\ t &= 50\end{aligned}$$

Check to see if the LHS equals the RHS.

$$\begin{aligned}7 \times 50 + 150 &= 5 \times 50 + 250 \\ 500 &= 500 \ ✓\end{aligned}$$

Answer

The price of each ticket is \$50.

18. Alice is a fitness enthusiast who tracks her daily steps. She aims to walk at least 13 204 steps a day. One day, she walks x steps in the morning and 7259 steps in the afternoon. If the inequality $x + 7259 \geq 13\,204$ represents her step count for the day, solve for x to find out the minimum number of steps Alice needs to walk in the morning to meet her daily goal.

Key points

- Alice aims to walk at least 13 204 steps a day.
- On one day, she walks x steps in the morning.
- On that same day, she walked 7259 steps in the afternoon.
- The inequality $x + 7259 \geq 13\,204$ can be used to represent her step count for that day.
- Solve for x, the minimum number of steps that Alice needs to walk in the morning to meet her goal.

Explanation

Isolate the unknown variable by applying the inverse operations. To keep the inequality true, apply the same operations on both sides of the inequality sign.

$$\begin{aligned}x + 7259 - 7259 &\geq 13\,204 - 7259 \\ x &\geq 5945\end{aligned}$$

Answer

Alice needs to walk at least 5945 steps in the morning to meet her daily step goal.

19. A company uses the formula $C = \frac{1}{5}(P - 3R)$ to calculate the expected per unit profit (C) from selling a certain product. P is the price they sell each unit for and R is the cost of raw materials per unit. If the company sells each unit for \$20 and the cost of raw materials per unit is \$2, what is the expected per unit profit of the product?

Key points

- A company uses the formula $C = \frac{1}{5}(P - 3R)$ to calculate the expected per unit profit (C) from selling a certain product. P is the price they sell each unit for and R is the cost of raw materials per unit.
- The company sells each unit for $20.
- The cost of raw materials per unit is $2.
- What is the expected per unit profit of the product?

Explanation

Substitute $P = 20$ and $R = 2$ into the equation and solve for C.

$$\begin{aligned} C &= \frac{1}{5}(20 - 3 \times 2) \\ &= \frac{1}{5}(20 - 6) \\ &= \frac{1}{5}(14) \\ &= \frac{14}{5} \\ &= 2.8 \end{aligned}$$

Answer

The expected per unit profit is $2.80.

20. A company is planning to produce two types of products: A and B. The profit they make from each unit of product A is $7, and from each unit of product B is $5. The company has a target to make a total profit of $3500. They also know that the number of units of product A they will sell is 200 more than the number of units of product B. How many units of each product does the company need to sell to meet their profit target?

Key points

- A company is planning to produce two products, A and B.
- The profit made for selling one unit of A is $7.
- The profit made for selling one unit of B is $5.
- They want to make a total of $3500 in profit.
- There will be 200 more A units sold than B units.
- How many units of each product does the company need to sell to meet their profit target?

Explanation

Let a be the amount of product A that should be sold.

Let b be the amount of product B that should be sold.

Write an equation to represent the profit made by selling products A and B.

$$3500 = 7a + 5b$$

Write an equation to represent the relationship between the number of A and B units sold.

$$a = b + 200$$

There are two simultaneous equations.

$$\begin{aligned} 3500 &= 7a + 5b \quad [1] \\ a &= b + 200 \quad [2] \end{aligned}$$

Substitute equation [2] into equation [1].

$$3500 = 7(b + 200) + 5b$$

Solve for b.

$$\begin{aligned} 3500 &= 7b + 1400 + 5b \\ 3500 &= 12b + 1400 \\ 3500 - 1400 &= 12b + 1400 - 1400 \\ 2100 &= 12b \\ 2100 \div 12 &= 12b \div 12 \\ b &= 175 \end{aligned}$$

Substitute $b = 175$ into [2] to solve for a.

$$\begin{aligned} a &= 175 + 200 \\ &= 375 \end{aligned}$$

Answer

The company should sell 375 units of product A and 175 units of product B.

Reasoning

21. a. Sam's profit for the day is $430.

b. Sam can expect to sell 180 glasses and make a profit of $470.

c. Sam's profit was $315 less than the previous days'.

d. Sam has to sell at least 4 orange juices to make a profit.

e. Suggested option 1: Sam should consider the customer demand prior to increasing the cost of each lemonade glass.

Suggested option 2: Sam should consider the weather conditions prior to increasing the cost of each lemonade glass.

Note: There are other possible options.

22. a. $x \leq 1$

b. $x \geq -1$

c. In part **a**, there was a positive coefficient before the unknown variable, and in part **b**, there was a negative coefficient before the unknown variable, which affected the direction of the solution to the inequality. The change in direction of the inequality is determined by the sign of the coefficient of the unknown variable, where a negative coefficient means the sign must be reversed.

Back to contents

4A Expanding binomial products

Student practice

Worked example 1

a. $y^2 + 7y + 12$ **b.** $p^2 - 5p + 6$

c. $x^2 - 5x - 14$ **d.** $3t^2 + 2t - 5$

Understanding worksheet

1. **a.** $5x$ **b.** 21 **c.** $2x^2$ **d.** $15x$

2. **a.** 5 **b.** 4 **c.** 3 **d.** –

3. polynomial; two; distributive; sums

Fluency

4. **a.** $y^2 + 3y + 2$ **b.** $r^2 + 6r + 8$
c. $x^2 + 8x + 15$ **d.** $d^2 + 10d + 24$
e. $n^2 + 16n + 60$ **f.** $a^2 + 14a + 33$
g. $m^2 + 13m + 42$ **h.** $x^2 + 20x + 96$

5. **a.** $v^2 - 4v + 3$ **b.** $c^2 - 6c + 8$
c. $x^2 - 7x + 10$ **d.** $y^2 - 7y + 12$
e. $m^2 - 12m + 27$ **f.** $w^2 - 13w + 42$
g. $p^2 - 13p + 40$ **h.** $x^2 - 20x + 99$

6. **a.** $x^2 + 2x - 8$ **b.** $p^2 + 2p - 15$
c. $e^2 + 8e - 9$ **d.** $x^2 - 3x - 18$
e. $z^2 + z - 20$ **f.** $y^2 - y - 30$
g. $x^2 - 2x - 63$ **h.** $r^2 - 5r - 104$

7. **a.** $2x^2 + 7x + 6$ **b.** $3y^2 + 5y + 2$
c. $2x^2 + 2x - 4$ **d.** $2t^2 - 7t + 5$
e. $2z^2 + 5z - 3$ **f.** $2x^2 + 3x - 20$
g. $4r^2 + 12r - 7$ **h.** $6p^2 - 19p + 10$

8. A

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. Sam wants to extend her square living room. She will be able to add two and three metres to the length and width respectively. Using x m to represent the current length and width, formulate a binomial product expression for the area of the new extended living room.

Key points

- The current length and width are both x m.
- Sam will add 2 m to the length and 3 m to the width.
- Formulate a binomial product expression for the new area.

Explanation

Write expressions for the new length and width of the living room.

New length $= x + 2$

New width $= x + 3$

New area = new length × new width

$$A = (x + 2)(x + 3)$$

Answer

The binomial product expression for the area of the new extended living room is $(x + 2)(x + 3)$.

11. Duyen is changing the layout of her farm in a computer game. She wants to alter the square cabbage patch so that one side is three blocks longer and the other is one block shorter. Using p to represent the number of blocks along one side of the current cabbage patch, write an expanded expression for the area of the cabbage patch after Duyen changes it.

Key points

- The cabbage patch is a square with side lengths of p blocks.
- Duyen adds three blocks to one side and takes away one block for the other side.
- Write an expanded expression for the new area.

Explanation

Write expressions for the new length and width of the cabbage patch.

New length $= p + 3$

New width $= p - 1$

New area = new length × new width

$$A = (p + 3)(p - 1)$$

Expand the brackets by using the distributive law.

$$A = p \times p + p \times (-1) + 3 \times p + 3 \times (-1)$$

$$A = p^2 - p + 3p - 3$$

Simplify the expression by collecting like terms.

$$A = p^2 + 2p - 3$$

Answer

An expanded expression for the area of the cabbage patch after Duyen changed it is $p^2 + 2p - 3$.

12. Two high school football teams have an equal number of players (n) each, including substitutes. On the day of a match three people from one team and two from the other were absent. Write an expanded expression for the number of handshakes that occurred at the end of the game if each present member of one team shook hands with each present member of the other team exactly once.

Key points

- The two teams have an equal number of players (n) each.
- On one team, there were three fewer people; on the other team, there were two fewer people.
- Each present member of one team shook hands with each present member of the other team exactly once.
- Write an expanded expression for the total number of handshakes.

Explanation

Write expressions for the number of present members on each team on the day of the match.

Number of present members on the first team $= n - 3$

Number of present members on the second team $= n - 2$

Total number of handshakes = number of present members on the first team × number of present members on the second team

$$= (n - 3)(n - 2)$$

Expand the brackets by using the distributive law.

$$= n \times n + n \times (-2) + (-3) \times n + (-3) \times (-2)$$

$$= n^2 - 2n - 3n + 6$$

Simplify the expression by collecting like terms.

$$= n^2 - 5n + 6$$

Answer

The expanded expression for the total number of handshakes is $n^2 - 5n + 6$.

13. The city council is extending the boundaries of a rectangular suburb which is currently x km long and y km wide. An additional 3 km will be added to the length and 4 km will be added to the width of the area. Express the new area of the suburb in expanded form.

Key points

- The current length of the rectangle is x km and the current width is y km.
- The new length would be 3 km longer than the current length, and the new width will be 4 km longer than the current width.
- Write an expression for the new area and expand the expression.

Explanation

Write expressions for the new length and width of the suburb.

New length $= x + 3$

New width $= y + 4$

New area = new length × new width

$$A = (x + 3)(y + 4)$$

Expand the brackets by using the distributive law.

$$A = x \times y + x \times 4 + 3 \times y + 3 \times 4$$

$$A = xy + 4x + 3y + 12$$

Answer

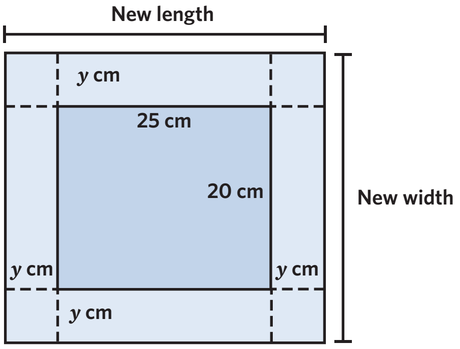
The new area of the suburb in expanded form is $xy + 4x + 3y + 12$.

14. Salma wants to frame her 25 cm by 20 cm diploma certificate and display it on the wall. The wooden border of the frame she chose is y cm wide. Write an expanded expression showing the total area of the entire picture including the frame.

Key points

- The certificate has a length of 25 cm and a width of 20 cm.
- The frame is y cm wide.
- Write an expanded expression for the total area, including the frame.

Explanation



Write expressions for the new length and width of the entire frame with the certificate inside.

New length $= y + 25 + y = 2y + 25$

New width $= y + 20 + y = 2y + 20$

New area = new length × new width

$$A = (2y + 25)(2y + 20)$$

Expand the brackets by using the distributive law.

$$A = 2y \times 2y + 2y \times 20 + 25 \times 2y + 25 \times 20$$

$$A = 4y^2 + 40y + 50y + 250$$

Simplify the expression by collecting like terms.

$$A = 4y^2 + 90y + 500 \text{ cm}^2$$

Answer

The expanded expression for the total area, including the frame is $4y^2 + 90y + 500 \text{ cm}^2$.

Reasoning

15. a. $A = x^2 + ax + bx + ab$

b. The area of this room's plan is $A = x^2 + 35x + 300$.

c. The area of this room's plan is $A = x^2 + 3x - 4$.

d. Suggested option 1: My dream home would have a games room.

Suggested option 2: My dream home would have a large garden with fruit trees.

Note: There are other possible options.

16. a. $x^2 + 7x + 10$

b. $10 + 7x + x^2$

c. The expanded and simplified expressions from parts **a** and **b** are equivalent because the commutative law states that the sum of values does not change when they are added in a different order.

Exam-style

17. C

18. a. $3x^2 + 8x - 3$

b. 8

19. $(3x - 8)(x - 8) = 3x^2 - 32x + 64$

20. $y^2 + 3y - 10$

Remember this?

21. C **22.** E **23.** B

4B Perfect squares and the difference of two squares

Student practice

Worked example 1

a. $y^2 + 4y + 4$ **b.** $p^2 - 8p + 16$

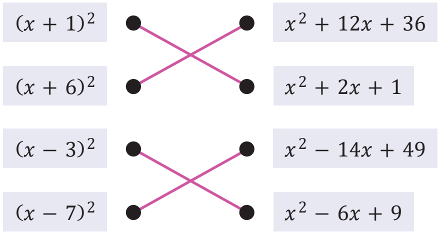
c. $4r^2 + 20r + 25$

Worked example 2

a. $y^2 - 16$ **b.** $4t^2 - 9$

Understanding worksheet

1. **Factorised form** **Expanded form**



2. **a.** 1 **b.** 25 **c.** 16 **d.** 49

3. squares; binomial; expanded; difference

Fluency

4. **a.** y^2+6y+9 **b.** $x^2+10x+25$
c. $a^2+20a+100$ **d.** t^2-6t+9
e. $x^2-12x+36$ **f.** $b^2-14b+49$
g. $121+22m+m^2$ **h.** $64-16n+n^2$

5. **a.** $4c^2+4c+1$ **b.** $9x^2+24x+16$
c. $16t^2+24t+9$ **d.** $4y^2-28y+49$
e. $16r^2-24r+9$ **f.** $25d^2-80d+64$
g. $100+60x+9x^2$ **h.** $81-126p+49p^2$

6. **a.** y^2-25 **b.** x^2-36
c. c^2-64 **d.** d^2-9
e. n^2-121 **f.** m^2-196
g. $81-x^2$ **h.** $144-y^2$

7. **a.** $4y^2-1$ **b.** $4x^2-9$
c. $9t^2-4$ **d.** $16a^2-9$
e. $25p^2-36$ **f.** $49b^2-64$
g. $25-36x^2$ **h.** $225-64d^2$

8. **a.** p^2-t^2 **b.** $x^2+2xy+y^2$
c. $4c^2-d^2$ **d.** $4a^2+4ab+b^2$
e. $9r^2-6qr+q^2$ **f.** $4m^2+12mn+9n^2$
g. $25x^2-81y^2$ **h.** $16k^2-40km+25m^2$

9. C

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. A square room has been extended, so that exactly 2 m is added to the length and width. Write a binomial product expression for the new area of the room, using x to represent the length and width of the room prior to extension.

Key points

- The room is a square with a side length of x metres.
- The room is extended so that 2 m is added to the length and width.
- Write a binomial product expression for the new area of the room.

Explanation

Write an expression for the new length of the square room.

New length $= x+2$

New area $=$ (new length)^2

$A=(x+2)^2$

Answer

A binomial product expression for the area of the extended room is $(x+2)^2$ m^2.

12. A large square park is surrounded by a 1 m wide footpath on all sides. Write an expanded simplified expression for the total area of the park and footpath, using p to represent the length and width of the park.

Key points

- The park is a square with a side length and width of p metres.
- The park is surrounded by a 1 m wide footpath on all sides.
- Write an expanded simplified expression for the total area of the park and footpath.

Explanation

Write an expression for the total length of the park and footpath.

Total length $= p+2$

Total area $=$ (total length)^2

$A=(p+2)^2$

$A=(p+2)(p+2)$

Expand the binomial product.

$A=p^2+2p+2p+4$

Simplify the expression by collecting like terms.

$A=p^2+4p+4$

Answer

An expanded expression for the total area of the park and footpath is p^2+4p+4 m^2.

13. Charlie is building a house in a simulation computer game. He uses the square room template to place a room inside the house. Charlie decides to add five units to the length and reduce the width by five units. Write an expanded simplified expression for the area of Charlie's room after alterations.

Key points

- The room is a square.
- Charlie adds five units to the length.
- Charlie removes five units from the width.
- Write an expanded simplified expression for the new area.

Explanation

Let x be the original length of the square room.

Write expressions for the new length and width of the room.

New length $= x+5$

New width $= x-5$

New area = new length × new width

$$A = (x + 5)(x - 5)$$

Expand the binomial product.

$$A = x^2 - 5x + 5x - 25$$

Simplify the expression by collecting like terms.

$$A = x^2 - 25$$

Answer

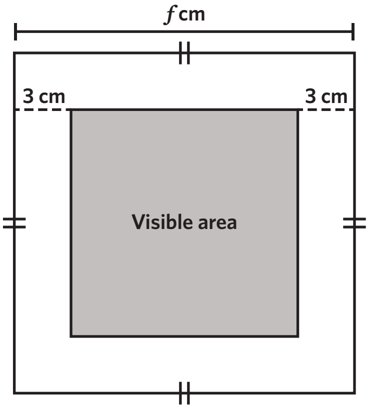
An expanded expression for the area of the room after Charlie changes it is $x^2 - 25$.

14. Dawn is putting a completed cross-stitch in a square frame that has a border that is 3 cm wide. Write an expanded simplified expression for the visible part of the picture, using f to represent the length and width of the entire frame.

Key points

- The frame is a square with a side length and width of f centimetres.
- The frame is 3 cm wide on each side.
- Write an expanded simplified expression for the visible area of the picture.

Explanation



Write an expression for the visible length of the cross-stich.

Length of visible area $= f - 6$

Visible area = (length of visible area)^2

$$A = (f - 6)^2$$

$$A = (f - 6)(f - 6)$$

Expand the binomial product.

$$A = f^2 - 6f - 6f + 36$$

Simplify the expression by collecting like terms.

$$A = f^2 - 12f + 36$$

Answer

An expanded expression for the visible part of the cross-stitch is $f^2 - 12f + 36$ cm^2.

15. An airport in the shape of a square is extending their runway to accommodate larger planes. They are planning on adding to the overall length of the airport in the direction of the runway, but reducing the width by an equal amount due to council regulations. Write an expanded simplified expression for the new area of the airport if they were to add 650 m to the length of the airport while reducing the width by the same amount.

Key points

- The airport is a square.
- 650 m is added to the length of the airport.
- 650 m is removed from the width of the airport.
- Write an expanded simplified expression for the new area of the airport.

Explanation

Let a be the original length and width of the square airport.

Write expressions for the new length and width of the airport.

New length $= a + 650$

New width $= a - 650$

New area = new length × new width

$$A = (a + 650)(a - 650)$$

Expand the binomial product.

$$A = a^2 - 650a + 650a - 422\,500$$

Simplify the expression by collecting like terms.

$$A = a^2 - 422\,500$$

Answer

An expanded expression for the area of the airport after the runway extension is $a^2 - 422\,500$ m^2.

Reasoning

16. **a.** One side length of the base is given by $30 - 2x$ cm.

b. The total area of the base is given by $(30 - 2x)(30 - 2x) = (30 - 2x)^2$ cm^2.

c. The expanded simplified expression for the area of the base is $900 - 120x + 4x^2$.

d. The area of the base is 484 cm^2 when $x = 4$ cm.

e. Suggested option 1: Homemade gifts show that time and effort were put into giving someone a present.

Suggested option 2: Homemade gifts can be stressful to make because of time constraints and quality.

Note: There are other possible options.

17. **a.** $x^2 + 10x + 25$

b. $4x^2 + 20x + 25$

c. The constant is not affected by changing the coefficient of x terms in binomial products. Only the terms involving x are affected by this.

Exam-style

18. D

19. **a.** $y^2 - 8y + 16$

b. 1

20. $A = (x - 6)^2 = x^2 - 12x + 36$

21. $81 - x^2$ m^2

Remember this?

22. A

23. D

24. E

4C Factorising algebraic expressions

Student practice

Worked example 1

a. $3a$

b. $4ab$

Worked example 2

a. $2(x + 5)$

b. $-4x(2x + 3)$

Back to contents

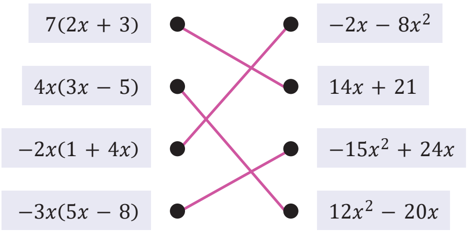
Worked example 3

a. $(4 + x)(x + y)$ **b.** $(6 - a)(3a - 1)$

Understanding worksheet

1. **a.** 4 **b.** $2x$ **c.** $2c$ **d.** $-3a$

2. **Factorised form** **Expanded form**



3. reverse; HCF; common; leading

Fluency

4. **a.** 2 **b.** $5t$ **c.** $6a$ **d.** $6x$
e. $3y$ **f.** $2xy$ **g.** $5ab$ **h.** $4pq$

5. **a.** $6(x + 2)$ **b.** $3y(1 + 8y)$
c. $7a(1 - 3a)$ **d.** $b(4a - 5)$
e. $2x(5x + 2y)$ **f.** $4y(3z + y)$
g. $5r(t - 7r)$ **h.** $14b(ac - 2b)$

6. **a.** $-5(a + 3)$ **b.** $-4(x - 4)$
c. $-3x(x + 2)$ **d.** $-3t(4 - t)$
e. $-xy(7y + 1)$ **f.** $-13a(b - 3a)$
g. $-12t(t - 2r)$ **h.** $-3xy(7 + y)$

7. **a.** $(5 + t)(t + r)$ **b.** $(3 + x)(x - 4)$
c. $(9 - a)(8a + 3)$ **d.** $(x - 2)(6x + 1)$
e. $(2x + 5)(7 - 4x)$ **f.** $(4x - y)(x + y)$
g. $(7t - 6)(5 - 3t)$ **h.** $(1 - x)(2y + 5)$

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. A number of adults and children attended a school production, with tickets costing \$5 per person. Using a to represent the number of adults and c to represent the number of children, write a factorised expression showing the total cost for the whole group.

Key points

- A number of adults and children attended a school production.
- Tickets cost \$5 per person.
- Write a factorised expression showing the total cost for a adults and c children.

Explanation

The cost of a adults paying \$5 for a ticket is $5 \times a = 5a$.

The cost of c children paying \$5 for a ticket is $5 \times c = 5c$.

Write an expression for the total cost of the group of adults and children.

$5a + 5c$

Factorise the expression for the total cost of the group of adults and children.

$5a + 5c = 5(a + c)$

Answer

$5(a + c)$ represents the total cost of the group of adults and children.

11. Junji is planning a concert for his band and is charging \$7 pre-sale and \$10 on the door for the tickets. Junji's band played two shows and sold out of every ticket on both nights. Write a factorised expression showing the total profits made by Junji's band over the two nights.

Key points

- Ticket prices are \$7 pre-sale and \$10 on the door.
- Both of the two shows sold out of every ticket.
- Write a factorised expression showing the total profits made by the band.

Explanation

Define the variables. Any variable may be chosen to represent the number of tickets.

Let p be the number of pre-sale tickets available for one show.

Let d be the number of tickets on the door available for one show.

The value of p tickets worth \$7 is $7 \times p = 7p$.

The value of d tickets worth \$10 is $10 \times d = 10d$.

$\therefore$ The profit made by the band from one sold-out show is $7p + 10d$.

Multiply the profit made by the band from one sold-out show by 2 to determine the total profits made by the band from two sold-out shows.

$2 \times (7p + 10d) = 2(7p + 10d)$

Answer

$2(7p + 10d)$ represents the total profits made by the band.

12. Tony is a fruit picker at an orchard where apples and pears grow. Every day, he picks exactly the same amount of each type of fruit and is paid \$3 for a kg of apples and \$5 for a kg of pears. Write a factorised expression showing Tony's total pay over a four-day work week.

Key points

- Tony picks exactly the same amount of each type of fruit every day.
- He is paid \$3 for a kg of apples.
- He is paid \$5 for a kg of pears.
- Write a factorised expression showing Tony's total pay for four days of work.

Explanation

Define the variables. Any variable may be chosen to represent the mass of apples and pears.

Let a be the number of kg of apples picked by Tony in a day.

Let p be the number of kg of pears picked by Tony in a day.

Tony is paid \$3 for each kg of apples he picks. Therefore, for a kg of apples, he is paid $3 \times a = 3a$.

Tony is paid \$5 for each kg of pears he picks. Therefore, for p kg of pears, he is paid $5 \times p = 5p$.

$\therefore$ Tony is paid $3a + 5p$ every day for picking a kg of apples and p kg of pears.

Back to contents

Multiply Tony's daily pay by 4 to determine Tony's total pay for four days of work.

$4 \times (3a + 5p) = 4(3a + 5p)$

Answer

$4(3a + 5p)$ represents Tony's total pay over a four-day work week.

13. A number of students from five classes and three teachers participated in an origami crane making competition. There is an equal number of students in each class and each person who participated made an average of 10 cranes. Write a factorised expression showing the total number of origami cranes made by the students and teachers.

Key points

- A number of students from five classes and three teachers participated in a competition making origami cranes.
- There are an equal number of students in each class.
- Each person made an average of 10 cranes.
- Write a factorised expression showing the total number of origami cranes made by the students and teachers.

Explanation

Let s be the number of students in each class. The total number of participants, including the three teachers is given by:

$5s + 3$

Each person made an average of 10 cranes and so the total number of cranes is given by:

$$\begin{aligned} 10 \times \text{number of participants} &= 10 \times (5s + 3) \\ &= 10(5s + 3) \end{aligned}$$

Answer

The total number of origami cranes made by the students and teachers is given by $10(5s + 3)$.

14. At an all ages music festival there were 50 children under 12 and twice as many teenagers as there were adults. Write a factorised equation showing total ticket sales of \$41 800 if each person paid \$95 to be there.

Key points

- 50 children under 12 and twice as many teenagers as there were adults attended a music festival.
- The total from ticket sales was \$41 800.
- Each ticket cost \$95.
- Write a factorised equation showing total ticket sales.

Explanation

Let x be the number of adults who attended the music festival.

Given that twice as many teenagers as adults attended, the number of teenagers that attended is $2x$.

Write an equation that represents the total proceeds from ticket sales, given that each ticket costs \$95.

Total proceeds = 95 × number of children attendees + 95 × number of adult attendees + 95 × number of teenage attendees

Substitute known values into the equation and factorise.

$$\begin{aligned} 41\,800 &= 95 \times 50 + 95 \times x + 95 \times 2x \\ &= 95(50 + x + 2x) \\ &= 95(50 + 3x) \end{aligned}$$

Answer

Total ticket sales can be shown by $41\,800 = 95(50 + 3x)$.

Reasoning

15. **a.** The factorised formula is $p = 0.3(2x - y)$.

b. The new price of the couch is given by $0.6(x - 25)$.

c. The new price of the computer is given by $0.3(2x - 129)$.

d. The new price of Amber's TV is given by $699 = 0.3(2x - 99.95)$.

e. Suggested option 1: An advantage of waiting for sales to purchase an item is that you save money because the prices are reduced.

Suggested option 2: A disadvantage of waiting for sales to purchase an item is that you may need the item immediately before sales have started.

Note: There are other possible options.

16. **a.** $-2(x - 4)$

b. $-2(x + 4)$

c. When a negative HCF has been divided out of a positive term, the remaining factor is always negative. When a negative HCF has been divided out of a negative term, the remaining factor is always positive.

Exam-style

17. D

18. **a.** $(x + 3)$

b. $(5 - x)(x + 3)$

19. $l = 5 + 3x$

20. $432 = 3(s + 2t)$

Remember this?

21. A

22. A

23. C

4D Factorising the difference of two squares

Student practice

Worked example 1

a. $(x + 2)(x - 2)$

b. $(3t + 4)(3t - 4)$

c. $(4x + 7y)(4x - 7y)$

Worked example 2

a. $2(x + 3)(x - 3)$

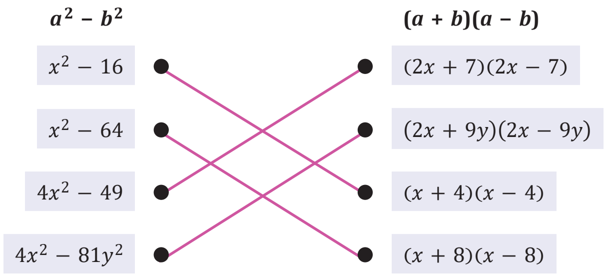
b. $(x + 5)(x - 1)$

Understanding worksheet

1.

$a^2 - b^2$	a	b
$x^2 - 25$	x	5
$4x^2 - 9$	$2x$	3
$9t^2 - 4y^2$	$3t$	$2y$
$(x - 2)^2 - 16$	$x - 2$	4

2.



3. binomial; subtracted; terms; formula

Back to contents

Fluency

4. **a.** $(x + 1)(x - 1)$ **b.** $(x + 4)(x - 4)$
c. $(a + 6)(a - 6)$ **d.** $(y + 9)(y - 9)$
e. $(p + 12)(p - 12)$ **f.** $(7 + r)(7 - r)$
g. $(10 + c)(10 - c)$ **h.** $(k + 50)(k - 50)$

5. **a.** $(2x + 1)(2x - 1)$ **b.** $(2t + 3)(2t - 3)$
c. $(3y + 5)(3y - 5)$ **d.** $(4y + 3)(4y - 3)$
e. $(5x + 9)(5x - 9)$ **f.** $(12n + 1)(12n - 1)$
g. $(10 + 7m)(10 - 7m)$ **h.** $(20 + 9x)(20 - 9x)$

6. **a.** $(2a + b)(2a - b)$ **b.** $(2x + 3y)(2x - 3y)$
c. $(3x + 4y)(3x - 4y)$ **d.** $(m + 2n)(m - 2n)$
e. $(7p + 3q)(7p - 3q)$ **f.** $(9k + 4m)(9k - 4m)$
g. $(8t + 9r)(8t - 9r)$ **h.** $(3b + 10a)(3b - 10a)$

7. **a.** $3(p + 1)(p - 1)$ **b.** $2(y + 4)(y - 4)$
c. $3(x + 2)(x - 2)$ **d.** $2(t + 5)(t - 5)$
e. $5(x + 6)(x - 6)$ **f.** $3(7 + r)(7 - r)$
g. $7(m + 2n)(m - 2n)$ **h.** $3(xy + 3)(xy - 3)$

8. **a.** $(x + 3)(x - 1)$ **b.** $(y + 7)(y + 1)$
c. $(r + 9)(r + 1)$ **d.** $(t - 5)(t - 7)$
e. $(p + 3)(p - 7)$ **f.** $(y + 12)(y + 2)$
g. $(x + 11)(7 - x)$ **h.** $(x + 2)(18 - x)$

9. B

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. Cindy has added a garage to her square yard. The sides of the yard are y metres long while the length and width of the garage are both 6 metres long. Express the new area of Cindy's yard, excluding the garage, as a difference of two squares.

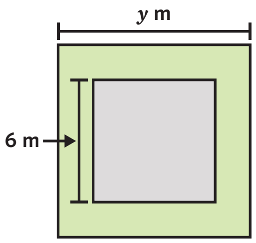
Key points

- Cindy's yard is a square with sides of length y metres.
- A garage of dimensions 6 metres by 6 metres is added to the yard.
- The new area of the yard, excluding the garage, needs to be expressed as a difference of two squares.

Explanation

The area of a square is given by the square of the length of its side. Therefore, the original area of the yard is y^2 m^2.

The area of the garage is $6 \times 6 = 36$ m^2.



The new area of the yard, excluding the garage, is the original area of the yard minus the area of the garage. This can be written as $y^2 - 36$.

Answer

The new area of Cindy's yard can be expressed as $y^2 - 36$ m^2.

12. Kevin has a square section in his backyard covered in grass. He redesigns it into a rectangle and now the total area of grass in his yard is given by $g^2 - 16$, where g is one side length of the old square patch of grass in metres. Determine how many metres were taken off the length and added to the width of the grass patch by factorising the given expression.

Key points

- Kevin has a square section that is covered in grass.
- The grass is redesigned, and the total area of the grass is now given by the expression $g^2 - 16$, where g is the length of one side of the old grass patch in metres.
- Determine how many metres were taken off the length and added to the width of the grass patch by factorising the given expression.

Explanation

The expression $g^2 - 16$ is a difference of squares, which can be factorised using the formula $a^2 - b^2 = (a + b)(a - b)$.

Identify a^2 and b^2 in the expression $g^2 - 16$ to determine the values of a and b.

$a^2 = g^2$ and $a = \sqrt{g^2} = g$

$b^2 = 16$ and $b = \sqrt{16} = 4$

Factorise the difference of two squares by substituting the values for a and b into the general formula.

$g^2 - 16 = (g + 4)(g - 4)$

Answer

4 metres were taken off the length and added to the width of the grass patch.

13. Two table tennis teams, A and B, have exactly t members each. One day, some players from the A team joined the players on the B team for a practice round robin tournament, where each member of one team plays against each member of the other team exactly once. If the total number of games that were played during the tournament is $t^2 - 9$, then how many players from the A team played for the B team?

Key points

- There are two table tennis teams, each with exactly t members.
- Some players from the A team joined the B team for a practice round robin tournament.
- Each member of one team plays against each member of the other team exactly once.
- The total number of games played during the tournament is given by the expression $t^2 - 9$.
- How many players from the A team played for the B team?

Explanation

The expression $t^2 - 9$ is a difference of squares, which can be factored using the formula $a^2 - b^2 = (a + b)(a - b)$.

Identify a^2 and b^2 in the expression $t^2 - 9$ to determine the values of a and b.

$a^2 = t^2$ and $a = \sqrt{t^2} = t$

$b^2 = 9$ and $b = \sqrt{9} = 3$

Factorise the difference of two squares by substituting the values for a and b into the general formula.

$t^2 - 9 = (t + 3)(t - 3)$

Since some players from the A team joined the B team, the number of players on the A team decreased and the number of players on the B team increased. The difference in the number of players on each team is represented by the 3 in $(t - 3)$ and $(t + 3)$.

Answer

Three players from the A team joined the B team for the round robin tournament.

14. An art gallery in the shape of a square has side lengths $2h$ metres long and contains an open courtyard directly in the centre of the building, accessible only from the gallery. Write a factorised expression for the total interior area of the gallery, excluding the open courtyard, if the area of the courtyard is 49 m^2.

Key points

- An art gallery is in the shape of a square with side lengths of $2h$ metres.
- There is an open courtyard directly in the centre of the gallery, accessible only from the gallery.
- The area of the courtyard is 49 m^2.
- Write a factorised expression for the total interior area of the gallery, excluding the open courtyard.

Explanation

The total area of the square gallery is given by the square of the side length, which is $(2h)^2 = 4h^2$ square metres.

The area of the courtyard is given as 49 m^2.

The interior area of the gallery, excluding the courtyard, is the difference between the total area of the gallery and the area of the courtyard. This can be written as $4h^2 - 49$.

The expression $4h^2 - 49$ is a difference of squares, which can be factorised using the formula $a^2 - b^2 = (a + b)(a - b)$.

Identify a^2 and b^2 in the expression $4h^2 - 49$ to determine the values of a and b.

$a^2 = (2h)^2$ and $a = 2h$

$b^2 = 49$ and $b = \sqrt{49} = 7$

Factorise the difference of two squares by substituting the values of a and b into the general formula.

$4h^2 - 49 = (2h + 7)(2h - 7)$.

Answer

The factorised expression for the total interior area of the gallery, excluding the open courtyard, is given by $(2h + 7)(2h - 7)$ m^2.

15. Nancy is redeveloping a square area into a playground. She initially adds two metres to each side, then decides to allocate 9 m^2 of this new larger area to a public bathroom. Express the total area of the planned playground, excluding the bathroom, as a fully factorised and simplified binomial product, using x to represent one side length of the original square area.

Key points

- Nancy is redeveloping a square area into a playground.
- She initially adds two metres to each side of the square.
- She then allocates 9 m^2 of this new larger area to a public bathroom.
- Express the total area of the planned playground, excluding the bathroom, as a fully factorised and simplified binomial product, using x to represent one side length of the original square area.

Explanation

One side length of the new larger area of the playground is $x + 2$ metres after Nancy adds two metres to it. The area is then given by $(x + 2)^2$.

Nancy reduces the playground area by 9 m^2 so that a public bathroom can be added. The area is now given by $(x + 2)^2 - 9$.

The expression $(x + 2)^2 - 9$ is a difference of squares, which can be factorised using the formula $a^2 - b^2 = (a + b)(a - b)$. Identify a^2 and b^2 to determine the values of a and b.

$a^2 = (x + 2)^2$ and $a = \sqrt{(x + 2)^2} = x + 2$

$b^2 = 9$ and $b = \sqrt{9} = 3$

Substitute the values for a and b into the general formula and simplify.

$$\begin{aligned}(x + 2)^2 - 9 &= (x + 2 + 3)(x + 2 - 3)\\ &= (x + 5)(x - 1)\end{aligned}$$

Answer

The total area of the planned playground, excluding the bathroom, is given by $(x + 5)(x - 1)$ m^2.

Reasoning

16. **a.** The total area of the frame is $x^2 - y^2$ cm^2.

b. If $y = 5$ cm, the total area of the frame is $x^2 - 25$ cm^2.

c. $x^2 - 25$ can be factorised to $(x + 5)(x - 5)$.

d. If the total area of the frame is 96 cm, the length of x is 11 cm.

e. Suggested option 1: Machines are faster than people and can produce more items.

Suggested option 2: Some handmade items are of better quality compared to those made by machines.

Note: There are other possible options.

17. **a.** The common binomial factor in the expression is $x + 3$.

b. Using the common binomial factor, the expression factorises to $(x + 3)(x - 3)$.

c. The factorised expression from part **b** produces a difference of two squares. This is because the expanded form of the expression showed an unsimplified difference of two squares including the common terms which add to zero after simplifying.

Exam-style

18. C

19. **a.** $3(x^2 - 36)$

b. $3(x + 6)(x - 6)$

20. $(6 + s)(6 - s)$

21. $(x + 13)(x - 5)$

Remember this?

22. D

23. B

24. A

4E Factorising by grouping

Student practice

Worked example 1

a. $(x + 2)(x + 5)$

b. $(x - 4)(x + 3)$

Worked example 2

a. $(x + 8)(x + 2)$

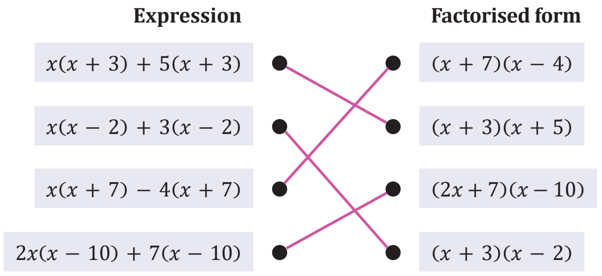
b. $(2x - 1)(x + 3)$

Understanding worksheet

1. **a.** x **b.** 4 **c.** – **d.** $2x$

Back to contents

2.



3. common; grouping; rearranged; binomial

Fluency

4. **a.** $(x + 3)(x + 5)$ **b.** $(a + 3)(a + 4)$
c. $(y + 6)(y + 3)$ **d.** $(p + 5)(p + 4)$
e. $(t + 3)(t + 10)$ **f.** $(2x + 3)(x + 6)$
g. $(2r + 5)(r + 7)$ **h.** $(3x + 1)(x + 8)$

5. **a.** $(x + 2)(x - 3)$ **b.** $(x + 5)(x - 3)$
c. $(y + 6)(y - 2)$ **d.** $(x + 2)(x - 3)$
e. $(r + 7)(r - 2)$ **f.** $(t - 2)(t - 5)$
g. $(2x + 5)(x - 3)$ **h.** $(2y - 7)(y - 1)$

6. **a.** $(x + 2)(x + 5)$ **b.** $(y + 2)(y + 7)$
c. $(p + 6)(p + 2)$ **d.** $(k + 5)(k + 4)$
e. $(m + 3)(m + 10)$ **f.** $(2x + 5)(x + 3)$
g. $(2n + 3)(n + 7)$ **h.** $(4x + 1)(x + 3)$

7. **a.** $(x + 4)(x - 2)$ **b.** $(y + 5)(y - 2)$
c. $(b + 4)(b - 5)$ **d.** $(x - 3)(x - 5)$
e. $(a - 6)(a - 7)$ **f.** $(2t - 5)(t + 3)$
g. $(3x - 1)(x - 4)$ **h.** $(5b - 4)(b - 6)$

8. A

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. The total area of a rectangular playground can be given by the expression $x(x + 4) + 2(x + 4)$ where x represents an unknown distance in metres. Factorise the expression by grouping in order to determine the dimensions (length and width) of the playground area in terms of x, assuming that length is greater than the width.

Key points

- The total area of the rectangular playground is given by $x(x + 4) + 2(x + 4)$.
- x represents an unknown distance in metres.
- Factorise the expression to determine the dimensions (length and width) of the playground area in terms of x, where length is the longest side.

Explanation

The area of a rectangle is given by $l \times w$, where l is the length and w is the width.

Factorise the expression for the total area of the playground to determine the expressions for its length and width.

$$A = l \times w$$

$x(x + 4) + 2(x + 4) = (x + 4)(x + 2)$

Length > width

$(x + 4) > (x + 2)$

$\therefore$ Length $= (x + 4)$ and width $= (x + 2)$

Answer

The length of the playground area is given by $x + 4$ m and the width is given by $x + 2$ m.

11. Oscar is a carpenter who has been cutting a square flat piece of wood with side lengths w cm long. After all the adjustments were made, the area of the wood was given by $w^2 - 4w - 3w + 12$. Factorise the expression to determine how many centimetres were cut from the sides of the piece of wood.

Key points

- The side lengths of the square flat piece of wood are given by w cm before cutting.
- The area of the wood is calculated by the expression $w^2 - 4w - 3w + 12$ after cutting.
- Factorise the expression to determine how many centimetres were cut from the sides of the piece of wood.

Explanation

Prior to cutting, the area of the wood was given by:

$A_1 = w^2$

After cutting, the area of the wood was given by:

$A_2 = w^2 - 4w - 3w + 12$

To factorise A_2 group pairs of terms together and factorise each pair by dividing out the HCF.

$A_2 = w(w - 4) + (-3(w - 4))$

$= w(w - 4) - 3(w - 4)$

Divide out the common binomial factor.

$A_2 = (w - 3)(w - 4)$

The factorised expression shows that 3 and 4 have been subtracted from w.

Answer

3 cm and 4 cm were cut from the sides of the piece of wood.

12. Suresh has added an extension to his small cottage with a square floor plan. After the alterations, the total area of the house can be given by $c^2 + 42 + 7c + 6c$ where c represents the original side lengths of the cottage, in metres. Determine how many metres were added to the length and width of the cottage's floor plan by factorising the expression.

Key points

- The total area of the cottage after extension is represented by $c^2 + 42 + 7c + 6c$.
- c represents the original side lengths of the cottage, in metres.
- Factorise the expression for area to determine how many metres were added to the length and width.

Explanation

Prior to extension, the area of Suresh's cottage was given by:

$A_1 = c^2$

After extension, the area of the cottage is given by:

$A_2 = c^2 + 42 + 7c + 6c$

Rearrange A_2 so that consecutive pairs of terms are unlike and have a common factor.

$A_2 = c^2 + 7c + 6c + 42$

Back to contents

Group pairs of terms together and factorise each pair by dividing out the HCF.

$A_2 = c(c + 7) + 6(c + 7)$

Divide out the common binomial factor.

$A_2 = (c + 6)(c + 7)$

The factorised expression shows that 6 and 7 were added to c.

Answer

Suresh added 7 m and 6 m to the length and width of his cottage.

13. At the start of the year, Tony had exactly the same number (n) of pairs of shoes and socks. By the end of the year, if he mixed and matched every single pair of shoes he owns with every single pair of socks exactly once, there would be a total of $n^2 - 18 + 6n - 3n$ combinations. How many pairs of shoes did Tony add to his collection over the course of the year?

Key points

- n represents the same number of pairs of shoes and socks at the start of the year.
- After mixing and matching every pair exactly once, the total number of combinations by the end of the year is represented as so, $n^2 - 18 + 6n - 3n$.
- How many pairs of shoes did Tony add to his collection over the course of the year?

Explanation

At the start of the year, the number of combinations of socks and shoes available to Tony is given by:

$C_1 = n \times n = n^2$

By the end of the year, the number of combinations is given by:

$C_2 = n^2 - 18 + 6n - 3n$

Rearrange C_2 and factorise it by grouping.

$$\begin{aligned} C_2 &= n^2 + 6n - 3n - 18 \\ &= n(n + 6) + (-3(n + 6)) \\ &= (n - 3)(n + 6) \end{aligned}$$

The factorised expression shows that 3 items were lost and 6 were added throughout the year.

Answer

Tony had added six pairs of shoes to his collection over the course of the year.

14. A farmhouse with a square floor plan f metres long on each side is located in the north-west corner of a large rectangular property. The total area of the property and farmhouse is given by $f^2 + 300f + 25f + 7500$ m^2. What is the length of the longest side of the property if the area of the farmhouse is 121 m^2?

Key points

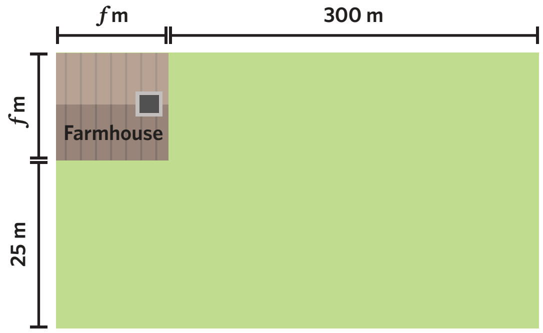
- A square house with side lengths f metres long is located in the north-west corner of a property.
- The total area of the property and farmhouse is represented by the expression $f^2 + 300f + 25f + 7500$.
- What is the length of the longest side of the property if the area of the farmhouse is 121 m^2?

Explanation

Factorise the expression for the area of the property to find expressions for its length and width.

$$\begin{aligned} f^2 + 300f + 25f + 7500 &= f(f + 300) + 25(f + 300) \\ &= (f + 25)(f + 300) \end{aligned}$$

The width is given by $f + 25$ m and the length (longest side) is given by $f + 300$ m.



Calculate the value of f using the total area of the farmhouse, $f^2 = 121$ m^2.

$f = \sqrt{f^2} = \sqrt{121} = 11$ m

Substitute $f = 11$ m into the expression for the length of the property and evaluate.

$f + 300 = 11 + 300 = 311$ m

Answer

The length of the longest side of the property is 311 m.

Reasoning

15. **a.** $10(x + 15)$

b. $10(x + 15) + x(x + 15) = (10 + x)(x + 15)$

c. The total area of the paddock is 13 334 m^2.

d. Hannah can only have 3 horses.

e. Suggested option 1: To ensure the animals are not living in crowded conditions.

Suggested option 2: To ensure the animals have enough room for physical exercise.

Note: There are other possible options.

16. **a.** $(x - 2)(x - 4)$

b. $(x - 4)(x - 2)$

c. The factorised expressions in parts **a** and **b** are equivalent, with the only difference being the order of the binomial expressions that are being multiplied, which does not affect the product. When rearranging expressions to factorise by grouping it is important to group unlike terms in a pair, but the order in which those terms are paired is not important.

Exam-style

17. B

18. **a.** $(p - 4)(p - 13)$

b. 22

19.
$$\begin{aligned} A_1 + A_2 &= 6x^2 + 3x + 4x + 2 \\ &= 3x(2x + 1) + 2(2x + 1) \\ &= (3x + 2)(2x + 1) \\ &= A \end{aligned}$$

20. $(4x + 5)(x - 2)$

Remember this?

21. D

22. C

23. D

Back to contents

4F Simplifying algebraic fractions – multiplication and division

Student practice

Worked example 1

a. $3x + 2$ **b.** $\frac{x - 6}{7}$ **c.** 3

Worked example 2

a. $5x$ **b.** $\frac{1}{3(x - 7)}$

Worked example 3

a. $\frac{3}{7x}$ **b.** $\frac{1}{28}$

Understanding worksheet

1. **a.** $\frac{4\cancel{(x + 2)}}{7} \times \frac{14}{\cancel{x + 2}}$

b. $\frac{9(x + 1)}{\cancel{x - 8}} \times \frac{3\cancel{(x - 8)}}{4}$

c. $\frac{\cancel{(y + 2)}(y - 7)}{2(y + 3)} \times \frac{y + 5}{\cancel{y + 2}}$

d. $\frac{3\cancel{(p - 5)}(p - 4)}{2\cancel{(p + 3)}} \times \frac{3(p - 4)\cancel{(p + 3)}}{\cancel{p - 5}}$

2. **a.** $\frac{3x}{8} \div \frac{15x}{14} = \frac{3x}{8} \times \frac{14}{15x}$

b. $\frac{2(x - 1)}{3x} \div \frac{x - 1}{9x} = \frac{2(x - 1)}{3x} \times \frac{9x}{x - 1}$

c. $\frac{2(y + 3)}{7y} \div \frac{y + 3}{3y} = \frac{2(y + 3)}{7y} \times \frac{3y}{y + 3}$

d. $\frac{7(x + 4)(x - 1)}{3(x + 1)} \div \frac{4(x - 1)}{21x}$

$= \frac{7(x + 4)(x - 1)}{3(x + 1)} \times \frac{21x}{4(x - 1)}$

3. numerator; division; common; factorised

Fluency

4. **a.** $x + 5$ **b.** $\frac{x + 2}{3}$

c. $3p - 5$ **d.** $2(2y - 3)$

e. $\frac{t + 7}{2}$ **f.** $\frac{2x - 9}{3}$

g. $2(x - 5)$ **h.** $3(2x + 7)$

5. **a.** $\frac{2}{3}$ **b.** 2 **c.** $\frac{x + 4}{2}$ **d.** 6

e. $\frac{3}{2}$ **f.** $\frac{7}{4}$ **g.** 2 **h.** $\frac{5y}{2}$

6. **a.** 9 **b.** $\frac{1}{6}$ **c.** $\frac{7}{9}$ **d.** $\frac{2y}{3}$

e. $\frac{2p}{45}$ **f.** $\frac{27}{x}$ **g.** $18y$ **h.** $\frac{14n}{15}$

7. **a.** 2 **b.** $x + 7$

c. $\frac{3}{y - 2}$ **d.** $2(t + 5)$

e. $\frac{3(p - 3)}{2}$ **f.** 2

g. $\frac{2(n - 2)}{9}$ **h.** $\frac{5(x + 3)}{2}$

8. **a.** $\frac{5}{4}$ **b.** 12 **c.** $10t$ **d.** $\frac{x}{8}$

e. $\frac{7p}{4}$ **f.** $\frac{1}{15y}$ **g.** $\frac{3n}{16}$ **h.** $\frac{9y}{4x}$

9. **a.** $\frac{2}{3}$ **b.** 3 **c.** $\frac{1}{15}$ **d.** 6

e. $\frac{2}{n + 4}$ **f.** $\frac{3}{2}$ **g.** $\frac{1}{x + 3}$ **h.** $\frac{m + 4}{12}$

10. A

Spot the mistake

11. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

12. Joan the electrician works out the total cost of a job by using the equation $C = 90x + 75$ where x is the number of hours spent on the job and \$75 is the callout fee. She has agreed to charge her friend Jilly a third of this amount. Formulate an algebraic fraction and simplify it to show the equation representing the cost of Jilly's job.

Key points

- The equation $C = 90x + 75$ can be used to calculate the total cost of an electrician job, where x is the number of hours spent on the job.
- Joan will charge her friend Jilly a third of the amount.
- Formulate an algebraic fraction and simplify it to show the equation representing the cost of Jilly's job.

Explanation

The equation for Jilly's job can be written as $\frac{C}{3} = J = \frac{90x + 75}{3}$.

Simplify J.

$J = \frac{3(30x + 25)}{3}$

$= 30x + 25$

Answer

An equation for the cost of Jilly's job is $J = 30x + 25$.

13. Dolph's profits for the entire week can be modelled by the expression $280x - 490$, where x represents the number of orders he receives. Formulate a simplified expression for the average profit made in one day of Dolph's 7-day work week.

Key points

- The expression $280x - 490$ represents Dolph's profits for the entire week, where x is the number of orders received.
- Dolph has a 7-day work week.
- Formulate a simplified expression for the average profit made in one day.

Explanation

There are seven days in a week. The expression for the average daily profit can be written as $\frac{280x - 490}{7}$.

Simplify the fraction.

$\frac{280x - 490}{7} = 40x - 70$

Answer

An algebraic expression for the average daily profit for Dolph's 7-day work week is $40x - 70$.

14. The total floor area of a chicken coop is given by the expression $6x^2 + 27x$ where x is an unknown length in metres. Formulate a simplified expression for the area per chicken, if there are a total of 12 chickens living in the coop altogether.

Back to contents

Key points

- The total area of the coop is given by $6x^2 + 27x$, where x is an unknown length.
- There are 12 chickens in the coop.
- Formulate an expression showing the area per chicken.

Explanation

There are 12 chickens in the chicken coop. The expression for the average area per chicken can be written as $\frac{6x^2 + 27x}{12}$.

Factorise and simplify the fraction.

$$\begin{aligned}\frac{6x^2 + 27x}{12} &= \frac{3x(2x+9)}{12}\\ &= \frac{3x \div 3(2x+9)}{12 \div 3}\\ &= \frac{x(2x+9)}{4}\end{aligned}$$

Answer

An algebraic expression for the area per chicken in the coop is $\frac{x(2x+9)}{4}$.

15. Geoff is working out the best investment plan for his recently acquired inheritance. He comes up with an equation, $G = \frac{900(y-5)}{21y} \times \frac{700y^2}{2(y-2)(y-5)}$, representing his savings after a number of years (y) of investing using a customised plan. Generate a simplified expression for G.

Key points

- The equation $G = \frac{900(y-5)}{21y} \times \frac{700y^2}{2(y-2)(y-5)}$ can be used to represent Geoff's savings, where y represents the number of years of investing using a customised plan.
- Simplify G.

Explanation

Simplify G.

$$\begin{aligned}G &= \frac{900(y-5)}{21y} \times \frac{700y^2}{2(y-2)(y-5)}\\ &= \frac{900\cancel{(y-5)}}{21y \div 7y} \times \frac{700y^2 \div 7y}{2(y-2)\cancel{(y-5)}}\\ &= \frac{900 \div 2}{3} \times \frac{100y}{2 \div 2(y-2)}\\ &= \frac{450}{3} \times \frac{100y}{y-2}\\ &= \frac{150}{1} \times \frac{100y}{y-2}\\ &= \frac{15\,000y}{y-2}\end{aligned}$$

Answer

The simplified formula for Geoff's savings is $G = \frac{15\,000y}{y-2}$.

16. Chelsea is analysing the value of an item she sells in her online store. She used the expression $D = \frac{t+3}{6(t-2)}$ to represent the demand and $E = \frac{10}{3(t-2)}$ to represent the total effort put into the production of the item. In both equations, t represents the number of the items currently available for sale. Determine the ratio $\frac{D}{E}$ representing the overall value of the item.

Key points

- Chelsea is analysing the value of an item she sells in her online store.
- The equation $D = \frac{t+3}{6(t-2)}$ is used to represent the demand for the item, where t represents the number of items currently available for sale.
- The equation $E = \frac{10}{3(t-2)}$ is used to represent the total effort put into the production of the item, where t represents the number of items currently available for sale.
- Determine the ratio $\frac{D}{E}$, the overall value of the item.

Explanation

The ratio $\frac{D}{E}$ can be expressed as $D \div E$.

Substitute the equations for D and E into the ratio.

$$\frac{D}{E} = \frac{t+3}{6(t-2)} \div \frac{10}{3(t-2)}$$

Convert the division to multiplication and simplify.

$$\begin{aligned}\frac{D}{E} &= \frac{t+3}{6(t-2)} \times \frac{3(t-2)}{10}\\ &= \frac{t+3}{6 \div 3\cancel{(t-2)}} \times \frac{3 \div 3\cancel{(t-2)}}{10}\\ &= \frac{t+3}{2} \times \frac{1}{10}\\ &= \frac{t+3}{20}\end{aligned}$$

Answer

The ratio $\frac{D}{E} = \frac{t+3}{20}$ can be used to represent the overall value of an item in Chelsea's online store.

Reasoning

17. **a.** $V_{medium} = \frac{2(\pi r^2 h)}{9}$

b. $V_{small} = \frac{2(\pi r^2 h)}{15}$

c. $\frac{V_{large}}{V_{small}} = \frac{\pi r^2 h}{3} \div \frac{2\pi r^2 h}{15} = \frac{5}{2} = 2.5$

$\therefore$ Sandy's claim is true.

d. Suggested option 1: Customers have different appetites and dietary preferences.

Suggested option 2: Different portion sizes can be priced accordingly, catering to customers with varying budgets.

Note: There are other possible options.

18. **a.** $\frac{8x}{15} \div \frac{4x}{5} = \frac{8x}{15} \times \frac{5}{4x} = \frac{2}{3}$

b. $\frac{15}{8x} \times \frac{4x}{5} = \frac{3}{2}$

c. The answers in parts **a** and **b** are reciprocals of each other. This demonstrates that the order in which the fractions are reciprocated matters when converting division to multiplication.

Exam-style

19. B

20. **a.** $\frac{2x}{(x-6)(x+1)} \times \frac{x-6}{8x}$

b. $\frac{1}{4(x+1)}$

21. $A = \frac{40x}{x-2}$

22. $\frac{2(x-3)}{21}$

Remember this?

23. D

24. D

25. B

Back to contents

4G Simplifying algebraic fractions – addition and subtraction

Student practice

Worked example 1

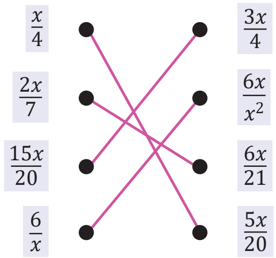
a. $\frac{5x}{6}$ **b.** $\frac{8x+14}{15}$ **c.** $\frac{17}{3x}$

Worked example 2

a. $-\frac{7x}{6}$ **b.** $\frac{x+13}{6}$ **c.** $\frac{5-4x}{x^2}$

Understanding worksheet

1. **Fraction** **Equivalent fraction**



2. **a.** 6 **b.** 14 **c.** $3x$ **d.** x^2

3. common; proportions; multiplied; LCD

Fluency

4. **a.** $\frac{5a}{4}$ **b.** $\frac{5x}{6}$ **c.** $\frac{11y}{12}$ **d.** $\frac{t}{2}$

e. $\frac{13x}{6}$ **f.** $-\frac{13m}{27}$ **g.** $\frac{25x}{18}$ **h.** $\frac{p}{5}$

5. **a.** $\frac{4t+11}{6}$ **b.** $\frac{5x+19}{6}$

c. $\frac{7p-6}{10}$ **d.** $\frac{10y+18}{21}$

e. $\frac{x+11}{6}$ **f.** $\frac{16r+33}{10}$

g. $\frac{2n+13}{24}$ **h.** $\frac{4x+3}{9}$

6. **a.** $\frac{7}{2x}$ **b.** $\frac{19}{3x}$ **c.** $\frac{22}{5t}$ **d.** $\frac{1}{2y}$

e. $\frac{7}{3x}$ **f.** $-\frac{2}{9r}$ **g.** $-\frac{17}{20n}$ **h.** $\frac{26}{21y}$

7. **a.** $\frac{1+2x}{x^2}$ **b.** $\frac{3y-2}{y^2}$

c. $\frac{4t+3}{t^2}$ **d.** $\frac{5-2x}{x^2}$

e. $\frac{3-7a}{a^2}$ **f.** $\frac{5-3p}{p^2}$

g. $\frac{7-4m}{2m^2}$ **h.** $\frac{5-12n}{2n^2}$

8. D

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. On Christmas day Chirag and Cindy ate pavlova together with some friends. Using p to represent the entire pavlova, Chirag ate a portion equivalent to $\frac{p}{4}$ and Cindy ate a portion equivalent to $\frac{p}{5}$. Simplify the expression showing how much Chirag and Cindy ate together in terms of p.

Key points

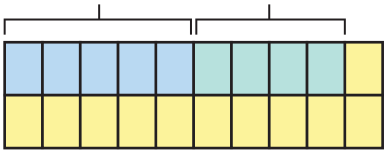
- p represents the entire pavlova.
- Chirag ate $\frac{p}{4}$.
- Cindy ate $\frac{p}{5}$.
- Sum the portions eaten and simplify.

Explanation

Add the fractions representing Chirag's and Cindy's portions.

$$\frac{p}{4}+\frac{p}{5}=\frac{5\times p}{5\times 4}+\frac{4\times p}{4\times 5}$$
$$=\frac{5p}{20}+\frac{4p}{20}$$
$$=\frac{5p+4p}{20}$$
$$=\frac{9p}{20}$$

Chirag ate $\frac{p}{4}=\frac{5p}{20}$ **Cindy ate $\frac{p}{5}=\frac{4p}{20}$**



$\frac{5p}{20}+\frac{4p}{20}=\frac{9p}{20}$ of the pavlova was eaten by Chirag and Cindy

Answer

Chirag and Cindy ate $\frac{9p}{20}$ together.

11. Ashley is raising funds to go on a basketball trip to New York City. Using x to represent the total cost of the trip, his mother will contribute $\frac{x}{5}$ towards the trip and his grandfather will contribute $\frac{2x}{3}$. Simplify the expression representing the total amount contributed by Ashley's mother and grandfather in terms of x.

Key points

- x is the total cost of the trip.
- His mother will contribute $\frac{x}{5}$ towards the trip.
- His grandfather will contribute $\frac{2x}{3}$.
- Sum the contributions and simplify.

Explanation

Add the fractions representing Ashley's mother's and grandfather's contributions.

$$\frac{x}{5}+\frac{2x}{3}=\frac{3\times x}{3\times 5}+\frac{5\times 2x}{5\times 3}$$
$$=\frac{3x}{15}+\frac{10x}{15}$$
$$=\frac{3x+10x}{15}$$
$$=\frac{13x}{15}$$

Answer

The total amount contributed by Ashley's mother and grandfather is $\frac{13x}{15}$.

12. After a family dinner, there is a third $\left(\frac{L}{3}\right)$ lasagne left over. Jimmy gets home later and eats a portion equivalent to $\frac{2L}{7}$. Simplify the expression for the remaining portion of lasagne in terms of L.

Key points

- $\frac{L}{3}$ was left over.
- $\frac{2L}{7}$ gets eaten.
- Write and simplify the expression for the remaining proportion.

Explanation

Subtract the portion eaten by Jimmy from the left over portion of lasagne.

$$\begin{aligned}\frac{L}{3}-\frac{2L}{7}&=\frac{7\times 2L}{7\times 3}-\frac{3\times 2L}{3\times 7}\\&=\frac{7L}{21}-\frac{6L}{21}\\&=\frac{7L-6L}{21}\\&=\frac{L}{21}\end{aligned}$$

Answer

The remaining portion of lasagne is $\frac{L}{21}$.

13. Sylvie withdraws half of her total savings (s) out of her account to take on holiday. During the holiday, Sylvie spends an amount equivalent to $\frac{3s}{8}$. Simplify the expression showing how much money she is bringing back with her from holiday in terms of s.

Key points

- Half of s was withdrawn.
- $\frac{3s}{8}$ was spent.
- Write and simplify an expression for the amount of money brought back from holiday.

Explanation

Write an expression for the amount of money withdrawn by Sylvie.

Half of $s=\frac{s}{2}$

Subtract the amount spent by Sylvie on holiday from the amount she withdrew.

$$\begin{aligned}\frac{s}{2}-\frac{3s}{8}&=\frac{4\times s}{4\times 2}-\frac{3s}{8}\\&=\frac{4s}{8}-\frac{3s}{8}\\&=\frac{4s-3s}{8}\\&=\frac{s}{8}\end{aligned}$$

Answer

Sylvie is bringing back $\frac{s}{8}$ from the holiday.

14. John and Enyd are getting married and their parents have agreed to pay for some of their wedding expenses. John's parents will pay $\frac{w}{4}$ towards the total cost of the wedding (w), while Enyd's parents have agreed to pay $\frac{2w}{5}$. Simplify the expression showing the remaining cost of the wedding in terms of w.

Key points

- $\frac{w}{4}$ of the cost was paid out of w.
- $\frac{2w}{5}$ of the cost was then paid out of w.
- Write and simplify the expression for the remaining unpaid cost.

Explanation

Subtract John's and Enyd's parents' contributions from the total cost of the wedding w.

$$\begin{aligned}w-\frac{w}{4}-\frac{2w}{5}&=\frac{20\times w}{20\times 1}-\frac{5\times w}{5\times 4}-\frac{4\times 2w}{4\times 5}\\&=\frac{20w}{20}-\frac{5w}{20}-\frac{8w}{20}\\&=\frac{20w-5w-8w}{20}\\&=\frac{7w}{20}\end{aligned}$$

Answer

The remaining cost of the wedding is $\frac{7w}{20}$.

Reasoning

15. a. Spencer has $\frac{2g}{3}$ left.

b. Spencer has $\frac{4g}{15}$ left.

c. Spencer's grandmother gave him $1181.

d. Suggested option 1: Spending money instead of saving it may allow for a more active social life through spending money on experiences and activities with people.

Suggested option 2: Spending money instead of saving it limits the ability to save up to eventually make larger purchases, such as holidays.

Note: There are other possible options.

16. a. $\frac{5x}{6}-\frac{2x}{9}=\frac{3\times 5x}{3\times 6}-\frac{2\times 2x}{2\times 9}=\frac{15x-4x}{18}=\frac{11x}{18}$

b. $\frac{5x}{6}-\frac{2x}{9}=\frac{9\times 5x}{9\times 6}-\frac{6\times 2x}{6\times 9}=\frac{45x-12x}{54}=\frac{33x}{54}=\frac{11x}{18}$

c. Both parts **a** and **b** simplify through converting both fractions to a common denominator. In part **a**, the expression is in its simplest form as both fractions have been converted using the LCM of the two denominators. In part **b**, the expression needed to be simplified further because the product of 6 and 9 is not the LCM of the two denominators. Regardless, the two final answers are the same.

Exam-style

17. D

18. a. $\frac{4+25x}{5x^2}$

b. $5\frac{4}{5}$

19.

$$\begin{aligned}\frac{2a}{5}+\frac{a}{7}&=\frac{7\times 2a}{7\times 5}+\frac{5\times a}{5\times 7}\\&=\frac{14a}{35}+\frac{5a}{35}\\&=\frac{14a+5a}{35}\\&=\frac{19a}{35}\end{aligned}$$

20.

$$\begin{aligned}\frac{5x}{12}-c&=\frac{3x}{4}\\\therefore c&=\frac{5x}{12}-\frac{3x}{4}\\c&=\frac{5x}{12}-\frac{3\times 3x}{3\times 4}\\c&=\frac{5x}{12}-\frac{9x}{12}\\c&=\frac{5x-9x}{12}\\c&=\frac{-4x}{12}\\c&=-\frac{x}{3}\end{aligned}$$

Remember this?

21. B

22. C

23. A

Back to contents

Chapter 4 extended application

1. **a.** $(x + 3)(x + 7)\ \text{m}^2$

b. $x^2 + 10x + 21\ \text{m}^2$

c. $4x\ \text{m}^2$

d. $4x^2 + 40x + 84\ \text{m}^2$

e. $10x - 25\ \text{m}^2$

f. Suggested option 1: How close Kiddoland is to their place of residence.

Suggested option 2: The facilities at KiddoLand compared to other play centres.

Note: There are other possible options.

2. **a.** $8x + 48\ \text{m}$

b. $(2x + 12)^2\ \text{m}^2$

c. $4x(x + 12)\ \text{m}^2$

d. $180\ \text{m}^2$

e. $x = 6\ \text{m}$

f. Suggested option 1: The ambiance, including the lighting, within the gallery.

Suggested option 2: The grouping of similar paintings throughout the gallery.

Note: There are other possible options.

3. **a.** $V_{women} = \pi r^3$

b. $V_{youth} = \dfrac{8\pi r^3}{9}$

c. $\dfrac{29\pi r^3}{27}$

d. $r \approx 12\ \text{cm}$

e. Suggested option 1: It is fair because they have different sized hands on average.

Suggested option 2: It is not fair because the court size and basket size is standard so all players should have the same ball size too.

Note: There are other possible options.

Chapter 4 review

Multiple choice

1. C **2.** E **3.** B **4.** C **5.** D

Fluency

6. **a.** $m^2 + 5m + 4$ **b.** $n^2 - 3n - 18$
c. $p^2 - 5p + 4$ **d.** $3z^2 - 14z - 49$

7. **a.** $z^2 + 6z + 9$ **b.** $w^2 - 6w + 9$
c. $9x^2 - 12x + 4$ **d.** $16y^2 + 8y + 1$

8. **a.** $z^2 - 16$ **b.** $9x^2 - 36$
c. $25y^2 - 4$ **d.** $4w^2 - 49$

9. **a.** $4(y + 5)$ **b.** $5z(z - 3)$
c. $-3(x - 6)$ **d.** $-9w(w + 3)$

10. **a.** $(3 + c)(c + d)$ **b.** $(2y - 5)(4 - y)$
c. $(5f + 3)(e + f)$ **d.** $(6 - 5z)(3z - 4)$

11. **a.** $(y + 4)(y - 4)$ **b.** $(5x + 1)(5x - 1)$
c. $(7w + 6z)(7w - 6z)$ **d.** $(x + 1)(x - 5)$

12. **a.** $(x + 2)(x + 3)$ **b.** $(x + 8)(x - 4)$
c. $(2x + 7)(x + 3)$ **d.** $(3x - 2)(x - 4)$

13. **a.** 1 **b.** $\dfrac{3}{2}$ **c.** $\dfrac{19x}{9}$ **d.** $\dfrac{2}{x + 3}$

14. **a.** $\dfrac{1}{y}$ **b.** $\dfrac{1}{12}$
c. $\dfrac{5}{2w}$ **d.** $\dfrac{a(b + c)^2}{d^2}$

15. **a.** $\dfrac{5x}{6}$ **b.** $\dfrac{2x + 3}{x^2}$
c. $\dfrac{9x + 17}{20}$ **d.** $\dfrac{31x + 19}{35}$

16. **a.** $-\dfrac{14x}{15}$ **b.** $\dfrac{4 - 3x}{x^3}$ **c.** $\dfrac{12 - x}{12}$ **d.** $\dfrac{7 - x}{8}$

Problem solving

17. A rectangular garden has a length of x m and a width of y m. The gardener plans to extend the garden by adding 2 m to the length and 4 m to the width. Write an expanded expression for the new area of the garden after the extension.

Key points

- A rectangular garden has a length of x m and a width of y m.
- The garden is to be extended by adding 2 m to the length and 4 m to the width.
- Write an expanded expression for the new area of the garden after the extension.

Explanation

Write expressions for the new length and width of the garden.

New length $= x + 2$

New width $= y + 4$

Write an expression for the new area of the garden after the extension.

$$\begin{aligned}\text{New area} &= \text{new length} \times \text{new width}\\ &= (x + 2)(y + 4)\end{aligned}$$

Expand the brackets using the distributive law.

$$\begin{aligned}(x + 2)(y + 4) &= x \times y + x \times 4 + 2 \times y + 2 \times 4\\ &= xy + 4x + 2y + 8\end{aligned}$$

Answer

The new area of the garden is $xy + 4x + 2y + 8\ \text{m}^2$.

18. A community garden is in the shape of a square. The local council decides to renovate the garden by extending all sides by 4 m. Write an expanded simplified expression for the new area of the garden, using g to represent the length and width of the garden before the renovation.

Key points

- A garden is in the shape of a square.
- All sides of the garden are extended by 4 m.
- Let g be the original length and width of the garden.
- Write an expanded simplified expression for the new area only.

Back to contents

Explanation

The garden is in the shape of a square, ∴ length = width

Write an expression for the new length of the garden.

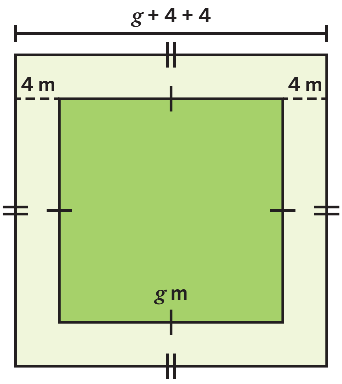
New length of garden $= g + 4 + 4$

$= g + 8$

Write an expression for the new area of the garden.

New area of garden $= (\text{new length})^2$

$= (g + 8)^2$



Expand the brackets using the distributive law.

$(g + 8)^2 = (g + 8)(g + 8)$

$= g^2 + 8g + 8g + 64$

Simplify the expression by collecting like terms.

$= g^2 + 16g + 64$

Answer

An expanded simplified expression for the new area of the garden is $g^2 + 16g + 64$ m^2.

19. A local musical band was performing two nights in a row. The profit they make from each ticket is represented by the expression $2x - 7$ where x represents an unknown amount of dollars. On the first night of their performance, they sold y tickets and on the second night of their performance, they sold 28 tickets. Write a factorised expression representing their total profits.

Key points

- A musical band performed over two nights.
- $2x - 7$ represents the profit they make from each ticket.
- x is an unknown amount of dollars.
- y tickets were sold on the first night.
- They sold 28 tickets on the second night.
- Write a factorised expression representing their total profits.

Explanation

Write an expression representing the profits made on a night.

Profit = number of tickets sold × profit from one ticket

Calculate the profits made on the first night when y tickets were sold.

Profit $= y(2x - 7)$

Calculate the profits made on the second night when 28 tickets were sold.

Profit $= 28(2x - 7)$

Write an expression representing the total profits made by the musical band.

Total profits = profits made on the first night + profits made on the second night

$= y(2x - 7) + 28(2x - 7)$

Factorise the expression.

$y(2x - 7) + 28(2x - 7) = (2x - 7)(y + 28)$

Answer

$(2x - 7)(y + 28)$ represents the total profits.

20. A landscape artist isn't satisfied with his square garden design. He adjusts the length by increasing it and adjusts the width by decreasing it, after which the expression for the new area is represented by $p^2 + 3p - 9p - 27$, where p represents the length of one side prior to adjustment. Factorise the expression to determine how many metres he increased the length and decreased the width by.

Key points

- The dimensions of a square garden are changed by increasing the length and decreasing the width.
- $p^2 + 3p - 9p - 27$ represents the new area.
- p is the original length and width.
- Factorise the expression to determine how many metres he increased the length and decreased the width by.

Explanation

Factorise the expression representing the new area.

$p^2 + 3p - 9p - 27 = p(p + 3) - 9(p + 3)$

$= (p + 3)(p - 9)$

The square garden initially had the same length and width, p.

Factorising the expression demonstrates that one side has increased by 3 m and the other side has decreased by 9 m.

The question states that the length increases and the width decreases.

Answer

The landscape artist increases the length by 3 m and decreases the width by 9 m.

21. A safety inspector is assessing the dimensions of a stage at a concert. The stage has an area of $4x^2 - 25$ square metres, where x represents an unknown length in metres. The rule is that the longer side of the stage must be at least 8 m more than the shorter side. Factorise the expression to determine whether the stage meets this requirement.

Key points

- A concert stage has an area of $4x^2 - 25$.
- x is an unknown length in metres.
- The longer side must be at least 8 m more than the shorter side.
- Factorise the expression to determine whether the stage meets this requirement.

Explanation

The expression $4x^2 - 25$ is a difference of squares, which can be factored using the formula $a^2 - b^2 = (a + b)(a - b)$.

Identify a^2 and b^2 in the expression $4x^2 - 25$ to determine the values of a and b.

$a^2 = 4x^2$ and $a = \sqrt{4x^2} = 2x$

$b^2 = 25$ and $b = \sqrt{25} = 5$

Factorise the difference of two squares by substituting the values for a and b into the general formula.

$4x^2 - 25 = (2x + 5)(2x - 5)$

The length of the longer side is represented by $(2x + 5)$ and the length of the shorter side is represented by $(2x - 5)$.

Calculate the difference between the longer side and the shorter side.

$$\begin{aligned}\text{Difference} &= \text{length of longer side} - \text{length of shorter side}\\ &= (2x+5)-(2x-5)\\ &= 2x+5-2x+5\\ &= 10\text{ m}\end{aligned}$$

$10\text{ m} > 8\text{ m}$

Answer

The stage meets the requirement.

22. Jackson is trying to decide whether he should purchase and renovate a house using all his cash savings (c). Assuming the house alone costs $\frac{2c}{5}$ and the renovations cost $\frac{2c}{7}$, how much money will Jackson have left, in terms of c, after purchasing and renovating the house?

Key points

- c is Jackson's current amount of cash.
- A house costs $\frac{2c}{5}$.
- Renovating this house costs $\frac{2c}{7}$.
- How much cash will Jackson have left in terms of c after purchasing and renovating the house?

Explanation

Write an expression for the amount of cash Jackson will have left after purchasing and renovating the house.

$$\begin{aligned}\text{Amount of cash left} &= \text{current amount of cash} - \text{cost of house}\\ &\quad - \text{cost of renovations}\\ &= c - \frac{2c}{5} - \frac{2c}{7}\end{aligned}$$

Simplify the expression.

$$\begin{aligned}c - \frac{2c}{5} - \frac{2c}{7} &= \frac{c\times 35}{1\times 35} - \frac{2c\times 7}{5\times 7} - \frac{2c\times 5}{7\times 5}\\ &= \frac{35c}{35} - \frac{14c}{35} - \frac{10c}{35}\\ &= \frac{35c-14c-10c}{35}\\ &= \frac{11c}{35}\end{aligned}$$

Answer

Jackson will have $\frac{11c}{35}$ left after purchasing and renovating the house.

23. Sam is a biologist studying two new populations of bacteria. The first population can be represented by the equation $p_1 = \frac{t(t-7)}{3(t+4)}$ and the second population can be represented by the equation $p_2 = \frac{t(t-7)}{(t+3)}$, where t represents the number of hours after cultivating the populations of bacteria. Write the simplified fraction $\frac{p_1}{p_2}$.

Key points

- $p_1 = \frac{t(t-7)}{3(t+4)}$
- $p_2 = \frac{t(t-7)}{(t+3)}$
- Write the simplified fraction $\frac{p_1}{p_2}$.

Explanation

The ratio $\frac{p_1}{p_2}$ can be expressed as $p_1 \div p_2$.

Substitute the expressions for p_1 and p_2 into the ratio.

$$\frac{p_1}{p_2} = \frac{t(t-7)}{3(t+4)} \div \frac{t(t-7)}{t+3}$$

Convert the division to multiplication and simplify.

$$\begin{aligned}\frac{t(t-7)}{3(t+4)} \div \frac{t(t-7)}{t+3} &= \frac{t(t-7)}{3(t+4)} \times \frac{t+3}{t(t-7)}\\ &= \frac{t\cancel{(t-7)}}{3(t+4)} \times \frac{t+3}{t\cancel{(t-7)}}\\ &= \frac{t\times(t+3)}{3t\times(t+4)}\\ &= \frac{\cancel{t}(t+3)}{3\cancel{t}(t+4)}\\ &= \frac{t+3}{3(t+4)}\end{aligned}$$

Answer

The simplified fraction $\frac{p_1}{p_2}$ is $\frac{t+3}{3(t+4)}$.

Reasoning

24. a. The factorised expression for the total area is $a(a+8)$.

b. The expanded expression for the total number of produced fruits is $xy + 40x + 50y + 2000$.

c. The factorised expression for the total number of trees is $(t+10)^2$.

d. The new apple yield in terms of x is $\frac{31x}{35}$.

e. Suggested option 1: An advantage of using equations is that it helps predict how many fruits will grow, making planning easier. A disadvantage is that they can't predict unexpected events like harsh weather.

Suggested option 2: An advantage of using equations is that it's easier to budget the costs of production. A disadvantage is that it can't predict changes in production due to other factors such as pests.

Note: There are other possible options.

25. a. The factorised expression is $(x+6)(x-6)$.

b. The simplified fraction is $x-6$.

c. Factorising the numerator and/or the denominator of algebraic fractions allows for any common factors to be divided out and for the fraction to be fully simplified.

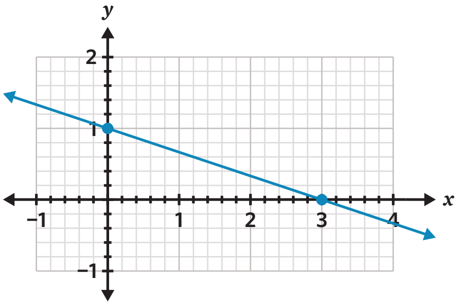
Back to contents

5A Graphing straight lines using intercepts

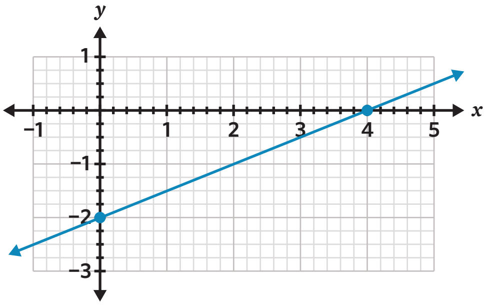
Student practice

Worked example 1

a.



b.



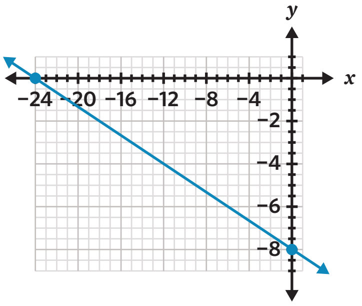
Worked example 2

a. $\left(\frac{1}{3},0\right)$ and (0,–1)

b. (2,0) and (0,4)

Worked example 3

a.



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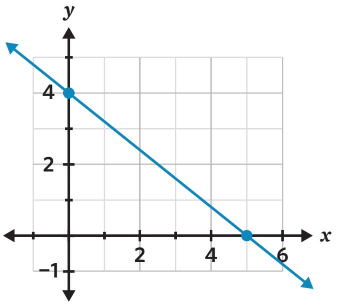
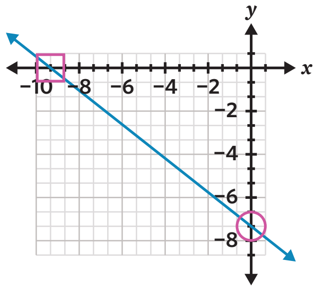


Understanding worksheet

1. a.



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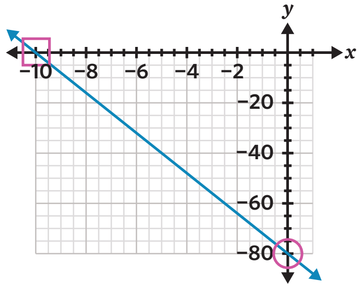

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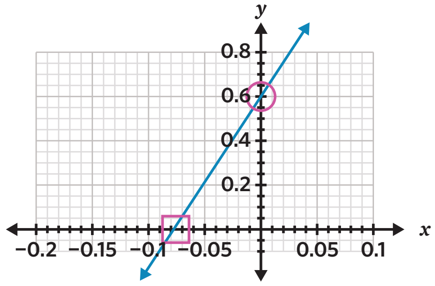

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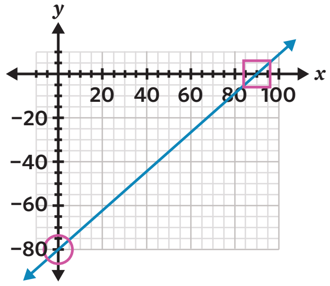
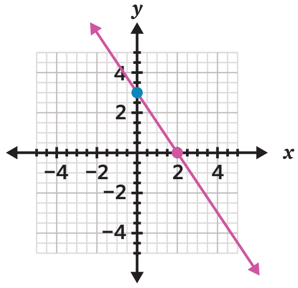


2. a.



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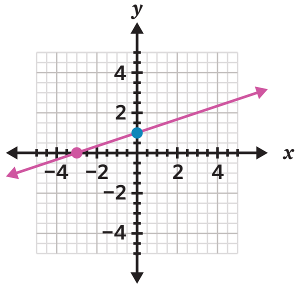
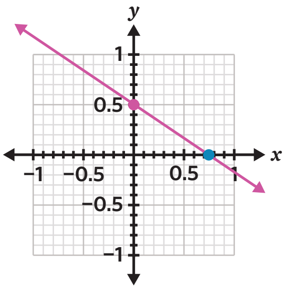


Back to contents

c.



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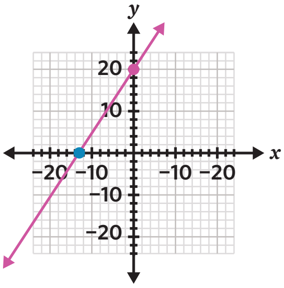
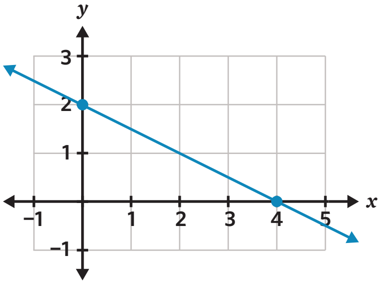


3. *y*-intercept; *x*-intercept; substituting; equations; Cartesian plane

Fluency

4. a.



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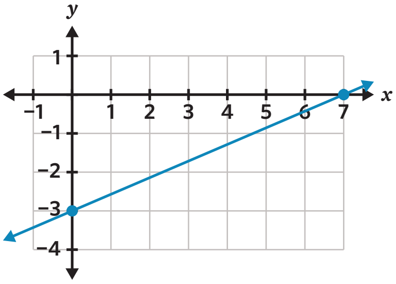

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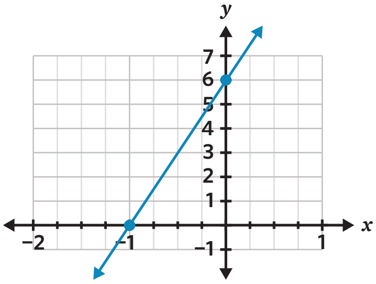

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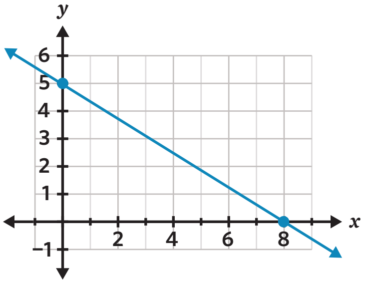

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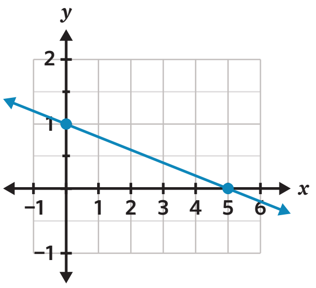
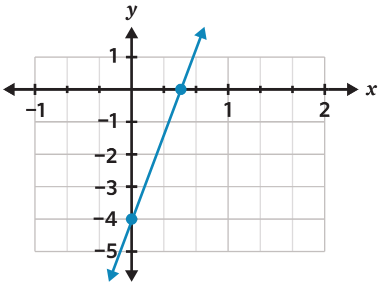
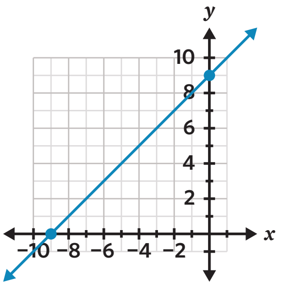


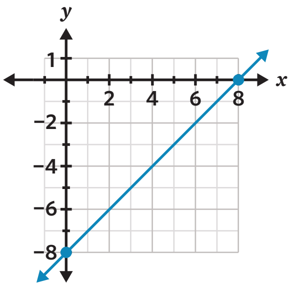
f.



g.



h.



5. **a.** (0,1) and (−1,0)

b. (0,−2) and $\left(\frac{1}{2},0\right)$

c. (0,−1) and (3,0)

d. (0,4) and (−10,0)

e. (0,−6) and (4,0)

f. (0,2) and (4,0)

g. (0,9.5) and (−23.75,0)

h. (0,−0.7) and (0.5,0)

6. **a.** (0,4) and (−2,0)

b. (0,−6) and (−2,0)

c. (0,2) and (−4,0)

d. (0,5) and $\left(\frac{5}{4},0\right)$

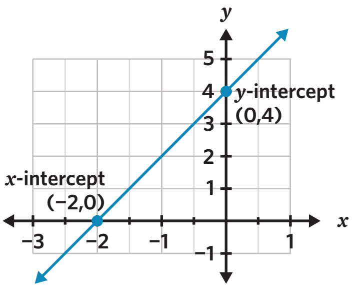
e. (0,4) and (4,0)

f. (0,1) and $\left(\frac{2}{5},0\right)$

g. $\left(0,\frac{1}{5}\right)$ and $\left(-\frac{1}{4},0\right)$

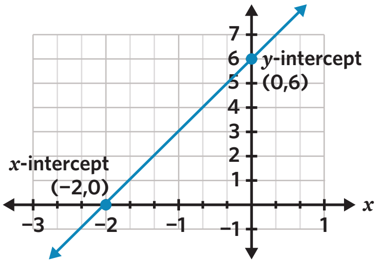
h. (0,−3) and (−2.5,0)

7. **a.**



Back to contents

b.



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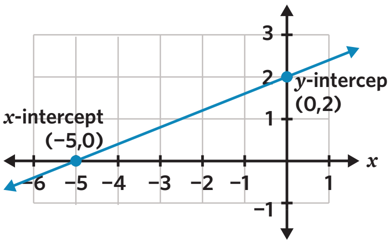

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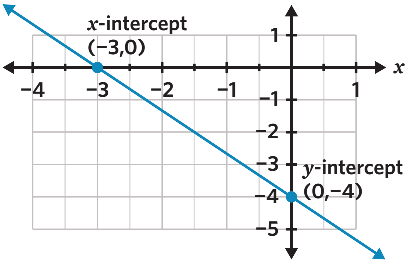

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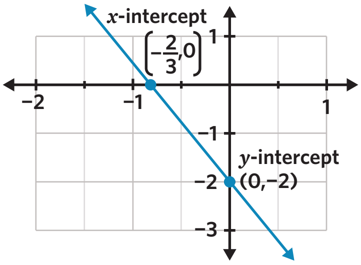

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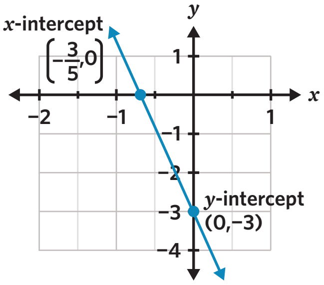

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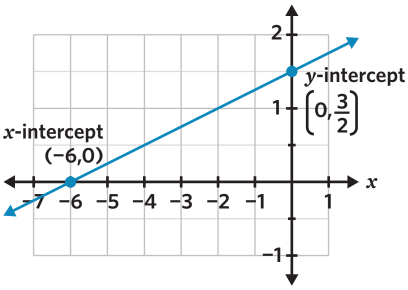

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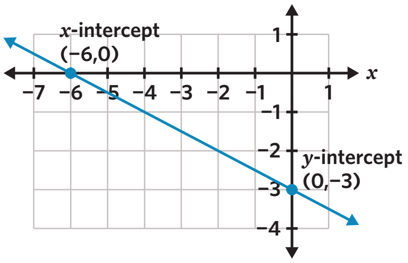
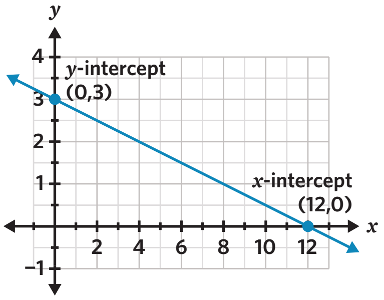


8. a.



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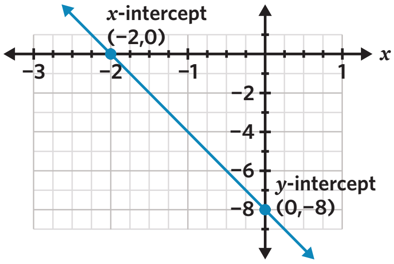

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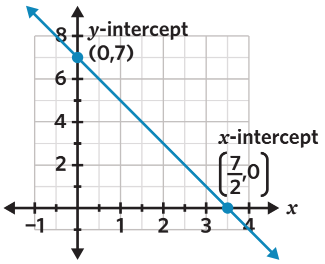

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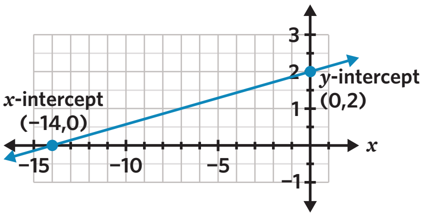

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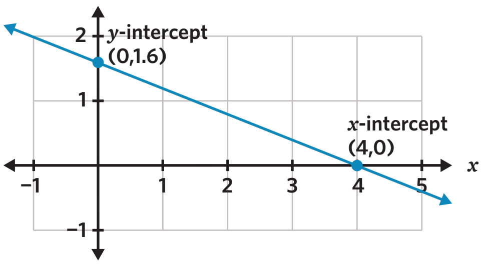

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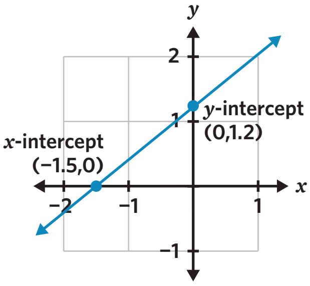

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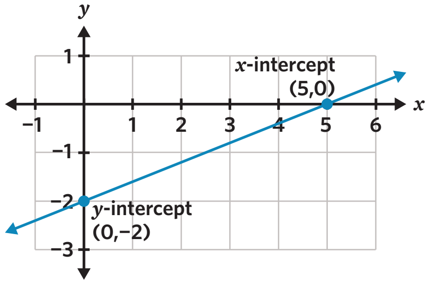

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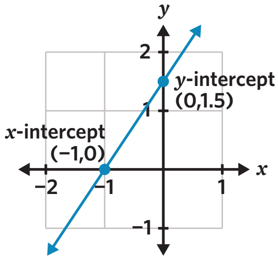


9. C

Spot the mistake

10. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. The declining population of a pest in a local park can be modelled using the equation $y = -20x + 350$. x represents the periods after the initial population and y represents the population. Sketch the graph showing the initial population and the time taken for the population to reach zero.

Key points

- Pest population represented by equation $y = -20x + 350$.
- x represents the periods after the initial population.
- y represents the population.
- Sketch the graph of the pest population and time until the population reaches zero.

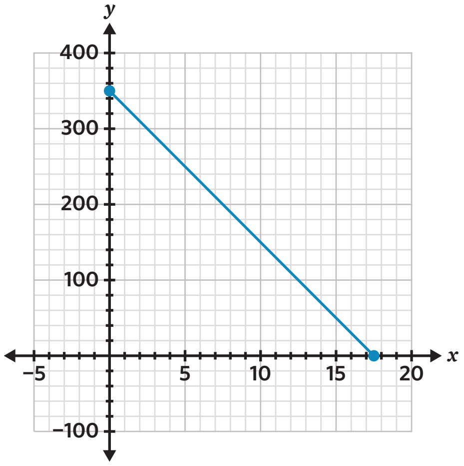
Explanation

x-intercept:	y-intercept:
let $y = 0$	let $x = 0$
$0 = -20x + 350$	$y = -20(0) + 350$
$-350 = -20x$	$y = 350$
$x = 17.5$	$\therefore$ y-intercept is 350
$\therefore$ x-intercept is 17.5	(0,350)
(17.5,0)	

Plot (17.5,0) and (0,350) on a Cartesian plane. Use a ruler to draw a straight line extending through the plotted axes intercepts. This determines the slope of the line.

Restrict the domain based on the context. The x-values and y-values cannot be negative as time and population cannot be negative, respectively.

Answer



12. The population (y) of a town can be modelled over time in years (x) using the linear equation $y = 200x + 250$. Sketch a graph showing the population of the town over different years showing the initial population of the town.

Key points

- Town population can be modelled by $y = 200x + 250$, where x is the number of years.
- Sketch the graph of the town population over different years showing the initial population.

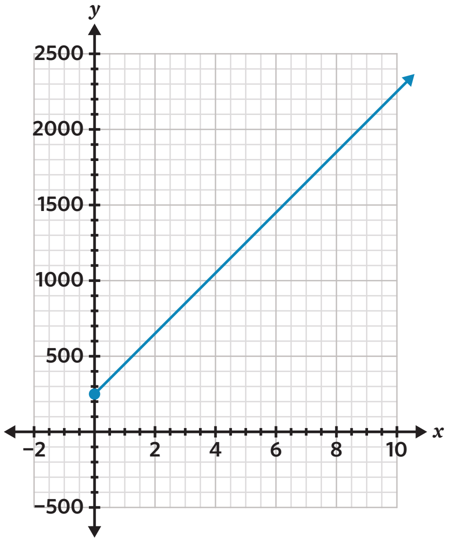
Explanation

x-intercept:	y-intercept:
let $y = 0$	let $x = 0$
$0 = 200x + 250$	$y = 200(0) + 250$
$-250 = 200x$	$y = 250$
$x = -1.25$	$\therefore$ y-intercept is 250
$\therefore$ x-intercept is -1.25	(0,250)
(−1.25,0)	

Plot (−1.25,0) and (0,250) on a Cartesian plane. Use a ruler to draw a straight line extending through the plotted axes intercepts. This determines the slope of the line.

Restrict the domain based on the context. The x-values and y-values cannot be negative as time and population cannot be negative, respectively. This means that the x-intercept will not be plotted on the graph.

Answer



13. A plant is known to grow at 3 cm per day after it has germinated and can be modelled using the equation $y = 3x + 1$. Sketch the graph that would represent the height of the plant from the day it germinates.

Key points

- A plant's height in centimetres is modelled by the equation $y = 3x + 1$, where x is the number of days after it has germinated.
- Sketch the graph that would represent the height of the plant from the day it germinates.

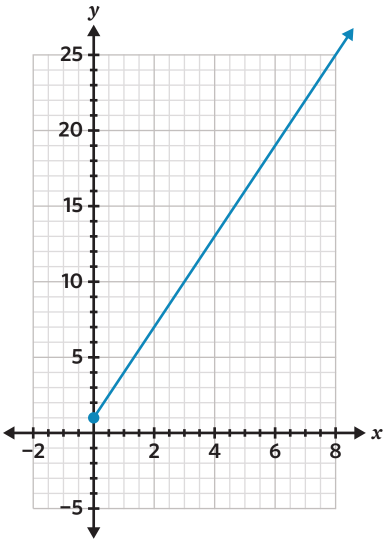
Explanation

x-intercept:	y-intercept:
let $y = 0$	let $x = 0$
$0 = 3x + 1$	$y = 3(0) + 1$
$-1 = 3x$	$y = 1$
$x = -\frac{1}{3}$	∴ y-intercept is 1
∴ x-intercept is $-\frac{1}{3}$	$(0,1)$
$(-\frac{1}{3},0)$	

Plot $(-\frac{1}{3},0)$ and $(0,1)$ on a Cartesian plane. Use a ruler to draw a straight line extending through the plotted axes intercepts. This determines the slope of the line.

Restrict the domain based on the context. The x-values and y-values cannot be negative as time and height cannot be negative, respectively. This means that the x-intercept will not be plotted on the graph.

Answer



14. A person has \$300 in a bank account and withdraws the same amount each week for 10 weeks leaving a balance of \$0. Sketch a linear graph to represent this situation, where the x-axis represents the weeks and the y-axis represents the balance in the account.

Key points

- Starting balance of \$300.
- Withdraws the same amount for 10 weeks leaving a \$0 balance.
- Sketch a linear graph to represent this situation, where the x-axis represents the weeks and the y-axis represents the account balance.

Explanation

Identify the known points.

Initial value at 0 weeks is the starting balance of \$300.

∴ y-intercept is $(0,300)$

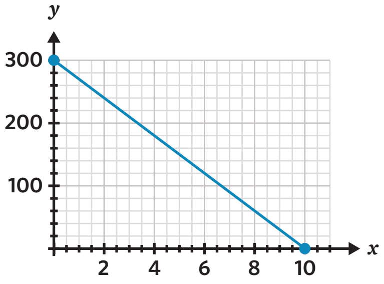
The balance is \$0 after 10 weeks.

∴ x-intercept is $(10,0)$

Plot $(10,0)$ and $(0,300)$ on a Cartesian plane. Use a ruler to draw a straight line extending through the plotted axes intercepts. This determines the slope of the line.

Restrict the domain based on the context. The x-values and y-values cannot be negative as time and balance in the bank account cannot be negative, respectively.

Answer



15. The water level in a tank decreases at a constant rate of 10 L per minute. The initial water level is 500 L. Sketch the graph showing the initial water level and the time taken for the tank to completely drain.

Key points

- Tank's water level decreases at a constant rate of 10 L per minute.
- Tank's initial water level is 500 L.
- Sketch the graph showing the tank's initial water level and the time taken for it to completely drain.

Explanation

Identify the equation. Let y represent the water level in litres and x represent time in minutes.

$y = -10x + 500$

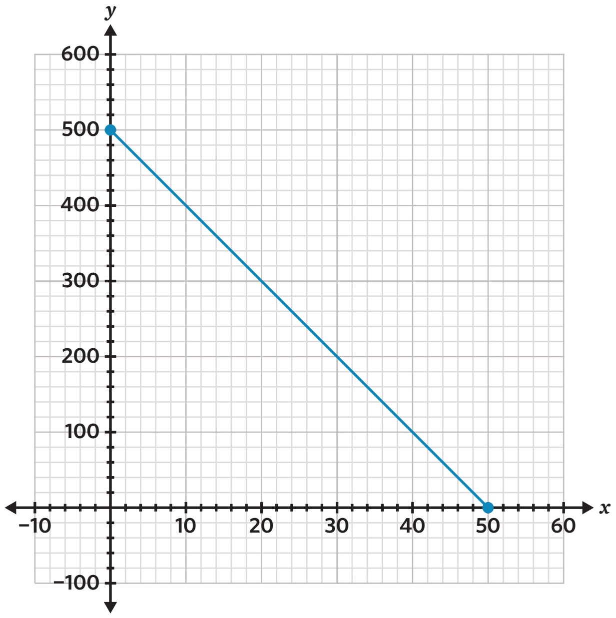
x-intercept:	y-intercept:
let $y = 0$	let $x = 0$
$0 = -10x + 500$	$y = -10(0) + 500$
$-500 = -10x$	$y = 500$
$x = 50$	∴ y-intercept is 500
∴ x-intercept is 50	$(0,500)$
$(50,0)$	

Plot $(50,0)$ and $(0,500)$ on a Cartesian plane. Use a ruler to draw a straight line extending through the plotted axes intercepts. This determines the slope of the line.

Restrict the domain based on the content. The x-values and y-values cannot be negative as time and amount of water in the tank cannot be negative, respectively.

Back to contents

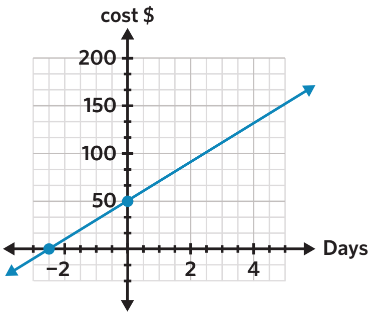
Answer



Reasoning

16. a. The x-intercept is $(-2.5,0)$ and the y-intercept is $(0,50)$.

b.



c. The y-intercept represents the $50 flat fee.

d. A reasonable scale is that x must be greater than 0 since the number of days that the car has been rented for can never be negative. The graph in part **b** should be sketched with this restricted domain and should not have the x-intercept plotted.

e. Suggested option 1: Hiring a car allows for a greater degree of flexibility in travel plans.

Suggested option 2: Driving directly to your destination can often be faster than using public transport.

Note: There are other possible options.

17. a. The x-intercept is $\left(\frac{-b}{a},0\right)$.

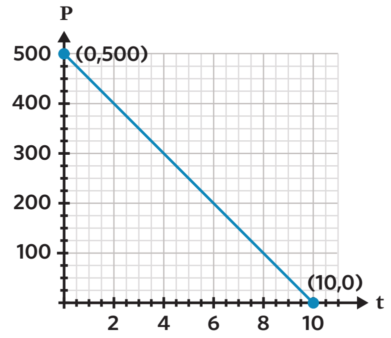
b. The y-intercept is $(0,b)$.

c. When in the form $y = mx + c$, x- and y-intercepts can be calculated using the m and c values.

Exam-style

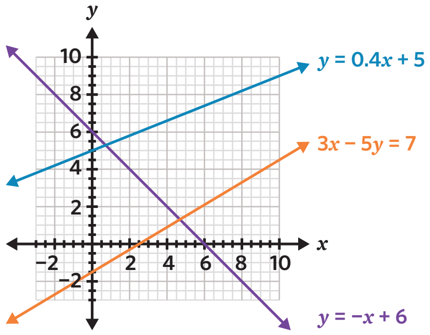
18. D

19. a.

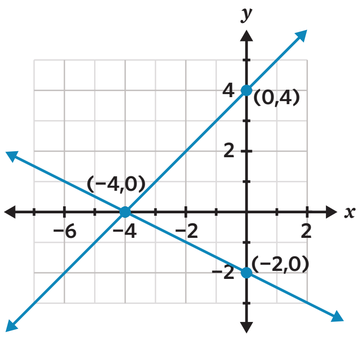


b. 8 years

20.



21.



Remember this?

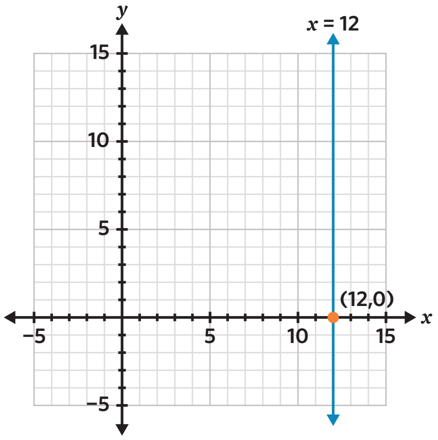
22. C **23.** D **24.** B

5B Lines with one intercept

Student practice

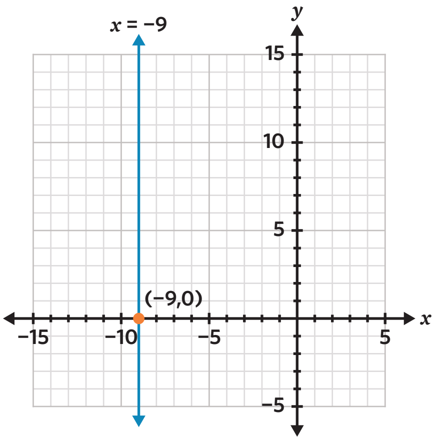
Worked example 1

a.



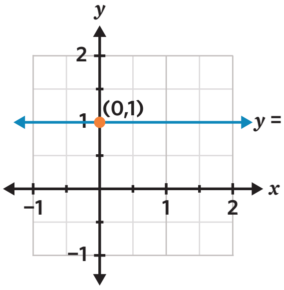
Back to contents

b.

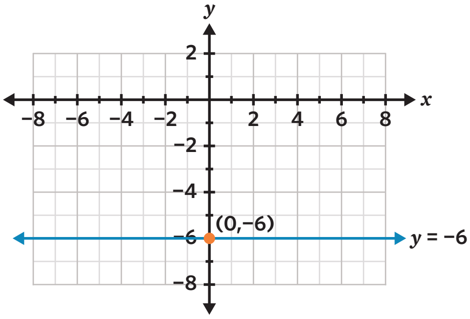


Worked example 2

a.

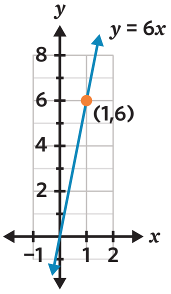


b.

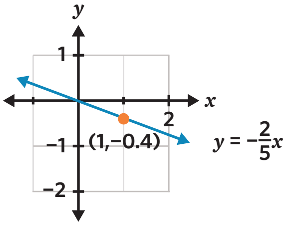


Worked example 3

a.



b.



Understanding worksheet

1. Vertical: III; IV

Horizontal: I; II

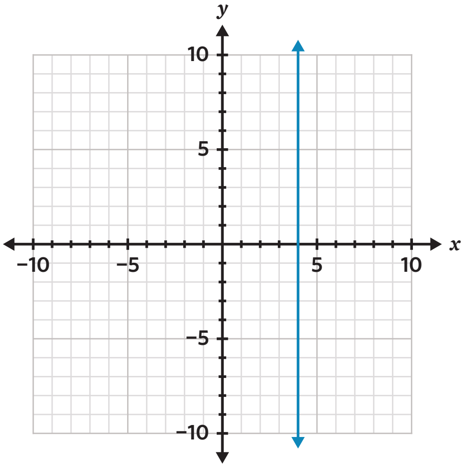
2.

	x-coordinate	y-coordinate
$y = 10x$	1	10
$y = -2x$	1	-2
$y = \frac{1}{2}x$	2	1
$y = -\frac{2}{5}x$	5	-2

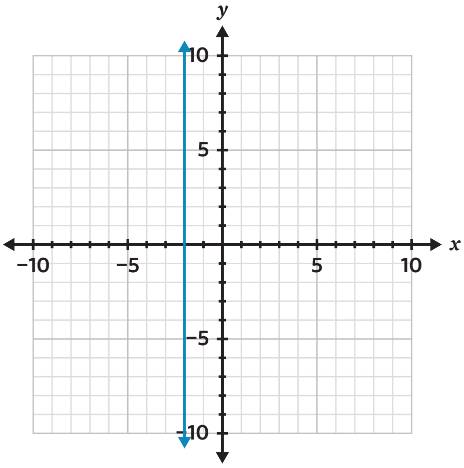
3. intercept; horizontal; vertical; origin

Fluency

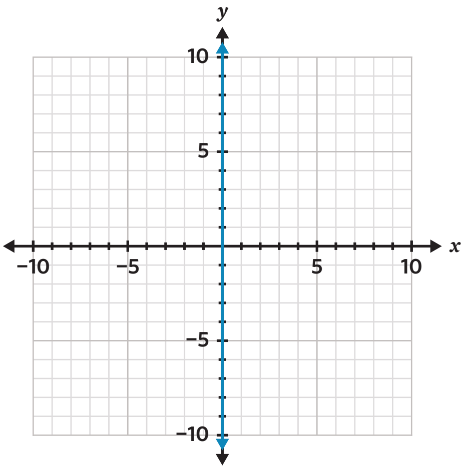
4. a.



b.

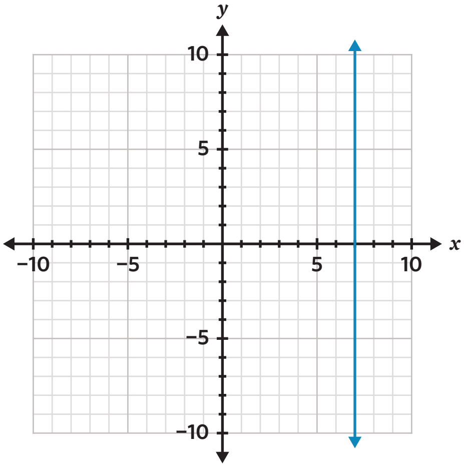


c.

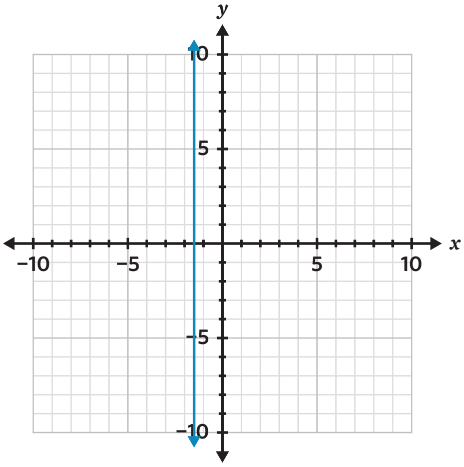


Back to contents

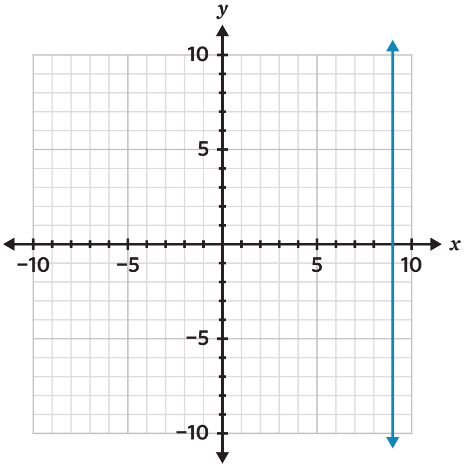
d.



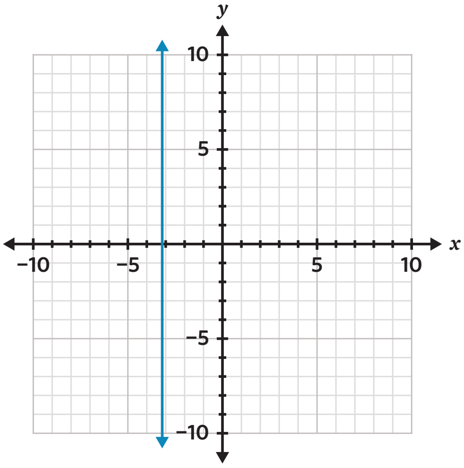
e.



f.



g.



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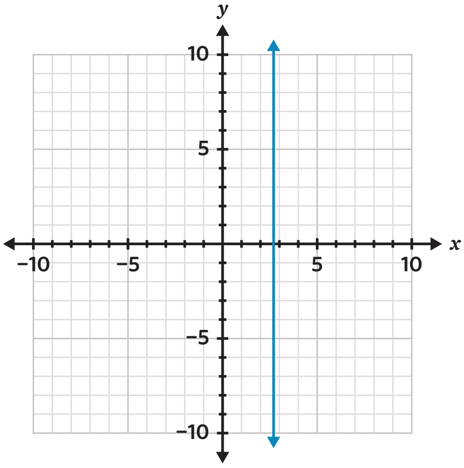
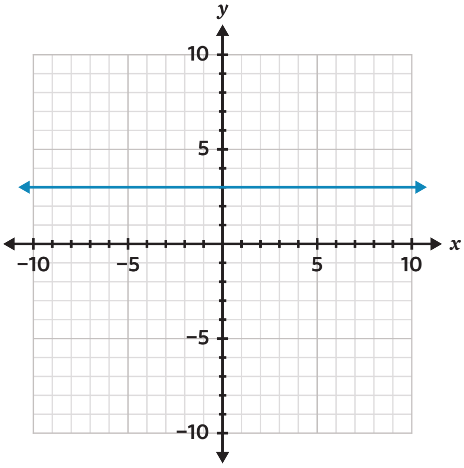
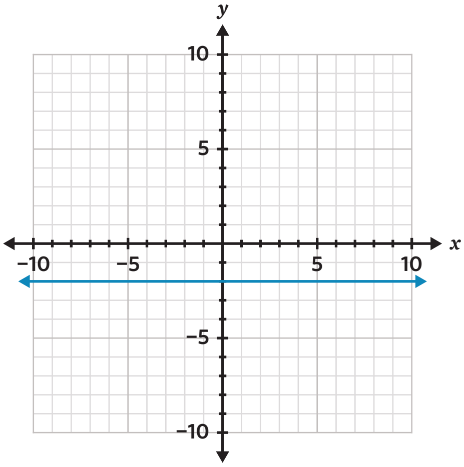


5. a.



b.



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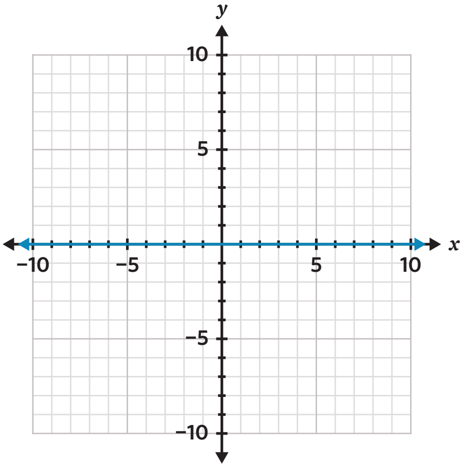

Back to contents

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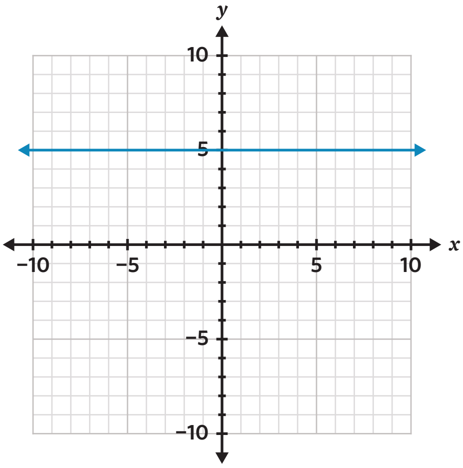

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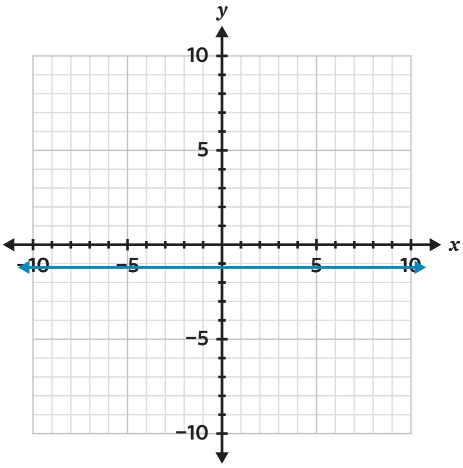

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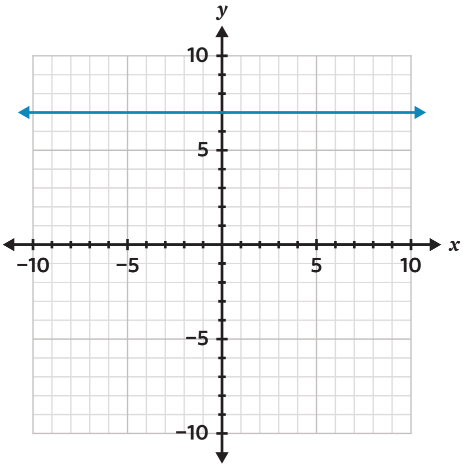

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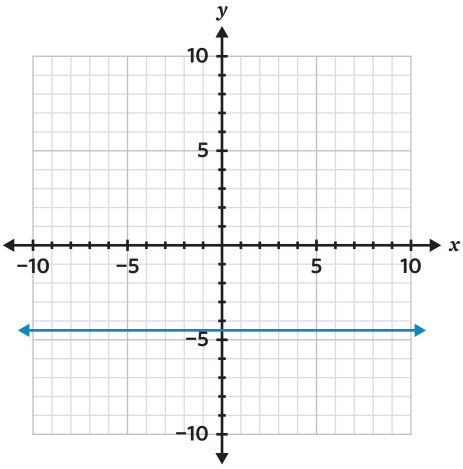

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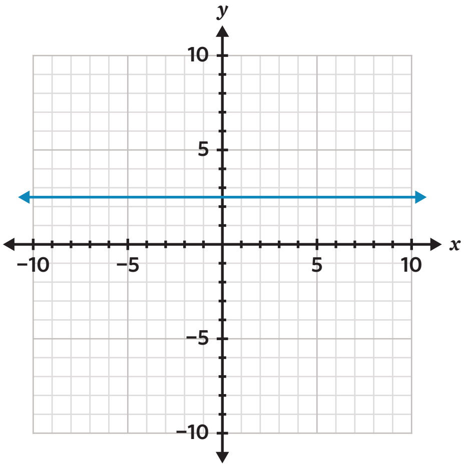

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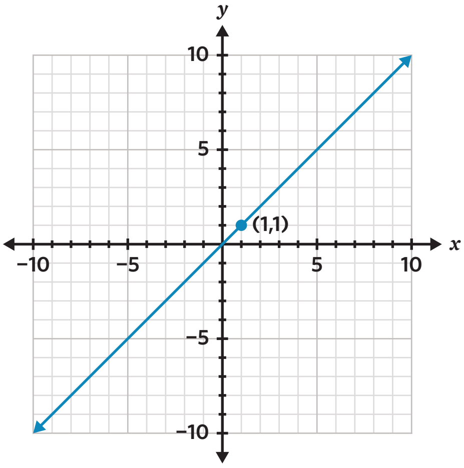
6. a.



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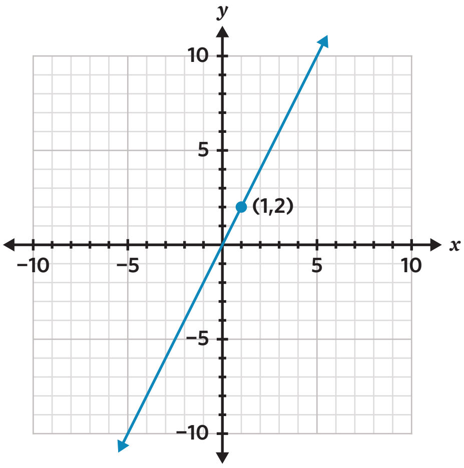

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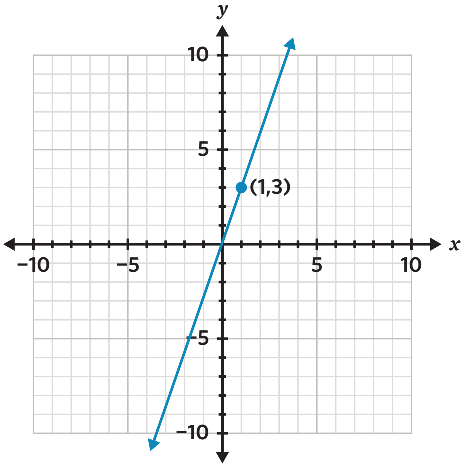

Back to contents

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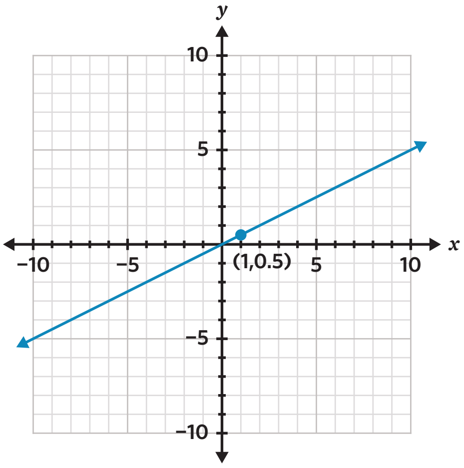

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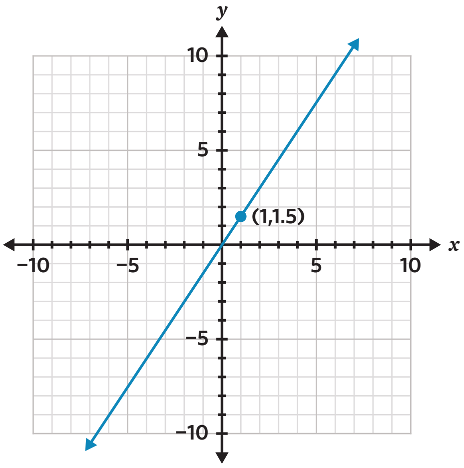

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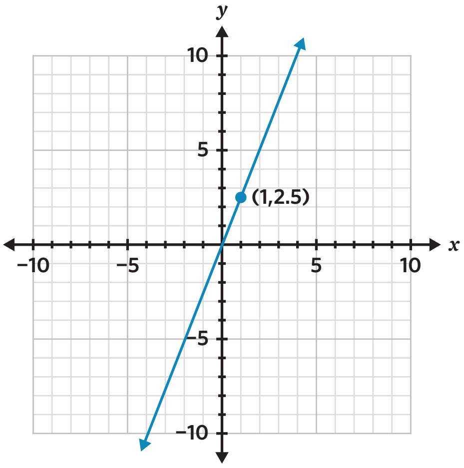

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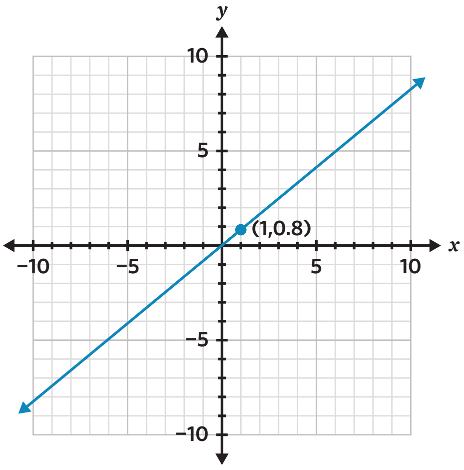

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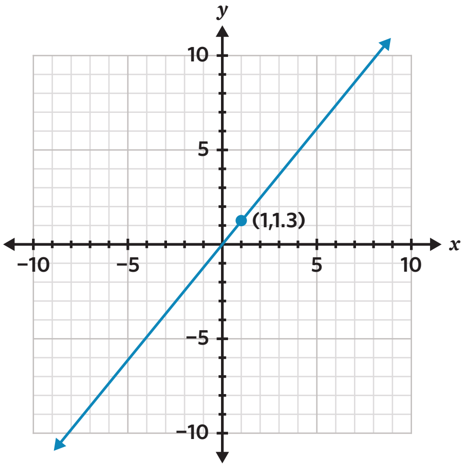
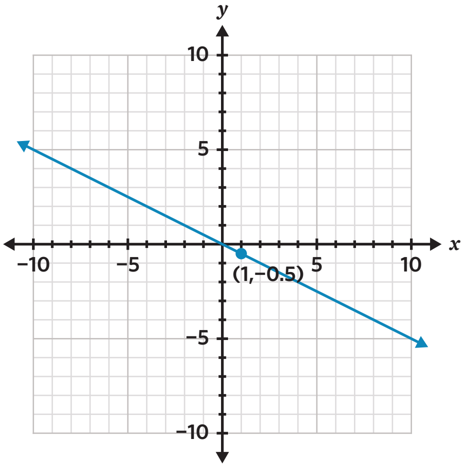


7. a.



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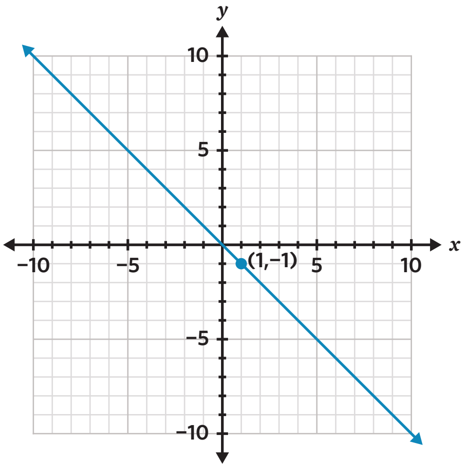

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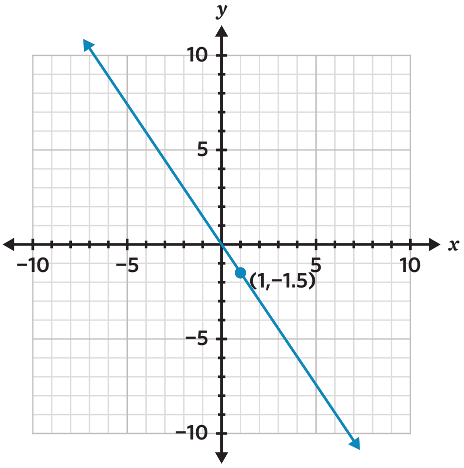

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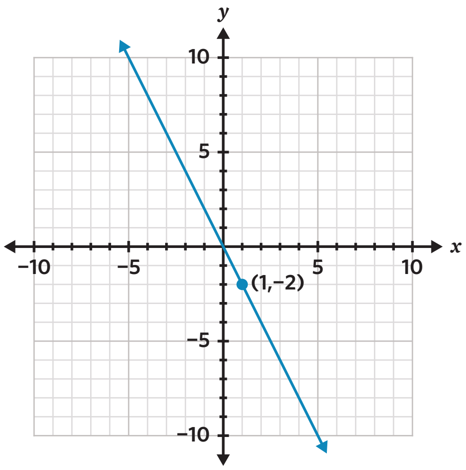

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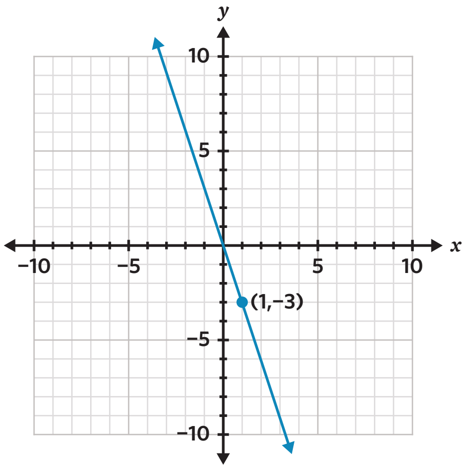

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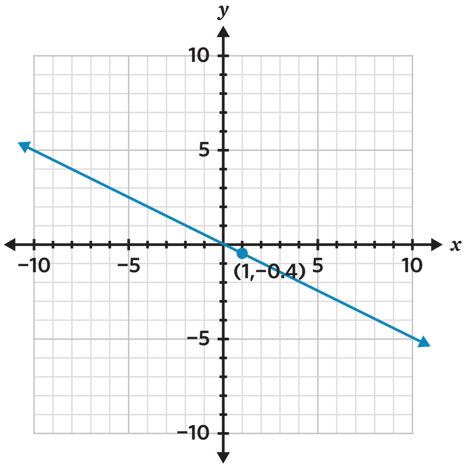

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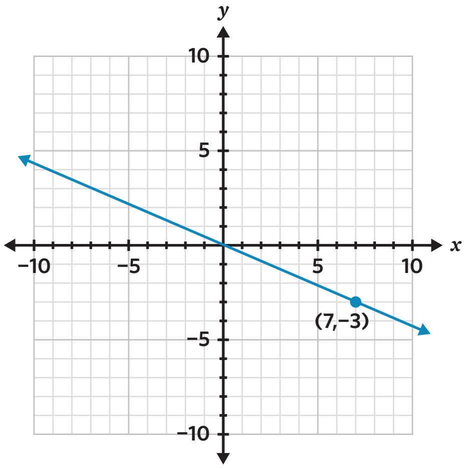

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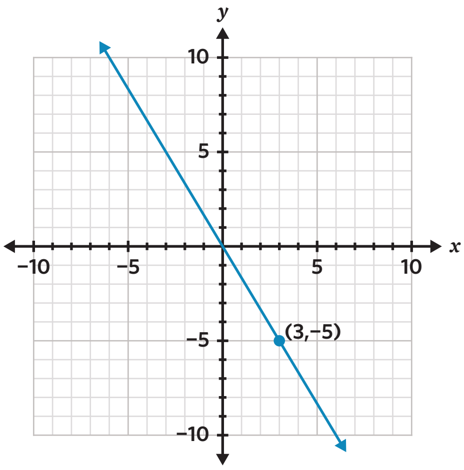


8. D

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. An IT help line is offering a service with a fixed fee (f) of \$5 per month with unlimited questions answered (q). Sketch a horizontal graph to represent this information, $0 \leq q$.

Key points

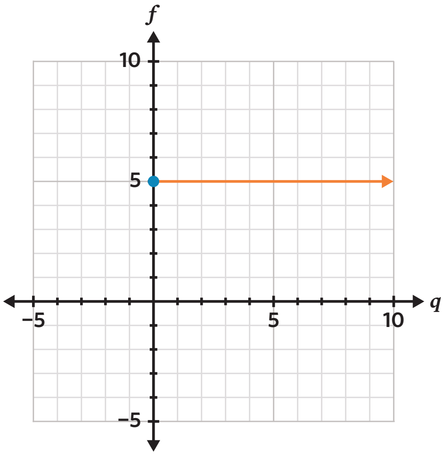
- The IT help line charges a fixed fee of \$5 per month.
- The number of questions answered (q) is unlimited.
- q is greater than or equal to 0.
- Sketch a horizontal graph to represent this information.

Explanation

The cost of the IT help line service is a fixed fee of \$5 per month, regardless of the number of questions answered.

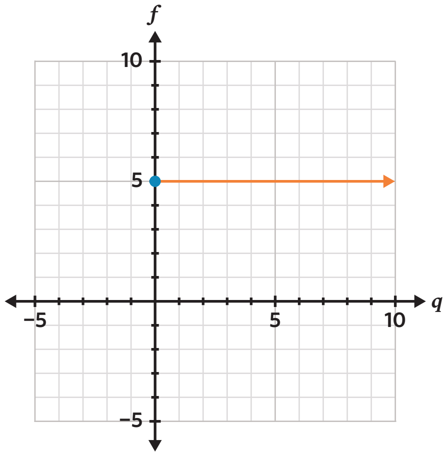
The graph would start at the point (0,5) on the y-axis, indicating that even with 0 questions answered, the cost is \$5. The line would then extend horizontally to the right, indicating that the cost remains \$5 no matter how many questions are answered.

This can be represented by the following graph:



Back to contents

Answer



11. An internet provider offers a fixed cost (c) of $50 per month regardless of the data (d) used. Sketch a horizontal graph to represent this information.

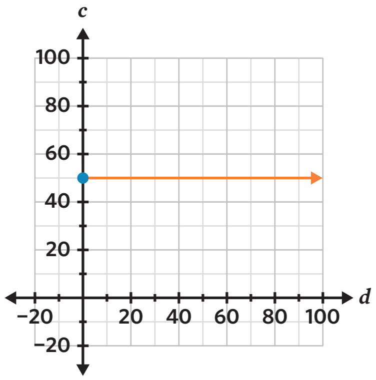
Key points

- The internet provider offers a fixed cost of $50 per month.
- The amount of data used (d) is unlimited.
- Sketch a horizontal graph to represent this information.

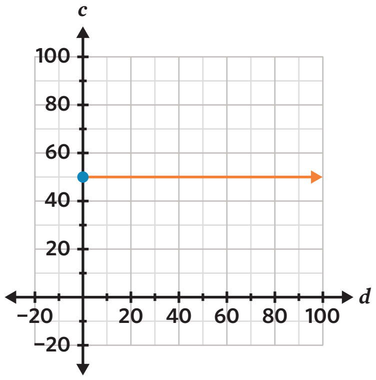
Explanation

The cost of the internet service is a fixed cost of $50 per month, regardless of the amount of data used. This means that the cost does not change with the amount of data used.

To represent this information on a graph, plot the cost (c) on the y-axis and the amount of data used (d) on the x-axis. Since the cost is constant, the graph will be a horizontal line at $50.



Answer



12. A tennis club charges $25 per lesson. Sketch a graph to represent this information showing the costs (c) for 0 to 10 lessons (l).

Key points

- The tennis club charges $25 per lesson.
- Sketch a graph to represent the cost for 0 to 10 lessons.

Explanation

Identify the cost per lesson:

The cost per lesson is $25.

Calculate the total cost for 10 lessons:

The total cost for 10 lessons is calculated by multiplying the cost per lesson by the number of lessons.

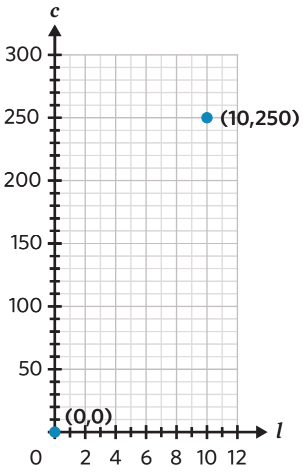
Total cost = cost per lesson × number of lessons

= $25 × 10

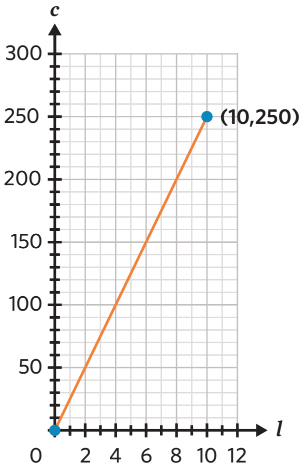
= $250

Sketch the graph:

To represent this information on a graph, plot the cost (c) on the y-axis and the number of lessons (l) on the x-axis. Since the cost is directly proportional to the number of lessons, the graph will be a straight line starting from the origin (0,0) and going up to the point (10,250).



Answer



13. A graphic designer uses a digital program to form a square. The program uses a Cartesian plane and the equations: $y = 0$, $x = 0$, $y = 1$, $x = 1$. What are the coordinates of each of the corners of the square?

Key points

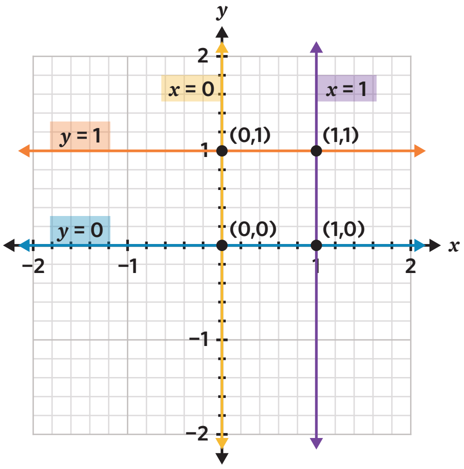
- The program uses a Cartesian plane.
- The equations used are: $y = 0, x = 0, y = 1, x = 1$.
- Determine the coordinates of each of the corners of the square.

Explanation

The given equations represent the four sides of the square on the Cartesian plane.

When plotted on the same Cartesian plane, the corners of the square are the points where these lines intersect.

Back to contents



Answer

The coordinates of the corners of the square are (0,0), (0,1), (1,0), and (1,1).

14. Three students are planning where they will meet during an orienteering challenge by plotting their paths on a map. They each walk along a different straight line each represented by one of these equations: $y = x, y = 7, x = 7$. What are the coordinates of the point they will meet?

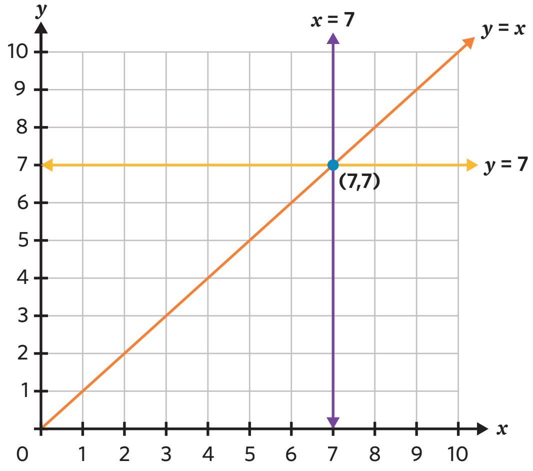
Key points

- The students are planning to meet at a point where their paths intersect.
- Each student's path is represented by a different equation: $y = x, y = 7, x = 7$.
- Find the coordinates of the point where these three lines intersect.

Explanation

The three equations given are $y = x, y = 7$, and $x = 7$.
Plot these equations on the same Cartesian plane and find the point of intersection.

The solution to these equations is the point where all three lines intersect.



Answer

The students will meet at the point (7,7).

Reasoning

15. a. An equation that could represent the horizontal sections of the A-line and D-line is $y = 0$.

b. An equation that when plotted would go through the vertical section of the A-line is $x = -10$.

c. An equation that when plotted would go through C-line stations 1–3 is $y = 10$.

d. An equation that when plotted would go through C-line stations 5–4 is $y = x$.

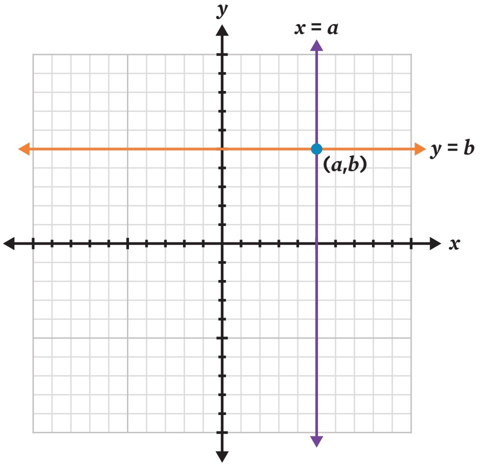
e. Suggested option 1: tram network

Suggested option 2: major city road grid

Note: There are other possible options.

16. a. The equation $x = a$ is vertical and the equation $y = b$ is horizontal.

b.

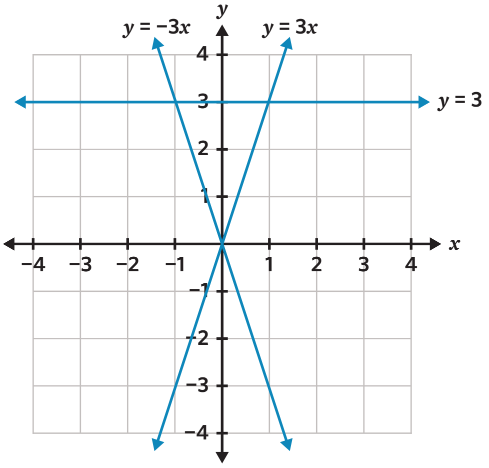


c. The vertical line represents the x-coordinate while the horizontal line represents the y-coordinate of the point of intersection between a vertical and a horizontal line.

Exam-style

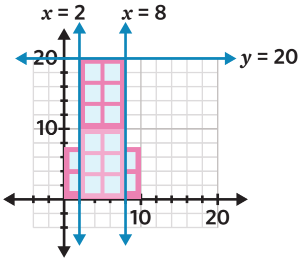
17. C

18. a.



b. The distance between the top two corners is 2 units.

19.



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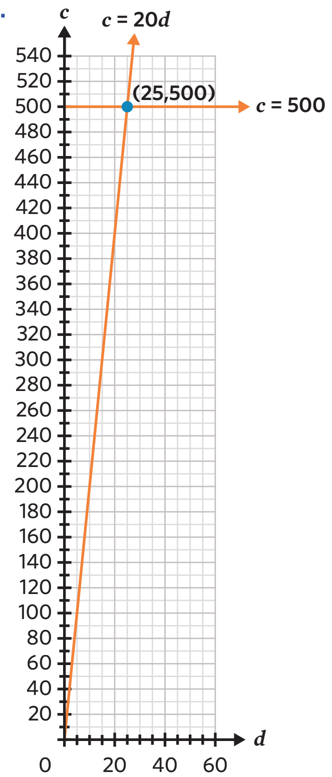


25 dogs

Remember this?

21. B

22. B

23. B

5C Gradient-intercept form

Student practice

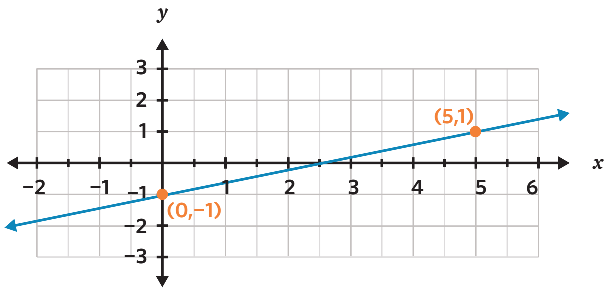
Worked example 1

a. Gradient $(m) = \frac{1}{4}$, y-intercept $(c) = 2$

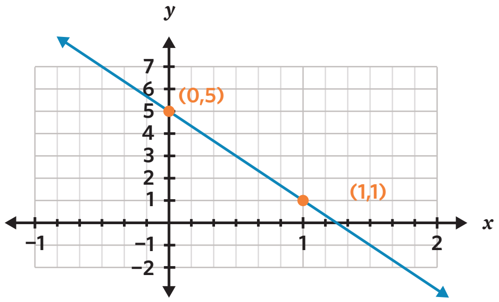
b. Gradient $(m) = -10$, y-intercept $(c) = 5$

Worked example 2

a.



b.



Worked example 3

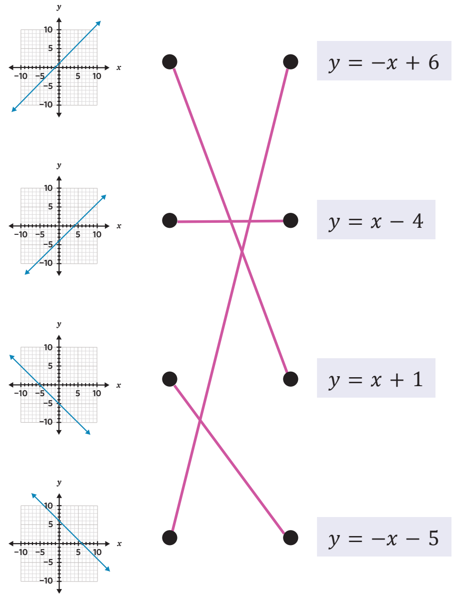
a. $y = -3x + 15$

b. $y = \frac{4}{7}x - 4$

Understanding worksheet

1. **a.** 4 **b.** −3 **c.** −5 **d.** 0.5

2. Graph Equation



3. gradient; sketch; equation; y-intercept

Fluency

4. **a.** Gradient $(m) = 2$, y-intercept $(c) = 3$

b. Gradient $(m) = -3$, y-intercept $(c) = -1$

c. Gradient $(m) = 0.5$, y-intercept $(c) = 2$

d. Gradient $(m) = -2$, y-intercept $(c) = 9$

e. Gradient $(m) = -1.2$, y-intercept $(c) = -3$

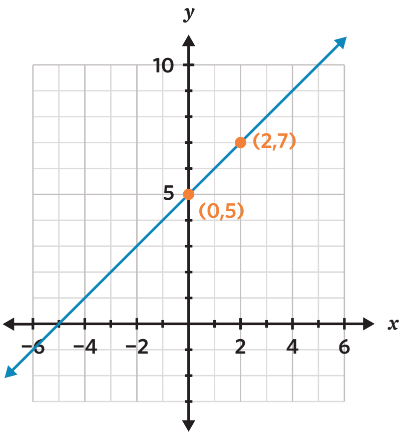
f. Gradient $(m) = -2.5$, y-intercept $(c) = 1$

g. Gradient $(m) = 0.4$, y-intercept $(c) = 0.1$

h. Gradient $(m) = 1.2$, y-intercept $(c) = 3$

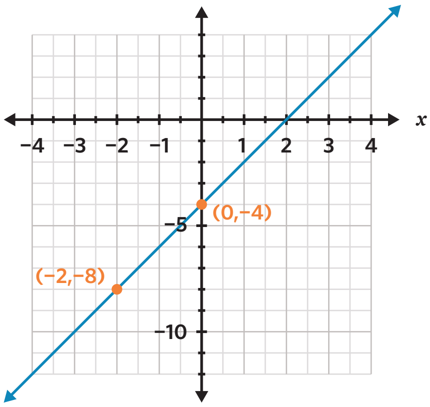
---

5. **a.**

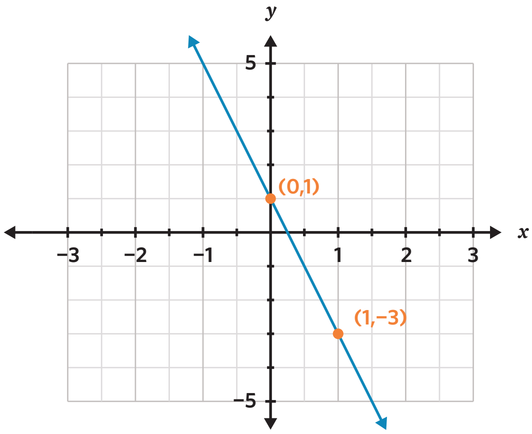


Back to contents

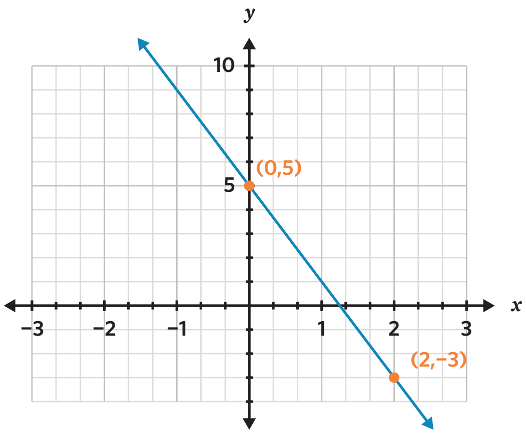
b.



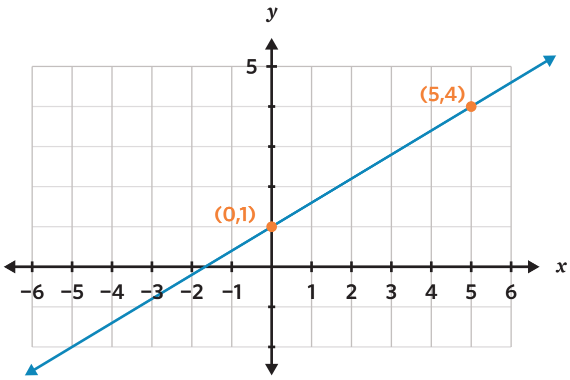
c.



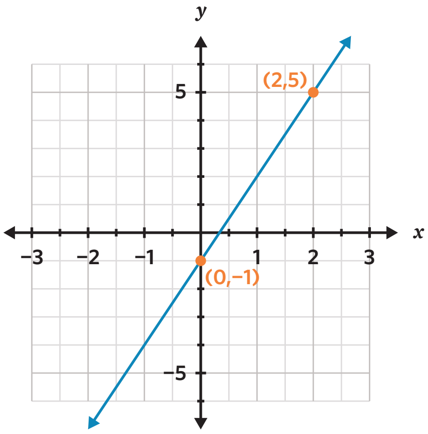
d.



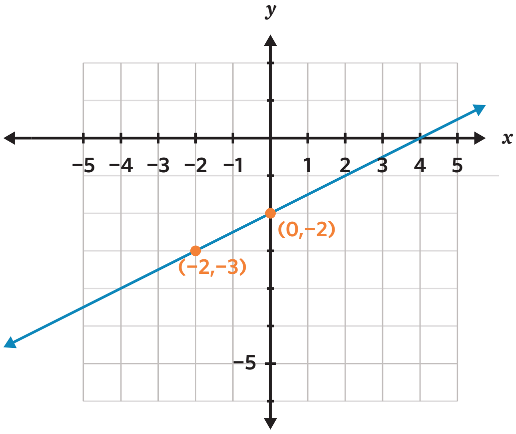
e.



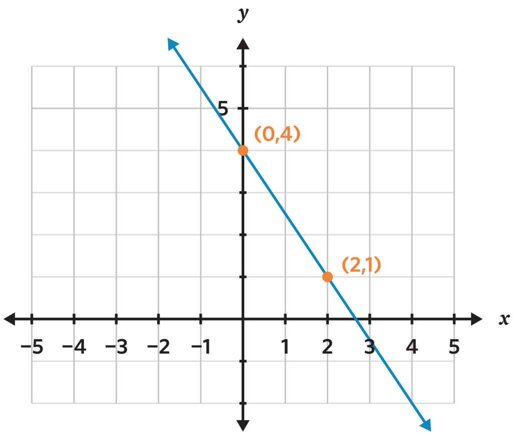
f.



g.



h.



6. **a.** $y = 2x + 1$ **b.** $y = x - 1.5$
c. $y = -3x + 6$ **d.** $y = x + 45$
e. $y = -2x + 100$ **f.** $y = 2x - 40$
g. $y = 50x - 1$ **h.** $y = -100x - 2$

7. **a.** $y = \frac{2}{3}x + 1$ **b.** $y = \frac{1}{2}x - 3$
c. $y = -\frac{5}{2}x + 5$ **d.** $y = -\frac{6}{5}x - 3$
e. $y = -\frac{8}{3}x + 6$ **f.** $y = \frac{17}{10}x - 17$
g. $y = \frac{9}{8}x - \frac{3}{2}$ **h.** $y = -\frac{3}{2}x + \frac{1}{2}$

8. B

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. An electricity company charges a monthly fee of $10 plus $0.25 per kilowatt hour used. This information can be graphed using the the rule $b = 0.25k + c$, where b is the amount of the monthly electricity bill, k is the number of kilowatt hours used. What is the value of c?

Key points

- An electricity company charges a monthly fee of $10 plus $0.25 per kilowatt hour used.
- This information is represented by rule $b = 0.25k + c$.
- b is the amount of the monthly electricity bill and k is the number of kilowatt hours used.
- What is the value of c?

Explanation

The equation $b = 0.25k + c$ represents the monthly electricity bill. It is in gradient-intercept form, which is written as $y = mx + c$, where m is the gradient, c is the y-intercept, and y is the subject.

The gradient is given as 0.25. This means that every kilowatt hour used costs $0.25.

The value of c, or the y-intercept, is the amount the electricity company charges as a fixed monthly fee, regardless of the number of kilowatt hours used.

The fixed monthly fee is $10. $\therefore c = 10$

Answer

The value of c is 10.

11. A student puts $30 in a moneybox and then adds $5 every week. Generate an equation representing the total amount of money in the moneybox (y) after (x) weeks.

Key points

- A student puts $30 in a moneybox.
- They then add $5 every week.
- Generate an equation representing the total amount of money in the moneybox (y) after (x) weeks.

Explanation

The gradient-intercept form is written as $y = mx + c$, where m is the gradient, c is the y-intercept, and y is the subject.

Determine the value of the y-intercept.

Initially, there is $30 in the moneybox. When $x = 0, y = 30$.
$\therefore$ The y-intercept is 30.

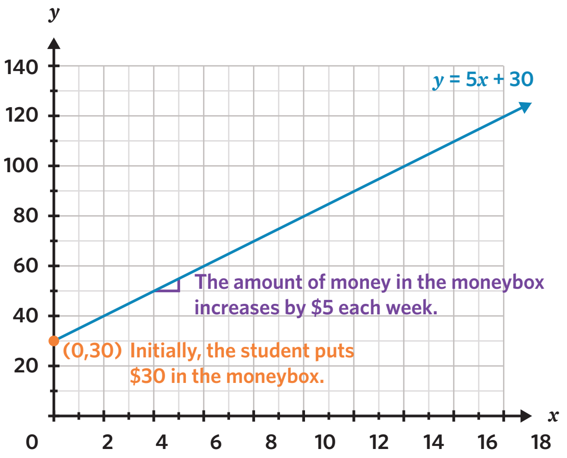
Determine the value of the gradient.

The student adds $5 to the moneybox each week. For every unit increase in x, y increases by 5.

$\therefore \frac{\text{rise}}{\text{run}} = \frac{5}{1}$, meaning the gradient is 5.

Substitute the values of the gradient and y-intercept into the gradient-intercept form.

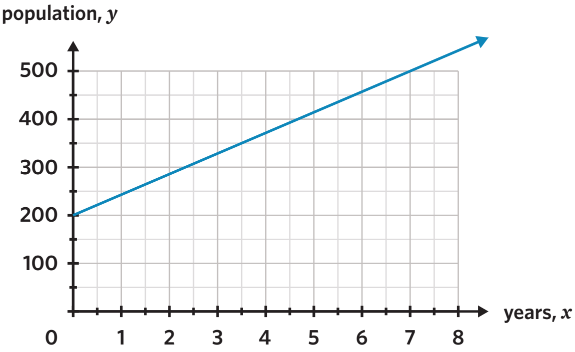
$y = 5x + 30$



Answer

$y = 5x + 30$ is the equation that represents the total amount of money in the moneybox (y) after (x) weeks.

12. The graph shows the population of a town over time.



Generate an equation that represents the population over time.

Key points

- The graph shows the population (y) over time (x).
- Generate an equation that represents the population over time.

Explanation

Look at the graph and determine the value of the y-intercept.

When $x = 0, y = 150$. This means that the population of the town was 150 people when it was initially recorded.

$\therefore$ The y-intercept is 150.

Look at the graph and determine the value of the gradient, or the degree of steepness of the line.

For every unit increase in x, y increases by 50. This means that every year, the population of the town increases by 50 people.

$\therefore \frac{\text{rise}}{\text{run}} = \frac{50}{1}$, meaning the gradient is 50.

Substitute the values of the gradient and y-intercept into the gradient-intercept form to generate an equation representing the population (y) over time (x).

$y = 50x + 150$

Answer

An equation that represents the population over time is $y = 50x + 150$.

13. A cup of hot chocolate is placed in a freezer and the temperature is measured each minute until it reaches −2°C. Sketch the graph if the initial temperature is 85°C and the temperature is dropping at a constant rate of 3°C each minute.

Key points

- The temperature of a cup of hot chocolate is measured each minute until it reaches −2°C.
- The initial temperature of the cup of hot chocolate is 85°C.
- The temperature is decreasing at a constant rate of 3°C each minute.
- Sketch the graph.

Explanation

Define the variables. Any pronumeral may be chosen to represent the variables.

Let x be the number of minutes since the cup of hot chocolate has been placed in the freezer.

Let y be the temperature of the cup of hot chocolate.

Identify the y-intercept.

Initially, the temperature of the cup of hot chocolate is 85°C. When $x = 0$, $y = 85$. ∴ The y-intercept is 85.

Determine the value of the gradient.

The temperature decreases by 3°C each minute. For every unit increase in x, y decreases by 3.

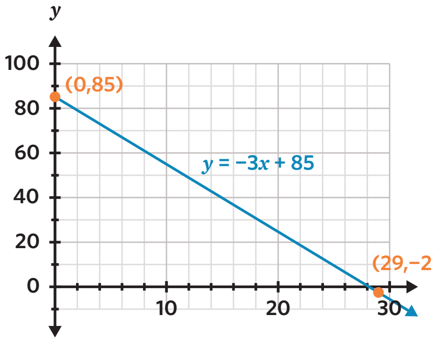
∴ $\frac{\text{rise}}{\text{run}} = \frac{-3}{1}$, meaning the gradient is −3.

Substitute the values of the gradient and y-intercept into the gradient-intercept form, written as $y = mx + c$, where m is the gradient and c is the y-intercept.

$y = -3x + 85$

Use this information to sketch the graph.

Answer



14. The cost of renting an e-scooter for one ride is advertised as $1 plus an additional fee of $1.20 per kilometre travelled. Generate an equation to represent the total cost (c) depending on the distance travelled (d).

Key points

- The cost of renting an e-scooter is $1 plus an additional fee of $1.20 per kilometre travelled.
- Generate an equation to represent the total cost (c) depending on the distance travelled (d).

Explanation

Determine the value of the y-intercept.

The fixed rate to rent the e-scooter is $1. When $d = 0$, $c = 1$.
∴ The y-intercept is 1.

Determine the value of the gradient.

Every kilometre travelled on the e-scooter costs $1.20. For every unit increase in d, c increases by 1.2.

∴ $\frac{\text{rise}}{\text{run}} = \frac{1.2}{1}$, meaning the gradient is 1.2.

Substitute the values of the gradient and y-intercept into the gradient-intercept form.

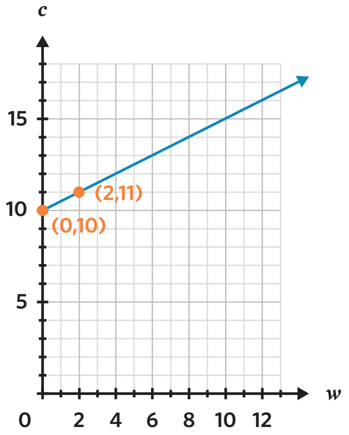
$c = 1.2d + 1$

Answer

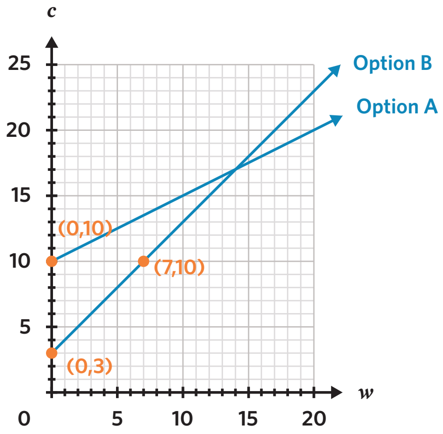
An equation that represents the total cost of the e-scooter (c) depending on the distance travelled (d) is $c = 1.2d + 1$.

Reasoning

15. a.



b.



c. Option B is cheaper for a parcel weighing 5 kg.

d. Sending a parcel weighing 14 kg costs the same amount of $17 for both options.

e. Suggested option 1: Parcels that are high priority should cost more that parcels that are low priority because the postal service must ensure they reach their destination quickly.

Suggested option 2: Parcels that weigh more should cost more because the postal service must transport more weight when delivering them.

Note: There are other possible options.

16. a. $y = -\frac{a}{b}x + \frac{c}{b}$

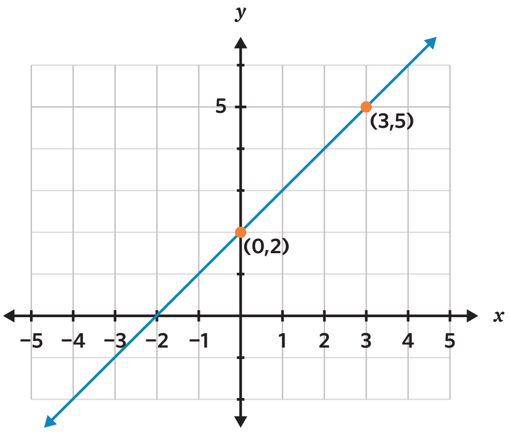
b. The gradient is $-\frac{a}{b}$ and the y-intercept is $\frac{c}{b}$.

c. To calculate the gradient and y-intercept from the form $ax + by = c$, the equation can be written in gradient-intercept form, $y = -\frac{a}{b}x + \frac{c}{b}$, where $-\frac{a}{b}$ is the gradient, $\frac{c}{b}$ is the y-intercept.

Exam-style

17. B

18. a.



b. Gradient $(m) = 1$

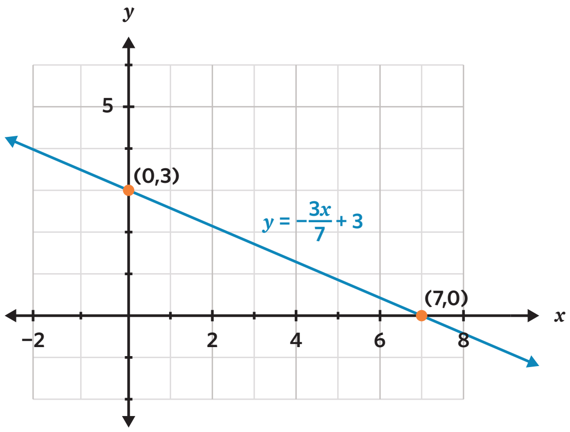
c. $y = x + 2$

19. A: $y = -\frac{15}{4}x + 6$

B: $y = \frac{6}{5}x + 6$

C: $y = -x - 5$

20.



Remember this?

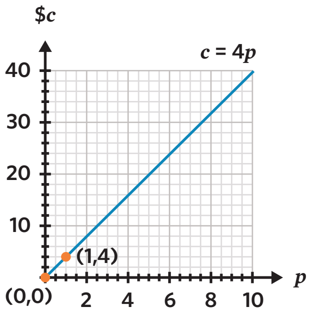
21. D **22.** D **23.** D

5D Gradient and direct proportion

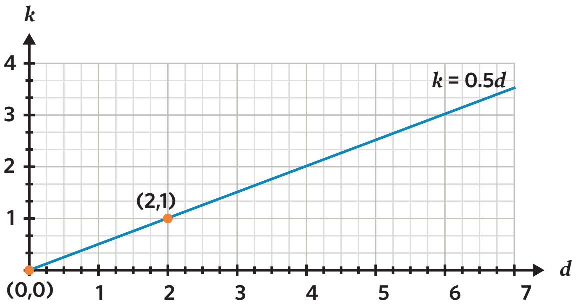
Student practice

Worked example 1

a.



b.



Worked example 2

a. $c = 22l$ **b.** $c = 3.8n$

Worked example 3

a. 5 hot drinks cost $22.50.

b. $p = 52b$; 4 buses are required for a school group of 208 students.

Understanding worksheet

1. **a.** 43 **b.** 20 000 **c.** 40 **d.** 2.55

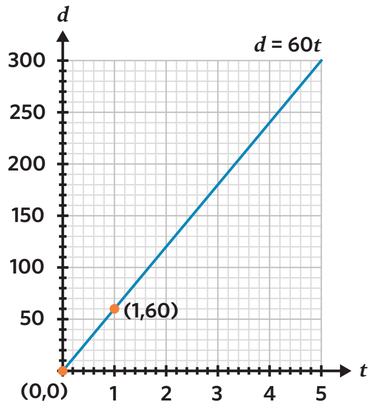
2. Direct proportion: II

Not direct proportion: I, III, IV

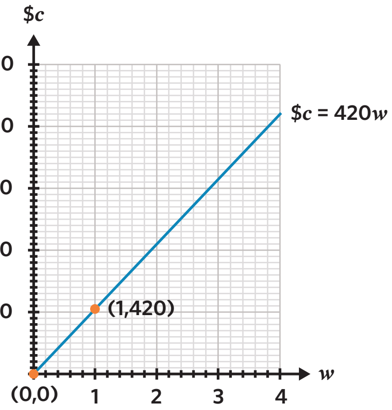
3. variables; rate; gradient; constant; proportional

Fluency

4. **a.**

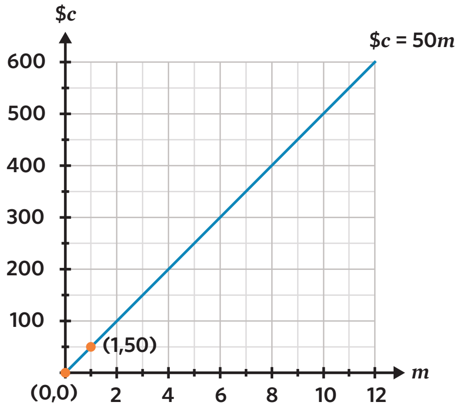


b.

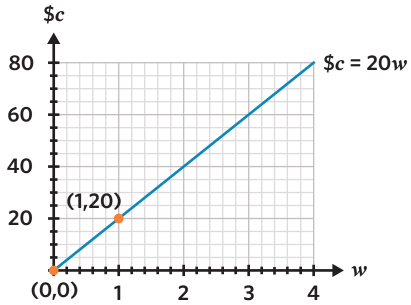


Back to contents

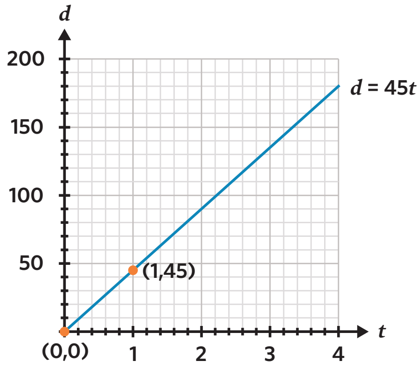
c.



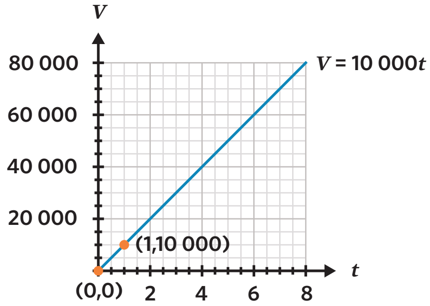
d.



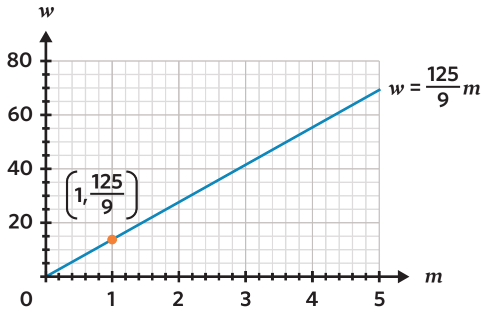
e.



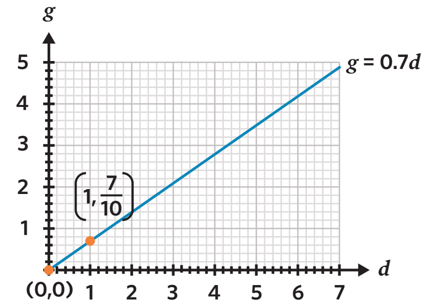
f.



g.



h.



5. **a.** $d = 38t$ **b.** $v = 12\,000t$
c. $w = 50t$ **d.** $h = 30t$
e. $d = 55t$ **f.** $c = 500n$
g. $c = 105t$ **h.** $c = 6.14t$

6. **a.** The profit on 20 items is $600.
b. The distance travelled in 30 minutes is 60 km.
c. The commission paid on $34 000 is $680.
d. The total number of messages that can be sent for $5 is 100.
e. The total fruit boxes packed by 15 employees is 450.
f. $v = 342t$; The amount of water flowing into the dam in 6 hours is 2052 L.
g. $w = 25t$; It will take the student 100 minutes to write a 2500 word essay.
h. $s = 12.50t$; 514 tickets are required to meet the sales target.

7. E

Spot the mistake

8. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. A student deposits $30 per month into a money box. Calculate the amount in the money box after 5 months.

Key points

- The rate of deposit is $30 per month.
- Calculate the amount in the money box after 5 months.

Explanation

$$\begin{aligned}\text{Total money} &= \text{rate of deposit} \times \text{number of months}\\ &= 30 \times 5\\ &= \$150\end{aligned}$$

Answer

The amount in the money box after 5 months is $150.

10. A leaf travels down a river at 2 km per hour. Calculate the distance a leaf would travel in 2 hours.

Key points

- The rate is 2 km per hour.
- Calculate the distance if the time is 2 hours.

Explanation

$$\begin{aligned}d &= s \times t\\ &= 2 \times 2\\ &= 4 \text{ km}\end{aligned}$$

Answer

A leaf would travel 4 km in 2 hours.

11. The tide is receding at 2 cm per minute. Write a rule connecting the distance (d) and time (t) and calculate the time, in minutes, it would take for the tide to recede 7 cm.

Key points

- The tide is receding at 2 cm per minute.
- Write a rule connecting the distance (d) and time (t) and calculate the time, in minutes, it would take for the tide to recede 7 cm.

Explanation

Distance = speed × time

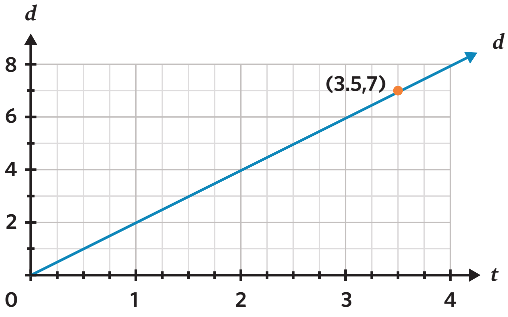
$d = 2t$

$d = 7$, solve for t

$7 = 2t$

$7 \div 2 = 2t \div 2$

$t = 3.5$



Answer

$d = 2t$; it would take 3.5 minutes or 3 minutes and 30 seconds for the tide to recede 7 cm.

12. A pond is being filled with water using two garden hoses. One hose has a flow rate of 20 L per minute and the other 30 L per minute. Calculate the time, in minutes, it will take to fill a 1000 litre pond.

Key points

- Two garden hoses are filling the pond.
- One has a flow rate of 20 L per minute.
- The other has a flow rate of 30 L per minute.
- Calculate the time, in minutes, it will take to fill a 1000 litre pond.

Explanation

Let t be the time it takes to fill a 1000 litre pond.

Volume = total flow rate × time

$1000 = (20 + 30) \times t$

$1000 = 50 \times t$

$1000 \div 50 = 50 \times t \div 50$

$20 = t$

$t = 20$

Answer

It takes 20 mins to fill a 1000 litre pond.

13. A kayaker is paddling at a rate of 3.5 km per hour. If the kayaker is paddling upstream against a river flowing at 1.5 km per hour, calculate how far they will travel in 3 hours.

Key points

- The rate is 3.5 km per hour.
- The rate is reduced by the river flowing at 1.5 km per hour against the kayaker.
- Calculate how far they will travel in 3 hours.

Explanation

Distance = speed × time

$= (3.5 - 1.5) \times 3$

$= 2 \times 3$

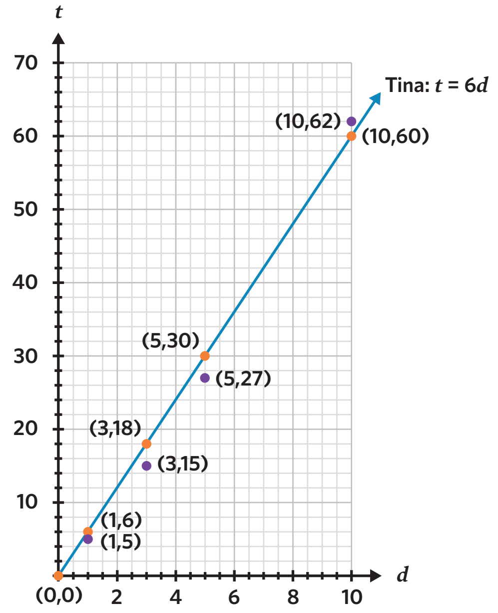
$= 6$ km

Answer

They will travel 6 km in 3 hours.

Reasoning

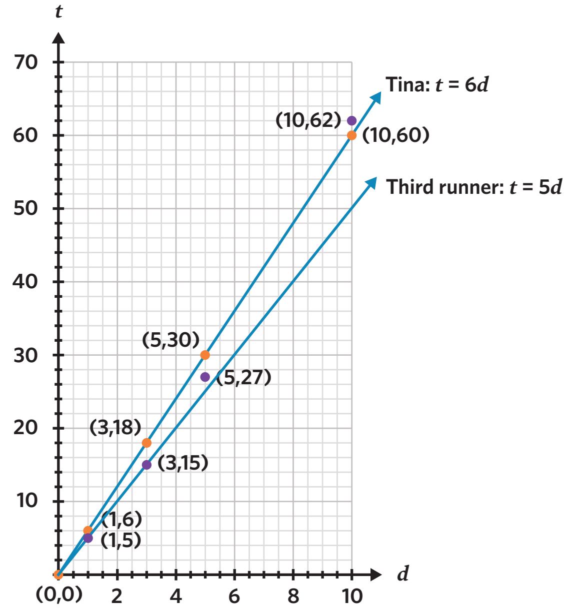
14. a.



b. $t = 6d$

c. $t = 5d$

d.



e. Suggested option 1: Runners can slowly increase the distance they run over months.

Suggested option 2: Runners can practise tempo running which is faster running over a shorter period of time.

Note: There are other possible options.

15. a.

Length (m)	0	1	2	3	4
Area (m^2)	0	1	4	9	16

b.

Length (m)	0	1	2	3	4
Perimeter (m)	0	4	8	12	16

c. The area is not directly proportional to the length as the rate of change of length and the area are not the same. The perimeter is directly proportional to the length as the rate of change of length to the perimeter is 4, which is constant.

Exam-style

16. C

17. a. 15 km/L
b. $d = 15l$
c. 600 km

18. $n = 3.5t$; 700 items can be produced in 200 hours

19. 200 m in 4.5 mins ≈ 44.44 m/min
50 m in 1.15 mins ≈ 43.48 m/min
∴ The first swimmer has a greater speed.

Remember this?

20. B **21.** D **22.** C

5E Midpoint and length of a line segment

Student practice

Worked example 1

a. (5,5) **b.** (−1,4.5)

Worked example 2

a. (14,16) **b.** (−2,−10)

Worked example 3

a. 13 **b.** 13.89

Understanding worksheet

1. a. 4 **b.** 9.5 **c.** 2.5 **d.** −2

2. a. *w* **b.** 80 **c.** 25 **d.** *z*

3. line; midpoint; average; distance

Fluency

4. a. (10,10) **b.** (−2.5,6)
c. (−4,3) **d.** (7.5,−12.5)
e. (−2,15.5) **f.** (1.5,9)
g. (−2.5,−7) **h.** (0,0)

5. a. (8,11) **b.** (−10,7)
c. (9,16) **d.** (34,−10)
e. (−3,−4) **f.** (10,15)
g. (28.5,10) **h.** (4,13)

6. a. 5 **b.** 10 **c.** 13 **d.** 13
e. 17 **f.** 61 **g.** 17 **h.** 26

7. a. 3.16 **b.** 9.85 **c.** 12.65 **d.** 13.93
e. 14.56 **f.** 10.82 **g.** 9.55 **h.** 21.93

8. B

Spot the mistake

9. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. Two hikers start walking towards each other from separate locations, $A(150,30)$ and $B(400,90)$. Both hikers will cover the same distance and meet in the middle point. Calculate the coordinates of their meeting point, if they were to walk in a straight line.

Key points

- Two hikers are walking towards each other.
- The first hiker is at point $A(150,30)$, and the second hiker is at point $B(400,90)$.
- Both hikers will cover the same distance and meet in the middle point.
- Calculate the coordinates of their meeting point, if they were to walk in a straight line.

Explanation

The meeting point is the midpoint of A and B.

$A(150,30)$, $B(400,90)$

$$M = \left(\frac{150 + 400}{2}, \frac{30 + 90}{2}\right)$$

$$M = \left(\frac{550}{2}, \frac{120}{2}\right)$$

$$M = (275,60)$$

Answer

The meeting point has the coordinates (275,60).

11. A cricket ball is hit and lands exactly between two players located at (25,30) and (15,27). Calculate the coordinates of the ball's landing point.

Key points

- A cricket ball lands on the midpoint of (25,30) and (15,27).
- Calculate the coordinates of the ball's landing point.

Explanation

Calculate the midpoint between (25,30) and (15,27).

$$M = \left(\frac{25 + 15}{2}, \frac{30 + 27}{2}\right)$$

$$M = \left(\frac{40}{2}, \frac{57}{2}\right)$$

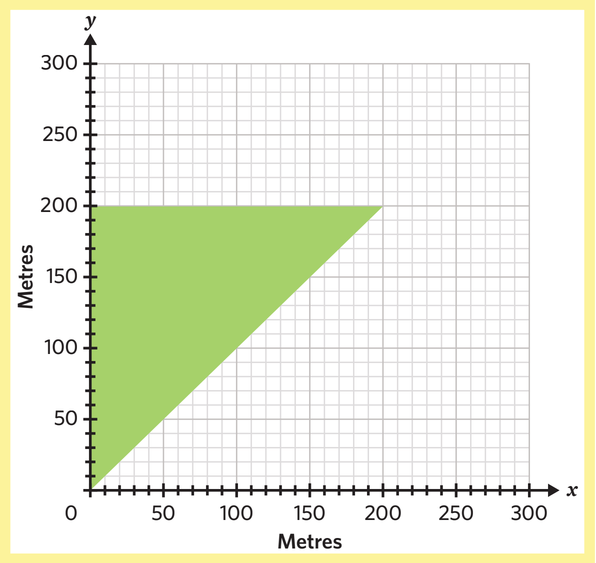
$$M = (20,28.5)$$

Answer

The balls landing point has the coordinates (20,28.5).

Back to contents

12. A dog park is to be fenced so the dogs do not have access to the road. Calculate the exact length, in metres, of fencing required to fence the perimeter of the park shown on the map in green. Round to one decimal place.



Key points

- The dimensions of the triangle is shown in the graph.
- Calculate the perimeter in metres. Round to one decimal place.

Explanation

Determine the length of the line segment connecting (0,0) and (200,200).

Vertical distance

$y_2 - y_1 = 200 - 0$

$= 200$

Horizontal distance

$x_2 - x_1 = 200 - 0$

$= 200$

$c^2 = a^2 + b^2$

$c^2 = 200^2 + 200^2$

$c^2 = 400 + 400$

$c^2 = 800$

$c \approx 282.84...$ m

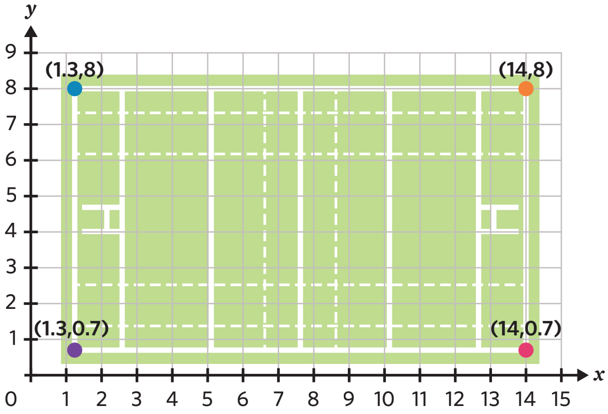
Perimeter $= 200 + 200 + 282.84...$

≈ 682.8 m

Answer

The perimeter is approximately 682.8 m.

13. A rugby game begins with a kickoff from the centre of the field. Using the information provided, calculate the coordinates of the location of the centre of this field. Round to one decimal place.



Key points

- A field has four points (1.3,0.7), (1.3,8), (14,0.7), (14,8).
- Calculate the coordinates of the location of the centre of this field. Round to one decimal place.

Explanation

The centre of the field is the midpoint between two points on a diagonal line.

Calculate the midpoint between (1.3,8) and (14,0.7).

$M = \left(\frac{1.3 + 14}{2}, \frac{8 + 0.7}{2}\right)$

$M = \left(\frac{15.3}{2}, \frac{8.7}{2}\right)$

$M = (7.65,4.35)$

$M \approx (7.7,4.4)$

Answer

The location of the centre of the field is (7.7,4.4).

14. A sailing race is set out using markers at locations $A(-10,5)$, $B(-70,-20)$ and $C(30,50)$. Calculate the distance around the course (1 unit = 1 km). Round to one decimal place.

Key points

- A sailing race is set out using markers at locations $A(-10,5)$, $B(-70,-20)$ and $C(30,50)$.
- Calculate the distance around the course (1 unit = 1 km). Round to one decimal place.

Explanation

Distance around the course $= \overline{AB} + \overline{BC} + \overline{AC}$

$\overline{AB} = \sqrt{(-70 - (-10))^2 + (-20 - 5)^2}$

$= \sqrt{(-60)^2 + (-25)^2}$

$= \sqrt{3600 + 625}$

$= \sqrt{4225}$

$= 65$

$\overline{BC} = \sqrt{(30 - (-70))^2 + (50 - (-20))^2}$

$= \sqrt{100^2 + 70^2}$

$= \sqrt{10\,000 + 4900}$

$= \sqrt{14\,900}$

$= 10\sqrt{149}$

$\overline{AC} = \sqrt{(30 - (-10))^2 + (50 - 5)^2}$

$= \sqrt{40^2 + 45^2}$

$= \sqrt{1600 + 2025}$

$= \sqrt{3625}$

$= 5\sqrt{145}$

$\overline{AB} + \overline{BC} + \overline{AC} = 65 + 10\sqrt{149} + 5\sqrt{145}$ km

≈ 247.3 km

Answer

The distance around the course is 247.3 km.

Reasoning

15. a. The distance between point A and point B is $2\sqrt{5} \approx 4.5$ km.

b. The coordinates of the end point C is (2,0).

c. $A(-6,4)$, $C(2,0)$

Vertical distance

$y_2 - y_1 = 0 - 4$

$= -4$

Back to contents

Horizontal distance

$x_2 - x_1 = 2 - (-6)$

$= 8$

$c^2 = a^2 + b^2$

$c^2 = (-4)^2 + 8^2$

$c^2 = 16 + 64$

$c^2 = 80$

$c = \sqrt{80}$

$c = 4\sqrt{5}$

$4\sqrt{5} = 2 \times 2\sqrt{5}$

∴ The distance between point A and C is twice the distance calculated in part **a**.

d. Suggested option 1: Topology maps are used by bushwalkers to understand how steep the climb is.

Suggested option 2: Topology maps are used by the government to assist with urban planning.

Note: There are other possible options.

16. **a.** $y_2 - y_1$ **b.** $x_2 - x_1$

c. $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

Exam-style

17. C

18. **a.** 80.22 **b.** 90.05 **c.** (75,0.5)

19. $B(3,0)$

$$\begin{aligned}\overline{AB} &= \sqrt{(-5-3)^2 + (4-0)^2}\\ &= \sqrt{(-8)^2 + 4^2}\\ &= \sqrt{64 + 16}\\ &= \sqrt{80}\\ &= 2\sqrt{20}\end{aligned}$$

20.

$$\begin{aligned}\overline{AB} &= \sqrt{(2-1)^2 + (8-2)^2}\\ &= \sqrt{1^2 + 6^2}\\ &= \sqrt{1 + 36}\\ &= \sqrt{37}\\ &\approx 6.08\\ \overline{BC} &= \sqrt{(12-2)^2 + (6-8)^2}\\ &= \sqrt{10^2 + (-2)^2}\\ &= \sqrt{100 + 4}\\ &= \sqrt{104}\\ &\approx 10.20\\ \overline{AC} &= \sqrt{(12-1)^2 + (6-2)^2}\\ &= \sqrt{11^2 + 4^2}\\ &= \sqrt{121 + 16}\\ &= \sqrt{137}\\ &\approx 11.70\end{aligned}$$

$11.70 > 10.20 > 6.08$

∴ $\overline{AC}$ is the longest leg.

Remember this?

21. C **22.** A **23.** C

5F Equations of lines

Student practice

Worked example 1

a. $y = -3x + 11$ **b.** $y = -4x + 2$

Worked example 2

a. $y = 3x + 2$ **b.** $y = 2x + 6$

Worked example 3

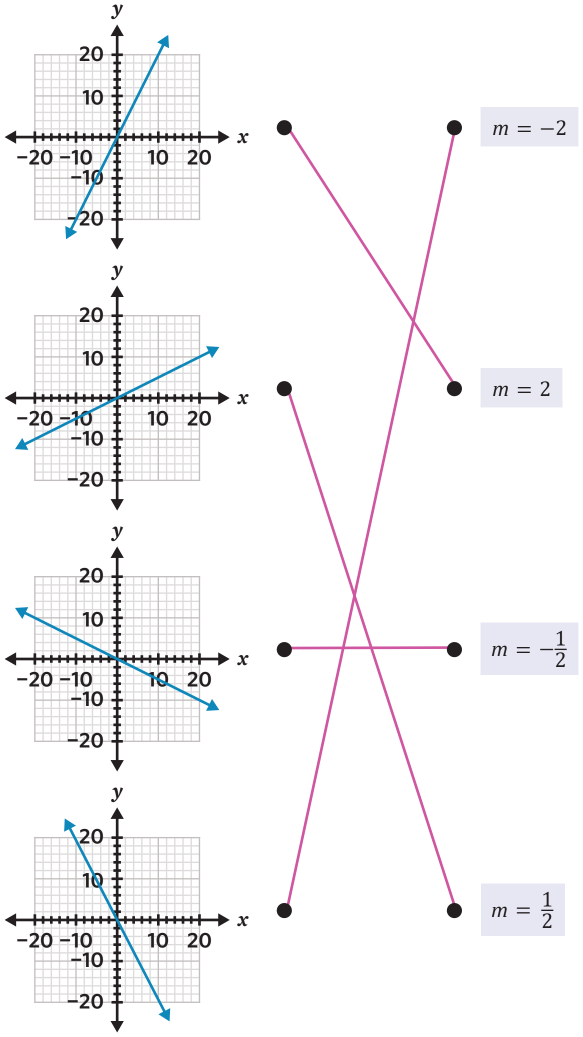
a. $y = -\frac{1}{5}x + 3$ **b.** $y = -\frac{3}{2}x + 9$

Understanding worksheet

1. **a.** parallel **b.** perpendicular

c. parallel **d.** perpendicular

2. Linear graph Gradient



3. parallel; equal; perpendicular; reciprocal; gradient

Fluency

4. **a.** $y = 7x - 14$ **b.** $y = -6x + 24$

c. $y = x + 3$ **d.** $y = 5x - 6$

e. $y = -3x + 6$ **f.** $y = 3x + 1$

g. $y = 4x - 7$ **h.** $y = 5x + 19$

Back to contents

5. **a.** $y = -2x - 3$ **b.** $y = \frac{4}{7}x - 4$
c. $y = -\frac{5}{4}x + \frac{27}{4}$ **d.** $y = \frac{3}{2}x - 5$
e. $y = -\frac{4}{5}x + 9$ **f.** $y = \frac{3}{2}x - 3$
g. $y = 12x - 87$ **h.** $y = -\frac{5}{4}x + \frac{15}{4}$

6. **a.** $y = 4x + 9$ **b.** $y = -2x + 7$
c. $y = -6x + 19$ **d.** $y = \frac{1}{2}x + 4$
e. $y = \frac{4}{5}x + \frac{1}{5}$ **f.** $y = -\frac{1}{4}x - \frac{1}{2}$
g. $y = -4.5x - 7.5$ **h.** $y = 0.3x + 2.4$

7. **a.** $y = -\frac{1}{2}x + 6$ **b.** $y = \frac{1}{4}x - 3$
c. $y = -\frac{3}{2}x + 1$ **d.** $y = 7x + 1$
e. $y = x + 1$ **f.** $y = -\frac{5}{4}x + 6$
g. $y = 0.4x - 3$ **h.** $y = -\frac{10}{3}x + \frac{50}{3}$

8. C

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. The council planted mature trees on the nature strip and conducted annual health checks, measuring their growth. Upon planting, a tree had a height of 1 m. After one year, its height increased to 1.2 m. Determine the equation that represents the tree's growth over time.

Key points

- Upon planting, a tree had an initial height of 1 m.
- After one year, the height of the tree increased to 1.2 m.
- Determine the equation that represents the tree's growth over time.

Explanation

Let x be the time, in years, and y be the height of the tree in metres.

Two points can be identified: (0,1) and (1,1.2)

Calculate the gradient, m, using the points.

$$m = \frac{1.2 - 1}{1 - 0}$$
$$= \frac{0.2}{1}$$
$$= 0.2$$

Identify the value for c.

The y-intercept is given: (0,1)

$\therefore c = 1$

Write the equation in the form $y = mx + c$.

$y = 0.2x + 1$

Answer

The equation $y = 0.2x + 1$ represents the tree's growth over time.

11. A fencing design uses parallel lines to represent the bottom, mid and top rails that are 60 cm apart. On sloping ground equations of the bottom and mid lines are $y = 0.5x$ and $y = 0.5x + 60$. State the equation used to represent the top rail.

Key points

- A fencing design uses parallel lines to represent the bottom, mid and top rails that are 60 cm apart.
- The equation of the bottom line is $y = 0.5x$.
- The equation of the mid line is $y = 0.5x + 60$.
- State the equation used to represent the top rail.

Explanation

Identify the gradient, m.

Parallel lines have the same gradient.

$m = 0.5$

Identify the value for c.

The top rail is 60 cm above the mid line so the y-intercept shifts up by 60.

$$c = 60 + 60$$
$$= 120$$

Write the equation in the form $y = mx + c$.

$y = 0.5x + 120$

Answer

$y = 0.5x + 120$

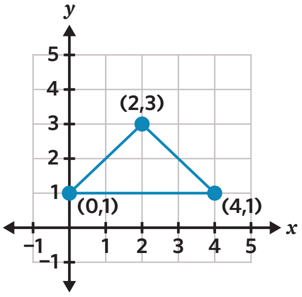
12. An architect drafted a design for a roof shaped like a right-angled triangle. The design outlines the roof's vertices at points (0,1), (2,3), and (4,1). Determine the equations for each of the sides of the triangle.

Key points

- The design for the roof is shaped like a right-angled triangle.
- The vertices are at points (0,1), (2,3) and (4,1).
- Determine the equations for each of the sides of the triangle.

Explanation

Draw the design for the roof using the coordinates of the vertices.



Determine the equation for the base of the roof.

$m = 0$ (horizontal line)

$c = 1$

$\therefore y = 1$

Determine the equation for the left slant of the roof.

$$m = \frac{3 - 1}{2 - 0}$$
$$= \frac{2}{2}$$
$$= 1$$

$c = 1$

$\therefore y = x + 1$

Determine the equation for the right slant of the roof.

$$\begin{aligned} m &= \frac{1-3}{4-2} \\ &= \frac{-2}{2} \\ &= -1 \end{aligned}$$

$1 = -1(4) + c$

$c = 5$

$\therefore y = -x + 5$

Answer

The equations for the sides of the roof are: $y = 1$, $y = x + 1$ and $y = -x + 5$.

13. A zip-line is anchored at the top of a platform and extends to the ground, 17.5 m away from the platform's base. Determine the linear equation for the zip-line, given that it has a gradient of −0.4.

Key points

- A zip-line extends from a platform to the ground 17.5 m away from the platform's base.
- The gradient of the zip-line is −0.4.
- Determine the linear equation for the zip-line.

Explanation

Identify the gradient, m.

$m = -0.4$

Substitute known x and y values from a given point and the gradient into $y = mx + c$.

$(x,y) = (17.5,0)$

$0 = -0.4(17.5) + c$

Simplify and solve to find c.

$0 = -7 + c$

$c = 7$

Write the equation in the form $y = mx + c$.

$y = -0.4x + 7$

Answer

The linear equation for the zip-line is $y = -0.4x + 7$.

14. The design of a bridge incorporates parallel and perpendicular lines. The segment of the bridge that spans the river begins at (0,70) and ends at (120,70). Determine the equations for the two pillars supporting the bridge, given that they are perpendicular to the span of the bridge.

Key points

- The segment of the bridge that spans the river begins at (0,70) and ends at (120,70)
- The two pillars supporting the bridge are perpendicular to the bridge.
- Determine the equations for the two pillars.

Explanation

Determine the gradient, m_1, of the span of the bridge.

$$\begin{aligned} m_1 &= \frac{70-70}{120-0} \\ &= \frac{0}{120} \\ &= 0 \end{aligned}$$

Determine the gradient, m_2, of the perpendicular pillars.

$$\begin{aligned} m_2 &= -\frac{1}{m_1} \\ &= -\frac{1}{0} \\ &= \text{undefined} \end{aligned}$$

The pillars have an undefined gradient which means they are represented by vertical lines.

The equation for a vertical line is expressed as $x = 0$ and $x = 120$.

Answer

$x = 0$ and $x = 120$

Reasoning

15. a. $m_{coastline} = \frac{8-4}{2-0} = 2$

$m_{Xero} = \frac{7-3}{3-1} = 2$

$\therefore m_{coastline} = m_{Xero}$

b. $m_{Xero} = \frac{7-3}{3-1} = 2$

$m_{Max} = \frac{3.5-1}{3-8} = -\frac{1}{2}$

$\therefore m_{Max} = -\frac{1}{m_{Xero}}$

c. $y = 2x - 2.5$

d. Suggested option 1: It can assist with navigation and bearing since sailors often use paths that are parallel to known routes or coastlines.

Suggested option 2: It can help avoid collisions as the sailor can recognise if a boat is on a perpendicular path that might intersect with their own path.

Note: There are other possible options.

16. a. $m = -\frac{2}{3}$

b. $m = -\frac{a}{b}$

c. $m = \frac{b}{a}$

Exam-style

17. C

18. a. $m_{AB} = \frac{9-5}{2-0} = \frac{4}{2} = 2$

$m_{CD} = \frac{6-2}{8-6} = \frac{4}{2} = 2$

$\therefore m_{AB} = m_{CD}$

b. $m_{AB} = \frac{9-5}{2-0} = \frac{4}{2} = 2$

$m_{BD} = \frac{6-9}{8-2} = \frac{-3}{6} = -\frac{1}{2}$

$\therefore m_{BD} = -\frac{1}{m_{AB}}$

c. They are parallel.

19. Line A: $y = x + 10$

Line B: $y = x - 10$

Line C: $y = -x + 30$

Line D: $y = -x + 10$

Lines A and B are parallel since $m_A = m_B = 1$.

Lines C and D are parallel since $m_C = m_D = -1$.

Lines A and B are perpendicular to lines C and D. This is because the negative reciprocal of 1 is −1.

20. $y = 2$, $y = -\frac{1}{4}x + 11$, $y = 4x + 28$

The two slanted sides of the triangle are perpendicular.

Remember this?

21. C **22.** D **23.** C

Back to contents

5G Graphical solutions to simultaneous equations

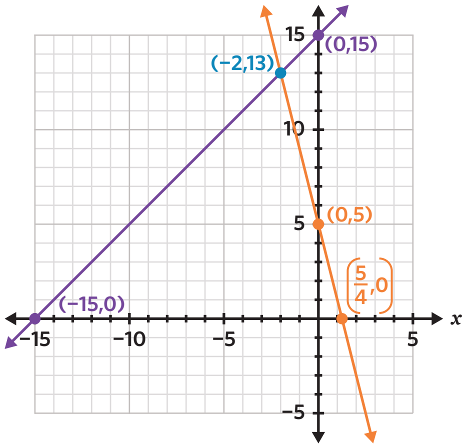
Student practice

Worked example 1

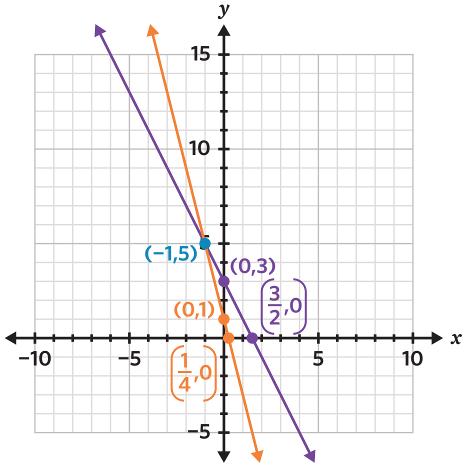
a. Yes **b.** No

Worked example 2

a.



b.



Worked example 3

a. Zero solutions **b.** One solution

Understanding worksheet

1. Zero solutions: II
One solution: I; III
Infinite solutions: IV

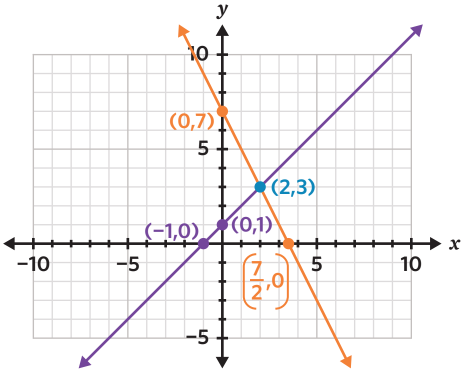
2. True: II
False: I; III; IV

3. infinite; solving; coordinates; parallel; intersection

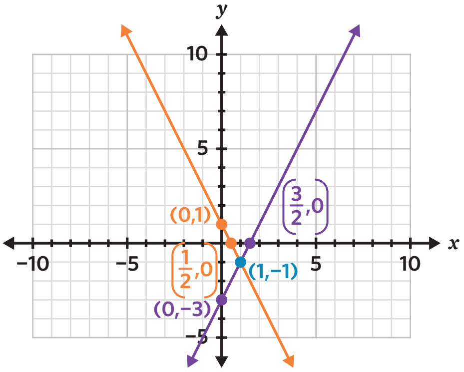
Fluency

4. **a.** Yes **b.** Yes **c.** No **d.** Yes
e. No **f.** No

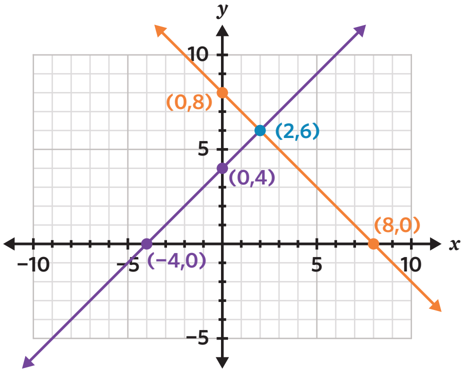
5. **a.**



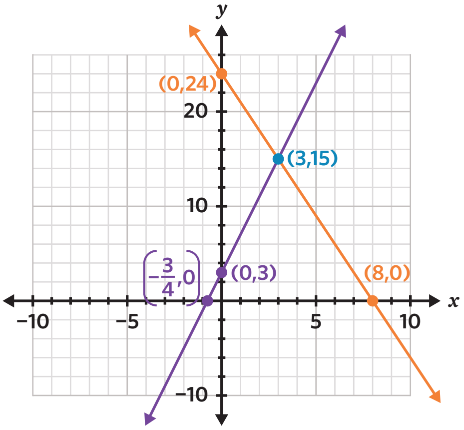
b.



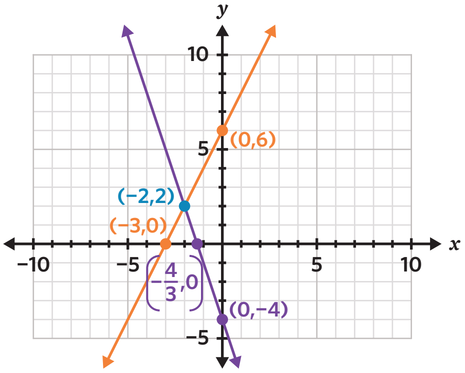
c.



d.

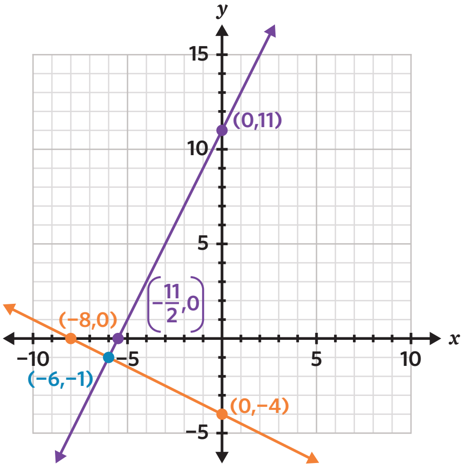


e.

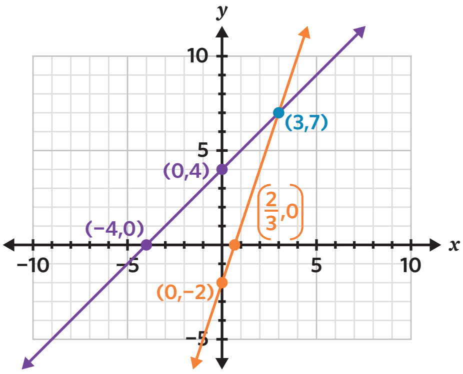


Back to contents

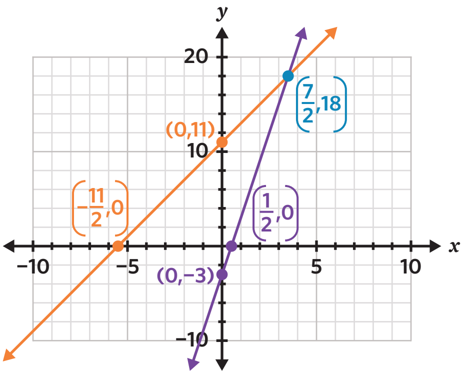
f.



g.



h.



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6. **a.** One solution
b. Infinite solutions
c. Zero solutions
d. One solution
e. Infinite solutions
f. Zero solutions

---

7. **a.** Infinite solutions
b. One solution
c. One solution
d. Zero solutions
e. Zero solutions
f. One solution

---

8. B

Spot the mistake

9. **a.** Student B is incorrect.
b. Student A is incorrect.

Problem solving

10. Two stalls are promoting strawberry punnets (p) using distinct pricing approaches. Stall A applies a fixed rate of \$6 per punnet, whereas Stall B employs a pricing structure of \$5 per punnet along with an additional \$2 per order. Using the equations $c = 6p$ and $c = 5p + 2$, verify if purchasing 2 punnets will result in an identical cost of \$12 at both stalls.

Key points

- Stall A is selling strawberry punnets at a fixed rate of \$6 per punnet, thus $c = 6p$.
- Stall B employs a pricing structure of \$5 per punnet along with an additional \$2 per order, thus $c = 5p + 2$.
- Determine if purchasing 2 punnets will result in an identical cost of \$12 at both stalls.

Explanation

Substitute the x-coordinate (number of punnets) into the first equation to verify the y-coordinate (the price).

$c = 6 \times 2$

$c = 12$

Substitute the x-coordinate (number of punnets) into the second equation to verify the y-coordinate (the price).

$c = 5 \times 2 + 2$

$c = 10 + 2$

$c = 12$

Since the coordinate (2,12) satisfies both equations, it is a solution.

Answer

Purchasing 2 punnets of strawberries will result in an identical cost of \$12 at both stalls.

11. A company computes its weekly costs using the equation $y = 150x + 300$ for its expenses, and its sales are described by $y = 200x$, where x is the number of weeks. They predict they will break even in less than 7 weeks. Plot the graphs on a single set of axes and state the number of weeks required before they break even.

Key points

- A company computes its weekly expenses using the equation $y = 150x + 300$, where x is the number of weeks.
- Its weekly sales are described by $y = 200x$, where x is the number of weeks.
- Plot the graphs on a single set of axes and state how many weeks it will take before they break even.

Explanation

Plot each graph by first determining the x- and y-intercepts.

$y = 150x + 300$

x-intercept, let $y = 0$

$0 = 150x + 300$

$0 = x + 2$

$x = -2$

$\therefore$ x-intercept is -2

$(-2,0)$

y-intercept, let $x = 0$

$y = 150(0) + 300$

$y = 300$

$\therefore$ y-intercept is 300

$(0,300)$

$y = 200x$

Back to contents

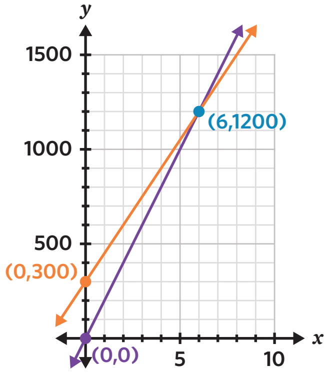
x-intercept, let $y = 0$

$0 = 200x$

$x = 0$

$\therefore$ x-intercept is (0,0) which implies that the y-intercept is also (0,0)

Plot the above graphs, remembering to not allow $x < 0$ as time cannot be negative.



Locate the intersection point and read the coordinates from the graph.

Verify that (6,1200) is the correct solution by substitution.

$1200 = 150(6) + 300$ ✓

$1200 = 200(6)$ ✓

Answer

There are 6 weeks before the company breaks even.

12. To assess the number of building permits to be granted, a city council employs a simplified model for forecasting housing needs. Supply is represented by $y = 5x + 80$, while $y = x + 100$ predicts demand, with x is the time in years and y indicating the count of houses. Use a graph to identify the point at which supply equals demand and give the corresponding timeframe and house count.

Key points

- Supply of houses in a city council is represented by $y = 5x + 80$.
- While demand of houses is given by $y = x + 100$.
- With x being the time in years and y indicating the count of houses.
- Use a graph to identify the point at which supply equals demand and state the number of years and count of houses.

Explanation

Plot each graph by first determining the x- and y-intercepts.

$y = 5x + 80$

x-intercept, let $y = 0$

$0 = 5x + 80$

$-5x = 80$

$x = -16$

$\therefore$ x-intercept is -16

$(-16,0)$

y-intercept, let $x = 0$

$y = 5(0) + 80$

$y = 80$

$\therefore$ y-intercept is 80

(0,80)

$y = x + 100$

x-intercept, let $y = 0$

$0 = x + 100$

$x = -100$

$\therefore$ x-intercept is -100

$(-100,0)$

y-intercept, let $x = 0$

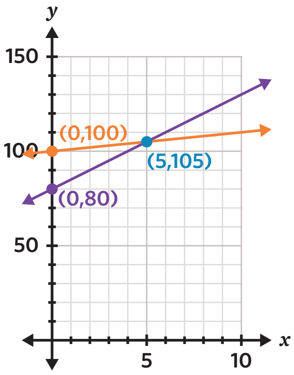
$y = (0) + 100$

$y = 100$

$\therefore$ y-intercept is 100

(0,100)

Plot the above graphs, remembering to not allow $x < 0$ as time cannot be negative.



Locate the intersection point and read the coordinates from the graph.

Verify that (5,105) is the correct solution by substitution.

$105 = 5(5) + 80$ ✓

$105 = 5 + 100$ ✓

Answer

Supply equals demand after 5 years with a house count of 105.

13. Store A sells notebooks for \$3 each, while Store B charges \$2.50 per notebook plus a \$2 fee for each order. This can be written as $c = 3n$ for Store A and $c = 2.50n + 2$ for Store B. Store A says that you need to buy 5 notebooks, costing \$15 in total, before Store B becomes cheaper. Plot both graphs and verify if Store A is correct.

Key points

- Store A sells notebooks for \$3 each thus $c = 3n$.
- Whereas Store B charges \$2.50 per notebook plus a \$2 fee for each order, thus $c = 2.50n + 2$.
- Store A says that you need to buy 5 notebooks, costing \$15 in total, before Store B becomes cheaper.
- Plot both graphs and determine if Store A is correct in this prediction.

Explanation

Plot each graph by determining points on the lines.

$c = 3n$ is an equation for a line with one intercept at (0,0).

Substitute $n = 1$ to determine another point.

$c = 3(1)$

$= 3$

Another point is (1,3).

Determine the x- and y-intercepts of $c = 2.50n + 2$.

$c = 2.50n + 2$

x-intercept, let $c = 0$

$0 = 2.50n + 2$

$-2.5n = 2$

$n = -\frac{4}{5}$

$\therefore$ x-intercept is $-\frac{4}{5}$

$\left(-\frac{4}{5},0\right)$

y-intercept, let $n = 0$

$c = 2.50(0) + 2$

$c = 2$

$\therefore$ y-intercept is 2

(0,2)

Plot the above graphs, remembering to not allow $n < 0$ as you can't buy a negative amount of books.

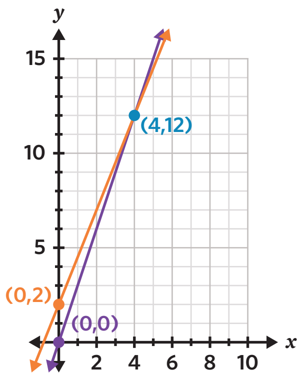
Verify if the point (5,15) sits on both lines

$15 = 3(5)$ ✓

$15 \neq 2.5(5) + 2$ ✓

$\therefore$ Store A is incorrect.

Answer



You need to buy 4 notebooks for $12 for store B to be cheaper.

14. An IT business uses the equations $y = 100x + 40$ to anticipate revenue and $y = -20x + 100$ to project costs, where x represents time in years and y represents thousands of dollars. Use a graph to determine the time in years the business will achieve a break-even scenario.

Key points

- An IT business uses the equation $y = 100x + 40$ to anticipate its revenue.
- The business uses the equation $y = -20x + 100$ to project its costs.
- Use a graph to determine the time in years it will take for the business to break-even.

Explanation

Plot each graph by first determining the x- and y-intercepts.

$y = 100x + 40$

x-intercept, let $y = 0$

$0 = 100x + 40$

$-100x = 40$

$x = -\frac{2}{5}$

$\therefore$ x-intercept is $-\frac{2}{5}$

$\left(-\frac{2}{5},0\right)$

y-intercept, let $x = 0$

$y = 100(0) + 40$

$c = 40$

$\therefore$ y-intercept is 40

(0,40)

$y = -20x + 100$

x-intercept, let $y = 0$

$0 = -20x + 100$

$20x = 100$

$x = 5$

$\therefore$ x-intercept is 5

(5,0)

y-intercept, let $x = 0$

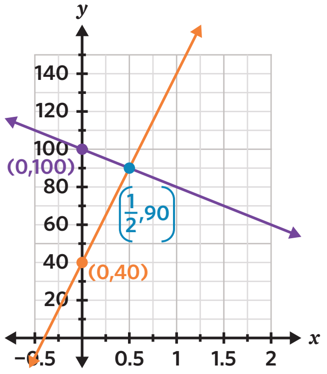
$y = -20(0) + 100$

$c = 100$

$\therefore$ y-intercept is 100

(0,100)

Plot the above graphs, remembering to not allow $x < 0$ as you can't have negative time.



Locate the intersection point and read the coordinates from the graph.

Verify that $\left(\frac{1}{2},90\right)$ is the correct solution by substitution.

$90 = 100\left(\frac{1}{2}\right) + 40$ ✓

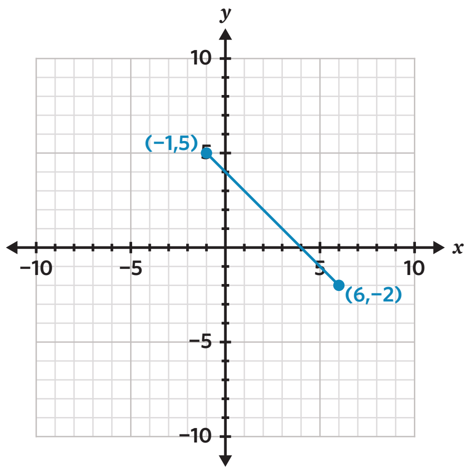
$90 = -20\left(\frac{1}{2}\right) + 100$ ✓

Answer

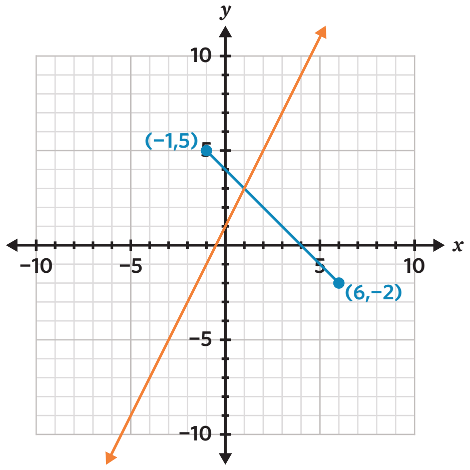
The business will achieve a break-even scenario in half a year.

Reasoning

15. a.



b.



Back to contents

c. A river crossing is required in this section of the road as seen by the intersection between the road and the river. The crossing happens at the coordinates (1,3).

d. $y = 2x + 2$

e. Suggested option 1: A tunnel under the river.

Suggested option 2: Boats that take cars as cargo.

Note: There are other possible options.

16. a. $y = 2x + 4$ and $y = 2x - 5$ have the same gradient $(m_1 = m_2)$ and different y-intercepts $(c_1 \neq c_2)$ which indicates that they are parallel lines, thus there are zero solutions.

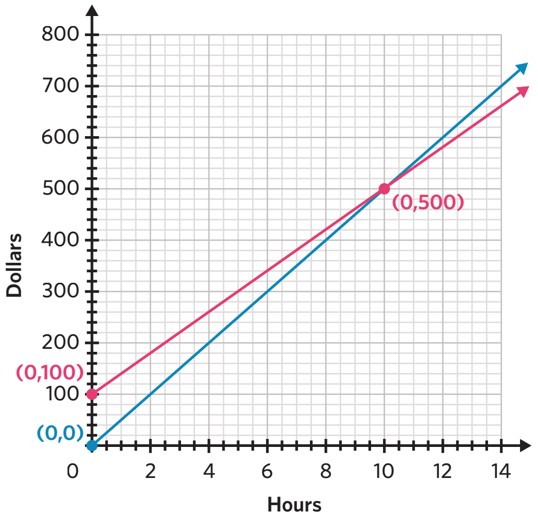
b. The lines $y = x + 4$ and $y = 2x + 4$ are both straight lines with different gradients and therefore they have one solution which is located at (0,4).

c. When two equations of the form $y = mx + c$ have the same c value, the solution to the simultaneous equations will be at their y-intercept (0,c).

Exam-style

17. A

18. a.



b. With 10 hours of work both cleaners charge the same amount.

19. Substitute the point $\left(-\frac{1}{2},3\right)$ into $y = -4x + 1$ and $y = 2x + 4$

$3 = -4\left(-\frac{1}{2}\right) + 1$ ✓

$3 = 2\left(-\frac{1}{2}\right) + 4$ ✓

Therefore $\left(-\frac{1}{2},3\right)$ is a solution to the pair of simultaneous equations.

Substitute the point (0,1) into $y = -4x + 1$ and $y = 0.5x + 1$

$1 = -4(0) + 1$ ✓

$1 = 0.5(0) + 1$ ✓

Therefore (0,1) is a solution to the pair of simultaneous equations.

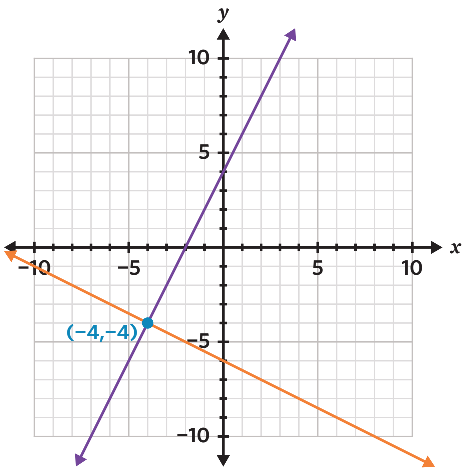
Substitute the point (−2,0) into $y = 0.5x + 1$ and $y = 2x + 4$

$0 = 0.5(-2) + 1$ ✓

$0 = 2(-2) + 4$ ✓

Therefore (−2,0) is a solution to the pair of simultaneous equations.

20.



The solution to the simultaneous equations (−4,−4).

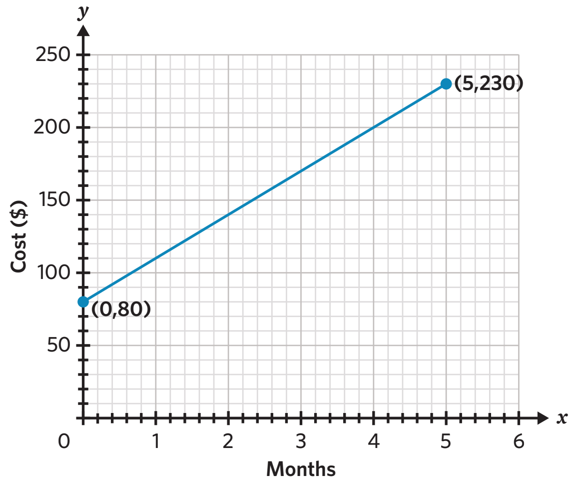
Remember this?

21. B **22.** C **23.** D

Chapter 5 extended application

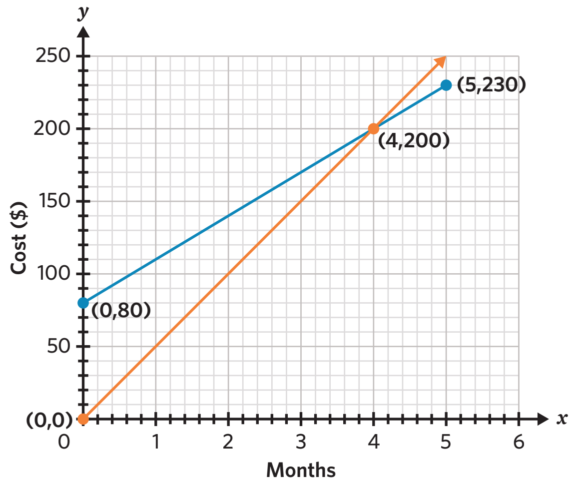
1. a. x-intercept: $\left(-\frac{8}{3},0\right)$ y-intercept: (0,80)

b.



c. The equation to model the cost of the new art gallery's subscription is $y = 50x$.

d.



The intersection point is (4,200). The point shows that the total payments for both subscriptions are equal at 4 months. After the fourth month, the new gallery's subscription is more expensive.

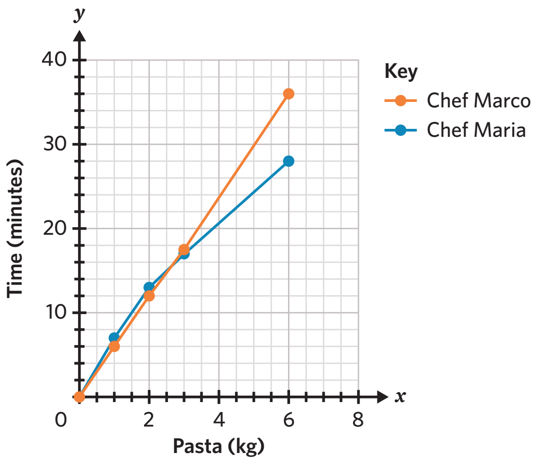
e. The equation of the third gallery is given by $y = 5x + 100$. This gallery has a signup fee of $100 for new members and a monthly fee of $5.

f. Suggested option 1: Visitors are more likely to return regularly if subscribing is cheaper than buying one-time tickets.

Suggested option 2: The gallery can keep track of their regular visitors.

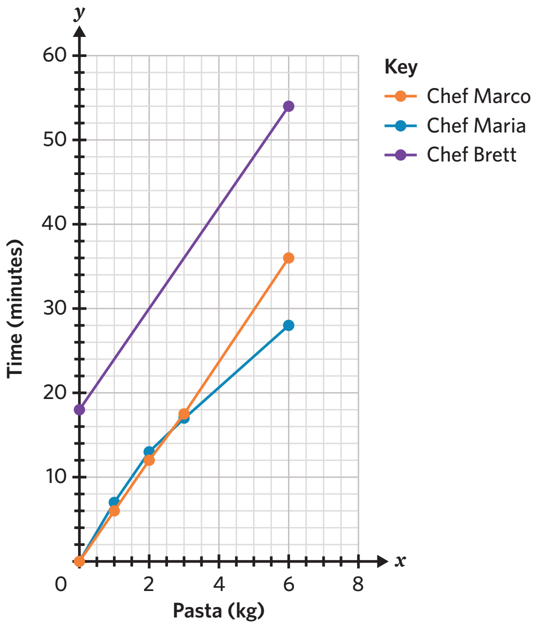
Note: There are other possible options.

2. **a.**



b. Chef Marco seems to be cooking at a constant rate that can be described by the equation $y = 6x$.

c.



d. Chef Brett and Chef Marco are cooking at the same rate of 1 kg of pasta every 6 minutes and therefore the lines that represent their pasta cooking are parallel and will never intersect.

e. Suggested option 1: The chefs could split up the jobs in the kitchen among them.

Suggested option 2: The chefs could coordinate to make the process run smoother.

Note: There are other possible options.

3. **a.** $y = \frac{1}{3}x + \frac{10}{3}$

b. The coordinates of the marker between Christie and Lauren are (8,6)

c. The distance that Christie needs to pass the ball for it to reach Lauren is 12.65 m

d. The distance that the coach will need to walk to get to Lauren is 16.12 m

e. $y = -3x + 30$

f. Suggested option 1: Warming up is important to prevent injury.

Suggested option 2: Warming up is important to go over strategy for the game.

Note: There are other possible options.

Chapter 5 review

Multiple choice

1.	D	**2.**	B	**3.**	D	**4.**	C	**5.**	D

Fluency

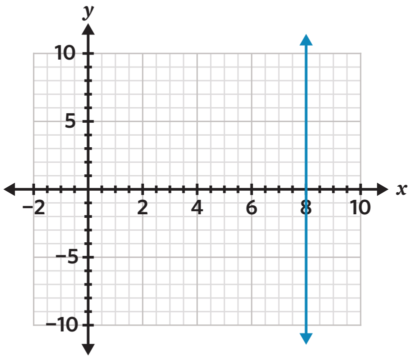
6. **a.** (2,0) and (0,6)

b. (10,0) and (0,–8)

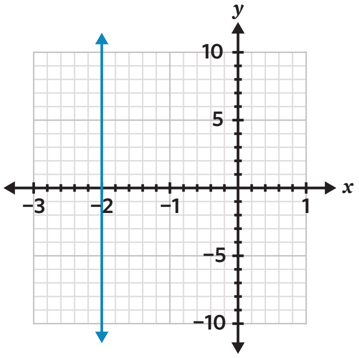
c. $\left(\frac{3}{5},0\right)$ and (0,–3)

d. $\left(\frac{3}{2},0\right)$ and (0,–1)

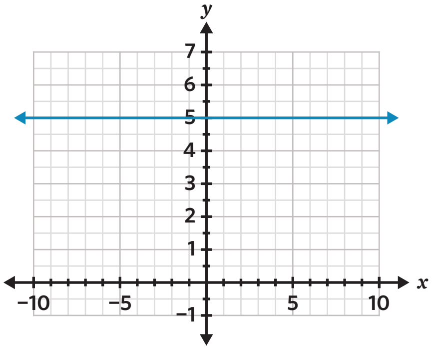
7. **a.**



b.

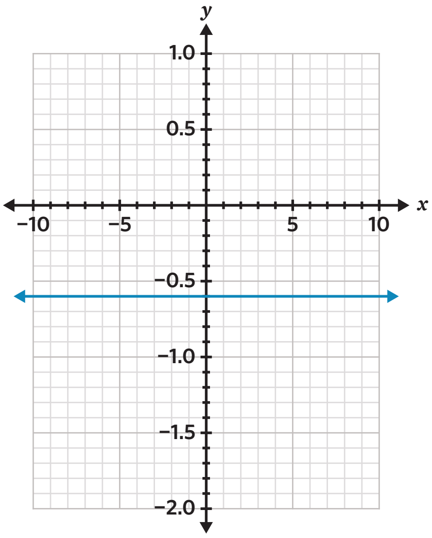


c.



Back to contents

d.



8. **a.** $m = \frac{-1}{2}$ and (0,–4) **b.** $m = 2.2$ and (0,–10)

c. $m = 1.4$ and (0,–1) **d.** $m = \frac{2}{5}$ and (0,3)

9. **a.** $y = 3x - 5$ **b.** $y = -4x + 6$

c. $y = \frac{2}{3}x - 9$ **d.** $y = -\frac{1}{5}x + 6$

10. **a.** $15 **b.** 400 toys

c. $g = 2.5d$; 15 cm **d.** $f = 0.07d$; 21 L

11. **a.** (3,3) **b.** (6,1)

c. $\left(\frac{3}{2},6\right)$ **d.** $\left(-\frac{11}{2},-1\right)$

12. **a.** 5 **b.** 2.24 **c.** 10 **d.** 23.35

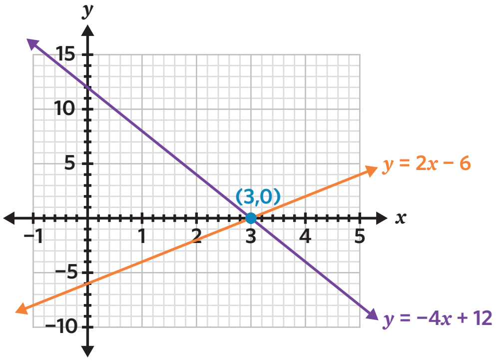
13. **a.** $y = 3x + 6$ **b.** $y = -x - 4$

c. $y = 8x - 29$ **d.** $y = -\frac{1}{2}x + 9$

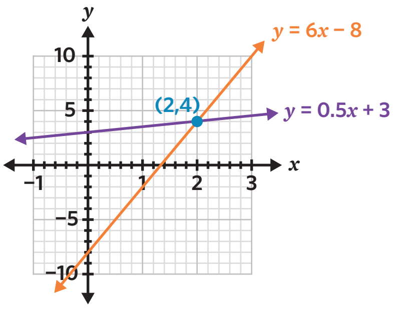
14. **a.** $y = \frac{1}{2}x + 11$ **b.** $y = \frac{1}{3}x - 6$

c. $y = -4x - \frac{37}{5}$ **d.** $y = -\frac{8}{3}x - 2$

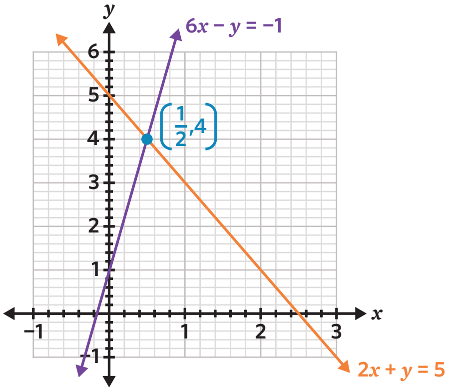
15. **a.**



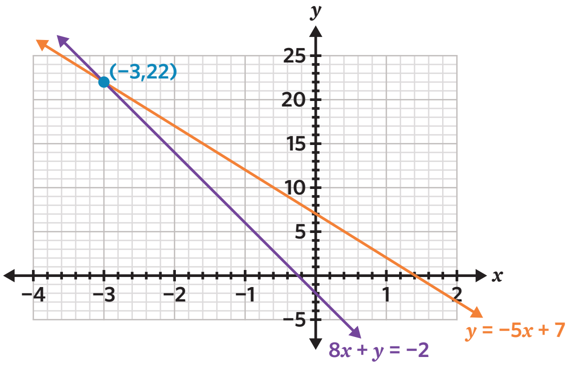
b.



c.



d.



Problem solving

16. A marine conservation team is tracking the declining population of a particular fish species. The team has modelled the population using the linear equation $y = -4x + 200$, where y represents the number of fish and x represents the number of years since the study began. Sketch the graph showing the fish population and the number of years until the population reaches zero.

Key points

- The marine conservation team has modelled the population of a particular fish species using the linear equation $y = -4x + 200$.
- Sketch the graph showing the fish population and the number of years until that population reaches zero.

Explanation

Calculate the x-intercept by letting $y = 0$.

$0 = -4x + 200$

$4x = 200$

$x = 50$

$\therefore$ x-intercept is 50

(50,0)

$\therefore$ The fish population reaches zero after 50 years.

Calculate the y-intercept by letting $x = 0$.

$y = -4(0) + 200$

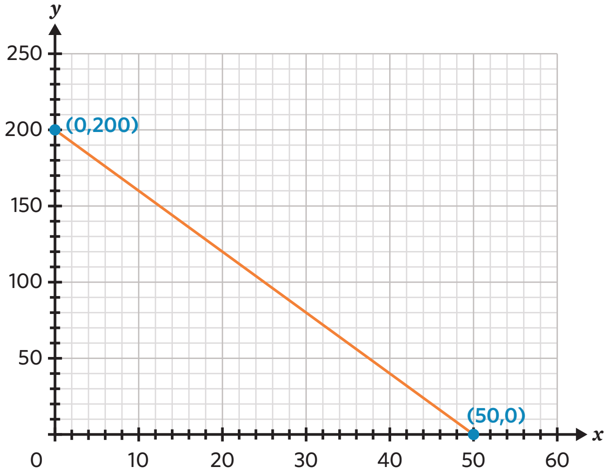
$y = 200$

$\therefore$ y-intercept is 200

(0,200)

When sketching the graph, ensure the line does not drop below the x-axis as the population can never be negative.

Answer



17. A local gym is planning to offer a new membership package that includes a fixed monthly fee of \$45 for unlimited access to the gym. The cost (c) remains the same regardless of how many times a member visits the gym in a month (v). Sketch a graph to represent this information, where $v \geq 0$.

Key points

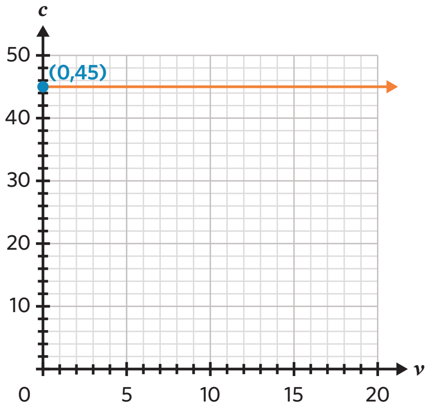
- A new membership package for a local gym costs a \$45 fixed fee for unlimited access to the gym.
- The cost (c) doesn't change regardless of how many times a member visits the gym in a month (v).
- Sketch a graph to represent this information for $v \geq 0$.

Explanation

The cost of the gym membership is a fixed cost of \$45 per month, regardless of the number of times the member visits the gym. This means the cost does not change with the number of visits.

To represent this information on a graph, plot the cost (c) on the horizontal axis and the number of visits (v) on the vertical axis. Since the cost is constant, the graph will be a horizontal line at \$45.

Answer



18. A coffee shop wants to explore the connection between coffee pricing and daily sales. They've observed a decrease of 20 cups sold for every \$1 increase in price. At the current rate of \$4.00 per cup, they sell 100 cups per day. Sketch the graph and formulate an equation depicting the total cups sold per day (c) dependent on the price (p).

Key points

- A coffee shop has observed a decrease of 20 cups sold for every \$1 increase in price.
- At the current rate of \$4.00 per cup, they sell 100 cups per day.
- Sketch the graph and formulate an equation depicting the total number of cups sold per day (c) dependent on the price (p).

Explanation

Determine the value of the gradient.

For every dollar increase the number of coffee cups decreases by 20.

$\therefore \frac{\text{rise}}{\text{run}} = -20$, meaning the gradient is -20.

Determine the y-intercept.

Substitute the point $(4,100)$ and the value of the gradient into the equation of a straight line: $y = mx + c$.

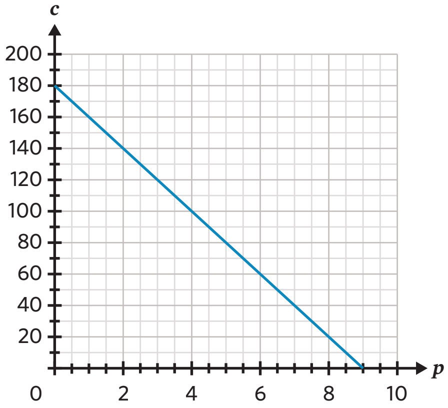
$100 = -20(4) + c$

$100 = -80 + c$

$c = 180$

$\therefore$ the y-intercept is $(0,180)$.

Sketch the graph.



Substitute the values of the gradient and the y-intercept into the gradient-intercept form, using the variable c (number of cups sold per day) for the vertical axis and the variable p (price) for the horizontal axis.

$c = -20p + 180$

Answer

The equation that represents the total number of cups sold per day at the coffee shop is $c = -20p + 180$.

19. A bakery is planning to bake cupcakes for a charity event. The bakery is able to prepare 72 cupcakes in half an hour. Calculate the number of cupcakes it can bake in 3.5 hours.

Key points

- A bakery is able to prepare 72 cupcakes in half an hour.
- Calculate the number of cupcakes the bakery can bake in 3.5 hours.

Explanation

Determine the baking rate.

72 cupcakes are baked in half an hour.

$\therefore$ 144 cupcakes can be baked in an hour.

$$\begin{aligned}\text{Total number of cupcakes} &= \text{baking rate} \times \text{time}\\ &= 144 \times 3.5\\ &= 504\end{aligned}$$

Answer

The bakery can bake 504 cupcakes in 3.5 hours.

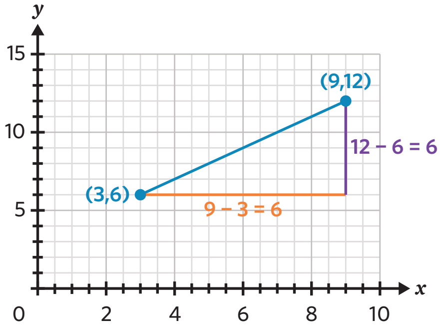
20. A city planner is working on a new park design. He wants to connect two points in the park, located at (3,6) and (9,12), with a pathway. Calculate the length of the pathway in metres, where 1 unit equals 10 m. Round to two decimal places.

Key points

- A city planner wants to connect two points in the park, located at (3,6) and (9,12).
- 1 unit equals 10 m.
- Calculate the length of the pathway he wants to create in metres, rounded to two decimal places.

Explanation

Determine the length of the line segment connecting (3,6) and (9,12).



$$\text{Distance} = \sqrt{(9-3)^2 + (12-6)^2}$$
$$= \sqrt{6^2 + 6^2}$$
$$= \sqrt{36 + 36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

1 unit = 10 m

$$6\sqrt{2} \times 10 = 60\sqrt{2}$$
$$= 84.852...$$
$$\approx 84.85 \text{ m}$$

Answer

The length of the pathway is approximately 84.85 m.

21. In a coastal city, two lighthouses are being built to guide ships. The first lighthouse's beam follows the equation $y = -3x + 12$. The second lighthouse's beam will be perpendicular to the first one, and the city wants it to reach a specific island located at the coordinates (3,17). Determine the equation of the second lighthouse's beam.

Key points

- The first lighthouse's beam to guide ships follows the equation $y = -3x + 12$.
- The second lighthouse's beam will be perpendicular to the first beam.
- The second beam needs to reach the coordinates (3,17).
- Determine the equation of the second lighthouse's beam with this information.

Explanation

Identify the gradient of the first lighthouse beam.

$m_1 = -3$

Determine the gradient of the second lighthouse beam.

$$m_2 = -\frac{1}{m_1}$$
$$= -\frac{1}{-3}$$
$$= \frac{1}{3}$$

Substitute known x and y values from a given point and the gradient into $y = mx + c$.

$(x,y) = (3,17)$

$17 = \frac{1}{3}(3) + c$

$17 = 1 + c$

$c = 16$

Write the equation in the form $y = mx + c$.

$y = \frac{1}{3}x + 16$

Answer

The equation of the second lighthouse's beam is $y = \frac{1}{3}x + 16$.

22. Two businesses are planning their production schedules. The first business's production is represented by the equation $y = 0.4x + 6$, where y is the number of products and x is the number of hours. The second business's production is represented by the equation $y = 0.9x + 1$. After how many hours will both businesses have produced the same number of products?

Key points

- The first business's production schedule is represented by the equation $y = 0.4x + 6$.
- The second business's production schedule is represented by the equation $y = 0.9x + 1$.
- Determine after how many hours will both businesses have produced the same number of products.

Explanation

Plot each graph by first determining the x- and y-intercepts.

$y = 0.4x + 6$

To determine the x-intercept, let $y = 0$

$0 = 0.4x + 6$

$-0.4x = 6$

$x = -15$

To determine the y-intercept, let $x = 0$

$y = 0.4(0) + 6$

$y = 6$

∴ x-intercept is $(-15,0)$ and y-intercept is $(0,6)$

$y = 0.9x + 1$

To determine the x-intercept, let $y = 0$

$0 = 0.9x + 1$

$-0.9x = 1$

$$x = -\frac{1}{-0.9}$$
$$= -\frac{10}{9}$$

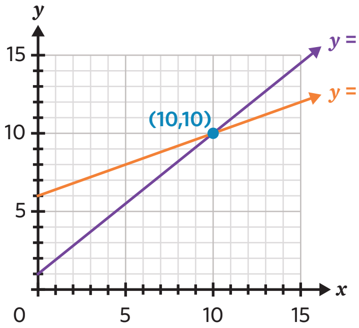
To determine the y-intercept, let $x = 0$

$y = 0.9(0) + 1$

$y = 1$

∴ x-intercept is $\left(-\frac{10}{9},0\right)$ and y-intercept is $(0,1)$

Sketch each graph, with $x > 0$ as it is not possible for the businesses to produce a negative quantity.



Locate the intersection point and read the coordinates from the graph.

The intersection point is (10,10).

Verify by substitution.

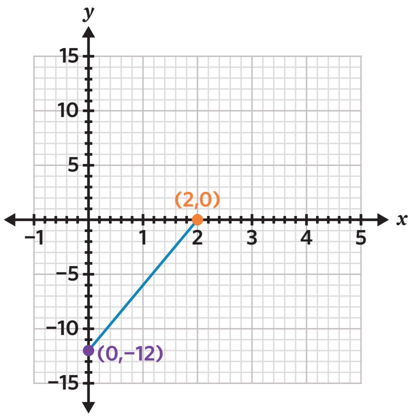
$10 = 0.4(10) + 6$	$10 = 0.9(10) + 1$
$10 = 10$ ✓	$10 = 10$ ✓

Answer

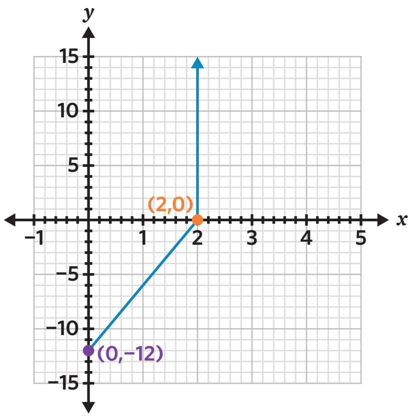
Both businesses will have produced the same number of products after 10 hours.

Reasoning

23. a.



b. $x = 2$



c. (1,–6)

d. $y = -\frac{1}{6}x - \frac{11}{4}$

e. Suggested option 1: There may be trees where she wants the paths to go.

Suggested option 2: It might be too costly to build.

Note: There are other possible options.

24. a. $m_1 = -4$

$m_2 = \frac{1}{4}$

$\therefore m_1 = -\frac{1}{m_2}$ so the lines are perpendicular

b. One solution

c. Perpendicular lines will always have one intersection point, which is the solution of the two equations.

Back to contents

6A Quadratic equations

Student practice

Worked example 1

a. $x^2 - 6x - 7 = 0$

b. $5x^2 + 3x + 3 = 0$

c. $3x^2 - 12x - 5 = 0$

Worked example 2

a. Solution

b. Not a solution

Worked example 3

a. $x = -7, x = 0$

b. $x = -6, x = 2$

c. $x = -\frac{2}{3}, x = \frac{5}{2}$

Understanding worksheet

1. **a.** 3 **b.** 1 **c.** −11 **d.** 0

2. Standard form: I, III, IV

Not in standard form: II

3. quadratic; zero; solutions; Null Factor Law

Fluency

4. **a.** $x^2 - 2x - 4 = 0$

b. $2x^2 - 3x - 1 = 0$

c. $x^2 + x - 5 = 0$

d. $x^2 + 4x - 3 = 0$

e. $x^2 + 8x + 7 = 0$

f. $5x^2 + 2x - 9 = 0$

g. $3x^2 - 10x + 5 = 0$

h. $x^2 + 4x - 9 = 0$

5. **a.** $2x^2 + 6x - 4 = 0$

b. $3x^2 + 3x + 5 = 0$

c. $2x^2 - 10x - 11 = 0$

d. $5x^2 - 7x - 15 = 0$

e. $3x^2 + 2x - 12 = 0$

f. $2x^2 + 3x + 8 = 0$

g. $8x^2 + 3x - 6 = 0$

h. $6x^2 + 15x - 10 = 0$

6. **a.** Solution

b. Not a solution

c. Not a solution

d. Solution

e. Not a solution

f. Solution

g. Not a solution

h. Solution

7. **a.** $x = -3, x = 0$

b. $x = 0, x = 5$

c. $x = -2, x = 2$

d. $x = -5, x = 3$

e. $x = 3, x = 7$

f. $x = -11, x = 10$

g. $x = -\frac{2}{3}, x = 8$

h. $x = -\frac{1}{4}, x = \frac{1}{2}$

8. **a.** $x = -\frac{1}{2}, x = 0$

b. $x = 0, x = \frac{1}{3}$

c. $x = -3, x = \frac{1}{2}$

d. $x = -\frac{3}{2}, x = 1$

e. $x = -\frac{5}{2}, x = -\frac{1}{3}$

f. $x = -\frac{3}{4}, x = \frac{5}{3}$

g. $x = \frac{3}{5}, x = \frac{11}{6}$

h. $x = -\frac{13}{7}, x = \frac{9}{4}$

9. D

Spot the mistake

10. **a.** Student A is incorrect.

b. Student A is incorrect.

Problem solving

11. The shape of a canal running through a city can be modelled by the equation $x^2 - 3x = d$ where x is the width of the canal and d is its depth, relative to the ground. Show that when the canal is 3 m wide, its depth is 0 m.

Key points

- The equation modelling a canal is $x^2 - 3x = d$.
- x represents the width of the canal.
- d represents the depth.
- Show that the canal is 0 m deep when it's 3 m wide.

Explanation

Substitute $x = 3$ into the equation $x^2 - 3x = d$, and solve for d.

$$(3)^2 - 3(3) = d$$
$$d = 0$$

Answer

When the canal is 3 m wide, its depth is 0 m because substituting $x = 3$ m results in $d = 0$ m.

12. A hammock has been tied between two trees. Its shape can be modelled by the equation $x(x - 4) = y$ where x is the distance from one of the trees and y is the depth of the hammock, relative to the points where it was tied. Calculate the depth of the hammock when the distance from one of the trees is 4 m.

Key points

- A hammock has been tied between two trees.
- Equation modelling the hammock is $x(x - 4) = y$.
- x represents the distance from one of the trees.
- y represents the depth of the hammock relative to the points where it was tied.
- Calculate the depth when $x = 4$ m.

Explanation

Substitute $x = 4$ into the equation $x(x - 4) = y$ and solve for y.

$$4(4 - 4) = y$$
$$y = 0$$

Answer

When the distance from one of the trees is 4 m, the depth of the hammock is 0 m.

13. Colin writes a cooking book. His profit per sold book can be modelled by the equation $10n - n^2 - 21 = s$, where n represents the retail price of one book and s is the profit from its sale, in dollars. Show that when the retail price of one book is $7, the profit per book is equal to zero.

Key points

- Colin's cooking book's profit equation is $10n - n^2 - 21 = s$.
- n is the retail price of one book.
- s is the profit from its sale in dollars.
- Show that when the retail price of one book is $7, the profit is equal to zero.

Explanation

Substitute $n = 7$ into the equation $10n - n^2 - 21 = s$.

$$s = 10(7) - (7)^2 - 21$$

Evaluate the equation for s.

$$s = 70 - 49 - 21$$
$$= 0$$

Answer

When the retail price of one book is $7, the profit per book is equal to zero because substituting $n = 7$ dollars produces $s = 0$ dollars.

14. The shape of a half-pipe at the skate park can be modelled by the equation $w(w - 6) = y$, where w is the width of the half-pipe and y is its depth, relative to the edges. Assuming that the half-pipe's edges are at the points where depth $y = 0$ m, how wide is the half-pipe?

Key points

- The equation modelling the shape of a half-pipe at a skate park is $w(w - 6) = y$.
- w is the width of the half-pipe.
- y is the depth of the half-pipe.
- Half-pipe's edges are at the points where depth $y = 0$ m.
- How wide is the half-pipe?

Explanation

Edges of the half-pipe are where $y = 0$, so solve for w when $w(w - 6) = 0$.

Apply the Null Factor Law by making each factor equal to zero and solving.

$w = 0$

and

$$w - 6 = 0$$
$$w - 6 + 6 = 0 + 6$$
$$w = 6$$

Calculate the distance between the two half-pipe's edges.

$6 - 0 = 6$ m

Answer

The width of the half-pipe is 6 m.

15. To calculate the best price for her homemade soaps, Jenny uses the equation $(2 - x)(x - 5) = p$, where x is the price per bar of soap in dollars and p is the potential profit from its sale. Solve the equation when $p = 0$ and identify the two retail prices per bar of soap that will result in zero profit according to Jenny's model.

Key points

- The equation modelling Jenny's profit from her homemade soaps is $(2 - x)(x - 5) = p$.
- x is the price per bar of soap.
- p is the potential profit.
- Solve the equation to find the two prices that will result in zero profit, according to the equation.

Explanation

Solve for x when $p = 0$, so $(2 - x)(x - 5) = 0$.

Apply the Null Factor Law by making each factor equal to zero and solving.

$$2 - x = 0$$
$$2 - x - 2 = 0 - 2$$
$$-x = -2$$
$$x = 2$$

and

$$x - 5 = 0$$
$$x - 5 + 5 = 0 + 5$$
$$x = 5$$

Answer

The two retail prices per bar of soap that will result in zero profit are $2 and $5, according to Jenny's model.

Reasoning

16. a. When m is equal to 1, v is equal to 0.

b. When m is equal to 4, v is equal to 0.

c. The views on Kenny's videos reduce to zero when they are not between one and four minutes long. When Kenny's videos are outside of those times, his model does not apply in context as it is not possible to view a video for a negative number of minutes.

d. Suggested option 1: Use narratives with relatable anecdotes and real-life examples to connect emotionally with viewers.

Suggested option 2: Incorporate live polls, Q&A sessions, or audience participation to create a dynamic and engaging experience.

Note: There are other possible options.

17. a. The solutions to $x(x + 9) = 0$ are $x = 0$ and $x = -9$.

b. The solutions to $5x(2x - 5) = 0$ are $x = 0$ and $x = \frac{5}{2}$.

c. A general solution to quadratic equations of the form $ax(bx + c) = 0$ where $a \neq 0$ and $b \neq 0$ is $x = 0$ and $x = -\frac{c}{b}$.

Exam-style

18. D

19. a.
$$1 = 5x - 4x^2$$
$$1 + 4x^2 = 5x$$
$$1 + 4x^2 - 5x = 0$$
$$4x^2 - 5x + 1 = 0$$

b.
$$4(1)^2 - 5(1) + 1 = 0$$
$$4 - 5 + 1 = 0$$

20. Substitute $x = 40$ m and calculate a.

$$\begin{aligned}(40 - x)(x - 50) &= (40 - 40)(40 - 50)\\ &= 0 \times (-10) = 0\\ &= a\end{aligned}$$

Substitute $x = 50$ m and calculate a.

$$\begin{aligned}(40 - x)(x - 50) &= (40 - 50)(50 - 50)\\ &= -10 \times 0 = 0\\ &= a\end{aligned}$$

Area cannot be 0 m^2.

21. $2x^2 - 5x - 3 = 0$

Substitute $x = 3$

$$2(3)^2 - 5(3) - 3 = 0$$
$$18 - 15 - 3 = 0$$

Remember this?

22. B

23. D

24. C

6B Factorising and solving monic quadratic equations

Student practice

Worked example 1

a. $x = -2, x = -1$

b. $x = -2, x = 4$

c. $x = -6, x = 1$

Back to contents

Worked example 2

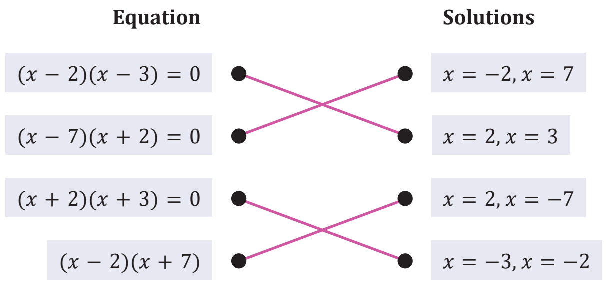
a. $x = 4$

b. $x = -5, x = 5$

Understanding worksheet

1. **a.** 4 **b.** -2 **c.** -2 **d.** -3

2.



3. trinomial; one; factorised; single

Fluency

4. **a.** $x = -3, x = -1$ **b.** $x = -4, x = -1$
c. $x = -5, x = -2$ **d.** $x = -6, x = -2$
e. $x = -7, x = -2$ **f.** $x = -9, x = -4$
g. $x = -14, x = -3$ **h.** $x = -4, x = -3$

5. **a.** $x = -1, x = 3$ **b.** $x = 1, x = 4$
c. $x = -2, x = 6$ **d.** $x = -10, x = 1$
e. $x = -5, x = 2$ **f.** $x = -2, x = 9$
g. $x = -7, x = 8$ **h.** $x = 7, x = 9$

6. **a.** $x = -4, x = 1$ **b.** $x = -2, x = 4$
c. $x = -3, x = 1$ **d.** $x = -3, x = 2$
e. $x = -4, x = 8$ **f.** $x = -9, x = 2$
g. $x = -2, x = 8$ **h.** $x = 3, x = 7$

7. **a.** $x = -2$ **b.** $x = -3$ **c.** $x = 1$ **d.** $x = 5$
e. $x = 6$ **f.** $x = 7$ **g.** $x = 9$ **h.** $x = 8$

8. **a.** $x = -2, x = 2$ **b.** $x = -6, x = 6$
c. $x = -7, x = 7$ **d.** $x = -10, x = 10$
e. $x = -14, x = 14$ **f.** $x = -y, x = y$
g. $x = -2y, x = 2y$ **h.** $x = -3p, x = 3p$

9. C

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. Over a number of weeks (w), Jenny's savings account balance (a) could be modelled by the equation $a = w^2 - 8w + 12$. Factorise and solve the equation to find the weeks during which Jenny's account balance was zero (when $a = 0$).

Key points

- Jenny's account balance is represented by the equation $a = w^2 - 8w + 12$.
- w represents the weeks.
- a represents the account balance.
- Factorise and solve the equation for when $a = 0$.

Explanation

Set up the equation to find the weeks when the account balance was zero.

$w^2 - 8w + 12 = 0$

Identify the values of b and c for the equation of the form $w^2 + bw + c = 0$.

$b = -8$

$c = 12$

Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.

$b = -6 + (-2) = -8$ ✓

$c = -6 \times (-2) = 12$ ✓

$\therefore p = -6$ and $q = -2$

Write the given expression in the form $(w + p)(w + q) = 0$ to factorise.

$w^2 - 8w + 12 = (w - 6)(w - 2)$

Solve the equation by applying the Null Factor Law.

$w - 6 = 0$

$w = 6$

and

$w - 2 = 0$

$w = 2$

Answer

Jenny's account balance was zero in the second and sixth weeks.

12. During a cold winter night, outside temperature can be modelled by the equation $t = h^2 - 6h + 8$ where h is the hours since midnight and t is the temperature in degrees Celsius. Identify the times during which the temperature was 0°C.

Key points

- Outside temperature represented by $t = h^2 - 6h + 8$.
- h is the hours since midnight.
- t is the temperature in degrees Celsius.
- Identify the times during which the temperature was 0°C.

Explanation

Substitute $t = 0$ to find the times when the temperature was 0°C.

$h^2 - 6h + 8 = 0$

Identify the values of b and c for the equation of the form $h^2 + bh + c = 0$.

$b = -6$

$c = 8$

Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.

$c = -4 \times (-2) = 8$ ✓

$b = -4 + (-2) = -6$ ✓

$\therefore p = -4$ and $q = -2$

Write the given expression in the form $(h + p)(h + q) = 0$ to factorise.

$h^2 - 6h + 8 = (h - 4)(h - 2)$

Solve the equation by applying the Null Factor Law.

$h - 4 = 0$

$h = 4$

and

$h - 2 = 0$

$h = 2$

Answer

The temperature was 0°C at 2:00 am and 4:00 am.

13. During a dive session, a diver's depth, d is measured relative to the surface, in metres. It can be modelled by the equation $d = m^2 - 6m$ where m is their horizontal distance from the edge of the pool. Factorise and solve the equation when $d = -9$ m to determine how far the diver was from the edge of the pool when they were at a depth of 9 m.

Key points

- A diver's depth can be modelled by the equation $d = m^2 - 6m$.
- m is the horizontal distance from the edge of the pool.
- d is the diver's depth relative to the surface, in metres.
- Substitute $d = -9$ and solve.

Explanation

Substitute $d = -9$ into the equation.

$-9 = m^2 - 6m$

Rearrange the equation to the form $m^2 + bm + c = 0$ and identify the values of b and c.

$-9 + 9 = m^2 - 6m + 9$

$0 = m^2 - 6m + 9$

$m^2 - 6m + 9 = 0$

$b = -6$

$c = 9$

Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.

$c = -3 \times (-3) = 9$ ✓

$b = -3 + (-3) = -6$ ✓

$\therefore p = -3$ and $q = -3$

Write the given expression in the form $(m + p)(m + q) = 0$ to factorise.

$m^2 - 6m + 9 = (m - 3)(m - 3)$

$= (\mathrm{m} - 3)^2$

Solve the equation by applying the Null Factor Law.

$m - 3 = 0$

$m = 3$

Answer

The diver was 3 m from the edge of the pool when they were 9 m deep.

14. The number of people (p) who visited a website within t minutes of it being live is given by the equation $t^2 - 5t = p$. Assuming that t may only be positive, how many minutes did it take for 24 people to visit the website?

Key points

- The number of people visiting a website represented by $t^2 - 5t = p$.
- p represents the number of people.
- t represents the number of minutes after the website was live.
- t can only be positive.
- How many minutes did it take for 24 people to visit the website?

Explanation

Substitute $p = 24$ into the equation.

$t^2 - 5t = 24$

Rearrange the equation to the form $t^2 + bt + c = 0$ and identify the values of b and c.

$t^2 - 5t - 24 = 24 - 24$

$t^2 - 5t - 24 = 0$

$b = -5$

$c = -24$

Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.

$c = -8 \times 3 = -24$ ✓

$b = -8 + 3 = -5$ ✓

$\therefore p = -8$ and $q = 3$

Write the given expression in the form $(t + p)(t + q) = 0$ to factorise.

$(t - 8)(t + 3) = 0$

Solve the equation by applying the Null Factor Law.

$t - 8 = 0$

$t = 8$

and

$t + 3 = 0$

$t = -3$

Select the positive value.

$t = 8$ ✓

Answer

It took 8 minutes for 24 people to visit the website.

15. A rectangular playground's area is given by the equation $x(x + 4) = 45$ where x is an unknown length in metres. Assuming that x must be a positive value, solve the equation to determine the length of x.

Key points

- Playground area represented by $x(x + 4) = 45$.
- x is the length in metres.
- x must be a positive value.
- Solve the equation to determine the length of x.

Explanation

Expand the LHS and rearrange the equation to the form $x^2 + bx + c = 0$ to identify the values of b and c.

$x(x + 4) = 45$

$x^2 + 4x = 45$

$x^2 + 4x - 45 = 45 - 45$

$x^2 + 4x - 45 = 0$

$b = 4$

$c = -45$

Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.

$c = 9 \times (-5) = -45$ ✓

$b = 9 + (-5) = 4$ ✓

$\therefore p = 9$ and $q = -5$

Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.

$(x + 9)(x - 5) = 0$

Back to contents

Solve the equation by applying the Null Factor Law.

$x + 9 = 0$

$x = -9$

and

$x - 5 = 0$

$x = 5$

Select the positive value.

$x = 5$ ✓

Answer

The length x is 5 m.

Reasoning

16. **a.** Substituting $p = 40$ and rearranging the equation to standard form gives us $t^2 + 6t - 40 = 0$.

b. The solutions to $t^2 + 6t - 40 = 0$ is $t = -10$ and $t = 4$.

c. The market has 40 visitors at 10:30 am.

d. Suggested option 1: People often take a break from work around the same time which leads to shops being busier.

Suggested option 2: People often aim to do errands as soon as possible during the day which leads to busier shops and markets in the mornings.

Note: There are other possible options.

17. **a.** The solutions to $x^2 - 64 = 0$ is $x = -8$ and $x = 8$.

b. The solutions to $x^2 - 81 = 0$ is $x = -9$ and $x = 9$.

c. Equations of the form $x^2 = n$, where $n > 0$, have pairs of solutions of the form $x = -\sqrt{n}$ and $x = \sqrt{n}$.

Exam-style

18. D

19. **a.** $x^2 - 9x + 20 = 0$

b. $(x - 4)(x - 5) = 0$

c. $x = 4, x = 5$

20. Jane's small business starts making money after 7 weeks.

21. $b = -11, c = 30$

Remember this?

22. B **23.** B **24.** C

6C Graphs of quadratic functions

Student practice

Worked example 1

a. Axis of symmetry: $x = -3$

Minimum turning point $(-3,0)$

x-intercept $(-3,0)$

y-intercept $(0,9)$

b. Axis of symmetry: $x = 1$

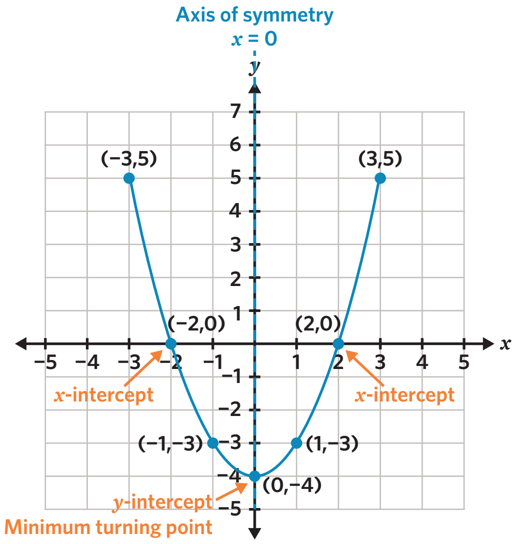
Maximum turning point $(1,4)$

x-intercepts $(-1,0)$, $(3,0)$

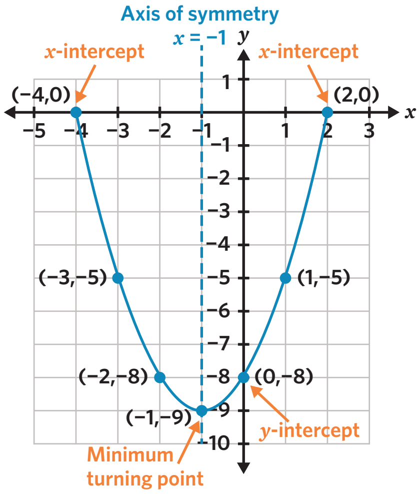
y-intercept $(0,3)$

Worked example 2

a.

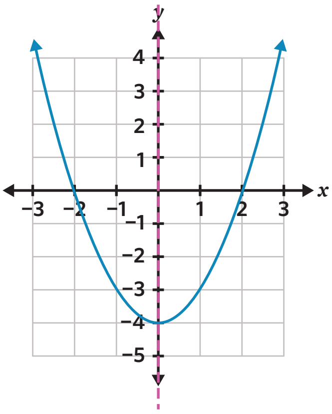


b.

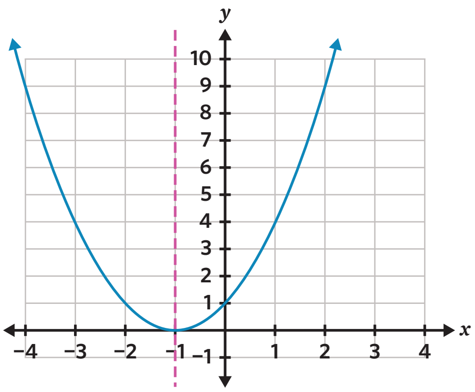


Understanding worksheet

1. **a.**

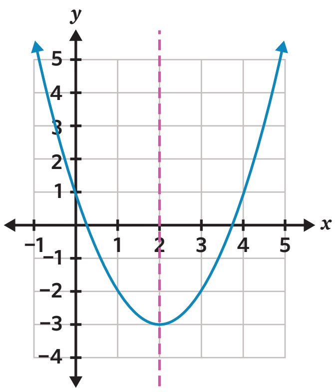


b.

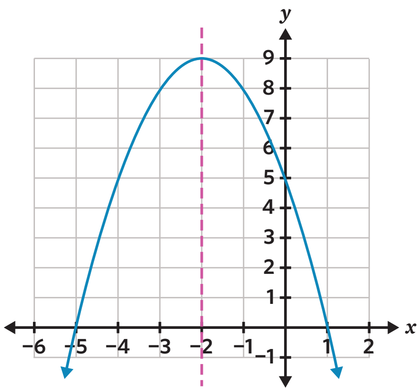


Back to contents

c.



d.



2. **a.** 4 **b.** 4 **c.** −1 **d.** 0

3. parabola; symmetry; two; minimum; maximum

Fluency

4. **a.** Axis of symmetry: $x = 0$
Minimum turning point (0,0)
x-intercept (0,0)
y-intercept (0,0)

b. Axis of symmetry: $x = 3$
Minimum turning point (3,0)
x-intercept (3,0)
y-intercept (0,9)

c. Axis of symmetry: $x = 0$
Minimum turning point (0,–4)
x-intercepts (−2,0), (2,0)
y-intercept (0,–4)

d. Axis of symmetry: $x = 3$
Minimum turning point (3,–4)
x-intercepts (1,0), (5,0)
y-intercept (0,5)

e. Axis of symmetry: $x = -2$
Minimum turning point (−2,−4)
x-intercepts (−4,0), (0,0)
y-intercept (0,0)

f. Axis of symmetry: $x = -2$
Minimum turning point (−2,–1)
x-intercepts (−3,0), (−1,0)
y-intercept (0,3)

g. Axis of symmetry: $x = 2$
Minimum turning point (2,–9)
x-intercepts (−1,0), (5,0)
y-intercept (0,–5)

h. Axis of symmetry: $x = 2$
Minimum turning point (2,1)
No x-intercepts
y-intercept (0,5)

5. **a.** Axis of symmetry: $x = 0$
Maximum turning point (0,0)
x-intercept (0,0)
y-intercept (0,0)

b. Axis of symmetry: $x = -2$
Maximum turning point (−2,0)
x-intercept (−2,0)
y-intercept (0,–4)

c. Axis of symmetry: $x = 0$
Maximum turning point (0,9)
x-intercepts (−3,0), (3,0)
y-intercept (0,9)

d. Axis of symmetry: $x = 3$
Maximum turning point (3,1)
x-intercepts (2,0), (4,0)
y-intercept (0,–8)

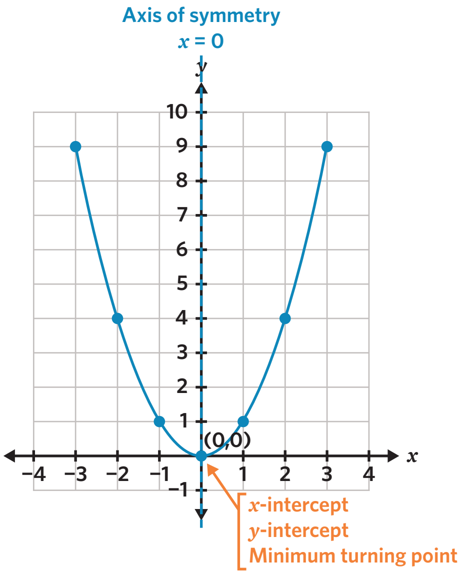
e. Axis of symmetry: $x = -1$
Maximum turning point (−1,1)
x-intercepts (−2,0), (0,0)
y-intercept (0,0)

f. Axis of symmetry: $x = -3$
Maximum turning point (−3,4)
x-intercepts (−5,0), (−1,0)
y-intercept (0,–5)

g. Axis of symmetry: $x = -1$
Maximum turning point (−1,9)
x-intercepts (−4,0), (2,0)
y-intercept (0,8)

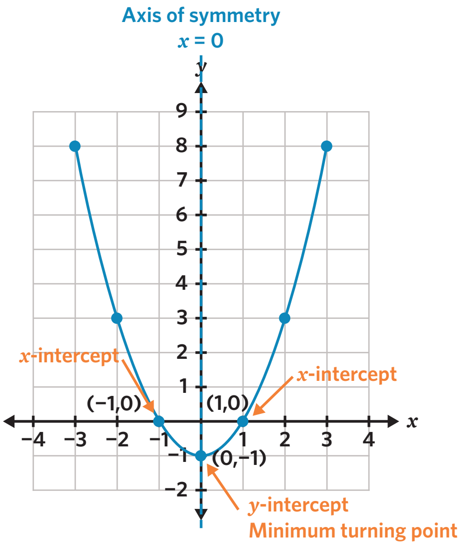
h. Axis of symmetry: $x = -3$
Maximum turning point (−3,–2)
No x-intercepts
y-intercept (0,–11)

6. **a.**

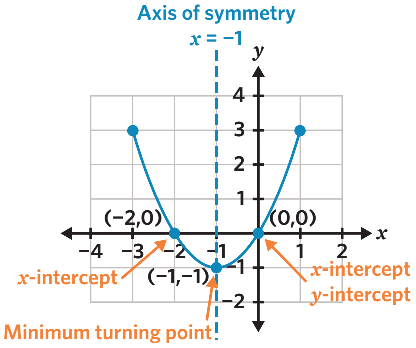


Back to contents

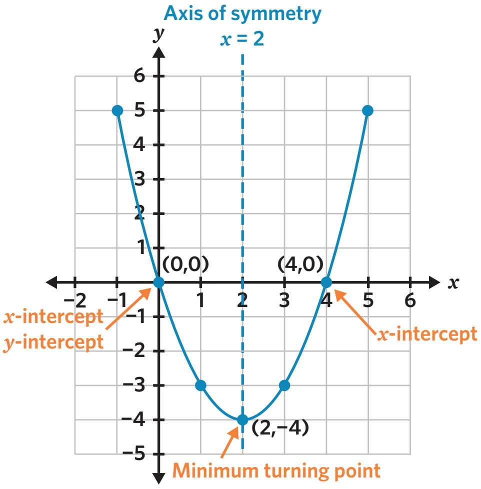
b.



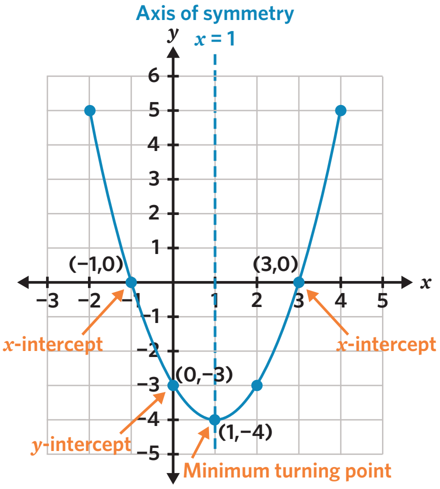
c.



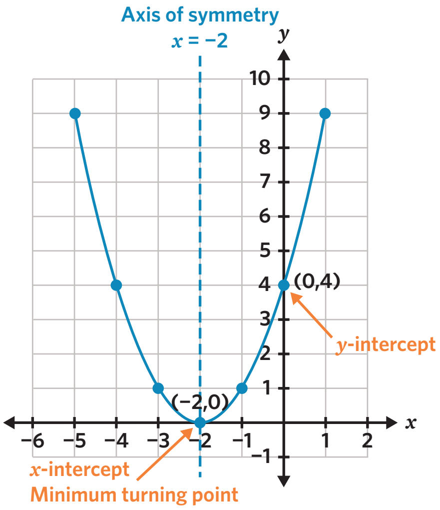
d.



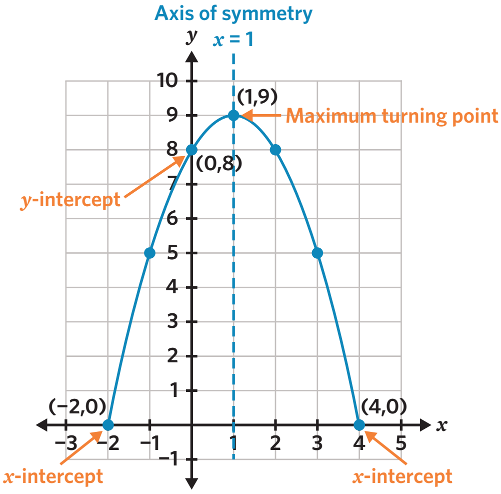
e.



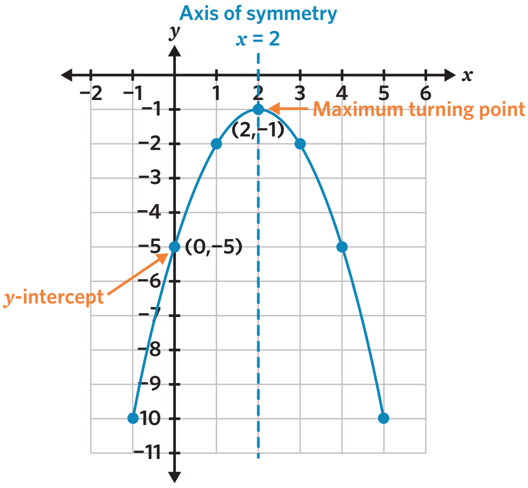
f.



g.



h.



7. D

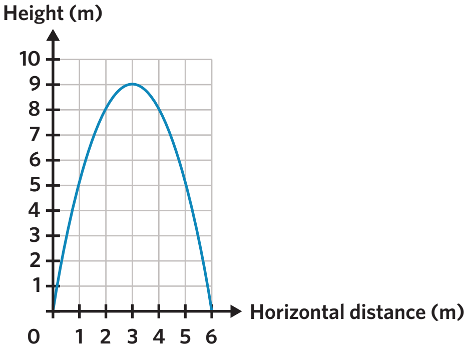
Spot the mistake

8. a. Student A is incorrect. **b.** Student B is incorrect.

Back to contents

Problem solving

9. A ball was thrown into the air and followed the path modelled by the given parabola. The horizontal and vertical axes respectively represent the ball's horizontal distance and height in metres, relative to its starting point at (0,0). Determine the ball's maximum height and its horizontal distance from the starting point when it occurred.



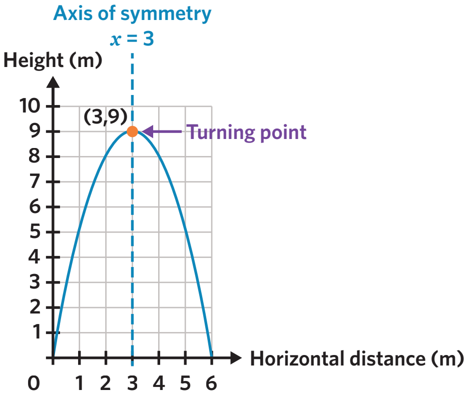
Key points

- The ball thrown into the air follows the given parabola's path.
- Horizontal and vertical axes represent the ball's distance and height in metres, respectively.
- Ball starts at (0,0).
- Determine the ball's maximum height and its horizontal distance from the starting point when it occurred.

Explanation

Identify the vertical line about which the graph is symmetrical and determine its equation.

Identify the turning point by the point where the axis of symmetry intersects with the parabola and determine its coordinates.

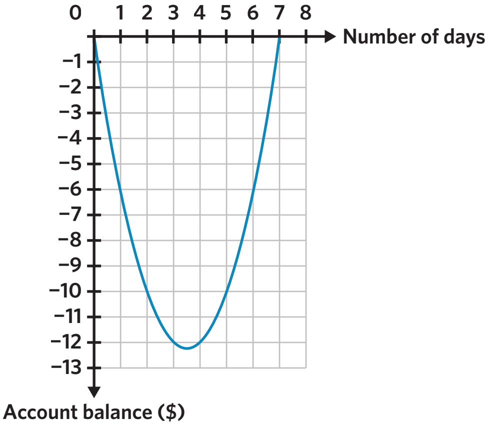


Maximum turning point (3,9).

Answer

The ball's maximum height was 9 m when its horizontal distance from the starting point was 3 m.

10. The balance of an account over a week can be modelled by the given parabola. Assuming that day number one was a Monday, on which days of the week was the account balance equal to −\$10?

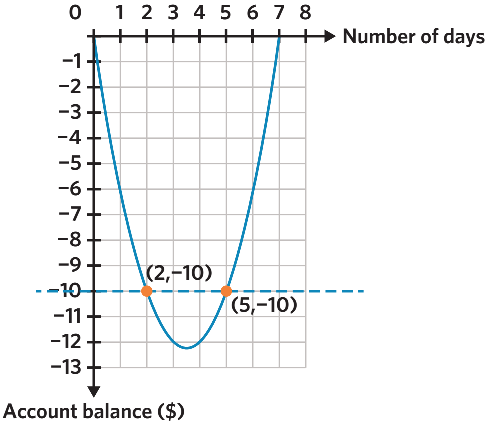


Key points

- A bank account's balance is modelled with the provided parabola.
- Day one is Monday.
- Determine the days of the week when the balance of the bank account was −\$10.

Explanation

Identify the points on the parabola where $y = -10$ and determine their coordinates.



Match the x-coordinates $x = 2$ and $x = 5$ with a day of the week, given it is Monday when $x = 1$.

Day 2 is Tuesday and day 5 is Friday.

Answer

The days that the balance of the bank account was −\$10 were Tuesday and Friday.

11. Jolene aims and throws a ball of paper in the rubbish bin from her desk in the classroom. The trajectory of the ball is given by the equation $y = -x^2 + 2x$ where x is the horizontal distance of the ball from Jolene's desk and y is the ball's height relative to the point from which it was thrown, both in metres. Plot the graph of the given quadratic function for $0 \leq x \leq 2$ and determine the horizontal distance from Jolene's desk to the bin.

Key points

- Trajectory of Jolene's ball of paper is given by the equation $y = -x^2 + 2x$.
- x is the horizontal distance of the ball from Jolene's desk.
- y is the ball's vertical height from the starting point.
- Plot $y = -x^2 + 2x$ for $0 \leq x \leq 2$, and determine the horizontal distance from Jolene's desk to the bin.

Explanation

Substitute each of the required x-values in the given equation to determine the y-values.

Back to contents

$-(0)^2 + 2(0) = -0 + 0 = 0$

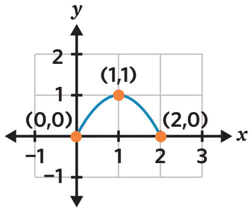
$-(1)^2 + 2(1) = -1 + 2 = 1$

$-(2)^2 + 2(2) = -4 + 4 = 0$

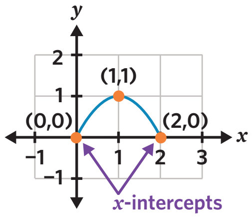
Draw a table, including all the required x-values and enter the y-values. Determine the coordinates of each point.

x	0	1	2
y	0	1	0
(x,y)	(0,0)	(1,1)	(2,0)

Plot the coordinates (x,y) from the table on a Cartesian plane and join them with a smooth curve.



To determine the horizontal distance from Jolene's desk to the bin, identify the x-intercepts of the parabola.



x-intercepts (0,0), (2,0)

Answer

The horizontal distance from Jolene's desk to the bin is 2 m.

12. Sally is building an enclosure for her rabbits. The possible area of the enclosure can be modelled by the equation $a = -m^2 + 6m$, where m is the length of one side in metres. Plot the quadratic function for $0 \leq m \leq 6$ and determine the maximum possible area of the rabbit enclosure.

Key points

- Total area of Sally's rabbit enclosure modelled by equation $a = -m^2 + 6m$.
- m is the length of one side in metres.
- Plot $a = -m^2 + 6m$ for $0 \leq m \leq 6$, and determine the maximum possible area.

Explanation

Substitute each of the required m-values in the given equation to determine the a-values.

$-(0)^2 + 6(0) = -0 + 0 = 0$

$-(1)^2 + 6(1) = -1 + 6 = 5$

$-(2)^2 + 6(2) = -4 + 12 = 8$

$-(3)^2 + 6(3) = -9 + 18 = 9$

$-(4)^2 + 6(4) = -16 + 24 = 8$

$-(5)^2 + 6(5) = -25 + 30 = 5$

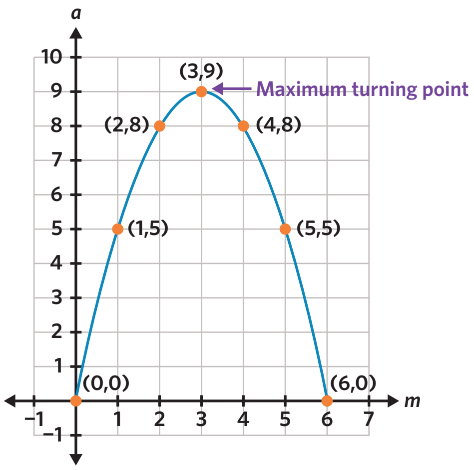
$-(6)^2 + 6(6) = -36 + 36 = 0$

Draw a table, including all the required m-values and enter the a-values. Determine the coordinates of each point.

m	0	1	2	3	4	5	6
a	0	5	8	9	8	5	0
(m,a)	(0,0)	(1,5)	(2,8)	(3,9)	(4,8)	(5,5)	(6,0)

Plot the coordinates (m,a) from the table on a Cartesian plane and join them with a smooth curve.

Identify the coordinates of the maximum turning point.



Maximum turning point (3,9)

Answer

The maximum possible area of the rabbit enclosure is 9 m^2.

13. Over three weeks, the average weekly temperature of a town could be modelled by the equation $t = w^2 - 2w - 3$ for $0 \leq w \leq 3$, where t is temperature in degrees Celsius and w is the number of weeks. Plot the graph of the function for the given number of weeks to determine the minimum average temperature over the three weeks.

Key points

- Temperature of a town over three weeks represented by $t = w^2 - 2w - 3$ for $0 \leq w \leq 3$.
- t is temperature in degrees Celsius.
- w is the number of weeks.
- Plot $t = w^2 - 2w - 3$ for $0 \leq w \leq 3$, and determine the minimum average temperature.

Explanation

Substitute each of the required w-values in the given equation to determine the t-values.

$(0)^2 - 2(0) - 3 = 0 - 0 - 3 = -3$

$(1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$

$(2)^2 - 2(2) - 3 = 4 - 4 - 3 = -3$

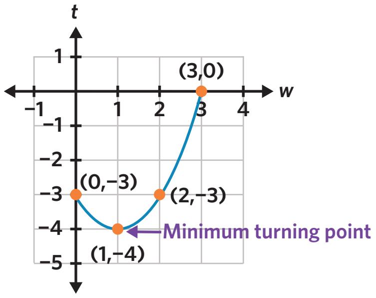
$(3)^2 - 2(3) - 3 = 9 - 6 - 3 = 0$

Draw a table, including all the required w-values and enter the t-values. Determine the coordinates of each point.

w	0	1	2	3
t	−3	−4	−3	0
(w,t)	(0,−3)	(1,−4)	(2,−3)	(3,0)

Plot the coordinates (w,t) from the table on a Cartesian plane and join them with a smooth curve.

Identify the coordinates of the minimum turning point.



Minimum turning point (1,−4)

Back to contents

Answer

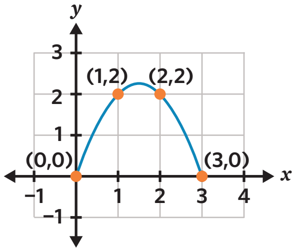
The minimum average temperature of the town over the three weeks is −4 degrees Celsius.

Reasoning

14. a.

x	0	1	2	3
y	0	2	2	0
(x,y)	(0,0)	(1,2)	(2,2)	(3,0)

b.



x-intercepts (0,0), (3,0)

y-intercept (0,0)

c. The previous high jump record was 1.93 m.

d. Suggested option 1: School production.

Suggested option 2: School swimming carnival.

Note: There are other possible options.

15. a. The equation for the axis of symmetry is $x = 1$, the minimum turning point is at (1,−4), the x-intercepts are at (−1,0) and (3,0), and the y-intercept is at (0,−3).

b. The equation for the axis of symmetry is $x = 1$, the maximum turning point is at (1,4), the x-intercepts are at (−1,0) and (3,0), and the y-intercept is at (0,3).

c. The differences in the key features between the two graphs is the nature of the turning point and the direction of the y-intercept. The first parabola has a minimum turning point and is concave up, while the second has a maximum and is concave down. The x-coordinates of the turning point are equal for both parabolas, while the y-coordinates are equal but have opposite directions. They y-intercepts have the same value but opposite directions.

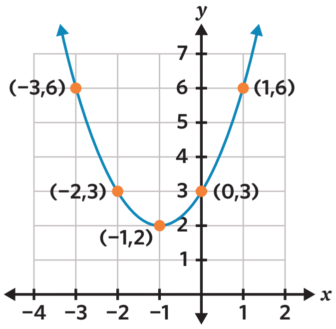
Exam-style

16. B

17. a.

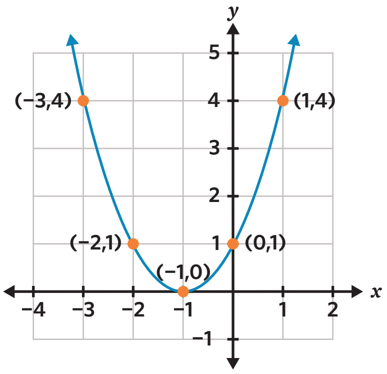
x	−3	−2	−1	0	1
y	6	3	2	3	6
(x,y)	(−3,6)	(−2,3)	(−1,2)	(0,3)	(1,6)

b.



18.

x	−3	−2	−1	0	1
y	4	1	0	1	4



19. (2.5,−6.25)

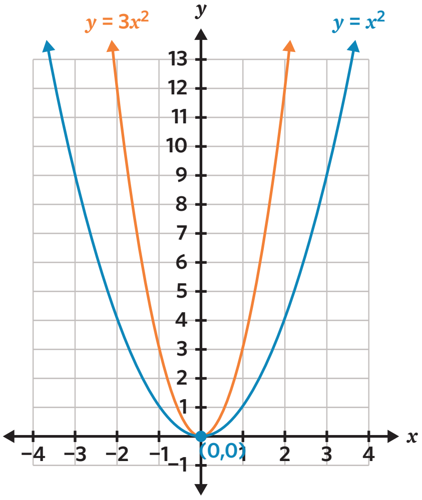
Remember this?

20. D **21.** B **22.** B

6D Sketching parabolas with dilations and reflections

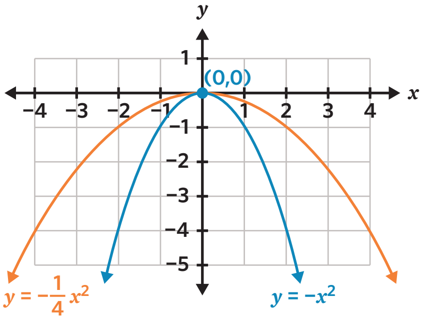
Student practice

Worked example 1



$y = 3x^2$ is narrower than $y = x^2$.

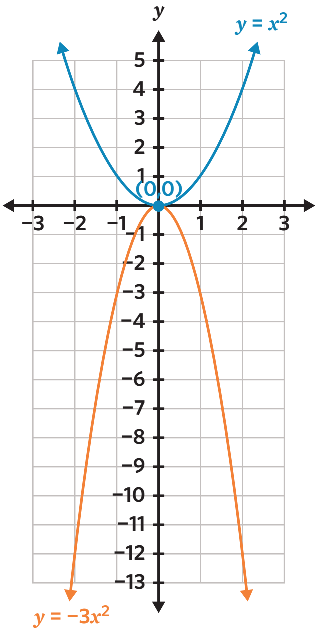
Worked example 2



$y = -\frac{1}{4}x^2$ is wider than $y = -x^2$.

Back to contents

Worked example 3



$y = -3x^2$ is inverted and is narrower than $y = x^2$.

Understanding worksheet

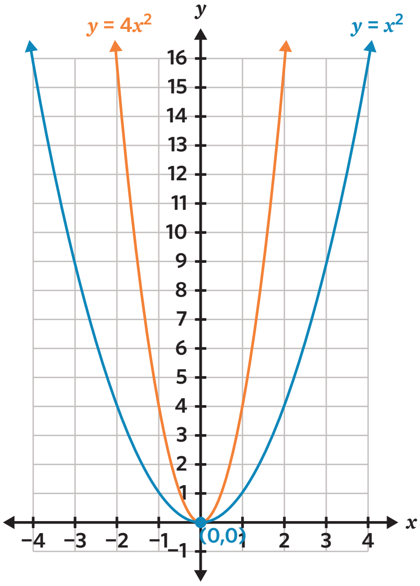
1. **a.** II **b.** III **c.** IV **d.** I

2. **a.** II **b.** I **c.** III **d.** IV

3. turning; basic; wider; narrower

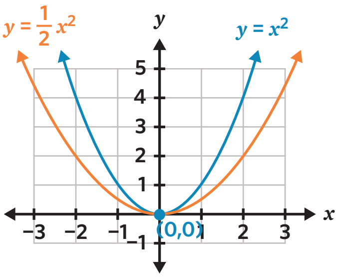
Fluency

4. **a.**



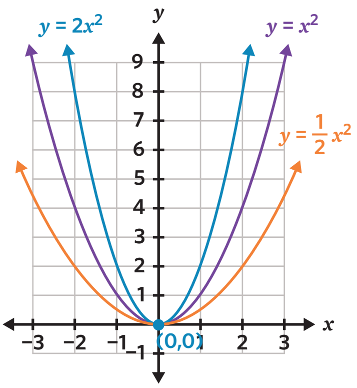
$y = 4x^2$ is narrower than $y = x^2$.

b.



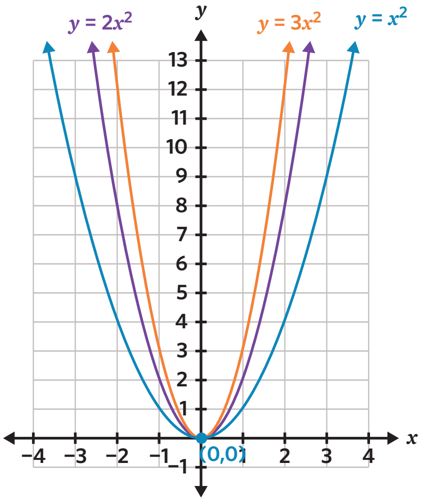
$y = \frac{1}{2}x^2$ is wider than $y = x^2$.

c.



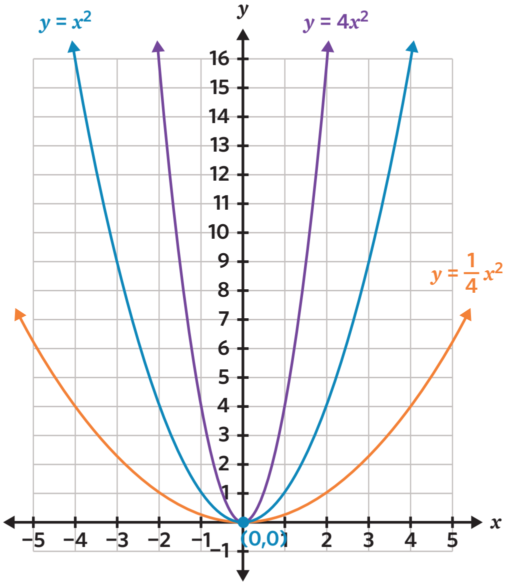
$y = \frac{1}{2}x^2$ is wider than $y = x^2$, which is wider than $y = 2x^2$.

d.



$y = 3x^2$ is narrower than $y = 2x^2$, which is narrower than $y = x^2$.

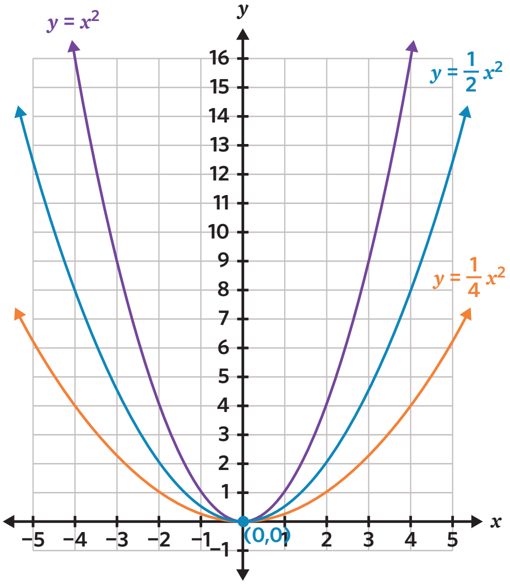
e.



$y = \frac{1}{4}x^2$ is wider than $y = x^2$, which is wider than $y = 4x^2$.

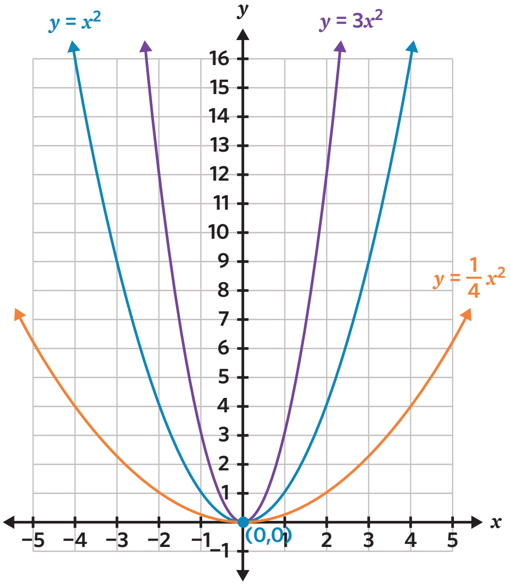
Back to contents

f.



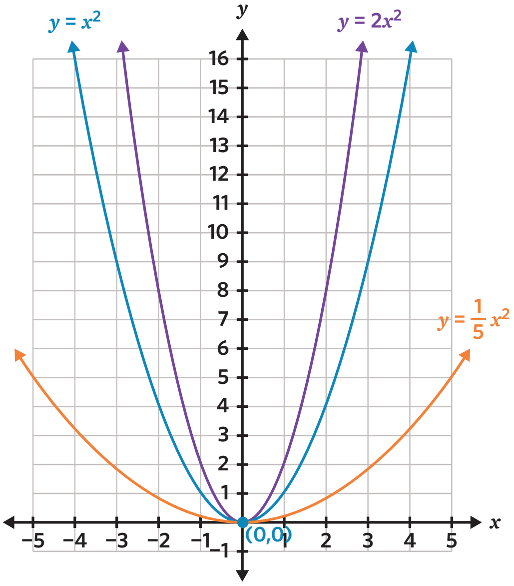
$y = \frac{1}{4}x^2$ is wider than $y = \frac{1}{2}x^2$, which is wider than $y = x^2$.

g.



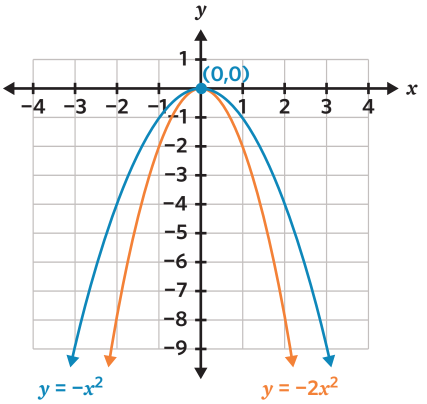
$y = \frac{1}{4}x^2$ is wider than $y = x^2$, which is wider than $y = 3x^2$.

h.



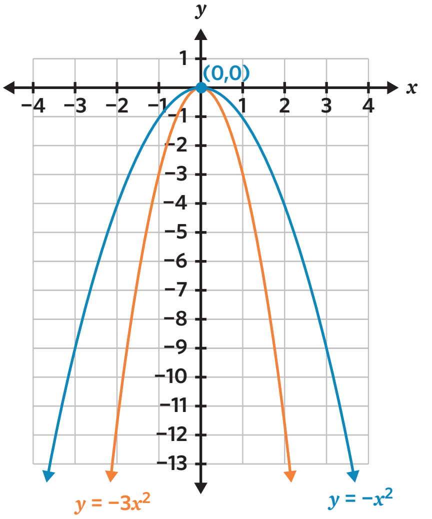
$y = \frac{1}{5}x^2$ is wider than $y = x^2$, which is wider than $y = 2x^2$.

5. a.



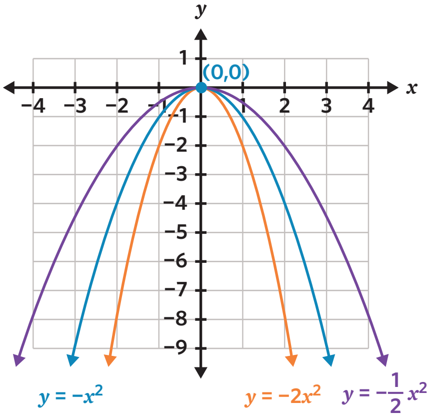
$y = -2x^2$ is narrower than $y = -x^2$.

b.



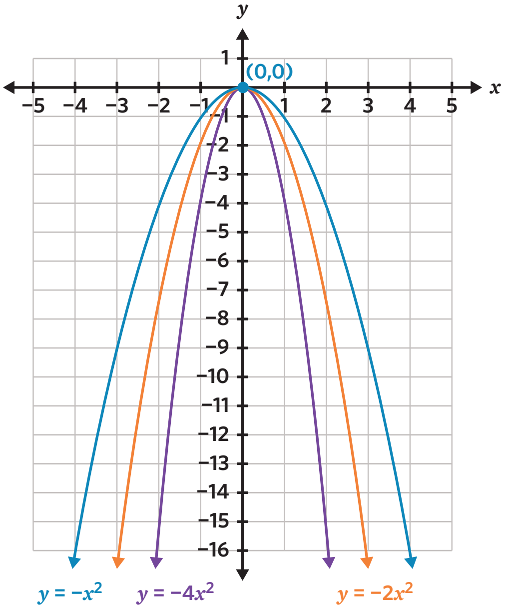
$y = -3x^2$ is narrower than $y = -x^2$.

c.



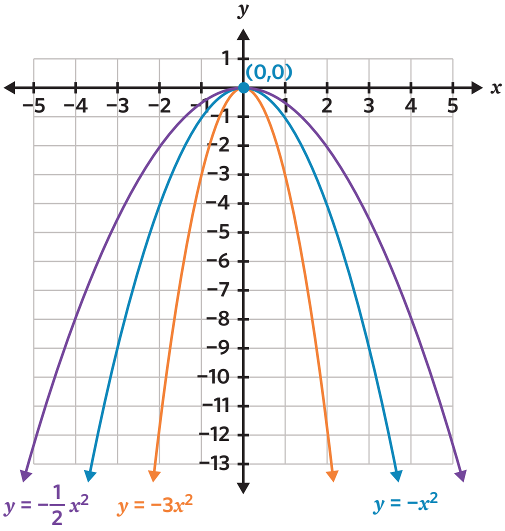
$y = -\frac{1}{2}x^2$ is wider than $y = -x^2$, which is wider than $y = -2x^2$.

d.



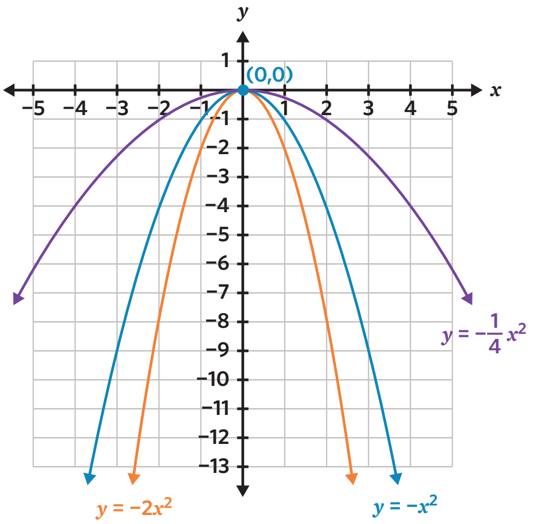
$y = -4x^2$ is narrower than $y = -2x^2$, which is narrower than $y = -x^2$.

e.



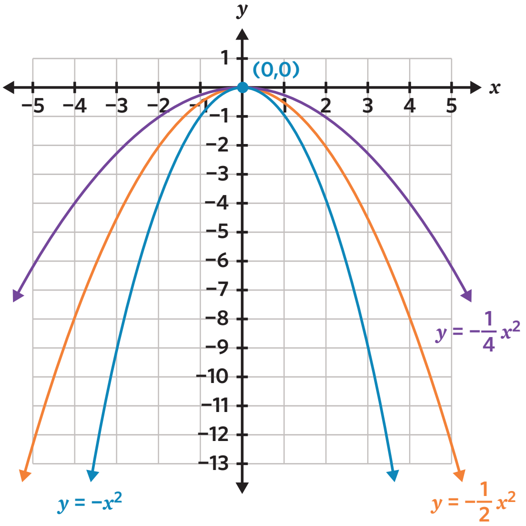
$y = -\frac{1}{2}x^2$ is wider than $y = -x^2$, which is wider than $y = -3x^2$.

f.



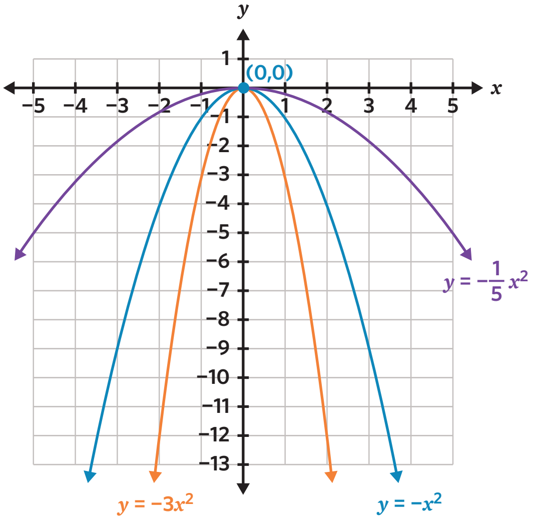
$y = -\frac{1}{4}x^2$ is wider than $y = -x^2$, which is wider than $y = -2x^2$.

g.



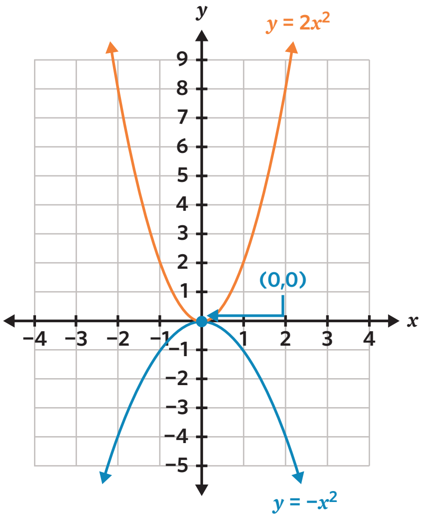
$y = -\frac{1}{4}x^2$ is wider than $y = -\frac{1}{2}x^2$, which is wider than $y = -x^2$.

h.



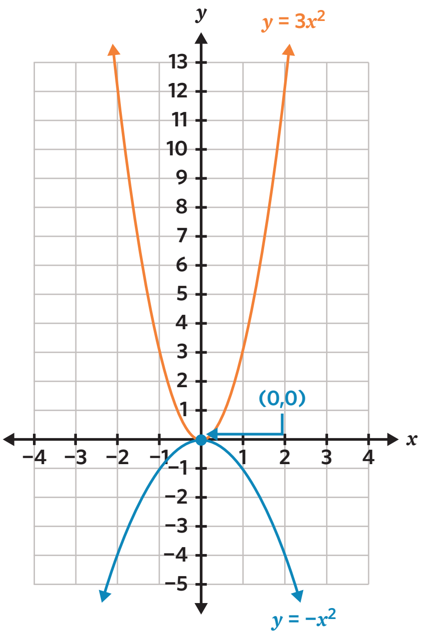
$y = -\frac{1}{5}x^2$ is wider than $y = -x^2$, which is wider than $y = -3x^2$.

6. a.



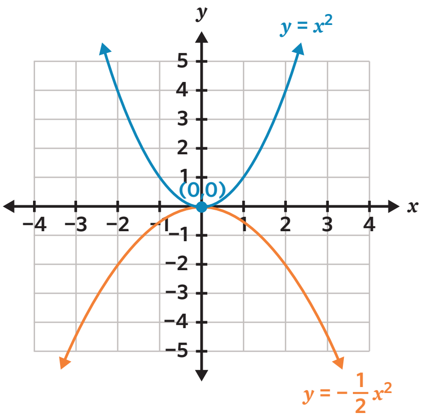
$y = -x^2$ is inverted and is wider than $y = 2x^2$.

b.



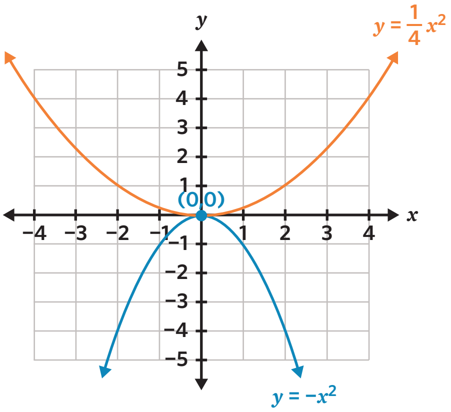
$y = -x^2$ is inverted and is wider than $y = 3x^2$.

c.



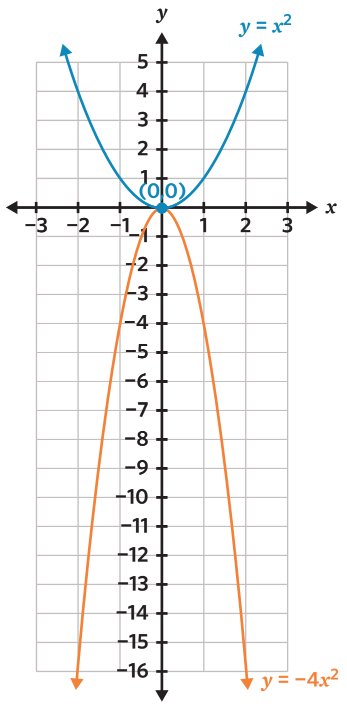
$y = -\frac{1}{2}x^2$ is inverted and is wider than $y = x^2$.

d.



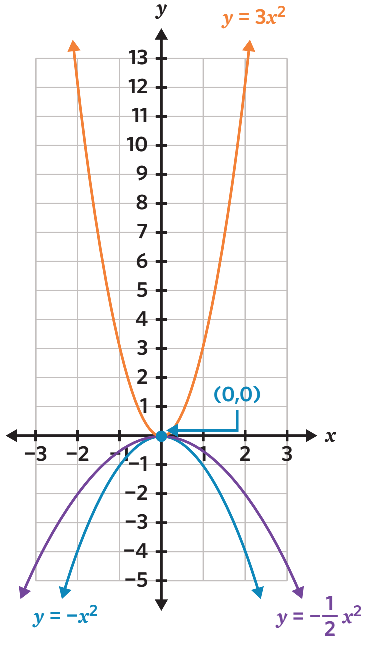
$y = -x^2$ is inverted and is narrower than $y = \frac{1}{4}x^2$.

e.



$y = -4x^2$ is inverted and is narrower than $y = x^2$.

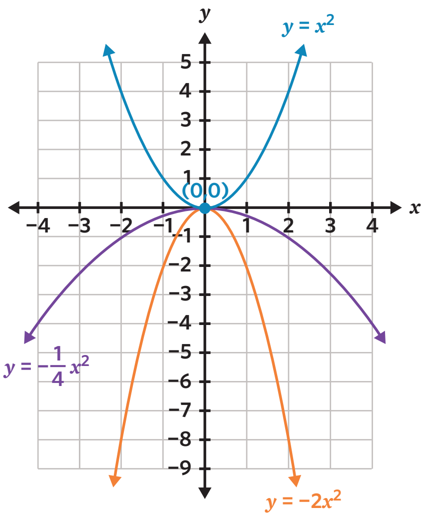
f.



$y = -\frac{1}{2}x^2$ and $y = -x^2$ are inverted and wider than $y = 3x^2$.

Back to contents

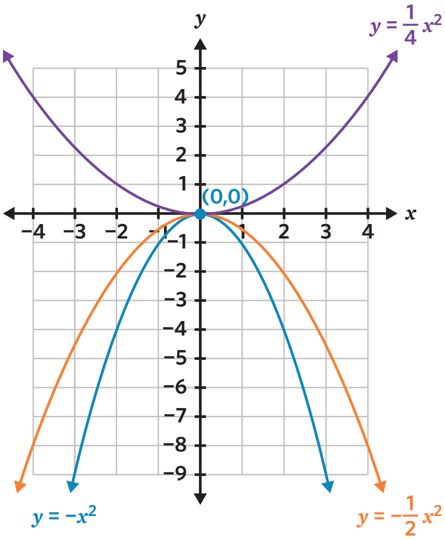
g.



$y = -\frac{1}{4}x^2$ and $y = -2x^2$ are inverted compared to $y = x^2$.

$y = -\frac{1}{4}x^2$ is wider than $y = x^2$, which is wider than $y = -2x^2$.

h.



$y = -\frac{1}{2}x^2$ and $y = -x^2$ are inverted and narrower than $y = \frac{1}{4}x^2$.

---

7. C

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. The shape of an archway can be modelled by the equation $y = -x^2$ where it is symmetrical about the line $x = 0$. During renovation, the archway was made wider and its equation was scaled by a factor of a third. Determine an equation which could be used to model the shape of the archway after renovation.

Key points

- The shape of an archway can be modelled by the equation $y = -x^2$.
- The archway was made wider and the equation was scaled by a third.
- Determine an equation which could be used to model the shape of the archway after renovation.

Explanation

For the equation $y = -ax^2$, the parabola dilates away from the y-axis and is wider than the basic parabola $y = -x^2$ when $0 < a < 1$.

Determine the value of a that scales the parabola by a factor of $\frac{1}{3}$.

$a = \frac{1}{3}$

Substitute this value of a into the equation for the shape of the archway before renovation.

$y = -\frac{1}{3}x^2$

Answer

The equation for the shape of the archway after renovation is $y = -\frac{1}{3}x^2$.

10. A water bird dives into the sea to catch a fish. The bird's path can be modelled by the equation $y = ax^2$ where it is symmetrical about the point at which the bird reaches its greatest depth and starts coming back up again immediately after. Determine the value of a if the point (1,3) lies on the graph.

Key points

- The bird's path can be modelled by the equation $y = ax^2$.
- The point (1,3) lies on the graph.
- Determine the value of a.

Explanation

The bird passes through the coordinates (1,3). Substitute $x = 1$ and $y = 3$ into the equation $y = ax^2$.

$3 = a \times 1^2$

Solve for a.

$a = 3$

Answer

The value of a is 3.

11. The shape of a bridge over a river can be modelled by the equation $y = -\frac{1}{4}x^2$. Determine an equation that could be used to model the reflection of the bridge in the river and sketch the parabola.

Key points

- The shape of a bridge over a river can be modelled by the equation $y = -\frac{1}{4}x^2$.
- Determine an equation that could be used to model the reflection of the bridge in the river and sketch the parabola.

Explanation

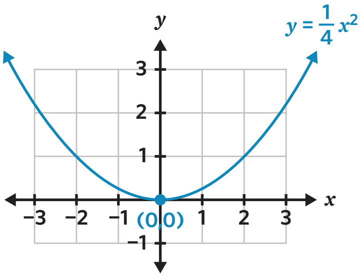
Reflecting the equation $y = -\frac{1}{4}x^2$ in the x-axis gives the equation $y = \frac{1}{4}x^2$.

Complete a table of values for $y = \frac{1}{4}x^2$ for $-3 \leq x \leq 3$.

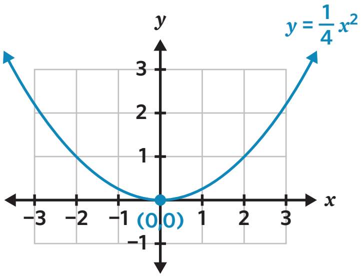
x	−3	−2	−1	0	1	2	3
$y = \frac{1}{4}x^2$	$\frac{9}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	$\frac{9}{4}$

To sketch the parabola $y = \frac{1}{4}x^2$, plot the sets of coordinates given by the table of values. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.

Back to contents



Answer



12. The shape of the cross-section of a small bowl in a dinner set is given by $y = \frac{1}{2}x^2$ where the axis of symmetry splits the bowl in two identical halves. In the same set, a large bowl has the same shape, but the equation modelling it has half the scale factor of the small bowl's. Determine an equation that could be used to describe the shape of a large bowl and sketch both equations on the same set of axes.

Key points

- The shape of the cross-section of a small bowl is given by $y = \frac{1}{2}x^2$.
- The large bowl has the same equation as the small bowl, but half the scale factor.
- Determine an equation that could be used to describe the shape of a large bowl and sketch both equations on the same set of axes.

Explanation

For the equation $y = ax^2$, the parabola dilates away from the y-axis and is wider than the basic parabola $y = x^2$ when $0 < a < 1$.

The parabola $y = \frac{1}{2}x^2$ is adjusted so that the scale factor is halved.

Adjust the value of a.

$$\begin{aligned}\text{New value of } a &= \text{old value of } a \times \frac{1}{2}\\ &= \frac{1}{2} \times \frac{1}{2}\\ &= \frac{1}{4}\end{aligned}$$

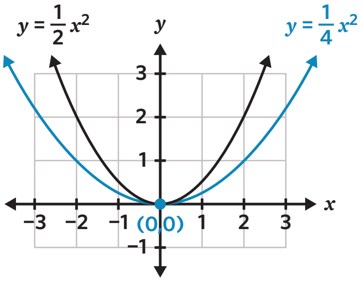
Substitute this value of a into the equation $y = \frac{1}{2}x^2$ to describe the shape of the large bowl.

$y = \frac{1}{4}x^2$

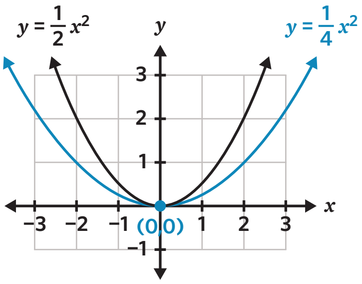
Complete a table of values for the two equations for $-2 \le x \le 2$.

x	-2	-1	0	1	2
$y = \frac{1}{4}x^2$	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1
$y = \frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

Sketch the parabolas for the small bowl $\left(y = \frac{1}{2}x^2\right)$ and the large bowl $\left(y = \frac{1}{4}x^2\right)$ on the same set of axes. Plot the sets of coordinates given by the table of values. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.



Answer



13. Three different parts of a roller coaster can be modelled by the quadratic equations $y = \frac{1}{3}x^2$, $y = -x^2$ and $y = 2x^2$. Sketch all three parabolas on the same set of axes and determine which of the three equations modelling parts of the roller coaster describes the steepest incline.

Key points

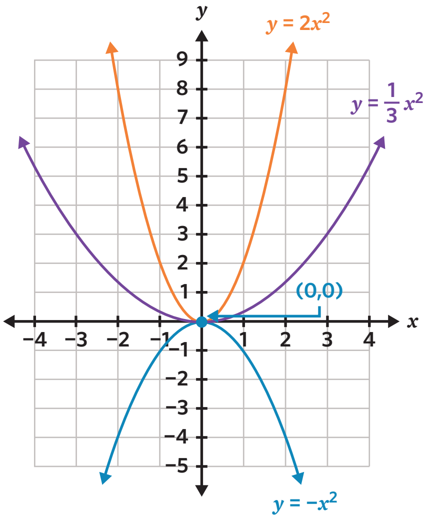
- Three different parts of a roller coaster can be modelled by the quadratic equations $y = \frac{1}{3}x^2$, $y = -x^2$ and $y = 2x^2$.
- Sketch all three parabolas on the same set of axes and determine which of the three equations describes the steepest incline.

Explanation

Complete a table of values for the three equations for $-2 \le x \le 2$.

x	-2	-1	0	1	2
$y = \frac{1}{3}x^2$	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$
$y = -x^2$	-4	-1	0	-1	-4
$y = 2x^2$	8	2	0	2	8

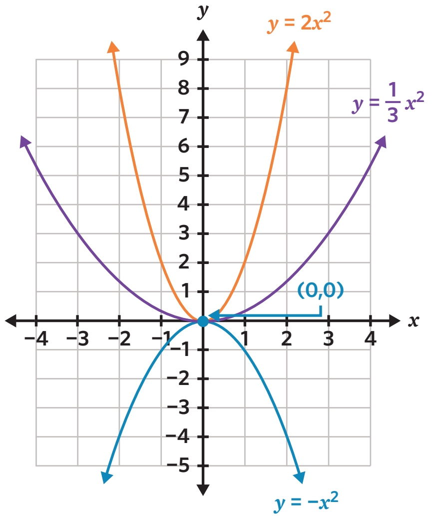
Sketch all three parabolas on the same set of axes. Plot the sets of coordinates given by the table of values. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.



Look at the graph to determine which of the three equations modelling parts of the roller coaster describes the steepest incline.

The steepest incline is described by the equation $y = 2x^2$ because it is the most narrow parabola.

Answer



$y = 2x^2$ describes the steepest incline.

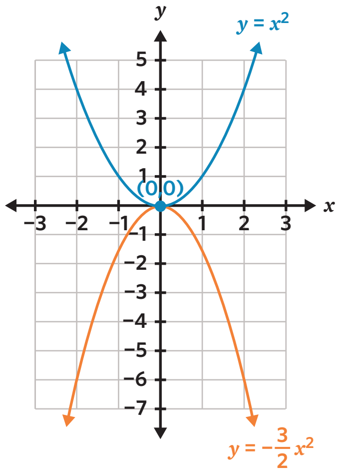
Reasoning

14. a. Giovanni's basic parabola has a turning point at (0,0), crosses the x-axis and y-axis at (0,0), and has an axis of symmetry at $x = 0$.

b. The equation of this parabola is $y = 3x^2$.

c. The equation of this parabola is $y = \frac{3}{2}x^2$.

d. The equation of this parabola is $y = -\frac{3}{2}x^2$.

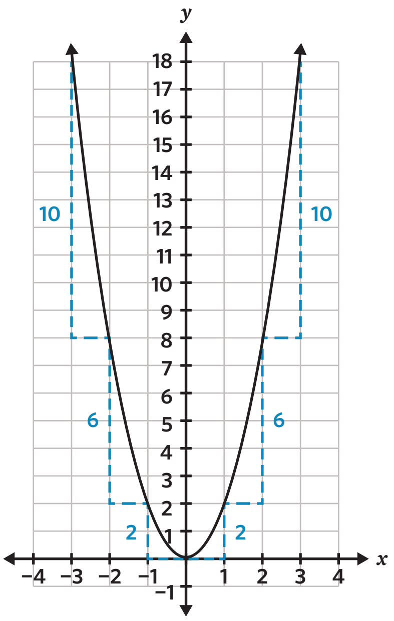


e. Suggested option 1: An advantage is that the art is created consistently and neatly.

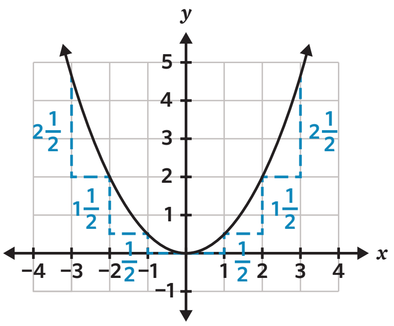
Suggested option 2: A disadvantage is that a machine that creates art, such as a 3D printer, may be expensive.

Note: There are other possible options.

15. a.



b.

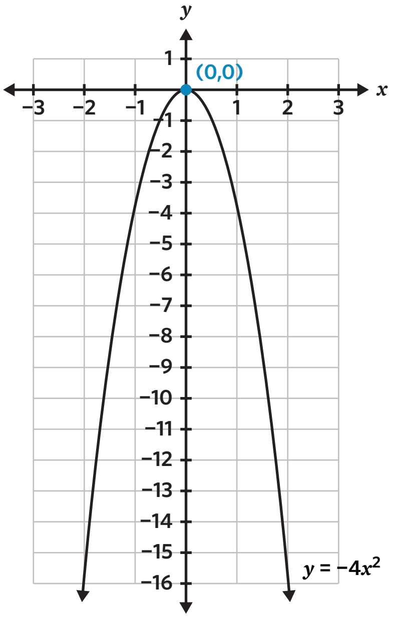


c. For a parabola of the form $y = ax^2$, the change in y-values with x-values on either side of the y-axis follows the pattern a, $3a$, $5a$, $7a$... where each coefficient of a is a consecutive odd number and the positive value of a is always used, regardless of parabola orientation.

Exam-style

16. E

17. a.

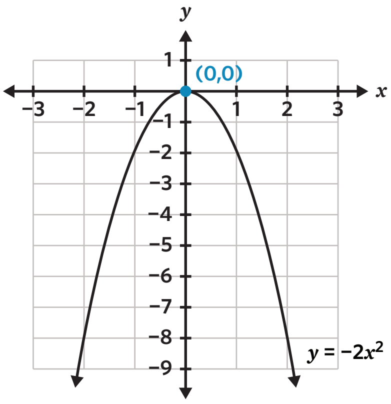


b. $y = -2x^2$

Back to contents

18. $a = \frac{1}{4}$

19.



Remember this?

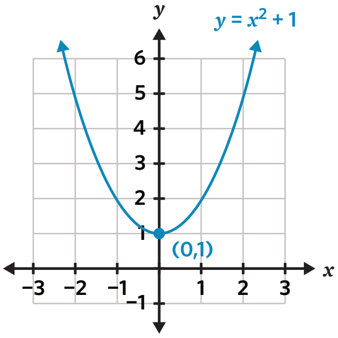
20. D **21.** C **22.** C

6E Sketching translations of parabolas

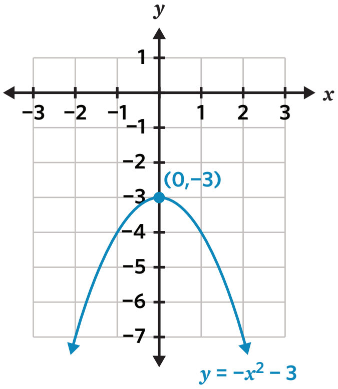
Student practice

Worked example 1

a.

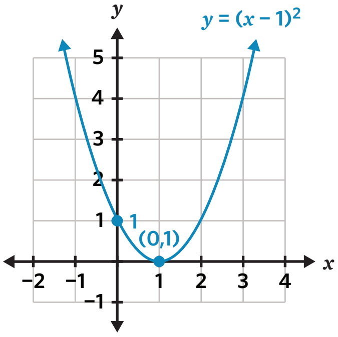


b.

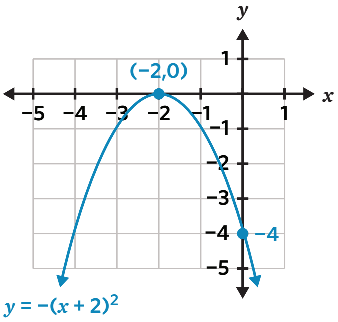


Worked example 2

a.

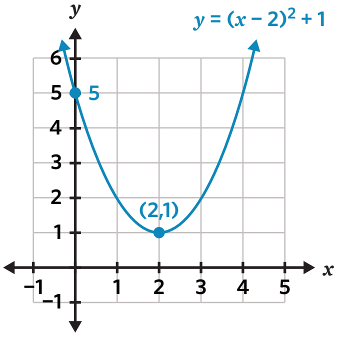


b.

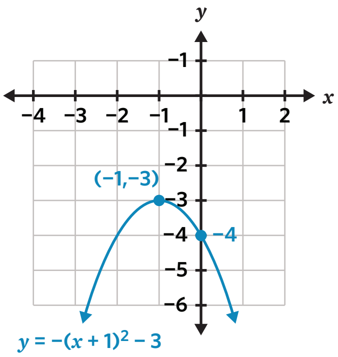


Worked example 3

a.

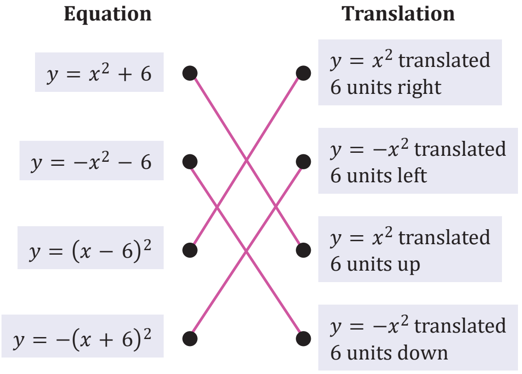


b.



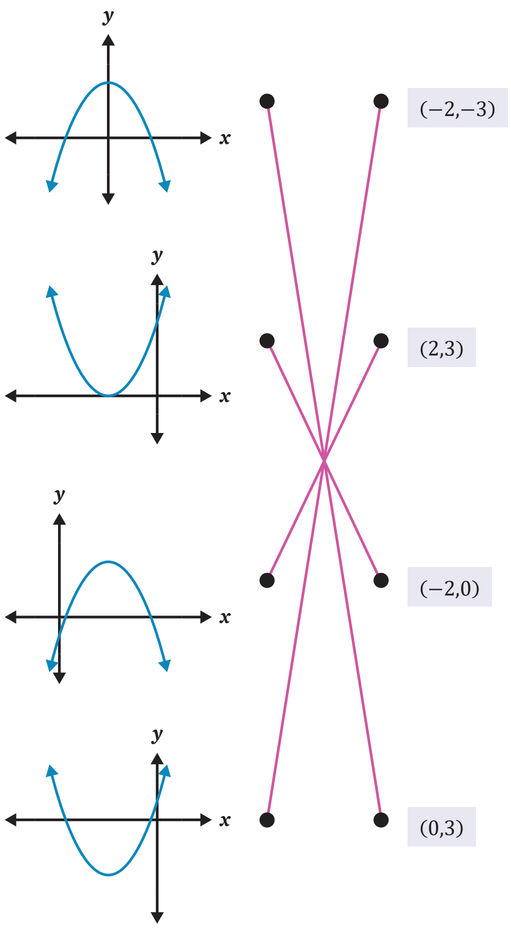
Understanding worksheet

1.



Back to contents

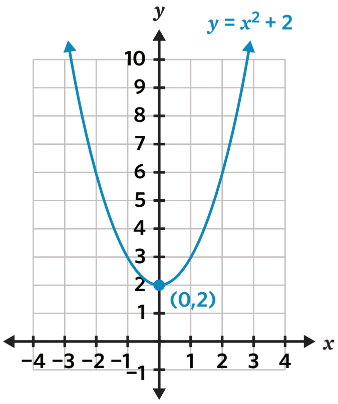
2. Graph Turning point



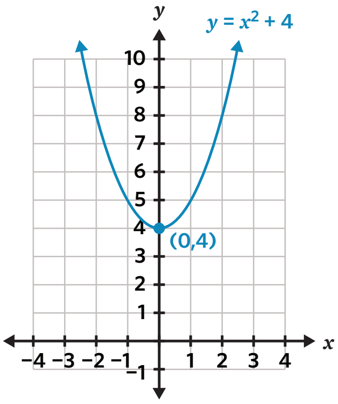
3. translated; point; units; coordinates

Fluency

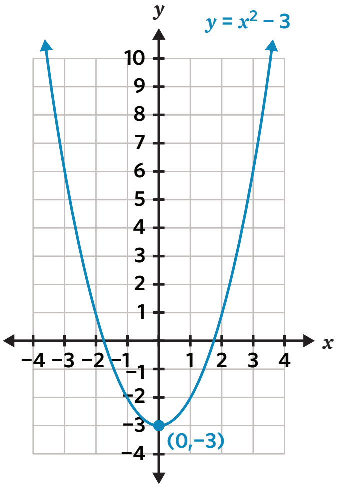
4. a.



b.



c.



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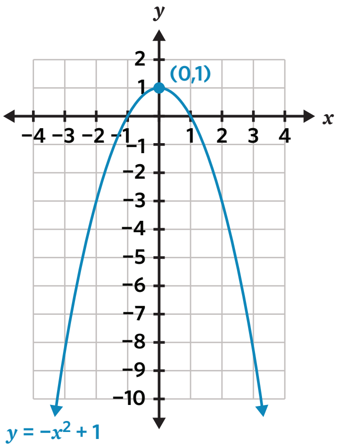

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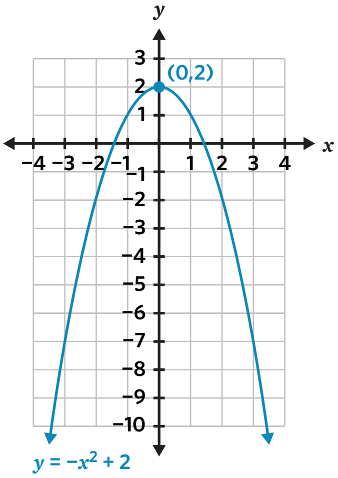
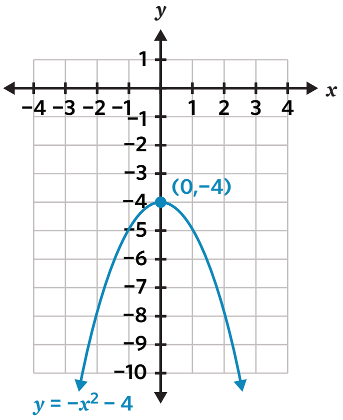
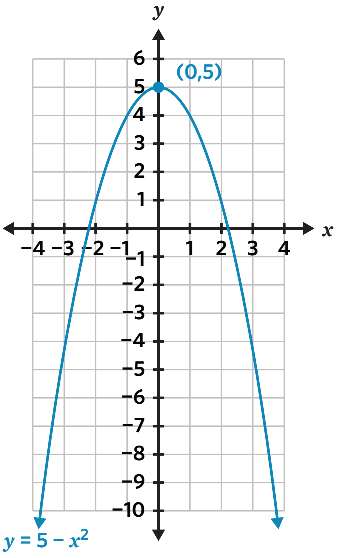


f.

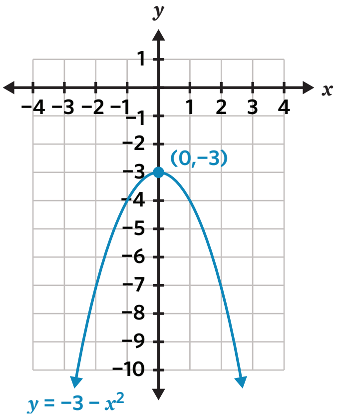


Back to contents

g.



h.



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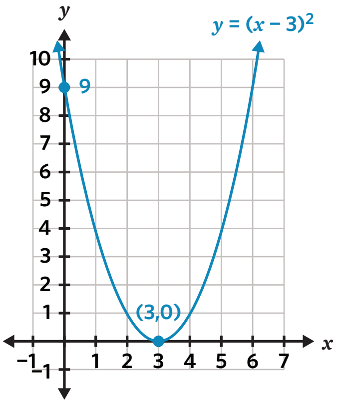
5. a.



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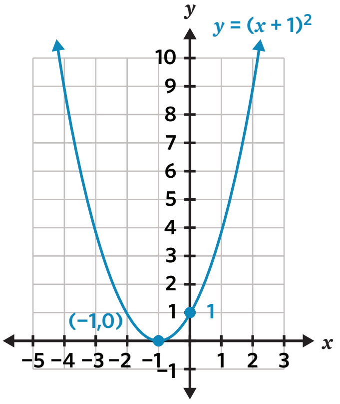

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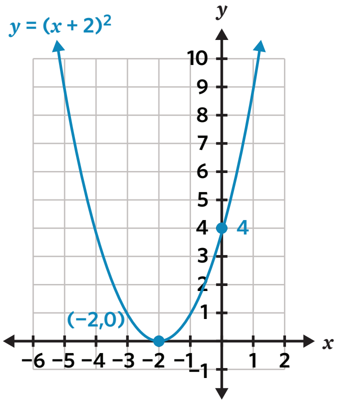

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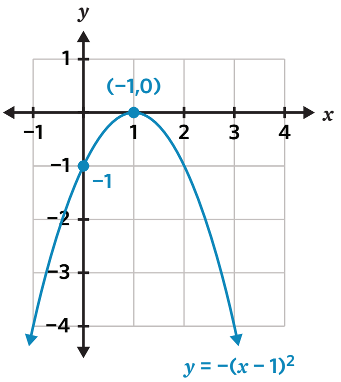

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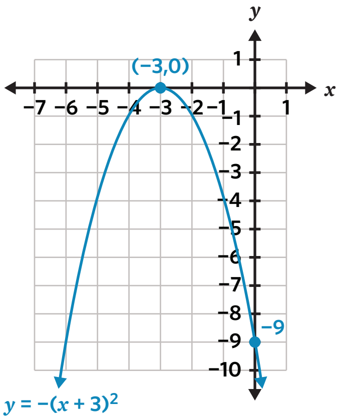

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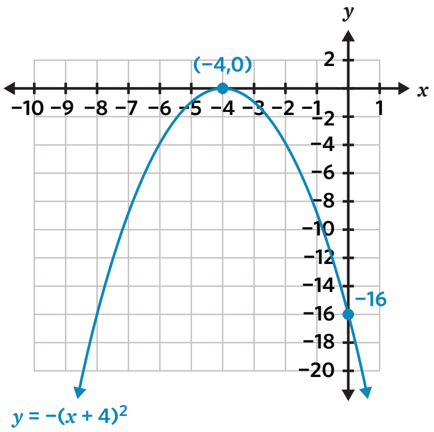

Back to contents

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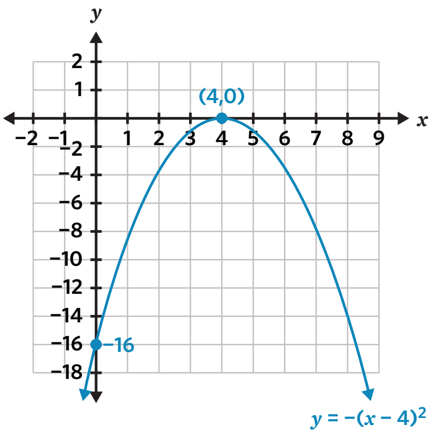

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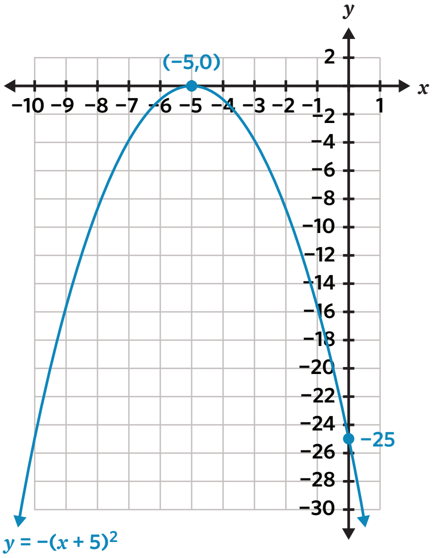
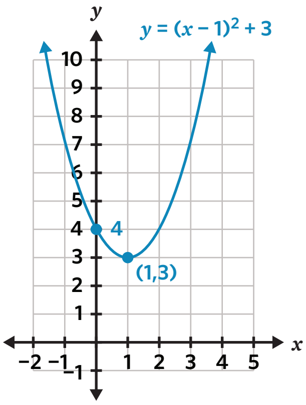


6. a.



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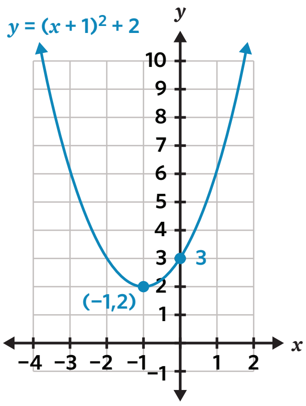

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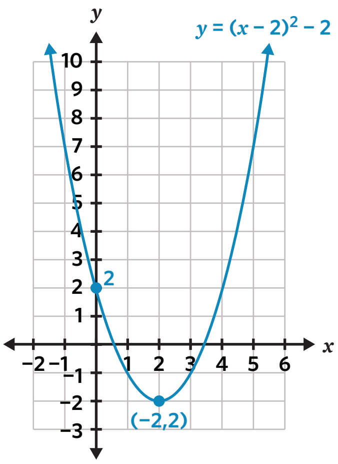

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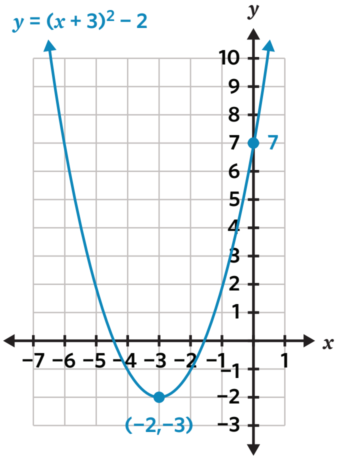

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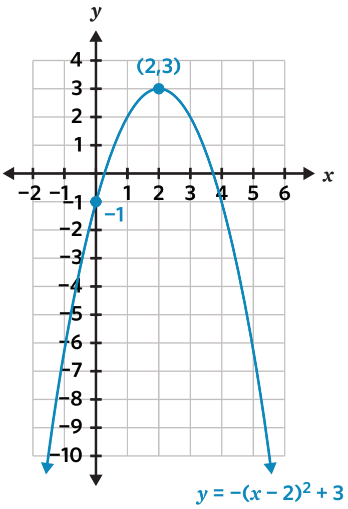

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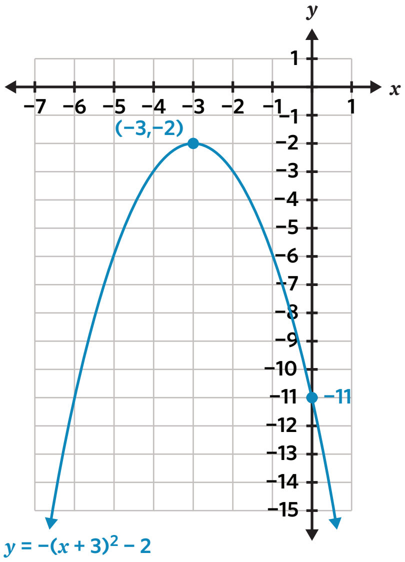

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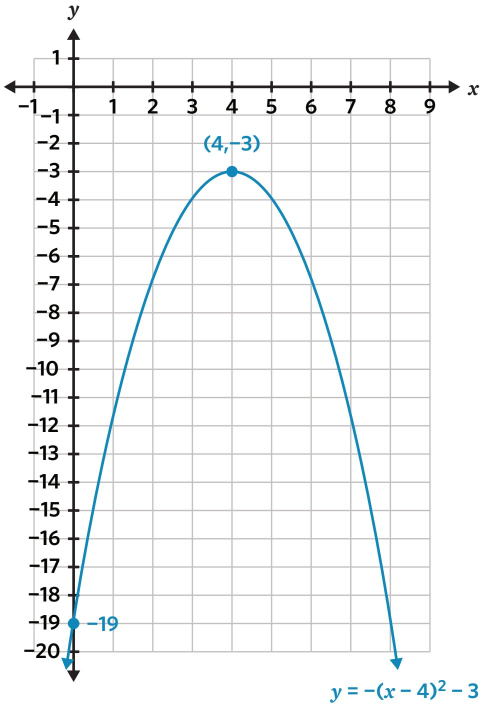

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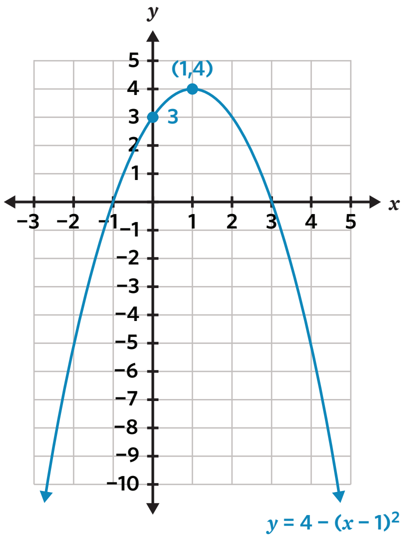


7. B

Spot the mistake

8. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. The height above sea level of a mountain can be modelled by the equation $y = -(x - 4)^2 + 10$, where the average sea level is represented by the line $y = 0$ (x-axis) and the values of x represent horizontal distance. Determine the coordinates of the mountain's summit.

Key points

- Equation for height of mountain is $y = -(x - 4)^2 + 10$.
- $y = 0$ represents average sea level.
- x-values represent horizontal distance.
- Determine the mountain summit's coordinates.

Explanation

The summit is the highest point of the mountain and so it will have the coordinates of the maximum turning point.

In the equation of the form $y = a(x - h)^2 + k$, determine the values of h and k.

For $y = -(x - 4)^2 + 10$

$h = 4$ and $k = 10$

Turning point (4,10)

Answer

The coordinates of the mountain's summit are (4,10).

10. Frances wants to install a pool in her backyard. The area of the pool can be modelled by the equation $y = -(x - 6)^2 + 36$ where x represents the length of one of the sides in metres. Determine how long Frances should make the side length of the pool in order to achieve the maximum possible area.

Key points

- Equation for pool area is $y = -(x - 6)^2 + 36$.
- x represents one side length in metres.
- Determine the x-value of the maximum possible area.

Explanation

Maximum area will be given by the highest y-value of the parabola (k), and the associated side length will be given by h.

In the equation of the form $y = a(x - h)^2 + k$, determine the values of h and k.

For $y = -(x - 6)^2 + 36$

$h = 6$ and $k = 36$

Turning point (6,36)

Answer

Francis should make the side of the pool 6 m long.

11. A flare is released from the top of a hill. The height of the flare following its release can be modelled by the equation $h = -(x - 5)^2 + 45$, where x is the horizontal distance in metres. Determine the height of the hill from which the flare was released.

Key points

- Equation for flare height is $h = -(x - 5)^2 + 45$.
- x is horizontal distance.
- Determine the initial height.

Back to contents

Explanation

The height of the hill from which the flare was released is given by the point where the parabola crosses the vertical axis (where $x = 0$).

Substitute $x = 0$ into the equation of the parabola to determine the h-intercept.

$$\begin{aligned} h &= -(0 - 5)^2 + 45 \\ &= -(-5)^2 + 45 \\ &= -25 + 45 \\ &= 20 \end{aligned}$$

h-intercept is 20

Answer

The height of the hill from which the flare was released was 20 m.

12. The number of students enrolled in a course over the first four weeks of the semester can be modelled by the equation $s = (w - 1)^2 + 20$ for $0 \leq w \leq 4$. Sketch the parabola and determine how many students were enrolled in the course at the beginning of the semester.

Key points

- Equation for number of students is $s = (w - 1)^2 + 20$.
- w between 0 and 4, inclusive.
- Sketch the graph and determine the initial number of students.

Explanation

In the equation of the form $s = a(w - h)^2 + k$, determine the values of h and k. This shows by how many units the graph of $s = aw^2$ must be translated horizontally and vertically.

For $s = (w - 1)^2 + 20$

$h = 1$ and $k = 20$

$\therefore s = w^2$ is translated 1 unit right and 20 units up

Determine the coordinates of the turning point (h,k).

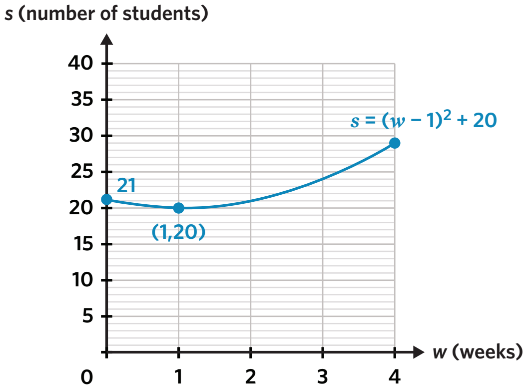
Turning point (1,20)

Substitute $w = 0$ into the equation of the parabola to determine the s-intercept.

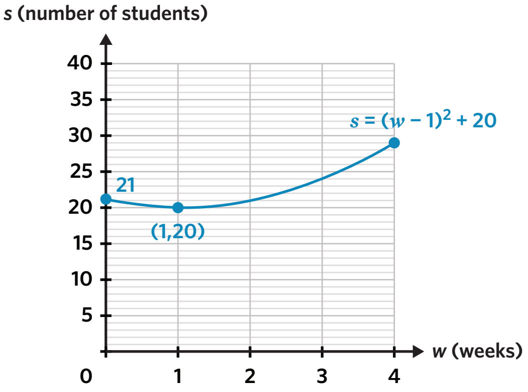
$$\begin{aligned} s &= (0 - 1)^2 + 20 \\ &= (-1)^2 + 20 \\ &= 1 + 20 \\ &= 21 \end{aligned}$$

s-intercept is 21

Sketch an upright parabola with an axis of symmetry at $w = h$, a turning point at (h,k) and an s-intercept at (0,21).



Answer



At the beginning of the semester, there were 21 students enrolled.

13. The parabolic cable of a suspension bridge can be modelled by the equation $y = \frac{1}{4}(x - 4)^2$ for $0 \leq x \leq 8$, where the y-axis represents a vertical metal beam to which the parabolic cable is attached. Sketch this part of the bridge and determine at what height the cable attaches to the vertical beam, given that the units are metres.

Key points

- Suspension bridge cable equation is $y = \frac{1}{4}(x - 4)^2$.
- x between 0 and 8, inclusive.
- y-axis represents the vertical metal beam height.
- Sketch the equation and determine the y-coordinate of the y-intercept.

Explanation

In the equation of the form $y = a(x - h)^2 + k$, determine the values of h and k. This shows by how many units the graph of $y = ax^2$ must be translated horizontally and vertically.

For $y = \frac{1}{4}(x - 4)$

$h = 4$ and $k = 0$

$\therefore y = x^2$ is translated 4 units right

Determine the coordinates of the turning point (h,k).

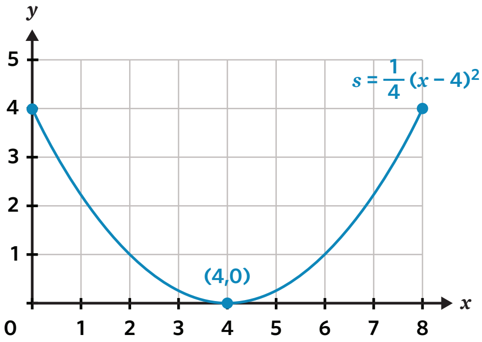
Turning point (4,0)

Substitute $x = 0$ into the equation of the parabola to determine the y-intercept.

$$\begin{aligned} y &= \frac{1}{4}(0 - 4)^2 \\ &= \frac{1}{4}(-4)^2 \\ &= \frac{1}{4} \times 16 \\ &= 4 \end{aligned}$$

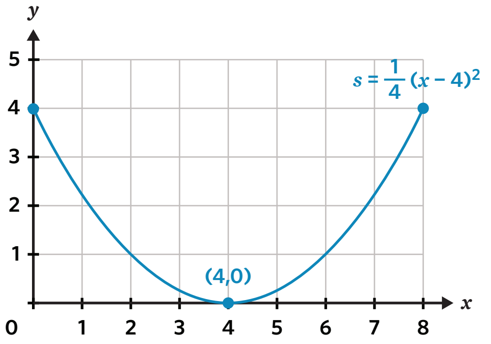
y-intercept is 4

Sketch an upright parabola with an axis of symmetry at $x = h$, a turning point at (h,k) and a y-intercept at (0,4).



Back to contents

Answer



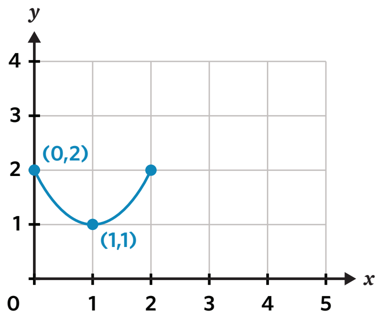
The cable attaches to the vertical beam at a height of 4 m.

Reasoning

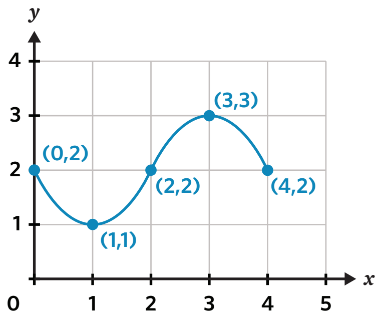
14. **a.** The coordinates of the skating bowl's turning point for $0 \leq x \leq 2$ are (1,1).

b. The coordinates of the skating bowl's turning point for $2 \leq x \leq 4$ are (3,3).

c.



d.

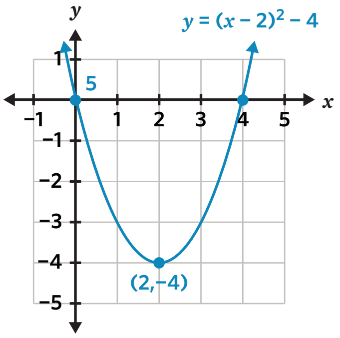


e. Suggested option 1: Public drinking fountains.

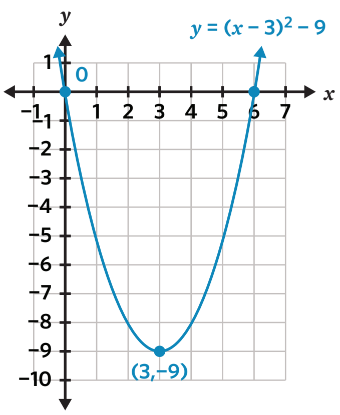
Suggested option 2: Public parks.

Note: There are other possible options.

15. **a.**



b.



c. The y-intercept of a parabola with the equation of the form $y = (x - h)^2 - h^2$ is 0.

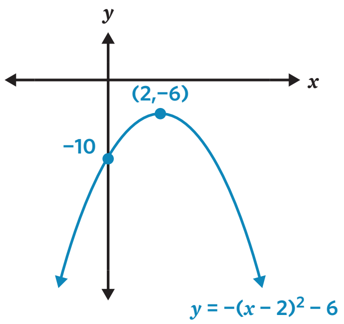
Exam-style

16. B

17. **a.** 44 rabbits

b. 40 rabbits in week 2

18.



19. $11 = (0 + h)^2 - 5$

$16 = h^2$

$h = 4$

$\therefore y = (x + 4)^2 - 5$ and the turning point is (−4,−5).

Remember this?

20. E

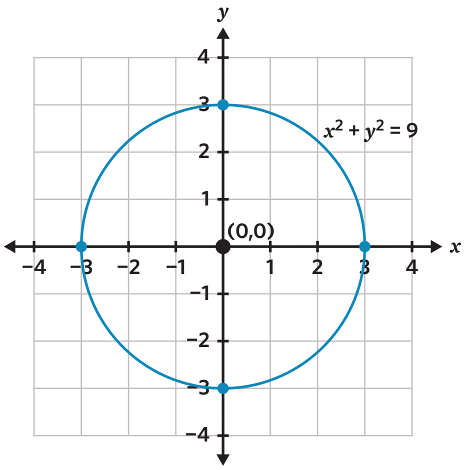
21. C

22. D

6F Non-linear graphs

Student practice

Worked example 1

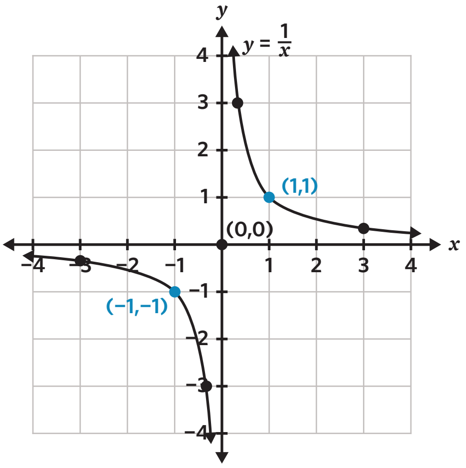


Back to contents

Worked example 2

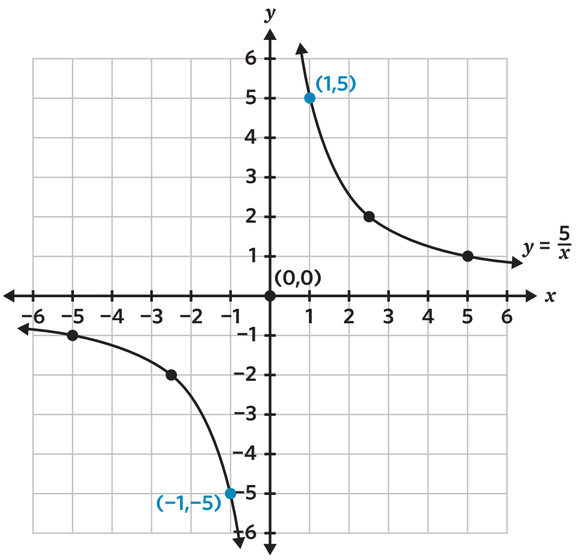
a.

x	$y = \frac{1}{x}$	(x,y)
-3	$-\frac{1}{3}$	$\left(-3,-\frac{1}{3}\right)$
-1	-1	$(-1,-1)$
$-\frac{1}{3}$	-3	$\left(-\frac{1}{3},-3\right)$
0	Undefined	Asymptote
$\frac{1}{3}$	3	$\left(\frac{1}{3},3\right)$
1	1	$(1,1)$
3	$\frac{1}{3}$	$\left(3,\frac{1}{3}\right)$



b.

x	$y = \frac{5}{x}$	(x,y)
-5	-1	$(-5,-1)$
$-\frac{5}{2}$	-2	$\left(-\frac{5}{2},-2\right)$
-1	-5	$(-1,-5)$
0	Undefined	Asymptote
1	5	$(1,5)$
$\frac{5}{2}$	2	$\left(\frac{5}{2},2\right)$
5	1	$(5,1)$



Understanding worksheet

1. **a.** 3 **b.** 5 **c.** 10 **d.** 0.4

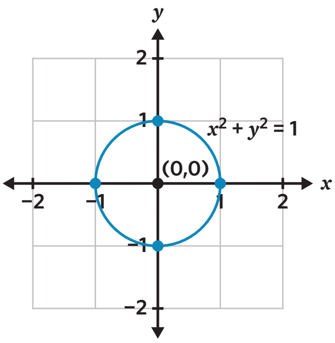
2.

x	-5	-2	-1	0	1	2	5
$y = \frac{10}{x}$	-2	-5	-10	Undefined	10	5	2

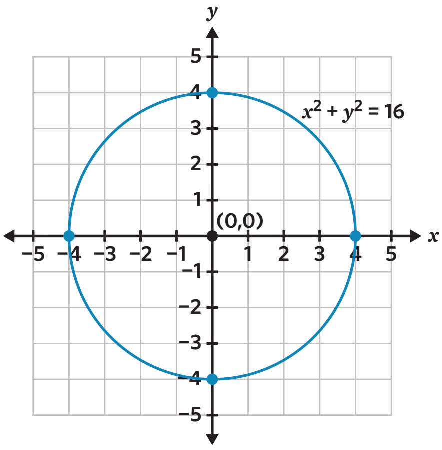
3. conic; continuous; discontinuous; asymptotes

Fluency

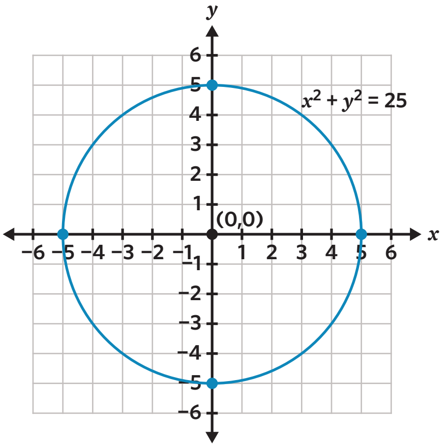
4. **a.**



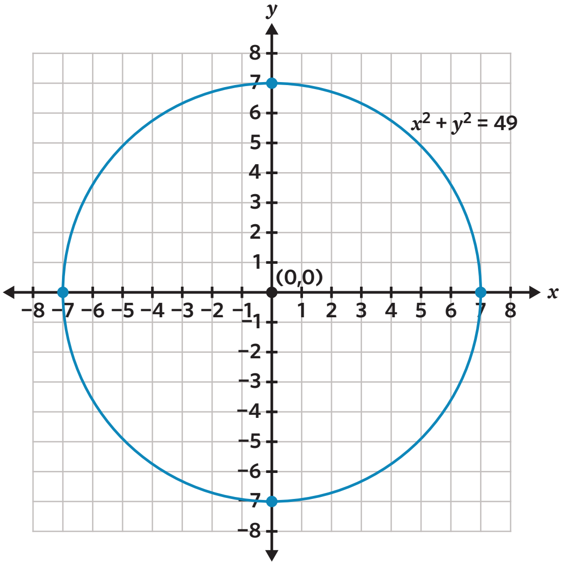
b.



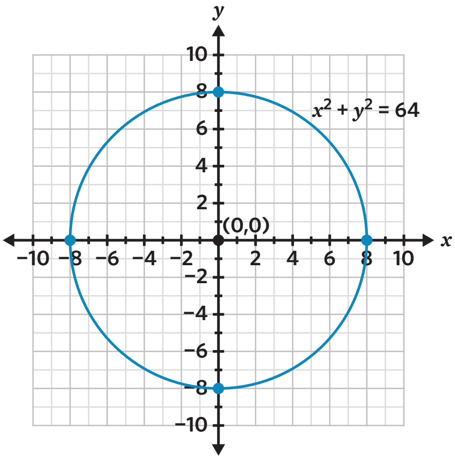
c.



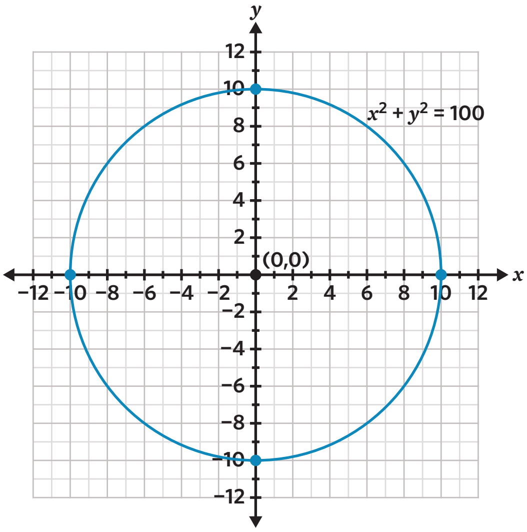
d.



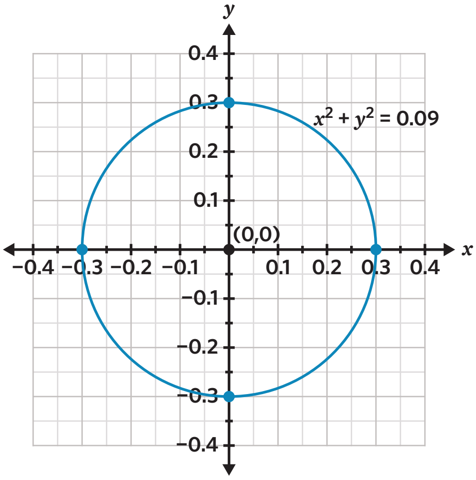
e.



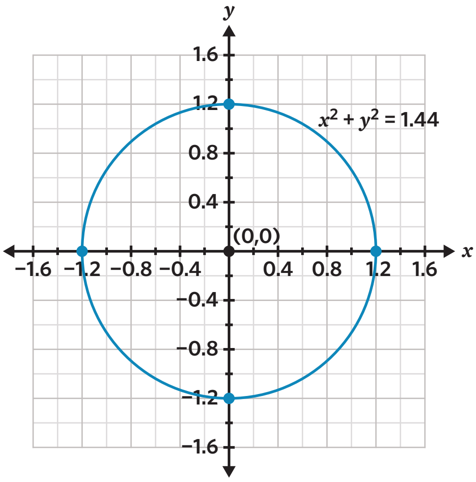
f.



g.

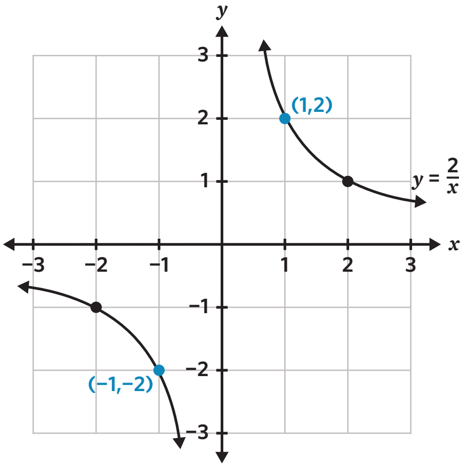


h.



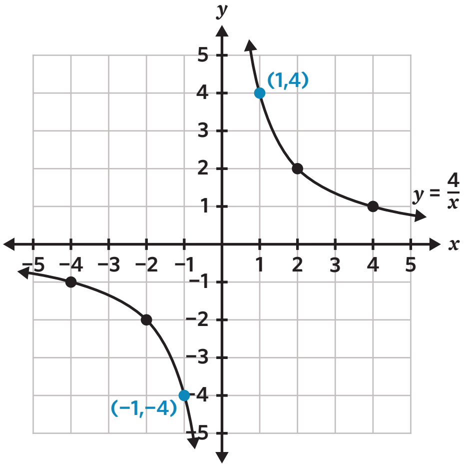
5. **a.**

x	$y = \frac{2}{x}$	(x,y)
−2	−1	(−2,−1)
−1	−2	(−1,−2)
0	Undefined	Asymptote
1	2	(1,2)
2	1	(2,1)



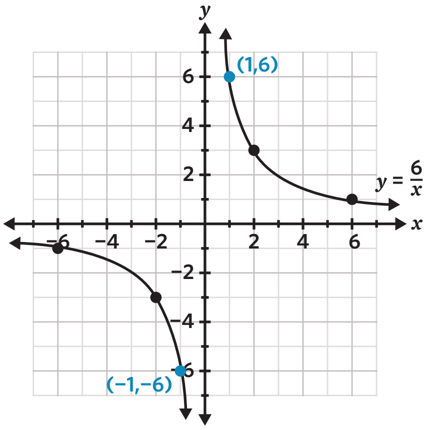
b.

x	$y = \frac{4}{x}$	(x,y)
−4	−1	(−4,−1)
−2	−2	(−2,−2)
−1	−4	(−1,−4)
0	Undefined	Asymptote
1	4	(1,4)
2	2	(2,2)
4	1	(4,1)



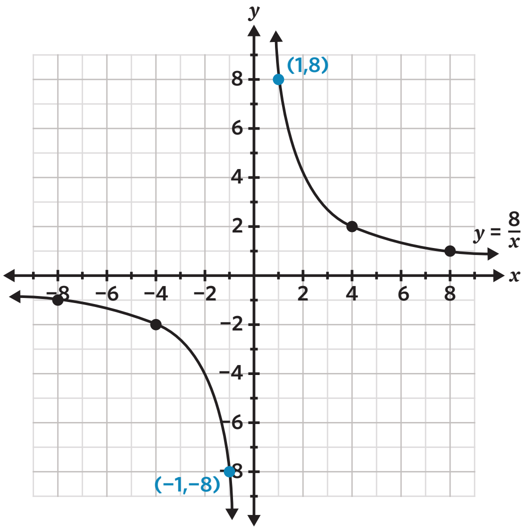
c.

x	$y = \frac{6}{x}$	(x,y)
−6	−1	(−6,−1)
−2	−3	(−2,−3)
−1	−6	(−1,−6)
0	Undefined	Asymptote
1	6	(1,6)
2	3	(2,3)
6	1	(6,1)



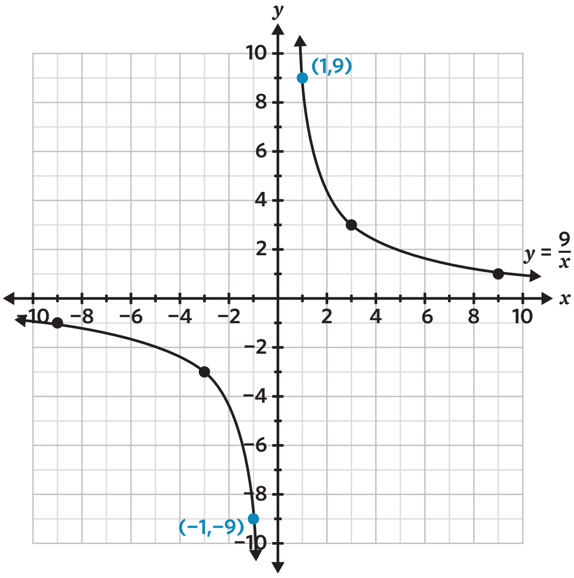
d.

x	$y = \frac{8}{x}$	(x,y)
−8	−1	(−8,−1)
−4	−2	(−4,−2)
−1	−8	(−1,−8)
0	Undefined	Asymptote
1	8	(1,8)
4	2	(4,2)
8	1	(8,1)



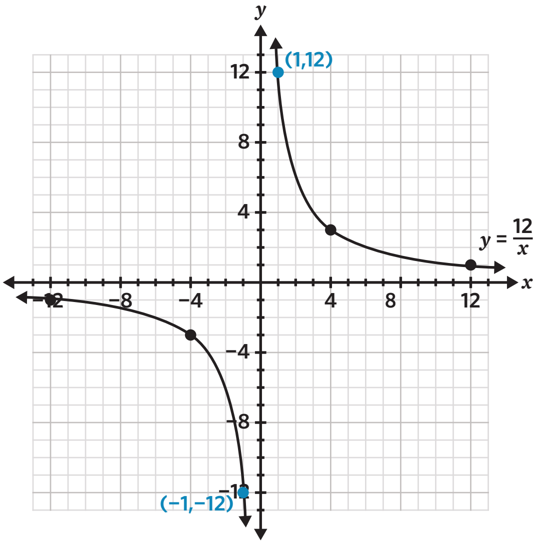
e.

x	$y = \frac{9}{x}$	(x,y)
−9	−1	(−9,−1)
−3	−3	(−3,−3)
−1	−9	(−1,−9)
0	Undefined	Asymptote
1	9	(1,9)
3	3	(3,3)
9	1	(9,1)



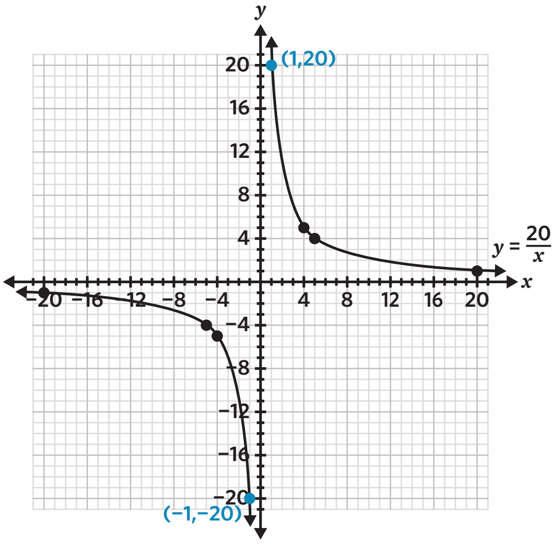
f.

x	$y = \frac{12}{x}$	(x,y)
−12	−1	(−12,−1)
−4	−3	(−4,−3)
−1	−12	(−1,−12)
0	Undefined	Asymptote
1	12	(1,12)
4	3	(4,3)
12	1	(12,1)



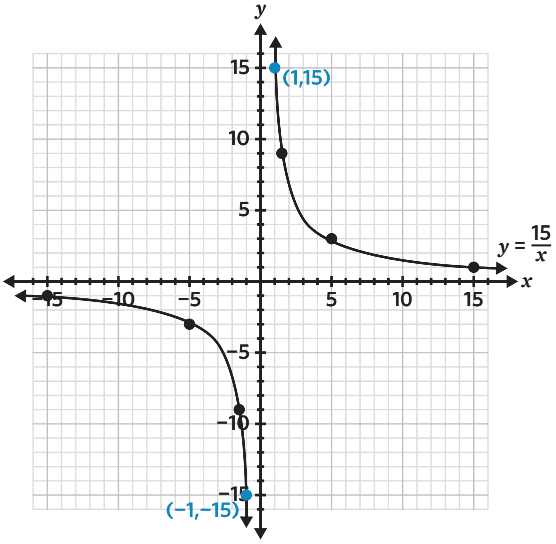
g.

x	$y = \frac{20}{x}$	(x,y)
−20	−1	(−20,−1)
−5	−4	(−5,−4)
−4	−5	(−4,−5)
−1	−20	(−1,−20)
0	Undefined	Asymptote
1	20	(1,20)
4	5	(4,5)
5	4	(5,4)
20	1	(20,1)



h.

x	$y = \frac{15}{x}$	(x,y)
−15	−1	(−15,−1)
−5	−3	(−5,−3)
$-\frac{5}{3}$	−9	$\left(-\frac{5}{3},-9\right)$
−1	−15	(−1,−15)
0	Undefined	Asymptote
1	15	(1,15)
$\frac{5}{3}$	9	$\left(\frac{5}{3},9\right)$
5	3	(5,3)
15	1	(15,1)



6. D

Spot the mistake

7. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

8. The shape of a roundabout in bird's eye view can be modelled by the equation $x^2 + y^2 = 16$. Determine the radius of the roundabout, in metres.

Key points

- Equation for the roundabout is $x^2 + y^2 = 16$.
- Determine the radius of the circle.

Explanation

A circle with a centre at (0,0) has the general equation $x^2 + y^2 = r^2$ where r is the radius.

Determine the length of the radius, r.

$$x^2 + y^2 = 16 = 4^2$$
$$4^2 = r^2$$
$$\therefore r = 4$$

Answer

The radius of the roundabout is 4 m.

9. At the outermost lane of a circular cycling track, the diameter of the track is equal to 100 m. Write an equation of the form $x^2 + y^2 = r^2$ modelling the complete circular path of a cyclist at the outermost lane of the track.

Key points

- Diameter of the circular track is 100 m.
- Determine the equation of the circle in the form $x^2 + y^2 = r^2$.

Explanation

Determine the length of the radius, r.

$$r = \frac{d}{2}$$
$$r = \frac{100}{2}$$
$$\therefore r = 50 \text{ m}$$

A circle with a centre at (0,0) has the general equation $x^2 + y^2 = r^2$ where r is the radius.

Substitute r into the equation.

$$x^2 + y^2 = 50^2$$
$$x^2 + y^2 = 2500$$

Answer

The equation modelling the circular path is $x^2 + y^2 = 2500$.

10. The shape of a bicycle wheel can be modelled by the equation $x^2 + y^2 = 1225$, where all sprockets extend directly from the centre of the wheel to a point on its rim. Determine the length of each sprocket, given that the units are centimetres.

Key points

- Bicycle wheel modelled by equation $x^2 + y^2 = 1225$.
- Sprockets extend from the centre to the rim.
- Determine the length of each sprocket.

Explanation

Recognise that the length of a sprocket is equal to the wheel's radius, r.

A circle with a centre at (0,0) has the general equation $x^2 + y^2 = r^2$ where r is the radius.

Determine the length of the radius, r.

$$x^2 + y^2 = 1225 = 35^2$$
$$35^2 = r^2$$
$$\therefore r = 35$$

Answer

The length of each sprocket is 35 cm.

11. A statue at the centre of a circular garden bed is surrounded by a path. The path's outer edge can be modelled by the equation $x^2 + y^2 = 36$, relative to the statue at the centre. Determine the width of the path if the shape of the circular garden bed can be modelled by the equation $x^2 + y^2 = 9$, relative to the statue at the centre.

Key points

- Outer edge of garden bed path modelled by $x^2 + y^2 = 36$.
- Garden bed modelled by $x^2 + y^2 = 9$.
- Determine the width of the path.

Explanation

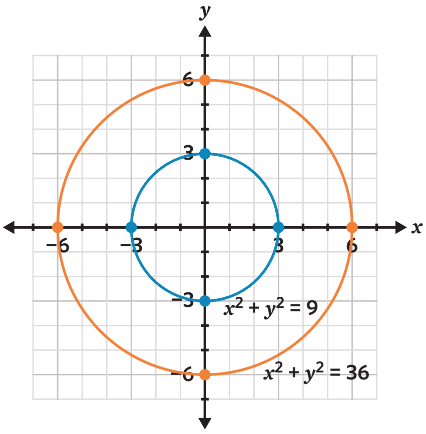
Determine the length of the radius, r, for the circular path.

$$x^2 + y^2 = 36 = 6^2$$
$$6^2 = r^2$$
$$\therefore r = 6$$

Determine the length of the radius, r, for the circular garden.

$$x^2 + y^2 = 9 = 3^2$$
$$3^2 = r^2$$
$$\therefore r = 3$$

Visually represent both equations.



The width of the path will be the difference between both radii.

Calculate the width.

$$6 - 3 = 3$$

Answer

The width of the path is 3 m.

12. A space station observed by astronomers from Earth follows a path modelled by $y = \frac{810\,000}{x}$ for $405 \le x \le 2000$ where x and y respectively represent the horizontal and vertical distance of the space station from the point of observation on Earth. Plot the hyperbolic path of the station using the given table of values and determine the coordinates of the space station when it is at its closest point to Earth.

x	405	900	1000	2000
$y = \frac{810\,000}{x}$				

Key points

- Space station modelled by equation $y = \frac{810\,000}{x}$.
- $405 \le x \le 2000$.
- x and y represent horizontal and vertical distance from Earth.
- Plot the hyperbolic path and determine the coordinates when it is closest to Earth.

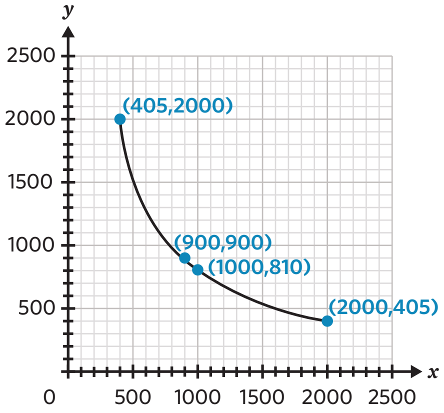
Explanation

Determine the y-coordinates of the space station using the provided table.

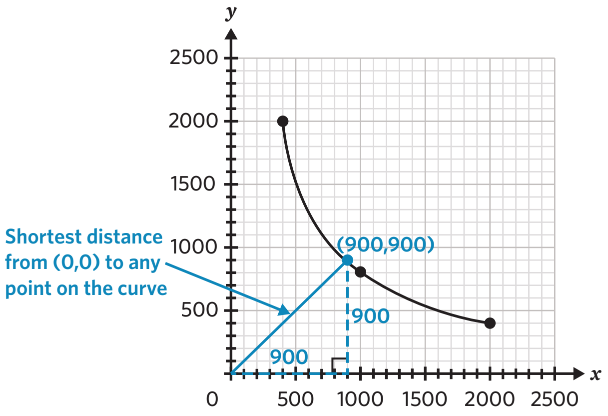
x	405	900	1000	2000
$y = \frac{810\,000}{x}$	2000	900	810	405

Back to contents

Plot the path of the space station on a Cartesian plane.



Use Pythagoras' theorem to determine the point closest to Earth (0,0).



$\sqrt{900^2 + 900^2} = 1272.79$

1272.79 units is the closest distance.

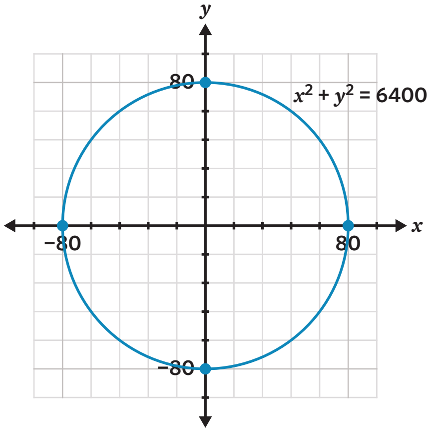
Answer

The coordinates of the closest point to Earth are (900,900).

Reasoning

13. a. An equation to represent The High Roller is $x^2 + y^2 = 80^2$.

b.



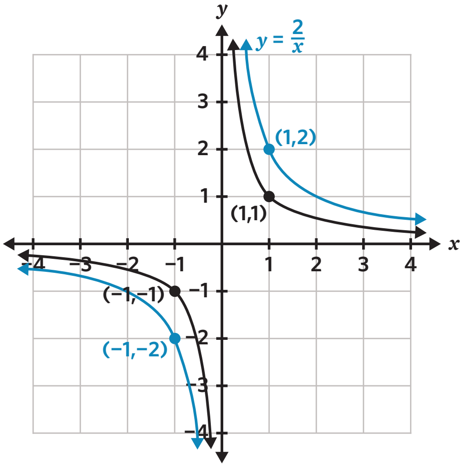
c. The height of a cabin that is 40 m from the centre horizontally in the upper half of the ferris wheel is approximately 157 m from the ground.

d. Suggested option 1: Sydney Opera House.

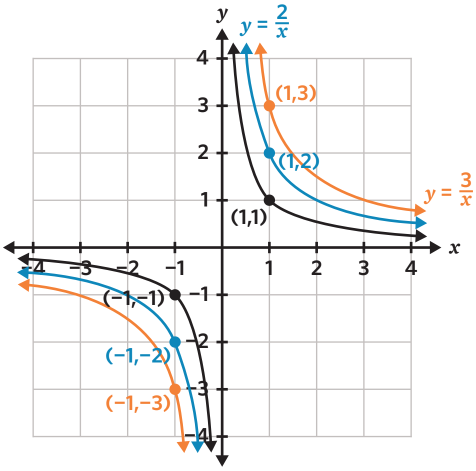
Suggested option 2: Hosier Lane.

Note: There are other possible options.

14. a.



b.



c. Increasing the value of a in $y = \frac{a}{x}$ dilates the hyperbola's branches further away from the asymptotes $x = 0$ and $y = 0$.

Exam-style

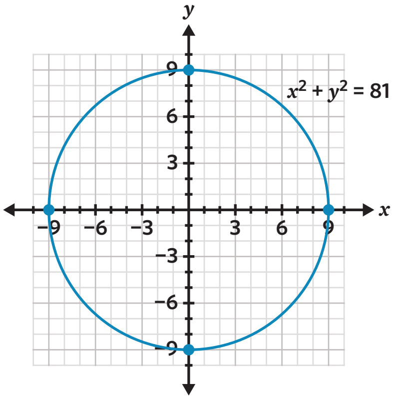
15. D

16. a. $a = 16$

b. $y = \frac{1}{4}$

17. $A(0,11)$, $B(11,0)$

18.



Remember this?

19. A **20.** D **21.** B

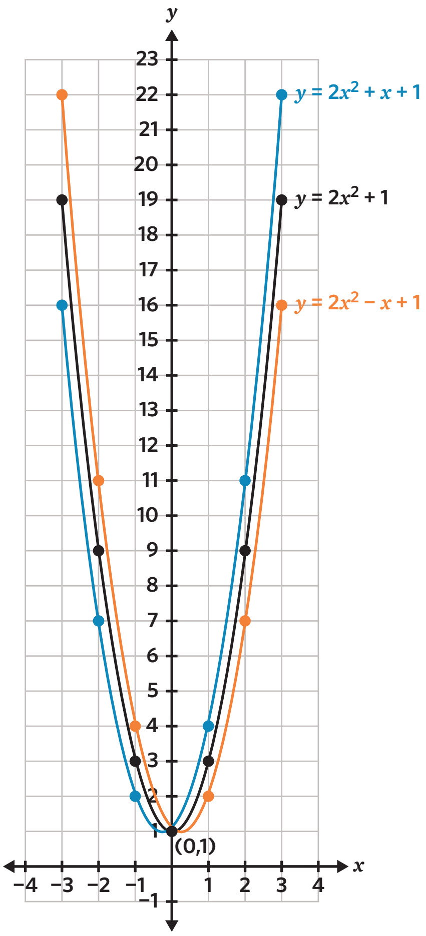
Back to contents

Chapter 6 extended application

1. **a.**

x	-3	-2	-1	0	1	2	3
$y = 2x^2 + 1$	19	9	3	1	3	9	19
$y = 2x^2 + x + 1$	16	7	2	1	4	11	22
$y = 2x^2 - x + 1$	22	11	4	1	2	7	16

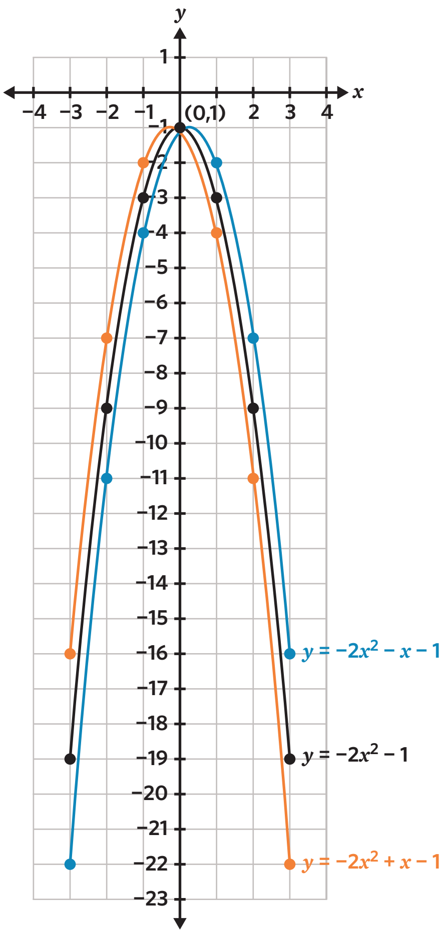
b.



c. A dilation by a factor of 2 from the y-axis and a vertical translation of 1 unit in the positive direction.

d. The new equations are $y = -2x^2 - 1$, $y = -2x^2 - x - 1$, and $y = -2x^2 + x - 1$.

e.



f. Suggested option 1: Parabolas have a balanced shape that looks pleasing and even.

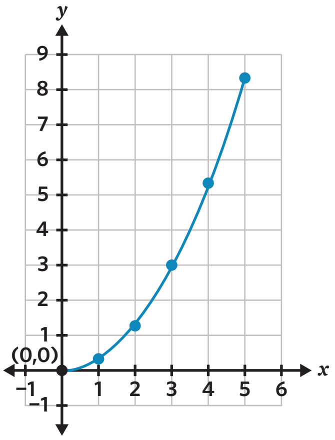
Suggested option 2: The smooth curve of a parabola helps connect different parts of a design smoothly.

Note: There are other possible options.

2. **a.**

x	0	1	2	3	4	5
$y = \frac{1}{3}x^2$	0	$\frac{1}{3}$	$\frac{4}{3}$	3	$\frac{16}{3}$	$\frac{25}{3}$

b.



c. 4.17 m

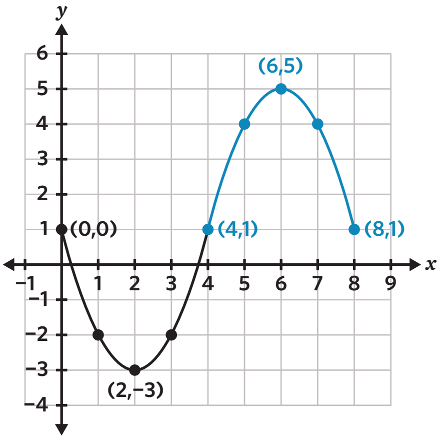
d. $y = -\frac{1}{100}x^2 + x$ is more suitable than $y = \frac{1}{100}x^2 + x$, as it consists of a maximum turning point which indicates that the ball rises and then begins to descend to ground level.

e. Suggested option 1: Playing golf lets you spend time outdoors and enjoy nature while playing a game.

Suggested option 2: Palying golf is a great way to hang out with friends and meet new people.

Note: There are other possible options.

3. a.



b. (10,–3)

c. Let $y = 141$ and solve for x.

$$141 = (x - 10)^2 - 3$$
$$144 = (x - 10)^2$$
$$\sqrt{144} = \sqrt{(x - 10)^2}$$
$$12 = (x - 10)$$
$$x = 22$$

When $y = 141$, $x = 22$. Given the x-axis represents the horizontal distance in tens of metres, this point is 220 m from the start of the section.

d. When $x = 2$, there is a turning point that is a minimum and when $x = 6$, there is a turning point that is a maximum. These points are important for safety as they model a smooth change in the vertical direction of the roller coaster.

e. Suggested option 1: Visitors should expect longer lines for rides and consider purchasing quick access tickets if they are available.

Suggested option 2: Visitors should expect the theme park to be crowded, so should designate a spot to meet if they lose their group.

Note: There are other possible options.

Chapter 6 review

Multiple choice

1. C **2.** B **3.** E **4.** C **5.** C

Fluency

6. a. Solution **b.** Solution

c. Not a solution **d.** Not a solution

7. a. $x = -2, x = 3$ **b.** $x = -1, x = \frac{4}{3}$

c. $x = -\frac{3}{2}, x = 5$ **d.** $x = -\frac{1}{4}, x = 2$

8. a. $x = -5, x = 1$ **b.** $x = 2, x = 3$

c. $x = 3, x = 5$ **d.** $x = -3, x = 3$

9. a. Axis of symmetry: $x = 0$

Minimum turning point: (0,0)

x-intercept: (0,0)

y-intercept: (0,0)

b. Axis of symmetry: $x = 0$

Minimum turning point: (0,5)

No x-intercepts

y-intercept: (0,5)

c. Axis of symmetry: $x = -2$

Minimum turning point: (−2,0)

x-intercept: (−2,0)

y-intercept: (0,4)

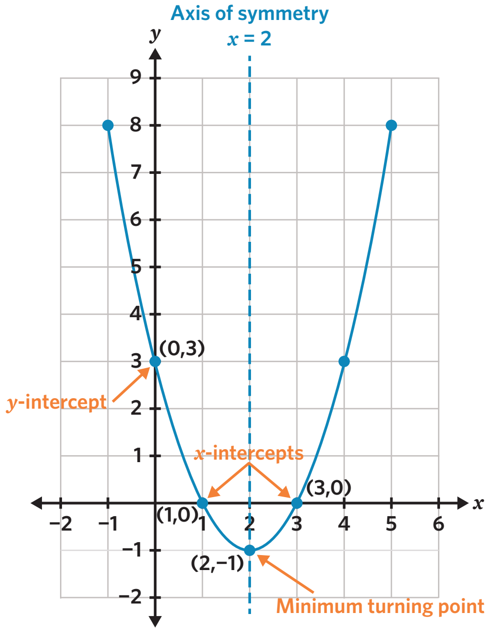
d. Axis of symmetry: $x = 3$

Maximum turning point: (3,4)

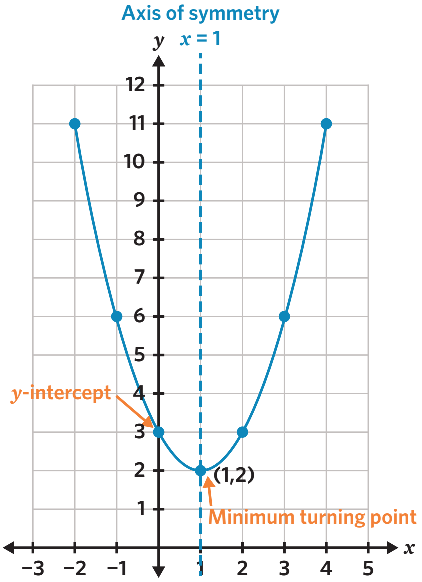
x-intercepts: (1,0), (5,0)

y-intercept: (0,–5)

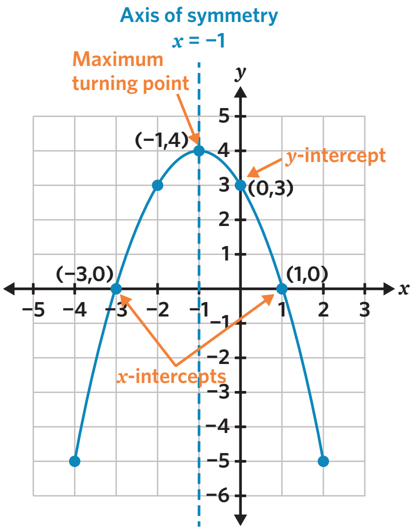
10. a.



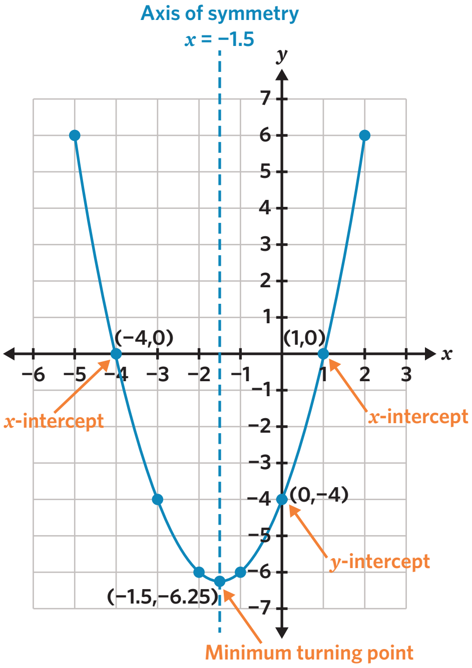
b.



c.



d.



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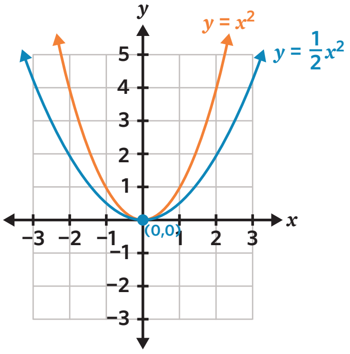
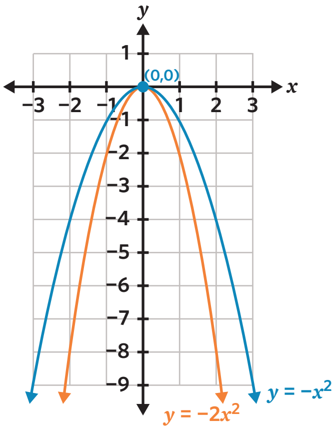
11. a.



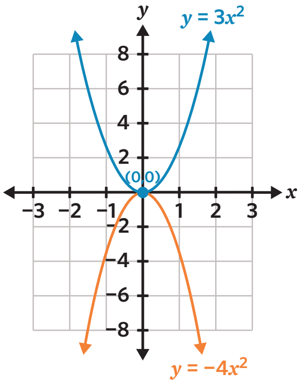
$y = \frac{1}{2}x^2$ is wider than $y = x^2$.

b.



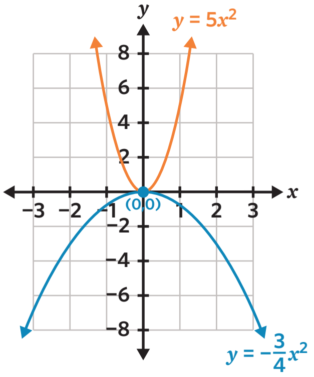
$y = -2x^2$ is narrower than $y = -x^2$.

c.



$y = -4x^2$ is inverted and is narrower than $y = 3x^2$.

d.



$y = -\frac{3}{4}x^2$ is inverted and is wider than $y = 5x^2$.

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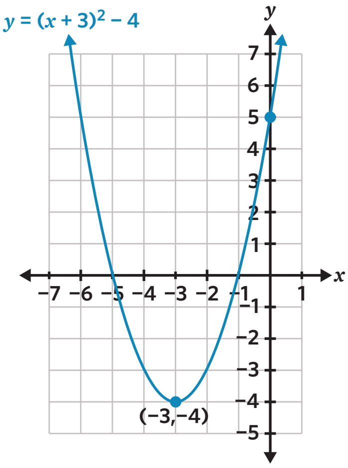
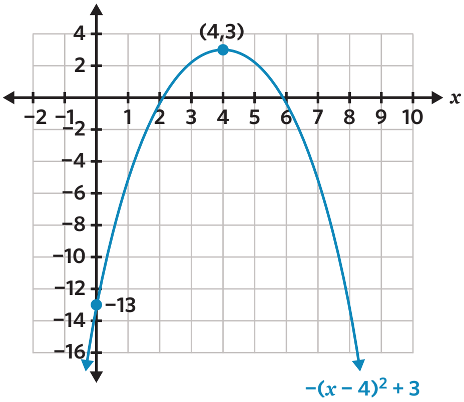
12. a.

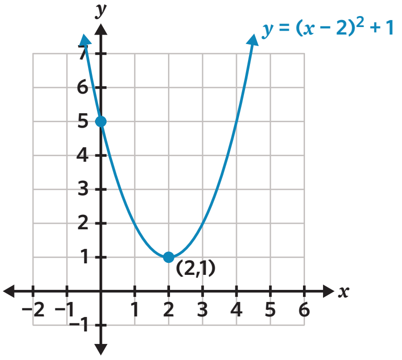


Back to contents

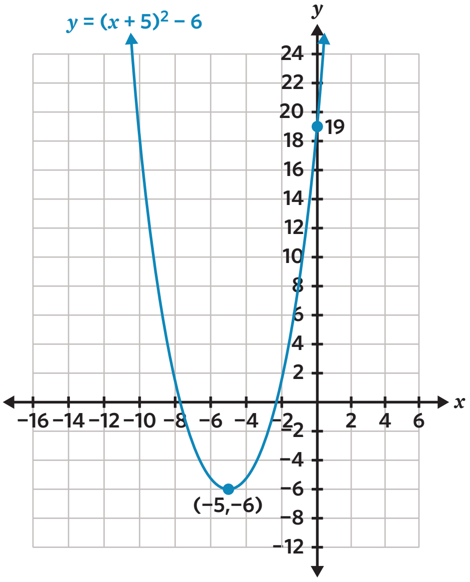
b.



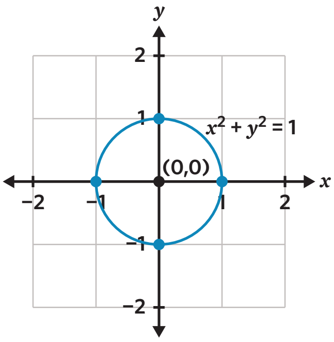
c.



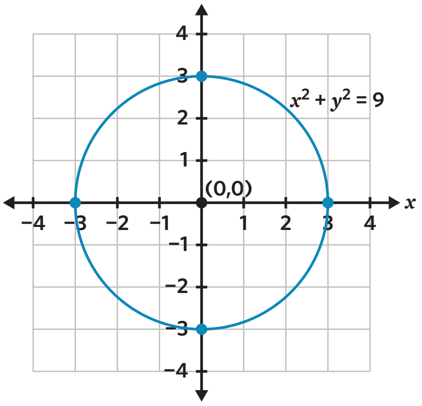
d.



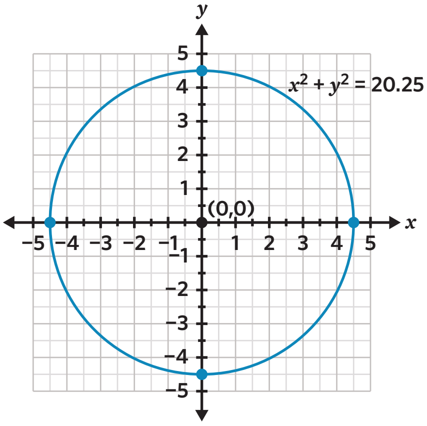
13. a.



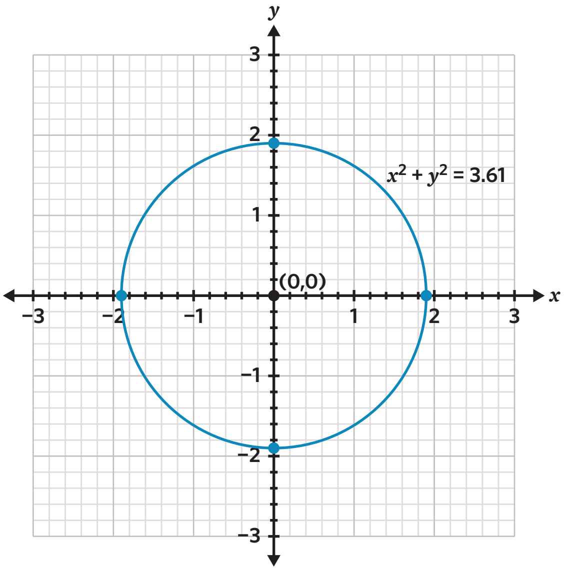
b.



c.



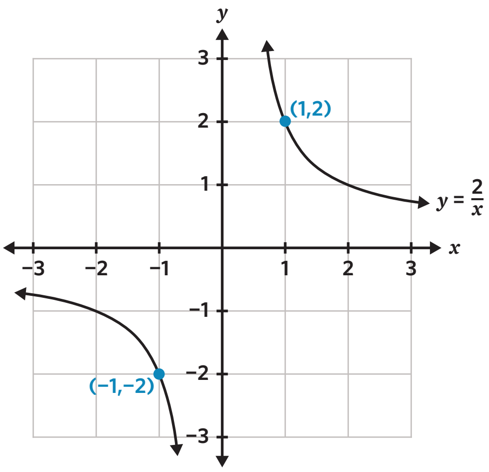
d.



14. a.

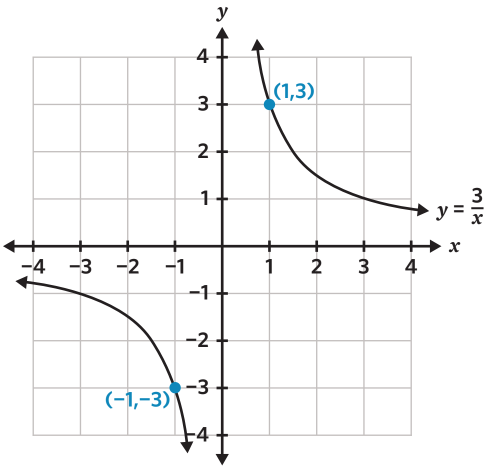
x	$y = \frac{2}{x}$	(x,y)
−2	−1	(−2,−1)
−1	−2	(−1,−2)
0	Undefined	Asymptote
1	2	(1,2)
2	1	(2,1)

Back to contents



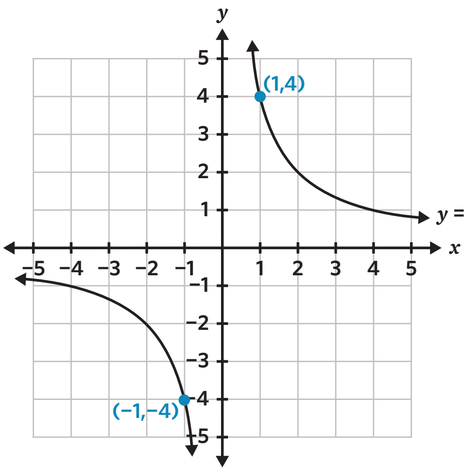
b.

x	$y = \frac{3}{x}$	(x,y)
−3	−1	(−3,−1)
−2	$-\frac{3}{2}$	$\left(-2,-\frac{3}{2}\right)$
−1	−3	(−1,−3)
0	Undefined	Asymptote
1	3	(1,3)
2	$\frac{3}{2}$	$\left(2,\frac{3}{2}\right)$
3	1	(3,1)



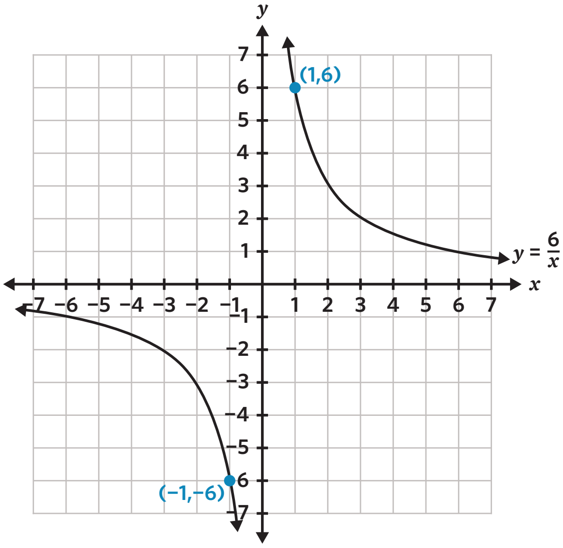
c.

x	$y = \frac{4}{x}$	(x,y)
−4	−1	(−4,−1)
−2	−2	(−2,−2)
−1	−4	(−1,−4)
0	Undefined	Asymptote
1	4	(1,4)
2	2	(2,2)
4	1	(4,1)



d.

x	$y = \frac{6}{x}$	(x,y)
−6	−1	(−6,−1)
−2	−3	(−2,−3)
−1	−6	(−1,−6)
0	Undefined	Asymptote
1	6	(1,6)
2	3	(2,3)
6	1	(6,1)



Problem solving

15. Sophie is designing a garden bridge that has a parabolic shape given by $y = -x(x - 2)$, where y is the height of the bridge in metres, and x is the horizontal distance from the start of the bridge, in metres. Assuming that the bridge starts at ground level, find the points where the bridge meets the ground.

Key points

- The bridge has a parabolic shape.
- The shape of the bridge is given by the equation $y = -x(x - 2)$.
- y is the height of the bridge in metres.
- x is the horizontal distance from the start of the bridge, in metres.
- Find the points where the bridge meets the ground.

Explanation

Solve for x when $y = 0$, so $-x(x - 2) = 0$.

Apply the Null Factor Law by making each factor equal to zero and solving for x.

$$x - 2 = 0 \quad \text{and} \quad -x = 0$$

$$x - 2 + 2 = 0 + 2 \qquad -x \times -1 = 0 \times -1$$

$$x = 2 \qquad x = 0$$

Answer

The bridge meets the ground 0 m and 2 m from the start of the path.

16. In the design of a new microscope, an engineer is working on the curvature of a lens that can be modelled by the equation $y = -x^2 + 3x$, where y is the height of the lens in millimetres, and x is the horizontal distance from the edge of the lens in millimetres. Factorise and solve the equation when $y = 2$ mm to determine the horizontal distance from the edge of the lens when the height is 2 mm.

Key points

- The curvature of the lens is modelled by the equation $y = -x^2 + 3x$.
- y is the height of the lens in millimetres.
- x is the horizontal distance from the edge of the lens in millimetres.
- Factorise and solve the equation when $y = 2$ mm.

Explanation

Substitute $y = 2$.

$2 = -x^2 + 3x$

Rearrange the equation to the form $m^2 + bm + c = 0$ and identify the values of b and c.

$2 - 2 = -x^2 + 3x - 2$

$0 = -x^2 + 3x - 2$

$x^2 - 3x + 2 = 0$

$b = -3$

$c = 2$

Determine a pair of factors of c that sum to b. Let this pair of factors be p and q.

$c = (-2) \times (-1) = 2$ ✓

$b = (-2) + (-1) = -3$ ✓

$\therefore p = -2$ and $q = -1$

Write the given expression in the form $(x + p)(x + q) = 0$ to factorise.

$-x^2 + 3x - 2 = (x - 2)(x - 1)$

Solve the equation by applying the Null Factor Law.

$$x - 2 = 0 \quad \text{and} \quad x - 1 = 0$$

$$x - 2 + 2 = 0 + 2 \qquad x - 1 + 1 = 0 + 1$$

$$x = 2 \qquad x = 1$$

Answer

The horizontal distance is 1 mm and 2 mm from the edge of the lens when the height is 2 mm.

17. A local park is designing a new water fountain. The height of the water stream, h, in metres, can be modelled by the equation $h = -x^2 + 4x$, where x is the time in seconds since the water was released. Plot the quadratic function for $0 \leq x \leq 4$ and determine the maximum height of the water stream.

Key points

- The height of the water stream over time is modelled by the equation $h = -x^2 + 4x$.
- h is the height of the water stream in metres.
- x is the time in seconds since the water was released.
- Plot the quadratic function for $0 \leq x \leq 4$ and determine the maximum height of the water stream.

Explanation

Substitute each of the required x-values in the given equation to determine the y-values.

$-(0)^2 + 4(0) = -0 + 0 = 0$

$-(1)^2 + 4(1) = -1 + 4 = 3$

$-(2)^2 + 4(2) = -4 + 8 = 4$

$-(3)^2 + 4(3) = -9 + 12 = 3$

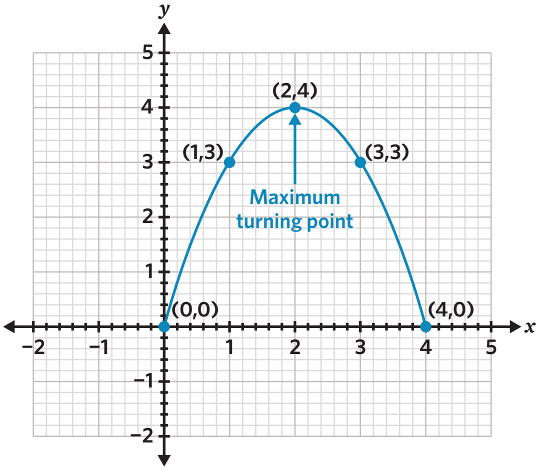
$-(4)^2 + 4(4) = -16 + 16 = 0$

Draw a table, including all the required x-values and enter the y-values. Determine the coordinates of each point.

x	0	1	2	3	4
y	0	3	4	3	0
(x,y)	(0,0)	(1,3)	(2,4)	(3,3)	(4,0)

Plot the coordinates (x,y) from the table on a Cartesian plane and join them with a smooth curve.

Identify the coordinates of the maximum turning point.



Maximum turning point (2,4).

Answer

The maximum height of the water stream is 4 m.

18. In a skatepark, there's a half-pipe ramp used for skateboarding. The shape of the cross-section of the half-pipe is given by the equation $y = \frac{1}{2}x^2$ where y is the height in metres and x is the horizontal distance in metres. The axis of symmetry splits the half-pipe into two identical halves. The skatepark is planning to build a new half-pipe with a similar shape. It can be described by an equation with a scaling factor that is a third of the original half-pipe. Determine an equation that could be used to describe the shape of the new half-pipe. Sketch the equations of both the existing and the new half-pipe on the same set of axes.

Back to contents

Key points

- The shape of the cross-section of the half-pipe is given by the equation $y = \frac{1}{2}x^2$.
- y is the height of the cross-section in metres.
- x is the horizontal distance of the cross-section in metres.
- The new half-pipe's equation's scaling factor is a third of the original.
- Determine an equation that could be used to describe the shape of the new half-pipe and sketch together with $y = \frac{1}{2}x^2$.

Explanation

For the equation $y = ax^2$, the parabola dilates away from the y-axis and is wider than the basic parabola $y = x^2$ when $0 < a < 1$.

The scaling factor of the parabola $y = \frac{1}{2}x^2$ is multiplied by a third. Adjust the value of a.

$$\begin{aligned}\text{New value of } a &= \text{old value of } a \times \frac{1}{3}\\ &= \frac{1}{2} \times \frac{1}{3}\\ &= \frac{1}{6}\end{aligned}$$

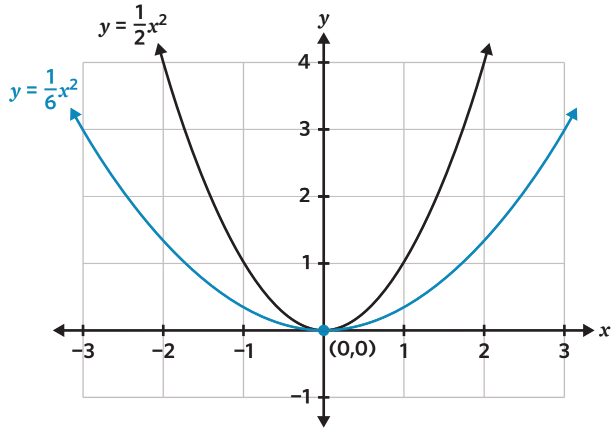
Substitute this value of a into the equation $y = ax^2$ to describe the shape of the new half-pipe cross-section.

$y = \frac{1}{6}x^2$

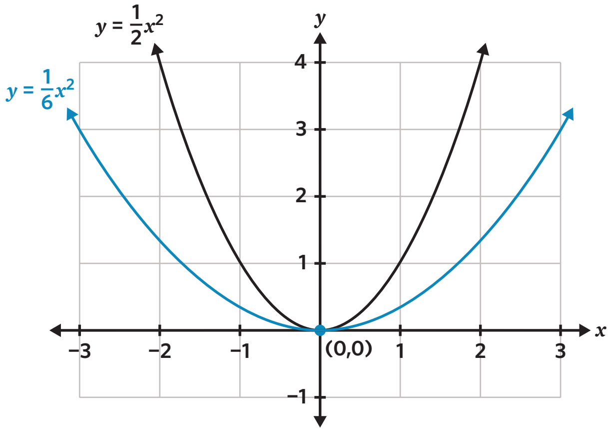
Complete a table of values for the two equations for $-2 \le x \le 2$.

x	-2	-1	0	1	2
$y = \frac{1}{6}x^2$	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{2}{3}$
$y = \frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

Sketch the parabolas for the existing half-pipe $\left(y = \frac{1}{2}x^2\right)$ and the new half-pipe $\left(y = \frac{1}{6}x^2\right)$ on the same set of axes. Plot the sets of coordinates given by the table of values. Join each set of coordinates with a smooth curve and add arrows to the ends. Mark the coordinates of the turning point.



Answer



19. In an amusement park, a section of a roller coaster track is being designed. The shape of the track can be modelled by the equation $y = \frac{1}{3}(x - 3)^2$ for $0 \le x \le 6$, where y is the height in metres and x is the horizontal distance in metres. Sketch the section of the roller coaster track and determine the distance at which the track reaches its lowest elevation.

Key points

- The shape of the track is modelled by the equation $y = \frac{1}{3}(x - 3)^2$.
- x is the horizontal distance in metres and is between 0 and 6, inclusive.
- y is the height in metres.
- Sketch the section of the roller coaster track and determine the height at which the track reaches its maximum elevation.

Explanation

In the equation of the form $y = a(x - h)^2 + k$, determine the values of h and k. This shows by how many units the graph of $y = ax^2$ must be translated horizontally and vertically.

For $y = \frac{1}{3}(x - 3)^2$

$h = 3$ and $k = 0$

$\therefore$ $y = x^2$ is translated 3 units right

Determine the coordinates of the turning point (h,k).

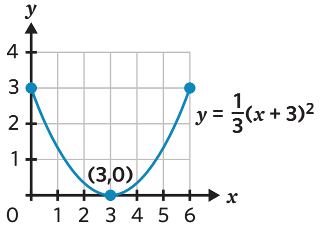
Turning point $(3,0)$

Substitute $x = 0$ into the equation of the parabola to determine the y-intercept.

$$\begin{aligned}y &= \frac{1}{3}(0 - 3)^2\\ &= \frac{1}{3}(-3)^2\\ &= \frac{1}{3} \times 9\\ &= 3\end{aligned}$$

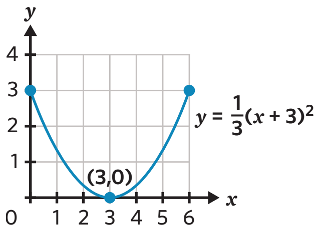
$\therefore$ y-intercept is 3

Sketch an upright parabola with an axis of symmetry at $x = h$, a turning point at (h,k) and a y-intercept at $(0,3)$.



Back to contents

Answer



The distance at which the roller coaster reaches its lowest elevation is 3 m.

20. In a wildlife reserve, the number of kangaroos expected to be observed by researchers is modelled by $y = \frac{5000}{x}$, where y represents the number of kangaroos and x is the distance in kilometres from the research centre for $1 \leq x \leq 5$. Sketch the graph and determine the number of kangaroos that will be observed when the research team is 4 km away from the research centre.

Key points

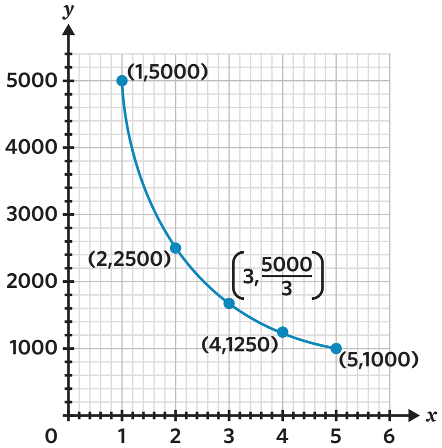
- The equation $y = \frac{5000}{x}$ models the number of kangaroos expected to be observed by researchers.
- y represents the number of kangaroos.
- x is the distance in kilometres from the research and is between 1 and 5, inclusive.
- Sketch the graph and determine the number of kangaroos 4 km away from the research centre.

Explanation

Complete the given table of values using the equation of the hyperbola and determine the coordinates of the points for $1 \leq x \leq 5$.

x	$y = \frac{5000}{x}$	(x,y)
1	5000	(1,5000)
2	2500	(2,2500)
3	$\frac{5000}{3}$	$\left(3,\frac{5000}{3}\right)$
4	1250	(4,1250)
5	1000	(5,1000)

Plot the coordinates (x,y) from the table on a Cartesian plane.

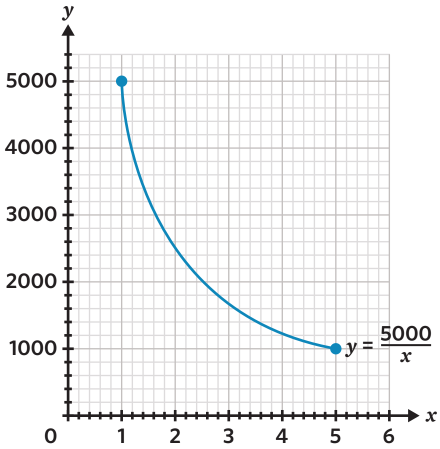


Calculate the number of kangaroos that will be observed 4 km away from the research centre.

Substitute $x = 4$ into the equation $y = \frac{5000}{x}$.

$y = \frac{5000}{(4)} = 1250$

Answer



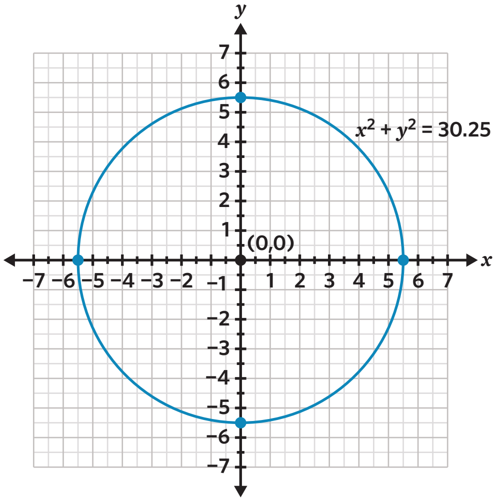
1250 kangaroos will be observed 4 km away from the research centre.

Reasoning

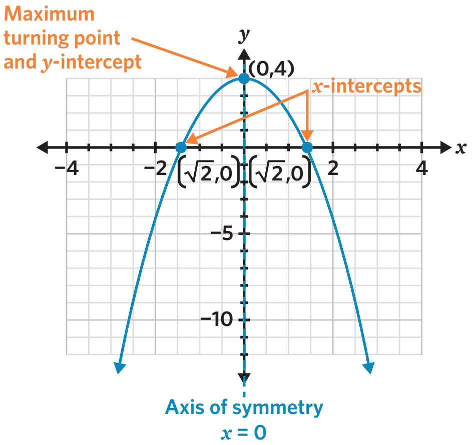
21. a. $y = -x^2 + 4x$

b. The slide intersects the ground when the horizontal distance is equal to 1 m.

c.



d.

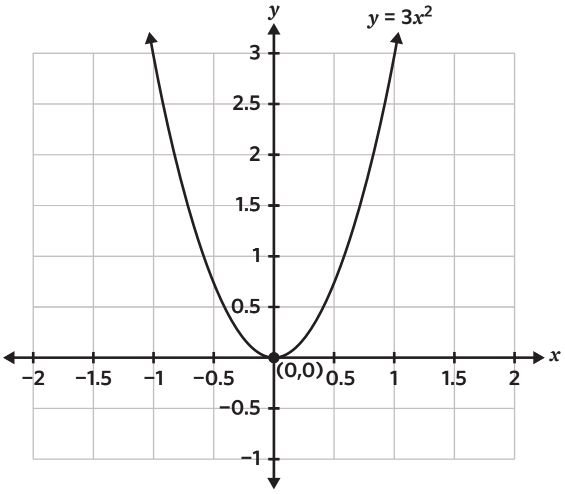


e. Suggested option 1: A flying fox that has a parabolic path.

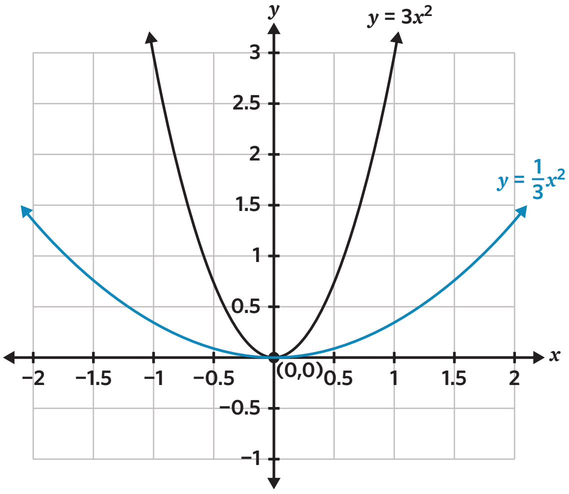
Suggestion option 2: A half-pipe that has a parabolic cross-section.

Note: There are other possible options.

22. a.



b.



c. Increasing the magnitude of a in $y = ax^2$ decreases the width of the parabola. Decreasing the magnitude of a increases the width of the parabola.

Back to contents

7A Length and perimeter

Student practice

Worked example 1

a. 329 mm **b.** 679.2 m

Worked example 2

a. 86.2 cm **b.** 140 mm

Worked example 3

a. 33 mm **b.** 6 cm

Understanding worksheet

1. **a.** ÷ **b.** 1000
c. 26 800 cm **d.** 1000

2. **a.** 12 **b.** 16 **c.** 14 **d.** 12

3. perimeter; measurement; length; convert.

Fluency

4. **a.** 3 m **b.** 153 mm
c. 7.85 km **d.** 1.275 m
e. 7500 mm **f.** 0.08459 km
g. 26.3 mm **h.** 8.56 km

5. **a.** 107 mm **b.** 14.8 cm
c. 10 km **d.** 14.5 cm
e. 15.4 m **f.** 18.6 cm
g. 244 mm **h.** 116.2 mm

6. **a.** 70 m **b.** 102 mm
c. 46 km **d.** 115.2 cm
e. 30 cm **f.** 50 cm
g. 180 m **h.** 158 mm

7. **a.** 30 mm **b.** 4.6 cm **c.** 2.7 km **d.** 61 cm
e. 40 mm **f.** 21.1 m **g.** 21.5 cm **h.** 20 cm

8. **a.** 23 cm **b.** 16 m
c. 17.95 m **d.** 18.65 km
e. 216 mm **f.** 190 mm
g. 3.954 km **h.** 1 545 000 cm

9. E

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. How many centimetres of purple ribbon does Jemma have if she has 4 different pieces that measure 21 cm, 1.7 m, 3520 mm and 98 cm.

Key points

- There are 4 different pieces that have the measurements of 21 cm, 1.7 m, 3520 mm and 98 cm.
- Determine the total length, in centimetres, of purple ribbon Jemma has.

Explanation

Convert all the measurements to be in cm as the unit of measurement.

1.7 m = 170 cm

3520 mm = 352 cm

Add all the values together to get the total.

21 + 170 + 352 + 98 = 641 cm

Answer

Jemma has a total of 641 cm of purple ribbon.

12. The perimeter of Ralph's rectangular dining table is 2.6 m. Each of the long sides are 95 cm. What is the length of the short sides in metres?

Key points

- The perimeter of the rectangular dining table is 2.6 m.
- The long sides of the table are 95 cm.
- Determine the length of the short sides in metres.

Explanation

Convert 95 cm to m.

95 cm = 0.95 m

Subtract the two long sides from the perimeter.

2.6 − 0.95 − 0.95 = 0.7

Divide 0.7 by 2 to calculate the value of the short sides.

0.7 ÷ 2 = 0.35

Answer

The length of the short sides of the dining table is 0.35 m.

13. Cassandra charges her customers $21.50 for each kilometre they travel in her horse and carriage. How much will a customer pay for a 5600 metre journey?

Key points

- The charges are $21.50 for each kilometre travelled.
- Determine how much a customer pays for a 5600 metre journey.

Explanation

Convert 5600 m to km.

5600 m = 5.6 km

Multiply the price per kilometre by the length of the journey.

21.50 × 5.6 = $120.40

Answer

A customer will pay $120.40 for a 5600 metre journey.

14. Peter's Pool Yard Company builds a pool that is 15 m by 11 m. The pool comes with a standard decking that can be assembled around the edges of the pool. The width of the decking is 55 cm. In metres, what is the length of fencing if it is built around the outer side of the decking?

Key points

- The pool has measurements of 15 m by 11 m.
- The width of the decking is 55 cm that goes around the pool.
- In metres, determine what the length of fencing is if it is built around the outer side of the decking.

Back to contents

Explanation

Convert 55 cm to m.

55 cm = 0.55 m

Add this value to the pool measurements.

$15 + 0.55 \times 2 = 16.1$

$11 + 0.55 \times 2 = 12.1$

The length of the fencing is the perimeter around the decking.

To determine the length of the fencing, add all of the sides together.

$(16.1 \times 2) + (12.1 \times 2) = 56.4$ m

Answer

The length of the fencing will be 56.4 m.

15. Phil made a mosaic around a 2.25 m perimeter of a kite shaped mirror. Two of the equal sides sum to 128 cm. Using a to represent one of the unknown sides, write an equation to represent the situation and calculate the missing side lengths in metres.

Key points

- The perimeter of the mirror is 2.25 m and it is in the shape of the kite.
- Two of the equal sides sum to 128 cm.
- Using a to represent one of the unknown sides.
- Write an equation to represent the situation and then calculate the missing side lengths in metres.

Explanation

Convert 128 cm to m.

128 cm = 1.28 m

Write an equation to represent the situation.

$2.25 = 2a + 1.28$

Solve for a.

$2.25 - 1.28 = 2a + 1.28 - 1.28$

$0.97 \div 2 = 2a \div 2$

$a = 0.485$ m

Answer

The equation to represent the situation is $2.25 = 2a + 1.28$ and the missing side lengths are equal and are 0.485 m.

Reasoning

16. **a.** The length of the baseline is 1097 cm.

b. The length of the entire side of the court from baseline to baseline is 23.76 m.

c. The value of b is 910 mm.

d. She will need 3656 ml of paint to paint all the lines of the court.

e. Suggested option 1: The cost of building the court.

Suggested option 2: The time it will take to build the court.

Note: There are other possible options.

17. **a.** 7 cm **b.** 70 mm

c. The shapes in part **a** and part **b** have the same dimensions but have different orientations, as well as the side lengths being marked with different units, making them seem as they are different. The perimeter of both shapes is the same, when the final answers are converted to the same units. For example, if we were to convert 70 mm into cm, we get the value of 7 cm which is equal to the answer from part **a**.

Exam-style

18. D

19. **a.** 646 mm
b. 1564 mm

20. 0.68425 m

21. $2450

Remember this?

22. B

23. C

24. C

7B Circumference and perimeter of a sector

Student practice

Worked example 1

a. $C = 8\pi$ cm; $C = 21.13$ cm

b. $C = 12\pi$ mm; $C = 37.70$ mm

Worked example 2

a. $a = \frac{55}{\pi}$ cm; $a \approx 17.51$ cm

b. $x = \frac{123}{\pi}$ mm; $x \approx 39.15$ mm

Worked example 3

a. $P = \frac{10}{9}\pi + 10$ cm; $P \approx 13.49$ cm

b. $P = 10\pi + 30$ units; $P \approx 61.42$ units

Understanding worksheet

1. **a.** Diameter **b.** Radius

c. Arc length **d.** Sector

2. **a.** $\frac{60}{360} \times 2\pi \times 5 + 2 \times 5$

b. $\frac{118}{360} \times 2\pi \times 12 + 2 \times 12$

c. $\frac{133}{360} \times 2\pi \times 9 + 2 \times 9$

d. $\frac{260}{360} \times 2\pi \times 13 + 2 \times 13$

3. circumference; radius; sector; arc

Fluency

4. **a.** $C = 9\pi$ cm; $C \approx 28.27$ cm

b. $C = 12\pi$ cm; $C \approx 37.70$ cm

c. $C = 9\pi$ mm; $C \approx 28.27$ mm

d. $C = 116\pi$ mm; $C \approx 364.42$ mm

e. $C = 7.5\pi$ km; $C \approx 23.56$ km

f. $C = 36\pi$ cm; $C \approx 113.10$ cm

g. $C = 5.3\pi$ km; $C \approx 16.65$ km

h. $C = 144\pi$ mm; $C \approx 452.39$ mm

5. **a.** $a = \frac{34}{\pi}$ cm; $a \approx 10.82$ cm

b. $x = \frac{30}{\pi}$ mm; $x \approx 9.55$ mm

c. $b = \frac{121}{\pi}$ km; $b \approx 38.52$ km

d. $x = \frac{78.5}{2\pi}$ cm; $x \approx 12.49$ cm

e. $j = \frac{88}{\pi}$ mm; $j \approx 28.01$ mm

f. $y = \frac{173}{2\pi}$ cm; $y \approx 27.53$ cm

g. $z = \frac{237.6}{2\pi}$ m; $z \approx 37.82$ m

h. $k = \frac{1696.4}{\pi}$ mm; $k \approx 539.98$ mm

---

6. **a.** $P = \frac{40}{9}\pi + 20$ m; $P \approx 33.96$ m

b. $P = \frac{8}{3}\pi + 8$ cm; $P \approx 16.38$ cm

c. $P = 4\pi + 16$ units; $P \approx 28.57$ units

d. $P = \frac{5}{3}\pi + 10$ units; $P \approx 15.24$ units

e. $P = 12\pi + 24$ cm; $P \approx 61.70$ cm

f. $P = 240\pi + 320$ mm; $P \approx 1073.98$ mm

g. $P = \frac{372}{5}\pi + 186$ mm; $P \approx 419.73$ mm

h. $P = \frac{85}{8}\pi + 17$ cm; $P \approx 50.38$ cm

---

7. D

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. A circular playpen for toddlers has a radius of 5.8 m. What is the circumference of the playpen in metres? Round your answer to two decimal places.

Key points

- A circular playpen has a radius of 5.8 m.
- Calculate the circumference of the playpen in metres, rounded to two decimal places.

Explanation

Apply the formula and express the circumference in terms of π.

$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 5.8 \\ &= 11.6\pi \end{aligned}$$

Calculate the circumference and round the value to two decimal places.

$C \approx 36.44$ m

Answer

The circumference of the playpen, to two decimal places, is approximately 36.44 m.

10. The diameter of a bike wheel is 66 cm. How many rotations does the wheel need to complete to travel 5 km?

Key points

- The diameter of a bike wheel is 66 cm.
- The wheel needs to travel 5 km.
- How many rotations does the wheel need?

Explanation

The distance per rotation is the circumference of the bike wheel.

$$\begin{aligned} \text{distance per rotation} &= \pi d \\ &= 66\pi \end{aligned}$$

Convert the required distance into centimetres.

5 km = 5000 m = 500 000 cm

The required number of rotations can be calculated as

$$\begin{aligned} \text{Number of rotations} &= \text{required distance} \div \text{distance per rotation} \\ &= 500\,000 \div 66\pi \\ &= 2411.438... \\ &\approx 2412 \text{ rotations} \end{aligned}$$

This is rounded up because we require a full rotation, and rounding down to 2411 rotations means that the wheel would not have travelled 5 km.

Answer

The wheel needs to make 2412 rotations to travel 5 km.

11. Joe's Pizza sells pizza by the slice. Joe makes large circular pizzas with a diameter of 18 cm and cuts them into 6 slices. What is the perimeter of one slice in centimetres? Round your answer to two decimal places.

Key points

- Joe makes large circular pizzas and sells them by the slice.
- The diameter of the circular pizzas is 18 cm.
- The large circular pizzas are cut into 6 slices.
- Calculate the perimeter of one pizza slice, rounded to two decimal places.

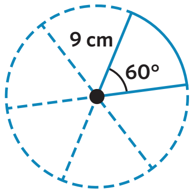
Explanation

Calculate the internal angle of each pizza slice.

$$\begin{aligned} \theta &= \frac{360}{6} \\ &= 60^\circ \end{aligned}$$

Calculate the radius of the circle.

$$\begin{aligned} r &= \frac{18}{2} \\ &= 9 \text{ cm} \end{aligned}$$



Each pizza slice is a sector with an angle of 60° and a radius of 9 cm.

Calculate the perimeter of the sector.

$$\begin{aligned} P &= \frac{60}{360} \times 2\pi \times 9 + 2 \times 9 \\ &= 3\pi + 18 \\ &\approx 27.42 \text{ cm} \end{aligned}$$

Answer

The perimeter of one pizza slice, to two decimal places, is approximately 27.42 cm.

12. The large hand on a clock measures 16 cm and reaches from the centre of a clock to its outer edge. How many centimetres, rounded to two decimal places, will the tip traverse in 25 minutes?

Key points

- The radius of the circle made by the large hand on a clock is 16 cm.
- Calculate the distance, in centimetres, the tip will traverse in 25 minutes, rounded to two decimal places.

Back to contents

Explanation

The large hand moves a full rotation in an hour (60 minutes).

In 25 minutes, the tip will traverse $\frac{25}{60}$ of the circle made by the large hand.

Calculate the arc length.

$$L = \frac{25}{60} \times 2\pi \times 16$$
$$\approx 41.89 \text{ cm}$$

Answer

The tip of the large hand on the clock will traverse approximately 41.89 cm in 25 minutes.

13. A roundabout in the centre of Paris has a diameter of 187 m. One sector of the roundabout is lined with hedges to create a dog play area. Two of the hedges form an angle of 129° at the centre of the roundabout. What is the perimeter of the dog play area rounded to two decimal places?

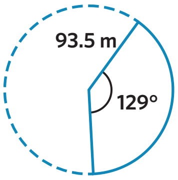
Key points

- The diameter of a roundabout is 187 m.
- Hedges are used to create a dog play area.
- An angle of 129° is formed at the centre of the roundabout.
- What is the perimeter of the dog play area, rounded to two decimal places?

Explanation

Calculate the radius of the circle.

$$r = \frac{187}{2}$$
$$= 93.5 \text{ m}$$



The dog play area is a sector with an angle of 129° and a radius of 93.5 m.

Calculate the perimeter of the sector.

$$P = \frac{129}{360} \times 2\pi \times 93.5 + 2 \times 93.5$$
$$\approx 397.51 \text{ m}$$

Answer

The perimeter of the dog play area, to two decimal places, is approximately 397.51 m.

Reasoning

14. a. The internal angle of the orange sector is 90°.

b. The difference between the perimeter of the adjusted blue sector and the original is approximately 6.98 mm.

c. The perimeter of the logo in Figure 2 is approximately 55.95 cm.

d. Suggested option 1: Millie could propose incorporating more vibrant and attention-grabbing colours in the logo.

Suggested option 2: Millie could recommend tweaking the visual elements of the logo to make it more eye-catching.

Note: There are other possible options.

15. a. 59.78 cm

b. 597.78 mm

c. The answers in parts **a** and **b** are equal because they have the same dimensions, despite being represented using different units. The angle that is labelled in Shape 2 is not the internal angle of the sector, hence this must be put into consideration when applying the formula $P = \frac{\theta}{360} \times 2\pi r + 2r$.

Exam-style

16. C

17. a. $P = \frac{10}{3}\pi + 16$ cm

b. $P = \frac{95}{12}\pi + 6$ cm

18. $r \approx 9$ cm

19. $\theta \approx 41.2°$

Remember this?

20. C **21.** D **22.** C

7C Area

Student practice

Worked example 1

a. 3.8 cm^2 **b.** 890 000 m^2

Worked example 2

a. 11 cm^2 **b.** 36 m^2 **c.** 50 cm^2

Worked example 3

a. 20 m^2 **b.** 35 m^2

Worked example 4

a. 113.10 mm^2 **b.** 8.73 cm^2

Understanding worksheet

1. a. 16 **b.** 12 **c.** 8 **d.** 6

2. a. mm^2 **b.** m^2 **c.** ha **d.** km^2

3. area; sector; circle; radius

Fluency

4. a. 1200 mm^2 **b.** 0.3 m^2
c. 370 000 cm^2 **d.** 0.02 km^2
e. 0.845 m^2 **f.** 180 000 000 cm^2
g. 89 000 m^2 **h.** 0.00009205 km^2

5. a. 60 km^2 **b.** 54 cm^2
c. 1100 mm^2 **d.** 37.5 cm^2
e. 8.25 m^2 **f.** 69.6 km^2
g. 38.25 m^2 **h.** 2.835 cm^2

6. a. 56 m^2 **b.** 4200 mm^2
c. 21.875 cm^2 **d.** 70 m^2
e. 84.5 km^2 **f.** 27.47 cm^2
g. 50.6 km^2 **h.** 19.09 mm^2

Back to contents

7. **a.** 28.27 cm^2 **b.** 153.94 cm^2
c. 16.76 cm^2 **d.** 69.81 m^2
e. 63.62 mm^2 **f.** 63.62 cm^2
g. 180.96 km^2 **h.** 141.86 cm^2

---

8. B

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. Kim has combined three adjoining properties she owns. If the areas of the properties are 3250 m^2, 2.5 hectares and 5 km^2, how many hectares of land does Kim own altogether?

Key points

- Three adjoining properties are combined.
- The areas of the properties are 3250 m^2, 2.5 hectares and 5 km^2.
- How many hectares of land does Kim own altogether?

Explanation

Convert all units to hectares.

$3250\ \text{m}^2 = 3250 \div 100^2 = 0.325$ hectares

$5\ \text{km}^2 = 5 \times 10^2 = 500$ hectares

Total area $= 0.325 + 2.5 + 500 = 502.825$ hectares

Answer

Kim owns 502.825 hectares of land altogether.

11. Jim is putting down parquetry flooring in a rectangular room. What is the cost of the flooring if the room measures 7.5 m by 11 m and the cost of each square metre is $220.60?

Key points

- The room is rectangular.
- The room has a width of 7.5 m and a length of 11 m.
- The cost of each square metre is $220.60.
- What is the cost of the flooring?

Explanation

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width}\\ &= 11 \times 7.5\\ &= 82.5\ \text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Total cost} &= \text{area} \times \text{cost per square metre}\\ &= 82.5 \times 220.60\\ &= \$18\,199.50\end{aligned}$$

Answer

The cost of the flooring is $18 199.50.

12. A clown wears two different large circular buttons. One with a radius of 150 mm and another with a diameter of 16 cm. What is the difference in area between the two buttons? Give the answer in cm^2, correct to 2 decimal places.

Key points

- There are two different large circular buttons.
- One circle has a radius of 150 mm.
- The other circle has a diameter of 16 cm.
- What is the difference in area between the two buttons in cm^2, correct to 2 decimal places?

Explanation

Convert all radius to cm.

150 mm = 150 ÷ 10 = 15 cm

$$\begin{aligned}\text{Area of small circle} &= \pi r^2\\ &= \pi \times 15^2\\ &= 225\pi\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of big circle} &= \pi r^2\\ &= \pi \times \left(\frac{16}{2}\right)^2\\ &= 64\pi\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Difference} &= \text{area of big circle} - \text{area of small circle}\\ &= 225\pi - 64\pi\\ &= 161\pi\\ &\approx 505.80\ \text{cm}^2\end{aligned}$$

Answer

The difference in area between the two buttons is 505.80 cm^2.

13. Steven's glass shop is replacing four identical triangular windows in a church. Each window has a base length of 3.4 m and a height of 6.3 m. How many square metres of glass does Steven need to replace all of the windows?

Key points

- There are four identical triangular windows.
- Each window has a base length of 3.4 m and a height of 6.3 m.
- How many square metres of glass does Steven need to replace all of the windows?

Explanation

$$\begin{aligned}\text{Area of a triangle} &= \frac{bh}{2}\\ &= \frac{3.4 \times 6.3}{2}\\ &= 10.71\ \text{m}^2\end{aligned}$$

Area of four triangles $= 10.71 \times 4 = 42.84\ \text{m}^2$

Answer

Steven needs to have 42.84 square metres of glass to replace all of the windows.

14. A school has increased the diameter of their circular play area from 50 m to 62 m. What is the percentage increase in the play area?

Key points

- The diameter of their circular play area is increased from 50 m to 62 m.
- What is the percentage increase in the play area?

Explanation

$$\begin{aligned}\text{New area} &= \pi r^2\\ &= \pi\left(\frac{62}{2}\right)^2\\ &= \pi \times 31^2\\ &= 961\pi\ \text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Old area} &= \pi r^2\\ &= \pi\left(\frac{50}{2}\right)^2\\ &= \pi \times 25^2\\ &= 625\pi\ \text{m}^2\end{aligned}$$

Back to contents

$$\begin{aligned}\text{Percentage increase in area} &= \frac{\text{new area} - \text{old area}}{\text{old area}} \times 100\\ &= \frac{961\pi - 625\pi}{625\pi} \times 100\\ &= \frac{336\pi}{625\pi} \times 100\\ &= \frac{336}{625} \times 100\\ &= 53.76\%\end{aligned}$$

Answer

The percentage increase in the play area is 53.76%.

Reasoning

15. **a.** The area of Rishmi's original piece of paper is 500 cm^2 to 1 significant figure.

b. The area of the remaining shape is 144π cm^2.

c. The percentage decrease in the area of the paper after Rishmi cuts the white hole is 19.75%.

d. Suggested option 1: Rishmi can try jigsaw puzzles.

Suggested option 2: Rishmi can try colouring-in.

Note: There are other possible options.

16. **a.** 52.5 cm^2

b. 35 cm^2; 17.5 cm^2

c. The quadrilateral $ABCE$ is made up of the three triangles $\triangle BCD$, $\triangle ADE$ and $\triangle ABD$, therefore the areas of the three triangles sum to the area of the quadrilateral.

Exam-style

17. E

18. **a.** 12.6π mm

b. 3.087π cm^2

19. 37.5%

20. Length = 16 cm, Width = 8 cm

Remember this?

21. D **22.** C **23.** B

7D Composite shapes

Student practice

Worked example 1

a. Perimeter = 40 km, Area = 60 km^2

b. Perimeter ≈ 52.85 cm, Area ≈ 168.55 cm^2

Understanding worksheet

1. **a.** Parallelogram **b.** Triangle

c. Rhombus **d.** Sector

2. **a.** 9 **b.** 25 **c.** 7 **d.** 135°

3. composite; regular; area; perimeter

Fluency

4. **a.** Perimeter = 70 m, Area = 180 m^2

b. Perimeter ≈ 27.42 m, Area ≈ 50.14 m^2

c. Perimeter = 59 m, Area = 137 m^2

d. Perimeter = 56 mm, Area = 192 mm^2

e. Perimeter ≈ 19.42 m, Area ≈ 26.14 m^2

f. Perimeter ≈ 53.53 cm, Area ≈ 108.82 cm^2

g. Perimeter = 90 mm, Area = 324 mm^2

h. Perimeter ≈ 50.21 cm, Area ≈ 136.83 cm^2

5. **a.** 10 475 m^2 **b.** 167 m^2

c. 72 m^2 **d.** 907 mm^2

e. 136 cm^2 **f.** 25.75 cm^2

g. 216.03 cm^2 **h.** 102.54 mm^2

6. C

Spot the mistake

7. **a.** Student A is incorrect. **b.** Student B is incorrect.

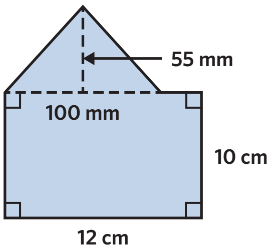
Problem solving

8. A rectangular tile with dimensions of 12 cm by 10 cm is glued together with a triangular tile with a base length of 100 mm and a height of 55 mm. Calculate the area of the adjoined tile in centimetres.

Key points

- The rectangular tile has dimensions of 12 cm by 10 cm.
- The glue triangular tile has a base length of 100 mm and a height of 55 mm.
- Calculate the area of the adjoined tile in centimetres.

Explanation



Before calculating the area, ensure all measurements are in the same unit. The triangular tile's dimensions are given in millimetres, so we need to convert them to centimetres.

1 cm = 10 mm

Base of triangle = 10 cm

Height of triangle = 5.5 cm

Calculate the area of the rectangle.

$$\begin{aligned}\text{Area of a rectangle} &= \text{length} \times \text{width}\\ &= 12 \times 10\\ &= 120\ \text{cm}^2\end{aligned}$$

Calculate the area of the triangle.

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 10 \times 5.5\\ &= 27.5\ \text{cm}^2\end{aligned}$$

Calculate the total of the shape.

Area of composite shape = rectangle area + triangle area

$= 120 + 27.5$

$= 147.5\text{ cm}^2$

Answer

The area of the adjoined tile is 147.5 cm^2.

9. A carpenter is making a wooden tabletop by joining a large rectangle with a rhombus on one side. The rectangle measures 1.2 m by 0.8 m. The length between opposite vertices of the rhombus are 0.9 m and 0.7 m. Calculate the total area of the tabletop, in square metres.

Key points

- A carpenter is making a tabletop made of a large rectangle and a rhombus.
- The rectangle measures 1.2 m by 0.8 m.
- The rhombus has diagonals measuring 0.9 m and 0.7 m.
- Calculate the total area of the tabletop in square metres.

Explanation

Calculate the area of the rectangle.

Area of a rectangle = length × width

$= 1.2 \times 0.8$

$= 0.96\text{ m}^2$

Calculate the area of the rhombus.

Area of a rhombus $= \frac{1}{2} \times$ diagonal 1 × diagonal 2

$= \frac{1}{2} \times 0.9 \times 0.7$

$= 0.315\text{ m}^2$

Calculate the total of the shape.

Area of composite shape = rectangle area + rhombus area

$= 0.96 + 0.315$

$= 1.275\text{ m}^2$

Answer

The total area of the tabletop is 1.275 m^2.

10. A school running track is made up of a rectangle that is 50 m long and 25 m wide. Adjoined to each of the 25 m sides is a semicircle with a diameter of 25 m. Calculate the perimeter, in exact form, of the running track, in metres.

Key points

- The rectangle that makes up part of the school's running track is 50 m long and 25 m wide.
- The 2 semicircles that make up part of the school's running track have a diameter of 25 m.
- Calculate the perimeter, in exact form, of the running track in metres.

Explanation

Calculate the perimeter of the rectangle.

Perimeter of a rectangle = 2 × length (since the semicircles are covering the other 2 sides)

$= 2 \times 50$

$= 100\text{ m}$

Calculate the perimeter of the 2 semicircles.

Perimeter of a semicircle $= \frac{1}{2} \times$ circumference (since the semicircles are half a circle)

$= \frac{1}{2} \times \pi \times 25$

$= \frac{25}{2}\pi\text{ m}$

Perimeter of 2 semicircles = 2 × perimeter of 1 semicircle

$= \frac{25}{2}\pi \times 2$

$= 25\pi\text{ m}$

Calculate the total of the shape.

Perimeter of composite shape = rectangle perimeter + semi circles perimeter

$= 100 + 25\pi\text{ m}$

Answer

The perimeter of the running track is $100 + 25\pi$ m.

11. Samantha is redecorating her room and uses an old rectangular rug and cuts a circular hole in its centre. The rectangular rug has dimensions of 2.5 m by 3 m, and the circle cut out has a radius of 150 cm. Calculate the area, in exact form, of the redecorated rug in square centimetres.

Key points

- Samantha is using a rectangular rug with a circular hole while redecorating her room.
- The rectangular rug has dimensions of 2.5 m by 3 m.
- The circle cut out has a radius of 150 cm.
- Calculate the area, in exact form, of the redecorated rug in square centimetres.

Explanation

Before calculating the area, ensure all measurements are in the same unit. The rectangle's dimensions are given in metres, so we need to convert them to centimetres.

1 m = 100 cm

Dimension = 250 cm by 300 cm

The given composite shape consists of a rectangle with a circular hole cut out in its centre. This means the area is the rectangle's area − the circle's area, since it was cut out.

Calculate the area of the rectangle.

Area of a rectangle = length × width

$= 250 \times 300$

$= 75\,000\text{ cm}^2$

Calculate the area of the circle.

Area of a circle $= \pi \times r^2$, where r is the radius.

$= \pi \times 150^2$

$= 22\,500\pi\text{ cm}^2$

Calculate the total of the shape.

Area of composite shape = rectangle area − circle area

$= 75\,000 - 22\,500\pi\text{ cm}^2$

Answer

The area of the redecorated rug is $75\,000 - 22\,500\pi\text{ cm}^2$.

12. A swimming pool has a rectangular main section that measures 12 m by 8 m. On one side, there is an adjoining large shallow pool that is two thirds of a full circle with a radius of 8 m. Calculate the perimeter, in exact form, and area of the entire pool.

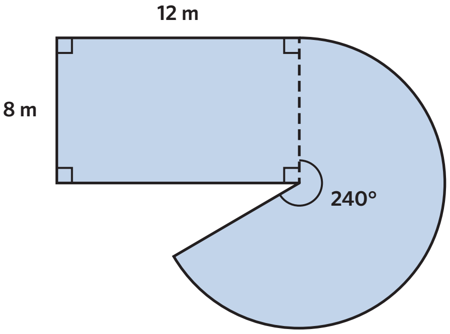
Key points

- The rectangular main section of the pool measures 12 m by 8 m.
- The adjoining large shallow pool is two thirds of a full circle with a radius of 8 m.
- Calculate the perimeter and area, in exact form, of the entire pool.

Back to contents

Explanation

The given composite shape consists of a rectangle and two thirds of a circle.



Calculate the perimeter of the rectangle (excluding the side with the circular pool).

$$\begin{aligned}\text{Perimeter of a rectangle} &= 2 \times \text{length} + \text{width}\\ &= 2 \times 12 + 8\\ &= 32 \text{ m}\end{aligned}$$

Calculate the perimeter of two thirds of the circle.

$$\begin{aligned}\text{Perimeter of a two thirds of a circle} &= \frac{2}{3} \times 2\pi \times r, \text{ where } r \text{ is the radius.}\\ &= \frac{2}{3} \times 2\pi \times 8\\ &= \frac{32\pi}{3} \text{ m}\end{aligned}$$

Calculate the total of the shape.

$$\begin{aligned}\text{Perimeter of composite shape} &= \text{rectangle perimeter} + \text{circular perimeter} + \text{radius}\\ &= 32 + \frac{32\pi}{3} + 8 \text{ m}\\ &= 40 + \frac{32\pi}{3} \text{ m}\end{aligned}$$

Calculate the area of the rectangle.

$$\begin{aligned}\text{Area of a rectangle} &= \text{length} \times \text{width}\\ &= 12 \times 8\\ &= 96 \text{ m}^2\end{aligned}$$

Calculate the area of two thirds of the circle.

$$\begin{aligned}\text{Area of two thirds of a circle} &= \frac{2}{3} \times \pi \times r^2, \text{ where } r \text{ is the radius.}\\ &= \frac{2}{3} \times \pi \times 8^2\\ &= \frac{128\pi}{3} \text{ m}^2\end{aligned}$$

Calculate the total of the shape.

$$\begin{aligned}\text{Area of composite shape} &= \text{rectangle area} + \text{circular area}\\ &= 96 + \frac{128\pi}{3} \text{ m}^2\end{aligned}$$

Answer

The perimeter of the pool is $40 + \frac{32\pi}{3}$ m and the area of the pool is $96 + \frac{128\pi}{3}$ m^2.

Reasoning

13. a. Anika will run approximately 372.79 m on her first lap.

b. Anika will run approximately 75.40 m further on her second lap if she runs on the outside of the track.

c. The track will require approximately 4925.84 m^2 of material to complete the entire track.

d. Suggested option 1: Penny Lane Secondary school can explore partnering with local businesses or community organisations for sponsorships or donations, which could offset some of the track's costs.

Suggested option 2: Involve the school's mathematics students in the track's design process. By turning it into a real-life project for the students, the school can tap into their problem-solving skills and potentially come up with cost-effective design alternatives while also fostering a sense of ownership and pride in the project.

Note: There are other possible options.

14. a. The calculated shaded area using the area formula for a triangle is 60 m^2.

b. The calculated shaded area using the area formula for a trapezium and rectangle is 60 m^2.

c. Part **a** only involves using the formula for a triangle whereas part **b** involved both the formula for a rectangle and trapezium. The answers are the same as a rectangle consists of a trapezium and a triangle or two triangles.

Exam-style

15. C

16. a. 24 mm

b. $12\sqrt{3}$ mm^2

17. $\frac{327}{100}$ m^2

18. 3 cm

Remember this?

19. C

20. B

21. C

7E Surface area of prisms and pyramids

Student practice

Worked example 1

a. 310 cm^2

b. 660 m^2

Worked example 2

161 cm^2

Understanding worksheet

1. a. Triangle

b. Square

c. Rectangle

d. Rectangle

2. a. 2

b. 8

c. 8

d. 4

3. net; faces; surface; prism

Fluency

4. a. 40 cm^2

b. 132 m^2

c. 118 mm^2

d. 246 cm^2

e. 430 cm^2

f. 5190 mm^2

g. 2.32 km^2

h. 458.6 m^2

Back to contents

5. **a.** 96 cm^2 **b.** 144 cm^2
c. 217 mm^2 **d.** 75 cm^2
e. 420 m^2 **f.** 98.55 m^2
g. 53.2 cm^2 **h.** 156 m^2

---

6. **a.** 230 mm^2 **b.** 548 m^2
c. 318 m^2 **d.** 208 cm^2
e. 696.8 mm^2 **f.** 4910 cm^2
g. 298.5 cm^2 **h.** 600 m^2

---

7. D

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. A large inflatable cube die has edge lengths of 15 cm. What is the total surface area of the die?

Key points

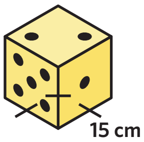
- A cube die has edge lengths of 15 cm.
- What is the total surface area of the die?

Explanation

Calculate the surface area of one face of the cube die.

$$\begin{aligned}\text{Area of one face} &= \text{side length} \times \text{side length}\\ &= 15 \times 15\\ &= 225\text{ cm}^2\end{aligned}$$

The total surface area is the sum of the area of the faces of a solid object. Given that the die is a cube, the total surface area is made up of 6 identical faces.



$$\begin{aligned}\text{Total surface area of a cube} &= 6 \times \text{area of one face}\\ &= 6 \times 225\\ &= 1350\text{ cm}^2\end{aligned}$$

Answer

The total surface area of the die is 1350 cm^2.

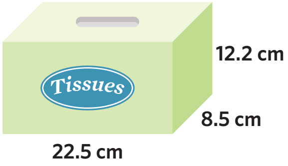
10. What is the total surface area of a standard rectangular tissue box with the dimensions 22.5 cm, 12.2 cm, and 8.5 cm?

Key points

- A tissue box has the dimensions 22.5 cm, 12.2 cm, and 8.5 cm.
- What is the total surface area of the tissue box?

Explanation

The tissue box is a rectangular prism. The diagram displays its dimensions.



Write the formula for the total surface area of a rectangular prism.

$$\begin{aligned}\text{Total surface area} &= 2 \times \text{base} + 2 \times \text{side} + 2 \times \text{end}\\ &= 2lw + 2lh + 2wh\end{aligned}$$

Substitute the side lengths into the formula to calculate the total surface area, with units.

Total surface area

$$\begin{aligned}&= 2(22.5 \times 8.5) + 2(22.5 \times 12.2) + 2(12.2 \times 8.5)\\ &= 382.5 + 549 + 207.4\\ &= 1138.9\text{ cm}^2\end{aligned}$$

Answer

The total surface area of the tissue box is 1138.9 cm^2.

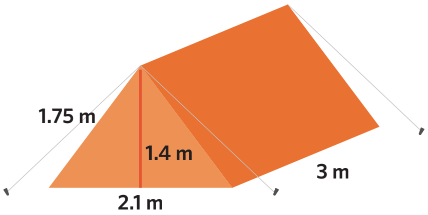
11. Abigail's camping tent has two triangular entries. Both entries have a base of 2.1 m, a height of 1.4 m, and slanted sides of 1.75 m. If the tent is 3 m long, what is its total surface area?

Key points

- Two triangular entries to Abigail's tent have a base of 2.1 m, a height of 1.4 m, and slanted sides of 1.75 m.
- The tent is 3 m long.
- What is the total surface area of the tent?

Explanation

Abigail's tent is a triangular prism. The diagram displays its dimensions.



Write the formula for the total surface area of a triangular prism.

$$\text{Total surface area} = 2 \times \text{end} + \text{side 2} + \text{side 1} + \text{side 3}$$

Substitute the side lengths into the formula to calculate the total surface area, with units.

$$\begin{aligned}\text{Total surface area} &= 2\left(\frac{2.1 \times 1.4}{2}\right) + (2.1 \times 3) + 2(1.75 \times 3)\\ &= 2.94 + 6.3 + 10.5\\ &= 19.74\text{ m}^2\end{aligned}$$

Answer

The total surface area of Abigail's tent is 19.74 m^2.

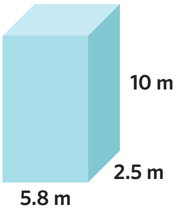
12. A rectangular water storage tank is 5.8 m long, 2.5 m wide, and 10 m deep. What is the total cost of the plastic for the tank if the plastic costs $5.80 per square metre?

Key points

- A rectangular tank is 5.8 m long, 2.5 m wide, and 10 m deep.
- The plastic for the tank costs $5.80 per square metre.
- What is the total cost of the plastic for the tank?

Explanation

The water storage tank is a rectangular prism. The diagram displays its dimensions.



Back to contents

Write the formula for the total surface area of a rectangular prism.

Total surface area $= 2 \times \text{base} + 2 \times \text{side} + 2 \times \text{end}$

$= 2lw + 2lh + 2wh$

Substitute the side lengths into the formula to calculate the total surface area, with units.

Total surface area $= 2(5.8 \times 2.5) + 2(2.5 \times 10) + 2(10 \times 5.8)$

$= 29 + 50 + 116$

$= 195\text{ m}^2$

The plastic for the tank costs $5.80 per square metre. Calculate the cost of the plastic for the tank.

Cost of plastic = total surface area of tank in squared metres × cost of plastic per square metre

$= 195 \times 5.8$

$= \$1131$

Answer

The cost of the plastic for the tank is $1131.

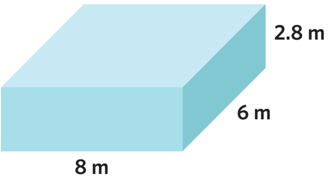
13. A green room is built for an immersive movie. The room is 8 m long, 6 m wide and 2.8 m tall. How much green material is required to only cover the walls of the room?

Key points

- A green room is 8 m long, 6 m wide and 2.8 m tall.
- How much green material is required to only cover the walls of the room?

Explanation

The green room is a rectangular prism. The diagram displays its dimensions.



The green material only covers the walls of the room. Therefore, do not include the surface area of the floor or ceiling when calculating the total surface area.

∴ Total surface area of green room walls $= 2 \times \text{side} + 2 \times \text{end}$

$= 2lh + 2wh$

Substitute the side lengths into the formula to calculate the total surface area of the green room walls, with units.

Total surface area $= 2(2.8 \times 8) + 2(2.8 \times 6)$

$= 44.8 + 33.6$

$= 78.4\text{ m}^2$

Answer

78.4 m^2 of green material is required to cover the walls of the green room.

Reasoning

14. **a.** 31.2 m^2 of hardened steel is required for the medium container.

b. The total surface area of the small container is 42.48 m^2.

c. The combined total surface area of a small and large container is 174.96 m^2.

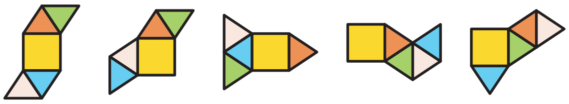
d. 23.4 m^2 more is required to construct a large flat rack container compared to a small flat rack container.

e. Suggested option 1: The weight of the goods to be placed in the shipping container.

Suggested option 2: How quickly the customer wants the goods delivered.

Note: There are other possible options.

15. **a.**



b. Both nets have an area of 160 cm^2.

c. The nets drawn in part **a** both have the same surface area, despite being arranged differently and therefore having different layouts. This demonstrates that the layout of the net does not impact the total surface area.

Exam-style

16. C

17. **a.** 700 cm^2

b. 6900 cm^2

18. 20 cm

19. 37 000 cm^2

Remember this?

20. D **21.** C **22.** A

7F Surface area of a cylinder

Student practice

Worked example 1

a. 138.23 cm^2 **b.** 245.04 m^2

Worked example 2

a. 44.27 cm^2 **b.** 356.50 cm^2

Understanding worksheet

1. **a.** 7 cm **b.** 2 **c.** π **d.** 2.5

2. **a.** 10 **b.** 2 **c.** 5 **d.** 6

3. net; face; circumference; cylinder

Fluency

4. **a.** 37.70 cm^2 **b.** 603.19 cm^2
c. 565.49 mm^2 **d.** 439.82 cm^2
e. 791.68 m^2 **f.** 3769.91 cm^2
g. 714.71 cm^2 **h.** 25.74 m^2

5. **a.** 402.12 m^2 **b.** 628.32 cm^2
c. 466.53 cm^2 **d.** 881.22 cm^2
e. 1306.90 cm^2 **f.** 70.32 m^2
g. 2342.06 cm^2 **h.** 1950.55 m^2

6. **a.** 64.84 cm^2 **b.** 94.83 cm^2
c. 83.90 cm^2 **d.** 182.52 cm^2
e. 228.80 cm^2 **f.** 168.07 m^2
g. 3599.04 mm^2 **h.** 136.45 m^2

Back to contents

7. **a.** 196.53 cm^2 **b.** 121.26 cm^2
c. 63.42 cm^2 **d.** 125.12 cm^2
e. 529.03 cm^2 **f.** 414.20 cm^2
g. 80.25 cm^2 **h.** 44.94 km^2

---

8. D

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. A standard kitchen roller has a radius of 40 mm and a length of 360 mm. How many square centimetres can a chef roll with one full roll, correct to two decimal places?

Key points

- Standard kitchen roller has a radius of 40 mm.
- Standard kitchen roller has a length of 360 mm.
- Determine how many square centimetres can a can be rolled in one full roll. Correct to two decimal places.

Explanation

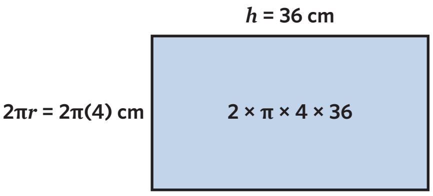
Identify the side lengths of the cylinder.

$h = 360 \text{ mm}, r = 40 \text{ mm}$

Convert values to centimetres.

$h = 36 \text{ cm}, r = 4 \text{ cm}$

Write the formula for the surface area of the curved rectangular side and substitute the side lengths to calculate the surface area, with units.



$$\begin{aligned} \text{SA} &= 2\pi rh \\ &= 2\pi(4)(36) \\ &= 288\pi \\ &\approx 904.78 \text{ cm}^2 \end{aligned}$$

Answer

The chef can roll approximately 904.78 cm^2 with one full roll.

11. Multiple 10 cent pieces are wrapped in cardboard and form a cylinder that is 200 mm tall. A 10 cent piece has a diameter of 23.6 mm. Assuming that none of the cardboard overlaps, how many square millimetres of cardboard are used for the roll, correct to two decimal places?

Key points

- Cylinder formed out of 10 cent pieces.
- The cylinder is 200 mm tall.
- The 10 cent piece has a diameter of 23.6 mm.
- Determine the surface area of the cylinder. Correct to two decimal places.

Explanation

Identify the height and the radius of the cylinder.

$h = 200 \text{ mm}, r = \frac{23.6}{2} = 11.8 \text{ mm}$

Write the formula for the total surface area of a cylinder and substitute the side lengths to calculate the total surface area, with units.

$$\begin{aligned} \text{TSA} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(11.8)^2 + 2\pi(11.8)(200) \\ &= 4998.48\pi \\ &\approx 15\,703.19 \text{ mm}^2 \end{aligned}$$

Answer

Approximately 15 703.19 mm^2 of cardboard is used for the roll.

12. The concrete road roller is a cylindrical shape and has a radius of 0.7 m and is 160 cm in length. How many square metres of the road does it level if it travels straight and rotates 30 times, correct to two decimal places?

Key points

- The cylinder has a radius of 0.7 m.
- The cylinder has a length of 160 cm.
- Determine how much surface area the cylinder covered if it rotated 30 times. Correct to two decimal places.

Explanation

Identify the side lengths of the cylinder.

$h = 160 \text{ cm}, r = 0.7 \text{ m}$

Convert values to the required unit of measure : metres.

$h = 1.6 \text{ m}, r = 0.7 \text{ m}$

Write the formula for the surface area of the curved rectangular side and substitute the side lengths to calculate the surface area, with units.

$$\begin{aligned} \text{SA} &= 2\pi rh \\ &= 2\pi(0.7)(1.6) \\ &= 2.24\pi \text{ m}^2 \end{aligned}$$

Multiply by 30.

$2.24\pi \times 30 \approx 211.12 \text{ m}^2$

Answer

The concrete road roller levels approximately 211.12 m^2 of road.

13. Kamal charges \$0.01 per square centimetre that he has to paint. How much does Kamal charge a customer for painting the external surfaces of 15 cylinder containers, each with a radius of 60 cm and a height of 1 m, rounded to the nearest cent?

Key points

- Kamal charges \$0.01 per square centimetre that he paints.
- Each cylinder has a radius of 60 cm.
- Each cylinder has a height of 1 m.
- Calculate the charge for painting the external surfaces of 15 cylinder containers. Rounded to the nearest cent.

Explanation

Identify the side lengths of the cylinder.

$h = 1 \text{ m}, r = 60 \text{ cm}$

Convert values to the required unit of measure : centimetres.

$h = 100 \text{ cm}, r = 60 \text{ cm}$

Write the formula for the total surface area of a cylinder and substitute the side lengths to calculate the total surface area, with units.

$$\begin{aligned} \text{TSA} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(60)^2 + 2\pi(60)(100) \\ &= 19\,200\pi \text{ cm}^2 \end{aligned}$$

Multiply by 15 for the number of cylindrical containers.

$19\,200\pi \times 15 = 288\,000\pi$ cm^2

Multiply by 0.01 to calculate the money charged.

$288\,000\pi \times 0.01 \approx \$9047.79$

Answer

Kamal charges $9047.79 for painting 15 cylindrical containers.

14. The butterfly greenhouse is shaped like a half-cylinder. It has a diameter of 5 m and is 16 m long. Not including the flooring, how many square metres of plastic is required for its cover, correct to two decimal places?

Key points

- The greenhouse is in the shape of half a cylinder.
- Has a diameter of 5 m.
- Has a length of 16 m.
- Don't include flooring in the surface area.
- Determine the surface area in square metres. Correct to two decimal places.

Explanation

Identify the height and radius of the cylinder.

$h = 16$ m

$d = 5$ m and $r = \frac{5}{2} = 2.5$ m

Write the formula for the surface area of a curved face and semi-circle face and substitute the side lengths to calculate the surface area, with units.

$$\begin{aligned} \text{SA} &= (\pi r^2) + \tfrac{1}{2}(2\pi rh) \\ &= (\pi \times 2.5^2) + \tfrac{1}{2}(2 \times \pi \times 2.5 \times 16) \\ &\approx 145.30\text{ m}^2 \end{aligned}$$

Answer

Approximately 145.30 m^2 of cover will be needed for the butterfly greenhouse.

Reasoning

15. a. Omari would need 1.79 m^2 of material.

b. Omari would need 1.17 m^2 more material.

c. Omari would need 1.26 m^2 of material.

d. Omari would need 0.94 m^2 of material.

e. Suggested option 1: Upholster only the visible sides of furniture so it isn't as expensive.

Suggested option 2: Choose cheaper material when upholstering the furniture.

Note: There are other possible options.

16. a. The length of the curved edges is 28.27 cm.

b. The total surface area of the rectangular curved face is 226.19 cm^2.

c. To calculate the total surface area of the rectangular curved face in part **b**, we multiply the circumference by the length of the cylinder. Since the formula for a circumference is $2\pi r$, it is also included in calculating the surface area of the rectangular curved face, which would be used to calculate a cylinder's total surface area.

Exam-style

17. C

18. a. 36π m^2

b. 204π m^2

19. 16π cm^2

20. 90.40 cm

Remember this?

21. B **22.** D **23.** E

7G Volume of a prism

Student practice

Worked example 1

a. 21 cm^3 **b.** 2 935 000 L

Worked example 2

a. 150 mm^3 **b.** 108 m^3

Worked example 3

a. 42 cm^3 **b.** 280 m^3

Understanding worksheet

1. a. 4 cm **b.** 8 mm **c.** 7 cm **d.** 8 cm

2. a. 22 **b.** 8 **c.** 12 **d.** 9

3. volume; cubic; capacity; cross-section

Fluency

4. a. 10 cm^3
b. 5000 mm^3
c. 0.0025 m^3
d. 8 000 000 000 mm^3
e. 0.015 m^3
f. 250 000 cm^3
g. 0.00005 m^3
h. 1 000 000 000 000 000 mm^3

5. a. 5000 kL **b.** 5 L
c. 1.5 kL **d.** 0.002 ML
e. 2 000 000 L **f.** 0.00075 ML
g. 50 000 000 mL **h.** 7 kL

6. a. 54 m^3 **b.** 105 cm^3
c. 12.8 m^3 **d.** 90 cm^3
e. 80 m^3 **f.** 44 km^3
g. 374 cm^3 **h.** 154.08 mm^3

7. a. 100 cm^3 **b.** 336 mm^3
c. 36 m^3 **d.** 512 km^3
e. 704 m^3 **f.** 126 cm^3
g. 864 cm^3 **h.** 756 mm^3

8. C

Spot the mistake

9. a. Student B is incorrect. **b.** Student A is incorrect.

7G ANSWERS

Back to contents

Problem solving

10. How much space does a rectangular shoebox take up in cubic centimetres if its dimensions are 30 cm by 15 cm by 10 cm?

Key points

- A rectangular shoebox has dimensions of 30 cm by 15 cm by 10 cm.
- How much space does it take up in cubic centimetres?

Explanation

Write the formula for the volume of a rectangular prism.

$\text{Volume} = l \times w \times h$

Substitute the dimensions to calculate the volume of the shoebox.

$\text{Volume} = 30 \times 10 \times 15$

$= 4500 \text{ cm}^3$

Answer

The rectangular shoebox takes up 4500 cm^3.

11. A chef bakes a cake that is 11 cm tall. The top of the cake has an area of 65 cm^2. What is the volume of the cake in cubic centimetres?

Key points

- A cake is 11 cm tall.
- The top of the cake has an area of 65 cm^2.
- What is the volume of the cake in cubic centimetres?

Explanation

Write the formula for the volume of the cake.

Volume = area of cross-section × height

Substitute the values to calculate the volume, with units.

$\text{Volume} = 65 \times 11$

$= 715 \text{ cm}^3$

Answer

The volume of the cake is 715 cm^3.

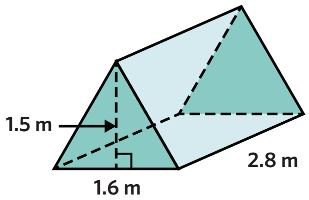
12. A tent shaped as a triangular prism measures 2.8 m in length, 1.6 m in width, and 1.5 m in height. What is the volume of the tent in cubic metres?

Key points

- A tent shaped as a triangular prism measures 2.8 m in length, 1.6 m in width, and 1.5 m in height.
- What is the volume of the tent in cubic metres?

Explanation

The tent is a triangular prism. The diagram displays its dimensions.



Write the formula for the volume of a triangular prism.

$$\text{Volume} = \frac{b \times h}{2} \times l$$

Substitute the values to calculate the volume, with units.

$$\text{Volume} = \frac{1.5 \times 1.6}{2} \times 2.8$$

$$= 3.36 \text{ m}^3$$

Answer

The volume of the tent is 3.36 m^3.

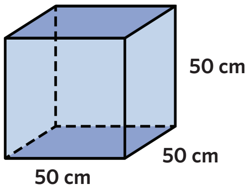
13. A creative ice sculpture is in the shape of a cube. How much liquid in millilitres is used to create the sculpture if all of its edges are 50 cm long. 1 cubic centimetre is equal to 1 millilitre.

Key points

- An ice sculpture is in the shape of a cube.
- All edges are 50 cm long.
- 1 cubic centimetre equals 1 millilitre.
- How much liquid in millilitres is used to create the sculpture?

Explanation

The ice sculpture is a cube. The diagram displays its dimensions.



Write the formula for the volume of a rectangular prism.

$\text{Volume} = l \times w \times h$

Substitute the dimensions to calculate the volume of the ice sculpture.

$\text{Volume} = 50 \times 50 \times 50$

$= 125\,000 \text{ cm}^3$

Convert 125 000 cm^3 to millilitres.

$1 \text{ cm}^3 = 1 \text{ mL}$

$125\,000 \text{ cm}^3 = 125\,000 \text{ mL}$

Answer

125 000 mL of water is used to create the ice sculpture.

14. Tracy's garden pond is empty and has a uniform cross-section from its lowest to highest point. The surface has an area of 35 m^2. If 60 mm depth of rain fell in a day, what is the total volume of rain contained in the garden pond in litres? 1 cubic metre is equal to 100 litres.

Key points

- An empty garden pond has a uniform cross-section from its lowest to highest point.
- The surface has an area of 35 m^2.
- 60 mm of rain fell in one day.
- 1 cubic metre equals 100 litres.
- What is the total volume of rain contained in the garden pond in litres?

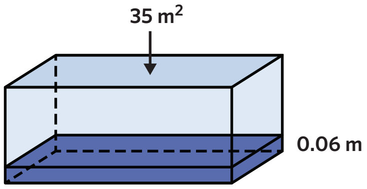
Explanation

Convert 60 mm to m to determine the depth of rain in metres.

$1 \text{ mm} = 0.001 \text{ m}$

$60 \text{ mm} = 0.06 \text{ m}$

Calculate the volume of rainwater contained in the garden pond.



Volume = area of cross-section × depth of rain

$= 35 \times 0.06$

$= 2.1 \text{ m}^3$

Convert 2.1 m^3 to litres.

$1 \text{ m}^3 = 100 \text{ L}$

$2.1 \text{ m}^3 = 210 \text{ L}$

Back to contents

Answer

210 L of rain would be contained in the garden pond.

Reasoning

15. **a.** The main deck cargo volume is 489.16 m^3.

b. The maximum weight of the cargo that can be loaded onto the main deck is 97 832 kg.

c. The new main deck cargo volume is 398.57 m^3.

d. 10.12% of the main deck's volume is now full.

e. Suggested option 1: Aid organisations should put essential resources, such as food and medical supplies, in the cargo hold rather than non-essential resources.

Suggested option 2: Aid organisations should completely fill the cargo hold to transport the maximum amount of essential supplies to people in need.

Note: There are other possible options.

16. **a.** The volume is 30 cm^3, calculated using the pink face as the cross-section.

b. The volume is 30 cm^3, calculated using the orange face as the cross-section.

c. The volume of the solid object is the same, despite being calculated from different cross-sections. This is because the volume of rectangular prisms is calculated by multiplying the length by the width by the height. The order that the values are multiplied by does not affect the final calculation because of the commutative law, therefore, different cross-sections can be used to calculate the volume of rectangular prisms.

Exam-style

17. D

18. **a.** 72 000 cm^3 **b.** 18 000 pebbles

19. 115 m^3

20. 104.49 cm^3

Remember this?

21. D **22.** A **23.** B

7H Volume of a cylinder

Student practice

Worked example 1

a. 280 mm^3 **b.** 82.5 cm^3

Worked example 2

a. 282.74 m^3 **b.** 183.78 cm^3

Worked example 3

a. 4.02 L **b.** 169.65 kL

Understanding worksheet

1. **a.** 86 **b.** 734 **c.** L **d.** 0.063

2. **a.** π **b.** 5 **c.** 14 **d.** $\frac{1}{2}$

3. volume; capacity; cylinder; cross-section

Fluency

4. **a.** 700 cm^3 **b.** 450 m^3
c. 200 cm^3 **d.** 350 cm^3
e. 132 mm^3 **f.** 591 cm^3
g. 428 m^3 **h.** 546 cm^3

5. **a.** 75.40 cm^3 **b.** 706.86 cm^3
c. 502.65 m^3 **d.** 2309.07 mm^3
e. 254.47 cm^3 **f.** 9047.79 mm^3
g. 1696.46 m^3 **h.** 16.60 m^3

6. **a.** 75.40 cm^3 **b.** 141.37 cm^3
c. 183.78 cm^3 **d.** 43.98 cm^3
e. 150.80 cm^3 **f.** 201.06 cm^3
g. 138.23 cm^3 **h.** 11.54 cm^3

7. **a.** 7853.98 mL **b.** 13 571 680.26 L
c. 165.67 L **d.** 169.65 L
e. 1017.88 kL **f.** 785.40 kL

8. D

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. A cylindrical container has a radius of 10 cm and height of 15 cm. Calculate the volume in cubic centimetres. Round the answer to two decimal places.

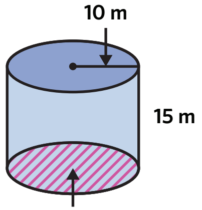
Key points

- A cylindrical container has a radius of 10 cm and height of 15 cm.
- Calculate the volume in cubic centimetres.

Explanation

Calculate the volume by substituting the dimensions into the formula for the volume of a cylinder.

$$\begin{aligned} \text{Volume} &= \text{area of cross-section} \times \text{height} \\ &= \pi r^2 \times h \\ &= \pi \times 10^2 \times 15 \\ &= 1500\pi \\ &\approx 4712.39 \text{ cm}^3 \end{aligned}$$



Area of cross-section = πr^2 = 100π

Answer

The volume of the cylindrical container is approximately 4712.39 cm^3

Back to contents

11. A gas cylinder has a diameter of 20 cm and is 75 cm tall. Calculate its volume in cubic centimetres. Round the answer to two decimal places.

Key points

- A gas cylinder has a diameter of 20 cm and height of 75 cm.
- Calculate its volume in cubic centimetres.

Explanation

Calculate the radius.

$r = 20 \div 2$

$= 10$ cm

Calculate the volume by substituting the dimensions into the formula for the volume of a cylinder.

$$\begin{aligned}\text{Volume} &= \text{area of cross-section} \times \text{height}\\ &= \pi r^2 \times h\\ &= \pi \times 10^2 \times 75\\ &= 7500\pi\\ &\approx 23\,561.94 \text{ cm}^3\end{aligned}$$

Answer

The gas cylinder's volume is approximately 23 561.94 cm^3

12. The Lismore Water Tower is located in rural Victoria and has a height of 31 m and a diameter of 14 m. How much water can it store, in litres, when it is three quarters full? Round the answer to two decimal places.

Key points

- The Lismore Water Tower has a height of 31 m and diameter of 14 m.
- How much water can it store, in litres, when it is three quarters full? Round the answer to two decimal places.

Explanation

Calculate the radius.

$r = 14 \div 2$

$= 7$ m

Calculate the volume by substituting the dimensions into the formula for the volume of a cylinder.

$$\begin{aligned}\text{Volume} &= \text{area of cross-section} \times \text{height}\\ &= \pi r^2 \times h\\ &= \pi \times 7^2 \times 31\\ &= 1519\pi \text{ m}^3\end{aligned}$$

Convert the volume to capacity.

1 m^3 = 1000 L

$1519\pi \times 1000$

Calculate three quarters of the tower's capacity.

$\frac{3}{4} \times 1\,519\,000\pi \approx 3\,579\,059.43$ L

Answer

The Lismore Water Tower can store approximately 3 579 059.43 L when it is three quarters full.

13. The Shark Dive Xtreme experience at the Melbourne Aquarium includes a 15 minute scuba dive with real sharks. The cylindrical tank for the dive has a depth of 8 m and a radius of 4 m. What is the capacity of the tank in litres, to the nearest whole number?

Key points

- The cylindrical tank has a depth of 8 m and radius of 4 m.
- What is the capacity of the tank in litres?

Explanation

Calculate the volume by substituting the dimensions into the formula for the volume of a cylinder.

$$\begin{aligned}\text{Volume} &= \text{area of cross-section} \times \text{height}\\ &= \pi r^2 \times h\\ &= \pi \times 4^2 \times 8\\ &= 128\pi \text{ m}^3\end{aligned}$$

Convert the volume to capacity.

1 m^3 = 1000 L

$128\pi \times 1000 \approx 402\,124$ L

Answer

The capacity of the tank is approximately 402 124 L.

14. 20 g of a sports electrolyte powder is required to be mixed with 500 mL of water to formulate a sport rehydration supplement. Approximately, how many grams of the electrolyte powder are required to be mixed with water to fill a cylindrical cooler with a diameter of 50 cm and a height of 25.56 cm?

Key points

- 20 g of a sports electrolyte powder is required to be mixed with 500 mL of water.
- A cylindrical cooler has a diameter of 50 cm and height of 25.56 cm.
- How many grams of the electrolyte powder are required to be mixed with water to fill the cylindrical cooler?

Explanation

Calculate the radius of the cylindrical cooler.

$r = 50 \div 2$

$= 25$ cm

Calculate the volume of the container by substituting the dimensions into the formula for the volume of a cylinder.

$$\begin{aligned}\text{Volume} &= \text{area of cross-section} \times \text{height}\\ &= \pi r^2 \times h\\ &= \pi \times 25^2 \times 25.56\\ &= 15\,975\pi \text{ cm}^3\end{aligned}$$

Convert the volume to capacity.

1 cm^3 = 1 mL

$15\,975\pi$ cm^3 = $15\,975\pi$ mL

Determine the amount of electrolyte powder required.

500 mL of water requires 20 g of powder.

$15\,975\pi \times \frac{20}{500} \approx 2007.48$ g

Answer

Approximately 2007.48 g of the electrolyte powder is needed.

Reasoning

15. **a.** The cylindrical storage tank has a capacity of 62 832 L.

b. 62 831.85 kg is the maximum weight of liquid the tank can hold when completely full.

c. The tank's capacity with the rod inside is 60 318.58 L.

d. The height of the tank is 6 m.

e. Suggested option 1: Cylindrical tanks could be grouped by size to optimise organisation.

Suggested option 2: If tanks are able to be stacked, and it is safe to do so, this could optimise the amount of available floor space.

Note: There are other possible options.

16. a. The base has an approximate area of 28 cm^2.

b. The cylinder has an approximate volume of 224 cm^3.

c. The base is a circle so its area can be determined from the formula πr^2. The cylinder comprises multiple 'layers' of the base so its volume can be calculated by multiplying the area of the circular base by the number of layers. Therefore πr^2 is used in the formula for the volume of a cylinder.

Exam-style

17. C

18. a. 120 mm

b. 150.80 cm^3

19. 1188.45 cm^3

20. 13 cm

Remember this?

21. B

22. C

23. B

Chapter 7 extended application

1. a. The unknown parallel side is 11 cm.

b. The perimeter is 150 cm and the area is 180 cm^2.

c. The perimeter is 48 cm and the area is 179.14 cm^2.

d. Design 1 = \$1899, Design 2 = \$1889.93, Design 3 = \$358.70

e. It would take Michelle 19 days to complete this work for her customer.

f. Suggested option 1: Mosaic art requires assembly of pieces rather than direct application of mediums.

Suggested option 2: Mosaic art considers the interplay between individual tesserae rather than continuous strokes or shapes.

Note: There are other possible options.

2. a. The diameter of the silo's base is approximately 9 m.

b. The perimeter is 16 m and the area is 16 m^2.

c. The surface area is 143 m^2.

d. The cylindrical silo has a capacity of 286 278 L.

e. The cylindrical silo can store 229 022 litres of milk when 80% full.

f. Suggested option 1: Dairy farmers can implement rotational grazing to improve soil health and reduce overgrazing.

Suggested option 2: Farmers can invest in methane digesters to convert cow manure into renewable energy.

Note: There are other possible options.

3. a. The total volume is 0.003 km^3.

b. The surface area of drill A and B is 178.80 m^2.

c. Drill A displaces 34.56 m^3 of earth and Drill B displaces 64.63 m^3 of earth.

d. To build the road, it will require approximately 398 196 869 L of cement.

e. It would be approximately \$23 493 cheaper to use Drill A.

f. Suggested option 1: Drill 2 because, while it may be costly, will get the task done faster.

Suggested option 2: Drill 1, because it may be slower, it will be a cheaper task.

Note: There are other possible options.

Chapter 7 review

Multiple choice

1. D **2.** B **3.** A **4.** D **5.** D

Fluency

6. a. 15.5 cm **b.** 27 mm **c.** 112 m **d.** 60 km

7. a. 31.42 m **b.** 25.13 mm
c. 42.73 cm **d.** 352.49 km

8. a. 45.71 cm **b.** 23.09 km
c. 18.08 mm **d.** 215.74 m

9. a. 21.35 mm^2 **b.** 112 cm^2
c. 640 km^2 **d.** 120.225 m^2

10. a. 153.94 km^2 **b.** 206.12 cm^2
c. 178.72 m^2 **d.** 465.66 mm^2

11. a. Perimeter = 65.00 m, area = 235.62 m^2

b. Perimeter ≈ 49.51 km, area ≈ 134.04 km^2

c. Perimeter = 69.46 cm, area = 181.50 cm^2

d. Perimeter ≈ 113.10 mm, area ≈ 922.86 mm^2

12. a. 160 cm^2 **b.** 453 km^2
c. 191.88 m^2 **d.** 703.8 mm^2

13. a. 24 km^2 **b.** 1188 m^2
c. 553.28 cm^2 **d.** 158.13 mm^2

14. a. 427.26 cm^2 **b.** 132.15 mm^2
c. 495.23 m^2 **d.** 82.13 km^2

15. a. 48 m^3 **b.** 575 mm^3
c. 155.82 km^3 **d.** 542.79 cm^3

16. a. 180 cm^3 **b.** 1000 mm^3
c. 1390.442 m^3 **d.** 874.944 km^3

17. a. 197.92 cm^3 **b.** 118.79 m^3
c. 337.56 cm^3 **d.** 895.67 km^3

Back to contents

Problem solving

18. Sophie measures the height of three different plants in her garden. What is the total height of the three plants, in centimetres, if the three individual measurements are 16 cm, 1441 mm, and 2.87 m?

Key points

- Sophie measures the height of three different plants.
- The three individual heights are 16 cm, 1441 mm, and 2.87 m.
- Determine the total height of the three plants.

Explanation

Convert all the measurements to centimetres.

16 cm = 16 cm

1441 mm = 144.1 cm

2.87 m = 287 cm

Add all the values together to get the total.

16 + 144.1 + 287 = 447.1 cm

Answer

The total height of the three plants is 447.1 cm.

19. Seamus bakes a circular cake that has a radius of 12.3 cm to celebrate his birthday. He shares the cake equally between six people. Each piece is the shape of a sector. What is the perimeter of the base of one piece of cake, rounded to the nearest centimetre?

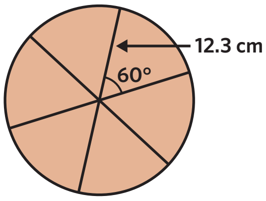
Key points

- A circular cake has a radius of 12.3 cm.
- He shares the cake equally between six people.
- Each piece is the shape of a sector.
- Determine the perimeter of the base of one piece of the cake.

Explanation

Calculate the internal angle of each cake piece.

$$\begin{aligned}\theta &= \frac{360}{6}\\ &= 60^\circ\end{aligned}$$



Each cake piece is a sector with an angle of 60° and a radius of 12.3 cm.

Calculate the perimeter of the sector.

$$\begin{aligned}P &= \frac{60}{360} \times 2\pi \times 12.3 + 2 \times 12.3\\ &\approx 37 \text{ cm}\end{aligned}$$

Answer

The perimeter of one cake slice is approximately 37 cm.

20. A standard bicycle wheel has a diameter of 622 mm. Tessa measures the diameter of her bicycle wheel as 58 cm. What is the difference between the area of a standard bicycle wheel and Tessa's bicycle wheel in squared centimetres? Round to 2 decimal places.

Key points

- Standard bicycle wheel's diameter is 622 mm.
- Tessa's bicycle wheel has a diameter of 58 cm.
- Determine the difference between Tessa's and the standard bicycle wheel area.

Explanation

Convert the diameter to centimetres.

622 mm = 622 ÷ 10 = 62.2 cm

Calculate the radius.

$$\text{Radius} = \frac{\text{diameter}}{2}$$

$$\frac{62.2}{2} \text{ cm} = 31.1 \text{ cm}$$

$$\begin{aligned}\text{Area of big circle} &= \pi r^2\\ &= \pi \times 31.1^2\\ &= 967.21\pi \text{ cm}^2\end{aligned}$$

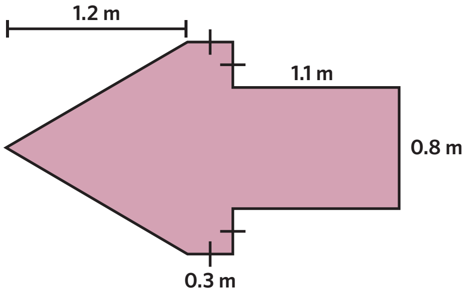
$$\begin{aligned}\text{Area of small circle} &= \pi r^2\\ &= \pi \times \left(\frac{58}{2}\right)^2\\ &= 841\pi \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Difference} &= \text{area of big circle} - \text{area of small circle}\\ &= 967.21\pi - 841\pi\\ &= 126.21\pi\\ &\approx 396.50 \text{ cm}^2\end{aligned}$$

Answer

The difference between the area of a standard bicycle wheel with Tessa's bicycle wheel is approximately 396.50 cm^2.

21. An arrow is to be painted on a road to indicate the direction of traffic.



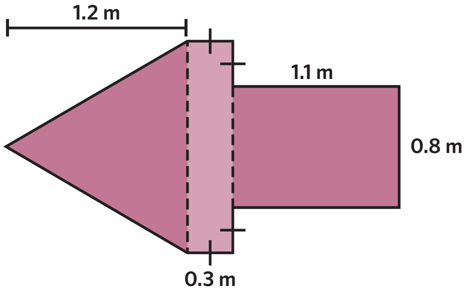
What is the total area to be painted in square metres?

Key points

- Composite shape of an arrow.
- Determine the total area of the composite shape.

Explanation

The given composite shape consists of a triangle and two rectangles.



Calculate the area of the right rectangle.

$$\begin{aligned}\text{Area of a rectangle} &= \text{length} \times \text{width}\\ &= 1.1 \times 0.8\\ &= 0.88 \text{ m}^2\end{aligned}$$

Calculate the area of the left rectangle.

$$\begin{aligned}\text{Area of a rectangle} &= \text{length} \times \text{width}\\ &= 0.3 \times (0.8 + 0.3 + 0.3)\\ &= 0.3 \times 1.4\\ &= 0.42 \text{ m}^2\end{aligned}$$

Calculate the area of the triangle.

$$\begin{aligned}\text{Area of a triangle} &= \text{base} \times \text{height} \div 2\\ &= 1.4 \times 1.2 \div 2\\ &= 0.84\ \text{m}^2\end{aligned}$$

Calculate the total area of the shape.

$$\begin{aligned}\text{Area of composite shape} &= \text{rectangle area} + \text{rectangle area} + \text{triangle area}\\ &= 0.88 + 0.42 + 0.84\ \text{m}^2\\ &= 2.14\ \text{m}^2\end{aligned}$$

Answer

The total area to be painted is 2.14 m^2.

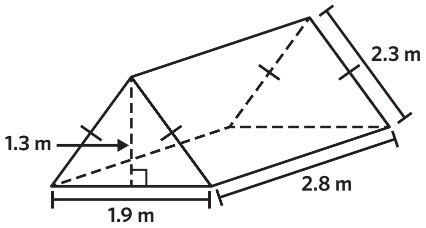
22. Jimmy wants to construct his own camping tent rather than purchasing one. He builds a metal frame shaped as a triangular prism with a base of 1.9 m, a height of 1.3 m, a slant height of 2.3 m, and a length of 2.8 m. How much tent fabric is needed to completely cover the metal frame?

Key points

- A metal frame for a tent is a triangular prism.
- Base of 1.9 m.
- Height of 1.3 m.
- Slant height of 2.3 m.
- Length of 2.8 m.
- Determine the surface area to determine the amount of fabric required.

Explanation

The metal frame for the tent is a triangular prism. The diagram displays its dimensions.



Write the formula for the total surface area of a triangular prism.

Total surface area = 2 × base + side 2 + side 1 + side 3

Substitute the side lengths into the formula to calculate the total surface area, with units.

$$\begin{aligned}\text{Total surface area} &= 2\left(\frac{1.9 \times 1.3}{2}\right) + (1.9 \times 2.8) + 2(2.3 \times 2.8)\\ &= 2.47 + 5.32 + 12.88\\ &= 20.67\ \text{m}^2\end{aligned}$$

Answer

Jimmy and Joey will need 20.67 m^2 of tent fabric.

23. Zoe is using stickers to decorate her cylindrical drink bottle that is 22.5 cm high and has a diameter of 9.4 cm. If it costs Zoe \$0.02 to cover one square centimetre of the bottle in stickers, how much will it cost to cover the entire drink bottle? Round to two decimal places.

Key points

- A cylindrical drink bottle is 22.5 cm high and has a diameter of 9.4 cm.
- It costs \$0.02 to cover one square centimetre of the bottle in stickers.
- How much will it cost to cover the entire drink bottle?

Explanation

Identify the height and radius of the cylinder.

$h = 22.5$ cm

$d = 9.4$ cm, $r = \frac{9.4}{2} = 4.7$ cm

Write the formula for the surface area of a curved face and circle faces and substitute the side lengths to calculate the total surface area, with units.

$$\begin{aligned}TSA &= 2\pi r^2 + 2\pi rh\\ &= 2\pi \times 4.7^2 + 2 \times \pi \times 4.7 \times 22.5\\ &= 44.18\pi + 211.5\pi\\ &= 255.68\pi\ \text{cm}^2\end{aligned}$$

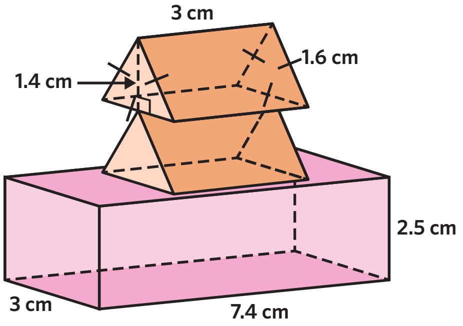
Calculate the total cost.

$255.68\pi \times 0.02 \approx \$16.06$

Answer

It will cost Zoe \$16.06 to cover her entire water bottle in stickers.

24. Tom is an artist who is creating a sculpture consisting of a rectangular prism with two identical triangular prisms stacked on top.



To enter a particular art competition, the volume of the sculpture must not exceed 60 000 mm^3. By how much does the volume of Tom's sculpture exceed the competition's limit in squared millimetres?

Key points

- The shape consists of two triangular prisms and one rectangular prism.
- The shape cannot exceed 60 000 mm^3.
- Determine by how much the volume of the sculpture exceeds 60 000 mm^3.

Explanation

Identify shapes that make up the sculpture.

2 triangular prisms and 1 rectangular prism make up the sculpture.

Calculate triangular prism volume.

$$\begin{aligned}\text{Volume} &= \text{area of cross section} \times \text{length}\\ &= \text{base} \times \text{height} \div 2 \times \text{length}\\ &= (1.6 \times 1.4 \div 2) \times 3\\ &= 3.36\ \text{cm}^3\end{aligned}$$

Calculate rectangular prism volume.

$$\begin{aligned}\text{Volume} &= \text{base} \times \text{width} \times \text{length}\\ &= 3 \times 2.5 \times 7.4\\ &= 55.5\ \text{cm}^3\end{aligned}$$

Calculate sculptures total volume.

$$\begin{aligned}\text{Total volume} &= 2 \times \text{triangular prisms volume} + \text{rectangular prism volume}\\ &= 2 \times 3.36 + 55.5\\ &= 62.22\ \text{cm}^3\end{aligned}$$

Convert to mm^3.

$62.22 \times 1000 = 62\,220\ \text{mm}^3$

Calculate the difference between the volume of the sculpture and the competition's limit.

$62\,220 - 60\,000 = 2220\ \text{mm}^3$

Answer

Tom's sculpture exceeds the competition's limit by 2220 mm^3.

25. Annabel wants to install a water trough shaped as a half cylinder in a paddock. Annabel needs the trough to hold 650 L of water when completely full so her horses have enough to drink. If the water trough has a radius of 65 cm, what is the minimum length it must be to hold this amount of water, rounded to two decimal places?

Key points

- The trough is shaped as a half cylinder.
- The trough needs to contain 650 L.
- The trough has a radius of 65 cm.
- Determine the minimum length for the trough.

Explanation

Convert 650 L to square centimetres.

$650\ \text{L} = 650\,000\ \text{cm}^3$

Write the formula for the volume of the solid object.

$\text{Volume} = \text{area of cross-section} \times \text{length}$

$\text{Volume} = \dfrac{\pi \times r^2}{2} \times \text{length}$

Substitute the values to calculate the volume, with units.

$650\,000 = \dfrac{\pi \times 65^2}{2} \times \text{length}$

$\text{Length} = 650\,000 \times 2 \div (\pi \times 65^2)$

$\text{Length} = 1\,300\,000 \div (\pi \times 4225)$

$\text{Length} \approx 97.94\ \text{cm}$

Answer

The minimum length of the trough is approximately 97.94 cm.

Reasoning

26. a. The total length of the inner edges of the walking track is 156 m.

b. The area covered by the door as it opens and closes is approximately 78.54 m^2.

c. The cost of repaving the path is $4343.65.

d. The total capacity of the fountain is 10 390.82 L.

e. Suggested option 1: The local council could add a flower garden to the park.

Suggested option 2: The local council could add a playground to the park.

Note: There are other possible options.

27. a. The volume of object 1 is 120 m^3.

b. The volume of object 2 is 120 m^3.

c. The volume calculated in parts **a** and **b** is the same, despite the measurements being multiplied in different orders. For calculations of volumes of rectangular prisms, the commutative law applies, meaning that the order in which the measurements of the three dimensions are multiplied does not affect the final answer.

Back to contents

8A Angles and parallel lines

Student practice

Worked example 1

a. $a^\circ = 36^\circ$ as a° and 54° form a complementary angle.

$b^\circ = 270^\circ$ as a°, b° and 54° form a revolution.

b. $a^\circ = 74^\circ$ as a° and 74° are vertically opposite angles.

$b^\circ = 106^\circ$ as b° and 74° are supplementary angles.

Worked example 2

a. $a^\circ = 144^\circ$ as a° and 36° are co-interior angles on parallel lines.

b. $a^\circ = 73^\circ$ as a° and 73° are corresponding angles.

$b^\circ = 107^\circ$ as a° and b° are supplementary angles.

Worked example 3

a. $a^\circ = 103^\circ$ as a° and 103° are alternate angles on parallel lines.

$b^\circ = 57^\circ$ as b° and 57° are corresponding angles on parallel lines.

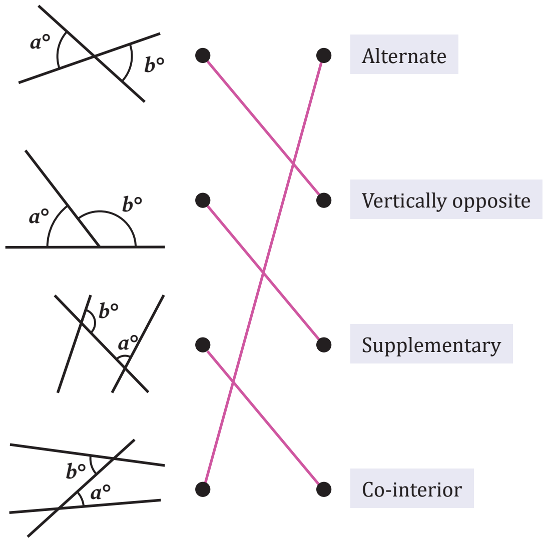
b. $b^\circ = 46^\circ$ as b° and 46° are alternate angles on parallel lines.

$a^\circ = 46^\circ$ as a° and b° are vertically opposite angles.

$c^\circ = 104^\circ$ as c° and $b^\circ + 58^\circ$ are alternate angles on parallel lines.

Understanding worksheet

1. **Angle** **Relationship**



2. **a.** Alternate **b.** Alternate

c. Co-interior **d.** Corresponding

3. complementary; supplementary; transversal; co-interior; equal

Fluency

4. **a.** $x^\circ = 42^\circ$ as x°, 21° and 27° form a complementary angle.

b. $y^\circ = 46^\circ$ as y°, 90°, 99° and 125° form a full revolution.

c. $y^\circ = 60^\circ$ as y° and 120° form a supplementary angle.

d. $x^\circ = 116^\circ$ as x° and 116° are vertically opposite angles.

$y^\circ = 64^\circ$ as y° and 116° form a supplementary angle.

e. $x^\circ = 53^\circ$ as x° and 37° form a complementary angle.

f. $y^\circ = 24^\circ$ as y° and 66° form a complementary angle.

$x^\circ = 24^\circ$ as x° and y° are vertically opposite angles.

g. $x^\circ = 25^\circ$ as x° and 25° are vertically opposite angles.

$y^\circ = 23^\circ$ as y°, 25° and 132° form a supplementary angle.

h. $x^\circ = 74^\circ$ as x°, 25° and 81° form a supplementary angle.

$y^\circ = 25^\circ$ as y° and 25° are vertically opposite angles.

5. **a.** $a^\circ = 51^\circ$ as a° and 51° are corresponding angles on parallel lines.

b. $a^\circ = 36^\circ$ as a° and 36° are corresponding angles on parallel lines.

$b^\circ = 36^\circ$ as b° and a° are vertically opposite angles.

c. $a^\circ = 65^\circ$ as a° and 65° are alternate angles on parallel lines.

d. $a^\circ = 105^\circ$ as a° and 75° are co-interior angles on parallel lines.

e. $a^\circ = 46^\circ$ as a° and 46° are alternate angles on parallel lines.

f. $a^\circ = 125^\circ$ as a° and 125° are corresponding angles on parallel lines.

$b^\circ = 55^\circ$ as b° and 125° are co-interior angles on parallel lines.

g. $a^\circ = 76^\circ$ as a° and 104° are co-interior angles on parallel lines.

h. $a^\circ = 60^\circ$ as the angle vertically opposite to a° and 120° are co-interior angles on parallel lines.

6. **a.** $a^\circ = 99^\circ$ as a° and 81° are co-interior angles on parallel lines.

$b^\circ = 99^\circ$ as b° and a° are corresponding angles on parallel lines.

b. $a^\circ = 65^\circ$ as a° and 65° are alternate angles on parallel lines.

$b^\circ = 37^\circ$ as $b^\circ + 78^\circ$ and 65° are co-interior angles on parallel lines.

c. $a^\circ = 75^\circ$ as a° and 75° are corresponding angles on parallel lines.

$b^\circ = 105^\circ$ as b° and 75° are co-interior angles on parallel lines.

$c^\circ = 105^\circ$ as c° and b° are corresponding angles on parallel lines.

$d^\circ = 75^\circ$ as d° and a° are alternate angles on parallel lines.

d. $b^\circ = 125^\circ$ as b° and 55° are co-interior angles on parallel lines.

$a^\circ = 55^\circ$ as a° and b° are co-interior angles on parallel lines.

e. $a^\circ = 86^\circ$ as a° and 86° are corresponding angles on parallel lines.

$b^\circ = 78^\circ$ as b° and 78° are alternate angles on parallel lines.

f. $a^\circ = 71^\circ$ as a° and 71° are alternate angles on parallel lines.

$b^\circ = 58^\circ$ as b° and 58° are alternate angles on parallel lines.

$c^\circ = 51^\circ$ as c°, b° and a° form a supplementary angle.

g. $a^\circ = 109^\circ$ as a° and 109° are alternate angles on parallel lines.

$c^\circ = 42^\circ$ as c° and 42° are alternate angles on parallel lines.

$b^\circ = 29^\circ$ as c°, b° and a° form a supplementary angle.

h. $a^\circ = 37^\circ$ as a° is vertically opposite to the angle which is the difference between the angles corresponding to 115° and 78°.

7. C

Back to contents

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

9. A botanic garden designer intends to incorporate a central fountain surrounded by 5 straight paths radiating outward. To achieve a harmonious design, the paths will be uniformly spaced. Calculate the angle formed between each adjacent pair of paths.

Key points

- There are 5 paths surrounding the central fountain, radiating outwards.
- The paths are uniformly spaced.
- Calculate the angle formed between each adjacent pair of paths.

Explanation

There are a total of five angles, all which form a full revolution. Since the angles are uniformly spaced, the angle formed between each adjacent path can be calculated by dividing 360° by 5.

$$\begin{aligned}\text{Angle formed between each adjacent path} &= \frac{360^\circ}{5}\\ &= 72^\circ\end{aligned}$$

Answer

The angle formed between each adjacent pair of paths will be 72°.

10. A bridge spans across a straight section of a double lane highway diagonally. The bridge forms an angle of 68° with one road and an angle of 112° with the other road. Illustrate a diagram that demonstrates how, in this section, the two roads are parallel to each other.

Key points

- A bridge spans across a double lane highway diagonally.
- The bridge forms an interior angle of 68° with one road and an interior angle of 112° with the other road.
- Illustrate a diagram that demonstrates how, in this section, the two roads are parallel to each other.

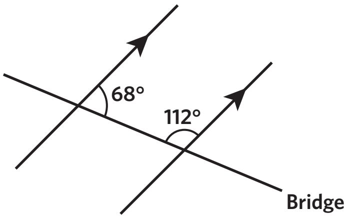
Explanation

In this section, the bridge acts as a transversal. For the two roads to be parallel to each other, the sum of the co-interior angles formed between them and the bridge must be equal to 180°.

$68^\circ + 112^\circ = 180^\circ$

The co-interior angles sum to 180° so the two roads must be parallel to each other.

Answer



11. An engineer is examining the alignment of two parallel lines of wiring. Instead of measuring the constant distance between the two wires, they use a ruler as a transversal and measure an interior angle as 53°. Provide a visual representation illustrating the measurements of all angles formed when the wires are parallel.

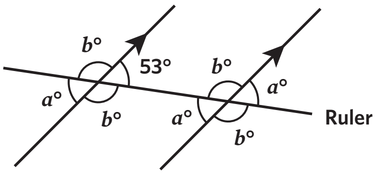
Key points

- The two wires are parallel.
- A ruler is used as a transversal.
- The interior angle is 53°.
- Provide a visual representation illustrating the measurements of all angles formed when the wires are parallel.

Explanation

Let angle a° denote the angles vertically opposite, alternate and corresponding to the measured interior angle 53°.

Let angle b° denote the angles supplementary and co-interior to the measured interior angle 53°.



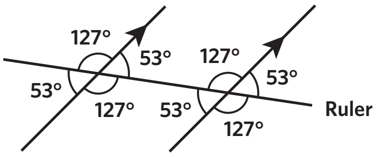
Vertically opposite angles, alternate angles on parallel lines, and corresponding angles on parallel lines are equal.

$\therefore a^\circ = 53^\circ$

Angles that form supplementary angles and co-interior angles on parallel lines sum to 180°.

$$\begin{aligned}\therefore b^\circ + 53^\circ &= 180^\circ\\ b^\circ &= 127^\circ\end{aligned}$$

Answer



12. At a roundabout, where five roads (*A*–*E*) intersect, the angles between the roads are as follows: Road *A* and *B* form a right angle (90°), Road *B* and *C* form a 65° angle, Road *C* and *D* form a 25° angle, Road *D* and *E* form a 155° angle, and Road *E* and *A* form a 25° angle. Determine which roads, if any, appear to continue straight through the roundabout.

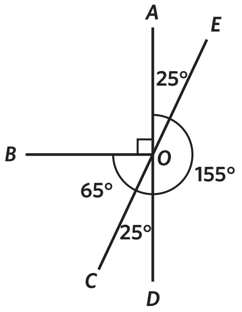
Key points

- At a roundabout five roads (*A*–*E*) intersect.
- Road *A* and *B* form a right angle (90°), Road *B* and *C* form a 65° angle, Road *C* and *D* form a 25° angle, Road *D* and *E* form a 155° angle, and Road *E* and *A* form a 25° angle.
- Determine which roads, if any, appear to continue straight through the roundabout.

Explanation

For a road to appear to continue straight through the roundabout, it shall form a supplementary angle with another road.

The roundabout and Roads *A*–*E* are visually represented by the following diagram.



Back to contents

$\angle AOD = 90° + 65° + 25°$

$= 180°$

$\angle COE = 25° + 155°$

$= 180°$

Road *B* does not form a supplementary angle with any other road.

Answer

Roads *AD* and *CE* appear to continue straight through the roundabout.

13. A city park is enclosed by four roads. The internal angles within the park are given as 110°, $a°$, $b°$ and 65°. Given that two of the roads forming the park are parallel, determine the values of $a°$ and $b°$.

Key points

- A city park is enclosed by four roads.
- The internal angles within the park are given as 110°, $a°$, $b°$ and 65°.
- Two of the roads forming the park are parallel.
- Determine the values of $a°$ and $b°$.

Explanation

Two of the roads are parallel, two sets of co-interior angles on a parallel line will be formed. The first set will consist of angles $a°$ and 110°, and the second set will consist of angles $b°$ and 65°.

Co-interior angles on a parallel line sum to 180°, hence:

$a° + 110° = 180°$

$a° = 70°$

$b° + 65° = 180°$

$b° = 115°$

Answer

Angle $a°$ is 70° and angle $b°$ is 115°.

Note: The values of angles $a°$ and $b°$ are interchangeable.

Reasoning

14. a. $x° = 17°$ as $x°$, 35° and 128° form a supplementary angle.

b. $y° = 35°$ as $y°$, 17° and 128° are internal angles of a triangle, which sum to 180°.

c. Level 1 and ground level are parallel, as $x°$ is equal to its alternate angle of 17°, and $y°$ is equal to its alternate angle of 35°.

d. Suggested option 1: Not adhering to these regulations could result in issues involving the health and safety of users.

Suggested option 2: Adhering to these regulations allows all users to comfortably navigate between floors.

Note: There are other possible options.

15. a. Alternate angles on parallel lines are equal.

b. Supplementary angles sum to 180°.

c. The external angle $d°$ can be calculated using $a° + c° = d°$.

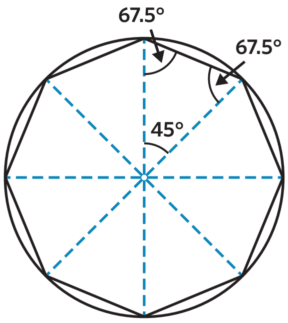
Exam-style

16. A

17. a. $x° = 20°$ **b.** $y° = 65°$

18. $b° = 110°$

19.



Remember this?

20. C **21.** A **22.** E

8B Congruent triangles

Student practice

Worked example 1

a. $x° = 23°$ **b.** $x° = 76°$

Worked example 2

a. $\therefore \triangle ABC \equiv \triangle DEF$ (**ASA**)

b. $\therefore \triangle ABC$ and $\triangle DEF$ are not congruent (**AAS**)

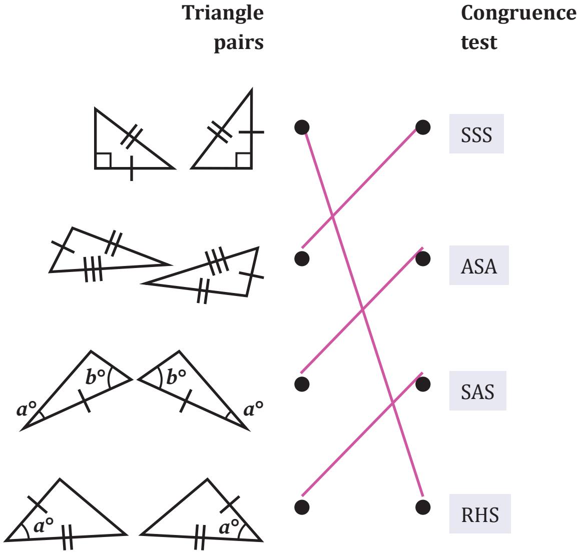
Worked example 3

a. $x° = 41°$ **b.** $x = 6$

Understanding worksheet

1. a. *EF* **b.** *EFD* **c.** *FE* **d.** *EFD*

2.



3. equal; corresponding; congruence; unknown

Fluency

4. a. $x° = 85°$ **b.** $x° = 34°$

c. $x° = 84°$ **d.** $x° = 140°$

e. $x° = 26°$ **f.** $x° = 36°$

g. $x° = 77°$ **h.** $x° = 45°$

5. **a.** $\triangle ABC \equiv \triangle EFD$ (SSS) **b.** $\triangle ABC \equiv \triangle FDE$ (SAS)
c. $\triangle ABC \equiv \triangle DEF$ (SAS) **d.** $\triangle ABC \equiv \triangle EDF$ (SSS)
e. $\triangle ABC \equiv \triangle FED$ (AAS) **f.** $\triangle ABC \equiv \triangle EDF$ (SAS)
g. $\triangle ABC \equiv \triangle EDF$ (RHS) **h.** $\triangle ABD \equiv \triangle CBD$ (SAS)

6. **a.** $\triangle ABC$ and $\triangle DEF$ are not congruent (SSS)
b. $\triangle ABC$ and $\triangle DEF$ are not congruent (ASA)
c. $\triangle ABC$ and $\triangle CDA$ are not congruent (AAS)
d. $\triangle ABC$ and $\triangle DFE$ are not congruent (AAS)
e. $\triangle ABC$ and $\triangle DEF$ are not congruent (RHS)
f. $\triangle ABC$ and $\triangle DEF$ are not congruent (SAS)
g. $\triangle ABC$ and $\triangle DEF$ are not congruent (ASA)
h. $\triangle ABC$ and $\triangle FDE$ are not congruent (ASA)

7. **a.** $x = 10$ **b.** $a° = 67°$
c. $a° = 43°$ **d.** $x = 19, y = 23$
e. a° = 71° **f.** $a° = 33°, b° = 22°$
g. $a° = 133°, b° = 26°$ **h.** $a° = 16°, b° = 60°$

8. D

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

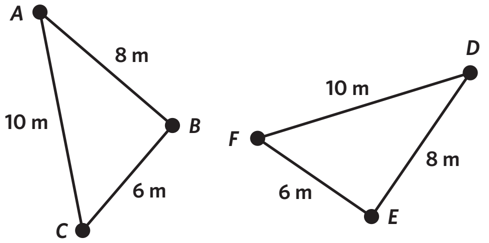
10. A landscape gardener is in the process of devising a layout for a garden bed by utilising three rope lengths: 10 m, 8 m, and 6 m. Using these ropes, the gardener generates two triangular garden beds for the client. Use a congruence test to determine if these two designs are different shapes.

Key points

- Two triangular garden beds.
- Triangles made of three side lengths: 10 m, 8 m, and 6 m.
- Determine if the triangles are congruent.

Explanation

Illustrate the two garden beds.



Identify the corresponding features.

$AB = DE$ (S)

$BC = EF$ (S)

$AC = DF$ (S)

$\therefore \triangle ABC \equiv \triangle DEF$ (SSS)

Answer

The two triangular garden beds are the same shape.

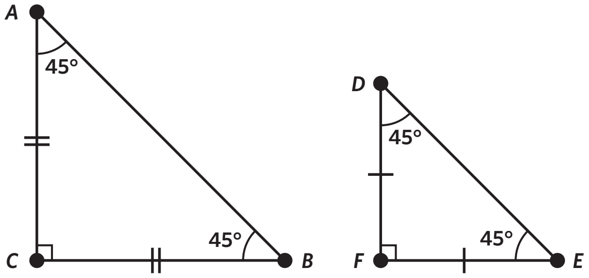
11. Alec and Sasha are making origami using identical square pieces of paper. Alec folds the paper in half once on the diagonal forming a right-angled triangle. Sasha also forms a right-angle triangle by folding the paper twice on the diagonal. Sasha says the two triangles are congruent because all the angles are the equal in both triangles. Explain using a congruence test if Sasha is correct.

Key points

- Alex creates a right-angled triangle by folding a paper diagonally in half.
- Sasha creates a right-angled triangle by folding a paper diagonally in half twice.
- Sasha claims that the two triangles are congruent due to the same angles.
- Explain whether Sasha's claim is true using a congruence test.

Explanation

Illustrate the two triangles.



Identify the corresponding features.

$\angle ABC = \angle DEF$ (A)

$\angle ACB = \angle DFE$ (A)

$\angle BAC = \angle EDF$ (A)

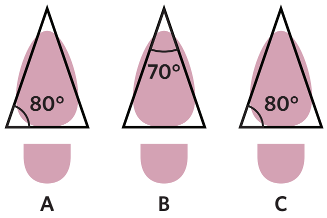
Only the angles are equal.

$\therefore \triangle ABC$ and $\triangle DEF$ are not congruent (SSS)

Answer

Sasha is not correct because the triangles are not congruent.

12. A detective is studying footprints found at three different scenes. The detective suspects they could be made by the same person since the isosceles triangles formed by the footprints have the same height when compared. Using a congruence test, explain which pairs of shoes, if any, could be worn by the same person.



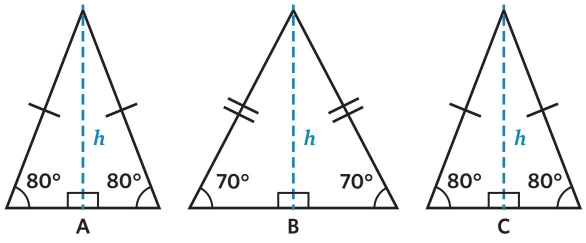
Key points

- The triangles are isosceles and have the same height.
- Use a congruence test to determine which two triangles are congruent.

Explanation

Identify corresponding features.

Triangles A and C share the 80° angle while triangle B does not.



Back to contents

Determine congruence.

△*A* and △*C* can both be split into two pairs of right-angled triangles that are all congruent by AAS.

$\therefore \triangle A \equiv \triangle C$ (ASA)

Answer

Shoes A and C could have come from the same person as the triangular shapes are congruent.

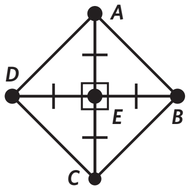
13. In a four-way tug of war, four teams compete against each other. Each team is equipped with an identical length of rope, and all four ropes converge at the centre. At the game's initiation, the angles formed between the ropes are all right angles. Utilise congruent triangles to demonstrate that for each team, the distances to the teams on the left and right are equivalent.

Key points

- Four teams playing tug of war.
- All ropes are equal length and converge in the centre.
- Angles between ropes are right angles.
- Demonstrate that the distance between the teams to the left and right are equal for all teams, using congruent triangles.

Explanation

Illustrate the tug of war game.



Identify shared characteristics.

All triangles have two equal sides and a shared 90° angle.

Determine whether the triangles are congruent.

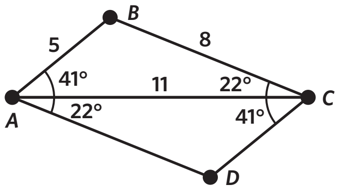
$\triangle ADE \equiv \triangle DCE \equiv \triangle CBE \equiv \triangle BAE$ by SAS.

$\therefore AB = BC = CD = DA$

Answer

The distance between each team's left and right team is equal since the triangles formed are congruent.

14. A long distance running course is being pegged out and is shown below. The instructions state that participants will need to go around the markings in the following order: A–B–C–A–D–C–D. What is the distance of the course?



Key points

- Long distance running course illustrated by diagram.
- Participants need to travel in the order A–B–C–A–D–C–D.
- Determine the distance of the entire course.

Explanation

Determine if the two triangles formed by the track are congruent.

Identify the shared features of the two triangles.

$\angle CAD = \angle ACB$ (A)

$\angle ACD = \angle BAC$ (A)

AC is shared (S)

$\therefore \triangle ABC \equiv \triangle ACD$ (ASA)

Identify the missing sides.

$AD = BC = 8$

$CD = AB = 5$

Calculate total track length.

$5 + 8 + 11 + 8 + 5 + 5 = 42$ units long.

Answer

The total distance of the course is 42 units.

Reasoning

15. a. An equation to represent the relationship between the lengths of the sliding pole and ladder is $AB = CD$.

b. An equation to represent the relationship between the lengths of the netting and floor section is $BC = AD$.

c. An equation to represent the relationship between the angles is $\angle ACD = \angle CAB$.

d. The floor and netting represented by *BC* and *AD*, respectively, are equal. The sliding pole and ladder represented by *AB* and *CD*, respectively, are equal. The ramp, represented by *AC*, is a shared side. Since all corresponding sides are of equal length, △*ABC* and △*CDA* are congruent.

e. Suggested option 1: Scaffolding is used to make the construction safer for workers.

Suggested option 2: Scaffolding is used to make construction easier for varied configurations.

Note: There are other possible options.

16. a. Triangle A and triangle B are congruent (RHS).

b. Triangle C is not congruent with triangle A and B since $\sqrt{10} \neq 3 \neq 4$ (SSS).

c. Two congruence tests could be RHS, SSS or SAS.

Exam-style

17. C

18. a. $AB = AC$

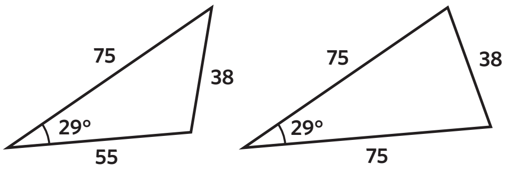
AD is shared

If $BD = CD$, $\triangle ABD \equiv \triangle ACD$ by the SSS congruence test.

b. $\angle BAD = \angle DAC$

19. Since $\triangle ABC \equiv \triangle ACD$ by ASA, $AB = CD$ and $BC = AD$.

20.



Triangles A and B would pass the congruence test using SSA even though they are not congruent triangles.

Remember this?

21. B **22.** C **23.** A

Back to contents

8C Quadrilaterals and other polygons

Student practice

Worked example 1

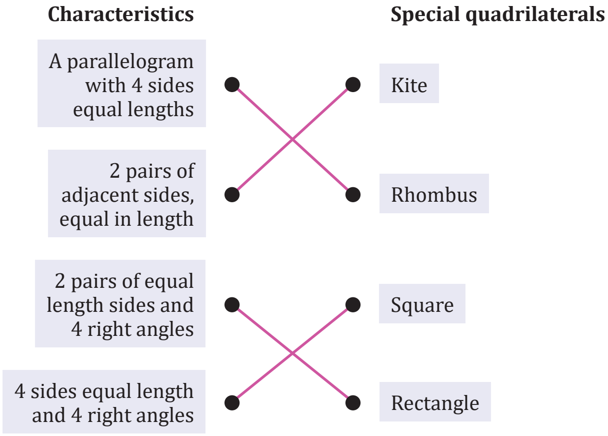
a. $a° = 47°$ **b.** $a° = 138°, b° = 42°$

Worked example 2

a. $a° = 150°$ **b.** $a° = 313°$

Understanding worksheet

1.



2. **a.** 8 **b.** 6 **c.** 10 **d.** 5

3. concave; quadrilaterals; sum; angles; interior

Fluency

4. **a.** $a° = 85°$ **b.** $a° = 206°$
c. $a° = 16°$ **d.** $a° = 77°$
e. $a° = 260°$ **f.** $a° = 49°, b° = 115°$
g. $a° = 246°, b° = 103°$ **h.** $a° = 14°, b° = 133°$

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5. **a.** $S = 540°, a° = 108°$
b. $S = 540°, a° = 71°$
c. $S = 720°, a° = 98°$
d. $S = 540°, a° = 108°$
e. $S = 720°, a° = 56°$
f. $S = 720°, a° = 113°$
g. $S = 1080°, a° = 135°$
h. $S = 540°, a° = 108°, b° = 72°$

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6. D

Spot the mistake

7. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

8. A builder is assessing whether a finished room can be considered a rectangle. After measuring all the dimensions of the room, the builder asserts that the room is rectangular due to two walls measuring 10 m and the other two measuring 5 metres long. Explain why this information does not confirm that the room is rectangular, provide visual representations of two potential room designs.

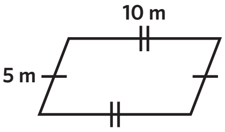
Key points

- A builder is determining if a finished room can be considered a rectangle and asserts that it is.
- He asserts this because there are two walls that are 10 m and the other two are 5 metres long.
- Explain why this information does not confirm that the room is rectangular, and provide visual representations of two potential room designs.

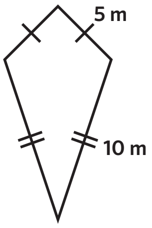
Explanation

A rectangle has two pairs of sides of equal length and four right angles. The description that there are two walls measuring 10 m and two others measuring 5 m does not explain the position of the sides nor whether there are right angles.

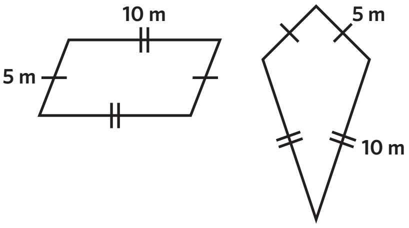
A room with these dimensions could be in the shape of a parallelogram with opposite sides being equal in length but without the right angles necessary in a rectangle.



A room with these dimensions could be in the shape of a kite with two pairs of adjacent sides that are equal in length.



Answer



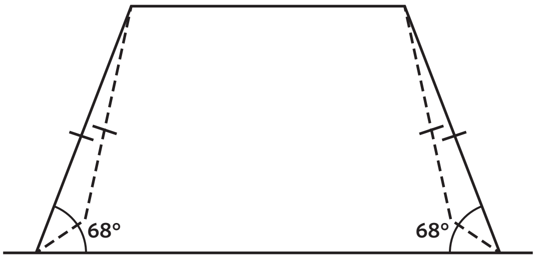
9. A swingset is built by connecting two isosceles triangles at the top with a single crossbar. To enhance stability, the triangles are inclined inward at the top, creating an angle of 68° with the ground. Given that the resulting shape is a trapezium, what is the angle formed between the crossbar and one of the triangles?

Key points

- A swingset is built by connecting two isosceles triangles at the top with a single crossbar.
- So the swingset is more stable, the triangles are inclined inward at the top, creating an angle of 68 degrees with the ground.
- The resulting shape from inclining is a trapezium, determine the angle formed between the crossbar and one of the triangles.

Explanation

Draw a diagram to represent the situation.



The internal angle sum of a trapezium is 360°. Given that the angles made with the ground and the triangles are both 68°, the angles made with the triangle and the crossbar must also be equal.

Solve for the unknown by adding the four angles and setting the sum equal to 360°.

$$68° + 68° + x° + x° = 360°$$
$$136° + 2x° = 360°$$
$$2x° = 224°$$
$$x° = 112°$$

Answer

The angle formed between the crossbar and one of the triangles is 112°.

10. An extendable table takes the form of a regular hexagon designed to seat 6 people when fully extended. By removing the central rectangular section and sliding the remaining triangles a smaller table is formed, capable of seating four people. What is the name of the shape of the smaller table, and what are the measurements of the angles formed at each vertex?

Key points

- An extendable table is in the form of a regular hexagon.
- By removing the central rectangular section and sliding the remaining triangles a smaller table is formed.
- Determine the name of the shape of the smaller table, and the measurements of the angles formed at each vertex.

Explanation

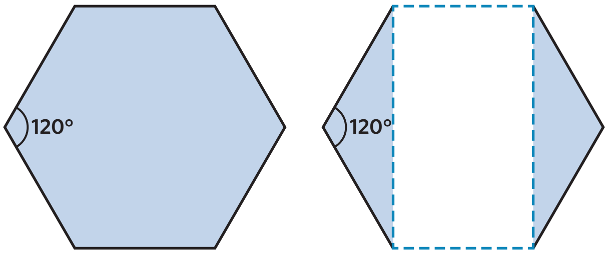
Calculate the internal angle sum of the regular hexagon.

$S = (n - 2) \times 180°$ when $n = 6$:

$S = (6 - 2) \times 180°$

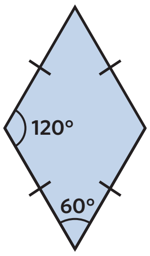
$S = 720°$

Draw the regular hexagon and remove the central rectangular section.



The triangles that are left are isosceles triangles with internal angles 120°, 30° and 30°.

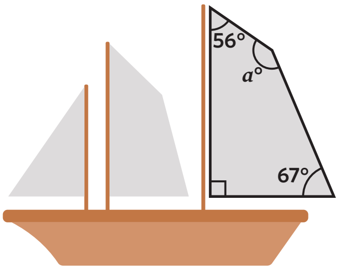
When the two triangles are combined a rhombus will be formed, where all side lengths are equal and pairs of opposite angles are equal. The size of the angles will be 120° and 60°.



Answer

The name of the shape of the smaller table is a rhombus with opposite and equal angles of 120° and 60°.

11. Sail boats often have triangular sails. Schooners are one of the various types of boats that use differently shaped sails. Calculate the internal angles of the sail shown.



Key points

- Calculate the internal unknown angle of the quadrilateral sail shown.

Explanation

The internal angles of a quadrilateral add to 360°.

$56° + 90° + 67° + a° = 360°$

Solve for the unknown, $a°$.

$$a° + 213° = 360°$$
$$a° = 360° - 213°$$
$$a° = 147°$$

Answer

The unknown internal angle of the sail shown is 147°.

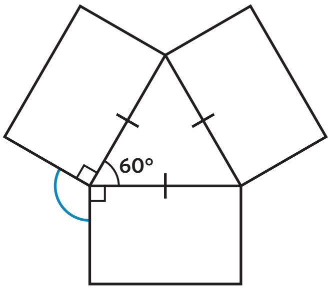
12. A tri-blade wind-turbine is formed by connecting congruent rectangles to the edges of an equilateral triangle. Determine the angle made between the blades.

Key points

- A tri-blade wind-turbine is formed by connecting congruent rectangles to the edges of an equilateral triangle.
- Determine the angle made between the rectangular blades.

Explanation

Draw a diagram to represent the situation.



The angle of a full revolution is 360° so to find the angle between the blades, solve for the unknown.

$$90° + 90° + 60° + x° = 360°$$
$$240° + x° = 360°$$
$$x° = 360° - 240°$$
$$x° = 120°$$

Answer

The angle made between the blades of the tri-blade wind turbine is 120°.

Reasoning

13. a. The total sum of the interior angles in the polygon is 720°.

b. The angle that completes the polygon is 270°.

c. The polygon is a concave polygon.

Back to contents

d. The total sum of the interior angles for the polygon representing the letter 'F' is 1440°.

e. Suggested option 1: To cut the material for the signs at the correct angle for the desired shape.

Suggested option 2: To plan for the amount of materials that need to be purchased for the signs.

Note: There are other possible options.

14. a. The interior angle $a°$ for this polygon is 108°.

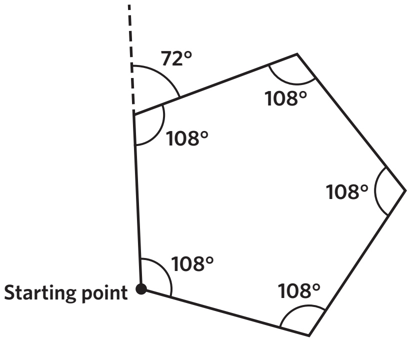
b. The exterior angle $b°$ for this polygon is 72°.

c. The formula for the rule for the exterior angle for any n-sided regular polygon is given by $180 - \frac{(n-2) \times 180}{n}$.

Exam-style

15. A

16. a.



b. 60°

17. $a° = 94°, b° = 135°$

18. A rhombus is not a suitable design for the roof of a building. A rhombus is symmetrical on its diagonals but not vertically or horizontally meaning the building needs to be built on an angle for the rhombus to cover it completely. A trapezium, square or rectangle would be a better suited quadrilateral for a roof.

Remember this?

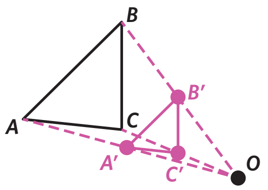
19. A **20.** B **21.** B

8D Enlargement and similar figures

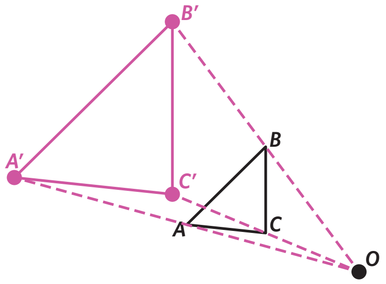
Student practice

Worked example 1

a.



b.



Worked example 2

a. Scale factor $= \frac{3}{2}, x = 30, y = 24$

b. Scale factor $= \frac{3}{4}, x = 36, y = 76$

Understanding worksheet

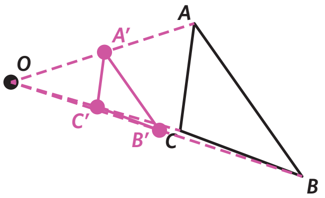
1. Less than 1: I; III
Equal to 1: IV
Greater than 1: II

2. a. EF **b.** EH **c.** FG **d.** NO

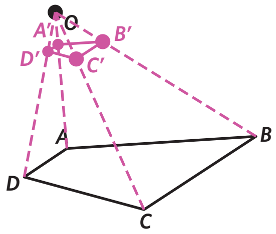
3. scale; enlargement; similar; corresponding; multiplied

Fluency

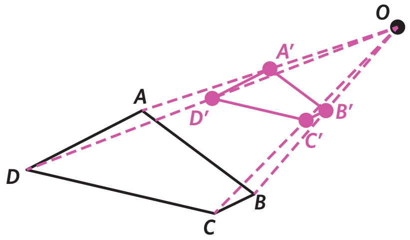
4. a.



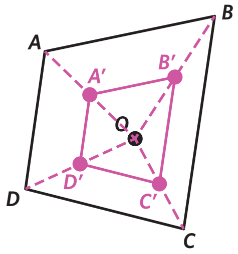
b.



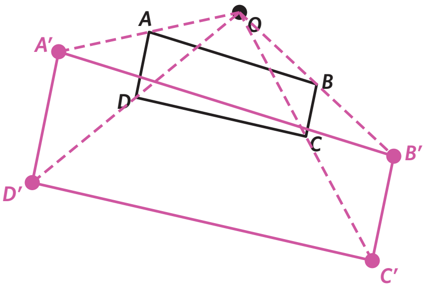
c.



d.

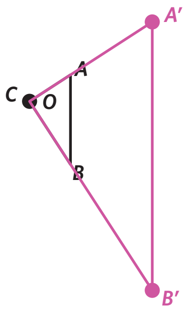


5. a.

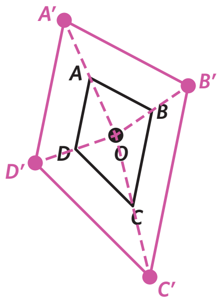


Back to contents

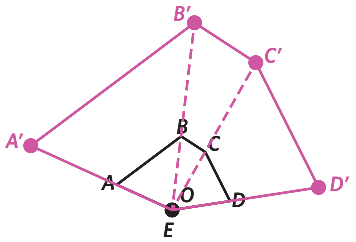
b.



c.



d.



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6. **a.** $x = 6$ **b.** $x = 12, y = 6$

c. $x = 24$ **d.** $x = 75$

e. $x = 29.5$

---

7. E

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. Max is creating a scale drawing of a car. The original wheels have a diameter of 86 cm. If Max is using a scale factor of $\frac{1}{10}$, determine the diameter of the wheels in the drawing.

Key points

- Original wheels have a diameter of 86 cm.
- Scale factor of $\frac{1}{10}$ for scale drawing.
- Determine the diameter of the wheels in the scale drawing.

Explanation

Understand that circles are similar figures.

Calculate the unknown diameter by multiplying the known diameter of the original by the scale factor.

Original length × scale factor = image length

$86 \times \frac{1}{10} = 8.6$ cm

Answer

The diameter of the wheels in the drawing is 8.6 cm.

10. An adventure park is planning to place some model dinosaurs around the park for decoration. They have a model Tyrannosaurus Rex that is 2 m tall. If an average adult Tyrannosaurus Rex is 12.4 m tall, calculate the scale factor if the actual dinosaur was used as the original, rounded to 1 decimal place.

Key points

- Model of Tyrannosaurus Rex is 2 m tall.
- Real, average length of a Tyrannosaurus Rex is 12.4 m tall.
- Calculate the scale factor.

Explanation

Calculate the scale factor by dividing the original length by the image length.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$\frac{2}{12.4} = 0.1612...$$

$$\approx 0.2$$

Answer

The scale factor of the model is approximately 0.2.

11. A team of architects are creating a model of an upcoming project. They have been provided with a rectangular plot of land measuring 4 km in length and 2 km in width. The model will be constructed on a platform that is 3 m long. Determine the scale factor they will be using for the model.

Key points

- Actual land's length is 4 km.
- Actual land's width is 2 km.
- The scale model's length is 3 m long.
- Determine the scale factor.

Explanation

Convert the length to metres.

$4 \times 1000 = 4000$ m

Calculate the scale factor by dividing the image length by the original length.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$= \frac{3}{4000}$$

Answer

The scale factor is $\frac{3}{4000}$.

12. The Burj Khalifa in Dubai stands at a height of 828 m, while the Merdeka 118 in Kuala Lumpur reaches a height of 677 m. Calculate the scale factor of Merdeka 118 in comparison to the Burj Khalifa, rounded to 2 decimal places.

Key points

- Burj Khalifa is 828 m tall.
- Merdeka 118 is 677 m tall.
- Calculate the scale factor.

Explanation

Calculate the scale factor by dividing the image length (Burj Khalifa) by the original length (Merdeka 118).

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$\frac{828}{677} \approx 1.22$$

Answer

The scale factor is approximately 1.22.

13. In a photograph of an old house, the windows appear to be 3 cm tall, and the first floor is 5 cm above the ground. If the original windows were 2.5 m tall, calculate the height, in metres, of the first floor from the ground, rounded to 2 decimal places.

Key points

- The window appears to be 3 cm tall in the photo.
- The first floor of the house is 5 cm above the ground in the photo.
- The original window height was 2.5 m.
- Calculate the actual height of the first floor from the ground.

Explanation

Convert the window height to centimetres.

$2.5 \times 100 = 250$ cm

Calculate the scale factor by dividing the original window height by the height in the photograph.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$= \frac{250}{3}$$

Calculate the image length by multiplying the original length in the photograph by the scale factor.

Image length × scale factor = original length

$$\frac{250}{3} \times 5 = \frac{1250}{3}$$

$$\approx 417 \text{ cm}$$

Convert to metres.

$417 \div 100 = 4.17$ m

Answer

The height of the first floor from the ground is approximately 4.17 m.

Reasoning

14. **a.** A scale factor of 2.

b. The length is 32 cm.

c. A scale factor of 10.

d. Suggested option 1: The image will become distorted.

Suggested option 2: The image will become misleading.

Note: There are other possible options.

15. **a.** The two circles are similar.

b. The two rectangles are not similar.

c. Two shapes are similar if the ratio of the distances between corresponding points is equal and constant.

Exam-style

16. E

17. **a.** $\frac{1}{250}$ **b.** $\frac{1}{400}$ **c.** 1.25 m

18. Scale factor: $\frac{1}{2}$

Coordinates for centre of enlargement = (3,–7)

19. Scale factor = $\frac{1}{40}$, door length = 5.5 cm

Remember this?

20. B **21.** C **22.** C

8E Similar triangles

Student practice

Worked example 1

a. $\triangle PQR \sim \triangle STU$ (AAA) **b.** $\triangle XYZ \sim \triangle TPY$ (SAS)

Worked example 2

a. The scale factor of $\triangle STU$ to $\triangle XVY$ is 4.

b. The scale factor of $\triangle ABC$ to $\triangle WXV$ is $\frac{4}{5}$.

Worked example 3

a. $x = 3.6$ cm **b.** $x = 7.1$

Understanding worksheet

1. **a.** SSS (side-side-side)

b. SAS (side-angle-side)

c. RHS (right angle-hypotenuse-side)

d. AAA (angle-angle-angle)

2. **a.** $\angle ABC = 49°$ **b.** $\angle DFE = 74°$

c. $\frac{AB}{FD} = \frac{63}{42}$ **d.** $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF} = \frac{3}{2}$

3. shape; equal; angles; vertices

Fluency

4. **a.** $\triangle ABC \sim \triangle PTR$ (SSS) **b.** $\triangle WXY \sim \triangle DEG$ (AAA)

c. $\triangle XYZ \sim \triangle SPT$ (RHS) **d.** $\triangle KNO \sim \triangle TYZ$ (SAS)

e. $\triangle ABC \sim \triangle OMN$ (SAS)

5. **a.** The scale factor of $\triangle KLM$ to $\triangle BAC$ is 3.

b. The scale factor of $\triangle TVW$ to $\triangle HFG$ is $\frac{3}{2}$.

c. The scale factor of $\triangle PQR$ to $\triangle UST$ is $\frac{3}{4}$.

d. The scale factor of $\triangle AOD$ to $\triangle COB$ is $\frac{2}{3}$.

e. The scale factor of $\triangle PQT$ to $\triangle PRS$ is $\frac{7}{4}$.

6. **a.** $x = 6$ **b.** $a \approx 69.3$

c. $y \approx 12.1$ cm **d.** $x \approx 68.6$

e. $x \approx 17.3$

7. A

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

9. A triangular area has been traced on a map. The dimensions of the drawn triangle are 2.4 cm, 3.5 cm, and 5.1 cm. The map has a scale of 150 km to 1 cm. What are the real-life dimensions of the triangular area, in km?

Key points

- The dimensions of the drawn triangle are 2.4 cm, 3.5 cm, and 5.1 cm.
- The map has a scale of 150 km to 1 cm.
- What are the real-life dimensions of the triangular area?

Explanation

The scale factor of the drawn map to real-life dimensions is 150.

1 cm = 150 km

Multiply the dimensions of the drawn triangle by the scale factor to calculate the real-life dimensions.

$2.4 \times 150 = 360$ km

$3.5 \times 150 = 525$ km

$5.1 \times 150 = 765$ km

Answer

The real-life dimensions of the triangular area are 360 km, 525 km, and 765 km.

10. Two triangles are similar and their longest sides are 81 and 54 units. What is the scale factor of the larger triangle to the smaller triangle?

Key points

- Two triangles are similar.
- The longest sides of the triangles are 81 and 54 units.
- What is the scale factor of the larger triangle to the smaller triangle?

Explanation

The two triangles are similar so the longest sides will be corresponding side lengths.

Calculate the ratio of the corresponding side lengths.

$$\frac{54}{81} = \frac{2}{3}$$

Answer

The scale factor of the large triangle to the smaller triangle is $\frac{2}{3}$.

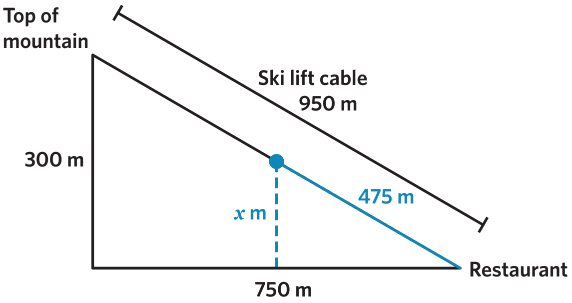
11. A ski lift cable extends from the top of the mountain to the restaurant. The top of the mountain is 300 m above the ground where the restaurant is located 790 m away, and the cable is approximately 950 m long. Calculate how high up from the ground a skier is halfway down the line, to the nearest metre.

Key points

- The top of the mountain is 300 m above the ground.
- The restaurant is located 790 m away.
- The ski lift cable, which extends between these points, is approximately 950 m long.
- Calculate how high up from the ground a skier is halfway down the line to the nearest metre.

Explanation

The right-angled triangles in the diagram are similar.



Form an equation for the side length ratio using corresponding side lengths and solve for the unknown.

$$\frac{x}{300} = \frac{475}{950}$$

$$x = \frac{475}{950} \times 300$$

$$= 150 \text{ m}$$

Answer

The skier is 150 m above the ground.

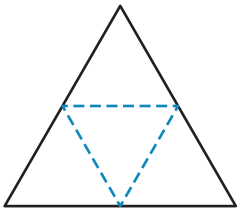
12. An equilateral triangle has been split into similar triangles by drawing lines between the midpoints of each side. What is the scale factor of the smaller triangles to the larger triangle?

Key points

- An equilateral triangle has been split by drawing lines between the midpoints of each side.
- What is the scale factor of the smaller triangles to the larger triangle?

Explanation

When the equilateral triangle is split by drawing lines between the midpoints of each side, 4 smaller similar triangles are formed.



Each smaller triangle's side lengths are exactly half the length of the larger triangle's side lengths. Therefore, double the side lengths of the smaller triangle to scale to the larger triangle.

Answer

The scale factor of the smaller triangles to the larger triangle is 2.

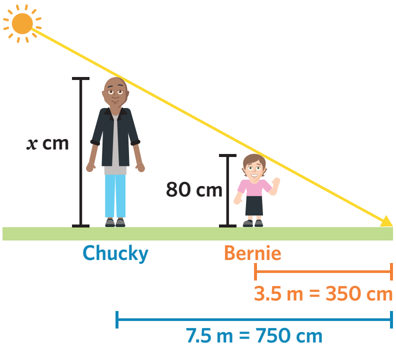
13. The sun is shining down on Bernie and Chucky at the same angle. They cast shadows on the ground that are perpendicular to each person's body. Bernie is 80 cm tall and her shadow is 3.5 m long. If Chucky's shadow is 7.5 m long, then how tall is Chucky, to the nearest centimetre?

Key points

- The is shining down on Bernie and Chucky at the same angle.
- The shadows are perpendicular to their bodies.
- Bernie is 80 cm tall.
- Bernie's shadow is 3.5 m long.
- Chucky's shadow is 7.5 m long.
- How tall is Chucky, to the nearest centimetre?

Explanation

The angle of the sun shining down on Bernie and Chucky is the same. The shadows are perpendicular to their bodies, which forms a right angle. All three angles are therefore equal, so the triangles are similar.



Form an equation for the side length ratio using corresponding side lengths.

$$\frac{\text{Chucky's height}}{\text{Bernie's height}} = \frac{\text{Chucky's shadow}}{\text{Bernie's shadow}}$$

Substitute the given values, in centimetres, into the side length ratio equation and solve for the unknown.

$$\frac{x}{80} = \frac{750}{350}$$

$$\begin{aligned} x &= \frac{750}{350} \times 80 \\ &= 171.428... \\ &\approx 171 \text{ cm} \end{aligned}$$

Answer

Chucky is 171 cm tall.

Reasoning

14. **a.** The width of the river is *BZ*.

b. $\triangle AOX$ and $\triangle BOZ$ are similar triangles.

c. AAA can be used to prove that $\triangle AOX$ and $\triangle BOZ$ are similar.

d. The width of the river is 12.7 m.

e. Suggested option 1: Similar triangles may be used by astronomers to estimate the distance between two celestial objects.

Suggested option 2: Similar triangles may be used by navigators to estimate the distance between two points on a map.

Note: There are other possible options.

15. **a.** The size of $\angle XYE$ is 57°.

b. AAA proves that $\triangle XYE$ is similar to $\triangle HPZ$.

c. It is not necessary to know all three internal angles of a pair of triangles in order to test them for similarity. This is because if two internal angles are known, the size of the third internal angle can be calculated, given that the sum of the internal angles in a triangle is always 180°. Therefore, the AAA test for similarity can be conducted when any two internal angles of each triangle in a similar pair are known.

Exam-style

16. D

17. **a.** $\triangle ABD$ and $\triangle ECD$

b. The scale factor of $\triangle ABD$ to $\triangle ECD$ is $\frac{2}{3}$.

c. $CE \approx 2.7$ cm

18. $x \approx 33.4$

19. 72 cm

Remember this?

20. D **21.** C **22.** B

Chapter 8 extended application

1. **a.** The types of angles that are formed when two parallel roads are intersected by a transversal road are corresponding angles, alternate angles, and co-interior angles. Corresponding angles are in matching positions and are equal, alternate angles are on either side of the transversal and are equal, and co-interior angles are in between the two parallel lines and the angles are supplementary.

b. The engineers can check whether the angle corresponding to 36° at intersection *A* is the same at intersection *B*. This is verified by subtracting 144° from 180°, producing 36° and proving the lines are parallel. The engineers can also check whether the angle alternate to 36° at intersection *A* is the same at intersection *B*. This is verified by subtracting 144° from 180°, producing 36° and again proving the lines are parallel. Lastly the properties of co-interior angles being supplementary can also be used to prove that the lines are parallel. Adding 36° to 144° produces 180°, therefore they are supplementary angles.

c. For a road to form another right angle it must be parallel to either road *AH* or road *IK*. As there are no parallel roads to *AH* or *IK* there are no other right angles in this system of roads.

d. $\angle CAB = 36°, \angle ABD = 144°, \angle DCA = 54°, \angle BDC = 126°$

e. $\angle ACK = 126°, \angle KCL = 54°, \angle LCJ = 48°, \angle JCD = 78°,$ $\angle ACD = 54°$

f. Suggested option 1: City planners should think about having enough greenery such as parks.

Suggested option 2: City planners should think about where the highest volume of traffic will be.

Note: There are other possible options.

2. **a.** The scale factor of the existing bridge to the new pedestrian bridge is $\frac{2}{3}$.

b. The new bridge will have a length of 40 m, a height of 10 m and a base height of 13 m.

c. The minimal conditions required to prove that two triangles are congruent in shape and size for this bridge are that corresponding sides are equal in length.

d. The total length of steel necessary for the construction of these diagonal segments for both sides of the pedestrian bridge is 256 m.

e. Suggested option 1: Architects should think about the strength of the material used to build the bridge.

Suggested option 2: Architects should think about accessibility for everyone, such as those in wheelchairs.

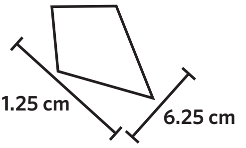
Note: There are other possible options.

3. **a.** The scale factor that relates the wingspan of Bird A to the wingspan of Bird B is $\frac{5}{4}$.

b. The length of the corresponding wing bone in Bird B is 2.5 cm.

c. The minimal conditions required to prove that the skeletal structures of Bird A and Bird B are similar are that the ratios between the lengths of corresponding sides of the structures are equal.

d. Wing of Bird B



The predicted surface area of the wing of Bird B is 35.156 cm^2.

e. Suggested option 1: If the birds do not have enough food in their habitat, they may not be able to grow as much, decreasing wingspan.

Suggested option 2: If the birds live amongst many predators whom they are regularly flying away from, they may have stronger wings, increasing wingspan.

Note: There are other possible options.

Back to contents

Chapter 8 review

Multiple choice

1. C **2.** D **3.** A **4.** C **5.** B

Fluency

6. **a.** $a° = 145°$ because $a°$ and 145° are corresponding angles on parallel lines.

b. $a° = 60°$ because $a°$ and 60° are alternate angles on parallel lines.

c. $a° = 93°$ because $a°$ and 87° are co-interior angles on parallel lines.

$b° = 87°$ because $b°$ and 87° are corresponding angles on parallel lines.

d. $a° = 68°$ because $a°$ and 68° are alternate angles on parallel lines.

$b° = 39°$ because $b°$, $a°$, and 73° form a supplementary angle of 180°.

7. **a.** $x° = 57°$ **b.** $x° = 47°$

c. $x° = 158°$ **d.** $x° = 63°$

8. **a.** $\triangle ABC$ and $\triangle DEF$ are not congruent (SSS).

b. $\triangle HIG$ and $\triangle JKL$ are congruent (SAS).

c. $\triangle NOM$ and $\triangle PQR$ are congruent (ASA).

d. $\triangle TUS$ and $\triangle VWX$ are not congruent (RHS).

9. **a.** $x° = 62°$ **b.** $x° = 227°$

c. $x° = 239°$ **d.** $x° = 21°$

10. **a.** $S = 540°, x° = 114°$ **b.** $S = 540°, x° = 108°$

c. $S = 720°, x° = 98°$ **d.** $S = 1080°, x° = 92°$

11. **a.** $x = 2$ **b.** $x = 14, y = 5$

c. $x = 2.25, y = 16$ **d.** $x \approx 5.45, y = 11$

12. **a.** $\triangle KLM \sim \triangle NOP$ (SSS) **b.** $\triangle ABC \sim \triangle UWV$ (AAA)

c. $\triangle EFD \sim \triangle GHI$ (RHS) **d.** $\triangle NOM \sim \triangle RPQ$ (SAS)

13. **a.** The scale factor of $\triangle XVW$ to $\triangle YZA$ is $\frac{1}{3}$.

b. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{7}{4}$.

c. The scale factor of $\triangle GHI$ to $\triangle KJI$ is $\frac{1}{2}$.

d. The scale factor of $\triangle LMN$ to $\triangle OPN$ is $\frac{2}{5}$.

Problem solving

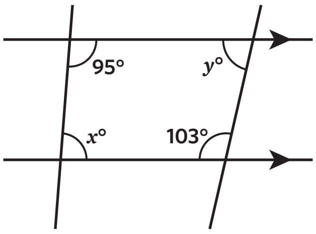
14. A courtyard is enclosed by four walls. The internal angles within the courtyard are given as 95°, $x°$, $y°$, and 103°. Given that two of the walls forming the courtyard are parallel, determine the values of $x°$ and $y°$.

Key points

- A courtyard is enclosed by four walls.
- The internal angles are 95°, $x°$, $y°$, and 103°.
- Two of the walls of the courtyard are parallel to each other.
- Determine the values of $x°$ and $y°$.

Explanation

Given that two of the walls forming the courtyard are parallel, two sets of co-interior angles on parallel lines are formed. The first set of co-interior angles is $x°$ and 95° and the second set of co-interior angles is $y°$ and 103°.



Co-interior angles on a parallel line sum to 180°.

$\therefore x° + 95° = 180°$

$x° = 85°$

$y° + 103° = 180°$

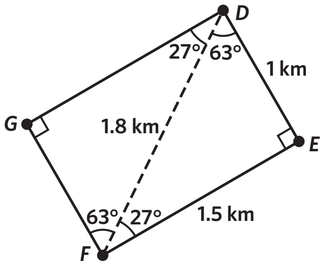
$y° = 77°$

Answer

$x°$ equals 85° and $y°$ equals 77°.

Note: The values of angles $x°$ and $y°$ are interchangeable.

15. The run and swim legs of a triathlon course are mapped out as shown below, with the line DF representing the shoreline.



The athletes are to first complete the swim leg, going from D–E–F followed by running back to the starting point D and completing the D–G–F–D run leg. They then complete their bike leg, which is 10 km. Determine the total distance of the triathlon.

Key points

- The diagram maps out the run and swim legs of a triathlon course.
- The swim leg goes from D–E–F.
- The run leg goes from F–D–G–F–D.
- The bike leg is 10 km.
- Determine the total distance of the triathlon.

Explanation

Determine if the two triangles formed by the triathlon course are congruent by identifying the shared features of the two triangles.

$\angle DFG = \angle FDE$ (Angle)

$\angle FDG = \angle DFE$ (Angle)

FD is shared (Side)

$\therefore \triangle DFG \equiv \triangle DFE$ (ASA)

Identify the known side lengths.

$DE = FG = 1$ km

$EF = DG = 1.5$ km

$DF = 1.8$ km

Calculate total distance of the triathlon course.

Total distance = distance of swim leg + distance of run leg + distance of bike leg

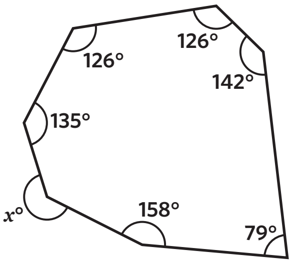
$= (1 + 1.5) + (1.8 + 1.5 + 1 + 1.8) + (10)$

$= 18.6$ km

Answer

The total distance of the triathlon course is 18.6 km.

16. A greenhouse roof is built in an irregular shape to maximise the sunlight exposure. Calculate the missing angle $x°$ from the greenhouse.



Key points

- The diagram shows the irregular shape of a greenhouse roof.
- Calculate the missing angle $x°$.

Explanation

$S = (n - 2) \times 180°$ where S is the internal angle sum and n is the number of sides of the shape.

Calculate the internal angle sum of the greenhouse roof, given that $n = 7$.

$S = (7 - 2) \times 180°$

$= 5 \times 180°$

$= 900°$

Use the internal angle sum to calculate the unknown internal angle.

Unknown internal angle $= 900° -$ all known internal angles

$= 900° - 135° - 126° - 126° - 142° - 79° - 158°$

$= 134°$

134° and $x°$ form a full revolution, which is 360°. Calculate $x°$.

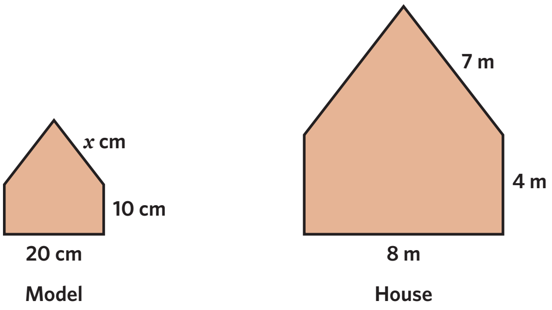
$x° = 360° - 134°$

$= 226°$

Answer

The missing angle $x°$ is 226°.

17. Tom is an architect who builds models of houses. The house he is currently working on has a front facade shaped as a pentagon. The dimensions of the facades of both the model and house are shown in the diagram. Determine the scale factor from the model to the house, and determine the value of x in centimetres.



Key points

- A house has a front facade shaped as a pentagon.
- The diagram shows the dimensions of the facades of both the model and house.
- Determine the scale factor from the model to the house, and determine the value of x in centimetres.

Explanation

Convert the lengths of the house to centimetres so all dimensions are the same unit of measurement.

1 m = 100 cm

∴ 8 m = 800 cm, 7 m = 700 cm, 4 m = 400 cm

Calculate the scale factor by dividing the image length (house) by the original length (model).

$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$= \frac{\text{house length}}{\text{model length}}$$

$$= \frac{400}{10}$$

$$= 40$$

The scale factor is 40.

When this scale factor is applied to x, the result is 700 cm. Represent this in an equation and solve for x.

$x \times 40 = 700$

$x = \frac{700}{40}$

$x = 17.5$ cm

Answer

The scale factor from the model to the house is 40. The value of x is 17.5 cm.

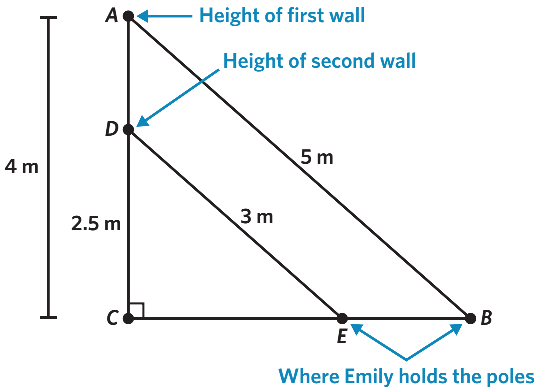
18. Emily wants to hang a banner on a wall that is 4 m high. She uses a pole that is 5 m long to reach the top of the wall. Later, she needs to hang a smaller banner on a different wall that is 2.5 m high. She uses a shorter pole that is 3 m long for this task. Determine if two similar triangles are formed from the poles.

Key points

- Emily wants to hang a banner on a wall that is 4 m high.
- The pole is 5 m long.
- Emily then uses another pole to reach a smaller banner on a different wall that is 2.5 m high.
- The pole is 3 m long.
- Determine if two similar triangles are formed from the poles.

Explanation

Create a diagram that represents this scenario.



If the hypotenuses and another pair of corresponding side lengths are in the same ratio, then two right-angled triangles are similar.

Determine if the two triangles in the diagram are similar by RHS.

$\angle ABC = \angle DEC = 90°$

$\frac{AB}{DE} = \frac{5}{3} = 1.\overline{6}$

$\frac{AC}{DC} = \frac{4}{2.5} = 1.6$

$\frac{5}{3} \neq \frac{4}{2.5}$

The hypotenuses and side lengths are not in the same ratio,
∴ the triangles are not similar.

Answer

The two triangles formed by the poles are not similar.

Reasoning

19. a. The unknown angle $x°$ equals 63°.

b. The triangles are congruent by SSS, meaning they have three corresponding sides of the same length.

c. Each internal angle of the hexagonal fountain should be 120°.

d. The length of the side in the scale model is 2 m.

e. Suggested option 1: It may cost Sarah more to purchase the materials for the actual construction of the park compared to the scale model.

Suggested option 2: It may take Sarah longer to actually construct the park compared to the scale model.

Note: There are other possible options.

20. a. The interior angle $x°$ is 135°.

b. The exterior angle $y°$ is 45°.

c. The sum of $x°$ and $y°$, which is 180°, is not specific to this polygon. Interior and exterior angles of polygons that are on a straight line are supplementary, meaning they sum to 180°.

Back to contents

9A Pythagoras' theorem

Student practice

Worked example 1

a. 5 **b.** 9.09

Worked example 2

a. 5 **b.** 6.55

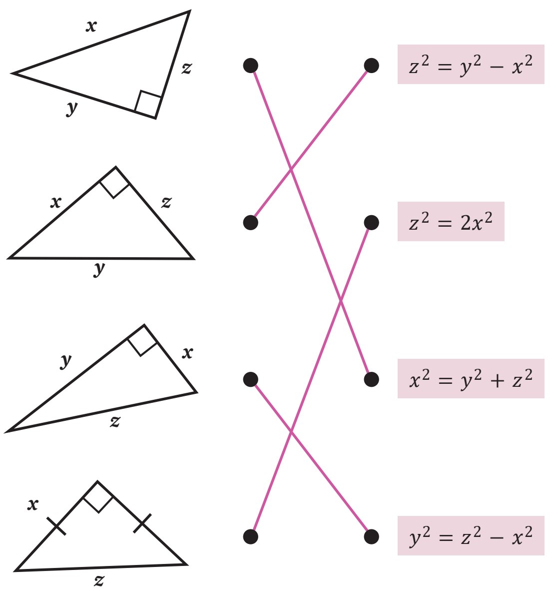
Worked example 3

a. $\sqrt{74}$ **b.** $\sqrt{55}$

Understanding worksheet

1. **a.** p **b.** m **c.** t **d.** a

2. **Triangle** **Side length relationship**



3. hypotenuse; lengths; square; sum; two

Fluency

4. **a.** 15 **b.** 17 **c.** 6.71 **d.** 8.22
e. 14.15 **f.** 13.51 **g.** 12.87 **h.** 0.64

5. **a.** 24 **b.** 16 **c.** 6.71 **d.** 6.12
e. 8.48 **f.** 2.4 **g.** 7.24 **h.** 15.04

6. **a.** $\sqrt{5}$ **b.** $\sqrt{2}$ **c.** $\sqrt{15}$ **d.** $\sqrt{91}$
e. $\sqrt{145}$ **f.** $\sqrt{795}$ **g.** $\sqrt{95}$ **h.** $\sqrt{69}$

7. **a.** 9.43 cm **b.** 14.96 cm
c. 13.75 cm **d.** 19.84 cm
e. 10.15 cm **f.** 2.99 cm
g. 6.09 cm **h.** 54.71 cm

8. C

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

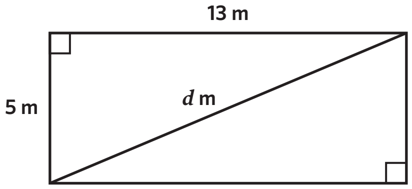
10. Jenny's backyard is rectangular with sides 13 m and 5 m long. She wants to install a fence running diagonally from one corner to the opposite corner of the yard. Calculate the length of the fence to the nearest metre.

Key points

- Jenny's backyard is rectangular in shape.
- The side lengths of the backyard are 13 m and 5 m long.
- She wants to install a fence running diagonally from one corner to the other corner of the backyard.
- Determine the length of this fence to the nearest metre.

Explanation

Draw a diagram to represent the situation.



To determine the diagonal distance apply Pythagoras' theorem formula.

$a = 5$
$b = 13$
$c = d$
$d^2 = 5^2 + 13^2$
$d^2 = 194$
$d = \sqrt{194}$
$= 13.928...$
≈ 14

Answer

The length of the diagonal fence in Jenny's backyard is 14 m.

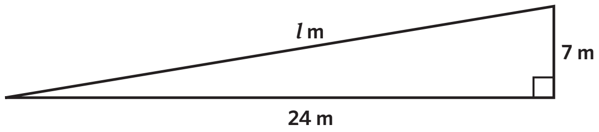
11. A swimming pool, 24 m long, has a sloped floor so that the increase in depth is gradual from 0 m on one side to its maximum depth of 7 m on the other side. Calculate the length of the sloped floor of the pool, in metres.

Key points

- A swimming pool is 24 m in length.
- The pool has a sloped floor.
- The depth increases gradually from 0 m on one side to its maximum depth of 7 m on the other side.
- Calculate the length of the sloped bottom of the pool in metres.

Explanation

Draw a diagram to represent the situation.



To calculate the length of the sloped floor apply Pythagoras' theorem formula.

$a = 24$

$b = 7$

$c = l$

$l^2 = 24^2 + 7^2$

$l^2 = 576 + 49$

$l^2 = 625$

$l = \sqrt{625}$

$\quad = 25$

Answer

The length of the sloped floor of the pool is 25 m.

12. A wheelchair ramp is shaped like a right-angled triangle, where the hypotenuse represents the ramp's length and the side which makes a right angle with the height represents the ramp's run. To meet requirements, a 1 m high wheelchair ramp's length must be at least 14 m. Determine the minimum ramp run, in metres, of a 1 m high wheelchair ramp, rounded to 2 decimal places.

Key points

- A wheelchair ramp is shaped like a right-angled triangle.
- The length of the ramp is considered to be the hypotenuse.
- The side which makes a right angle with the height is called the ramp's run.
- A 1 m high wheelchair ramp's length must be at least 14 m.
- Determine the minimum ramp run, in metres, of a 1 m high wheelchair ramp, rounded to 2 decimal places.

Explanation

To calculate the minimum ramp run (r), of a 1 m high wheelchair ramp Pythagoras' theorem formula must be used.

$a = \text{ramp height} = 1$

$b = \text{ramp run} = r$

$c = \text{ramp length} = 14$

$14^2 = 1^2 + r^2$

$r^2 = 14^2 - 1^2$

$r^2 = 196 - 1$

$r^2 = 195$

$r = \sqrt{195}$

$\quad = 13.964...$

$\quad \approx 13.96$

Answer

The minimum ramp run of a 1 m high wheelchair ramp is 13.96 m.

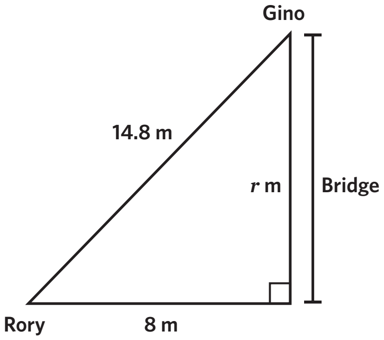
13. Rory is standing on a river bank when he sees his friend Gino on the other side at the end of a bridge. The direct distance between the two friends is 14.8 m at this point. Rory walks a total of 8 m to the end of the bridge along his side of the river and walks across it to meet Gino. Determine the distance, in metres, across the river, rounded to 1 decimal place.

Key points

- The direct distance between two friends Gino and Rory is 14.8 m.
- Rory walks 8 m to the end of the bridge along his side of the river and then walks across it to meet Gino.
- Determine the distance, in metres, across the river, rounded to 1 decimal place.

Explanation

Draw a diagram to represent the situation.



To calculate the distance across the river (r), use Pythagoras' theorem formula.

$a = 8$

$b = r$

$c = 14.8$

$14.8^2 = 8^2 + r^2$

$r^2 = 14.8^2 - 8^2$

$r^2 = 219.04 - 64$

$r^2 = 155.04$

$r = \sqrt{155.04}$

$\quad = 12.45...$

$\quad \approx 12.5$

Answer

The distance that Rory walks across the river to reach Gino is 12.5 m.

14. Sam is casting a shadow on the ground. The direct distance between the top of Sam's head and the top of his shadow's head is 2.3 m. Given that Sam's shadow is 1.7 m long, calculate Sam's height, in metres, correct to 2 decimal places.

Key points

- The direct distance between the top of Sam's head and the shadow he is casting is 2.3 m.
- Sam's shadow is 1.7 m in length.
- Determine Sam's height in metres, correct to 2 decimal places.

Explanation

To calculate Sam's height (h), identify the base, height and hypotenuse that represent this situation and use Pythagoras' theorem formula.

$a = \text{length of Sam's shadow} = 1.7$

$b = \text{Sam's height} = h$

$c = \text{distance between Sam's head and the shadow} = 2.3$

$2.3 = 1.7^2 + h^2$

$h^2 = 2.3^2 - 1.7^2$

$r^2 = 5.29 - 2.89$

$r^2 = 2.4$

$r = \sqrt{2.4}$

$\quad = 1.549...$

$\quad \approx 1.55$

Answer

Sam's height is 1.55 m.

Back to contents

Reasoning

15. **a.** The hypotenuse in $\triangle ACE$ is AC.

b. $x^2 + y^2 = c^2$

c. The length of the hypotenuse of $\triangle ACE$ is 7.81 m.

d. The total length of timber needed for the entire frame is 51 m.

e. Suggested option 1: More freedom and creativity in the design of the house.

Suggested option 2: It is expensive to hire specialised workers for aspects of the build.

Note: There are other possible options.

16. **a.** $A = \frac{ab}{2}$

b. $b - a$

c. $A_{big\ square} = A_{triangles} + A_{small\ square}$

$$\begin{aligned} c^2 &= 4 \times \frac{ab}{2} + (b - a)^2 \\ &= 2ab + (b^2 - 2ab + a^2) \\ &= a^2 + b^2 \end{aligned}$$

Exam-style

17. B

18. **a.** 14.3 m **b.** 15.4 m

19. 48 cm^2

20. Let the width of the room be w.

$$\begin{aligned} w^2 &= 13.5^2 - 11^2 \\ &= 182.25 - 121 \\ &= 61.25 \\ w &= \sqrt{61.25} \\ &= 7.826... \\ &\approx 7.83 \text{ m} \end{aligned}$$

Remember this?

21. B **22.** C **23.** C

9B Trigonometric ratios

Student practice

Worked example 1

a. $\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

b. $\sin\theta = \frac{21}{41}$

$\cos\theta = \frac{35}{41}$

$\tan\theta = \frac{3}{5}$

Worked example 2

a. $\tan36° = \frac{x}{53}$ **b.** $\cos\theta = \frac{44}{53}$

Understanding worksheet

1. **a.** z **b.** x **c.** y **d.** x

2. **a.** x **b.** z **c.** x **d.** z

3. hypotenuse; adjacent; side; tangent; angles

Fluency

4. **a.** $\sin\theta = \frac{15}{17}$

$\cos\theta = \frac{8}{17}$

$\tan\theta = \frac{15}{8}$

b. $\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{5}{13}$

$\tan\theta = \frac{12}{5}$

c. $\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

d. $\sin\theta = \frac{x}{10}$

$\cos\theta = \frac{x}{5}$

$\tan\theta = \frac{1}{2}$

5. **a.** $\cos52° = \frac{11}{y}$ **b.** $\sin34° = \frac{7}{x}$

c. $\tan18° = \frac{1}{x}$ **d.** $\tan48° = z$

e. $\sin45° = \frac{1}{x}$ **f.** $\tan62° = \frac{5}{x}$

g. $\cos60° = \frac{x}{3y}$ **h.** $\tan60° = y$

6. **a.** $\tan\theta = \frac{45}{34}$ **b.** $\cos\theta = \frac{25}{32}$

c. $\sin\theta = \frac{2}{3}$ **d.** $\tan\theta = \frac{25}{11}$

e. $\sin\theta = \frac{25}{29}$ **f.** $\sin\theta = \frac{13}{18}$

g. $\cos\theta = \frac{15}{29}$ **h.** $\tan\theta = \frac{13}{17}$

7. B

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. Council stipulates that the tangent ratio of an access ramp cannot exceed $\frac{1}{14}$. Create a triangle to illustrate the steepest ramp, and label the reference angle and any known side lengths.

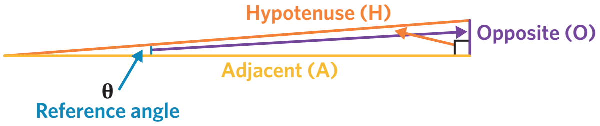
Key points

- Ramp's tangent ratio cannot exceed $\frac{1}{14}$.
- Create a triangle to represent the steepest ramp and label all sides using the reference angles.

Explanation

Create a right-angled triangle and identify a reference angle.

Label the sides using the reference angle.



Identify the provided sides from $\tan\theta = \frac{1}{14}$.

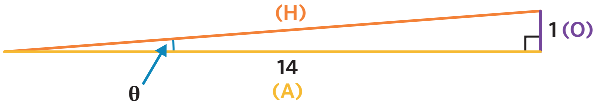
$\tan\theta = \frac{1}{14}$

$\tan\theta = \frac{O}{A}$

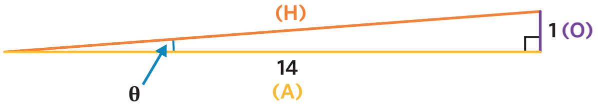
$O = 1$

$A = 14$

Add the known sides to the triangle.



Answer



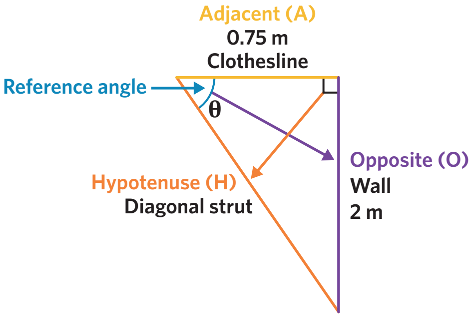
10. A clothesline is attached to a wall, making a 90° angle. The clothesline is 2 m high off the ground and protrudes 0.75 m from the wall and has a diagonal strut that attaches to the ground at the base of the wall. Write a trigonometric ratio to represent the situation if the wall is opposite the reference angle.

Key points

- Clothesline makes a 90° angle with the wall.
- The clothesline is 2 m high off the ground.
- The clothesline is 0.75 m from the wall and has a diagonal strut attached to the base.
- The wall is opposite the reference angle.
- Write a trigonometric ratio for the scenario.

Explanation

Create a triangle to represent the provided measurements and label the angle opposite the wall as the reference angle. Label the sides in terms of the reference angle.



Identify the known sides.

O = 2

A = 0.75

Identify the trigonometric ratio that uses O and A.

$$\tan\theta = \frac{O}{A}$$

Substitute the known values.

$$\tan\theta = \frac{2}{0.75} = \frac{8}{3}$$

Answer

The trigonometric ratio to represent the situation is $\tan\theta = \frac{8}{3}$.

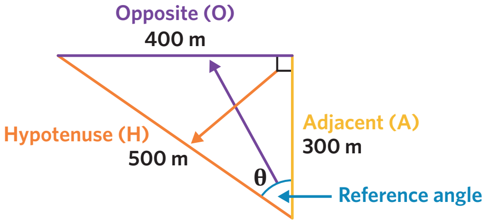
11. In a triangular race course, participants first run 300 m in a straight line, make a 90° turn to the left, then continue for 400 m, before returning directly to the starting point, 500 m away. Illustrate the course, and with the starting point as the reference angle, write the three trigonometric ratios for the formed triangle.

Key points

- In the race, participants run 300 m forward.
- They turn 90° left.
- They run left 400 m.
- They run 500 m back to the starting point.
- Draw the triangle and, using the starting point as a reference angle, write three trigonometric ratios.

Explanation

Create a triangle to represent the provided measurements and label the angle at the starting point as the reference angle. Label the sides in terms of the reference angle.



Identify the values of O, A and H.

O = 400 m

A = 300 m

H = 500 m

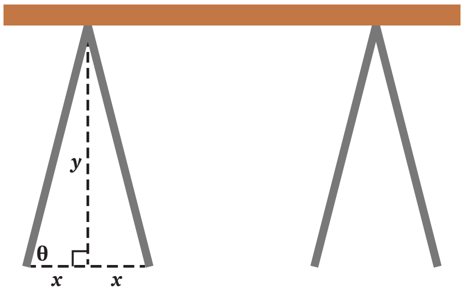
Substitute the values into the trigonometric ratios.

$$\sin\theta = \frac{O}{H}$$
$$= \frac{400}{500} = \frac{4}{5}$$
$$\cos\theta = \frac{A}{H}$$
$$= \frac{300}{500} = \frac{3}{5}$$
$$\tan\theta = \frac{O}{A}$$
$$= \frac{400}{300} = \frac{4}{3}$$

Answer

The three trigonometric ratios are $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, and $\tan\theta = \frac{4}{3}$.

12. A trestle table is held up by A-frames as shown below. Each A-frame has 1 m length arms. Write the cosine ratio using θ as the reference angle.

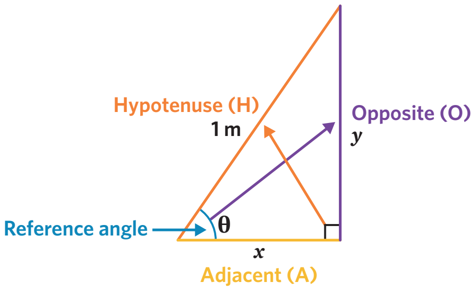


Key points

- The provided diagram illustrates the A-frames.
- The A-frames' arms are 1 m in length.
- Determine the cosine ratio for θ.

Explanation

Illustrate half of an A-frame as a right-angled triangle. Label all the sides in reference to θ.



Back to contents

Determine the sides that make up the cosine ratio.

$\cos\theta = \frac{A}{H}$

Identify the values of A and H.

$A = x$

$H = 1$

Substitute the value into the cosine ratio.

$\cos\theta = \frac{x}{1}$

$= x$

Answer

The cosine ratio for θ is x.

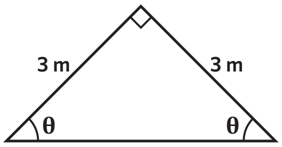
13. A frame for training a plant up a wall is constructed using a right-angled isosceles triangle, with two side lengths of 3 m. Express the tangent ratio for this specific triangle.

Key points

- The triangle is a right-angled isosceles triangle.
- Two side lengths of 3 m.
- Determine the tangent ratio.

Explanation

Illustrate the right-angled isosceles triangle.



Using either of the acute angles as the reference angle, the opposite and adjacent sides will be 3 m.

Identify what sides make up the tangent ratio.

$\tan\theta = \frac{O}{A}$

Identify the value of O and A.

$O = 3$

$A = 3$

Substitute the values into the ratio.

$\tan\theta = \frac{O}{A}$

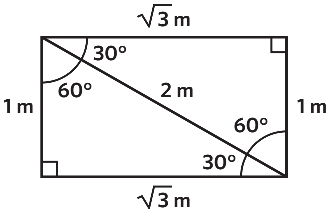
$= \frac{3}{3} = 1$

Answer

The tangent ratio for the frame is $\tan\theta = 1$.

Reasoning

14. a.



b. $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

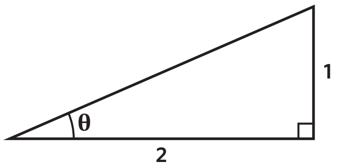
c. $\tan 45° = 1$

d. Suggested option 1: Art gallery.

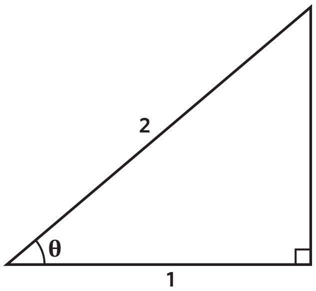
Suggested option 2: Skate park.

Note: There are other possible options.

15. a.



b.



c. A triangle representing $\cos\theta = \frac{2}{1}$ is not possible because the hypotenuse must always be larger than the opposite and adjacent sides.

Exam-style

16. D

17. a. $\cos\theta = \frac{4}{5}$

b. $\cos\theta = \frac{4}{5}$

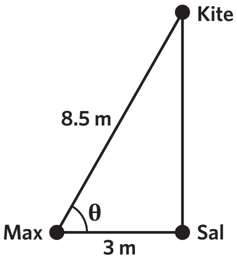
c. The answers from parts **a** and **b** are the same. The cosine ratio for all similar triangles are equal from the same reference angle.

18. $\sin\angle CAB = \frac{41}{76}$

$\cos\angle CAB = \frac{16}{19}$

$\tan\angle CAB = \frac{41}{64}$

19.



$\cos\theta = \frac{6}{17}$

Remember this?

20. B **21.** A **22.** C

9C Calculating unknown side lengths

Student practice

Worked example 1

a. 0.2588 **b.** 0.9205 **c.** 0.2126

Worked example 2

a. $x \approx 0.52$ **b.** $x \approx 3.28$

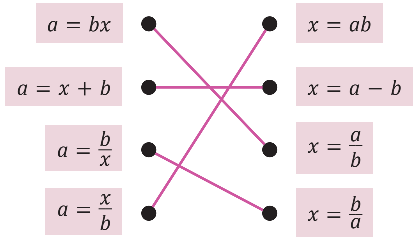
Worked example 3

a. $x \approx 30.71$ **b.** $x \approx 78.56$

Back to contents

Understanding worksheet

1. **Equation** **Transposed equation**



2. **a.** y **b.** y **c.** x **d.** 9

3. right; trigonometric; reference; denominator; subject

Fluency

4. **a.** 0.6428 **b.** 0.9877 **c.** 0.8693 **d.** 0.1822
e. 0.8480 **f.** 2.7475 **g.** 0.4337 **h.** 0.9728

---

5. **a.** $x \approx 0.87$ **b.** $x \approx 4.45$
c. $x \approx 5.36$ **d.** $x \approx 30.01$
e. $x \approx 0.28$ **f.** $x \approx 58.67$
g. $x \approx 7.61$ **h.** $x \approx 0.99$

---

6. **a.** $x \approx 2.14$ **b.** $x = 10.00$
c. $x \approx 5.43$ **d.** $x \approx 29.70$
e. $x \approx 1.21$ **f.** $x \approx 1.02$
g. $x \approx 87.28$ **h.** $x \approx 0.53$

---

7. **a.** $x \approx 4.29$ **b.** $x \approx 22.46$
c. $x \approx 7.06$ **d.** $x \approx 11.03$
e. $x \approx 0.41$ **f.** $x \approx 2.84$

---

8. **a.** $x \approx 10.24$ **b.** $x \approx 21.73$
c. $x \approx 5.36$ **d.** $x \approx 13.24$
e. $x \approx 9.40$ **f.** $x \approx 5.60$

---

9. C

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

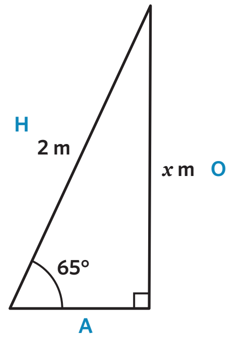
11. A person fishing off the beach uses a rod-stand that holds the rod at an angle of 65° from the ground. If the rod is 2 m long, what is the vertical height of the tip of the rod? Round to two decimal places.

Key points

- Rod-stand holds the fishing rod at an angle of 65° from the ground.
- The fishing rod is 2 m long.
- Determine the vertical height of the fishing rod's tip.

Explanation

Label the right-angled triangle using the reference angle.



Identify the trigonometric ratio and substitute the relevant values into the equation.

$\sin\theta = \frac{O}{H}$

$\sin 65° = \frac{x}{2}$

Transpose the equation to make x the subject.

$x = 2\sin 65°$

Evaluate using a calculator, rounding to the required number of decimal places.

$x \approx 1.81$ m

Answer

The tip of the fishing rod is 1.81 m high.

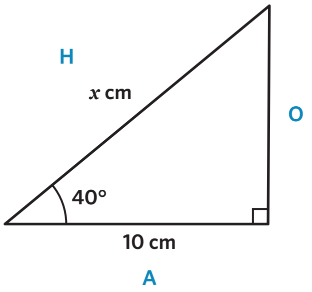
12. A rectangle of origami paper is folded on the diagonal, then opened back out forming two triangles. The longest side of the rectangle paper is 10 cm and makes an angle of 40° with the diagonal. Calculate the length of the diagonal to the nearest centimetre.

Key points

- Rectangle paper is folded along the diagonal making two triangles.
- Rectangle length is 10 cm.
- Diagonal and length makes an angle of 40°.
- Determine the diagonal length.

Explanation

Label the right-angled triangle using the reference angle.



Identify the trigonometric ratio and substitute the relevant values into the equation.

$\cos\theta = \frac{A}{H}$

$\cos 40° = \frac{10}{x}$

Transpose the equation to make x the subject.

$x = \frac{10}{\cos 40°}$

Evaluate using a calculator, rounding to the nearest centimetre.

$x \approx 13$ cm

Answer

The diagonal length is 13 cm.

Back to contents

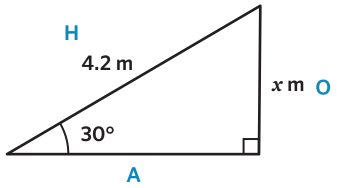
13. On a construction site, a ramp is assembled to provide wheelbarrows access to an area normally accessed via stairs only. The ramp, made from a spare 4.2 m long plank of wood, forms a 30° angle with the ground. Calculate the exact vertical height of the stairs.

Key points

- A ramp was built as an alternative to stairs.
- The ramp is made of a plank 4.2 m long.
- The plank forms a 30° angle with the ground.
- Determine the height of the ramp and stairs.

Explanation

Label the right-angled triangle using the reference angle.



Identify the trigonometric ratio and substitute the relevant values into the equation.

$\sin\theta = \frac{O}{H}$

$\sin 30° = \frac{x}{4.2}$

Transpose the equation to make x the subject.

$x = 4.2\sin 30°$

Evaluate using a calculator.

$x = 2.1$ m

Answer

The vertical height of the stairs is 2.1 m.

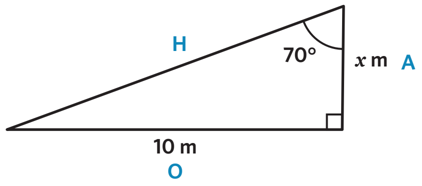
14. A sound engineer is determining the optimal speaker placement for even sound distribution. The speaker will be wall-mounted, angled downward, on a bracket that makes a 70° angle with the wall, and should deliver the best sound quality on the ground at a horizontal distance of 10 m from the wall. How high up the wall should the speaker be positioned? Round to one decimal place.

Key points

- Speaker should be angled downward, making a 70° angle with the wall.
- Speaker should be 10 m from the base of the wall.
- Determine the height of the speaker on the wall.

Explanation

Label the right-angled triangle using the reference angle.



Identify the trigonometric ratio and substitute the relevant values into the equation.

$\tan\theta = \frac{O}{A}$

$\tan 70° = \frac{10}{x}$

Transpose the equation to make x the subject.

$x = \frac{10}{\tan 70°}$

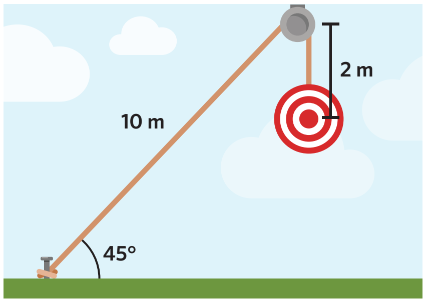
Evaluate using a calculator, rounding to the required number of decimal places.

$x \approx 3.6$ m

Answer

The speaker should be positioned at a height of 3.6 m on the wall.

15. In an archery competition, an elevated target is suspended above the ground via a pulley system. The angle made with the rope and the ground is 45° and the distance from the anchor point on the ground to the top of the pulley is 10 m. Calculate the height of the bullseye in metres if it is hanging 2 m from the pulley, correct to two decimal places.



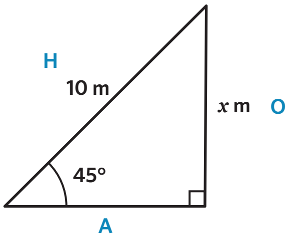
Key points

- The rope makes an angle of 45° with the ground.
- The rope between the pulley system and the ground is 10 m.
- The bullseye is hanging 2 m from the pulley.
- Determine the height of the bullseye.

Explanation

Determine the height of the pulley system.

Label the ring-angled triangle using the reference angle.
Let x represent the height of the pulley system, in metres.



Identify the trigonometric ratio and substitute the relevant values into the equation.

$\sin\theta = \frac{O}{H}$

$\sin 45° = \frac{x}{10}$

Transpose the equation to make x the subject.

$x = 10\sin 45°$

Determine the height of the bullseye by subtracting the distance between the pulley and the bullseye from the height of the system.

$10\sin 45° - 2 = 5.071...$

≈ 5.07 m

Answer

The height of the bullseye is 5.07 m.

Reasoning

16. **a.** The depth of the ocean at the boat's anchor point is 12.02 m.

b. There has been 13.88 m of the anchor chain released.

c. The two anchors are 5.08 m apart.

d. Suggested option 1: Increased stability against shifting currents and winds.

Suggested option 2: Reduced risk of drifting if one anchor fails to hold.

Note: There are other possible options.

Back to contents

17. **a.** $\cos 58° = \frac{25}{x}$

$x \approx 47.2$

b. $\sin 32° = \frac{25}{x}$

$x \approx 47.2$

c. If more than one angle and at least two sides are known, more than one reference angle and trigonometric ratio can be used to calculate an unknown value.

Exam-style

18. D

19. **a.** 100 m **b.** 173.21 m

20. 1.56 m

21. $\sin 30° = 0.5$

$\cos 60° = 0.5$

Remember this?

22. B **23.** B **24.** A

9D Calculating unknown angles

Student practice

Worked example 1

a. 36.87° **b.** 57.26°

Worked example 2

a. 14° **b.** 73°

Understanding worksheet

1. **a.** 0.5 **b.** 2 **c.** 0.9 **d.** 1.2

2. **a.** *c* **b.** *b* **c.** *a* **d.** *b*

3. inverse; trigonometric; angle; lengths

Fluency

4. **a.** 23.58° **b.** 72.08° **c.** 2.86° **d.** 19.88°
e. 75.52° **f.** 68.20° **g.** 60.00° **h.** 44.43°

5. **a.** 34° **b.** 65° **c.** 62° **d.** 14°
e. 10° **f.** 19° **g.** 53° **h.** 70°

6. D

Spot the mistake

7. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

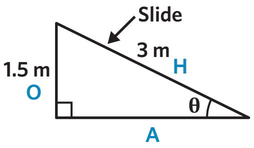
8. In a playground, there is a slide attached to a platform that is 1.5 m above the ground. The slide itself is 3 m long. What angle does the slide make with the ground?

Key points

- A slide is attached to a platform that is 1.5 m above the ground.
- The slide is 3 m long.
- Determine the angle that the slide makes with the ground.

Explanation

Label the sides of the right-angled triangle formed by the slide, using the reference angle (the angle the slide makes with the ground).



Identify the relevant trigonometric ratio and substitute the values into the equation.

$\sin\theta = \frac{\text{O}}{\text{H}}$

$\sin\theta = \frac{1.5}{3}$

Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

$\theta = \sin^{-1}\left(\frac{1.5}{3}\right)$

Evaluate using a calculator.

$\theta = 30°$

Answer

The slide makes a 30° angle with the ground.

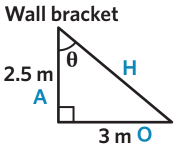
9. A sound engineer is looking at where to place a speaker for optimal sound distribution. They know the speaker will be placed on a wall bracket at a height of 2.5 m and angled down so that the best sound will be heard 3 m from the base of the wall. Calculate the angle the bracket should make with the wall, correct to the nearest degree.

Key points

- A sound engineer is placing the speaker on a wall bracket at a height of 2.5 m.
- The speaker will be angled down so that the best sound will be heard 3 m from the base of the wall.
- Calculate the angle the bracket should make with the wall.

Explanation

Label the sides of the right-angled triangle using the reference angle (the angle the wall bracket makes with the wall).



Identify the relevant trigonometric ratio and substitute the values into the equation.

$\tan\theta = \frac{\text{O}}{\text{A}}$

$\tan\theta = \frac{3}{2.5}$

Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

$\theta = \tan^{-1}\left(\frac{3}{2.5}\right)$

Evaluate using a calculator, rounding to the nearest degree.

$\theta = 50.194...$

$\approx 50°$

Answer

The angle bracket should make a 50° angle with the wall.

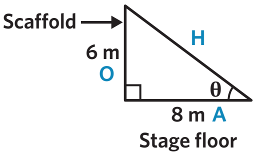
10. In a theatre production, a spotlight is mounted on a scaffold 6 m above the stage floor. The spotlight can be rotated to focus on different areas of the stage. What angle does the spotlight's beam make with the stage floor when it is aimed at a point 8 m away from the base of the scaffold, correct to 1 decimal place?

Key points

- A spotlight is mounted on a scaffold 6 m above the stage floor.
- Determine the angle that the spotlight's beam makes with the stage floor when it is aimed at a point 8 m away from the base of the scaffold.

Explanation

Label the sides of the right-angled triangle formed by the spotlight's beam, using the reference angle (the angle the spotlight beam makes with the stage floor).



Identify the relevant trigonometric ratio and substitute the values into the equation.

$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{6}{8}$

Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

$\theta = \tan^{-1}\left(\frac{6}{8}\right)$

Evaluate using a calculator, rounding to the required number of decimal places.

$\theta = 36.869...$

$\approx 36.9°$

Answer

The spotlight's beam makes a 36.9° angle with the stage floor when aimed at a point 8 m from the base of the scaffold.

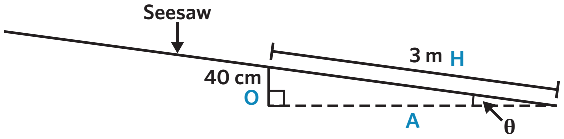
11. A seesaw, supported by a stand 40 cm tall, has a total length of 6 m. When one side is on the ground, determine the angle formed between the seesaw and the ground, correct to 2 decimal places.

Key points

- A seesaw is supported by a stand 40 cm tall and has a total length of 6 m.
- When one side is on the ground, determine the angle formed between the seesaw and the ground.

Explanation

Label the sides of the right-angled triangle formed by the seesaw, using the reference angle (the angle formed between the seesaw and the ground).



Identify the relevant trigonometric ratio and substitute the values into the equation.

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{0.4}{3}$

Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

$\theta = \sin^{-1}\left(\frac{0.4}{3}\right)$

Evaluate using a calculator, rounding to the required number of decimal places.

$\theta = 7.662....$

$\approx 7.66°$

Answer

The angle formed between the seesaw and the ground is 7.66°.

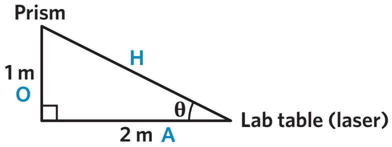
12. In a science experiment, a laser beam on a table is directed at a prism placed 2 m away. The prism is then raised 1 m above the lab table. Determine the angle the laser beam makes with the lab table when it enters the prism, correct to 2 decimal places.

Key points

- A laser beam is directed at a prism placed 2 m away.
- The prism is positioned 1 m above the lab table.
- Determine the angle the laser beam makes with the lab table when it enters the prism.

Explanation

Label the sides of the right-angled triangle formed by the laser beam and prism, using the reference angle (the angle the laser beam makes with the lab table).



Identify the relevant trigonometric ratio and substitute the values into the equation.

$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{1}{2}$

Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

Evaluate using a calculator, rounding to the required number of decimal places.

$\theta = 26.565...$

$\approx 26.57°$

Answer

The laser beam makes a 26.57° angle with the lab table when it enters the prism.

Reasoning

13. a. The angle from point B to point A is $\theta \approx 53.13°$.

b. The angle from point C to point A is $\alpha \approx 26.57°$.

c. The angle from the new point D to point A is 20.56°.

d. Suggested option 1: Hiking/camping in the wilderness without a GPS.

Suggested option 2: Marine navigation for sailors and boaters.

Note: There are other possible options.

14. a. The two equal angles in the right-angled isosceles triangle are 45°.

b. $\sin 45° = \frac{1}{x}$ $\quad$ $\cos 45° = \frac{1}{x}$

$x = \frac{1}{\sin 45°}$ $\quad$ $x = \frac{1}{\cos 45°}$

c. From part **b**, given a triangle with a hypotenuse of x and a reference angle of 45°, the length of the opposite side is equal to the length of the adjacent side. Since $\sin 45° = \frac{1}{x}$ and $\cos 45° = \frac{1}{x}$, then $\frac{1}{x} = \sin 45° = \cos 45°$.
Therefore $\sin 45° = \cos 45°$.

9D ANSWERS

Back to contents

Exam-style

15. B

16. **a.** 67° **b.** 2.28 m

17. 11.89°

18. Strut A makes a 70° angle with the wall and strut B makes a 50° angle with the wall.

Remember this?

19. D **20.** C **21.** B

9E Bearings

Student practice

Worked example 1

a. 107° T; 287° T **b.** 302° T; 122° T

Worked example 2

a. *N* 63° *E* **b.** *S* 29° *W*

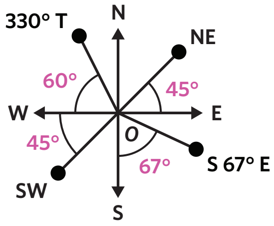
Worked example 3

a. 5.66 km **b.** 4.45 km

Understanding worksheet

1. **a.** 240 **b.** 270 **c.** 350 **d.** 103

2.



3. clockwise; compass; opposite; internal

Fluency

4. **a.** 030° T; 210° T **b.** 045° T; 225° T
c. 076° T; 256° T **d.** 120° T; 300° T
e. 050° T; 230° T **f.** 126° T; 306° T
g. 157° T; 337° T **h.** 167° T; 347° T

5. **a.** 200° T; 020° T **b.** 240° T; 060° T
c. 308° T; 128° T **d.** 210° T; 030° T
e. 234° T; 054° T **f.** 216° T; 036° T
g. 297° T; 117° T **h.** 339° T; 159° T

6. **a.** *N* 50° *E* **b.** *S* 27° *W*
c. *N* 34° *W* **d.** *S* 36° *W*
e. *S* 69° *E* **f.** *N* 52° *W*
g. *S* 30° *W* **h.** *S* 14° *W*

7. **a.** 2.00 km **b.** 4.68 km
c. 2.82 km **d.** 4.33 km
e. 5.52 km **f.** 7.02 km
g. 3.23 km **h.** 7.49 km

8. D

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. A ranger walks on a bearing of 058° T from a forest hut to his car. Determine the true bearing of the direction the ranger should take in order to return back to the hut from the car.

Key points

- A ranger walks from a forest hut to his car on a bearing of 058° T.
- Determine the true bearing of the direction the ranger should take in order to return back to the hut from the car.

Explanation

Identify if the true bearing of the car from the hut is greater than or less than 180°.

058° < 180°

The bearing is less than 180°, so adding 180° will result in the opposite direction.

The bearing of the hut from the car:

058° + 180° = 238° T

Answer

The true bearing of the direction the ranger should take in order to return back to the hut from the car is 238° T.

11. The bearing of a ship at sea from its docking point on the shore is 290° T. Determine the true bearing of the direction the ship must sail in order to reach the dock from its position at sea at the time of observation.

Key points

- The true bearing of a ship at sea from its docking point on the shore is 290° T.
- Determine the true bearing of the direction the ship must sail in order to reach the dock from its position at sea at the time of observation.

Explanation

Identify if the true bearing of the ship from the dock is greater than or less than 180°.

290° > 180°

The bearing is greater than 180°, so subtracting 180° will result in the opposite direction.

The bearing of the dock from the ship:

290° − 180° = 110° T

Answer

The true bearing of the direction the ship must sail in order to reach the dock from its position at sea at the time of observation is 110° T.

Back to contents

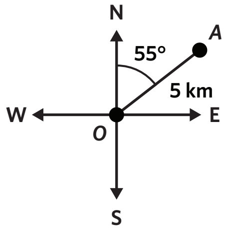
12. A hiker walks 5 km on a bearing of N 55° E from point O to point A. Determine how far east point A is from point O, correct to 1 decimal place.

Key points

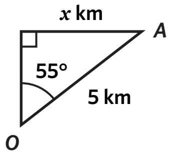
- A hiker walks 5 km on a compass bearing of N 55° E from point O to point A.
- Determine how far east point A is from point O, correct to 1 decimal place.

Explanation

Draw a diagram to represent the situation.



Form a right-angled triangle and label the hypotenuse with the distance between points O and A. Label an internal angle with the given bearing and label the required distance east with a pronumeral.



Solve for the pronumeral using trigonometric ratios.

$$\begin{aligned} \sin 55° &= \frac{x}{5} \\ x &= 5\sin 55° \\ &= 4.095... \\ &\approx 4.1 \end{aligned}$$

Answer

Point A is 4.1 km east of point O.

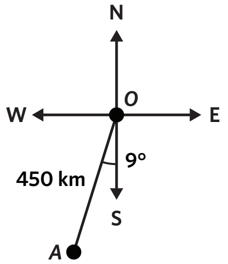
13. A plane flies on a bearing of 189° T for 450 km. Determine how far south, to the nearest kilometre, the plane flew from its starting point.

Key points

- A plane flies for 450 km on a bearing of 189° T.
- Determine how far south, to the nearest kilometre, the plane flew from its starting point.

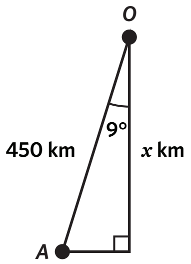
Explanation

Draw a diagram to represent the situation. Let O be the plane's starting position and A the destination.



Form a right-angled triangle and label the hypotenuse with the distance between points O and A.

Mark an internal angle on the right-angled triangle and label the required distance south with a pronumeral.



Solve for the pronumeral using trigonometric ratios.

$$\begin{aligned} \cos 9° &= \frac{x}{450} \\ x &= 450\cos 9° \\ &= 444.459... \\ &\approx 444 \end{aligned}$$

Answer

The plane flew 444 km south from its starting point.

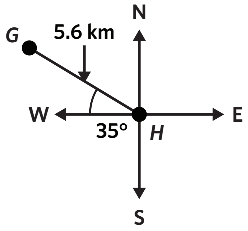
14. Rowan leaves his house and walks due west, then turns and walks due north until he gets to the grocery store. The store is on a true bearing of 305° and a direct distance of 5.6 km from Rowan's house. Determine how far north Rowan walked on his way to the store from his house. Round the answer to 1 decimal place.

Key points

- Rowan leaves his house and walks due west.
- Rowan then turns and walks due north until he gets to the grocery store.
- The store is on a true bearing of 305° and a direct distance of 5.6 km from Rowan's house.
- Determine how far north Rowan walked on his way to the store from his house. Round the answer to 1 decimal place.

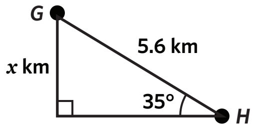
Explanation

Draw a diagram to represent the situation. Let H be the location of Rowan's house and G the location of the grocery store.



Form a right-angled triangle and label the hypotenuse with the distance between points H and G.

Mark an internal angle on the right-angled triangle and label the required distance north with a pronumeral.



Solve for the pronumeral using trigonometric ratios.

$$\begin{aligned} \sin 35° &= \frac{x}{5.6} \\ x &= 5.6\sin 35° \\ &= 3.212... \\ &\approx 3.2 \end{aligned}$$

Answer

Rowan walked 3.2 km north on the way to the store from his house.

Back to contents

Reasoning

15. **a.** The true bearing of the boat from the cruise ship is 055° T.

b. The true bearing of the lighthouse from the boat is 288° T.

c. The boat is 28 km east from the cruise ship.

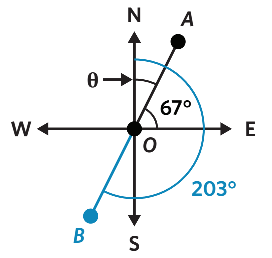
d. The direct distance between the lighthouse and the cruise ship is 45 km.

e. Suggested option 1: Ships and boats need to be able to communicate in case they need help from each other.

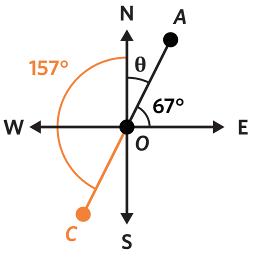
Suggested option 2: Ships and boats need to be able to communicate to ensure there are no crashes.

Note: There are other possible options.

16. **a.**



b.



c. Point *B* and point *C* are the same. This demonstrates that adding and subtracting 180° from a bearing produces the same direction.

Exam-style

17. D

18. **a.** 077° T

b. 50 km

19. 1532 km

20. 2.6 km

Remember this?

21. B

22. A

23. D

9F Elevation and depression

Student practice

Worked example 1

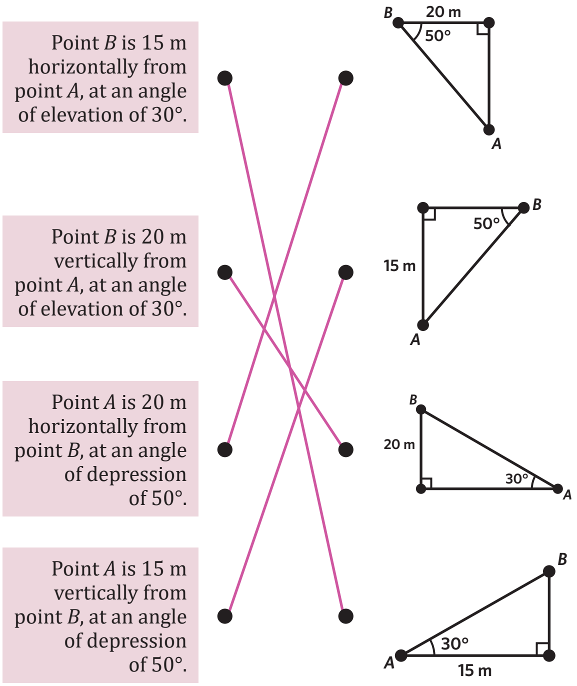
a. 7 m **b.** 22 m

Worked example 2

24°

Understanding worksheet

1. **Description** **Right-angled triangle**



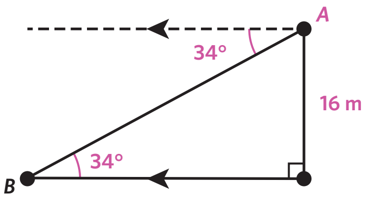
Point *B* is 15 m horizontally from point *A*, at an angle of elevation of 30°.

Point *B* is 20 m vertically from point *A*, at an angle of elevation of 30°.

Point *A* is 20 m horizontally from point *B*, at an angle of depression of 50°.

Point *A* is 15 m vertically from point *B*, at an angle of depression of 50°.

2.



3. elevation; depression; equal; alternate

Fluency

4. **a.** 6 m **b.** 63 m **c.** 23 m **d.** 44 m
e. 23 m **f.** 59 m **g.** 62 m **h.** 37 m

5. **a.** 18° **b.** 23° **c.** 31° **d.** 29°
e. 48° **f.** 38° **g.** 20° **h.** 16°

6. A

Spot the mistake

7. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

8. From a point on the ground 50 m away, the angle of elevation to the top of a building is 30°. Determine the height of the building to the nearest metre.

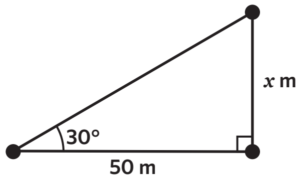
Key points

- A point on the ground is 50 m away from a building.
- The angle of elevation to the top of the building from this point is 30°.
- Determine the height of the building to the nearest metre.

Back to contents

Explanation

Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the top of the building and the ground. Label two sides of the triangle using the given information and mark the angle of elevation opposite the side representing the height.



Solve for the unknown length by using a trigonometric ratio. Round the answer to the nearest metre.

$$\tan 30° = \frac{x}{50}$$
$$x = 50\tan 30°$$
$$= 28.868...$$
$$\approx 29 \text{ m}$$

Answer

The height of the building is approximately 29 m.

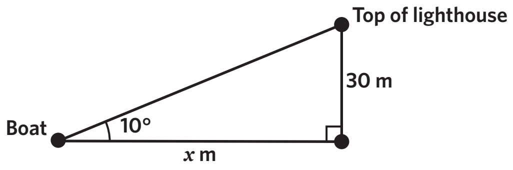
9. From a boat out at sea, the angle of elevation to the top of a lighthouse is 10°. Determine the distance from the boat to the lighthouse if the lighthouse and the small island it is positioned on have a combined height of 30 m. Round the answer to the nearest metre.

Key points

- The angle of elevation from a boat out at sea to the top of a lighthouse is 10°.
- The lighthouse and the small island it is positioned on have a combined height of 30 m.
- Determine the distance from the boat to the lighthouse, rounded to the nearest metre.

Explanation

Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the boat and the top of the lighthouse. Label two sides of the triangle using the given information and mark the angle of elevation opposite the side representing the height.



Solve for the unknown length by using a trigonometric ratio. Round the answer to the nearest metre.

$$\tan 10° = \frac{30}{x}$$
$$x = \frac{30}{\tan 10°}$$
$$= 170.138...$$
$$\approx 170 \text{ m}$$

Answer

The distance from the boat to the lighthouse is approximately 170 m.

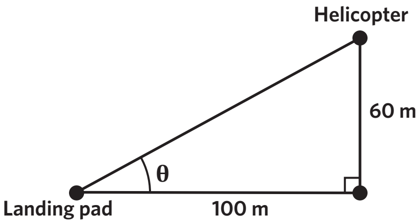
10. A helicopter is at a height of 60 m and a horizontal distance of 100 m from a landing pad. Determine the angle of elevation of the helicopter from the landing pad to the nearest degree.

Key points

- A helicopter is at a height of 60 m.
- The helicopter is at a horizontal distance of 100 m from a landing pad.
- Determine the angle of elevation of the helicopter from the landing pad to the nearest degree.

Explanation

Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the helicopter and the landing pad. Label the shorter sides of the triangle with the given vertical and horizontal distances.



Solve for the angle of elevation by using a trigonometric ratio. Round the answer to the nearest degree.

$$\tan\theta = \frac{60}{100}$$
$$\theta = \tan^{-1}\left(\frac{60}{100}\right)$$
$$= 30.964...$$
$$\approx 31°$$

Answer

The angle of elevation of the helicopter from the landing pad is approximately 31°.

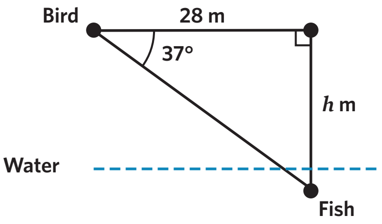
11. A sea bird flying over the water spots a fish at a horizontal distance of 28 m at an angle of depression of 37°. Determine the height of the bird above the surface of the water if the fish was at a depth of 2 m when it was seen by the bird. Round the answer to the nearest metre.

Key points

- A sea bird flying over the water spots a fish at a horizontal distance of 28 m.
- The angle of depression is 37°.
- The fish was at a depth of 2 m when it was seen by the bird.
- Determine the height of the bird above the surface of the water, rounding to the nearest metre.

Explanation

Draw a right-angled triangle where the hypotenuse represents the direct line of vision between the bird and the fish. Label two sides of the triangle using the given information and mark the angle of depression opposite the side representing the height of the bird above the fish.



Solve for the unknown length by using a trigonometric ratio.

$$\tan 37° = \frac{h}{28}$$
$$h = 28\tan 37°$$

Back to contents

Taking into account the fish being 2 m underwater, the height of the bird above the surface of the water is given by;

$28\tan 37° - 2$

$= 19.099...$

≈ 19 m

Answer

The height of the bird above the surface of the water is approximately 19 m.

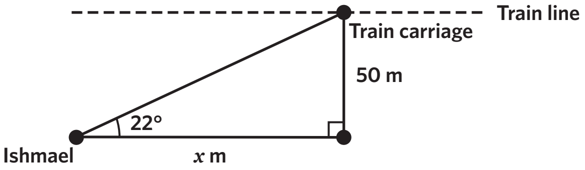
12. Ishmael observes a sky train carriage directly above him at a height of 50 m. A few seconds later, Ishmael observes the same carriage again, this time at an angle of elevation of 22°. Determine the distance travelled by the sky train between the two observations, to the nearest metre.

Key points

- Ishmael observes a sky train carriage directly above him at a height of 50 m.
- Ishmael observes the same carriage again, this time at an angle of elevation of 22°.
- Determine the distance travelled by the sky train between the two observations, to the nearest metre.

Explanation

Draw a right-angled triangle where the hypotenuse represents the direct line of vision between Ishmael and the sky train carriage. Label the shorter sides of the triangle with the given vertical and horizontal distances.



$$\tan 22° = \frac{50}{x}$$

$$y = \frac{50}{\tan 22°}$$

$$= 123.754...$$

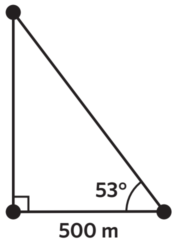
$$\approx 124 \text{ m}$$

Answer

The distance travelled by the sky train between the two observations is approximately 124 m.

Reasoning

13. a.



b. The height of the plane above the water is 668 m.

c. The height of the crow's nest of boat B above the water is 6.8 m.

d. The angle of depression of boat B from the plane is 52°.

e. Suggested option 1: The crow's nest is placed high for the 360° unobstructed view of the boat's surroundings.

Suggested option 2: The crow's nest is placed high so that a telescope or binoculars can be used to find land.

Note: There are other possible options.

14. a. 27°

b. 63°

c. The angles of elevation and depression between two points are less than 45° when the horizontal distance is greater than the vertical distance, equal to 45° when the vertical and horizontal distances are equal, and greater than 45° when the horizontal distance is less than the vertical distance.

Exam-style

15. E

16. a. 41° **b.** 23.85 m

17. 23°

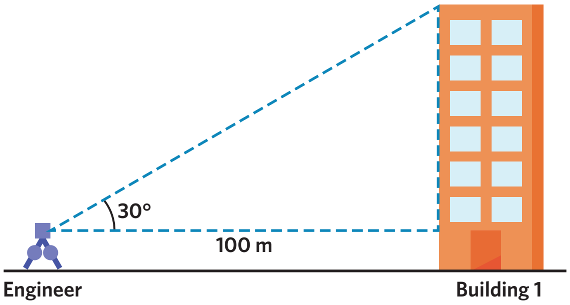
18. 27.7 m

Remember this?

19. B **20.** A **21.** A

Chapter 9 extended application

1. a.



b. The height of the tower is 59 m.

c. The reading will be 115 m for the distance to the top of the tower.

d. The height of the second tower is 126 m.

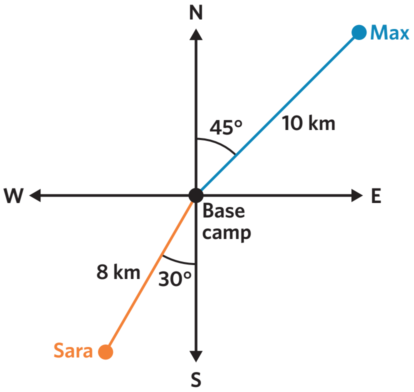
e. The horizontal distance between the two towers is 49 m.

f. Suggested option 1: They could've used a measuring wheel.

Suggested option 2: They could've used a measuring tape.

Note: There are other possible options.

2. a.



b. Max is located 7.07 km north of base camp.

c. Sara is located 6.93 km south of base camp.

d. The east-west distance between Max and Sara is 11.07 km.

e. Sara is now 7 km from base camp on a bearing of *S* 8° *E*.

Back to contents

f. Suggested option 1: Using a map to navigate.

Suggested option 2: Carrying a flare so location is known if lost.

Note: There are other possible options.

3. a. Aircraft Charlie is 30 km east of the control room.

b. Aircraft Delta is 43 km west of the control room.

c. The north-south distance between Aircraft Charlie and Aircraft Delta is 43 km.

d. Aircraft Charlie's new true bearing from the control room is 085° T.

e. The angle of elevation from the control room to Aircraft Delta's position is 89°.

f. Suggested option 1: Determining the order of takeoffs and landings.

Suggested option 2: Contact pilots regarding aircraft safety.

Note: There are other possible options.

Chapter 9 review

Multiple choice

1. C **2.** D **3.** A **4.** B **5.** A

Fluency

6. a. 9.49 **b.** 8.94 **c.** 4.93 **d.** 13.62

7. a. $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

b. $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

c. $\sin\theta = \frac{2}{x}$, $\cos\theta = \frac{1}{3}$, $\tan\theta = \frac{6}{x}$

d. $\sin\theta = \frac{b}{4}$, $\cos\theta = \frac{3b}{4}$, $\tan\theta = \frac{1}{3}$

8. a. $x \approx 15.71$ **b.** $x \approx 20.14$

c. $x \approx 13.69$ **d.** $x \approx 18.35$

9. a. 36.87° **b.** 79.70° **c.** 42.27° **d.** 7.59°

10. a. 47° **b.** 62° **c.** 38° **d.** 44°

11. a. 108° T; 288° T **b.** 243° T; 063° T

c. 054° T; 234° T **d.** 260° T; 080° T

12. a. 4.77 km **b.** 6.62 km **c.** 2.74 km **d.** 9.58 km

13. a. 23 m **b.** 21 m **c.** 4 m **d.** 179 m

14. a. 25° **b.** 9° **c.** 27° **d.** 19°

Problem solving

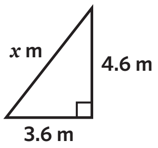
15. Lani is sailing her boat, and decides to anchor it 4.6 m above the sea floor. The boat can drift a maximum distance of 3.6 m horizontally from the anchor. Calculate the length of the anchor's rope, correct to two decimal places.

Key points

- The boat is 4.6 m above the sea floor.
- The boat is 3.6 m horizontally from the anchor.
- Determine the length of the anchor's rope.

Explanation

Draw a diagram to represent the situation.



To calculate the length of the anchor's rope apply Pythagoras' theorem formula.

$a = 3.6$

$b = 4.6$

$c = x$

$x^2 = 3.6^2 + 4.6^2$

$x^2 = 12.96 + 21.16$

$x^2 = 34.12$

$x = \sqrt{34.12}$

≈ 5.84

Answer

The length of the anchor's rope is approximately 5.84 m.

16. In a professional baking contest, participants are required to design a triangular cake with a tangent ratio $\frac{2}{9}$. Create a right-angled triangle to illustrate the cake, and label the reference angle and any known side lengths.

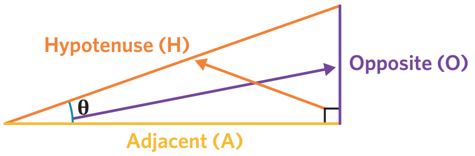
Key points

- Triangular cake's tangent ratio cannot exceed $\frac{2}{9}$.
- Create a triangle to represent the cake and label all sides using the reference angles.

Explanation

Create a right-angled triangle and identify a reference angle.

Label the sides using the reference angle.



Identify the provided sides from $\tan\theta = \frac{2}{9}$.

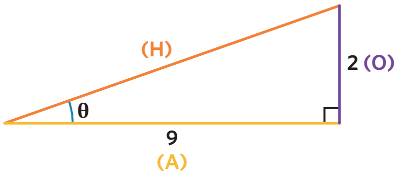
$\tan\theta = \frac{2}{9}$

$\tan\theta = \frac{O}{A}$

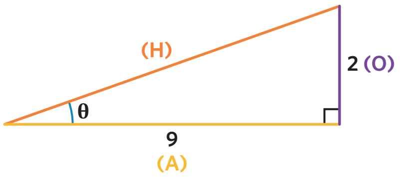
$O = 2$

$A = 9$

Add the known sides to the triangle.



Answer



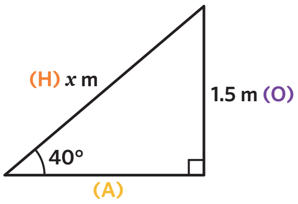
17. After a storm, the top part of a tree has snapped at the trunk, fallen over, and now makes a 40° angle with the horizontal ground. The lower part of the tree's trunk is still standing at a right angle with the ground and is 1.5 m long. Calculate the total height of the tree prior to the storm, correct to 2 decimal places.

Key points

- The top part of a tree snapped and is at a 40° angle with the horizontal.
- The lower part of the tree is 1.5 m long and at a right angle with the horizontal.
- Calculate the total height of the entire tree.

Explanation

Using the reference angle, label the sides of the right-angled triangle; opposite, adjacent and hypotenuse.



Identify the trigonometric ratio and substitute the relevant values into the equation.

$$\sin\theta = \frac{O}{H}$$

$$\sin 40° = \frac{1.5}{x}$$

Transpose the equation to make x the subject.

$$x = \frac{1.5}{\sin 40°}$$

The total height of the tree can be determined by adding the length of the hypotenuse (x) to 1.5 m.

$$\text{Height} = x + 1.5$$

$$= \frac{1.5}{\sin 40°} + 1.5$$

Evaluate using a calculator, rounding to two decimal places.

Height ≈ 3.83 m

Answer

The total height of the tree prior to the storm was approximately 3.83 m.

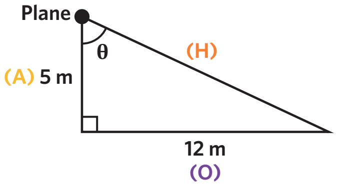
18. Mason is flying his remote controlled plane. He starts to descend for landing when the plane is 5 m above the ground. The runway is 12 m away from the point directly below the plane. Form a right-angled triangle showing this scenario and calculate the angle with opposite and adjacent side lengths of 12 m and 5 m, respectively, rounded to the nearest degree.

Key points

- Plane's height is 5 m when descending.
- Plane is 12 m away from the runway directly.
- Form a right-angled triangle showing the scenario and calculate the angle with opposite and adjacent side lengths of 12 m and 5 m, respectively.

Explanation

Label the sides of the right-angled triangle using the reference angle (the angle the wall bracket makes with the wall).



Identify the relevant trigonometric ratio and substitute the values into the equation.

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{12}{5}$$

Transpose the equation, using the inverse trigonometric ratio, to make θ the subject.

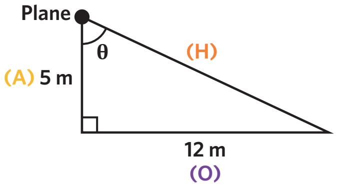
$$\theta = \tan^{-1}\left(\frac{12}{5}\right)$$

Evaluate using a calculator, rounding to the nearest degree.

$$\theta = 67.3801....$$

$$\approx 67°$$

Answer



The angle with opposite and adjacent side lengths of 12 m and 5 m, respectively, is 67°.

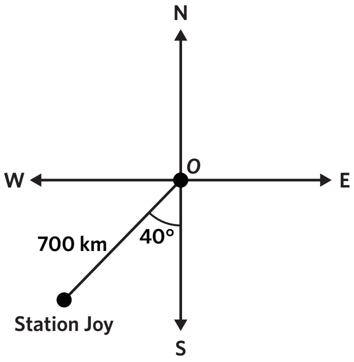
19. Patrick is an astronaut piloting his spacecraft on a bearing of $S\ 40°\ W$ for 700 km to research an orbiting space station, Station Joy. Calculate how far west Station Joy is from the spacecraft's starting point, rounded to 2 decimal places.

Key points

- Station Joy is travelling on a bearing of $S\ 40°\ W$.
- Station Joy travelled for 700 km.
- Determine how far west Station Joy has travelled.

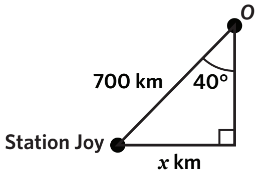
Explanation

Draw a diagram to represent the situation.



Form a right-angled triangle and label the hypotenuse with 700 km.

Label the required distance west with a pronumeral.



Solve for the pronumeral using a trigonometric ratio.

$\sin 40° = \frac{x}{700}$

$x = 700\sin 40°$

Evaluate using a calculator, rounding to two decimal places.

$x \approx 449.95$ km

Answer

Station Joy is approximately 449.95 km west from the starting point.

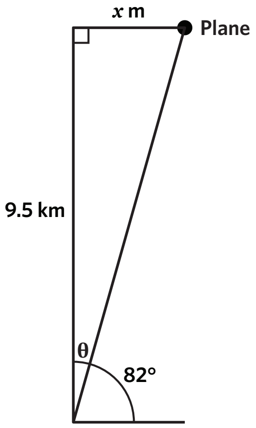
20. Timmy spots a plane directly above him in the sky at a height of 9.5 km. After a few seconds, Timmy notices that the plane is now at an angle of elevation of 82°. Determine the distance travelled by the plane between the two observations, in kilometres, rounded to two decimal places.

Key points

- Timmy observes a plane 9.5 km directly above him.
- Later, Timmy observes the same plane at an angle of elevation of 82°.
- Determine the distance travelled by the plane between the two observations.

Explanation

Draw a right-angled triangle where the hypotenuse represents the direct line of vision between Timmy and the plane. Label the shorter sides of the triangle with the given vertical and horizontal distances and label the reference angle.



Determine the reference angle.

$\theta + 82° = 90°$

$\theta = 90° - 82°$

$\theta = 8°$

Determine the distance travelled.

$\tan 8° = \frac{x}{9.5}$

$x = 9.5\tan 8°$

Evaluate using a calculator, rounding to two decimal places.

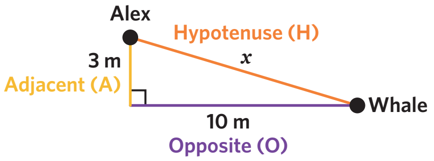
$x \approx 1.34$ km

Answer

The distance travelled by the plane between the two observations is approximately 1.34 km.

Reasoning

21. a.



The hypotenuse is approximately 10.44 m.

b. $\sin\theta = \frac{3}{10.44}$

$\cos\theta = \frac{10}{10.44}$

$\tan\theta = \frac{3}{10}$

c. The angles of elevation and depression between Alex and the whale are approximately 9°.

d. Alex needs to turn 155° clockwise to align his boat towards the whales.

e. Suggested option 1: Alex should ensure that he does not disturb marine life while gathering data.

Suggested option 2: Alex should ensure that he does not pollute the area in which he is gathering data.

Note: There are other possible options.

22. a. The value of a is approximately 0.50 when $c = 1$.

b. The value of b is approximately 0.87 when $c = 1$.

c. The values of a and b are approximately 5.0 and 8.7, respectively, when $c = 10$. In parts **a** and **b**, when increasing the length of the hypotenuse by a factor of 10, the lengths of sides a and b both increased by a factor of 10. When the length of the hypotenuse is scaled by a factor, the lengths of the opposite and adjacent sides are also scaled by the same factor.

11A Measures of centre and spread

Student practice

Worked example 1

a. Mean = 8.8
Median = 8
Mode = 2, 6 and 8

b. Mean = 23.7
Median = 24
Mode = 16

Worked example 2

a. $Q_1 = 5$
$Q_3 = 16$

b. $Q_1 = 4$
$Q_3 = 17$

Worked example 3

a. Range = 19
IQR = 11

b. Range = 20
IQR = 12.5

Understanding worksheet

1. **a.** 2, 3, 5, 7, 11
b. 4, 7, 9, 12, 15, 19
c. 1, 5, 9, 13, 15, 20, 25
d. 1, 4, 6, 8, 11, 13, 16, 19

2. **a.** 15, 25, 10, 1, 20
b. 7, 95, 15, 22, 25, 35
c. 23, 7, 19, 43, 13, 41, 31, 29, 70
d. 3, 50, 38, 45, 41, 110, 52, 48, 39

3. summary; median; typical; interquartile range; spread

Fluency

4. **a.** Mean = 12.7
Median = 14
Mode = 14

b. Mean = 15.4
Median = 15.5
Mode = 14 and 16

c. Mean = 12.2
Median = 10
Mode = none

d. Mean ≈ 2.7
Median = 2.5
Mode = 1

e. Mean ≈ 25.9
Median = 27
Mode = 19, 23 and 28

f. Mean ≈ 159.1
Median = 157.5
Mode = 145

g. Mean ≈ 0.3
Median = 0
Mode = 0

h. Mean ≈ 16.2
Median = 16.4
Mode = none

5. **a.** $Q_1 = 4.5$
$Q_3 = 17$

b. $Q_1 = 6$
$Q_3 = 17$

c. $Q_1 = 8$
$Q_3 = 18.5$

d. $Q_1 = 1$
$Q_3 = 3.85$

e. $Q_1 = 16$
$Q_3 = 35$

f. $Q_1 = 17$
$Q_3 = 27$

g. $Q_1 = -4.5$
$Q_3 = 5$

h. $Q_1 = 14.25$
$Q_3 = 18.4$

6. **a.** Range = 16
IQR = 12.5

b. Range = 17
IQR = 11

c. Range = 15
IQR = 10.5

d. Range = 7
IQR = 2.85

e. Range = 43
IQR = 19

f. Range = 20
IQR = 10

g. Range = 18
IQR = 9.5

h. Range = 7.5
IQR = 4.15

7. **a.** No outliers; mean = 16.6; range = 26
b. Outlier = 40; median = 14.5; mode = none; IQR = 7
c. No outliers; mean ≈ 11.4; range = 33
d. No outliers; mean ≈ 5.1; range = 9.4
e. No outliers; mean ≈ −0.9; range = 28

8. C

Spot the mistake

9. **a.** Student A is incorrect.
b. Student A is incorrect.

Problem solving

10. Dylan records the times (in seconds) of nine athletes in a 100 m sprint. The times are 10.8, 11.2, 10.5, 11.1, 10.8, 10.9, 10.6, 11.3 and 10.7. What are the mean and median of the athletes' times in this sprint? Round to one decimal place.

Key points

- There are nine athletes.
- Their times are 10.8, 11.2, 10.5, 11.1, 10.8, 10.9, 10.6, 11.3, and 10.7 seconds.
- What are the mean and median of their times to one decimal place?

Explanation

Organise the data set in ascending order.

10.5, 10.6, 10.7, 10.8, 10.8, 10.9, 11.1, 11.2, 11.3

Calculate the mean by adding all the times together and dividing by the total number of athletes.

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of all times}}{\text{number of athletes}}\\ &= \frac{97.9}{9}\\ &\approx 10.9\end{aligned}$$

Identify the median by the 'middle' value in the ordered data set.

Median = 10.8

Answer

The mean is approximately 10.9 seconds and the median is 10.8 seconds.

11. A meteorologist records the daily temperatures (in degrees Celsius) for a week. The temperatures are displayed in the following table.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temp.	22	24	23	19	25	21	20

What are the range and interquartile range of temperatures recorded for the week?

Back to contents

Key points

- The daily temperatures for a week are displayed in the table.
- What is the range and interquartile range of the temperatures?

Explanation

Organise the data set in ascending order.

19, 20, 21, 22, 23, 24, 25

Calculate the range by subtracting the minimum value from the maximum value.

$$\begin{aligned}\text{Range} &= \text{maximum} - \text{minimum}\\ &= 25 - 19\\ &= 6\end{aligned}$$

Determine the first quartile by identifying the middle value of the first half of the ordered set.

19, 20, 21, 22, 23, 24, 25

$Q_1 = 20$

Determine the third quartile by identifying the middle value of the second half of the ordered set.

19, 20, 21, 22, 23, 24, 25

$Q_3 = 24$

Calculate the interquartile range by subtracting the first quartile from the third quartile.

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 24 - 20\\ &= 4\end{aligned}$$

Answer

The range is 6°C and the interquartile range is 4°C.

12. Karen is a researcher and wants to compare the median length of stay for two different wards in a hospital. The wards treat the same types of patients. Ward A has 10 patients and Ward B has 13 patients. The length of stay (in days) for both wards are:

Ward A: 4, 5, 6, 2, 3, 3, 3, 5, 8, 45

Ward B: 3, 4, 4, 5, 6, 7, 9, 10, 12, 15, 20, 25, 80

Which ward has a shorter median length of stay?

Key points

- Ward A has 10 patients with lengths of stay of 4, 5, 6, 2, 3, 3, 3, 5, 8, and 45 days.
- Ward B has 13 patients with lengths of stay of 3, 4, 4, 5, 6, 7, 9, 10, 12, 15, 20, 25, and 80 days.
- Which ward has a shorter median length of stay?

Explanation

Organise the data set for ward A in ascending order.

2, 3, 3, 3, 4, 5, 5, 6, 8, 45

Identify the median for ward A by the 'middle' value in the ordered data set. Because there is an even number of data values, add the two middle numbers together and divide the result by two.

$$\begin{aligned}\text{Median} &= \frac{4+5}{2}\\ &= 4.5\end{aligned}$$

Organise the data set for ward B in ascending order.

3, 4, 4, 5, 6, 7, 9, 10, 12, 15, 20, 25, 80

Identify the median for ward B by the 'middle' value in the ordered data set.

Median = 9

Compare the medians for ward A and ward B to determine which is lower, indicating a shorter median length of stay.

$4.5 < 9$

Ward A < Ward B

Answer

Ward A has a shorter median length of stay than ward B.

13. A business owner wants to know the number of items sold in a given week. The owner knows that the total revenue generated from the sales is \$12 000 and the average selling price of each item is \$25. How many items were sold during that week?

Key points

- The total revenue generated from the sales is \$12 000.
- The average selling price of each item is \$25.
- How many items were sold?

Explanation

The mean is calculated by adding all the values together and dividing by the total number of values.

$$\text{Mean selling price} = \frac{\text{total revenue from sales}}{\text{number of items sold}}$$

Substitute the known values into the equation.

$$25 = \frac{12\,000}{\text{number of items sold}}$$

Solve for the number of items sold.

$$\begin{aligned}\text{Number of items sold} &= \frac{12\,000}{25}\\ &= 480\end{aligned}$$

Answer

480 items were sold during the week.

14. A car dealer wants to analyse the prices of the used cars they sold on a particular day. The data is shown below:

\$16 800, \$19 400, \$16 200, \$18 200, \$15 750, \$14 500, \$66 000, \$17 100, \$21 500

They calculate some summary statistics of the data:

Summary statistic	Mean	Median	Range	IQR
Value	\$22 827.78	\$17 100	\$51 500	\$4475

They realise that the value \$66 000 was actually the price of a new car sold on the same day. How much would each of the mean, median, range, and interquartile range change after this outlier is removed from the data set?

Key points

- The prices of used cars sold on a particular day are \$16 800, \$19 400, \$16 200, \$18 200, \$15 750, \$14 500, \$66 000, \$17 100, and \$21 500.
- Summary statistics were calculated.
- \$66 000 was the price of a new car and was not supposed to be included in the data set.
- How much would each of the mean, median, range, and interquartile range change after this outlier is removed from the data set?

Explanation

Remove 66 000 from the data set. Organise the data set in ascending order.

14 500, 15 750, 16 200, 16 800, 17 100, 18 200, 19 400, 21 500

Calculate the mean by adding all the prices together and dividing by the total number of used cars.

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of all prices}}{\text{number of used cars}}\\ &= \frac{139\,450}{8}\\ &= 17\,431.25\end{aligned}$$

Identify the median by the 'middle' value in the ordered data set. Because there is an even number of data values, add the two middle numbers together and divide the result by two.

$$\text{Median} = \frac{16\,800 + 17\,100}{2}$$

$$= 16\,950$$

Calculate the range by subtracting the minimum value from the maximum value.

$$\text{Range} = \text{maximum} - \text{minimum}$$

$$= 21\,500 - 14\,500$$

$$= 7000$$

Determine the first and third quartiles by identifying the middle value of the first and second half of the ordered set, respectively.

14 500, 15 750, 16 200, 16 800, 17 100, 18 200, 19 400, 21 500

$$Q_1 = \frac{15\,750 + 16\,200}{2} = 15\,975$$

$$Q_3 = \frac{18\,200 + 19\,400}{2} = 18\,800$$

Calculate the interquartile range by subtracting the first quartile from the third quartile.

$$\text{IQR} = Q_3 - Q_1$$

$$= 18\,800 - 15\,975$$

$$= 2825$$

Compare the summary statistics including and excluding the outlier of 66 000.

Summary statistic	Mean	Median	Range	IQR
Value including outlier	\$22 827.78	\$17 100	\$51 500	\$4475
Value excluding outlier	\$17 431.25	\$16 950	\$7000	\$2825

Calculate how much each summary statistic changed by subtracting the value excluding the outlier from the value including the outlier.

Change in mean = 22 827.78 − 17 431.25 = 5396.53

Change in median = 17 100 − 16 950 = 150

Change in range = 51 500 − 7000 = 44 500

Change in IQR = 4475 − 2825 = 1650

Answer

When the outlier was removed from the data, the mean changed by \$5396.53, the median changed by \$150, the range changed by \$44 500, and the interquartile range changed by \$1650.

Reasoning

15. a. The mean of the train travel times is 70.4 minutes. The mean of the bus travel times is 77.5 minutes.

b. The range of the train travel times is 10 minutes. The range of the bus travel times is 45 minutes.

c. The train is typically quicker than the bus. This is because it has a lower mean, indicating that the typical values are lower in the dataset.

d. The train is more consistent than the bus. This is because it has a lower range, indicating that the data values are less spread out.

e. Suggested option 1: Yolanda should consider whether the train station or bus stop is closer to her house.

Suggested option 2: Yolanda should consider whether her friends travel on the train or bus so she can spend time with them while travelling to school.

Note: There are other possible options.

16. a. The mean before the new values are added is 35. The mean after the new values are added is 35.

b. The mean after the new values are added is 35.

c. The mean does not change when the new values are added because the new values sum to 70 and therefore have a mean of 35.

Exam-style

17. C

18. a. Range = 17.8

b. Median = 0

19. Difference in the means is approximately 2.2 mm

20. Outlier = 70. Given the presence of an outlier, the median, mode and interquartile range are the appropriate measures of centre and spread, respectively. Median = 76.5. Mode = 78. Interquartile range = 3.

Remember this?

21. D **22.** D **23.** A

11B Stem-and-leaf plots

Student practice

Worked example 1

a.

Stem	Leaf	Key
0	4	0 | 4 = 0.4
1	1 6	
2	3 5 6	
3	3 7 8	
4	1 4 5 9	
5	2 2 7 8	
6	3 5	
7	4	

b. Median = 42.5, mode = 5.2. The data is negatively skewed.

Worked example 2

a.

Class A	Stem	Class B	Key
	0	5 6	1 | 8 = 18°C
9 8 5 2 2 0	1	0 1 2 3 5 5 8	
8 3 2 1 0	2	0 1	

b. Class A has the highest favourite temperature value. Class B has the lowest favourite temperature value. Class A has a median favourite temperature of 19°C, compared to class B which has a median favourite temperature of 13°C. Class A's favourite temperature lies between 10 and 28°C, and Class B between 5 and 21°C.

Understanding worksheet

1.

Stem	Leaf	Key
5	6 8	2 | 4 = 24
6	0 2 4 6 7 8	
7	0 2 4 6 8	
8	0 0 4 6 8	
9	0	
10	0	

2. **a.** 58 **b.** 90 **c.** 70 **d.** 76

3. stem; key; leaf; ordered; median

Fluency

4. **a.**

Stem	Leaf	Key
0	7	1 | 7 = 17
1	7 8	
2	1 2 4 5 7 7 7	
3	0 0 3 4 8 9	
4		
5	2	

b.

Stem	Leaf	Key
1	4 4	1 | 4 = 14
2	0 2 2 2 5 9	
3	0 0 2 3 4 4 6 8	
4	1 1 4 4 7 9 9	
5	0	

c.

Stem	Leaf	Key
2	1 2 8	2 | 1 = 2.1
3	2 4 5 9	
4	1 2 6 9	
5	0 2 7	

d.

Stem	Leaf	Key
31	5	31 | 5 = 31.5
32		
33	0 1 3 6 8 9	
34	0 2 3 7 7 8 9	
35	0 3 6 7	
36	5 5	

e.

Stem	Leaf	Key
15	4 5 6 6 6 6 8	15 | 4 = 154
16	1 4 4 4 4	
17	0 2 4	
18	2 2	
19	9	

5. **a.** The median is 27.

The mode is 27.

The data is negatively skewed with potential outlier.

b. The median is 33.5.

The mode is 22.

The data is approximately symmetric.

c. The median is 4.0.

There is no mode.

The data is symmetrical.

d. The median is 34.5.

The modes are 34.7 and 36.5.

The data is positively skewed with a potential outlier.

e. The median is 164.

The modes are 156 and 164.

The data is positively skewed.

---

6. **a.**

Set A	Stem	Set B	Key
9 9 9 4 1	2	3	2 | 3 = 23
7 4 2	3	1 4 7 7	
7 6 2 1	4	1 2 3 5 5 6 9	
4 3	5	0 3	

b.

Set X	Stem	Set Y	Key
9 9 6 5	0	3 4 6 7 8	0 | 5 = 0.5
9 8 6 5	1	2 3 5 7	
4 3 2 1	2	4 4 9 9	
8 2 1	3	1 5 7	
2 2	4	1 8	
7 6	5	7	
1	6	2	
9	7	5	

c.

Lakers	Stem	Celtics	Key
8 8 6 2 2 0 0	7	3 8	7 | 0 = 70
8 8 8 7	8	0 1 2 7	
7 4 3	9	0 2 8 9 9	
4 0	10	1 5 7 8 8	
9 6	11	3 3 3 4	
3 1	12	5 8 9	
6 5 2	13	7	
8	14		

d.

Model I	Stem	Model II	Key
	10	2 5 7 8	10 | 2 = 10.2
	11	0 0 1 2	
	12	2 4 6	
9 7 5 2 0	13	1 3	
8 2 0	14	3 5	
9 7 6	15		
4 3	16		
3 1	17		

Back to contents

7. **a.** Set B has the higher median at 42.5, and Set A has the lower median at 35.5. The data in set A ranges from 21 to 54, whereas the data in set B ranges from 23 to 53.

b. Set Y has the higher median at 2.4, and Set X has the lower median at 2.3. The data in set X ranges from 0.5 to 7.9, whereas the data in set Y ranges from 0.3 to 7.5.

c. Celtics have the higher median at 103, and Lakers have the lower median at 93.5. The scores for Lakers ranges from 70 to 148, whereas the scores for Celtics ranges from 73 to 137.

d. Model I has the higher median at 14.8, and Model II has the lower median at 11.2. The speed for Model I ranges from 13.0 to 17.3, whereas the speed in Model II ranges from 10.2 to 14.5.

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. John has recently taken up running and he has been recording the number of kilometres he runs every day for over a fortnight. Here is his data: 8, 12, 15, 9, 25, 10, 9, 5, 35, 14, 38, 42, 9, 26, 27. Construct a stem-and-leaf plot to display John's running data.

Key points

- The data is: 8, 12, 15, 9, 25, 10, 9, 5, 35, 14, 38, 42, 9, 26, 27.
- Construct a stem-and-leaf plot to display John's running data.

Explanation

Identify the stem as the tens values, and leaves as the unit values. Arrange the data in ascending order from smallest to largest. Include a key.

Stem	Leaf
0	5 8 9 9 9
1	0 2 4 5
2	5 6 7
3	5 8
4	2

Key

1 | 0 = 10 km

Answer

Stem	Leaf
0	5 8 9 9 9
1	0 2 4 5
2	5 6 7
3	5 8
4	2

Key

1 | 0 = 10 km

11. Mary is a teacher who has recorded the marks of a recent maths test out of 50. The marks are: 39, 17, 25, 46, 38, 19, 46, 50, 6, 18, 29, 20, 47. Construct a stem-and-leaf plot using Mary's data, and then determine the median score.

Key points

- The marks are: 39, 17, 25, 46, 38, 19, 46, 50, 6, 18, 29, 20, 47.
- Construct a stem-and-leaf plot using Mary's data.
- Determine the median score.

Explanation

Identify the stem as the tens values, and leaves as the unit values. Arrange the data in ascending order from smallest to largest. Include a key.

Stem	Leaf
0	6
1	7 8 9
2	0 5 9
3	8 9
4	6 6 7
5	0

Key

1 | 7 = 17 marks

Identify the median by referring to the 'middle' value in the ordered stem-and-leaf plot.

Median = 29

Answer

Stem	Leaf
0	6
1	7 8 9
2	0 5 9
3	8 9
4	6 6 7
5	0

Key

1 | 7 = 17 marks

The median score is 29.

12. At a local play centre, the number of cars that enter the play centre car park in one day were recorded for two weeks: 120, 121, 150, 128, 135, 148, 140, 154, 142, 130, 136, 145, 138, 154. Construct a stem-and-leaf plot of the data, determine the median and mode, and comment on whether the data is skewed or symmetrical.

Key points

- The data is 120, 121, 150, 128, 135, 148, 140, 154, 142, 130, 136, 145, 138, 154.
- Construct a stem-and-leaf plot of the data.
- Determine the median and mode.
- Comment on whether the data is skewed or symmetrical.

Explanation

Identify the stem as the hundreds and tens values, and leaves as the unit values. Arrange the data in ascending order from smallest to largest. Include a key.

Stem	Leaf
12	0 1 8
13	0 5 6 8
14	0 2 5 8
15	0 4 4

Key

12 | 0 = 120 cars

Identify the median for the number of cars by referring to the 'middle' value in the ordered stem-and-leaf plot. As there is an even number of data values, add the two middle numbers together and divide the result by two.

Median $= \frac{138 + 140}{2} = 139$

Identify the mode by referring to the data value with the highest frequency in the 'leaf' column.

The mode is 154.

The data is symmetrical.

Back to contents

Answer

Stem	Leaf	Key
12	0 1 8	12 | 0 = 120 cars
13	0 5 6 8	
14	0 2 5 8	
15	0 4 4	

The median number of cars is 139.

The mode is 154.

The data is symmetrical.

13. A small company recently conducted a survey asking their employees how many hours they worked last Monday. The results were as follows: 8.5, 9.0, 6.5, 8.0, 9.5, 8.0, 8.0, 7.5, 8.5, 9.0, 7.5, 8.5, 9.0, 9.5, 9.5. Construct a stem-and-leaf plot of the data, determine the median and mode, and comment on whether the data is skewed or symmetrical.

Key points

- The data is 8.5, 9.0, 6.5, 8.0, 9.5, 8.0, 8.0, 7.5, 8.5, 9.0, 7.5, 8.5, 9.0, 9.5, 9.5.
- Construct a stem-and-leaf plot of the data.
- Determine the median and mode.
- Comment on whether the data is skewed or symmetrical.

Explanation

Identify the stem as the unit values, and leaves as the tenth values. Arrange the data in ascending order from smallest to largest. Include a key.

Stem	Leaf	Key
6	5	8 | 5 = 8.5 hours
7	5 5	
8	0 0 0 5 5 5	
9	0 0 0 5 5 5	

Identify the median by referring to the ‘middle’ value in the ordered stem-and-leaf plot.

Median = 8.5

Identify the mode by referring to the data value(s) with the highest frequency in the ‘leaf’ column.

The modes are 8.0, 8.5, 9.0 and 9.5.

The data is negatively skewed.

Answer

Stem	Leaf	Key
6	5	8 | 5 = 8.5 hours
7	5 5	
8	0 0 0 5 5 5	
9	0 0 0 5 5 5	

The median is 8.5.

The modes are 8.0, 8.5, 9.0 and 9.5.

The data is skewed.

14. A sports event was conducted where two teams competed. The team scores (out of 50) were recorded as follows:

Team A: 24, 38, 10, 40, 45, 27, 35, 38, 17, 44, 40, 45, 43, 38

Team B: 33, 45, 47, 50, 46, 48, 47, 45, 50, 37, 48, 39, 41, 50

Construct back-to-back stem-and-leaf plots for the two teams. Calculate the mode and median for each team, and compare and contrast the performance of the two teams based on your analysis of the plots.

Key points

- Team A’s scores are: 24, 38, 10, 40, 45, 27, 35, 38, 17, 44, 40, 45, 43, 38.
- Team B’s scores are: 33, 45, 47, 50, 46, 48, 47, 45, 50, 37, 48, 39, 41, 50.
- Construct back-to-back stem-and-leaf plots for the two teams.
- Calculate the mode and median for each team.
- Compare and contrast the performance of the two teams based on your analysis of the plots.

Explanation

Identify the stem as the tens values, and leaves as the unit values. Arrange the data in ascending order from smallest to largest. Include a key.

Team A	Stem	Team B	Key
	0		1 | 0 = score of 10
7 0	1		
7 4	2		
8 8 8 5	3	3 7 9	
5 5 4 3 0 0	4	1 5 5 6 7 7 8 8	
	5	0 0 0	

Identify the median for Team A and B by referring to the ‘middle’ value in the ordered stem-and-leaf plot. As there is an even number of data values, add the two middle numbers together and divide the result by two.

Median for Team A $= \frac{38 + 38}{2} = 38$

Median for Team B $= \frac{46 + 47}{2} = 46.5$

Identify the mode for both Team A and Team B by referring to the data value(s) with the highest frequency in the ‘leaf’ column.

The mode for Team A is 38.

The mode for Team B is 50.

Team B has a median score of 46.5, compared to Team A that has a median score of 38. Team A scores generally lie between 10 and 45, and Team B between 33 and 50.

Answer

Team A	Stem	Team B	Key
	0		1 | 0 = score of 10
7 0	1		
7 4	2		
8 8 8 5	3	3 7 9	
5 5 4 3 0 0	4	1 5 5 6 7 7 8 8	
	5	0 0 0	

The median score for Team A is 38.

The median score for Team B is 46.5.

The mode score for Team A is 38.

The mode score for Team B is 50.

Team B has a median score of 46.5, compared to Team A that has a median score of 38. Team A scores generally lie between 10 and 45, and Team B between 33 and 50.

Back to contents

Reasoning

15. a.

Location A	Stem	Location B	Key
8 8 5 5 5 5 2	8	0 0	8 | 0 = 80 customers
8 5 5 0 0	9	2 5 5 5	
0 0	10	0 0 5 5 8 8	
0 0	11	0 0 0 2 2 5 8	
0 0	12		
0	13		

b. Location A has a median number of 95 customers. Location B has a median number of 105 customers.

c. The mode for location A is 85 customers. The mode for location B is 95 and 110 customers. Location A has a higher frequency for the mode.

d. Location A has the highest number of customers. Location B has the lowest number of customers. Location A has a median number of 95 customers, compared to Location B which has a median number of 105 customers. Location A generally has between 82 and 130 customers, and Location B between 80 and 118 customers.

e. Suggested option 1: The fast-food chain can run advertisements on social media.

Suggested option 2: The fast-food chain can sponsor events.

Note: There are other possible options.

16. a.

Stem	Leaf	Key
2	6 9	2 | 6 = 2.6
3	1 7	
4	3	
5	2 5	

The median for set A is 3.7.

b.

Stem	Leaf	Key
2	5	2 | 5 = 2.5
3	0 2	
4	0 5	
5	0 0 3 5 7	

The median for set B is 4.75.

c. The median for set A is the middle value because set A has an odd number of values. The median for set B is the mean of the two middle values because set B has an even number of values.

Exam-style

17. C

18. a. 36 cm

b. Median $= \dfrac{32 + 33}{2} = 32.5$ cm

19. The median is 22.0 km.

The mode is 22.8 km.

The data is approximately symmetrical.

20. The median is 29.5.

The median value of 31.5 is incorrect because the leaf is not ordered from smallest to largest.

Remember this?

21. D **22.** B **23.** C

11C Grouped data

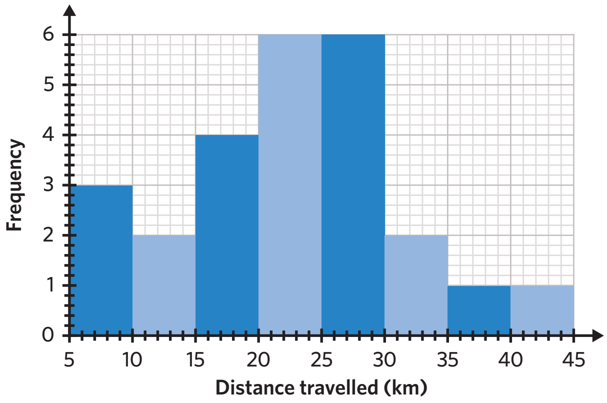
Student practice

Worked example 1

a.

Distance travelled (km)	Frequency	Percentage frequency
5–<10	3	12%
10–<15	2	8%
15–<20	4	16%
20–<25	6	24%
25–<30	6	24%
30–<35	2	8%
35–<40	1	4%
40–<45	1	4%
Total	**25**	**100%**

b.



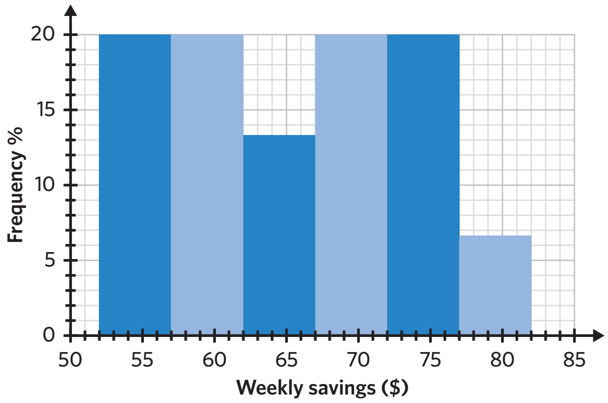
Approximately symmetric

Worked example 2

a.

Weekly saving ($)	Frequency	Percentage frequency
52–<57	3	20%
57–<62	3	20%
62–<67	2	$13.\dot{3}\%$
67–<72	3	20%
72–<77	3	20%
77–<82	1	$6.\dot{6}\%$
Total	**15**	**100%**

b.



Bimodal

Understanding worksheet

1.

Hours of study	Frequency	Percentage frequency
0–<3	1	10%
3–<6	4	40%
6–<9	3	30%
9–<12	2	20%
Total	**10**	**100%**

2. **a.** 1 **b.** 23 **c.** 40–<45 **d.** 19

3. histogram; frequency; axis; shape; percentage

Fluency

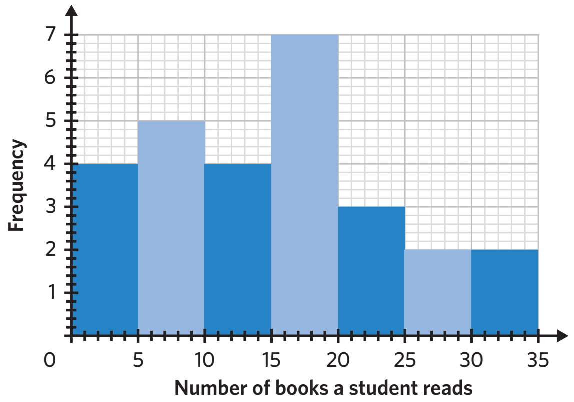
4. **a.**

Number of books a student reads	Frequency
0–<5	4
5–<10	5
10–<15	4
15–<20	7
20–<25	3
25–<30	2
30–<35	2
Total	**27**

b.

Number of books a student reads	Frequency	Percentage frequency
0–<5	4	14.81%
5–<10	5	18.52%
10–<15	4	14.81%
15–<20	7	25.93%
20–<25	3	11.11%
25–<30	2	7.41%
30–<35	2	7.41%
Total	**27**	**100%**

c.



d. Approximately symmetric

e. 9

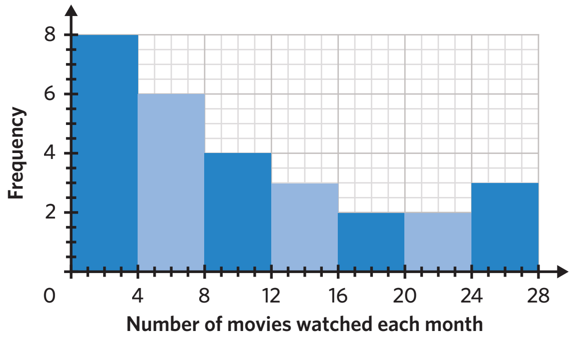
f. 25.93%

5. **a.**

Number of movies watched each month	Frequency	Percentage frequency
0–<4	8	28.57%
4–<8	6	21.43%
8–<12	4	14.29%
12–<16	3	10.71%
16–<20	2	7.14%
20–<24	2	7.14%
24–<28	3	10.71%
Total	**28**	**99.99%***

*Due to rounding.

b.



c. Positively skewed

d. 14

e. 46.43%

6. **a.**

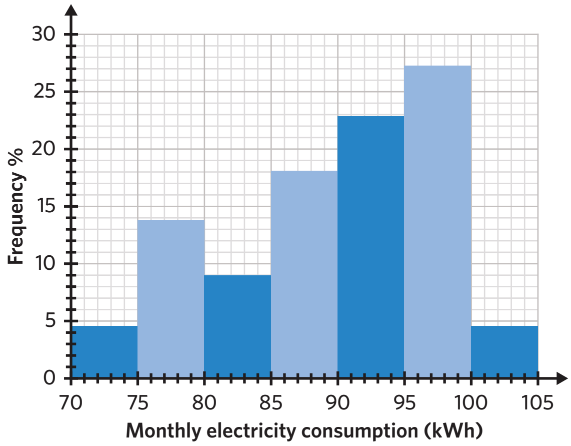
Monthly electricity consumption (kWh)	Frequency
70–<75	1
75–<80	3
80–<85	2
85–<90	4
90–<95	5
95–<100	6
100–<105	1
Total	**22**

b.

Monthly electricity consumption (kWh)	**Frequency**	**Percentage frequency**
70–<75	1	4.55%
75–<80	3	13.64%
80–<85	2	9.09%
85–<90	4	18.18%
90–<95	5	22.73%
95–<100	6	27.27%
100–<105	1	4.55%
Total	**22**	**100.01%***

*Due to rounding.

c.



d. Negatively skewed

e. 4

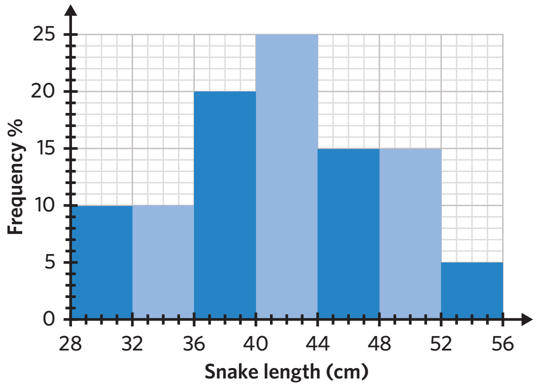
f. 50%

---

7. a.

Snake length (cm)	**Frequency**	**Percentage frequency**
28–<32	2	10%
32–<36	2	10%
36–<40	4	20%
40–<44	5	25%
44–<48	3	15%
48–<52	3	15%
52–<56	1	5%
Total	**20**	**100%**

b.



c. Approximately symmetric

d. 16

e. 35%

---

8. D

Spot the mistake

9. a. Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. Sarah conducted a survey at school to find out the number of books students read in a month. She collected the following data: 8, 5, 12, 9, 7, 4, 10, 6, 8, 11, 5, 9, 10, 7, 6

Complete the grouped frequency table for the number of books read. Round to two decimal places where necessary.

Number of books read	**Frequency**	**Percentage frequency**
0–<5		
5–<10		
10–<15		
Total		

Key points

- Sarah conducted a survey to find out the number of books students read in a month.
- Data: 8, 5, 12, 9, 7, 4, 10, 6, 8, 11, 5, 9, 10, 7, 6.
- Complete the grouped frequency table.

Explanation

Go through the list of values and calculate the frequency for each class interval occurrence in the middle column.

Number of books read	**Frequency**	**Percentage frequency**
0–<5	1	
5–<10	10	
10–<15	4	
Total	**15**	

Convert each frequency to a percentage by dividing by the total and multiplying by 100.

Number of books read	**Frequency**	**Percentage frequency**
0–<5	1	$\frac{1}{15} \times 100 = 6.67\%$
5–<10	10	$\frac{10}{15} \times 100 = 66.67$
10–<15	4	$\frac{4}{15} \times 100 = 26.67\%$
Total	**15**	**100.01%***

Answer

Number of books read	Frequency	Percentage frequency
0–<5	1	6.67%
5–<10	10	66.67%
10–<15	4	26.67%
Total	**15**	**100.01%***

*Due to rounding

11. During the school's sports day, the PE teacher recorded the time taken (in seconds) by students to complete a 100 m sprint. The data is as follows: 15.2, 14.8, 16.5, 15.0, 15.4, 16.2, 14.9, 15.1, 16.0, 15.3. Using the data, construct a grouped frequency table with class intervals of 0.5, starting with interval 14.5–<15.

Key points

- The PE teacher recorded the time taken (in seconds) by students to complete a 100 m sprint.
- Data: 15.2, 14.8, 16.5, 15.0, 15.4, 16.2, 14.9, 15.1, 16.0, 15.3.
- Construct a grouped frequency table with class intervals of 0.5, starting with interval 14.5–<15.

Explanation

Go through the list of values and calculate the frequency for each class interval occurrence in the middle column.

Time taken for 100 m sprint (seconds)	Frequency	Percentage frequency
14.5–<15	2	$\frac{2}{10} \times 100 = 20\%$
15–<15.5	5	$\frac{5}{10} \times 100 = 50\%$
15.5–<16	0	$\frac{0}{10} \times 100 = 0\%$
16–<16.5	2	$\frac{2}{10} \times 100 = 20\%$
16.5–<17	1	$\frac{1}{10} \times 100 = 10\%$
Total	**10**	**100%**

Convert each frequency to a percentage by dividing by the total and multiplying by 100.

Answer

Time taken for 100 m sprint (seconds)	Frequency	Percentage frequency
14.5–<15	2	20%
15–<15.5	5	50%
15.5–<16	0	0%
16–<16.5	2	20%
16.5–<17	1	10%
Total	**10**	**100%**

12. A local gym recorded the number of hours members exercised in a month. The data is as follows:

15, 20, 18, 22, 17, 19, 21, 15, 20, 18, 22. Using the data, construct a frequency histogram with class intervals of 3 starting with interval 15–<18, and then use the shape of the histogram to comment on the distribution of the data.

Key points

- A gym recorded the number of hours members exercised in a month.
- Data: 15, 20, 18, 22, 17, 19, 21, 15, 20, 18, 22.
- Construct a frequency histogram with class intervals of 3 starting with interval 15–<18. Comment on the distribution of the data.

Explanation

Construct a grouped frequency table from the data.

Number of hours exercised in a month	Frequency	Percentage frequency
15–<18	3	27.27%
18–<21	5	45.45%
21–<24	3	27.27%
Total	**11**	**99.99%***

*Due to rounding.

Use the class intervals from the frequency table to determine how to scale the horizontal (x) axis.

Minimum value = 15

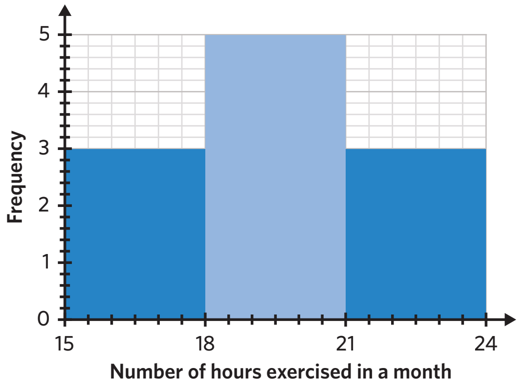
Maximum value = 24

Identify the smallest and largest frequency value from the frequency table to determine how to scale the vertical (y) axis.

Minimum frequency = 3

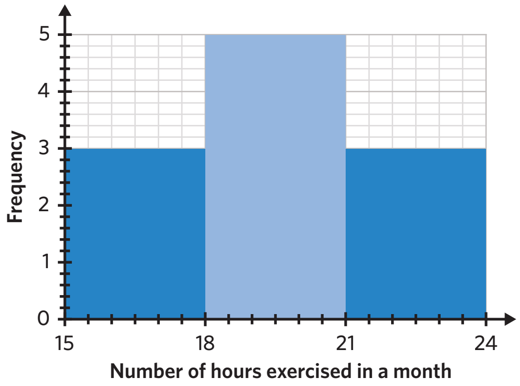
Maximum frequency = 5

Draw the bars of the histogram using the frequencies and class intervals in the table. Label the histogram's axes.



The overall shape of the histogram is symmetric.

Answer



The distribution is symmetric.

13. A basketball coach recorded the number of points scored by each player in a season. The data is displayed in the grouped frequency table.

Points scored	Frequency	Percentage frequency
0–<10	5	27.78%
10–<20	8	44.44%
20–<30	3	16.67%
30–<40	2	11.11%
Total	**18**	**100%**

Construct a percentage frequency histogram to represent the data and analyse the performance of the players. Is the data symmetric, skewed, or bimodal?

Key points

- A basketball coach recorded the number of points scored by each player in a season.
- The data is displayed in the grouped frequency table.
- Construct a percentage frequency histogram. Is the data symmetric, skewed, or bimodal?

Explanation

Identify the smallest and largest data value from the frequency table to determine the scale of the horizontal (x) axis.

Minimum value $= 0$

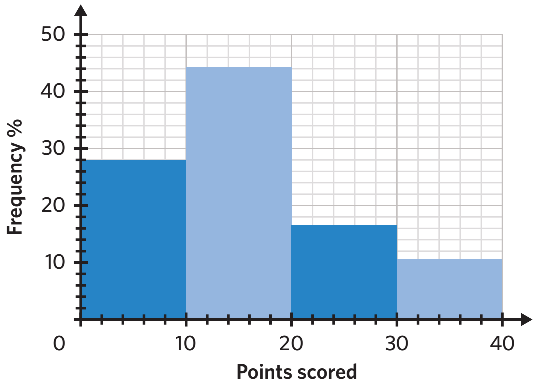
Maximum value $= 40$

Identify the smallest and largest percentage frequency value from the frequency table to determine the scale of the vertical (y) axis.

Minimum percentage frequency $= 11.11\%$

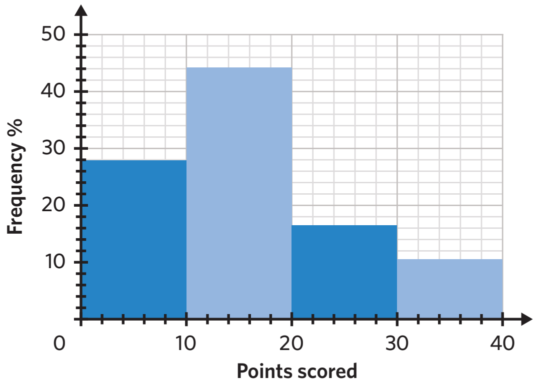
Maximum percentage frequency $= 44.44\%$

Draw the bars of the histogram using the percentage frequencies and class intervals given in the table. Label the histogram's axes.



The 'tail' of the histogram closer to positive values indicates the direction of the skew.

Answer



The distribution is positively skewed.

14. A local farmer recorded the number of apples picked from his orchard over 16 days. The data is as follows: 52, 50, 55, 60, 45, 52, 50, 55, 53, 60, 45, 53, 50, 55, 60, 51. Construct a frequency table with class intervals of 3 units, starting with the interval 45–<48, and then create a frequency histogram using the frequency table. Use the shape of the histogram to comment on the distribution of the data.

Key points

- A farmer recorded the number of apples picked from his orchard.
- Data: 52, 50, 55, 60, 45, 52, 50, 55, 53, 60, 45, 53, 50, 55, 60, 51.
- Construct a frequency table with class intervals of 3 units, starting with the interval 45–<48, and then create a frequency histogram. Comment on the shape of the distribution.

Explanation

Construct a grouped frequency table from the data.

Number of apples	Frequency	Percentage frequency
45–<48	2	12.5%
48–<51	3	18.75%
51–<54	5	31.25%
54–<57	3	18.75%
57–<60	0	0%
60–<63	3	18.75%
Total	**16**	**100%**

Identify the smallest and largest data value from the frequency table to determine the scale of the horizontal (x) axis.

Minimum value $= 45$

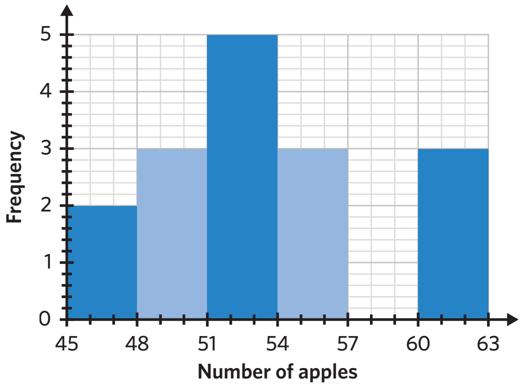
Maximum value $= 63$

Identify the smallest and largest frequency value from the frequency table to determine the scale of the vertical (y) axis.

Minimum frequency $= 0$

Maximum frequency $= 5$

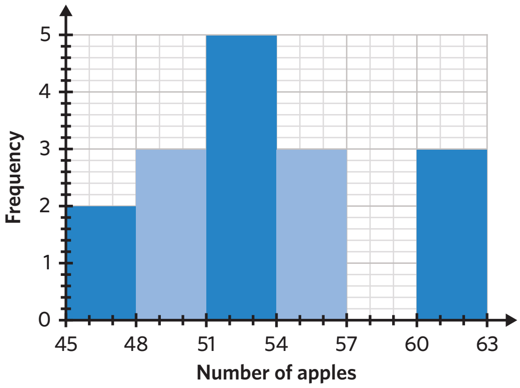
Draw the bars of the histogram using the frequencies and class intervals in the frequency table. Label the histogram's axes.



The overall shape of the histogram is approximately symmetric.

Answer

Number of apples	Frequency	Percentage frequency
45–<48	2	12.5%
48–<51	3	18.75%
51–<54	5	31.25%
54–<57	3	18.75%
57–<60	0	0%
60–<63	3	18.75%
Total	**16**	**100%**

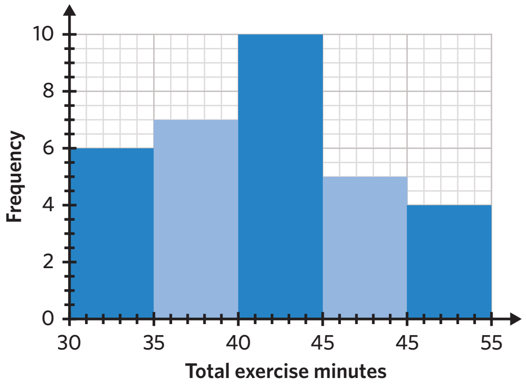


The distribution is approximately symmetric.

Reasoning

15. **a.** The frequency percentage for 45–<50 minutes of exercise each day is 15.63%.

b.



c. The distribution is approximately symmetric.

d. The percentage of students who complete 35–<50 minutes of exercise each day is 68.75%.

e. Suggested option 1: Walking

Suggested option 2: Lifting

Note: There are other possible options.

16. **a.** The frequency percentage for the class interval 10–<15 is 60%.

b. The frequency value for the class interval 15–<20 is 2.

c. No it's not possible to determine the frequency value in part **b** without first calculating the frequency percentage in part **a**. This is because the frequency percentage is first needed to determine the frequency total which is then used to calculate the frequency of the interval in part **b**.

Exam-style

17. D

18. **a.** Negatively skewed

b. 66.67%

19. 60%

20.

Height (m)	Frequency	Percentage frequency
14.5–<15.0	4	25%
15.0–<15.5	5	31.25%
15.5–<16.0	3	18.75%
16.0–<16.5	4	25%
Total	**16**	**100%**

Remember this?

21. E **22.** E **23.** A

11D Boxplots

Student practice

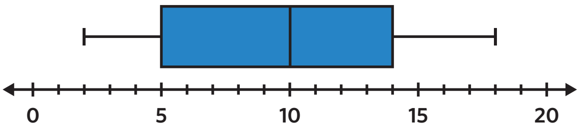
Worked example 1

a. Minimum $= 2$, $Q_1 = 5$, median $= 10$, $Q_3 = 14$, maximum $= 18$

Lower fence $= -8.5$

Upper fence $= 27.5$

b.



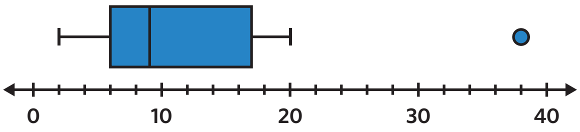
Worked example 2

a. Minimum $= 2$, $Q_1 = 6$, median $= 9$, $Q_3 = 17$, maximum $= 38$

Lower fence $= -10.5$

Upper fence $= 33.5$

b.



Worked example 3

a. **Set A:** Minimum $= 5$, $Q_1 = 9$, median $= 16$, $Q_3 = 25$, maximum $= 29$

Lower fence $= -15$

Upper fence $= 49$

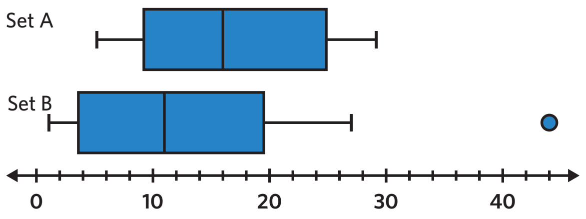
Set B: Minimum $= 1$, $Q_1 = 3.5$, median $= 11$, $Q_3 = 19.5$, maximum $= 44$

Lower fence $= -20.5$

Upper fence $= 43.5$

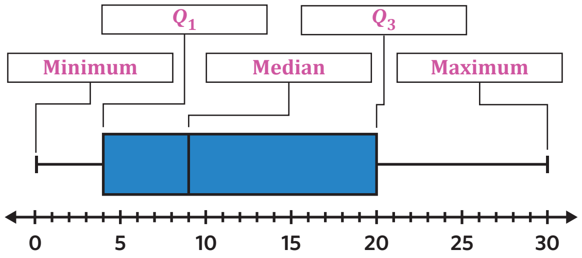
Back to contents

b.



Understanding worksheet

1.



2. **a.** 2 **b.** 30

c. 15 and 30 **d.** 25

3. median; whiskers; 50%; outlier

Fluency

4. **a.** Minimum = 5, Q_1 = 10 , median = 15, Q_3 = 25, maximum = 30

Lower fence = −12.5

Upper fence = 47.5

b. Minimum = 1, Q_1 = 5, median = 11, Q_3 = 17, maximum = 21

Lower fence = −13

Upper fence = 35

c. Minimum = 3, Q_1 = 6, median = 11, Q_3 = 16, maximum = 19

Lower fence = −9

Upper fence = 31

d. Minimum = 2, Q_1 = 5, median = 10, Q_3 = 17, maximum = 20

Lower fence = −13

Upper fence = 35

e. Minimum = 2, Q_1 = 7, median = 13.5, Q_3 = 19.5, maximum = 24

Lower fence = −11.75

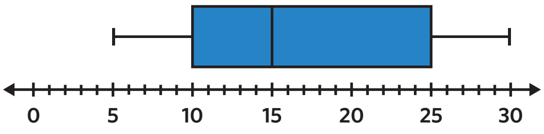
Upper fence = 38.25

f. Minimum = 2, Q_1 = 6.5, median = 17.5, Q_3 = 25.5, maximum = 27

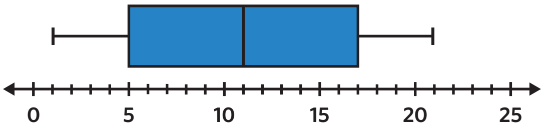
Lower fence = −22

Upper fence = 54

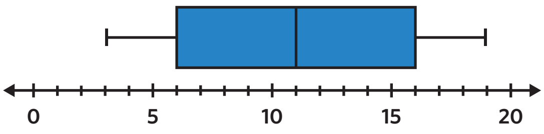
5. **a.**



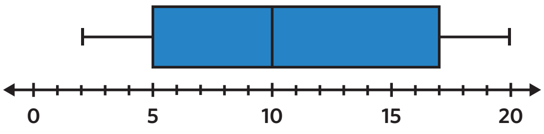
b.



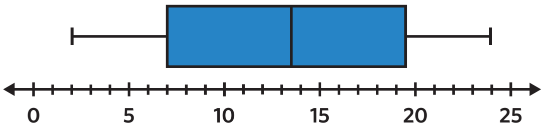
c.



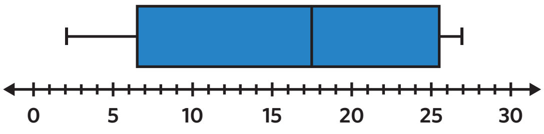
d.



e.



f.



6. **a.** Minimum = 0, Q_1 = 9, median = 10, Q_3 = 23, maximum = 45

Lower fence = −12

Upper fence = 44

b. Minimum = 1, Q_1 = 18, median = 19, Q_3 = 28, maximum = 30

Lower fence = 3

Upper fence = 43

c. Minimum = 6, Q_1 = 20.5, median = 27, Q_3 = 29, maximum = 33

Lower fence = 7.75

Upper fence = 41.75

d. Minimum = 12, Q_1 = 18, median = 22, Q_3 = 29, maximum = 46

Lower fence = 1.5

Upper fence = 45.5

e. Minimum = 4, Q_1 = 14.5, median = 15.5, Q_3 = 19, maximum = 25

Lower fence = 7.75

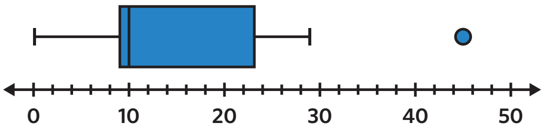
Upper fence = 25.75

f. Minimum = 15, Q_1 = 16.5, median = 25.5, Q_3 = 28.5, maximum = 47

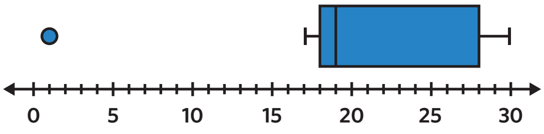
Lower fence = −1.5

Upper fence = 46.5

7. **a.**

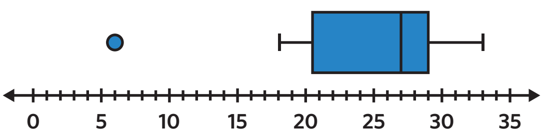


b.

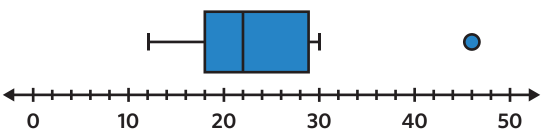


Back to contents

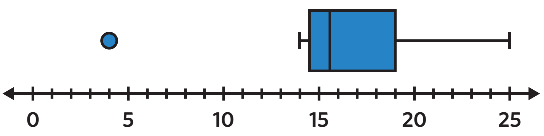
c.



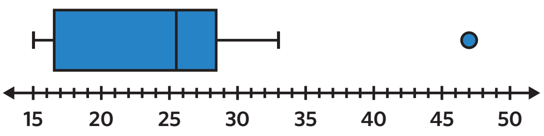
d.



e.



f.



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8. **a.** **Set A:** Minimum $= 5$, $Q_1 = 8$, median $= 19$, $Q_3 = 28$, maximum $= 33$

Lower fence $= -22$

Upper fence $= 58$

Set B: Minimum $= 1$, $Q_1 = 2$, median $= 19$, $Q_3 = 26$, maximum $= 39$

Lower fence $= -34$

Upper fence $= 62$

b. **Set A:** Minimum $= 4$, $Q_1 = 8$, median $= 17$, $Q_3 = 27$, maximum $= 31$

Lower fence $= -20.5$

Upper fence $= 55.5$

Set B: Minimum $= 0$, $Q_1 = 11.5$, median $= 17$, $Q_3 = 22$, maximum $= 36$

Lower fence $= -4.25$

Upper fence $= 37.75$

c. **Set A:** Minimum $= 5$, $Q_1 = 9.5$, median $= 15$, $Q_3 = 20.5$, maximum $= 38$

Lower fence $= -7$

Upper fence $= 37$

Set B: Minimum $= 1$, $Q_1 = 6$, median $= 16.5$, $Q_3 = 21$, maximum $= 30$

Lower fence $= -16.5$

Upper fence $= 43.5$

d. **Set A:** Minimum $= 5$, $Q_1 = 7$, median $= 14.5$, $Q_3 = 24$, maximum $= 50$

Lower fence $= -18.5$

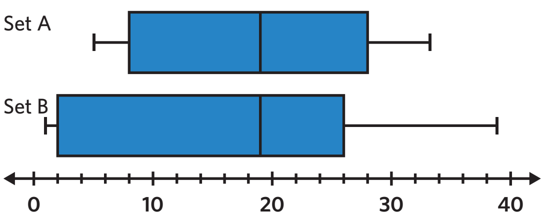
Upper fence $= 49.5$

Set B: Minimum $= 0$, $Q_1 = 18$, median $= 21$, $Q_3 = 26$, maximum $= 38$

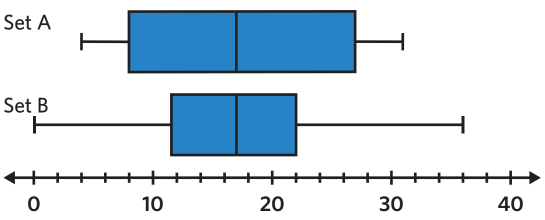
Lower fence $= 6$

Upper fence $= 38$

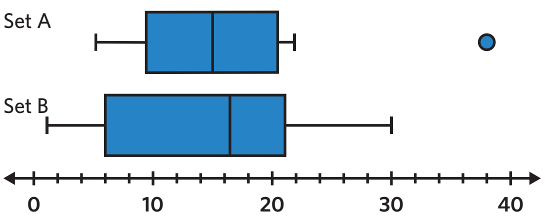
9. **a.**



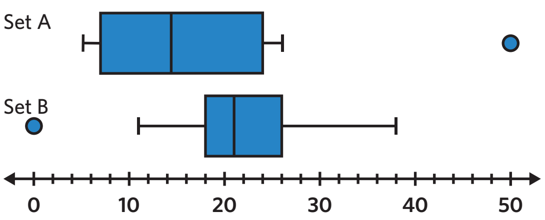
b.



c.



d.



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10. E

Spot the mistake

11. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

12. A tech enthusiast is studying the price range of different iPhone models released over the past few years in AUD. The prices she noted down are: \$799, \$899, \$999, \$1099, \$1599, \$749, \$1199, \$849, \$949, \$1099, \$1199. Prove that \$1599 is not considered an outlier.

Key points

- The prices of different iPhone models released over the past few years are: \$799, \$899, \$999, \$1099, \$1599, \$749, \$1199, \$849, \$949, \$1099, \$1199.
- Prove that \$1599 is not considered an outlier.

Explanation

Organise the data set in ascending order and identify the first and third quartiles.

Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3.

$$\begin{aligned} \text{IQR} &= 1199 - 849 \\ &= 350 \end{aligned}$$

Determine the upper fence to prove that \$1599 is not considered an outlier.

$$\text{Upper fence} = 1199 + 1.5 \times 350$$
$$= 1724$$

$1599 < 1724$

∴ \$1599 is not considered an outlier.

Answer

\$1599 is less than the upper fence (\$1724) and therefore is not considered an outlier.

13. A local marathon recorded the finish times (in hours) of its participant: 3.5, 2.4, 4, 3.8, 5.2, 3.9, 4.5, 4.2, 4.7, 4.1, 5, 5.1, 3.7, 4.2. Determine if there are any outliers in this data.

Key points

- The recorded finish times (in hours) of a local marathon were: 3.5, 2.4, 4, 3.8, 5.2, 3.9, 4.5, 4.2, 4.7, 4.1, 5, 5.1, 3.7, 4.2.
- Determine if there are any outliers in this data.

Explanation

Organise the data set in ascending order and identify the first and third quartiles.

2.4, 3.5, 3.7, 3.8, 3.9, 4, 4.1, 4.2, 4.2, 4.5, 4.7, 5, 5.1, 5.2

$Q_1 = 3.8$

$Q_3 = 4.7$

Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3.

$$\text{IQR} = 4.7 - 3.8$$
$$= 0.9$$

Determine the lower and upper fences to identify if there are any outliers in the data.

$$\text{Lower fence} = 3.8 - 1.5 \times 0.9$$
$$= 2.45$$

$$\text{Upper fence} = 4.7 + 1.5 \times 0.9$$
$$= 6.05$$

There is one value less than 2.45 but there are no values greater than 6.05.

∴ 2.4 is an outlier.

Answer

There is one outlier (2.4) in this data.

14. A local library has conducted a survey to determine how many books each visitor borrows every week. Generate a boxplot for 1, 2, 1, 3, 2, 1, 3, 4, 2, 1, 2, 3, 3. The data has a lower fence value of −2 and an upper fence value of 6.

Key points

- The number of books each visitor borrowed at a local library were recorded: 1, 2, 1, 3, 2, 1, 3, 4, 2, 1, 2, 3, 3.
- The data has a lower fence value of −2 and an upper fence value of 6.
- Generate a boxplot for the given data.

Explanation

Organise the data set in ascending order to determine the five-number summary.

1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4

Minimum = 1

$Q_1 = 1$

Median = 2

$Q_3 = 3$

Maximum = 4

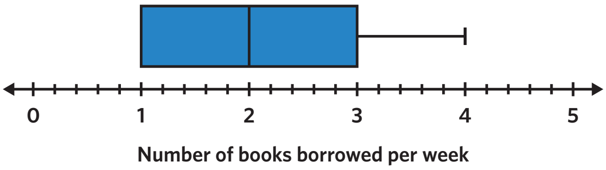
Identify if there are any outliers in the data.

There are no data values less than −2 and/or greater than 6.

∴ There are no outliers.

Plot the five-number summary on a number line and construct the box plot.

Answer



15. During a cricket tournament, a team's individual scores for eleven matches were: 280, 290, 315, 270, 330, 310, 295, 275, 340, 320, 205. The coach believes that the lowest score (205) is a statistical outlier and wants to verify this. Determine the five-number summary, and then create a boxplot to display and confirm this.

Key points

- A team's individual scores for eleven matches were: 280, 290, 315, 270, 330, 310, 295, 275, 340, 320, 205.
- The coach believes that the lowest score (205) is a statistical outlier and wants to verify this.
- Determine the five-number summary, and then create a boxplot to display and confirm if 205 is a statistical outlier.

Explanation

Organise the data set in ascending order to determine the five-number summary.

205, 270, 275, 280, 290, 295, 310, 315, 320, 330, 340

Minimum = 205

$Q_1 = 275$

Median = 295

$Q_3 = 320$

Maximum = 340

Calculate the interquartile range (IQR) and determine the lower and upper fences to identify if there are any outliers in the data.

$$\text{IQR} = 320 - 275$$
$$= 45$$

$$\text{Lower fence} = 275 - 1.5 \times 45$$
$$= 207.5$$

$$\text{Upper fence} = 320 + 1.5 \times 45$$
$$= 387.5$$

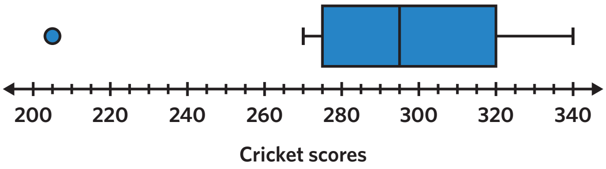
$205 < 207.5$

∴ 205 is an outlier.

There are no data values greater than 387.5 therefore there are no other outliers.

Plot the five-number summary on a number line and construct the box plot.

Answer



Back to contents

16. For a geography project, John collected data on the daily temperatures (in degrees Celsius) of his town for the month of January and March.

January: 25, 27, 24, 29, 31, 38, 26, 30, 24, 25, 32, 28, 31, 24, 33, 29, 30, 31, 26, 27, 28, 29, 33, 25, 27, 28, 30, 30, 24, 26, 27.

March: 23, 21, 20, 32, 34, 25, 23, 21, 24, 25, 27, 22, 25, 32, 27, 31, 26, 38, 19, 21, 20, 24, 23, 20, 20, 19, 19, 18, 23, 20, 20.

Determine the five-number summaries and if there are any outliers, then display the data using parallel boxplots.

Key points

- John collected data on the daily temperatures (in degrees Celsius) for January and March.
- January data: 25, 27, 24, 29, 31, 38, 26, 30, 24, 25, 32, 28, 31, 24, 33, 29, 30, 31, 26, 27, 28, 29, 33, 25, 27, 28, 30, 30, 24, 26, 27
- March data: 23, 21, 20, 32, 34, 25, 23, 21, 24, 25, 27, 22, 25, 32, 27, 31, 26, 38, 19, 21, 20, 24, 23, 20, 20, 19, 19, 18, 23, 20, 20
- Determine the five-number summaries and if there are any outliers, then display the data using parallel boxplots.

Explanation

Organise the data sets in ascending order to determine the five-number summaries.

January:

24, 24, 24, 24, 25, 25, 25, 25, 26, 26, 26, 27, 27, 27, 27, 28, 28, 28, 29, 29, 29, 30, 30, 30, 30, 31, 31, 31, 32, 33, 33, 38

Minimum = 24

Q_1 = 26

Median = 28

Q_3 = 30

Maximum = 38

March:

18, 19, 19, 19, 20, 20, 20, 20, 20, 20, 21, 21, 21, 22, 23, 23, 23, 23, 24, 24, 25, 25, 25, 26, 27, 27, 31, 32, 32, 34, 38

Minimum = 18

Q_1 = 20

Median = 23

Q_3 = 26

Maximum = 38

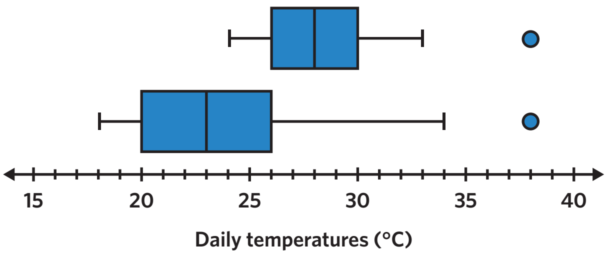
Calculate the interquartile range (IQR) for each data set and determine the lower and upper fences to identify if there are any outliers in the data.

January:	**March:**
IQR = 30 − 26	IQR = 26 − 20
= 4	= 6
Lower fence = 26 − 1.5 × 4	Lower fence = 20 − 1.5 × 6
= 20	= 11
Upper fence = 30 + 1.5 × 4	Upper fence = 26 + 1.5 × 6
= 36	= 35
∴ There is one outlier (38).	∴ There is one outlier (38).

Plot the five-number summaries on a number line and construct the box plots. The outliers are displayed by using a ‘●’ or a ‘✕’.

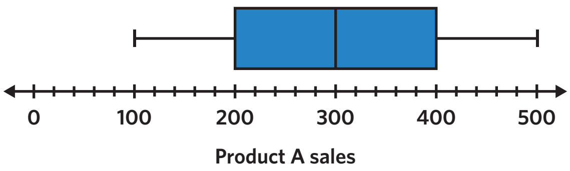
Answer

January:	**March:**
Minimum = 24	Minimum = 18
Q_1 = 26	Q_1 = 20
Median = 28	Median = 23
Q_3 = 30	Q_3 = 26
Maximum = 38	Maximum = 38
There is one outlier in January (38).	There is one outlier in March (38).

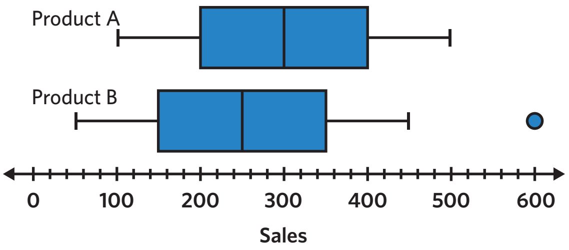


Reasoning

17. a.



b.



c. **Product A:** IQR = 200, range = 400

Product B: IQR = 200, range = 550

d. Upper fence = 680

∴ 700 is an outlier.

e. Suggested option 1: Beanica could use a marketing campaign that includes special promotions, discounts or bundled offers to boost sales.

Suggested option 2: Beanica could collect customer feedback to understand why Product D is underperforming so that improvements can be made.

Note: There are other possible options.

18. a. Minimum = 15, Q_1 = 21, median = 25, Q_3 = 31, maximum = 40

Lower fence = 6

b. Minimum = 5, Q_1 = 19, median = 25, Q_3 = 31, maximum = 40

Lower fence = 1

c. The additional value (5) in Set B directly impacts the minimum value. It also lowers the first quartile and lower fence.

Back to contents

Exam-style

19. C

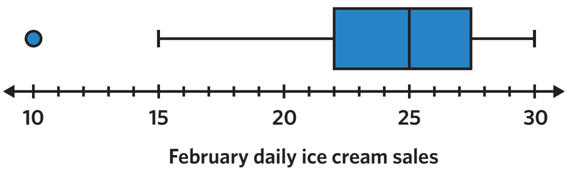
20. a. Lower fence $= 22 - 1.5 \times (27.5 - 22)$

$= 13.75$

Minimum $= 10 < 13.75$

$\therefore$ There is an outlier (10).

b.

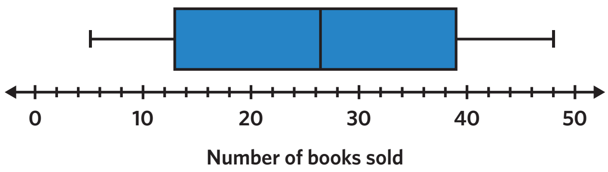


21. Lower fence $= 9.25$

Upper fence $= 15.25$

There are no values less than 9.25 and none greater than 15.25. Therefore the minimum and maximum values are not outliers.

22.



Remember this?

23. C **24.** B **25.** A

Chapter 11 extended application

1. a. Mean $= 9.99$, median $= 9.90$, no mode

b. Range $= 1.37$, IQR $= 0.25$

c.

Stem	Leaf	Key
9	6 7 8 8 8 9 9 9	10 | 1 = 10.1
10	0 0 1 1 3	
11	0	

d. 11.00 seconds is an outlier in this data set as it is greater than the upper fence value of 10.27 seconds.

e. Suggested option 1: The athlete running the race.

Suggested option 2: The weather conditions on the day of the race.

Note: There are other possible options.

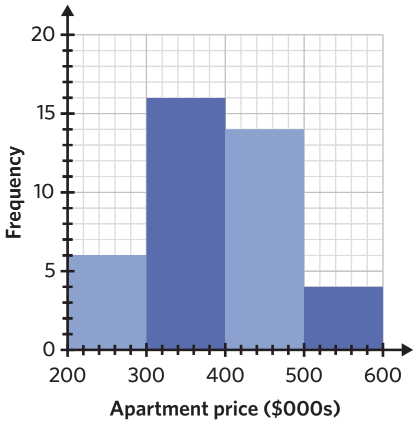
2. a. Apartment prices are measured using currency, such as dollars. Currency is a continuous variable that can be displayed along a continuous scale with no discrete gaps.

b.

House prices (000s)	Frequency	Percentage frequency
\$200–<\$300	6	15%
\$300–<\$400	16	40%
\$400–<\$500	14	35%
\$500–<\$600	4	10%
Total	**40**	**100%**

c. The class interval with the least frequent apartment prices is \$500–<\$600. The class interval with the most frequent apartment prices is \$300–<\$400.

d.



e. Approximately symmetric

f. Suggested option 1: The features of the property, such as the number of bathrooms or bedrooms.

Suggested option 2: The size of the block of land that the property is on.

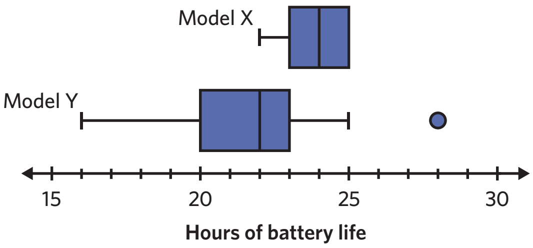
Note: There are other possible options.

3. a. Model X: Minimum $= 22$, $Q_1 = 23$, median $= 24$, $Q_3 = 25$, maximum $= 25$

Model Y: Minimum $= 16$, $Q_1 = 20$, median $= 22$, $Q_3 = 23$, maximum $= 28$

b. The data set for Model Y has an outlier of 28 hours as it is greater than the upper fence value of 27.5 hours.

c.



d. The spread of the data for Model X and Model Z, which have ranges of 3 hours, is smaller than the spread of the data for Model Y, which has a range of 12 hours.

e. A consumer may prefer to use Model Z rather than Model Y because the battery life is more consistent. Model Z has a range of 3 hours, whereas Model Y has a range of 12 hours. Therefore, Model Z has a significantly less variable battery life than Model Y, meaning that the consumer can better estimate when their smartphone will run out of battery.

f. Suggested option 1: Consumers may consider the storage of the smartphone.

Suggested option 2: Consumers may consider the price of the smartphone.

Note: There are other possible options.

Chapter 11 review

Multiple choice

1. D **2.** B **3.** E **4.** D **5.** C

Back to contents

Fluency

6. **a.** Mean = 10.1
Median = 11
Mode = 11

b. Mean = 35.1
Median = 34
Mode = 22, 32

c. Mean = 27.1
Median = 12
Mode = 12

d. Mean = 1.6
Median = 3.2
Mode = None

7. **a.** Range = 16
IQR = 10.5

b. Range = 8
IQR = 4

c. Range = 24
IQR = 16.5

d. Range = 6
IQR = 2.4

8. **a.**

Stem	Leaf
0	2 4 6
1	0 2 8
2	0 2 7 8
3	0 2
4	4 9 9
5	1

Key
3 | 0 = 30

b.

Stem	Leaf
2	2 3 8 9
3	5 6
4	2 3 9
5	0 6 7
6	3 4
7	0 1

Key
3 | 5 = 35

c.

Stem	Leaf
0	9
1	0 1 2 3 4 5 6
2	3 8
3	1 2 4 5 7
4	6

Key
3 | 1 = 31

d.

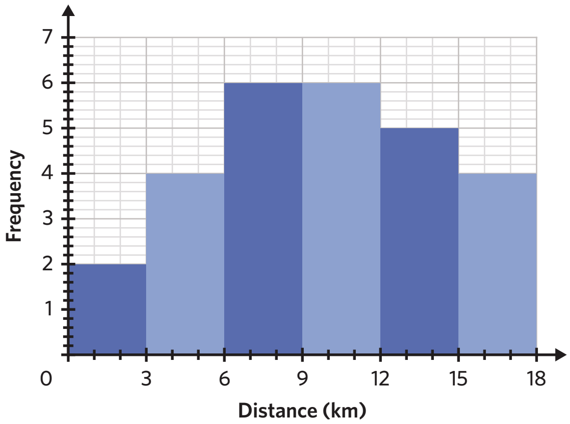
Stem	Leaf
1	4 8
2	4 7
3	0 1 8 9
4	0 6
5	0 5 7
6	3 7
7	2 9

Key
3 | 1 = 3.1

9. **a.**

Distance (km)	Frequency	Percentage frequency
0–<3	2	7.41%
3–<6	4	14.81%
6–<9	6	22.22%
9–<12	6	22.22%
12–<15	5	18.52%
15–<18	4	14.81%
Total	**27**	**100%**

b.



c. Approximately symmetric

d. 33.33%

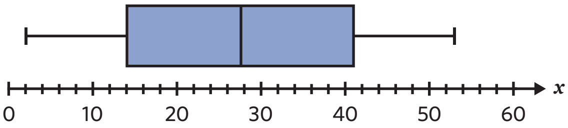
10. **a.** Minimum = 12, Q_1 = 28, median = 47.5, Q_3 = 66, maximum = 85
Lower fence = −29
Upper fence = 123

b. Minimum = 12, Q_1 = 17, median = 21.5, Q_3 = 26, maximum = 30
Lower fence = 3.5
Upper fence = 39.5

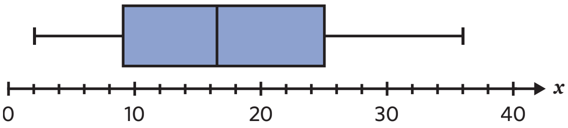
c. Minimum = 1.5, Q_1 = 2.7, median = 4.1, Q_3 = 5.8, maximum = 7.8
Lower fence = −2.0
Upper fence = 10.5

d. Minimum = −11, Q_1 = −5, median = 4.5, Q_3 = 11, maximum = 15
Lower fence = −29
Upper fence = 35

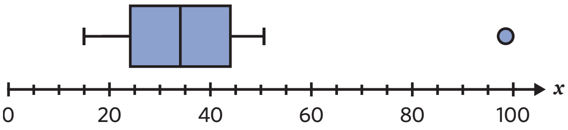
11. **a.**



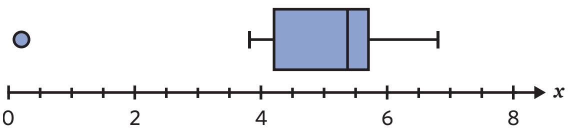
b.



c.



d.



Problem solving

12. A bakery owner is analysing the weights of two types of bread loaves: whole wheat and multigrain. The weights, in grams, of both types of loaves are as follows:

Whole wheat: 300, 320, 310, 330, 290, 280, 315

Multigrain: 350, 365, 360, 380, 370, 355, 385, 395

Calculate the range and interquartile range of weights for both types of bread loaves.

Key points

- The weight in grams of whole wheat loaves are: 300, 320, 310, 330, 290, 280, 315.
- The weight in grams of multigrain loaves are: 350, 365, 360, 380, 370, 355, 385, 395.
- Calculate the range and interquartile range of weights for both types of bread loaves.

Explanation

Organise the data set for whole wheat loaves in ascending order.

280, 290, 300, 310, 315, 320, 330

Calculate the range.

Range = maximum value − minimum value

$= 330 - 280$

$= 50$

Determine the first and third quartiles and calculate the interquartile range.

$Q_1 = 290$

$Q_3 = 320$

$\text{IQR} = Q_3 - Q_1$

$= 320 - 290$

$= 30$

Organise the data set for multigrain loaves in ascending order.

350, 355, 360, 365, 370, 380, 385, 395

Calculate the range.

Range = maximum value − minimum value

$= 395 - 350$

$= 45$

Determine the first and third quartiles and calculate the interquartile range.

$Q_1 = 357.5$

$Q_3 = 382.5$

$\text{IQR} = Q_3 - Q_1$

$= 382.5 - 357.5$

$= 25$

Answer

For whole wheat loaves, the range is 50 and the IQR is 30.
For multigrain loaves, the range is 45 and the IQR is 25.

13. Taura is a cyclist who is tracking the distances, in kilometres, that she covers during her rides over a period of two weeks. The data she collects is: 18, 22, 15, 28, 20, 30, 23, 25, 35, 14, 40, 42, 19, 26. Construct a stem-and-leaf plot to display Taura's cycling distances and comment on whether the data is skewed or symmetrical.

Key points

- Taura's cycling distances in kilometres are: 18, 22, 15, 28, 20, 30, 23, 25, 35, 14, 40, 42, 19, 26.
- Construct a stem-and-leaf plot to display Taura's cycling distances and comment on whether the data is skewed or symmetrical.

Explanation

Identify the stem and leaves in the data set.

The stem represents the tens values, while the leaves represent the ones values of each number in the data set.

Arrange the data in ascending order.

14, 15, 18, 19, 20, 22, 23, 25, 26, 28, 30, 35, 40, 42

Ensure that the stem ranges from 1 to 4 so it is long enough to include all values in the set. Enter each data value in order from smallest to largest.

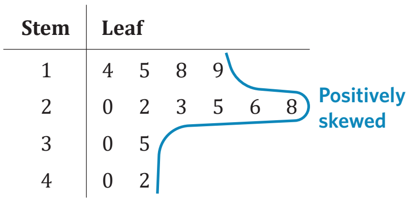
Stem	Leaf
1	4 5 8 9
2	0 2 3 5 6 8
3	0 5
4	0 2

Include a key to show how the data in the stem-and-leaf plot is read.

Key: 2 | 3 = 23 km

Determine whether the data is skewed or symmetrical.

The data is positively skewed because the majority of data values are around lower data values.



Stem	Leaf
1	4 5 8 9
2	0 2 3 5 6 8
3	0 5
4	0 2

Answer

Stem	Leaf
1	4 5 8 9
2	0 2 3 5 6 8
3	0 5
4	0 2

Key

2 | 3 = 23 km

The data is positively skewed.

14. A music teacher recorded the number of practice hours completed by her students in a month. The data is displayed in the grouped frequency table, rounded two decimal places where necessary.

Number of practice hours	Frequency	Percentage frequency
0–<3	10	27.78%
3–<6	15	41.67%
6–<9	7	19.44%
9–<12	3	8.33%
12–<15	1	2.78%
Total	**36**	**100%**

Construct a percentage frequency histogram to represent the data. Is the data symmetric, skewed, or bi-modal?

Key points

- The number of monthly practice hours completed by music students is displayed in a grouped frequency table.
- Construct a percentage frequency histogram to represent the data and determine whether the data is symmetric, skewed, or bi-modal.

Explanation

Use the class intervals from the frequency table to determine how to scale the horizontal (x) axis.

The horizontal (x) axis should be scaled from 0 to 15.

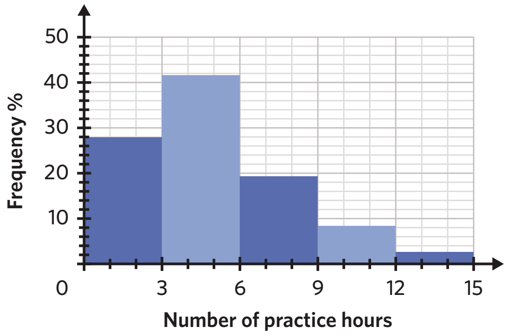
Identify the smallest and largest percentage frequency value from the frequency table to determine how to scale the vertical (y) axis.

Minimum percentage frequency $= 2.78\%$

Maximum percentage frequency $= 41.67\%$

∴ An appropriate scale for the vertical (y) axis would be from 0 to 42.

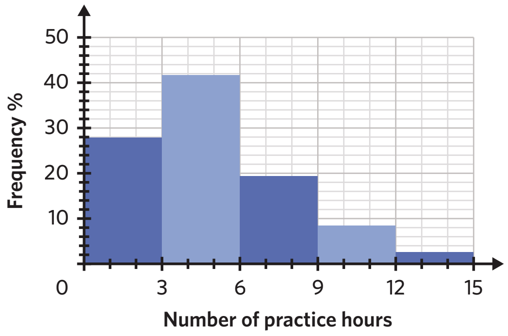
Draw the bars of the histogram using the percentage frequencies and class intervals given in the frequency table. Label the histogram's axes.



Determine the shape of the histogram to comment on the distribution of the data.

The data is positively skewed because the tail of the distribution is to the right.

Answer



The data is positively skewed.

15. For a weather study, Daisy collected data on the daily humidity levels (%) of her town for the month of July. The humidity levels were: 60, 65, 70, 55, 75, 80, 62, 68, 72, 58, 64, 69, 73, 57, 81, 76, 74, 59, 66, 67, 63, 71, 79, 11, 78, 61, 70, 65, 62, 52, 55. Determine if there are any outliers and then create a boxplot to display the data.

Key points

- The daily humidity levels (%) in Daisy's town over July were: 60, 65, 70, 55, 75, 80, 62, 68, 72, 58, 64, 69, 73, 57, 81, 76, 74, 59, 66, 67, 63, 71, 79, 11, 78, 61, 70, 65, 62, 52, 55.
- Determine if there are any outliers and then create a boxplot to display the data.

Explanation

Organise the data set in ascending order.

11, 52, 55, 55, 57, 58, 59, 60, 61, 62, 62, 63, 64, 65, 65, 66, 67, 68, 69, 70, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81

State the five-figure summary.

Minimum $= 11$

$Q_1 = 60$

Median $= 66$

$Q_3 = 73$

Maximum $= 81$

Calculate the interquartile range.

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 73 - 60\\ &= 13\end{aligned}$$

Calculate the lower and upper fences.

$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR}\\ &= 73 + 1.5 \times 13\\ &= 92.5\end{aligned}$$

$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR}\\ &= 60 - 1.5 \times 13\\ &= 40.5\end{aligned}$$

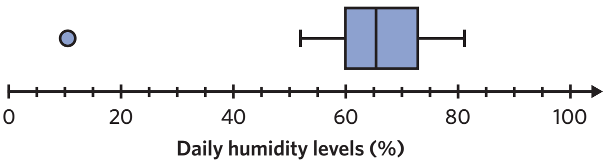
Identify if there are any outliers in the data that are less than the lower fence or greater than the upper fence.

$11 < 40.5$

∴ 11 is an outlier.

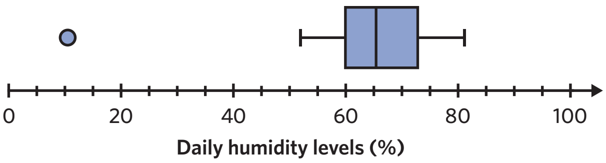
Plot the five-number summary and the next minimum data value on a number line and construct the boxplot. An outlier is displayed by using a '○' or a '✕'.

The box is formed using the Q_1, median, and Q_3 values. The whiskers extend from the box to the minimum and maximum values. Outliers are not included in the whiskers.



Answer

11 an outlier.



Reasoning

16. a.

Branch Pawsome	Stem	Branch Petopia	Key
8 5	0	5 9	2 | 3 = 23kg
8 5 2	1	3 5 7 9	
8 7 6 4 2 0	2	1 3 5 6 7 9	
8 6 4 2 0	3	1 3 5 7 9	
5 2 0	4	1 3 5	

b. Branch Pawsome has a higher median dog weight.

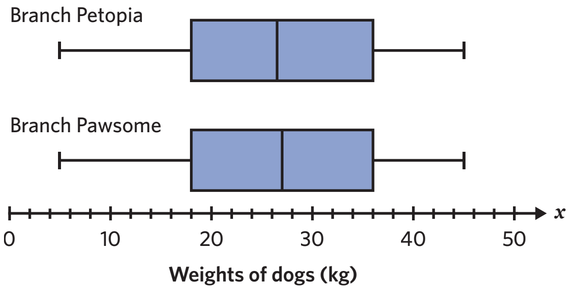
Back to contents

c. Branch Pawsome:

Minimum $= 5$, $Q_1 = 18$, median $= 27$, $Q_3 = 36$, maximum $= 45$

Branch Petopia:

Minimum $= 5$, $Q_1 = 18$, median $= 26.5$, $Q_3 = 36$, maximum $= 45$



d. The interquartile range for both Branch Pawsome and Branch Petopia is 18.

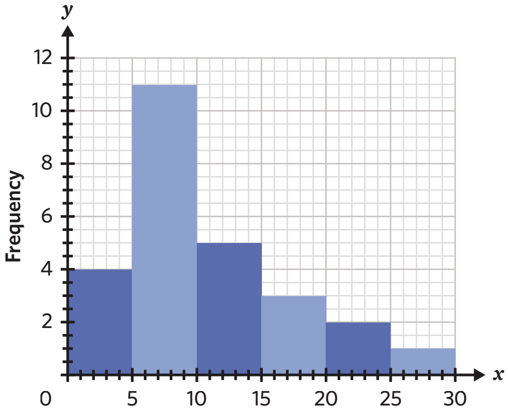
e. Suggested option 1: If the pet store sells products specifically for the breed of their dog.

Suggested option 2: The price of the products sold at the pet store.

Note: There are other possible options.

17. a. The mean is 9.9.

b.



c. The mean cannot be accurately calculated from the histogram, but rather it requires the raw data. Only an approximation/estimate of the mean can be obtained from the histogram.

Back to contents

10A Venn diagrams and two-way tables

Student practice

Worked example 1

a. 16 **b.** 21 **c.** $\frac{3}{25}$

Worked example 2

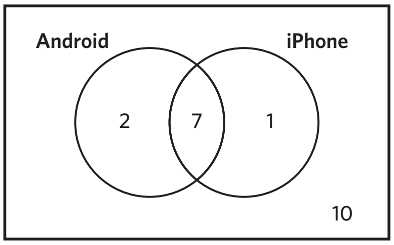
a. 14 **b.** 21 **c.** $\frac{5}{26}$

Worked example 3

a.

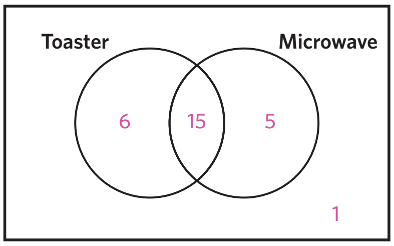
	Owns an iPhone	Doesn't own an iPhone	Total
Owns an android	7	2	**9**
Doesn't own an android	1	10	**11**
Total	**8**	**12**	**20**

b.



Understanding worksheet

1.



2.

	Watch	No watch	Total
Ring	17	4	**21**
No ring	2	7	**9**
Total	**19**	**11**	**30**

3. two-way table; Venn diagram; inclusive; exclusive

Fluency

4. **a.** 100 **b.** 59 **c.** 11 **d.** 30
e. 45 **f.** 25

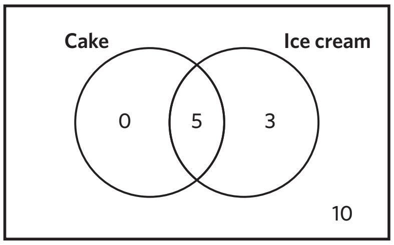
5. **a.** $\frac{9}{20}$ **b.** $\frac{1}{2}$ **c.** $\frac{1}{5}$ **d.** $\frac{1}{20}$
e. $\frac{3}{4}$ **f.** $\frac{7}{20}$

6. **a.** 25 **b.** 12 **c.** 8 **d.** 9
e. 1 **f.** 17

7. **a.** $\frac{8}{25}$ **b.** $\frac{21}{50}$ **c.** $\frac{12}{25}$ **d.** $\frac{4}{25}$
e. $\frac{1}{10}$ **f.** $\frac{9}{10}$

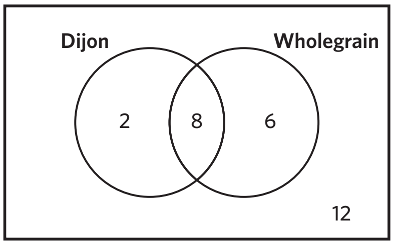
8. **a.**

	Likes ice cream	Doesn't like ice cream	Total
Likes cake	5	0	**5**
Doesn't like cake	3	10	**13**
Total	**8**	**10**	**18**



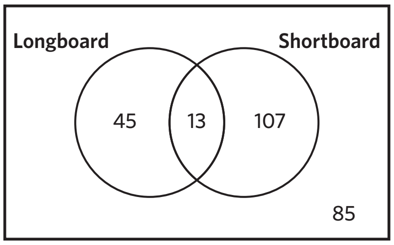
b.

	Wholegrain mustard	Not wholegrain mustard	Total
Dijon mustard	8	2	**10**
Not dijon mustard	6	12	**18**
Total	**14**	**14**	**28**



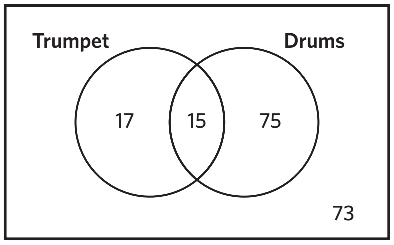
c.

	Owns a shortboard	Doesn't own a shortboard	Total
Owns a longboard	13	45	**58**
Doesn't own a longboard	107	85	**192**
Total	**120**	**130**	**250**



d.

	Can play the drums	Can't play the drums	Total
Can play the trumpet	15	17	**32**
Can't play the trumpet	75	73	**148**
Total	**90**	**90**	**180**



9. B

Spot the mistake

10. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

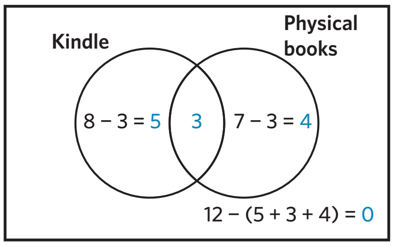
11. In a book club of 12 friends, 8 members read books on a Kindle, 7 read a physical book, and 3 read both from a Kindle and physical books. Construct a Venn diagram to represent the situation.

Key points

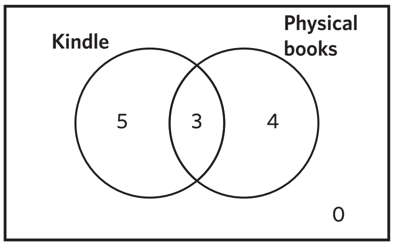
- There's a book club of 12 people.
- 8 of the members read books on a Kindle.
- 7 members read physical books.
- 3 of the members read both types of books.
- Construct a Venn diagram to represent the situation.

Explanation

Draw a rectangle and circles, and write the labels on the Venn diagram. Fill in the information that is known and determine the remaining information.



Answer



12. Eighty Minecraft players were surveyed regarding their in-game building material usage. Of that group, 28 said that they used wood, 42 said that they used stones, and 10 said that they did not use either stone or wood. Construct a two-way table to represent the scenario.

Key points

- There were 80 minecraft players surveyed.
- 28 said they use wood as their in-game building material.
- 42 said they use stones for the same purpose.
- 10 said that they used neither wood nor stones.
- Construct a two-way table to represent the scenario.

Explanation

Draw a grid to make a two-way table and fill in the headings. Fill in the information that is known and determine the remaining information.

	Wood	Not wood	Total
Stone	$42 - 42 = 0$	$52 - 10 = 42$	**42**
Not stone	$38 - 10 = 28$	10	**$80 - 42 = 38$**
Total	**28**	**$80 - 28 = 52$**	**80**

Answer

	Wood	Not wood	Total
Stone	0	42	**42**
Not stone	28	10	**38**
Total	**28**	**52**	**80**

13. Among a group of 50 students, 25 like maths, 30 like science, and 10 like both. What's the probability of selecting a student from the group who likes maths only?

Key points

- There's a group of 50 students.
- 25 of them like maths.
- 30 of them like science.
- 10 like both maths and science.
- Find the probability of selecting a student from the group who likes maths only.

Explanation

Calculate the number of people who like maths only by subtracting the number of people who like both maths and science from the number of people who like maths.

$$\text{Number of students that only like maths} = 25 - 10$$
$$= 15$$

To find the probability of selecting a student who likes maths only, divide by the total number of students.

$$\Pr(\text{likes maths only}) = \frac{15}{50} = \frac{3}{10}$$

Answer

The probability of selecting a student from this group that only likes maths is $\frac{3}{10}$.

14. At a young artists studio, 17 artists focus solely on sculpture, 14 focus only on painting, 5 focus on neither and 9 embrace both forms of artistic expression. If a new artist joins the club, what is the likelihood they will focus on sculpture or painting?

Key points

- 17 artists at a studio focus solely on sculpture.
- 14 focus only on painting.
- 5 focus on neither form of artwork.
- 9 embrace both sculpture and painting.
- If someone joins the art club, find the likelihood that they will focus on sculpture or painting.

Back to contents

Explanation

Calculate the number of people who focus on sculpture or painting by adding the number of artists that focus solely on sculpture to the total number that only focus on painting and those that focus on both.

Number of artists who focus on sculpture or painting
$= 17 + 14 + 9 = 40$

If the number of artists that focus on neither is added to this number then that is the total number of young artists at the studio.

Total number of artists $= 40 + 5 = 45$

To find the probability of a new artist focussing on sculpture or painting, divide the number of students interested in sculpture or painting by the total number of artists.

Pr(focuses on sculpture or painting) $= \frac{40}{45} = \frac{8}{9}$

Answer

The probability of a new artist that joins the studio focusing on sculpture or painting is $\frac{8}{9}$.

15. Of the 70 puzzle enthusiasts completing a puzzle making course, 40 are interested in Sudoku, 25 are interested in crosswords, and 10 are puzzle enthusiasts who are interested in both. Determine the probability that a randomly selected puzzle lover only favours crosswords.

Key points

- There are 70 puzzle enthusiasts.
- 40 of them like Sudoku.
- 25 of them like crosswords.
- 10 of them like both Sudoku and crosswords.
- Determine the probability that a randomly selected puzzle lover prefers crosswords only.

Explanation

Calculate the number of people who are interested in crosswords only by subtracting the number of people who are interested in both crosswords and Sudoku from the number of people interested in crosswords.

Number of puzzle enthusiasts that only like crosswords $= 25 - 10$
$= 15$

To find the probability of a randomly selected puzzle enthusiast preferring crosswords only, divide by the total number of people.

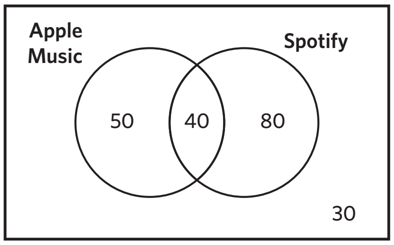
Pr(prefers crosswords only) $= \frac{15}{70} = \frac{3}{14}$

Answer

The probability of a randomly selected puzzle lover only favouring crosswords is $\frac{3}{14}$.

Reasoning

16. a.



b. The number of participants who use Spotify, Apple Music, or both is 170.

c. The probability that a randomly chosen participant uses Spotify only is $\frac{2}{5}$.

d. 25% of participants use only Apple Music.

e. Suggested option 1: The cost of the streaming service.

Suggested option 2: The range of music on the platform.

Note: There are other possible options.

17. a. A, B or both.

b. Both A and B.

c. In part **a**, the shaded area included all of A and B, including the areas representing A or B only. This was indicated by the words 'or both'. In part **b**, the shaded area included only the intersection of A and B. The word 'both' was used to indicate that only A and B must be included.

Exam-style

18. E

19. a. 11
b. 24%

20. 85.00%

21. 18.67%

Remember this?

22. B

23. D

24. B

10B Using set notation

Student practice

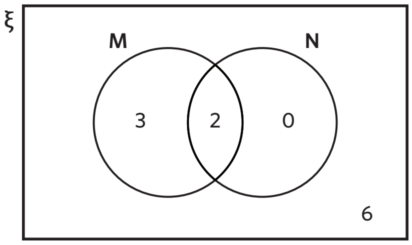
Worked example 1

a. ξ = {5, 6, 7, 8, 9, 10 11, 12, 13, 14, 15}

b. M = {6, 8, 10, 12, 14}

c. N = {8, 12}

d.



Worked example 2

a. A ∪ B = {1, 2, 3, 5, 7}

b. A ∩ B = {1, 5}

c. B' = {3, 4, 6, 7, 8}

Worked example 3

a. $\frac{9}{10}$

b. $\frac{1}{5}$

Understanding worksheet

1. a. B **b.** ∪ **c.** A' **d.** ∩

2. a. ∪ **b.** ∩ **c.** C' **d.** 3

3. sample space; subset; complement; intersection; union

Fluency

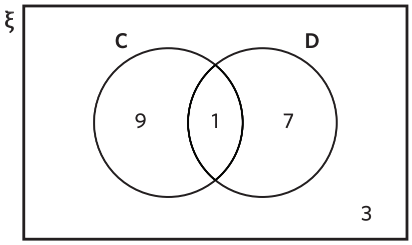
4. a. ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

b. C = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

c. D = {2, 3, 5, 7, 11, 13, 17, 19}

Back to contents

d.



e. $C \cup D = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$

f. $C \cap D = \{2\}$

g. $C' = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

h. $D' = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

---

5. **a.** $F \cup G = \{31, 32, 34, 36, 37, 39\}$

b. $F \cap G = \{32, 34\}$

c. $n(G) = 5$

d. $G' = \{33, 35, 36, 38, 40\}$

e. $F' = \{31, 33, 35, 37, 38, 39, 40\}$

f. $F' \cup G = \{31, 32, 33, 34, 35, 37, 38, 39, 40\}$

---

6. **a.** $\frac{3}{10}$ **b.** $\frac{3}{5}$ **c.** $\frac{9}{10}$ **d.** 0

e. $\frac{7}{10}$ **f.** $\frac{2}{5}$

---

7. **a.** $\frac{1}{4}$ **b.** $\frac{9}{20}$ **c.** $\frac{11}{20}$ **d.** $\frac{3}{20}$

e. $\frac{3}{4}$ **f.** $\frac{11}{20}$

---

8. **a.** $\frac{19}{25}$ **b.** $\frac{1}{5}$ **c.** $\frac{16}{25}$ **d.** $\frac{9}{25}$

e. $\frac{3}{5}$ **f.** $\frac{2}{5}$

---

9. D

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. A school holiday program consists of 12 students who were all born in different months of the year. The program is split into Group 1 and Group 2. Group 1 consists of all the students that were born in the first half of the year and Group 2 consists of all the students that were born in the second half of the year. List the sets of Group 1 and Group 2.

Key points

- The sample space of students consists of 12 students who were all born in different months.
- Group 1 is all the students who were born in the first half of the year.
- Group 2 is all the students who were born in the second half of the year.
- List the sets of Group 1 and Group 2.

Explanation

Identify the sample space.

ξ = {January, February, March, April, May, June, July, August, September, October, November, December}

Write the set for Group 1, which is the first six months of the year.

Group 1 = {January, February, March, April, May, June}

Write the set for Group 2, which is the second six months of the year.

Group 2 = {July, August, September, October, November, December}

Answer

Group 1 = {January, February, March, April, May, June}

Group 2 = {July, August, September, October, November, December}

12. A kennel has a group of different breeds of dogs. The group includes a Bulldog, Poodle, Boxer and Beagle. List the set of A if it is equal to {dog breeds with the letter o}.

Key points

- The sample space of dogs contains a Bulldog, Poodle, Boxer and Beagle.
- A is equal to {dog breeds with the letter o}.
- List the set of A.

Explanation

Identify the sample space.

ξ = {Bulldog, Poodle, Boxer, Beagle}

Identify the dog breeds with the letter o.

Bulldog ✓

Poodle ✓

Boxer ✓

Beagle ✗

List using set notation.

A = {Bulldog, Poodle, Boxer}

Answer

The set A = {Bulldog, Poodle, Boxer}.

13. A group of surgeons in a hospital are surveyed about their country of birth. The universal set is represented as ξ = {names of all surgeons}. The subsets of the universal set are represented as A = {names of surgeons born in Australia} and N = {names of surgeons not born in Australia}. Explain in words why $A \cap N = \emptyset$.

Key points

- The universal set is the names of all surgeons.
- Subset A represents the names of surgeons born in Australia.
- Subset N represents the names of surgeons not born in Australia.
- Explain why $n(A \cap N) = \emptyset$.

Explanation

Identify that the complement of the subset A is N.

$N = A'$

Identify that the intersection of complementary subsets is always ∅ as it is the empty set.

$n(A \cap A') = 0$ and $A \cap A' = \emptyset$

Answer

$A \cap N = \emptyset$ because A and N are complements of each other.

14. A group of 150 people attended a seminar on programming languages. Among them, 100 were proficient in Python (P), 75 were proficient in Java (J), and 40 were proficient in both languages. What is the probability of randomly selecting an individual from the group who is not proficient in Python?

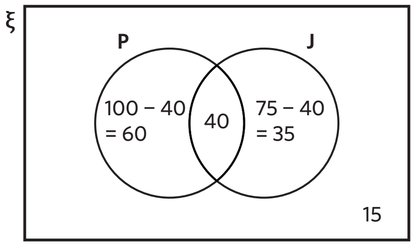
Key points

- Sample space contained 150 people.
- 100 people were proficient in Python.
- 75 people were proficient in Java.
- 40 people were proficient in both.
- Determine the probability of selecting someone not proficient in Python.

Back to contents

Explanation

Represent the sample space visually.



Identify n(P’).

$n(P') = 15 + 35$

$= 50$

Calculate Pr(P’).

$Pr(P') = \frac{50}{150}$

$= \frac{1}{3}$

Answer

The probability of randomly selecting an individual from the group who is not proficient in Python is $\frac{1}{3}$.

15. Anita’s Gelato had a group of customers complete a poll on their favourite flavour. 103 customers said they like pistachio (P), 98 customers said they like stracciatella (S), while 27 customers said they liked both pistachio and stracciatella. Six customers liked neither flavour. What is the probability of randomly selecting a customer that completed the poll who satisfies the condition P ∪ S?

Key points

- 103 customers said they like pistachio gelato.
- 98 customers said they like stracciatella gelato.
- 27 customers said they like both.
- 6 said they like neither.
- Determine the probability of randomly selecting a customer that completed the poll who satisfied the condition of P ∪ S.

Explanation

Represent the outcomes using a two-way table.

	Likes P	Does not like P	Total
Likes S	27		98
Does not like S		6	
Total	103		

Calculate the remaining outcomes.

	Likes P	Does not like P	Total
Likes S	27	98 − 27 = 71	98
Does not like S	103 − 27 = 76	6	76 + 6 = 82
Total	103	71 + 6 = 77	103 + 77 = 180

Determine P ∪ S.

$P \cup S = P + S - (P \cap S)$

$= 103 + 98 - 27$

$= 174$

Calculate Pr(P ∪ S).

$Pr(P \cup S) = \frac{174}{180}$

$= \frac{29}{30}$

Answer

The probability of randomly selecting a customer that completed the poll who satisfied the condition of P ∪ S is $\frac{29}{30}$.

Reasoning

16. a. There is a total of 18 teams in the sample space.

b. The set B ∩ C is equal to {Geelong Cats, Brisbane Lions, Port Adelaide Power, Western Bulldogs}.

c. The probability that the fan selects a new team that played in both the 2020 and 2021 finals is $\frac{4}{17}$.

d. Suggested option 1: Increase the funding for lower-performing teams to provide them with more resources to improve.

Suggested option 2: Facilitate mixed team training sessions so that lower-performing teams can learn from higher-performing teams.

Note: There are other possible options.

17. a. $n(C) + n(D) = 9$

b. $n(C \cup D) = 8$

c. In parts **a** and **b** n(C) + n(D) is greater than n(C ∪ D) because there is an element in the intersection (C ∩ D). n(C) + n(D) may equal to n(C ∪ D) when there are no elements in the intersection of C and D.

Exam-style

18. A

19. a. A ∪ B = {2, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17, 18, 19}

b. $\frac{1}{20}$

20. 20%

21. (M ∪ R)’ = {JACKIE, PETER}

Remember this?

22. D **23.** C **24.** D

10C Using arrays for two-step experiments

Student practice

Worked example 1

a.

		Second selection			
		G	G	S	Y
First selection	G	(G,G)	(G,G)	(G,S)	(G,Y)
	G	(G,G)	(G,G)	(G,S)	(G,Y)
	S	(S,G)	(S,G)	(S,S)	(S,Y)
	Y	(Y,G)	(Y,G)	(Y,S)	(Y,Y)

ξ = {(G,G), (G,G), (G,S), (G,Y), (G,G), (G,G), (G,S), (G,Y), (S,G), (S,G), (S,S), (S,Y), (Y,G), (Y,G), (Y,S), (Y,Y)}

b. $\frac{1}{8}$

c. $\frac{1}{2}$

Back to contents

Worked example 2

a.

		Second selection			
		G	**P**	**B**	**W**
First selection	**G**	X	(G,P)	(G,B)	(G,W)
	P	(P,G)	X	(P,B)	(P,W)
	B	(B,G)	(B,P)	X	(B,W)
	W	(W,G)	(W,P)	(W,B)	X

ξ = {(G,P), (G,B), (G,W), (P,G), (P,B), (P,W), (B,G), (B,P), (B,W), (W,G), (W,P), (W,B)}

b. $\frac{1}{2}$ **c.** $\frac{1}{6}$

Understanding worksheet

1.

		Second selection		
		B	**G**	**Y**
First selection	**B**	(B,B)	(B,G)	(B,Y)
	G	(G,B)	(G,G)	(G,Y)
	Y	(Y,B)	(Y,G)	(Y,Y)

2.

		Second selection		
		R	**B**	**W**
First selection	**R**	X	(R,B)	(R,W)
	B	(B,R)	X	(B,W)
	W	(W,R)	(W,B)	X

3. two-step; array; replacement; probability

Fluency

4. a.

		Second selection			
		Y	**Y**	**W**	**B**
First selection	**Y**	(Y,Y)	(Y,Y)	(Y,W)	(Y,B)
	Y	(Y,Y)	(Y,Y)	(Y,W)	(Y,B)
	W	(W,Y)	(W,Y)	(W,W)	(W,B)
	B	(B,Y)	(B,Y)	(B,W)	(B,B)

ξ = {(Y,Y), (Y,Y), (Y,W), (Y,B), (Y,Y), (Y,Y), (Y,W), (Y,B), (W,Y), (W,Y), (W,W), (W,B), (B,Y), (B,Y), (B,W), (B,B)}

b. $\frac{1}{4}$ **c.** $\frac{1}{8}$ **d.** $\frac{1}{8}$ **e.** $\frac{7}{16}$ **f.** $\frac{3}{8}$

5. a.

		Fair six-sided dice					
		1	**2**	**3**	**4**	**5**	**6**
Coin toss	**H**	(H,1)	(H,2)	(H,3)	(H,4)	(H,5)	(H,6)
	T	(T,1)	(T,2)	(T,3)	(T,4)	(T,5)	(T,6)

ξ = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

b. $\frac{1}{12}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{6}$ **e.** $\frac{1}{2}$ **f.** $\frac{1}{4}$

6. a.

		Second fair six-sided dice		
		1	**2**	**3**
First fair six-sided dice	**1**	(1,1)	(1,2)	(1,3)
	2	(2,1)	(2,2)	(2,3)
	3	(3,1)	(3,2)	(3,3)
	4	(4,1)	(4,2)	(4,3)
	5	(5,1)	(5,2)	(5,3)
	6	(6,1)	(6,2)	(6,3)

		Second fair six-sided dice		
		4	**5**	**6**
First fair six-sided dice	**1**	(1,4)	(1,5)	(1,6)
	2	(2,4)	(2,5)	(2,6)
	3	(3,4)	(3,5)	(3,6)
	4	(4,4)	(4,5)	(4,6)
	5	(5,4)	(5,5)	(5,6)
	6	(6,4)	(6,5)	(6,6)

ξ = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

b. $\frac{1}{36}$ **c.** $\frac{1}{6}$ **d.** $\frac{5}{18}$ **e.** $\frac{5}{12}$ **f.** $\frac{5}{12}$

7. a.

		Second selection		
		G	**P**	**W**
First selection	**G**	X	(G,P)	(G,W)
	P	(P,G)	X	(P,W)
	W	(W,G)	(W,P)	X

ξ = {(G,P), (G,W), (P,G), (P,W), (W,G), (W,P)}

b. $\frac{1}{6}$ **c.** 0 **d.** $\frac{2}{3}$ **e.** $\frac{1}{3}$ **f.** 1

8. a.

		Second selection			
		M	**E**	**E**	**T**
First selection	**M**	X	(M,E)	(M,E)	(M,T)
	E	(E,M)	X	(E,E)	(E,T)
	E	(E,M)	(E,E)	X	(E,T)
	T	(T,M)	(T,E)	(T,E)	X

ξ = {(M,E), (M,E), (M,T), (E,M), (E,E), (E,T), (E,M), (E,E), (E,T), (T,M), (T,E), (T,E)}

b. $\frac{1}{6}$ **c.** $\frac{1}{2}$ **d.** $\frac{2}{3}$ **e.** $\frac{1}{4}$ **f.** $\frac{1}{6}$

9. B

Spot the mistake

10. a. Student A is incorrect. **b.** Student B is incorrect.

Back to contents

Problem solving

11. Phil is trying to predict the winner of the Australian Open men's and women's competitions by placing the female players' names Keys (K) and Swiatek (S) in one hat and male players' names Djokovic (D) and Sinner (S) in another hat. Create an array to show all possible outcomes of selecting from the women's names first followed by the men.

Key points

- Phil is trying to predict the winner of the competitions so he places the female players' names Keys (K) and Swiatek (S) in one hat.
- Phil then places male players' names Djokovic (D) and Sinner (S) in another hat.
- Create an array to show all possible outcomes of selecting from the women's names from the hat first followed by the men.

Explanation

Draw the array, label the possible outcomes for each event. Write all pairs of possible outcomes.

		Second selection	
		D	S
First selection	K	(K,D)	(K,S)
	S	(S,D)	(S,S)

Answer

		Second selection	
		D	S
First selection	K	(K,D)	(K,S)
	S	(S,D)	(S,S)

12. Frederick has two boxes of coloured spray paint. One box has black and purple spray, and the second box has black, green and yellow spray. If he randomly chooses one spray from each box, what is the probability he selected at least one black spray?

Key points

- Frederick has two boxes of coloured spray paint.
- One of his boxes has black and purple spray paint.
- His second box has black, green and yellow spray paint.
- If he randomly chooses one spray paint from each box, find the probability that he selected at least one black spray.

Explanation

Draw an array for selecting two cans of spray paint from one box with black (B) and purple (P) spray paint, followed by another box with black (B), green (G), and yellow (Y) spray paint. Fill in with all possible outcomes.

		Second selection		
		B	G	Y
First selection	B	(B,B)	(B,G)	(B,Y)
	P	(P,B)	(P,G)	(P,Y)

Determine the number of possible outcomes.

Possible outcomes $= n(\xi) = 6$

Determine the number of desired outcomes.

At least one black spray $= \{(B,B), (B,G), (B,Y), (P,B)\}$

n(at least one black spray) $= 4$

The probability that he selected at least one black spray is given by the simplified fraction of the number of desired outcomes over the total number of possible outcomes.

Pr(at least one black spray) $= \frac{4}{6} = \frac{2}{3}$

Answer

The probability that Frederick selected at least one black spray from the two boxes is $\frac{2}{3}$.

13. The names Frank, Gigi, Ibrahim, Davis, Micahel and Carlotta are placed in a hat so a teacher can select pairs randomly for a group project. The teacher takes a name, does not replace it and then takes another name to form a pair. What is the probability that the first pair selected is Davis and Carlotta?

Key points

- The names Frank, Gigi, Ibrahim, Davis, Micahel and Carlotta are placed in a hat so a teacher can select pairs randomly for a group project.
- The teacher takes a name, does not replace it and then takes another name to form a pair.
- Find the probability that the first pair selected is Davis and Carlotta.

Explanation

Draw an array for the situation without replacement, fill in all possible outcomes.

		Second selection					
		F	G	I	D	M	C
First selection	F	X	(F,G)	(F,I)	(F,D)	(F,M)	(F,C)
	G	(G,F)	X	(G,I)	(G,D)	(G,M)	(G,C)
	I	(I,F)	(I,G)	X	(I,D)	(I,M)	(I,C)
	D	(D,F)	(D,G)	(D,I)	X	(D,M)	(D,C)
	M	(M,F)	(M,G)	(M,I)	(M,D)	X	(M,C)
	C	(C,F)	(C,G)	(C,I)	(C,D)	(C,M)	X

Determine the number of possible outcomes.

Possible outcomes $= n(\xi) = 30$

The number of desired outcomes will be 2 as the pair of Davis and Carlotta can be made by Davis being picked first or Carlotta being picked first.

The probability that the first pair selected will be Davis and Carlotta is given by the simplified fraction of the number of desired outcomes over the total number of possible outcomes.

$\Pr((D,C) \text{ or } (C,D)) = \frac{2}{30} = \frac{1}{15}$.

Answer

The probability that the first pair selected for the group project is Davis and Carlotta is $\frac{1}{15}$.

14. Each turn in a game of backgammon a player rolls two fair six-sided dice. What is the probability that the sum of the two dice is greater than 10?

Key points

- In the game of backgammon a player rolls two fair six-sided dice.
- What is the probability that the sum of the two dice is greater than 10?

Explanation

Draw an array for the situation, fill in all possible outcomes and identify the cells where pairs of numbers have a sum greater than 10.

		Second fair six-sided dice		
		1	**2**	**3**
First fair six-sided dice	**1**	(1,1)	(1,2)	(1,3)
	2	(2,1)	(2,2)	(2,3)
	3	(3,1)	(3,2)	(3,3)
	4	(4,1)	(4,2)	(4,3)
	5	(5,1)	(5,2)	(5,3)
	6	(6,1)	(6,2)	(6,3)

		Second fair six-sided dice		
		4	**5**	**6**
First fair six-sided dice	**1**	(1,4)	(1,5)	(1,6)
	2	(2,4)	(2,5)	(2,6)
	3	(3,4)	(3,5)	(3,6)
	4	(4,4)	(4,5)	(4,6)
	5	(5,4)	(5,5)	(5,6)
	6	(6,4)	(6,5)	(6,6)

Determine the number of possible outcomes.

Possible outcomes = n(ξ) = 36.

Determine the number of desired outcomes from the highlighted array above.

n(sum of the two dice is greater than 10) = 3

The probability that the sum of the two dice is greater than 10 is given by the simplified fraction of the number of desired outcomes over the total number of possible outcomes.

Pr(sum of the two dice is greater than 10) $= \frac{3}{36} = \frac{1}{12}$.

Answer

The probability that the sum of the two dice is greater than 10 is $\frac{1}{12}$.

15. Zac has a king of hearts (red), diamonds (red), clubs (black) and spades (black) in his pocket. If he picks two cards from his pocket at random and without replacing the first card selected, what is the probability both cards are red?

Key points

- Zac has a king of hearts (red), diamonds (red), clubs (black) and spades (black) in his pocket.
- He picks two cards from his pocket at random and without replacing the first card selected.
- Determine the probability that both cards are red.

Explanation

Draw an array to represent the situation, fill in all possible outcomes.

		Second fair six-sided dice			
		R	**R**	**B**	**B**
First fair six-sided dice	**R**	X	(R,R)	(R,B)	(R,B)
	R	(R,R)	X	(R,B)	(R,B)
	B	(B,R)	(B,R)	X	(B,B)
	B	(B,R)	(B,R)	(B,B)	X

Determine the number of possible outcomes.

Possible outcomes = n(ξ) = 12

Determine the number of desired outcomes.

Both R = {(R,R),(R,R)}

n(both R) = 2

The probability that both cards selected are red is given by the simplified fraction of the number of desired outcomes over the total number of possible outcomes.

Pr(R,R) $= \frac{2}{12} = \frac{1}{6}$

Answer

The probability that both cards selected from Zac's pocket are red is $\frac{1}{6}$.

Reasoning

16. a.

		Second selection		
		7S	**6C**	**10H**
First selection	**7S**	X	(7S,6C)	(7S,10H)
	6C	(6C,7S)	X	(6C,10H)
	10H	(10H,7S)	(10H,6C)	X
	9D	(9D,7S)	(9D,6C)	(9D,10H)
	2H	(2H,7S)	(2H,6C)	(2H,10H)
	8C	(8C,7S)	(8C,6C)	(8C,10H)
	2S	(2S,7S)	(2S,6C)	(2S,10H)
	5D	(5D,7S)	(5D,6C)	(5D,10H)

		Second selection		
		9D	**2H**	**8C**
First selection	**7S**	(7S,9D)	(7S,2H)	(7S,8C)
	6C	(6C,9D)	(6C,2H)	(6C,8C)
	10H	(10H,9D)	(10H,2H)	(10H,8C)
	9D	X	(9D,2H)	(9D,8C)
	2H	(2H,9D)	X	(2H,8C)
	8C	(8C,9D)	(8C,2H)	X
	2S	(2S,9D)	(2S,2H)	(2S,8C)
	5D	(5D,9D)	(5D,2H)	(5D,8C)

		Second selection	
		2S	**5D**
First selection	**7S**	(7S,2S)	(7S,5D)
	6C	(6C,2S)	(6C,5D)
	10H	(10H,2S)	(10H,5D)
	9D	(9D,2S)	(9D,5D)
	2H	(2H,2S)	(2H,5D)
	8C	(8C,2S)	(8C,5D)
	2S	X	(2S,5D)
	5D	(5D,2S)	X

Back to contents

b. Pr(selected two cards that are 5 or below) $= \frac{6}{56} = \frac{3}{28}$

c. Pr(selected the same card twice) $= \frac{8}{64} = \frac{1}{8}$

d. Suggested option 1: Go Fish.

Suggested option 2: Snap.

Note: There are other possible options.

17. a. Pr(R,A) for the experiment with replacement is $\frac{1}{9}$.

b. Pr(R,A) for the experiment without replacement is $\frac{1}{6}$.

c. Having replacement means that there is a larger sample space for the experiment, this is because all the double letters are included. Therefore Pr(R,A) is less in an experiment with replacement rather than without.

Exam-style

18. A

19. a. 32

b. $\frac{1}{8}$

20. $\frac{13}{18}$

21. $\frac{4}{5}$

Remember this?

22. B

23. D

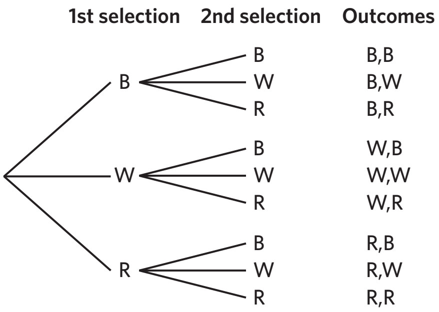
24. C

10D Tree diagrams

Student practice

Worked example 1

a.



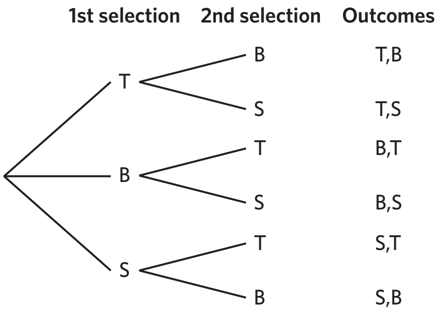
ξ = {(B,B), (B,W), (B,R), (W,B), (W,W), (W,R), (R,B), (R,W), (R,R)}

b. $\frac{1}{9}$

c. $\frac{1}{3}$

Worked example 2

a.



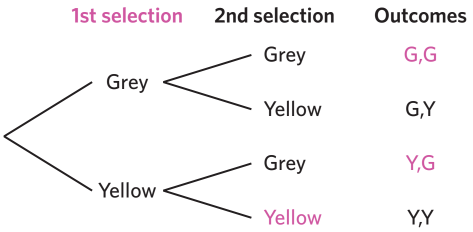
ξ = {(T,B), (T,S), (B,T), (B,S), (S,T) (S,B)}

b. $\frac{1}{6}$

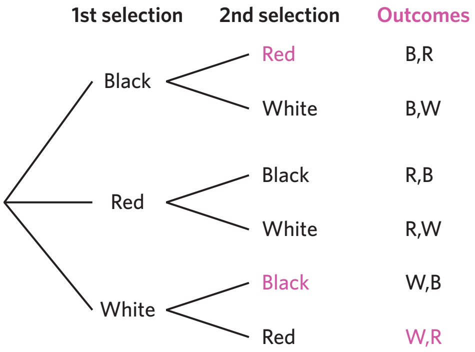
c. $\frac{2}{3}$

Understanding worksheet

1.



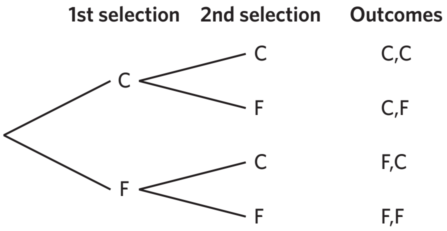
2.



3. two-step; tree diagram; events; replacement

Fluency

4. a.



ξ = {(C,C), (C,F), (F,C), (F,F)}

b. $\frac{1}{4}$

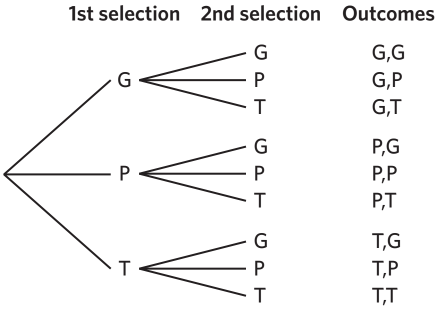
c. $\frac{3}{4}$

d. $\frac{1}{2}$

e. 4

f. $\frac{1}{2}$

5. a.



ξ = {(G,G), (G,P), (G,T), (P,G), (P,P), (P,T), (T,G), (T,P), (T,T)}

b. $\frac{1}{9}$

c. $\frac{1}{9}$

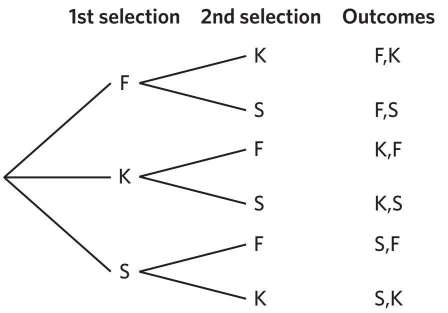
d. $\frac{5}{9}$

e. $\frac{1}{3}$

f. $\frac{2}{3}$

Back to contents

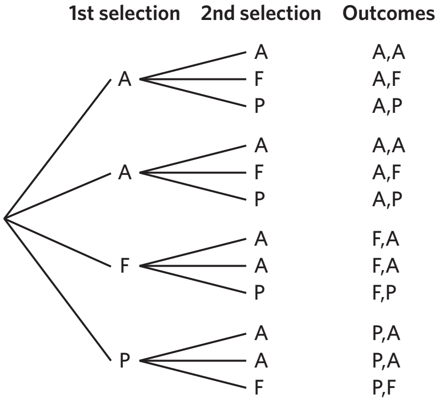
6. **a.**



ξ = {(F,K), (F,S), (K,F), (K,S), (S,F), (S,K)}

b. $\frac{1}{6}$ **c.** $\frac{2}{3}$ **d.** $\frac{1}{3}$ **e.** $\frac{2}{3}$ **f.** 1

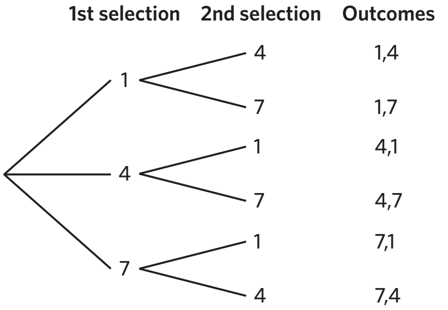
7. **a.**



ξ = {(A,A), (A,F), (A,P), (A,A), (A,F), (A,P), (F,A), (F,A), (F,P), (P,A), (P,A), (P,F)}

b. $\frac{1}{12}$ **c.** $\frac{1}{6}$ **d.** $\frac{1}{4}$ **e.** $\frac{5}{6}$ **f.** $\frac{1}{2}$

8. **a.**



ξ = {(1,4), (1,7), (4,1), (4,7), (7,1), (7,4)}

b. $\frac{1}{6}$ **c.** $\frac{2}{3}$ **d.** $\frac{2}{3}$ **e.** $\frac{1}{3}$ **f.** $\frac{2}{3}$

9. B

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. There are two egg (E) sandwiches and one cheese (C) sandwich left on a shelf at a canteen. Sashin randomly selects two sandwiches. Construct a tree diagram and list the sample space for this scenario.

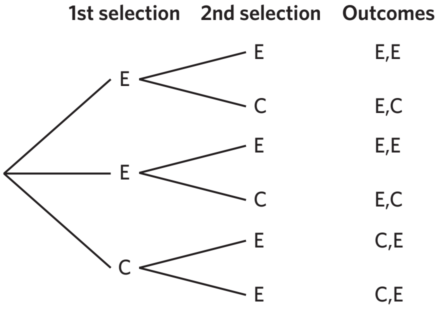
Key points

- At a canteen there are two egg (E) sandwiches left.
- As well as one cheese (C) sandwich left.
- Sashin randomly selects two sandwiches from the canteen.
- Construct a tree diagram and list the sample space for this scenario.

Explanation

This is a two-step experiment without replacement as once a sandwich has been chosen it doesn't get replaced.

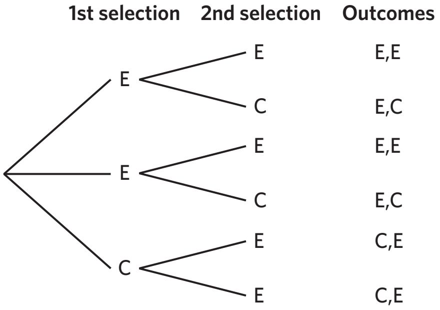
Label the first and second event and draw a branch for each possible outcome for each event.



List the sample space.

ξ = {(E,E), (E,C), (E,E), (E,C), (C,E), (C,E)}

Answer



ξ = {(E,E), (E,C), (E,E), (E,C), (C,E), (C,E)}

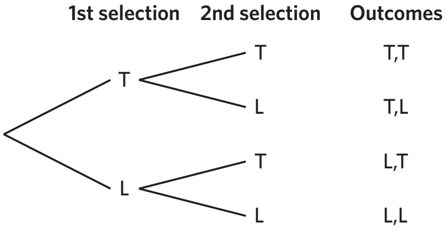
12. Ben takes the train to and from work. There is an equal chance that the trains will be on time (T) or late (L). Calculate the probability that the two trains he rides on the same day will both be late.

Key points

- There is an equal chance that the trains will be on time (T) or late (L).
- Determine the probability that the two trains he rides on the same day will both be late.

Explanation

Label the first and second event, first and second trains of the day, and draw a branch for each possible outcome for each event.



Determine the number of possible outcomes.

n(ξ) = 4

Determine the number of desired outcomes, both trains being late.

Both trains late = {(L,L)}

n(both trains late) = 1

Back to contents

Write the probability as a simplified fraction of the number of desired outcomes out of the total number of possible outcomes.

$\text{Pr(L,L)} = \frac{1}{4}$

Answer

The probability that the two trains Ben takes on the same day are both late is $\frac{1}{4}$.

13. Each question on the Learner's permit test contains answer options A, B, C or D. Mitchel randomly selects the answers for the last two questions. What is the probability that he selected C then D?

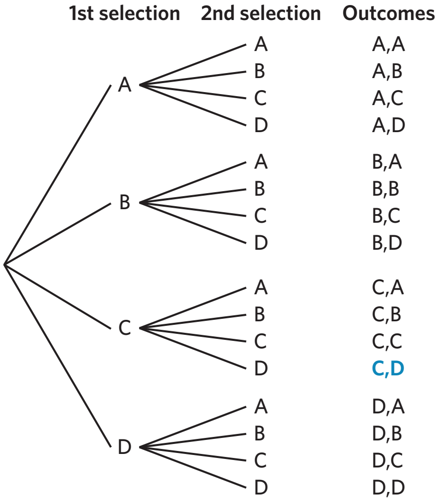
Key points

- Questions in a Learner's permit test have options A, B, C or D.
- Mitchel randomly selects the answers for the last two questions from the multiple choice options.
- Find the probability that Mitchel selected C then D.

Explanation

This is a two-step experiment with replacement as the four multiple choice options are always available.

Label the first and second event, first and second questions, and draw a branch for each possible outcome for each event.



Determine the number of possible outcomes.

$n(\xi) = 16$

Determine the number of desired outcomes.

C followed by D = {(C,D)}

n(C followed by D) = 1

Write the probability as a simplified fraction of the number of desired outcomes out of the total number of possible outcomes.

$\text{Pr(C,D)} = \frac{1}{16}$

Answer

The probability that Mitchel selected C then D as his answers to the last two questions of the Learner's permit test is $\frac{1}{16}$.

14. Louisa is picking names out of a hat to create pairs for a badminton competition. If there are 6 different names in the hat, what is the probability that the first pair she picks is Darren and Jessica?

Key points

- Louisa is picking names out of a hat to create pairs
- There are 6 different names in the hat.
- Determine the probability that the first pair she picks is Darren and Jessica.

Explanation

This is a two-step experiment without replacement as once a name is picked it cannot be chosen again.

Determine the number of possible outcomes without replacement.

$n(\xi) = 6 \times 5 = 30$

Determine the number of desired outcomes.

Darren and Jessica = {(D,J), (J,D)}

n(Darren and Jessica) = 2

Write the probability as a simplified fraction of the number of desired outcomes out of the total number of possible outcomes.

$\text{Pr(Darren and Jessica)} = \frac{2}{30} = \frac{1}{15}$

Answer

The probability that the first badminton pair Louisa picks is Darren and Jessica is $\frac{1}{15}$.

15. Rishmi plays a card game in which there is an equal chance of a win, loss or draw. What is the probability that she draws three consecutive matches?

Key points

- In a card game there is an equal chance of a win, loss or draw.
- What is the probability that she draws three consecutive matches?

Explanation

This is a three-step experiment with replacement, Rishmi is able to consecutively achieve the same outcome.

Determine the number of possible outcomes without replacement.

$n(\xi) = 3 \times 3 \times 3 = 27$

Determine the number of desired outcomes.

Three consecutive draws = {(D,D,D)}

n(three consecutive draws) = 1

Write the probability as a simplified fraction of the number of desired outcomes out of the total number of possible outcomes.

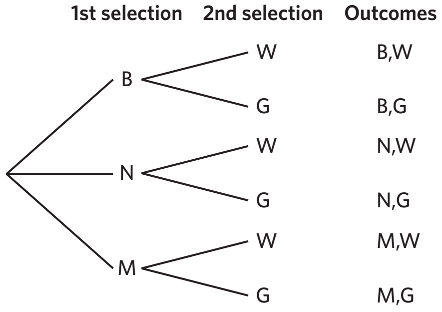
$\text{Pr(D,D,D)} = \frac{1}{27}$

Answer

The probability that Rishmi draws three consecutive matches is $\frac{1}{27}$.

Reasoning

16. a.



ξ = {(B,W), (B,G), (N,W), (N,G), (M,W), (M,G)}

b. The probability that Erica selects the maroon suit and grey shirt combination is $\frac{1}{6}$.

c. The probability that Erica packs the navy suit for her work trip is $\frac{2}{3}$.

d. Suggested option 1: To project a professional image.

Suggested option 2: So there is no bias based on personal clothing.

Note: There are other possible options.

Back to contents

17. **a.** $\Pr(A,A) = \frac{1}{9}$ for the experiment with replacement.

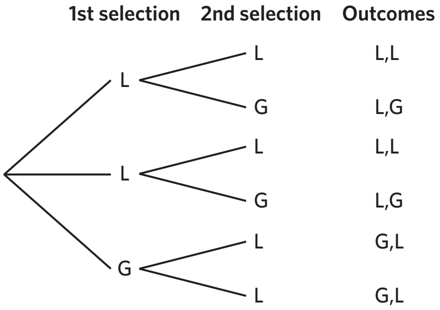
b. $\Pr(A,A) = \frac{1}{6}$ for the experiment without replacement.

c. Replacement can decrease the probability of a certain outcome occurring as it increases the total number of possible outcomes. In this case, conducting the experiment without replacement makes the desired outcome (A,A) impossible, so the probability is 0.

Exam-style

18. D

19. **a.**



$\xi = \{(L,L), (L,G), (L,L), (L,G), (G,L), (G,L)\}$

b. $\frac{2}{3}$

20. $\frac{2}{9}$

21. $\frac{1}{12}$

Remember this?

22. D **23.** B **24.** E

10E Experimental probability

Student practice

Worked example 1

a. $\frac{3}{10}$ **b.** 36

Worked example 2

a. 50 **b.** $\frac{1}{5}$ **c.** 36

Understanding worksheet

1. Theoretical probability: II; IV

Experimental probability: I; III

2.

Number on die	7	8	9	10	11	12
Relative frequency	4	3	3	4	2	2

3. theoretical; experimental; trials; frequency

Fluency

4. **a.** $\frac{1}{4}$ **b.** 55 **c.** $\frac{9}{20}$ **d.** 189

e. $\frac{11}{20}$ **f.** 143

5. **a.** $\frac{1}{5}$ **b.** 120 **c.** $\frac{7}{10}$ **d.** 140

e. $\frac{1}{10}$ **f.** 4

6. **a.** 50 **b.** $\frac{3}{25}$ **c.** 27 **d.** $\frac{1}{2}$

e. 500 **f.** $\frac{16}{25}$ **g.** 48 **h.** $\frac{43}{50}$

7. **a.** 200 **b.** $\frac{1}{5}$ **c.** 100 **d.** $\frac{3}{8}$

e. 300 **f.** $\frac{23}{50}$ **g.** 460 **h.** $\frac{27}{50}$

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. James' hockey team has won three games and lost nine games when playing at an opponents' home venue. What is the relative frequency of losing games played at the opponents' home venue for James' hockey team? Give the answer as a decimal.

Key points

- James' hockey team has won three games and lost nine games.
- What is the relative frequency of losing games played at the opponents' home venue?

Explanation

Determine the number of trials and the frequency of the event.

Trials: $3 + 9 = 12$

Frequency: 9

Write the relative frequency as a fraction of the frequency of the event out of the total number of trials.

$$\begin{aligned}\Pr(\text{losing}) &= \frac{9}{12}\\ &= \frac{3}{4}\\ &= 0.75\end{aligned}$$

Answer

The relative frequency of losing games played at the opponents' home venue for James' hockey team is 0.75.

11. Derek is a chess enthusiast and has been tracking his wins, draws, and losses over the past month. He has won 6 games, drawn 8 and lost 11. What is the experimental probability of Derek winning his next game? Give the answer as a decimal.

Key points

- Derek has won 6 chess games, drawn 8 and lost 11.
- What is the experimental probability of Derek winning his next game?

Explanation

Determine the number of trials and the frequency of the event.

Trials: $6 + 8 + 11 = 25$

Frequency: 6

Write the experimental probability as a fraction of the frequency of the event out of the total number of trials.

$$\begin{aligned}\Pr(\text{winning}) &= \frac{6}{25}\\ &= 0.24\end{aligned}$$

Answer

The experimental probability of Derek winning his next game is 0.24.

12. A jeweller noticed that 2 out of 40 diamond rings had a minor flaw. If they are planning to showcase 500 rings in a jewellery exhibition, how many might be expected to have a flaw based on the experimental probability?

Key points

- 2 out of 40 diamond rings had a minor flaw.
- The jeweller is planning to showcase 500 rings in a jewellery exhibition.
- How many diamond rings might be expected to have a flaw based on the experimental probability?

Explanation

Determine the experimental probability of a diamond ring having a flaw.

Trials: 40

Frequency: 2

$$\begin{aligned}\text{Pr(flaw)} &= \frac{2}{40}\\ &= \frac{1}{20}\end{aligned}$$

Multiply the experimental probability by the number of trials.

$$\begin{aligned}\text{Expected number of occurrences} &= \text{Pr(flaw)} \times \text{number of trials}\\ &= \frac{1}{20} \times 500\\ &= 25\end{aligned}$$

Answer

25 diamond rings are expected to have a flaw.

13. A cinema showcases three types of movies: action, drama, and animation. The experimental probability of a moviegoer watching an action movie is $\frac{4}{7}$. If the cinema has 3500 viewers in a day, how many are predicted to watch an action movie?

Key points

- The experimental probability of a moviegoer watching an action movie is $\frac{4}{7}$.
- The cinema has 3500 viewers in a day.
- How many viewers are predicted to watch an action movie?

Explanation

Multiply the experimental probability of a moviegoer watching an action movie by the number of trials.

$$\begin{aligned}\text{Expected number of occurrences} &= \text{Pr(action)} \times \text{number of trials}\\ &= \frac{4}{7} \times 3500\\ &= 2000\end{aligned}$$

Answer

2000 viewers are predicted to watch an action movie.

14. After assessing traffic flow, a traffic analyst stated that the experimental probability of heavy traffic congestion at morning time in New York was 70%. If the analyst observed heavy traffic congestion on 7 mornings, over how many mornings in total was the traffic flow assessed?

Key points

- The experimental probability of heavy traffic congestion during rush hours in New York was 70%.
- This was the case for 7 mornings.
- Over how many mornings in total was the traffic flow assessed?

Explanation

Determine the number of trials (mornings that traffic flow was assessed).

$$\begin{aligned}\text{Pr(heavy congestion)} &= \frac{\text{frequency of heavy congestion}}{\text{total number of trials}}\\ 70\% &= \frac{7}{\text{total number of trials}}\\ \frac{7}{10} &= \frac{7}{\text{total number of trials}}\\ \text{total number of trials} &= 7 \div \frac{7}{10}\\ &= 10\end{aligned}$$

Answer

Traffic flow was assessed over 10 mornings in total.

Reasoning

15. a. There were 80 athletes in study 1 and 500 athletes in study 2.

b. The experimental probability of the sport drink increasing hydration is 0.6875 in study 1 and 0.6 in study 2.

c. 125 athletes are predicted to feel a decrease in hydration based on study 1 and 300 are predicted to feel a decrease based on study 2.

d. Study 2 because it has a larger sample size which generally provides more reliable and generalisable results.

e. Suggested option 1: The taste should be considered to ensure that consumers find the drink palatable.

Suggested option 2: Nutritional benefits should also be considered as the drink should help maintain electrolyte balance and other beneficial substances in the body.

Note: There are other possible options.

16. a. 0.3

b. 0.5

c. The theoretical probability of tossing heads is $\frac{1}{2}$. The relative frequency in part **b** is equal to the theoretical probability while the relative frequency in part **a** is further from the theoretical probability. This is because as the number of trials increases, the experimental probability becomes closer to the theoretical probability (more accurate).

Exam-style

17. B

18. a. $\frac{1}{20}$

b. 25 phones

19. 300 birds

20. 40 cards

Remember this?

21. E

22. A

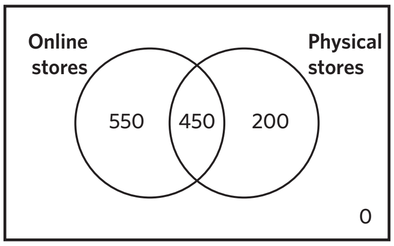
23. B

Back to contents

Chapter 10 extended application

1. **a.** n(Online stores ∩ Physical stores) = 450

b.



c.

	Online stores	Not online stores	Total
Physical stores	450	200	**650**
Not physical stores	550	0	**550**
Total	**1000**	**200**	**1200**

d. Pr(Physical stores ∩ Not online stores) = $\frac{1}{6}$

Pr(Online stores) = $\frac{5}{6}$

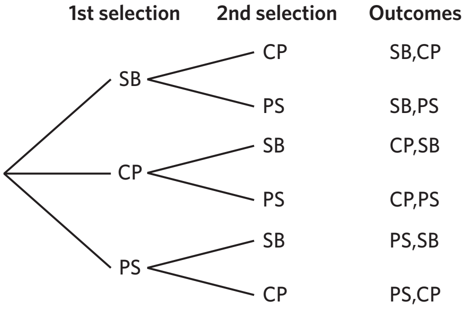
e. It is much more common for shoppers to use online stores currently.

f. Suggested option 1: A shopper may be influenced to purchase items in a physical store if there is a discount that is only offered in-store.

Suggested option 2: A shopper may be influenced to purchase items in a physical store if they need to try an item on, such as clothing.

Note: There are other possible options.

2. **a.**



ξ = {(SB,CP), (SB,PS), (CP,SB), (CP,PS), (PS,SB), (PS,CP)}

b.

		Second selection		
		SB	**CP**	**PS**
First selection	**SB**	X	(SB,CP)	(SB,PS)
	CP	(CP,SB)	X	(CP,PS)
	PS	(PS,SB)	(PS,CP)	X

c. $\frac{1}{3}$

d. $\frac{1}{9}$

e. Suggested option 1: To cater to the different artistic strengths and weaknesses of different students.

Suggested option 2: To expose the students to different types of artistic mediums.

Note: There are other possible options.

3. **a.** Stellar Films studied 150 movies. Galaxy Studios studied 295 movies.

b. Stellar Films:
Low: 0.13, Moderate: 0.53, High: 0.33

Galaxy Studios:
Low: 0.17, Moderate: 0.29, High: 0.54

c. Stellar Films: 130

Galaxy Studios: 170

d. Stellar FIlms appears to be more successful in avoiding low-revenue movie releases as when producing 1000 movies, 130 are predicted to be low-revenue for Stellar Films in contrast to 170 for Galaxy Studios.

e. Suggested option 1: The genre of the movie.

Suggested option 2: The duration of the movie.

Note: There are other possible options.

Chapter 10 review

Multiple choice

1. C **2.** A **3.** D **4.** C **5.** A

Fluency

6. **a.** 10 **b.** 16 **c.** $\frac{5}{11}$ **d.** 2

7. **a.** 20 **b.** 25 **c.** $\frac{5}{28}$ **d.** 3

8. **a.** A ∪ B = {21, 23, 24, 25, 26, 27, 29}

b. A ∩ B = {23}

c. B’ = {22, 24, 26, 28, 30}

d. A ∪ B’ = {22, 23, 24, 26, 28, 30}

9. **a.** $\frac{15}{16}$ **b.** $\frac{1}{4}$ **c.** $\frac{7}{16}$ **d.** $\frac{1}{16}$

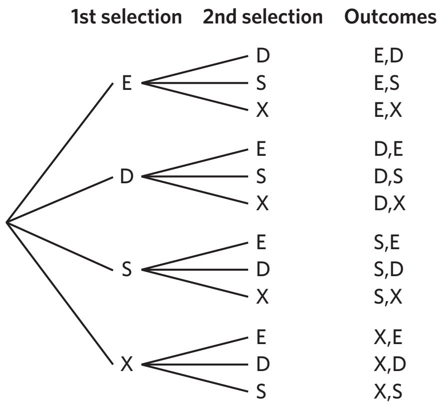
10. **a.**

		Second selection				
		R	**R**	**W**	**W**	**B**
First selection	**R**	X	(R,R)	(R,W)	(R,W)	(R,B)
	R	(R,R)	X	(R,W)	(R,W)	(R,B)
	W	(W,R)	(W,R)	X	(W,W)	(W,B)
	W	(W,R)	(W,R)	(W,W)	X	(W,B)
	B	(B,R)	(B,R)	(B,W)	(B,W)	X

ξ = {(R,R), (R,W), (R,W), (R,B), (R,R), (R,W), (R,W), (R,B), (W,R), (W,R), (W,W), (W,B), (W,R), (W,R), (W,W), (W,B), (B,R), (B,R), (B,W), (B,W)}

b. $\frac{1}{10}$ **c.** $\frac{3}{5}$ **d.** $\frac{2}{5}$

11. **a.**



(ξ) = {(E,D), (E,S), (E,X), (D,E), (D,S), (D,X), (S,E), (S,D), (S,X), (X,E), (X,D), (X,S)}

b. $\frac{1}{12}$ **c.** $\frac{1}{2}$ **d.** $\frac{1}{2}$

---

12. **a.** $\frac{1}{4}$ **b.** 75 **c.** $\frac{5}{12}$ **d.** 225

---

13. **a.** 100 **b.** $\frac{3}{20}$ **c.** 42 **d.** $\frac{49}{50}$

Problem solving

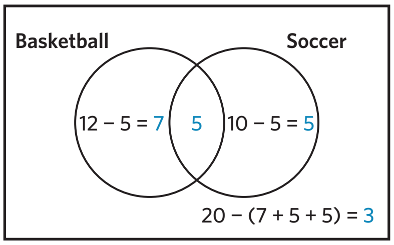
14. In a school sports club consisting of 20 students, 12 students play basketball, 10 students play soccer, and 5 students play both basketball and soccer. Construct a Venn diagram to represent the situation.

Key points

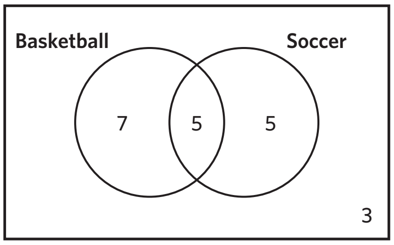
- There is a school sports club consisting of 20 students.
- 12 students play basketball.
- 10 students play soccer.
- 5 students play both basketball and soccer.
- Construct a Venn diagram to represent the situation.

Explanation

Draw a rectangle, circles and write the labels on the Venn diagram. Fill in the information that is known and determine the remaining information.



Answer



15. The music club at a school has a group of different musical instruments. The group includes a Violin, Trumpet, Flute, and Guitar. List the set M if it is equal to {musical instruments that contain the letter 'u'}.

Key points

- The music club at a school has a Violin, Trumpet, Flute, and Guitar.
- List the set M if it is the {musical instruments that contain the letter 'u'}.

Explanation

Identify the sample space.

ξ = {Violin, Trumpet, Flute, Guitar}

Identify which musical instruments contain the letter 'u'.

- Violin ✗
- Trumpet ✓
- Flute ✓
- Guitar ✓

M = {Trumpet, Flute, Guitar}

Answer

The set M is {Trumpet, Flute, Guitar}.

16. In a community gardening club, there are six members: Anna, Bruce, Clara, Derek, Emily, and Frank. The club coordinator randomly select pairs to work on different sections of the garden. The coordinator picks a name, does not replace it, and then picks another name to form a pair. What is the probability that the first pair selected is Anna and Derek?

Key points

- There are six members of a community gardening club Anna, Bruce, Clara, Derek, Emily, and Frank.
- The club coordinator randomly select pairs from the gardening club.
- The coordinator picks a name, does not replace it, and then picks another name to form a pair.
- Determine the probability that the first pair selected is Anna and Derek.

Explanation

Draw an array for the situation without replacement, fill in all possible outcomes.

		Second selection					
		A	**B**	**C**	**D**	**E**	**F**
First selection	**A**	X	(A,B)	(A,C)	(A,D)	(A,E)	(A,F)
	B	(B,A)	X	(B,C)	(B,D)	(B,E)	(B,F)
	C	(C,A)	(C,B)	X	(C,D)	(C,E)	(C,F)
	D	(D,A)	(D,B)	(D,C)	X	(D,E)	(D,F)
	E	(E,A)	(E,B)	(E,C)	(E,D)	X	(E,F)
	F	(F,A)	(F,B)	(F,C)	(F,D)	(F,E)	X

Determine the number of possible outcomes.

n(ξ) = 30

Determine the number of desired outcomes.

n(contains both A and D) = 2

Determine the probability of Anna and Derek being the first partners chosen.

$\Pr((A,D) \text{ or } (D,A)) = \frac{2}{30} = \frac{1}{15}.$

Answer

The probability that the first pair selected is Anna and Derek is $\frac{1}{15}$.

Back to contents

17. In a local chess tournament, there are 7 players: Alice, Bob, Charlie, David, Emily, Fiona, and Greg. The tournament organiser is randomly selecting two players to face off in the first match. What is the probability that the first match is between Alice and Fiona?

Key points

- There are 7 players in a local chess tournament: Alice, Bob, Charlie, David, Emily, Fiona, and Greg.
- First two players to face off are randomly chosen.
- Determine the probability that the first match is between Alice and Fiona.

Explanation

Determine the nature of the experiment.

Two-step experiment without replacement as a player cannot play against themselves in the tournament.

Determine the number of possible outcomes without replacement.

$n(\xi) = 7 \times 6 = 42$

Determine the number of desired outcomes.

$n((A,F) \text{ or } (F,A)) = 2$

Determine the probability of the desired outcomes.

$\Pr((A,F) \text{ or } (F,A)) = \frac{2}{42} = \frac{1}{21}$

Answer

The probability that the first match chosen is Alice and Fiona is $\frac{1}{21}$.

18. On a popular music streaming platform, there are three main genres that users listen to: Pop, Jazz, and Classical. The experimental probability of a user listening to a Jazz track is $\frac{3}{8}$. If the platform has 480 000 users on a particular day, how many are predicted to listen to Jazz?

Key points

- There are three main genres that users listen to on a popular music streaming platform Pop, Jazz, and Classical.
- The experimental probability of a user listening to a Jazz track is $\frac{3}{8}$.
- If the platform has 480 000 users on a particular day. Determine how many are predicted to listen to Jazz.

Explanation

Multiply the experimental probability of the event by the number of trials.

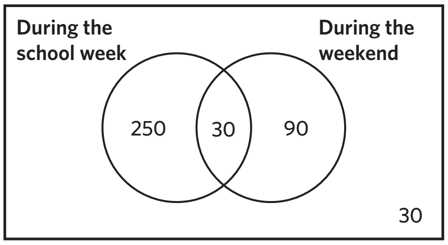
$$\begin{aligned}\text{Expected number of occurrences} &= \Pr(\text{Jazz}) \times \text{number of users}\\ &= \frac{3}{8} \times 480\,000\\ &= 180\,000\end{aligned}$$

Answer

180 000 users are predicted to listen to Jazz on a popular music streaming platform.

Reasoning

19. a.



340 students prefer only one option.

b. $\Pr(\text{prefers homework on weekends}) = \frac{3}{10}$

c. $n(D \cap W) = 30$

d.

		Second selection	
		D	W
First selection	D	(D,D)	(D,W)
	W	(W,D)	(W,W)

e. Suggested option 1: To have it done sooner rather than later.

Suggested option 2: To have more free time over the weekend.

Note: There are other possible options.

20. a. 14

b. 14

c. In parts **a** and **b** $n(B) + n(C)$ is equal to $n(B \cup C)$ because there is no element in the intersection $(B \cap C)$. $n(B) + n(C)$ may not equal to $n(B \cup C)$ when there are elements in the intersection of B and C.

Back to contents

Glossary

A

Acute An acute angle is greater than 0° but less than 90° in size.

Adjacent When two objects are adjacent, they are positioned directly next to each other or share a common boundary or side.

Adjacent angles Adjacent angles are angles that share a vertex and a common side.

Angle The angle between line segments AB and BC is denoted $\angle ABC$.

Apex A apex is the vertex at the top (furthest from the base).

Arc An arc is a part of the circumference of a circle.

Area The area is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.

Array An array in probability is a structured representation, resembling a table, used to systematically display all possible outcomes resulting from multiple successive events or choices.

Asymptote An asymptote is a line that the graph of a function approaches, but does not touch at any point.

Australian Tax Office (ATO) The Australian Tax Office (ATO) manages taxation on behalf of the government.

B

Binomial expression A binomial expression is a polynomial expression which contains two terms.

C

Capacity Capacity is the maximum amount of fluid or substance a container can hold.

Cartesian plane A Cartesian plane is a set of two perpendicular number lines that intersect at the origin.

Centimetre (cm) A centimetre (cm) is one-hundredth of a metre.

Circumference The circumference is the perimeter of a circle.

Class interval A class interval defines the data range of the groups into which a set of data has been organised.

Complement A complement is a set of all elements in the universal set that are not in a given set.

Composite shape A composite shape consists of two or more regular shapes.

Composite solid A composite solid is a 3D object made up of two or more objects combined together.

Compound period The compound period of a loan or investment is how often interest is calculated.

Concave polygon A concave polygon has at least one internal angle greater than 180°.

Cone A cone is a 3D object with a circular base joined to the apex by a curved surface.

Congruent Two triangles are congruent if each has the same three corresponding side lengths and the same three corresponding angles.

Continuous A continuous function has a graph without any breaks or abrupt changes in values.

Convex polygon A convex polygon has all internal angles less than 180°.

Corresponding Corresponding sides or angles appear in the same position in similar or congruent shapes.

Cost price The cost price is how much a product or service costs to produce or provide.

Critical digit The critical digit is the digit to the right of the decimal place or significant figure that a number is being rounded to; it determines whether the number is rounded up or down.

Cross-section A cross-section is a surface that is created when making a straight cut through a 3D shape.

Cylinder A cylinder is a 3-dimensional object that has 2 flat, circular bases that are connected by a curved surface.

D

Decimal notation A number written in decimal notation can be expressed using digits separated by a decimal point.

Diameter The diameter is a straight line between two points on the circumference of a circle that passes through the centre.

Discontinuous A discontinuous function has a graph which is not connected or contains abrupt changes in values.

Distributive law The distributive law for multiplication means that multiplying a number by a group of numbers is the same as multiplying the number by the sum of the other numbers.

Double pay Hours worked at double pay is equal to 2 times the hourly rate.

Due A due bearing refers to an exact direction, such as directly north, east, west, or south.

E

Elimination method The elimination method is the process of eliminating one of the variables by adding the two equations.

Equate To equate expressions is to make them equal to each other

Equivalent fractions Equivalent fractions have different numerators and denominators but represent and are equal to the same value.

Exact form A value in exact form is a number that is not expressed as a decimal approximation, rather it is left in fraction or surd form.

Exclusive Exclusive is excluding the value(s).

Exclusive 'or' Exclusive 'or' suggests an element belongs to two separate categories but not both.

Experiment An experiment is a series of trials conducted to examine the results of chance activities.

Expression An expression is a number of terms grouped together by operations.

Back to contents

F

Face A face is a flat surface of a solid object.

Figure A figure, when talking about numbers, is another term for a digit.

Financial year In Australia, the financial year is between July of one year to June of the next year.

Formula A formula is a rule written using mathematical symbols and pronumerals that are connected using an equals sign.

Frequency Frequency refers to how many times an event occurs.

G

General formula A general formula is a rule or mathematical relationship between values that is expressed using symbols or pronumerals.

Gradient The gradient (slope) is the degree of steepness of a line of a line.

Graph theory Graph theory is the study of the relationship between lines (edges) and points (vertices).

Gross income Gross income is the money earned from a salary or wage before tax.

Grouping Grouping involves pairing a sum or difference of two terms in an expression so that they share a common factor.

H

Hectare A hectare is a metric unit of area equal to 10 000 square metres.

Hemisphere A hemisphere is half of a sphere, where every point on its curved surface is an equal distance away from its centre.

Highest common factor The highest common factor (HCF) is the largest number, term or expression that is a common factor of two or more terms or expressions.

Horizontal A line is said to be horizontal if two points on the line have the same y-coordinate points.

Horizontal line A horizontal line is parallel with the horizon.

Included angle An included angle is an angle between two sides of a triangle.

I

Inclusive Inclusive is including the value(s).

Inclusive 'or' Inclusive 'or' suggests an element that belongs to two separate categories as well as both.

Index notation Index notation (or form) is a way of representing repeated factors of the same number.

Inequality Inequality is a statement when one value or algebraic expression is less than or greater than another.

Intersect Intersect is when an edge crosses/overlaps two or more lines.

Intersection point The intersection point is the point at which two lines cross.

Intersection The intersection is the set of elements that two or more sets have in common.

Inverse operations Inverse operations are mathematical operations that are the reverse of each other. The inverse of multiplication is division and the inverse of division is multiplication; the inverse of addition is subtraction and the inverse of subtraction is addition.

Inverted parabola An inverted parabola (concave down) with an equation of the form $y = ax^2$ is produced when $a < 0$.

Investment An investment is a sum of money that generates income through interest.

Irrational numbers Irrational numbers cannot be written as a simple fraction and contain an infinite number of non-recurring decimal digits.

K

Kilometre (km) A kilometre (km) is one thousand metres.

L

Leading zero A leading zero is any zero digit that comes before the first non-zero digit in a number.

Lender A lender is a financial institution that loans money.

Line A line extends in both directions with no end.

Line of vision Line of vision is the implied straight line along which an observer looks at an object.

Line segment A line segment is part of a line connecting two points that has definite ends.

Linear rule A linear rule is an equation used for a straight line.

Loan A loan is a sum of money borrowed and paid back with interest.

Loss A loss occurs when a product or service is sold for less than it cost.

Lowest common denominator (LCD) The lowest common denominator (LCD) is the lowest common multiple (LCM) of the denominators of two or more fractions.

Lowest common multiple (LCM) The lowest common multiple (LCM) is the smallest number that is a multiple of two or more numbers.

M

Maximum The maximum is the largest value in a data set.

Metre (m) A metre (m) is a standardised unit measuring length.

Millimetre (mm) A millimetre (mm) is one-thousandth of a metre.

Minimum The minimum is the smallest value in a data set.

Monic quadratic trinomial A monic quadratic trinomial is a quadratic expression of the form $ax^2 + bx + c$ where $a = 1$.

Multi-step equation A multi-step equation is an equation that can be solved by applying more than two inverse operations.

N

Net A net is a 2D representation of a solid object.

Node A node represents a point where a decision or event occurs, branching out into different possible outcomes or paths.

Null Factor Law The Null Factor Law states that if the product of two values is zero then either or both of the two values is zero.

Numerical data Numerical data is quantitative data that can either be discrete or continuous.

O

One-step equation A one-step equation is an equation that can be solved by applying a single inverse operation.

Opposite When two objects are opposite, they are situated on opposite sides of a specified or implied boundary or facing in contrasting directions.

Origin The origin, (0,0), is the point of intersection of the x and y axes on a Cartesian plane.

Overtime Overtime is any time worked in excess of reasonable work hours and usually attracts time-and-a-half or double pay.

P

Parabola A parabola is the graph of a quadratic function with the general equation $y = ax^2 + bx + c$ where a, b, c are known values and $a \neq 0$.

Parallel Parallel lines never touch and are always the same distance apart.

PAYG PAYG stands for Pay-As-You-Go and means that tax is withheld each pay cycle rather than paid at the end of the financial year.

Per annum (p.a.) Per annum (or p.a.) means every year, and is commonly used to refer to interest rates.

Perfect square A perfect square, also known as a square number, in a given number system is the product of a number multiplied by itself.

Perpendicular Perpendicular lines form a right angle (90°).

Plane Plane is when two or more points are joined together to form a flat surface.

Polynomial expression A polynomial expression must contain variables with positive integer exponents and may contain a constant.

Principal The principal is the initial amount loaned out or invested.

Prism A prism is a 3-dimensional object which has two identical polygon faces on either end, connected by rectangular faces. A prism has the same cross-section when cut anywhere along its length.

Product The product is the result when two or more values are multiplied together.

Profit A profit occurs when a product or service is sold for more than it cost.

Proportional relationship If two quantities have a proportional relationship they can be written as a ratio.

Q

Quadratic surd A quadratic surd has the order $n = 2$.

Quartile A quartile is a statistical measure that divides a data set into four equal parts.

Quotient The quotient is the result when a value (dividend or numerator) is divided by another value (divisor or denominator).

R

Radical A radical symbol $\sqrt{}$ is used to denote the root of a number.

Radicand The radicand is the number or expression under the radical.

Radius The radius is the direct distance from the centre of a circle to any point on the circumference.

Range The range is a statistical measure of the spread of data. It is the difference between the maximum and minimum values.

Rate A rate is used to compare quantities in different units.

Ratio A ratio is a proportional relationship between two or more quantities with the same unit.

Reciprocal of a fraction The reciprocal of a fraction can be obtained by swapping the values or expressions in the numerator with those in the denominator.

Reiprocal of a number The reciprocal of a number is 1 divided by that number.

S

Sale price The sale price is how much a product or service is sold for.

Sample space A sample space is a set of all possible outcomes or results of an experiment. This is also referred to as the universal set.

Sector A sector is part of a circle that is enclosed by two radii and an arc.

Simultaneous equations Simultaneous equations is a set of two or more equations, each containing two or more variables whose values can simultaneously satisfy both or all of the equations in the set.

Skewed data Skewed data refers to data sets where the values are not symmetrically distributed around the mean.

Solution A solution is a value, or values, that when substituted into an equation, make that equation true.

Solving equations Solving equations is a process to find the value of the unknown by performing a series of inverse operations.

Sphere A sphere is a round 3D object in which every point on its surface is an equal distance away from its centre.

Subject The subject of a formula is the variable isolated on one side of the equal sign.

Subset A subset is a set that contains only elements that are also in the sample space, without any additional elements.

Substitution Substitution is the process of replacing a variable or an unknown with a given value.

Substitution method The substitution method is the process of adjusting one equation so that the value of one variable is defined in terms of the other.

Surds Surds are roots that are irrational numbers.

Symmetrical data Symmetrical data refers to data sets where the values are evenly distributed around the mean, such that the left and right halves of the data distribution are mirror images of each other.

System of equations A system of equations consists of two or more equations that share the same variables.

Back to contents

T

Tax deductions Tax deductions are certain expenses that reduce taxable income, and therefore the amount of income tax paid.

Time-and-a-half Hours worked at time-and-a-half pay is equal to 1.5 times the hourly rate.

Total surface area (TSA) Total surface area (TSA) is the sum of the area of all faces of a solid object.

Trailing zero A trailing zero is any zero digit that comes after the last non-zero digit in a number.

Translation A translation of a graph requires all points to move the same distance and in the same direction horizontally and/or vertically

Transpose An equation can be transposed to make any variable the subject.

Tree diagram A tree diagram is a visual representation used to display all possible outcomes for a series of sequential events or decisions.

Trinomial A trinomial is a polynomial expression containing three different terms, one of which may be a constant.

Truncating Truncating means shortening a number, usually through rounding.

Two-step equation A two-step equation is an equation that can be solved by applying two inverse operations.

Two-step experiments Two-step experiments describe a probability event that is made up of two actions.

Two-way table A two-way table displays the frequency (count) of two categories, written in a table format with rows and columns.

U

Undefined An undefined value or expression does not have mathematical meaning, such as division by zero.

Union The union is the set of all elements that are in either of two or more sets.

Upright parabola An upright parabola (concave up) with an equation of the form $y = ax^2$ is produced when $a > 0$.

V

Variable A variable is a letter used to represent a value that is unknown or may vary. This is also known as a pronumeral or an unknown.

Venn diagram A Venn diagram provides a visual representation of similarities and differences between two or more sets of information.

Vertex The vertex or turning point of a parabola is the point at which the axis of symmetry intersects with the parabola.

Vertical A line is said to be vertical if two points on the line have the same x-coordinate points.

Vertical line A vertical line is perpendicular (at a 90° angle) with the horizon.

W

Withheld tax Withheld tax is money taken from gross income by an employer and paid to the ATO.