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MATHEMATICS ESSENTIAL

UNITS 1 & 2

CAMBRIDGE SENIOR MATHEMATICS FOR WESTERN AUSTRALIA

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Cambridge Senior Maths for Western Australia
Mathematics Essential 1&2

SBN 978-1-009-25785-5

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

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First published 2023
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover design by Denise Lane (Sardine Design)
Text design by Jane Pitkethly
Typeset by diacriTech
Printed in Australia by Ligare Pty. Ltd.

A catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-009-25785-5 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

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Practice Practical Applications tasks and Statistical Investigations

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Introduction

Cambridge Mathematics Essential for Western Australia Units 1 & 2 is a new resource aligned specifically to the Western Australian Mathematics Essential Year 11 syllabus. Covering both Units 1 and 2 in one resource, it has been written with practical contexts and worded questions as its priority alongside ample practice offered through worked examples and exercises. This course follows on from the K–10 Australian Curriculum and expands upon the fundamental skills and knowledge developed in earlier levels.

Unit 1 begins by building a foundation of basic calculation skills and giving students ample practice at checking the reasonableness of their solutions. This foundation is then utilised to develop understanding of percentages and rates, building upon the non-calculator skills necessary for the Year 11 and 12 Essential course. Chapters 3 and 4 move to the study of linear measure, area measure, mass, volume, capacity, and units of energy, including the use of formulas. The application of the mathematical thinking process is developed through the Mathematical Thinking Process Tasks at the end of chapters. Interpreting and drawing graphs is introduced in Chapters 5 and 6 and linked to the statistical investigation process in preparation for Unit 2.

Unit 2 begins with representing and comparing data and learning to apply the statistical investigation process. Percentages and rates are revised and extended, and ratios are introduced. Time and motion and their application within the mathematical thinking process complete Unit 2.

Worked examples utilising appropriate technology are provided throughout, with screenshots and detailed user instructions for both scientific calculators and Excel spreadsheets.

The integration of the features of the textbook and the new digital components of the package, powered by Cambridge HOTmaths, are illustrated on pages x to xiv.

About Cambridge HOTmaths

Cambridge HOTmaths is a comprehensive, award-winning mathematics learning system – an interactive online maths learning, teaching and assessment resource for students and teachers, for individuals or whole classes, for school and at home. Its digital engine or platform is used to host and power the Interactive Textbook and the Online Teaching Suite, and selected topics from HOTmaths' own Years 9 and 10 courses area are available for revision of prior knowledge. All this is included in the price of the textbook.

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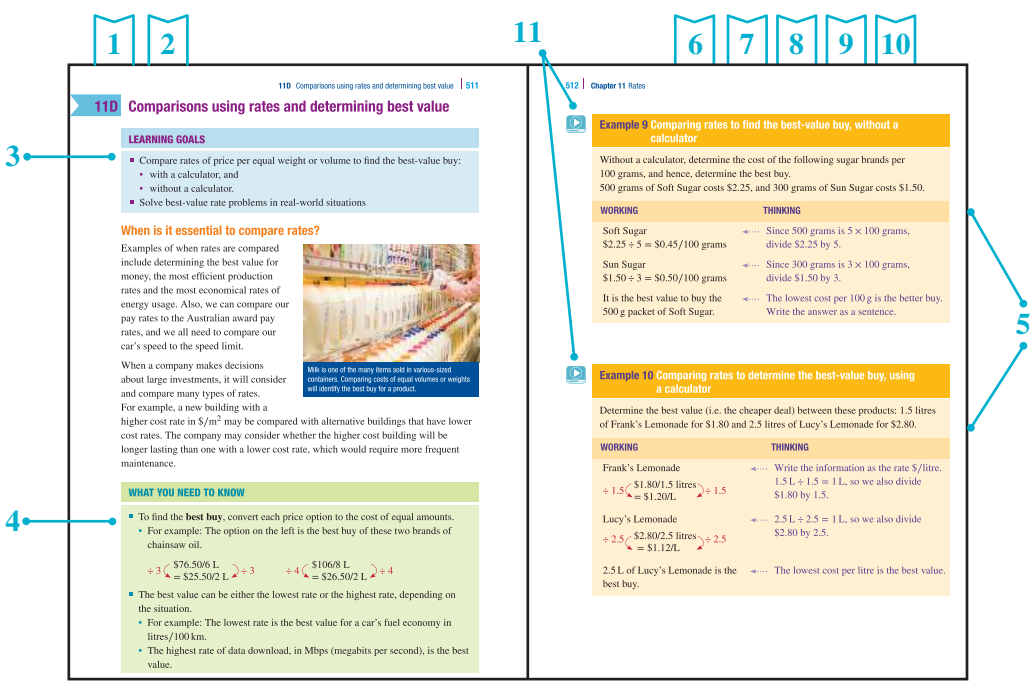
Overview

Overview of the print textbook

1. **Syllabus references** are listed at the beginning of each chapter.
2. **Pre-tests** provide a check of requisite knowledge and skills. Links are provided to access HOTmaths lessons from earlier years as revision for those students who need it.
3. **Learning goals** based on the syllabus are given for each section.
4. **What you need to know** boxes list important concepts and principles in concise and accessible format.
5. **Worked examples** detail thinking and the solution in a logical sequence, and are linked to exercises. Video versions are provided in the Interactive Textbook to encourage independent learning.

Print book features — numbers refer to points above



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6 Exercises are divided into:
   - **Fundamentals –** integrating the Fundamental topic: Calculations throughout the topic, as required by the syllabus.
   - **Applications**: questions that model real-life contexts.

7 **Mathematical thinking process tasks** and **Statistical investigation process tasks** are provided at the end of chapters, and are, set out in stages with a flowchart based on the approach to problem-solving and mathematical thinking outlined by SCSA.

8 **Chapter checklists** comprise short questions assessing achievement of learning goals.

9 **Review** exercises contain assessment-style questions.

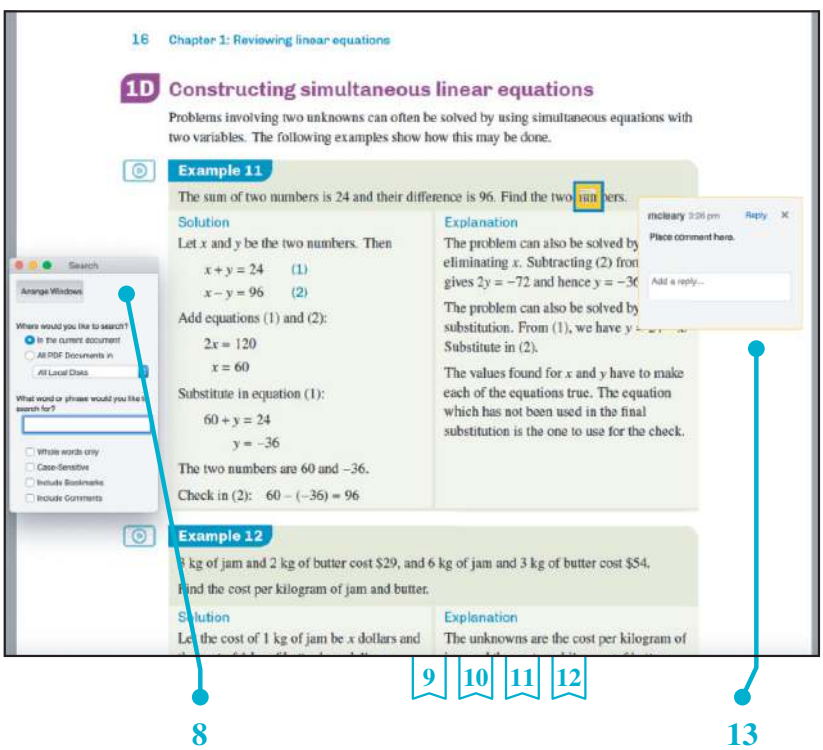
10 Examples and questions using **spreadsheets** are integrated throughout the text, with accompanying Excel files in the Interactive Textbook.

11 Additional linked resources in the Interactive Textbook and Online Teaching Suite such as practice assessment items are indicated by icons or notes in the text.

Overview of the downloadable PDF textbook

12 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

13 PDF annotation and search features are enabled.



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ISBN 978-1-009-25785-5

Overview of the Interactive Textbook

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate purchase.

14 The material is formatted for on screen use with a convenient and easy-to-use navigation system and links to all resources.

15 **Definitions** pop up for key terms in the text and are also provided in a printable online **glossary**, while the HOTmaths dictionary is also accessible.

16 All worked examples have **video versions** to encourage independent learning.

17 The **Desmos scientific calculator**, **graphics calculator** and **geometry tool** are also available for students to use for their own calculations and exploration.

18 **Spreadsheets** are provided in Excel format.

19 **Quick quizzes** containing automarked multiple-choice questions enable students to check their understanding.

20 Previous years' **HOTmaths lessons** are provided for revision of prior knowledge and skills, linked below Pre-Tests.

21 **Workspaces** for all questions, which can be enabled or disabled by the teacher, allow students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing done on paper.

22 **Self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite, so that teachers can review student self-assessment and provide feedback or adjust marks.

23 **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher.

24 **Practice assessment** items are provided in downloadable PDF and Word files.

25 **Messages** from the teacher to students can be sent to assign tasks and tests.

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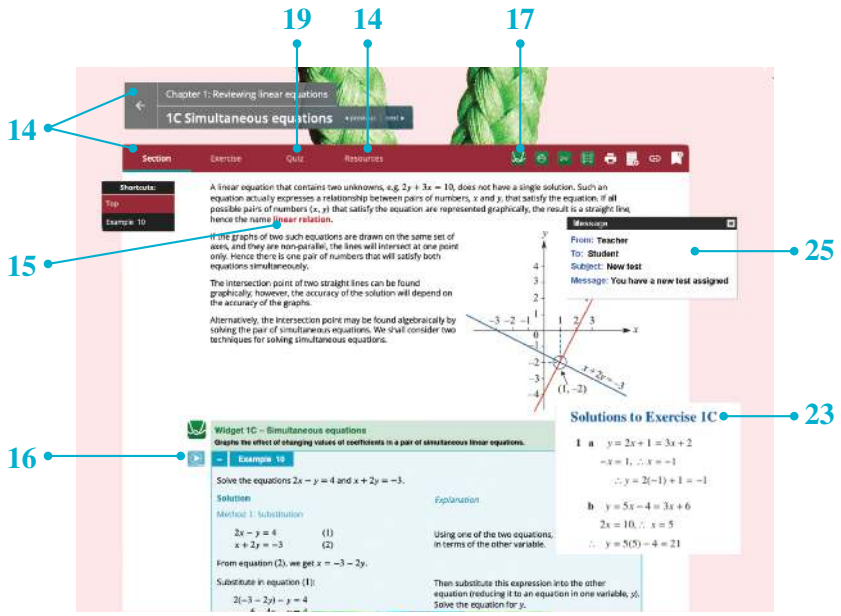
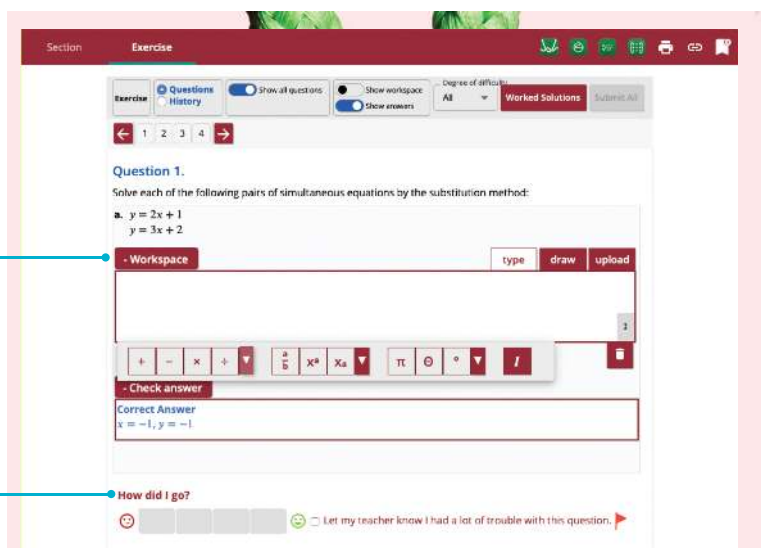
INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

A selection of features is shown. Numbers refer to the descriptions on pages xiii–xiv. HOTmaths platform features are updated regularly. Screenshots below show the Mathematics Methods title in the series - not all features of Essential Mathematics are shown.



WORKSPACES AND SELF-ASSESSMENT



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Overview of the Online Teaching Suite powered by the HOTmaths platform

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the teacher resources are in one place for easy access. The features include:

26 The HOTmaths learning management system with class and student analytics and reports, and communication tools.
27 Teacher's view of a student's working and self-assessment.
28 A HOTmaths-style test generator.
29 Chapter test worksheets as PDFs and editable Word documents.
30 Assessment practice items as PDFs and editable Word documents.
31 Editable curriculum grids and teaching programs.

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1 Basic calculations and checking and making sense of all calculations

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In this chapter

Syllabus reference

Unit 1 Topic 1.1: Basic calculations, percentages and rates

Checking and making sense of all calculations

- use leading digit approximation to obtain estimates of calculations
- check results of calculations for accuracy
- understand the meaning and magnitude of numbers involved, including fractions, percentages and the significance of place value after the decimal point
- ascertain the reasonableness of answers, in terms of context, to arithmetic calculations
- round up or round down answers to the accuracy required, including to the required number of decimal places

Basic calculations

- choose and use addition, subtraction, multiplication and division, or combinations of these operations, to solve practical problems
- apply arithmetic operations according to their correct order
- convert between fractions, decimals and percentages, using a calculator when appropriate
- apply approximation strategies for calculations if appropriate
- use mental and/or flexible written strategies when appropriate
- use a calculator appropriately and efficiently for multi-step calculations

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Pre-test

1 Calculate using mental strategies.

a 21×12 **b** 13×15

2 Show if the answer given for each calculation is reasonable or not.

a $39 + 44 = 83$

b $156 - 29 = 127$

c $22 \times 37 = 514$

3 Which is larger?

a $\frac{3}{9}$ or $\frac{5}{9}$ **b** $\frac{1}{5}$ or $\frac{1}{3}$ **c** 75% or 64% **d** 4.26 or 4.73

4 Round each number as required.

a 8644.629 to the nearest ten

b 101 358.2 to the nearest thousand

c 153.4634 to 3 decimal places (3 dp)

d 86.63 to 0 dp

5 There are 7 red balls, 8 blue balls, 2 yellow balls and a black ball in your ball bag. How many balls are in your ball bag?

6 Evaluate.

a $15 + 3 \times 7$ **b** $14 - 20 \div 5$

c $(3 \times 3) + (12 \div 4)$ **d** $40 \div 4 + 2^3$

7 Convert the following percentages to fractions and then write each fraction in its simplest form.

a 50% **b** 89%

8 Convert the following fractions to percentages (1 dp).

a $\frac{5}{7}$ **b** $\frac{70}{65}$

9 Convert the following decimals to fractions and then write each fraction in its simplest form.

a 0.6 **b** 0.85

10 Convert the following decimals to percentages (1 dp).

a 0.9 **b** 0.321

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1A Comparing fractions, decimals and percentages

LEARNING GOALS

- Compare fractions to identify which is the smaller or larger
- Compare percentages to identify which is the smaller or larger
- Compare decimals to identify which is the smaller or larger

Why is it essential to be able to compare fractions, decimals and percentages?

Fractions, decimals and percentages are used to quantify numerical information and are found in many everyday situations. Being able to understand and compare the three number forms means you are able to make judgements or interpretations about the quantified information.

WHAT YOU NEED TO KNOW

- **Fractions** are written with the **numerator** above the **denominator**, separated by a quotient line:

$$\frac{\text{Numerator}}{\text{Denominator}}$$

  The denominator is how many parts one whole is broken up into, and the numerator is how many of those parts we have.
  - If the numbers on the top of two fractions are the same, then the fraction with the smaller number on the bottom is larger, or the fraction with the larger number on the bottom is smaller.
  - If the numbers on the bottom of two fractions are the same, then the fraction with the larger number on the top is larger, or the fraction with the smaller number on the top is smaller.
- **Percentages** are written with a number followed by the percentage symbol %. 28% means 28 out of 100 or $\frac{28}{100}$, 52% means 52 out of 100 or $\frac{52}{100}$.
- **Decimals** are numbers that are written after (to the right of) the decimal point and have a value that is less than one whole. The further the numbers are to the right of the decimal point, the smaller the value of the numbers. The place value of each number, as we move to the right, is 10 times smaller each time, as shown in the table below.

Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths
1000s	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s	$\frac{1}{1000}$s

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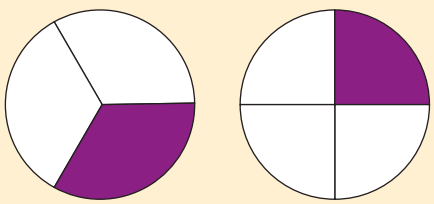
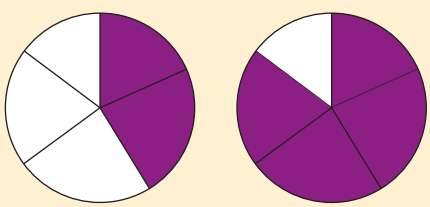
For example, the number 5256.895 can be thought of as:
5 thousands + 2 hundreds + 5 tens + 6 ones (decimal point) + 8 tenths + 9 hundredths + 5 thousandths
0.01 is ten times larger than 0.001 ($0.001 \times 10 = 0.01$) and 0.3 is 3 times larger than 0.1 ($0.1 \times 3 = 0.3$) and 0.05 is 3 hundredths more than 0.02 ($0.05 - 0.02 = 0.03$)

Example 1 Understand the meaning and magnitude of fractions

a If Billy has $\frac{1}{3}$ of the cake and Jac has $\frac{1}{4}$ of the cake, who has the most cake?

b If Billy has $\frac{2}{5}$ of the money and Jac has $\frac{4}{5}$ of the money, who has the most money?

c If Billy has $\frac{2}{3}$ of a pizza and Jac has $\frac{3}{4}$ of another pizza, who has the most pizza?

WORKING	THINKING
a  $\frac{1}{3}$ $\frac{1}{4}$	Use a diagram to decide which is bigger: $\frac{1}{3}$ or $\frac{1}{4}$. Or Use a calculator. See Calculator activity 1A *Compare fractions to identify which is the smaller or larger*
The shaded area of $\frac{1}{3}$ is bigger than the shaded area of $\frac{1}{4}$, so $\frac{1}{3}$ is bigger, and Billy has the most cake.	
b  $\frac{2}{5}$ $\frac{4}{5}$	Use a diagram to decide which is bigger: $\frac{2}{5}$ or $\frac{4}{5}$.
The shaded area of $\frac{4}{5}$ is bigger than the shaded area of $\frac{2}{5}$, so $\frac{4}{5}$ is bigger, and Jac has the most money.	

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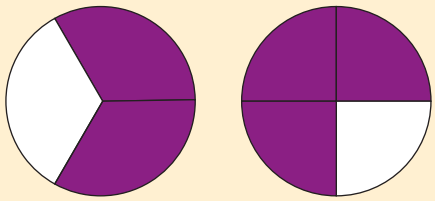
c



$\frac{2}{3}$ $\frac{3}{4}$

The shaded area of $\frac{3}{4}$ is bigger than the shaded area of $\frac{2}{3}$, so $\frac{3}{4}$ is bigger, and Jac has the most pizza.

◄····· Use a diagram to decide which is bigger: $\frac{2}{3}$ or $\frac{3}{4}$.

Calculator activity 1A Compare fractions to identify which is the smaller or larger

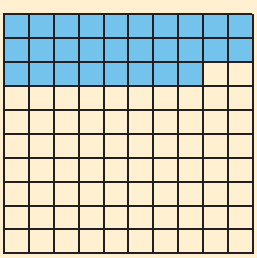
Solve the following using a calculator.
Which is bigger: $\frac{1}{3}$ or $\frac{1}{4}$?

1 ÷ 3 =	0.33333
1 ÷ 4 =	0.25

$0.33333 - 0.25 = 0.0833$, so $\frac{1}{3}$ is bigger.

Example 2 Understand the meaning and magnitude of percentages

If Billy has 28% of the cake and Jac has 52% of the cake, who has the most cake?

WORKING	THINKING
 28%	◄····· Use a diagram to decide which is bigger: 28% or $\frac{28}{100}$, Or

... *Continued*

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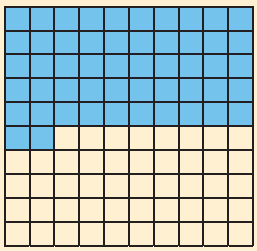
WORKING	THINKING
 52%	52% or $\frac{52}{100}$.
The shaded area of 52% is bigger than the shaded area of 28%, so 52% is bigger.	If the value of the number before the % symbol is larger, then it is a larger percentage value.

Example 3 Understand the meaning, magnitude and significance of place value after the decimal point

a Which number is larger: 47.1 or 47.2?
b Which number is larger: 47.1 or 47.01?

WORKING	THINKING
a (47.2 − 47.1 = 0.1) 47.2 is bigger.	47.1 is 4 tens + 7 ones + 1 tenth, 47.2 is 4 tens + 7 ones + 2 tenths, so 47.2 is bigger by one tenth.
b (47.1 − 47.01 = 0.09) 47.1 is bigger	47.1 is 4 tens + 7 ones + 1 tenth, 47.01 is 4 tens + 7 ones + 0 tenths + 1 hundredth, so 47.1 is bigger by nine hundredths.

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Exercise 1A

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a If the numbers on the top of two fractions are the same, then the fraction with the _____ number on the bottom is _____, or the fraction with the _____ number on the bottom is _____.

b If the numbers on the bottom of two fractions are the same, then the fraction with the _____ number on the top is _____, or the fraction with the _____ number on the top is _____.

c If the value of the number before the % symbol is smaller, then it is a _____ percentage value.

d 0.02 is _____ times larger than 0.001 and 0.5 is _____ times larger than 0.1, and 0.08 is _____ hundredths more than 0.03.

Examples 1, 2 & 3

2 Which is larger?

a $\frac{2}{11}$ or $\frac{4}{11}$

b $\frac{1}{5}$ or $\frac{1}{2}$

c $\frac{5}{12}$ or $\frac{7}{9}$

d 76% or 23%

e 5.17 or 5.54

f 256.023 or 256.23

g 56.752% or 56.759%

3 Which is smaller?

a $\frac{2}{9}$ or $\frac{4}{9}$

b $\frac{1}{6}$ or $\frac{1}{8}$

c $\frac{3}{7}$ or $\frac{7}{9}$

d 43% or 82%

e 3.67 or 3.34

f 583.034 or 583.340

g 43.934% or 43.582%

ISBN 978-1-009-25785-5

1B Rounding numbers

LEARNING GOAL

- Follow mathematical conventions when rounding numbers

Why is it essential to be able to round numbers?

When we round numbers, we are simplifying them so it is easier and faster to do mental calculations. In science we round numbers to a certain number of decimal places to reflect the **accuracy** of the data recording or measuring processes. To ensure consistency in the rounding of numbers from all mathematicians, there is a specific convention that must be followed, which we will learn about in this section.

WHAT YOU NEED TO KNOW

- To round to a particular place value, we need to remember the following rule:

  If number to the right is 4 or less, the digit stays the same. If it is 5 or more, we add one to the digit.

- If the digit we rounded to is a decimal (to the right of the decimal point), we then remove all the digits after it. If the digit was a whole number (to the left of the decimal point), then we replace all whole numbers after it with zeros and drop the decimal part.

Example 4 Rounding numbers to a given place value

a Round 1638.835 to the nearest 100.
b Round 1638.835 to 2 decimal places (2 dp), or round 1638.835 to the nearest hundredth.

WORKING	THINKING
a 1638.835 rounded to the nearest 100 is 1600.	To the nearest 100 means we round the digit in the hundreds position. We need to decide if the 6 in the hundreds place stays the same or changes to a 7. To do this we look at the number to the right in the tens place. In this case the number in the tens place is a 3. If we look at the rule, 3 is *less than* 4 so the 6 stays the same and we replace the whole numbers after the 6 with zeros (0's).

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b 1638.835 rounded to 2 decimal places or rounded to the nearest hundredth is 1638.84.

◄···· To 2 decimal places means we round the digit in the hundredths position. We need to decide if the 3 in the hundredths place, or second decimal place, stays the same or changes to a 4. To do this we look at the number to the right in the thousandths place. In this case the number in the thousandths place is a 5. If we look at the rule, 5 is *5 or more* so we add 1 to the 3 and it becomes a 4. We then remove all the digits after the hundredths position.

Exercise 1B

FUNDAMENTALS

Example 4

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a When we round numbers, we are _____ them so it is easier and faster to do _____ calculations.

b In science we round numbers to a certain number of _____ to reflect the _____ of the data recording or measuring process.

c To round numbers, we need to remember the following rule:
4 or less the number stays the _____, *5 or more we* _____ *to the number.*

2 Round each number as required.

a 1863.629 to the nearest ten

b 94763.2 to the nearest thousand

c 254.983 to the nearest hundred

d 349.17 to the nearest whole number

3 Round each number as required.

a 1863.629 to 1 decimal place (1 dp)

b 947.637 to 2 dp

c 254.9834 to 3 dp

d 52.53 to the nearest whole number

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1C Choosing operations

LEARNING GOAL

- Choose the correct mathematical operation to find the correct value when using calculations to obtain a solution to a mathematical problem

Why is it essential to be able to choose the correct mathematical operations?

In 'real life', solving problems with mathematics rarely involves a set of numbers with symbols like '+' and '×' mixed in with them. Instead they are often presented as sentences and paragraphs, and we have to interpret the problem to choose the correct operation to use.

WHAT YOU NEED TO KNOW

- When you are given a practical situation, problem or question that you need to use mathematics for to find out the solution, you need to decide what mathematics you are going to use to calculate the solution.
- The mathematical processes that you can use are called Mathematical Operations. These include, but are not limited to, the operators; Addition (+), Subtraction (−), Multiplication (×) and Division (÷). You need to know when to use which.
- **Addition** is the most basic operation of arithmetic. In its simplest form, addition combines two quantities into a single quantity, or *sum*.
- **Subtraction** is the opposite of addition. Instead of adding quantities together, we are removing one quantity from another to find the *difference* between the two.
- **Multiplication** also combines multiple quantities into a single quantity, called the *product*. In fact, multiplication can be thought of as a repetitive addition. Specifically, the product of x and y is the result of x added together y times.
- **Division** is the opposite of multiplication. Instead of multiplying quantities together to result in a larger value, you are splitting a quantity into a smaller value, called the *quotient*.

Example 5 Addition

If there are 25 desks and 27 chairs in a classroom, how many pieces of furniture are in the room?

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WORKING	THINKING
$25 + 27 = 52$	To get the total, you combine how many of each type of furniture together by using addition.
There are 52 pieces of furniture in the room.	You now have one value: 'Number of pieces of furniture in the room.'

Example 6 Subtraction

If there are 25 desks and 27 chairs in a classroom, how many extra chairs are in the room?

WORKING	THINKING
$27 - 25 = 2$	To find the number of extra chairs, you need to reduce the number of chairs by the number of desks, using subtraction.
There are 2 extra chairs in the room.	You now have one value: 'Number of extra chairs in the room.'

Example 7 Multiplication

If there are 5 classrooms in the building and 25 chairs in each classroom, how many chairs are in the building?

WORKING	THINKING
$25 + 25 + 25 + 25 + 25 = 125$	We could add 25 together 5 times.
or	
$25 \times 5 = 125$	Or we could multiply 25 by 5.
There are 125 chairs in the building.	Note that both methods give you the same result of 125 but in many cases, particularly when you have large quantities or many groups, multiplying can be much faster.

Example 8 Division

If there are 34 classrooms and 748 chairs in the school spread equally across the classrooms, how many chairs are in each classroom?

... *Continued*

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WORKING	THINKING
$748 \div 34 = 22$ There are 22 chairs in each classroom.	◄···· We split 748 chairs into 34 groups by using division.

Exercise 1C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a Addition is the _____operation of arithmetic. In its simplest form, addition _____ two quantities into a single quantity, or *sum*.

b Subtraction is the _____ of addition. Instead of adding quantities together, we are removing one quantity from another to find the _____ between the two.

c Multiplication also combines multiple quantities into a single quantity, called the _____. In fact, multiplication can be thought of as _____. Specifically, the product of x and y is the result of x added together y times.

d Division is the opposite of _____. Instead of multiplying quantities together to result in a larger value, you are _____ a quantity into a smaller value, called the *quotient*.

Example 5 **2** There are 16 coloured pencils, 7 pens, 3 highlighters and a ruler in your pencil case. How many objects are in your pencil case?

Example 8 **3** **a** There are 768 imported cars to be delivered. Each truck can hold six vehicles. How many trips will it take a truck to deliver the imported cars?

b These cars are to be equally shared between four car dealerships. How many cars are expected to be delivered to each car dealership?

Example 7 **4** **a** Three rows of VIP seats are to be set out with 24 seats in each row. How many VIP reserved signs need to be made?

b Nathan's mum will make seventeen flower arrangements required on the night. Each arrangement consists of 10 roses. How many roses must she order?

5 The Economics class held a Hot Dog Fundraising Day. Hot dogs were $1.50 each. What amount was raised if 167 students ordered hot dogs, and $97.00 was spent on buying the buns, sausages and sauce.

Example 6 **6** Jason, his parents and younger brother, Thomas, went to the movies last Tuesday night. Calculate the change from $100 if adult tickets cost $29.50 and children's tickets cost $16.00.

ISBN 978-1-009-25785-5

1D Order of operations

LEARNING GOAL

- Follow mathematical conventions when using multiple mathematical operations in one calculation

Why is it essential to have an order of operations?

Consider the calculation '$2 + 7 \times 3$'. Is the correct answer 23 or 27? To ensure consistency in the outcome of calculations when using multiple mathematical operations, there is a convention that must be followed. We call this convention the order of operations.

WHAT YOU NEED TO KNOW

- In maths, **order of operations** are the rules that state the order in which the multiple operations in an expression should be calculated. A way to remember the order of the operations is BIMDAS, where each letter stands for a mathematical operation.

B	Brackets
I	Indices
M	Multiplication
D	Division
A	Addition
S	Subtraction

- The **BIMDAS rules** that state the order in which the operations in an expression should be calculated are:
  - **Brackets**: These are calculated before all other operators. The first step is to calculate all the operations within the brackets. Work out all groupings from inside to out. (Whatever is in brackets is a grouping.)
  - **Indices**: Calculate the value of all indices.
  - **Multiplication** and **Division**: Next, moving from left to right, multiply and/or divide, whichever comes first.
  - **Addition** and **Subtraction**: Lastly, moving from left to right, add and/or subtract, whichever comes first.

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Example 9 Order of operations

a Evaluate $2 + 7 \times 3$. **b** Evaluate $4 - 8 \div 2 \times 3$. **c** Evaluate $12 \div (2 + 4) + 3^2$.

WORKING	THINKING
a $2 + 7 \times 3$ $2 + 21$ $= 23$	BIMDAS tells you to do the multiplication first. Then do the addition.
b $4 - 8 \div 2 \times 3$ $4 - 4 \times 3$ $4 - 12$ $= -8$	BIMDAS tells you to do the division first. Then do the multiplication. Then do the subtraction.
c $12 \div (2 + 4) + 3^2$ $12 \div 6 + 3^2$ $12 \div 6 + 9$ $2 + 9$ $= 11$	BIMDAS tells you to do the addition in the bracket first. Then evaluate the indice. Then do the division. Then do the addition.

Exercise 1D

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

In maths, the order of operations is the set of rules that specify the order in which the _____in an expression should be calculated.
A way to remember the order of the operations is_____.

In relation to the order of operations, the BIMDAS rules state that:

- **i** The first step is to calculate all the operations within the _____.
  Work out all groupings from inside to out.
- **ii** Calculate the value of all _____.
- **iii** Next, moving from _____, multiply and/or _____, whichever comes first.
- **iv** Lastly, moving from left to right, _____ and/or subtract, whichever comes _____.

Example 9 **2** Evaluate.

a $14 + 2 \times 5$ **b** $28 - 14 \div 2$ **c** $32 \div 2 \times 4$ **d** $24 - 4 + 8$

3 Calculate.

a $25 \div (2 + 3)$ **b** $12 + (8 - 2)$ **c** $(4 \times 2) + (6 \div 3)$ **d** $10 \div 2 + 2^3$

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1E Is my answer reasonable?

LEARNING GOAL

- Make valid judgements of the 'reasonableness' of the outcome of a calculation

Why is it essential to ensure your answer is reasonable?

When carrying out calculations, either mentally or with technology, we need to make sure the outcome we have calculated is 'reasonable' or makes sense.

WHAT YOU NEED TO KNOW

You can use estimation or approximation to check if the answer you have obtained is reasonable or sensible. One way of approximating is '**Leading Digit Approximation**'. In leading digit approximation we round the first digit of a number to make calculations simpler.

Example 10 Using Leading Digit Approximation to obtain estimates of calculations

a Estimate 47 + 64.
b Estimate 219 × 386.

WORKING	THINKING
a 47 *becomes* 50	The leading digit is the first number. 47 starts with the number 4, and so we round 47 to 50 because the '7' is greater than 5.
64 *becomes* 60	64 starts with the number 6, and so we round 64 to 60.
47 + 64 *becomes* 50 + 60 50 + 60 = 110 47 + 64 is approximately 110, or we estimate 47 + 64 to be about 110.	Using leading digit approximation, the calculation becomes easier to do using mental mathematics.

... *Continued*

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WORKING		THINKING
b 219 *becomes* 200	◄····	The leading digit is the first number. 219 starts with the number 2, so we round 219 to 200.
386 *becomes* 400 219×386 *becomes* 200×400	◄····	386 starts with the number 3, and we round 386 to 400 because the '8' is greater than 5.
$2 \times 4 = 8$ $200 \times 400 = 80\,000$ 219×386 is approximately 80 000, or we estimate 219×386 to be about 80 000.	◄····	To calculate the estimation, use mental mathematics. Multiply the leading digits. Then put the total number of zeros after the 8 to get 80 000.

Example 11 Checking results of calculations for accuracy

a There are 42 cricket bats and 57 baseball bats in the storeroom. Mrs Batty said there are 99 bats. Could Mrs Batty be correct?

b Billy said $219 \times 386 = 84\,543$. Show if Billy could be right or wrong.

WORKING		THINKING
a		You can use Leading Digit Approximation to find out.
$40 + 60 = 100$	◄····	Round 42 to 40 and 57 to 60.
$40 + 50 = 90$	◄····	Then round 42 to 40 and 57 to 50.
This shows that $42 + 57$ is more than 90 and less than 100, so Mrs Batty could be correct.		
b		You can use Leading Digit Approximation to find out.
$200 \times 400 = 80\,000$	◄····	Round 219 to 200 and 386 to 400.
$300 \times 300 = 90\,000$	◄····	Then round 219 to 300 and 386 to 300.
This shows that 219×386 is more than 80 000 and less than 90 000, so Billy could be right.		

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Exercise 1E

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.
You can use _____ or _____ to check if the answer you have obtained is _____ or sensible. One way of approximating is '_____Approximation'.

Example 10

2 Estimate.

a $57 + 62$ **b** $112 - 34$ **c** $236 + 872$ **d** $1176 + 2953$

3 Estimate.

a 24×49 **b** 39×128 **c** 745×302 **d** 1154×289

Example 11

4 Show if the answer given for each calculation is reasonable or not.

a $42 + 53 = 95$
b $265 - 23 = 242$
c $402 + 289 = 799$
d $1203 + 420 = 1925$

5 Show if the answer given for each calculation is reasonable or not.

a $23 \times 38 = 574$
b $47 \times 311 = 14\,617$
c $154 \times 889 = 15\,669$
d $542 \times 378 = 204\,876$

APPLICATIONS

6 John, the store manager, counts 57 long bolts and 121 short bolts. He writes down on the stock monitoring sheet that there are 198 bolts in the storeroom. Show if John wrote down the correct number of bolts.

7 Lucy owns a dress shop. She orders 234 boxes each containing 24 dresses. She then orders 5616 coat hangers to hang the dresses on the dress racks in her store. Did she order enough coat hangers?

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1F Converting between number forms

LEARNING GOAL

- Convert between fractions, decimals and percentages

Why is it essential to be able to convert between fractions, decimals and percentages?

Converting between fractions, decimals and percentages is perhaps the most common type of calculation you may need to do in your life! What's more, in order to complete calculations to carry out the Mathematical Thinking and Statistical Investigation Processes involving fractions, decimals and percentages, you need to be able to convert between them.

WHAT YOU NEED TO KNOW

- The word **per cent** means **per 100**. The symbol % also means **per 100**.
  For example: 75 *per cent* = 75% = 75 parts out of 100 parts.
- **To convert a percentage to a fraction**, add a denominator of 100.
  For example: 45 *per cent* $= \frac{45}{100}$ $\quad 45\% = \frac{45}{100}$
  This can also be used in reverse.
  For example: $\frac{1}{3} = 1 \div 3 \times 100 = 33.3$ *per cent* $\quad \frac{1}{3} = 1 \div 3 \times 100 = 33.3\%$
- **A fraction is converted to a decimal** when the numerator (top number) is divided by the denominator (bottom number).
  For example: $\frac{1}{4} = 1 \div 4 = 0.25$
  This can also be used in reverse.
  For example: $0.25 = \frac{25}{100} = \frac{1}{4}$ or $0.2 = \frac{2}{10} = \frac{1}{5}$
- **A percentage is converted to a decimal** when the word per cent or the symbol % is changed to divide by 100.
  For example: 45 *per cent* $= 45 \div 100 = 0.45$ $\quad 45\% = 45 \div 100 = 0.45$
  This can also be used in reverse.
  For example: $0.333 \times 100 = 33.3$ *per cent* $\quad 0.333 \times 100 = 33.3\%$

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- Common percentages and their equivalent fractions and decimals are shown in the table below. It is useful to memorise these.

%	1	5	10	$12\frac{1}{2}$	20	25	$33\frac{1}{3}$	50	$66\frac{2}{3}$	75
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.01	0.05	0.1	0.125	0.2	0.25	0.333	0.5	0.667	0.75

Example 12 Converting percentages to fractions

Convert the following percentages to fractions in their simplest form.

a 30% **b** 76%

WORKING	THINKING
a $30\% = \frac{30}{100}$	The % symbol is changed to a denominator of 100.
$\frac{30}{100} = \frac{3}{10}$	Divide the numerator and denominator by the highest common factor (HCF) of 10.
b $76\% = \frac{76}{100}$	The % symbol is changed to a denominator of 100.
$\frac{76}{100} = \frac{19}{25}$	Divide the numerator and denominator by the HCF of 4.

Example 13 Converting fractions to percentages

Convert the following fractions to percentages.

a $\frac{2}{5}$ **b** $\frac{35}{45}$

WORKING	THINKING
a $\frac{2}{5} = 2 \div 5 \times 100$ $= 40\%$	Divide 2 by 5 and multiply by 100 to get the percentage.
b $\frac{35}{45} = 35 \div 45 \times 100$ $= 77.8\%$	Divide 35 by 45 and multiply by 100 to get the percentage (1 dp).

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Example 14 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{2}{5}$ **b** $\frac{35}{45}$

WORKING	THINKING
a $\frac{2}{5} = 2 \div 5$ $= 0.4$	Divide 2 by 5 to get the decimal (1 dp).
b $\frac{35}{45} = 35 \div 45$ $= 0.78$	Divide 35 by 45 to get the decimal (2 dp).

Example 15 Converting decimals to fractions

Convert the following decimals to fractions.

a 0.3 **b** 0.335

WORKING	THINKING
a $0.3 = \frac{3}{10}$	0.3 is, using place value, 3 tenths.
b $0.335 = \frac{335}{1000}$	0.335 is, using place value, 335 thousandths.
$= \frac{67}{200}$	Simplify by dividing by 5 (HCF).

Example 16 Converting percentages to decimals

Convert the following percentages to decimals.

a 68% **b** 42.7%

WORKING	THINKING
a $68 \div 100 = 0.68$	Divide the percentage by 100.
b $42.7 \div 100 = 0.427$	Divide the percentage by 100.

Example 17 Converting decimals to percentages

Convert the following decimals to percentages.

a 0.39 **b** 0.594

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WORKING	THINKING
a $0.39 \times 100 = 39\%$	◂···· Multiply the decimal by 100.
b $0.594 \times 100 = 59.4\%$	◂···· Multiply the decimal by 100.

Exercise 1F

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words or numbers in each of the following.

a The word per cent means per____. The symbol ____ also means per 100.

b A fraction is converted to a decimal when the_____ (top number) is _____the denominator (bottom number).

c A percentage is converted to a_____ when the word per cent or the symbol % is changed to_____.

d

%	1	5	10	$12\frac{1}{2}$	20	25	$33\frac{1}{3}$	50	iii	75
Fraction	$\frac{1}{100}$	i	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.01	0.05	0.1	ii	0.2	0.25	0.333	0.5	0.667	0.75

Example 12 **2** Convert the following percentages to fractions and then use division to write each fraction in its simplest form.

a 60% **b** 39% **c** 154% **d** 23.9%

Example 13 **3** Convert the following fractions to percentages (1 dp).

a $\frac{3}{7}$ **b** $\frac{65}{85}$ **c** $\frac{102}{200}$ **d** $\frac{157}{75}$

Example 14 **4** Convert the following fractions to decimals (2 dp).

a $\frac{4}{7}$ **b** $\frac{65}{95}$ **c** $\frac{102}{150}$ **d** $\frac{125}{75}$

Example 15 **5** Convert the following decimals to fractions.

a 0.2 **b** 0.72 **c** 1.25 **d** 0.755

Example 16 **6** Convert the following percentages to decimals (2 dp).

a 55% **b** 62.8% **c** 145% **d** 5.6%

Example 17 **7** Convert the following decimals to percentages (1 dp).

a 0.8 **b** 0.41 **c** 1.025 **d** 0.451

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1G Calculator use or mental mathematics?

LEARNING GOALS

- Develop mental calculation strategies using partitioning
- Choose when it is appropriate to use mental calculations or a calculator

Why is it essential to be able to choose between calculator use or mental mathematics?

You will need to carry out calculations that use Mathematical Thinking and Statistical Investigation Processes. These calculations can be done mentally or using a calculator. Depending on the calculation, it may be faster to do the calculations mentally than to use a calculator. You will need to decide which method is appropriate for each calculation.

WHAT YOU NEED TO KNOW

- **Mental Mathematics Strategies** involve breaking one difficult calculation down into several easy calculations.
  For example, 12×16 is not easy to do in one step mentally, but if we break it down into calculations we know, then it becomes easy.
  We can break 12×16 into:
  $10 \times 16 = 160$ and $2 \times 16 = 32$, and we then add $160 + 32 = 192$.
  If adding 160 and 32 is difficult, we can break this down into $160 + 30 = 190$ and $190 + 2 = 192$.
  Check the answer with a calculator.
- We can also break down more complex calculations involving decimals.
  For example, 0.7×15 is not easy to do in one step mentally, but if we break it down into calculations we know, then it becomes easy.
  We can break 0.7×15 into:
  $0.5 \times 15 = 7.5$ and $0.1 \times 15 = 1.5$, and we then add $7.5 + 1.5 + 1.5 = 10.5$ (because $0.7 = 0.5 + 0.1 + 0.1$).
  Check the answer with a calculator.

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Example 18 Mental calculations

Mentally calculate 13×21.

WORKING	THINKING
$13 = 1 \text{ ten} + 3 \text{ ones}$	Partition 13 into 10 and 3 as multiplying by ten is easy.
$10 \times 21 = 210$ $21 = 2 \text{ tens and } 1 \text{ one}$ $3 \times 20 = 60 \quad 3 \times 1 = 3$	Multiply 10 by 21 (need to still do 3 times 21). Partition 21 into 20 and 1. Multiply 3 by 20 and multiply 3 by 1.
$210 + 60 + 3 = 273$ $13 \times 21 = 273$	Sum the results of the three multiplications.

Calculator activity 1G Example 1 Multiplying on a calculator

Solve the following using a calculator.

a 13×21

[1] [3] [×] [2] [1] [=] 273

Example 19 Mental calculations

Mentally calculate 0.75×28.

WORKING	THINKING
$0.75 = 5 \text{ tenths} + (2 \times 1 \text{ tenths}) + 5 \text{ hundredths}$	Partition 0.75 into 0.5, 2×0.1 and 0.05.
$0.5 \times 28 = 14$ $0.1 \times 28 = 2.8$ $0.05 \times 28 = 1.4$	Multiply 0.5×28. 0.1×28 0.05×28 (0.05×28 is half of 0.1×28)
$14 + (2 \times 2.8) + 1.4$ $14 + 5.6 + 1.4 = 21$ $0.75 \times 28 = 21$	Sum the results of the three multiplications.
or	
	$0.75 = \frac{3}{4}$, so the result is three lots of four parts.
$28 \div 4 = 7$	Divide 28 by 4 (one part is 7).
$3 \times 7 = 21$ so $0.75 \times 28 = 21$	Multiply one part (one part is 7) by 3.

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Calculator activity 1G Example 2 Multiplying on a calculator

Solve the following using a calculator.

a 0.75×28

[0] [.] [7] [5] [×] [2] [8] [=] 21

Exercise 1G

FUNDAMENTALS

Example 18

1 Break down the following calculations as much as you need to get the final results. Check your answers using a calculator.

a 17×12
b 23×11
c 45×15
d 12×12
e 21×37
f 19×18
g 96×13
h 65×32

Example 19

2 Break down the following calculations as much as you need to get the final results. Check your answers using a calculator.

a 0.7×12
b 0.75×11
c 0.4×15
d 0.95×12
e 0.05×24
f 0.13×50
g 0.8×43
h 0.32×16

3 Break down the following calculations as much as you need to get the final results. Check your answers using a calculator.

a 10.1×13
b 12.3×11
c 17.7×15
d 45.95×12
e 12.4×16
f 23.75×36

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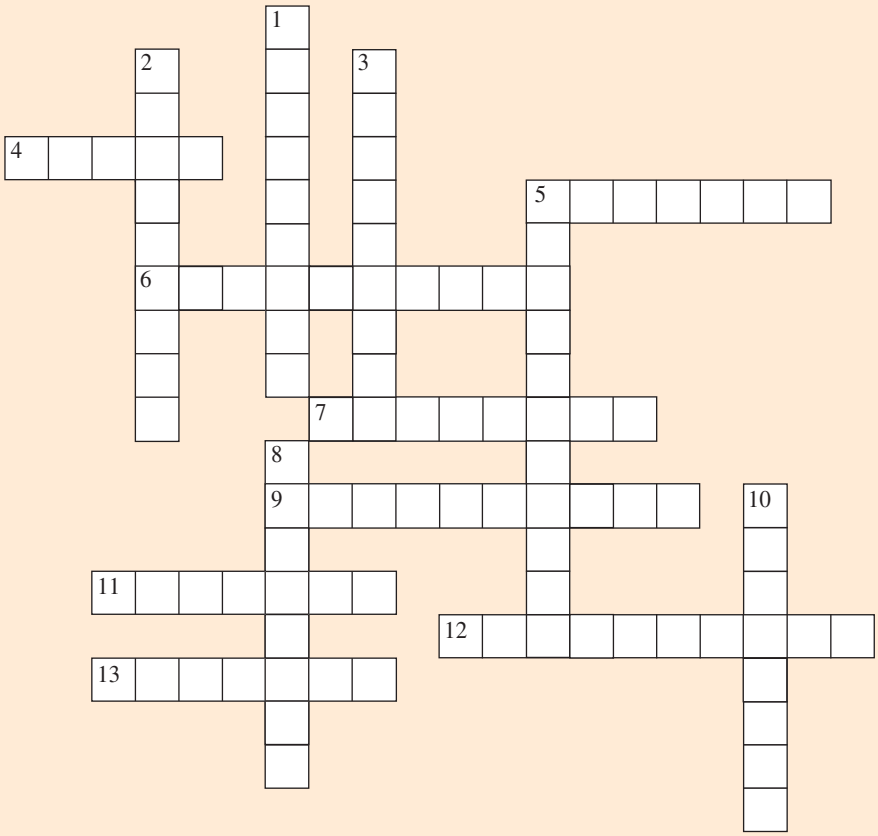
Puzzle



Across

4 Expressed in convenient units rather than exactly, for example, to the nearest whole number or multiple of ten

5 Relating to or denoting a system of numbers and arithmetic based on the number ten, tenth parts and powers of ten

6 A rough calculation of the value, number, quantity or extent of something

7 An expression that indicates the quotient of two quantities, such as 1/3

9 Having sound judgement; fair and sensible

11 An exponent or other superscript number appended to a quantity

12 A fraction, decimal or amount out of one hundred

13 To be able to change from one number form to another

Down

1 A process in which a number or quantity is altered according to set formal rules, such as those of addition or multiplication

2 Each of one hundred equal parts into which something is or may be divided

3 The number above the line in a fraction showing how many of the parts indicated

5 The number below the line in a fraction

8 Each of a pair of marks used to enclose figures so as to separate them

10 A plan of action designed to achieve a long-term or overall aim

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Chapter checklist

☐ I can compare fractions, decimals and percentages.

Which is bigger?

1 **a** $\frac{3}{13}$ or $\frac{5}{13}$ **b** $\frac{7}{12}$ or $\frac{8}{10}$

2 26% or 29%

3 756.023 or 756.23

☐ I can correctly follow mathematical conventions when rounding numbers.

Round each number as required.

4 580.001 to the nearest hundred

5 725.3697 to 3 decimal places (3 dp)

☐ I can choose the correct mathematical operation to obtain a solution to a mathematical problem.

6 There are 12 coloured pencils, 9 pens, 4 highlighters and a ruler in your pencil case. How many objects are in your pencil case?

7 5 bunches of flowers are to be made, each with 7 flowers. How many flowers are needed?

☐ I can correctly follow mathematical conventions when using multiple mathematical operations in one calculation.

8 Evaluate.

a $18 + 3 \times 5$ **b** $12 - 14 \div 7$

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☐ I can make valid judgements of the 'reasonableness' of the outcome of a calculation.

9 Gregory, a barista, counts 24 medium cups and 184 small cups. He tells the manager there are 208 cups in the coffee shop. Show if Gregory told the manager a reasonable number of cups.

10 Stacey owns a florist shop. She orders 154 bunches of roses, each containing 25 flowers. She then orders 3000 singles rose boxes. Did she order enough single rose boxes?

☐ I can convert between fractions, decimals and percentages.

11 Convert $\frac{1}{5}$ to a decimal and a percentage.

12 Convert 47% to a decimal and a fraction.

13 Convert 0.25 to a fraction and a percentage.

☐ I can use mental calculation strategies.

14 Evaluate.

a 34×27 **b** 0.3×21

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Chapter review

Section 1G

1 Calculate using mental strategies.

a 29×12

b 23×14

c 0.8×30

d 0.95×20

2 Estimate using mental strategies.

a $46 + 94$

b $122 - 42$

c 757×341

d 1052×487

Section 1E

3 Show if the answer given for each calculation is reasonable or not.

a $61 + 34 = 95$

b $206 - 22 = 184$

c $174 \times 279 = 48\,546$

d $542 \times 348 = 138\,131$

4 Lisa needs more than 200 plastic flowers to finish making an order of hair clips. She orders 12 boxes of 25 plastic flowers. Show if Lisa ordered enough flowers.

5 Greta counts 157 Red Belly Robins and 206 Yellow Belly Robins. She reports there are 363 Robins in total. Show if Greta's answer is reasonable.

Section 1A

6 Which is larger?

a $\frac{2}{13}$ or $\frac{4}{13}$

b $\frac{1}{6}$ or $\frac{1}{12}$

c $\frac{5}{12}$ or $\frac{7}{9}$

d 69% or 58%

e 3.39 or 3.41

f 554.093 or 554.93

g 56.452% or 56.454%

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Section 1B **7** Round each number as required.

a 1594.429 to the nearest ten

b 62648.5 to the nearest thousand

c 725.4764 to 3 decimal places (3 dp)

d 48.44 to 0 dp

Section 1C **8** A farmer is expecting 7 trucks of hay to arrive on their farm. Each truck contains 48 bales of hay. The hay shed can fit 330 bales of hay. Will all the hay fit in the hay shed?

Section 1D **9** Evaluate.

a $12 + 4 \times 4$

b $28 - 16 \div 2$

c $(5 \times 2) + (12 \div 3)$

d $40 \div 2 + 2^3$

Section 1F **10** Convert the following percentages to fractions and then write each fraction in its simplest form.

a 20%

b 155%

11 Convert the following fractions to percentages (1 dp).

a $\frac{2}{6}$

b $\frac{38}{25}$

12 Convert the following fractions to decimals (2 dp).

a $\frac{5}{13}$

b $\frac{65}{45}$

13 Convert the following decimals to fractions and then write each fraction in its simplest form.

a 0.7

b 2.25

14 Convert the following percentages to decimals (2 dp).

a 28%

b 34.9%

15 Convert the following decimals to percentages (1 dp).

a 0.7

b 1.099

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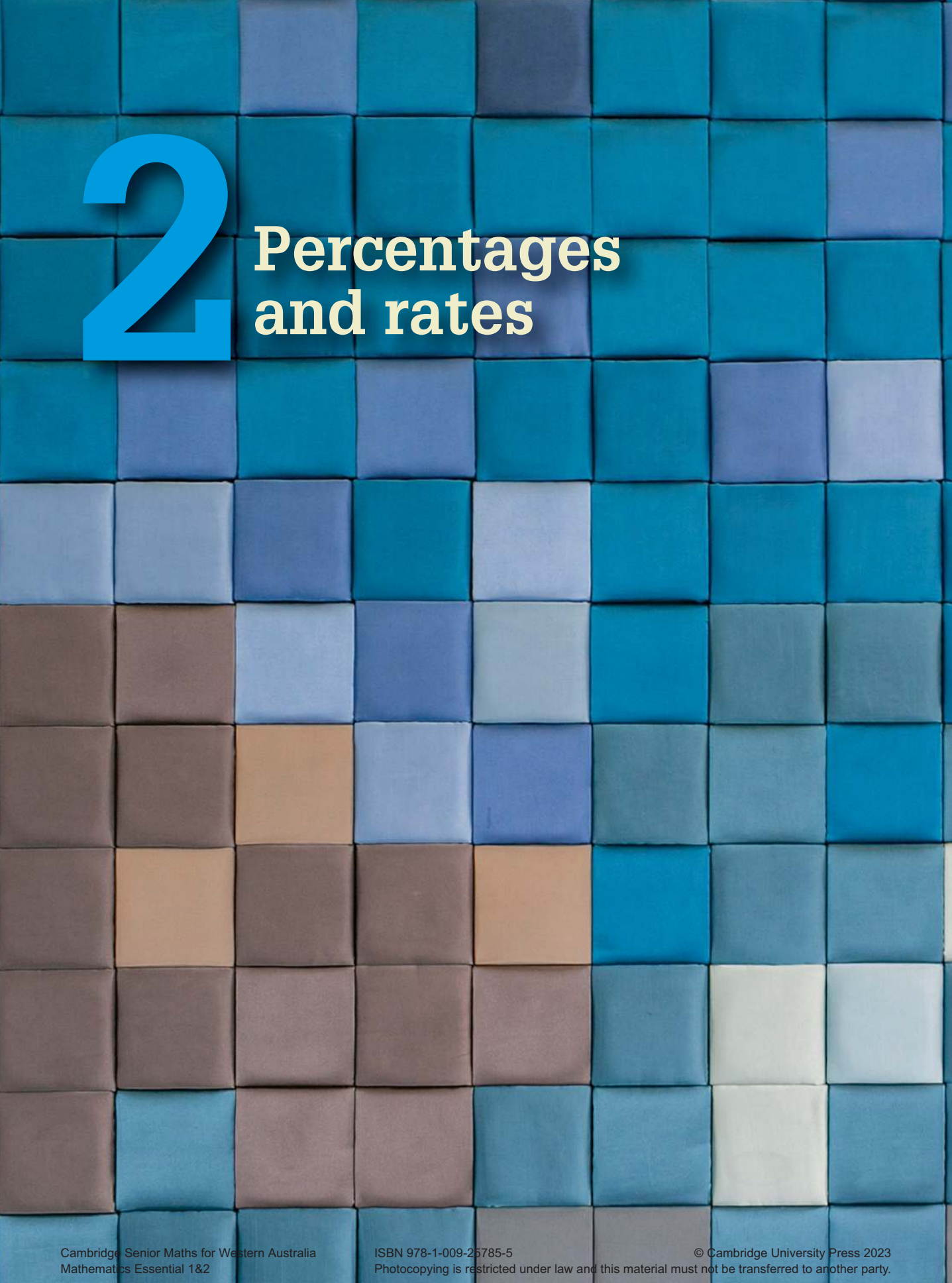

2 Percentages and rates

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In this chapter

Syllabus reference

Unit 1 Topic 1.1: Basic calculations, percentages and rates

Percentages

- calculate a percentage of a given amount, using mental/written strategies or technology when appropriate
- determine one amount expressed as a percentage of another
- apply percentage increases and decreases in situations

Rates (no inverse proportion)

- identify common usage of rates
- convert units of rates occurring in practical situations to solve problems
- use rates to make comparisons

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Pre-test

1 Change the following to the simplest fraction.

- **a** one half
- **b** two fifths
- **c** five hundredths
- **d** nine tenths
- **e** one hundred and twenty-five thousandths

2 Change the following decimals to the simplest fraction.

- **a** 0.3
- **b** 0.75
- **c** 0.99
- **d** 1.25
- **e** 0.76

3 What do the following units measure?

- **a** metres (m)
- **b** dollars ($)
- **c** hours (Hr)
- **d** kilograms (kg)
- **e** litres (L)
- **f** teaspoons (tsp)
- **g** feet (ft)
- **h** pounds (lb)

4 Give two units of measurement that can be used to calculate the following.

- **a** speed
- **b** fuel usage
- **c** cost of fruit
- **d** energy usage
- **e** heart rate

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2A Using percentages

LEARNING GOALS

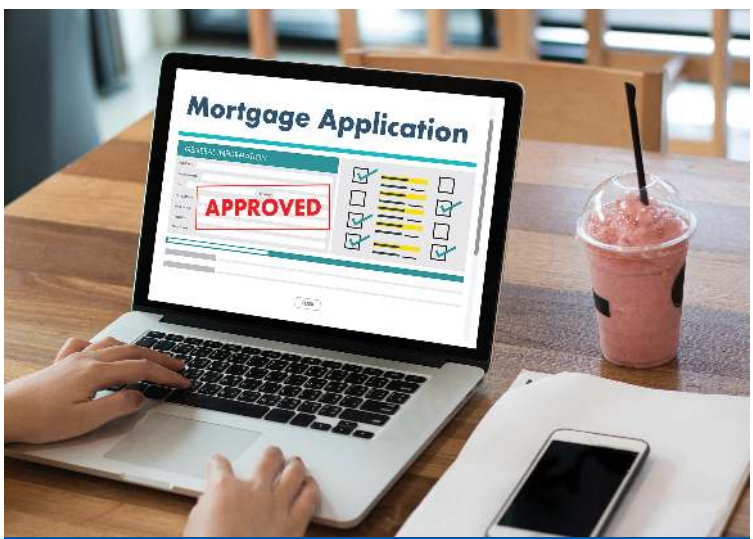
- Convert percentages to fractions
- Convert simple percentages to equivalent divisions and multiplications with whole numbers
- Calculate the percentage of a quantity with and without a calculator
- Apply percentage calculations to real-world situations

Why are percentage calculations essential?

We can more usefully compare parts of a whole that are written as percentages than those that are written as fractions with different denominators. This is because all percentages are fractions with a denominator of 100; for example, $\frac{25}{100} = 25\%$.

A test result of $\frac{37}{46}$ is more easily compared to a result of $\frac{19}{24}$ when they are both expressed as a percentage, which is 80% in both cases.

Our whole economy is built on the exchange of money. Many money calculations use rates that are universally written as percentages. Examples include retail mark-up rates and discount rates, inflation rates, wage increase rates, tax rates and bank interest rates for investments and loans, including mortgage rates for house loans.



A 4% per annum (p.a.) mortgage rate on a $500 000 home loan initially costs around $20 000 p.a. in interest plus the loan repayments.

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WHAT YOU NEED TO KNOW

- The word **per cent** means **per 100**. The symbol % also means **per 100**.
  - For example: 75 per cent = 75% = 75 parts out of 100 parts
- A percentage is converted to a fraction when the word per cent or the symbol % is changed to a denominator of 100.
  - For example: 45 per cent $= \frac{45}{100}$ $\qquad$ $45\% = \frac{45}{100}$
- Common percentages and their equivalent fractions are shown in the table. It is useful to memorise these.

Percentage	1%	5%	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%	$66\frac{2}{3}\%$	75%
Equivalent fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

- Many percentages, when written as fractions, simplify to whole number divisions and multiplications.
  - For example: $5\% = \frac{5}{100} = \frac{1}{20} = 1 \div 20$. Finding 5% is equivalent to $\div 20$.
  - For example: $90\% = \frac{90}{100} = \frac{9}{10} = \div 10 \times 9$. Finding 90% is equivalent to $\div 10 \times 9$.
- A percentage of a quantity can be calculated without a calculator by first simplifying the percentage fraction and then calculating that fraction of the given quantity.
  - For example: 75% of 80 marks $= \frac{75}{100} \times 80 = \frac{3}{\cancel{4}_1} \times \cancel{80}^{20} = 60$ marks.
- Many percentages of a quantity can be calculated mentally using equivalent division and multiplication of whole numbers. It is easier to complete the division before the multiplication.
  - For example: 75% of 120 kg $= 120 \div 4 \times 3 = 90$ kg.
  - For example: 90% of \$210 $= 210 \div 10 \times 9 = \$189$.
- A calculator can be used to calculate a percentage of a quantity.
  - For example: 87.5% of 238 cm. Enter: 87.5 ÷ 100 × 238. Answer is 208.25 cm.

Example 1 Converting percentages to fractions

Convert the following percentages to fractions and then use division to write each fraction in its simplest form.

a 30% $\qquad$ **b** 76%

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WORKING	THINKING
a $30\% = \frac{\cancel{30}^{3}}{\cancel{100}_{10}}$ $= \frac{3}{10}$	The % symbol is changed to a denominator of 100. Divide the numerator and denominator by the highest common factor (HCF) of 10.
b $76\% = \frac{\cancel{76}^{19}}{\cancel{100}_{25}}$ $= \frac{19}{25}$	The % symbol is changed to a denominator of 100. Divide the numerator and denominator by the HCF of 4.

Example 2 Converting simple percentages to equivalent divisions and multiplications, using whole numbers

In each of the following, write a sentence to describe how simple division, and multiplication if necessary, can be used to calculate the given percentage of a quantity.

a 10% **b** 25% **c** 80% **d** 150%

WORKING	THINKING
a To find 10% of a quantity, we divide the quantity by 10.	$10\% = \frac{10}{100} = \frac{1}{10}$ To calculate 10% of a quantity, we multiply by 1 and divide by 10, which is equivalent to simply dividing by 10.
b To find 25% of a quantity, we divide the quantity by 4.	$25\% = \frac{25}{100} = \frac{1}{4}$ To calculate 25% of a quantity, divide by 4.
c To find 80% of a quantity, we divide the quantity by 5 and then multiply by 4.	$80\% = \frac{80}{100} = \frac{4}{5}$ To calculate 80% of a quantity, we multiply by 4 and then divide by 5 (i.e. 4 lots of 20%). Tip: It is easier to divide first then multiply.
d To find 150% of a quantity, divide by 2 then multiply by 3.	$150\% = \frac{150}{100} = \frac{3}{2}$ So to calculate 150% of a quantity, we multiply by 3 and divide by 2.

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Example 3 Calculating a percentage of a quantity, without using a calculator

In each the following, use simple division, and multiplication if necessary, to calculate the percentages of the given amounts, without using your calculator.

a 10% of $140

b 25% of $600

c 125% of 500 kg

WORKING	THINKING
a $\$140 \div 10 = \$14$	10% simplifies to $\frac{1}{10}$, so we divide $140 by 10.
b $\$600 \div 4 = \$150$	25% simplifies to $\frac{1}{4}$, so we divide $600 by 4.
c $500\,\text{kg} \div 4 \times 5 = 625\,\text{kg}$	$125\% = 5 \times 25\%$, so we divide 500 kg by 4 and then multiply by 5.

Example 4 Calculating a percentage of a quantity, using a calculator

Calculate the given percentage of each quantity, using your calculator. Round each answer to two decimal places.

a 35% of $1050

b 72% of 86.9 kg

WORKING	THINKING
a $\frac{35}{100} \times 1050 = \$367.50$	Enter this into your calculator: $35 \div 100 \times 1050$
b $\frac{72}{100} \times 86.9 = 62.57\,\text{kg}$	Enter this into your calculator: $72 \div 100 \times 86.9$ As the third decimal place of 62.568 is 8, which is ≥ 5, round the second decimal place up to 7.

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Example 5 Applying percentage calculations in real-world situations

Naomi's secret to her cake recipe is that 5% of the baked cake is sugar. If the cake weighs 300 grams when cooked, calculate the weight of sugar that Naomi uses in her cake.

WORKING	THINKING
$5\% \text{ of } 300\text{ g} = \frac{5}{\cancel{100}_{1}} \times \cancel{300}^{3}$ $= 15\text{ g}$	Identify that the sugar is 5% of 300 grams. Calculate 5% of 300 grams.
Naomi uses 15 grams of sugar.	Answer the question in words.

Exercise 2A

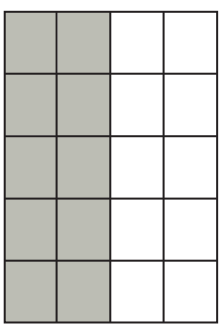
FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

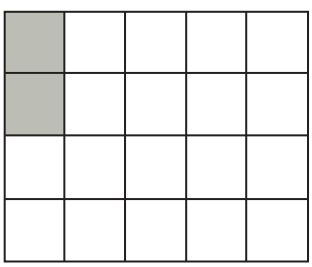
- **a** The word per cent means per _____, and the symbol % also means per ____.
- **b** When the word _____ or the symbol ___ is changed to a denominator of ____, the percentage is converted to a _____.
- **c** A percentage of a quantity can be calculated mentally by first _____ the percentage fraction and then calculating that _____ of the given quantity.
- **d** Many percentages of a quantity can also be calculated mentally using equivalent _____ and _____ of whole numbers.

2 In each of the following, determine the percentage of the total area that is shaded.

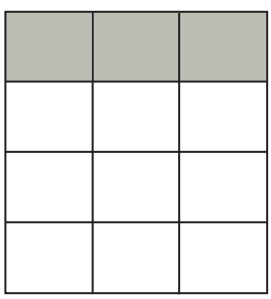
a



b



c



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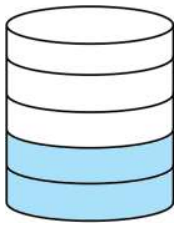
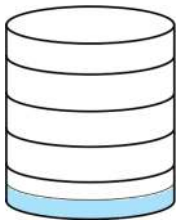
3 In each of the following, determine the percentage of the cylinder that is shaded blue.

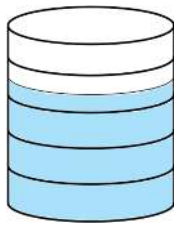
a



b



c



4 A child's drawing of a tiger is shown. Estimate the percentage of the total area of the picture that is shaded green.

5 Sketch a circle that has 75% shaded blue.

Example 1

6 Convert the following percentages to fractions and then divide by the highest common factor (HCF), to write each fraction in its simplest form.

a 10%
b 20%
c 25%
d 40%
e 50%
f 75%
g 100%
h 15%
i 60%
j 24%
k 95%
l 88%

Hint
Divide the numerator and denominator by the HCF.

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Example 2

7 In each of the following, write the percentage as a simplified fraction and use this to complete the sentence to describe how simple division, and multiplication if necessary, can be used to calculate the given percentage of a quantity.

Hint It is useful to memorise these equivalent percentage calculations.

a To find 1% of a quantity, we divide the quantity by _____.

b To find 10% of a quantity, we divide the quantity by _____.

c To find 50% of a quantity, we divide the quantity by _____.

d To find 25% of a quantity, we divide the quantity by _____.

e To find 100% of a quantity, we divide the quantity by _____.

f To find $33\frac{1}{3}$% of a quantity, we divide the quantity by _____.

g To find 20% of a quantity, we divide the quantity by _____.

h To find 80% of a quantity, we divide the quantity by _____ and multiply by _____.

i To find $66\frac{2}{3}$% of a quantity, we divide the quantity by _____ and multiply by _____.

j To find 75% of a quantity, we divide the quantity by _____ and multiply by _____.

k To find 150% of a quantity, we divide the quantity by _____ and multiply by _____.

Example 3

8 Without using your calculator, use simple division and multiplication with whole numbers for the following percentage calculations.

Hint Use ÷ and × of whole numbers to calculate each %.

a 10% of $200

b 20% of $1000

c 25% of $80

d 50% of $842

e 1% of $7000

f 75% of $800

g 150% of $4000

h 120% of $550

Example 4

9 Calculate the following percentages of the given quantities, using your calculator. Round your answers correct to one decimal place.

a 5% of 280 kg

b 11% of $432

c 12% of 620 km

d 37% of 128 litres

e 80% of 640 mL

f 150% of $12 500

g 300% of 140 kg

h 146% of 232 tonnes

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APPLICATIONS

10 A survey finds that 78% of high school students own a mobile phone. A school has 1180 students.

a Calculate 78% of 1180.

b Use your result from part **a** to state approximately how many students at the school would be expected to have mobile phones.

11 Tanya earns $84 000 annually and receives a 7% pay rise.

a Calculate the amount of extra money Tanya receives.

b Determine the amount that Tanya earns annually after receiving this pay increase.

12 Eddie bought a new TV marked at $780 and was given a 15% discount.

a Calculate the discount he received on the TV.

b Determine how much he paid for the TV.

13 Refer to the nutritional label shown.

a Identify the number of grams in one serve (i.e. the serving size).

b Identify the percentage of one serve that is fat (total fat).

c Calculate the number of grams in a single serve that is fat.

d Identify the percentage of one serve that is carbohydrate (total carbohydrate).

NUTRITION INFORMATION

Serves per pack: 6		**Serving Size 150 g**
Avg Quantity per serve		Avg Quantity per 100 g
Energy	146.4 kJ	97.7 kJ
Proteins	10 g	6.7 g
Fat, Total	1 g	0.67 g
– Saturated Fat	0 g	0 g
Carbohydrate	20 g	13.3 g
– Dietary Fibre	14 g	9.4 g
– Sugars	6 g	4 g
Sodium	6 mg	4 mg

e Calculate the number of grams in a single serve that is carbohydrate.

f Identify the percentage of one serve that is dietary fibre.

g Calculate the number of grams in a single serve that is dietary fibre.

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Example 5

14 Ryan scores 60% on his Maths test. If the test was out of 50 marks, calculate how many marks Ryan obtained.

15 Jake drives 25% of the 1184 km distance from Longreach to Port Douglas. Calculate the distance that Jake drives.

16 In Western Australia, there are 28 000 students studying Year 11 and 12, where 30% of these students study Essential Maths. Calculate the number of students in this group who study Essential Maths.

17 April's taxable income for the past financial year was $136 000 and she must pay 2% of this income as a Medicare levy. Calculate how much Medicare levy April must pay.

18 Henry deposited $8000 into his bank account and after one year received 12% in interest.

a Calculate how much interest Henry received after one year.

b What is Henry's account balance when the interest is added to his deposit?

19 Three health companies – GoodFeeling, IMGood and HappyHealth – employ 6000, 7360 and 9430 people, respectively. The percentages of women employed in the three companies are 62%, 57% and 43%, respectively. Determine which company has the highest number of female employees.

Hint 'Respectively' means in the same order.

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2B Determining an amount as a percentage of another

LEARNING GOALS

- Express one quantity as a percentage of another quantity for two amounts with the same units and with different units
- Calculate one quantity as a percentage of another quantity in real-world contexts

Why is it essential to calculate one amount as a percentage of another?

As noted in the previous section, percentages make things easier to compare than fractions with different denominators. First though, we need to express one quantity as a percentage of another.

For example, a person calculates percentages to compare the various proportions of their total income spent on rent, food, transport and other living expenses.

A chef calculates percentages to record and compare each ingredient's share of the total cost and profit per recipe.

A business calculates percentages to compare the proportion of its total cost spent on expenses such as rent, wages, buying new stock and paying tax. A business also calculates the percentage each part of the business contributes to its overall profit.

WHAT YOU NEED TO KNOW

- To express one quantity (A) as a percentage of another quantity (B):
  1. Write each quantity in the same units, if necessary.
  2. Write quantity A as a fraction of quantity B: $\frac{A}{B}$.
  3. Multiply the fraction by 100 to calculate the percentage.
- When we calculate a percentage, the answer includes the symbol %.
  - For example: Write 45 marks as a percentage of 75 marks.

$$\frac{45}{75} \times 100 = 60\%$$

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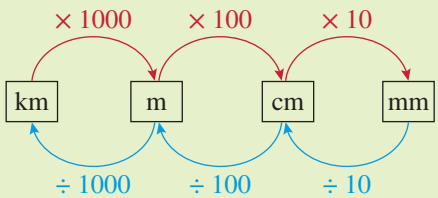
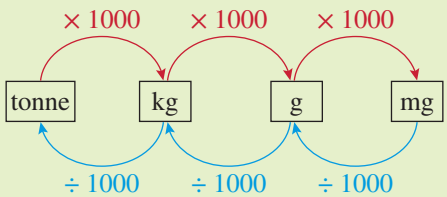
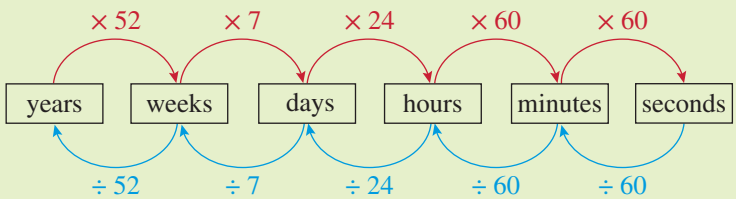
- Larger units × factor to find smaller units. Smaller units ÷ factor to find larger units.
- Conversion of **length** units



- Conversion of **mass** units



- Conversion of **time** units



- Other unit conversions:
  - 1000 mL = 1 litre
  - 100 cents = $1

Example 6 Expressing one quantity as a percentage of another quantity when both quantities have the same units

Use an appropriate calculation to express each of the following as percentages, giving answers to one decimal place.

a 30 km out of 240 km **b** 11 litres out of 84 litres

WORKING	THINKING
a $\frac{30}{240} \times 100 = 12.5\%$	Write 30 out of 240 as a fraction. Then multiply by 100 to calculate the percentage.

... *Continued*

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b $\frac{11}{84} \times 100 = 13.0952\%$
$= 13.1\%$

Write 11 out of 84 as a fraction. Then multiply by 100 to calculate the percentage.
As the second decimal place is a 9, round up the first decimal place to 1.

Example 7 Expressing one quantity as a percentage of another quantity when each quantity has different units

For each of the following, first express the quantities in the same units and then calculate the first amount as a percentage of the second amount. Round your answers to one decimal place where appropriate.

a 750 m out of 5 km

b 18 hours out of 1 week

WORKING	THINKING
a $5\text{ km} = 5 \times 1000\text{ m}$ $= 5000\text{ m}$	Convert the larger unit (km) to the smaller unit (m). [km ($\times 1000$) → m]
$\frac{750}{5000} \times 100 = 15\%$	Write 750 out of 5000 as a fraction, then multiply by 100.
b $1\text{ week} = 1 \times 7 \times 24$ $= 168\text{ h}$	Convert the larger unit (weeks) to the smaller unit (hours). [week ($\times 7$) → days ($\times 24$) → hours]
$\frac{18}{168} \times 100 = 10.7142\%$ $= 10.7\%$	Write 18 out of 168 as a fraction, then multiply by 100. As the second decimal place is 1, which is < 5, round the answer down to the first decimal place of 7.

Example 8 Calculating one quantity as a percentage of another quantity in a real-world context

a Beth solved 52 problems of the 60 problems presented in her university exam. Calculate the percentage of problems that Beth solved. Round your answer to one decimal place.

b Levi has a student savings account with $20. Levi receives 98 cents in interest. Calculate the percentage interest rate for this account. Round your answer to one decimal place.

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WORKING	THINKING
a $\frac{52}{60} \times 100 = 86.66\%$ $= 86.7\%$	Write 52 out of 60 as a fraction. Then multiply by 100 to write this fraction as a percentage. As the second decimal is ≥ 5, round up the first decimal place to 7.
b $\$20 \times 100 = 2000$ cents	Convert the larger unit ($) to the smaller unit (cents).
$\frac{98}{2000} \times 100 = 4.9\%$	Write 98 out of 2000 as a fraction. Then multiply by 100 to write this fraction as a percentage.

Exercise 2B

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

a To express one quantity (A) as a percentage of another quantity (B), we:

i Write each quantity in the _____ units, if necessary.

ii Write quantity A as a _____ of quantity B: $\frac{\square}{\square}$.

iii Multiply the fraction by _____: $\frac{\square}{\square} \times$ _____ ____.

b When we calculate a percentage, the answer includes the symbol _____.

2 Without a calculator, multiply by 100 to express each of the following fractions as percentages.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{10}$ **d** $\frac{2}{5}$

e $\frac{7}{20}$ **f** $\frac{27}{1000}$ **g** $\frac{35}{200}$

> **Hint** Divide by the HCF to simplify.
> e.g. $\frac{3}{50} \times 100$
> $= \frac{3}{\cancel{50}_{1}} \times \cancel{100}^{2}$
> $= 6\%$

Example 6

3 Use an appropriate calculation to express each of the following as percentages, giving answers to one decimal place where appropriate.

> **Hint** Multiply the fraction by 100 to calculate the percentage.

a 5 kg out of 40 kg **b** 6 litres out of 60 litres

c $24 out of $80 **d** 40 MB out of 320 MB

e 14 grams out of 91 grams **f** 51 litres out of 310 litres

g $325 out of $125 **h** 112.5 m out of 90 m

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Example 7

4 For each of the following:

i Use multiplication or division to express the quantities with the same units.

Hint Convert each larger unit to the smaller unit

ii Use an appropriate calculation to express the first amount as a percentage of the second amount. (Give answers to one decimal place where appropriate.)

a 500 g out of 4 kg
b 750 mL out of 3 L
c 1200 m out of 8 km
d 30 minutes out of 2 hours
e 42 cents out of $10
f 5 days out of 3 weeks
g 90 cm out of 50 m
h 880 mm out of 3.2 m
i 480 seconds out of $\frac{1}{2}$ an hour
j 20 hours out of 1 week

APPLICATIONS

5 Becky drives 695 km out of the 1930 km from Geraldton to Broome. Becky's friends, Olly and Kate, share the remaining driving equally.

a Calculate the percentage of the trip driven by Becky.

b Use your result from part **a** to determine what percentage of the trip is driven by Olly and by Kate.

6 Brian receives a $240 discount on a lawn mower because it is the demonstration model. It was originally $699.

a Write the discount as a fraction of the original cost of the mower.

b Use the fraction from part **a** to determine the percentage discount that Brian receives.

Example 8a

7 Anna scored 60 marks out of 80 marks on her Science exam. Calculate the percentage that Anna got correct.

8 A 'pound' cake weighs approximately 450 grams. The cake contains 250 grams of flour. Calculate the percentage of the pound cake that is flour.

Hint Start with a fraction, then times by 100.

9 Of the 120 students in Year 11, 55 students study Design. Calculate the percentage of Year 11 students studying Design.

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10 A holiday costs $1100 for accommodation, $580 for airfares and $720 for food. Calculate each one of the following costs as a percentage of the total holiday costs.

a accommodation

b airfares

c food

Example 8b

11 Carlene ran 800 m of a 2-km race.

a Use multiplication to convert 2 kilometres to metres.

b Calculate the percentage of the race that Carlene ran.

12 In the mixing of cement, 2 tonnes of cement includes 600 kg of sand. Determine the percentage of the cement mixture that is sand.

Hint First convert to the same units.

13 In a 400-m relay swimming event, Otto swam 400 m in 48 seconds. If the total relay team took 3.5 minutes, determine the percentage of the total relay time that Otto swam.

14 Trish is charged a credit card fee of 90 cents for a $36 online purchase of movie tickets. Calculate the percentage of the purchase price that is the credit card fee.

15 Noah uses 225 mL of green cordial to make up 3 L of cordial drink. Calculate the percentage of the drink that is the green cordial.

16 Dana uses her dishwasher for 1.5 hours every two days. Calculate the percentage of time in a fortnight that Dana uses her dishwasher.

17 In a 2-kg packet of mixed nuts, 400 grams are pistachios, 420 grams are almonds, 540 grams are cashews and the rest are walnuts. Determine the percentage of walnuts in the packet.

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2C Applying percentage increases and decreases, including mark-ups and discounts

LEARNING GOALS

- Apply percentage increases to calculate price mark-up amounts
- Apply percentage discounts to calculate price reductions
- Calculate the Goods and Services Tax (GST)
- Calculate selling prices that include the GST
- Apply percentage increases and decreases in various real-world situations

Why is it essential to understand percentage increases and decreases?

We live in a society that seems to always have something on sale. Understanding how stores mark-up products to make **profits** and also use discounts to entice people to buy helps us to determine which items are the best value for money.

Store managers, retail assistants and customers all use percentages to calculate mark-up and discounted prices.

WHAT YOU NEED TO KNOW

- The **cost price** is the amount of money that a business pays for a product.
- When a business sells a product, they make money (i.e. a profit) by having marked up (i.e. increased) the cost price.
- The **mark-up** is usually given as a percentage increase of the cost price.
- To calculate the **marked price** from a given cost price, the mark-up amount, in $ (found from the percentage increase), is added to the cost price of the item.

$$\text{Mark-up amount in \$} = \% \times \text{ cost price}$$
$$\text{Marked price} = \text{cost price} + \text{mark-up amount}$$

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- The marked price is also called the selling price and the retail price.
  - For example: A laptop with cost price \$600 is marked up by 37.5%.

$$\text{Mark-up} = \frac{37.5}{100} \times 600 = \$225$$

$$\text{Marked price} = \$600 + \$225 = \$825$$

- A **discount** is usually given as a percentage decrease of the marked price.
- To calculate the **discounted price** or sale price of an item, the amount of discount is subtracted from the marked price.

Discount amount in \$ = % × marked price
Discounted price = marked price − discount

  - For example: A handsaw with a marked price of \$50 is discounted by 20%.

$$\text{The discount is } \frac{20}{100} \times 50 = \$10.$$

$$\text{The discounted price is } \$50 - \$10 = \$40.$$

- The Goods and Services Tax **(GST)** is a 10% government tax added to the selling price of goods and services.
  - For example: A gas bill of \$275.87 has GST $= \$275.87 \times \frac{10}{100}$
$= \$27.587 = \$27.59$
The final amount to pay is \$275.87 + \$27.59 = \$303.46.
- Round all money answers to two decimal places.

Example 9 Applying a percentage increase to calculate the mark-up amount

An electric bike costs a business \$3700 and is marked up by 8.25%. Calculate:
a the mark-up amount
b the marked price of the electric bike

WORKING	THINKING
a $\text{Mark-up} = \frac{8.25}{100} \times \$3700$ $= \$305.25$	$8.25\% = \frac{8.25}{100}$ Find this fraction of the cost price.
b $\text{Marked price} = \$3700 + \$305.25$ $= \$4005.25$	Marked price = cost price + mark-up Add the amount of increase to the original cost. Round to two decimal places for all money answers.

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Example 10 Applying a percentage decrease to calculate the discounted price

A set of junior golf clubs has a marked price of $387 and is discounted by 13%. Apply this percentage decrease to calculate:

a the discount, in dollars

b the discounted price of the product

WORKING	THINKING
a $\text{Discount} = \frac{13}{100} \times \$387$ $= \$50.31$	Calculate 13% of $387.
b $\text{Discounted price} = \$387 - \$50.31$ $= \$336.69$	Discounted price = marked price – discount amount Round to two decimal places for all money answers.

Example 11 Calculating the GST percentage increase

Calculate the GST applicable on a frying pan that has a selling price of $29.85.

WORKING	THINKING
$\text{GST} = \frac{10}{100} \times \$29.85$ $= \$2.99$	10% of $29.85 is $29.85 ÷ 10 = $2.985 = $2.99. Round to two decimal places for all money answers.

Example 12 Applying a percentage increase or decrease in a real-world context

Mandy purchases a Rubik's cube on eBay for $12, including postage. She then marks it up by 80% to sell at her garage sale. Determine the selling price of Mandy's Rubik's cube at the garage sale.

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WORKING	THINKING
$\text{Mark-up} = \frac{80}{100} \times \$12$ $= \$9.60$	Find 80% of $12.
$\text{Marked price} = \$12 + \$9.60$ $= \$21.60$	Add the new amount to the original value.
Mandy will sell the Rubik's cube for $21.60.	Write the answer as a sentence.

Exercise 2C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** The _____ price is the amount that a business pays for a product.
- **b** A business makes a _____ by _____ up the _____ _____.
- **c** The _____-__ is usually given as a _____ increase of the cost price.
- **d** The _____ price is also called the selling price and the _____ price.
- **e** A discount is usually given as a percentage _____ of the _____ price.
- **f** To calculate the _____ price or sale price of an item, the amount of discount is _____ from the marked price.
- **g** The Goods and Services Tax, also known as the _____, is a __ % government tax added to the selling price of goods and services.

Example 9

2 For each of the following, apply the given percentage increase to calculate:

i the mark-up amount **ii** the final marked price

- **a** Mark-up a cost price of $1200 by 10%.
- **b** Mark-up a cost price of $400 by 15%.
- **c** Mark-up a cost price of $160 by 12.5%.
- **d** Mark-up a cost price of $35 by 20%.
- **e** Mark-up a cost price of $2500 by 28%.
- **f** Mark-up a cost price of $58 by 150%.
- **g** Mark-up a cost price of $350 000 by 3.69%.
- **h** Mark-up a cost price of $5250 by 9.75%.
- **i** Mark-up a cost price of $199 by 82.6%.

Hint The mark-up amount is a % of the cost price.

Hint Marked price = cost price + mark-up

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Example 10

3 For each of the following, apply the given percentage decrease to calculate:

i the discount amount **ii** the final sale price

a Discount a marked price of $125 by 20%.

Hint: Sale price = marked price – discount

b Discount a marked price of $695 by 10%.

c Discount a marked price of $58 by 12%.

d Discount a marked price of $500 by 15%.

e Discount a marked price of $240 by 25%.

f Discount a marked price of $78 by 4.5%.

g Discount a marked price of $299 by 17.25%.

h Discount a marked price of $28 800 by 8.9%.

Example 11

4 Australia's GST is currently 10% of the selling price of goods or services rendered. Without using a calculator, calculate the GST applicable to the following amounts.

a $50 **b** $285

c $3000 **d** $56

e $174 **f** $69.90

g $16 999 **h** $426 500

Hint: Finding 10% is the same as ÷ 10.

APPLICATIONS

Example 12

5 At DenimStop, Lillie wishes to buy a pair of jeans marked at $69.95. If the store offers her a 15% discount, calculate:

a the discount amount that Lillie will receive

b the final selling price of the jeans

Hint: Sale price = marked price – discount

6 Keith wants to surprise his partner with a new gold bracelet, which has a marked price of $295. The store has a sale offering 30% off the marked price.

a Calculate the discount that Keith will receive.

b What is the bracelet's final selling price?

7 James runs a mowing business. He wants to keep $45 for each average mowing job, yet he must also charge the customer GST and send this tax to the Australian Taxation Office.

a Calculate the amount of GST payable on $45.

b Determine how much James should charge for each average mowing job if the price he quotes includes the GST.

Prices given to customers must, by law, include the GST in the price.

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8 **a** In New Zealand, the GST is 15%. If a bungee jump in New Zealand costs $99 plus GST, calculate the total cost for one bungee jump.
b If a bungee jump in Australia costs $105 plus GST, compare the overall cost of a jump in Australia with a jump in New Zealand.

9 Telio imports 50-inch televisions from Taiwan at a cost of $295 each and then sells them in his stores. Telio marks up these televisions by 150%. Determine the marked price of each television.

10 Rick bought two tickets for a total of $240 to a sell-out World Cup final. Unfortunately, he can't go and decides to sell the tickets and make a profit of 25%. Determine the selling price of the tickets.

11 A cabinet maker makes bookshelves, each costing her $80 for materials. The cabinet maker sells the bookshelves to a furniture retail business, adding a 75% mark-up on her material costs. The retailer sells each bookshelf to customers with a 120% mark-up on the cost price. Calculate the retailer's marked price of each bookshelf.

Hint The price is being increased each time.

12 Erik wishes to buy a new red baseball cap online. He can buy one from either the Square Shop, where it is marked at $48 and a 20% discount is being offered, or the Triangle Shop, where it is marked at $42 and a 12% discount is being offered. Both online stores are offering free delivery.
a At which online store is the cap cheaper?
b Determine how much Erik will save buying it from the cheaper online store compared with the other store.

13 Letisha is planning a holiday on a cruise ship. At Go-quick Travel, she sees a 7-night cruise for $890 and they are offering a 10% discount if she books today. At Time-off Travel, Letisha sees the same cruise advertised at $829 with a 5% discount if she books today.
a Determine which travel agent has the best deal.
b Calculate how much Letisha will save by booking with Time-off Travel if she books today.

14 The local supermarket purchases a box of 60 specialty chip packets for $48. The store manager marks up the packets by 120%. In the last week before the use-by date, the store manager reduces the price by 50%, as any that have not sold by the use-by date must be thrown away.
Determine what profit, if any, the supermarket made if it sold 18 of the packets at full price, 28 at the discounted price and had to throw away the rest.

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2D Understanding and using rates

LEARNING GOALS

- Identify a rate
- Determine suitable units for common rates
- Write rates and simplify them
- Calculate average rates of change
- Apply rates in real-life situations

Why are rates essential?

Rate measurements and calculations provide important information for many choices and plans made in our day-to-day lives. For example, using your pay rate $(\$/\text{hour})$ to predict income over longer periods; comparing a vehicle's rate of movement (km/h) to the **speed** limit; applying the rate at which a car uses fuel (litres/100 km) to calculate range and fuel cost; considering the rate of data download charge ($/GB) when choosing a data plan; and reading the rate of energy per serving on product labels (kJ/100 g) when choosing a brand and/or portion size.

Comparing fuel efficiency rates (i.e. litres per 100 km) is an important consideration when choosing a car.

WHAT YOU NEED TO KNOW

- **Rates** show the relationship between different types of quantities with **different units**.
  - For example: a car's speed is a rate usually written in kilometres per hour (km/h).
- Rates are always written with two different units, unlike simplified ratios which are written without units.
  - For example: rate $\$50/10\,\text{m} = \$5/\text{m}$
  - For example: ratio 60 cents : \$1 = 60 : 100 = 3 : 5
- The word **per** is used instead of the words 'divided by' or instead of the dividing symbol '/' (also known as a solidus).

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- When simplifying rates, we find the amount of the first quantity per 1 unit of the second quantity.
  - For example: 32 km walked over 8 hours would be simplified to how many kilometres walked in 1 hour. Dividing both parts by 8 gives 4 km/h.
- It is conventional to write rates in their simplest form.
- The **average rate** is the change in the first quantity divided by the change in the second quantity.
  - For example: Jason's report increased from 500 words to 2000 words from 8 p.m. to 8:40 p.m. Average typing rate = change in words/change in time.

$$\div 40 \left(\begin{array}{l} = 1500 \text{ words/40 min} \\ = 37.5 \text{ words/min} \end{array}\right) \div 40$$

Example 13 Identifying a rate

Determine whether or not each of the following is a rate.

a increasing 2 kg to5 kg

b $12 per 2 kg

c 250 mL

WORKING		THINKING
a This is a ratio but not a rate.	◄····	Look for the word **per** to identify a rate.
b This is a rate.	◄····	This gives a number of dollars **per** number of kilograms.
c This is not a rate.	◄····	This is a measurement, not a rate.

Example 14 Determining suitable units for common rates

Recall the typical units for the following rates.

a the speed of a bicycle

b the cost of fish

WORKING		THINKING
a km/h	◄·····	A cyclist usually travels some kilometres every hour, so typical units are km per h. Use the symbol / for per.
b $/kg	◄·····	Fish are generally weighed in kg and sold for dollars, so typical units are $ per kg. Use the symbol / for per.

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Example 15 Writing a rate and simplifying it

Use division to write each of the following as simplified rates.

a $34.95 is the cost of a 5 kg piece of ham.

b Monica took 32 minutes to drive 8 laps in her go-cart.

WORKING	THINKING
a $\div 5$ ($\$34.95/5\text{ kg}$ / $\$6.99/\text{kg}$) $\div 5$	Convert the rate to $ per kg. Divide both sides by 5. Round to two decimal places for money.
b $\div 8$ ($32\text{ min}/8\text{ laps}$ / 4 min/lap) $\div 8$	Convert the rate to minutes per lap. Divide both sides by 8.

Example 16 Calculating an average rate of change

Calculate the average rate of change in the following situations.

a Joseph walked 150 km in 5 days.

b When Blake was aged 9 he wore a size 5 shoe, and at age 17 he wore a size 11 shoe.

WORKING	THINKING
a $\div 5$ ($150\text{ km}/5\text{ days}$ / $= 30\text{ km/day}$) $\div 5$	The distance travelled was 150 km over 5 days. Divide distance by the 5 days to get the average km per day.
b size $11 - $ size $5 = 6$ sizes $17 - 9 = 8$ years $\div 8$ ($6\text{ sizes}/8\text{ years}$ / $= 0.75\text{ sizes/year}$) $\div 8$	The increase in shoe size is $11 - 5 = 6$ sizes. Divide the size change by 8 years to calculate average size change per year.

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Example 17 Applying rates in a real-world situation

Kirra cycles 42 km in 72 minutes. Skye finishes a 30 km cycling race in 58 minutes. Calculate the rate of change of distance (i.e. speed) in km/minute, correct to one decimal place, for both cyclists. Who has the fastest rate?

WORKING	THINKING
Kirra $\div 72$ (42 km/72 min = 0.6 km/min) $\div 72$	Write the rate from the information given. Calculate the rate per minute by dividing both sides by 72 and rounding to one decimal place.
Skye $\div 58$ (30 km/58 min = 0.5 km/min) $\div 58$	Write the rate from the information given. Calculate the rate per minute by dividing both parts by 58 and rounding to one decimal place.
Kirra has the fastest rate.	Identify that Kirra's cycling rate is a larger number and write your answer as a short sentence.

Exercise 2D

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** Rates show the relationship between _____ types of quantities with _____ units.
- **b** Rates are always written with two ____ units. Simplified ratios are written _____ units.
- **c** A simplified rate gives the amount of the first quantity per _____ unit of the second quantity.
- **d** The average rate is the _____ in the first quantity divided by the _____ in the second quantity.

Example 13

2 Determine whether or not each of the following is a rate.

Hint A rate compares amounts with different units.

- **a** 80km/hour
- **b** 5 litres/minute
- **c** increase from 120 km to 500 km
- **d** \$ 28/kg
- **e** 25 cm^2
- **f** 1 point/minute
- **g** 30 degrees to 90 degrees
- **h** 60 000 cans/hour

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Example 14

3 Recall the typical units for the following rates.

a The speed of a 100 m sprinter
b The cost of buying grapes
c The cost of petrol
d A person's typing speed
e The calorie content in a breakfast cereal
f The speed of a car
g The cost of buying land
h A cricket team's run rate

Example 15

4 Use division to write each of the following as simplified rates. Round your answer to one decimal place if needed.

a 300 km travelled in 5 hours
b $7 for 4 kg of apples
c $180 000 for 600 m^2 of land
d 12 tries in six games
e 660 mL of rainfall in 30 days
f 5000 revolutions of a jet engine rotor in 30 seconds
g 48 customers served at a drive-thru in 1.5 hours
h 20 000 bottles filled in 7 hours at a factory
i 250 km travelled in 4.5 hours
j 80 jumps in 60 seconds

Hint A simplified rate is written per 1 unit of the second quantity.

Example 16

5 Calculate the average rate of change in the following situations. Round your answer to one decimal place if needed.

a John drives 1200 km in 3 days.
b Stormy the kitten gains 4 kg over 6 months.
c Tim runs 20 km in 2 hours.
d Brenda eats 12 lollies in 5 minutes.
e Sam the snail travels 1.2 metres in 3 hours.
f In her netball match, Melanie scores 20 goals in 15 minutes.
g Jonah notices the temperature was 22°C at 6 a.m. and then 34°C at 2 p.m.
h Luther was 100 cm tall in 2015 and is 149 cm tall in 2022.

Hint An average rate is the change in the first quantity divided by the change in the second quantity.

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APPLICATIONS

Example 17

6 Kane cycles 50 km in 108 minutes. Archie finishes a 42-km cycling race in 90 minutes.

a Calculate Kane's rate of change of distance (i.e. speed) in km/minute, correct to two decimal places.

b Calculate Archie's rate of change of distance (i.e. speed) in km/minute, correct to two decimal places.

c Determine who has the fastest rate of movement.

7 Round your answers to the following to two decimal places where necessary.

a A car uses 26 litres of fuel to travel 208 km.

i Use division to express this as a simplified rate in km/litre.

ii Use division to express this as a simplified rate in litres/km.

b i Calculate 208 ÷ 100.

ii Use division to express the information as a simplified rate in litres/100 km.

8 Amanda purchases 4 kg of chips and 16 litres of soft drink for a party. She has invited 32 guests to her party, where the chips and drink will be shared equally.

a i Calculate the number of grams in 4 kg.

ii Calculate how many grams of chips each guest receives. Express this as a rate with a unit of grams/person.

b i Calculate the number of millilitres in 16 litres.

ii Calculate how many millilitres of soft drink each person receives. Express this as a rate with appropriate units.

9 If sausages cost $8 for 20 sausages, calculate the cost per sausage.

10 Jarvi was 51 cm when he was born. He grows at a rate of 8 cm/year until he is 16 years old. Determine Jarvi's height when he turns 16.

11 The Fremantle Dockers had a club membership of 8000 in 2020 and have a club membership of 17 000 in 2022. Calculate the average rate of membership growth of the club for this time period.

12 Marissa finds a leaking tap and places a 6-litre bucket underneath it to catch the leaking water. The bucket fills in 4 hours. Calculate how long it will take to fill a 15-litre bucket.

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13 Brooke's Boat Hire company charges $20 to hire a boat and then $12 per hour that the boat is used. Jessica's Jetty hires out boats at a flat rate of $16 with no initial cost.

a Calculate the cost of hiring a boat from each company for the lengths of time given in the table.

Time (in hours)	Cost of hire from Brooke's Boat Hire	Cost of hire from Jessica's Jetty
1		
2		
3		

b Determine the number of hours when each company's hiring cost is the same.

c i Determine which company, Brooke's Boat Hire or Jessica's Jetty, has the best offer for hiring a boat for the whole day (8 hours).

ii Calculate how much you would save when hiring a boat for a whole day from the company with the best offer.

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Mathematical thinking process task

SCHOOL SURVEY: FAVOURITE SPORT

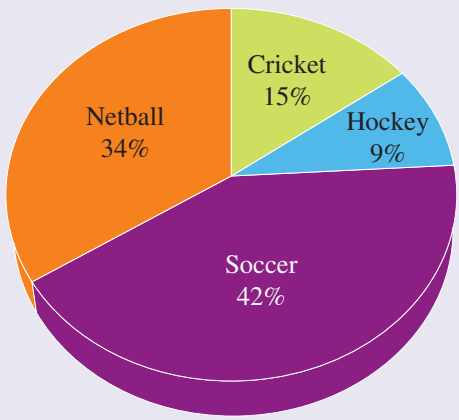
Background: Percentages are useful in analysing all kinds of statistics such as the results of surveys where we want to compare the sizes of different parts of the results.

A survey is conducted at two different schools, Apply School and Weval School, asking students to choose their favourite sport from a choice of netball, cricket, hockey or soccer. The results are given in percentages on the pie charts shown.

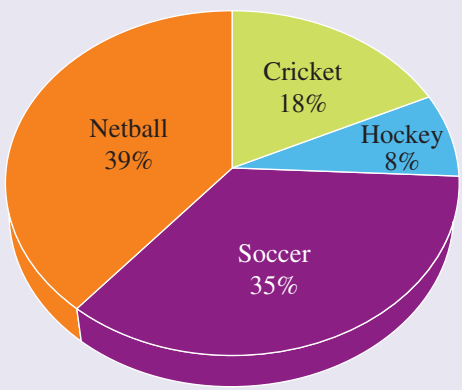
Task: You are to use your knowledge of percentages to analyse the pie charts and write a report discussing the popularity of the sports, from least popular to most popular, based on the total number of students in the two schools.

The explanation you provide must include the total number of students who voted for each sport, as well as the overall percentage of students who voted for that sport.

Apply High School – 400 students



Weval High School – 600 students



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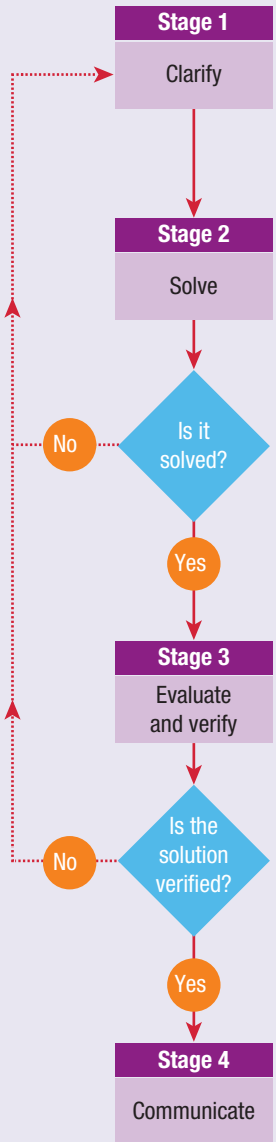
To complete this task, follow the mathematical thinking process workflow diagram below and use the steps listed as a guide.



Stage 1: Clarify
Interpret the task and gather the key information and identify the mathematics which could help to complete the task

1. For each sport, identify the percentage of students at Apply High who voted for that sport.
2. For each sport, identify the percentage of students at Weval High who voted for that sport.

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

3. For each sport, calculate the number of students at Apply High who voted for that sport.
4. For each sport, calculate the number of students at Weval High who voted for that sport.
5. For each sport, determine the total number of students who voted for that sport.
6. Calculate the overall percentage of student votes for each sport.

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources, and verify the reasonableness of the solution

7. Determine the least popular sport to the most popular sport, according to total number of students.
8. Determine the least popular sport to the most popular sport, according to overall percentage.
9. Compare the orders of popularity to confirm that both orders agree with each other.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

10. Comment on your findings in a short report, discussing the popularity of the sports in order from least popular to most popular.

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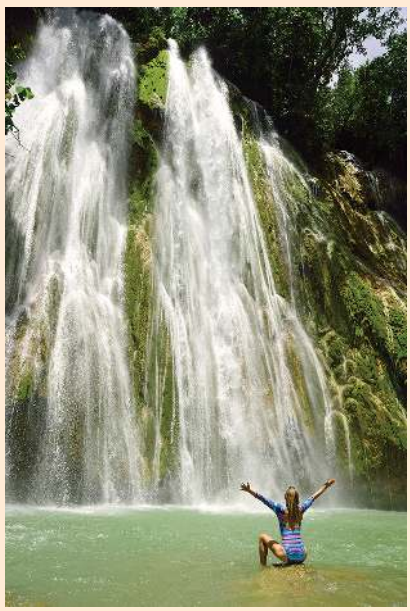
Puzzle



An interesting percentage fact is that about 70% of an adult human is water.

Another interesting percentage fact is that 95% describes the amount of 'what'?

To find out 'what', solve the following questions and use the answer to fill in the question letter above the answer in the grids below.

Calculate the following percentages using your calculator.

A $\$400 \times 3\%$

E $840\,\text{kg} \times 15\%$

Use an appropriate calculation to express the first quantity as a percentage of the second.

R 3 cm out of 20 cm

N 120 kg out of 200 kg

Calculate the following mark-up or discount for each of the following.

L \$600 is discounted by 20%

E \$36 is marked up by 250%

Calculate the GST on the following amounts.

Y \$650

H \$180

Determine the overall change that has occurred when:

I 80 litres is increased by 30% and then increased again by 50%.

S \$100 is increased by 10% and then decreased by 20%.

J Calculate 12.5% of 88.

T Calculate, as a percentage, 210 cm out of 8.4 m.

W Calculate, as a percentage, 72 marks out of 80.

90	12	25	90	15	156	60		12

11	90	120	120	65	126	156	88	18

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Chapter checklist

☐ I understand the fundamentals of percentages.

1. Convert 35% to a simplified fraction.
2. Calculate 12.5% of $4320.
3. Cody scored 72% on his Design Arts test. Calculate how many marks he scored out of 50.

☐ I can determine an amount as a percentage of another amount.

4. Calculate $32 out of $80 as a percentage.
5. Calculate 500 grams out of 2.5 kg as a percentage.
6. Raelene ran 400 m of a 1.6 km relay race. Determine the percentage of the race that she ran.

☐ I can apply a percentage increase or decrease, including mark-ups and discounts.

7. Calculate the mark-up on $1350 when marked up by 30%.
8. Calculate the discount on $2700 when discounted by 12.5%.
9. Calculate the GST on a price of $598 000.

☐ I can recognise rates.

10. Identify which of the following are examples of rates: 20 km/h, 15 m^2, 24 pings per millisecond, $1000/tonne, $7 to 10 cents.
11. What typical unit would be used to describe the speed of a bicycle.

☐ I can simplify and calculate average rates.

12. Write as a rate and use division to simplify: 400 apples in 10 boxes.
13. Calculate the average growth rate of a tree that has grown 30 cm in 6 months.

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Chapter review

Section 2A

1 There are 155 students in Years 11 and 12 at Crib Island High School. According to a recent survey, 60% of the students have a part-time job. Determine how many students have part-time jobs.

2 Rex earns $72 500 a year and must pay 2% of his taxable income as a Medicare levy. Calculate how much Medicare levy Rex must pay.

Section 2B

3 Sandy scores 36 out of 40 on her Biology multiple-choice test. Calculate the percentage of questions that Sandy got correct.

Section 2B

4 Reggie uses 750 mL of orange juice to make up 2.5 litres of fruit punch. Calculate the percentage of the punch that is orange juice.

5 Zoe plays video games for 21 hours in 1 week. Calculate the percentage of time in a week that she plays video games.

Section 2C

6 Dianne buys a house for $180 000. She sells it many years later for a profit of 40%. Calculate the selling price of the house.

7 Ali buys electric motors at $72 each. He then adds on a mark-up of 160% when he sells them. Calculate the profit on each motor as a percentage of the selling price.

Section 2C

8 Aimee bought two tickets for $240 to see the West Australian Ballet perform. Unfortunately, she can't go and decides to sell the tickets at a discount of 22%. Calculate Aimee's selling price of the ballet tickets.

Section 2D

9 A tray of mangoes costs $30 and each tray has 12 mangoes. Calculate the cost per mango.

10 If a car uses 32 litres to travel 420 km, calculate its fuel economy in L/100 km.

11 Calculate the average rate of change in the following situations. Round your answer to one decimal place if needed.

a 750 mL of fluid is used up in 14 minutes

b 6 kg of flour is used in 18 days

c 160 m^3 of vegetation is cut in 2 hours (Show the average rate per minute.)

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3 Formulas, linear measure and area measure

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In this chapter

Syllabus references

Unit 1 Topic 1.2: Using formulas for practical purposes

- identify common uses of formulas to describe practical relationships between quantities
- substitute values for the variables in a mathematical formula, in given form, to calculate the value of the subject of the formula

Unit 1 Topic 3: Measurement

Linear measure

- choose and use appropriate metric units of length, their abbreviations, conversions between them and appropriate levels of accuracy, such as mm for building and other trade contexts and cm for textiles
- estimate lengths
- convert between metric units of length and other length units for simple practical purposes, for example, 1 inch $\approx$ 2.54 cm
- calculate perimeters of familiar shapes, including triangles, squares and rectangles and composites of these

Area measure

- choose and use appropriate metric units of area, their abbreviations and conversions between them
- estimate the areas of different shapes
- convert between metric units of area and other area units
- calculate areas of rectangles and triangles and composites of these shapes

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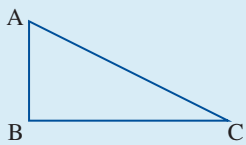
Pre-test

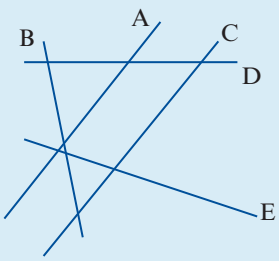
1 Determine the number of:
- **a** sides in a rectangle
- **b** corners in a rectangle

2 Identify:
- **a** the unit of measure for angles
- **b** the number of angles in a triangle

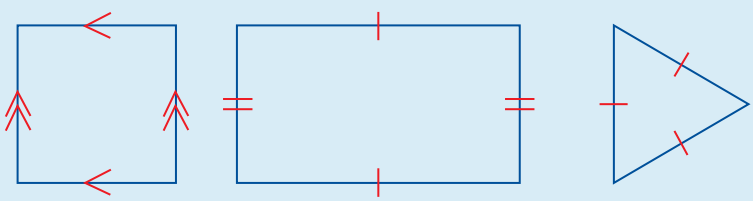
3 Identify:
- **a** the right angle in the diagram shown
- **b** the size of a right angle in degrees



4 Determine which of the following lines are parallel lines.



5 In these geometry diagrams:
- **i** explain what is meant by the marks on the sides
- **ii** identify the shape



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3A Use of formulas and substituting into formulas

LEARNING GOALS

- Understand that formulas quantify mathematical relationships
- Substitute values for pronumerals in formulas, and calculate the value for a single unknown pronumeral

Why is it essential to be able to use formulas?

There are many **formulas** across most academic disciplines. To be able to use them efficiently will empower you to use the mathematical thinking and statistical investigation processes to solve problems involving formulas and **variables** from these academic disciplines.

WHAT YOU NEED TO KNOW

A formula is a mathematical relationship, connecting two or more variables.

For example:
$C = 55t + 100$ is a formula for relating the cost, C dollars, of hiring a plumber for t hours. C and t are the variables.

$A = L^2$ is a formula for finding the area of a square, where A is the area and L is the side length of the square. A and L are the variables.

By substituting all known variables into a formula, we can find the value of an unknown variable.

Example 1 Using a formula

The cost of hiring a surfboard is given by the formula:

$$C = 40t + 10$$

where C is the cost, in dollars, and t is the time in hours.
How much will it cost to hire a surfboard for 2 hours?

... *Continued*

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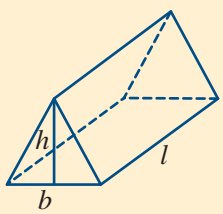
WORKING		THINKING
$C = 40t + 10$ $C = 40 \times 2 + 10$	◄	Write the formula. To determine the cost of hiring a surfboard for 2 hours, substitute $t = 2$ into the formula.
$C = 80 + 10$ $C = 90$	◄	Calculate the value for C.
The cost of hiring a surfboard for 2 hours is \$90.	◄	Communicate your answer.

Example 2 Using a formula

The formula for the volume of a triangular prism is:

$$V = \frac{1}{2}bhl.$$



where V = volume, b = base length of the base triangle, h = the perpendicular height of the base triangle and l = the length of the prism.

Find the volume of a triangular prism when $b = 10$ cm, $h = 13$ cm and $l = 25$ cm.

WORKING		THINKING
$V = \frac{1}{2}bhl$	◄	Write down the formula.
$V = \frac{1}{2} \times 10 \times 13 \times 25$	◄	Substitute the known variables: $b = 10$ cm, $h = 13$ cm and $l = 25$ cm.
$V = 1625 \text{ cm}^3$	◄	Calculate the volume (V).
The volume of the triangular prism is 1625 cm^3.	◄	Communicate your answer.

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Exercise 3A

Example 1

1 The cost of hiring a dance hall is given by the rule $C = 50t + 1200$. Where C is the total cost, in dollars, and t is the number of hours for which the hall is hired. Find the cost of hiring the hall for:

a 4 hours **b** 6 hours **c** 4.5 hours

2 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the formula $d = v \times t$. Find the distance travelled by a car, travelling at a speed of 95 km/h for 4 hours.

3 Taxi fares are calculated using the formula:

$$F = 1.3K + 4$$

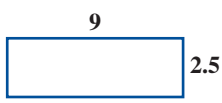
where K is the distance travelled, in kilometres, and F is the cost of the fare in dollars. Find the costs of the following trips.

a 5 km **b** 8 km **c** 20 km

Example 2

4 If the perimeter of a rectangle is given by $P = 2(L + W)$, find the value of P for the following rectangles.

a $L = 4$ and $W = 3$ **b** $L = 15$ and $W = 8$ **c**



5 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$. Use this formula to convert the following temperatures to degrees Celsius. Give your answers correct to one decimal place.

a 50° F **b** 0° F **c** 212° F **d** 92° F

6 Suggested cooking times for roasting m kilograms of meat are given in the table.

Meat type	Formula
Chicken (well done)	$T = 45m + 20$
Lamb (medium)	$T = 55m + 25$
Lamb (well done)	$T = 65m + 30$
Beef (medium)	$T = 55m + 20$
Beef (well done)	$T = 65m + 30$

T = cooking time m = kg of meat

a Find the cooking time, to the nearest minute, that it will take to cook:

i a 2-kg chicken

ii 2.25 kg of beef (well done)

iii a piece of lamb, weighing 2.4 kg (well done)

iv 2.5 kg of beef (medium)

b At what time should you put a 2-kg leg of lamb into the oven so that it is ready to eat, served medium, at 7:30 p.m?

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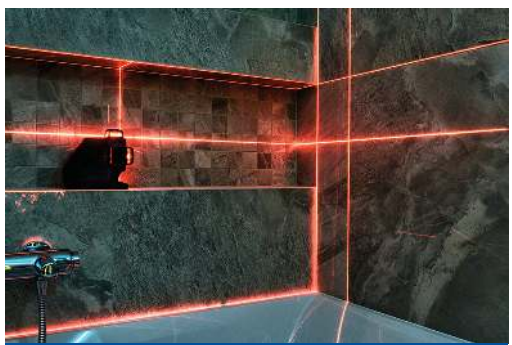
3B Using and converting linear measurements and estimating lengths

LEARNING GOALS

- Use the abbreviations of mm, cm, m and km to represent units of length
- Convert between units of length
- Identify the most appropriate unit of length for measurement
- Estimate the lengths of objects

Why is it essential to understand units of measure?

Understanding units of measure and the **conversion** between them is essential as it is not only used in many jobs but also in daily life. The distance people drive or walk per day is often calculated for either working out the best route or calculating the number of calories burned. Measurements are also used in many industries, such as construction, town planning, fashion and sports.



Modern measuring tools include lasers, being used here to check measurements in a bathroom.

WHAT YOU NEED TO KNOW

- **Units of length** include millimetres, centimetres, metres and kilometres, and these can be abbreviated to mm, cm, m and km.
- **Millimetres** are used for small objects of measure, such as an insect or a bolt. They are also used in scale drawings or in construction and manufacturing industries (e.g. to measure lengths of timber and pipes). Anything greater than 2000 mm in these industries is usually measured in metres. For example, a hardware store uses millimetres to identify the width or thickness of timber, pipes, posts and rods, but uses metres to identify and measure the lengths required of each item.
- **Centimetres** are often used for medium-sized objects, such as the length of a desk or the height of a person.
- **Metres** are typically used for larger objects, such as the length of a room (e.g. the real estate industry use metres but the building industry uses millimetres) or the height of a building.

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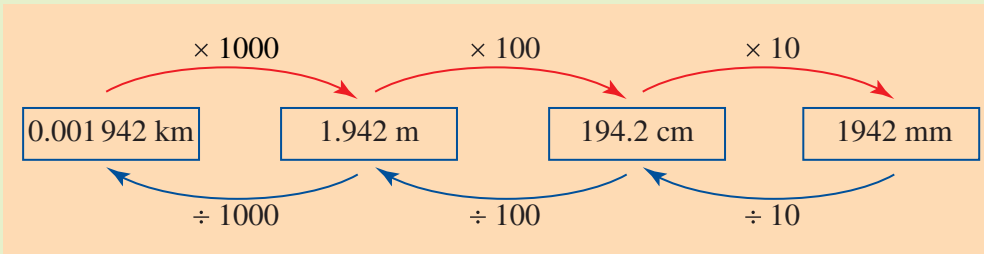
- **Kilometres** are used when calculating long distances (e.g. the distance for a cross-country run or the distance from one city to the next).
- When solving a problem using units of length, ensure that the measurements are in the same units.
- To convert between units:
  - use multiplication when converting from a larger unit to a smaller unit (e.g. converting from km to m).
  - use division when converting from a smaller unit to a larger unit (e.g. converting from cm to m).
- Refer to the following conversion chart when solving problems that require length conversions.



- **Estimating** lengths is the application of your knowledge and skills of measuring lengths to approximately guess the length of an object. Estimation mainly takes place through physical, mental and comparative techniques.
  - Physical – knowing the length of a hand span or walking stride can help to estimate the length of an object. For example, if an adult's stride is around 1 metre, then the length of a room can be estimated, based on the number of steps.
  - Mental – recalling the approximate length of an item and comparing this knowledge to a similar item that is being estimated. For example, remembering that the length of a ruler is 30 cm can then help to estimate the length of a desk.
  - Comparative – knowing the height of a particular item and using this measurement to compare to the height of another object nearby. For example, the height of a house may be used to estimate the height of a nearby tree.

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Example 3 Converting between units of measurement

Convert the following measurements into the units given in brackets.

a 6.5 km (m)

b 25 mm (cm)

c 0.07 m (mm)

WORKING	THINKING
a $6.5\text{ km} \times 1000 = 6500\text{ m}$	Converting from a larger unit to a smaller unit, so multiply. There are 1000 m in each km.
b $25\text{ mm} \div 10 = 2.5\text{ cm}$	Converting from a smaller unit to a larger unit, so divide. There are 10 mm in each cm.
c $0.07\text{ m} \times 100 \times 10 = 70\text{ mm}$	Converting from a larger unit to a smaller unit twice, so multiply. There are 100 cm in each m and 10 mm in each cm.

Example 4 Applying metric units of length

Blake works as a costume designer and he is making costumes for a dance concert. He has worked out that each dancer will need 82 cm of material. There are 8 dancers in the troupe. Determine how many metres of material Blake will need to purchase.

WORKING	THINKING
$82\text{ cm} \times 8 = 656\text{ cm}$	Calculate how many centimetres of material Blake will require by multiplying the length of material by the number of dancers.
$656\text{ cm} \div 100 = 6.56\text{ m}$	Converting from a smaller unit to a larger unit, so divide by 100 as there are 100 cm in each m.
Blake will need to purchase 6.56 m of material.	Communicate your answer in a sentence.

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Example 5 Estimating lengths

Estimate the length (diameter) of the diamond in the ring below, given that the following line is 1.5 cm long and the photo of the ring is actual size.

WORKING	THINKING
1.5 cm ÷ 2 = 0.75 cm	Compare the length of the line to the diameter of the diamond. The diamond is around half the size of the line, so divide the length by 2 to estimate the diameter.
The diamond has a length of approximately 0.75 cm.	Communicate your answer using words.

Exercise 3B

FUNDAMENTALS

Example 3

1 Convert the following measurements into the units given in brackets.

a 5 m (cm)
b 7 cm (mm)
c 20 mm (cm)
d 2 km (m)
e 3.7 m (cm)
f 1.7 km (m)
g 490 cm (m)
h 7 m (mm)
i 1.2 km (mm)
j 3000 cm (km)

Hint If converting from a larger unit to a smaller unit, multiply. If converting from a smaller unit to a larger unit, divide.

2 Calculate the sum of the following measurements. Express your answer in the units given in the brackets.

a 10 mm, 2 cm (cm)
b 3 m, 200 cm (m)
c 1.5 cm, 10 mm (mm)
d 500 m, 2 km (km)
e 150 cm, 3.5 m (cm)
f 200 m, 1.5 km (m)

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3 A good way to estimate the length of an object is to compare it to the length of a familiar object. Use a ruler to measure the span of your hand from your thumb to your pinkie (outside finger) in centimetres. Use this measurement to estimate the length of the following items around your classroom.

a length of your desk
b height of your chair
c length of your pen
d width of your book
e length of your eraser
f length of your calculator
g length of your pencil case
h length of the whiteboard

4 Determine the most appropriate unit of length (mm, cm, m or km) for the following measurements.

a length of a car
b width of a pencil
c length of a ladybeetle
d length of a desk
e flight distance from Perth to Port Hedland
f length of a fingernail
g length of an arm
h height of a giraffe
i distance around the Earth
j distance from London to Cairo

APPLICATIONS

Example 4

5 Jade is an estimator for a building company. A plan shows that a wall in a family room is 5860 mm long. Jade needs the measurement in metres to estimate the cost of the wall panelling. Determine the length of the wall in metres.

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6 Lachlan is training for a fun run and he runs 4.5 km per day. Determine how many metres Lachlan runs per day.

7 Harrison is looking at house plans to decide which house to build. On one of the house plans, he has measured the length of the master bedroom as 4.2 m. Determine the length of the master bedroom in millimetres.

8 At the age of two, Kelly's son is 90 cm tall. Kelly is told that he will grow to be twice that height. Determine the adult height of Kelly's son in metres.

9 Allen is building a garden box for one of his clients. A length of timber is 250 cm long, and Allen needs 6 lengths of timber. Determine how many metres of timber, in total, Allen needs to purchase for the garden box.

10 Choose the correct estimation for each of the measurements shown in the table.

Measurement	Correct estimation		
Length of a diamond in a ring	2.5 cm	6 mm	28 mm
Length of a mobile phone	220 mm	0.4 m	12 cm
Height of an old gum tree	32 m	0.8 km	30 000 cm
Height of a 12-storey high skyscraper	0.1 km	36 m	80 m
Distance from Perth to Port Hedland	368 000 m	3200 km	1300 km

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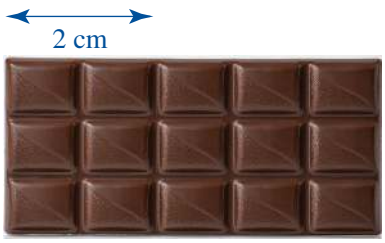
Example 5

11 Estimate the length of the following images, given that the blue line is 2 cm long.

a



b

c

12 Estimate the height of the palm tree near the front door, given the height of the front wall on the house is 3 m.

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3C Calculating perimeters of familiar shapes

LEARNING GOALS

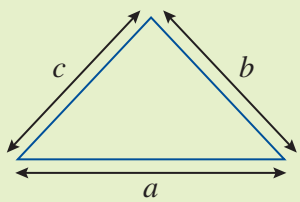
- Calculate the perimeters of familiar shapes, including triangles, squares, rectangles and polygons

Why is it essential to understand how to calculate perimeters?

Calculating perimeters can be used in industries such as construction and design. In construction, perimeter can be used to calculate the total length around the property when building a fence. Perimeter is used to calculate the amount of timber required to frame a painting. Or it can also be used in fashion to work out the length of trimming required around the hem of a skirt.

WHAT YOU NEED TO KNOW

- **Perimeter** is the **distance** around the outside of a shape. For example, the perimeter of the triangle on the right would be the total of $a + b + c$.



Perimeter is used to calculate the amount of fencing required around a property.

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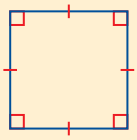
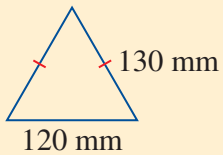
Example 6 Calculating the perimeter of 2D shapes

Calculate the perimeter of the following shapes.

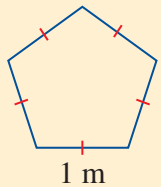
a



b

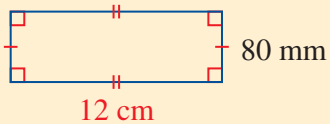


c



For the following question, calculate the perimeter in the units highlighted in red.

d



WORKING		THINKING
a $8 + 8 + 8 + 8 = 32\,\text{cm}$	◄	Add all the side lengths together.
or		
$4 \times 8 = 32\,\text{cm}$	◄	Alternatively, as there are 4 equal sides, multiply the side length by 4.
b $130 + 130 + 120 = 380\,\text{mm}$	◄	Add all the side lengths together.
or		
$2 \times 130 + 120 = 380\,\text{mm}$	◄	Alternatively, as there are 2 equal sides, multiply the equal side length by 2 and then add the base of the triangle.
c $1 + 1 + 1 + 1 + 1 = 5\,\text{m}$	◄	Add all the sides together.
or		
$5 \times 1 = 5\,\text{m}$	◄	Alternatively, as there are 5 equal sides, multiply the side length by 5.
d $80\,\text{mm} \div 10 = 8\,\text{cm}$	◄	Convert mm to cm first. There are 10 mm in every 1 cm, so divide 80 by 10.
$8 + 8 + 12 + 12 = 40\,\text{cm}$	◄	Add all the side lengths together.
or		
$2 \times 8 + 2 \times 12 = 40\,\text{cm}$	◄	Alternatively, as there are 2 lots of equal side lengths, multiply both side lengths by 2 and add them together.

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Example 7 Applying perimeter to practical problems

Griffin has 30 m of flexible fencing. He would like to use all of the fencing to make the largest square pen for his pigs. Calculate the side length of the largest pen that Griffin can make, using all of the fencing. Round your answer to two decimal places.

WORKING	THINKING
$P = 4L$	Use the formula $P = 4L$.
$30 = 4L$	Substitute into the formula the known measurements; that is, the perimeter of 30 m.
$L = \frac{30}{4}$ $= 7.5$ m	To find the length, rearrange the equation by dividing by 4.
The side length will be 7.5 m.	Communicate your answer in a sentence.

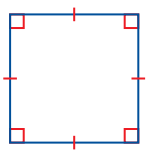
Exercise 3C

FUNDAMENTALS

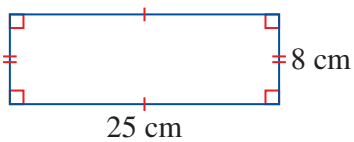
Example 6

1 Calculate the perimeter of the following shapes.

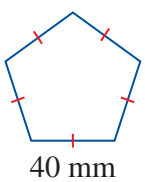
a



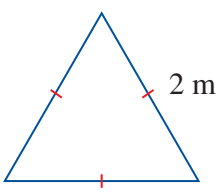
b



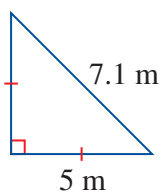
c



d



e



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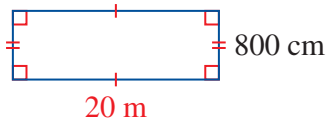
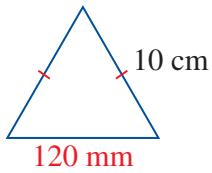
2 Calculate the perimeters for the following shapes, using the units indicated in red.

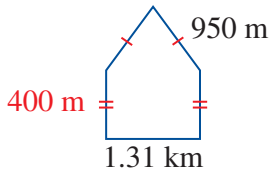
a



b



c



APPLICATIONS

For the following questions, write your answers correct to two decimal places where appropriate.

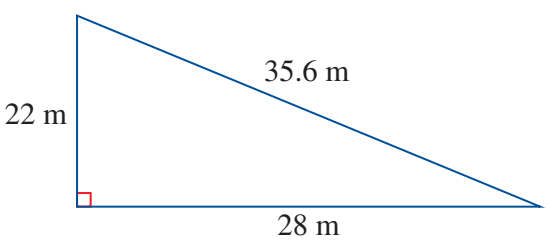
Example 7

3 Quinn is baking her mother a cake for her birthday. Quinn uses a 20-cm square baking dish and plans on decorating the cake by wrapping a ribbon around it. Determine how much ribbon Quinn will require for the cake.

4 Lyn lives on a large property and has a new dog that has a reputation for running away. She has decided that she will put in an electric dog fence along the perimeter of her property. The shape of the property is a rectangle. It is 80 metres long and 70 metres wide. Determine how many metres of electric dog fencing Lyn will need to purchase.

5 Josh is renovating his bathroom and has purchased a square mirror. The measurements on the box state that the mirror has a side length of 1 m. He would like to wrap a string of lights around the whole mirror. How long does the string of lights need to be?

6 Jarrah is building a fence for one of his yards to help keep his stock away from the road. The yard is triangular, as per the diagram shown. The fence design that he is planning has three railings of timber.



a Calculate the perimeter of the yard.

b Determine how many metres of timber Jarrah requires to construct the fence.

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3D Calculating perimeters of familiar composite shapes

LEARNING GOALS

- Identify composite shapes
- Calculate perimeters of composite shapes

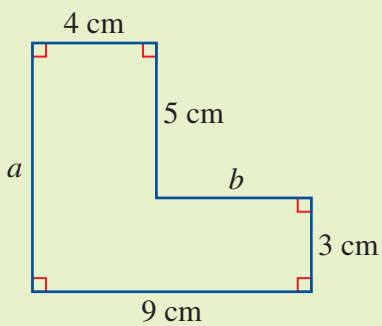
Why is it essential to calculate perimeters of composite shapes?

Often in life, shapes are more intricate in detail and sometimes form a composite shape. They are made by combining two or more familiar shapes. For example, the shape of a house plan can be L-shaped and is made up of two rectangles. The face of a building is another example as it may be a triangle and a rectangle combined.

In a quilt, simple shapes can be sewn together to make more complex composite shapes.

WHAT YOU NEED TO KNOW

- A **composite shape** is a shape that is made up of two or more basic shapes.
- Some composite shapes do not mark the length of every side, as it is possible to calculate these lengths using either addition or subtraction.
  - For example:
    The length of the side marked a in the diagram is equal to the sum of the lengths of the opposite sides. As $5 + 3 = 8$ cm, $a = 8$ cm.
    The side marked b is equal to the side length 9 cm minus the side length 4 cm. As $9 - 4 = 5$ cm, $b = 5$ cm.



The perimeter of the composite shape is $8 + 4 + 5 + 5 + 3 + 9 = 34$ cm.

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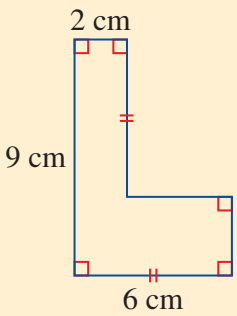
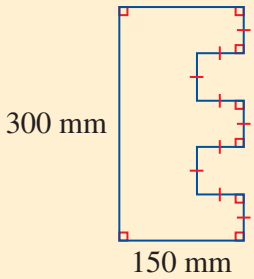
Example 8 Calculating the perimeter of composite shapes

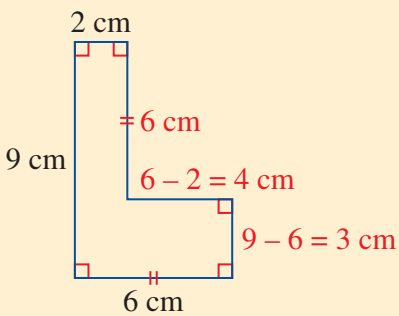
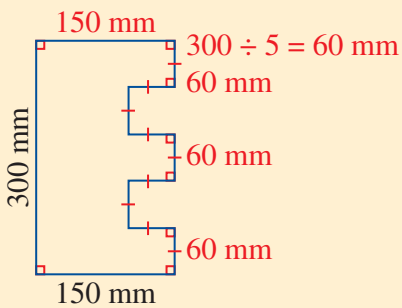
Calculate the perimeter of the following composite shapes.

a



b



WORKING	THINKING
a 	Two side lengths are unmarked. These lengths are calculated by subtracting the opposite side lengths of the shape.
$P = 2 + 6 + 4 + 3 + 6 + 9$ $P = 30\,\text{cm}$	Add all the side lengths together to calculate the total perimeter.
b 	There are 5 small lengths that equal the total of the opposite side length. Divide 300 by 5 to calculate the length of one of these small lengths. There are 9 of these small lengths, in total, so multiply the answer by 9.
$P = (9 \times 60) + (2 \times 150) + 300$ $P = 1140\,\text{mm}$	Add the lengths of the remaining longer sides. Although not marked, we know that there are two side lengths of 150 mm because all the angles are right angled.

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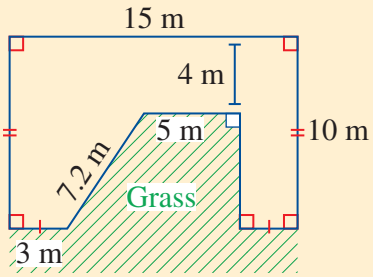
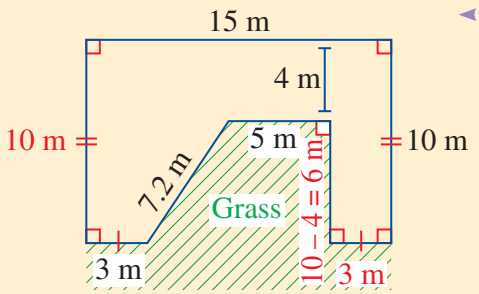
Example 9 Applying perimeter of composite shapes to practical problems

Noah has designed a garden bed for his new property, shown on the right. He would like to place timber edging along the entire perimeter to hold the soil in place. Calculate how much timber Noah requires for his garden bed.



Note: The dimensions are measured around the 'outside' of the garden bed after the timber has been added. So you do not have to consider the thickness of the timber.

WORKING	THINKING
	There are two sides that are marked as 3 m. There are two sides that are marked as 10 m. There is only one length that is not given. We can calculate this length by subtracting 4 m from the opposite side length of 10 m. This tells us that the unmarked length is 6 m.
$P = 15 + 10 + 3 + 6 + 5 + 7.2 + 3 + 10$ $P = 59.2$ m Noah requires 59.2 metres of timber to build his garden bed.	Add all the side lengths together to calculate the total perimeter. Communicate your solution in a sentence.

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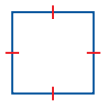
Exercise 3D

FUNDAMENTALS

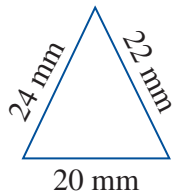
Where appropriate, write your answers to two decimal places.

1 Calculate the perimeter of the following common shapes.

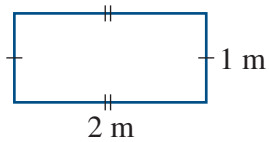
a



b



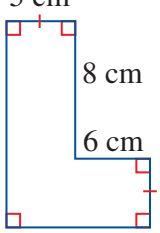
c



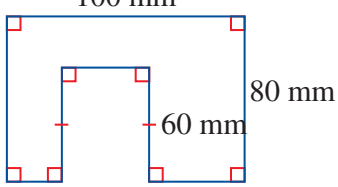
Example 8

2 Calculate the perimeter of the following composite shapes.

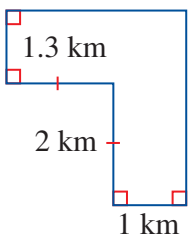
a



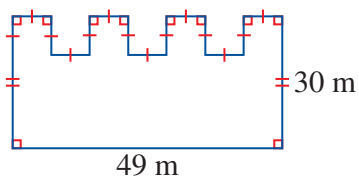
b



c



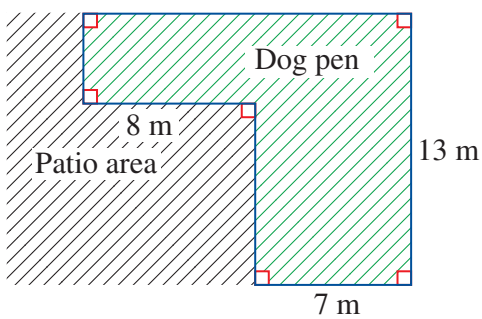
d



APPLICATIONS

Example 9

3 Russell needs to fence a large patch of grass area next to his patio to create a dog pen with dimensions shown in the diagram. Calculate the total length of fencing that Russell needs to purchase.



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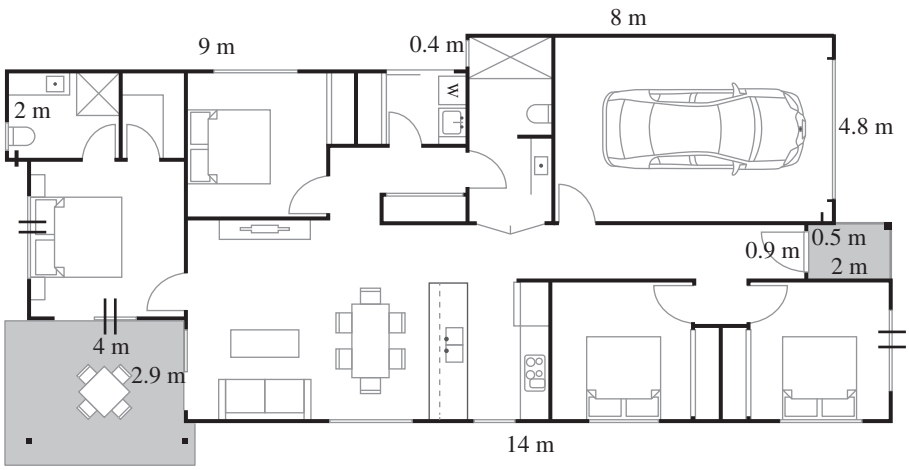
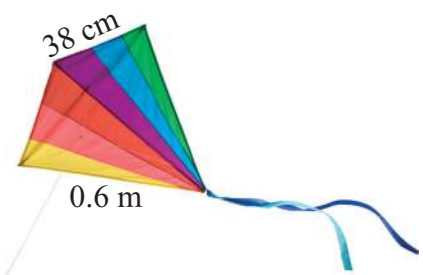
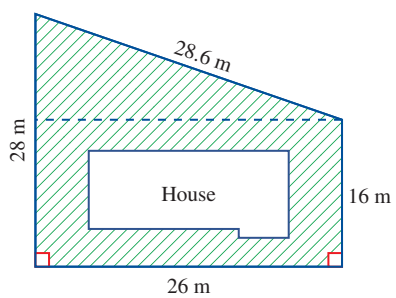
4 Valerie is renovating her house and she would like to purchase some paint to change the colour of the outside of her house. Her local hardware store has asked her for the perimeter and height of the external walls of her house. She knows the external walls are 3 metres high. Valerie is using her house plan to work out the perimeter. Calculate the perimeter of Valerie's house using the plan shown.



5 Mathew has cut and sewn a pattern for a kite in the shape of the image shown. He now needs to buy dowel (wooden rods) to insert along the edges of the kite before he can fly it. Calculate how many metres of dowel Mathew must purchase.



6 Shekila's property contains her house, situated in the front rectangular section of the property as shown in the diagram. She wishes to enclose the back triangular section of her yard with fencing. Calculate the length of fencing that Shekila will require.



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3E Using and converting between the metric area units

LEARNING GOALS

- Understand the use and appropriate choice of the metric units of area, including square millimetres, square centimetres, square metres, square kilometres and hectares
- Use abbreviations such as mm^2, cm^2, m^2, km^2 and ha to represent units of area
- Convert between units of area

Why is it essential to understand units of area and convert between them?

Calculating the area of a shape is a skill used in many careers, including landscaping, building, painting, clothes making and interior design. It is important to convert between units of area so that we can compare sizes of objects that may have been measured in different units.

Tiling often involves conversion of units between square metres and square millimetres.

WHAT YOU NEED TO KNOW

- Calculating the **area** of a shape is a measure of the space enclosed by the boundaries of a 2D shape.
- For squares and rectangles, area is length multiplied by width (also known as breadth).
- If the shape is upright, the term height might be used in place of length or width.
- **Units of area** include square millimetres, square centimetres, square metres, square kilometres and hectares, and these can be abbreviated to mm^2, cm^2, m^2, km^2 and ha.
  - **Square millimetres** are typically used in construction or electrical cabling.
  - **Square centimetres** are often used for medium-sized objects, such as the surface area of a desk.

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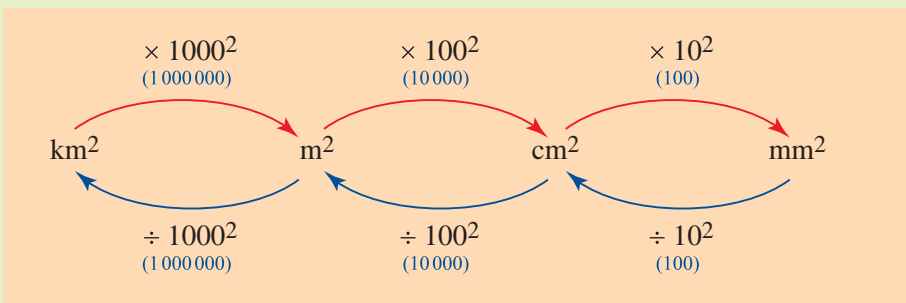
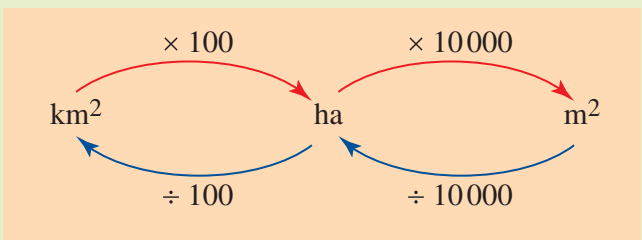
- **Square metres** are typically used for larger objects, such as floors and floor coverings, walls or small blocks of land, such as a plot for a single house.
- A **hectare** is an area equal to a square with sides of 100 m. Its area is 100 m × 100 m = 10 000 square metres (m^2). This unit of area is typically used for land bigger than an average house plot, such as farm paddocks.
- **Square kilometres** are typically used for very large areas of land, such as the area of a city or region.

- When a **2D** shape is referred to as being 'l units square' (e.g. 60 cm square), it means that it is a square with sides of 'l units' (e.g. 60 cm). When a 3D solid is referred to as being 'l units cube' (e.g. 100 cm cube), it means that it is a cube with sides of 'l units' (e.g. 100 cm).
- When calculating area, always ensure that the area units are in squares of the linear units used to measure the dimensions (i.e. cm and cm^2).
- To convert between units of area:
  - use multiplication when converting from a larger unit to a smaller unit (e.g. from km^2 to m^2)
  - use division when converting from a smaller unit to a larger unit (e.g. from cm^2 to m^2).
- Refer to the following **conversion** chart when solving problems that require conversion.

Area conversion



Area conversion with hectares



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Example 10 Converting between units of area

Convert these area measurements to the units given in brackets.

a $0.72\,\text{km}^2$ (m^2)
b $393\,\text{mm}^2$ (cm^2)
c $34.5\,\text{km}^2$ (ha)
d $800\,\text{m}^2$ (ha)

WORKING	THINKING
a $0.72 \times 1000^2 = 720\,000\,\text{m}^2$	As there are 1000^2 square metres in a square kilometre and we are converting from km^2 to a smaller unit of m^2, we need to multiply 0.72 by 1000^2.
b $393 \div 10^2 = 3.93\,\text{cm}^2$	As there are 10^2 square millimetres in a square centimetre and we are converting from mm^2 to a larger unit of cm^2, we need to divide 393 by 10^2.
c $34.5 \times 100 = 3450$ ha	As there are 100 hectares in a square kilometre and we are converting from km^2 to a smaller unit of ha, we need to multiply 34.5 by 100.
d $800 \div 10\,000 = 0.08$ ha	As there are 10 000 square metres in a hectare and we are converting from m^2 to a larger unit of ha, we need to divide 800 by 10 000.

Example 11 Converting between units, requiring more than one step

Convert these area measurements to the units given in brackets.

a $5\,000\,000\,\text{cm}^2$ (km^2)
b $0.07\,\text{m}^2$ (mm^2)

WORKING	THINKING
a $5\,000\,000 \div 10\,000\,000\,000 = 0.0005\,\text{km}^2$	As there are 100^2 square centimetres in a square metre and 1000^2 square metres in a square kilometre and we are converting from square centimetres to a larger unit of square kilometres, we need to divide $5\,000\,000$ by $(100^2 \times 1000^2) = 10\,000\,000\,000$.

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b $0.07 \times 1\,000\,000$
$= 70\,000\ \text{mm}^2$

As there are 100^2 square centimetres in a square metre and 10^2 square millimetres in a square centimetre and we are converting from square metres to a smaller unit of square millimetres, we need to multiply 0.07 by $(100^2 \times 10^2) = 1\,000\,000$.

Example 12 Applying unit conversion to practical problems

Alana has just purchased some new land that is 1.32 ha. Determine the area of Alana's new property in square metres.

WORKING	THINKING
$1.32 \times 10\,000 = 13\,200\ \text{m}^2$	As there are 10 000 square metres in a hectare and we are converting from ha to a smaller unit of m^2, we need to multiply 1.32 by 10 000.
Alana's new property is $13\,200\ \text{m}^2$.	Communicate your answer in a sentence.

Exercise 3E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Use _____ when converting from a larger unit of area to a _____ unit of area.

b Use division when converting from a _____ unit of area to a _____ unit of area.

c One hectare is equal to _____ square metres.

d One square metre is equal to _____ hectares.

Example 10

2 Convert the following measurements to the units given in brackets.

a $5\ \text{m}^2$ (cm^2)
b $7\ \text{cm}^2$ (mm^2)
c $200\ \text{mm}^2$ (cm^2)
d $2\ \text{km}^2$ (m^2)
e $5\ \text{km}^2$ (ha)
f $3.7\ \text{m}^2$ (cm^2)
g $1.7\ \text{km}^2$ (m^2)
h $490\ \text{cm}^2$ (m^2)
i $527\ \text{m}^2$ (ha)
j $5\ \text{mm}^2$ (cm^2)
k $500\ \text{m}^2$ (km^2)
l $3.75\ \text{km}^2$ (ha)

Example 11

3 Convert the following measurements to the units given in brackets.

a $7\ \text{m}^2$ (mm^2)
b $0.4\ \text{km}^2$ (cm^2)
c $2800\ \text{mm}^2$ (m^2)
d $1.2\ \text{km}^2$ (mm^2)
e $3000\ \text{cm}^2$ (km^2)

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APPLICATIONS

Example 12

4 Anne's property is 2.5 ha. Determine the area of land Anne owns in square kilometres.

5 Perth covers an area of 6418 km^2. Determine the area of Perth in square metres.

6 Molly is making a new dress for her daughter's tiny doll. She has calculated that she needs 19 cm^2 of material. Determine the amount of material that Molly needs in square millimetres.

7 Ki is replacing the cover on an old chair. He calculates that he requires 1600 cm^2 of material, which is sold in square metres. Determine the amount of material that Ki needs in square metres.

8 Harley's horses each have a paddock of 16 180 m^2 to roam. Determine the total area that her two horses have in hectares.

9 Kirra works for Surf Life Saving Australia. Part of Kirra's role requires her to wax the rescue boards. She has some boards that have an approximate total surface area of 9200 cm^2 that requires waxing. Determine the total surface area that Kirra will have to wax in square metres.

10 Jana is laying a small patch of turf in her backyard. She measures the patch to be 3400 mm long by 2100 mm wide, which she calculates to be an area of 7 140 000 mm^2. The turf is sold in square metres. Determine the amount of turf that Jana requires in square metres.

11 Damien is designing a platform to be able to bear a certain weight and is using a computer program to calculate the strength of the beams he will use. The program requires him to input the area of the cross-section of the beam. He calculated the area of the cross-section of the beam to be 0.018 m^2 but he did not realise that the program requires the area to be in mm^2. Determine the area of the cross-section of the beam in mm^2.

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3F Estimating and calculating areas of triangles, squares and rectangles

LEARNING GOALS

- Calculate the area of various familiar shapes including:
  - triangles
  - squares
  - rectangles
- Estimate the area of familiar shapes

Why is it essential to understand how to calculate the area of a shape?

There are many real-life situations where you would need to calculate the area of various shapes. This skill is used in careers such as fashion, construction, landscaping and painting. Calculating the area allows a landscaper to find the amount of turf required or a painter to work out how much paint is needed. It is also a good way to compare the sizes of properties when you are looking at buying or leasing (renting) some land.

Real estate agents' signs and ads for small to medium properties for sale or lease usually state the area in square metres – larger blocks such as farms would be in hectares.

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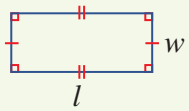
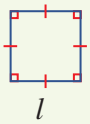
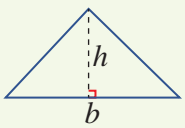
WHAT YOU NEED TO KNOW

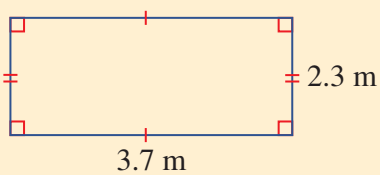
- The **formulas** for the areas of some common shapes are given in the table below.

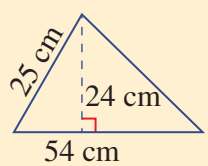
Shape	Area formula
Rectangle 	$A = l \times w$
Square 	$A = l^2$ or $l \times l$
Triangle 	$A = \frac{1}{2}(b \times h)$ or $\frac{b \times h}{2}$

- Units of area include square millimetres, square centimetres, square metres, square kilometres and hectares, and these can be abbreviated to mm^2, cm^2, m^2, km^2 and ha.

Example 13 Calculating the area of common 2D shapes

Calculate the area of the following common 2D shapes.

a 

b 

Round your answers to 2 decimal places.

WORKING	THINKING
a $A = lw$	Calculate the area of the rectangle using $A = lw$.
$A = l \times w$	
$A = 3.7 \times 2.3$	Substitute the values for l and w.
$A = 8.51\ m^2$	

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b $A = \frac{1}{2}bh$ ◄ Calculate the area of the triangle using $A = \frac{1}{2}bh$.

$A = \frac{1}{2}(b \times h)$

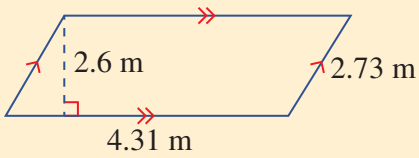
$A = \frac{1}{2}(54 \times 24)$ Substitute the values for b and h.
Note: The height must be the *perpendicular* height, meaning it meets the base at a right angle (i.e. 24 is used here).

$A = 648\,\text{cm}^2$

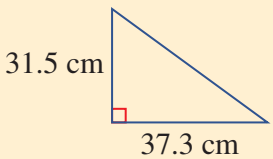
Example 14 Estimating the area of common 2D shapes

Estimate the area of the following common 2D shapes by rounding the dimensions to the nearest whole number.

a



b



WORKING	THINKING
a $2.6\,\text{m} \approx 3\,\text{m}$ $4.31\,\text{m} = 4\,\text{m}$	◄ Begin by rounding your dimensions to the nearest whole number. Look at the first decimal place. If it is 0.5 or more, then round your whole number up. If it is less than 0.5, then drop the decimal place. Note: The perpendicular height of the parallelogram is used (i.e. 2.6 m for this shape).
$A = b \times h$	◄ Calculate the area using $A = b \times h$.
$A = 4 \times 3$ $A = 12\,\text{m}^2$	◄ Substitute the values for b and h using the whole numbers to estimate the area of the parallelogram.
b $37.3 \approx 37\,\text{cm}$ $31.5 \approx 32\,\text{cm}$	◄ Begin by rounding your dimensions to the nearest whole number.
$A = \frac{1}{2}(b \times h)$	Calculate the area of the triangle, using $A = \frac{1}{2}(b \times h)$.
$A = \frac{1}{2}(37 \times 32)$ $A = 592\,\text{cm}^2$	Substitute the rounded values for b and h.

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Example 15 Applying the area of a shape to practical problems

Michelle is painting the top surface of a rectangular stage that has a length of 2.4 m and a width of 1.1 m. Calculate the area that Michelle will be painting.

WORKING	THINKING
$A = L \times W$	Given that the surface is rectangular, calculate the area using $A = L \times W$.
$A = 2.4 \times 1.1$ $A = 2.64\ \text{m}^2$	Substitute the values for L and W.
Michelle will be painting an area that is $2.64\ \text{m}^2$.	Communicate your answer in a sentence.

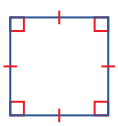
Exercise 3F

FUNDAMENTALS

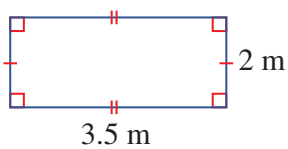
Example 13

1 Calculate the area of the following common 2D shapes.

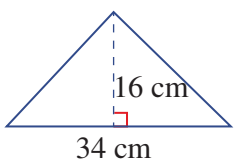
a



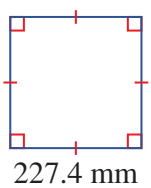
b



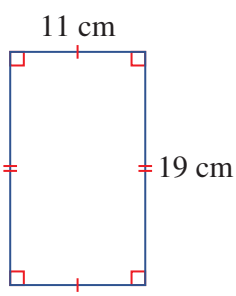
c



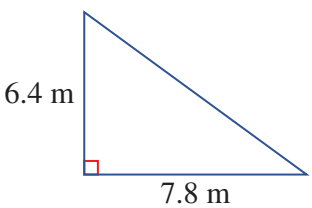
d



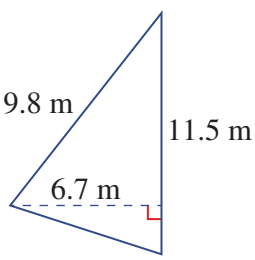
e



f



g



Hint The height must be the perpendicular height for triangles and parallelograms.

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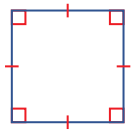
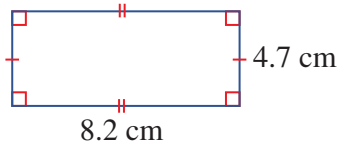
Example 14

2 Estimate the area of the following common 2D shapes by rounding the dimensions to the nearest whole number.

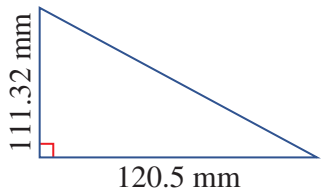
a



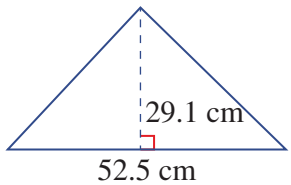
b



c



d



Hint To estimate, look at the first decimal place. If it is 0.5 or more, then round your whole number up. If it is less than 0.5, then drop the decimal place.

APPLICATIONS

Example 15

3 Jarred is covering a display board with blue paper. The board is a rectangle and it has a length of 3.5 m and height of 1.5 m. Calculate the area that Jarred will be covering in blue paper.

4 Haley is tiling the floor of her 2.8 metre square bathroom. Calculate the area that Haley is tiling.

5 Chelsea is painting the triangular section on the top of her house. The triangle has a base of 8.2 m and a perpendicular height of 3.2 m. Calculate the area that Chelsea is painting so that she can determine the amount of paint required.

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6 Dane has been commissioned to paint a mural on the side of a building. The mural will be rectangular and has a width of 5.7 m and a height of 3 m. Calculate the area of the mural.

7 Graham has just purchased some rectangular-shaped land that is 220 m by 100 m. He knows that he can have 10 sheep per hectare, and he would like to have as many sheep as possible on his land.

a Calculate the area of Graham's land in hectares.

b Determine how many sheep Graham can keep on his new property.

Hint 1 hectare is 10 000 m^2.

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3G Calculating areas of familiar composite shapes

LEARNING GOALS

- Apply your knowledge of regular shapes to decompose composite shapes
- Calculate the area of composite shapes

Why is it essential to understand how to calculate the area of composite shapes?

As in the previous chapter, there are many real-life situations where you would need to calculate the area of composite shapes. These shapes can be found in construction, painting, town planning, fashion and even cooking. The majority of shapes around the home are actually composite shapes.

Understanding area and composite shapes can help with clothing design and manufacture.

WHAT YOU NEED TO KNOW

- A **composite shape** is a shape that is made up of two or more common shapes. Addition and/or subtraction can be used to find the area of composite shapes.
- You will also need to recall the area formulas of the shapes from the previous section.

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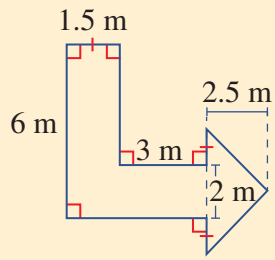
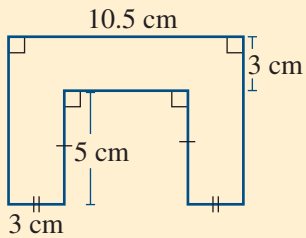
Example 16 Calculating the area of composite shapes, with addition

Calculate the area of the following composite shapes, correct to 2 decimal places if needed.

a



b



WORKING	THINKING
a There are 2 rectangles and one triangle.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate the area of the rectangles, using the formula.
$A = 6 \times 1.5 = 9\,\text{m}^2$	Substitute the values for l and w.
$A = 3 \times 2 = 6\,\text{m}^2$	
$A = \frac{1}{2} \times b \times h$	Calculate the area of the triangle, using the formula.
$h = 2.5\,\text{m}$, $b = 1.5 + 2 + 1.5$ $= 5\,\text{m}$	The unlabelled parts of the base have tick marks, indicating they are the same as the side labelled 1.5 m.
$A = \frac{1}{2} \times (5 \times 2.5)$ $= 6.25\,\text{m}^2$	Substitute the values for b and h.
$A = 9 + 6 + 6.25$ $A = 21.25\,\text{m}^2$	Add all the shape areas together to find the total area of the composite shape.
b There are 3 rectangles.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate the area of all rectangles, using the formula.
$A = 10.5 \times 3 = 31.5\,\text{cm}^2$	Substitute the values for l and w.
$A = 5 \times 3 = 15\,\text{cm}^2$	
$A = 5 \times 3 = 15\,\text{cm}^2$	
$A = 31.5 + 15 + 15$ $A = 61.5\,\text{cm}^2$	Add all the shape areas together to find the total area of the composite shape.

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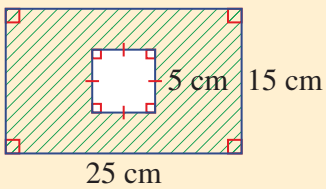
Example 17 Calculating the area of composite shapes, with subtraction

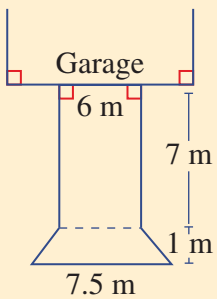
For the following composite shape, calculate the area of the shaded section, correct to 2 decimal places if needed.



WORKING	THINKING
There is one rectangle and one square that is cut out from the centre.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate the area of the rectangle using $A = l \times w$.
$A = 25 \times 15 = 375\ \text{cm}^2$	Substitute the values for l and w.
$A = l \times l$	Calculate the area of the square using $A = l \times l$.
$A = 5 \times 5 = 25\ \text{cm}^2$	Substitute the value for l.
$A = 375 - 25 = 350\ \text{cm}^2$	Subtract the square from the rectangle to find the shaded area.

Example 18 Applying the area of composite shapes to practical problems

A driveway, as shown in the diagram, is to be concreted. Calculate the area of the driveway, correct to 2 decimal places.



... Continued

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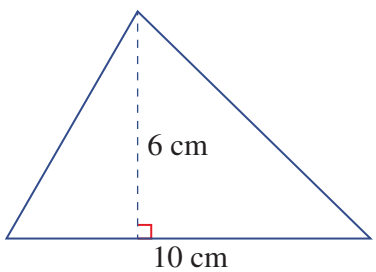
WORKING	THINKING
There is one rectangle and one trapezium.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate the area of the rectangle using $A = l \times w$.
$A = 7 \times 6 = 42\,\text{m}^2$	Substitute the values for l and w.
$A = l \times w$	Decompose the trapezium into a rectangle and 2 triangles. Calculate the area of the rectangle inside the trapezium with $A = l \times w$.
$A = 6 \times 1 = 6\,\text{m}^2$	Substitute the values for l and w.
$A = \frac{1}{2} \times b \times h$	Calculate the area of the two triangles inside the trapezium with $A = \frac{1}{2} \times b \times h$.
$A = \frac{1}{2} \times 0.75 \times 1 = 0.375\,\text{m}^2$	Substitute the values of b and h.
$A = 42 + 6 + (2 \times 0.375)$ $A = 48.75\,\text{m}^2$	Add the four areas together to calculate the total area of the composite shape.
The area of the driveway is $48.75\,\text{m}^2$.	Communicate your answer using words.

Exercise 3G

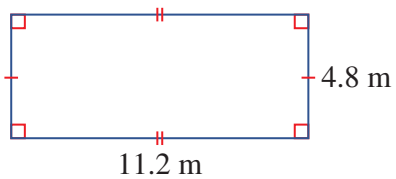
FUNDAMENTALS

1 Calculate the area of the following common 2D shapes.

a



b



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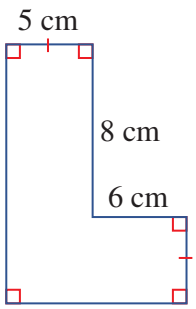
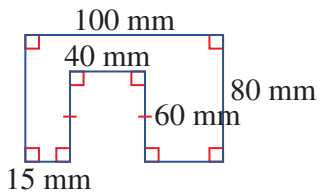
Example 16

2 Determine the area of the following composite shapes by first decomposing the shapes into regular shapes and then adding the areas of the shapes.

a

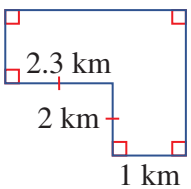


b

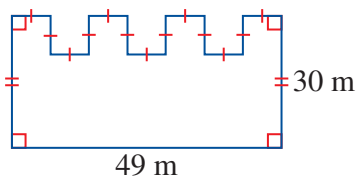


Hint Redraw the shapes into common shapes to help you to calculate the total area.

c



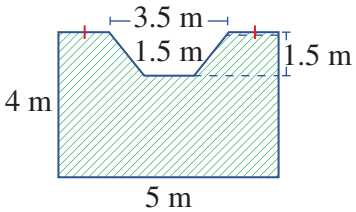
d



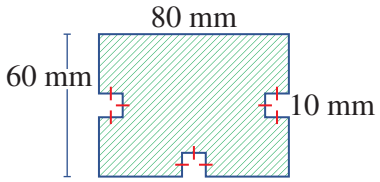
Example 17

3 Determine the area of the shaded section in the following composite shapes by first decomposing the shape into regular shapes and then subtracting the sections that are not shaded. All angles that look like right angles are 90°.

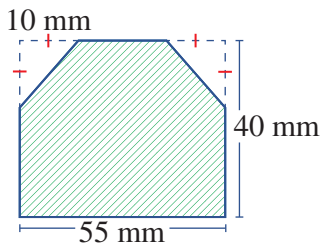
a



b



c



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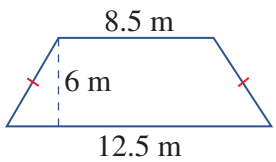
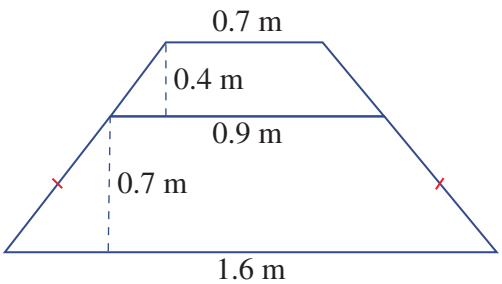
APPLICATIONS

Example 18

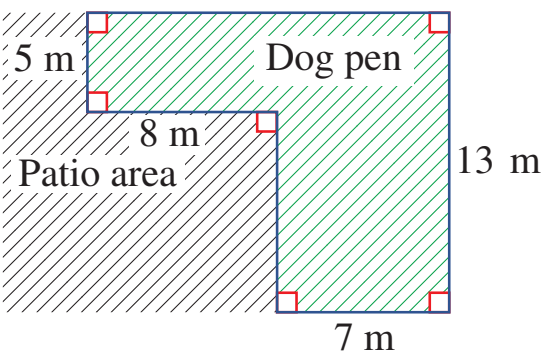
4 Ben is planning on making a new shade sail to run from the roof line of his house, across his deck, and on to some posts that are near his pool. The design of the sail is shown opposite. Calculate the area of the material that Ben needs for the sail.



5 Hans has been commissioned to re-paint a fading sign of a pizza restaurant. He is calculating how much red paint he will require for the section of the sign shown. He has discovered that the sign makes two trapeziums and has measured the dimensions shown. Calculate the total area that will require red paint.



6 Ruby is laying a large patch of grassed area next to her patio, as shown in the diagram below, to create a dog pen. Calculate the total area of turf that Ruby needs to purchase.



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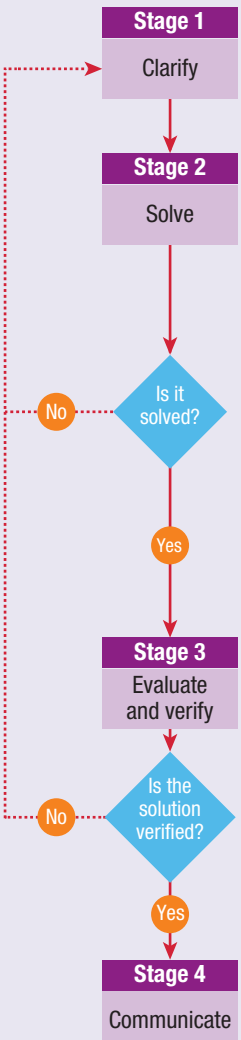
Mathematical thinking process task

Background: House and land plans provide great applications of area measurement.

Task: Your task is to design a house and land package for a new estate area. You need to show the positioning and outline of the house, along with the measurements for the external walls. You must show the outline of the block of land along with some landscaping ideas, including fencing, gardens and other outdoor items (i.e. a pool or fountain). Your design does not need to be to scale; however, you must show all relevant measurements. You will need to include the area of the land, house, fenced area and the composite-shaped backyard feature.

Use the following approach to the mathematical thinking process task.



Stage 1: Clarify
Interpret the task, gather the key information and identify the mathematics which could help to complete the task

1. Research various house and land designs.
2. Research various features of backyards.

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

3. Sketch the bird's eye view of your land, ensuring that you include measurements, and consider the functionality of your design. Could this be in the shape of a parallelogram or trapezium?
4. Add the outline of your house. Could you make it a composite shape? Include any measurements.
5. Add the fencing design of your house and land package.
6. Add the backyard feature.
7. Calculate the area for the land, house, fenced area and backyard feature.

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources and verify the reasonableness of the solution

8. Check that you have included as many measurements as possible in your design.
9. Check that your composite shapes have at least two familiar shapes.
10. Check that you have shown your working for your area calculations.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

11. Share your design with a classmate and have them check your calculations.
12. Write a report to communicate your findings.

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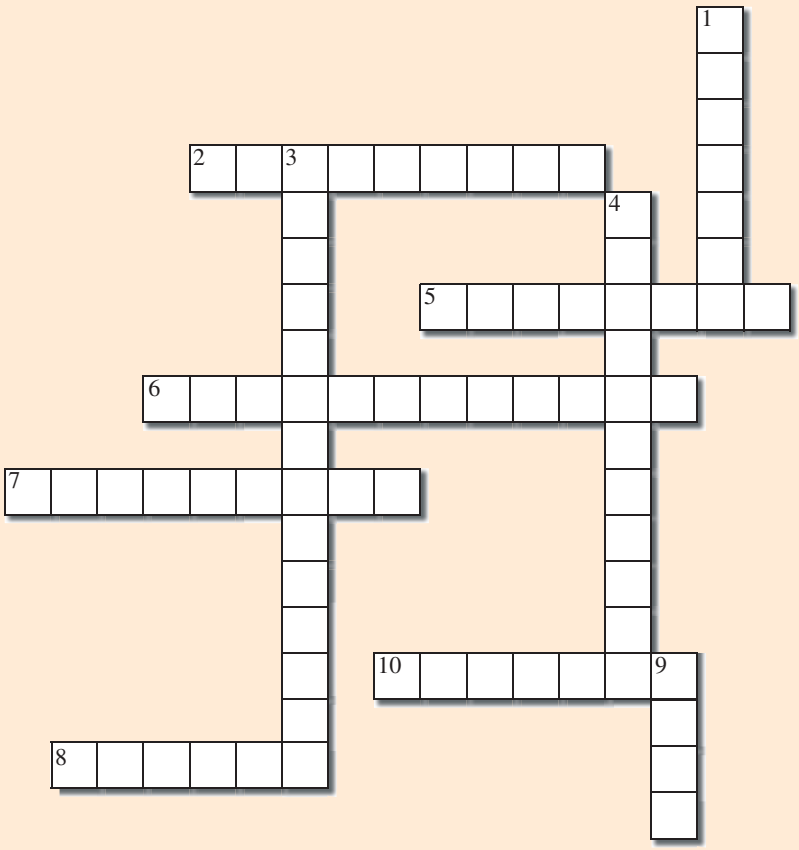
Puzzle



Across

2 $A = l \times w$: This is the formula for the area of a?
5 You can do this when calculating areas to find an approximate answer quickly
6 Replace variables with numerical values in a formula
7 The continuous line forming the boundary of a closed geometrical figure
8 $A = l^2$ or $l \times l$: This is the formula for the area of a ...?
10 A mathematical relationship connecting two or more variables

Down

1 Change units of measurement from one to the other
3 A shape that is made up of two or more basic shapes
4 2D means 2?
9 A measure of the space enclosed by the boundaries of a 2D shape

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Chapter checklist

☐ I can use units of length and their abbreviations and identify the most appropriate metric units for length.

1 Use abbreviations to identify the most appropriate unit of length to measure each of these:
- **a** length of a guinea pig
- **b** height of a large tree
- **c** length of a bolt
- **d** distance between towns

2 List some techniques that you can use to help you estimate lengths.

3 Choose the best estimate for the following measurements.
- **a** Thickness of a garden hose
  - **i** 12 mm **ii** 100 mm **iii** 3 cm
- **b** Length of an adult male foot
  - **i** 15 cm **ii** 30 cm **iii** 50 cm
- **c** Perimeter of a house
  - **i** 300 m **ii** 0.8 km **iii** 62 m
- **d** Distance from Perth to Gold Coast
  - **i** 10 000 m **ii** 2500 km **iii** 4000 km

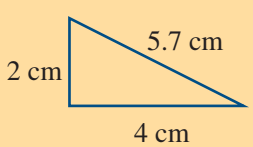
☐ I can convert between units of length.

4 Convert the following measurements into the units given in brackets.
- **a** 4 cm (mm)
- **b** 30 mm (cm)
- **c** 4.5 km (m)
- **d** 7.3 m (cm)

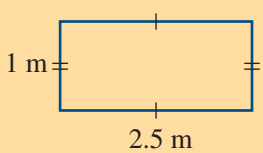
☐ I can calculate and estimate the perimeter of various familiar shapes.

5 Calculate the perimeter of each of the following familiar 2D shapes.

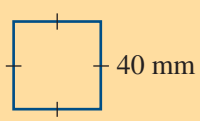
a



b



c



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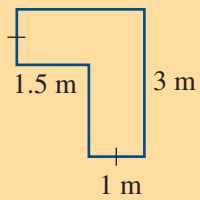
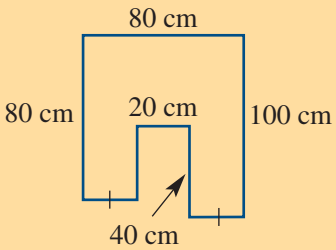
☐ I can calculate the perimeter of composite shapes by decomposing them into common shapes.

6 Calculate the perimeter of the following composite shapes. Where appropriate, write your answers correct to two decimal places.

a



b



☐ I can use units of area and their abbreviations and identify the most appropriate metric units for area.

7 Determine the abbreviation for:

a square millimetres

b square kilometres

8 Write the units out in full.

a cm^2

b m^2

c ha

9 State the most appropriate choice of units to measure the following areas.

a surface area of a classroom desk

b suburban block of land

c horse paddock

d outback cattle station

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☐ I can convert between units of area.

10 Convert the following measurements into the units given in brackets.

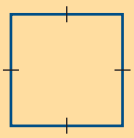
a $6\,m^2$ (cm^2) **b** $9\,cm^2$ (mm^2) **c** $350\,mm^2$ (cm^2)

d $2.7\,km^2$ (m^2) **e** $6\,km^2$ (ha) **f** $7\,m^2$ (mm^2)

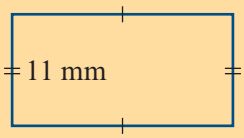
☐ I can calculate and estimate the area of various familiar shapes.

11 Calculate the area of the following common 2D shapes.

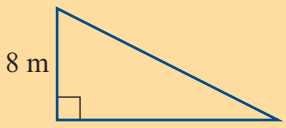
a



b

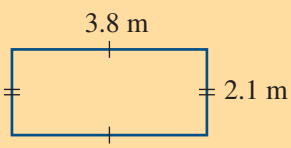


c

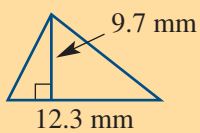


12 Estimate the area of the following common 2D shapes by first rounding to the nearest whole number.

a



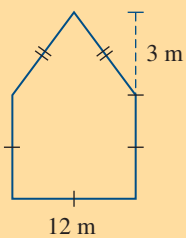
b



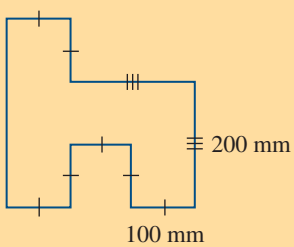
☐ I can calculate the area of composite shapes by decomposing them into common shapes.

13 Calculate the area of the following composite shapes.

a



b



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Chapter review

1 A warehouse receives a shipment of widgets, but they have all been labelled with their lengths in the wrong units for their particular uses. Convert the measurements to the units shown in brackets so that they can be correctly labelled.

a 38 m (cm)

b 15 cm (mm)

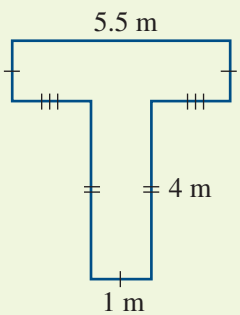
c 44 mm (cm)

d 1.79 m (cm)

2 Beatrix is planning to sew lace around the perimeter of a 45 cm square cushion that she has just completed. Determine how many metres of lace Beatrix will require.

3 Tara works at a school in maintenance. She has been asked to build a new sandpit that requires a 6 m by 8 m rectangular frame (measured around the outside of the timber frame). Determine how many metres of timber Tara needs to order.

4 Lee-Ann is planning to concrete a path to her front door and underneath her patio. She needs to first construct a timber frame to hold the concrete in place. The diagram below shows Lee-Ann's plan for the concrete. Calculate how much timber Lee-Ann needs for the timber frame.



5 Daniel is painting a 19.2 m^2 feature wall in his house. Determine the area of Daniel's wall in square centimetres.

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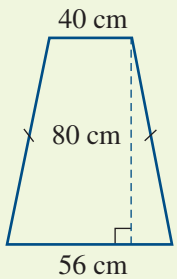
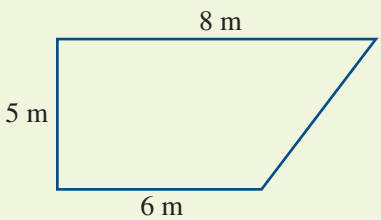
6 Lyla's horse paddock is 27 190 m^2 where her 3 horses roam. Determine the area of the horse paddock in hectares.

7 Ben has calculated that his front door is 1 810 000 mm^2. Determine the area of the front door in square metres.

8 Emily has designed an A-line skirt that she is now planning on sewing. The diagram below shows the pattern for the front panel of the skirt. Calculate the area of material that Emily will need to make the front of the skirt.



9 Mason is polishing a lounge-room floor, as shown in the diagram below. Calculate the area of the floor that Mason is polishing



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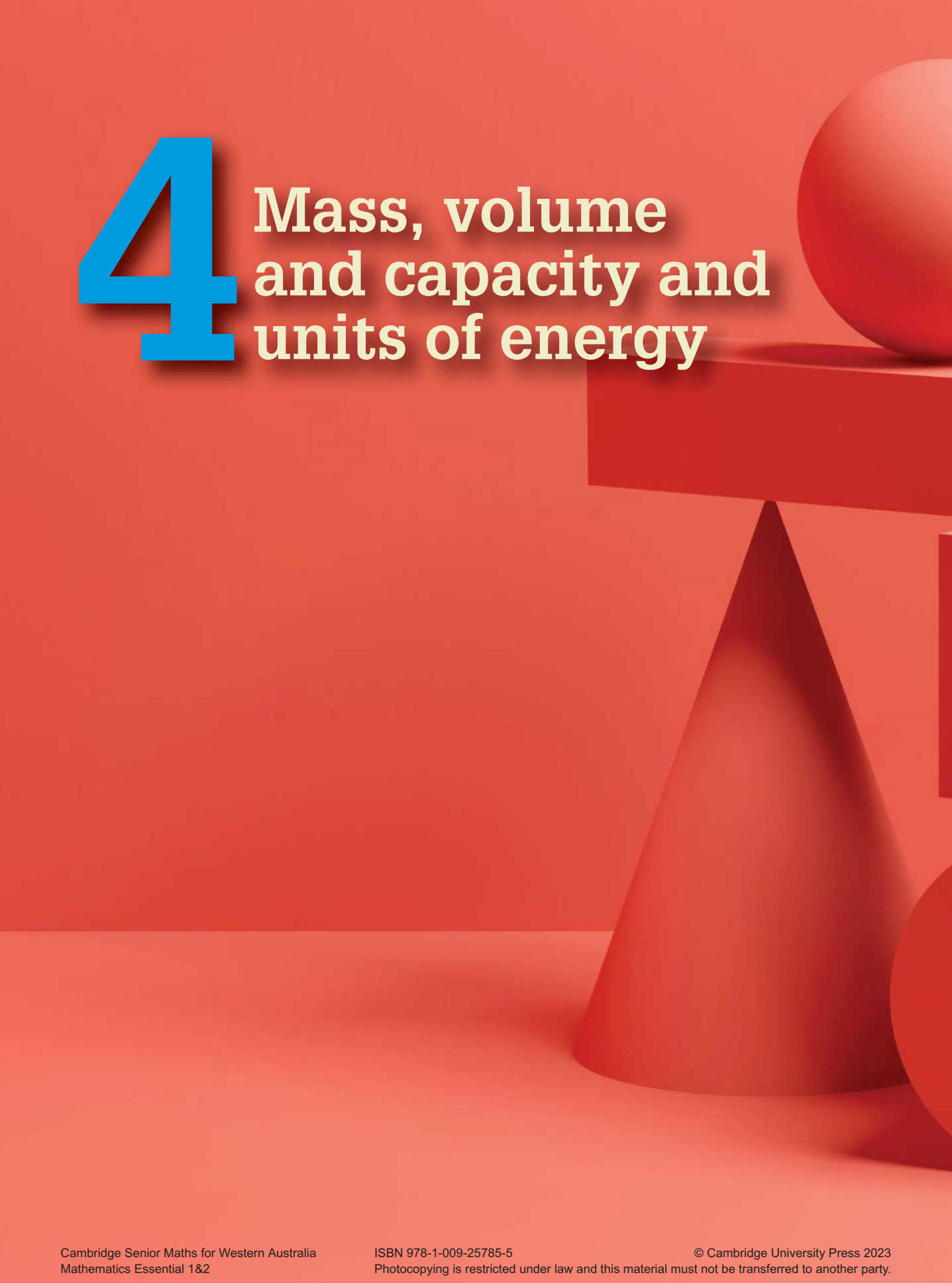
4 Mass, volume and capacity and units of energy



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In this chapter

Syllabus reference

Unit 1 Topic 1.3: Measurement

Mass

- choose and use appropriate metric units of mass, their abbreviations and conversions between them
- estimate the mass of different objects

Volume and capacity

- choose and use appropriate metric units of volume, their abbreviations and conversions between them
- understand the relationship between volume and capacity, recognising that 1 cm^3 = 1 mL and 1 m^3 = 1 kL
- estimate volume and capacity of various objects
- calculate the volume and capacity of cubes and rectangular and triangular prisms

Units of energy

- use units of energy to describe consumption of electricity, such as kilowatt hours
- use units of energy used for foods, including kilojoules and calories
- use units of energy to describe the amount of energy expended during activity
- convert from one unit of energy to another, such as calories/kilojoules

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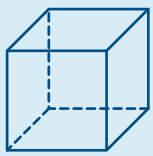
Pre-test

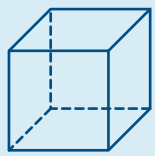
1 Explain the meaning of:

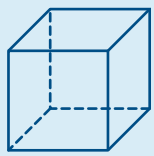
a 'milli-' as used in the unit 'millimetre'

b 'kilo-' as used in the unit 'kilometre'

2 Each cube represents a unit of volume. They are not drawn to the same scale. Write down the full name of each unit and its abbreviation.


1 mm


1 cm


1 m

3 How many millilitres are in a litre?

4 'Mass' refers to:

A the capacity of an object

B the amount of matter in a solid, liquid or gas

C the amount of space a solid, liquid or gas occupies

D the pull of gravity

5 Identify which of these units can be used to measure mass: litres, grams, square metres, kilograms, hectares, cubic centimetres, tonnes.

6 Identify which of these units can be used to measure energy: joules, square metres, litres, kilowatts, calories, hectares, cubic centimetres.

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4A Using and converting between metric units of mass

LEARNING GOALS

- Understand the metric units of mass (milligrams, grams, kilograms, metric tonnes)
- Use abbreviations for the metric units of mass (mg, g, kg, t)
- Convert between the metric units of mass:
- Recognise appropriate choice of units

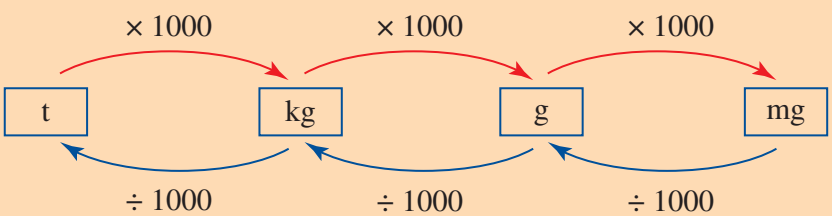
Why is it essential to use and understand the metric units of mass?

Understanding how to use and convert mass units and estimate mass is essential in careers but also in many aspects of life. In the transport industry, it is important to know the mass of things being loaded onto trucks and the trucks' maximum load capacity. In the health and medical industry, the application is crucial when administering medication. In the cooking industry, it is important to be able to convert between mass units in order to follow recipes.

A huge variety of scales for measuring mass are used in everyday life.

WHAT YOU NEED TO KNOW

- Mass is the amount of matter in an object. In everyday life we measure mass by its weight, which is the force exerted on it by gravity, and so mass and weight are used interchangeably.
  - 1 tonne = 1000 kilograms
  - 1 kilogram = 1000 grams
  - 1 gram = 1000 milligrams
- Abbreviations for units of mass are metric tonne (t), kilogram (kg), gram (g) and milligram (mg).
- Conversion between units of mass:



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Example 1 Converting between units of mass

Convert these mass measurements into the units given in brackets.

a 0.7 kg (g)
b 793 500 mg (g)
c 5 t (kg)
d 2.3 kg (mg)
e 6000 mg (kg)

WORKING	THINKING
a $0.7 \times 1000 = 700$ g	As there are 1000 grams in a kilogram, multiply 0.7 by 1000 to convert to the smaller unit.
b $793\,500 \div 1000 = 793.5$ g	As there are 1000 milligrams in a gram, divide 793 500 by 1000 to convert to the larger unit.
c $5 \times 1000 = 5000$ kg	As there are 1000 kilograms in a tonne, multiply 5 by 1000 to convert to the smaller unit.
d $2.3 \times 1000 = 2300$ g $2300 \times 1000 = 2\,300\,000$ mg	As there are 1000 grams in a kilogram, multiply 2.3 by 1000 to convert to the smaller unit. As there are 1000 milligrams in a gram, multiply 2300 by 1000 to convert to the smaller unit.
e $6000 \div 1000 = 6$ g $6 \div 1000 = 0.006$ kg	As there are 1000 milligrams in a gram, divide 6000 by 1000 to convert to the larger unit. As there are 1000 grams in a kilogram, divide 6 by 1000 to convert to the larger unit.

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Example 2 Applying the conversion of units of mass to practical problems

Doug's new caravan weighs 1.93 tonnes.

a Determine the mass of the caravan in kilograms.

b If Doug added 50 kg of food and water plus 35 kg of linen and clothing to his caravan before heading away on a trip, determine the new weight of the caravan in tonnes.

WORKING	THINKING
a $1.93 \times 1000 = 1930$ kg	As there are 1000 kilograms in a tonne, multiply 1.93 by 1000 to convert to the smaller unit.
Doug's caravan weighs 1930 kilograms.	Communicate your answer in a sentence.
b $1930 + 50 + 35 = 2015$ kg	Use the weight of the caravan in kilograms and add the weight of the food and water and the linen and clothing.
$2015 \div 1000 = 2.015$ t	As there are 1000 kilograms in a tonne, divide by 1000 to convert to the larger unit.
Doug's caravan now weighs 2.015 tonnes.	Communicate your answer in a sentence.

Exercise 4A

FUNDAMENTALS

Example 1

1 Convert these mass measurements into the units given in brackets.

a 0.85 kg (g)
b 973 400 mg (g)
c 2300 g (kg)
d 7.5 t (kg)
e 32 g (mg)
f 3570 kg (t)
g 4.7 kg (mg)
h 0.085 t (g)
i 78 000 mg (kg)
j 3 560 000 g (t)

APPLICATIONS

Example 2

2 Shae has had a new baby who weighs 3.45 kg. Determine the weight of Shae's baby in grams.

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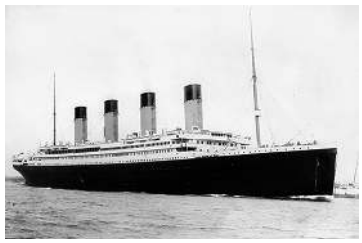
3 Jesse works in a food laboratory and needs to know the weight of a steak in milligrams to use in a formula. The supplied steak is labelled as being 450 g. Determine the weight of the steak in milligrams.

4 The Titanic ship weighed 52 310 tonnes. Scott needs to know the weight of the ship in kilograms to compare it to a table of all the materials used in its construction. Calculate the weight of the Titanic in kilograms.



5 Esther has an ant farm that has around 420 worker ants inside and each ant weighs 3 milligrams. Calculate the total weight of the ants in grams.

6 Jude's caravan weighs 2.9 tonne.

a Determine the mass of the caravan in kilograms.

b If Jude packs the van with 20 kg of food, 60 kg of water and 40 kg of linen and clothing, determine the total weight of the caravan in kilograms.

7 Grace has packed 90 small packets of chips that weigh 45 grams each, ready for a school camp. Determine the total weight of all the packets of chips in kilograms.

8 Isaiah has returned from an overseas holiday. When he departed, his bags weighed 25.2 kg. After a three-week holiday in Europe, his bags weighed 28.9 kg. Determine the weight in grams of the items that Isaiah purchased overseas.

9 Ivanna is moving house and has hired a shipping container to store her household items. The shipping container weighs 3.7 tonnes and she loads it with 9940 kilograms of furniture and homewares. Calculate, in tonnes, the total weight of the shipping container once it has been packed with Ivanna's items.

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4B Choosing appropriate units, estimating mass and recognising the need for milligrams

LEARNING GOALS

- Recognise the appropriate choices of units
- Estimate the mass of various objects
- Recognise the need for milligrams

Why is it essential to estimate the mass of an object?

There are many circumstances when you have to be sure that something, such as a boat or trailer, is not overloaded, which may cause it to collapse or sink. For example, when loading a small boat with baggage and people, for safety reasons you must be careful not to overload the boat, bearing in mind its maximum safe carrying capacity. If scales are not available, you will have to estimate the mass of the baggage and people loaded on board.

The boat has a label saying the maximum permitted load is 360 kg, so these people need to estimate how much weight they are putting in the boat.

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WHAT YOU NEED TO KNOW

- There are four main types of units that are used for **mass**. Below is a table that provides examples of items that are measured in each of the metric units of mass.

Mass unit	Examples
Milligrams – commonly used for measuring extremely light and small objects	Medicine, vitamins, strand of hair, mosquito or fruit fly
Grams – commonly used for measuring lightweight items	Cotton ball, small business card, insect or small animal
Kilograms – used for measuring heavier items	1 litre of water (= 1 kg), whole pineapple, baseball bat, baby, adult or large animal
Tonnes – used for measuring very heavy items	Car, plane, ship, truck, train, tractor or blue whale

- Why milligrams? Milligrams are necessary as they help with precise measurement of very light objects. They are particularly important in medicine.
- Conversion of units of mass: see page 117 for chart.
- When estimating the mass of an object, it is important to keep in mind some typical weights to which you can compare the object. Below is a list of common items with their average weight.
  - Mosquito 2.5 mg
  - Vitamin C tablet 1000 mg (1 g)
  - Single paperclip 1 g
  - A4 sheet of paper 5 g
  - Mouse 19 g
  - One cup of butter 225 g
  - 1000 paper clips 1 kg
  - 1 litre of water 1 kg
  - Supermarket bag of groceries 5–10 kg
  - Large suitcase packed with clothes and personal items 20 kg
  - Newborn baby 3.5 kg
  - Labrador dog 30 kg
  - Average-sized adult human 65–75 kg
  - Small car 1 t
  - Adult polar bear 1 t
  - Shipping container 3.5 t
  - Blue whale 150 t
  - Mid-sized aeroplane 60 t

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Example 3 Choosing the appropriate unit of mass

Decide on the appropriate mass units that would be used when weighing the following.

a 

b 

c 

d 

WORKING		THINKING
a grams	◄	Grams are a relatively small unit of mass. A teaspoon of sugar would be quite light.
b kilograms	◄	Large dogs look like they could weigh around half the size of humans. Kilograms would be the most appropriate unit of mass.
c tonnes	◄	Large ships would be extremely heavy, therefore tonnes would be the most appropriate unit of mass.
d milligrams	◄	Small butterflies are very light-weight, therefore milligrams would be the best unit of mass.

Example 4 Estimating mass

Select the mass which is the most accurate for the following objects.

a A bunch of flowers are around ... i 800 mg ii 890 g iii 8 kg
b A car tyre is around ... i 750 g ii 7.2 kg iii 1 t
c A train carriage is around ... i 2.8 t ii 45 t iii 980 kg

WORKING	THINKING
a ii 890 g b ii 7.2 kg c ii 45 t	Think of things that you know the weight of and then use that knowledge to estimate the weight of these items.

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Exercise 4B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a A _____ is a very small mass unit of measure that is for _____ lightweight objects, such as _____, vitamins and _____.

b _____ are the units used to measure relatively light objects, such as a paperclip or a mouse.

c One litre of _____ has a mass of one _____.

d Extremely large and heavy items can be measured using the unit _____, for example an aeroplane.

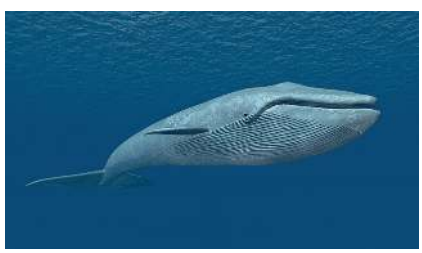
Example 3

2 Decide on the appropriate mass unit (mg, g, kg or t) that would be used when weighing the following objects.

a

b

c



d

Example 4

3 In your surrounds, look for the following items to pick up and estimate their mass. Compare your estimation with a classmate, and if scales are available, check the actual weight of the items.

a pencil
b classroom desk
c stapler
d chair
e book
f full pencil case
g eraser
h shoe
i an item of jewellery

Hint A millilitre of water weighs 1 g, and a litre of water weighs 1 kg. Sometimes it helps to imagine a volume of water about the same size as the object, and then estimate what that would weigh. Then adjust the estimate up or down, taking into account whether it's made from a material that is 'heavier' or 'lighter' than water.

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APPLICATIONS

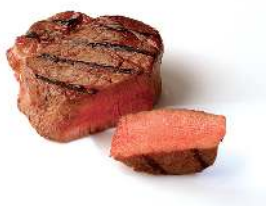
Example 3

4 Decide on the appropriate mass units that would be used when weighing the following:

a

b

c



d

e

f

g

h

i

j

k

l

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Example 4

5 Select which mass is most likely accurate for the following objects.

a	A watermelon weighs around ...	**i**	100 g	**ii**	9 kg	**iii**	1 kg
b	A butterfly weighs around ...	**i**	2 g	**ii**	1 kg	**iii**	60 mg
c	An Olympic pool weighs around ...	**i**	2400 t	**ii**	980 kg	**iii**	2 t
d	A calculator weighs around ...	**i**	0.5 t	**ii**	2 kg	**iii**	65 g
e	An elephant weighs around ...	**i**	1000 g	**ii**	1 t	**iii**	100 kg
f	A smart phone weighs around ...	**i**	200 g	**ii**	1.5 kg	**iii**	900 g
g	A skateboard weighs around ...	**i**	3 kg	**ii**	12 kg	**iii**	600 g
h	A tennis ball weighs around ...	**i**	800 g	**ii**	1.3 kg	**iii**	59 g
i	A cricket bat weighs around ...	**i**	45 g	**ii**	1 kg	**iii**	3 t
j	A cow weighs around ...	**i**	780 kg	**ii**	2 t	**iii**	1230 g

6 Use the images from question **4** and estimate a mass for each object.

Use the following information for questions **7** to **10** and utilise a search engine for your estimations.

Big Ted's Truck Hire have different-sized moving trucks that they hire out: a 500-kg ute, 1-tonne truck, 3-tonne truck and a 10-tonne truck.

7 Graham is moving a washing machine and a dryer. Estimate their weight and hence determine what size vehicle he must hire.

8 Leanne is moving a large concrete water fountain from her front yard. It comes apart for transportation but requires four people to lift each section. Estimate the weight of the fountain and hence determine what size vehicle she will need to hire.

9 Melanie is moving the furniture from her two-bedroom apartment, which includes a fridge, washing machine, two beds, two-piece lounge suite, dining table and chairs, two chests of drawers and all the contents of the kitchen. Estimate the weight of her furniture and hence determine the size of the truck she will need to hire.

10 Cassie and Ashley are moving house. They have a four-bedroom house and three children with lots of toys, including a trampoline, and lots of furniture as they have two living rooms. Estimate the weight of their house contents and hence determine the vehicle they will need to hire.

11 Luan is using a canoe to transport himself, a supermarket bag of groceries and 10 L of drinking water to an island picnic site. The canoe is labelled to take a maximum load of 100 kg. If Luan weighs 75 kg, decide whether he is likely to be under or over the limit.

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4C Using and converting between metric volume and capacity units

LEARNING GOALS

- Understand the use and appropriate choice of the metric units of volume including cubic millimetres, cubic centimetres and cubic metres
- Use abbreviations such as mm^3, cm^3 and m^3 to represent units of volume
- Convert between units of volume
- Understand the relationship between volume and capacity including the conversions between millilitres, litres, kilolitres and megalitres

Why is it essential to understand units of volume and capacity?

There are many careers, including pool installation, landscaping, cooking, storage hire or hairdressing, where volume and **capacity** can be used. The production, storage, distribution and use of materials, whether gas, liquid or **solid**, and their containers, requires measurement and conversion of volume and capacity.

It is an essential skill to be able to convert between the units of volume and capacity so that the capacity (or amount of gas, solid or liquid) can be calculated using the measurements of an object.

Measuring volume is a vital skill in laboratories.

WHAT YOU NEED TO KNOW

- Calculating the **volume** of a solid involves measuring the amount of space taken up by a 3D shape.
- **Units of volume** include cubic millimetres, cubic centimetres, cubic metres and cubic kilometres, which can be abbreviated to mm^3, cm^3, m^3 and km^3.
- To convert between units of volume:
  - use multiplication when converting from a larger unit to a smaller unit (e.g. from m^3 to cm^3)
  - use division when converting from a smaller unit to a larger unit (e.g. from cm^3 to m^3).

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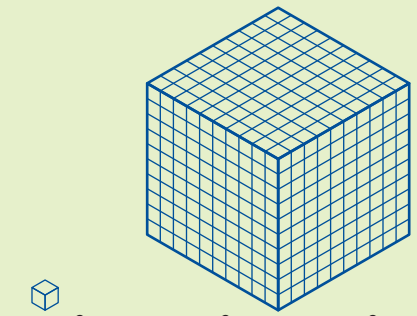
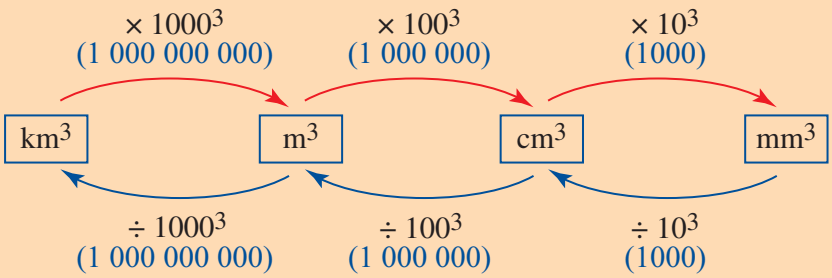
- If one linear unit is 10 times longer than another, then its cubic unit is 10^3 (i.e. 1000 times bigger).

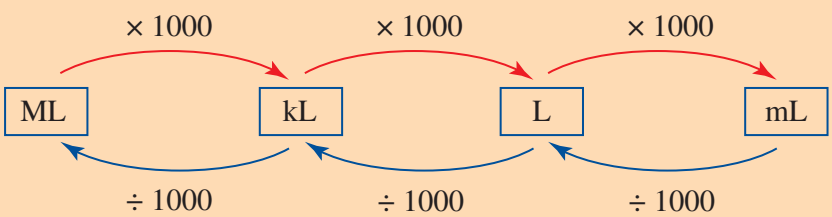


1 mm^3 1 cm^3 (1000 mm^3)

- Refer to the following conversion chart when solving problems that require conversion of units of volume.



- **Capacity** is the same as volume, but in everyday contexts we use capacity for the volume that a container (3D solid) can hold of a substance (solid, liquid or gas).
- Units of capacity include megalitres, kilolitres, litres and millilitres, which can be abbreviated to ML, kL, L and mL.
- Capacity is measured in units based on the litre. Use the following conversion chart for problems that require conversion of units of capacity.



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- To convert between units of volume and capacity:
  - $1\text{ cm}^3 = 1\text{ mL}$
  - $1000\text{ cm}^3 = 1000\text{ mL} = 1\text{ L}$
  - $1\text{ m}^3 = 1000\text{ L} = 1\text{ kL}$
  - $1000\text{ m}^3 = 1000\text{ kL} = 1\text{ ML}$
  - $1\,000\,000\text{ L} = 1\text{ ML}$

Example 5 Converting between units of volume

Convert these volume measurements into the units given in brackets.

a 0.72 m^3 (mm^3)

b 10.5 cm^3 (mm^3)

c 393 mm^3 (cm^3)

d 800 cm^3 (m^3)

WORKING	THINKING
a $0.72 \times 1\,000\,000\,000 = 720\,000\,000\text{ mm}^3$	As there are 1000^3 cubic millimetres in a cubic metre and we are converting from m^3 to a smaller unit of mm^3, we need to multiply 0.72 by 1000^3, which is 1 000 000 000.
b $10.5 \times 10^3 = 10\,500\text{ mm}^3$	As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from cm^3 to a smaller unit of mm^3, we need to multiply 10.5 by 10^3.
c $393 \div 10^3 = 0.393\text{ cm}^3$	As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from mm^3 to a larger unit of cm^3, we need to divide 393 by 10^3.
d $800 \div 100^3 = 0.0008\text{ m}^3$	As there are 100^3 cubic centimetres in a cubic metre and we are converting from cm^3 to a larger unit of cubic metres, we need to divide 800 by 100^3.

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Example 6 Converting between units of capacity

Convert these capacity measurements into the units given in brackets.

a 2125 mL (L)

b 0.5 L (mL)

c 2.8 ML (L)

WORKING	THINKING
a $2125 \div 1000 = 2.125$ L	To convert millilitres to litres, going from a smaller to a bigger unit, divide by 1000.
b $0.5 \times 1000 = 500$ mL	To convert litres to millilitres, going from a bigger to a smaller unit, multiply by 1000.
c $2.8 \times 1\,000\,000 = 2\,800\,000$ L	To convert megalitres to litres, going from a bigger to a smaller unit, multiply by 1 000 000.

Example 7 Converting between units of volume and units of capacity

Convert these volume and capacity measurements into the units given in brackets.

a 325 cm^3 (mL)

b 175 kL (m^3)

c 2 L (cm^3)

d 1825 cm^3 (L)

WORKING	THINKING
a $325\ cm^3 = 325$ mL	There is 1 millilitre in 1 cubic centimetre.
b 175 kL $= 175\ m^3$	There is 1 cubic metre in 1 kilolitre.
c 2 L $\times 1000 = 2000$ mL $= 2000\ cm^3$	As there are 1000 mL in a litre, multiply by 1000 to convert to mL, and then use 1 mL = 1 cm^3.
d $1825\ cm^3 = 1825 \div 1000$ $= 1.825$ L	As there are 1000 cubic centimetres in a litre, divide by 1000 to convert to litres.

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Example 8 Applying unit conversion between volume and capacity

Mackenzie works at a plant nursery and has to fill 640 pots with soil. The pots have a capacity of 750 mL. The soil is measured in cubic metres. Determine the approximate amount of soil, in cubic metres, Mackenzie needs to fill the pots.

WORKING	THINKING
$750\text{ mL} = 750\text{ cm}^3$	Convert 750 mL to cm^3.
$750\text{ cm}^3 \times 640 = 480\,000\text{ cm}^3$	Multiply this by 640 pots.
$480\,000\text{ cm}^3 \div 1\,000\,000 = 0.48\text{ m}^3$	There are 100^3 cubic centimetres in a cubic metre, and we are converting from cm^3 to a larger unit, so divide 480 000 by 100^3, which is 1 000 000.
Mackenzie needs 0.48 m^3 of soil.	Communicate your answer by writing down the result.

Exercise 4C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Use multiplication when converting from a _____ unit to a _____ unit, for example, when converting from km^3 to m^3.
- **b** Use _____ when converting from a smaller unit to a larger unit, for example, when converting from _____ to m^3.
- **c** Capacity is a measure of a 3D shapes ability to _____ a substance, such as a _____, _____ or gas.
- **d** A capacity of 1 _____ is equivalent to 1 cubic centimetre.
- **e** One litre is equivalent to _____ cubic centimetres.
- **f** One cubic metre is equivalent to one _____ (unit of capacity).
- **g** One megalitre is equivalent to 1000 _____ (unit of capacity).

Hint A litre is equivalent to a cube with a side length of 10 cm (100 mm). You can work out the number of cubic centimetres or cubic millimetres in a litre by 'cubing' the side length (raising it to the power of 3).

Hint Ten one-litre cubes (side length 10 cm each) will fit exactly along each side of a one-cubic-metre cube (side length 100 cm).

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Example 5

2 Convert these volume measurements to the units given in brackets.

a 0.86 km^3 (m^3)
b 18.4 cm^3 (mm^3)
c 2.7 m^3 (cm^3)
d 276 mm^3 (cm^3)
e 1.38 km^3 (m^3)
f 68 cm^3 (mm^3)
g 129 cm^3 (m^3)
h 2.7 m^3 (mm^3)

Hint If converting from a larger unit to a smaller unit, multiply. If converting from a smaller unit to a larger unit, divide.

Example 6–7

3 Convert these volume and capacity measurements to the units given in brackets.

a 265 cm^3 (mL)
b 412 kL (m^3)
c 6 L (cm^3)
d 2128 cm^3 (L)
e 27 mL (cm^3)
f 1247 mL (kL)
g 2.7 ML (kL)
h 27.5 m^3 (kL)
i 267 kL (ML)
j 0.8 L (mL)
k 628 mL (L)
l 3.2 ML (L)

APPLICATIONS

Example 8

4 Carlene has calculated that to spread mulch to a depth of 5 cm over a garden bed she needs to buy 800 000 cm^3 of mulch. She finds that mulch is sold in cubic metres. Calculate the number of cubic metres of mulch Carlene should order.

5 Tessa has 1089 m^3 of bitumen that she can use to pave the driveway on her farm. To work out how thick the bitumen layer can be she needs to convert this volume to cubic centimetres. Determine how many cm^3 are in 1089 m^3.

6 Lee-Ann's pool holds 12 000 L of water. Unfortunately, the pool tiles are damaged so she has decided to fill the pool in with soil and make a big garden instead. Calculate the number of cubic metres of soil Lee-Ann will need in order to fill the pool.

7 Jarred works in a factory that makes sugar cubes. A sugar cube with a size of 1 cm^3 is made from 1 mL of liquid sugar. The factory produces 5000 cubes per day. Calculate the number of millilitres of liquid sugar that the factory requires each day

8 An Olympic swimming pool has 2.5 megalitres of water. Calculate the number of litres in an Olympic swimming pool.

9 A large fuel truck carries around 10 kilolitres of fuel. Calculate how many litres of fuel a large truck can carry

10 If all the water in Sydney Harbour was poured into a cubic tank with edges of 1 km, it would fill it to a depth of 562 m. Calculate the number of megalitres of water in Sydney Harbour.

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4D Estimating and calculating the volume and capacity of prisms

LEARNING GOALS

- Calculate the volume and capacity of the following regular objects:
  - cubes
  - rectangular prisms
  - triangular prisms
  - cylinders
- Estimate the volume and capacity of the above regular objects

Why is it essential to know how to calculate volume and capacity?

Among its many uses, calculating volume and capacity is essential for packing, storing and filling containers. For example, when packing a crate into a storage container, we must calculate how much space the crate takes up and how much space is available. Another situation could be when calculating the volume of liquid that can be held by a container or a pool.

This pool is in the shape of a rectangular prism, and you will be able to calculate how much water is required to fill it.

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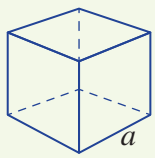
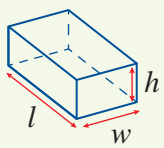
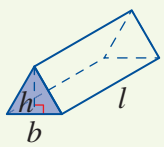
WHAT YOU NEED TO KNOW

- Right **prisms** have two faces on opposite sides to which the other faces join at **right angles**, hence the name. A cylinder is not a prism because it includes a curved surface, but it is related because it has a curved surface joining two circular faces at right angles.
- You will need the rules for area from the previous section.
- The general rule for calculating the volume is the area of the base multiplied by the height. Specific rules for calculating the volume of regular solids are found in the table below.

Solid	Rule
Cube 	$V = a^3$ (V: volume, a: edge length)
Rectangular prism 	$V = l \times w \times h$ (V: volume, l: length, w: width, h: height)
Triangular prism 	$V = \frac{1}{2}(b \times h) \times l$ (V: volume, b: width of base, h: **perpendicular** height, l: length)

- Linear measurements must be in the same units.
- The volume units will then be cubic units of the linear units used.
- When calculating capacity from linear dimensions, calculate the volume in cubic units and then convert to units of capacity.

Example 9 Converting volume to capacity units

Convert to the capacity units in the brackets. Round to one decimal place if necessary.

a $1.44\,m^3$ (kL)

b $1000\,cm^3$ (L)

c $356\,720\,mm^3$ (mL)

d $2\,035\,752.04\,cm^3$ (L)

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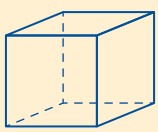
WORKING	THINKING
a 1.44 kL	◂····· Use 1 m^3 = 1 kL to convert to kL.
b 1000 ÷ 1000 = 1 L	◂····· Given that 1000 cm^3 = 1 L, divide by 1000 to convert to litres.
c 356 720 ÷ 10^3 = 356.7 cm^3 = 356.7 mL	◂····· Convert the answer to cm^3 by dividing by 10^3. Use 1 cm^3 = 1 mL to convert to mL.
d 2 035 752.04 ÷ 1000 = 2035.8 L	◂····· Given that 1000 cm^3 = 1 L, divide the answer by 1000 to convert to litres.

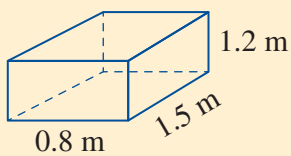
Example 10 Calculating the volume of solids

Calculate the volume of the following solids.

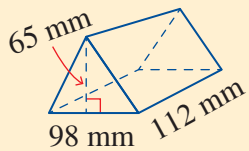
a



b



c



WORKING	THINKING
a $V = a^3$ $V = 10^3$ $V = 1000\ cm^3$	◂····· As the solid is a cube, apply the formula $V = a^3$. Substitute the value for a, which is 10.
b $V = l \times w \times h$ $V = 0.8 \times 1.5 \times 1.2$ $V = 1.44\ m^3$	◂····· As the solid is a rectangular prism, apply the formula $V = l \times w \times h$. Substitute the values for $l = 1.5$, $w = 0.8$ and $h = 1.2$. Note: it does not matter which order you multiply these values in.
c $V = \frac{1}{2}(b \times h) \times l$ $V = \frac{1}{2}(98 \times 65) \times 112$ $V = 356\,720\ mm^3$	◂····· As the solid is a triangular prism, apply the formula $V = \frac{1}{2}(b \times h) \times l$. Substitute the values for $b = 98$, $h = 65$ and $l = 112$.

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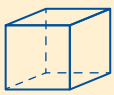
Example 11 Estimating volume and capacity

Estimate the volume and capacity of the following shapes by first rounding each measurement to the nearest whole number.

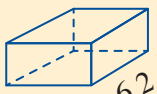
a



3.2 cm

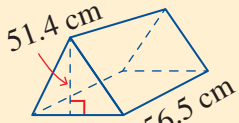
Convert to mL

b



Convert to kL

c



Convert to L

WORKING	THINKING
a 3.2 rounds to 3	Given that the first decimal place is less than 5, round down to the nearest whole number, which equals three.
$V = a^3$	The solid is a cube so apply the formula $V = a^3$.
$V = 3^3 = 27\,\text{cm}^3$	
The capacity is 27 mL.	Use $1\,\text{cm}^3 = 1\,\text{mL}$ for capacity.
b 4.8 rounds to 5 6.2 rounds to 6 3.4 rounds to 3	Check the value of the first decimal place. If the value is less than 5 then round down to the nearest whole number. If the first decimal place is 5 or more, round the whole number up by one.
$V = l \times w \times h$ $V = 5 \times 6 \times 3 = 90\,\text{m}^3$	The solid is a rectangular prism so apply the formula $V = l \times w \times h$.
The capacity is 90 kL.	Use $1\,\text{m}^3 = 1\,\text{kL}$ for capacity.
c 45.2 rounds to 45 51.4 rounds to 51 56.5 rounds to 57	Estimate the values given.
$V = \frac{1}{2}(b \times h) \times l$ $V = \frac{1}{2}(45 \times 51) \times 57$ $V = 65\,407.5\,\text{cm}^3$	The solid is a triangular prism so apply the formula $V = \frac{1}{2}(b \times h) \times l$.
The capacity is $65\,407.5 \div 1000 = 65.41\,\text{L}$	Use $1000\,\text{cm}^3 = 1\,\text{L}$ for capacity.

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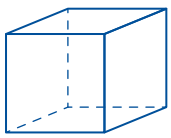
Exercise 4D

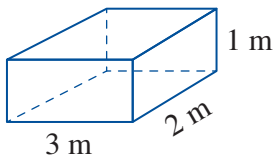
FUNDAMENTALS

1 Calculate the volume of the following solids.

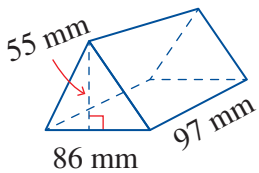
a



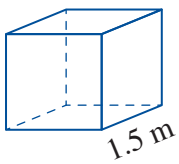
b



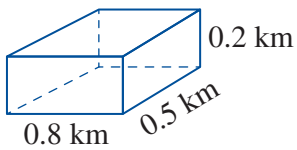
c



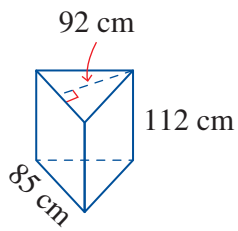
d



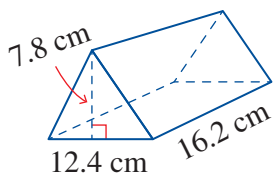
e



f

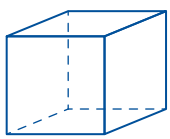


g



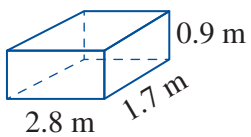
Example 9, 10 **2** Calculate the volume and then the capacity of the following solids.

a



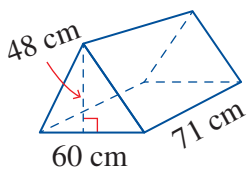
Convert to mL

b



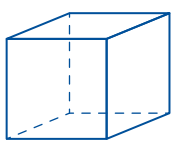
Convert to kL

c



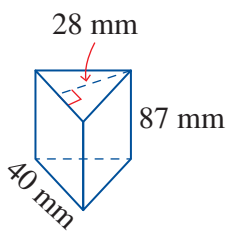
Convert to L

d



Convert to ML

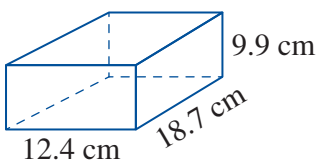
e



Convert to mL

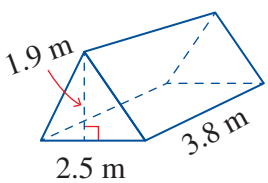
Example 11 **3** Estimate the volume and then the capacity of the following solids.

a



Convert to L

b



Convert to kL

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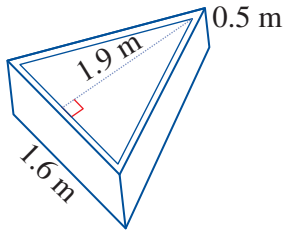
APPLICATIONS

4 Mim has built a triangular garden bed as shown. She plans on filling the garden bed with soil. Determine how many cubic metres of soil Mim will require.



5 Lisa has a 45-cm cubic cardboard box. Calculate the volume of the box.

6 A Toblerone box is 30.6 cm long and the base of the triangle is 5.4 cm with a height of 5 cm. Determine the capacity of the box in millilitres.

7 Kyösti's box of cereal is shown. Calculate the capacity of the box in litres.

8 Rochelle is installing a new pool in her backyard. It will be a flat-bottomed pool with a depth of 1.4 metres. The length of the pool will be 12 metres and the width will be 8 metres. Calculate how much water Rochelle's pool will hold in kilolitres.

ISBN 978-1-009-25785-5

4E Calculating energy and energy being used

LEARNING GOALS

- Convert between the food energy units of kilojoules (kJ) and calories
- Use nutritional label data to calculate the kilojoules and calories in food products
- Calculate the energy used from exercise

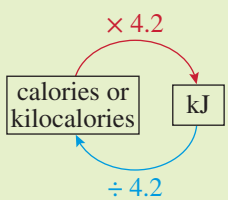
What is the essential reason for counting calories or kilojoules?

Nutrient-dense foods, such as bananas, salmon, kale, blueberries and sweet potato, offer more nutrients for fewer calories.

One of the widely accepted ways to maintain a healthy body weight uses food-energy tracking or calorie counting. Our body requires a certain amount of energy every day and one way of keeping track of how much energy you feed your body is by calorie counting. There are other accepted scientific ways of weight control; for example, eating meals with increased protein and reduced carbohydrates or eating fewer calories on only two days a week, commonly called the 5:2 diet plan. Each technique reduces the body's energy intake (which is measured in calories or kilojoules) compared to the energy used by activity.

WHAT YOU NEED TO KNOW

- **Food energy** is measured in everyday units of **calories** (cal) and **scientific** (SI) units of **kilojoules** (kJ).
- For historical reasons a food energy calorie is also called a **kilocalorie** (keal).
  - So 60 food calories = 60 keal.
- 1 calorie or kilocalorie equals approximately 4.2 kilojoules (kJ).
  - For example: 60 calories = 60×4.2 kJ = 252 kJ



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- Government regulation requires **nutrition information labels** on food products. This label includes energy in kilojoules (kJ) per serving and per 100 g or 100 mL.
- Your energy requirement varies with gender, age, weight and activity level. For average-sized 17-year-old students with low activity levels, the daily energy requirements are about 13 000 kJ for males and 10 500 kJ for females. These daily requirements increase for people who are more active.
- Weight loss occurs when the body uses more energy than the food and drink calories consumed. One school of thought for dieting is the use of calorie counting.
- **Units of Electrical Energy** The unit of energy for electricity is the '**watt**' **(W)**. Most commonly the unit used is the '**kilowatt**' **(kW)** equal to **1000 W**. Electricity used in houses and businesses is measured in '**kilowatt hours**' (kWh). This is how many kW are used in one hour of energy usage.
- The cost of using electrical energy is calculated by multiplying the number of kWh by a '**tariff**' or the amount of money charged per kWh.

Australian nutrition information labels state energy in units of kilojoules (kJ).

Example 12 Converting between kilojoules (kJ) and calories

Convert the following energy units, rounding the result to the nearest whole number.

a 1350 kilojoules (kJ) to calories

b 620 calories to kilojoules (kJ)

WORKING	THINKING
a $1350 \div 4.2 = 321$ calories	Kilojoules ÷ 4.2 gives calories.
b $623 \text{ calories} \times 4.2 = 2617$ kJ	Calories × 4.2 gives kilojoules.

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Example 13 Using nutritional data to calculate the kilojoules and calories in a food product

For the following packets of food, calculate the total energy in units of kilojoules (kJ) and calories. Round each answer to the nearest whole number.

a Liquorice Lollies have 2786 kJ per 100 g. Calculate the total energy in a 275 g packet in calories.

b Frozen potato chips have 193 calories per 100 g. Calculate the total energy in a 750 g bag in kilojoules.

WORKING	THINKING
a 2786 kJ/100 g	Write the information as a rate. Divide both parts by 100 to find kJ per gram.
= 27.86 kJ/g	Keep the decimal places.
= 7662 kJ in a 275 g packet	Multiply both parts by 275 to find the kJ in a 275 g packet.
= 1824 calories in a 275 g packet	Divide kJ by 4.2 to give calories and round to the nearest whole number.
b 193 cal/100 g	Write the information as a rate. Divide both parts by 100 to find calories per grams.
= 1.93 cal/g	Keep the decimal places.
= 1448 calories in a 750 g bag	Multiply both parts by 750 to find the calories in a 750 g bag.
= 6080 kJ in a 750 bag	Multiply calories by 4.2 to give kJ and round to the nearest whole number.

Example 14 Calculating the calories used by an activity

Use the table shown to calculate the calories used by the listed activities. Round each answer to the nearest whole number.

a A 72-kg person dancing for one hour.

b A 67-kg person cycling at 15 km/h for 2.5 hours.

Exercise energy chart Cal/kg of body mass per 30 min of exercise	
Activity	**Energy used (cal/kg/30 min)**
Cycling at 15 km/h	2.76
Dancing	2.20
Handball	5.07

... Continued

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WORKING	THINKING
a Dancing energy rate per 30 min:	
$= 2.2$ cal/kg	State the dancing energy rate per kg per half hour.
$= 158.4$ cal/72 kg person	Multiply both parts by 72 for a 72-kg person. Keep the decimal places.
Dancing energy rate per hour:	There are two half hours in 1 hour.
$= 317$ cal/72 kg person	Multiply by 2 and round to the nearest whole number.
b Cycling energy rate per 30 min:	
$= 2.76$ cal/kg	State the cycling energy rate per kg per half hour.
$= 184.92$ cal/67 kg person	Multiply both parts by 67 for a 67-kg person. Keep the decimal places.
Cycling energy used in 2.5 hours:	There are five half hours in 2.5 hours.
$= 925$ cal/67 kg person	Multiply by 5 and round to the nearest whole number.

Example 15 Solving an energy problem in a real-world context

Andy drinks a 350 mL chocolate milk, which has 185 calories per 100 mL. If Andy weighs 92 kg and wishes to burn off the milk's energy by cycling, determine approximately how long he will need to ride.

WORKING	THINKING
185 cal/100 mL $= 1.85$ cal/mL $350 \text{ mL} \times 1.85 = 647.5$ calories	First, write the information as a rate. Now divide by 100 to find cal per mL. Multiply by 350 to find the calories in the 350 mL milk drink.
Andy burns 2.76 $\times$ 92 cal per kg in 30 min	Use the table to identify the calories per kg used per 30 min.
$= 253.92$ cal/kg/30 min ≈ 8.5 cal/kg/min $\therefore 647.5 \div 8.5 = 76$ min	Divide the number by 30 to calculate the calories per kg burned per minute. Divide the total calories by the calories/kg used per minute to determine the number of minutes needed to exercise. Round your answer to the nearest whole number.
Andy would need to cycle for approximately 76 minutes.	Write your answer as a sentence.

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Example 16 Calculating the cost of electricity

A clothes dryer uses 537 kWh of electricity per year. If the tariff is 32.729 c/kWh, how much does it cost to run the clothes dryer for a year?

WORKING	THINKING
$537 \times \$0.32729 = \$175.75473$	Convert the tariff to $ and multiply the energy used in kWh by the tariff in dollars.
$= \$175.75$	Round your answer to 2 dp since it is in dollars.
The clothes dryer costs $175.75 to run for one year.	Communicate your answer.

Example 17 Calculating the cost of electricity.

TWB mining company runs a ball mill to crush the iron ore. The ball mill uses 185 000 W when it is running. TWB run the ball mill 18 hours a day. The company is charged a tariff of 25.783 c per kWh by the power company. How much does it cost TWB to run the ball mill each day?

WORKING	THINKING
$185\,000 \div 1000 = 185$ $185 \times 18 = 3330$	Convert W to kW by dividing by 1000, then multiply by 18 to get the total electricity used per day.
$3330 \times 0.25783 = 858.5739$ $= \$858.57$	Multiply the energy used in kWh by the tariff in dollars. Round your answer to 2 dp since it is in dollars.
The ball mill costs $858.57 per day to run.	Communicate your answer.

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Exercise 4E

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words and units in the following statements.

a Food energy is measured in everyday units of _____ and scientific units of _____ (__).

b Calories also can be referred to as _____ (___).

c Kilojoule conversions:
Kilojoules _ __ = calories and calories____= kilojoules.

d Food nutrition information labels state energy in units of _____ per serving and per _____ or _____.

e Weight loss occurs when the body uses _____ energy than it _____.

Example 12

2 **a** Convert the following kilojoules to calories, rounding to the nearest whole number.

i 1200 kJ **ii** 455 kJ **iii** 75 kJ **iv** 986 kJ

b Convert the following calories to kilojoules, rounding to the nearest whole number.

i 546 calories **ii** 58 calories **iii** 314 calories **iv** 182 calories

Example 13

3 For the following foods, calculate the total energy in units of kilojoules (kJ) and calories. Round each answer to the nearest whole number.

a Flakies cereal has 1590 kJ per 100 g. Calculate the total energy in a 400 g carton.

b Cool Bubbly has 319 kJ per 100 mL. Calculate the total energy in a 600 ml bottle.

c Ozzie Pies have 445 kJ per 100 g. Calculate the total energy in a 600 g packet.

d Ozzie Sausage Rolls have 1220 kJ per 100 g. Calculate the total energy in a 250 g box.

e Popcorn has 451 cal per 100 g. Calculate the total energy in a 250 g bucket.

f Potato chips have 522 cal per 100 g. Calculate the total energy in a 350 g packet.

g Blueberries have 56 cal per 100 g. Calculate the total energy in a 125 g punnet.

h Almonds have 580 cal per 100 g. Calculate the total energy in a 750 g packet.

Example 14

4 Use the table shown on the next page to calculate the calories used in the listed activities. Round each answer to the nearest whole number.

a A 50-kg boy running at 13 km/h on average for half an hour.

b A 75-kg woman walking at 5 km/h on average for 30 min.

c An 80-kg man jogging at 8 km/h on average for 45 min.

d A 64-kg boy hiking with a 15 kg load for 2.5 hours.

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e An 88-kg woman swimming at 25 m/min for 20 min.
f A 64-kg girl using a rowing machine for 15 min.
g A 62-kg teenager stair climbing for 10 min.
h A 70-kg boy playing soccer for 1.5 hours.
i A 56-kg girl skipping rope for 18 min.
j A 95-kg man hiking with no load for 4 h and 12 min.

Exercise energy chart Cal/kg of body mass per 30 min of exercise	
Activity	**Energy used (cal/kg/30 min)**
Cycling at 15 km/h average	2.76
Hiking with 15-kg load	5.18
Hiking, no load	3.42
Jogging at 8 km/h average	4.08
Rowing machine	3.97
Running at 13 km/h average	6.72
Skipping rope	6.28
Soccer	4.30
Stair climbing	3.09
Swimming at 25 m/min	2.65
Walking at 5 km/h average	1.76

APPLICATIONS

Example 15

5 For lunch, Hudson ate four slices of a Hawaiian cheesy pizza, which has 190 calories per slice, two chocolate chip cookies, each having 214 calories, and drank a 750 mL chocolate milk, which states 88 calories per 100 mL on its nutritional label. Hudson weighs 78 kg and wishes to burn off his lunch calories by stair climbing, in training for the Step Up stair challenge. Complete the following, rounding each answer to the nearest whole number.

a How many calories are in Hudson's lunch?
b What percentage is Hudson's lunch of his daily allowance of 2500 calories?
c Refer to the exercise energy chart in Question **4** to calculate the number of calories that Hudson's body would burn per minute of stair climbing.
d For approximately how many minutes will Hudson need to climb stairs to burn off these calories?

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6 Eva joined her friends at an outdoor cafe for brunch. She ate a feta and spinach omelette with smoked salmon (2764 kJ), one-third of a bowl of potato wedges (3639 kJ per bowl) and a small banana smoothie (1047 kJ). Eva weighs 65 kg and would like to burn off her breakfast energy by jogging around the local park. Complete the following, rounding each answer to the nearest whole number.

a How many calories are in Eva's brunch?

b What percentage is this breakfast of Eva's daily allowance of 2000 calories?

c Refer to the exercise energy chart in Question **4** to calculate the number of calories that Eva's body would use per minute of jogging.

d For approximately how many minutes will Eva need to jog to burn off this food energy?

Example 17

7 Calculate the annual cost of the appliances below if the tariff is 19.492 c/kWh.

a Washing machine which uses 427 kWh annually

b TV which uses 27 kWh annually

c Pool pump which uses 826 kWh annually

8 Hot Plastic Formers use an injection moulding machine. The machine does 15 moulds per hour. Each mould uses 500 W of energy and the company moulded for 16 hours per day.

a How much energy does the mould use per day?

b How much does it cost to run the machine if the tariff is 34.569 c/kWh?

Example 16

9 For breakfast, Zane ate 1 cup of Loops cereal, 1 cup (250 mL) of full-cream milk, 1 cup of scrambled eggs and two slices of toast. Use the nutrition information in the table to determine the number of calories that remain out of his daily allowance of 2500 calories.

	Serving size	Energy/serving
Loops cereal	½ cup	490 kJ
full-cream milk	125 mL	370 kJ
scrambled eggs	½ cup	791 kJ
toast	1 slice	386 kJ

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10 Beatrice makes up a stir-fry dinner that has approximately 1640 kJ in total for the four serves in the entire meal. She then adds four servings of sauce, which has the nutrition label shown. If Beatrice's husband eats two whole serves of the meal, how many calories did he consume?

Nutritional information for sauce Servings per package: 4 Serving size: 10 g		
Average quantity	per serving	per 100 g
Energy	110 kJ	1100 kJ

11 Larna eats a 50 g chocolate bar which, according to its nutrition label, has 420 calories per 100 g. Larna is 65 kg and wishes to exercise for no more than 30 min to burn off the energy gained from the chocolate bar. Referring to the exercise energy chart in Question **4**, which activity, out of cycling and hiking, would you recommend to Larna?

12 Haku is a sumo wrestler and weighs 150 kg. The doctor has advised Haku that he needs to lose weight to lower his blood pressure and reduce the risk of dying from a heart attack. Haku decides to exercise daily at the gym by cycling for 30 min on the stationary bike, stair climbing for 30 min and jogging on the treadmill.

Referring to the exercise energy chart in Question **4**, calculate how many minutes per day Haku will need to jog so his body burns 2000 cal/day from all his exercise in the gym.

13 Joe is a man who loves to exercise and loves to eat. Even though the recommended intake for men is 2500 calories, Joe prefers to eat 4200 calories and burn the extra calories through exercise. His weight is 79 kg. Referring to the exercise energy chart in Question **4**, create a daily exercise program for Joe that includes at least three different exercises to be completed in less than 3 hours.

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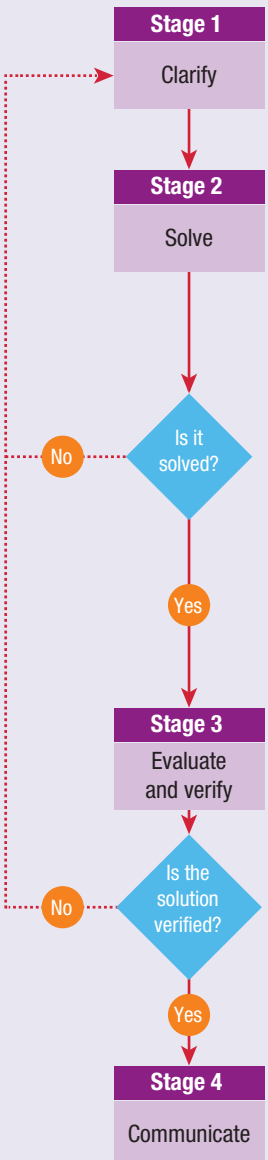
Mathematical thinking process task

Background: You have been instructed by your manager to find out what the electricity cost of the staff lunchroom is per year. You are required to provide a breakdown of the cost of each appliance in the lunchroom.

To complete this task, follow the mathematical thinking workflow diagram below and use the steps listed as a guide.

Approach to problem-solving and modelling task:



Stage 1: Clarify
Interpret the task, gather the key information and identify the mathematics which could help to complete the task

1. To prepare the report, identify the individual annual electricity cost of each appliance and sum to get the total annual cost.
2. Measure the electricity consumption, in watts, of each appliance.
3. Measure the hours per day each appliance is using electricity at that wattage.
4. Use the calculations involving watts (W), kilowatts (kW), hours of usage (h), kilowatt-hours (kWh) and electricity tariffs (c/kWh).

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

5. For each appliance:
   - Convert the usage in W to kW
   - Multiply the kW by the hours (h) to obtain kWh per day.
   - Multiply the kWh per day by 365 days to get annual usage in kWh.
   - Multiply the annual kWh by the tariff to get the annual cost.
6. Sum the annual cost of each appliance to get the total annual cost of the lunchroom.

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources and verify the reasonableness of the solution

7. Using the make and model of each appliance, google the average annual electricity cost of each appliance which the manufacturers will have on their websites.
8. Sum these costs and see if they are similar to your calculations.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

9. Prepare a detailed report outlining the costs of each appliance and the total cost to present to your manager.

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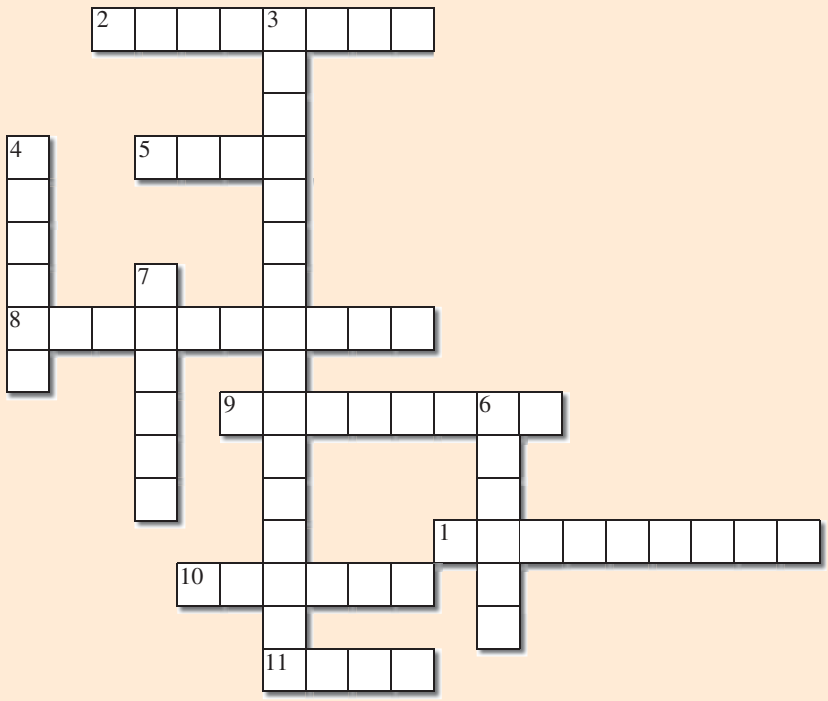
Puzzle



Across

2 Food energy is measured in everyday units of c ___
5 The unit of energy for electricity is the ?
6 The amount of money charged per kWh
8 Food energy is measured in everyday units of k ___
9 The volume that a container (3D solid) can hold of a substance
10 In everyday life we measure mass by its ?
11 The amount of matter in an object

Down

1 The basic unit of mass
3 $V = l \times w \times h$ is the formula for the volume of a ?
4 1 tonne is equal to ?
7 A measure of the amount of space taken up by a 3D shape

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Chapter checklist

☐ I can use abbreviations for units of volume.

1 Express the following metric units of volume in their abbreviated forms.

a cubic millimetres **b** cubic centimetres **c** cubic metres

☐ I can convert between units of volume.

2 Convert the following measurements into the units given in brackets.

a $1.74\,cm^3$ (mm^3) **b** $2.67\,m^3$ (cm^3)

c $2340\,mm^3$ (cm^3) **d** $0.0001\,km^3$ (cm^3)

☐ I can understand the relationship between volume and capacity, including the conversions to millilitres, litres, kilolitres and megalitres.

3 Convert these volume and capacity measurements into the units given in brackets.

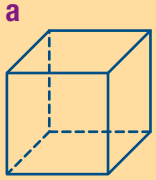
a $75\,cm^3$ (mL) **b** 2.5 L (cm^3)

c $6172\,cm^3$ (L) **d** 3.2 ML (kL)

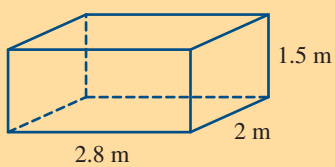
☐ I can calculate the volume and capacity of regular objects.

4 Calculate the volume of the following solids to 1 dp.

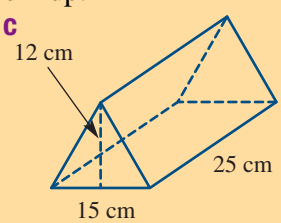
a



b

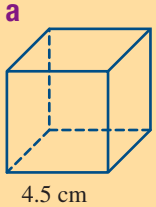


c

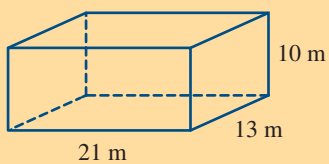


5 Calculate the capacity of the following solids to 2 dp.

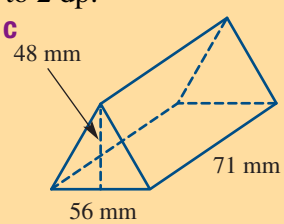
a



b



c



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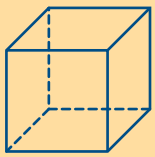
☐ I can estimate the volume and capacity of regular objects.

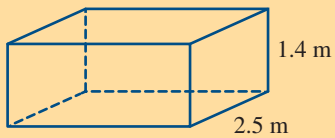
6 Estimate the volume and capacity of the following solids to 2 dp.

a



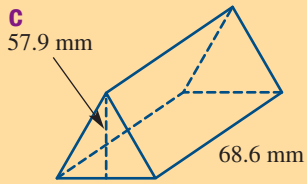
Convert to L

b



Convert to kL

c



Convert to mL

☐ I can use abbreviations for units of mass and convert between units of mass.

7 Convert the following metric units of mass to their abbreviated forms.

a milligrams
b grams
c kilograms
d tonnes

8 Convert these mass measurements into the units given in brackets.

a 0.027 kg (g)
b 973 400 mg (g)
c 2300 g (kg)
d 7.5 t (kg)
e 3570 kg (t)
f 94 000 mg (kg)

9 Decide on the appropriate mass units that would be used when weighing the following.

a a horse

b a train

c a flower

d a sesame seed

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10 Use the images on the previous page and estimate the mass for each item.

☐ I can understand different units of energy and can calculate energy from food labels.

11 If chicken nuggets have 782 kJ per 100 g serving, calculate the total kilojoules in a 750-g box.

12 Using the table given, calculate the calories burned by a 67-kg person cycling for 1.5 hours at 15 km/h on average.

Exercise energy chart
Calories per kg for 30 min of exercise

Activity	Energy used (cal/kg/30 min)
Cycling at 15 km/h average	2.76
Hiking with 15-kg load	5.18
Hiking, no load	3.42
Jogging at 8 km/h average	4.08
Rowing machine	3.97
Running at 13 km/h average	6.72
Skipping rope	6.28
Soccer	4.30
Stair climbing	3.09
Swimming at 25 m/min	2.65
Walking at 5 km/h average	1.76

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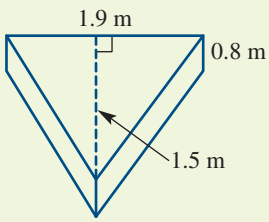
Chapter review

1 Brian has spread 2987 cm^3 of soil. Calculate the volume of soil in cubic metres.

2 Kendra has a spa which holds 1400 L of water. Calculate the capacity of the spa in cubic centimetres.

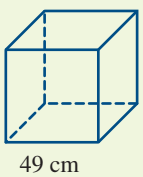
3 Danni has a 50-cm cubic fish tank. Calculate the capacity of the tank in litres.

4 Wai-Lim is building a triangular fish pond as shown. Calculate the capacity of the fish pond in kilolitres.

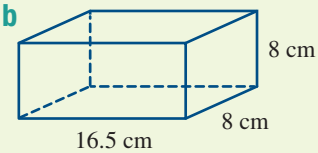


5 Calculate the volume of plastic you would need to make the following solids.

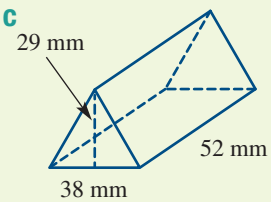
a



b



c



6 Andrew has a box of 80 bolts and each bolt weighs 120 g. Calculate the total weight of the box of bolts in kilograms.

7 Ronslee is packing for an overseas holiday. She has carry-on luggage that weighs 660 g plus luggage that goes in the hold that weighs 22.7 kg. Determine the total weight of Ronslee's luggage in kilograms.

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8 Decide on the appropriate units for weighing the following living things.

a

b

c

d

9 Select which mass is most likely accurate for the following objects;

1 A banana weighs around ...

i 1 kg **ii** 130 g **iii** 27 mg

2 A grasshopper weighs around ...

i 3 g **ii** 1 kg **iii** 60 mg

3 A hippopotamus weighs around ...

i 3 t **ii** 692 kg **iii** 999 mg

4 A blade of grass weighs around ...

i 0.5 t **ii** 100 mg **iii** 65 g

10 Yasmin decides she is only going to eat Sugar Sweet Circle Cereal for the day, which has a nutrition label that reads 520 kJ per 100 g. Calculate how many grams she can have for the day if her recommended kilojoule daily allowance is 8000 kJ.

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11 Leon, who weighs 86 kg, eats all of a 250 g block of chocolate, which has a nutrition label indicating 232 calories per 100 g. Use the table's information to calculate how long Leon will have to skip rope to burn the calories consumed.

Exercise energy chart Cal/kg of body mass per 30 min of exercise	
Activity	**Energy used (cal/kg/30 min)**
Cycling at 15 km/h average	2.76
Hiking with 15-kg load	5.18
Hiking, no load	3.42
Jogging at 8 km/h average	4.08
Rowing machine	3.97
Running at 13 km/h average	6.72
Skipping rope	6.28
Soccer	4.30
Stair climbing	3.09
Swimming at 25 m/min	2.65
Walking at 5 km/h average	1.76

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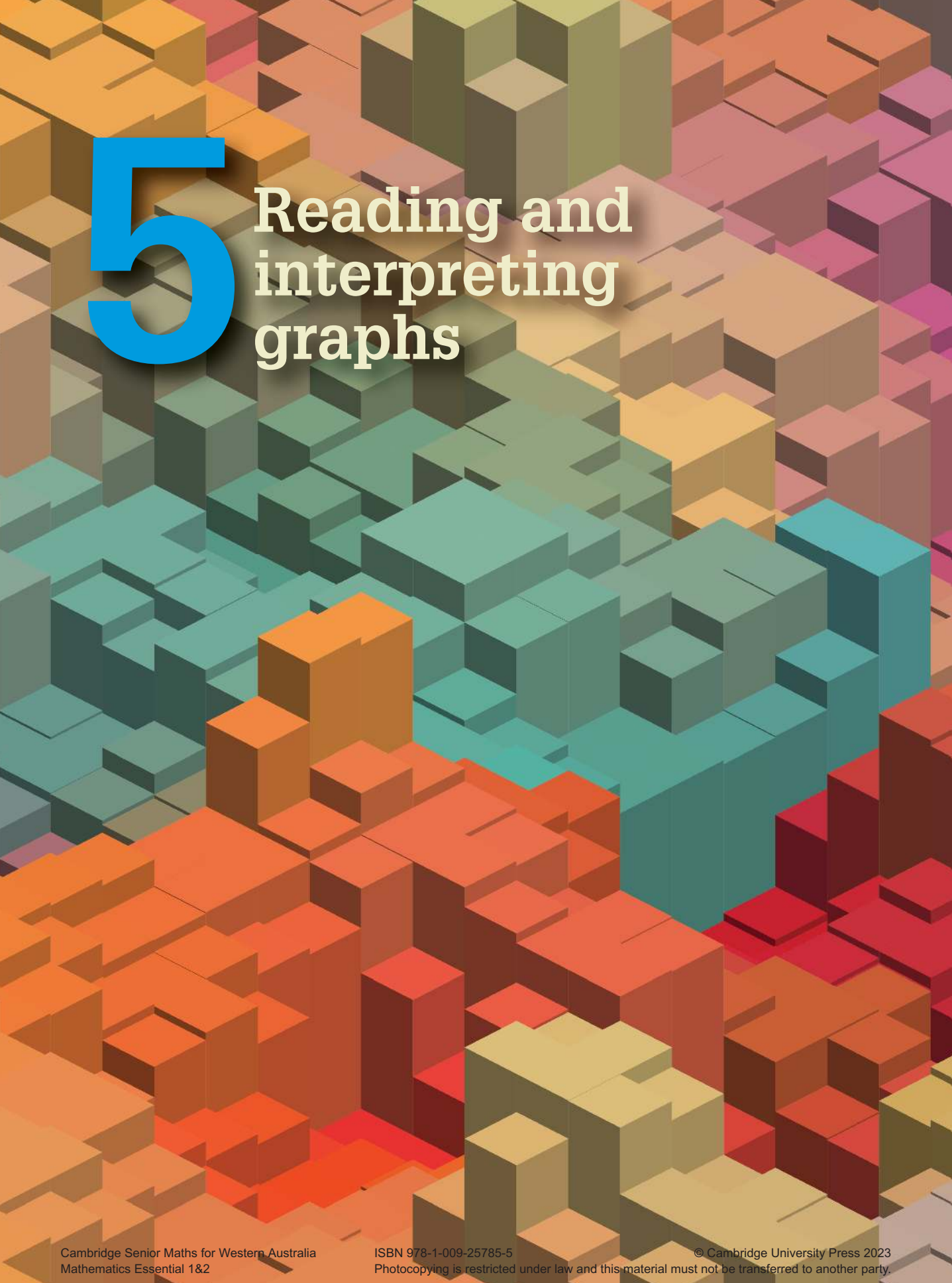

5 Reading and interpreting graphs

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In this chapter

Syllabus reference

Unit 1 Topic 1.4: Graphs

Reading and interpreting graphs

- interpret information presented in graphs, such as conversion graphs, line graphs, step graphs, column graphs and picture graphs
- interpret information presented in two-way tables
- discuss and interpret graphs found in the media and in factual texts

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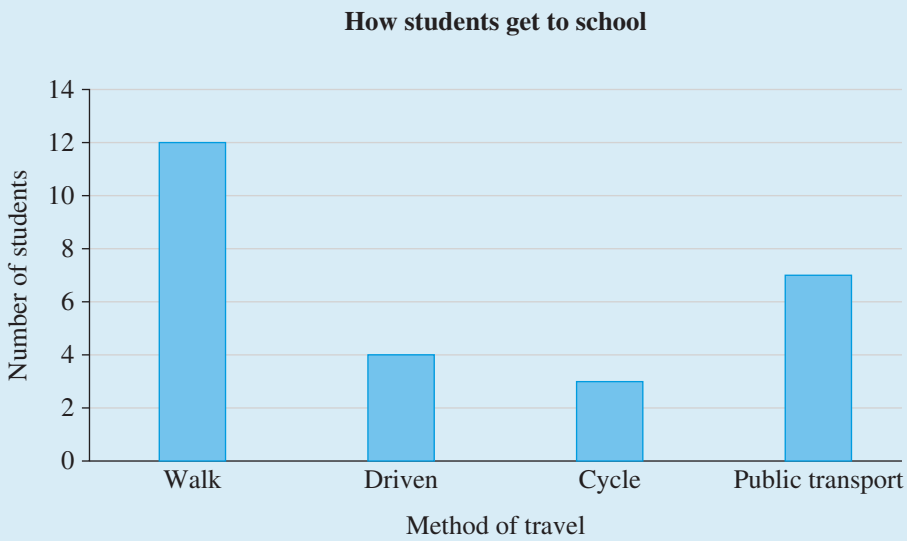
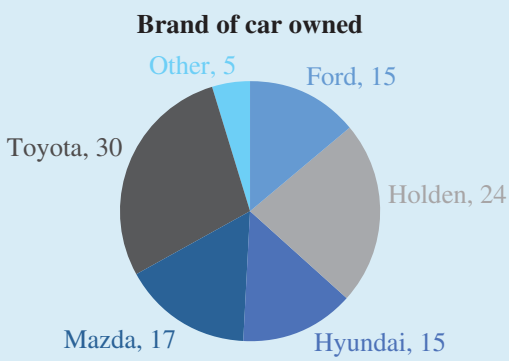
Pre-test

1 A class of 26 students are asked how they usually get to school. The results are shown in the following graph.



a How many students are driven to school?
b How many students cycle to school?
c What is the most common method of getting to school for this class of students?

2 A group of 106 young adults are asked what brand of car they own. The results are shown in the following pie chart.



a How many young adults own a Mazda?
b What brand is the most common?
c What is the difference between the number of people who own a Hyundai and the number of people who own a Holden?

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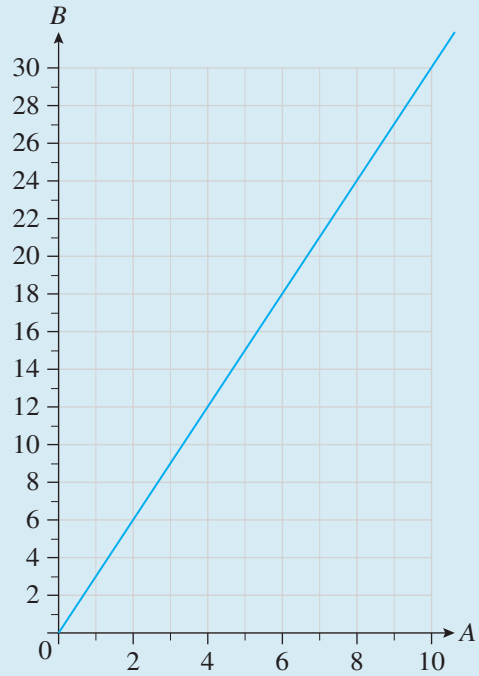
3 The graph below shows the relationship between two units, A and B.



a What is the value of B when A is 2?
b What is the value of B when A is 7?
c What is the value of A when B is 12?
d What is the value of A when B is 27?

4 Construct a two-way table from the data below of the number of boys and girls in two different classes. Don't forget to include totals for each row and column of the table.
- There are 16 boys and 10 girls in Class B.
- There are 13 girls and 12 boys in Class A.

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5A Step graphs and column graphs

LEARNING GOALS

- Read and interpret information presented in step graphs
- Read and interpret information presented in column graphs

Why are step and column graphs essential?

Every day we are bombarded by data and information. It is much simpler to understand groups of data when it is expressed in the form of a graph. Graphs are used in our everyday life; step graphs are used to determine how much parking we must pay and how much postage to pay, and your electricity bill comes with column graphs, comparing usage from the same time last year. It is therefore essential that you learn how to read and interpret these graphs.

Column graphs are often used in the media during an election to better present the required information to the viewers.

WHAT YOU NEED TO KNOW

- A **step graph** is a type of line graph that looks like steps and consists of a series of horizontal line segments with vertical gaps from one segment to the next.
- A closed circle on a line segment represents an **included value**.
- An open circle on a line segment represents an **excluded value**.
- **Column graphs** display the quantity of data in various categories. When the columns are horizontal it is usually called a **bar graph**.
- The categories are labelled at the base of the columns.
- Categories are mostly given word names, but some categories are **integers**.
  - For example: Labels for hotel star ratings are the integers 5, 4, 3, 2 and 1.

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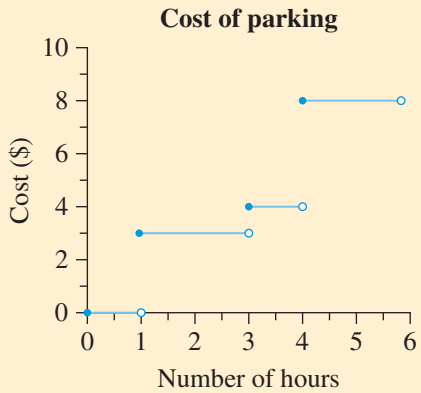
Example 1 Reading and interpreting information from a step graph

Lilly parks her car at a shopping centre. The parking costs are shown in the following step graph.

a Determine how much it will cost Lilly to park for 2 hours.

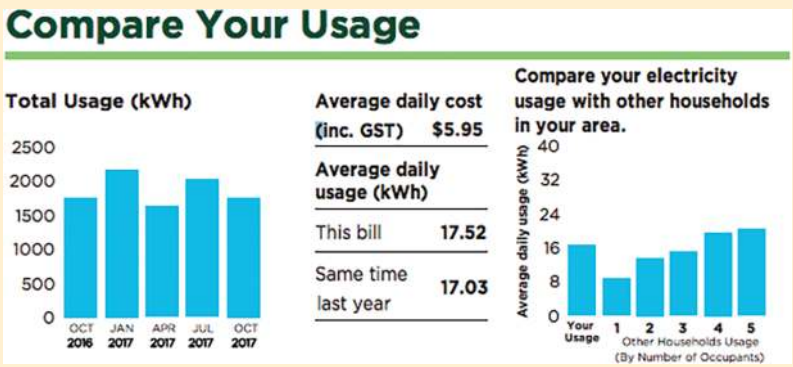
b Lilly runs into a friend and they have a coffee. She has now parked for a total of 4 hours. Calculate how much extra Lilly must pay for parking.



WORKING	THINKING
a $3	Go from 2 hours up to the line and read off the vertical scale.
b $5 extra	The cost for 4 hours is $8 because at 4 hours you must go up to the closed circle. So the difference is $8 - 3 = 5$.

Example 2 Applying information from practical graphs to real-life situations

Shown below are column graphs from the Heart family's electricity bill. The Heart's household comprises three family members.



a Identify the quarter in which the Heart family has used the most energy.

b Give some reasons why the Heart family might have used the most energy in these months.

c Explain how the Heart's average daily usage compares with other households in their area.

... *Continued*

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WORKING	THINKING
a The quarter starting at January has the most energy usage.	Determine the highest columns on the 'Total Usage' column graph.
b January to March • They used more air conditioners. • They used more fans. • More family members at home during the holidays using electrical appliances.	January is a hot month.
c The Heart family uses more energy than the average one- to three-person household in their area, but less energy than the average four- or five-person household.	Read the comparison graph on the right and compare the column heights. The Heart family uses, on average, about 1 kWh per day more energy than the average three-person household in their area.

Exercise 5A

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

Hint Use 'What you need to know' (WYNTK) and the examples as a guide.

a A step graph consists of a series of _____ line segments.

b A step graph is a type of _____ graph.

c A line segment on a step graph with a closed circle indicates an _____ value.

d A line segment on a step graph with an open circle indicates an _____ value.

e Column graphs display the quantity of data in various _____.

f When the columns are horizontal it is usually called a _____ graph.

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Example 1

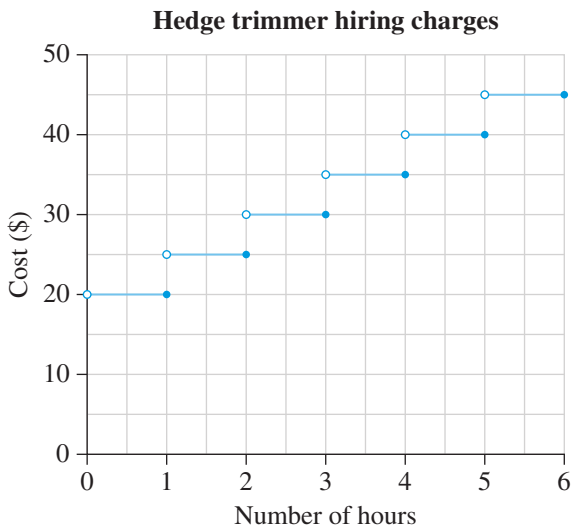
2 A hedge trimmer can be hired for a fee of \$15 plus a charge of \$5 per hour or part thereof, as seen in the following graph.

a Determine the cost of hiring the hedge trimmer for:

i 1 h **ii** 4 h 10 min **iii** $2\frac{1}{2}$ h

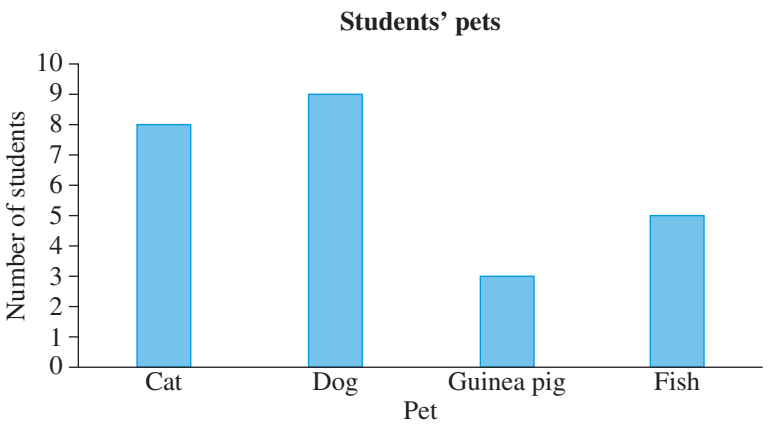
b Why would there be an initial charge of \$15?

Hint A *closed* circle on a line segment represents an *included* value. An *open* circle on a line segment represents an *excluded* value.



Example 2

3 A group of students who own pets are asked what type of pet they have. The results are shown in the column graph.



a Determine how many students said they owned a cat.

b Determine how many students said they owned a fish.

c Identify the most popular response.

d Identify the least popular response.

Hint Find 'Cat' along the horizontal axis, go up the column, then go across to find the 'Number of students' on the vertical axis.

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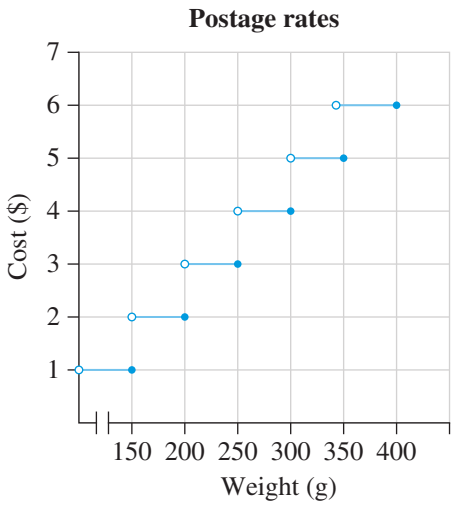
APPLICATIONS

4 Don wishes to mail a package to his friend who is currently overseas. The cost of sending a package depends on its weight. These costs are given in the step graph.

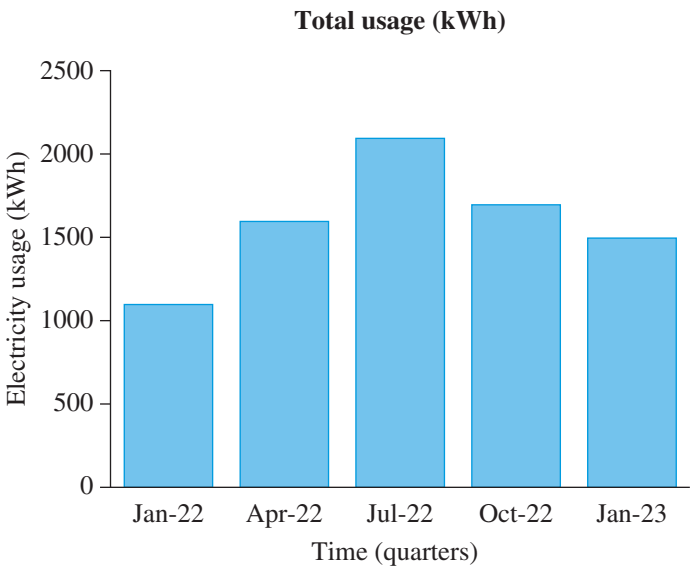


a The parcel weighs 395 grams. How much will it cost to post?

> **Hint** Find the weight along the horizontal axis, go up to the step, then go across to the vertical axis and read the cost.

b Don has only $2.50 in change. What is the size of the largest parcel he can post?

5 Fran and her housemate, Cindy, share the cost of the utilities. A section of their latest electricity bill is shown. The graph shows their electricity usage over a year, divided into quarters, e.g. the bill for Jan-22 covers the Oct-21 to Jan-22 period.



a Identify the time period when Fran and Cindy used the most energy.

> **Hint** What would happen in these months to require more energy?

b Approximately how many kWh did Fran and Cindy use in the months of April to July 2022?

c There seems to be a big difference in the energy used in the month of January in 2022 compared to January in 2023. Give some possible reasons for this.

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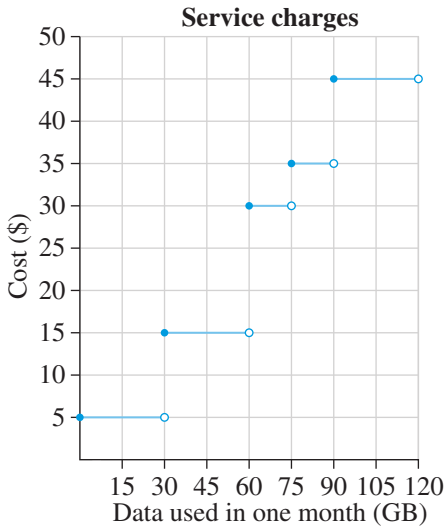
6 A certain media streaming service charges its customers according to how much data they use each month. A summary of the charges is displayed in the step graph shown.



a Determine how much it would cost if 45 GB of data is used in one month.

b How much would it cost if 60 GB of data is used in one month?

c How much would it cost if 80 GB of data is used in one month?

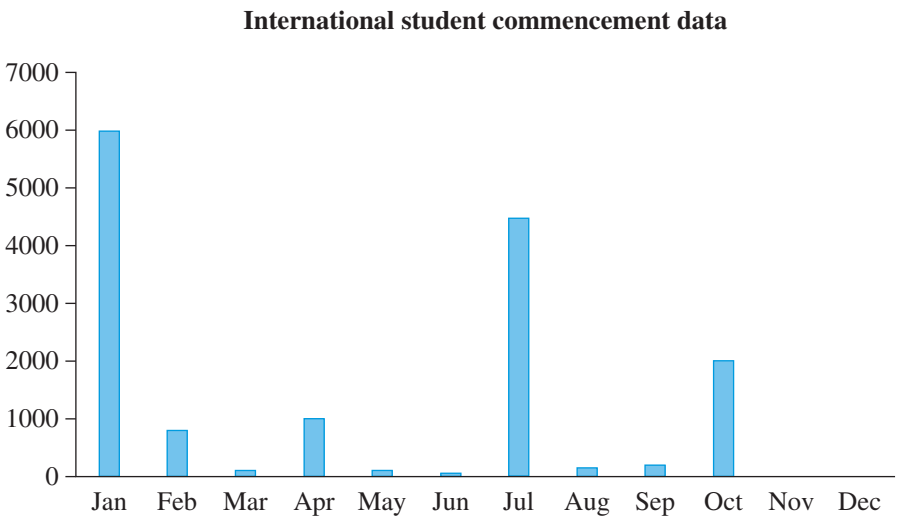
d If a customer receives a bill of $30 for the month, how much data might they have used in that month?

e If a customer receives a bill of $45 for the month, how much data might they have used in that month?

f Calculate the difference between the monthly cost of a customer who uses 17 GB in one month and someone who uses 63 GB in one month.

Hint Which operation (+ − × ÷) is used for the word 'difference'?

7 The column graph shows international school student commencements in a monthly time series format.



Use the graph to complete the following.

a Suggest how the vertical axis should be labelled.

b Which month has the most commencements?

c Explain why you think the most commencements occur in this month.

d Approximately how many students, in total, commenced in February, March and April?

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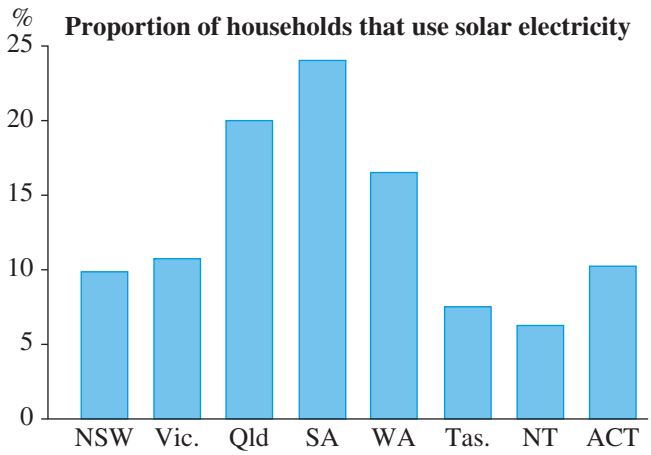
8 The following graph shows the proportion of households that use solar electricity in the different states and territories of Australia.



a Approximately what proportion of households use solar electricity in Queensland?

b Approximately what proportion of households use solar electricity in Tasmania?

c Identify which state or territory has the highest proportion of households that use solar electricity.

d Identify which state or territory has the lowest proportion of households that use solar electricity.

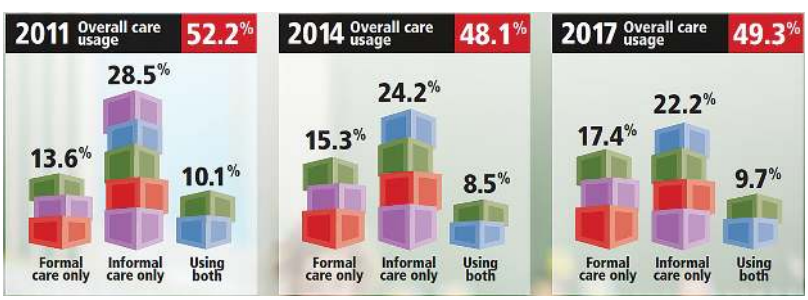
e It has been suggested that many people in South Australia use solar electricity owing to the frequent occurrence of power outages during the summer months. Does this graph support this suggestion? Justify your answer.

9 A summary of children aged 0 to 12 years who usually attended child care in 2011, 2014 and 2017 is shown.

a Determine the percentage of children aged 0 to 12 years that attended only formal care in 2017.

b Determine the percentage of children aged 0 to 12 years that attended only informal care in 2014.

c Explain how the percentage of children aged 0 to 12 years attending only informal care changed over the years of 2011, 2014 and 2017.



Source: Childhood Education and Care (www.abs.gov.au/childcare)

ISBN 978-1-009-25785-5

5B Pie graphs and picture graphs

LEARNING GOALS

- Read and interpret information presented in pie graphs
- Read and interpret information presented in picture graphs

How are pie and picture graphs essential?

Pie graphs (circle graphs or sector graphs) and picture graphs (pictograms or pictographs) give an excellent visual representation of statistical information. They are commonly used to compare sets of data. Being able to read and interpret them is an essential skill.

Pie and picture graphs are used in everyday life. It is essential that we learn how to read and interpret this useful data.

WHAT YOU NEED TO KNOW

- A **pie graph** is a circle divided up into sectors, with each sector representing a percentage of the whole.
- Pie graphs are useful for displaying data where it is important to compare the sizes of the parts that make up the whole.
- Pie graphs are commonly used in the media to display the size of the components that add up to the total value of a category.
- A **picture graph** is a pictorial display of data, using symbols, **icons** or pictures, to represent different quantities.
- A common type of picture graph is like a **dotplot**, with icons replacing dots.
- A picture graph must include a key/legend to show the value of the icons used.
- The size of the icons should relate to the quantities they represent. Icons representing the same quantity should be the same size.
- A fraction of the symbol used will represent that fraction of the value of the symbol.
  - For example: Half of the symbol represents half of the value of the symbol. So, if the whole symbol represents 10, then half of the symbol represents 5.
- Picture graphs and pie charts can communicate information visually, overcoming differences in language, culture and education.

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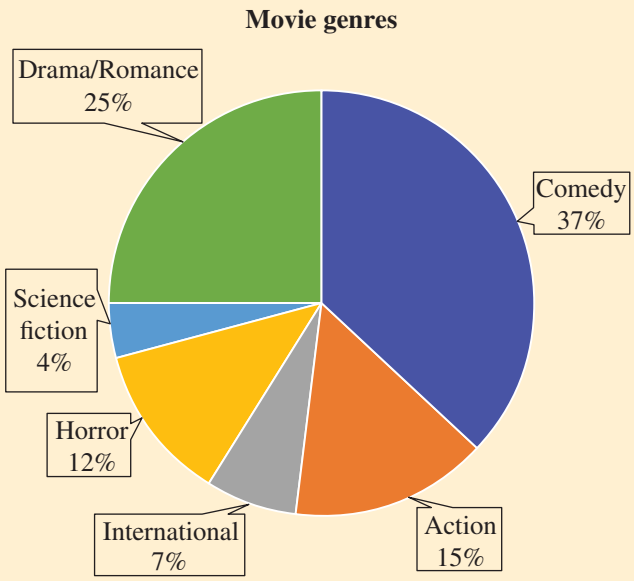
Example 3 Reading and interpreting information presented in a pie graph

A Year 11 class is surveyed about their preferred movie genre (a category such as thriller or romance).



a Identify the most favoured movie genre.

b Calculate the total percentage of Horror, Drama/Romance and Action.

WORKING	THINKING
a The most favoured movie genre is Comedy.	Pie graphs are a good way to show relative sizes. At a glance, it is easy to see which movie genres are the most preferred and which are the least. The largest sector represents Comedy (37%), therefore it is the most preferred genre in the class.
b Sum = 12% + 25% + 15% = 52% The total percentage is 52%.	List each genre and find the sum. Horror = 12% Drama/Romance = 25% Action = 15%

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Example 4 Reading and interpreting information presented in a picture graph

A soccer team records their goals for the season.

a Identify the player who scored the most goals.

b Calculate the percentage of the goals that were scored by Adrian for the season. Round your answer to 2 dp.

WORKING	THINKING
a $4\frac{1}{2}$ balls are shown for Catarina.	The key shows that a complete ball represents 10 goals and half a ball represents 5 goals.
$4 \times 10 = 40$	
$1 \times 5 = 5$	
$40 + 5 = 45$	Therefore, Catarina has scored the most number of goals, scoring 45 goals for the season.
Catarina scored the most goals.	
b Percentage $= \frac{\text{amount}}{\text{total}} \times 100$	Remember from previous chapters: Percentage $= \frac{\text{amount}}{\text{total}} \times 100$
Percentage $= \frac{25}{150} \times 100$	Adrian scored 25 goals. The total amount of goals scored by the team during the season is 150 goals (this number is found by adding the number of each player's goals together: $35 + 20 + 10 + 15 + 25 + 45 = 150$). Substitute these values into the formula to find the percentage and round to 2 dp.
$= 16.67\%$	
The percentage of Adrian's goals for the season is 16.67%.	

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Exercise 5B

FUNDAMENTALS

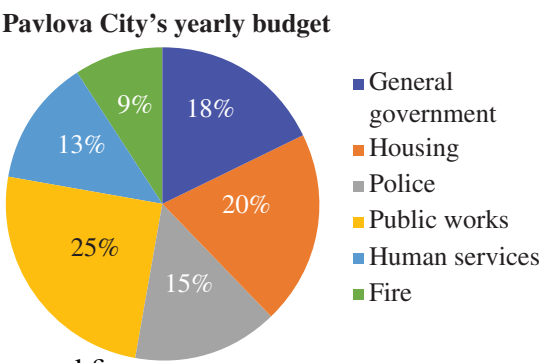
1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

Hint Use WYNTK and the examples as a guide.

- **a** A pie graph is a circle divided into _____.
- **b** Each sector of a pie graph represents a _____ of the whole.
- **c** A picture graph is a pictorial display of data using _____.
- **d** When interpreting a picture graph, a _____ is needed.
- **e** Picture graphs and pie charts are an excellent way to communicate _____.

Example 3

2 Use the pie chart to answer the following questions.



- **a** Identify which section of the budget receives the least amount.

Hint The smaller the sector, the smaller the amount it represents.

- **b** Calculate the total percentage of the budget allocated to police, housing and fire.
- **c** If the total budget for Pavlova City is $3 000 000, calculate how much public works would receive.

Example 4

3 The picture graph shows how many cars each salesperson at Freddie's Car Sales has sold in the past month.

Salesperson	
Mari	6 cars
Graeme	3 cars
Charlie	2 cars
Freddie	5 cars
Kylie	4 cars
Nik	3 cars

Key: = 5 cars

Hint A picture graph includes a key to explain what each picture represents.

- **a** Identify who sold the most cars.
- **b** Determine how many cars Freddie sold.
- **c** Calculate the total percentage of the cars that Kylie and Charlie sold for the week.
- **d** Explain how 2 cars being sold could be represented in this graph. Use an illustration.
- **e** Explain how this type of graph could be used to help with sales.

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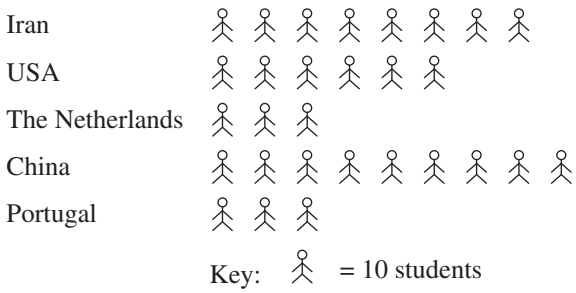
4 The picture graph shows the number of international students enrolled at an Australian school and the country they are from.

a Determine the number of students from China.

b Calculate the percentage of international students from Iran.

International students enrolled



APPLICATIONS

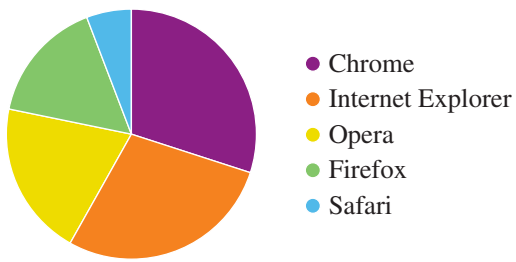
5 A pie graph is best used when aiming to work out the composition of something. If you have categorical data, then using a pie graph would work well as each slice can represent a different category.

a List some advantages of using a pie graph.

b List some disadvantages of using a pie graph.

c Suggest what informative component is missing from the pie graph shown.

Web browser market share



6 A group of students are surveyed about whether they have a part-time job and, if yes, what type of job they have. The survey's results are shown in the pie graph below.

Hint Each sector in a pie graph represents a percentage of the whole.

a Identify how many students have a part-time job as a retail assistant.

b Identify how many students have a part-time job as a cleaner.

c What is the most common response?

d What is the most common type of part-time job?

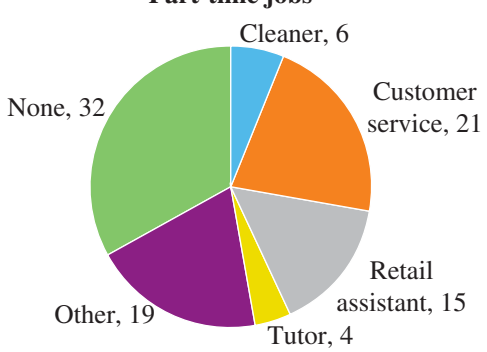
e What is the least common response?

f Calculate how many of the students surveyed have a part-time job in total.

g Calculate the difference between the number of students who have a part-time job and the number of students who do not.

h In total, how many students participated in the survey?

Part-time jobs



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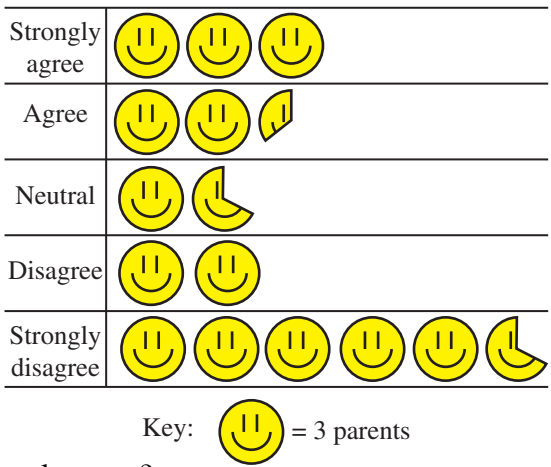
7 A group of parents are surveyed regarding the statement: 'Every child should be forced to play at least one sport outside of school.'

Parents are able to give one of the following responses: strongly agree, agree, neutral, disagree, strongly disagree or they can choose not to respond. The results of the parents' responses are shown in the picture graph.



a Determine the number of parents who strongly agreed with the statement.

b Determine the number of parents who agreed with the statement.

c Identify the response that six parents selected.

d What is the difference in the number of parents who selected strongly disagree and the number of parents who selected strongly agree?

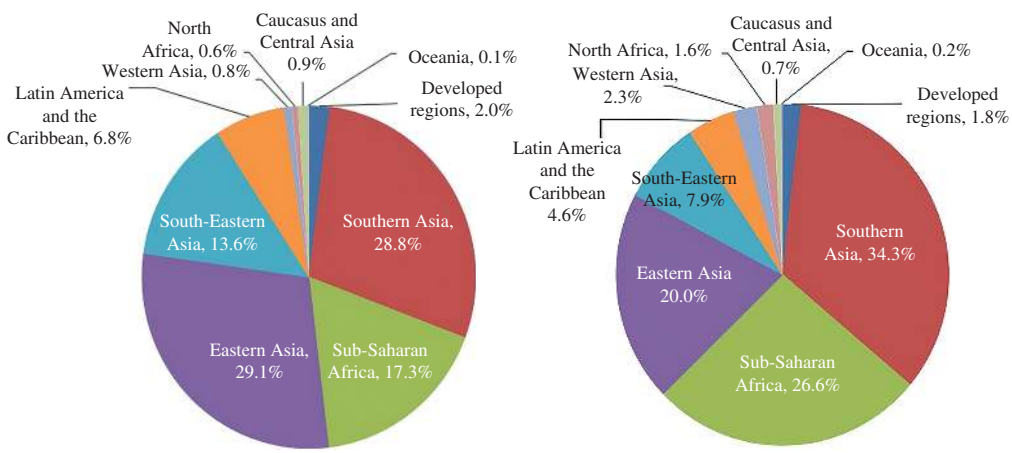
e In total, how many parents either strongly agreed or strongly disagreed?

f In total, how many parents gave a response?

g Given that 50 parents were asked, calculate how many chose not to give a response.

8 Read these graphs.

Changing distribution of world hunger, 1990–2014



1990–92 Total = 1015 million **2012–14 Total = 805 million**

a i List in order the four regions that have changed the most.

ii By how many people did each region change from 1990–92 to 2012–14?

b Is world hunger getting worse or better? Justify your answer.

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5C Line graphs and conversion graphs

LEARNING GOALS

- Read and interpret information presented in line graphs
- Read and interpret information presented in conversion graphs

Why are line and conversion graphs essential?

Line and conversion graphs are used in everyday life, by many different people. Line graphs are especially useful and much used in the media, business and science for showing how a measurement, such as price, temperature or mass, varies over time. Conversion graphs are used in cooking (e.g. converting teaspoons to millilitres), measurement (e.g. converting pounds to kilograms or calories to kilojoules), money (e.g. converting Australian dollars to Euros) and especially carpentry (e.g. converting inches to centimetres), to name just a few. Learning how to read and interpret line and conversion graphs is an essential skill that you will need throughout life.

Nurses use weight vs height conversion graphs as an indication of the health of babies and toddlers.

WHAT YOU NEED TO KNOW

- A **continuous variable** is any characteristic or quantity that can be measured.
  - For example: time, height, money, distance, volume and energy.
- A **line graph** displays data points (x, y) connected by straight line segments.
  - For example: A line graph could show the consumption of household energy in kilowatt hours in a 24-hour period.
- When time is one of the variables, it is usually shown on the horizontal axis.
- When the variables are **continuous**, estimates can be made between the points or beyond the points, to make a prediction.
- **Conversion graphs** are specific line graphs used to convert a measurement using one type of unit to the same measurement expressed in another type of unit.
  - For example: A conversion graph can be used to convert calories to kilojoules.

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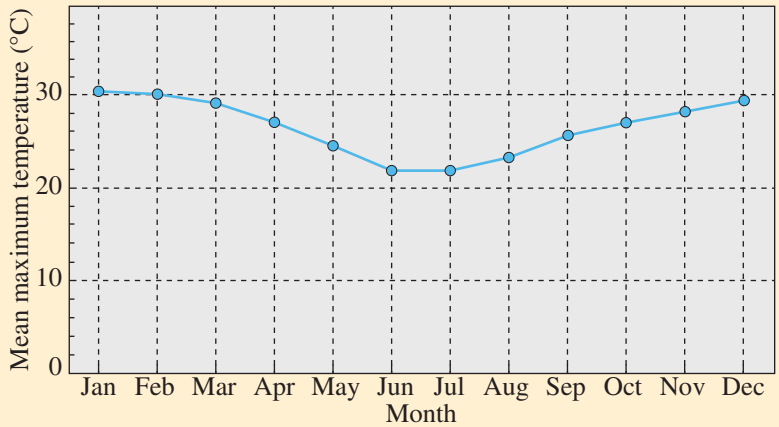
Example 5 Interpreting and analysing data from real-life graphs

The graph shows Broome's maximum temperature (°C) averaged for each month of the years 2014 to 2022.



a Approximately, what is Broome's average maximum temperature for February?
b On average, what are the two coldest months in Broome?
c Approximately, what is the difference in Broome's average maximum temperature between the months of February and June?
d If someone planning to visit Broome for a holiday preferred to visit when the temperature is between 24°C and 28°C, when would be the best months for them to travel?

WORKING	THINKING
a Approximately 30°C	Start on the x-axis and look for the 'Feb' label. Move vertically up to the graph's line and then left across to the y-axis to estimate the temperature.
b June and July	By finding the lowest values on the graph, the corresponding months for these temperatures can be found.
c February = 30°C June = 22°C. Difference = 30°C – 22°C = 8°C	Reading off the graph, February is 30°C and June is 22°C, so the difference is 8°C.
d April, May, August, September, October or November would be the best months to travel because they have an average maximum temperature between 24°C and 28°C.	Find the months that have an average maximum temperature between 24°C and 28°C.

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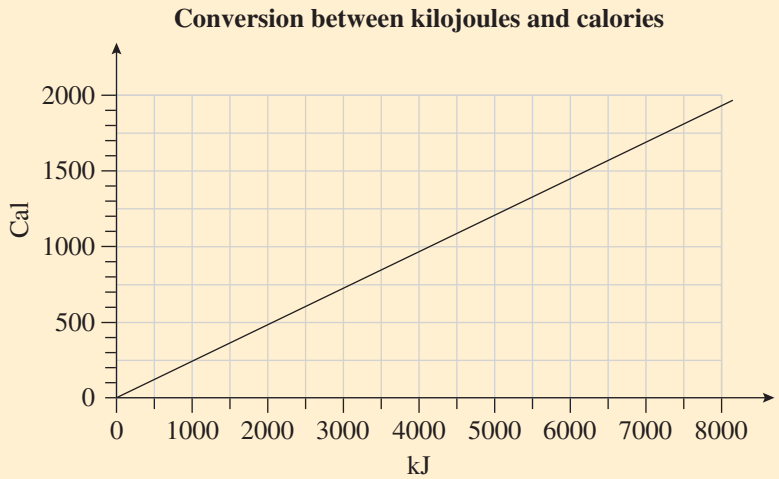
Example 6 Reading and interpreting information from a conversion graph

The following conversion graph shows the conversion between kilojoules (kJ) and calories (Cal).



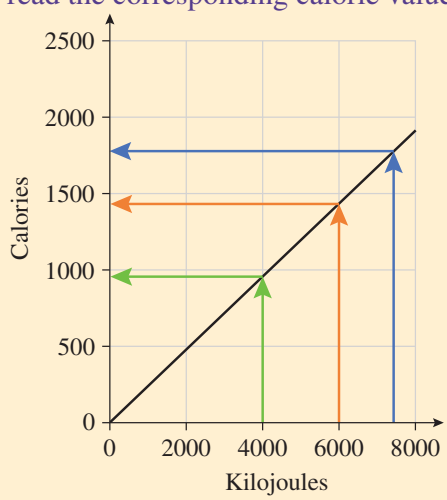
a Using the conversion graph, convert the following kilojoules to calories.

i 7500 kJ **ii** 6000 kJ **iii** 4000 kJ

b Using the conversion graph, convert the following from calories to kilojoules.

i 250 Cal **ii** 1550 Cal **iii** 1200 Cal

WORKING	THINKING
a i 7500 kJ ≈ 1800 Cal **ii** 6000 kJ ≈ 1450 Cal **iii** 4000 kJ ≈ 950 Cal	Identify the location of the kilojoule value on the horizontal axis. Move vertically up to the graph line. Move horizontally left to the vertical axis and read the corresponding calorie value.



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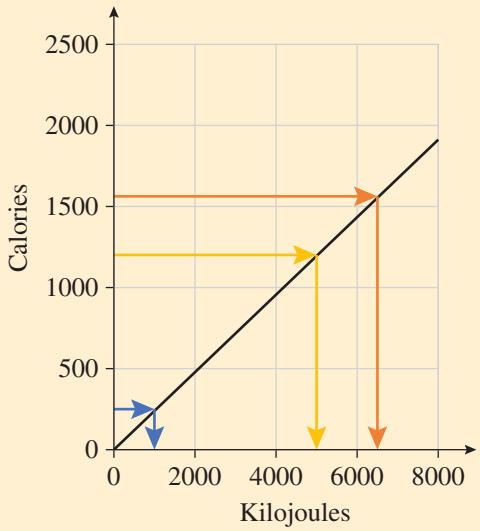
WORKING	THINKING
b i 250 Cal ≈ 1000 kJ **ii** 1550 Cal ≈ 6500 kJ **iii** 1200 Cal ≈ 5000 kJ	Identify the location of the calorie value on the vertical axis. Move horizontally to the graph line. Move vertically down to the horizontal axis and read the corresponding kilojoule value.



Reading and interpreting graphs is an essential skill for most everyday jobs.

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Exercise 5C

FUNDAMENTALS

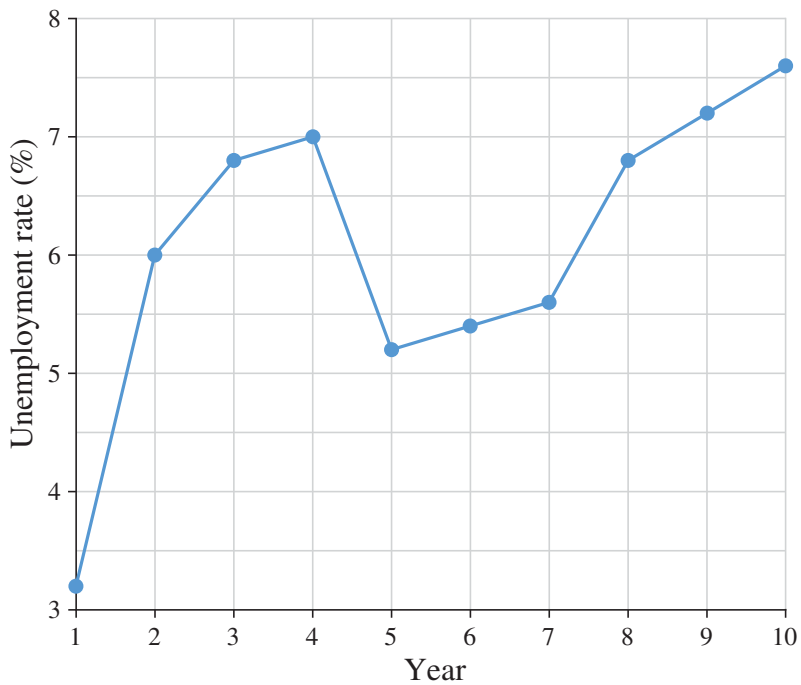
1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

Hint Use WYNTK and the examples as a guide.

- **a** Time, height, money, distance, volume and energy are all examples of continuous _____.
- **b** A line graph displays data points connected by _____ line segments.
- **c** When time is one of the variables, it is usually shown on the _____ axis.
- **d** Conversion graphs can be used to _____ units of measurement.

Example 5

2 This line graph shows the percentage of unemployed individuals in a town over a period of 10 years.



- **a** Identify the variable that the horizontal axis is measuring.
- **b** Identify the variable that the vertical axis is measuring.
- **c** Construct a table showing the changing unemployment rate percentage from Year 1 to Year 10.
- **d** Decide whether the unemployment rate has increased or decreased overall.

Hint Is the overall trend of the line going up or down?

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Example 6

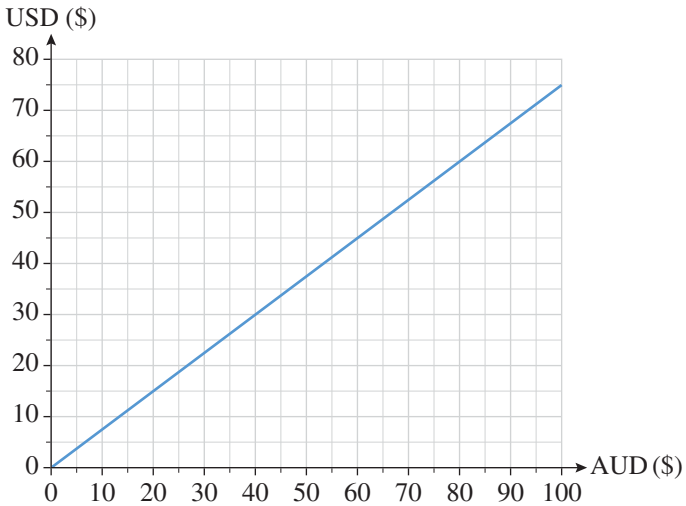
3 The following conversion graph shows the conversion between Australian dollars (AUD) and US dollars (USD).

a Determine the value of AUD$20 in USD.

b Determine the value of AUD$80 in USD.

c Determine the value of USD$30 in AUD.

d Determine the value of USD$75 in AUD.



Hint Move vertically up the USD axis. Move horizontally to the conversion line, then down to the corresponding AUD dollars.

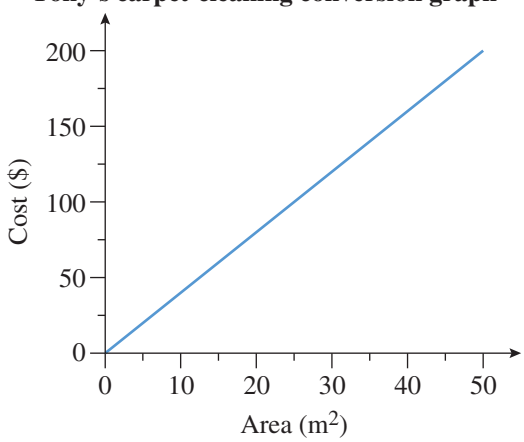
APPLICATIONS

4 Tony has just started his own carpet cleaning business. He constructs his own conversion graph to provide quotes to his customers.

a A carpet has an area of 30 m^2. Use Tony's conversion graph to determine the cost of cleaning this carpet.

b Tony quotes $150 to clean a customer's carpet. Use the graph to determine the area of the carpet.

Tony's carpet-cleaning conversion graph



Hint Find the amount ($) along the vertical axis and move horizontally to the conversion line. Then move down to find the corresponding area (m^2).

c A rectangular carpet has a length of 8.6 m and a width of 5 m. How much would Tony charge to clean this carpet?

Hint First calculate the area of the carpet.

d List some other uses for a conversion graph.

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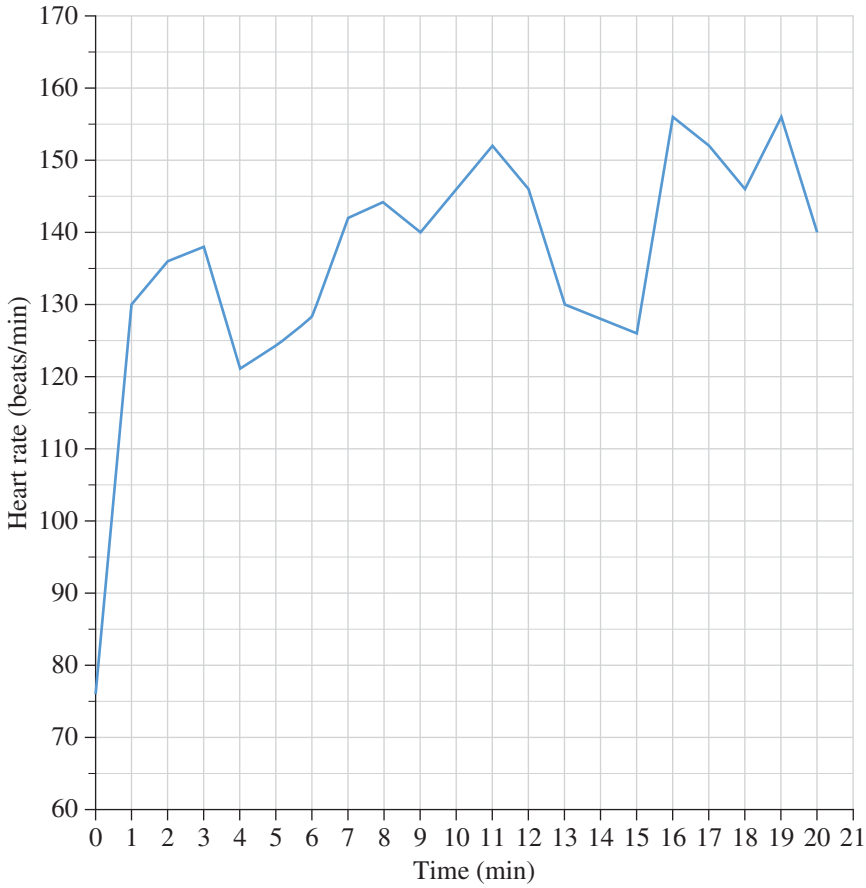
5 Sandra wears a heart rate monitor while going for a run. A graph of her heart rate, in beats per minute, against time, in minutes, is shown.



a Determine Sandra's heart rate when she first starts her run, that is, at 0 minutes.
b Determine Sandra's heart rate after 9 minutes of running.
c Determine Sandra's heart rate after 18 minutes of running.
d By how much does Sandra's heart rate increase between 15 and 16 minutes of running?
e Suggest a reason for the increase in heart rate in part **d**.
f By how much does Sandra's heart rate decrease between 3 and 4 minutes of running?
g Suggest a reason for the decrease in heart rate in part **f**.

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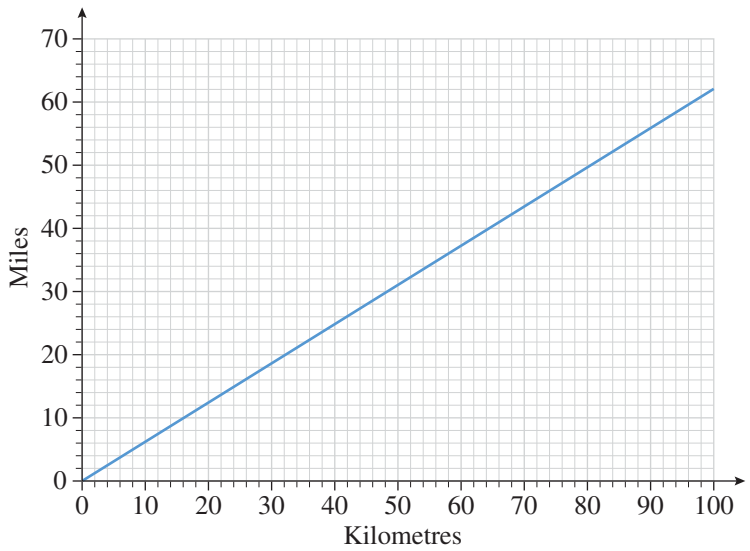
6 The conversion graph between kilometres and miles is shown.



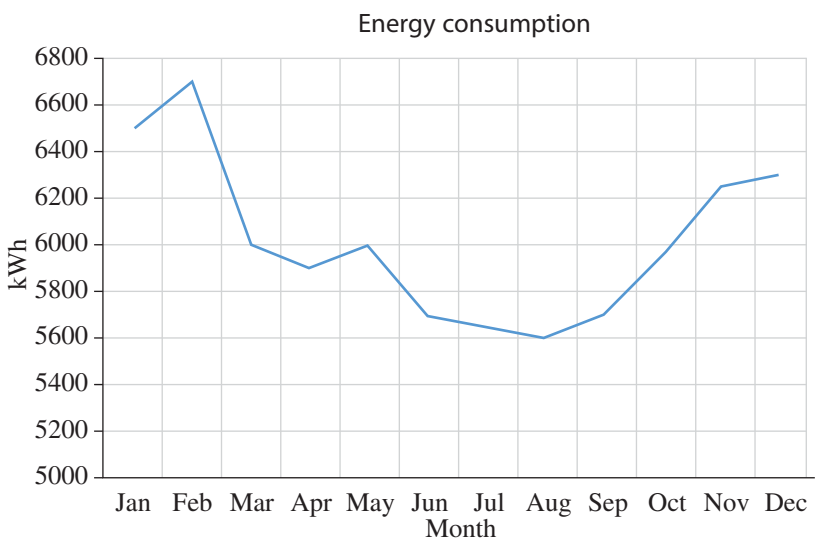
a Using the conversion graph, convert the following kilometres to miles. Estimate your answers to the nearest mile where necessary.

i 10 km **ii** 60 km **iii** 42 km **iv** 98 km

b Using the conversion graph, convert the following miles to kilometres. Estimate your answers to the nearest kilometre where necessary.

i 10 miles **ii** 30 miles
iii 34 miles **iv** 42 miles

7 The line graph below shows the energy consumption, in kilowatt hours (kWh), per month for the Singh family, who live in Western Australia.



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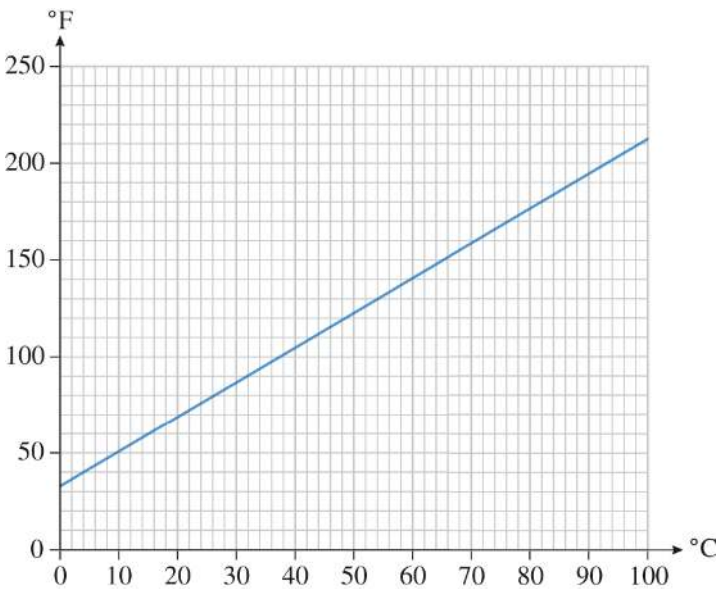
a Determine how many kilowatt hours (kWh) were used by the Singhs in March.
b Determine how many kilowatt hours were used by the family in September.
c Determine how many kilowatt hours were used by the Singhs in November.
d Identify the month in which the Singh family consumed the most energy.
e Suggest a reason for the result in part **d**.
f Calculate the difference in energy consumption between the months of May and June

8 In Australia, temperature is measured in degrees Celsius (°C), whereas in the USA, temperature is measured in degrees Fahrenheit (°F) . The temperature conversion graph between degrees Celsius and degrees Fahrenheit is given below.



Hint In the USA, temperature is measured in degrees Fahrenheit (°F).

a In Australia, water boils at 100°C. How would this be written in a cookbook published in America?
b Water freezes at 0°C. What is this in degrees Fahrenheit?
c On a particular day in New York, the maximum temperature is recorded as 100°F. Determine this temperature in degrees Celsius.

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5D Two-way tables

LEARNING GOAL

- Read and interpret information presented in two-way tables

Why are two-way tables essential?

It can be very difficult to analyse results from a survey when they are presented as lists of data values. A two-way table is a way of summarising these possible results into a more manageable format. Two-way tables are also used to compare the values between different variables. If an association exists between two variables, a two-way table can help identify that **association**.

Advertising companies employ data analysts to determine the age group that is most responsive to a particular ad. Two-way tables can analyse survey results.

WHAT YOU NEED TO KNOW

- As given in its title, a **two-way table** displays data from **two** categories of variables that are often connected to each other.
  - For example: Data from a car dealership about their cars' fuel and wheel-drive type.

	Uses petrol	Uses diesel	Total
4-wheel drive	15	55	70
2-wheel drive	30	20	50
Total	45	75	120

- Each category is labelled at the top and on the left side of the table, with the information appearing in the four (or more) internal cells of the table.
- The 'Total' of each row appears at the right, and the 'Total' of each column appears at the bottom. The total of all the data appears at the bottom right of the table.
- The percentage of a value can be found by dividing it by the total and then multiplying it by 100.

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Example 7 Reading and interpreting a two-way table

The two-way table shows whether people are likely to purchase a product after viewing advertisements on videos.

Video advertisement survey				
	Advertisement A	**Advertisement B**	**Advertisement C**	**Total**
Likely to purchase	65	38	18	121
Unlikely to purchase	27	15	37	79
Total	92	53	55	200

a Identify how many people participated in the survey.
b Did all the survey participants watch all three advertisements?
c How many survey participants watched advertisement A?
d Of the people who watched advertisement A, how many are likely to purchase the product?
e What percentage of those who watched advertisement A are likely to purchase the product? Round your answer to 1 dp.
f Explain which advertisement, A, B or C, is more likely to persuade the viewer to purchase the product.

WORKING	THINKING
a The total number of survey participants is 200.	Two-way tables should include two sets of 'Total' cells: both vertical and horizontal. Determine the corresponding total in the bottom far right of the table.
b No, not all of the survey participants watched all of the advertisements: 92 people watched advertisement A, 53 people watched advertisement B and 55 people watched advertisement C.	Reading the bottom row of the table, you can see a total of 92 people watched advertisement A, 53 people watched advertisement B, and 55 people watched advertisement C.

... Continued

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c 92 people watched advertisement A.

◄····· Read the bottom row to see the total number of survey participants who watched advertisement A.

d 65 people of those who watched advertisement A are likely to buy the product.

◄····· Read through the row that states how many people are likely to purchase after viewing each advertisement.

e $\frac{65}{92} \times 100 = 70.7\%$

Of the people surveyed who watched advertisement A, 70.7% are likely to buy the product.

◄····· 65 people out of the 92 who watched advertisement A are likely to buy the product. Calculate this as a percentage and round to 1 dp. Show working.

f Advertisement A: 70.7% are likely to buy the product.

◄····· Advertisement A has the highest number of people likely to purchase the product, but it also has the highest number of people who viewed the video. Advertisements B and C may have had proportionately more viewers who are likely to buy. For each advertisement, work out the percentage of viewers who are likely to buy.
This value was worked out in part **e**, so write down the answer from part **e** for Advertisement A.

Advertisement B:
$\frac{38}{53} \times 100 = 71.7\%$
are likely to buy the product.

Advertisement B: Calculate the percentage to 1 dp, showing your working.

Advertisement C:
$\frac{18}{55} \times 100 = 32.7\%$
are likely to buy the product.

Advertisement C: Calculate the percentage to 1 dp, showing your working.

Hence, advertisement B is more likely to persuade the viewer to purchase the product.

Write down which advertisement has the highest percentage of viewers likely to purchase the product.

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Exercise 5D

FUNDAMENTALS

1 Copy and complete the following two-way tables by calculating the values of each total.

Hint Use WYNTK and the examples as a guide.

a

	A	B	Total
C	8	5	
D	2	7	
Total			22

b

	A	B	Total
C	13	48	
D	32	25	
Total			118

2 Copy and complete the following two-way tables using the total values to fill in the missing values.

Hint Each category is labelled, at the top and the left side of the table, with the information appearing in the internal cells of the table.

a

	A	B	Total
C	4		11
D		6	15
Total	13	13	26

b

	A	B	Total
C		24	41
D	20		25
Total	37	29	66

APPLICATIONS

3 In a particular children's hospital, a nurse is given information regarding the number of patients on the two wards that he manages. This information is summarised in the two-way table below.

	Ward B	Ward C	Total
Boys	7	9	16
Girls	13	8	21
Total	20	17	37

a Identify how many boys, in total, are in wards B and C.
b Identify how many patients, in total, are girls.
c Determine the number of boys in ward C.
d Determine the number of girls in ward B.
e Calculate how many more girls than boys were in ward B.
f Calculate how many more boys were in ward C than ward B.

Hint The 'Total' of each row appears at the right, and the 'Total' of each column appears at the bottom.

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Example 7

4 A survey asks some Year 11 students, 'What are your plans with regards to TAFE and travel after high school?'

Comparing TAFE and travel			
	TAFE	**Not TAFE**	**Total**
Travel	43	10	53
Not travel	24	5	29
Total	67	15	82

a Using the two-way table, determine how many students participated in the survey.
b What is the total number of survey participants who have plans to go to TAFE?
c How many survey participants have plans to go to TAFE, but are not planning to travel?
d Calculate what percentage of the whole survey group are not travelling and not going to TAFE.

Hint The 'Total' of each row appears at the right, and the 'Total' of each column appears at the bottom.

5 A survey asks some people, 'If you could have a new vehicle, would you want a sport utility vehicle (SUV) or a sports car?'

Comparing sport utility vehicle (SUV) and sports car			
	SUV	**Sports car**	**Total**
Male	11	106	117
Female	63	20	83
Total	74	126	200

a Using the two-way table, identify how many people participated in the survey.
b Which gender prefers SUVs to sports cars?
c Identify which vehicle is preferred overall.
d Determine how many females prefer sports cars.
e What percentage of males prefer SUVs?

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6 A survey asks some Year 11 students information about their test preparation.

Year 11 test preparation			
	Passed test	**Failed test**	**Total**
Completed review sheet	84	1	85
Did not complete review sheet	5	6	11
Total	89	7	96

a Using the two-way table, calculate the percentage of survey participants that failed the test.

b Calculate the percentage of survey participants who completed the review sheet and passed the test.

c Calculate the percentage of survey participants who did not complete the review sheet and failed.

d Calculate the percentage of students who sat the test and did not complete the review sheet?

Hint The percentage of a value can be found by dividing it by the total and then multiplying it by 100.

7 The table gives the statistics of those people who were on the ship RMS *Titanic*.

Survival statistics of RMS *Titanic*					
	First class	**Second class**	**Third class**	**Crew**	**Total**
Survived	203	118	178	212	711
Died	122	167	528	696	1513
Total	325	285	706	908	2224

a Calculate the percentage of passengers on the RMS *Titanic* that survived and were in first class.

b Calculate the percentage of passengers on the RMS *Titanic* that survived and were in third class.

c Calculate the percentage of crew members on the RMS *Titanic* that survived.

d Calculate the percentage of people who were on the RMS *Titanic* that died.

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5E Misleading graphs and tables

LEARNING GOAL

- Identify and interpret graphs that could possibly be misleading

Why is it essential to know about misleading graphs?

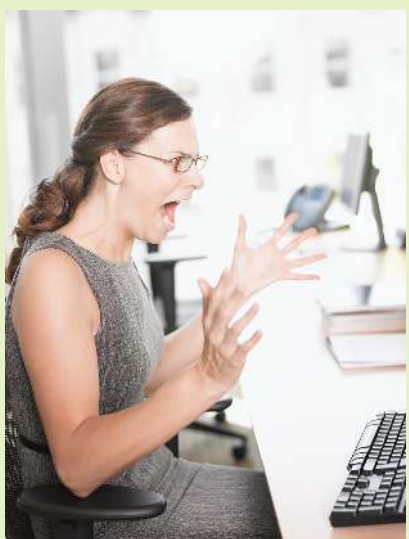
At times, when data is presented graphically, it can end up being **misleading** to the reader. This could be either accidental or intentional. A misleading graph could make certain data look better or worse than it actually is by changing the scale or using certain images. Misleading graphs can also persuade the reader to form certain opinions about the data being presented by having leading statements as titles. Because of this, it is essential that you are able to identify and interpret misleading graphs by knowing what important features to look for.

Recognising the features of misleading graphs is a key analytical skill to help you avoid making a false conclusion.

WHAT YOU NEED TO KNOW

The following features are important when analysing whether or not a graph is misleading.

- **Title**
  - Is the title biased? Does it favour a particular viewpoint?
- **Axis or sector labels**
  - Are all axes and sectors labelled or have some been intentionally left out?
- **Axis scales**
  - Are the scales used for the axes acceptable, or have they been chosen to zoom in to a particular feature of the graph?
  - Do the scales start at zero, or are they **broken scales**?
- **Uniform size of a symbol in a pictograph**
  - Are the symbols (pictures or icons) the same size where they present the same quantities?



It is important to be able to interpret misleading graphs, so you can identify incorrect data.

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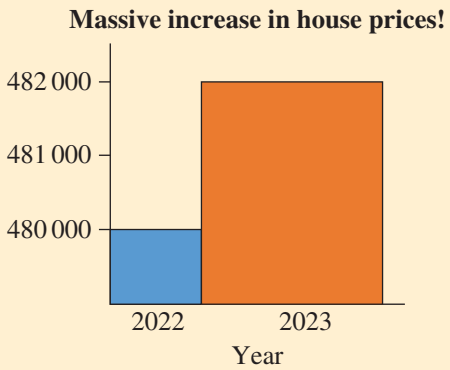
Example 8 Reading and interpreting a misleading graph

The graph shown has been used to entice people to buy now, as house prices are drastically increasing.
Describe how this graph is misleading in terms of:

a the title **b** the labels
c its source **d** its size
e the actual prices **f** the scale used



WORKING	THINKING
a The graph is misleading due to the title using biased language (e.g. using the word massive).	The title uses biased words, such as massive.
b The *y*-axis label has not been included. The prices could be average house prices or higher house prices or lower house prices, which makes the graph misleading.	The *y*-axis does not have a label.
c The source of the data has not been given. The graph is misleading because the graph could have been fabricated.	The source has not been given.
d The width of the columns in the graph are different, where 2023 is larger. The graph is misleading, as the 2023 column is not only taller but it is also wider.	Measure the width of the columns; one is wider than the other.
e The 2022 price was $480 000 and the 2023 price was $482 000, being a difference of only $2000.	Read the graph to see how much house prices have actually risen.
f This graph is misleading, as it has used scale manipulation and omitted baselines to create the impression of a larger difference than is really there.	Omitting baselines, or not starting at zero, is one of the most common ways data is manipulated in graphs. This misleading tactic is frequently used to emphasise the difference between groups.

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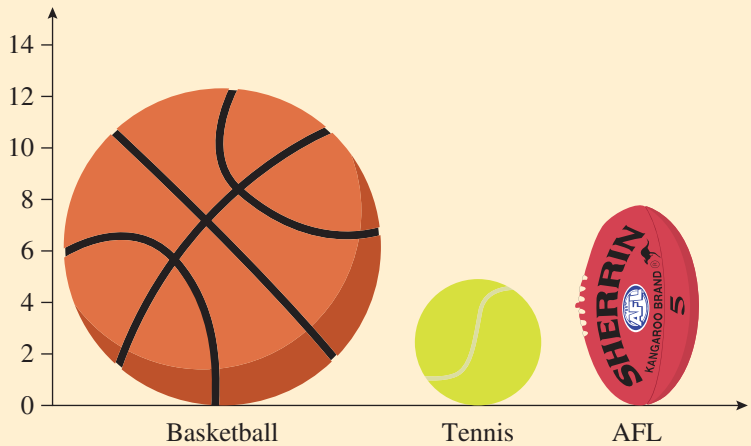
Example 9 Identifying and interpreting graphs that could be misleading

The graph below is a picture graph, created by James, summarising the results of a survey he conducted where he asked students which sport they preferred out of basketball, tennis and AFL.



a Describe what conclusions a reader could make from this graph.
b Identify and interpret some of the features that make this a misleading graph.
c Describe how James should change his graph so it is not misleading.

WORKING	THINKING
a Basketball is by far the most popular choice.	The basketball looks at least four times bigger than the AFL football, suggesting that basketball is more than four times as popular.
b The size of the balls used to display the data is misleading. Basketball is the most popular choice, but not by as much as the images suggest.	From looking at the scale, 12 survey participants selected basketball and 8 selected AFL. This is not as big a difference as shown by the difference in the size of the balls.
c Change the graph to a picture graph using multiple images of the balls, each of the same size. Or create a column graph and have the images of the balls within the columns so that the images are all the same width.	When creating a graph, be sure that all columns are the same width and nothing looks bigger than it should.

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Exercise 5E

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

Hint Use WYNTK and the examples as a guide.

a A graph that has altered features to skew the interpretation of the data is known as a _____ graph.

b A graph title with a particular viewpoint may be _____.

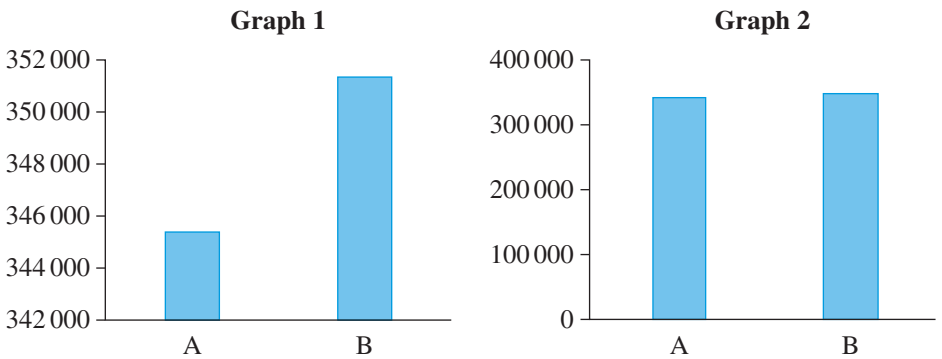
c A scale that does not start at zero is known as a _____ scale.

d A picture graph must have images or icons that are the same _____.

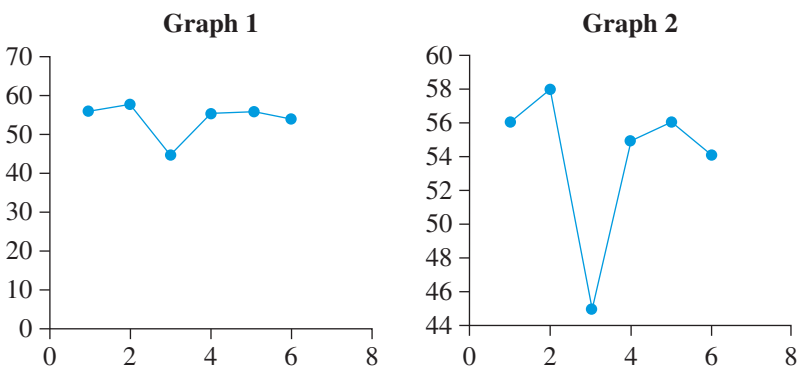
Example 8

2 The following are pairs of graphs that have been drawn from the same data. Identify the misleading and the non-misleading graph, giving a reason for your answer.

a



b



Hint Which sections of each graph could be biased?

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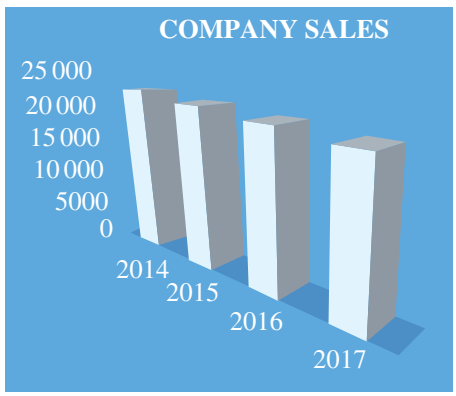
APPLICATIONS

Example 9

3 The graph shows a company's sales of a particular product for the years 2014 to 2017.



- **a** Decide if it is possible to easily see if the sales in 2017 are more or less than in previous years?
- **b** What conclusions could someone make from this graph?
- **c** Identify and interpret some of the features that make this a misleading graph.

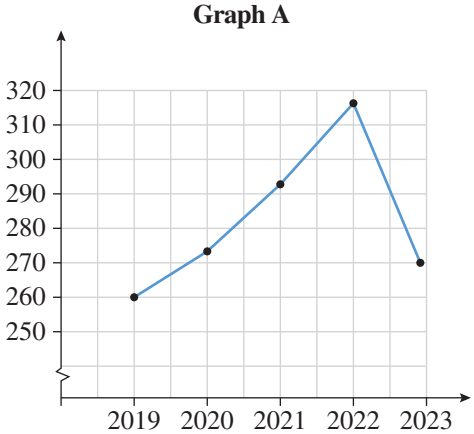
4 The graph shows Australia's unemployment rate between February 2017 and November 2017.

- **a** Describe what conclusions a reader could make from this graph.
- **b** Identify and interpret some of the features that make this a misleading graph.

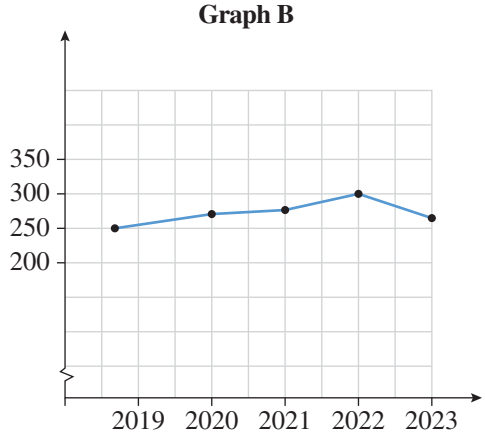
5 The two graphs below, illustrating the same data, are being considered by a high school to show prospective students and their parents.

Year 11 enrolment numbers at Top Heights High School

Graph A



Graph B



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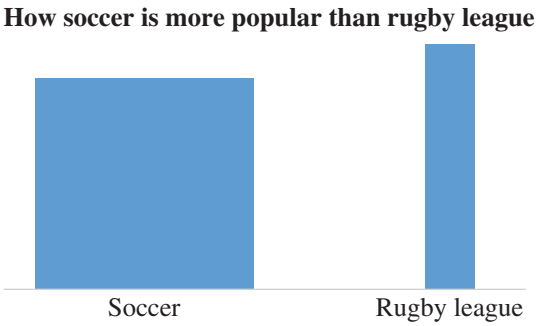
a By comparing the graphs, decide whether or not both graphs convey the same message.
b Determine which graph gives a more accurate representation of the data.
c Describe the way in which graph B could be misleading.
d Explain why graph B would be considered by the school to use at a meeting with prospective students and their parents.

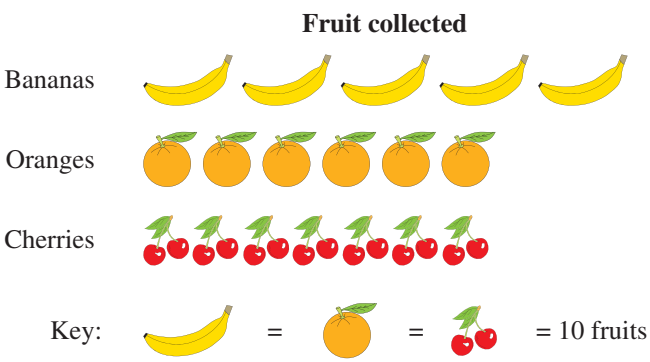
6 John is trying to convince his friend that soccer is more popular than rugby league. He conducts a survey, asking people which sport they prefer out of soccer and rugby league. John summarises his data in a column graph, as shown on the right.



a What conclusions could you make from this graph?
b Identify and interpret some of the features that make this a misleading graph.
c Give some possible reasons for the graph being drawn this way.

Hint What message is John trying to send to his friend?

7 The following graph is presented at a pickers' meeting.



a Determine how many of each fruit was collected.
b In what ways is this graph misleading?
c Redraw the graph to present the results more accurately.

Hint Use a grid to ensure the symbols are the same size.

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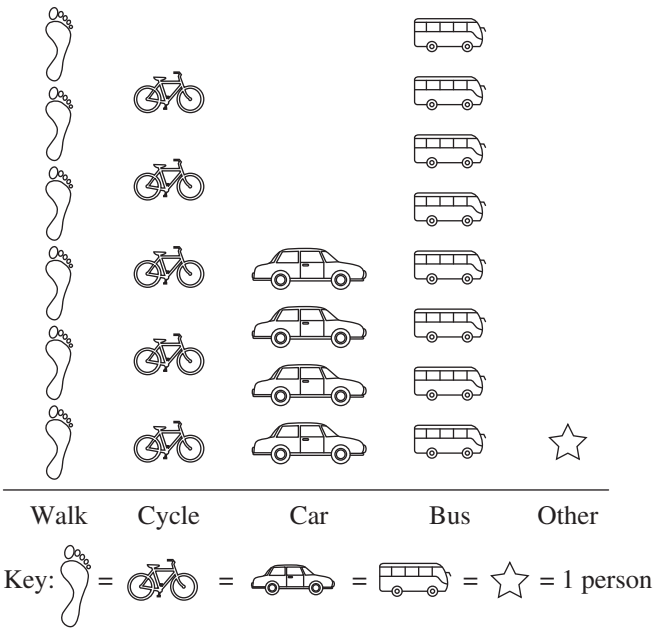
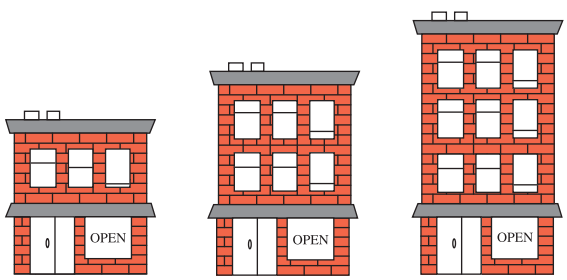
8 Sarah is doing a research project for school, looking into how students get to school. She summarises her results in a picture graph shown below.



a Describe what conclusions could be made from this graph.

b Identify and interpret some of the features that make this a misleading graph.

9 The graph below reports on the proportion of businesses in a city that have a social media presence.



a Quickly reading the graph, what message is being presented?

b Read the graph more thoroughly; what is actually being displayed? Justify your answer.

c Describe the ways this graph could be misleading.

Hint What is the purpose of the graph and how could it be biased?

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5F Interpreting tables and graphs

LEARNING GOALS

- Read and interpret tables, then justify your responses
- Read and interpret graphs, then justify your responses

Why is it essential to know how to interpret tables and graphs?

Tables and graphs are regularly found in the media and in factual texts. They are useful instruments for helping us make decisions in the home or at work. It is essential to learn the skills required to not only read and interpret tables and graphs, but also to be able to justify your reasoning when writing about them.

It is important to know how to interpret tables and graphs.

WHAT YOU NEED TO KNOW

- To analyse tables and graphs, you need to know what the words read, interpret and justify mean.
- **Read**: how to read a graph or table, take in the information and ensure you understand it. For this you have to know how graphs and tables organise and present information. You must be able to describe their characteristics and features, and state any relevant facts they show.
- **Interpret**: use the information to recognise any trends and draw conclusions from the information. You must be able to understand the meaning of the graph or table.
- **Justify**: give detailed reasons or evidence to support your responses and conclusions.

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Example 10 Reading, interpreting and justifying a table

Interpreting tables found in factual texts

The Government of Western Australia is currently concerned with the amount of people failing the provisional licence test. The statistics on the right were taken from a randomly selected driving school over a week.

WA learner driver test overview			
	Male	**Female**	**Total**
Passed	17	14	31
Failed	8	6	14
Total	25	20	45

a Calculate the percentage of people from the driving school who failed their test. Round your answer to 1 dp.

b Determine whether a greater proportion of males or females failed the test.

c Interpret the answer to **b** in terms of whether gender is a significant reason for people failing the test. Justify your interpretation.

WORKING	THINKING
a $\frac{14}{45} \times 100 = 31.1\%$ In a week, 31.1% of the people from this driving school failed their test.	Read all the information. Calculate the percentage who failed.
b $\frac{8}{25} \times 100 = 32\%$ 32% of males from this driving school failed in a week. $\frac{6}{20} \times 100 = 30\%$ 30% of females from this driving school failed in a week. A greater proportion of males failed during this week.	Read all the information. Determine the proportion using the percentage calculations of both males and females who failed.
c Almost the same proportion of males and females failed their provisional licence test during the week. 32% of males failed and 30% of females failed. Even though the percentage of males who failed is higher, it is only by 2%, which is not a significant amount. This table is showing that gender is not a significant reason for people failing their provisional driving test.	Interpret what the proportions reveal. Justify your responses.

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Example 11 Reading, interpreting and justifying graphs

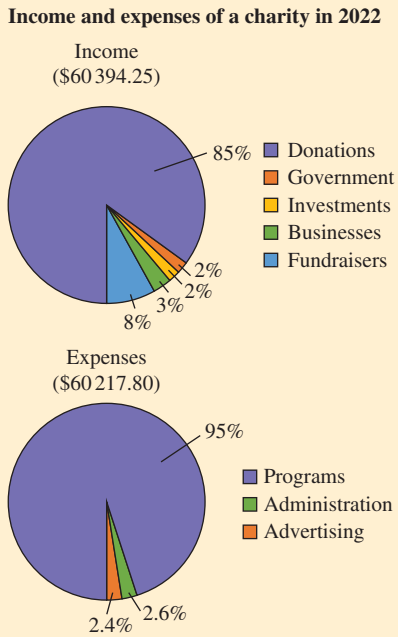
Interpreting graphs found in factual texts

The pie graphs to the right were recently used in a charity's pamphlets. The 'programs' refer to the charitable work carried out to support people in need.

a Identify the greatest income source.

b Calculate the difference in value between income and expenses.

c Describe what the graphs show overall, and compare the income and expenses of the charity. Suggest whether the graphs indicate that the charity's 2022 performance was satisfactory, in relation to its financial situation and supporting people in need.



WORKING	THINKING
a The greatest income source is donations, making up 85% of income.	Read all the information. Identify the graph that is relevant to income, and identify the largest sector of the pie chart.
b $60 394.25 − $60 217.80 = $176.45 The difference between income and expenses is $176.45.	Identify the total income and expenses. Calculate the difference between income and expenses.
c The pie graphs show income and expenses for a charity in 2022. The total amount of income was enough to cover the expenses. 95% of the charity's spending was on programs supporting people in need. Both of these indicate good performance as the charity did not lose money and spent nearly all of its money supporting people in need, which is what a charity should do. Very little was spent on administration and advertising.	Describe what the graphs show overall. Interpret the difference between income and expenses. Interpret how the charity's income was used. Decide whether the charity's performance was good, bad or in between, and justify your decision.

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Exercise 5F

FUNDAMENTALS

Example 10

1 The table shows the number of males and females in a police force, according to rank.

	Male	Female	Total
Constable	87	56	143
Sergeant	25	7	32
Inspector	8	11	19
Chief Inspector	3	3	6
Total	123	77	200

a Calculate the percentage of females in the police force, according to the data.

b What percentage of Sergeants are male, according to the data?

c What percentage of Inspectors are female, according to the data?

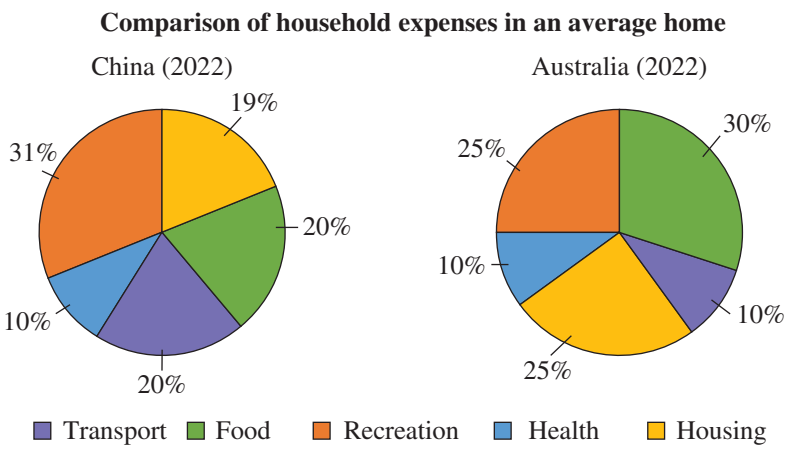
d What percentage of Chief Inspectors are female, according to the data?

Hint The percentage of a value can be found by dividing it by the total and then multiplying it by 100.

e Explain your interpretation of this table in terms of the number of males compared with females in the police force.

Example 11

2 The pie graphs show the average household expenditures in China and Australia in the year 2022.



a Which sector has the least expenditure in both countries?

b What percentage does each country spend on food?

c Summarise the information by selecting and reporting the main features, and make comparisons where relevant.

Hint When comparing, list all of the things that are the same (compare), then list all of the things that are different (contrast).

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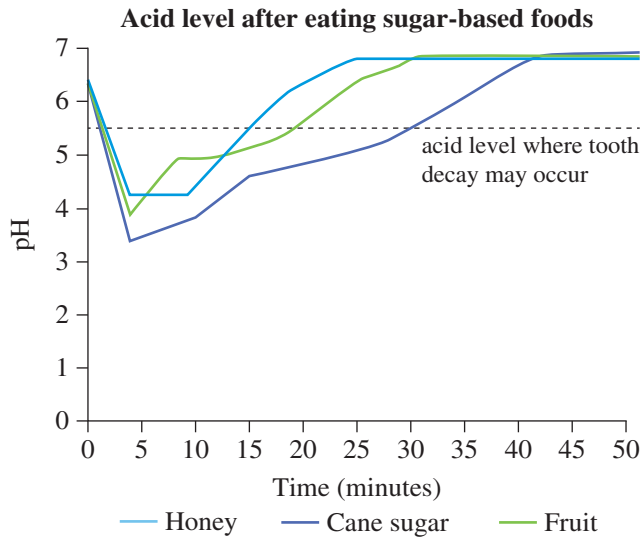
APPLICATIONS

3 Use the graph on the right to answer these questions. Acid level is measured by the pH scale, where the lower the pH reading, the higher the acid level. Higher acid levels contribute more to tooth decay.



a How many minutes after eating a food with cane sugar is acid level at its worst (i.e. the most acidic)?

b How long after eating each of the following is the acid level unlikely to contribute to tooth decay?

i honey **ii** cane sugar **iii** fruit

c What is the pH level of eating fruit after 12 minutes?

Hint What colour line is 'fruit'?

d What have you gathered from interpreting this graph?

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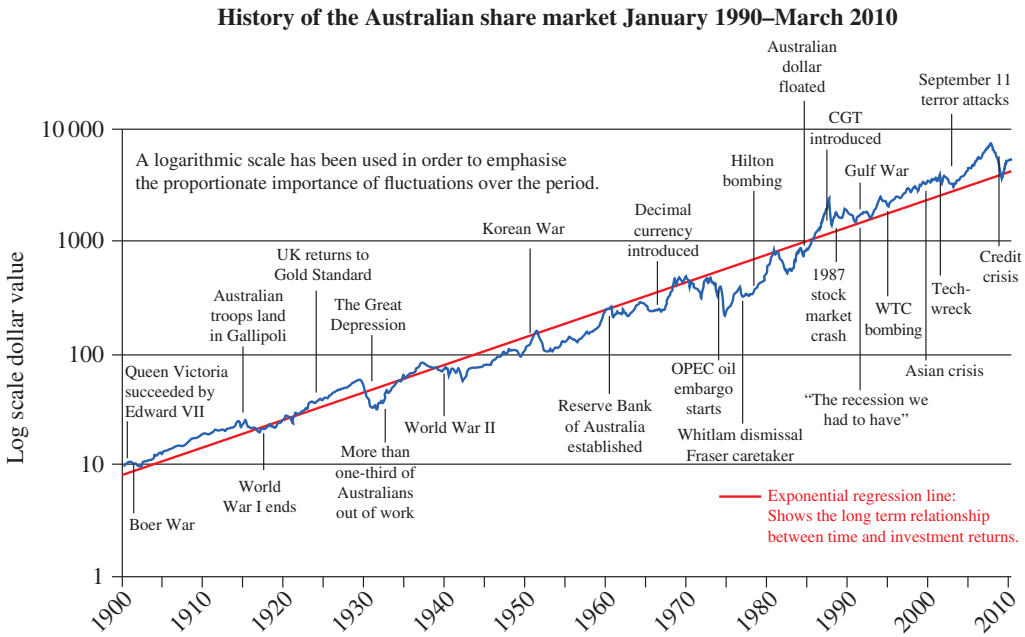
4 Study the graph given and answer the following.



Source: MLC Investments Commercial & Industrial Index Monthly Averages (1875–1936), Sydney All Ordinaries Index Monthly Averages (1936–1979), All Ordinaries Share Price Index Monthly Averages (1980–1999), All Ordinaries Share Price Index Month End Price (2000–2009).

a What is the overall trend of the Australian share market, as shown in the graph?

b What four events caused the share market to fall?

c What are some current events that could affect today's share market?

d After interpreting the graph, what condition would you expect today's share market to be in?

> **Hint** Expand on the information from the graph into what's happening in today's market.

5 A survey asked 1220 people in a population if they are satisfied with their body image or dissatisfied, and if they are dissatisfied, whether they think they are overweight or underweight. The results are shown in the two-way table. Use the information to complete the following.

	Body image			
	Satisfied	**Dissatisfied (overweight)**	**Dissatisfied (underweight)**	**Total**
Female	550	170	40	760
Male	300	80	80	460
Total	850	250	120	1220

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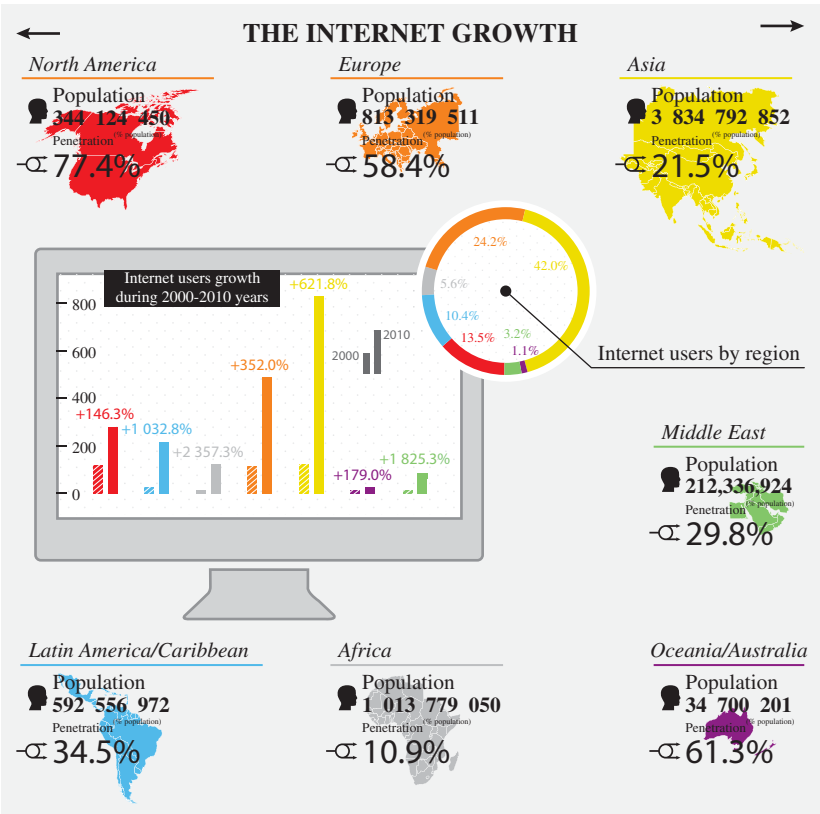
a Convert the counts of all female body images to percentages.

b Convert the counts of all male body images to percentages.

c Interpret the data to write a paragraph as to whether these body image categories are related to gender. Justify your answer.

6 Use the statistics infographic to complete the following.



a Use the infographic to list the world's regions of internet penetration in order from highest to lowest.

b According to the infographic, what are the world's regions of highest growth percentage in terms of internet users between the years 2000 and 2010, in order from highest to lowest?

c Interpret the data to write a paragraph discussing internet penetration in Asia.

Hint Don't be overwhelmed by all of the data. Explain one graph at a time, in a sentence.

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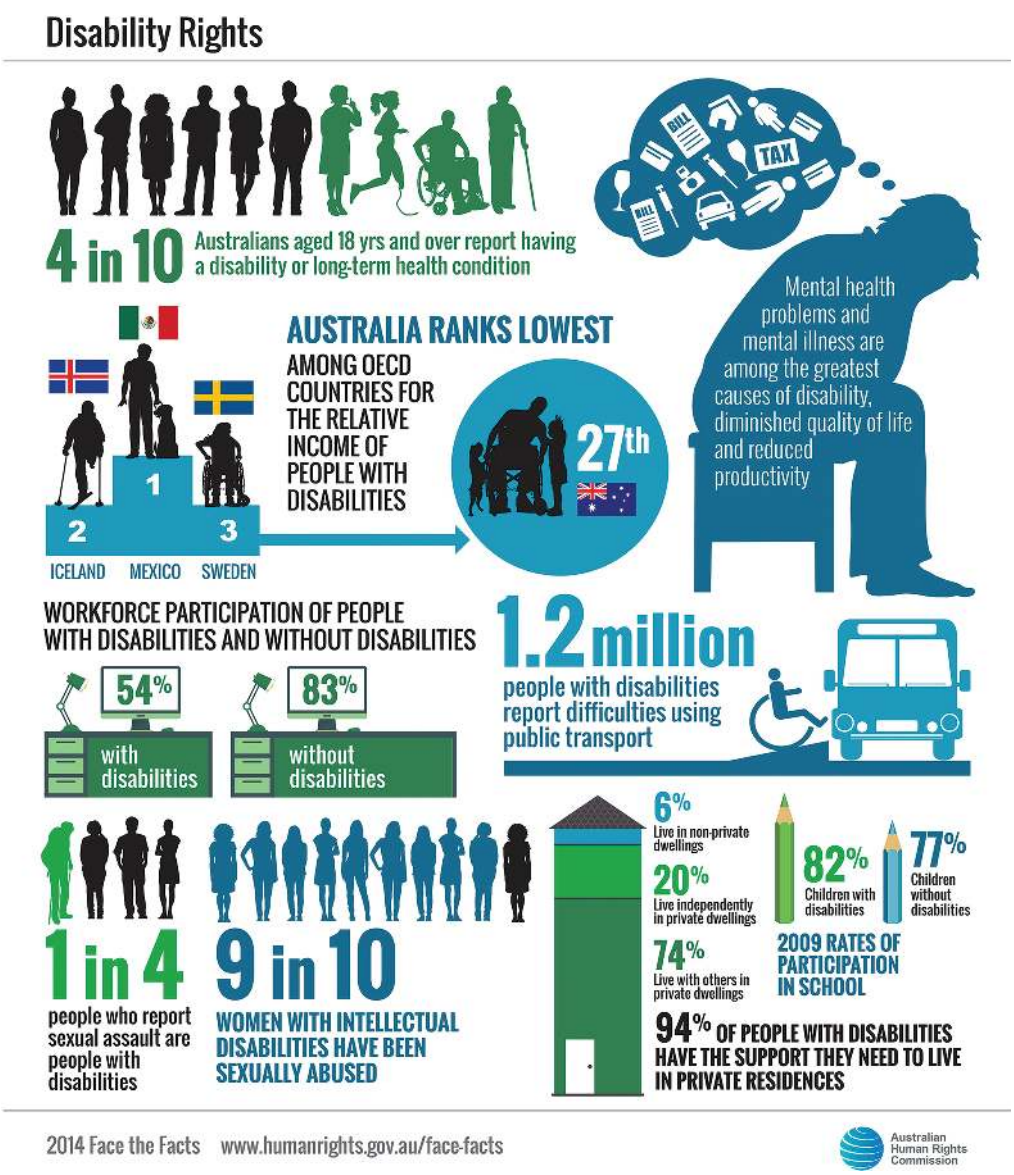

Use the statistics infographic to complete the following.

a According to the infographic, what is the percentage of Australians aged 18 years and over who report having a disability or long-term health condition?

b What percentage of Australians who report sexual assault are people with disabilities?

c Interpret the data to write a paragraph discussing the issues affecting people with disabilities in Australia.

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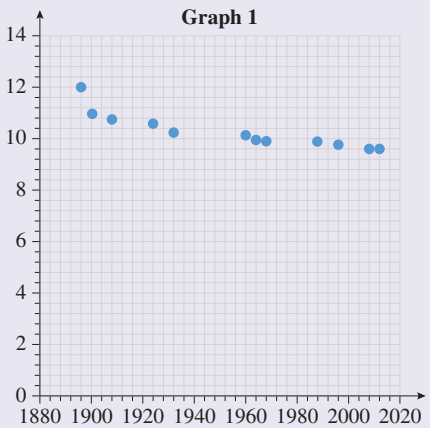
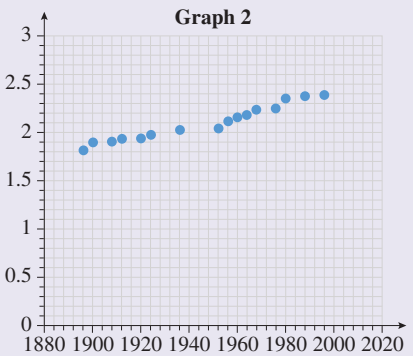
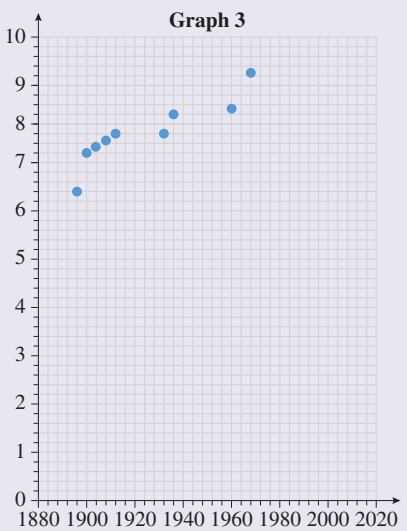
Statistical investigation process task

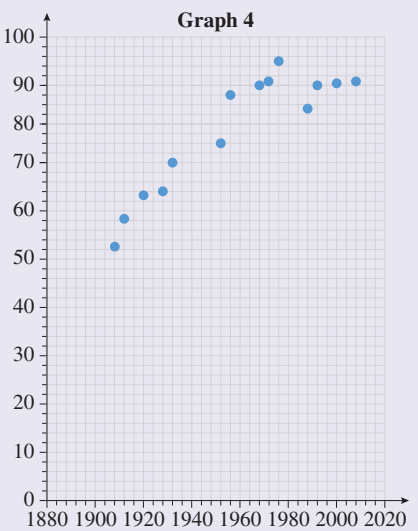
READING AND INTERPRETING GRAPHS

Background: Graphs of values over time can be very useful for displaying changes and trends – the general direction in which something is heading, for example, is it getting worse, or better? The record times or distances achieved in athletics are a good example. These six graphs for six different Olympics athletics events show when a new Olympic record was set and what its value was.









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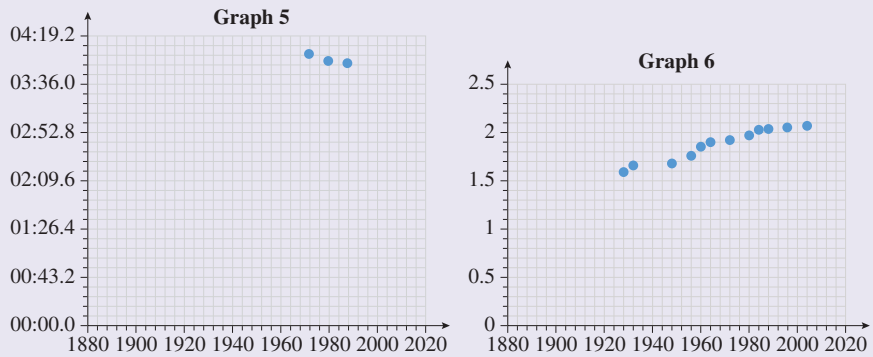

Task: These six graphs illustrate how Olympic records have changed over time in six different athletic events. Unfortunately, some parts of the information have been lost in transit.

Your task is to research Olympic athletic events. You must compare and contrast all events and conclude which event is the correct one for each graph. Determine what the units should be for the vertical axes. Explain why some graphs show a decreasing trend and some an increasing trend. Lastly, are there any unusual features on any of the graphs? Follow the investigation process workflow diagram on the next page and use the steps listed as a guide.

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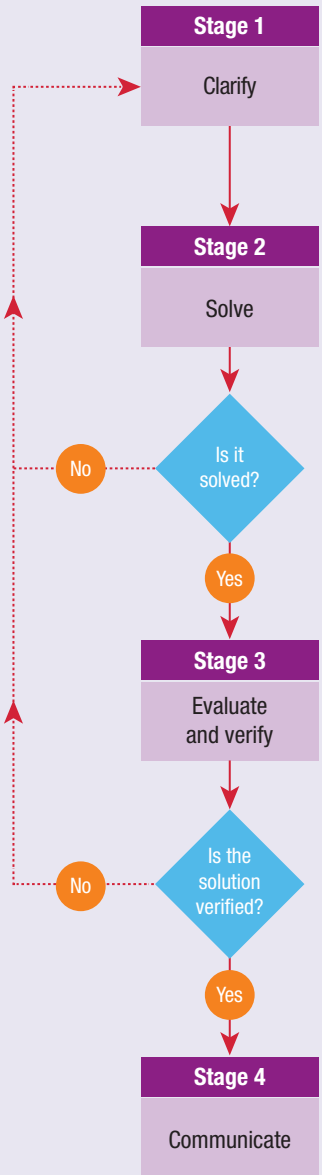

Stage 1: Clarify
Clarify the problem and pose one or more questions that can be answered with data

Decide which details are missing in the graphs.

1 Describe what you think is needed for each graph.
2 Research the athletics events held at the Olympics.
3 Research the type of score measured for each event.
4 Research the records of each event.

Stage 2: Solve
Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

Once you have obtained your data, complete the following.

5 Suggest the event for each graph and what the vertical axis title and units should be.
6 Describe the trends in the records for each event.
7 Suggest any improvements which could be made to any of the graphs.

Stage 3: Evaluate and verify
Interpret the results of this analysis and relate the interpretation to the original question

8 Check that the Olympic records for each event match the graph and that you can justify your responses with reasons.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

9 Communicate your findings in a short report giving your responses to all the questions and justifying them with your reasons.

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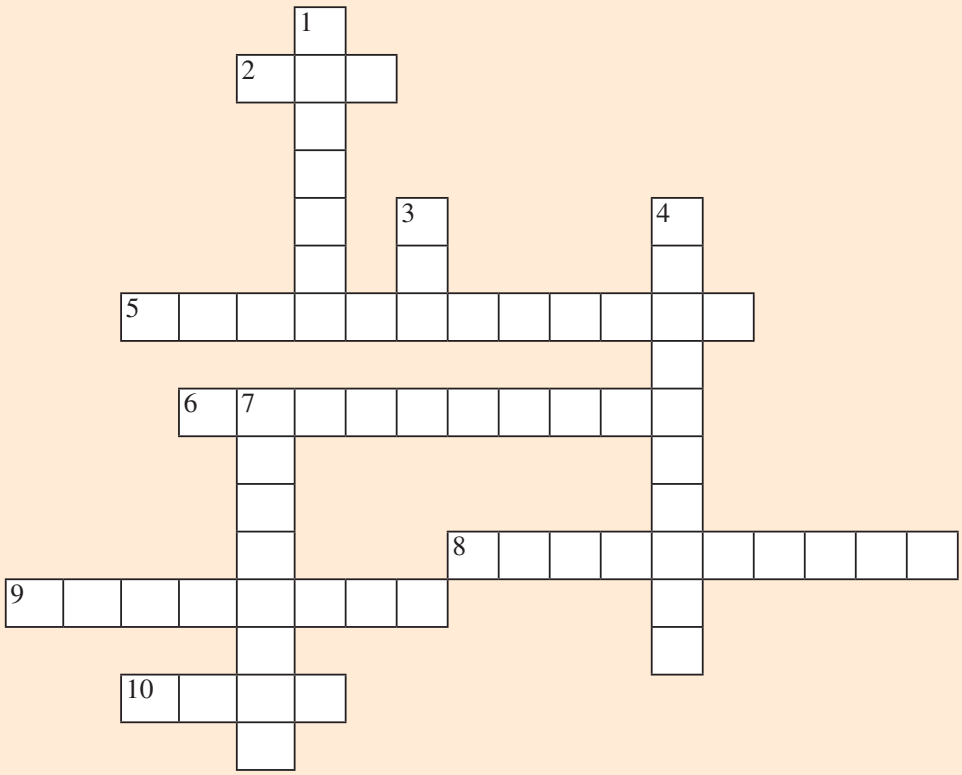
Puzzle



Across

2 Picture graphs must include a _____ to show the value of the icons used

5 Conversion graphs can be used to convert _____

6 A graph that has a scale that doesn't start at zero or has certain information missing may be a _____ graph

8 A quantity that can be measured is known as a _____ variable

9 An open circle on a line segment in a step graph represents an _____ value

10 A graph that consists of a series of horizontal line segments is called a _____ graph

Down

1 A pie graph is a circle divided up into _____

3 Another name for a column graph is a _____ chart

4 Column graphs display the quantity of data in various _____

7 A closed circle on a line segment in a step graph represents an _____ value

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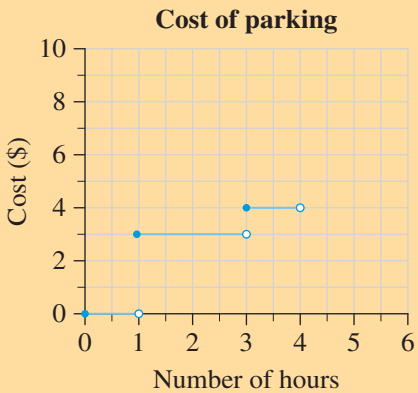
Chapter checklist

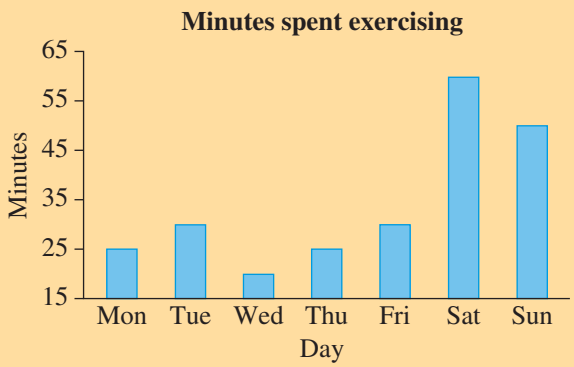
☐ I can read and interpret information presented in step graphs.

1 Given the step graph shown, what would be the cost for 3 hours of parking?



☐ I can read and interpret information presented in column graphs.

2 According to the column graph, how many minutes were spent exercising on Monday?



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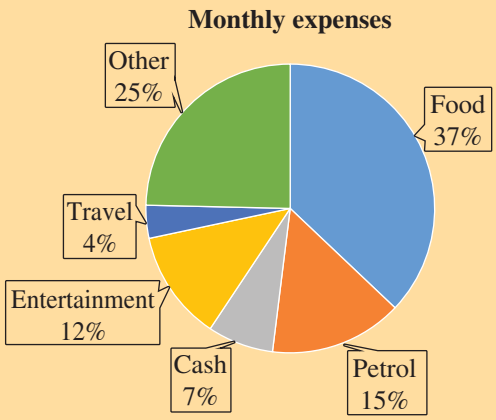
I can read and interpret information presented in pie graphs.

3 Given the pie graph shown, what percentage of the monthly expenses is spent on entertainment?



I can read and interpret information presented in picture graphs.

4 Referring to the picture graph, how many magpies were seen on the way home on Friday?

Day	Number of magpies seen on way home
Mon	
Tues	
Wed	
Thurs	
Fri	

Key: = 2 magpies

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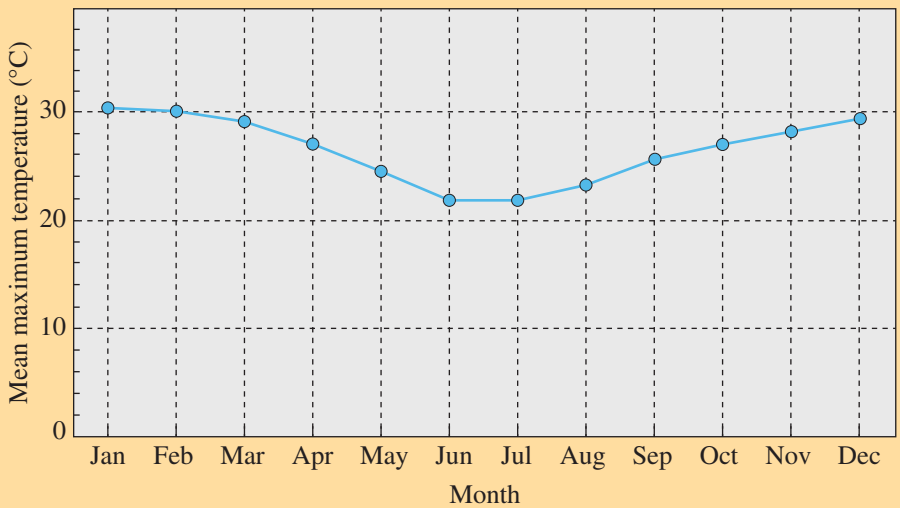
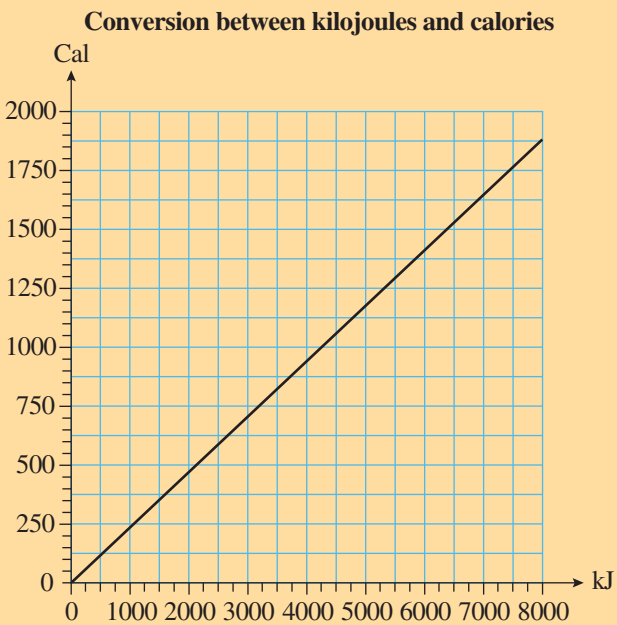
☐ I can read and interpret information presented in line graphs.

5 According to the line graph, what was the mean maximum temperature in June?



☐ I can read and interpret information presented in conversion graphs

6 Using the conversion graph, convert 5000 kilojoules to calories.



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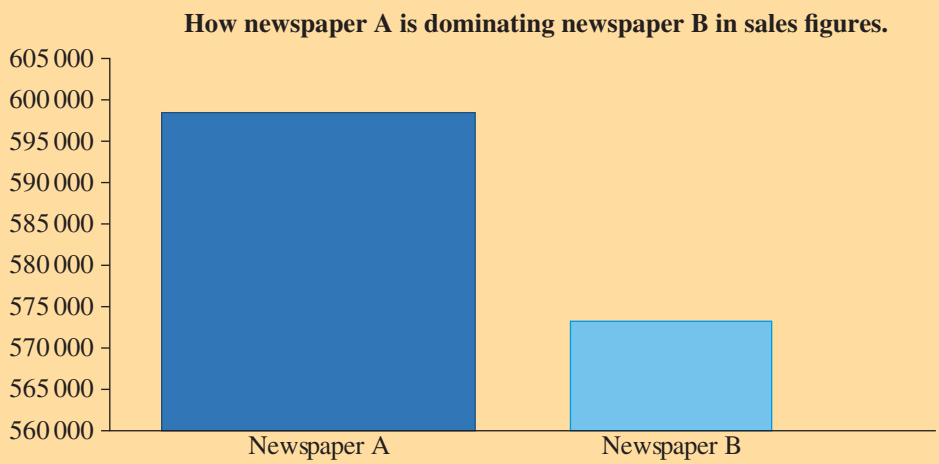
I can read and interpret information presented in two-way tables.

7 Given the two-way table, how many of the 2-wheel drive vehicles use diesel?

	Uses petrol	Uses diesel	Total
4-wheel drive	15	55	70
2-wheel drive	30	20	50
Total	45	75	120

I can identify and interpret graphs that could be possibly misleading.

8 For the column graph shown, what are some of the features that make it a misleading graph?



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☐ I can read and interpret information presented in tables and graphs.

9 Which gender prefers sports utility vehicles over sports cars?

Comparing sports utility vehicles and sports cars			
	SUV	**Sports car**	**Total**
Male	11	106	117
Female	63	20	83
Total	74	126	200

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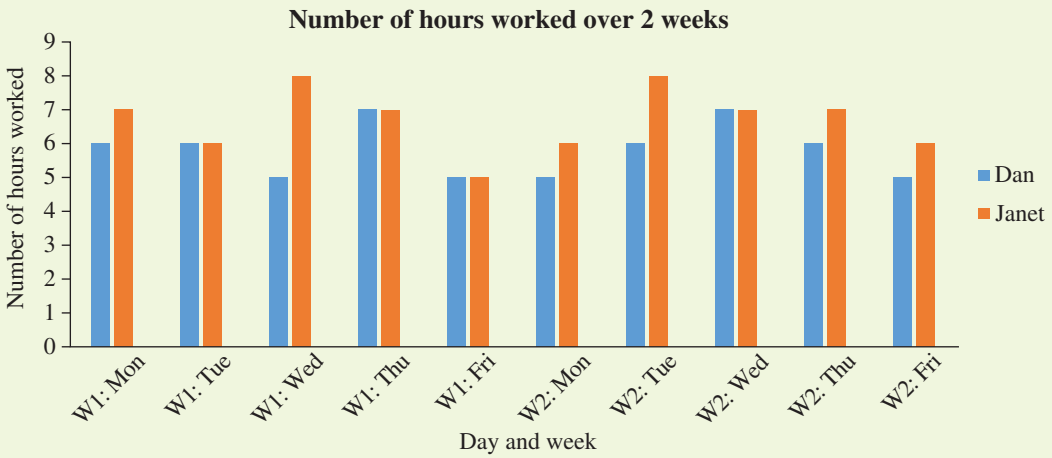
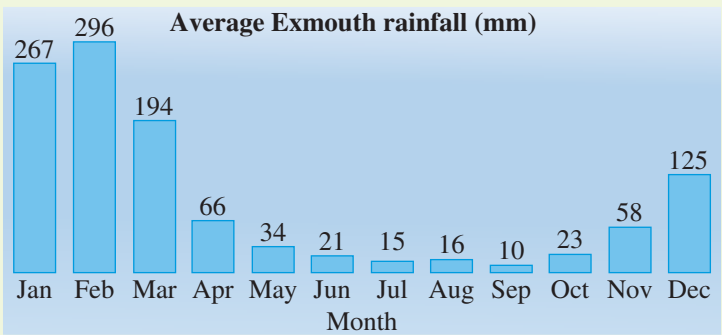
Chapter review

1 Johan runs a cafe and has two workers, Dan and Janet. The number of hours worked in the cafe over a 2-week period by both Dan and Janet is shown on the graph.



a How many hours did Dan work on the Monday of the second week?
b How many hours did Janet work on the Wednesday of the first week?
c On what days did both Dan and Janet work the same number of hours?
d Using the graph, who works more hours at the cafe? Justify your answer.

2 The following column graph shows the average monthly rainfall in Exmouth, in millimetres.



a On average, how many millimetres of rainfall occurs during November?
b On average, how many millimetres of rainfall occurs during May?
c Which month has the highest rainfall on average?
d Exmouth can be said to have a wet season and a dry season. Which months do you think would be classified as the wet season? Justify your answer.

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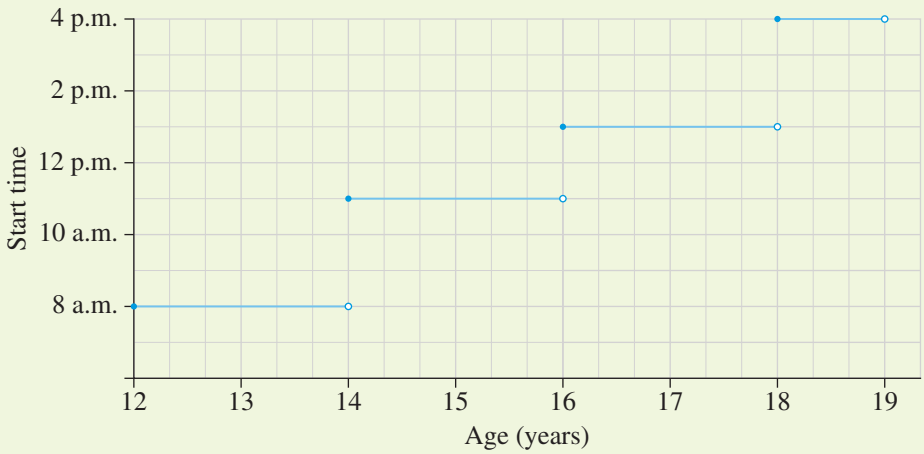
3 The step graph shows the event start times for an athletics competition, according to the participant's age.

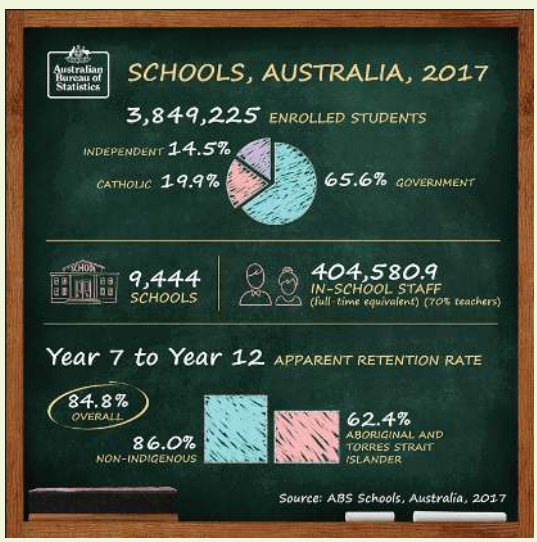


a What is the event start time for a competitor who is 13 years old?

b What is the event start time for a competitor aged 16 years?

c Andrea's event starts at 11 a.m. How old could she be?

4 Information regarding schools in Australia in 2017 is shown.



a What percentage of students attended government schools?

b What percentage of students attended independent schools?

c What percentage of students attended Catholic schools?

d What percentage of non-Indigenous students stayed in school from Year 7 to Year 12?

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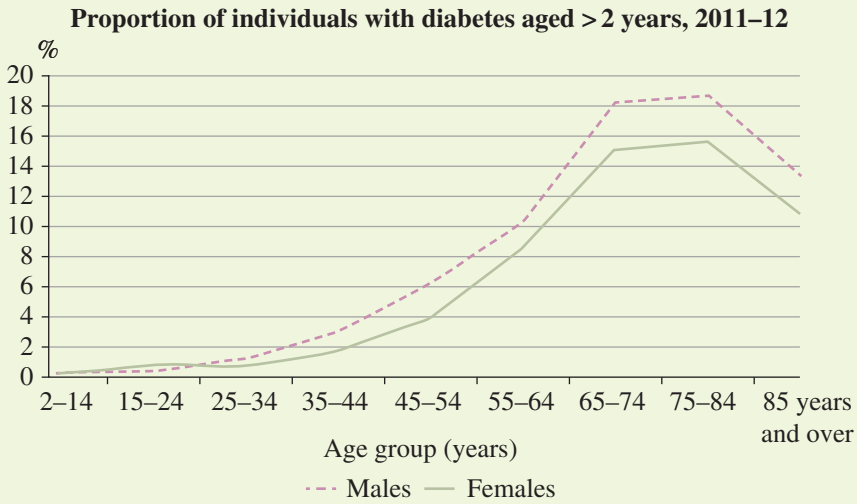
5 The chart shows the proportion of people with diabetes in Australia between 2011 and 2012.



a Approximately what percentage of males aged between 45 and 54 years had diabetes during 2011–12?

b Approximately what percentage of females aged between 54 and 74 years had diabetes during 2011–12?

c Which age group had the highest proportion of people with diabetes during 2011–12?

d Using the information in the graph, were males or females more likely to have diabetes during 2011–12? Justify your answer.

6 A carpenter is trying to decide whether to replace their circular saw or their nail gun, as they can currently afford to replace only one. They keep records over 3 weeks of how many times they use each tool for specific jobs. The results are shown in the two-way table.

	Circular saw	Nail gun	Total
Week 1	7	6	13
Week 2	3	12	15
Week 3	9	3	12
Total	19	21	40

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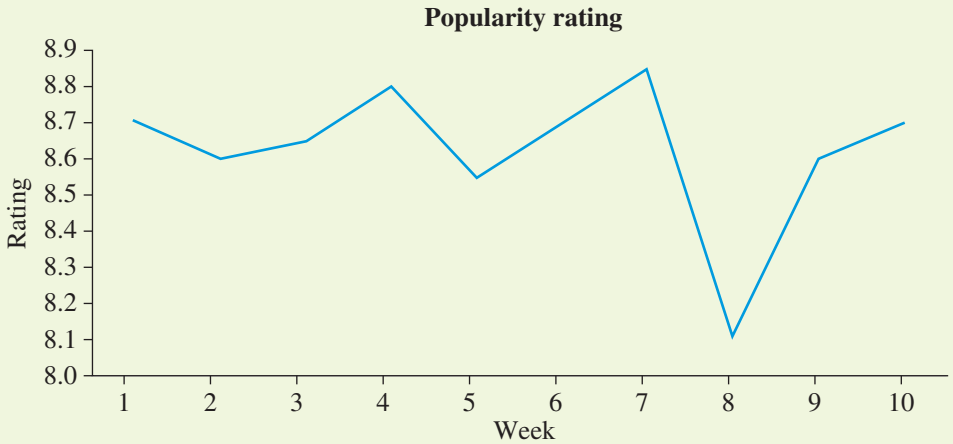
a How many times does the carpenter use their circular saw in Week 3?
b How many times does the carpenter use their nail gun in Week 1?
c What percentage of Week 2 does the carpenter use their nail gun?
d In total, what percentage of the 3 weeks does the carpenter use their circular saw?

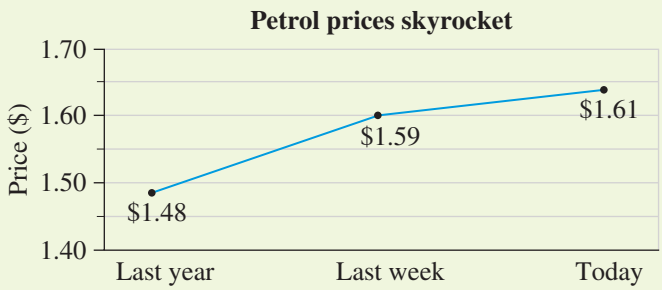
7 The popularity rating of a politician running for Prime Minister over a period of 10 weeks is shown.



a What conclusions can the reader make from this graph?
b Identify and interpret some of the features that make this a misleading graph.
c Give some reasons why this graph might have been produced to look the way that it does.

8 Interpret each of the following graphs and any accompanying text.
i Summarise the information provided by the graph.
ii What aspects of this graph might mislead a reader?
iii List the changes that should be made to make the graph and its title more accurate.

a



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b

9 Interpret the given table.

a Describe, in a few words, what the table shows.

b Which month and year had the highest number of visitors?

c Which month and year had the lowest number of visitors?

Short-term visitors to Australia

	2015	2016
Jan	303 800	546 200
Feb	319 100	722 200
Mar	313 500	674 500
Apr	294 200	551 400
May	244 800	483 000
Jun	261 400	488 300
Jul	329 700	627 100
Aug	304 900	573 800
Sep	268 600	571 500
Oct	320 700	624 000
Nov	342 900	668 900
Dec	422 300	897 700

Source: ABS 3401.0 – Overseas Arrivals and Departures, Australia, Feb 2016

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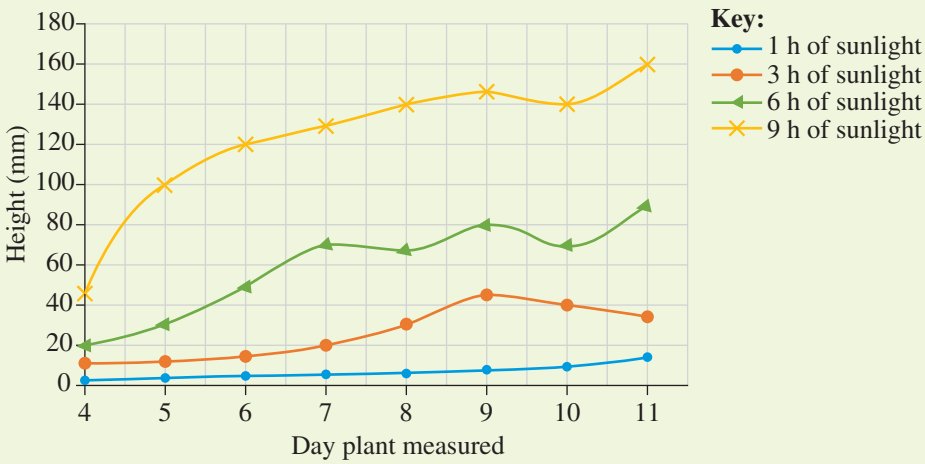
10 Interpret the graph, answering the following questions.



a Describe the experiment that was carried out, according to the graph.

b Did the plants grow at a constant rate (i.e. by the same amount or proportion each day)? If not, in which amount of sunlight was the fastest growth recorded?

c According to this experiment, which amount of sunlight is best for plant growth?

d What improvements would you suggest to the graph, if any?

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6 Drawing graphs

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In this chapter

Syllabus reference

Unit 1 Topic 1.4: Graphs

Drawing graphs

- determine which type of graph is the best one to display a dataset
- use spreadsheets to tabulate and graph data
- draw a line graph to represent any data that demonstrates a continuous change, such as hourly temperature

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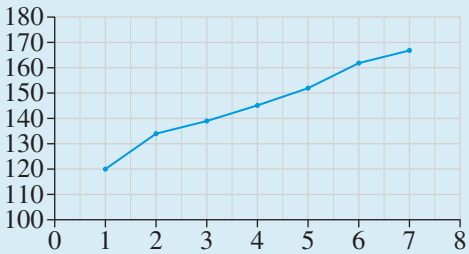
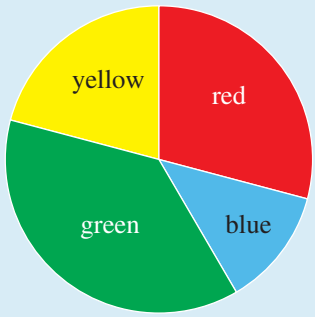
Pre-test

1 Label the following diagrams according to the type of graph or table they are, choosing from: pie graph, column graph, two-way table, line graph, histogram, dot plot and stem-and-leaf diagram.

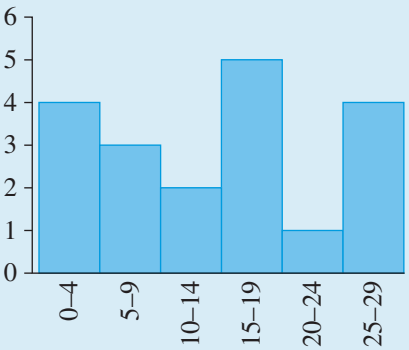
a



b



c



d

stem	leaf
1	0 4
2	2 6 7 9 9
3	2 3 4 7
4	1 1 7 9
5	2 6
6	8 8

2 The data of a survey that was carried out is shown.

Letter	A	E	I	O	U
Frequency	14	10	11	7	13

a What is the frequency for the letter A?

b What is the frequency for the letter O?

c Hand draw a column graph of this data, with 'Letter' on the horizontal axis and 'Frequency' on the vertical axis.

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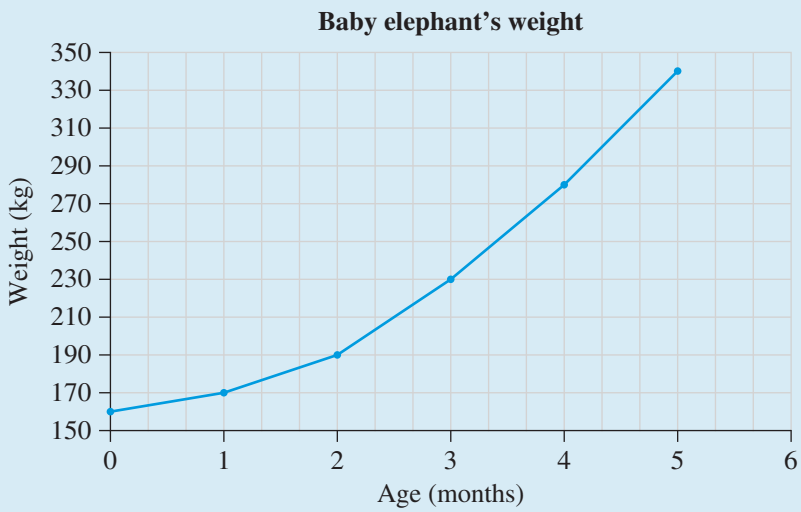
3 A group of young adults were asked what state or territory they were born in. The responses are shown. Copy and complete the following to tabulate the data.

Vic.	Qld	ACT	Qld	NSW	NSW
Qld	Qld	Vic.	SA	Qld	SA
WA	Vic.	NSW	Vic.	Qld	Tas.
SA	Qld	Qld	SA	WA	Qld
WA	NSW	Qld	Qld	Qld	Qld
Qld	Qld	Vic.	WA	NT	Vic.
SA	NSW	Qld	WA	NT	Qld

State	Frequency
NSW	5
Qld	
Vic.	
SA	
WA	
NT	
ACT	
Tas.	

4 The following is a graph of the weight of an elephant for the first 5 months of its life.



a How much did the elephant weigh at 1 month?
b How much did the elephant weigh at 3 months?
c How much did the elephant weigh at 4 months?

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6A Determining the best graph for a dataset

LEARNING GOAL

- Determine which type of graph is best for displaying a dataset

Why is determining the best graph for a dataset essential?

Choosing the right graph is essential to correctly presenting information.

At some point throughout your career, you may be asked to present some collected data to other people for them to interpret. Deciding how information should be presented can be a daunting task. You may ask yourself, 'What are the important features? How can this be presented visually to best summarise the information easily?' Because of this, it is essential to understand how to choose the best **graph**, depending on what data you are trying to present.

WHAT YOU NEED TO KNOW

- A **dataset** is a collection of related sets of information.
- Types of data:
  - **Categorical data** is the type where the responses are not numbers; for example, the types of livestock a farmer cares for.
  - **Numerical data** is the type where the responses are numbers; for example, the temperature of water in a hot water boiler.
  - **Discrete data** is numerical data where the responses are whole numbers; for example, the number of screws required to build a timber frame for a house.
  - **Continuous data** is numerical data that can take any value, including fractions and decimals, and is usually a measurement; for example, the weight of a newborn baby.

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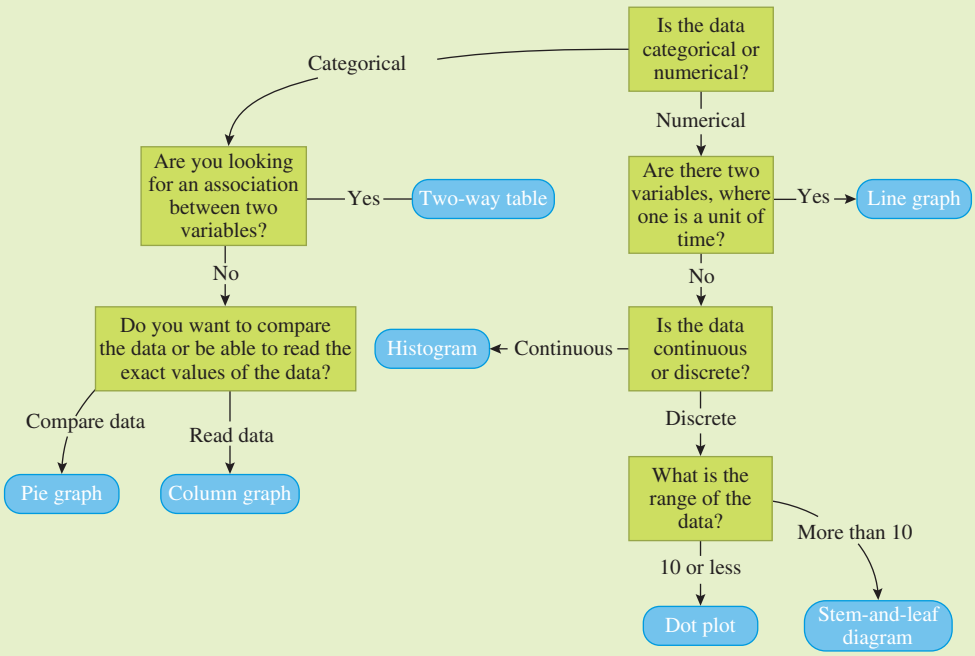
- In order to decide which graph to use, follow the flow diagram below, based on the types of data being used.



Example 1 Selecting and drawing a pie graph

James works as a graphic designer for a town council. He is given the task to produce a display of household water usage to be used in a TV advertisement promoting ways of saving water.

Household water usage	
Home area	**Percentage of water use**
Kitchen	20%
Laundry	18%
Toilet	22%
Bathroom	40%

a What features of a pie graph make it suitable for a quick comparison of data?

b Calculate the pie graph angles for each of the listed household water-use areas. Round each angle to the nearest whole number.

c Draw a pie graph showing household water usage.

... Continued

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WORKING | **THINKING**

a James would use a pie graph, as it is colourful and attracts attention, it arranges information quickly, and it gives a visual comparison of each part's proportion of the total.

What is James wanting to make?
A TV advertisement that shows information quickly and simply.

What type of dataset is this?
A visual comparison of water usage as a proportion of the total household consumption.
Which type of graph best suits arranging particular parts in relation to the whole?
The graph that best suits this data is a pie graph.

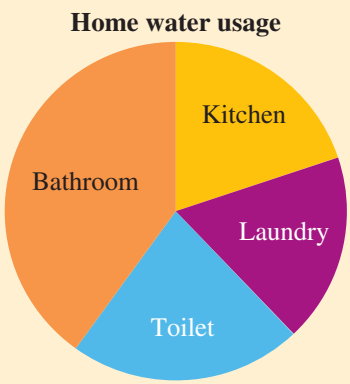
b

Home water usage		
Home area	**Percentage**	**Angle**
Kitchen	20%	72°
Laundry	18%	65°
Toilet	22%	79°
Bathroom	40%	144°

Add a third column to the table.
Each angle = % × 360°

$$\text{Laundry angle} = \frac{18}{100} \times 360 = 64.8$$
$$= 65°$$

c



Draw a circle and mark the angles using a protractor and ruler.
Colour and label the sectors and write the title.

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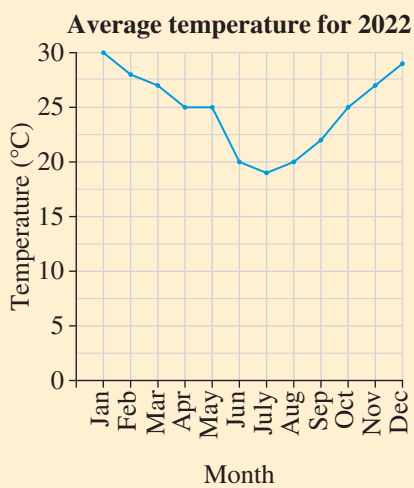
Example 2 Identify varying characteristic of datasets

Simon is asked to record how his city's average high temperature varies from month to month for 2022, in a report to his management, to see if the office should purchase an air conditioner.

January 30, February 28, March 27, April 25, May 25, June 20, July 19, August 20, September 22, October 25, November 27, December 29

a What type of graph will best suit Simon's report?

b Draw the graph.

WORKING	THINKING
a Line graphs are best to compare changes over the same period of time for more than one group.	*What is Simon wanting to show?* The variation in weather. *What type of dataset is this?* It is comparing varying temperatures. *Which type of graph best suits comparing varying data?* The graph that best suits this data is a line graph.
b 	It is best to use graph paper when drawing a line graph. Give your graph a title. Write labels for the vertical axis: Temperature (°C), in intervals of 5. Write labels for the horizontal axis: Month, writing the months of the year. Find each month's temperature and put a dot at the correct location.

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Exercise 6A

FUNDAMENTALS

1 Olivia is setting up a children's afterschool program. She has recently completed a survey of school-aged children, asking them what is their favourite activity. Olivia plans to include this information in her advertising flyer. The survey results are:

Dance = 30, Music = 40, Art = 25, Cricket = 20, Football = 35

a What type of graph should Olivia use?
b Explain why she should use this graph.
c Draw the graph.

Hint Use WYNTK and the examples to help you.

2 For each of the following, determine which type of graph would be best for displaying the dataset. Keep in mind there may be more than one possible answer.

a

Time	8 a.m.	9 a.m.	10 a.m.	11 a.m.	12 p.m.	1 p.m.
y	13	17	16	18	18	20

b

A	B	B	D	D
D	C	B	A	C
A	A	A	C	A
D	A	D	B	C

3 Sonia is recording rainfall to assist with her gardening project. She needs to include a visual component.

Jan 124 mm, Feb 70 mm, March 50 mm, April 48 mm, May 98 mm, June 210 mm, July 202 mm, August 185 mm, September 250 mm, October 252 mm, November 194 mm, December 160 mm

a What type of graph should Sonia use?
b Explain why she should use this graph.
c Draw the graph.

Hint How would this particular graph best display the information?

APPLICATIONS

Example 1

4 Jeff works at the Perth Zoo and has been given a list of how many snakes there are in other zoos in the area. He has been asked to plot a dataset, shown in the table on the next page, of how many snakes there are in the 15 local zoos.

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20	28	46	30	32
41	26	42	36	28
36	38	46	37	38

a What type of graph should Jeff use?
b Explain why he should use this graph.
c Draw the graph.

Hint Think about what type of data this is.

Example 2

5 As part of Brandon's role as an electrician, he must drive to clients' homes to provide quotes for jobs. As this is part of his job, he is able to claim some expenses on tax, such as petrol. His accountant has asked him to keep track of how far he travels each day for a month, in order to predict how much he will be able to claim at the end of the financial year. The distances driven each day by Brandon, for 30 days, can be seen in the following table.

28.1 km	31.3 km	21.2 km	17.0 km	13.8 km	21.3 km
26.4 km	5.2 km	5.8 km	8.9 km	27.5 km	24.8 km
22.4 km	7.6 km	31.1 km	7.7 km	20.9 km	12.3 km
13.6 km	14.4 km	23.1 km	9.5 km	12.0 km	15.5 km
8.0 km	22.6 km	12.8 km	22.5 km	27.7 km	11.0 km

a What type of graph should Brandon use?
b Explain why he should use this graph.
c Draw the graph.

6 Sam works for an online car sales company and has been given datasets of sales and prices for various makes of cars. She has been asked to give a visual representation of the data.

Make	Sales	Transaction price
Australian Honda	150 000	$27 500
Fiat Chrysler	220 000	$35 000
Ford Motor Company	235 000	$36 500
General Motors	280 000	$38 000
Nissan Australia	130 000	$25 000
Toyota Motor Company	245 000	$32 500
Volkswagen Group	50 000	$40 500

a What type of graph should Sam use?
b Explain why she should use this graph.
c Draw the graph.

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7 At a miners' union meeting, people are asked what is being mined at the mine site where they work, choosing from minerals, petroleum, natural gas or other. The results are shown in the table.

natural gas	petroleum	natural gas	minerals	petroleum
minerals	minerals	natural gas	other	other
natural gas	natural gas	other	minerals	other

a What type of graph should be used in order to read exactly how many people gave each response?
b Explain why you would use this graph.
c Draw the graph.

8 Henriette is helping out at a newspaper for her school's work experience program and has been given some data. She has been asked to give a visual representation of the data.

Males aiming for a High School Certificate = 16
Males aiming to go to TAFE = 46
Total amount of males recorded = 65
Females aiming to go to TAFE = 51
Females aiming for an apprenticeship = 3
Total aiming for a High School Certificate = 28
Total aiming for an apprenticeship = 6

a What type of graph should Henriette use?
b Explain why she should use this graph.
c Draw the graph.

9 A farmer is concerned with the health of one of her cows, which is pregnant. The vet suggests that she keep a record of the cow's weight during the pregnancy. The farmer weighs the cow at the beginning of each month during the pregnancy. The results are given below.

Month	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Weight (kg)	710	717	725	730	738	745	752	762	771	775

a What type of graph should the farmer use?
b Explain why she should use this graph.
c Draw the graph.

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6B Using spreadsheets for tables and graphs

LEARNING GOALS

- Tabulate data using a spreadsheet
- Graph data using a spreadsheet

Why is using spreadsheets for tables and graphs essential?

After it has been determined what is the best graph for displaying a dataset, the data then needs to be entered into a table, using a **spreadsheet**. This will allow the user to then graph this data using the spreadsheet, adjusting and modifying it to suit the information's purpose. It is very common that data may be added to or even altered at times. Using a spreadsheet is essential because it allows for these kinds of changes to be made easily, and any graphs are then updated automatically.

Using a spreadsheet allows changes to be made easily to your data

WHAT YOU NEED TO KNOW

- When inputting data into a spreadsheet, the format of the cell can be of significance.
  - Check that the 'Format' of the cell is what you need.
- When graphing data from a spreadsheet, remember to select a graph that has been assessed as being the best graph for a dataset.

Note: *A guide to spreadsheets is provided as an online appendix in the Interactive Textbook.*

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Example 3 Tabulating data using a spreadsheet

A group of accountants were asked which of their roles takes the most time, choosing from conducting financial investigations, preparing reports and undertaking audits. The responses of this questionnaire are shown below.

conducting financial investigations
undertaking audits
undertaking audits
conducting financial investigations
conducting financial investigations
undertaking audits
preparing reports
conducting financial investigations
preparing reports
conducting financial investigations
conducting financial investigations
undertaking audits
conducting financial investigations
conducting financial investigations
undertaking audits
conducting financial investigations
undertaking audits

Tabulate this data using a spreadsheet.

WORKING

	A	B
1	**Accountant's role**	**Frequency**
2	Conducting financial investigations	8
3	Preparing reports	2
4	Undertaking audits	6

THINKING

Within a spreadsheet, create three row headings: 'Conducting financial investigations', 'Preparing reports' and 'Undertaking audits'.

Title these roles as 'Accountant's role'.

Title the column next to it as 'Frequency' (i.e. the number of responses).

Count how many there are of each response and write the number in the table.

Format the table by considering borders, shading, text alignment, font, text size, etc.

Example 4 Graphing data using a spreadsheet

Alix is asked to create a graph that shows the most frequent age group of her company's customers. Graph this data on a histogram, using a spreadsheet.

Age group	Tally	Frequency
13–16	𝍸 | | |	8
17–20	𝍸 𝍸	10
21–24	𝍸 | | | |	9
25–28	𝍸	5
29–32	𝍸 | | |	8

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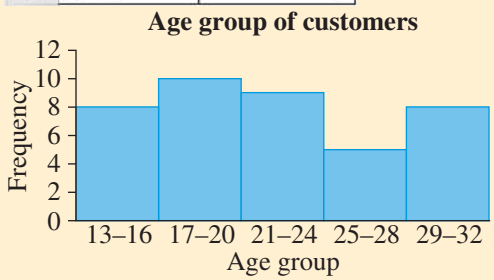
WORKING

	A	B
1	Age group	Frequency
2	13-16	8
3	17-20	10
4	21-24	9
5	25-28	5
6	29-32	8



THINKING

Insert all the age groups and their frequencies into two columns.

Select the data and choose to insert a chart.

Choose a histogram from the given options.

Alter the chart as required. The 'bin width', which can be found under 'axis options', may need to be changed to a value of 5.

Add elements to the chart, such as a 'Chart title' and 'Axis titles'.

Exercise 6B

FUNDAMENTALS

1 Tabulate the following sets of data, using a spreadsheet.

a

red	green	blue	red	green
blue	red	green	red	green
blue	red	green	green	green

Hint Use WYNTK and the examples to help you.

b

agree	neutral	strongly agree	strongly disagree
strongly agree	agree	strongly disagree	agree
strongly agree	agree	disagree	agree
neutral	disagree	disagree	agree

2 Graph the following data as a column graph, using a spreadsheet.

Section	Frequency
A	12
B	9
C	17

Hint For help, see the online guide to spreadsheets in the Interactive Textbook.

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APPLICATIONS

Example 4

3 A report has shown the following information relating to Australian greenhouse gas emissions by economic sector in 2018. Use the information to draw a pie graph, using a spreadsheet.

Economic sector	Percentage
Agriculture	9%
Transportation	27%
Commercial & Residential	12%
Electricity	29%
Industry	23%

Example 3

4 During an international flight, a flight attendant walks through the plane and surveys a selection of passengers about which meal they would prefer, choosing from chicken, beef and vegetarian. The survey's responses, which are shown below, are to be presented to the airline's catering company.

chicken	chicken	vegetarian	chicken	chicken	chicken
vegetarian	chicken	beef	beef	beef	vegetarian
vegetarian	beef	vegetarian	chicken		

a What type of graph should the flight attendant use to present the data?
b Explain why the attendant should use this graph.
c Draw the graph.

5 Emilia is preparing for a fitness competition and has been asked by her trainer to create a poster of how she spends her training time, including a visual representation of her data.
Emilia has calculated that she spends her training time in the following ways.

Cardio = 55 %
Weights = 30 %
Food preparation = 6 %
Stance practice = 9 %

a What type of graph should Emilia use?
b Explain why she should use this graph.
c Draw the graph.

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6 The table shows the number of young people aged 16–24 years with an anxiety disorder.

a Graph the data as a pie graph, using a spreadsheet.

b From the graph, which anxiety disorder is the most common in this age group?

Anxiety disorder	Number of young people
Panic disorder	55 700
Agoraphobia	71 600
Social phobia	138 300
Generalised anxiety disorder	195 300
Obsessive-compulsive disorder	56 300
Post-traumatic stress disorder	34 300

7 Samantha is planning a holiday and is trying to save some money. At the start of each week she writes down how much money she has saved. Her results are presented in the table.

Week	1	2	3	4	5	6	7	8	9
Money saved ($)	45	52	63	77	88	103	126	158	188

a Graph the data in the table above as a column graph, using a spreadsheet.

b If Samantha needs to save $200 by Week 10 for her holiday, does your graph suggest she would be able to achieve this? Justify your answer.

8 Zaran is collecting information for his new business. He has recently completed a survey of customers, shown in the table on the right, asking them what their main reason was for using social media. Zaran plans to include this information in his advertising flyer.

a What type of graph should Zaran use?

b Explain why he should use this graph.

c Draw the graph.

Reason	Frequency
Chat/send messages	79
Watch videos	57
Listen to music	48
Check up on friends	36
Update status	31
Follow influencers	29
Work/study	33
Comment on the news	20
Play games	35
Follow a brand	27
Meet new people	35
Participate in contests	10
Comment on products	2
Buy/sell	45
Contact customer service	3

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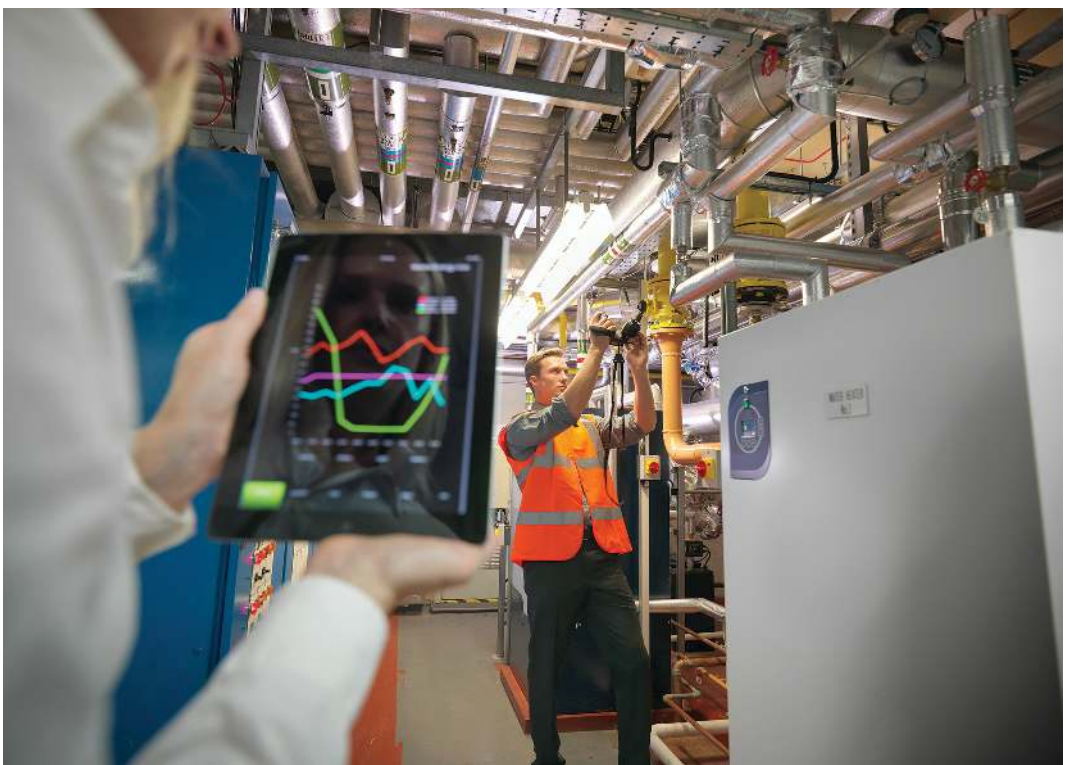
6C Graphing continuous change

LEARNING GOALS

- Draw a line graph by hand to display data that demonstrates a continuous change
- Draw a line graph, using a spreadsheet, to display data that demonstrates a continuous change

Why is graphing continuous change essential?

In many instances, things will change continuously over time. Examples include weight, temperature, distance of a moving object from its start and the public popularity of political figures. If we want to understand these changes, it is essential to be able to graph these relationships so we are then able to analyse and interpret them. This may then allow us to make certain predictions.



A plumbing technician using a digital tablet and camera to check the efficiency of a hotel's hot water supply.

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WHAT YOU NEED TO KNOW

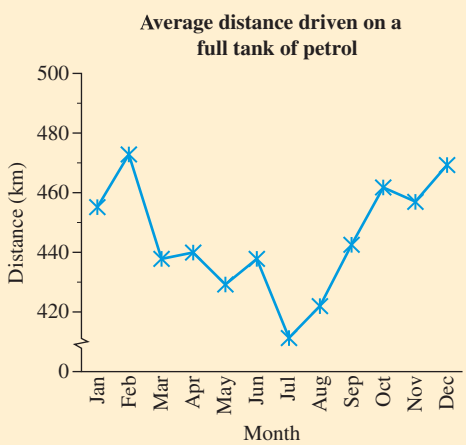
- In a line graph of change over time, the ***x*-axis** represents time (e.g. a date or time elapsed from the start).
- The ***y*-axis** has a scale and indicates the measurement (e.g. weight, temperature, distance or popularity survey results).
- A number of **data series** can be plotted in the same graph on the same axes, and this is particularly useful for analysing and comparing the **trends** in different datasets.

Example 5 Drawing a line graph by hand to display continuous change

Over a one-year period, Charlotte took note of how many kilometres she was able to drive in her car on a full tank of petrol, on average, for each month. Her results are shown in the table.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Distance (km)	455	473	438	440	429	438	411	422	443	462	457	470

a Construct a line graph by hand of the data collected.
b Describe the change over time shown by the graph.

WORKING	THINKING
a 	Label the months on the horizontal axis, and label the distance driven, in kilometres, on the vertical axis. Mark an × at each data point. Join up the data points with straight lines. Be sure to use a ruler when drawing straight lines.
b Charlotte's car travels further on a full tank of petrol during the warmer summer months. The colder the month, the less far it travels.	It can be seen that the number of kilometres travelled on a full tank of petrol goes down during the colder months.

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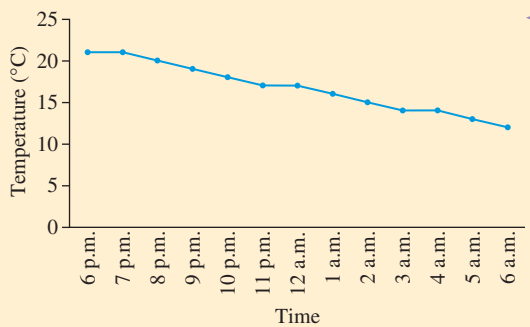
Example 6 Drawing a line graph using a spreadsheet to display continuous change

Create a line graph of the data given below, using technology of your choice. Here we will use a spreadsheet.

Time	Temperature (°C)
6 p.m.	21
7 p.m.	21
8 p.m.	20
9 p.m.	19
10 p.m.	18
11 p.m.	17
12 a.m.	17
1 a.m.	16
2 a.m.	15
3 a.m.	14
4 a.m.	14
5 a.m.	13
6 a.m.	12

WORKING



THINKING

Input the data into a spreadsheet.

Select the data and choose to insert a chart.

Choose an appropriate chart from the options given.

Add elements to the chart, such as a 'Chart title' and 'Axis titles'.

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Exercise 6C

FUNDAMENTALS

1 The table shows the data between two variables, x and y.

x	1	2	3	4	5
y	4	7	6	9	11

a When x is 2, what is the value of y?
b When x is 5, what is the value of y?
c Hand draw a line graph of the data collected, with x on the horizontal axis and y on the vertical axis.

Hint Find the number 2, then see which number corresponds in the y row.

2 For each of the following, use technology of your choice to create a line graph of the data.

a

Year	Temperature (°C)
Jan	35°C
Feb	32°C
Mar	30°C
Apr	27°C
May	23°C
June	15°C
July	5°C
Aug	10°C
Sept	20°C
Oct	23°C
Nov	28°C
Dec	30°C

b

x	1	2	3	4	5	6	7
y	231	263	288	294	344	351	374

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APPLICATIONS

Example 5

3 The average rainfall per month in Kalgoorlie is presented in the table.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average rainfall (mm)	27	31	29	17	20	20	19	16	11	13	18	19

a Hand draw a line graph of this data, with the month on the horizontal axis and the average rainfall on the vertical axis.

b Describe the change over time shown by the graph.

Hint It always looks best when you use a ruler when drawing straight lines.

Example 6

4 Cameron wishes to keep track of his spending habits, so he writes down how much money he spends on food each week for 10 weeks. His results are shown in the table.

XLS

a Using technology of your choice, create a line graph of this data. Label the week number on the horizontal axis and the money spent on food on the vertical axis.

Hint Use Example 6 to help you.

b Describe the change over time shown by the graph.

Week	Money spent on food ($)
1	96
2	112
3	65
4	158
5	143
6	88
7	165
8	173
9	92
10	151

5 The table shows a dataset of males and females who had the flu over a year.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Females	0	0	900	1000	3000	3500	3000	2900	2500	2500	2500	2450
Males	0	0	500	550	2000	2600	2500	2000	1750	1750	1500	1500

a Hand draw a line graph of this data, showing the month on the horizontal axis and the number of people on the vertical axis. You will end up drawing two lines on the same graph: one for females and one for males.

b Describe the change over time shown by the graph.

Hint Make sure you check which data goes along the vertical axis and which data goes along the horizontal axis.

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6 A plumbing technician is assessing the efficiency of a hot-water boiler system in an apartment complex. She takes the following water temperature measurements, in degrees Celsius, over a 20-minute period.

Time	10:11	10:12	10:13	10:14	10:15	10:16	10:17	10:18	10:19
Temperature of water (°C)	61.0	61.5	62.0	61.0	62.5	63.5	61.0	60.5	62.5

Time	10:20	10:21	10:22	10:23	10:24	10:25	10:26	10:27	10:28
Temperature of water (°C)	61.0	60.0	59.5	61.0	63.0	60.5	62.0	59.5	61.5

a Using technology of your choice, create a line graph of this data. Label the time on the horizontal axis and the temperature on the vertical axis.

Hint Try using different technology for each dataset to give you practice.

b A hot-water boiling system is said to be working efficiently when it is able to maintain water at 63°C and vary by only 2°C. What conclusions can be made from the graph about the hot-water boiling system?

7 The table shows a dataset of global water use, in gigalitres (GL), from 1960 to 2010, by sectors.

Year	**Agriculture (GL)**	**Industrial use (GL)**	**Domestic use (GL)**
1960	550	100	100
1970	650	150	140
1980	990	170	150
1990	1700	400	140
2000	2200	500	200
2010	3000	1100	400

a Using technology of your choice, create a line graph of this data. Label the year on the horizontal axis and water use (in GL) on the vertical axis.

b What conclusions can be made about water usage in the agricultural, industrial and domestic sectors?

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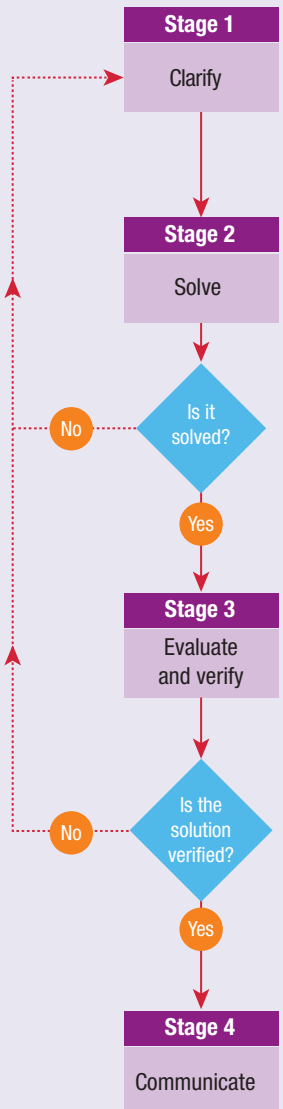
Statistical investigation process task

DRAWING GRAPHS

Background: Graphs are useful for comparing data, and pie charts are particularly useful for comparing the parts of a whole.

Task: You want to save up to buy a game (or other item) and you need to analyse your spending to see where you can save. Choose a game or item that costs considerably more than the amount you usually save per month.

To complete this task, follow the investigation process workflow diagram below, and use the steps listed as a guide.



Stage 1: Clarify

Clarify the problem and pose one or more questions that can be answered with data

1. Establish what your current regular monthly income is, in total, and the amount you need to save to buy the game (or other item).
2. Determine all your current regular monthly expenses. They must be the same as or less than monthly income (otherwise they would not be 'regular'). If less, this is your current monthly savings.

Stage 2: Solve

Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

3. Create a pie graph of your monthly expenses, using Excel, with income as your total, and include monthly savings, even if they are zero.
4. Create a second pie graph of your ideal monthly expenses for your target budget. Work out which particular expenses you can reduce, and see how you can maximise your monthly savings. Compare the pie graphs for regular expenses and target budget to see if the latter is realistic and achievable.
5. Work out how long it will take to buy the game.

Stage 3: Evaluate and verify

Interpret the results of this analysis and relate the interpretation to the original question

6. Is your target budget realistic and achievable?
7. Will it allow you to save up enough to buy the game in a reasonable amount of time?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

8. Write a report that explains (compared to your current expenses) which expenses you will reduce, how much you will save per month, and how long it will take to buy the game.

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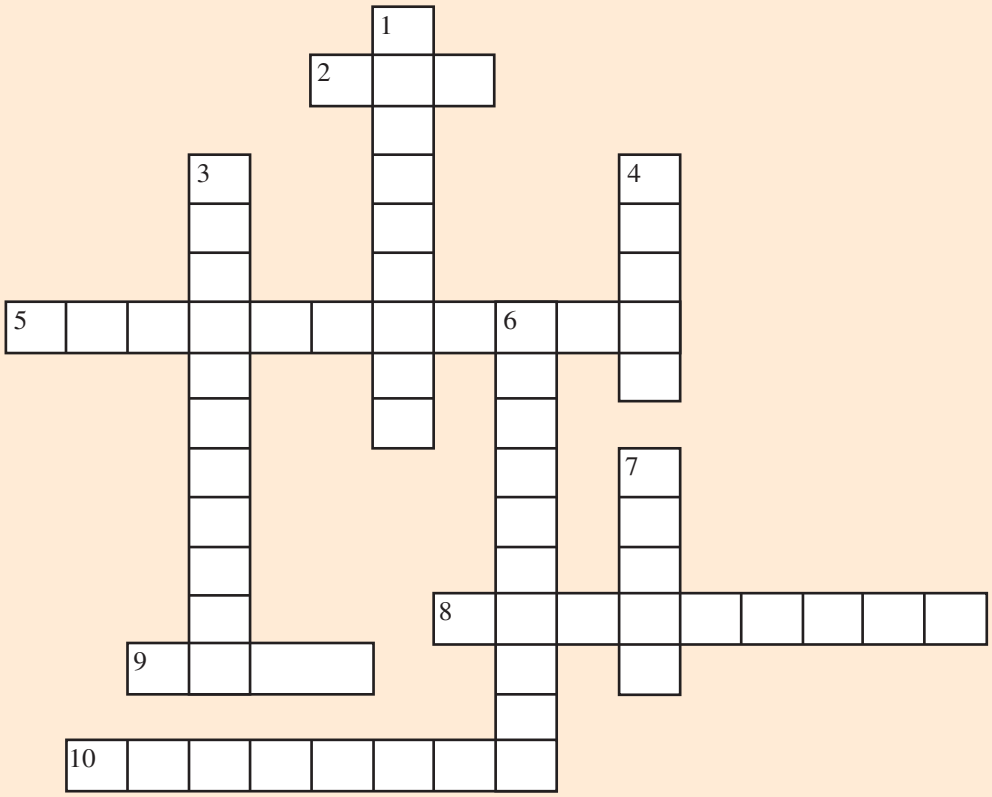
Puzzle



Across

2 Categorical data, where the user wishes to compare the values of the categories, should be displayed as a _____ graph

5 Profession, such as plumber or nurse, is an example of _____ data

8 Number of tools owned is an example of _____ data

9 Numerical data that is discrete, where the range is more than 10, should be displayed as a _____-and-leaf diagram

10 When using technology, features such as the chart title, axes labels or scales are known as _____ of the graph

Down

1 Numerical data that is continuous should be displayed as a _____

3 Data can be tabulated in a _____ using technology

4 Discrete data is numerical data where the responses are _____ numbers

6 _____ data is numerical data that can take any value, usually a measurement

7 When joining lines by hand, always use a _____

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Chapter checklist

☐ I can determine which type of graph is best for displaying a dataset.

1 Given the responses below to a survey, determine which type of graph would be best for displaying the dataset.

agree	neutral	disagree	agree	strongly disagree
strongly agree	neutral	disagree	strongly disagree	agree
disagree	strongly disagree	agree	disagree	strongly disagree
disagree	strongly agree	strongly disagree	strongly disagree	strongly disagree

☐ I can tabulate data using a spreadsheet.

2 Given the responses below to a survey, tabulate this data using a spreadsheet.

agree	neutral	disagree	agree	strongly disagree
strongly agree	neutral	disagree	strongly disagree	agree
disagree	strongly disagree	agree	disagree	strongly disagree
disagree	strongly agree	strongly disagree	strongly disagree	strongly disagree

☐ I can graph data using a spreadsheet.

3 Use the following dataset to create a pie graph using a spreadsheet.
Carly works at a new cafe and has been asked by the chef to create a graph for the following dataset.

Salad choices

Taco 10% *Spinach* 17% *Garden* 13% *Chef* 25% *Caesar* 35%

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☐ I can draw a line graph by hand to display a continuous change.

4 Hand draw a line graph of the data presented in the table.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Distance (km)	455	473	438	440	429	438	411	422

Month	Sep	Oct	Nov	Dec
Distance (km)	443	462	457	568

☐ I can draw a line graph that displays continuous change, using technology.

5 Create a line graph of the following dataset, using technology.
Pepper is currently working at a nursery. She has been asked to create a graph showing the following dataset of soil temperatures.

Month	10 cm deep (°C)	20 cm deep (°C)
January	23.6	25.7
February	22.8	24.7
March	21.2	23.0
April	18.1	20.0
May	14.3	16.1
June	11.2	12.8
July	10.0	11.8
August	11.0	12.3
September	14.1	15.9
October	17.9	19.0
November	19.9	21.1
December	21.9	23.9

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Chapter review

1 A group of crop farm workers are asked about what type of crops they work on. Their responses are shown below.

fruit	nuts	nuts	fruit	grains
vegetables	vegetables	fruit	fruit	fruit
grains	vegetables	fruit	nuts	vegetables
fruit	grains	nuts	grains	fruit
vegetables	fruit	grains	fruit	grains

Determine which type of graph would be best for displaying this dataset, if the purpose of collecting the data is to compare the proportion of types of crops being farmed.

2 The heights, in cm, of the members of the Opals basketball team are given.

188	178	170	191	185	172	178	188
178	187	192	165	196	180	185	203

Determine which type of graph would be best for displaying this dataset.

3 The average rainfall per month in Perth is presented in the table.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average rainfall (mm)	13	13	16	30	76	112	117	90	60	23	18	8

a Hand draw a line graph of this data.

b What conclusions can be made from the graph?

4 An infant's length, in cm, is measured at the start of each month after they are born. The lengths are given in the table.

Age (months)	0	1	2	3	4	5	6	7	8	9	10	11	12
Length (cm)	47	54	57	60	62	64	66	68	69	71	72	74	75

Using technology of your choice, create a line graph of this data. Label the age on the horizontal axis and the length on the vertical axis.

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5 A survey is conducted asking how people feel about the redevelopment of a local swimming pool. The responses from the survey are given below.

very positive	positive	positive	positive
neutral	neutral	very negative	very negative
positive	positive	negative	negative
positive	very negative	very negative	positive
neutral	positive	very negative	very negative

Tabulate this data using a spreadsheet.

6 A group of professional tradespeople are asked how many hours they worked in the previous week. The results are shown below.

39	42	35	47	41	33	32
48	39	41	40	50	47	37
39	41	36	39	42	45	33
52	36	44	38	51	47	42
39	37	33	51	38	42	39

Choose an appropriate graph for this data, and graph it using technology of your choice.

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7 Classifying and presenting data

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In this chapter

Syllabus reference

Unit 2 Topic 2.1: Representing and comparing data

Classifying data

- identify examples of categorical data
- identify examples of numerical data

Data presentation and interpretation

- display categorical data in tables and column graphs
- display numerical data as frequency distributions, dot plots, stem-and-leaf plots and histograms
- recognise and identify outliers
- compare the suitability of different methods of data presentation in real-world contexts

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Pre-test

1 The favourite colours are recorded for a class of students.

Colour	Vote
red	5
blue	8
green	13
purple	11

a How many favourite colours are there?
b How many students are in the class?
c How many students voted green as their favourite colour?
d Do more than half the class have blue or green as their favourite colour?
e How many more students have purple rather than blue as their favourite colour?

2 The following table shows the frequency of the number of siblings of the students in the same class as above.

No. of siblings	Frequency
0	12
1	18
2	5
3 or more	2

a Does the number of students in this table match the number of students in Question **1**?
b How many students have no siblings?
c How many students have two or more siblings?
d How many students have two or fewer siblings?
e How many more students have one sibling than no siblings?

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7A Classifying data

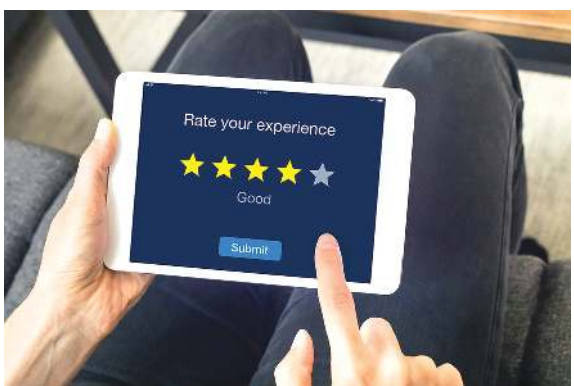
LEARNING GOAL

- Identify data as either categorical or numerical

Why is it essential to know how to classify data?

We live in a society that is driven by data. We are constantly being asked to 'rate our experience' at restaurants and shops. Data about the way we shop, what we are buying and what we are interested in is constantly being collected by store loyalty card programs and the web browsers we use to make a search. The way we use data depends on its type. If we surveyed people's favourite pizza topping, the data would consist of names (i.e. words) such as *pepperoni, ham and pineapple, meatlovers*, etc. However, we would not be able to calculate an 'average' for this data, as there are no numbers on a ham and pineapple pizza! But we can still display the responses in a graph or table.

On the other hand, if we surveyed the number of siblings our classmates have, the data would consist of numbers.



We are constantly being asked to rate our experience with shops, restaurants and hotels. This data is used to ensure that products we are interested in are marketed back to us.

WHAT YOU NEED TO KNOW

- **Data** is a collection of information that could be either words or numbers.
  - There are two main types of data: categorical (sometimes called qualitative) and numerical (sometimes called quantitative).
- **Categorical data** is data that can be grouped into separate **categories**. The data collected will consist of **words** and each category has a word name.
  - For example: favourite pizza toppings or car colours.
- **Numerical data** uses **numbers** for data that has been counted or measured.
  - For example: the number of siblings or the height of students.

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Example 1 Identifying data as categorical or numerical

Classify the following data as categorical or numerical.

a weight of a baby
b income
c favourite football team
d star rating of a restaurant
e type of car

WORKING		THINKING
a numerical	◄	The weight of a baby would be measured in grams. This is a **number**; e.g. 3567 grams.
b numerical	◄	Income is stated in dollars. This is a **number**; e.g. $ 45 000.
c categorical	◄	The data responses for teams would be **words**; e.g. Broncos or Cowboys.
d numerical	◄	The data would be the **number** of stars; e.g. 5 stars.
e categorical	◄	The data for type of car would be **words**; e.g. hatchback.

Exercise 7A

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a **Data** is a collection of information that could be _____ or _____.
b Data that consists of **words** is called _____.
c Data that consists of **numbers** is called _____.

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2 A cricket app polls fans about various aspects of the game via their device. In a Test match against England, six questions and the results of the poll (i.e. the data that was collected) are summarised on the following page. For each question polled, decide if the data that was collected is numerical (numbers) or categorical (words).

a Do the Australians have enough runs?
Yes 18% No 82%

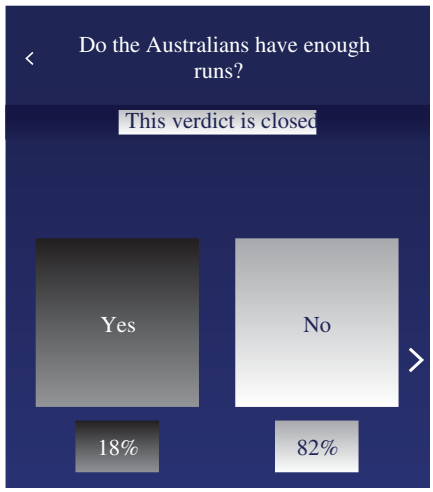
b What total should Australia be happy with?
336 runs

c Should England take the new ball?
Yes 74% No 26%

d Who's your favourite Marsh brother?
S Marsh 55% M Marsh 45%

e Should Australia make any changes for Game 2 at the Gabba?
Yes 84% No 16%

f What lead does Australia need to declare?
286 runs



APPLICATIONS

Example 1

3 Classify each type of data as categorical or numerical. Give two possible responses for each.

Hint Categorical data has word names.

a how you travelled to school today
b how many brothers/sisters you have
c your preferred style of music
d your favourite take-away food

4 Traffic Police collect data to monitor road safety. Classify each type of data as categorical or numerical. Give two possible responses for each.

Hint Numerical data has been counted or measured.

a cause of an accident
b during a shift, the number of drivers detected using a mobile phone while driving
c speed of cars as they pass a speed camera
d type of drug detected in a roadside test
e blood alcohol content (BAC) of drivers at a random breath test
f result of random roadside drug test

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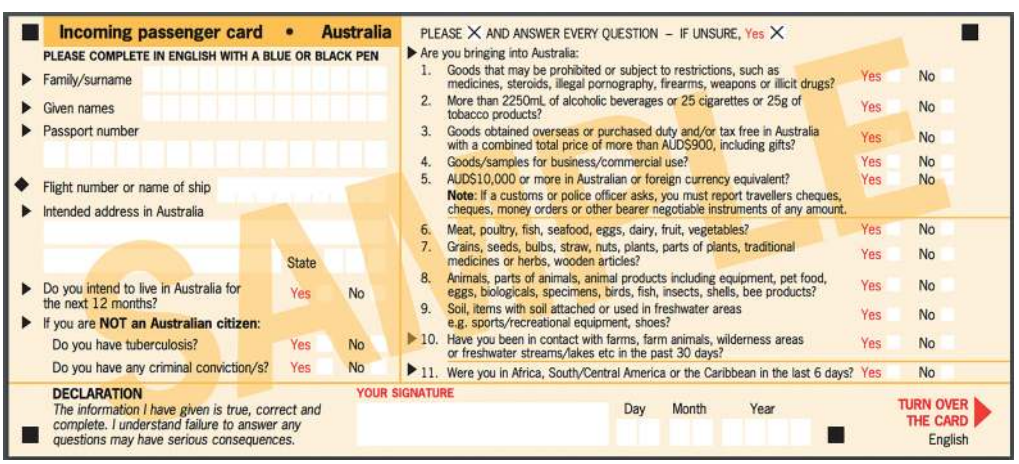
5 A group of people are surveyed about pet ownership. For each question, decide if the survey collects numerical or categorical data. Give two possible responses to each question.

a Do you own a pet?
b How many pets do you have?
c What type of pet do you own?
d How much does your pet weigh?
e How old is your pet?
f Where does your pet sleep?
g How much do you spend on pet food each week?
h How often do you feed your pet each day?
i How many legs does your pet have?
j What is the main colour of your pet?

6 Travellers coming into Australia must complete both sides of an Incoming Passenger card, shown below and on the next page.



Incoming passenger card • Australia

PLEASE COMPLETE IN ENGLISH WITH A BLUE OR BLACK PEN

- Family/surname
- Given names
- Passport number
- Flight number or name of ship
- Intended address in Australia

State

- Do you intend to live in Australia for the next 12 months? Yes No
- If you are **NOT an Australian citizen**:
  - Do you have tuberculosis? Yes No
  - Do you have any criminal conviction/s? Yes No

PLEASE ✕ AND ANSWER EVERY QUESTION – IF UNSURE, Yes ✕

Are you bringing into Australia:

1. Goods that may be prohibited or subject to restrictions, such as medicines, steroids, illegal pornography, firearms, weapons or illicit drugs? Yes No
2. More than 2250mL of alcoholic beverages or 25 cigarettes or 25g of tobacco products? Yes No
3. Goods obtained overseas or purchased duty and/or tax free in Australia with a combined total price of more than AUD$900, including gifts? Yes No
4. Goods/samples for business/commercial use? Yes No
5. AUD$10,000 or more in Australian or foreign currency equivalent? Yes No
   Note: If a customs or police officer asks, you must report travellers cheques, cheques, money orders or other bearer negotiable instruments of any amount.
6. Meat, poultry, fish, seafood, eggs, dairy, fruit, vegetables? Yes No
7. Grains, seeds, bulbs, straw, nuts, plants, parts of plants, traditional medicines or herbs, wooden articles? Yes No
8. Animals, parts of animals, animal products including equipment, pet food, eggs, biologicals, specimens, birds, fish, insects, shells, bee products? Yes No
9. Soil, items with soil attached or used in freshwater areas e.g. sports/recreational equipment, shoes? Yes No
10. Have you been in contact with farms, farm animals, wilderness areas or freshwater streams/lakes etc in the past 30 days? Yes No
11. Were you in Africa, South/Central America or the Caribbean in the last 6 days? Yes No

DECLARATION
The information I have given is true, correct and complete. I understand failure to answer any questions may have serious consequences.

YOUR SIGNATURE Day Month Year

TURN OVER THE CARD

English

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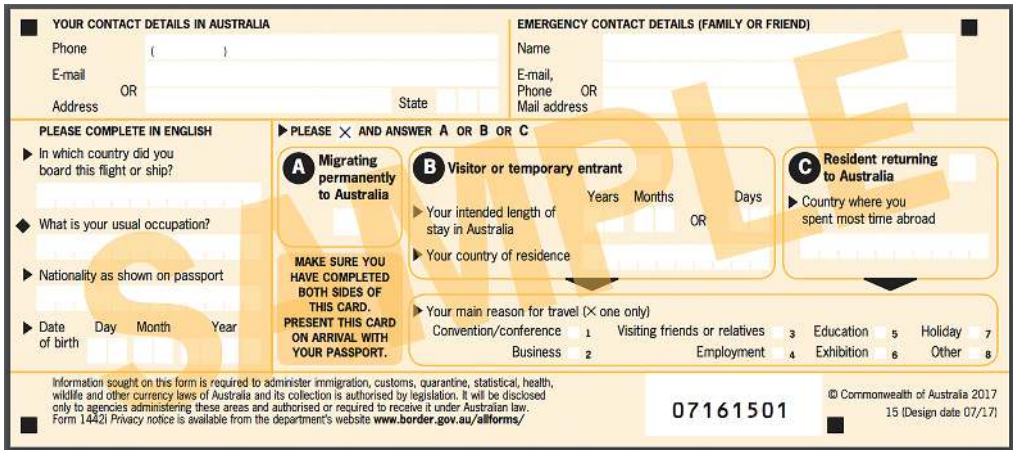
YOUR CONTACT DETAILS IN AUSTRALIA
Phone ( )
E-mail OR Address
State
EMERGENCY CONTACT DETAILS (FAMILY OR FRIEND)
Name
E-mail, Phone OR Mail address
PLEASE COMPLETE IN ENGLISH
In which country did you board this flight or ship?
What is your usual occupation?
Nationality as shown on passport
Date of birth Day Month Year
PLEASE × AND ANSWER A OR B OR C
A Migrating permanently to Australia
B Visitor or temporary entrant
Years Months Days
Your intended length of stay in Australia
OR
Your country of residence
C Resident returning to Australia
Country where you spent most time abroad
MAKE SURE YOU HAVE COMPLETED BOTH SIDES OF THIS CARD. PRESENT THIS CARD ON ARRIVAL WITH YOUR PASSPORT.
Your main reason for travel (× one only)
Convention/conference 1
Visiting friends or relatives 3
Education 5
Holiday 7
Business 2
Employment 4
Exhibition 6
Other 8
Information sought on this form is required to administer immigration, customs, quarantine, statistical, health, wildlife and other currency laws of Australia and its collection is authorised by legislation. It will be disclosed only to agencies administering these areas and authorised or required to receive it under Australian law. Form 1442i Privacy notice is available from the department's website www.border.gov.au/allforms/
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15 (Design date 07/17)
SAMPLE

For each of the following, classify the data collected as categorical or numerical.

a Migrating permanently to Australia

b Your intended length of stay in Australia

c Your country of residence

d Your main reason for travel

e Country where you spent most time abroad

7 You are asked by your year level coordinator to survey your classmates about their wishes for the school formal next year.

a Design two questions that collect numerical data.

b Design two questions that collect categorical data.

8 A Physical Education class is given the task of surveying the Year 11 group about their participation in team sports.

a Design two questions that collect numerical data.

b Design two questions that collect categorical data.

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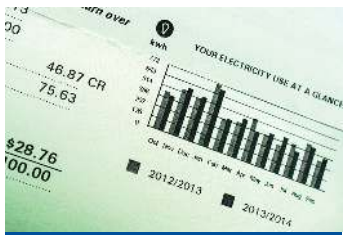
7B Categorical data: tables and graphs

LEARNING GOALS

- Organise categorical data into a frequency table
- Present categorical data in a column graph drawn by hand
- Identify an appropriate scale to use for a column graph
- Create a column graph, using Excel, to present categorical data

How is the organisation and presentation of categorical data essential?

Organising data after it has been collected allows us to quickly see trends in the data and make conclusions about it.



Readers glance at graphs within a report before reading the whole report, to give them an overview of what they are about to read.

WHAT YOU NEED TO KNOW

- A **frequency table** displays data by showing how much data has been collected in each category.
  - **Tally marks** allow you to count data as it is collected. For ease of counting, tally marks are grouped in sets of five by putting a diagonal mark across a group of four: 𝍸
  - The **frequency** column is the subtotal for the category.
  - The **total** gives a way of checking that all the data has been recorded.
- A **column graph** is used to represent categorical data.
  - The horizontal axes will show the different categories.
  - The vertical axes will show the frequency. It is sensible to start at zero and count up in factors of 10; that is, count up in increments of 1, 2, 5 or 10. If the frequency is bigger, you could count in 10s, 20s or 50s, or even 100s, 200s or 500s for really large datasets.
  - Generally, aim for your graph to have between 5 and 10 steps on the frequency when deciding on which scale to use.
  - The columns do not touch each other.
  - Use a ruler.
  - Label the axes and give your graph a title.

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Example 2 Organising categorical data into a frequency table and presenting it in a hand-drawn column graph

Forty people are asked which is their favourite WAFL (Western Australian Football League) team. The responses are given below.

Lions	Demons	Tigers	Tigers	Demons	Demons	Lions	Demons
Demons	Tigers	Lions	Demons	Lions	Demons	Lions	Demons
Demons	Lions	Demons	Lions	Lions	Lions	Demons	Lions
Tigers	Lions	Demons	Demons	Demons	Lions	Demons	Lions
Tigers	Demons	Tigers	Demons	Tigers	Demons	Lions	Tigers

a Construct a frequency table to record the data, using the headings 'Category', 'Tally' and 'Frequency'.

b Construct a column graph for the data.

WORKING

a

Category	Tally	Frequency
Lions	𝍸 𝍸 ||||	14
Tigers	𝍸 |||	8
Demons	𝍸 𝍸 𝍸 |||	18
Total	**40**	**40**

THINKING

Construct the table with the headings as asked and list the teams in the category column.
Work through the data, in order. Place a tally mark next to the category. Mark every fifth one with a diagonal line through the previous four to group the five together and make counting easier.
Count the tally marks and record the frequency for each category.
Check that the frequency total adds up to 40.

... *Continued*

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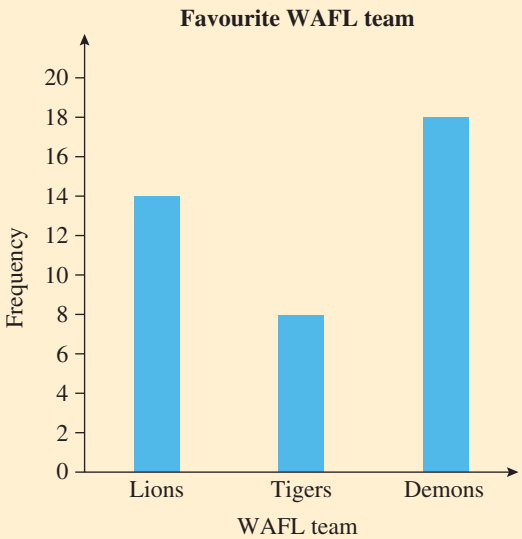
b Counting up in 1s will take 18 steps
→ too many steps. ☒
Counting up in 2s will take nine steps
→ OK. ☑
0, 2, 4, …, 16, 18, 20



◄···· We want 5 to 10 steps and to count only in 1s, 2s, 5s or 10s. Identify the most suitable option.
The largest frequency is 18. Draw up a set of axes with the frequency going up in 2s to just past 18.
For each category, draw a column so its height matches the frequency.
Leave a space between each column.
Label each column along the bottom axis.
Label the axes.
Give your graph a heading.

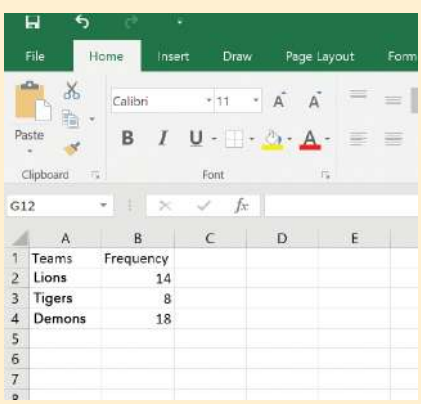
Example 3 Creating a column graph, using Excel, to present categorical data

Use Excel to produce a graph of the data that was collected in **Example 2**.

Category	Tally	Frequency
Lions	𝍸 𝍸 ||||	14
Tigers	𝍸 |||	8
Demons	𝍸 𝍸 𝍸 |||	18
Total	**40**	**40**

WORKING **THINKING**

a



◄···· Type the information from the frequency table into the columns, as shown.

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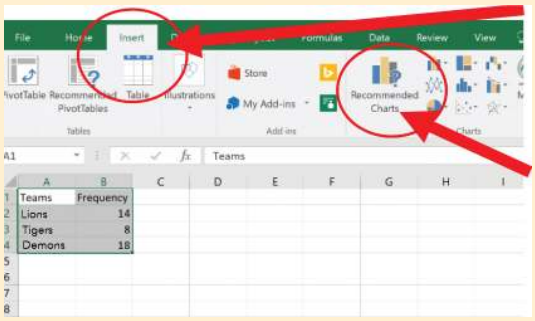
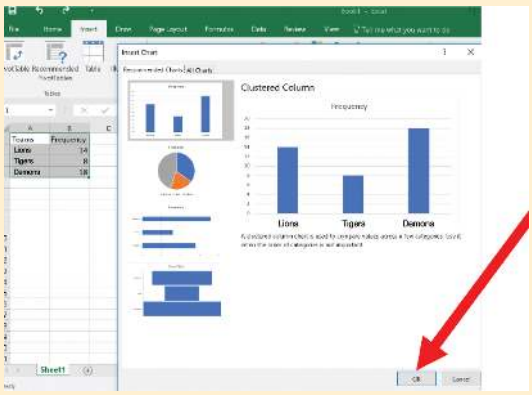
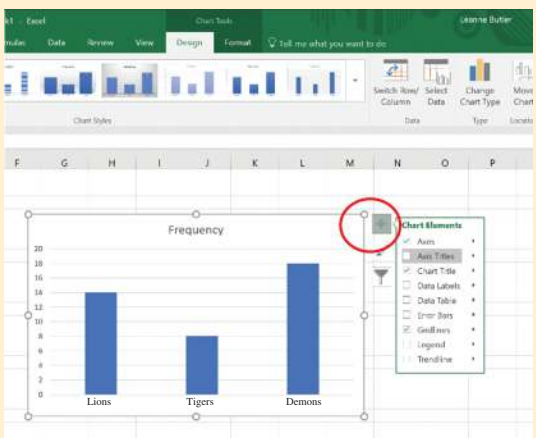
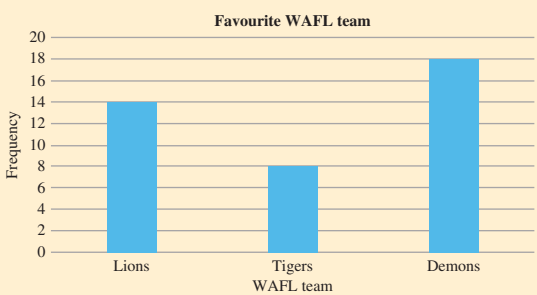
WORKING	THINKING
	Highlight the data. Click on the 'Insert' tab. Then click on 'Recommended Charts'.
	Select the type of graph you would like. In this case, we will pick the first one. Click on 'OK'.
	You should now have the start of your graph. We need to get some axes labels. Click anywhere on your graph, and three icons will appear at the right of the graph. Select '+' to add chart elements. ☒ Axis Titles to add labels to your horizontal and vertical axes.
	Type in the title and axes labels, as shown.

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Exercise 7B

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** A _____ **table** displays data by showing how much data has been collected in each category.
- **b** **Tally marks** allow you to _____ data as it is collected.
- **c** For ease of counting, tally marks are _____ in sets of five by putting a diagonal mark across a group of four.
- **d** The _____ column is the subtotal for the category.
- **e** The **total** gives a way of _____ that all the data has been recorded.
- **f** A **column graph** is used to represent _____ data.
- **g** Generally, aim to make your graph have between ____ and ____ steps on the frequency when deciding on which scale to use.

Example 2

2 Some data is collected about people's preferred snack.

Snack preference	Tally	Frequency
packet of chips	𝍸 | | |	
muesli bar	𝍸 𝍸 𝍸 |	
hot dog	𝍸 𝍸	
potato wedges	| | | |	
pizza slice	𝍸 𝍸 𝍸 𝍸 | |	

- **a** Complete the frequency column.
- **b** Determine how many people prefer:
  - **i** potato wedges
  - **ii** a muesli bar
- **c** Identify the largest frequency.
- **d** Determine how many steps it will take to reach the highest frequency:
  - **i** if you count up in 1s
  - **ii** if you count up in 2s
  - **iii** if you count up in 5s
- **e** Decide upon the most suitable scale to count with to reach the highest frequency.

Hint A suitable scale has 5 to 10 steps.

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3 The data presented at right shows the favourite colour of a group of people.

Colour	Tally	Frequency
pink	𝍸 𝍸 ||||	14
blue	𝍸 |||	8
yellow	𝍸	5
green	|	1
red	||||	4
orange	𝍸 ||||	9
purple	𝍸 ||	7
Total	**48**	**48**

a Determine how many more people chose pink than yellow.

b Calculate how many people chose yellow, blue or pink.

c Calculate how many people chose red or purple.

d Which is the most popular colour?

e Determine how many steps it would take to reach the highest frequency:

i if you count up in 1s

ii if you count up in 2s

iii if you count up in 5s

f Decide upon the most suitable scale to count with to reach the highest frequency.

g Construct a column graph to represent the data.

Hint Label each axis and give your graph a title.

APPLICATIONS

4 The types of vehicles passing through an intersection are recorded over 2 minutes. The observations yield the following results.

car	motorbike	truck	utility	motorbike
taxi	car	car	motorbike	car
utility	car	utility	car	truck
car	utility	bus	car	utility

a Copy and complete the frequency table to organise the data.

Category	Tally	Frequency
car		
truck		
utility		
motorbike		
bus		
taxi		
Total		

b Which is the most common vehicle passing through the intersection?

c Construct a column graph for the data.

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5 A vet has the following appointments during her morning at the vet surgery.

a Construct a frequency table to record the animal data, using the headings 'Animal', 'Tally' and 'Frequency'.

b What is the most common type of animal treated by the vet during the session?

c Construct a column graph for the data.

Time	Name	Animal
8:30 a.m.	Fifi	cat
8:45 a.m.	Bonnie	dog
9 a.m.	Buster	dog
9:15 a.m.	Rosie	dog
9:30 a.m.	Shadow	dog
9:45 a.m.	Rattles	cat
10 a.m.	Kitty	cat
10:15 a.m.	Sherby	turtle
10:30 a.m.	Star	dog
10:45 a.m.	Misty	cat
11 a.m.	Winnie	dog
11:15 a.m.	Sunny	dog
11:30 a.m.	Hiro	dog
11:45 a.m.	Ginger	guinea pig

6 When planning for a school camp, the year leader surveyed the students about what they would like on the menu. The options are: lasagne (L), chicken schnitzel and chips (C), and roast beef (B).
The students' responses are given below.

L	C	B	C	B	C	C	L	B	C
C	B	C	C	B	L	C	C	L	C
B	C	L	C	C	C	B	B	L	L
C	C	B	B	B	L	C	C	B	C
C	C	C	C	B	C	L	C	C	B

a Construct a frequency table to record the data, using the headings 'Meal', 'Tally' and 'Frequency'.

b Which is the preferred meal for this year group?

c Construct a column graph for the data.

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Example 3

7 The results of a political poll are presented in the table shown.

a Organise the information for Party into a frequency table.

b Use Excel to construct a column graph to display the preferred political party amongst the survey group.

c Organise the information for Gender into a frequency table.

d Use Excel to construct a column graph to display the distribution of gender amongst the group surveyed.

Age	Party	Gender	Income ($)
20	ALP	Male	45 000
25	LNP	Male	39 000
45	IND	Male	56 000
35	ALP	Female	49 000
50	LNP	Female	41 000
55	IND	Female	42 000
39	ALP	Male	58 000
48	LNP	Male	55 000
30	IND	Male	46 000
27	ALP	Female	42 000
47	LNP	Female	37 000
21	IND	Female	25 000
48	ALP	Male	75 000
24	IND	Male	43 000
28	IND	Female	40 000
40	LNP	Female	31 000

8 Visitors to Australia for the 12 months ending September 2017 were from the following regions.

Region	Frequency
Oceania (New Zealand, PNG…)	1 536 000
North-West Europe (UK, France…)	1 378 300
South-East Europe (Italy, Spain…)	220 500
Americas (Canada, USA, Mexico…)	1 069 600
North Africa and the Middle East (Israel, UAE…)	124 100
South-East Asia (Thailand, Indonesia…)	1 372 400
North-East Asia (China, Japan, Korea…)	2 507 800
Southern and Central Asia (India, Pakistan…)	380 200
Sub-Saharan Africa (South Africa…)	86 400
Total	**8 675 300**

a Modify the table and round the visitor numbers for each category to the nearest 100 000.

b Use an Excel spreadsheet to construct a column graph, showing where visitors to Australia come from.

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7C Numerical data: frequency distribution tables and histograms

LEARNING GOALS

- Sort numerical data into frequency distribution tables, displaying discrete data and grouped data
- Create hand-drawn histograms to display discrete data and grouped data
- Determine appropriate groupings for datasets with a large range
- Create a histogram, using Excel, to display grouped data

Why are frequency distribution tables and histograms essential?

Frequency distribution tables and histograms are powerful tools for displaying numerical data and communicating results and ideas. Visual presentations make understanding data easy and can be effective in communicating large amounts of information. Readers glance at graphs within a report before reading the whole report to give them an overview of what they are about to read.

Visual presentations are used widely to display and compare the key features of data.

WHAT YOU NEED TO KNOW

- A **frequency distribution table** displays data by showing how much data has been collected for each number or group of numbers.
  - From the previous section on frequency tables, we know there are three columns for **data, tally** and **frequency**. Adding a **total** row gives us a way of checking that all the data has been recorded.
  - When the range of data from the smallest score to the biggest score is large (i.e. more than 10), it is appropriate to group the data into intervals of 2s, 5s or 10s so that there are only between 5 and 10 lines in the table used to organise the data.

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- A **histogram** is similar to a column graph, but because we are using numbers, we can join the columns together, so there are no gaps.
  - Grouped numerical data vs frequency is always presented using a histogram.
  - The data scores are shown on the horizontal axis.
  - The frequency is shown on the vertical axis. Aim to make your graph have between 5 and 10 steps on the frequency scale.
  - The columns touch each other.
  - There needs to be a gap between the vertical axis and the first column.
  - Use a ruler.
  - Label each axis and give your graph a title.

Example 4 Sorting discrete numerical data into a frequency distribution table, and creating a hand-drawn histogram to display discrete data

A group of students are surveyed about how many pets they have. The results are shown below.

3	2	4	2	1	2	1	1	3	6
2	1	1	1	2	2	4	1	1	0
2	2	4	3	1	1	0	2	1	1

a Organise the data into a frequency distribution table.
b What is the most common number of pets to own?
c Determine the vertical scale.
d Construct a histogram for the discrete data.

WORKING

a

Pets	Tally	Frequency
0	||	2
1	𝍸 ||	12
2	𝍸 ||||	9
3	|||	3
4	|||	3
5		0
6	|	1
Total	**30**	**30**

THINKING

◄····· Discrete data means individual values.
Smallest number = 0, largest number = 6, so seven rows are required for the table. This is within a useable range of 5–10, so set up the table with numbers from 0 to 6 for the number of pets.
Complete your table as was done in the previous section.

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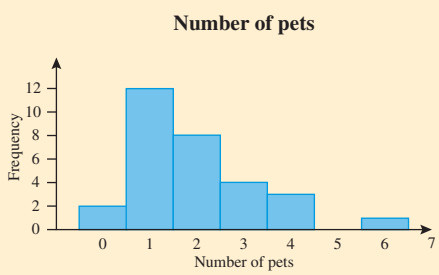
b Owning one pet is the most common.

◄····· 12 is the largest frequency.

c The largest frequency is 12.
Counting up in 1s will take 12 steps → too many steps. ☒
Counting up in 2s will take six steps → OK. ☑

◄····· We need between 5 and 10 steps on the vertical axis, so a suitable scale must be found.

d



◄····· Start at 0 and count up in 2s on the vertical axis to reach 12.
Mark the number of pets on the horizontal axis at even spaces from the axes and each other.
Note that 0 is part of the dataset and is not placed at the very beginning of the axis. Even though there were no responses for five pets, this still needs to be shown.
Complete the columns using a ruler. The data label should be in the *middle* of the column and there should be *no gaps* between the columns.

Example 5 Sorting discrete numerical data into groups, and creating a frequency distribution table and a histogram to display grouped data

The date of birth is recorded for a group of students. The results are given below.

1	6	10	15	2	31	26	30	23	3
28	6	22	15	25	25	8	3	15	30
10	17	29	1	29	4	10	12	26	23
11	4	7	7	12	22	17	20	2	14
3	22	17	11	7	25	3	25	19	29

a Determine suitable groupings for the data.
b Construct a grouped frequency distribution table.
c Create a histogram for the data.

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WORKING | **THINKING**

a Smallest number = 1
Largest number = 31
Groups of 5

◄··· A frequency table needs 5–10 rows.
It is easier to count up in 1s, 2s or 5s.
Counting up in 1s will take 31 steps → too many steps. ☒
Counting up in 2s will take 16 steps → too many steps. ☒
Counting up in 5s will take 7 steps → OK. ☑

b

Date of birth	Tally	Frequency
1 – 5		
6 – 10		
11 – 15		
16 – 20		
21 – 25		
26 – 30		
31 – 35		
Total		

◄··· Create a table with the seven rows with groupings of 5. Since we are counting in 5s, the groups will end with 5, 10, 15, ..., etc.

Date of birth	Tally	Frequency
1 – 5	𝍸	10
6 – 10	𝍸 ||||	9
11 – 15	𝍸 |||	8
16 – 20	𝍸	5
21 – 25	𝍸 ||||	9
26 – 30	𝍸 |||	8
31 – 35	|	1
Total	**50**	**50**

◄··· Work through the data, placing a tally mark in the appropriate group.
Complete totals to check all the data has been recorded.

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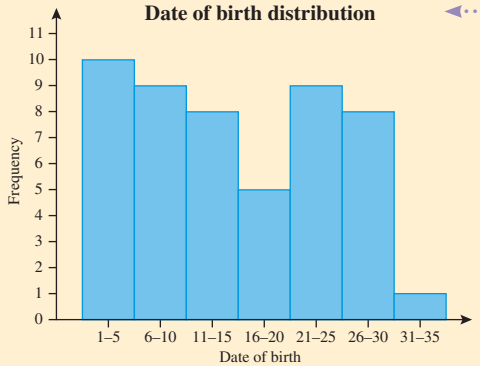
c



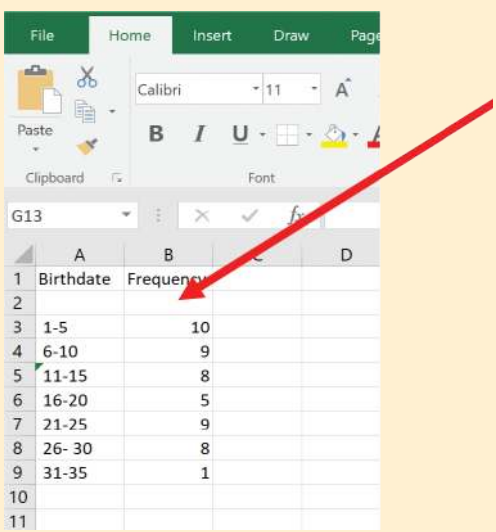
Leave a gap at the start of the horizontal axis, and then label your evenly spaced columns in 5s to match the groups from the frequency distribution table.
Maximum frequency is 10, so it will be okay to go up in 1s on the vertical axis.
Draw columns to match data from the frequency distribution table.
Label axes and give your graph a title.

Example 6 Creating a histogram to display grouped data, using Excel

Use the data from **Example 5** to construct a histogram using Excel.

Date of birth	Tally	Frequency
1 – 5	𝍸 𝍸	10
6 – 10	𝍸 ||||	9
11 – 15	𝍸 |||	8
16 – 20	𝍸	5
21 – 25	𝍸 ||||	9
26 – 30	𝍸 |||	8
31 – 35	|	1
Total	**50**	**50**

WORKING



THINKING

Type data from your frequency distribution table into Excel.

Tip 1: Leave an empty row at the beginning of the table. This will create a space at the start of the histogram so it does not touch the vertical axis.

Tip 2: To avoid Excel autocorrecting your group range to a date, press the space bar before you type the group name.

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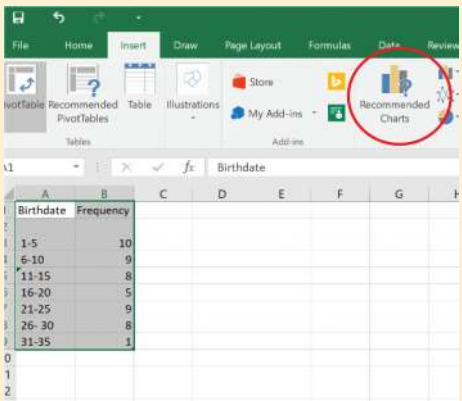
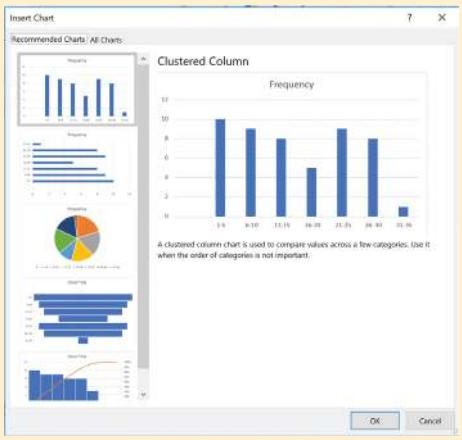
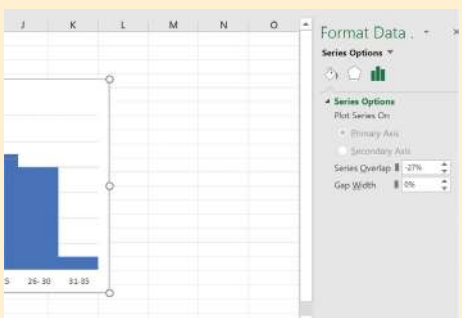


Highlight your data.
Go to the 'Insert' tab.
Click on 'Recommended Charts'.



We will select the first graph.
You will now have a basic column graph.

Histograms do not have gaps between columns. To fix that, *right click* anywhere on one of the columns. Select 'Format Data Series' from the menu that appears.



Change the gap width to 0%.

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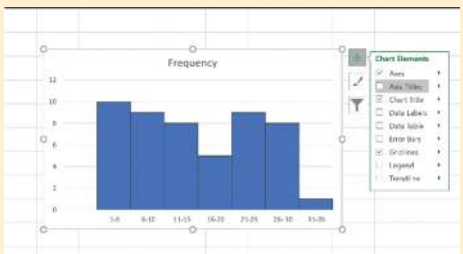
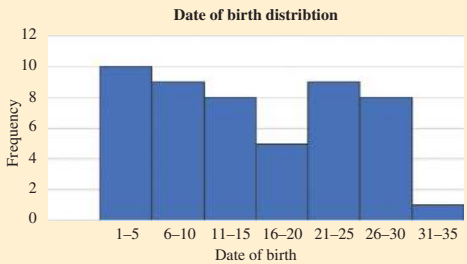


To add axis titles, click anywhere on the graph.
Select the '+' that appears at the top right.
☒ Axis Titles



Type in your axes labels to complete the graph.

Exercise 7C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

a A histogram is like a _____ graph. However, because we are using numbers, we can join the columns together so there are ____ gaps.

b The data scores are shown on the _____ axis.

c The _____ is shown on the vertical axis.

d The columns _____ each other.

e There must be a _____ with the vertical axis.

Example 4

2 A group of people are asked how many times they have visited the dentist in the past year. The responses are presented below.

1	0	2	1	1
1	1	5	2	3
0	1	2	2	3

a Organise the data according to the given frequency distribution table.

b Which is the most common response?

c Decide whether it is best to count up in 1s, 2s or 5s to reach the highest frequency.

d Use your answer from part **c** to present the data in a histogram.

Hint You need 5 to 10 steps from zero to the highest frequency.

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Visits	Tally	Frequency
0		
1		
2		
3		
4		
5		
Total		

APPLICATIONS

Example 5

3 This table show the times, in minutes, it takes a group of Year 11 students to run a 5 km cross-country course.

Time (minutes)	Frequency
10–14	1
15–19	5
20–24	13
25–29	12
30–34	7
35–39	2

a How many students participated in the race?

b Identify the most common time by students to complete the race.

c Determine whether it is more suitable to count up in 1s, 2s or 5s to reach the highest frequency.

d Use your answer from part **c** to construct a histogram for the data.

Hint There are no gaps between the columns of a histogram.

4 A group of 30 students are asked how many hours of homework and study they have done in the past week. Their responses are:

1	11	9	10	7	12	11	9	5	10
12	10	11	0	12	10	3	5	14	11
9	12	7	22	10	11	12	11	9	7

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a Identify the largest amount of time spent on homework from the students surveyed.
b Decide whether it's best to count up in 1s, 2s or 5s to reach the largest amount of homework time.
c Determine suitable groups for the time spent doing homework.
d Organise the data into a frequency distribution table, using groups.
e Create a histogram for the grouped data.

Hint Name the table columns 'Hours', 'Tally' and 'Frequency'.

Example 6

XLS

5 The age of the actors at the time when they were awarded the Best Actor and Best Actress at the Academy Awards from 1993 to 2022 are listed below.

Year	Best Actress	Age (years)	Film	Best Actor	Age (years)	Film
1993	Emma Thompson	33	*Howards End*	Al Pacino	52	*Scent of a Woman*
1994	Holly Hunter	36	*The Piano*	Tom Hanks	37	*Philadelphia*
1995	Jessica Lange	45	*Blue Sky*	Tom Hanks	38	*Forrest Gump*
1996	Susan Sarandon	49	*Dead Man Walking*	Nicolas Cage	32	*Leaving Las Vegas*
1997	Frances McDormand	39	*Fargo*	Geoffrey Rush	45	*Shine*
1998	Helen Hunt	34	*As Good As It Gets*	Jack Nicholson	60	*As Good As It Gets*
1999	Gwyneth Paltrow	26	*Shakespeare In Love*	Roberto Benigni	46	*Life Is Beautiful*
2000	Hilary Swank	25	*Boys Don't Cry*	Kevin Spacey	40	*American Beauty*
2001	Julia Roberts	33	*Erin Brockovich*	Russell Crowe	36	*Gladiator*
2002	Halle Berry	35	*Monster's Ball*	Denzel Washington	47	*Training Day*
2003	Nicole Kidman	35	*The Hours*	Adrien Brody	29	*The Pianist*
2004	Charlize Theron	28	*Monster*	Sean Penn	43	*Mystic River*

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Year	Best Actress	Age (years)	Film	Best Actor	Age (years)	Film
2005	Hilary Swank	30	*Million Dollar Baby*	Jamie Foxx	37	*Ray*
2006	Reese Witherspoon	29	*Walk the Line*	Philip Seymour Hoffman	38	*Capote*
2007	Helen Mirren	61	*The Queen*	Forest Whitaker	45	*The Last King of Scotland*
2008	Marion Cotillard	32	*La Vie En Rose*	Daniel Day-Lewis	50	*There Will Be Blood*
2009	Kate Winslet	33	*The Reader*	Sean Penn	48	*Milk*
2010	Sandra Bullock	45	*The Blind Side*	Jeff Bridges	60	*Crazy Heart*
2011	Natalie Portman	29	*Black Swan*	Colin Firth	50	*The King's Speech*
2012	Meryl Streep	62	*The Iron Lady*	Jean Dujardin	39	*The Artist*
2013	Jennifer Lawrence	22	*Silver Linings Playbook*	Daniel Day-Lewis	55	*Lincoln*
2014	Cate Blanchett	44	*Blue Jasmine*	Matthew McConaughey	44	*Dallas Buyers Club*
2015	Julianne Moore	54	*Still Alice*	Eddie Redmayne	33	*The Theory Of Everything*
2016	Brie Larson	26	*Room*	Leonardo DiCaprio	41	*The Revenant*
2017	Emma Stone	28	*La La Land*	Casey Affleck	41	*Manchester By The Sea*
2018	Frances McDormand	61	*Three Billboards Outside Ebbing, Missouri*	Gary Oldman	60	*Darkest Hour*
2019	Olivia Colman	45	*The Favourite*	Rami Malek	38	*Bohemian Rhapsody*
2020	Renée Zellweger	51	*Judy*	Joaquin Phoenix	46	*Joker*
2021	Frances McDormand	64	*Nomadland*	Anthony Hopkins	84	*The Father*
2022	Jessica Chastain	45	*The Eyes Of Tammy Faye*	Will Smith	53	*King Richard*

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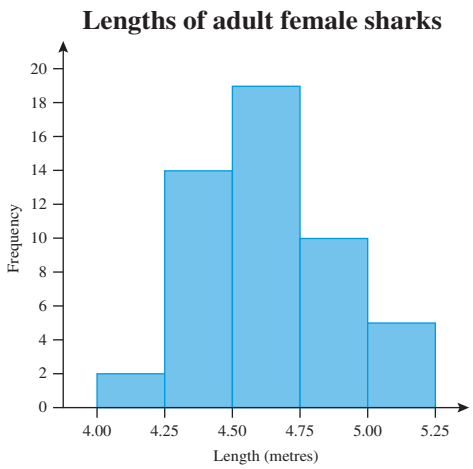
a Identify the youngest and oldest Best Actress from this list, and determine suitable groupings for a frequency distribution table, showing the age of the Best Actress at the time when she received her Academy Award.
b Construct a frequency distribution table for Age of Best Actress.
c Create a histogram for Age of Best Actress.
d Using the same age groupings as those used in part **b**, construct a frequency distribution table for Age of Best Actor.
e Create a histogram for Age of Best Actor.
f Comment on any similarities between the histograms.
g Comment on any differences between the histograms.

Hint Grouped numerical data vs frequency is presented using a histogram.

6 Decide if each of the following datasets might produce one or more gaps when represented in a histogram. Give a reason for your decision.
a the ages of people in a Year 11 class
b the ages of people in a high school
c the ages of people in a movie theatre
d the ages of people participating in a fun run

7 Here is a histogram that summarises the lengths of a group of sharks caught, measured and released as part of a research study.
Decide if each of the following statements is true or false, according to the histogram.



a A total of five sharks were measured.
b A total of 50 sharks were measured.
c The longest shark measured was 19 m long.
d The longest shark measured was over 5 m long.
e Five of the sharks measured were over 5 m long.

Hint A histogram has no gaps between columns, so it can display continuous data.

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7D Dot plots and stem-and-leaf plots

LEARNING GOALS

- Create a dot plot to present numerical data
- Identify suitable groupings for a stem-and-leaf plot
- Create a stem-and-leaf plot to present numerical data

How are dot plots and stem-and-leaf plots essential?

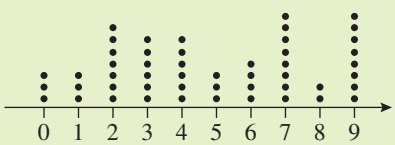
Dot plots give a quick visual representation of the distribution of data. Stem-and-leaf plots help to organise all the data into numerical order and display how it is distributed. Both these displays keep the original scores and arrange a lot of data into an ordered visual display that is much easier to follow than a list.

Many Japanese train timetables use stem-and-leaf to display 24-hour departure times. The 'stems' are hours, and each 'leaf' represents a train's departure time, in minutes past the hour.

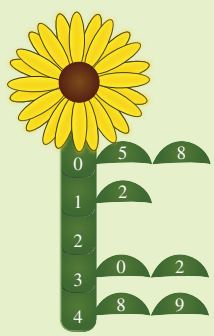
WHAT YOU NEED TO KNOW

- **Dot plots** are used for small sets of data that have a small range. Each piece of data is represented by a dot. For example:

Number of unread emails in Inbox



- A **stem-and-leaf plot** looks a bit like the stem of a plant with leaves coming off the side. It displays each data value with a stem number and a corresponding leaf number.



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- The key tells you how to interpret the stem-and-leaf parts.
  For example, 4|8 means 48, where 4 is the stem and 8 is the leaf.
- The 'leaves' data is in numerical order to allow statistical calculations to be performed easily. For example, finding a median requires the data to be in order.
- There are no commas between leaf numbers.
- The complete stem-and-leaf plot resembles a histogram with groups, but all the data is retained and shown.

The tens digits are called the *stems*.

Stem	Leaf
0	5 8
1	2
2	
3	0 2
4	8 9

The ones digits are called the *leaves*.

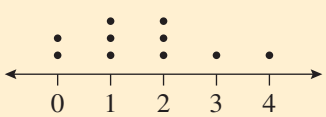
Key 4|8 = 48

Example 7 Creating a dot plot to present numerical data

A group of students are asked how many brothers and sisters they each have. The results are given below.

2 1 0 3 2 1 0 1 2 4

a How many students participated in the survey?
b Construct a dot plot for the data.
c Calculate how many brothers and sisters there are in total.

WORKING	THINKING
a 10 students participated in the survey.	There are 10 pieces of data.
b  Number of brothers and sisters	Draw a line with arrows at each end. Evenly space data labels from the lowest score to the highest score. Place a dot above the appropriate data label for each piece of data.
c Total = 2 + 1 + 0 + 3 + 2 + 1 + 0 + 1 + 2 + 4 = 16	Each piece of data represents how many siblings the student has. Add these numbers together.

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Example 8 Creating a stem-and-leaf plot to present numerical data

Below is the maximum temperature at Port Hedland Airport recorded on the first day of each month in 2022.

2022	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1st	41	43	37	28	31	18	18	22	26	19	28	40

a Organise the data into a stem-and-leaf plot.

b What was the highest temperature recorded on the first day of any month in 2022?

WORKING | **THINKING**

a

Stem	Leaf
1	
2	
3	
4	

The stem will be the first part of the number (i.e. the tens digit). The smallest number is 18 and the largest number is 43, so the stem (i.e. the tens digit) will need to go from 1 to 4.

Stem	Leaf
1	8 8 9
2	8 2 6 8
3	7 1
4	1 3 0

The leaf will be the second part of the number (i.e. the units digit). Work your way through the list of data, placing the leaf of each piece of data next to the appropriate stem.
Note: Since the data is not in order on each branch, it is called an **unordered** stem-and-leaf plot.

Stem	Leaf
1	8 8 9
2	2 6 8 8
3	1 7
4	0 1 3

Key: 4 | 1 means 41

Organise the data on each branch into numerical order. This is now an **ordered** stem-and-leaf plot. There are no commas between the leaf numbers. Include the key.

b The highest temperature recorded was 43°C.

The last score on the bottom line will be the maximum.

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Exercise 7D

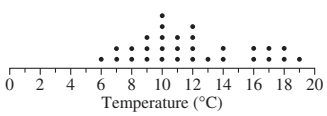
FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

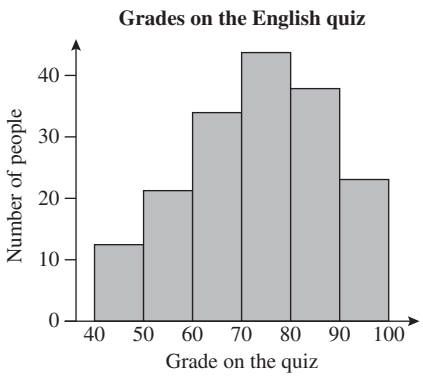
- **a** Dot _____ give a quick visual representation of the _____ of data.
- **b** Dot plots are used for small sets of ____ with a small _____. Each piece of data is represented by a _____.
- **c** Stem-and-_____ plots help to organise all the data into _____ and show how it is distributed.
- **d** A stem-and-leaf plot displays each value with a ____ number and a corresponding _____ number.

2 Match each of the following graphs (**i** to **iv**) with its correct description (**A** to **D**).

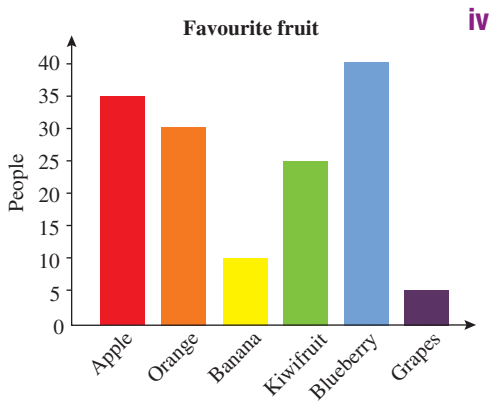
i



ii



iii



iv

Stem	Leaf
6	9
7	
8	7 8 8 9
9	0 6 7 7
10	0

Key: 6|8 means 68

- **A** column graph
- **B** stem-and-leaf plot
- **C** histogram
- **D** dot plot

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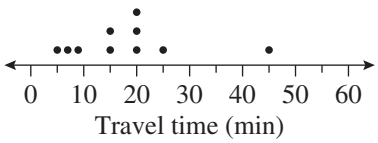
APPLICATIONS

Example 7

3 A number of students are surveyed about how long it takes them to travel to school. The results are shown in this dot plot.



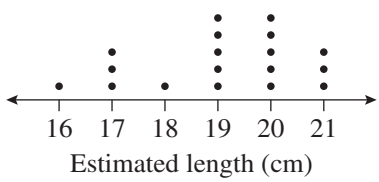
a How many students participated in the survey?
b Organise the data as a list of whole numbers.
c Identify the most common travel time taken.
d Identify the longest travel time taken by a student from the group surveyed.
e Identify the shortest travel time taken.

4 Shannon records the number of attempts it takes each of his 12 classmates to successfully shoot a basketball goal. Use Shannon's data (shown below) to create a dot plot.

Hint First, rule a line and draw arrows, then evenly space the data labels.

1 2 1 3 1 4 4 3 1 2 5 2

5 A teacher draws a line on a piece of paper that is 20 cm long and asks each of her students to estimate the length of the line. She uses their answers to construct the following dot plot.



a How many students are in the class?
b How many students' estimates are within 1 cm of the correct answer?
c How far off the correct answer is the least successful student's estimate?

The number of various real-time share sales in a stock market are displayed using many small LED lights as dots.

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6 A dog breeder records how many puppies are born per litter. The past ten litters resulted in the following numbers of puppies.

4 3 4 8 3 5 5 4 4 6

a Construct a dot plot to display the numbers of puppies born per litter.
b Determine the most common litter size.
c Comment on any values of the data that appear to be unusual.

7 The heart rates of students are taken after doing 1 minute of jumping jacks.

127, 113, 99, 84, 138, 147, 151, 121, 99, 61, 99, 86, 122, 148, 124, 114, 92, 124, 88, 129, 144

a Identify the lowest number and the highest number.
b Determine a suitable range of values to use for the stem.
c Organise the data in an ordered stem-and-leaf plot.

Hint Use tens for the stem; e.g. 148 has 14 tens.

Example 8

8 The maximum temperature, in °C, recorded on the first day of each month for Kalgoorlie in 2021 is shown.

33 31 28 25 20 17 16 18 22 25 28 31

a Organise the data into an ordered stem-and-leaf plot.
b Determine the highest temperature recorded on the first day of any month during 2021.
c Identify the lowest daily maximum temperature recorded on the first day of the month during 2021.

Hint Create an unordered stem-and-leaf plot first.

9 Use the data from Question **5** Exercise 7C (age of actors at the time when they were awarded Best Actor and Best Actress at the Academy Awards 1993–2022), to answer the following.

a Organise the data for Best Actress into an ordered stem-and-leaf plot.
b Organise the data for Best Actor into an ordered stem-and-leaf plot.
c Compare and comment on any differences in the distribution of the data between actresses and actors.

Hint Always include a key in a stem-and-leaf plot.

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7E Outliers

LEARNING GOAL

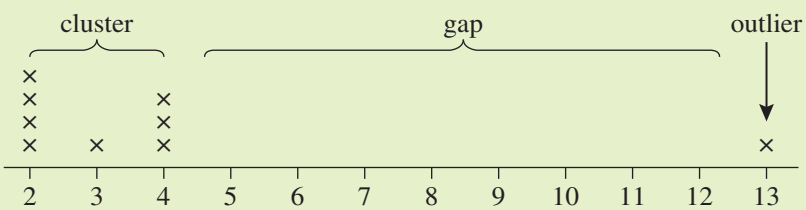
- Identify outliers in small and large datasets

Why is identifying outliers essential?

An outlier is a piece of data with a value appearing to be inconsistent with the remainder of that set of data. When we collect data, not all the data will have the same characteristics. Outliers can affect the calculation of the data average. Being able to identify outliers means that we can correct any abnormalities created when including them in our dataset. It could also be that the piece of data was not correctly recorded and is a mistake that we need to correct.

An outlier is a piece of data with a value appearing to be inconsistent with the remainder of that set of data.

WHAT YOU NEED TO KNOW



Number of hours of music practice per day

- An **outlier** is a piece of data that appears to be inconsistent with the remainder of that set of data. It sits a long way above or below the rest of the data, like a student practising an instrument for 13 hours a day when the next highest result recorded was 4.

If we look at the leading top 10 Test Cricket batting averages, in the table given on the next page, we can see Sir Donald Bradman's average of 99.94 sits far away from the other nine leading batsmen, who range only from 58.45 to 63.75. Therefore, Sir Don's *outstanding* batting average is an **outlier** in this set of data.

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Highest batting averages							
Player	**Span**	**Mat**	**Inns**	**No**	**Runs**	**Hs**	**Ave**
DG Bradman (AUS)	1928–1948	52	80	10	6996	334	99.94
AC Voges (AUS)	2015–2016	20	31	7	1485	269*	61.87
RG Pollock (SA)	1963–1970	23	41	4	2256	274	60.97
GA Headley (WI)	1930–1954	22	40	4	2190	270*	60.83
H Sutcliffe (ENG)	1924–1935	54	84	9	4555	194	60.73
SPD Smith (AUS)	2010–2022	87	154	18	8161	239	60.00
E Paynter (ENG)	1931–1968	20	31	5	1540	243	59.23
KF Barrington (ENG)	1955–1968	82	131	15	6806	256	58.67
ED Weekes (WI)	1948–1958	48	81	5	4455	207	58.61
WR Hammond (ENG)	1927–1947	85	140	16	7249	336*	58.45

Example 9 Identifying an outlier in a dataset

You record the amount of cash you have in your pocket each day for 12 days. The results are presented in the table.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Total money ($)	1.40	0.45	4.25	101.20	2.50	1.80	0.25	0.70	3.30	1.00	9.05	0.10

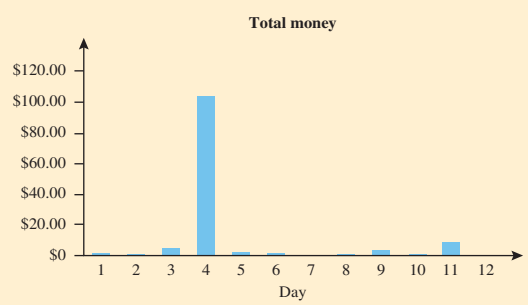
a Construct a column graph showing the amount of money in your pocket over the 12 days.

b Identify any outliers within this dataset.

c Explain a likely possibility for the outlier in the data.

WORKING

a



THINKING

Days are marked on the horizontal axis and the amount of money is marked on the vertical axis.

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b $101.20 is an outlier. ◄···· The rest of the data is less than $10. Therefore, $101.20 is an unusual amount.

c If you had drawn money out from the ATM, it is possible to have more than $100 in your pocket. ◄···· Outliers are not always a mistake and can have a logical explanation.

Example 10 Identifying an outlier in a large dataset

The historical monthly rainfall data recorded for a town is presented below. It shows monthly total amounts of rainfall together with calculated statistics, such as mean, median, highest totals and lowest totals.

Historic monthly rainfall

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
1996						0.0	39.0	57.6	48.4	49.8	71.2	104.0	
1997	37.8	229.8	22.2	1.0	59.8	9.4	9.6	2.6	74.2	132.0	128.8	102.4	809.6
1998	63.8	69.2	3.8	69.4	99.2	32.2	105.0	84.0	119.0	32.6	95.4	47.4	821.0
1999	69.1	245.6	104.6	8.0	48.6	29.8	77.8	33.6	27.0	78.6		139.0	
2000	95.8	45.4	32.6	37.8	51.6	40.8	6.4	6.2	2.0	43.8	68.4	131.0	561.8
2001	38.4	265.2	37.0	32.6	12.8	9.0	31.8	6.6	15.2	108.2	165.2	100.8	822.8
2002	14.0	60.2	159.0	10.2	9.0	54.6	0.2	67.6	22.2	16.6	68.4		
2003	9.2	145.2	86.7	59.8	28.6	62.0	14.2	19.8	4.2	94.4	23.2	118.8	666.1
2004	198.7	90.3	117.4	31.8	8.4	1.2	11.4	22.8	28.0	26.3	99.4	116.2	741.9
2005	91.6	14.2	35.4	9.4	25.6	91.6	1.4	12.4	13.0	122.8	120.4	32.4	570.2
2006	84.2	52.4	21.4	22.4	29.4	6.4	49.4	38.8	60.8	5.4	90.0	70.2	530.8
2007	26.2	56.8	5.4	17.0	13.8	55.6	0.0	55.2	34.4	67.6	63.4	62.8	458.2
2008	35.2	97.6	44.2	20.0	7.2	46.0	72.6	8.2	31.8	34.8	110.4	53.6	561.6
2009	87.6	63.4	15.2	34.4	49.4	25.2	3.8	4.2	25.6	45.0	32.0	47.6	433.4
2010	55.6	156.6	116.2	27.0	27.8	11.4	29.2	88.4	92.6	81.0	76.0	399.2	1161.0
2011	413.0	108.4	120.4	42.8	69.6	11.0	13.4	66.6	22.4	89.0	74.4	125.8	1156.8
2012	98.2	106.0	86.8	33.2	14.8	103.0	28.2	7.2	27.6	59.4	45.4	57.2	667.0
2013	416.0	184.2	113.0	41.0	51.4	61.2	27.8	4.8	16.8	43.2	80.0	39.6	1079.0
2014	12.0	12.0	190.0	7.0	35.6	25.4	13.4	30.2	18.0	12.6	54.6	251.2	662.0
2015	49.2	39.0	42.4	35.6	134.2	18.0	15.2	29.4	25.0	64.6	91.4	102.4	646.4
2016	82.2	79.6	70.2	6.4	19.0	109.6	40.4	54.8	104.0	29.0	25.0	33.8	654.0
2017	93.4	21.2	313.6	20.6	14.4	35.8	26.4	2.0	1.0	150.4	26.0	104.0	808.8
2018	8.6	189.8	51.0	23.0	7.6	13.8							

a Examine the data for June and decide if any of the data are outliers.

b Examine the data for December and decide if any of the data are outliers.

... Continued

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WORKING	THINKING
a No outliers	All data points lie close together, so no outliers.
b Outliers: 399.2 mm and 251.2 mm	The mean rainfall for December is 106.6 mm and all readings, except two, are less than 140 mm of rain. The readings in 2010 and 2014 are considerably higher than 106.6 mm or 140 mm, so they are outliers.

Exercise 7E

FUNDAMENTALS

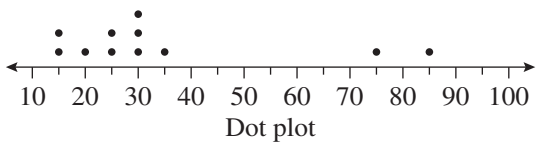
1 Demonstrate your understanding of terms by completing this sentence, selecting the words from this list.

above long outlier below

An _____ is a piece of data that appears to be inconsistent with the remainder of that set of data. It sits a _____ way _____ or _____ the rest of the data.

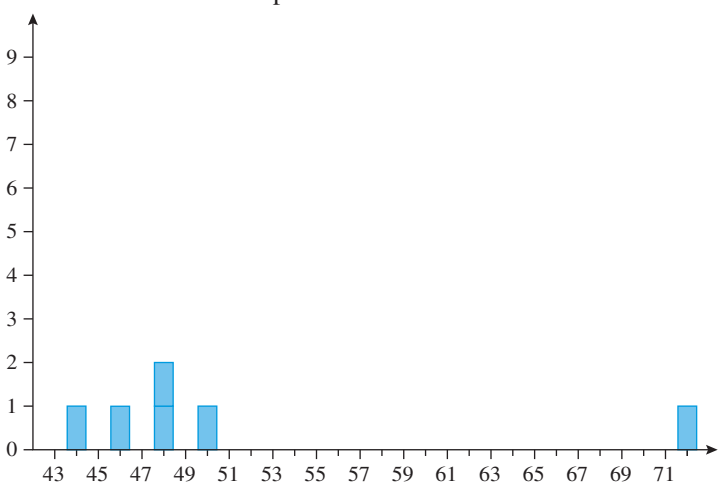
2 Identify any outliers in the following graphical displays.

a



Dot plot

b



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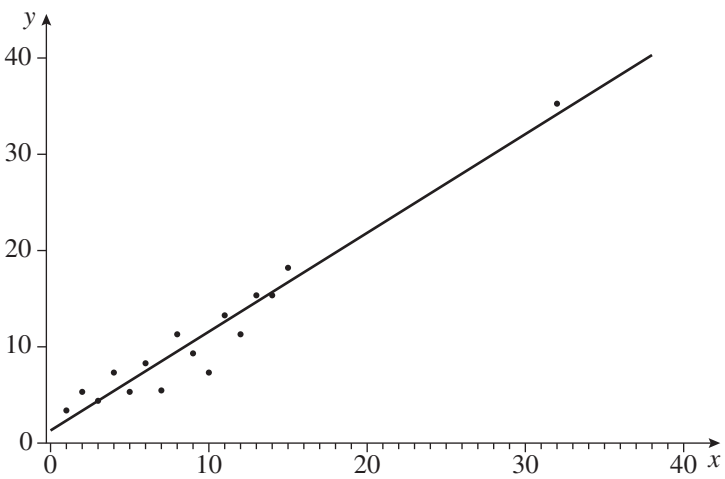
c



APPLICATIONS

Example 9

3 The following dataset represents the number of new computer accounts registered during ten consecutive days.

43, 37, 50, 51, 58, 105, 52, 45, 45, 10

a Reorganise the data in order from smallest to largest.
b Present the data as a dot plot.
c Identify any outliers.
d Offer a possible explanation for any unusual data in the set.

Hint An outlier is a long way above or below the rest of the data.

4 The following data shows the distance jumped by students participating in the long jump event at an athletics carnival.

Student	A	B	C	D	E	F	G	H	I	J
Distance (m)	3.2	2.7	3.1	2.8	2.8	2.7	31	3.3	2.6	3.0

a Create a column graph to display the data.
b Identify any outliers.
c Offer a possible explanation for any unusual data in the set.

5 The following data shows the numbers of baskets of apples picked by a backpacker over a week.

a Create a column graph to display the data.
b Identify any outliers.
c Offer a possible explanation for any unusual data in the set.

Day	**Number of baskets**
Sunday	14
Monday	17
Tuesday	16
Wednesday	20
Thursday	5
Friday	22
Saturday	18

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6 The following data represents how many fortnights the casual staff at a cafe worked over the past year.

10, 19, 20, 21, 22, 22, 23, 24, 24, 25, 26, 26

a Create a dot plot to organise the data.
b Identify any outliers.

Example 10

7 The table shows the medal tally for North and South American nations at the 2016 Summer Olympic Games.

a Identify any outliers within the Total column of the table.
b Identify any outliers within the Gold, Silver and Bronze columns of the table.
c Comment on the results obtained in parts **a** and **b**.

Hint Outliers can have a logical explanation.

Nation	Gold	Silver	Bronze	Total
Argentina	3	1	0	4
Bahamas	1	0	1	2
Brazil	7	6	6	19
Canada	4	3	15	22
Colombia	3	2	3	8
Cuba	5	2	4	11
Jamaica	6	3	2	11
Mexico	0	3	2	5
Puerto Rico	1	0	0	1
United States	46	37	38	121
Venezuela	0	1	2	3

8 Referring to **Example 10**, examine the monthly rainfall data recorded, and decide if there are any outliers for:

a January
b February
c March
d April
e May
f July
g August
h September
i October
j November

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7F Comparing how data is presented

LEARNING GOALS

- Analyse various presentation styles and identify whether key data is displayed or hidden
- Analyse the suitability of graphs for their purpose and compare the impressions given
- Analyse and compare various presentation styles and identify which features appeal to the intended readers

Why is comparing the presentation of data essential?

As we have seen in this chapter, there are many ways to present data and to make sense of collected data, as well as make it interesting to the reader. The way in which you choose to present the information, as a table or a graph, will emphasise different characteristics of the data.

Presentation graphs are chosen according to which key features of the data are to be emphasised.

WHAT YOU NEED TO KNOW

- Data, when collected in raw (i.e. original) form, is difficult to understand. It needs to be collated in either an effective table or a graph to allow us to make sense of it.
- A **table** is important when all information requires equal attention. It allows the reader to focus on the information that interests them within the table.
- **Graphs** show trends in data and allow the reader to make comparisons between groups.
- Choosing a presentation method for your data that is *simple* to understand is key to communicating the message you are wanting to get across.
- We should be careful when grouping data that we do not lose some of the key features of the data that need to be displayed.
  The following examples show different types of tables and graphs, and the key information that is communicated and lost by each presentation style.

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Presenting the same information in different ways

In a round of golf, 20 players had the following scores over 18 holes.

68	72	74	64	68	72	66	68	71	67
68	68	71	67	69	63	68	69	73	72

In its raw form, we cannot immediately know much about this data. It would be good to easily identify what the leader scored and what most players scored. We need to organise the data.

Frequency distribution table with discrete scores

Score	Tally	Frequency
63	|	1
64	|	1
65		0
66	|	1
67	||	2
68	𝍸 |	6
69	||	2
70		0
71	||	2
72	|||	3
73	|	1
74	|	1
Total	**20**	**20**

Some key information clearly seen

We can now see that the leader scored 63 (lowest score) and there is lots of data around 68, although the data is spread out over 12 rows.

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Frequency distribution table with grouped scores

Score	Tally	Frequency
63 – 64	||	2
65 – 66	|	1
67 – 68	𝍸 |||	8
69 – 70	||	2
71 – 72	𝍸	5
73 – 74	||	2
Total	**20**	**20**

Some key information lost

If we group the data so that we have fewer rows to work with, we start to lose some detail.
It is no longer clear that 68 is the most common score; it could be 67 or 68.
We can also no longer see that the leader had the lowest score of 63; it could be a 63 or 64.

A stem-and-leaf plot, grouped in 10s

Stem	Leaf
6	3 4 6 7 7 8 8 8 8 8 8 9 9
7	1 1 2 2 2 3 4

Some key information is not clear

This plot is not very effective at telling us how far the leader is from the rest of the field. This is because of the small number of stems.

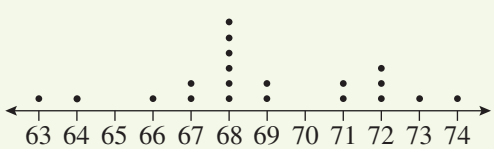
A stem-and-leaf plot, grouped in 5s

Stem	Leaf
6	3 4
6	6 7 7 8 8 8 8 8 8 9 9
7	1 1 2 2 2 3 4
7	

Some key information not easily noticed

We could make the groupings smaller on the stem by having low and high 60s, as shown.
This does separate the leaders a little but the range of scores is not very clear.

Dot plot



Golf scores for the round

Original data clearly seen

A dot plot gives us an easy representation of how the players were distributed and how good the best players were in comparison to everyone else. We can also see that 'most' players scored around 68 for the round. Having a dot for each piece of data gives us a visual for how many players there are.

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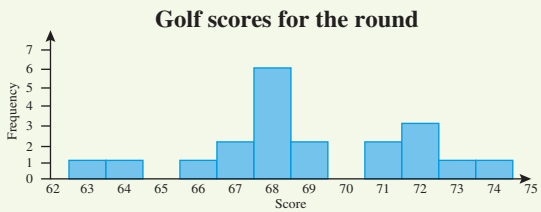
Histogram with discrete scores	Some key information clearly seen
	A histogram displaying **discrete** numerical data gives a visual comparison for the number of players achieving each score. We can still see the leader scored 63.
Histogram with grouped scores	**Some key information lost**
	In a histogram displaying **grouped** numerical data, the individual data is lost. For example, the graph has completely lost the leader's score of 63.

Example 11 Analysing the suitability of an infographic and a column graph for their purpose and comparing the impressions given

The infographic shows lists the amount of energy per point expended by some of the world's top tennis players.

a Create a column graph to display this data.

b Identify a possible reason why Australian Open Tennis chose to present this data in a table rather than as a column graph.

c Decide if a table is the most suitable method for displaying this type of information. Justify your response with reasons.

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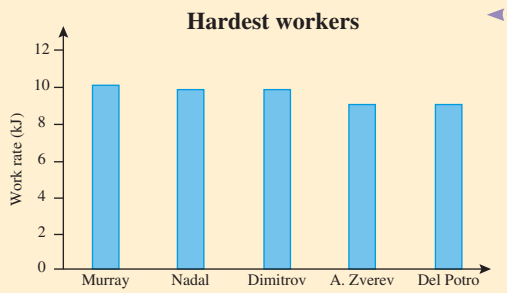
WORKING | **THINKING**

a



◄···· Even though all the numbers are close together, make sure you still start at zero on the vertical axis, otherwise the difference between the players will look greater than it actually is.

b When presenting the information as a column graph, the difference between the energy used per point by the players does not appear as great as it does when the data is arranged in a table.
Australian Open Tennis is wanting to emphasise that Murray uses more energy than his rivals, and a table shows that more clearly.

◄···· The difference between Murray and Nadal in energy used per point is only 0.3 kJ. To put that in perspective, a banana has 335 kJ. It will take 1116 points in a game of tennis for Murray to use more energy than Nadal. That extra energy can be gained by eating a single banana.

c No, a table is not a suitable method for displaying this information.
By presenting the information as a table rather than a graph, the creators of the table have tried to exaggerate the difference between the energy used by the players.

◄···· Make sure you always give a reason for your decision.

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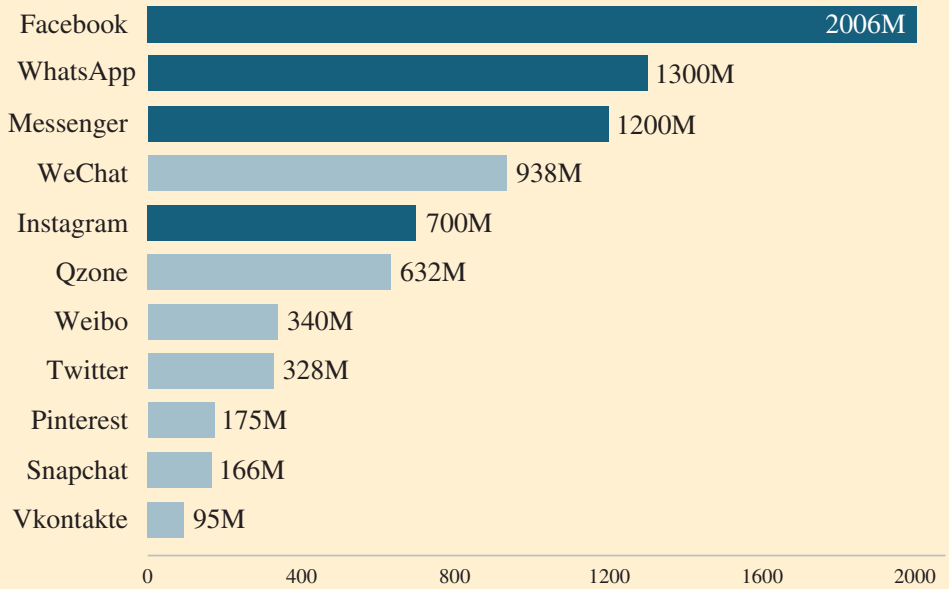
Example 12 Analysing the features of a column graph and comparing its effectiveness to a table and a dot plot

This column graph shows the worldwide number of monthly users of popular social media sites.



a Consider who this graph would be targeted at.
b Create a table to display this data.
c Explain why the producers of this graph have presented it as a column graph, rather than a table.
d Explain whether this information could be presented in a dot plot.
e Assess whether a column graph is a suitable method for presenting this information and explain your answer.

WORKING	THINKING
a Marketing companies would use this information to help businesses connect with potential customers.	What is the purpose? Who would benefit? How would they use it?

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b

Social media platform	Monthly users (millions)
Facebook	2006
WhatsApp	1300
Messenger	1200
WeChat	938
Instagram	700
Qzone	632
Weibo	340
Twitter	328
Pinterest	175
Snapchat	166
Vkontakte	95

◄···· Note: We could almost have used billions for the units. A billion is 1000 million or 1 000 000 000!!

c Presenting the information as a column graph is more visually appealing than a table. The differences are quickly identifiable.

◄···· Always give a reason to justify your decision.

d A dot plot would be difficult to use for this information because the numbers are so large and would require an enormous scale; e.g. • = 200 million.

◄···· Always give a reason to justify your decision.

e A column graph is a suitable method for displaying this information. The graph is accurate, visually appealing and shows the intended information.

◄···· Always give a reason to justify your decision.

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Exercise 7F

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

a A **table** is important when all _____ requires _____ attention.

b **Graphs** show _____ in data and allow readers to make comparisons _____ groups.

c Choosing a presentation method for your data that is _____ to understand is key to conveying the _____ you are wanting to get across.

d We need to be _____ when grouping data that we do not _____ some of the key _____ of the data.

2 A group of Year 11 students are each asked the current age of their mother. Their results are given below.

46	48	40	47	38	41	40	46	44	45
45	45	44	45	44	44	46	48	45	45

a Organise the information into a frequency distribution table without grouping.

b Create each of the following displays for this data.

i dot plot **ii** stem-and-leaf plot **iii** histogram

c Decide which presentation method is best suited for the data and justify your choice.

APPLICATIONS

Example 11

3 The infographic shows information about the digital activity in Australia, in January 2017.

a Identify the technique that has been used to present this information.

b Is it possible to have 128% of the population with mobile subscriptions? Explain how.

c Construct a frequency table to display the numbers of people in each category and the total number of people. Ignore percentages.

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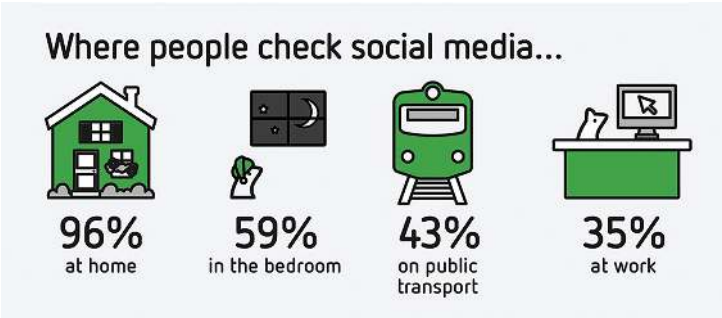
d Construct a column graph to display the numbers in each category.

e Decide whether the frequency table or the column graph is the most effective way to present this information. Explain your choice.

4 The following information comes from a survey of people who were asked where they were when they checked their social media. They were allowed to select more than one location (hence, the percentages add to greater than 100).



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a Describe the technique that has been used to present this information.

b Construct a frequency table to display this information, with frequencies given as percentages.

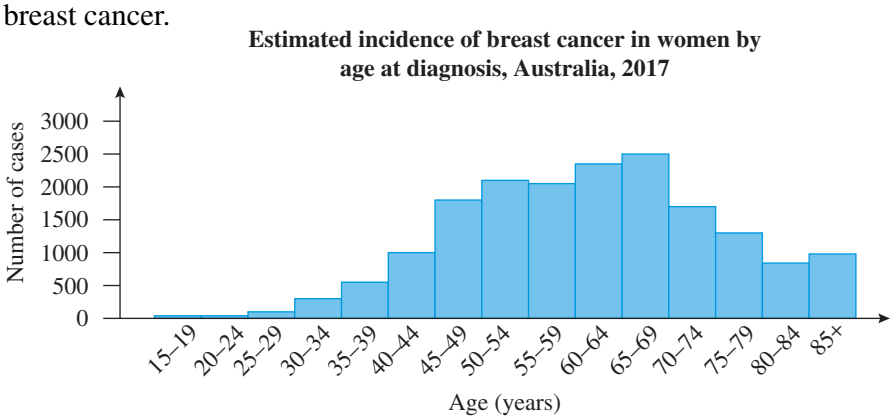
c Construct a column graph to display this data.

d Could this information be presented as a dot plot? Explain why or why not.

e Decide which of the displays is the most suitable way to present this information. Explain your answer.

Example 12

5 The graph below shows the age of women when they were diagnosed with breast cancer.



a Describe the technique that has been used to present this information.

b Organise the information in a grouped frequency table.

c Could this information be presented as a dot plot? Explain why or why not.

d What is the most suitable way to present this information? Why?

Hint Is the data grouped or discrete?

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6 The graph shown appeared in the *Australian Financial Review* in August 2017. It shows the price that various Australian states and other countries pay for their electricity.

a Describe the technique that has been used to present this information.

b Organise the information in a frequency table, rounding the prices to the nearest whole number.

c Could this information be presented as a dot plot? Explain why or why not.

d Create a column graph, counting up in 5s on the frequency axis, to display the same data.

e Compare your column graph to the one produced in the newspaper. Identify any similarities or differences.

f Assess the suitability of the method used by the newspaper to present the information. Write sentences to summarise your assessment.

g Explain what you believe to be the most suitable way to present this information.

h The graph at right shows the share price for AGL during the same time period as the prices presented above. AGL is a major electricity, gas and solar energy provider. The graph appeared on the same web page as the story, relating to the graph above of electricity prices. Explain what you think is the message that the newspaper is trying to convey about energy companies in Australia.

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7 The following table shows key trends associated with methamphetamine use between the periods 2009–10 and 2013–14.

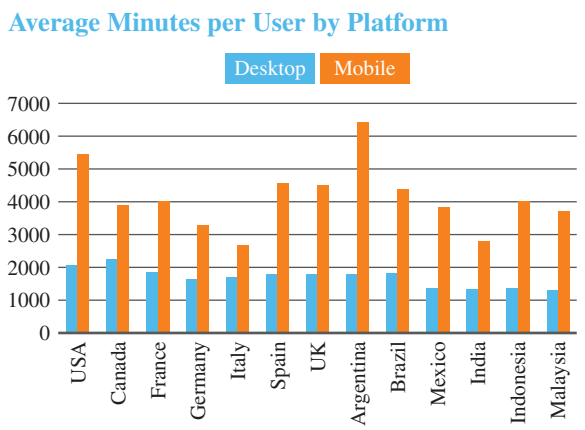
Selected key trends in methamphetamine use, availability and treatment, 2009–10 to 2013–14

	2009–10	2013–14
Arrests	14 000	26 300
Seizures	10 500	26 800
Detections	700	2400
Recent users reporting lifetime crystal use	190 000	280 000
Treatment episodes	10 000	28 900

a Describe the technique that has been used to present this information.
b Explain why you think the information has been presented in this way.
c Construct a column graph with two columns for the respective time frames.
d Discuss if there are any other ways this information could be presented.
e Explain which is the most suitable way of presenting this information.

8 The following graph shows time spent, in minutes per user, on either desktop or mobile devices.
a Describe the technique that has been used to present this information.
b Explain why you think the information was presented in this way.
c Construct a table to display this information.
d Could this information be presented as a dot plot? Explain why or why not.
e Explain what you believe to be the most suitable way to present this information.

Mobile users consume more than 2x minutes vs. desktop users



When looking at each region's desktop users and mobile users separately, mobile users universally consume more digital minutes per person – more than double in the majority of countries.

Argentina continues to deliver the largest number of mobile minutes per user, while Canada has the highest level of per-user desktop consumption.

Source: www.smartinsights.com/mobile-marketing/mobile-marketing-analytics/ mobile-marketing-statistics/

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9 The following graph shows how people from across the world access the internet.

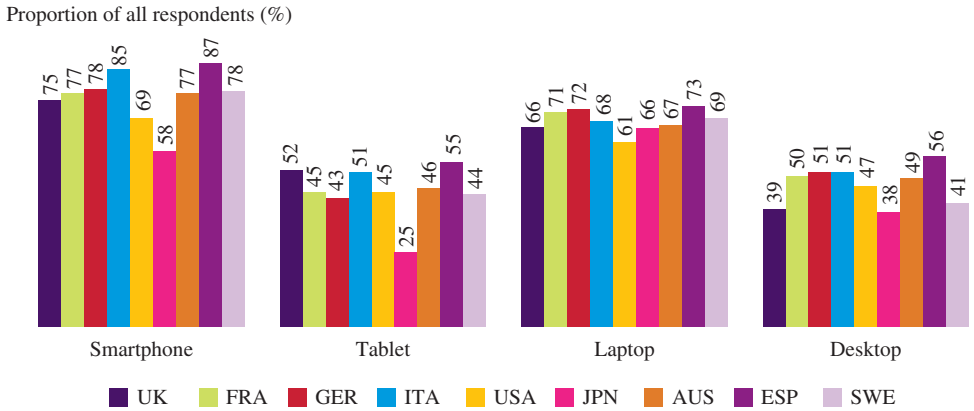
a Describe the technique that has been used to present this information.

b Give a possible explanation for why it has been presented in this way.

c Organise the information into a table.

d Discuss if there are any other ways this information could be presented.

e Explain the most suitable way to present this information.



Source: www.smartinsights.com/mobile-marketing/mobile-marketing-analytics/mobile-marketing-statistics/

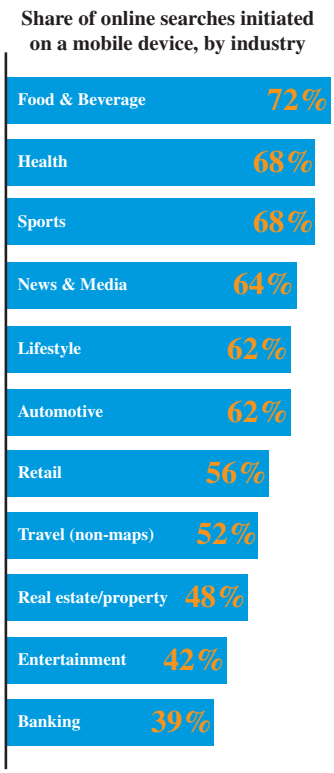
10 The following graph shows the industries that consumers search for using their mobile device.

a Describe the technique that has been used to present this information.

b Give a possible explanation for why the information has been presented in this way.

c Discuss if there are any other ways this information could be presented.

d Explain the most suitable way to present this information.



Based on top search variations resulting in a click to the indicated industry, April 10 - May 7, 2016.

Source: www.smartinsights.com/mobile-marketing/mobile-marketing-analytics/mobile-marketing-statistics/

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Statistical investigation process task

Background: Interpreting and presenting data are key skills in all aspects of life but especially in education, business, management, and media and communications. Two stimulus items are presented here as the basis of this task.

STIMULUS 1

The following article appeared in *The Courier-Mail*, 10 January 2018.

Southerners adding tough competition to find job in Queensland

JOHN McCARTHY, *The Courier-Mail*

INTERSTATE interlopers are adding to tough competition to find a job in Queensland, as the typical wait to find a job in the Sunshine State stretches to six months, the worst of all states.

Australian Bureau of Statistics data shows that, despite nation-leading jobs growth, the search for a job is the longest in Queensland at 26 weeks. Brisbane is 24 weeks.

In NSW, it takes a mere 13 weeks and Victoria 12 weeks. For the long-term unemployed, the wait in Queensland for a job could be as high as 126 weeks, or almost two and a half years, compared with the national median of 106 weeks.

Some areas of Queensland are better than others for finding work.

The coal town of Mackay is one. It takes a mere five weeks to land a job there, but in Toowoomba, the median time is 64 weeks and Wide Bay is 61 weeks.

The time it takes varies widely throughout the year, but the delays in Queensland have been occurring since June last year when the wait time suddenly spiked in the unadjusted data.

"The interstate migration would be a sensible explanation," QEAS economist Nick Behrens said. That's backed up by ABS data that shows Queensland recorded the highest quarterly increase in interstate migration, since 2007, at 5038 people last year.

For the June year, Queensland had the highest net gain with 17 400 people.

Mr Behrens said that a good measure of the labour market was how long it took to get a job and the data showed that Queensland had a long way to go, despite having *strong jobs growth*.

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He said another explanation could be that people who had not been in the market for a job, such as students or people doing home duties, have been lured back because of the *strong growth in jobs*.

Job wait times

Brisbane: 24 weeks
Ipswich: 40 weeks
Logan: 16 weeks
Moreton Bay North: 47 weeks
Cairns: 22 weeks
Maranoa: 22 weeks
Fitzroy: 47 weeks
Gold Coast: 14 weeks
Mackay: 5 weeks
Outback: 45 weeks
Sunshine Coast: 14 weeks
Toowoomba: 64 weeks
Townsville: 37 weeks
Wide Bay: 61 weeks

STIMULUS 2

A snapshot of employment by industry in 2016, in Mackay and across Queensland, revealed the following information.

Employment by industry, Mackay (R) LGA and Queensland, 2016

Industry	Mackay (R) LGA		Queensland		Specialisation ratio
	Number	%	Number	%	Number
Agriculture, forestry and fishing	1844	3.5	60 608	2.8	1.23
Mining	6516	12.4	49 997	2.3	5.28
Manufacturing	3425	6.5	128 787	6.0	1.08
Electricity, gas, water and waste services	516	1.0	23 883	1.1	0.88
Construction	3922	7.4	191 338	9.0	0.83
Wholesale trade	1969	3.7	56 370	2.6	1.42
Retail trade	5252	10.0	211 778	9.9	1.00
Accommodation and food services	3297	6.3	156 670	7.3	0.85

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Industry	Mackay (R) LGA		Queensland		Specialisation ratio
	Number	%	Number	%	Number
Transport, postal and warehousing	3343	6.3	108 083	5.1	1.25
Information media and telecommunications	289	0.5	25 265	1.2	0.46
Financial and insurance services	701	1.3	54 286	2.5	0.52
Rental, hiring and real estate services	863	1.6	42 500	2.0	0.82
Professional, scientific and technical services	2161	4.1	133 652	6.3	0.66
Administrative and support services	1532	2.9	75 336	3.5	0.82
Public administration and safety	2318	4.4	140 164	6.6	0.67
Education and training	3933	7.5	192 143	9.0	0.83
Healthcare and social assistance	5647	10.7	276 945	13.0	0.83
Arts and recreation services	406	0.8	33 667	1.6	0.49
Other services	2604	4.9	83 470	3.9	1.26
Total	**52 732**	**100.0**	**2 136 455**	**100.0**	**1.00**

Source: ABS, *Census of Population and Housing, 2016, General Community Profile – G51* and unpublished data.

Some key points of interest have been highlighted in italics in the newspaper article.

Task: Your task is to write a follow-up article that includes graphs and is encouraging to prospective workers moving to Queensland on the promise of 'strong jobs growth'.

Some things to include:

- The regions in Queensland that have the lowest job wait times.
- The industries that have the highest workforce in those regions.

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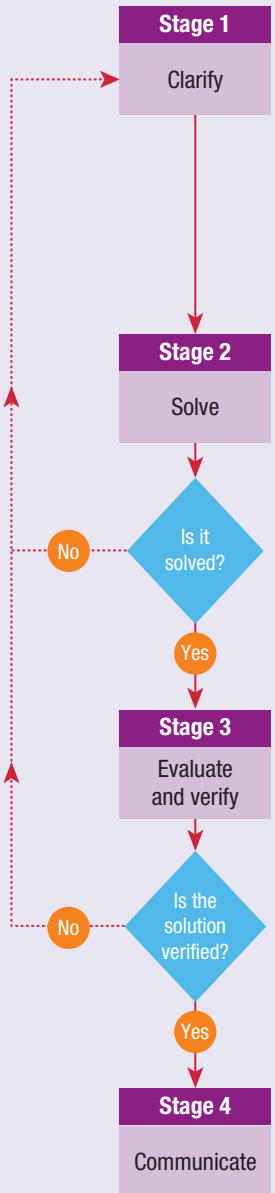
To complete this task, follow the investigation process workflow below and use the steps listed as a guide.



Stage 1: Clarify
Clarify the problem and pose one or more questions that can be answered with data

1. Identify any information available.
2. Decide upon the technology that you will use to produce your graphs.

Stage 2: Solve
Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

3. Identify regions in Queensland with the lowest job wait times.
4. Identify industries with the highest workforce in those regions.
5. Use technology to create graphs to support your reasons for people to move to Queensland for work.

Stage 3: Evaluate and verify
Interpret the results of this analysis and relate the interpretation to the original question

Once you have developed your graphs, consider:

6. How reasonable are they?
7. Have you presented them in the most suitable way?
8. Will they be appealing to prospective people considering moving to Queensland?

Stage 4: Communicate
Communicate findings in a systematic and concise manner

Write a report with an introduction, body and conclusion.

9. Communicate your main point of the article.
10. Include the evidence in the form of statements, graphs and tables of regions and industries that have good employment prospects for people moving to Queensland.
11. Explain the evidence. Use a sentence starter like ‘This means…’.
12. In the conclusion, link back to your main point.

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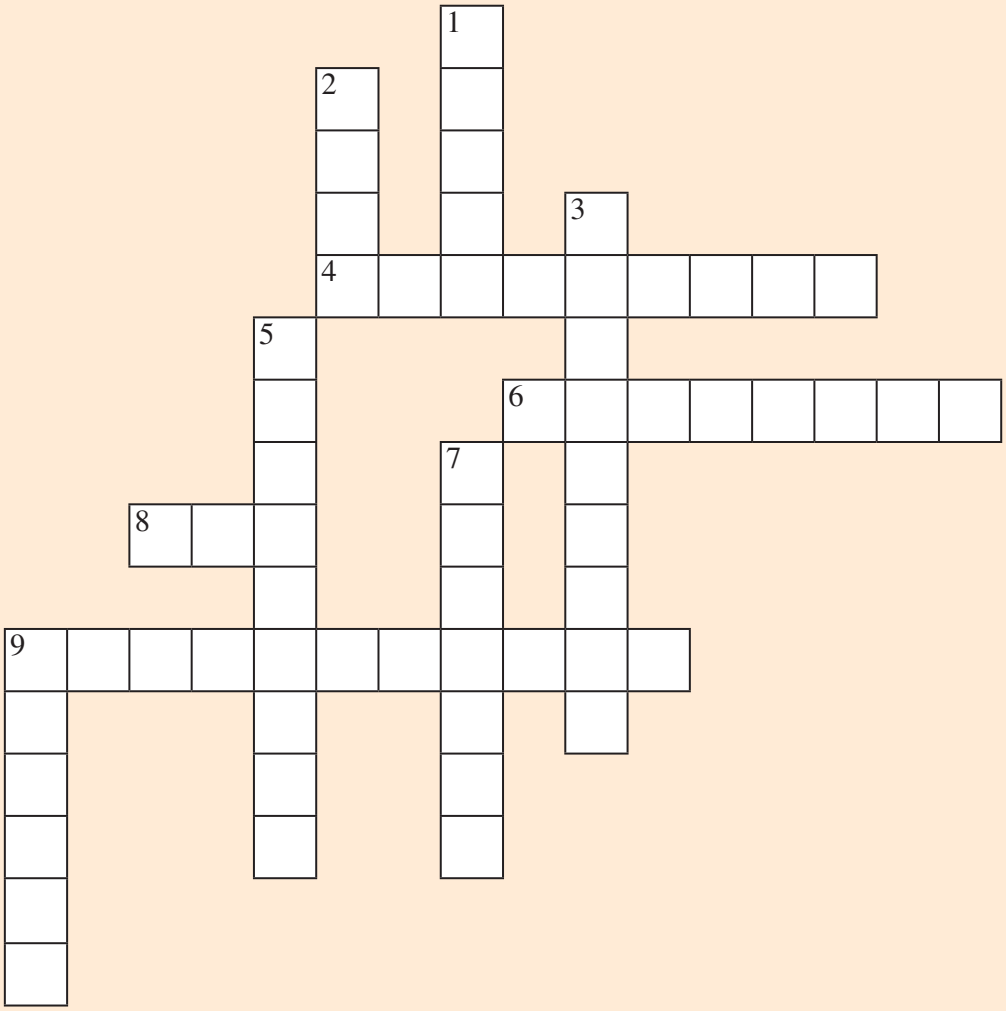
Puzzle



Across

4 How many times data occurs
6 Frequency is shown on the ____ axes on a column graph
8 A graph that is made up of points in columns is called a ____ plot
9 Data that can be categorised into groups (words)

Down

1 A graph should always have a ___ explaining what it displays
2 Stem-and-___ plot
3 Data that consists of numbers
5 A ___ is like a column graph but is for numerical data and has no gaps
7 Highest score
9 A ____ graph is used to represent categorical data

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Chapter checklist

☐ I can understand the difference between categorical and numerical data.

For each of the following, choose from either categorical or numerical.

1 **a** Data collected on the number of people attending the home games for a football team would be _____ data.

b Data collected on the favourite football team of people attending a football game would be _____ data.

☐ I can display categorical data in frequency tables and column graphs.

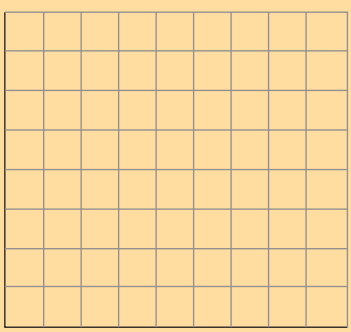
2 The main sport played by a group of students is given below.

netball	rugby league	cricket	rugby league	netball
rugby league	cricket	netball	netball	cricket
cricket	netball	rugby league	netball	netball
cricket	netball	cricket	rugby league	hockey

a Organise the data into a frequency table.

Sport	Tally	Frequency
Netball		
Rugby league		
Cricket		
Hockey		
Total		

b Construct a column graph, displaying the data.



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☐ I can display numerical data as a frequency distribution table, dot plot, stem-and-leaf plot and histogram.

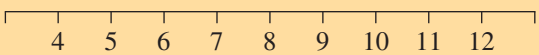
3 The amount of time (in hours) a group of students spends preparing for a test is presented below.

6	11	5	6	5	10	6	9	5
5	5	4	12	8	5	6	5	6
7	6	6	7	6	9	5	6	4

a Organise this information into a frequency distribution table.

Hours of study	Tally	Frequency
4		
5		
6		
7		
8		
9		
10		
11		
12		
Total		

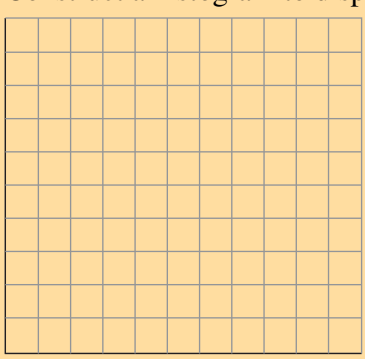
b Construct a dot plot to display this information.



c Construct a stem-and-leaf plot to display this information.

Stem	Leaf
0	
1	

d Construct a histogram to display this information.



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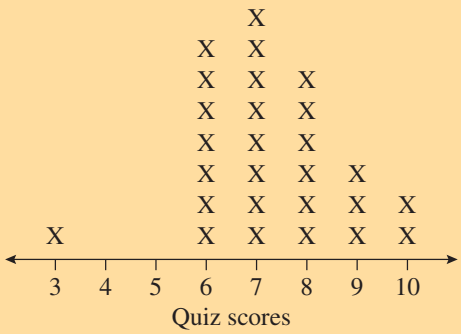
I can recognise and identify outliers from a dataset.

4 Identify any outliers in this dataset.



I can compare the suitability of different methods of data presentation in real-world contexts.

5 **a** Decide whether the information about the number of peas in a pod could be presented as a:

i dot plot

ii stem-and-leaf plot

iii column graph

iv histogram

Peas in pod	Frequency
3	5
4	10
5	28
6	36

b Explain why each method could/could not be constructed for this data.

c Decide which would be the most suitable way to present the information in a graph.

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Chapter review

1 Classify each type of data as categorical or numerical. Give two possible responses to each question.

- **a** How many pets do you own?
- **b** What type of pet do you have?
- **c** What books do you own?
- **d** How many books do you own?
- **e** Price paid for fuel
- **f** Type of fuel your car uses

2 You are asked by the Student Representative Council to survey students about the school canteen.

- **a** Design two questions that collect numerical data.
- **b** Design two questions that collect categorical data.

3 The students of a Year 11 home room class are surveyed about which maths subject they are studying. Their responses are:

Essential	Methods	Applications	Essential	Methods
Methods	Essential	Applications	Methods	Applications
Applications	Applications	Essential	Applications	Applications
Applications	Essential	Methods	Methods	Essential
Methods & Specialist	Applications	Methods & Specialist	Methods	Methods & Specialist

- **a** Organise the information into a frequency table.
- **b** Identify the subject with the highest frequency.
- **c** Determine how many steps it would take to reach the highest frequency:
  - **i** if we count up in 1s
  - **ii** if we count up in 2s
  - **iii** if we count up in 5s
- **d** Decide upon the most suitable scale to count with to reach the highest frequency.
- **e** Construct a column graph to represent this data.

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4 The ages of the 50 employees at a fast-food restaurant are listed below.

18	19	19	25	20	21	18	22	18	19
25	18	21	24	25	22	18	23	24	19
18	21	23	20	24	23	19	21	23	20
20	21	22	24	23	25	21	20	22	20
19	19	18	21	21	19	24	21	21	21

a Organise the data into a frequency distribution table.
b Construct a dot plot to display this data.
c Organise the data into a stem-and-leaf plot.
d Construct a histogram for this data.
e Explain which graph is the most suitable way to present this information.

5 Identify any outliers in the following sets of data. Account for any unusual data if possible.

a

47	48	69	79	65	66	4	78	91
53	68	89	67	92	77	76	90	73

b

Stem	Leaf
0	3
1	
2	
3	2 5 5 7 8 9
4	0 2 3 6
5	4 7
6	5
7	
8	
9	
10	
11	5
12	6

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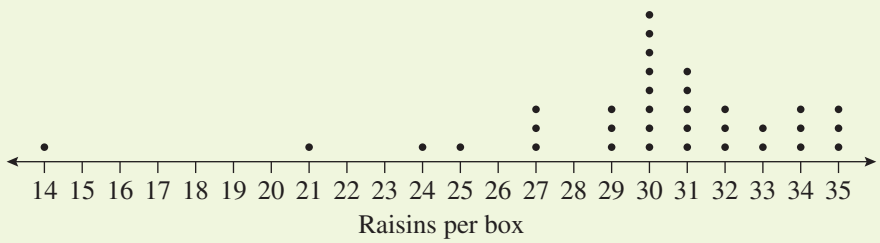
c



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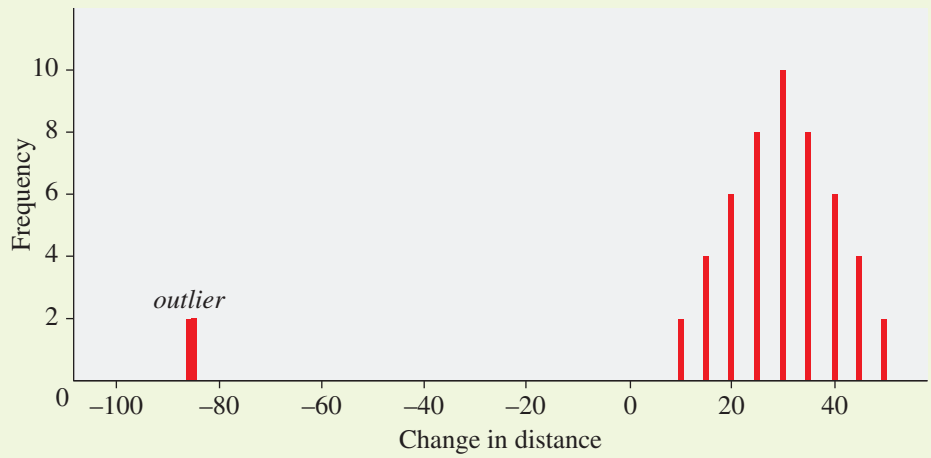

Questions **6** and **7** use the following information about nutritional breakdown and major nutrients of common foods.

Nutritional breakdown of selected foods by calories and major nutrient

	Calories per serving size	Major nutrient
Proteins		
chicken breast	280	niacin (vit. B3)
round steak	300	vit. B12
T-bone steak	412	vit. B12
lobster	160	vit. B12
shrimp	66	vit. B12
sirloin steak	260	vit. B12
turkey	230	niacin (vit. B3)
salmon	300	vit. B12
pork	420	vit. B1
spinach	21	vit. A
swordfish	270	niacin (vit. B3)
tuna	270	niacin (vit. B3)

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	Calories per serving size	Major nutrient
Carbohydrates		
tomato	32	vit. C
potato	300	vit. C
sweet potato	82	vit. A
cucumber	8	potassium
pumpkin	41	vit. A
beans (black)	136	potassium
beans (kidney)	109	potassium
corn	69	vit. B6
pasta	320	vit. B1
whole-wheat bread	67	magnesium
brown rice	116	vit. B6
Fruits		
apple	160	vit. C
orange	65	vit. C
Vegetables		
celery	14	potassium
broccoli	20	vit. C
onion	30	potassium
green peppers	12	vit. C
cabbage	16	vit. K
carrots	26	vit. A
peas	57	vit. C
lettuce	10	vit. A
cauliflower	14	vit. C
mushrooms	45	niacin (vit. B3)
asparagus	24	vit. C

Note: Serving size = $\frac{1}{2}$ cup, or 8 oz or 227 grams.

Source: https://labwrite.ncsu.edu/res/gh/gh-tables.html

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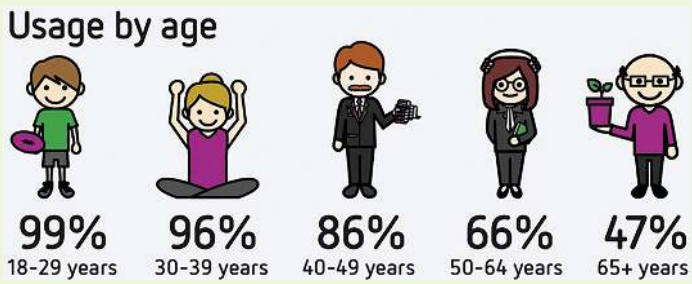
6 Refer to the data for calories per serve.

- **a** Determine if this data is categorical or numerical.
- **b** Identify the lowest and highest calorific foods.
- **c** Determine a suitable grouping for the calories, and organise the data into a grouped frequency distribution table.
- **d** Construct a histogram to display this data.

7 Refer to the major nutrients listed for each food.

- **a** Determine if this data is categorical or numerical.
- **b** Organise the information into a frequency table.
- **c** Construct a column graph to display this data.

8 The following information shows the usage of social media by age.



- **a** Describe the technique that has been used to present this information.
- **b** Explain why you think the information has been presented in this way.
- **c** Construct a column graph to display this information.
- **d** Could this information be presented as a dot plot? Explain why or why not.
- **e** Explain what you believe to be the most suitable way to present this information.

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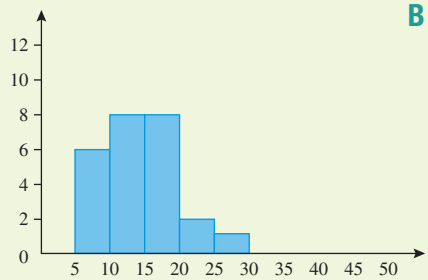
9 Five sets of data have been displayed below as both a histogram (**A** to **E**) and a dot plot (**i** to **v**) but not in the same order.

A



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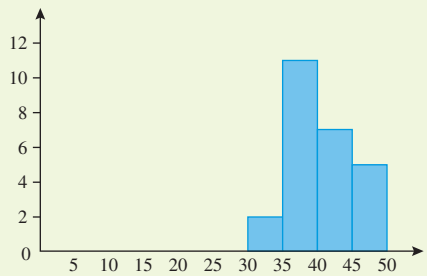

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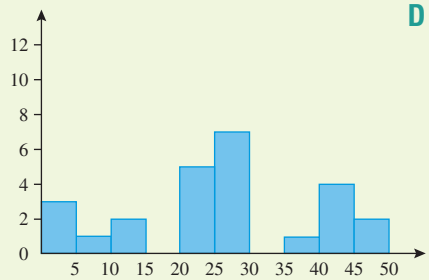

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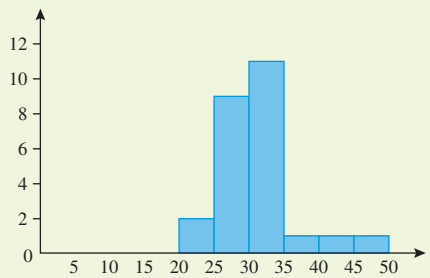
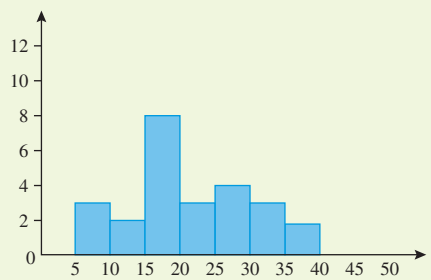
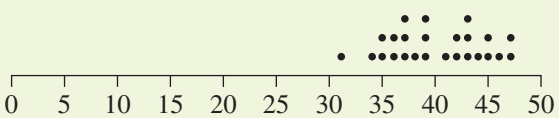


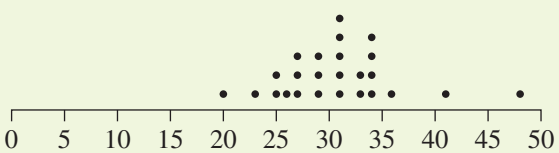
E



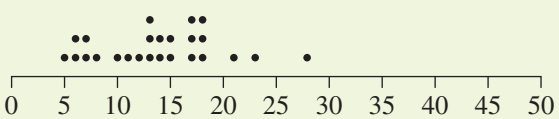
i



ii



iii



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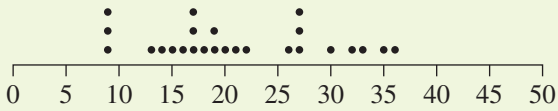
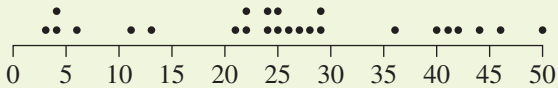
iv



v

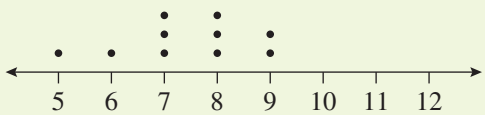


a Determine which dot plot shows the same data as histogram A.
b Determine which dot plot shows the same data as histogram B.
c Determine which dot plot shows the same data as histogram C.
d Determine which dot plot shows the same data as histogram D.
e Determine which dot plot shows the same data as histogram E.

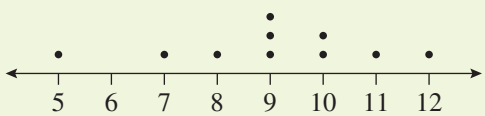
10 Three sets of data about ten Year 6 students were used to create three dot plots.
The person who created these dot plots forgot to label them.
For each dot plot, identify the appropriate label from options **i**, **ii** and **iii**.

i Age (in years)
ii Numbers of hours of sleep on nights before school days
iii Numbers of hours of sleep on nights before non-school days

A



B



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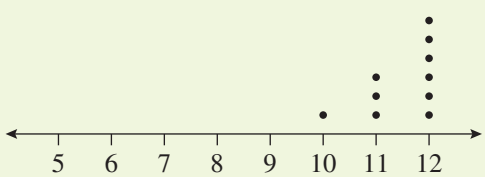

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8 Summarising and interpreting data

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In this chapter

Syllabus reference

Unit 2 Topic 2.1: Representing and comparing data

Summarising and interpreting data

- identify the mode and calculate other measures of central tendency, the arithmetic mean and the median, using technology when appropriate
- investigate the suitability of measures of central tendency in various real-world contexts
- investigate the effect of outliers on the mean and the median
- calculate and interpret quartiles
- use informal ways of describing spread, such as spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers
- interpret statistical measures of spread, such as the range, interquartile range and standard deviation
- investigate real-world examples from the media, illustrating inappropriate uses of measures of central tendency and spread

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Pre-test

1 Evaluate the following correct to 2 decimal places.

a $x = \frac{145}{7}$ **b** $y = \frac{3}{6}$ **c** $x = \frac{1}{3}$

2 Arrange the following numbers into order from lowest to highest, and identify the median (middle number).

11, 13, 7, 9, 4, 1, 17, 19, 21, 6, 5, 10, 13

3 Evaluate the following.

a $\frac{22 + 31}{2}$ **b** $\frac{43 + 45}{2}$ **c** $\frac{56 + 71}{2}$

4 Evaluate x, and write your answer as a percentage.

a $x = \frac{4}{16}$ **b** $x = \frac{7}{16}$

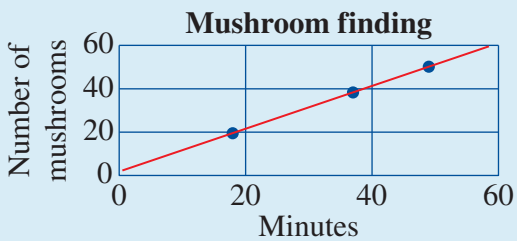
5 Determine the value of x that satisfies each percentage.

a $\frac{x}{8} = 12.5\%$ **b** $\frac{x}{36} = 25\%$

6 Evaluate the following squares.

a 2^2 **b** 7^2 **c** 0.5^2

7 From this graph, estimate how many minutes it took to collect 24 mushrooms.



8 Identify the smallest, largest and most common number in this stem-and-leaf plot.

Stem-and-leaf plot

4	1
5	2 7 8
6	5 6
7	0 5 8 8 8
8	0 0
9	5

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8A Identifying and calculating the measures of central tendency

LEARNING GOALS

- Identify the mode from a dataset
- Calculate the mean from a dataset
- Calculate the median from a dataset

Why is it essential to understand how to determine the measures of central tendency?

There are three main measures of central tendency: the mode, the mean and the median. The aim of each measure is to pinpoint the location of the centre of a data distribution with numerical data.

The measures can be used as a summary of the data, and any conclusion drawn from the data should refer to them. People that might use measures of central tendency as part of their job are car salespeople, real estate salespeople, scientists, business administrators and anyone in a vocation that requires the use of statistics.

Measures of central tendency help us to make sense of a large data distribution.

WHAT YOU NEED TO KNOW

- A **measure of central tendency** describes a set of data by identifying the central position within that dataset. There are three main measures of central tendency: the mode, the mean and the median.
- The **mode** reveals the most frequent value in the dataset, i.e. the value that occurs most often.
  - A dataset can also have two modes (it is **bimodal**) or no mode at all.
  - An example for using the mode is when a dress shop wants to know the most popular size when ordering stock.

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- The **mean** is the average; it is equal to the sum of all the values in the dataset divided by the number of values in the dataset.
  - The symbol for mean in mathematics is $\bar{x}$. This is pronounced as 'x bar'.
  - The formula for identifying the mean is $\bar{x} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$.
  - An example for using the mean is when a teacher wants to calculate the average marks for a class, as this value summarises how well the class is going as a whole.
- The **median** is the middle value of a dataset when the dataset is sorted in ascending order, meaning from the smallest value to the largest value.
  - The median is the number that falls exactly in the middle of the data.
  - The median is the $\dfrac{n+1}{2}$th value when ranked in order from smallest to largest, where n is the number of pieces of data. So if a dataset has 21 values, then the median will be the 11th value because $\dfrac{21+1}{2} = 11$.
  - If the dataset has an even number of values, then the median is the **average** of the two middle values.

Odd number of values	Even number of values
4, 6, (7), 9, 11 Median = 7	4, 6, (7, 9,) 11, 14 Median $= \dfrac{7+9}{2}$ $= 8$

  - An example for using the median is when a real estate agent uses the median for house prices in a particular suburb, especially if the suburb includes either some very expensive or very cheap properties that might have a big effect on the mean.

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Example 1 Identifying the mode from a dataset

Eralia is a dressmaker and needs to make some more dresses for her pop-up shop. She recalls the sizes of the dresses she sold over the past three months:
Size 6: 15 dresses
Size 8: 5 dresses
Size 10: 6 dresses
Size 12: 12 dresses
Size 14: 2 dresses
Size 16: 22 dresses
Identify the mode for the dataset.

WORKING	THINKING
The mode is size 16 which had 22 dresses.	The mode is the most common value. The dresses purchased the most are size 16, and 22 of them were sold.

Example 2 Calculating the mean from a dataset

The following are the term grades for a student's tests.

67%, 78%, 65%, 72%, 64%, 76%, 78%, 80%, 82%, 85%

Calculate the mean for the student's grades.

WORKING	THINKING
$\bar{x} = \frac{67+78+65+72+64+76+78+80+82+85}{10}$ $\bar{x} = 74.7$	Mean $= \bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$
The mean of the student's tests for the term is 74.7%.	Add all the values and divide by the number of values (with this dataset, there are 10).

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Example 3 Calculating the median from a dataset

The following are the sale prices of some houses in a particular area.

$345 000, $300 000, $450 000, $290 000, $390 000, $670 000, $345 000, $410 000

Calculate the median of the house sales.

WORKING	THINKING
290 000, 300 000, 345 000, 345 000, 390 000, 410 000, 450 000, 670 000	The data must first be ordered from smallest to largest. The median is the middle value.
The number of scores is 8. So, $n = 8$	Determine the number of scores. This is an even number of scores, so we need to average the middle two scores.
The median score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score. The median will lie between the 4th and 5th scores. The 4th score = 345 000 The 5th score = 390 000	
Median = (345 000 + 390 000) ÷ 2 = 367 500	Add the scores and divide by 2.
The median for the house sales is $367 500.	Communicate your answer in a sentence.

Exercise 8A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a The aim of measures of central tendency is to pinpoint the location of the _____ of a data distribution.

b There are three main measures of central tendency: the _____, the _____ and the _____.

c _____ is the most frequent value in the data.

d A dataset can have _____ modes (which means it is _____), or it can have no mode at all.

e The mean is the _____; it is equal to the _____ of all the values in the dataset, _____ by the number of values in the dataset.

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f The formula for identifying the mean is _____.

g The median is the _____ value of a dataset when the dataset is sorted in _____ order.

h If the dataset has an _____ number of values, then the median is the average of the _____ middle values.

Example 1–3

2 For each dataset, identify:

i the mean

ii the median

iii the mode

a 6, 4, 3, 5, 6, 2, 7, 6, 5, 9, 5, 4

b 85, 85, 95, 55, 75, 85, 75, 85, 55

c 6.7, 8.5, 8.6, 9.2, 7.4, 7.5, 7.9, 8.0

d 12, 18, 19, 11, 23, 24, 21, 18, 35

e 1, 3, 2, 2, 3, 2, 2, 1, 3, 3, 0, 3, 3, 2

Hint The mean is the average.

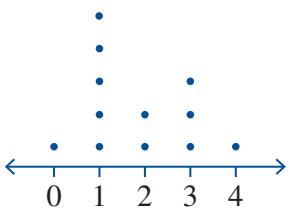
$$\bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$$

Hint The median is the middle score. Place all of your data in order, from smallest to largest, before finding the median.

3 Identify the mode for the following datasets.

Hint The mode is the most frequent.

a **Number of children**

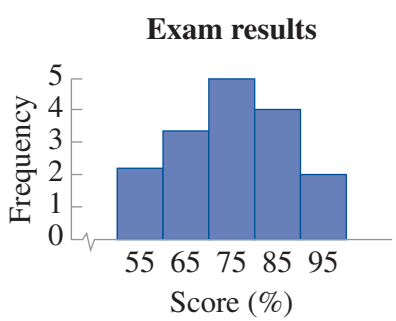


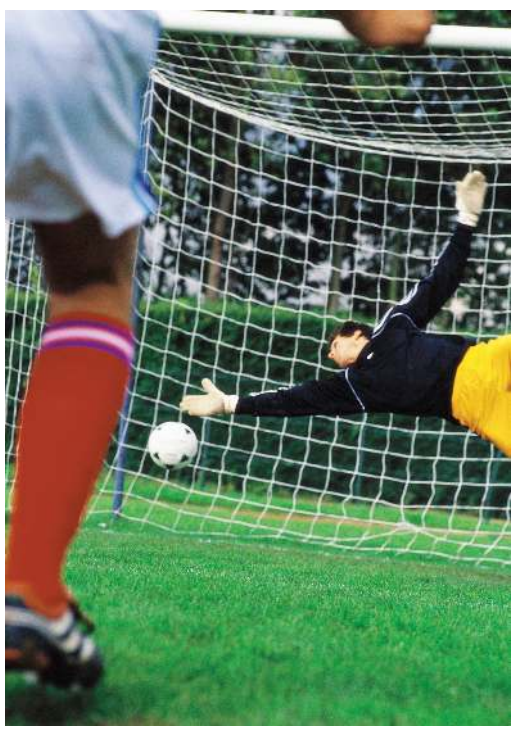
b **Number of goals**

0	9
1	2 2 5
2	3 5
3	0

2|3 = 23

c





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APPLICATIONS

4 A doctor is researching anaemia and he has recorded the following systolic pressure values from a group of patients' blood pressure readings.
115, 115, 107, 128, 122, 113, 108, 130, 115, 170, 120, 106

a Calculate the mean number of systolic pressure values.

b Identify the median number of systolic pressure values.

c Determine the mode.

Hint Mean is average; median is the middle; mode is the most frequent.

5 Markus wants to purchase a new laptop. The following are the amounts recorded from a selection of options.
$700, $900, $200, $1200, $400, $300, $900, $1000, $700, $850, $2600

a Calculate the mean price for a laptop.

b Identify the median price for a laptop.

c Determine the mode.

6 Micaela is a hockey coach and she is preparing for a new season. The following dataset is her team's scores for the previous season.
0, 4, 1, 1, 2, 1, 2, 1, 5, 4, 3, 5, 2, 1, 0, 2, 1, 0, 3, 2, 3, 3, 2, 1, 0, 2, 1, 2, 4, 0

a Calculate the mean score.

b Identify the median score.

c Determine the mode.

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7 For the data in the following stem-and-leaf plots, identify:

i the mean
ii the mode
iii the median

Hint A stem-and-leaf plot splits each value into a stem (the first digit/s) and a leaf (usually the last digit). For example, 22|5 is 225.

a

0	8 9
1	2 2 3 5 7
2	4 6 9

2|4 = 24

b

20	0 0 1 3
21	2 3 4 5 5
22	4 5 5 5 5 8 9
23	6 8
24	0 0 3 4

22|5 = 225

c

4	0 1 3 8
5	2 2 2 3 7 9
6	6 7
7	0 1 3 5

5|2 = 52

d

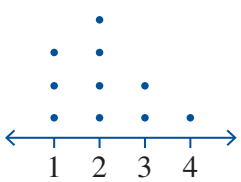
10	0 0 0 0 1 7
11	1 2 2 3 4 5 9
12	1 1 2 2 4 6 8
13	0 0 7

13|7 = 137

8 For the data in the following dot plots, identify:

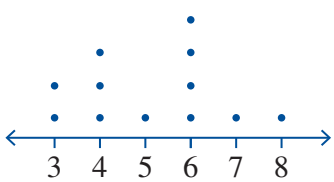
i the mean **ii** the mode **iii** the median

a



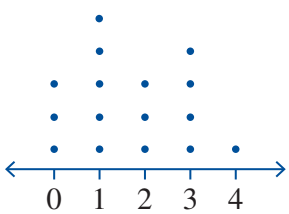
Number of goals

b



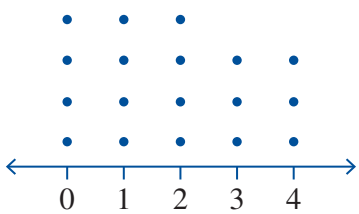
Number of strokes

c



Number of pets

d



Number of musical instruments

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8B Investigating the suitability of measures of central tendency

LEARNING GOALS

- Investigate the suitability of measures of central tendency in various real-world contexts
- Investigate real-world examples from the media, illustrating inappropriate uses of measures of central tendency

Why is it essential to determine the suitability of measures of central tendency?

Deciding which measure of central tendency to use in real-world contexts is important when trying to understand the distribution of a dataset. Measures of central tendency are the most typical representation of the collected data and have their own purposes.

Deciding which measure of central tendency to use can be important when interpreting datasets.

WHAT YOU NEED TO KNOW

- The mode reveals the most frequent value in the dataset.
  - Advantages of using the mode:
    - Simple to understand (the value that occurs most often).
    - Not affected by extreme values **(outliers)**.
  - Disadvantages of using the mode:
    - Not based on all the values in a dataset.
    - Sometimes the data has more than one mode, and sometimes there is no mode at all.
- The mean is the average of the dataset.
  - Advantages of using the mean:
    - All the data is taken into account.
    - Easy to understand (the value you would have if all the data points were equal) and calculate.
  - Disadvantages of using the mean:
    - Outliers (extreme values) can distort the results.
    - If the data is in the form of percentages or ratios, it could be challenging to calculate the mean.

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- The median is the middle value of a dataset, when the dataset is sorted in order from the smallest value to the largest value.
  - Advantages of using the median:
    - Simple to understand (the data point in the middle, with an equal number of greater and lesser values) and easy to calculate.
    - Not affected by outliers.
  - Disadvantages of using the median:
    - The median is based only on the middle value of an ordered dataset and does not include values from the other data points at all.
    - Need to remember that, if there are an even number of data points, to take the average of the middle two.
  - For example: For the dataset 1, 2, 1, 2, 20 (outlier):

    The mean *including* the outlier

    $$\frac{1+2+1+2+20}{5} = 5.2$$

    The mean with the outlier is 5.2.

    The mean *not including* the outlier

    $$\frac{1+2+1+2}{4} = 1.5$$

    The mean without the outlier is 1.5.

    An outlier does not affect the median so much:

    The median *including* the outlier:

    1 1 (2) 2 20

    The median with the outlier is 2.

    The median *not including* the outlier:

    1 (1 2) 2

    The median without the outlier is 1.5.

- The **misuse** of measures of central tendency may arise from the following issues:
  - The mean may be misleading when there are outliers or extreme values.
  - The median does not consider the exact value of each observation and is capable of misleading when all information is required.
  - The mode should not be used if the data is continuous, such as the heights of people in a basketball team, as it is not likely to have any one value that is more frequent than any other, or if the most frequent value is far away from the rest of the data.
- To sum up, measures of central tendency can be very useful for understanding a dataset, but each measure has the potential to give a misleading representation of the data if used in the wrong context. That's why it is important to consider the characteristics of the data before choosing an appropriate measure.
  - Is the data continuous? If so, be wary of using the mode.
  - Does the data have extremely large or small values, compared to the rest of the data? If so, be wary of using the mean.
  - Is it possible to logically order the data? If not, don't use the median.

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Example 4 Investigating the suitability of measures of central tendency in various real-world contexts

Henry's goal is to achieve 50% correct on his weekly maths test across 10 weeks. He has recorded his grades out of 20 questions for the past 8 weeks. He has two weeks to go.

4, 6, 9, 11, 8, 10, 12, 6

a Calculate the mean grade.

b Identify the median grade.

c Determine the mode grade.

d Which is a better measure to assist with Henry's preparation to reach his goal? Give a reason.

WORKING	THINKING
a $\bar{x} = \dfrac{4+6+9+11+8+10+12+6}{8}$ $\bar{x} = 8.25$ The mean of grades is 8.25.	◂···· Mean $= \bar{x} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$ Add all the values and divide by the number of values (with this dataset, there are 8).
b 4, 6, 6, (8), (9), 10, 11, 12	◂···· For the median, the data must first be ordered from smallest to largest. The median is the middle value.
The number of scores is 8. So $n = 8$. The median score will be $\dfrac{8+1}{2} = \dfrac{9}{2} = 4.5$th score. The median will lie between the 4th and 5th scores. The 4th score = 8 The 5th score = 9	◂···· Determine the number of scores. This is an even number of scores, so we need to average the middle two scores.
Median $= (8 + 9) \div 2$ $= 8.5$	◂···· Add the scores and divide by 2.
The median of the grades is 8.5.	◂···· Communicate your answer in a sentence.
c The mode of the grades is 6.	◂···· The mode is the most common value.

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d The better measure would be mean, as it is the average score of all grades.
The median score does not take into account all grades recorded. The mode has nothing to do with how well the other grades have contributed to his results.

◄···· Consider the pros and cons of each measure of tendency, and determine the purpose of the real-world central measure.

Worksheet 8B Investigating a real-world example from the media illustrating inappropriate uses of measures of central tendency. See the Interactive Textbook for this activity.

Exercise 8B

FUNDAMENTALS

Example 4

1 Describe the advantages and disadvantages of each measure of central tendency.

Measure of central tendency	Advantages	Disadvantages
Mean		
Median		
Mode		

APPLICATIONS

2 Verity is a shoe designer and she is preparing for a new pop-up shop. Verity has recorded the sizes of shoes purchased over the past week.
12, 7, 7, 7, 7, 8, 6, 7, 7, 10, 10, 5, 9, 9, 10, 9, 11

a Calculate the mean shoe size.
b Identify the median shoe size.
c Determine the mode shoe size.
d Which is a better measure to assist with Verity's preparation? Give a reason.

Hint Mean is average; median is the middle; mode is the most frequent.

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3 Noah is trying to improve his test scores and has been recording his results out of 40 questions in preparation for passing the term.
26, 13, 8, 2, 30, 17

Hint Give examples from your findings in your reasons.

a Calculate the mean result.
b Identify the median result.
c Determine the mode result.
d Clarify which measure would best assist Noah with his preparation to improve his test scores. Give a reason.

4 The following back-to-back stem-and-leaf plot displays the homework results over the term of two students.

a For each student, identify:
i the mean **ii** the mode **iii** the median

b Compare the performance of the two students, using the measures of central tendency.

Hint Include examples of central tendency in your comparison.

Homework scores (%)

Jamie		Scarlett
8 5	6	7
6 4	7	5 5
5 5 2	8	8 9 9
8 6 5	9	5 5 5 9

7|5 = 75%

5 Mitchell is a town planner and he needed to determine the number of children in families from two local high schools. He chose 300 students from the two schools and asked how many siblings each student had. Determine the median number of siblings for all the students surveyed.

Hint Think of a way you could do this without writing out each number.

Number of siblings	Town High School	Rangeville High School
0	118	142
1	82	108
2	59	31
3	31	9
4	10	10

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6 Ben and his father were trying to work out a fair way to determine how much per month Ben should have for pocket money. Ben surveyed 13 of his friends (one being the daughter of an extremely wealthy family) and wrote out a list of how much pocket money they received each month. Ben's father says he should go by the median, Ben's younger sister says he should go by the mode and Ben says the mean is the fairest solution.

Friend	Monthly pocket money ($)
1	140
2	50
3	40
4	70
5	2350
6	140
7	120
8	50
9	100
10	50
11	75
12	110
13	50

a Calculate the mean.
b Identify the median.
c Determine the mode.
d Why do you think each person suggested their particular measure of central tendency? Give a reason explaining which is the fairest in this situation.

7 Kenzi, a maths teacher, needs to choose who to give the maths award to in her class. She records all the results for her top three students.

Student A	33	45	23	24	47	48	46	25	45
Student B	30	43	20	26	50	50	49	30	47
Student C	30	42	20	23	48	50	48	32	43

Determine which student should win the maths award. Justify your answer, using the mean, median and modes for the students and explaining which of these is the fairest for her class.

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8C Calculating and interpreting measures of spread and outliers

LEARNING GOALS

- Calculate and interpret statistical measures of spread using the range
- Calculate and interpret statistical measures of spread using interquartile range
- Calculate and interpret statistical measures of spread using standard deviation
- Investigate real-world examples from the media, illustrating inappropriate uses of measures of spread
- Identify an outlier using a formula
- Investigate the effect of outliers on the mean
- Investigate the effect of outliers on the median

Why is it essential to understand how to calculate and interpret statistical measures of spread and outliers?

Being able to describe the spread of data using everyday language goes some way to understanding a data distribution, but the next step towards working mathematically is to *measure* the spread. When the spread of values in a dataset is large, then the mean is not as effective a representation of the data as when the spread of data is small. Understanding how to calculate and interpret the spread, using range, interquartile range and standard deviation, allows us to analyse data with a greater degree of precision.

An outlier lies beyond the other observations.

The same goes for outliers. Intuitively, an outlier is a score or observation that lies beyond the 'obvious edge' of a dataset, but in this section, we will learn to determine outliers (and whether there are any in the first place) using a precise formula.

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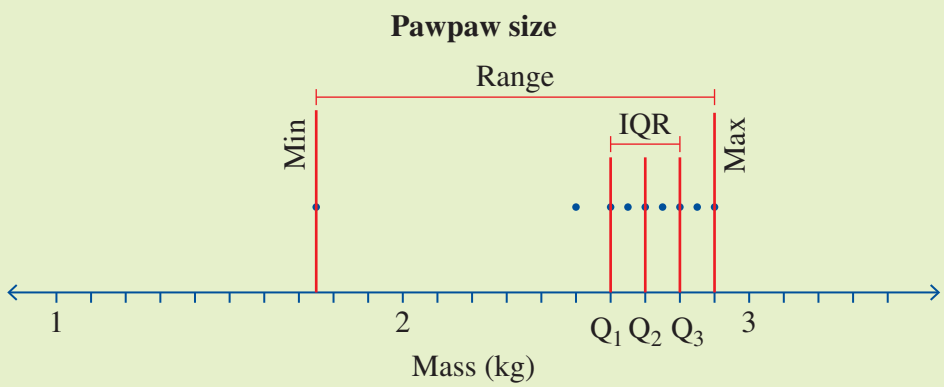
WHAT YOU NEED TO KNOW

- The range, interquartile range and standard deviation are useful measures of spread to compare datasets.
- The simplest measurement of spread is the **range**.
  - The range represents the limits of a dataset and only gives basic details about the spread.
  - It explains how wide or narrow the spread is, especially when compared to another dataset.
  - The range is found by subtracting the minimum value in a dataset from the maximum value.
  - The range can be distorted by extreme values (outliers), which are values that are very large or very small compared to the rest of the dataset.
- The **interquartile range** (IQR) is the range of the central half of the data, either side of the median. It is calculated by subtracting the lower quartile (Q_1) from the upper quartile (Q_3): $IQR = Q_3 - Q_1$.
  - If the IQR is small compared to the range, the dataset likely contains outliers. In such cases, the IQR is often a better measure of spread as it excludes outliers.
  - If the IQR is half the value of the range or more, the data is likely to be evenly spread or tightly packed.
  - There may be good reasons for examining only the data within the IQR. For example, if you are studying how to improve the shelf life of pawpaws and customers do not want fruit that is too small or too large, you include only pawpaws from the IQR for size in the results of your study.



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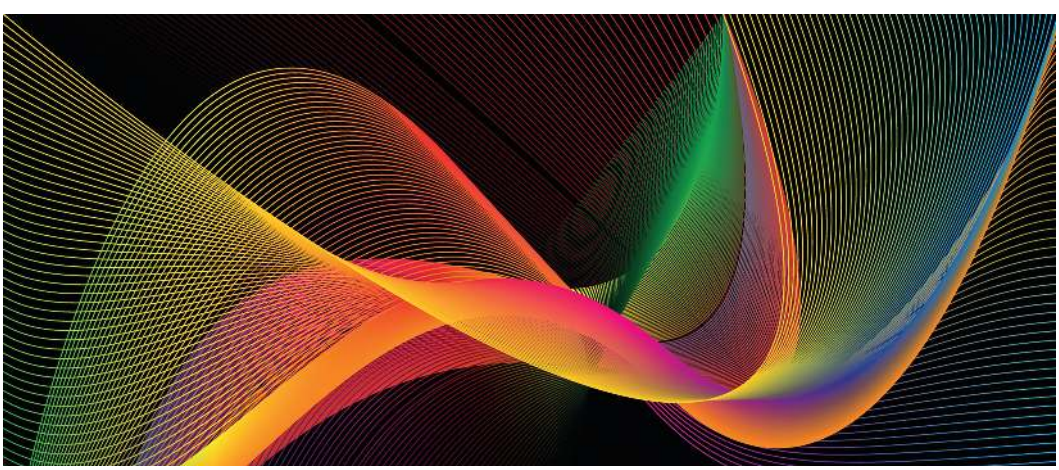
- The most important purpose of **standard deviation** from the mean is to understand how spread out a dataset is. It is a measure of the average distance of data away from the mean.
  - A low standard deviation means that most of the numbers are tightly packed around the mean.
  - A high standard deviation means that the numbers are spread out from the mean.
- The **variability** of a dataset is the amount by which data points differ from the mean and from each other, similar to the spread, and which can also be measured by the range, IQR and standard deviation.
- As we have learned previously, an outlier is an observation that appears to be inconsistent with the remainder of that set of data.
- Outliers can be determined mathematically using the following formula:

A data point is an outlier if it is either:
less than $(Q_1 - 1.5 \times \text{IQR})$ or
greater than $(Q_3 + 1.5 \times \text{IQR})$.

  - The mean can be affected by outliers; therefore, it should not be used when outliers are present.
  - An outlier may be due to an inconsistency in the measurement or it may indicate experimental error. They are, therefore, sometimes removed from the dataset, but this needs to be justified.
  - The mean is usually affected more by an outlier than the median, especially for small datasets.
- Misuse of measures of spread may arise in the following cases.
  - Standard deviation may be misleading when there are extremes in the data (extremely high or low values) as the average is skewed.
  - The range may be misleading to measure spread when there are outliers or extreme values.



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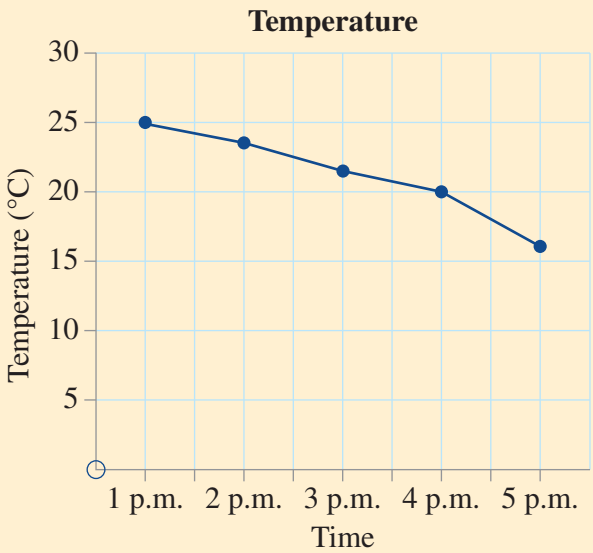
Example 5 Calculating and interpreting statistical measures of spread, using the range

Kari has measured some of the day's temperature.



a Identify the maximum and minimum temperatures recorded in the dataset.
b Calculate the range.
c Interpret the range of the dataset.

WORKING	THINKING
a The maximum temperature is 25°C. The minimum temperature is 16°C.	Identify the maximum (the highest) and minimum (the lowest) temperatures.
b 25° − 16° = 9° The range of the data is 9°C.	Range = Highest value – Lowest value
c The range of the data represents the limits of the dataset. Given the data was collected for four hours during the afternoon, it may not give an accurate measure of spread of the day's temperature range. However, it might be useful if compared with measurements taken during the same part of the day across several days, weeks or months.	Refer to the definition of range.

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Example 6 Calculating and interpreting statistical measures of spread, using interquartile range

The test results of ten students are 5, 7, 10, 5, 6, 7, 9, 4, 6, 9.

a Arrange the data in order from the smallest value to the largest value.

b Identify the median (Q_2), lower quartile (Q_1) and upper quartile (Q_3).

c Calculate the interquartile range.

d Interpret the interquartile range.

WORKING	THINKING
a 4 5 5 6 6 7 7 9 9 10	◄···· Put values in order from lowest to highest.
b 4 5 5 6 (6 7) 7 9 9 10	◄···· To determine the median of a distribution:
$Q_2 = \frac{6+7}{2}$	• The number of scores is 10, so $n = 10$.
$Q_2 = \frac{13}{2}$	• The median score will be $\frac{10+1}{2} = \frac{11}{2} = 5.5$th score.
$Q_2 = 6.5$	• The median will lie between the 5th and 6th scores.
(4 5 (5) 6 6) 7 7 9 9 10	The lower quartile is the middle number of the lower half. Place brackets around the lower half of the dataset.
$Q_1 = 5$	◄···· Identify the median of the lower half.
4 5 5 6 6 (7 7 (9) 9 10)	The upper quartile is the middle number of the upper half. Place brackets around the upper half of the dataset.
$Q_3 = 9$	◄···· Identify the median of the upper half.
c $IQR = Q_3 - Q_1$ $IQR = 9 - 5$ $IQR = 4$	◄···· The interquartile range is the difference between Q_3 and Q_1.
d The IQR is the range of the central half of the data. The IQR value of 4 compared to the range of 6 indicates that there are no outliers in the data, and it is fairly evenly spread across the range.	◄···· Refer to the definition and usefulness of the interquartile range.

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Example 7 Calculating and interpreting statistical measures of spread, using standard deviation, without using technology

*Note: In the assessment for this course, you will not be asked to calculate standard deviation without using technology. It is done here to help you learn what the standard deviation is. It is suggested that you work through part **a** in order to see what the technology does when it calculates standard deviation.*

A dog breeder records the number of pups in each of their dogs' litters:
2, 4, 4, 4, 5, 5, 7, 9.

a Calculate the standard deviation, without using technology.

b Interpret the standard deviation of the dataset.

WORKING	THINKING
a Step one: The mean for the eight litters is $\bar{x} = \frac{2+4+4+4+5+5+7+9}{8}$ $\bar{x} = 5$	◂···· *Step one*: Calculate the mean.
Step two: $2 - 5 = -3$ $\quad 5 - 5 = 0$ $4 - 5 = -1$ $\quad 5 - 5 = 0$ $4 - 5 = -1$ $\quad 7 - 5 = 2$ $4 - 5 = -1$ $\quad 9 - 5 = 4$	◂···· *Step two*: Find the difference of each number from the mean.

... *Continued*

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Step three:

$(-3)^2 = 9$ $\quad 0^2 = 0$

$(-1)^2 = 1$ $\quad 0^2 = 0$

$(-1)^2 = 1$ $\quad 2^2 = 4$

$(-1)^2 = 1$ $\quad 4^2 = 16$

◄···· *Step three*: Square the difference of each number from the mean. This makes all of them positive so they don't cancel each other out. It also magnifies larger differences and minimises smaller differences.

Step four:

$$\frac{9+1+1+1+0+0+4+16}{8} = 4$$

◄···· *Step four*: Calculate the mean of the squared differences.

Step five:

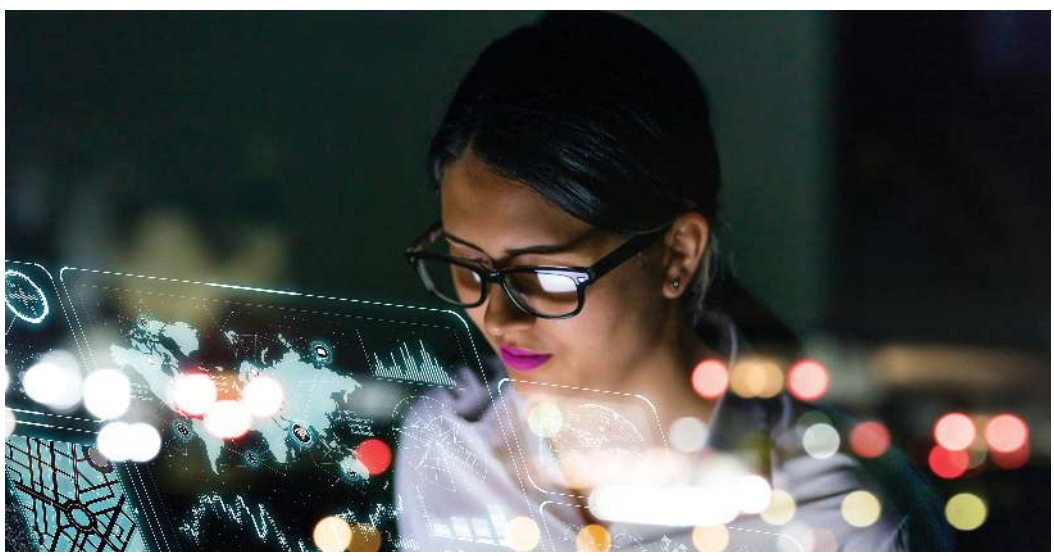
$$\sqrt{4} = 2$$

The standard deviation from the mean for the litter of puppies is 2.

◄···· *Step five*: Finally, calculate the square root of the answer. This counteracts the squaring from step three and allows the standard deviation to be expressed in the original units.

b The standard deviation is 2 from the mean of 5. This indicates neither a very narrow nor a very wide dispersion of the data. The standard deviation would be most useful for comparing to another dataset.

◄···· Refer to the definition of standard deviation.



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Example 8 Calculating and interpreting statistical measures of spread with technology

Sally recorded the heights of her friends in centimetres:
160, 171, 158, 167, 163
Calculate the mean, standard deviation, range and IQR using a calculator.
The instructions given below are for a Casio fx.

WORKING	THINKING
	Clear any data already in the calculator. Press Mode [setup] > 2 > 1. A table appears. Type individual data values followed by the = key to enter the data, then click AC[off]. For these statistics press these keys:
The mean is 163.8. Standard deviation for the heights of Sally's friends is 4.71 from the mean.	Mean, $\bar{x}$: Shift > 1 > 4 > 2 > enter Standard deviation, SX: Shift > 1 > 4 > 4 > enter Minimum value, minX: Shift > 1 > 5 > 1 > enter Maximum value, maxX: Shift > 1 > 5 > 2 > enter Median, med: Shift > 1 > 5 > 4 > enter Quartile 1, Q1: Shift > 1 > 5 > 3 > enter Quartile 3, Q3: Shift > 1 > 5 > 5 > enter
Range = 171 − 158 = 13	For the range, subtract the minimum value from the maximum value.
IQR = 169 − 159 = 10	For the IQR, subtract Q3 from Q1.

Calculator activity 8C: Calculating statistical measures of spread with scientific calculators.

Spreadsheet activity 8C: Calculating statistical measures of spread using a spreadsheet:
These technology activities are in the Interactive Textbook.

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Example 9 Identifying an outlier using a formula and demonstrating the effect on the mean and median

The following is a dataset representing the number of wedge-tailed eagles spotted each day on a property.

5, 1, 2, 17, 3, 1, 4, 5, 3, 4, 3

($Q_1 = 2$; $Q_2 = 3$; $Q_3 = 5$; IQR = 3)

a Determine whether any numbers are outliers, using the given formula.

> A data point is an outlier if it is either:
> less than $(Q_1 - 1.5 \times \text{IQR})$ or
> greater than $(Q_3 + 1.5 \times \text{IQR})$.

b Give a possible reason for the outlier.

c Calculate the mean:

i with the outlier

ii without the outlier

d Compare your answers to **c i** and **ii** and consider if the outlier should be included or removed.

e Calculate the median:

i with the outlier

ii without the outlier

f Compare your answers to **e i** and **ii** and consider if the outlier should be included or removed.

WORKING	THINKING
a $Q_1 - 1.5 \times \text{IQR} = 2 - 1.5 \times 3$ $= 2 - 4.5$ $= -2.5$	A data point is an outlier if it is either: • less than $Q_1 - 1.5 \times \text{IQR}$ or • greater than $Q_3 + 1.5 \times \text{IQR}$
$Q_3 + 1.5 \times \text{IQR} = 5 + 1.5 \times 3$ $= 5 + 4.5$ $= 9.5$ 17 is an outlier.	There is no number in the dataset less than –2.5, but the data point 17 is greater than 9.5.
b There may have been an abundance of prey on the property or a large dead animal.	*What possible reason could there be for this larger number of wedge-tailed eagles?*

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c **i** with the outlier ◄··· Calculate the mean, including 17.

$$\frac{5+1+2+17+3+1+4+5+3+4+3}{11}$$

The mean, with the outlier = 4.36.

ii without the outlier ◄··· Calculate the mean, removing 17 from the dataset.

$$\frac{5+1+2+3+1+4+5+3+4+3}{10}$$

The mean, without the outlier = 3.1.

d $4.36 - 3.1 = 1.26$ ◄··· Compare the answers with/without the outlier and determine whether the data is affected.

With the difference between the two means being larger than some of the actual observations, it is reasonable to determine that the outlier greatly affects the data and should be removed.

e **i** with the outlier ◄··· Calculate the median, including 17.

1, 1, 2, 3, 3, (3,) 4, 4, 5, 5, 17

The median, with the outlier = 3.

ii without the outlier ◄··· Calculate the median, removing 17 from the dataset.

1, 1, 2, 3, (3, 3,) 4, 4, 5, 5

The median, without the outlier = 3.

f Both datasets have a median of 3. ◄··· Compare the answers with/without the outlier and determine whether the data is affected.

Therefore, the outlier does not affect the median and should be included in the dataset.

Worksheet 8C Investigating a real-world example from the media illustrating inappropriate uses of measures of spread. See the Interactive Textbook for this activity.

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Exercise 8C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a The simplest measurement of spread is the _____.

b The range is the _____ between the _____ value and the _____ value.

c The _____ _____ (IQR) is calculated using the central section of data, either side of the median.

d Standard deviation is a _____ of the _____ distance of data away from the mean.

e An outlier is a score that lies _____ the obvious edge of a dataset.

f When a small set of data has an outlier, the _____ is usually affected more by the outlier than the _____.

2 For the dataset shown, calculate the range, IQR and standard deviation.
2, 11, 5, 15, 17, 12, 7, 2, 11, 3

3 Complete the questions below for the following dataset.
15, 17, 14, 22, 0, 25, 13, 19, 16, 20, 15, 14, 17, 11, 20

a Calculate Q_1, Q_3 and the IQR.

b Calculate the value of the formula $Q_1 - 1.5 \times \text{IQR}$, and determine if any data value in the dataset is less than this.

c Calculate the value of the formula $Q_3 + 1.5 \times \text{IQR}$, and determine if any data value in the dataset is greater than this.

d State whether there are any outliers in the data.

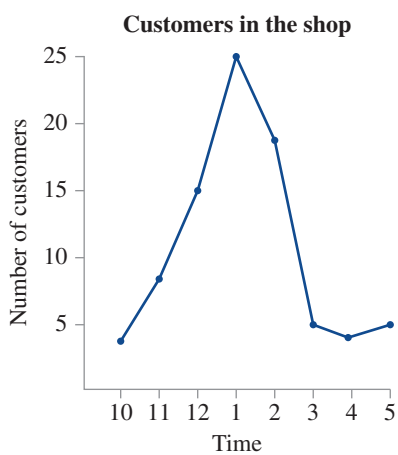
APPLICATIONS

Example 5

4 Ken owns a shop and is deciding whether he needs to hire an assistant. He records the number of customers in his shop on an hourly basis.

a Identify the maximum and minimum values.

b Calculate the range of the distribution.



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Example 6

5 A maths teacher has recorded some of her students' results.

Results	%
Amy	85
Fari	40
Grace	37
Sarah	80
Blake	75
Craig	100
Skye	20

a Arrange the data in order from lowest to highest.
b Identify the median (Q_2), lower quartile (Q_1) and upper quartile (Q_3).
c Calculate the interquartile range of the spread.

Example 7

6 Rose was trying to explain to her friends that they need to eat more vegetables. She recorded how many different types of vegetables were in each of her friends' refrigerators.
2 3 5 6 9
Without the use of technology:
a calculate the mean
b calculate the standard deviation

Example 8

7 Cino and his friends have recorded their current weights (kg).
90 120 72 78 85
Using a calculator:
a calculate the mean
b calculate the standard deviation
c identify the minimum, Q_1, median, Q_3 and the maximum values
d calculate the range
e calculate the IQR

8 Sheree has recorded the heights (in cm) of some children in her class.
150 140 130 102 105 163 110 152 145 143 147 139 140
Using a calculator:
a calculate the mean
b calculate the standard deviation
c identify the minimum, Q_1, median, Q_3 and the maximum values
d calculate the range
e calculate the IQR

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9 Rodney records the weekly weather temperatures for the local newspaper. Calculate the mean and standard deviation, using a spreadsheet.
25°C 27°C 32°C 31°C 29°C 32°C 30°C 29°C

10 A couple are recording home loan rates in preparation for buying a home. Calculate the mean and standard deviation, using a spreadsheet.
2.5% 3% 4.5% 3.5% 2.7% 3.4% 2.7% 4% 3.7% 2.8%

Example 9

11 Cooper has been practising shooting baskets every day in preparation for his basketball tournament. He recorded the number of successful shots he made each day from 20 shots.
13 20 17 2 17 20 15

a Find the mean of the successful basketball shots.
b Identify any outliers, using the formula.
c Explain whether the outliers affect the mean.
d Determine whether outliers need to be removed.

12 Faith is researching the purchase of a new car. Her top seven car prices are:
$45 000 $34 000 $40 000 $120 000 $52 000 $27 000 $30 000

a Find the median of the car prices.
b Identify any outliers, using the formula.
c Explain whether the outliers affect the median.
d Determine whether outliers need to be removed.

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8D Determining quartiles, deciles and percentiles

LEARNING GOALS

- Calculate quartiles from a dataset
- Interpret quartiles from a graph
- Interpret deciles from a graph
- Interpret percentiles from a graph

Why is understanding quartiles, deciles and percentiles essential?

The mean and median both describe the 'centre' of a distribution; however, there are other parts of the distribution that could be of more interest. To identify these, data can be divided into smaller equal parts, such as quartiles, deciles and percentiles. These can be used to break down the data and describe the information contained in the data more clearly.

When data is put into specific parts, such as quartiles, deciles and percentiles, the information begins to make more sense and you can see the overall 'picture' more clearly.

WHAT YOU NEED TO KNOW

- Parts or groups in an ordered dataset are divided by values that collectively are known as **quantiles**. ('Ordered' means the dataset is sorted in order from the smallest value to the largest value.) Quantiles are not groups; they are the boundaries between the groups.
  - Quantiles that divide ordered data into four equal parts are called **quartiles** (like 'quarters'). There are three quartiles, Q_1, Q_2 and Q_3, which are shown on the diagram on the next page.
  - Quantiles that divide ordered data into 10 equal parts are called **deciles** (think of 'decimal', which relates to tenths). There are nine deciles, as shown on the diagram on the next page. (They can be called D_1, D_2, D_3 etc.)
  - Quantiles that divide ordered data into 100 equal parts are called **percentiles** (think of 'percentage', which relates to hundredths). There are 99 percentiles, as shown on the diagram on the next page. (They can be called P_1, P_2, P_3 etc.)

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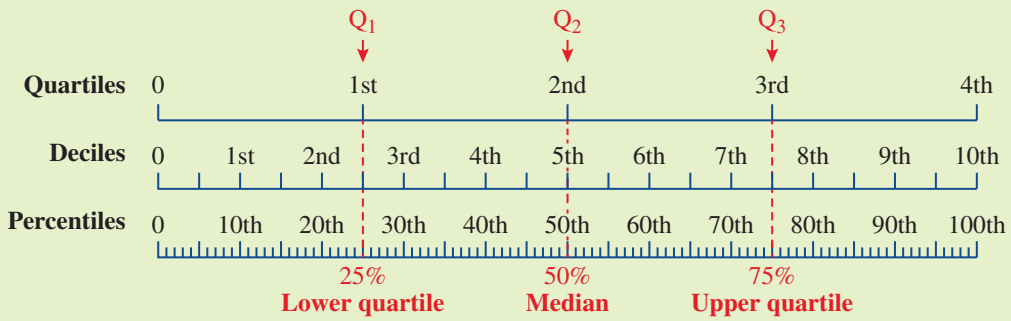
- Quartiles, deciles and percentiles can be compared and converted with this diagram. The first quartile lies between the 2nd and 3rd decile, and it is the same as the 25th percentile. The first decile is the same as the 10th percentile, and so on.



- There are the same number of data points between the quartiles. So, from 0 to Q_1 there are the same number of data points as there are between Q_1 and Q_2, and so on. Remember that quantiles divide the number of data points (not their values) into equal groups. The same principles apply to the deciles and percentiles: there are an equal number of data points between each one.
- Q_2 is the **median** of the whole dataset, the data point that splits the ordered data into two equally sized groups. It is the same as the 5th decile and the 50th percentile. Then, Q_1 is the median of the lower half of the data, and Q_3 is the median of the upper half of the data. Q_1 is also called the lower quartile, and Q_3 is also called the upper quartile. Q_1 is the same as the 25th percentile, and Q_3 is the same as the 75th percentile.
- Cumulative frequency graphs are often used with quartiles, deciles and percentiles. **Cumulative frequency** is the running total of the **frequency distribution** of the dataset, which is shown in a **frequency table**. The frequency table for a dataset of scores of 1 to 6 is shown on the following page. (These scores could be any kind of data, such as throws of a single die in a board game.) Next to the frequency table is a table showing how we could add up the running totals of the frequencies in a cumulative frequency column.

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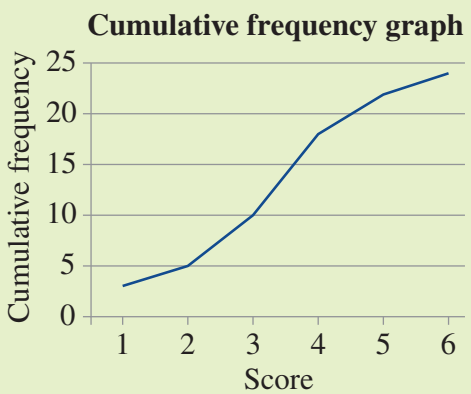
Score	Frequency
1	3
2	2
3	5
4	8
5	4
6	2
Total	24

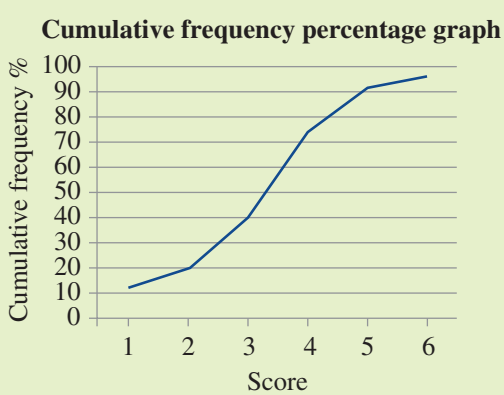
Cumulative frequency	Cumulative frequency %
3	= 3/24 × 100 = 12.5%
3 + 2 = 5	20.8%
5 + 5 = 10	41.7%
10 + 8 = 18	75.0%
18 + 4 = 22	91.7%
22 + 2 = 24	100.0%

- The cumulative frequencies can also be calculated as **cumulative frequency percentages** of the final total (which is 24 in this case). This is shown in the fourth column.
- The frequency table shows that the dataset contains 24 data points (the total of the frequencies and the last value of the cumulative frequency). As this is an even number, the median is the average of the two middle values: the 12th and 13th.

■ Graphs of cumulative frequency and cumulative frequency percentage:

- The cumulative frequency graph is represented by the cumulative frequency on the vertical axis and scores on the horizontal axis (shown in graph on left).
- If you plot the cumulative frequency percentage against the score, you get the same graph, but now the vertical axis is in percentages (shown in graph on right).





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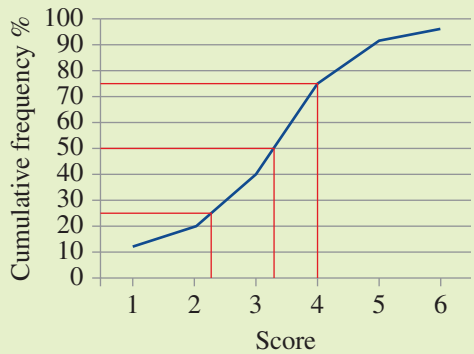
- A very useful characteristic of a cumulative frequency graph is that the quartiles can be easily marked on both axes, as shown below.

Cumulative frequency graph

For the cumulative frequency graph, the maximum value is 24. On the vertical axis, Q_1 is a quarter of the maximum value (6), Q_2 is half the maximum value (12) and Q_3 is three-quarters the maximum value (18).



Cumulative frequency percentage graph

It is even easier to mark Q_1, Q_2 and Q_3 on the cumulative frequency percentage graph. On the vertical axis, Q_1 is at 25% on the vertical axis, Q_2 is at 50% and Q_3 is at 75%.

  - Vertical lines can now be drawn from the intersection of each quartile with the graph, down to the horizontal axis, for the scores. This gives the Q_1, Q_2 and Q_3 of the scores, i.e. $Q_1 \approx 2$, $Q_2 \approx 3$, $Q_3 \approx 4$ on both graphs.
  - The same method can be applied to finding deciles and percentiles of the scores.
- Interpretation of quartiles, deciles and percentiles mainly involves determining where a particular value lies in relation to them and the percentage of scores that are above or below a particular quartile, decile or percentile. For example:
  - 75% of scores are above Q_1, and 25% are below it.
  - 40% of scores are above the sixth decile, and 60% are below it.
  - 70% of scores are above the thirtieth percentile and 30% are below it.
- Further interpretation depends on the subject of the data, and whether higher or lower scores are 'better'. For example, if the subject is 'marks in a test', it is better to be in the 80th percentile than the 20th, but if the data is 'the amount of time it takes to download 1 gigabyte', it is better to be in the 20th percentile than the 80th.

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Example 10 Calculating quartiles from a dataset

Consider this dataset.
9, 10, 7, 7, 8, 6, 12, 28, 6

a Identify the median (2nd quartile: Q_2).
b Determine the lower quartile (1st quartile: Q_1).
c Determine the upper quartile (3rd quartile: Q_3).
d Interpret what the quartiles mean, in relation to the fraction and percentage of scores that lie above and below each one.

WORKING	THINKING
a 6, 6, 7, 7, (8,) 9, 10, 12, 28 The number of scores is 9. The middle value is the 5th, which is 8.	The data must first be ordered from smallest to largest.
The median (Q_2) is 8.	The median is the middle value.
b (6, (6, 7,) 7,) 8, 9, 10, 12, 28 Average of 6 and 7 = $\frac{6+7}{2} = 6.5$	The lower quartile is the middle number of the lower half of the data. Place brackets around the lower half of the dataset, excluding the median, and identify its middle data point or points (ringed). There are an even number of scores, so the middle value is the average of the two middle scores (ringed).
The lower quartile, Q_1, is 6.5.	This score is Q_1.
c 6, 6, 7, 7, 8, (9, (10, 12,) 28) Average of 10 and 12 = $\frac{10+12}{2} = 11$	The upper quartile is the middle number of the upper half. Place brackets around the upper half of the dataset, excluding the median, and identify its middle data point or points (ringed). There are an even number of scores, so the middle value is the average of the two middle scores (ringed).
The upper quartile, Q_3, is 11.	This score is Q_3.

... Continued

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d A quarter or 25% of the data has a value that is less than or equal to 6.5, and three-quarters or 75% of the data has a value that is more than or equal to 6.5.

◄···· One quarter is 25%, a half is 50%, three-quarters is 75%.
25% of the data has a value that is less than or equal to Q_1, and 75% of the data has a value that is more than or equal to Q_1.

Half or 50% of the data has a value that is less than or equal to 8, and half or 50% of the data has a value that is more than or equal to 8.

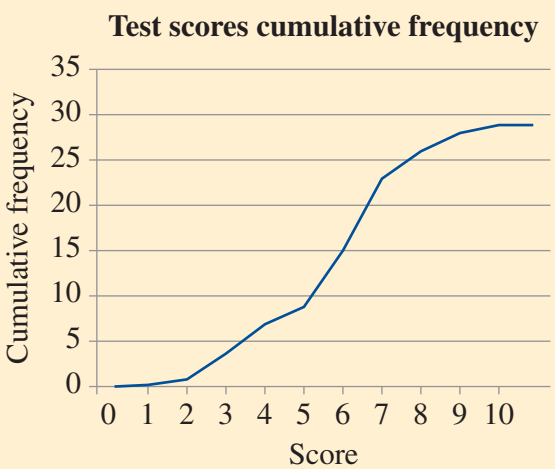
◄···· 50% of the data has a value that is less than or equal to Q_2, and 50% of the data has a value that is more than or equal to Q_2.

Three-quarters or 75% of the data has a value that is less than or equal to 11, and a quarter or 25% of the data has a value that is more than or equal to 11.

◄···· 75% of the data has a value that is less than or equal to Q_3, and 25% of the data has a value that is more than or equal to Q_3.

Example 11 Determining the quartiles from a cumulative frequency graph

The graph below shows the cumulative frequency of test scores (out of ten) in a class's test results. Determine the lower quartile, Q_1, median, Q_2 and upper quartile, Q_3, of the test scores.



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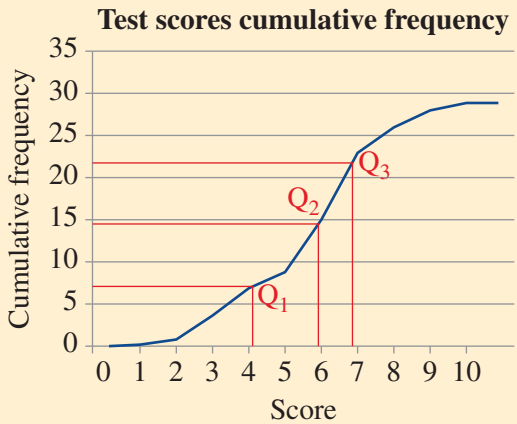
WORKING	THINKING
The cumulative frequency total is 29.	Write down the cumulative frequency total.
$Q_1 = 7.25$	Q_1 (one quarter of the total)
$Q_2 = 14.5$	Q_2 (half of the total)
$Q_3 = 21.75$	Q_3 (three quarters of the total)
	Draw horizontal lines at 7.25, 14.5, and 21.75 on the cumulative frequency axis, and label them Q_1, Q_2 and Q_3. Draw vertical lines from where these horizontal lines meet the graph to the horizontal axis.
Q_1 = score 4 Q_2 = score 6 Q_3 = score 7	Read off the values of the vertical lines Q_1, Q_2 and Q_3 on the horizontal axis, and round to the nearest whole number.
The lower quartile of the test scores is 4. The median of the test scores is 6. The upper quartile of the test scores is 7.	Communicate your answers in sentences.

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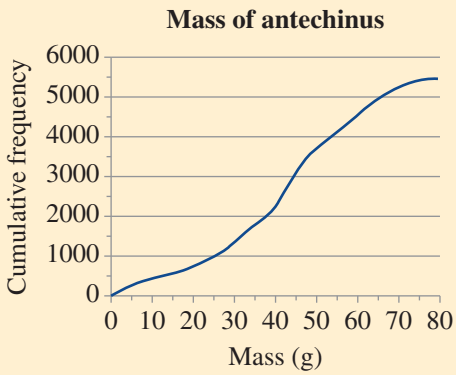
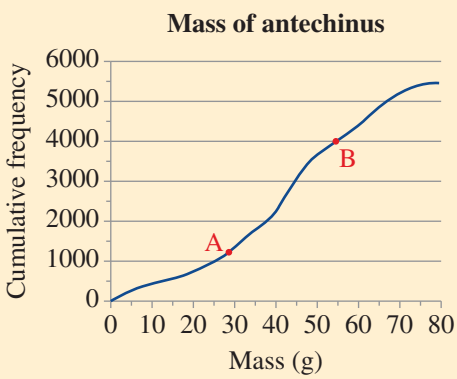
Example 12 Interpreting the deciles from a cumulative frequency

The cumulative frequency graph shows the results of a survey of the mass of marsupial mice (antechinus) in a population.
Two more antechinus specimens have been weighed, with mass of A (30 g) and B (55 g).



a Determine the deciles of mass between which these two specimens lie.

b Interpret what this means in terms of the proportion of the population that are lighter or heavier than the two new specimens.

WORKING	THINKING
a 	Mark the position on the graph for A (30 g) and B (55 g).
Maximum cumulative frequency is 5300.	Read the maximum cumulative frequency on the graph.
Position of A on vertical axis is 1250. $1250 \div 5300 = 0.24$	Determine the position of A on the vertical axis.
A lies between the 2–3 deciles.	Decide which deciles A lies between.
Position of B on the vertical axis is 4000. $4000 \div 5300 = 0.75$	Determine the position of B on the vertical axis.
B lies between the 7–8 deciles.	Decide which deciles B lies between.

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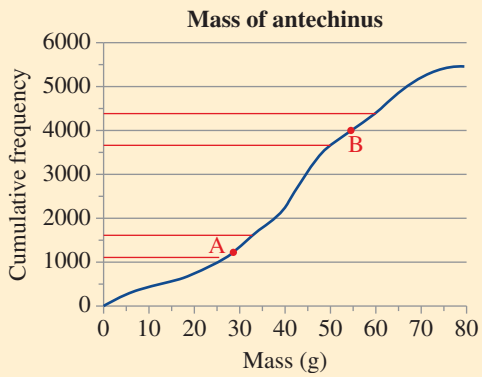
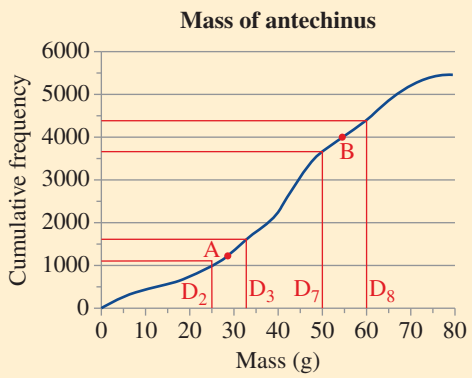
WORKING	THINKING
So we should mark D_2, D_3, D_7 and D_8. $D_2 = 2/10 \times 5300 = 1060$ $D_3 = 3/10 \times 5300 = 1590$ $D_7 = 7/10 \times 5300 = 3710$ $D_8 = 8/10 \times 5300 = 4240$	Calculate the values for D_2, D_3, D_7 and D_8 on the vertical axis.
	Draw the positions of D_2, D_3, D_7 and D_8 as horizontal lines against the vertical axis.
	Where the horizontal lines meet the graph, draw vertical lines down to the horizontal axis, and label as D_2, D_3, D_7 and D_8.
Specimen A lies between the second and third deciles, and specimen B lies between the seventh and eighth deciles.	Communicate your solution in words.
b Specimen A is heavier than two-tenths of the population and is lighter than seven-tenths of the population. Specimen B is heavier than seven-tenths of the population and is lighter than two-tenths of the population.	Two-tenths of the data lies below the second decile, and seven-tenths lies above the third decile (10 – 3 = 7). Data lower than a decile are lighter and data above a decile are heavier.

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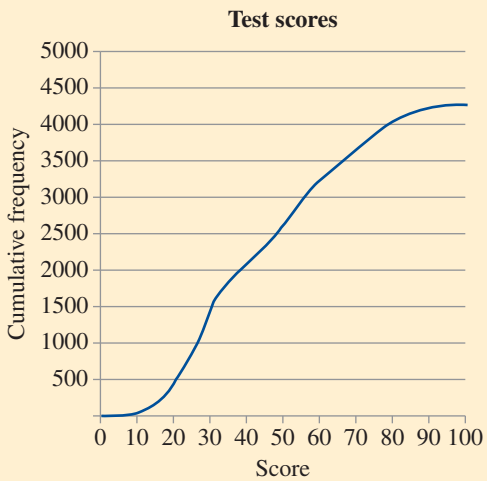
Example 13 Interpreting the percentiles from a cumulative frequency

The cumulative frequency graph on the right shows the results of scores in a test.
Student A had a score of 44 and Student B had a score of 80.

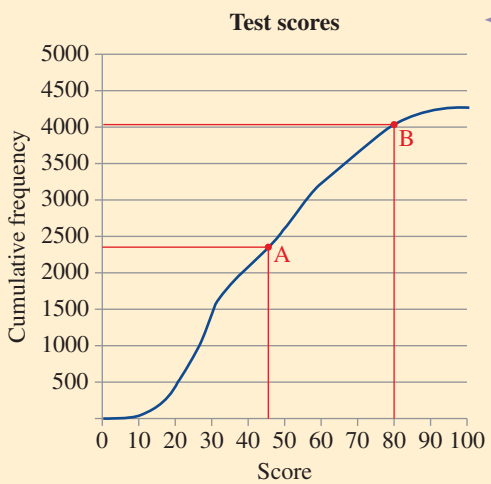
a Determine the percentiles of the scores which these two scores lie between.

b Interpret what this means in terms of the percentage of students who did better or worse on the test than students A and B.



WORKING

a



Maximum cumulative frequency is 4250.

A is about 2350 on the vertical axis.
B is about 4000 on the vertical axis.

$$A = \frac{2350}{4250} \times 100\% = 55.3\%$$

$$B = \frac{4000}{4250} \times 100\% = 94.1\%$$

THINKING

Mark the position on the graph for A at a score of 44.
Draw a vertical line from A to the horizontal axis.
Draw a horizontal line from A to the vertical axis.
Mark the position on the graph for B at a score of 80.
Draw a vertical line from B to the horizontal axis.
Draw a horizontal line from B to the vertical axis.

Read the maximum cumulative frequency on the graph.

Read the position of A and B on the vertical axis.

Calculate the vertical axis reading of A as a percentage of the maximum cumulative frequency.
Calculate the vertical axis reading of B as a percentage of the maximum cumulative frequency.

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WORKING	THINKING
Student A's score of 44 lies between the 55th and 56th percentiles. Student B's score of 80 lies between the 94th and 95th percentiles.	Percentiles are equivalent to percentage points, so 55% is the 55th percentile. 55.3% lies between the 55th and 56th percentiles, and 94.1% lies between the 94th and 95th percentiles.
b Student A scored better than 55% of students and worse than 44% of students. Student B scored better than 94% of students and worse than 5% of students.	55% of scores lie below the 55th percentile and 44% of scores lie above the 56th percentile (100 – 56 = 44). Scores lower than a percentile are worse and scores above a percentile are better.

Exercise 8D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Datasets can be divided into parts or groups by values that collectively are known as _____.

b Cumulative frequency is the _____ _____ of the frequency distribution of the dataset.

c Quartiles are _____ values that split the data into _____ equally sized groups.

d A quartile is a _____, not a group of numbers.

e Deciles of a distribution are the _____ values that split the data into _____ equally sized groups.

f Percentiles of a distribution are the _____ values that split the dataset into _____ equal parts.

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2 Copy and complete this table to show cumulative frequency, cumulative frequency percentage and the totals.

Score	Frequency	Cumulative frequency	Cumulative frequency %
1	1		
2	6		
3	25		
4	19		
5	31		
6	12		
Total			

3 Draw a graph of cumulative frequency against score for the data in question **2**.

4 On the graph in question **3**, draw horizontal and vertical lines to show Q_1, Q_2 and Q_3, and determine the approximate value of the score for each of them.

APPLICATIONS

Example 10

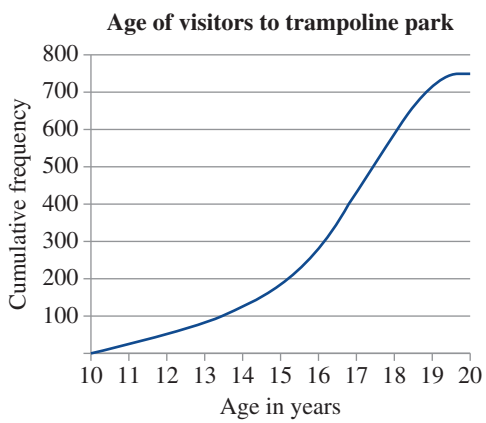
5 The dataset below is the number of matches won by teams in a league.
12, 10, 2, 4, 6, 7, 6, 9, 9, 8, 5

a Identify the median (2nd Quartile → Q_2).
b Determine the lower quartile (1st Quartile → Q_1).
c Determine the upper quartile (3rd Quartile → Q_3).
d Interpret what the quartiles mean in relation to the fraction of matches won that lie above and below each one.

Hint Determine the median, and then the medians for the upper and lower halves of the data.

Example 11

6 Caroline is a trampoline park operator. She has recorded the ages of the visitors to the park to plan new equipment for the following year. Caroline has made a cumulative frequency graph of the results.



a Determine the median of the ages (2nd Quartile → Q_2).
b Determine the lower quartile of the ages (1st Quartile → Q_1).
c Determine the upper quartile of the ages (3rd Quartile → Q_3).

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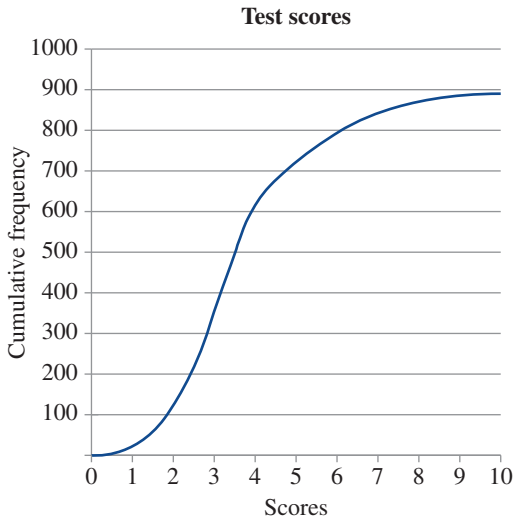
Example 12

7 This cumulative frequency graph shows the scores of students sitting a science test.
Andrew scored $4\frac{1}{2}$ and Beni scored 9.

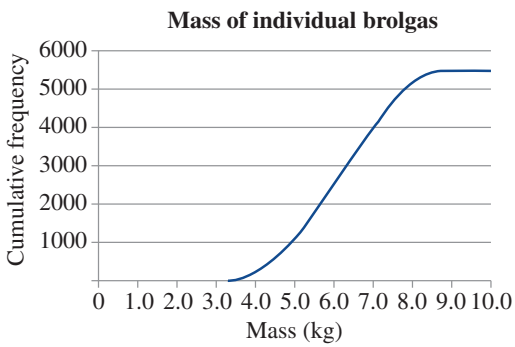
a Determine the deciles of the scores between which these students' scores lie.

b Interpret what this means in terms of the proportion of the students that did better or worse than Andrew and Beni.



Example 13

8 The mass of individual brolgas arriving at a site has been recorded, and the results are shown in the cumulative frequency graph. Two new brolgas visit. Brolga A weighs 4.2 kg and brolga B weighs 7.5 kg.



a Determine the percentiles of the masses between which the masses of brolga A and brolga B lie.

b Interpret what this means in terms of the percentage of the brolgas that are heavier or lighter than brolga A and brolga B.

9 Answer the following questions using the quartile information from the trampoline park graph in question **6**.

a 75% of the visitors are younger than Virat. Determine his age.

b Zara is younger than three-quarters of the visitors to the trampoline park. Determine how old she is.

c What percentage of the visitors to the park are between the ages of Zara and Virat?

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10 When receiving results for an exam, is it better to receive results with a high or low percentile? Explain your answer.

11 Troy is currently in his doctor's waiting rooms. He has been there for 32 minutes, which is the 85th percentile of waiting times. Is this good or bad? Explain your answer.

12 Katrina and Elliot are looking at purchasing a house. Their real estate agent has told them that the most expensive house they can afford is in the 25th percentile. Their research has shown that the 25th percentile of houses in the area in which they're looking at is currently $350 000. Does this mean they can afford 25% of the houses in their area or 75% of the houses?

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8E Describing the spread of data

LEARNING GOAL

- Use everyday language to describe the spread of data, including spread out, dispersed, tightly packed, clustered, gaps, more/less dense regions and outliers.

Why is it essential to be able to use everyday language to describe spread in data?

One very useful way to describe the graphs of datasets is to talk about the **spread** of the data. The spread describes the relationship with measures of central tendency and explains how the data is distributed. We can use the language of spread to describe how well the mean, for example, represents the data. Being able to describe the spread of data also allows us to compare and contrast sets of data which are related to each other.

Datasets are easier to understand when you are able to identify and describe the spread.

WHAT YOU NEED TO KNOW

- Data needs to be displayed to show spread. A good kind of graph to show **spread** is a histogram or a column graph.
- Two distributions differ in spread if the values of the data in one distribution are more spread out or dispersed than the values of the data in the other distribution.

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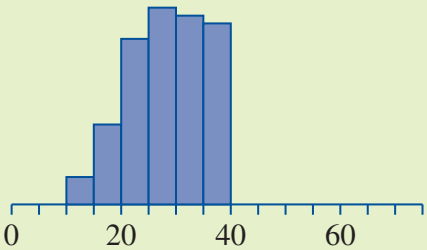
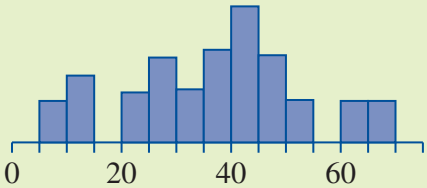
Histogram A

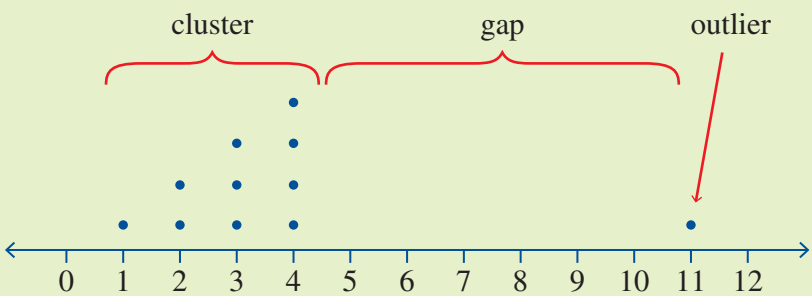


Histogram B



- *Words describing a distribution that is not spread out*
  - not spread out
  - tightly packed
  - clustered
  - more dense
  - narrowly (or tightly) distributed
  - narrowly dispersed
- *Words describing a distribution that is spread out*
  - spread out
  - loosely packed
  - dispersed
  - less dense
  - widely distributed
  - widely dispersed

- The spread also describes how a dataset is distributed around the mean or median. The location of the median can be estimated, and when the data is displayed, using histograms, the total area of the columns either side of the median will be equal. Looking at the histograms above, we would say 'Histogram A shows data that is tightly packed around the median, whereas histogram B shows data that is loosely packed around the median'.
- A **cluster** is produced when several data points lie in a group.
- A **gap** is a section that contains no data.
- An outlier has a value that is much greater than or much less than other data in the set.



- A large dataset may have a distribution that is mixed, with some parts or regions being tightly packed or clustered and other parts being loosely packed.

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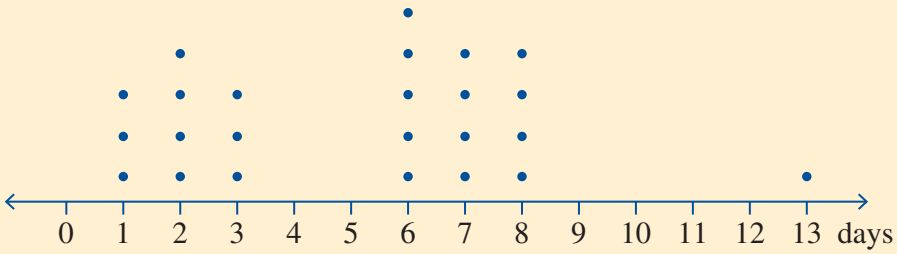
Example 14 Describing the spread in a dataset

An organic store is conducting research into the shelf life (in days) of a variety of berries.

Shelf life for a variety of berries



Expand on each description below in as much detail as you can.

a The distribution has an outlier.

b The distribution has gaps.

c The distribution has clusters.

WORKING	THINKING
a The distribution has an outlier at day 13, which is much greater than the other observations.	Does the distribution have an outlier? Is there a data point situated away from the other data points?
b There are two gaps in the dot plot, between 3 and 6 as well as between 8 and 13.	Does the distribution have a gap? Is there a section that contains no data?
c The distribution has clusters between 1 to 3 days and 6 to 8 days.	Is there a cluster of points?

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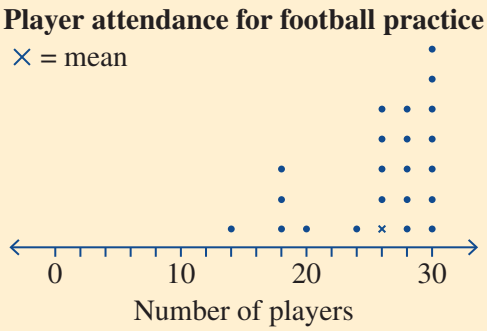
Example 15 Describing the spread from a dot plot

A coach is recording how many players are attending football practice over the season.



Describe the distribution, making use of the terms below where possible.

- spread out
- tightly packed
- loosely packed
- dispersed
- more/less dense regions
- clusters
- gaps
- outliers

WORKING	THINKING
The data is tightly packed around the mean. The dot plot has a small dispersion. There are two clusters. One between 18 and 20, and the other between 24 and 30. The data has two gaps. One between 14 and 18, the other between 20 and 24. 14 could be considered an outlier.	Go through each term and see if they correspond to the distribution. Where possible, include examples from the dataset.

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Exercise 8E

FUNDAMENTALS

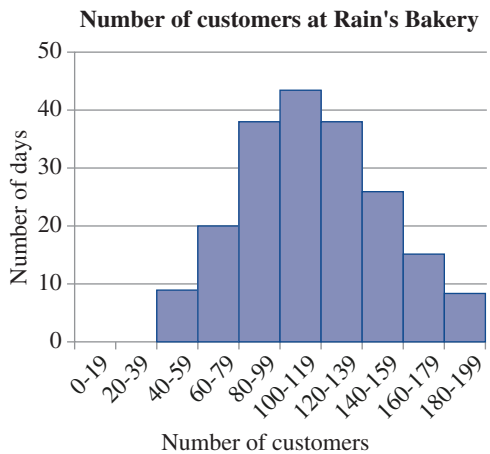
1 Determine the missing words in the following sentences.

- **a** Data needs to be _____ to show the spread.
- **b** The spread is used to describe the _____ between datasets.
- **c** The best kinds of graphs to show spread are a _____ or a _____ graph.
- **d** The spread can be described by how it is distributed around the _____.
- **e** A _____ is produced when several data points lie in a group.
- **f** A _____ is a section that contains no data.

APPLICATIONS

Example 14

2 Rain has just opened her new bakery and has recorded the number of customers who purchased cakes from her shop.



Expand on each description below in as much detail as you can.

- **a** The distribution has an outlier.
- **b** The distribution has a gap.
- **c** The data is tightly packed around the mean.

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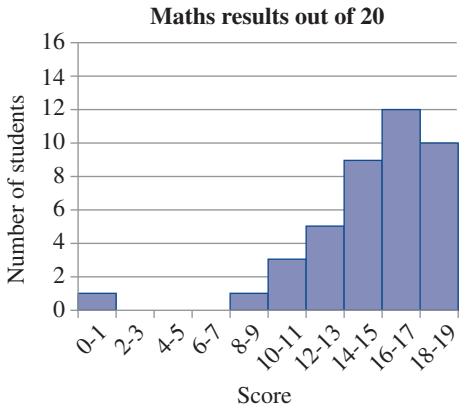
Example 15

3 A teacher is recording her students' test scores (out of 20). Describe the distribution, making use of the terms below where possible.

- spread out
- widely scattered
- dispersed
- tightly packed
- clusters
- gaps
- more/less dense regions
- outliers



Hint A gap is a section that contains no data.

4 An apiarist was doing some research on his honey farm. He counted how many bees were seen in January and February and compiled the data in a back-to-back stem-and-leaf plot. Describe and compare the spread for both months.

Bees seen in a day

Jan		Feb
1 1 0 0	10	0 1
	11	0 1
5 3 3 2 1 0 0 0	12	0 1 3 4
9 8 8 7 3 2 0 0 0	13	1 7 8 9
9 8 6 5 4 2 1 1 0	14	1 2 3 3
	15	
	16	1 3 5 5 5 6
9	17	0 2 2 3 3 4

17|3 = 173

Hint A stem-and-leaf plot is like a column graph turned on its side. This is a back-to-back stem-and-leaf plot, so it's like a double column graph, with one dataset on the left and one on the right.

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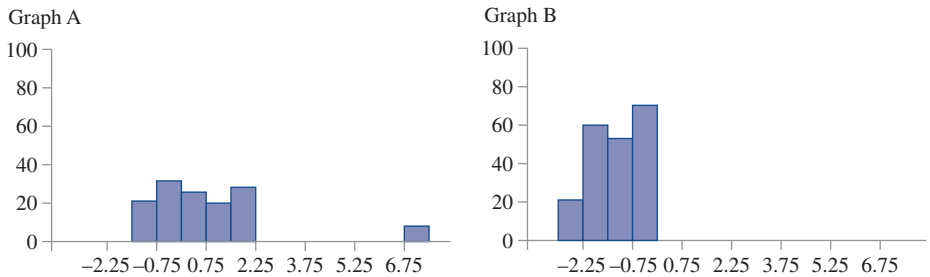
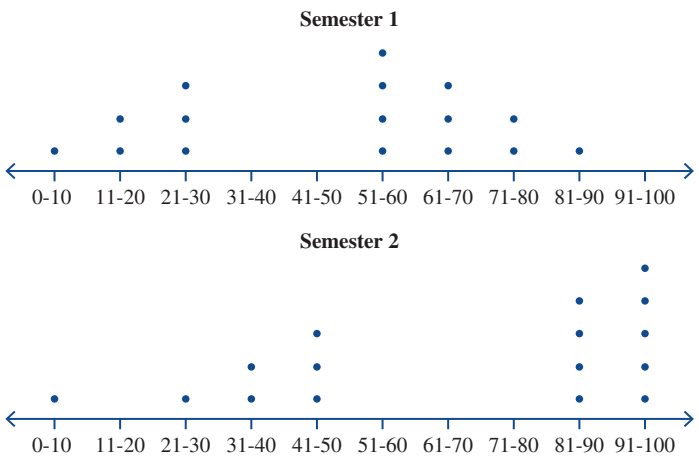
5 Two graphs were used in research. Describe and compare the spread for both graphs.



6 A teacher uses dot plots to display how her students are progressing after each semester. Describe and compare the spread for both semesters.



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8F Investigating real-world examples from the media

LEARNING GOALS

- Acknowledge that data is misrepresented in the media to support a point of view
- Recognise when and how data is being misrepresented

Why is it essential to recognise when data is being misrepresented?

With so much information being presented to us, through media, the internet and print materials, we need to be able recognise when the data is being misrepresented so we can filter out the misinformation from the valid information.

WHAT YOU NEED TO KNOW

- Data and statistics are regularly used in the media, which includes TV, radio, the internet and print materials. They are used both to inform and misinform.
- If the data is being misrepresented or manipulated, then it is being used to support a point of view that the misrepresentation of the data supports.
- Your understanding of the statistical investigation process task allows you to see the manipulation and misrepresentation of the data and thus recognise the misinformation being presented to you.
- The misrepresentations can occur through misleading graphs, incomplete reporting of the dataset, reporting on incomplete data studies, using small datasets and using **biased** data collection methods.
  - For example, a common misuse of statistics is strategically picking the time period to show a result. This is a case of misleading statistics that can be done on purpose, to achieve a specific result, or accidentally. Picking only a good-performing month to build a sales report will portray a misleading picture about the overall sales performance. Whether done on purpose or not, the time periods we choose to portray will affect the way viewers perceive the data. For instance, showing a value over 3 months can produce radically different trends compared to showing it over a year.
- It is up to the viewer or reader (you) to filter out the misinformation from the valid information, using your knowledge of data and how to collect and interpret it.

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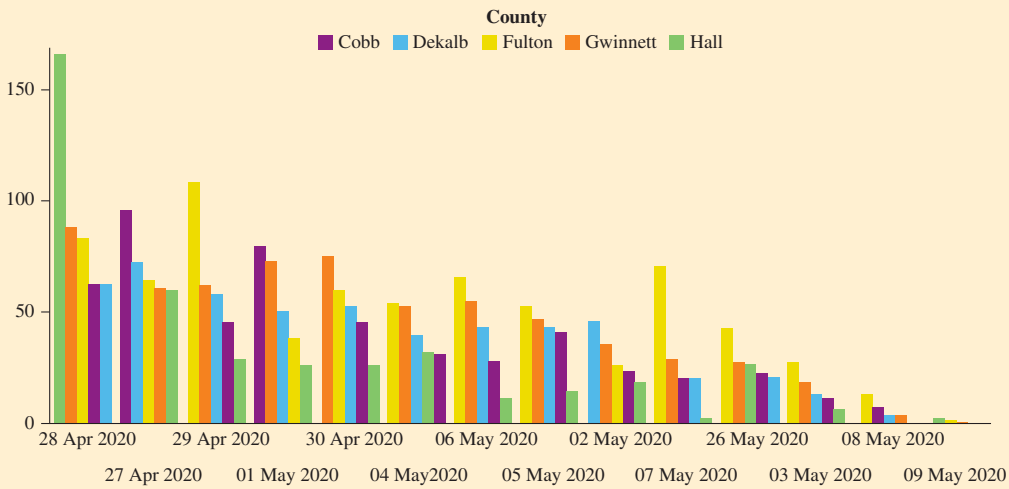
Example 16 Misrepresentation of data through graphs

In May 2020, around 5 months after COVID-19 started spreading around the world, the US Georgia Department of Public Health posted a graph that aimed to show the top 5 counties that had the highest COVID-19 cases in the past 15 days and the number of cases over time.

Top 5 Counties with the Greatest Number of Confirmed COVID-19 Cases

The chart below represents the most impacted counties over the past 15 days and the number of cases over time.



Source: Vox.

Identify some mistakes that make the information in this figure misleading

WORKING

If we take a closer look at this graph, we can find a few mistakes that make the information very misleading.

First, the x-axis does not have a label, even though, according to the graph, it is meant to show the number of cases over time, this doesn't happen.

Second, the dates under the bars are not ordered chronologically. Instead, we see the dates between April and May interspersed, with the aim of making viewers of this graph believe that the cases are gradually decreasing.

Third, the counties are not always depicted in the same order but, instead, in descending order of cases. This has the same aim of making it seem like the cases are dropping.

The graph was later republished with organised dates and counties. You can see the updated version on the following page.

... Continued

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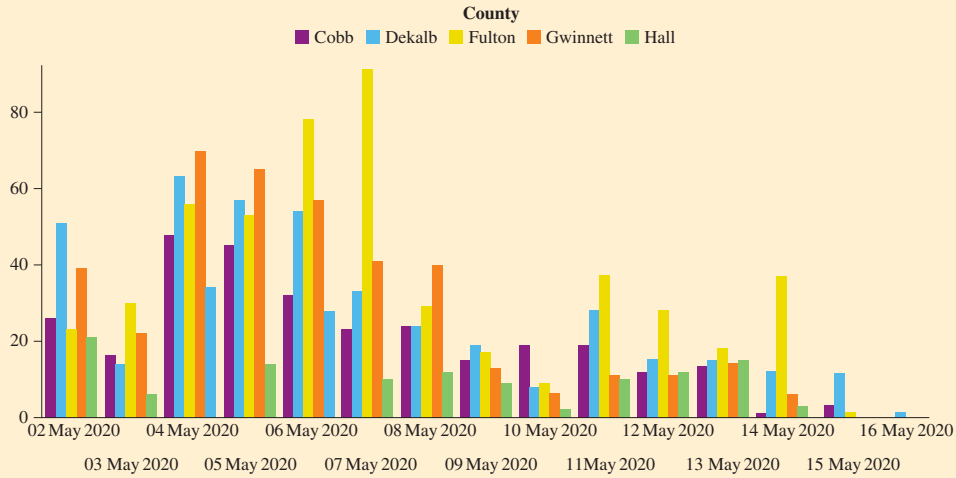
Top 5 Counties with the Greatest Number of Confirmed COVID-19 Cases

The chart below represents the most impacted counties over the past 15 days and the number of cases over time.

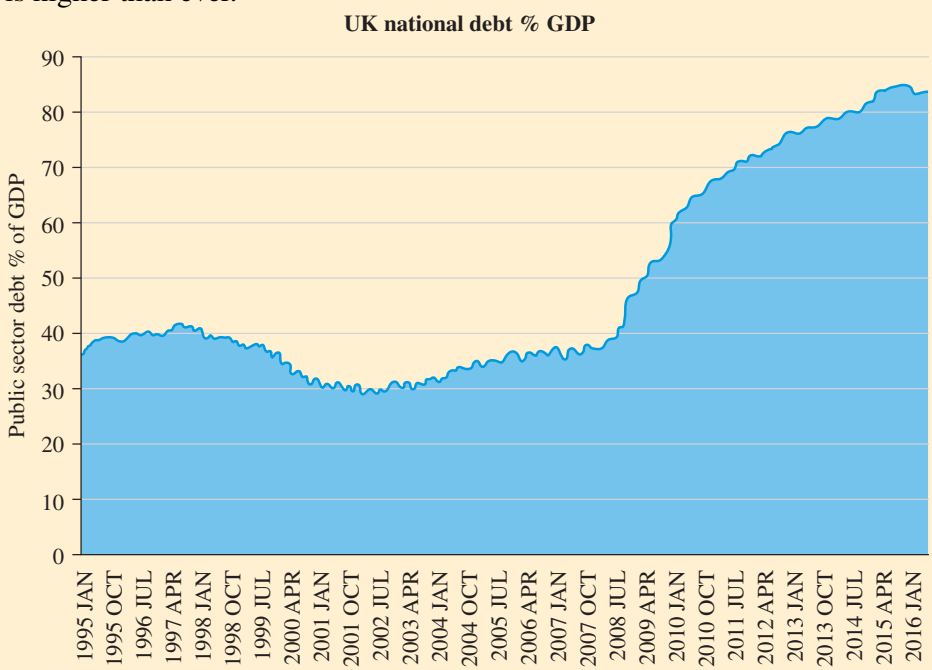


Source: Business Insider.

The corrected graph shows a different perspective on the information being presented.

Example 17 Strategically picking the time period

Look at the graph below which shows the UK national debt from 1995 to 2016. Explain why it is misleading to claim from this graph that the UK's national debt is higher than ever.



Source: www.economicshelp.org

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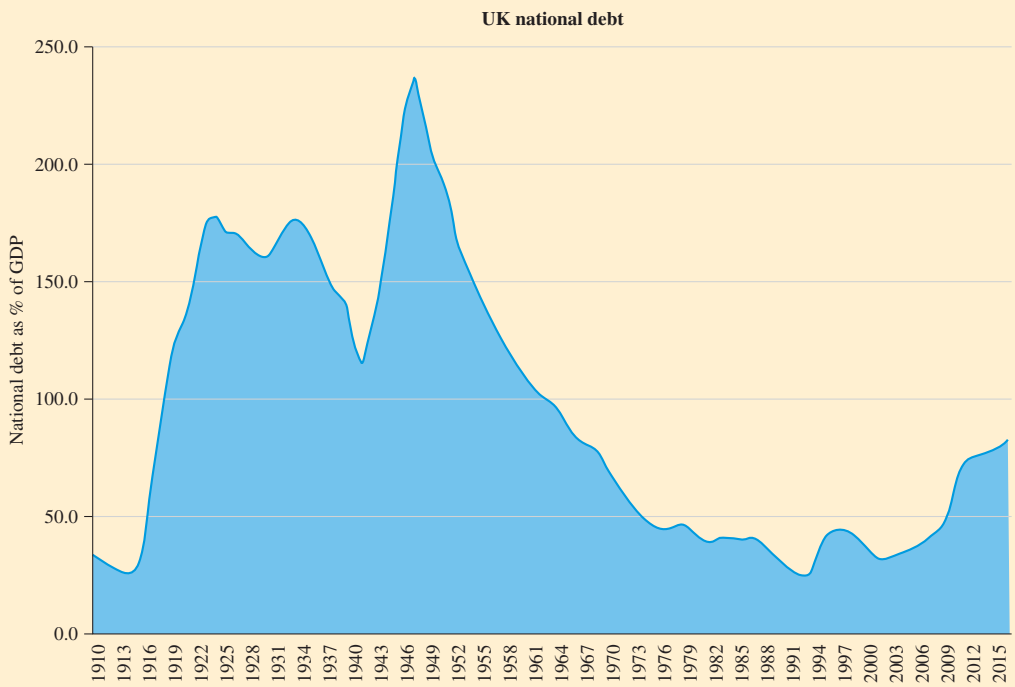
WORKING

This is because the graph does not show data for all years, so it is only possible to claim the debt is higher now relative to the last 20 years of data. When you look at a longer time period, such as 1910 to 2015 (image below), we realise that the debt is actually very low compared to other years. This is a clear example of how the time period that we choose to portray can significantly change the way people will perceive the information.



Source: https://www.datapine.com/blog/misleading-statistics-and-data/

Exercise 8F

Example 16–17

1 Over the next week, observe the data being presented to you for all forms of media and record 2 cases of misinformation and 2 cases of valid information.
For each case, note:

- the topic
- the type of media
- the misinformation or valid information
- what aspects of the presented data made you choose misinformation or valid information.

This will be discussed next week in class, led by your teacher.

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Statistical investigation process task

Background: Being able to organise and analyse data to present statistics and describe the data is a highly regarded skill in the workplace and for many interests that people have outside of work.

Task: Your task is to use statistics to report on a topic or area that is of interest to you. You will need to gather one or two datasets (we suggest they should contain 20 to 50 values) relevant to your interest. You will need to:

1. organise the data into either a stem-and-leaf plot or a graph of your choice
2. determine the mean, median and mode
3. determine the range, interquartile range and standard deviation
4. describe the centre and spread.

With this information, create a report, including your calculations, to communicate and perhaps promote your interest, demonstrating that statistics promote expertise. Some ideas for data that you could collect are:

- sports or digital game results or scores that you or your team have achieved
- results or scores of a sportsperson, team or gamer that you support
- data from things that you or your family create or collect as a hobby, pastime or business
- data collected from reliable internet sources to do with places, people, animals or plants that interest you.

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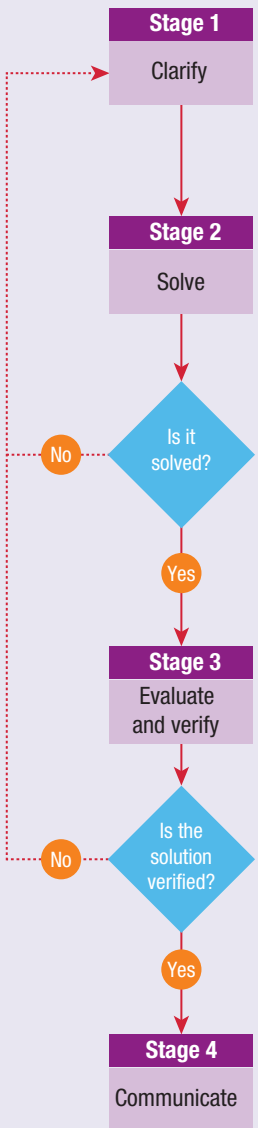
Use the following approach to your investigation process.



Stage 1 Clarify
Clarify the problem and pose one or more questions that can be answered with data

1 What is of particular interest?
2 What data do you need?
3 Do you already have some data?
4 How can you find the data?

Stage 2 Solve
Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

5 Collect the required amount of data.
6 Produce the required plots and/or graphs.
7 Complete the calculations and descriptions required.

Stage 3 Evaluate and verify
Interpret the results of this analysis and relate the interpretation to the original question

8 Are they reasonable?
9 Have you presented them in the most suitable way?
10 Are all statistics measured correctly?

Stage 4 Communicate
Communicate findings in a systematic and concise manner

11 Communicate your findings in a short report that justifies your findings. Include:
Introduction (What is your particular interest?)
Body (Include evidence in the form of statements, graphs and statistics.)
Conclusion (How have statistics enabled you to be more of an expert?)

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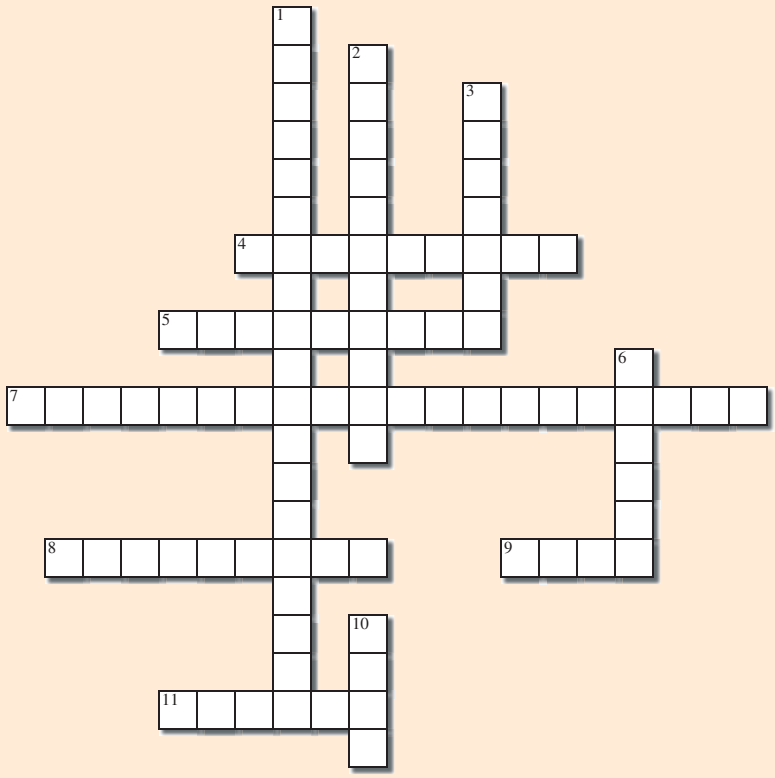
Puzzle



Across

4 Parts or groups in an ordered dataset are divided by values that collectively are known as?

5 Quantiles that divide ordered data into four equal parts are called?

7 The running total of the frequency distribution of the dataset

8 A good kind of graph to show spread is a _____ or a column graph.

9 The sum of all the values in the dataset divided by the number of values in the dataset

11 A good kind of graph to show _____ is a histogram or a column graph.

Down

1 It is calculated by subtracting the lower quartile from the upper quartile

2 Quantiles that divide ordered data into 100 equal parts are called?

3 Quantiles that divide ordered data into 10 equal parts are called?

6 The middle value of a dataset, when the dataset is sorted in order from the smallest value to the largest value

10 The most frequent value in the dataset

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Chapter checklist

☐ I can identify the mode from a dataset.

1 Identify the mode from this dataset.
5 4 8 7 3 6 7 5 8 7 5 9 4 9 7 7 8 9 10 2

☐ I can calculate the mean and median from a dataset.

2 Calculate the mean and median from the dataset in question **1**.

☐ I can investigate the suitability of measures of central tendency in various real-world contexts, and their inappropriate use in the media.

3 Determine the best measure of central tendency for this house price data.
$145 000, $360 000, $1 700 000, $650 000, $170 000, $300 000, $390 000

4 **a** List the measures of central tendency that are not much affected by extreme values.

b Explain which measure of central tendency is best to use when you want the value of all the data points in the dataset to be included in its calculation.

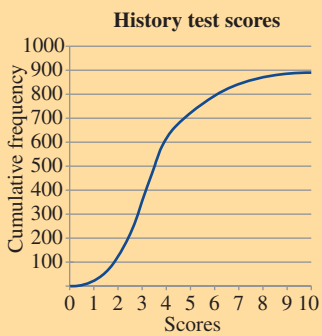
5 List the features of datasets that could make the mean, median and mode misleading as measures of spread.

☐ I can calculate quartiles from a dataset.

6 Calculate the 1st, 2nd and 3rd quartiles for this dataset:
5 4 8 7 3 6 7 5 8 7 5 9 4 9 7 7 8 9 10 2

☐ I can interpret quartiles from a graph.

7 From this graph of history test scores, determine the lowest score needed to be in the top quarter of the history class.



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☐ I can interpret deciles from a graph.

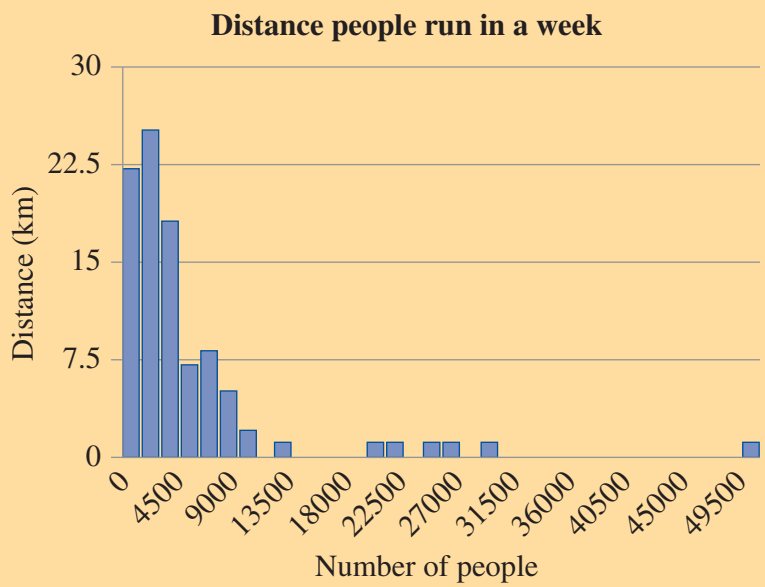
8 On the history test score graph from question **7**, my score is better than 3 tenths of the class. Determine the value of my score.

☐ I can interpret percentiles from a graph.

9 On the history test score graph in question **7**, determine the percentile of a score of 8, and explain what percentage of the class did better than this score.

☐ I can describe the spread of data, including the terms spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions and outliers.

10 Use as many of the terms listed to describe the spread of data in this graph.



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☐ I can calculate statistical measures of spread using the range, interquartile range and standard deviation.

11 Calculate the range, interquartile range and standard deviation of the data.
6, 7, 5, 24, 25, 27, 3, 0, 0, 1, 45

☐ I can interpret statistical measures of spread using the range, interquartile range and standard deviation.

12 Explain what the range, interquartile range and standard deviation of the dataset in question **11** tells us about the spread of the data.

☐ I can identify outliers and investigate their effect on the mean and the median.

13 The following is a dataset recording the number of crows found in a field each day.
50 49 40 50 107 45 37 35 20 10 6 18
Identify any outliers and then calculate the mean and median with and without the outlier.

☐ I can investigate real-world examples from the media illustrating inappropriate uses of measures of spread.

14 Give examples of features of datasets that could make the standard deviation and range misleading as measures of spread.

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Chapter review

Section 8A

1 A netball coach was preparing for a new season. The following dataset is her team's scores for the previous season.
56 76 45 78 67 48 65 45 65 60 65 59 79
Use the dataset to:

a identify the mode

b calculate the mean

c calculate the median

2 Explain which measure, or measures, of central tendency are the best to use in the following situations.

a You don't want the measure of central tendency to be distorted by extreme values.

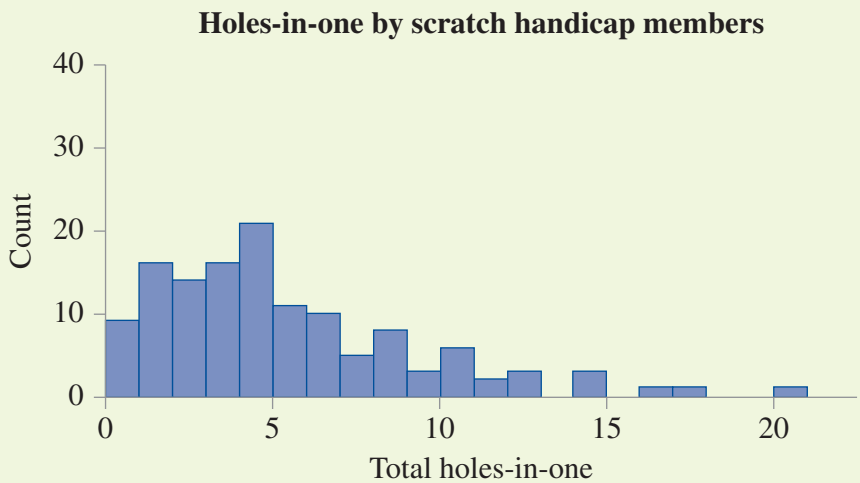
b You want to include the value of all the points in the measure.

c You want a measure that has an equal number of data points above it as below it.

d You want a measure that tells you which value occurs most often in the dataset.

Section 8E

3 A golf club recorded how many holes-in-one its scratch handicap members achieved during their memberships at the club.



Use everyday language to describe the measure of spread.

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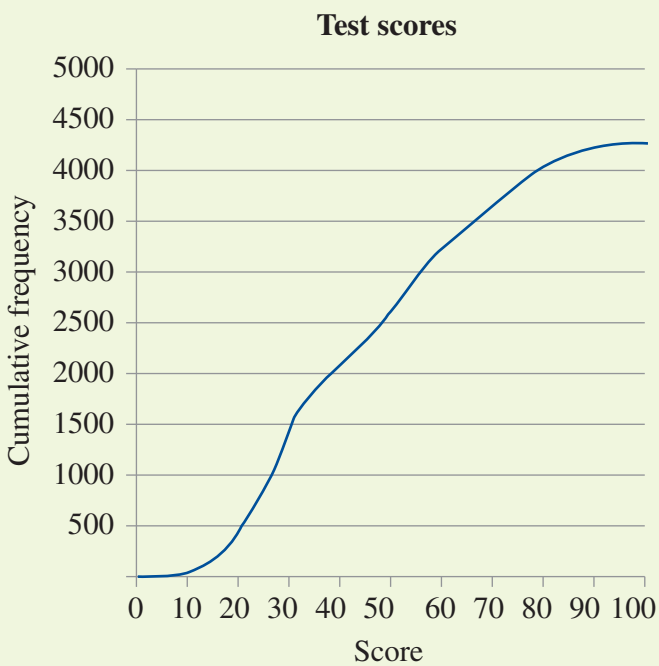
Section 8A **4** Abel's goal is to achieve a 50% grade on his mathematics tests. He has recorded his grades out of 20 for the past 8 weeks. He has two weeks to go.
5, 8, 9, 3, 2, 4, 12, 15

a Calculate the mean grade.
b Determine the median grade.
c Determine the mode grade.
d Which is a better measure to assist with his preparation to reach his goal? Give a reason.

Section 8D **5** The test scores for a class are displayed in the cumulative frequency graph.



a Calculate the median of the test scores (2nd Quartile → Q_2).
b Determine the lower quartile of the test scores (1st Quartile → Q_1).
c Determine the upper quartile of the test scores (3rd Quartile → Q_3).
d Determine which test score will a quarter of the test scores lie below.

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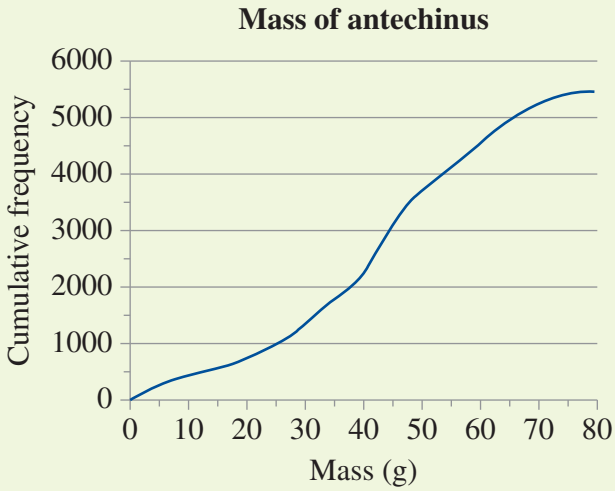
6 The cumulative frequency graph below shows the results of a survey of the mass of marsupial mice (antechinus) in a population.



a Estimate the 90th percentile of the mass.

b An individual antechinus weighs 25 g. What percentage of the populations are heavier than the antechinus?

Section 8C

7 Tom is recording the following rainfall (in mm) for the past number of days. Determine the range, interquartile range and standard deviation.
3, 7, 5, 2, 25, 2, 3, 0, 0, 1, 45

8 Tom has recorded the following maximum temperatures (in °C) for the past number of days. Using a calculator, determine the range, interquartile range, mean, five-number summary (the minimum value, Q_1, median, Q_3, and the maximum value) and standard deviation.
18 18 16 22 22 25 23 27 20 25 25 27

9 Tom has recorded the following minimum temperatures (in °C) for the past number of days. Using technology, determine the range, mean and standard deviation.
4 7 4 7 8 5 5 7 8 7 5 7

10 The following is a dataset recording the number of dolphins seen from a cruiseship each day.
35 49 36 50 107 43 37 34 20 10 6 7

a For the dataset, determine:

i the median (Q_2)

ii the upper quartile (Q_3)

iii the lower quartile (Q_1)

iv IQR

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b Determine whether any numbers are outliers. Remember that:

> A data point is an outlier if it is either:
> less than $Q_1 - 1.5 \times \text{IQR}$ or
> greater than $Q_3 + 1.5 \times \text{IQR}$

c Give a possible reason for the outlier.

d Determine whether outliers need to be removed.

Section 8A **11** Mitchell works five days a week, Tuesday to Saturday, in sales. He receives a set wage as well as commission from his sales. Mitchell is currently saving to buy a house and has a set budget, requiring his commission to be an average of $50 per working day. He has recorded his commission for the week so far.

Day	Commission ($)
Tuesday	80
Wednesday	20
Thursday	30
Friday	40
Saturday	

Calculate the minimum commission that Mitchell needs to earn on Saturday to meet his weekly budget.

Section 8C **12** Jackie works in administration and is concerned with her current salary. She surveys some colleagues working in administration in other companies and finds that her salary is in the 78th percentile. Should she be concerned? Explain your answer.

Section 8E **13** Consider the three datasets *A, B, C*.

$A = \{8, 10, 13, 7, 12\}$

$B = \{10, 10, 10, 10, 10\}$

$C = \{1, 1, 10, 19, 20\}$

a Calculate the standard deviation of each dataset.

b Determine which dataset has the largest standard deviation.

c Is it possible to answer question **b** without completing question **a**?

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9 Comparing datasets

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In this chapter

Syllabus reference

Unit 2 Topic 2.1: Representing and comparing data

Comparing datasets

- compare back-to-back stem plots for different datasets
- complete a five-number summary for different datasets
- construct and interpret box plots, using a five-number summary
- compare the characteristics of the shape of histograms, using symmetry, skewness and bimodality

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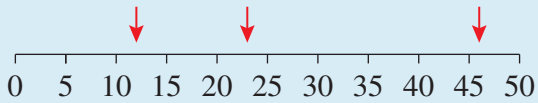
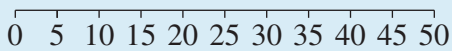
Pre-test

1 Identify the numbers marked by the arrows on this number line.



2 Given the following group of numbers, determine the range (largest number in the data minus smallest number) of the data and show this on the number line provided.
17, 34, 19, 41, 29



3 Organise the following numbers into a stem-and-leaf plot.
41, 52, 57, 58, 65, 66, 70, 75, 78, 78, 78, 80, 80, 95

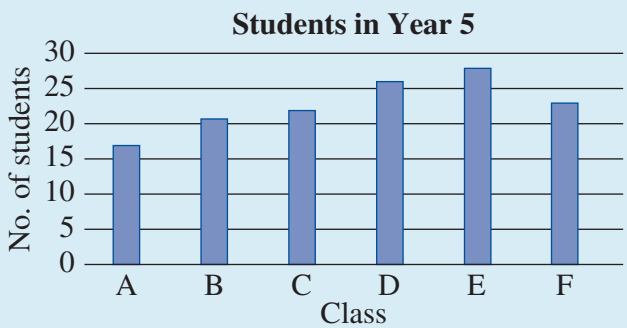
4 The following table shows the number of students in every year for a school.

Year	7	8	9	10	11	12
Students	212	225	210	223	195	165

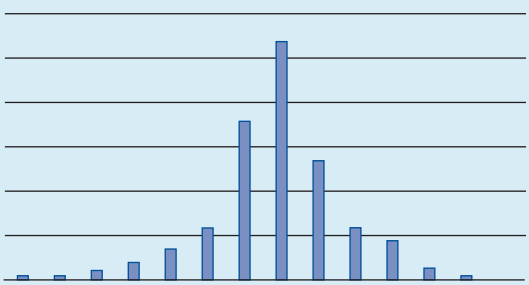
The most useful scale that should be used to plot the data is:

A increments of 1
B increments of 0.5
C increments of 10
D increments of 100

5 The following graph shows the number of Year 5 students in every class. Identify which class has the most students and which has the least.



6 Estimate the position of the median on this column graph.



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9A Completing a five-number summary

LEARNING GOALS

- Explore the structure of a five-number summary
- Create a five-number summary, without using technology
- Create a five-number summary, using technology

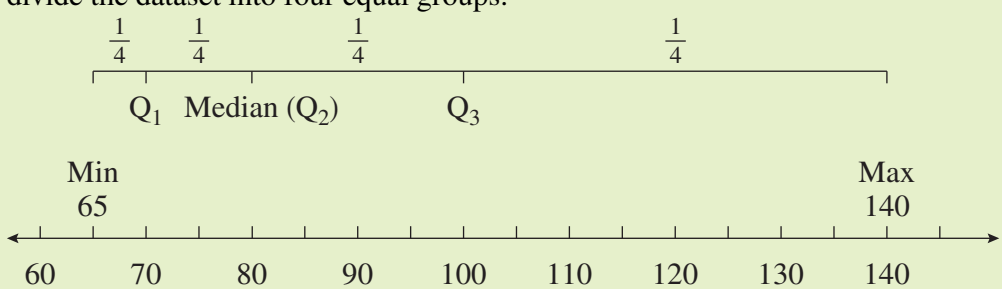
Why are five-number summaries essential?

In our world, we are surrounded by sets of data for a wide range of things. Sometimes it is overwhelming, looking at so much data and too many numbers.
The five-number summary contains the five most important numbers of the data. It is essential that you are able to recognise the structure of five-number summaries and are able to create them, so that you can reduce the overwhelming sets of numbers to just five important ones.

Too many numbers are overwhelming. It is essential to be able to recognise and create five-number summaries.

WHAT YOU NEED TO KNOW

- To find the **five-number summary**, the dataset must first be placed in ascending order.
- The median is the $\frac{n+1}{2}$ th score, when ranked in order from smallest to largest, where n represents the number of data pieces.
- The quartiles of a ranked (ordered) set of data values are the three points that divide the dataset into four equal groups.



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- A five-number summary is made up of the:
  1. minimum score (the smallest number in the set of data)
  2. lower quartile, Q_1 (the median of the lower half of the data)
  3. median, Q_2 or M (the number that falls exactly in the middle)
  4. upper quartile, Q_3 (the median of the upper half of the data)
  5. maximum score (the largest number in the set of data).
- The interquartile range (IQR) is another useful measure of spread: $IQR = Q_3 - Q_1$
- When identifying the upper and lower halves of a dataset with an odd number of data points, the upper and lower halves do not include the median. With an even number of data points, they do include the median.

Example 1 Exploring the structure of a five-number summary

Sally runs two health classes and has asked for everyone's weight. The lists of weights are shown below.

Green class: 72 kg, 65 kg, 80 kg, 140 kg, 70 kg, 100 kg, 94 kg

Blue class: 65 kg, 80 kg, 100 kg, 105 kg, 96 kg, 79 kg, 90 kg, 110 kg

Sally requires an idea of the dataset distribution.

a Sort the data, in order, from smallest to largest for each class.

b Determine the minimum and maximum values for each class.

c Determine the median for each class.

d Determine the lower quartile, Q_1, for each class.

e Determine the upper quartile, Q_3, for each class.

f State the five-number summary for each class.

WORKING	THINKING
a 65, 70, 72, 80, 94, 100, 140 65, 79, 80, 90, 96, 100, 105, 110	Put the numbers in each set in order from smallest to largest.
b $\boxed{65,}$ 70, 72, 80, 94, 100, $\boxed{140}$ Minimum = 65 Maximum = 140 $\boxed{65,}$ 79, 80, 90, 96, 100, 105 $\boxed{110}$ Minimum = 65 Maximum = 110	Find the minimum and maximum (smallest number and largest number) data values.

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c 65, 70, 72, 80, 94, 100, 140 ◄····· The median is the middle number.
To determine the median of an odd distribution:

Median = 80

- The number of scores is 7. So $n = 7$.
- The median score will be $\frac{7+1}{2} = \frac{8}{2} = 4$th score.

65, 79, 80, 90, 96, 100, 105, 110

To determine the median of an even distribution:

Median = (90 + 96) ÷ 2
= 186 ÷ 2
Median = 93

- The number of scores is 8. So $n = 8$.
- The median score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score.
- The median will lie between the 4th and 5th scores.

d (65, 70, 72,) 80, 94, 100, 140 ◄····· Put brackets around the numbers below the median (for an odd number, do not include the median). This helps in finding Q_1.
Find the median of the lower half.

(65, 70, 72,) 80, 94, 100, 140
$Q_1 = 70$

(65, 79, 80, 90,) 96, 100, 105, 110 ◄····· Put brackets around the numbers below the median (for an even number, include the median). This helps in finding Q_1.
Find the median of the lower half.

(65, 79, 80, 90,) 96, 100, 105, 110
$Q_1 = (79 + 80) \div 2$
$Q_1 = 159 \div 2$
$Q_1 = 79.5$

e 65, 70, 72, 80, (94, 100, 140) ◄····· Put brackets around the numbers above the median (for an odd number, do not include the median). This helps in finding Q_3.
Find the median of the upper half.

65, 70, 72, 80, (94, 100, 140)
$Q_3 = 100$

65, 79, 80, 90, (96, 100, 105, 110) ◄····· Put brackets around the numbers above the median (for an even number, include the median). This helps in finding Q_3.

65, 79, 80, 90, (96, 100, 105, 110)
$Q_3 = (100 + 105) \div 2$ ◄····· Find the median of the upper half.
$Q_3 = 205 \div 2$
$Q_3 = 102.5$

... Continued

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f The five-number summary for the Green class is:

Minimum = 65

Q_1 = 70

Median = 80

Q_3 = 100

Maximum = 140

The five-number summary for the Blue class is:

Minimum = 65

Q_1 = 79.5

Median = 93

Q_3 = 102.5

Maximum = 110

◄····· State the five-number summary.

Example 2 Creating a five-number summary from a stem-and-leaf plot

A charity has collected the following amounts of donations in the first hour of an event. Their collector placed the amounts in a stem-and-leaf plot.

First hour takings

0	5 5 5 6 8 9
1	0 0 5 5
2	0 5 5 7
3	0 5

2|5 = 25

a Decide if the data is in order.
b Determine the minimum and maximum scores.
c Determine the median.
d Determine the lower quartile, Q_1.
e Determine the upper quartile, Q_3.
f State the five-number summary.

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WORKING	THINKING
a Yes, the stem-and-leaf plot has the data in order.	Does this stem-and-leaf plot display the data in order?
b **First hour takings** 0 | 5 5 6 8 9 1 | 0 0 5 5 2 | 0 5 5 7 3 | 0 5 Minimum = 5 Maximum = 35	Find the minimum and maximum (first number and last number) data value.
c **First hour takings** 0 | 5 5 5 6 8 9 1 | 0 0 5 5 2 | 0 5 5 7 3 | 0 5 The 8th score = 10 The 9th score = 15 Median = (10 + 15) ÷ 2 Median = 12.5	Find the median. The median is the middle number. • There are 16 terms, so $n = 16$. • The median score will be $\frac{16+1}{2} = \frac{17}{2} = 8.5$th score. • The median will lie between the 8th and 9th scores.
d **First hour takings** 0 | 5 5 5 6 8 9 1 | 0 0 | 5 5 2 | 0 5 5 7 3 | 0 5 The 4th score = 6 The 5th score = 8	Find lower quartile, Q_1. • There are 8 terms in the bottom half, so $n = 8$. • The lower quartile score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score. • The lower quartile will lie between the 4th and 5th scores.
$Q_1 = (6 + 8) \div 2$ $Q_1 = 7$	Find the median of the lower half.

... *Continued*

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e **First hour takings**

0	5 5 5 6 8 9
1	0 0 | 5 5
2	0 (5) (5) 7
3	0 5

The 4th score after the median = 25
The 5th score after the median = 25

$Q_3 = (25 + 25) \div 2$
$Q_3 = 25$

◄ Find upper quartile, Q_3.
- There are also 8 terms in the top half, so $n = 8$
- The upper quartile score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score.
- The upper quartile will lie between the 4th and 5th scores after the median.

◄ Find the median of the upper half.

f The five-number summary for the data is:
Minimum = 5
$Q_1 = 7$
Median = 12.5
$Q_3 = 25$
Maximum = 35

◄ State the five-number summary.

Example 3 Creating a five-number summary using a calculator

Tom recorded the height of his friends in centimetres:
160, 171, 158, 167, 163
Create the five-number summary, using a calculator.
Note: this example is based on a Casio fx-82. For examples using a TI-30XB and Sharp EL531TH, see the Interactive Textbook (a link is below this example).

WORKING	THINKING
Reset All Press [AC] key	◄ Reset your calculator to remove all past data records. To clear, press: SHIFT CLR 9 3 =
Clear? 1 : Setup 2 : Memory 3 : All	

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WORKING	THINKING
X 1 160 2 170 3 158	Press Mode [setup] > 2 > 1. A table appears. Type individual data values followed by the = key to enter the data. After entering the five data points, click AC[off].
minX 158	Press Shift > 1 > 5 > 1 > enter for the minimum value.
Q1 159	Press shift > 1 > 5 > 3 > enter for quartile 1.
med 163	Press shift > 1 > 5 > 4 > enter for the median value.
Q3 169	Press shift > 1 > 5 > 5 > enter for quartile 3.
maxX 171	Press Shift > 1 > 5 > 2 > enter for the maximum value.
The five-number summary is: Minimum = 158 Q_1 = 159 Median = 163 Q_3 = 169 Maximum = 171	State the five-number summary.

Calculator activity 9A for TI and Sharp calculators See the Interactive Textbook for this activity on using a TI-30XB and Sharp EL531TH to create a five-number summary.

Spreadsheet activity 9A See the Interactive Textbook for this activity on using a spreadsheet to create a five-number summary.

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Exercise 9A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a To find the five-number summary, the dataset must first be placed in order from _____ to _____.

b The smallest number in the set of data is also called the _____.

c The lower and upper quartiles are represented by the symbols _____ and _____.

d The _____ can also be represented by the symbol Q_2. It is the number that falls exactly in the _____, when all the numbers have been placed from smallest to largest.

e The largest number in the set of data is also called the _____.

2 Write each dataset in order from smallest to largest.

a 2, 1, 0, 5, 2, 2, 0, 7, 4, 2, 9, 1, 0, 2, 3, 3

b 8.9, 8.7, 9, 7.7, 8.6, 9.6, 8.7, 8.5, 7.9, 9.2

c 45, 65, 46, 43, 42, 48, 46, 42, 49, 41, 47, 45

d \$45.90, \$34.70, \$35.80, \$36.50, \$36.00, \$36.30

3 Determine the minimum and maximum scores in each dataset.

a 7, 5, 9, 4, 6, 8, 5, 4, 7, 8, 6, 9, 6, 4, 7, 3, 7, 8, 5

b 8.9, 8.0, 8.7, 8.6, 8.6, 8.8, 8.6, 8.9, 8.5, 8.2

c 23, 26, 26, 21, 26, 27, 23, 21, 15, 28, 26, 24, 21

d 0.87, 0.76, 0.56, 0.88, 0.76, 0.65, 0.55, 0.76, 0.5

4 Determine $\frac{n+1}{2}$ when n is:

a 9 **b** 7 **c** 6

d 30 **e** 27 **f** 120

Hint: n is the number. Then add 1 and divide the answer by 2.

5 Write out the data represented by this stem-and-leaf plot in order from smallest to largest.

Stem	Leaf
0	6 7 4 7
1	3 6 2 6 3 7
2	9 7
3	
4	8 6 5
5	4 0
6	7
7	2
8	4 6 3

8|6 = 86

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APPLICATIONS

Example 1

6 A group of students was asked to record the number of pets each student owned. The data below shows the results.

0, 7, 4, 2, 0, 1, 0, 2, 3, 3, 0, 2, 1, 0, 3, 2, 2

a Order the data.

b Determine the minimum and maximum values.

c Determine the median.

d Determine the lower quartile, Q_1.

e Determine the upper quartile, Q_3.

f State the five-number summary.

> **Hint** Minimum (smallest number)
> Q_1 (median of lower half)
> Q_2 (middle value)
> Q_3 (median of upper half)
> Maximum (largest number)

7 Tony collects data on wedge-tailed eagles in Kalgoorlie. Below are the number of eagles he spotted over the past 12 days.

6, 4, 3, 5, 6, 2, 7, 6, 5, 9, 5, 4

a Order the data.

b Determine the minimum and maximum values.

c Determine the median.

d Determine the lower quartile, Q_1.

e Determine the upper quartile, Q_3.

f State the five-number summary.

8 The following number of bikes were recorded, over 20 days, on a street where the residents were asking for a bike way.

12, 9, 15, 19, 12, 21, 8, 12, 11, 10, 29, 12, 17, 28, 10, 15, 16, 34, 12, 18

a Order the data.

b Determine the minimum and maximum values.

c Determine the median.

d Determine the lower quartile, Q_1.

e Determine the upper quartile, Q_3.

f State the five-number summary.

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Example 2

9 An apple farmer recorded the number of apples picked, per hour, by a new group of pickers.

Apples picked by new pickers

9	8
10	1 3 5 7 9
11	2 4 6 8 9
12	7 7 7 8
13	4 5 5 7 8 9
14	5 5 6 7 8 9
15	0 0 0 5 5 5 7

11|4 = 114

a Order the data.
b Determine the minimum and maximum values.
c Determine the median.
d Determine the lower quartile, Q_1.
e Determine the upper quartile, Q_3.
f State the five-number summary.

Example 3

10 The following datasets are the measurements of the leg lengths of animals in centimetres. State the five-number summary for each of the following datasets, using technology.
a 24, 67, 54, 87, 56, 32, 76, 45, 31, 53
b 78, 65, 98, 68, 98, 65, 105, 45, 32, 48, 27, 41
c 84, 99, 48, 34, 93, 27, 12, 36, 73, 112, 117, 38, 96
d 4.7, 8.98, 9.56, 4.83, 2.76, 3.95, 2.98, 2.74, 1.05, 4, 3, 6.5, 6.74, 2.84

11 A doctor is researching diabetes, and she has recorded the following blood pressure numbers from a group of patients. Create a five-number summary, without the use of technology, to assist with her research.
128, 122, 113, 108, 115, 115, 107, 130, 115, 107, 120, 106

12 A student wanted to purchase a smartphone. The following dataset is the amounts recorded from a selection of options. Create a five-number summary, without the use of technology, to assist with the student's decision.
$200, $1200, $400, $300, $900, $700, $900, $1000, $700, $850

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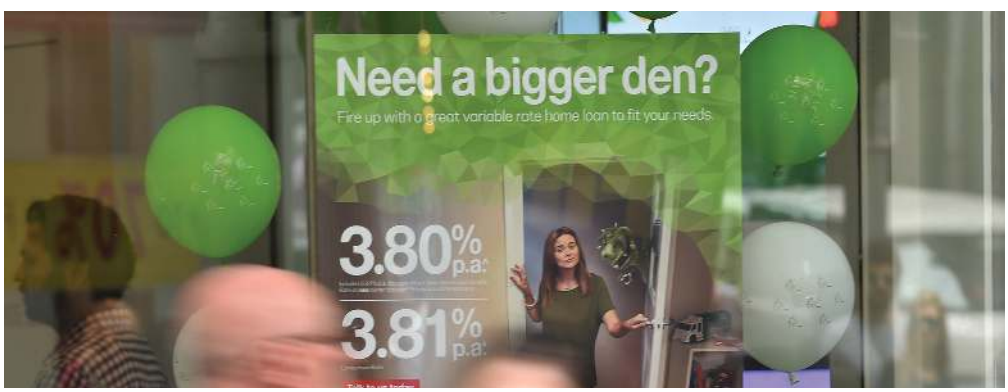
13 A soccer coach was preparing for a new season. The following dataset is their team's scores for the previous season. Create a five-number summary, without the use of technology, to assist with their preparation.
1, 5, 4, 3, 5, 2, 1, 0, 2, 1, 0, 3, 2, 0, 4, 1, 1, 2, 1, 2, 3, 3, 2, 1, 0, 2, 1, 2, 4, 0

14 A couple were deciding on a bank for a loan. The following dataset is the various interest rates on offer by different banks.
3.5%, 6.4%, 4.6%, 3.7%, 3.7%, 5.8%, 7.3%, 5.3%, 7%, 5.4%, 4%, 3.2%, 6%, 6.5%, 6.4%, 3.6%, 7%, 8.4%, 6%, 7%, 4.5%, 5.6%, 7%, 5.4%

a Organise the data into a stem-and-leaf plot.

b Create a five-number summary, with the use of technology, to assist with their decision.



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15 The following datasets are the scores from quizzes in a mathematics class. Determine the dataset that the following five-number summary corresponds to.
Minimum = 15
Q_1 = 20
Median = 22
Q_3 = 27
Maximum = 32

A 15, 20, 21, 32, 27, 16, 30
B 28, 15, 21, 32, 26, 22, 19
C 22, 27, 20, 23, 24, 32, 15
D 32, 20, 15, 22, 20, 22, 27

16 The following datasets are the scores from quizzes in a science class. Determine the dataset that the following five-number summary corresponds to.
Minimum = 23
Q_1 = 31
Median = 36
Q_3 = 45
Maximum = 51

A 22, 21, 45, 47, 52, 45, 36, 23, 51, 40, 45, 36, 45, 30, 46, 46, 23
B 36, 37, 40, 42, 33, 33, 32, 45, 32, 23, 23, 51, 45, 24, 30, 46, 50
C 51, 50, 42, 34, 22, 27, 20, 23, 24, 32, 15, 50, 51, 51, 37, 45, 23
D 36, 23, 32, 36, 45, 32, 20, 15, 22, 20, 22, 27, 51, 50, 50, 51, 36

17 Will is recording the number of cars passing per 20 minutes.
20, 20, 19, 27, 45, 30, 34, 45, 36, 44, 28, 26, 45, 45, 40, 46, 25, 28

a Arrange the data into a stem-and-leaf plot.
b State the five-number summary, with the use of technology.

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9B Constructing box plots

LEARNING GOAL

- Construct box plots using a five-number summary

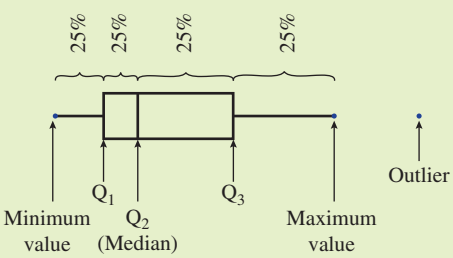
Why are box plots essential?

Understanding five-number summaries is just the beginning of coping with increasing data in our world. Box plots are the visual representation of the five-number summary of a dataset. They are particularly useful for presenting the information in a large dataset in a simplified way. Being able to construct box plots is essential in understanding how data can be visually represented.

Understanding how to construct a box plot will lessen the feeling of being overwhelmed with large datasets.

WHAT YOU NEED TO KNOW

- **Box plots** are the visual representation of the five-number summary of a dataset.
- They are drawn against a scale.
- Box plots can be drawn vertically or horizontally.
- They are divided into four sections, with a quarter (25%) of the data in each section.



Note: *Compare this diagram to the one of quartiles at the bottom of the first page in section 9A, and note the similarities.*

- Box plots are also known as box-and-whisker plots, with a box surrounding the Q_1, median and Q_3, and whiskers extending to the minimum and maximum scores.
- Outliers are also used in box plots. They are an observation that appears to be inconsistent with the remainder of that set of data; a surprising observation.

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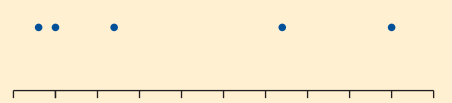
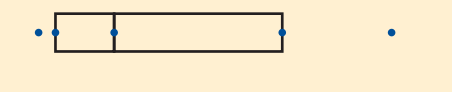
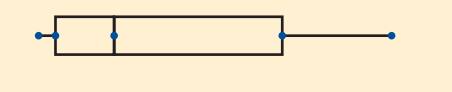
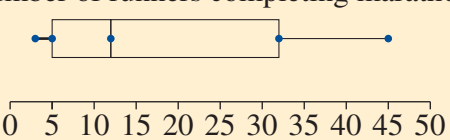
Example 4 Constructing box plots using a five-number summary

Greg records the number of runners, from each team, who complete a marathon.
3, 6, 8, 4, 12, 20, 45, 36, 28

a State the five-number summary for the dataset.

b Construct a box plot using the five-number summary, without the use of technology.

WORKING	THINKING
a (3, 4, 5, 6, 8) 12 (20, 28, 32, 36, 45)	Write the dataset in ascending order.
Minimum = 3	Find the minimum and maximum values.
$Q_1 = 5$	
Median = 12	Find the median.
$Q_3 = 32$	Put the remaining numbers either side in brackets. Find the Q_1 and Q_3.
Maximum = 45	
b (3, 4, 5, 6, 8) 12 (20, 28, 32, 36, 45)	Choose an appropriate scale to represent the data (e.g. go up only by 1s, 2s or 5s)
	Draw a number line representing the data. Place a dot above each of the five numbers.
	Extend the dots to small lines for the median, Q_1 and Q_3.
	Create the 'box'.
	Draw lines (whiskers) out to the minimum and maximum values.
Number of runners completing marathon	Give your box plot a title.
	

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Exercise 9B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Box plots are the _____ representation of the five-number summary of a _____.

b Box plots are drawn against a _____.

c Box plots are also known as box-and- _____ plots.

d The upper 25% of the dataset is above _____.

e Outliers are an observation that appears to be _____ with the remainder of that set of data; a _____ observation.

2 Choose an appropriate scale for the following data.

Hint Do increments of 2, 5 or 10 suit the data best?

a 0, 5, 3, 1, 5, 4, 2, 4, 0, 1, 3, 3, 4, 7, 3, 4, 2

b 14, 14, 21, 16, 15, 17, 14, 18, 14, 23, 19, 16, 16, 17, 18, 14

c 29, 12, 17, 28, 10, 15, 16, 34, 12, 18, 34, 43, 32, 46, 34, 32, 39, 34, 32, 19

d 0, 52, 4, 11, 0, 0, 7, 8, 0, 2, 18, 0, 0, 4, 0, 0, 5, 13, 2, 13, 1, 1, 14, 1, 12

3 Draw a number line for each dataset. Represent each data value, using a dot above the line.

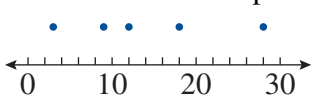
a 5, 8, 7, 9, 6

b 0.7, 0.4, 0.8, 0.2, 0.6

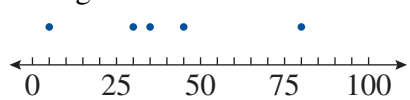
c 95, 56, 32, 67, 75

4 Record the numbers represented on the following number lines.

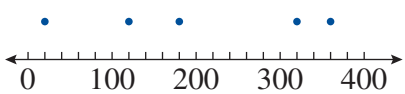
a



b



c



Hint To work out the value of the interval between the tick marks, divide the difference between two labelled values by the number of tick-mark intervals between them.

APPLICATIONS

Example 4

5 A group of students were asked to record the number of hours of homework they completed each week. The data below shows the results.

0, 5, 3, 1, 5, 4, 2, 4, 0, 1, 3, 3, 4, 7, 3, 4, 2

a Create a five-number summary.

b Construct a box plot.

Hint Remember to put the data in order from smallest to largest.

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6 A takeaway store recorded the ages of their staff.

14, 14, 21, 16, 15, 17, 14, 18, 14, 23, 19, 16, 16, 17, 18, 14

a Create a five-number summary.

b Construct a box plot.

Hint Organise your scale and number line first.

7 The following number of cars were recorded on a residential street, over 20 days, where the residents were requesting a traffic-calming speed hump.

29, 12, 17, 28, 10, 15, 16, 34, 12, 18, 34, 43, 32, 46, 34, 32, 39, 34, 32, 19

a Create a five-number summary.

b Construct a box plot.

8 A librarian is interested in the number of books people borrow from a library. She selected a sample of 25 people and recorded the number of books each person had borrowed in the previous year. Here are her results:

Hint Outliers are observations that appear to be inconsistent with the remainder of that set of data.

0, 52, 4, 11, 0, 0, 7, 8, 0, 2, 18, 0, 0, 4, 0, 0, 5, 13, 2, 13, 1, 1, 14, 1, 12

a Identify any possible outliers and write down their values.

b Construct a box plot of the data, showing outliers.

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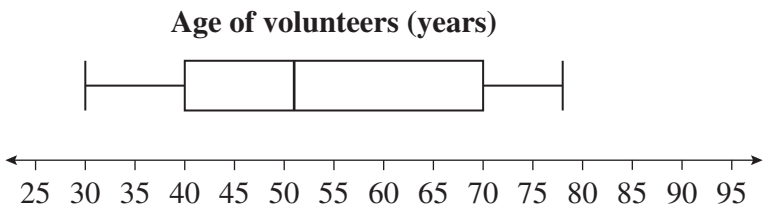
9 The time taken, in minutes, for a group of students to complete a maths problem is:
10, 8, 60, 6, 6, 14, 15, 6, 7, 6, 5, 7, 8, 6, 18, 9, 7, 10, 5, 8, 6, 14, 11, 5

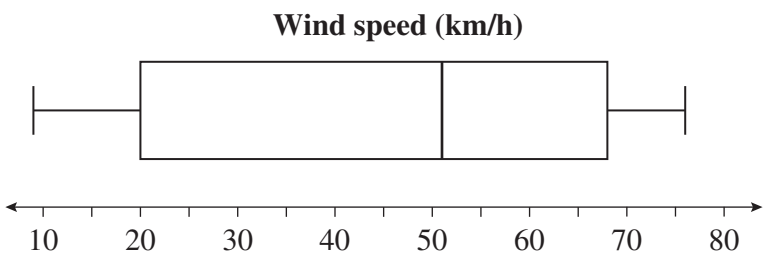
a Identify any possible outliers and write down their values.

b Construct a box plot of the data, showing outliers.

10 The box plot below shows the age of people volunteering for a particular charity. Create a five-number summary.



11 A group of drone enthusiasts meet at a park every day. To monitor wind speed, they have recorded the wind speed (in km/h) over a month. Create a five-number summary of their recorded data.



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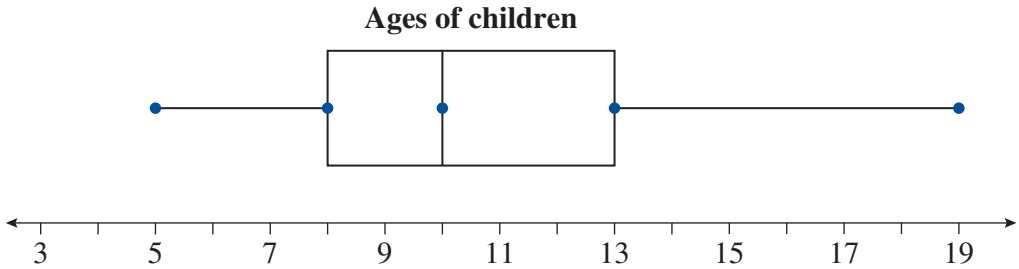
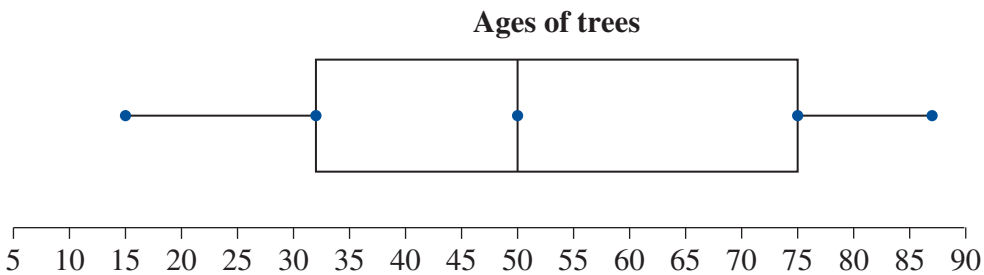
12 The following box plot represents the children's ages in a large family.



a What age are 75% of the children older than?
b Is it possible to determine how many children there are from this data?
c What percentage of the children are between 13 and 19?
d How old is the youngest child?
e The family are going to the agricultural show. Children's tickets are for children up to 13 years old. What percentage of the children qualify for a child's ticket?

13 A local arborist determined the age of 100 trees in their suburb.



a What age are 75% of the trees older than?
b What is the range of the data?
c What percentage of the trees are between 15 and 32 years old?
d How old is the oldest tree?

14 The stem-and-leaf plot represents a sample of 50 scores in a golf tournament.

Golf scores

5	8 9 9 9
6	2 3 3 4 4 7 7 8 9 9 9 9
7	0 0 0 0 1 2 2 2 5 7 8 8 8 8
8	0 1 1 1 3 4 6 6 7 8 9 9 9
9	2 3 3 4 8 8 9

7|0 represents a score of 70

a Create a five-number summary of the dataset.
b Construct a box plot.

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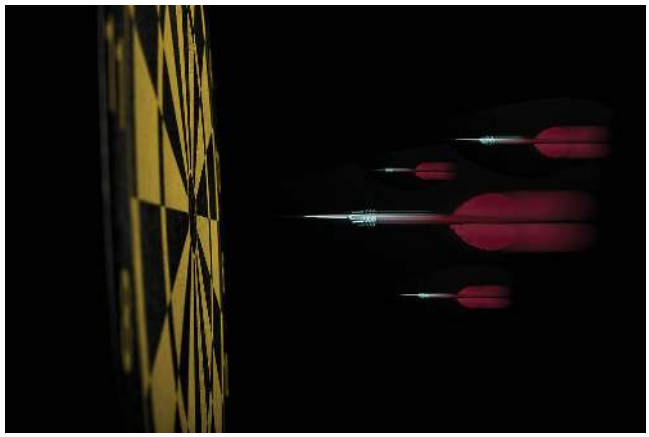
15 A group of dart players throw one dart each at a dart board and their scores are recorded.

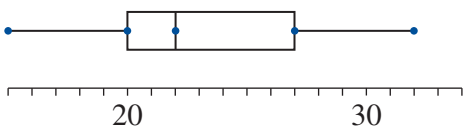
Name	Scores
Tony	8
Jarrah	22
Steve	28
Joni	14
Ted	30
Tai	13
Nick	20
Gregor	24
Davina	18
Brian	15
Min	7



a Create a five-number summary.

b Construct a box plot.

16 The following datasets are the scores from quizzes in a science class. Determine the dataset that the following box plot corresponds to.



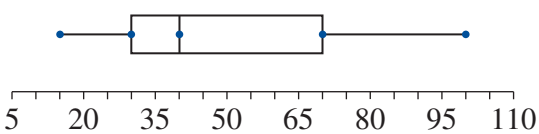
A 22, 27, 20, 23, 24, 32, 15

B 15, 20, 21, 32, 27, 16, 30

C 32, 20, 15, 22, 20, 22, 27

D 15, 32, 26, 28, 20, 20, 22

17 The following datasets are the scores from quizzes in a history class. Determine the dataset that the following box plot corresponds to.



A 95, 63, 15, 76, 45, 57, 31

B 20, 43, 67, 32, 69, 110, 100

C 71, 45, 50, 100, 34, 15, 36

D 58, 53, 80, 100, 43, 41, 79

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9C Comparing datasets

LEARNING GOALS

- Compare parallel box plots for different datasets
- Compare back-to-back stem-and-leaf plots for different datasets

Why is comparing datasets essential?

Understanding data summaries and being able to visually represent information is essential to living in our current overwhelming world of numbers and statistics. Being able to compare data is also essential, as it provides the information to make decisions effectively. It's difficult to make sound judgements and decisions without comparing data.

Comparing data is an essential skill to help us make better decisions.

WHAT YOU NEED TO KNOW

- **Parallel box** plots can be used to compare two or more groups.
- When comparing box plots, they must always be placed against the same axis. This enables the median, spread and possible outliers of the data to be easily identified and compared.
- The range can be used as a measure of spread in a dataset, but it is extremely sensitive to the presence of outliers and should only be used with care.
- The IQR should be compared and contrasted to the range.
- The variability of a dataset is the amount by which data points differ from the mean and from each other, similar to the spread, and which can also be measured by the range, IQR and standard deviation.

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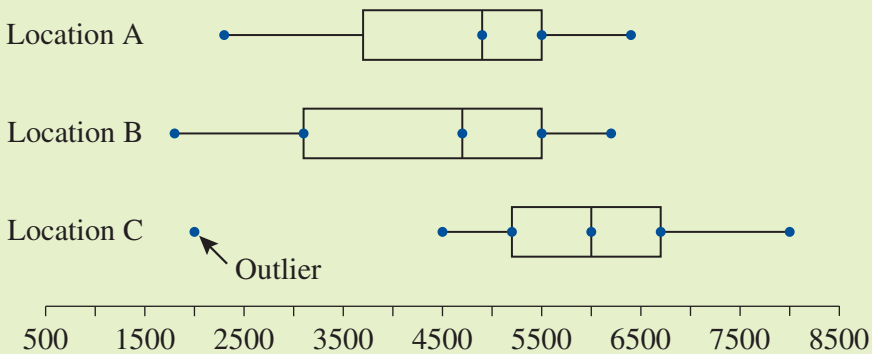
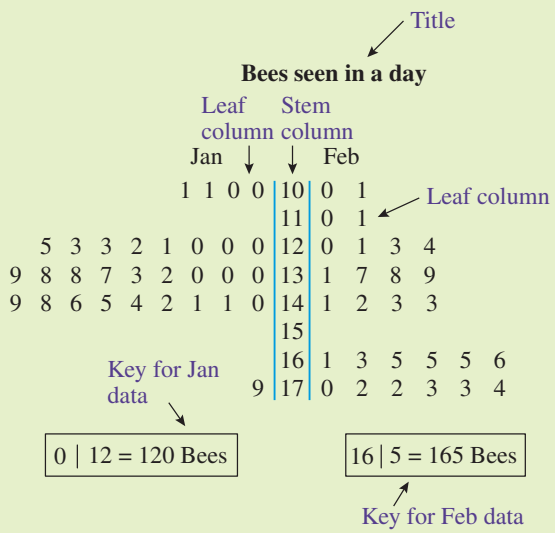
- Outliers are identified by a mark located beyond the 'whiskers' of the box plot. An outlier is an observation that is numerically distant from the rest of the data.

Comparison of the prices paid at auction in three different locations



- **Back-to-back stem-and-leaf plots** have a single stem with two sets of leaves. Each set of leaves is separated for the two groups being compared. The leaves for one set of data are on one side and the leaves for the second set of data are on the other side.
- When comparing back-to-back stem-and-leaf plots, the leaves must always be placed against the corresponding stem. This enables the shape, median, spread and possible outliers of the data to be easily identified and compared.
  - The shape of a back-to-back stem-and-leaf plot can be seen by looking at the shape and length of the leaves. This shows whether a dataset is symmetric (roughly the same on each side when cut down the middle) or skewed (lopsided).
  - A symmetric back-to-back stem-and-leaf plot shows the median roughly in the middle of the spread.
- Outliers are identified by a leaf located beyond the main cluster of data. An outlier is an observation that is numerically distant from the rest of the data.



Jan					Stem	Feb				
			1	1	0	0	10	0	1	
							11	0	1	
	5	3	3	2	1	0	12	0	1	3	4
9	8	8	7	3	2	0	13	1	7	8	9
9	8	6	5	4	2	1	14	1	2	3	3
							15				
							16	1	3	5	5	5	6
					9		17	0	2	2	3	3	4

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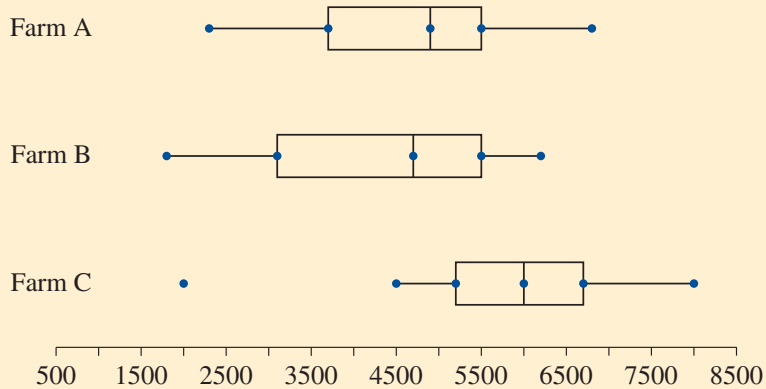
Example 5 Comparing parallel box plots for different datasets

The parallel box plots below represent the weights (in kgs) of bananas grown at three different farms.

Comparison of the weights (kg) of bananas from three different farms



a Compare the medians of all datasets.
b Compare the range of all datasets.
c Compare the spread, using the IQR which can be read directly from the box plot (it is the width of the boxes).
d Locate any outliers.
e Write a paragraph comparing the farms.

WORKING	THINKING
a Median for Farm A = 4900 Median for Farm B = 4700 Median for Farm C = 6000 The farm with the highest median is Farm C, with 6000 kg of bananas.	Locate the median (middle value) for each dataset by comparing the vertical line in the middle of the boxes.
b Farm A 6800 − 2300 = 4500 Farm B 6200 − 1800 = 4400 Farm C 8000 − 4500 = 3500 Farm A has the largest range, with 4500 kg of bananas; and Farm C has the smallest range, with 3500 kg of bananas.	To calculate the range, find the difference between the minimum and maximum of each dataset.

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c Farm A
$5500 - 3700 = 1800$
Farm B
$5500 - 3100 = 2400$
Farm C
$6700 - 5200 = 1500$
The farm with the largest spread between the interquartile range is Farm B, whereas the smallest spread is Farm C, with 1500 kg of bananas.

◄····· Compare the spread by determining the width of the boxes.
$IQR = Q_3 - Q_1$

d There are no outliers for Farms A and B; however, Farm C has an outlier at 2000 kg of bananas.

◄····· Record any marks beyond the 'whiskers'.

e The farm with the lightest weight is Farm B (1800 kg). The farm with the heaviest weight is Farm C (8000 kg). Farm A has the largest range, with 4500 kg of bananas. The smallest range is Farm C, with 3500 kg of bananas. The farm with the highest median is Farm C, with 6000 kg of bananas. Farm C is the only farm to have an outlier at 2000 kg of bananas.

◄····· Compare and contrast the median, spread and outliers of each farm.

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Example 6 Comparing back-to-back stem-and-leaf plots for different datasets

The back-to-back stem-and-leaf plot compares female life expectancy across 15 countries for the years 1960 and 2015.

Female life expectancy
(in years)

Key: 5|6 = 65 years; 7|2 = 72 years

1960		2015
9 9	4	8
9 9 5 3	5	9
5 5 2 1 1 1 0	6	8 9
5 5	7	2 3 4 5 5 9
	8	2 3 3 8
	9	0

a Compare the median of both datasets.
b Compare the range of values of both datasets.
c Compare the spread between the IQR (interquartile range) of both datasets.
d Write a paragraph comparing both datasets.

WORKING | **THINKING**

a **Female life expectancy**
(in years)

1960		2015
9 9	4	8
9 9 5 3	5	9
5 5 2 1 1 (1) 0	6	8 9
5 5	7	2 3 4 (5) 5 9
	8	2 3 3 8
	9	0

Locate the median (middle value) for each dataset, using $(n+1)/2$.

1960
$(15 \text{ numbers} + 1)/2 = 8\text{th number}$
Median for $1960 = 61$ years
2015
$(15 \text{ numbers} + 1)/2 = 8\text{th number}$
Median for $2015 = 75$ years

The median life expectancy of females in 2015 was 14 years higher than in 1960.

Compare the medians for each year.

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b **1960**

$75 - 49 = 26$

2015

$90 - 48 = 42$

2015 has the wider range, with 42 years, compared to 26 years for 1960.

◄···· To calculate the range, find the difference between the minimum and maximum of each dataset.

◄·· Compare the range for each year.

c **Female life expectancy**

(in years)

1960		2015
9 9	4	8
9 9 (5) 3	5	9
5 (5) 2 1 1 (1) 0	6	8 (9)
5 5	7	2 3 4 (5) 5 9
	8	2 (3) 3 8
	9	0

1960

$65 - 55 = 10$

2015

$83 - 69 = 14$

The spread of life expectancy of females in 1960 (IQR = 10 years) is smaller than the spread in 2015 (IQR = 14 years).

◄···· To compare the spread, find the quartiles.
Quartiles are found by finding the median of the lower half of the data and the median of the upper half of the data.

◄···· Then find IQR: $IQR = Q_3 - Q_1$

d The year with the lowest life expectancy was 2015; however, this was only by one year. The year with the highest life expectancy was 2015, by fifteen years. The median life expectancy for the particular countries has increased by fourteen years. The variability in life expectancy has increased, as demonstrated by the larger IQR and range in 2015.

◄···· Compare and contrast the median, range and spread summaries of each year.

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Exercise 9C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Parallel box plots can be used to compare _____ or more groups.

b When comparing box plots, they must always be placed against the _____ axis.

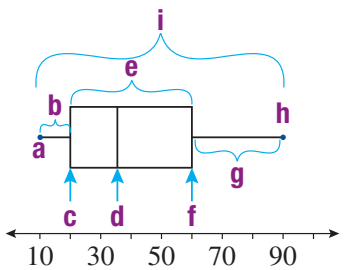
c The range can be used as a _____ of spread in a dataset, but it is extremely _____ to the presence of _____ and should only be used with care.

d An _____ is an observation that is numerically distant from the rest of the data.

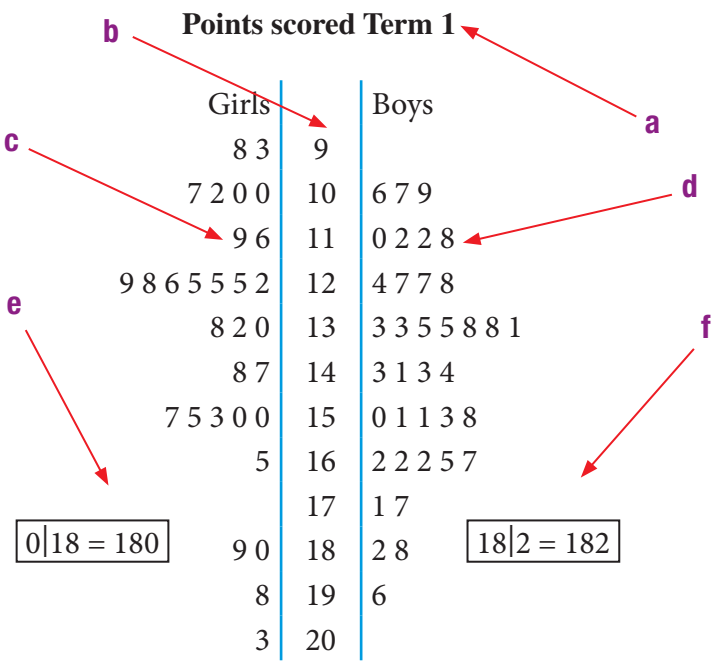
e Back-to-back stem-and-leaf plots have a _____ stem with _____ sets of leaves.

f Each set of leaves is _____ for the two groups being _____.

2 Draw the box plot and label the parts.



3 Label the parts of this back-to-back stem-and-leaf plot.



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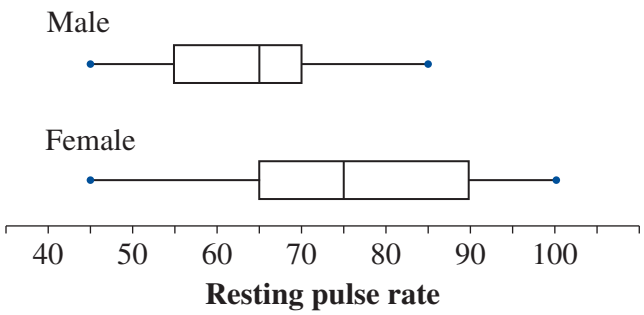
APPLICATIONS

Example 5

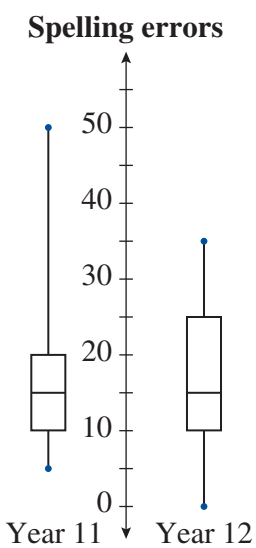
4 A group of 50 male and 50 female students were asked to record their resting pulse rates. The datasets below show the results.



a Compare the medians of both datasets.
b Compare the range of both datasets.
c Compare the spread, using the interquartile range.
d Write a statement comparing both datasets.

Hint Q_1 is the median of the lower half of the data; Q_3 is the median of the upper half of the data.

5 One hundred Year 11 students and one hundred Year 12 students sit the same writing task, with their spelling errors being recorded.
The datasets below show the results.



a Compare the medians of both datasets.
b Compare the range of both datasets.
c Compare the spread, using the IQR.
d Write a statement comparing both datasets.

Hint IQR = $Q_3 - Q_1$

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Example 6

6 A sample of Year 7 and Year 12 students were asked to record the amount of homework they completed, in hours, each week. The datasets below show the results.

Homework time (hours per week)

Year 7		Year 12
9 8 6 6 5 4 3 3	0	5 5 7 9
5 5 5 4 4 0 0	1	0 0 2 4 7 8 9
	2	0 1 1 1

3|0 = 3 hours per week

1|0 = 10 hours per week

a Compare the medians of both datasets.
b Compare the range of both datasets.
c Compare the spread, using the interquartile range.
d Write a statement comparing both datasets.

7 The parallel stem-and-leaf plot below displays the number of burpees a group of Year 11 students could do in a minute at the beginning of a netball season and at the end of the season.

Burpees per minute

Beginning of Netball season		End of Netball season
	0	
9 8	1	
8 3 1 0 0	2	0 5 5 7
6 5 5 3 2 2	3	0 1 3 7 9 9 9
5 0	4	9 9
	5	0 0

0|4 = 40 burpees per min

5|0 = 50 burpees per min

a Compare the medians of both datasets.
b Compare the range of both datasets.
c Compare the spread of the two datasets, using the interquartile range.
d Write a statement comparing the two datasets.

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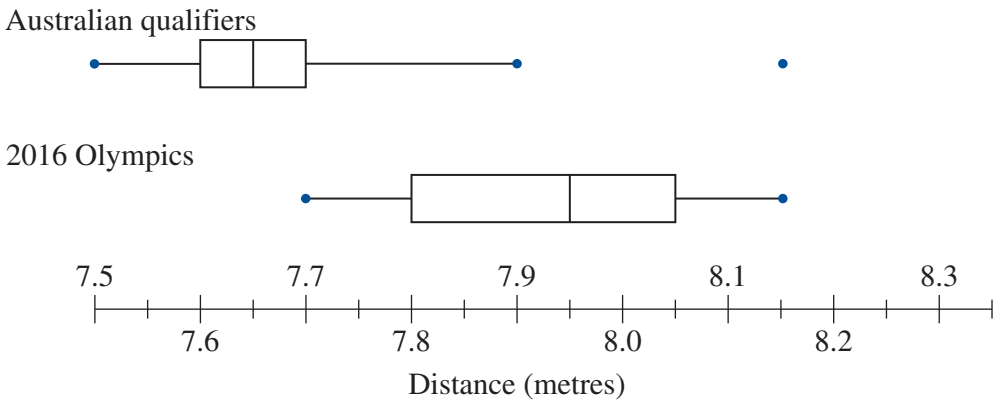
8 The following parallel box plots show the long jump distances of the Australian qualifiers, compared to the distances achieved at the 2016 Olympic games.

Comparison of long jump distances in metres



Explain how the following conclusions can be made from this data.

a The Australian qualifiers' long jumps were generally shorter than the Olympic long jumps.

b The Olympic jumps are spread out more than the Australians.

c Most of the Olympic long jumps were longer than all of the Australians' jumps.

9 A popular cafe is currently concerned by a new cafe in town that is stating that they have the fastest service around. The following datasets are the delivery times (in minutes) for both cafes. Compare the range and measures of central tendency of the datasets for the two cafes.

Comparison of delivery times
(in minutes)

Popular cafe		New cafe
9 8	0	
9 8 6 5 4 3 3 2	1	0 0 1 2 4 5 6 6 8 9
8 3 3 1	2	0 1 2 5
0	3	0

0|3 = 30 min

2|5 = 25 min

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10 A high school had their annual 10-pin bowling championships. The back-to-back stem-and-leaf plot shows the highest score of each player, by gender. The mean of the boys' highest scores was 143.3 and the mean of the girls' highest scores was 137.4. Compare the range and measures of central tendency of the datasets for the boys and the girls.

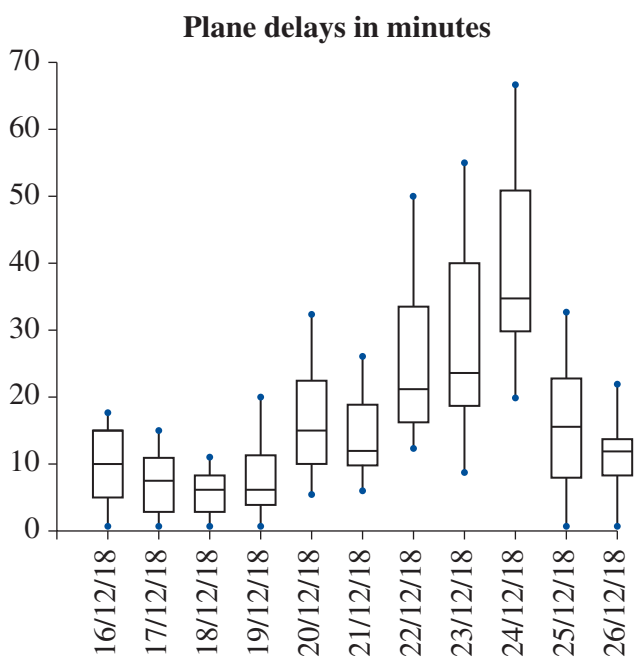
U18 bowling scores

Girls		Boys
8 3	9	
7 2 0 0	10	6 7 9
9 6	11	0 2 2 8
9 8 6 5 5 5 2	12	4 7 7 8
8 2 0	13	3 3 5 5 8 8
8 7	14	3 3 4
7 5 3 0 0 0	15	0 1 1 3 8
5	16	2 2 2 5 7
	17	1 7
9 0	18	2 8
8	19	6
3	20	

3|20 = 203 score

19|6 = 196 score

11 The following set of box plots were used to compare the delay times of plane flights (in minutes) during the 2018 Christmas holidays. Compare the delay times for each dataset.



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9D Comparing the characteristics of histograms

LEARNING GOALS

- Compare the characteristics of the shape of histograms using symmetry
- Compare the characteristics of the shape of histograms using skewness
- Compare the characteristics of the shape of histograms using bimodality

Why is comparing the characteristics of the shape of histograms essential?

A histogram is a graphical method of displaying data, using columns that reflect the distribution and frequency of each data group. As a histogram gives information in a compact and organised manner, it provides quick communication for informed decisions. Understanding how to quickly compare the characteristics of the shape of histograms, using symmetry, skewness and bimodality, is an essential skill used to interpret data.

Comparing the shape of histograms is an essential skill to help us easily interpret data.

WHAT YOU NEED TO KNOW

- A **histogram** is a special type of column graph:
  - The data scores are shown on the horizontal x-axis and are organised into groups, each specified by a range.
  - The y-axis is the frequency (it may also be labelled as the number of things being measured or scored as this is equivalent to frequency).
  - There are no gaps between the columns.
  - Usually a histogram has vertical columns, but they may be horizontal.
- Spread is the measure of the range of the distribution.
- The distribution is also characterised by the shape of the columns. The main ways to describe the shape are symmetry, skewness, number and height of peaks and whether it is uniform.

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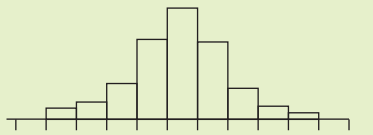
- **Symmetric** (mathematical term for symmetrical) shape is when the columns form a mirror image. (see Histogram 1)
  - The **axis** is the line through the distribution showing each half.
  - The **centre**, or half-way mark, of the shape is found 'by eye', where approximately half of the data is on one side and the rest on the other.

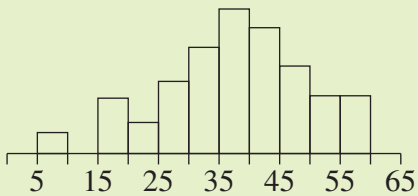
Histogram ①



Symmetric

- **Approximately symmetric** is identified where the shape is approximately the same on both sides. (see Histogram 2)
- The mode of a distribution is the observation that appears most often. It is the value that is most likely to be recorded.

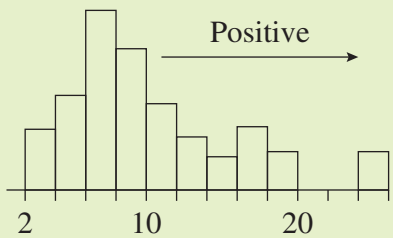
Histogram ②



Approximately symmetric

- **Positively skewed** is when the distribution's peak is higher on the left, with the tail stretching to the right. This is because the shorter columns are towards the positive section of the distribution. (see Histogram 3)

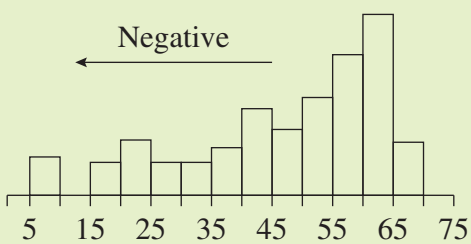
Histogram ③



Skewed

- **Negatively skewed** is when the distribution's peak is higher on the right, with the tail stretching to the left. This is because the shorter columns are towards the negative section of the distribution. (see Histogram 4)

Histogram ④



Skewed

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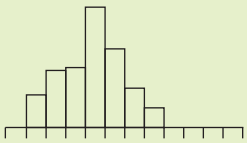
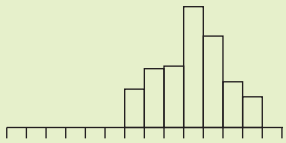
- **Comparing location**: Histograms 5 and 6 are similar in shape; however, they are in different locations on the axis.

Histogram ⑤

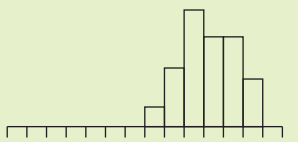


Histogram ⑥

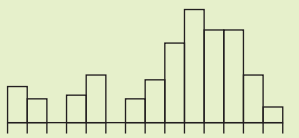


- **Comparing spread**: Histograms 7 and 8 are both centred in a similar location; however, Histogram 8 is more spread out than Histogram 7.

Histogram ⑦

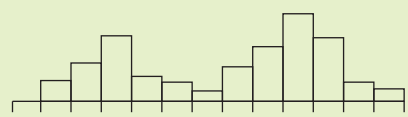


Histogram ⑧



- **Bimodal** is having two modes, which means there is not just one data value that occurs with the highest frequency, but two data values having high frequencies. (see Histogram 9)

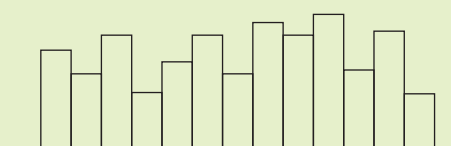
Histogram ⑨



Bimodal

- **Uniform** is also known as 'multimodal distribution' and has a fairly constant or even frequency distribution. (see Histogram 10)

Histogram ⑩



Uniform

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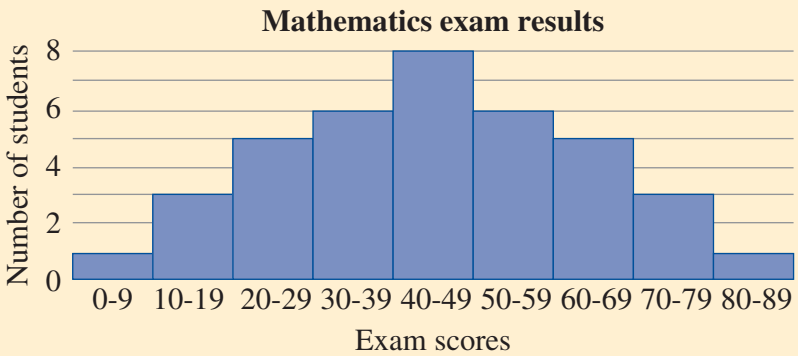
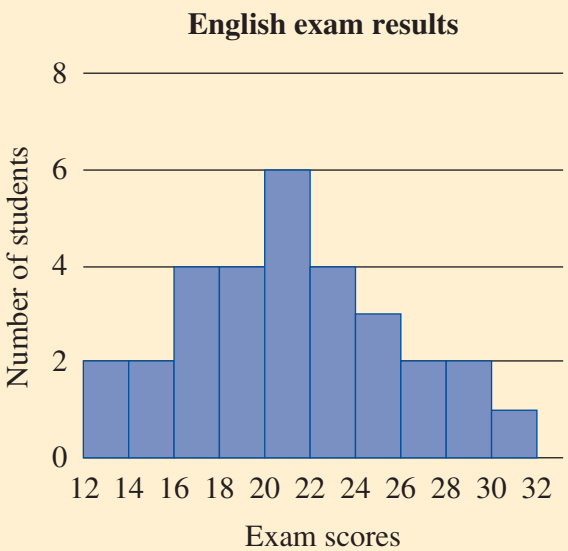
Example 7 Comparing the characteristics of the shape of histograms, using symmetry

The two histograms below display the results obtained by a class on their Maths exam (out of 89) and their English exam (out of 32).





a Compare the shape of both histograms.
b Compare the centre of both histograms.
c Compare the spread of both histograms.

WORKING	THINKING
a The Maths exam histogram is a symmetric shape, whereas the English exam histogram is approximately symmetric.	Which shape best describes each histogram?

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b The Maths exam histogram has a centre of 44.5 out of 89.

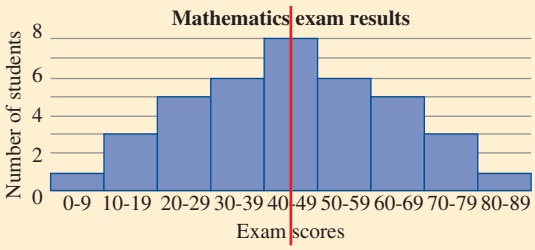
$\frac{44.5}{89} \times 100 = 50\%$

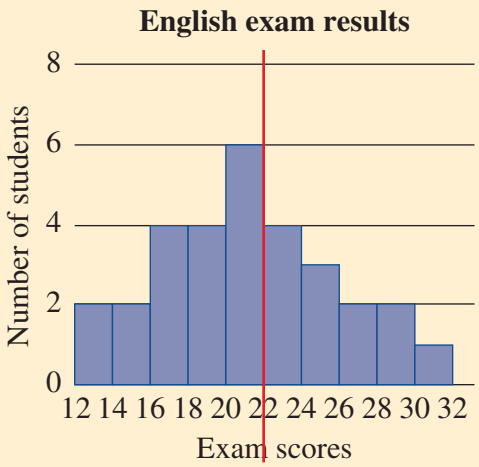
The English exam histogram has a centre of 22 out of 32.

$\frac{22}{32} \times 100 = 68.75\%$

◄····· Where is the centre column?

Use your eye to find the centre.





Therefore, the English exam has a higher centre.

To be able to compare fairly, convert both histograms' centres to a percentage.

c The spread for the Maths exam histogram is

$89 - 0 = 89$

The spread for the English exam histogram is

$32 - 12 = 20$

Therefore, the spread for the Maths exam histogram is greater than for the English exam histogram.

◄····· How are the columns spread across the *x*-axis?

Spread is measured by the range of the distribution.

Find the range of the distributions.

Maximum value – minimum value

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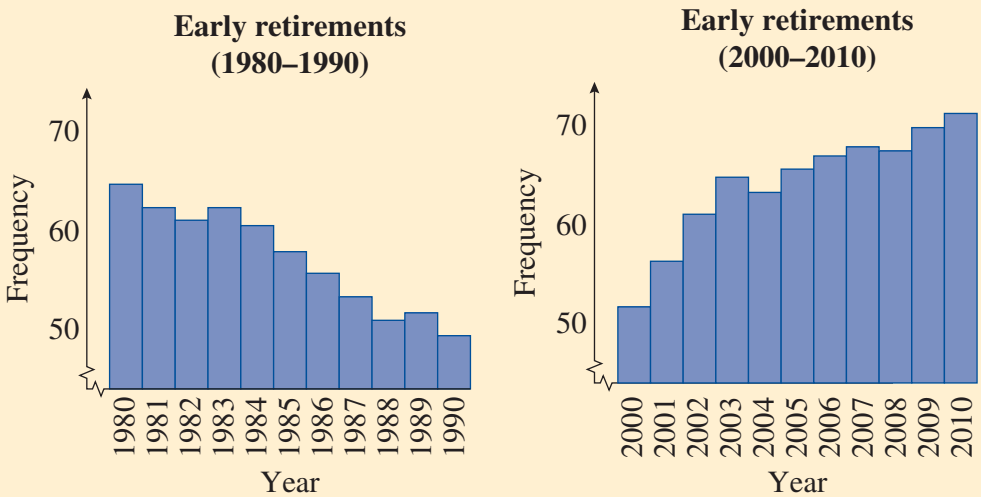
Example 8 Comparing the characteristics of the shape of histograms, using skewness

The two histograms below compare the frequency of early retirements from a company between 1980–1990 and 2000–2010.



a Compare the shape of both histograms.
b Compare the centre of both histograms.
c Compare the spread of both histograms.

WORKING	THINKING
a The 1980–1990 histogram is positively skewed, as the distribution's peak is higher on the left, with the tail stretching to the right. The 2000–2010 histogram is negatively skewed, as the distribution's peak is higher on the right, with the tail stretching to the left. This is because the shorter columns are towards the left of the distribution.	Which shape best describes each histogram?

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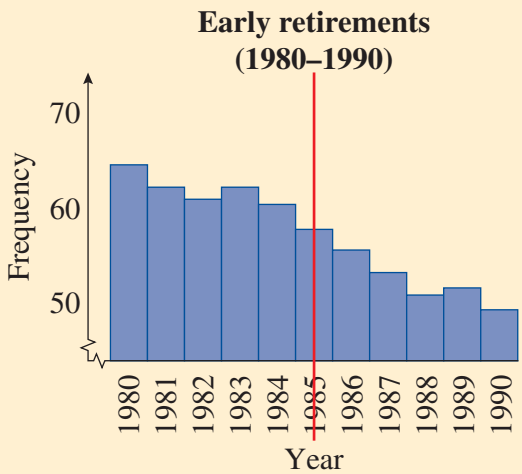
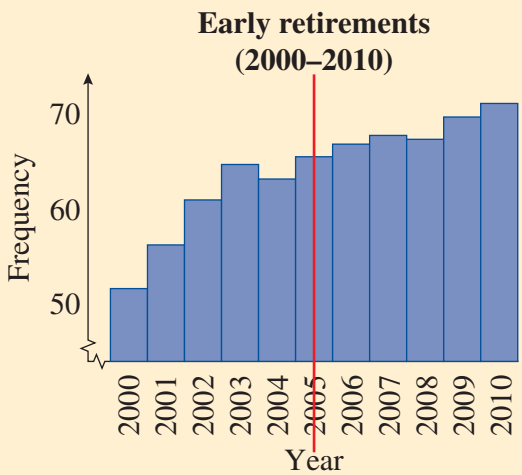
b The 1980–1990 histogram has a centre of 1985.
The 2000–2010 histogram has a centre of 2005.
The centre for both histograms is the 6th year of 11 years. Therefore, the centre is the same for both histograms.

◄··· Where is the centre column?
Use your eye to find halfway along the horizontal axis.





c The spread for the 1980–1990 histogram is across the whole axis (11 years).
The spread for the 2000–2010 histogram is across the whole axis (11 years).
Even though the histograms cover different years, the range is the same size – 11 years.
Therefore, the spread of both histograms is the same.

◄··· How are the columns spread across the *x*-axis?
Spread is the measure of the range of the distribution.
Find the range of the distributions.

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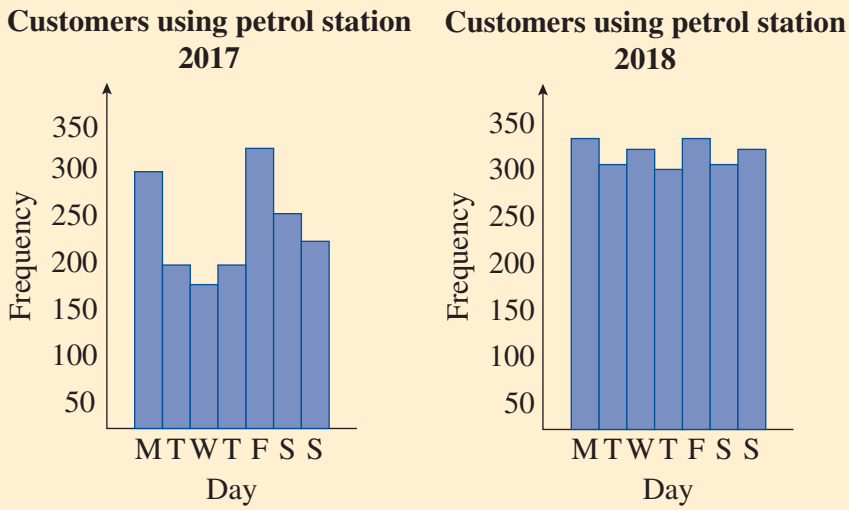
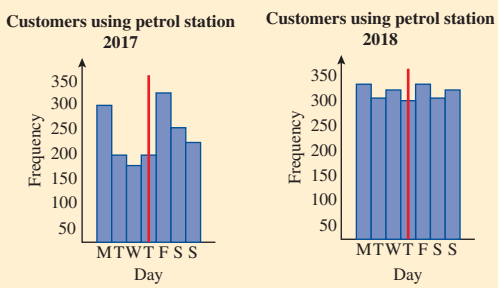
Example 9 Comparing the characteristics of the shape of histograms, using bimodality

The two histograms below compare the frequency of people using a petrol station in 2017 and 2018.



a Compare the shape of both histograms.
b Compare the centre of both histograms.
c Compare the spread of both histograms.

WORKING	THINKING
a The 2017 histogram's shape is bimodal, as it is double-peaked at Monday and Friday. The 2018 histogram's shape is a uniform distribution.	Which shape best describes each histogram?
b The 2017 histogram has a centre of Thursday. The 2018 histogram has a centre of Thursday. Therefore, both histograms have the same centre.	Where is the centre column? Use your eye to find halfway along the horizontal axis.



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c The spread for the 2017 histogram is across the whole axis (1 week).

How are the columns spread across the x-axis?

The spread for the 2018 histogram is across the whole axis (1 week).

Spread is the measure of the range of the distribution.

Therefore, the spread of both histograms is the same.

Find the range of the distributions.

Exercise 9D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Symmetric shape is when the columns form a _____ image.

b Approximately symmetric is identified where the shape is _____ the same on both sides.

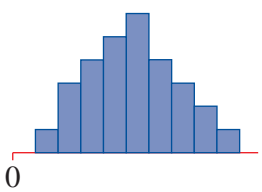
c Positively skewed is when the distribution's peak is higher on the _____, with the tail stretching to the _____.

d _____ skewed is when the distribution's peak is higher on the right, with the tail stretching to the _____.

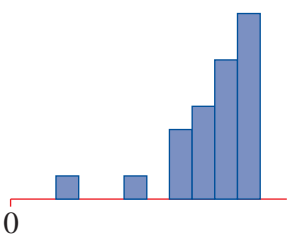
e _____ is having two modes, which means there is not just one data value that occurs with the highest frequency, but _____ data values having high frequencies.

2 Describe the shape of each of the following histograms.

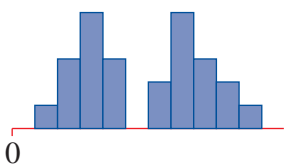
a



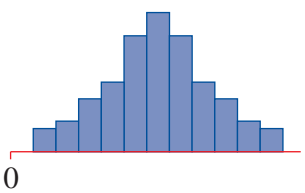
b



c



d



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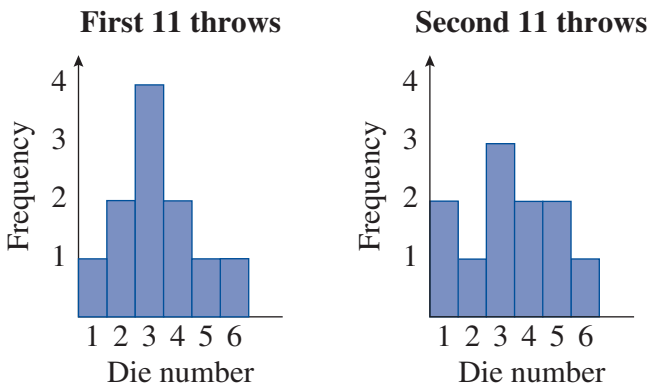
3 Draw a histogram that is:

- **a** symmetric
- **b** approximately symmetric
- **c** positively skewed
- **d** negatively skewed
- **e** bimodal
- **f** uniform

APPLICATIONS

Example 7

4 The scores of the first 11 throws and the second 11 throws of a die are shown in the histograms.



- **a** Compare the shape of the histograms.
- **b** Compare the centre and spread of both histograms.

Hint When finding the centre, use your eye to find halfway along the horizontal axis.

Example 8

5 A wind farm compared the wind speed over two days. The histograms on the next page show the results.

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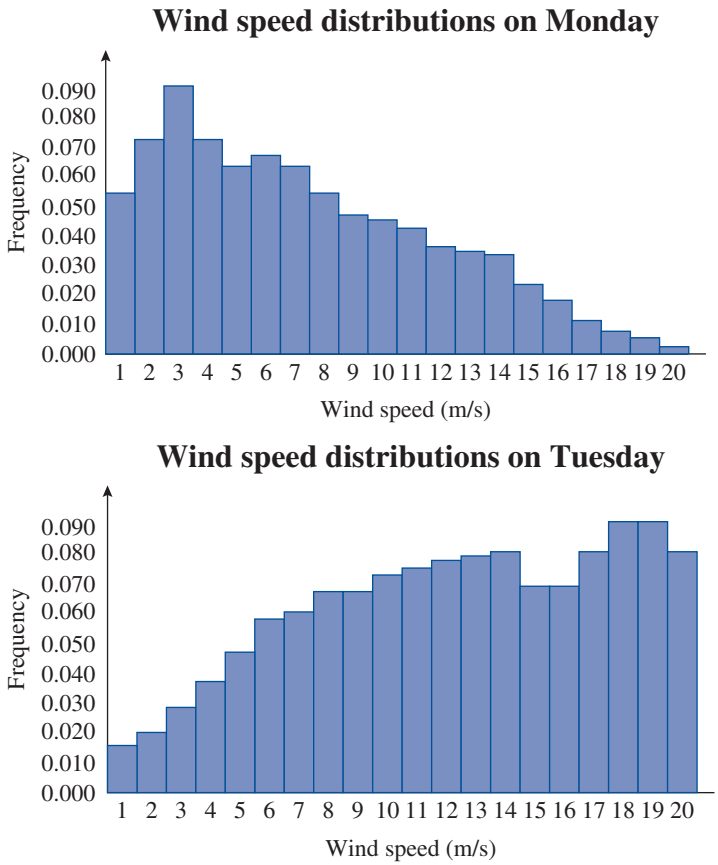


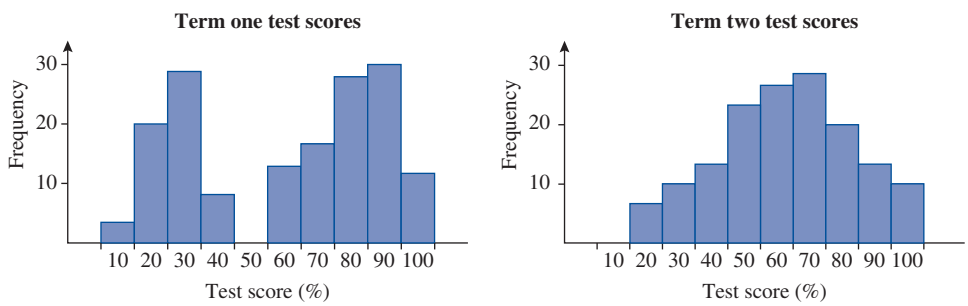
a Compare the shape of the histograms.

b Compare the centre and spread of both histograms.

Hint The spread describes the distribution.

Example 9

6 A teacher uses histograms to display the results of his students' mathematics test scores. The histograms below show the results.



a Compare the shape of the histograms.

b Compare the centre and spread of both histograms.

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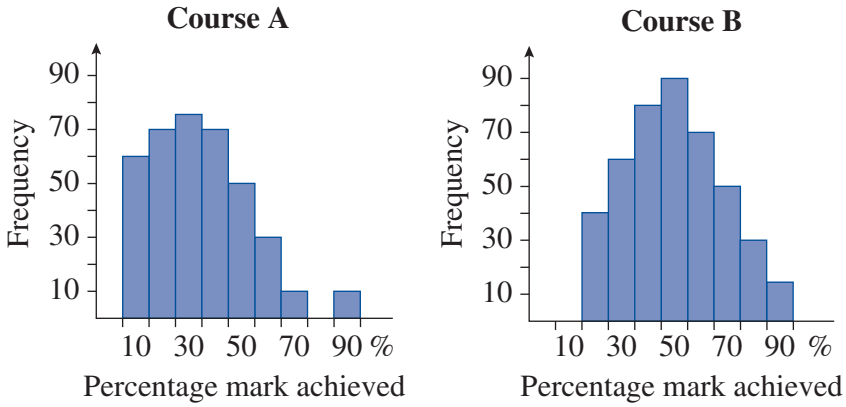
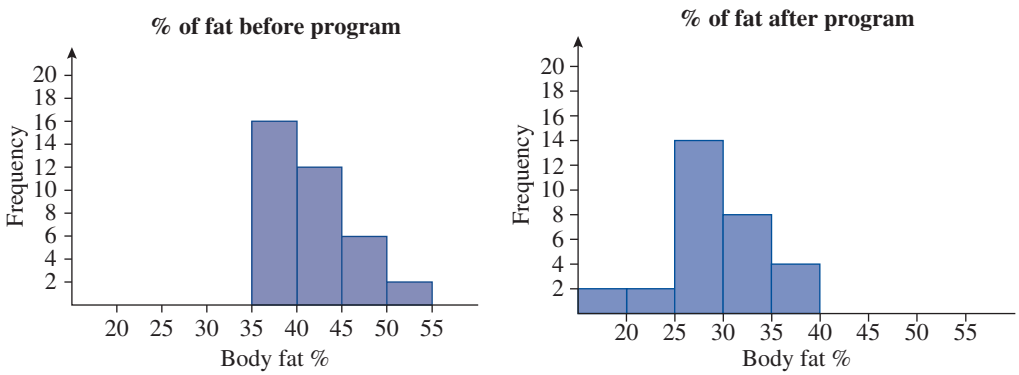
7 A Year 12 student is trying to choose between two TAFE courses. She uses the following histograms to help her decision.



a Write a statement comparing the shape of the histograms.

b Explain which course would give the student a greater chance of passing. Justify your answer.

8 A personal trainer is advertising their new program using the following histograms. Compare the shape of the histograms.



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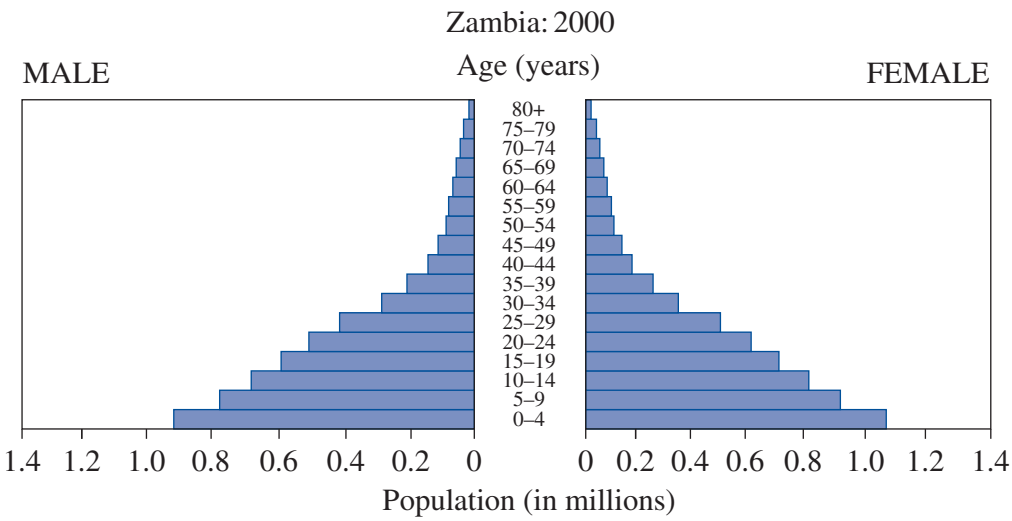
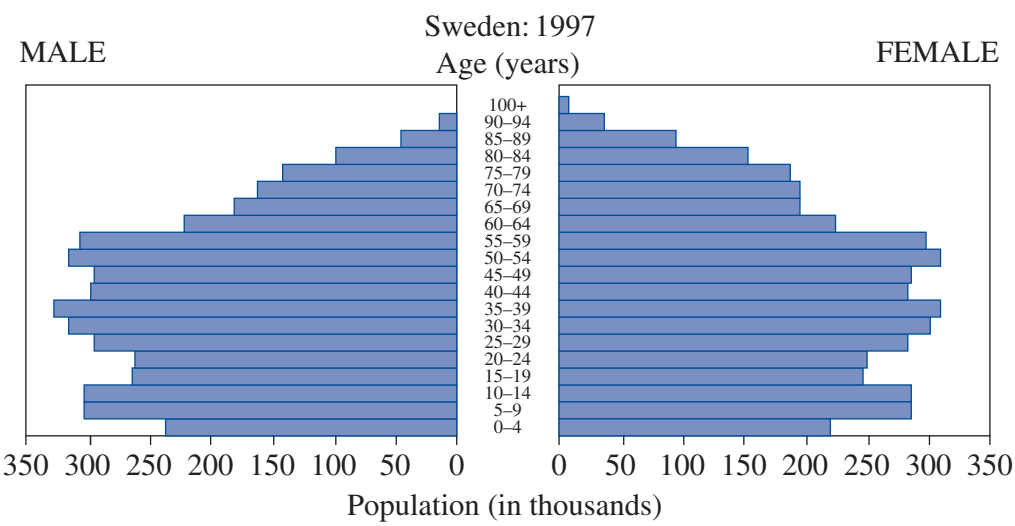
9 The two population distribution histograms below display the numbers of people in a population by age and gender in Zambia and Sweden.

Population distribution of Zambia by age and sex, 2000



Source: U.S. Census Bureau [Internet]. Washington, DC: IDB Population Pyramids [cited 2004 Sep 10]. Available from http://www.census.gov/ipc/www/idb/

Population distribution of Sweden by age and sex, 1997



Source: U.S. Census Bureau [Internet]. Washington, DC: IDB Population Pyramids [cited 2004 Sep 10]. Available from http://www.census.gov/ipc/www/idbpyr.html

a Compare the shape of the male and female histograms for Zambia.

b Compare the shape of the male and female histograms for Sweden.

c What conclusions can be made about the difference in population distribution by age and sex in Zambia and Sweden?

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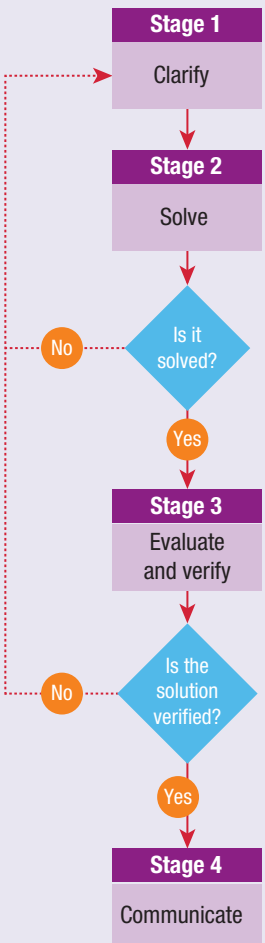
Statistical investigation process task

Background: Various articles have been published about the difference in total earnings between males and females in Australia.

Task: Your task is to find and compare datasets for male and female income in Australia.

From your sources, you need to compile the datasets, display them in an appropriate way and write a short report comparing the datasets. Write a conclusion that addresses whether there is any difference between male and female incomes in Australia.

Use the following approach to your investigation process and modelling tasks.



Stage 1: Clarify
Clarify the problem and pose one or more questions that can be answered with data

1. Research the data sources available.
2. Research the best way to display the data, such as parallel box plots or a back-to-back stem-and-leaf plot.

Stage 2: Solve
Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

3. Examine the datasets and the information that comes with them thoroughly to ensure understanding of each dataset.
4. Organise the two datasets (male total earnings and female total earnings).
5. Construct displays of the two datasets.
6. Decide what comparisons of the two datasets are relevant.

Stage 3: Evaluate and verify
Interpret the results of this analysis and relate the interpretation to the original question

7. Check how reasonable they are.
8. Have you presented them in the most suitable way?
9. Is there enough information to contrast?

Stage 4: Communicate
Communicate findings in a systematic and concise manner

10. Communicate your findings in a short report that introduces the datasets, the displays and the comparison of the datasets.
11. Write a conclusion about any differences between male and female total earnings.

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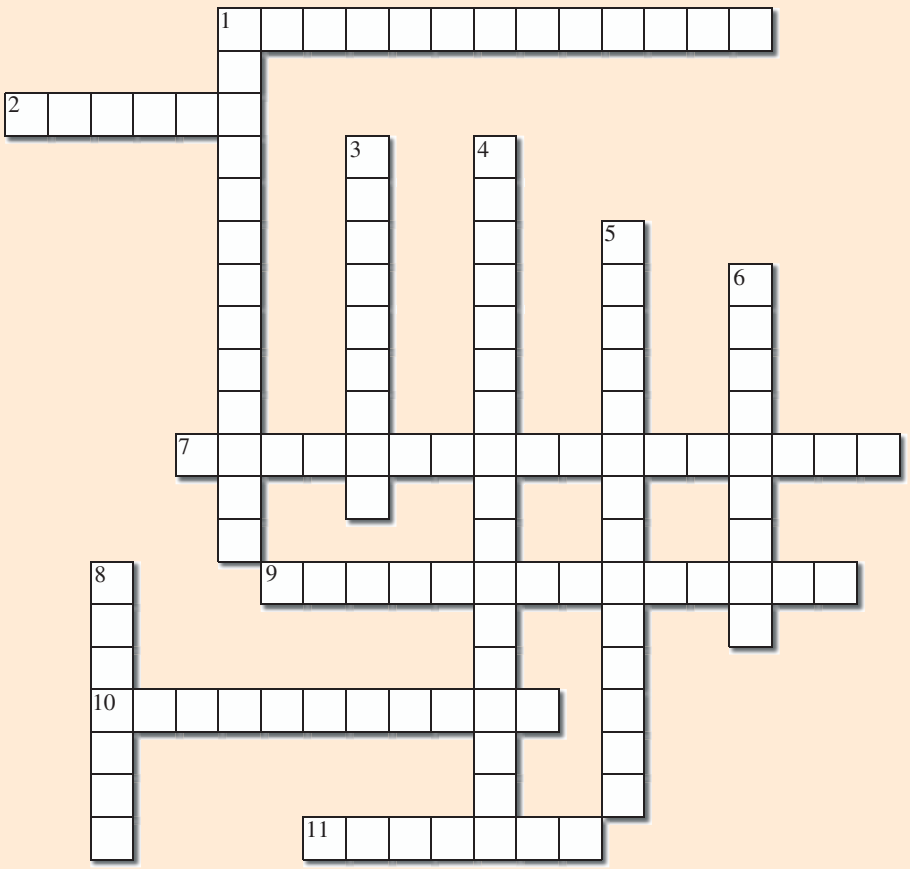
Puzzle



Across

1 The largest number in the set of data
2 The number that falls exactly in the middle (Q2)
7 When the distribution's peak is higher on the left, with the tail stretching to the right
9 The median of the upper half of the data (Q3)
10 To find the _____ summary, the dataset must first be placed in ascending order
11 Having two modes, which means there is not just one data value that occurs with the highest frequency, but two data values having high frequencies

Down

1 The smallest number in the set of data
3 The visual representation of the five-number summary of a dataset
4 When the distribution's peak is higher on the right, with the tail stretching to the left
5 The median of the lower half of the data (Q1)
6 When the columns form a mirror image
8 Also known as 'multimodal distribution' and which has a fairly constant or even frequency distribution

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Chapter checklist

☐ I can recognise the structure of a five-number summary.

1 List the names of the numbers in a five-number summary, in order, from smallest to largest.

2 Write a definition of each of the five numbers.

☐ I can create a five-number summary.

3 Create a five-number summary of the following data.
4, 8, 7, 4, 9, 13, 9, 8, 7

☐ I can create a five-number summary, using technology.

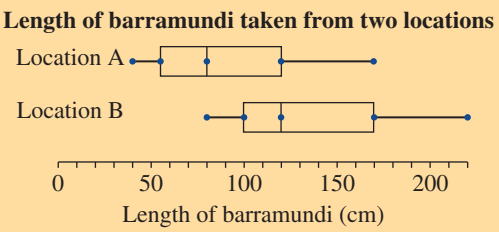
4 Create a five-number summary of the data from question **3**, using technology.

☐ I can construct box plots, using a five-number summary.

5 Construct a box plot for the following dataset.
3, 4, 2, 5, 3, 2, 1, 6, 5

☐ I can compare datasets, using parallel box plots.

6 Compare the length of barramundi found in location A and B from these parallel box plots.



☐ I can compare datasets, using back-to-back stem-and-leaf plots.

7 Compare the number of days till ripening of tomato batches A and B in the back-to-back stem-and-leaf plot at the right.

Number of days to tomatoes ripening

Batch A		Batch B
9	0	
9 8 2 1 0 0	1	1 2 3 3 7 8
	2	0
	3	

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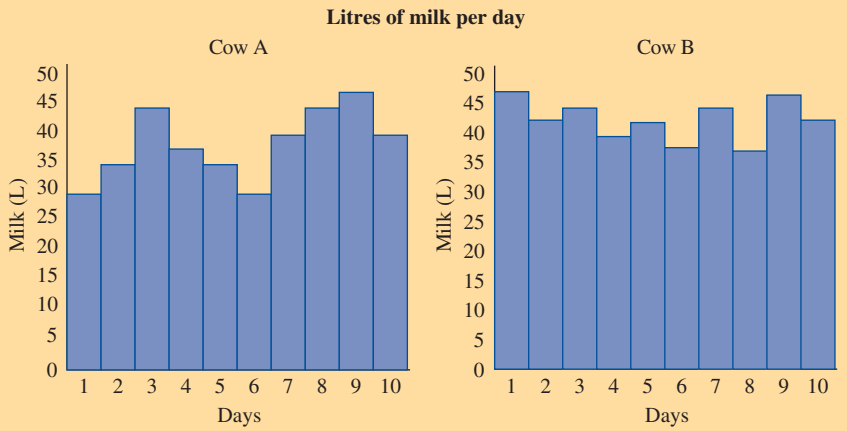
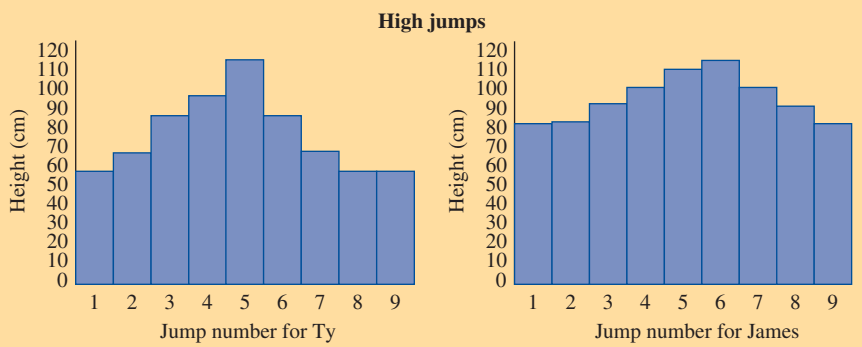
I can compare the characteristics of the shapes of histograms.

8 Compare the shapes of the following histograms, using symmetry, skewness or bimodality as appropriate.

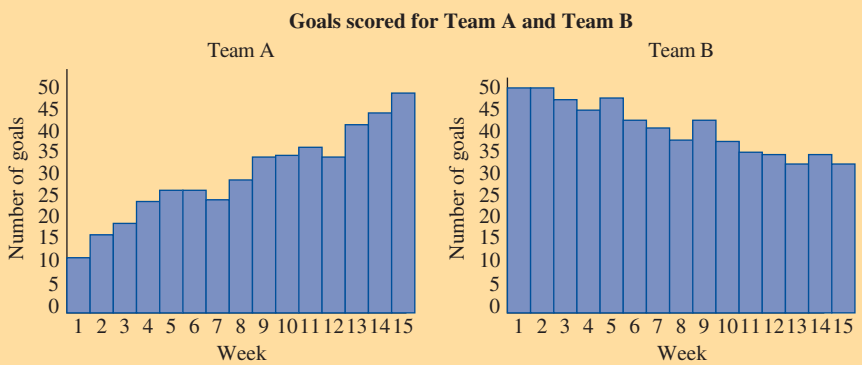
a



b



c



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Chapter review

1 Christine owns a dog shelter and she has recorded the current weights of the dogs in her care. Determine the minimum, Q_1, median, Q_3 and maximum of her current dataset below.

7 kg, 5 kg, 8 kg, 14 kg, 7 kg, 10 kg, 4 kg, 5 kg, 3 kg, 4 kg, 7 kg, 5 kg, 4 kg

Section 9A **2** A soccer coach was preparing for a new season. The following dataset is her team's scores for the previous season. Create a five-number summary, without technology, to assist with their preparation.

2, 1, 0, 3, 2, 0, 4, 1, 1, 2, 1, 2, 3

3 Steve is a diabetic. His blood glucose levels are recorded in mmol/L.

4, 8, 7, 4, 9, 13, 9, 8, 7

Create a five-number summary of his latest results, using technology.

Section 9B **4** A group of Year 11 students were asked to record the number of hours of homework they completed each week. The data below shows the results.

8, 9, 11, 4, 2, 4, 1, 3, 3, 6, 10, 10

a Create a five-number summary, using technology.
b Construct a box plot.

5 Sam records the number of players from each football team who are selected to play in the Western Australia representative team. Construct a box plot from Sam's dataset.

3, 4, 2, 5, 3, 2, 1, 6, 5

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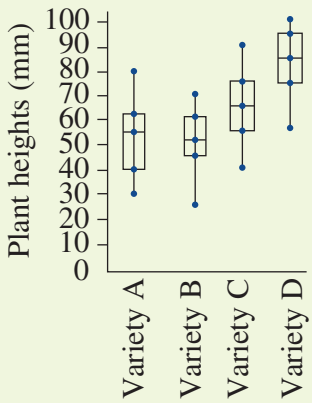
Section 9C **6** The following set of box plots were used to compare the heights of different varieties of a plant.

Distribution of plant heights (mm)



Compare the distributions of the plant varieties' heights, concluding which is tallest and commenting on the variability.

7 The following back-to-back stem-and-leaf plot represents the distributions of life expectancies for males in 15 countries in both 1988 and 2022.

Male life expectancy
(in years)

1988		2022
9	4	
8 6 5	5	5 5
9 9 7 5 0	6	0 0 5 5
7 5 5	7	5 5 5 9 9
5 5 0	8	5 5 5
	9	0

9|4 = 49 years

5|5 = 55 years

a Compare these distributions in terms of median, range and IQR.

b Write a paragraph, concluding with whether male life expectancy, in these countries, changed between 1988 and 2022.

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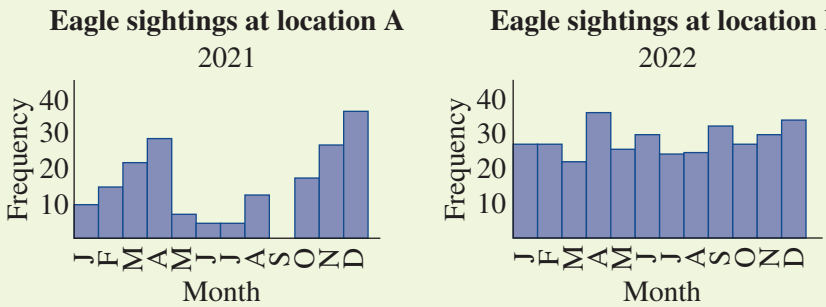
Section 9D

8 An avid bird watcher has been recording wedge-tailed eagle sightings at a particular location for each month in 2021 and 2022. Compare the shape of the two histograms.



9 Complete the following for each of the tables shown.

i Draw the histogram.

ii Determine the shape that the histogram would produce.

a

Digital article sizes	
Bytes per article	**Frequency**
500	1500
1000	3000
1500	5000
2000	5500
2500	6000
3000	7000
3500	7500
4000	7500
4500	9000
5000	8500

b

Drug dispensing times	
Times (minutes)	**Frequency**
0–10	3
11–20	4
21–30	5.5
31–40	7.5
41–50	5.5
51–60	4

c

Petal lengths	
Petal length (cm)	**Frequency**
1	7.5
2	20
3	10
4	12.5
5	27.5
6	12.5

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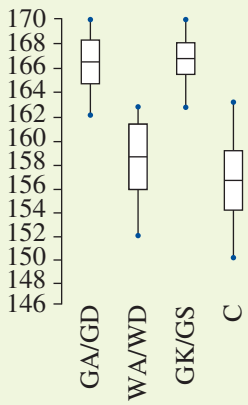
10 The following set of box plots show the heights (in cm) of 100 netball players, grouped according to their positions on their teams. Compare the heights, for the grouped positions, and their variability. Relate the height of the players and the variability to positions that involve goal and those that do not.

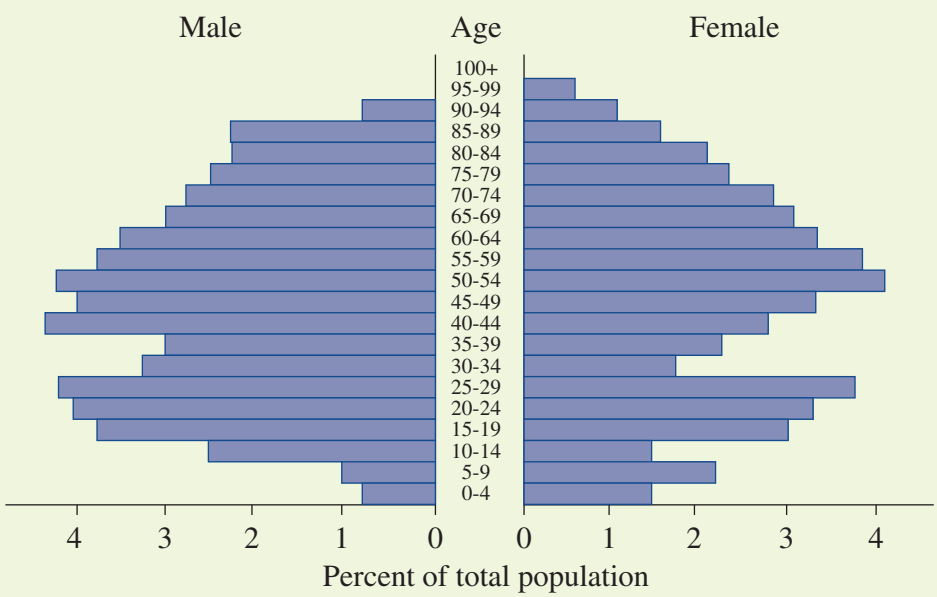
Height (in cm) by position for netball teams



Abbreviations
GA: Goal attack
GD: Goal defence
GS: Goal shooter
GK: Goal keeper
WA: Wing attack
WD: Wing defence
C: Centre

11 Histograms were used to display a population distribution by age and sex from data supplied by a recent census. Compare the shape of the histograms.

Population distribution by age and sex



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10 Percentage calculations and ratios

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In this chapter

Syllabus references

Unit 2 Topic 2.2: Percentages

Percentage calculations

- review calculating a percentage of a given amount
- review one amount expressed as a percentage of another

Applications of percentages

- determine the overall change in a quantity following repeated percentage changes; for example, an increase of 10% followed by a decrease of 10%
- calculate simple interest

Unit 2 Topic 2.3: Rates and ratios

Ratios

- identify common uses of ratios to express comparisons of quantities in practical situations
- use diagrams or concrete materials to show simple ratios, such as 1 to 4, 1:1:2
- understand the relationship between simple fractions, percentages and ratios
- express a ratio in its simplest form
- determine the ratio of two quantities in context
- divide a quantity in a given ratio; for example, share $12 in the ratio 1 to 2
- use ratios to describe simple scales

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Pre-test

1 Calculate the answers to each of the following, without using a calculator.

a 5×8 **b** 7×4 **c** 3×9

d $24 \div 4$ **e** $36 \div 9$ **f** $56 \div 7$

2 Express each fraction in its simplest form.

a $\frac{3}{6}$ **b** $\frac{14}{21}$ **c** $\frac{15}{20}$

d $\frac{12}{48}$ **e** $\frac{50}{70}$ **f** $\frac{250}{400}$

3 Use multiplication to fill in the missing numbers to make equivalent fractions.

a $\frac{3}{4} = \frac{\square}{12}$ **b** $\frac{2}{9} = \frac{6}{\square}$ **c** $\frac{2}{5} = \frac{12}{\square}$

4 Use multiplication or division, as appropriate, to convert the given quantity to the units shown in the brackets.

a 25 cm (m) **b** 1.5 days (h)

c 2500 mL (L) **d** 0.75 m (mm)

e 2.3 kg (g) **f** 1 hour (s)

Hint Write both amounts in the same units.

5 Express the first quantity as a fraction of the second.

a 500 m, 2 km **b** 8 weeks, 1 year **c** 2 L, 4000 mL

d 6000 mm, 15 m **e** 10 h, 1 day **f** 25 mm, 50 cm

6 Calculate:

a $\frac{1}{2}$ of \$30 **b** $\frac{3}{4}$ of \$100 **c** $\frac{5}{8}$ of \$64

7 Determine the highest common factor (HCF) of each pair of numbers.

a 12, 18 **b** 20, 100 **c** 15, 36

Hint The HCF is the largest shared factor.

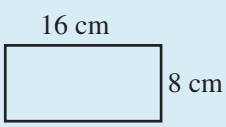
8 Determine the lowest common multiple (LCM) of each pair of numbers.

a 4, 6 **b** 10, 12 **c** 15, 30

Hint The LCM is the smallest shared multiple.

9 For each of these rectangles, determine the simplified ratio of length : width.

a



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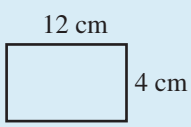

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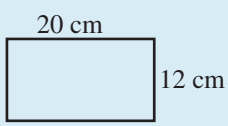

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Mathematics Essential 1&2
ISBN 978-1-009-25785-5

10A Review of using percentages

LEARNING GOAL

- Review the use of percentages

WHAT YOU NEED TO KNOW

- The word per cent means per 100. The symbol % also means per 100.
  - For example: 75 *per cent* 75% 75 parts out of 100 parts
- A percentage is converted to a fraction when the word per cent or the symbol % is changed to a denominator of 100.
  - For example: 45 *per cent* $= \frac{45}{100}$ $45\% = \frac{45}{100}$
- Many percentages of a quantity can be calculated mentally, using equivalent division and multiplication of whole numbers. It is easier to complete the division before the multiplication.
  - For example: 75% of 120 kg $= 120 \div 4 \times 3 = 90$ kg
  - For example: 90% of \$210 $= 210 \div 10 \times 9 = \$189$
- A calculator can be used to calculate a percentage of a quantity.
  - For example: 87.5% of 238 cm. Enter: $87.5 \div 100 \times 238$.
    Answer = 208.25 cm.
- To express one quantity (A) as a percentage of another quantity (B):
  1. Write each quantity in the same units, if necessary.
  2. Write quantity A as a fraction of quantity B: $\frac{A}{B}$.
  3. Multiply the fraction by 100 to calculate the percentage.
  4. When we calculate a percentage, the answer includes the symbol %.
  - For example: Write 45 marks as a percentage of 75 marks.

$$\frac{45}{75} \times 100 = 60\%$$

- The cost price is the amount of money that a business pays for a product.
- When a business sells a product, they make money (i.e. a profit) by having marked up (i.e. increased) the cost price. The mark-up is usually given as a percentage increase of the cost price. The marked price is also called the selling price and the retail price.

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- To calculate the marked price from a given cost price, the mark-up amount, in $ (found from the percentage increase), is added to the cost price of the item.

$$\text{Mark-up amount in \$} = \% \times \text{cost price}$$

$$\text{Marked price} = \text{cost price} + \text{mark-up amount}$$

  - For example: A laptop with a cost price of $600 is marked up by 37.5%.
    Mark-up $= \frac{37.5}{100} \times 600 = \$225$
    Marked price $= \$600 + \$225 = \$825$
- A discount is usually given as a percentage decrease of the marked price. To calculate the discounted price or sale price of an item, the amount of discount is subtracted from the marked price.

$$\text{Discount amount in \$} = \% \times \text{marked price}$$

$$\text{Discounted price} = \text{marked price} - \text{discount}$$

  - For example: A handsaw with a marked price of $50 is discounted by 20%.
    The discount is $\frac{20}{100} \times 50 = \$10$
    The discounted price is $\$50 - \$10 = \$40$.

Example 1 Calculating a percentage of a quantity, without using a calculator

a 10% of $140 **b** 25% of $600 **c** 125% of 500 kg

WORKING	THINKING
a $\$140 \div 10 = \$14$	10% simplifies to $\frac{1}{10}$, so we divide $140 by 10.
b $\$600 \div 4 = \$150$	25% simplifies to $\frac{1}{4}$, so we divide $600 by 4.
c $500\text{ kg} \div 4 \times 5 = 625\text{ kg}$	125% simplifies to $\frac{5}{4}$, so we divide 500 kg by 4 and then multiply by 5.

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Example 2 Calculating a percentage of a quantity, using a calculator

Calculate the given percentage of each quantity, using your calculator. Round each answer to two decimal places.

a 35% of \$1050 **b** 72% of 86.9 kg

WORKING	THINKING
a $\frac{35}{100} \times 1050 = \$367.50$	Enter this into your calculator: $35 \div 100 \times 1050$
b $\frac{72}{100} \times 86.9 = 62.57 \text{ kg}$	Enter this into your calculator: $72 \div 100 \times 86.9$

Example 3 Expressing one quantity as a percentage of another quantity

Use an appropriate calculation to express each of the following as percentages, giving answers to one decimal place.

a 30 km out of 240 km **b** 18 hours out of 1 week

WORKING	THINKING
a $\frac{30}{240} \times 100 = 12.5\%$	Write 30 out of 240 as a fraction. Then multiply by 100 to calculate the percentage.
b $1 \text{ week} = 1 \times 7 \times 24$ $= 168 \text{ h}$	Convert the larger unit (weeks) to the smaller unit (hours). [week $(\times 7) \rightarrow$ days $(\times 24) \rightarrow$ hours]
$\frac{18}{168} \times 100 = 10.7143\%$ $= 10.7\%$	Write 18 out of 168 as a fraction, then multiply by 100. As the second decimal place is 1, which is < 5, round the answer down to the first decimal place of 7.

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Example 4 Applying a percentage increase and decrease to calculate the new price

a An electric bike costs a business $3700 and is marked up by 8.25%. Calculate the marked price of the electric bike

b A set of junior golf clubs has a marked price of $387 and is discounted by 13%. Calculate the marked price of the junior golf clubs

WORKING	THINKING
a $\frac{8.25}{100} \times 3700 = \$305.25$	Find the amount of the mark-up.
$3700 + 305.25 = 4005.25$	Add the mark-up to the original price.
b $\frac{13}{100} \times 387 = \$50.31$	Find the amount of the discount.
$387 - 50.31 = 336.69$	Subtract the discount from the original price.

Exercise 10A

FUNDAMENTALS

Example 1

1 Without using your calculator, use simple division and multiplication with whole numbers for the following percentage calculations.

a 10% of $200 **b** 20% of $1000 **c** 25% of $80 **d** 50% of $842
e 1% of $7000 **f** 75% of $800 **g** 150% of $4000 **h** 120% of $550

Example 2

2 Calculate the following percentages of the given quantities, using your calculator. Round your answers correct to one decimal place.

a 5% of 280 kg **b** 11% of $432
c 12% of 620 km **d** 37% of 128 litres
e 80% of 640 mL **f** 150% of $12 500
g 300% of 140 kg **h** 146% of 232 tonnes

Example 3

3 Use an appropriate calculation to express each of the following as percentages, giving answers to one decimal place where appropriate.

a 5 kg out of 40 kg **b** 6 litres out of 60 litres
c $24 out of $80 **d** 40 MB out of 320 MB
e 42 cents out of $10 **f** 5 days out of 3 weeks
g 90 cm out of 50 m **h** 880 mm out of 3.2 m

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Example 4

4 For each of the following, apply the given percentage increase or decrease to calculate the final marked price.

a mark-up a cost price of $1200 by 10%
b mark-up a cost price of $400 by 15%
c mark-up a cost price of $160 by 12.5%
d mark-up a cost price of $35 by 20%
e discount a marked price of $240 by 25%
f discount a marked price of $78 by 4.5%
g discount a marked price of $299 by 17.25%
h discount a marked price of $28 800 by 8.9%

APPLICATIONS

5 Jake drives 25% of the 1184-km distance from Geraldton to Exmouth. Calculate the distance that Jake drives.

6 Brian receives a $240 discount on a lawn mower because it is the demonstration model. It was originally $699.

a Write the discount as a fraction of the original cost of the mower.
b Use the fraction from part **a** to determine the percentage discount that Brian receives.

7 At DenimStop, Lillie wishes to buy a pair of jeans marked at $69.95. If the store offers her a 15% discount, calculate the final selling price of the jeans.

8 Toni imports 50-inch televisions from Taiwan at a cost of $295 each and then sells them in her stores. Toni marks up these televisions by 150%. Determine the marked price of each television.

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10B Determining overall change from multiple percentage changes

LEARNING GOALS

- Calculate new amounts after multiple percentage changes
- Determine overall price changes in real-world contexts

Why is it essential to understand overall change from multiple percentage changes?

Our economy relies on a network for distributing manufactured or farmed products, which are then made available to consumers. Distribution channels include producers, importers, wholesalers, retailers, agents, dealers and the internet. It is important to realise that, at every level of exchange within the network, all those involved have expenses to cover, requiring various profit margins.

If a forklift driver earns a 2% pay increase, giving them a $1000 p.a. gross pay increase, and this amount is decreased by 32.5% tax, their net pay increase is $675.

When a worker receives a pay rise, the accountant first increases the wage by the given percentage, and then this new pay amount is decreased by the tax percentage, resulting in the final net (i.e. 'take-home') pay.

WHAT YOU NEED TO KNOW

- Multiple percentage changes can be applied to calculate the overall change from an original amount.

$$\text{Overall change in \$} = \text{final amount in \$} - \text{original amount in \$}$$

- Each percentage change, which can be either a percentage increase or a percentage decrease, must be calculated step-by-step.

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- The first percentage change is calculated using the original amount and then, after that, each percentage change is calculated on the previous changed amount.
- Consecutive percentage changes can be any combination of percentage increases and decreases.
  - For example: \$400 is increased by 50% and then decreased by 10 %.

    Increase = 50% of \$400 = \$200 ← + \$ amount for % increase.
    New amount = \$400 + \$200 = \$600
    Decrease = 10% of \$600 = \$60 ← – \$ amount for % decrease.
    Final amount = \$600 – \$60 = \$540
    Overall change = final amount – original amount
    = \$540 – \$400 = \$140
- If the overall change is negative, then a loss has been made.

Example 5 Calculating new amounts from multiple percentage changes

Determine the overall change in the following.

a \$130 is increased by 15% and then that amount is increased by 22%. State the amount of loss or gain.

b \$1000 is increased by 20% and then that amount is decreased by 20%. State the amount of loss or gain.

WORKING	THINKING
a First increase:	Write 'First increase'.
$\frac{15}{100} \times \$130 = \$19.50$	Calculate 15% of \$130.
\$130 + \$19.50 = \$149.50	Add that amount to the original amount.
Second increase:	Write 'Second increase'.
$\frac{22}{100} \times \$149.50 = \$32.89$	Calculate 22% of the new amount of \$149.50.
\$149.50 + \$32.89 = \$182.39	Add that amount to the \$149.50.
Overall change:	Write 'Overall change'.
\$182.39 – \$130 = \$52.39	Overall change = final amount – original amount
A gain of \$52.39.	The positive result indicates a gain of \$52.39.

... Continued

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b Increase: ◄····· Write 'Increase'.

$\frac{20}{100} \times \$1000 = \$200$ Calculate 20% of $1000.

$\$1000 + \$200 = \$1200$ Add that amount to the original amount.

Decrease: Write 'Decrease'.

$\frac{20}{100} \times \$1200 = \$240$ Calculate 20% of the new amount of $1200.

$\$1200 - \$240 = \$960$ Subtract the amount $240 from $1200.

Overall change: Write 'Overall change'.

$\$960 - \$1000 = -\$40$ Overall change = final amount – original amount

Overall, a loss of $40. The negative result shows there is a loss of $40.

Example 6 Determining overall price changes in real-world contexts

a Riley made a cubby house, which cost him $180 in timber, paint and nails. He sold it, making a profit of 125%. Calculate Riley's profit and his selling price.

b George bought Riley's cubby house but left it out in the weather, causing the paint to fade and the wood to deteriorate. George then sold it at a loss of 60%. Calculate the amount of money George lost and his selling price.

c Determine the overall change in price of the cubby house, from the amount it cost Riley to build to George's selling price.

WORKING	THINKING
a $\text{Riley's profit} = \frac{125}{100} \times \$180$	◄···· Use words at each step to indicate what is being calculated.
$= \$225$	
$\text{Riley's selling price} = \$180 + \$225$	Calculate 125% of $180. Then add that amount to the original amount.
$= \$405$	
b $\text{George's loss} = \frac{60}{100} \times \$405$	◄···· Calculate 60% of the new amount of $405.
$= \$243$	
$\text{George's selling price} = \$405 - \$243$	Then subtract that amount from $405.
$= \$162$	

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c Overall change in price $= \$162 - \$180$
$= -\$18$

◄···· Overall change = final amount – original amount
The negative result shows there is a loss of $18.

George sold the cubby at a loss of $18, compared to Riley's cost of the materials to build it.

Write your answer as a sentence.

Exercise 10B

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

a Multiple _____ changes can be applied to calculate the overall _____ from the original amount.

b _____ merely add and subtract all the percentages.

c Each percentage _____ or _____ must be calculated _____ by _____.

d The first percentage change is calculated using the _____ amount, and then each _____ change is calculated on the _____ changed amount.

e Overall change = _____ _____ – _____ _____

Example 5

2 Determine the overall change that has occurred when:

a $40 is increased by 60% and then increased further by 25%.

b $122 is increased by 20% and then increased further by 12%.

c $546 is increased by 24% and then increased further by 7.5%.

d $11 800 is increased by 35% and then increased further by 55%.

e $300 is increased by 20% and then decreased by 30%.

f $340 000 is increased by 6.5% and then decreased by 9.75%.

g $548 is decreased by 12% and then increased by 10.75%.

h $250 is decreased by 20% and then increased by 10%.

Shipping costs increase the price of imported goods by a percentage.

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3 **a** Increase $100 by 20%.
b Increase $100 by 10% and then increase that result by a further 10%.
c Explain why increasing $100 by 10% and then by 10% again is not the same as increasing $100 by 20%.

APPLICATIONS

4 One apple costs the farmer 22 cents to produce, ready for sale. The farmer sells the apple with a price increase of 75% to the retailer, and the retailer sells it to the customer with a price increase of 175%.
a Calculate the cost of the apple for the retailer.
b Calculate the price of the apple to the customer.

5 Heidi was struggling with Maths and scored 52 out of 100 in her exam. She studied hard and improved her mark by 28% of her previous test result. Heidi saw that she could do the work and was so pleased with her increased result that she studied even harder and had extra help from the teacher. She improved a further 38% on her second test result.
a Calculate Heidi's final mark out of 100.
b Determine how many marks, out of 100, Heidi improved overall.

6 Mr and Mrs Capps built their house in 1960 for $15 000. They sold it in 2020 with an increase of 3200%.
a Determine the price that the house sold for in 2020.
b In 2022 the house was renovated and then sold for a profit of 21%. Calculate the selling price of the renovated house.
c Determine by what amount the house has increased in value from when it was built in 1960 to the 2022 sale.

7 Hannah is a childcare worker in Geraldton, earning a gross wage of $42 000 per annum.
a Hannah is given a 1.5% pay increase. Calculate her gross wage increase.
b Hannah's employer must pay 32.5% tax on her wage increase. Calculate the actual net increase in pay that Hannah receives.

Hint Gross income is the amount before tax. Net income is the amount after tax.

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Example 6

8 Nikita purchased a bicycle from jBay for $328, including postage, and tried to sell it for a profit of 50% at the markets. By the end of the day, she had discounted her price on the bike by 30%. How much profit or loss did Nikita make?

9 A prawn farmer grows and produces prawns at a cost of $4/kg. The farmer sells the prawns to the fish market with a percentage increase of 70%. The fish market sells them to the retailer with a mark-up of 35%, and the retailer sells the prawns to the customer with a mark-up of 110%. Calculate the overall cost increase, from farmer to customer, of 1 kg of prawns.

10 Fenton's water tank contains 12 000 litres of water. It doesn't rain for a month, and after using the water for his garden, the tank's water volume has decreased by 81%. Calculate the overall change in water volume of the tank, in litres.

11 Giselle is a grape farmer and has harvested 63 tonnes of grapes. Unfortunately, a heat wave meant that she lost 17.24% of the grapes due to spoilage. The truck carrying the rest to market rolled over and another 47% were destroyed. Determine how many tonnes of grapes Giselle lost altogether.

12 Gary has $17 400 worth of shares in insurance stocks. The market is very volatile and the shares initially increase by 28%. A bad credit report sees the shares decrease by 40% and news of a merger with a larger company means the shares increase by a further 22.8%. Calculate the overall percentage increase or decrease in the value of Gary's shares.

Company shares are bought and sold daily on the stock market, resulting in many percentage changes.

ISBN 978-1-009-25785-5

10C Calculating simple interest for different rates and time periods

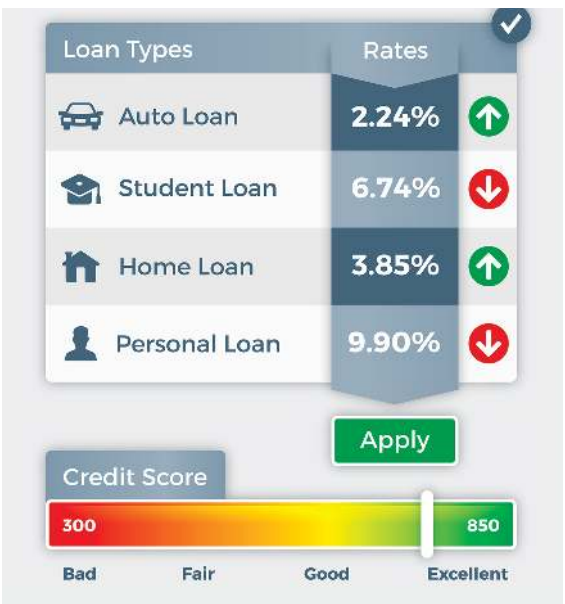
LEARNING GOALS

- Convert time periods to years
- Convert interest rates to rates p.a. (**per annum**; i.e. per year)
- Calculate simple interest, using the simple interest formula
- Calculate simple interest in real-world contexts

Why is an understanding of simple interest essential?

Using **simple interest** is essential to understanding how **interest** rates work in the world of finance. The knowledge of how simple interest is calculated shows how money lenders make profits using interest rates. It is also required knowledge for developing an understanding of compound interest in the future.

Calculating the total cost of any loan must include repaying the loan plus the interest to be paid. Applying the skills of interest calculations is a priority for personal financial success.



Interest rates can vary depending on what the loan is used for and the borrower's credit rating.

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WHAT YOU NEED TO KNOW

- The simple interest formula

$$I = Pin$$

where I is the simple interest, in $.
P is the **principal** amount or initial amount, in $.
i is the interest rate p.a. (i.e. per year), as a decimal.
n is the number of years.
It is more common for interest rates to be given as a percentage interest rate per annum. You can use the figure for the percentage interest rate in the formula if you divide the percentage interest rate per annum, i, by 100. The formula then becomes:

$$I = \frac{Pin}{100}$$

where now i = percentage interest rate per annum.

- Conversion of time units

1 year = 4 quarters = 12 months = 26 **fortnights** = 52 weeks = 365 days

Example 7 Converting time periods to years

Calculate the value of n, in years, if the time period is 21 months.

WORKING	THINKING
$n = 21 \div 12$ $= 1.75$ years	Recall that 12 months = 1 year. Divide the time period by 12 to get the number of years.

Example 8 Converting interest rates to rates per annum

Calculate the value of i, the interest rate per year, if the interest rate per time period is 0.25% per week.

WORKING	THINKING
$i = 0.25\% \times 52$ $= 13\%$ p.a.	Recall that 52 weeks = 1 year. Interest rate per week × 52 = rate p.a. (per year)

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Example 9 Calculating simple interest, using the formula

Calculate the simple interest, I, to be paid on a loan of \$420 000 over a total of 25 years at 0.3% per fortnight.

WORKING	THINKING
$P = 420\,000$	Start by listing P, i and n.
$i = 0.3\% \times 26$ $= 7.8\%$ p.a	Convert the interest per period to interest p.a. (per year). Recall that 26 fortnights = 1 year, so multiply 0.3 by 26.
$n = 25$ years	
$I = \frac{Pin}{100}$	Write the formula.
$I = \frac{420\,000 \times 7.8 \times 25}{100}$	Substitute $P = \$420\,000$, $i = 7.8\%$ and $n = 25$ years into the formula.
$I = \$819\,000$	Calculate the simple interest.

Example 10 Calculating simple interest in real-world contexts

Nadia invests \$17 400 of her savings for 9 months in a term deposit that is paying an interest rate of 7.45% p.a. Calculate how much simple interest she will receive.

WORKING	THINKING
$P = \$17\,400$	Start by listing the values for P, i and n.
$i = 7.45\%$	
$n = 9 \div 12 = 0.75$	To convert months to years, divide by 12.
$I = \frac{Pin}{100}$	Write the formula.
$I = \frac{17\,400 \times 7.45 \times 0.75}{100}$	Substitute $P = \$17\,400$, $i = 7.45\%$ and $n = 0.75$ years into the formula.
$I = \$972.23$	Calculate the interest and round your answer to two decimal places.

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Exercise 10C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words or symbols in the following statements.

i When using the simple interest formula, $I = \frac{Pin}{100}$:

a I is the simple _____ in __.

b P is the _____ amount or _____ amount, in $.

c i is the interest _____.

d n is the number of _____.

e **p.a.** is short for ____ _____.

ii 1 year = __ quarters = __ months = __ fortnights = __ weeks = ___ days

Example 7

2 Calculate the value of n, in years, if the time period is:

a 6 quarters

b 3 quarters

c 18 months

d 27 months

e 52 fortnights

f 65 fortnights

> **Hint** n = number of years
> 4 quarters = 1 year
> 12 months = 1 year
> 26 fortnights = 1 year
> 52 weeks = 1 year
> 365 days = 1 year

Example 8

3 Calculate the value of i, the interest rate per year, if the interest rate per time period is:

a 1.5% per month

b 0.75% per month

c 0.125% per fortnight

d 0.875% per fortnight

e 0.2% per week

f 0.7% per week

> **Hint** i is the rate p.a. (per year).

Example 9

4 Use the formula $I = \frac{P\,i\,n}{100}$ to calculate the simple interest to be paid on the following loans.

a $4000 over 2 years at 8.5% p.a.

b $8400 over 3 years at 4% p.a.

c $15 600 over 5 years at 3.75% p.a.

d $38 000 over 7 years at 4.5% p.a.

e $5500 over 4 years at 8% p.a.

f $13 200 over 5 years at 6.5% p.a.

> **Hint** Start by writing:
> P =
> i =
> n =
> $I = \frac{Pin}{100}$

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5 Use the formula $I = \frac{P\,i\,n}{100}$ to calculate the simple interest to be paid on a loan amount of:
- **a** $21 000 over a total of 4.5 years at 0.6% per month
- **b** $3800 over a total of 3 years at 2% every 6 months
- **c** $5200 over a total of 4 years at 2% per quarter

6 Calculate the simple interest earned on an amount of $420 000 invested for 25 years at 0.3% per fortnight.

7 Calculate the simple interest earned on:
- **a** $2700 invested over a total of 30 months at 12.75% p.a.
- **b** $10 800 invested over a total of 13 weeks at 15% p.a.
- **c** $20 000 invested over a total of 13 fortnights at 9% p.a.

8 Calculate the simple interest to be paid on $4000 borrowed for 39 weeks at 3.76% p.a.

APPLICATIONS

Example 10

9 Howard borrows $25 000 to be repaid over 3.5 years at a simple interest rate of 13.5% p.a. Using the simple interest formula, calculate how much Howard will need to repay in total.

10 Rachel invests $8200 for 18 months, earning 3.8% p.a. Use the simple interest formula to calculate the amount of interest Rachel will receive on her investment.

11 Simon borrows $320 000 to buy a house. Use the simple interest formula to calculate an estimate of the total interest he will pay for a loan period of 25 years at a rate of 0.5% per month.

12 Alexandrea obtains a short-term loan of $1800 to pay for her car to be repaired. Determine the total she will need to repay in one month's time if the simple interest rate on this loan is 28% p.a.

13 Jay buys some new recliner chairs for his home theatre. They cost $5800 and Jay takes up the store's offer of a 30-month loan at 19.6% p.a. to pay for them. Use the simple interest formula to calculate the total amount Jay will pay for the lounge chairs.

Taking out a loan to buy items greatly increases the purchase price.

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14 Ethan invests $68 000, earning a simple interest rate of 0.35% per month. Calculate how much interest he will earn in:

a 1 year **b** 1 day

15 Rhiannon wishes to buy a car worth $6200. She is offered two options for finance.
Option 1: A short-term loan of $6200 with repayment over 33 months at 11.5% p.a.
Option 2: A longer-term loan of $6200 with repayment over 5 years at 7.9% p.a.
Determine which of the two loans is the cheapest option for Rhiannon if she borrows $6200.

16 Penny inherits $20 000 and invests it in a bank term deposit for 6 months. The following spreadsheet compares the two different interest rates that the bank offers, with interest paid either every fortnight or every month. Copy the information into a spreadsheet and fill in the rest of the document.

a The formula for C3 is =B3∗6%/26. The formula for B4 is =B3+C3. Use this information to deduce the formulas used in F3 and E4.

b Create this spreadsheet and fill down the columns to complete it.

c Use the spreadsheet to determine which is the better investment option.

	A	B	C	D	E	F	G
1	**Penny's Term Deposit**						
2		**Amount**	**Interest 6%**			**Amount**	**Interest 6.1%**
3		$20,000.00	$46.15			$20,000.00	$101.67
4	fortnight 1	$20,046.15	$46.26		Month 1	$20,101.67	$102.18
5	fortnight 2				Month 2		
6	fortnight 3				Month 3		
7	fortnight 4				Month 4		
8	fortnight 5				Month 5		
9	fortnight 6				Month 6		
10	fortnight 7				Term Deposit Total		
11	fortnight 8						
12	fortnight 9						
13	fortnight 10						
14	fortnight 11						
15	fortnight 12						
16	fortnight 13						
17	Term deposit total						
18							

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Mathematics Essential 1&2
ISBN 978-1-009-25785-5

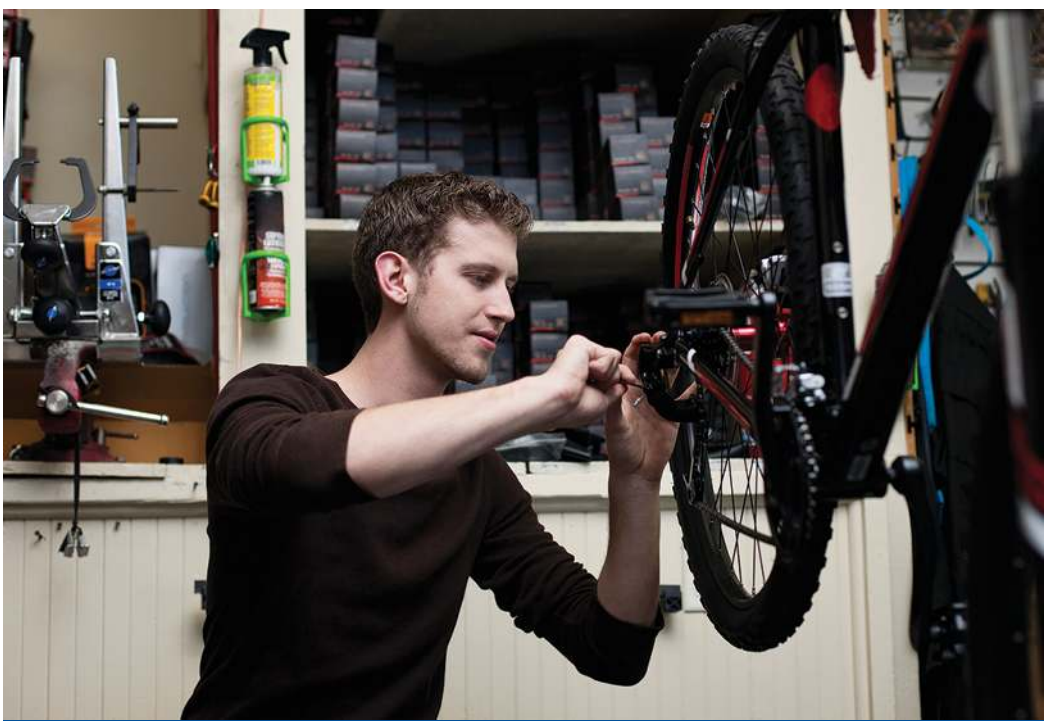
10D Understanding ratios and fractions

LEARNING GOALS

- Consider the relationship between ratios and fractions
- Understanding ratio order using words and numbers
- Recognise that ratios compare two or more quantities
- Identify ratios in real-world circumstances

Why are ratios essential?

Ratios are found often in many aspects of life. They help with comparing prices in supermarkets, mixing oil and petrol to create fuel for cars, mowers and boats, and with administering medicine in hospitals. You use ratios whenever you're deciding how to break up a restaurant bill between you and your friends.



Bicycle designers, mechanics and competitors use gear ratios to calculate the roll-out distance per pedal revolution. Altering gear ratios can increase acceleration and maximum speed.

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WHAT YOU NEED TO KNOW

- Ratios show the relationship between two or more related quantities.
  - For example: A boat's outboard motor fuel mix of oil to petrol is a 1 : 40 mix; that is, 1 mL of oil to 40 mL of petrol.
- A **ratio** compares amounts with the same units but it is written without units.
- The **colon ':'** is the mathematical symbol that operates the same way as the horizontal line in a fraction.
- The ratio $a : b$ can be read as the ratio of 'a to b' or 'a is to b', which is the same as $\frac{a}{b}$.
- The order of the writing of a ratio is important. The word '**respectively**' means 'in the same order'.
- The numbers in a given ratio follow the same order as they are listed in a sentence.
- Ratios should be written in the same units before they are simplified.
- When converting a ratio to a fraction, the first number of the ratio is the numerator and the second number is the denominator. Reverse this procedure when writing a fraction as a ratio.

$$\frac{\text{numerator} \quad \rightarrow 5}{\text{denominator} \rightarrow 8} = 5 : 8$$

Example 11 Converting between ratios and fractions

a Express $34 : 63$ as a fraction.

b Express $\frac{19}{32}$ as a ratio

WORKING		THINKING
a $34 : 63 = \frac{34}{63}$	◄	The first part of the ratio is the numerator, and the second part of the ratio is the denominator.
b $\frac{19}{32} = 19 : 32$	◄	The numerator is the first part of the ratio, and the denominator is the second part of the ratio.

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Example 12 Understanding the order of a ratio

Use words and numbers to write the given ratio, then interpret the ratio.

a When making bread, Julie uses flour and liquid in the ratio of 5 : 3.

b At a school camp, the ratio of students to teachers is 13 : 2.

WORKING	THINKING
a Flour : liquid = 5 : 3 For every 5 parts of flour there are 3 parts of liquid.	The question lists flour first, then liquid.
b Students: teachers = 13 : 2 For every 13 students there are 2 teachers.	The question lists students first, then teachers.

Example 13 Writing ratios that compare two or more quantities

A fruit bowl contains 3 bananas, 6 apples and 5 oranges. Use this information to identify the following ratios.

a bananas to oranges

b apples to bananas to oranges

c oranges to the total number of pieces of fruit

WORKING	THINKING
a 3 : 5	Write the number of bananas (3) to the number of oranges (5).
b 6 : 3 : 5	Write the number of apples (6) to the number of bananas (3) to the number of oranges (5).
c 5 : 14	Write the number of oranges (5) to the total number of fruit (3 + 6 + 5 = 14).

Example 14 Writing ratios in a real-world context

A batch mix of garden fertiliser includes 7 kg of nitrogen, 4 kg of phosphorus and 2 kg of potassium.

a Determine the ratio of nitrogen to potassium.

b Determine the ratio of phosphorus to the other listed nutrients.

WORKING	THINKING
a 7 : 2	Units are not included.
b 4 : 9	Nitrogen and potassium = 7 + 2 = 9

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Exercise 10D

FUNDAMENTALS

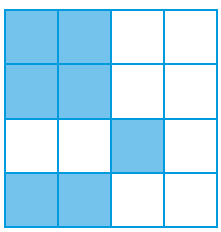
1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

- **a** The _____ of the writing of a ratio is important.
- **b** The numbers in a given ratio follow the _____ order that the items are listed in a sentence.
- **c** Ratios should be written in the same _____ before they are simplified.
- **d** A simplified ratio compares amounts with the _____ units but it is written _____ units.

2 For each diagram, complete the following tasks.

- **i** Determine the ratio of shaded parts to unshaded parts.
- **ii** Determine the fraction of shaded parts to total parts.
- **iii** Determine the ratio of unshaded parts to total parts

a



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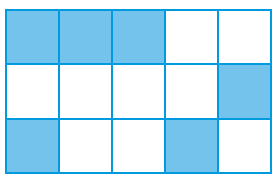
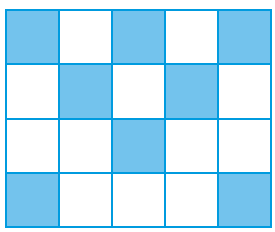


c



Example 11

3 Express the following fractions as ratios.

a $\frac{2}{3}$ **b** $\frac{8}{13}$ **c** $\frac{22}{37}$

d $\frac{45}{78}$ **e** $\frac{15}{2}$ **f** $\frac{9}{4}$

g $\frac{14}{19}$ **h** $\frac{7}{12}$

Hint: numerator $\rightarrow 7$, denominator $\rightarrow 12$ $= 7 : 12$

Example 12

4 Express the following ratios as fractions.

a 3 : 5 **b** 6 : 11

c 31 : 211 **d** 42 : 77

e 7 : 15 **f** 8 : 17

g 9 : 5 **h** 57 : 36

Example 13

5 **a** When students attend a swimming excursion, there are regulations that state that the student to teacher ratio should be 12 : 1. Use words and numbers to write the given ratio, then interpret the ratio.

b When making a cake mix, flour and sugar are used in the ratio 3 : 2. Use words and numbers to write the given ratio, then interpret the ratio.

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6 Use the information to write the following as ratios.

a 5 rainy days to 8 sunny days
b 3 cups of sugar to 3 cups of flour
c 11 eggs to 14 eggs
d 7 tries to 12 tries
e 8 cats to 11 dogs
f 4 cups of milk to 5 cups of water

Hint A ratio compares amounts with the same units but it is written without units.

APPLICATIONS

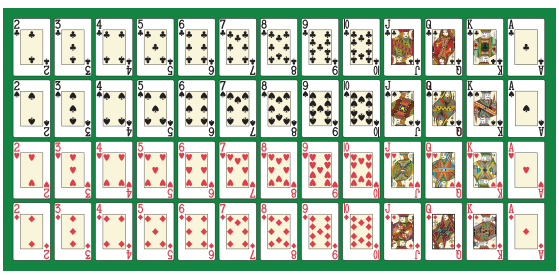
7 In the past month, it was rainy for 8 days, cloudy for 9 days and sunny for 14 days. Determine the ratio of:

a rainy days to cloudy days
b cloudy days to sunny days
c sunny days to non-sunny days
d rainy days to cloudy days to sunny days

Hint The order of numbers is important in a ratio.

Example 14

8 In a standard deck of 52 playing cards there are four suits, each with the following 13 cards: ace, numbers 2 to 10, jack, queen and king. Spades and clubs are black cards, whereas diamonds and hearts are red cards.
Determine the ratio of:



a black cards to red cards
b spades to red cards
c hearts to non-hearts
d aces to non-aces

9 In a new housing estate, the town planners make a requirement that for every 10 000 m^2 of development there must be 750 m^2 of green space. Determine the ratio of development area to total green space, in simplest form.

Hint Start with the words: 'Development area to green area =...'.

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10 When children are learning to play tennis, the recommended ratio of children to coaches for each age level are those given in the table.

Age (years)	Ratio
4–6	5 : 1
7–9	6 : 1
10–12	7 : 1
13+	1 : 1

a Konrad is 8 years old. Identify the ratio of children to coaches for his age group.

b Use this ratio to determine how many other children Konrad would expect to be with him during his training session.

11 A group of New York City residents are concerned about the increasing amount of shadow in Central Park, cast from the many new, super tall buildings.

a Consider the image and **assess** (i.e. form an **estimate** of) the proportion of the park that is in shadow from buildings. Write this estimate as a whole number ratio of shadow area to park area.

b Use your estimated ratio to **propose** (i.e. make a suggestion about) approximately what percentage of this park is currently in the shadow of a building.

Sheep Meadow in Central Park, New York City

12 Symbolise the following ratios using a picture or diagram.

a 1 : 3 **b** 4 : 9

13 Measure the length of your hand, from the wrist to the end of your pointer finger, and measure your foot, from the heel to the tip of your toes. Compare these two measurements, using a ratio.

Hint Ratios are written using whole numbers and without units.

ISBN 978-1-009-25785-5

10E Simplifying ratios and fractions

LEARNING GOALS

- Write fractions in simplest form by dividing the numerator and denominator by the HCF
- Write whole number ratios in simplest form by division of each part with the HCF
- Simplify fraction ratios by multiplication with the LCM
- Simplify ratios by first converting quantities to the same units
- Simplify ratios in real-world situations

Why is it essential to simplify ratios?

It is through the simplification of ratios that we can usefully compare them. Examples include comparing win-to-loss ratios of sporting events, comparing the aspect ratios of images or devices, comparing map scale ratios; and comparing price-to-weight ratios when calculating the cheapest way to purchase items.

We can also use simplification of ratios when comparing statistics. For example, in Victoria the ratio of males to females simplifies to 51 : 49.

Surveyors calculate 'closure ratio'. A 40 cm discrepancy after an 8 km traverse gives a closure ratio of 40 cm : 8 km = 1 : 20 000.

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WHAT YOU NEED TO KNOW

- The **highest common factor (HCF)** is the largest number that can be divided exactly (i.e. with no remainder) into two or more numbers.
  - For example: For 12 and 18, the HCF = 6.
  - The HCF is *smaller* than or equal to each given number.
- The **lowest common multiple (LCM)** is the lowest whole number that is exactly divisible by two or more numbers. The LCM is the lowest shared multiple.
  - For example: For 20 and 30, the LCM = 60.
  - The LCM is *greater* than or equal to each given number.
- The **lowest common denominator (LCD)** is the lowest whole number that is exactly divisible by two or more denominators. The LCD is the LCM of the denominators.
- A **fully simplified fraction** is found by dividing both the numerator and the denominator by the HCF.
  - For example: The HCF of 27 and 36 is 9. $\frac{27 \div 9}{36 \div 9} = \frac{3}{4}$
- A **fully simplified ratio** is found by dividing each part by the HCF.
  - For example: $18 : 24$ (HCF of 18 and 24 is 6.)
    $= 18 \div 6 : 24 \div 6$
    $= 3 : 4$
- Ratios written in simplest form use whole numbers only.
- Ratios should be expressed in simplest form, as it is a mathematical convention.
- If a ratio is given with one or more fractions, simplify by multiplying each part of the ratio by the LCD. This converts the ratio into whole numbers.

  $\frac{5}{12} : \frac{9}{8} = \frac{5}{12} \times 24 : \frac{9}{8} \times 24$ (LCD of 12 and 8 is 24.)
  $= 10 : 27$
- Ratios should be written in the same units before they are simplified.
- Smaller units $\boxed{\div \text{ factor}}$ to fewer larger units. Larger units $\boxed{\times \text{ factor}}$ to more smaller units.

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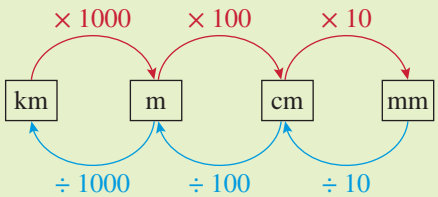
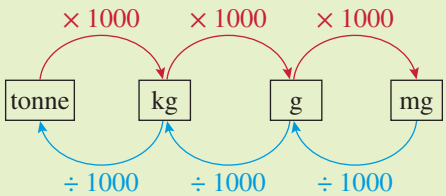
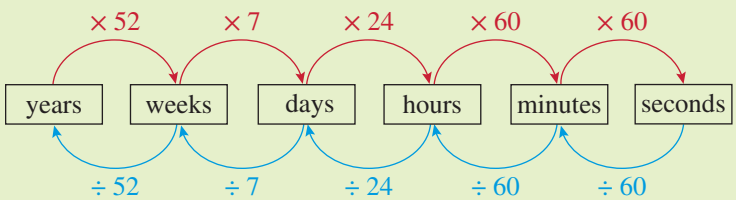
- Conversion of **length** units



- Conversion of **mass** units



- Conversion of **time** units



- Other unit conversions:
  - 1000 mL = 1 litre
  - 100 cents = $1

Example 15 Simplifying fractions

Use division to fully simplify these fractions.

a $\frac{12}{15}$ **b** $\frac{88}{77}$ **c** $\frac{7}{21}$

WORKING	THINKING
a $\frac{12 \div 3}{15 \div 3} = \frac{4}{5}$	The HCF of 12 and 15 is 3. Divide both numbers by 3.
b $\frac{88 \div 11}{77 \div 11} = \frac{8}{7}$	The HCF of 88 and 77 is 11. Divide both numbers by 11.
c $\frac{7 \div 7}{21 \div 7} = \frac{1}{3}$	The HCF of 7 and 21 is 7. Divide both numbers by 7.

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Example 16 Simplifying ratios

Determine the simplest form of each of the following ratios.

a 15 : 35

b 64 : 48

c 18 : 27 : 45

d $\frac{3}{7} : \frac{2}{3}$

e $3\frac{3}{5} : 2\frac{1}{4}$

WORKING	THINKING
a $15 : 35 = 3 : 7$ (÷ 5)	The HCF of 15 and 35 is 5. Divide all parts by 5.
b $64 : 48 = 4 : 3$ (÷ 16)	The HCF of 64 and 48 is 16. Divide all parts by 16.
c $18 : 27 : 45 = 2 : 3 : 5$ (÷ 9)	The HCF of 18, 27 and 45 is 9. Divide all parts by 9.
d $\frac{3}{7} : \frac{2}{3} = \frac{3}{7} \times 21 : \frac{2}{3} \times 21$ $= 9 : 14$	The LCM of 7 and 3 is 21. Multiply each fraction by 21.
e $3\frac{3}{5} : 2\frac{1}{4} = 3\frac{3}{5} \times 20 : 2\frac{1}{4} \times 20$ $= 72 : 45$ $= 8 : 5$	The LCM of 4 and 5 is 20. Multiply each fraction by 20. The HCF of 72 and 45 is 9. Divide both parts by 9.

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Example 17 Simplifying ratios with different units

Convert the following ratios to the same unit, as shown in the brackets, and then determine the simplest form of each ratio.

a 6 m : 300 cm (cm) **b** 1 h : 25 min (min) **c** 40 mL : 12 litres (mL)

WORKING	THINKING
a 6 m : 300 cm = 600 : 300 ÷ 300 = 2 : 1	6 m = 600 cm. When units are the same, they are not included in the ratio. The HCF of 600 and 300 is 300. Divide both parts by 300.
b 1 h : 25 min = 60 : 25 ÷ 5 = 12 : 5	1h = 60 min. Units are not used now. The HCF of 60 and 25 is 5. Divide both parts by 5.
c 40 mL : 12 L = 40 : 12 000 ÷ 40 = 1 : 300	12 L = 12 000 mL. Units are not used now. The HCF of 40 and 12 000 is 40. Divide both parts by 40.

Example 18 Finding the simplest ratio in a real situation

A fuel container has 60 mL of oil and 3 litres of petrol poured into it. Identify the ratio of oil to petrol, and then determine the simplest ratio of oil to petrol that is mixed in this fuel can.

WORKING	THINKING
oil : petrol 60 mL : 3 L × 1000 60 mL : 3000 mL 60 : 3000 ÷ 60 1 : 50	Identify the ratio of oil : petrol = 60 mL : 3 litres. Convert to the same smaller unit of mL. Write the ratio as whole numbers, without units. Simplify the ratio.

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Exercise 10E

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a The highest common factor (_____) is the largest number that divides _____ into two or more numbers.

b A ratio is fully simplified by _____ each part by the _____.

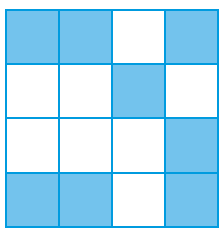
c A fraction is fully simplified by _____ both the numerator and the denominator by the _____.

d The lowest common denominator (_____) is the lowest whole number that is_____ divisible by two or more denominators.

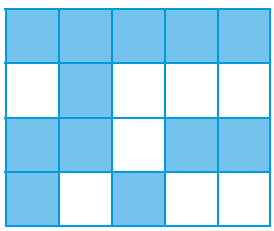
e When a ratio includes fractions, multiply each part of the ratio by the _____.

2 For the diagrams below, determine the ratio of shaded parts to unshaded parts, in simplest form.

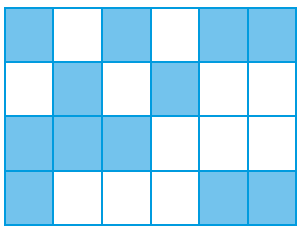
a



b



c



3 Identify which of the following is not written in simplest form and then simplify.

a **i** 1 : 7 **ii** 4 : 22 **iii** 3 : 13 **iv** 5 : 22

b **i** 2 : 31 **ii** 5 : 23 **iii** 13 : 39 **iv** 16 : 61

c **i** 5 : 8 **ii** 7 : 24 **iii** 8 : 3 **iv** 35 : 25

4 Determine the highest common factor (HCF) of these pairs of numbers.

a 10, 15 **b** 21, 28 **c** 15, 24

d 12, 24 **e** 35, 56 **f** 72, 60

g 81, 36 **h** 24, 18 **i** 60, 36

j 66, 88 **k** 70, 105 **l** 75, 125

> **Hint** Knowing your times tables helps with finding the HCF.

Example 15

5 Use division to simplify the following fractions.

a $\frac{5}{10}$ **b** $\frac{3}{12}$ **c** $\frac{6}{18}$

d $\frac{8}{12}$ **e** $\frac{14}{21}$ **f** $\frac{13}{25}$

g $\frac{72}{108}$ **h** $\frac{105}{120}$ **i** $\frac{105}{168}$

> **Hint** Divide both the numerator and the denominator by the HCF.

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Example 16a, b

6 Determine the simplest form of each of the following ratios.

a 2 : 6 **b** 15 : 30 **c** 8 : 24
d 12 : 48 **e** 9 : 15 **f** 12 : 18
g 14 : 35 **h** 15 : 45 **i** 40 : 180
j 68 : 17 **k** 81 : 24 **l** 330 : 550

Hint Divide each part of a ratio by the HCF.

Example 16c

7 Use division by a HCF to simplify the following ratios.

a 5 : 15 : 30 **b** 4 : 16 : 20 **c** 7 : 21 : 35
d 8 : 12 : 18 **e** 21 : 28 : 70 **f** 20 : 45 : 80
g 14 : 35 : 49 **h** 26 : 39 : 52 **i** 100 : 250 : 350

8 Identify the lowest common multiple (LCM) of the following pairs of numbers.

a 2, 3 **b** 5, 7 **c** 8, 12
d 21, 42 **e** 10, 20 **f** 10, 25
g 6, 9 **h** 16, 24 **i** 8, 10

Hint The LCM is the lowest whole number that is exactly divisible by both numbers.

Example 16d, e

9 Calculate the simplest form of each of the following ratios by multiplying both fractions by their lowest common denominator (LCD).

a $\frac{1}{2} : \frac{1}{3}$ **b** $\frac{1}{3} : \frac{1}{4}$ **c** $\frac{2}{5} : \frac{3}{4}$
d $\frac{3}{7} : \frac{2}{3}$ **e** $2\frac{1}{2} : 3\frac{1}{3}$ **f** $3\frac{1}{4} : 2\frac{1}{5}$
g $1\frac{2}{3} : 2\frac{1}{4}$ **h** $5\frac{3}{5} : 7\frac{5}{6}$ **i** $2\frac{1}{2} : \frac{25}{4}$

Hint The LCD is the lowest whole number that is exactly divisible by both denominators.

Example 17

10 Write each ratio in cents, and then determine the fully simplified form of each ratio.

a \$3.00 : 40 cents **b** 50 cents : \$7.50
c \$2.75 : 25 cents **d** 75 cents : \$4.25
e \$1.80 : \$3.00 **f** \$1.20 : \$4.00

Hint A ratio in simplest form uses whole numbers only and does not include units.

11 Convert the following ratios to the same unit, as shown in the brackets, and then determine the simplest form of each ratio.

a 6 hours : 1 day (hours) **b** 26 weeks : 2 years (weeks)
c 4 weeks : 1 year (weeks) **d** 26 weeks : 3 years (months)
e 20 hours : 3 days (hours) **f** 26 weeks : 4 years (years)
g 60 hours : 1 week (hours) **h** 450 minutes : 2 days (minutes)

12 Convert the following ratios so that both units match the unit shown in the brackets, and then use division to write each ratio in simplest form.

a 25 cm : 12 mm (mm) **b** 2000 m : 12 km (km)
c 200 m : 4 km (m) **d** 5 mm : 1.5 cm (mm)
e 2 km : 400 m (m) **f** 5 m : 30 cm (cm)
g 2000 m : 12 km (km) **h** 2000 m : 12 km (m)

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APPLICATIONS

Example 18

13 A two-stroke motor has a ratio of 5 mL of oil for every 2 litres of petrol.

a Identify the ratio of oil to petrol.

b Convert the units to mL for both quantities in the ratio.

c Determine the simplified ratio of oil to petrol.

Oil and petrol are mixed in a specific ratio for lawn mower fuel.

14 A radio station offers to donate $25 to a charity for every $200 raised by its listeners.

a In simplest form, determine the ratio of the money donated by the station to the money donated by the listeners.

b Express this information as a simplified ratio of money donated by the listeners to money donated by the station.

15 A country real estate agent has a large block of land for sale. The land has a length of $1\frac{3}{4}$ km and a width of $1\frac{1}{6}$ km.

Hint When writing a ratio, follow the order given in the question.

a Rewrite the ratio of length to width, $1\frac{3}{4} : 1\frac{1}{6}$, using improper fractions.

b Determine the lowest common denominator of the fractions in this ratio.

c Multiply by the lowest common denominator to write this ratio in simplest form.

16 Old MacDonald had a farm with 42 sheep, 30 cows and 54 chickens. Identify the ratio of the numbers of sheep to cows to chickens, and then determine the simplified form of this ratio.

Hint Start with words and show steps.

17 In one week, the local fish and chip shop made the following fish sales: 30 pieces of cod, 45 pieces of flake and 12 pieces of coral trout. Write the numbers of fish sales as a ratio, and then determine the simplified ratio.

Hint Start your answer by writing the words: cod to flake to trout = ...

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18 Research has found that, in Central Australia on a typical hot summer day, the water required by animals greatly varies. A wild camel can consume 200 litres of water, a red kangaroo 200 mL of water and an emu 15 litres of water. Determine the simplified ratio for the water requirements of a camel to a kangaroo to an emu.

> **Hint** Write the ratio in:
> - words
> - numbers with different units
> - numbers with the same units
> - simplified numbers with no units.

19 Harriett has just landed a new job, and her contract states that, for every 7 years of work, she will receive 7 weeks of long service leave. Express the ratio of years of work to weeks of long service leave, and then simplify.

> **Hint** Ratios in simplest form are written using whole numbers without units.

20 A school has 105 teachers and 750 students. In simplest form, determine the ratio of teachers to students.

21 Ken is not happy that his children watch so much television. He has promised his children that, for every 1.5 hours of home chores completed over the holidays, they are allowed to watch 45 minutes of television. Determine a correct simplified ratio for the ratio of time spent on home chores to television time.

> **Hint** Start with words and include working steps.

22 Jessica is in Year 11, and she has calculated that she has 12 weeks of holidays and attends school for 1344 hours that year. Use this information to determine a simplified ratio that shows the relationship of holidays to school days for Jessica's Year 11.

23 A recipe for muffins for 20 people consists of 5 cups of sugar, 15 cups of flour and 10 eggs.

a Determine the simplified ratio of sugar to flour to eggs.

b Determine how many people the simplified recipe would feed.

Chefs use ratios to scale recipes for different portion numbers.

ISBN 978-1-009-25785-5

10F Equivalent ratios and fractions

LEARNING GOALS

- Calculate equivalent fractions
- Calculate equivalent ratios
- Apply equivalent ratios in real-world situations

Where is it essential to use equivalent ratios?

Equivalent ratios maintain a fixed proportion of components when varying the total quantity. For example, bakers need to follow the same recipe ratio when mixing ingredients for either one loaf of bread or baking many loaves.

Nurses use ratios to calculate dosage.

There are many work situations, as well as for cooking, where people use equivalent ratios to calculate the correct quantities for a mixture. Farmers need to keep fixed proportions when mixing chemicals for fertiliser or poison, tradespeople mix ingredients in certain ratios for concrete or fuel, and jewellers calculate precise quantities of alloys when combining metals. Pharmacists and nurses use ratios to calculate medical dosages. For example, if a doctor orders 750 mg of a drug that has a stock strength of 300 mg/8 mL, then $300 : 8 = 750 : x$, giving the dosage $x = 20$ mL.

WHAT YOU NEED TO KNOW

- When both the numerator and the denominator are multiplied or divided by the same number, the resulting fraction is called an **equivalent fraction**.
  - For example: $\frac{75}{100}$, $\frac{3}{4}$ and $\frac{6}{8}$ are all equivalent fractions.

    $\frac{75 \div 25}{100 \div 25} = \frac{3}{4}$ and $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.
- When each value in the ratio is multiplied or divided by the same number, the resulting ratio is called an **equivalent ratio**.
  - For example: 200 : 500 and 2 : 5 and 6 : 15 are all equivalent ratios.

$$200 : 500 \overset{\div 100}{=} 2 : 5 \qquad 2 : 5 \overset{\times 3}{=} 6 : 15$$
$$\underset{\div 100}{} \qquad \underset{\times 3}{}$$

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Example 19 Calculating equivalent fractions

Use multiplication to complete each of these equivalent fractions.

a $\frac{1}{2} = \frac{5}{\square}$ **b** $\frac{2}{3} = \frac{\square}{18}$ **c** $\frac{5}{7} = \frac{\square}{21} = \frac{35}{\square}$

WORKING	THINKING
a $\frac{1 \times 5}{2 \times 5} = \frac{5}{10}$	$1 \times 5 = 5$, $2 \times 5 = 10$
b $\frac{2 \times 6}{3 \times 6} = \frac{12}{18}$	$3 \times 6 = 18$, $2 \times 6 = 12$
c $\frac{5}{7} \overset{\times 7}{\underset{\times 7}{\times 3}} = \frac{15}{21} = \frac{35}{49}$	$7 \times 3 = 21$, $5 \times 3 = 15$ $5 \times 7 = 35$, $7 \times 7 = 49$

Example 20 Calculating equivalent ratios

Use multiplication or division to determine the missing numbers in each of the following equivalent ratios.

a $3 : 7 = 12 : \square$ **b** $8 : 20 = \square : 35$ **c** $3 : 5 : \square = \square : 40 : 80$

WORKING	THINKING
a $3 : 7 \overset{\times 4}{\underset{\times 4}{=}} 12 : 28$	$3 \times 4 = 12$ $7 \times 4 = 28$
b $8 : 20 = \square : 35$ $\div 4$, $\times 7$ $2 : 5 = 14 : 35$ $\times 7$	$20 \times \boxed{?} = 35$. As $\boxed{?}$ is not a whole number, first we must simplify the ratio given on the left. $\frac{8}{4} : \frac{20}{4} = 2 : 5$ $5 \times 7 = 35$, $2 \times 7 = 14$
c $3 : 5 : 10 = 24 : 40 : 80$ $\times 8$, $\div 8$ $\times 8$	$5 \times 8 = 40$, $3 \times 8 = 24$ $80 \div 8 = 10$

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Example 21 Solving real-world problems, using equivalent ratios

a Caterina generally wins two games of cards out of every five games she plays. Identify the ratio of games won to games played, and use this to determine how many games she will need to play to win 12 games.

b Ashley's two-stroke outboard motor requires an oil-to-petrol mix of 1 : 50. If he has a jerry can with 30 L of petrol, calculate how many millilitres of oil Ashley should add to the jerry can.

WORKING	THINKING
a games won : games played $= 2 : 5$ $= 12 : 30$ $\times 6$	games won : games played $= 2 : 5 = 12 : \square$ Keep the same order. $2 \times 6 = 12$, $5 \times 6 = 30$
Caterina will need to play 30 games.	Write your answer in words.
b oil : petrol $= 1 : 50$ $= \square : 30$	oil : petrol $= 1 : 50 = \square : 30$
$\times 600$: $1 : 50 = 600 : 30\,000$ mL; $\times 600$	$50 \times \square = 30$ is difficult to solve; hence, convert 30 L to mL. [L $(\times 1000) \rightarrow$ mL] $50 \times 600 = 30\,000$, $1 \times 600 = 600$
Ashley will need 600 mL of oil.	Write your answer using words.

Exercise 10F

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a Equivalent fractions are found by multiplying or _____ the numerator and the _____ by the _____ number.

b When each value in a ratio is _____ or divided by the _____ number, the resulting ratio is called an _____ ratio.

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Example 19

2 Use multiplication to complete these equivalent fractions.

a $\frac{1}{2} = \frac{\square}{20}$ **b** $\frac{2}{3} = \frac{\square}{21}$ **c** $\frac{7}{8} = \frac{35}{\square}$

d $\frac{4}{7} = \frac{20}{\square}$ **e** $\frac{8}{11} = \frac{\square}{88}$ **f** $\frac{5}{12} = \frac{\square}{36}$

g $\frac{11}{12} = \frac{\square}{24} = \frac{55}{\square}$ **h** $\frac{13}{25} = \frac{\square}{50} = \frac{130}{\square}$ **i** $\frac{2}{7} = \frac{6}{\square} = \frac{\square}{49}$

j $\frac{5}{6} = \frac{\square}{30} = \frac{100}{\square}$ **k** $\frac{7}{12} = \frac{\square}{60} = \frac{49}{\square}$ **l** $\frac{3}{8} = \frac{40}{\square} = \frac{\square}{80}$

3 Demonstrate your understanding of equivalent ratios by writing three equivalent ratios for each of the following.

a 2 : 3 **b** 3 : 5 **c** 11 : 9 **d** 4 : 6 : 8

Example 20

4 Use multiplication or division to complete the following equivalent ratios.

a 1 : 4 = 5 : □ **b** 3 : 5 = □ : 30

c 2 : 7 = 8 : □ **d** □ : 4 = 6 : 24

e 7 : □ = 21 : 33 **f** □ : 9 = 20 : 36

g 12 : 15 = 20 : □ **h** 8 : 20 = □ : 35

i 22 : □ = 55 : 100 **j** 3 : 4 : 5 = 9 : □ : □

k 3 : 5 : □ = □ : 25 : 40 **l** □ : 8 : 12 = 12 : 24 : □

m 2 : 3 : 7 = □ : 9 : □ **n** 5 : 4 : 1 = 25 : □ : □

o □ : □ : 9 = 21 : 35 : 27

5 **a** Explain why the ratios 2 : 5 and 8 : 11 are not equivalent ratios.

b Determine a ratio that is equivalent to the ratio 2 : 5.

APPLICATIONS

Example 21

6 David is making his mum's favourite vinaigrette dressing. The recipe uses 3 tablespoons of olive oil and 1 tablespoon of apple cider vinegar. If David is making a larger quantity and is using 12 tablespoons of olive oil, use a calculation involving equivalent ratios to deduce how many tablespoons of apple cider vinegar he should add.

Hint First write: oil to vinegar = ... Keep the same order for the number ratio.

7 Lynn has noticed that, when she is driving through the city, she gets two red lights for every three green lights. If Lynn was stopped at 10 red lights during her journey, use a calculation involving equivalent ratios to determine how many green lights she passed through.

Hint Start by writing the ratio order in words.

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8 For safety reasons, the recommended nurse-to-patient ratio is 1 : 4. If there are 32 patients on a particular ward, use equivalent ratios to calculate how many nurses must be working on the ward for the hospital to be compliant with the safety recommendations.

9 Ryan is planning to build a new in-ground spa. His current spa has a diameter-to-circumference ratio of 1 : 3.14. His new spa will have a diameter of 1.8 m. So that Ryan can plan for the surrounding tiles, use the diameter : circumference ratio to calculate the circumference of the new spa.

Hint Use equivalent ratios to find the solution.

10 When fishing in the Pallinup River, Isabelle catches four legal-sized fish for every 12 undersized fish, which she has to throw back as they are too small. Using equivalent ratios, determine how many undersized fish Isabelle needed to throw back if she catches nine legal-sized fish.

Hint Always write your final answer using words.

11 Jonah scored 8 goals and 14 points in a game of Aussie Rules, and his brother Luther scored with the same ratio of goals to points. If Luther scored 21 points, use equivalent ratios to calculate how many goals he scored.

12 Concrete is to be made from a mixture of sand, gravel and cement in the ratio of 4 : 3 : 1. If 10 kg of sand is used, calculate how much gravel will be needed, using equivalent ratios.

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13 Gavin's brush cutter requires a petrol to two-stroke oil mix of 25 : 1. Use equivalent ratios to determine how much oil he must add to his 4 litres of petrol.

14 Priya has a 48 000 L chlorine swimming pool. The instructions on the liquid chlorine container state that the daily amount required is 400 mL for every 10 000 L of water.
Calculate how much chlorine Priya should add to her pool each day.

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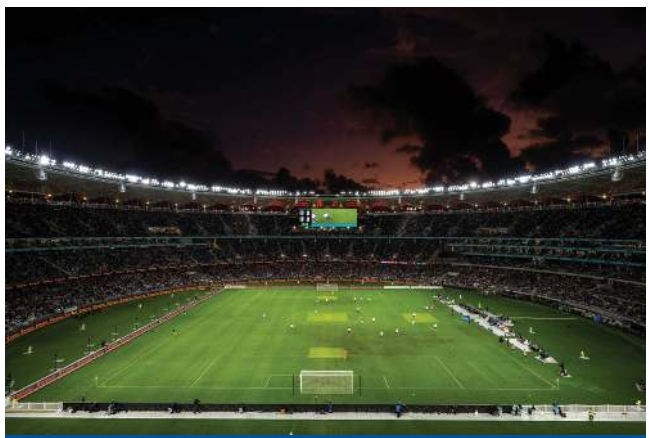
10G Dividing quantities in a given ratio

LEARNING GOALS

- Master the skill of dividing up a quantity into a given ratio
- Solve real-world problems that require dividing a quantity in a given ratio

Who would find it essential to divide up an amount in a given ratio?

Anyone who mixes materials, such as a concreter, a chemist, a cook, a farmer and many other workers, need to divide up an amount in a given ratio. People regularly use ratios to calculate the amount of each ingredient needed to add up to a certain total quantity. Solicitors also use ratios to divide up an inheritance according to the instructions in the will.



Food stock managers for sporting events use proportions to calculate suppies. If three in 25 fans buy pies, then 7800 pies are required for 65 000 fans at Optus Stadium.

WHAT YOU NEED TO KNOW

- When dividing up a quantity using a ratio, first calculate the sum of the ratio parts.
  - For example: the ratio $3 : 7$ has a sum of $3 + 7 = 10$ parts.
- Dividing (i.e. sharing) an amount using ratios is made simpler using fractions.

  $$\text{Proportion or fraction of the total amount} = \frac{\text{one part of a ratio}}{\text{sum of ratio parts}}$$

  - For example: to share \$200 in the ratio of $3 : 7$ means $\frac{3}{10} \times 200 : \frac{7}{10} \times 200$, which gives \$60 and \$140 as the two shares.

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Example 22 Dividing quantities in a given ratio

For each of the following, calculate the total number of parts in the ratio. Then use appropriate proportions (fractions) to divide the quantity into this ratio.

a \$3600 in the ratio of 2 : 7 **b** 420 kg in the ratio of 1 : 2 : 4

WORKING	THINKING
a $2 + 7 = 9$	Sum of parts $= 2 + 7 = 9$
$\frac{2}{9} \times 3600 = 800$	Proportions (i.e. fractions) are $\frac{2}{9}$ and $\frac{7}{9}$.
$\frac{7}{9} \times 3600 = 2800$	
\$800 and \$2800	Write units in the final amounts.
b $1 + 2 + 4 = 7$	Sum of parts $= 1 + 2 + 4 = 7$
$\frac{1}{7} \times 420 = 60$	Proportions (i.e. fractions) are $\frac{1}{7}$, $\frac{2}{7}$ and $\frac{4}{7}$.
$\frac{2}{7} \times 420 = 120$	
$\frac{4}{7} \times 420 = 240$	
60 kg, 120 kg and 240 kg	Write units in the final amounts.

Example 23 Dividing a quantity in a given ratio to solve a real-life situation

Jasmine found an old piece of jewellery, weighing 176 grams. It is a mixture of gold and nickel in the ratio 3 : 8. Calculate the weight of the gold in this jewellery piece to the nearest gram.

WORKING	THINKING
Total weight = 176 g	Summarise the information from the question.
Gold : nickel = 3 : 8	Write the ratio in words and numbers, keeping the same order as in the question.
$\text{Gold} = \frac{3}{11} \times 176 = 48$	Sum of parts $= 3 + 8 = 11$ Gold proportion $= \frac{3}{11}$
There are 48 grams of gold.	Write your answer in words, using units.

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Exercise 10G

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a When dividing up a quantity using a ratio, first calculate the _____ of the _____ parts.

b Each proportion of the total amount = $\frac{\square \text{ part of a ratio}}{\square \text{ of the ratio parts}}$.

c When dividing up a quantity using a ratio, each share = fraction of _____ amount × _____ amount.

2 Calculate the sum of the parts in these ratios.

a 1 : 5 **b** 3 : 5 **c** 9 : 11 **d** 3 : 5 : 7

3 At the local skate park, the ratio of bike riders to scooter riders is 3 : 7. Determine the fraction of the riders that are:

a bike riders

b scooter riders

Hint First calculate the sum of the ratio parts.

4 A jar contains jelly beans and snakes in the ratio 5 : 6. Determine the fraction of the lollies that are:

a snakes

b jelly beans

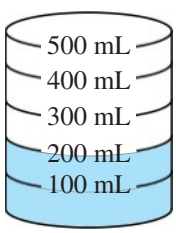
5 A litter of new kittens has ginger and tortoise shell colours in the ratio 2 : 5. Determine the fraction of the litter that is:

a ginger coloured

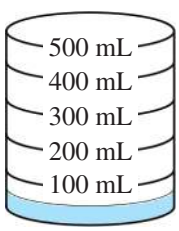
b tortoise-shell coloured

6 When completely full, each container holds 500 mL.

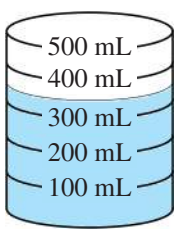
a



b



c



For each container, calculate the simplified ratio of volume of liquid to volume of the full container.

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Example 22

7 Calculate the total number of parts in the given ratio. Then use appropriate proportions (fractions) to divide $100 in the ratio of:

Hint: Start by finding the total parts.

a 1 : 9 **b** 1 : 3 **c** 3 : 7 **d** 2 : 5

8 Calculate the total number of parts in the given ratio. Then use appropriate proportions (fractions) to divide $1200 in the ratio of:

a 1 : 2 **b** 1 : 5 **c** 5 : 7 **d** 7 : 3

9 Calculate the total number of parts in the ratio. Then use appropriate proportions (fractions) to divide each of the following in the ratio given.

Hint: $\frac{\text{ratio part}}{\text{total of parts}} \times \text{amount}$

a $40 in the ratio 1 : 3
b $90 in the ratio 1 : 2
c $150 in the ratio 2 : 3
d $220 in the ratio 4 : 7
e 32 kg in the ratio 3 : 5
f 450 kg in the ratio 7 : 2
g 48 litres in the ratio 5 : 7
h 132 litres in the ratio 8 : 3
i 200 g in the ratio 4 : 1
j 49 lollies in the ratio 5 : 2

10 Calculate the total number of parts in the ratio. Then use appropriate proportions (fractions) to divide each of the following in the ratio given.

Hint: Remember, your final answer needs units.

a $300 in the ratio 1 : 2 : 3
b 270 kg in the ratio 2 : 3 : 5
c 64 000 litres in the ratio 1 : 3 : 4
d $1 250 000 in the ratio 12 : 8 : 5
e 300 m in the ratio 3 : 3 : 4
f 80 chocolates in the ratio 1 : 4 : 3
g 180 plants in the ratio 10 : 5 : 3
h $1500 in the ratio 20 : 7 : 3

APPLICATIONS

Example 23

11 Graham and Jean run a sheep farm, and after breeding season, they have calculated that the ewe-to-lamb ratio is 20 : 19. They currently have a total of 5265 sheep on their property.

a Calculate the total number of parts in the ratio.
b Determine what fraction of the sheep are ewes and what fraction are lambs.
c Calculate how many ewes and lambs they have in total.

Hint: First write the ratio order in words. Keep the same order for the numbers.

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12 During the wet season, it rains five days for every four sunny days. The wet season runs for a total of 90 days.

- **a** Identify the ratio of rainy days to sunny days.
- **b** Calculate the total number of parts in the ratio.
- **c** Determine what fraction of days are sunny.
- **d** Calculate how many sunny days there are during the wet season.

13 In one year recently, there were 2170 accidents on the roads. Use the ratio of fatal to non-fatal accidents of 1 : 9 to:

- **a** Calculate the total number of parts in the ratio.
- **b** Determine what proportion of accidents were fatal and the proportion of accidents that were non-fatal.
- **c** Calculate how many accidents were fatalities compared to non-fatalities.

14 Ariana, Kacey and Sarah inherited $55 000 from their grandmother, who left them an inheritance in the ratio of 5 : 4 : 2, respectively.

- **a** Use this ratio to explain who gets more of the inheritance.
- **b** Calculate the total number of parts in this ratio.
- **c** Determine what fraction of the inheritance Ariana receives.
- **d** Determine what fraction of the inheritance Sarah receives.
- **e** Use your answer to part **c** to calculate Ariana's share of the inheritance.
- **f** Use your answer to part **d** to calculate Sarah's share of the inheritance.
- **g** Use your answers to parts **e** and **f** to determine how much more Ariana will receive than Sarah.

Hint Write your final answer to part g in words, using units.

15 Cassie, Jarred, Andrew and Anneliese went to the movies, and Jarred decided to pay for Cassie and Anneliese, as well as his own ticket. All four tickets cost $56.

- **a** Use the information to write down how many tickets Jarred paid for and how many Andrew paid for.
- **b** Write the ratio of Jarred's tickets to Andrew's tickets.
- **c** Use this ratio to calculate how much Jarred paid.

16 Megan and Luke own a coffee shop together. They split the profits in the ratio 5 : 4. If the shop made a profit of $114 732 in one year, calculate how much money Megan and Luke each receive.

17 Brock, Jasper and Ruth win a 1-kg block of chocolate, which they split in the ratio 1 : 2 : 3. Calculate how many grams of chocolate each individual receives.

Hint 1 kg = 1000 g

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18 Great Aunty Rose passed away and left her inheritance in the ratio of 1 : 1 : 3 to her sisters, Gretel, Helga and Marie, respectively. Marie cared for Aunty Rose in her last few years and this is why she has the larger share. Rose's estate is worth $600 000; this is the total amount of inheritance that her sisters will receive. Calculate how much each sister would receive.

Hint The word 'respectively' means 'in the same order'.

19 Melanie is organising the cordial drinks for the end-of-year function.

a If the ratio of cordial to water is 2 : 18, respectively, calculate the amount of cordial and water that is needed to fill a 50-litre drink dispenser.

b Calculate the total cost of purchasing the cordial if it costs $2.55 per 2-L bottle of cordial.

20 Brooke runs a local hair salon, and she wants to calculate the cost price for one hair-colouring procedure. The mixture she makes has a 2 : 1 ratio of developer to colour.

a Brooke estimates that she needs around 450 mL of mixture to colour long hair. Use the ratio given to calculate how many millilitres of developer Brooke would need.

b The developer costs $3 per 100 mL, and the colour costs $2 per 100 mL. Determine the cost price of the products for one long-hair colouring procedure.

Hairdressers use ratios in their daily work.

21 Ezekiel is concreting his driveway and has worked out that it will take 9 cubic metres (m^3) of concrete mix to make his driveway. The concrete mix ratio is 1 : 2 : 3 of cement to sand to gravel, respectively.

Cement costs $150 per m^3, sand costs $60 per m^3 and gravel costs $80 per m^3. Calculate the cost of materials for Ezekiel's driveway.

Concrete is made from cement: sand: gravel mixed in a specific ratio.

22 Olivia is splitting chocolate between her Maths and English teachers in the ratio of 3 : 2 (Maths is her favourite subject.) Olivia's Maths teacher receives 900 g of chocolate. Calculate how many grams of chocolate Olivia gives to her teachers in total.

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10H Scale values using ratios

LEARNING GOALS

- Convert a scale distance to an actual distance
- Convert an actual distance to a scaled distance
- Determine scale ratios
- Finding the actual and scaled distances in real-world problems

When would it be essential to use scale values?

Scale models and scale drawings are used to find design errors and improvements for inventions, new designs (e.g. cars, buildings) and engineering projects.

A map is a scale copy of the landscape. The scale on a map or a house plan can be used to calculate actual distances.

Professional model makers use scale ratios to build architectural and engineering models for commercial and government use.

WHAT YOU NEED TO KNOW

- A **scale drawing** or **model** has exactly the same shape as the actual object but it is a different size. The scale ratio should be included with a drawing or model.
- The **scale** on a drawing, diagram, map or model is written as a scale ratio.
  Scale ratio = drawing (or model) length : actual length
- A scale ratio of 1 : 100 = drawing (or model) : actual, means that the actual (real life) lengths are 100 times larger than the drawing lengths.
  - For example: A house plan scale of 1 : 100 means that 2 cm on the plan = 200 cm in real life.
- A scale ratio of 100 : 1 = model: actual, means that the model lengths are 100 times larger than the actual object. (100 : 1 = 1 : 1/100)
  - For example: If a plastic model of a plant cell has scale 100 : 1, then 1 cm on the model = $\frac{1}{100}$ cm in the actual plant cell.
- The **scale factor** is the second number in a scale ratio that starts with 1.
  - For example: For the scale 1 : 1000, the scale factor is 1000.

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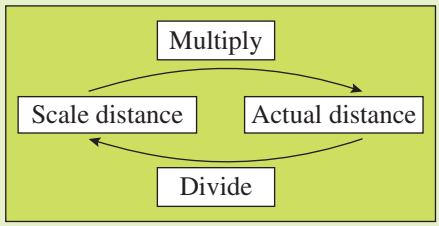
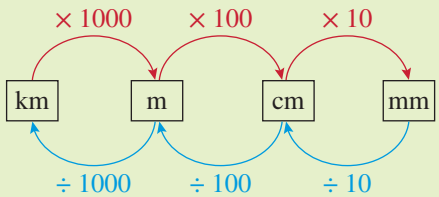
- To convert a scaled length to the actual length, multiply by the scale factor.
- To convert an actual length to the scaled length, divide by the scale factor.



- It is important to accurately **convert length units** when calculating with scales.



Example 24 Converting a scale distance to an actual distance

A map has a scale of 1 : 50 000.
Use the given scale to calculate the actual distance for the each of the following scaled distances.

a 6 cm **b** 150 mm **c** 2.75 cm

WORKING	THINKING
a Actual distance = 6 cm × 50 000 = 300 000 cm = 3000 m = 3 km	Multiply 6 cm by the scale factor of 50 000, and always keep the same units after multiplying by the scale factor (i.e. keep cm). Then convert cm to km, which is a more suitable unit. [cm (÷ 100) → m (÷ 1000) → km]
b Actual distance = 150 mm × 50 000 = 7 500 000 mm = 7500 m = 7.5 km	Multiply 150 mm by the scale factor of 50 000, giving an answer in mm. Then convert mm to m to km. [mm (÷ 1000) → m (÷ 1000) → km]
c Actual distance = 2.75 cm × 50 000 = 137 500 cm = 1375 m = 1.375 km	Multiply 2.75 cm by the scale factor of 50 000 to give an answer in cm. Then convert cm to m to km. [cm (÷ 100) → m (÷ 1000) → km]

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Example 25 Converting an actual distance to a scaled distance

A suburban development plan has a scale of 1 : 2000.
Use the given scale to calculate the scaled distance for the following actual distances.

a 60 m **b** 250 cm **c** 1.24 km

WORKING	THINKING
a Scaled distance = 60 m ÷ 2000 = 6000 cm ÷ 2000 = 3 cm	The scale 1 : 2000 shows a scale factor of 2000. Convert 60 m to 6000 cm, as 6000 is more easily divided by 2000. [m (× 100) → cm] Divide 6000 cm by 2000 and keep the units in cm.
b Scaled distance = 250 cm ÷ 2000 = 2500 mm ÷ 2000 = 1.25 mm	Convert 250 cm to 2500 mm, as 2500 is more easily divided by 2000. [cm (× 10) → mm] Divide by the scale factor of 2000.
c Scaled distance = 1.24 km ÷ 2000 = 1240 m ÷ 2000 = 0.62 m = 62 cm	Convert 1.24 km to 1240 m, as 1240 is more easily divided by 2000. [km (× 1000) → m] Divide 1240 m by 2000 and keep the units in m. Convert 0.62 m to cm. [m (× 100) → cm]

Example 26 Determining scale ratios

Determine the scale ratio for a map scale when a map length of 12 cm represents an actual distance of 36 km.

WORKING	THINKING
Scale ratio = 12 cm : 36 km = 12 cm : 3 600 000 cm = 12 : 3 600 000 = 1 : 300 000	Scale ratio = drawing length: actual length. Include the units for both distances. First convert both lengths to the same units, then write the scale ratio without units. [km (× 1000) → m (× 100) → cm] [HCF = 12; hence, divide both ratio parts by 12.]

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Example 27 Finding the height of a model from a real-life measurement

The TV transmission towers in Perth are approximately 200 m in height.

a Use an appropriate calculation to write this height in centimetres.
The tower height is reduced using a scale of 1 : 500 to make a scale model.

b Identify which number in this ratio represents the actual height of the tower.

c Use the scale to calculate the height of the model tower.

WORKING	THINKING
a $200\text{ m} = 200 \times 100\text{ cm}$ $= 20\,000\text{ cm}$	◄··· 1 m = 100 cm
b 500 represents the height of the tower.	◄··· 1 : 500 = model : actual tower
c Model tower height $= 20\,000\text{ cm} \div 500$ $= 40\text{ cm}$	◄··· The tower is *larger* than the model, so *divide* by the scale factor to find the model height.

Example 28 Finding the actual and scaled distances from a real-life situation

a A scale model of an Indigenous bark canoe is 20 cm long. If it was built at a scale of 1 : 15, calculate the length of the actual canoe in cm and m.

b The beach on K'gari (Fraser Island) is 120 km long. Calculate the scaled map distance of this beach on a map that has a scale ratio of 1 : 1 000 000.

WORKING	THINKING
a Actual length $= 20\text{ cm} \times 15$ $= 300\text{ cm}$ $= 3\text{ m}$	◄··· The actual canoe is *larger* than a model. So *multiply* the scale factor to give 300 cm. Then convert cm to m. [cm (÷ 100) → m]
The canoe is actually 3 m long.	Answer the question in words.

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b 120 km = 120 000 m
Scaled distance = 120 000 m ÷ 1 000 000
= 0.12 m
= 12 cm

◄··· First convert km to m for easier division.
A map is *smaller* than the real beach, so *divide* 120 000 m by the scale factor; the answer will also be in m. Then convert 0.12 m to cm as sensible map measurement units.
[m (× 100) → cm]

The map distance will be 12 cm.

Answer the question in words.

Exercise 10H

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in each of the following.

a Scale ratio on a drawing =_____length : _____length.

b A house plan with scale 1 : 100 means __ cm on the plan is _____ ___ on the house.

c The scale factor is the _____ number in a ratio that starts with 1.

d A drawing length is _____ than the actual distance, so _____ the drawing length by the scale factor to find the actual distance.

e An actual length is _____ than a drawing length, so _____ the actual length by the scale factor to find the drawing length.

Example 24

2 Use the given scale to calculate the actual distance for each of the scaled distances. Give your answer in appropriate units.

Hint Multiplying by a scale factor does not change the units.

a Scale 1 : 20 000
i 5 cm **ii** 150 mm **iii** 22 cm

b Scale 1 : 50 000
i 3 cm **ii** 200 mm **iii** 15 cm

c Scale 1 : 4000
i 50 mm **ii** 25 cm **iii** 1.6 cm

d Scale 1 : 2500
i 18 cm **ii** 2.75 cm **iii** 22.5 cm

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Example 25

3 Use the given scale to calculate the scaled distance for each of the actual distances.

Hint A scaled distance is *smaller* than the actual distance, so *divide* by the scale factor.

a Scale 1 : 100
i 500 m **ii** 3000 mm **iii** 2 km

b Scale 1 : 300
i 18 m **ii** 6 km **iii** 21 m

c Scale 1 : 20 000
i 4 km **ii** 200 m **iii** 14 km

d Scale 1 : 5000
i 30 m **ii** 2.5 m **iii** 3.75 km

Example 26

4 Determine the scale ratio for each of the following.

Hint Start with drawing length : actual length, including the units.

a A length of 24 mm on a scale drawing represents an actual distance of 4800 cm.
b A length of 15 cm on a scale drawing represents an actual distance of 30 km.
c A length of 2.5 mm on a scale drawing represents an actual distance of 75 km.
d A length of 75 cm on a scale drawing represents an actual distance of 900 m.
e An actual length of 5550 m is represented by 37 cm on a scaled drawing.
f An actual length of 5376 cm is represented by 38.4 mm on a scaled drawing.

5 A map has a scale of 1 : 20 000. Use this scale to calculate how many centimetres, in real life, is represented by a length of 2 cm on the map.

Hint The real distance is *greater* than a map distance, so *multiply* by the scale ratio.

Example 27

6 A car is 5 metres long.
a Use an appropriate calculation to write this length in centimetres.
A scale model of the car is to be built, using a scale of 1 : 50.
b Identify which number in the ratio represents the actual length of the car and explain why.
c Use the scale to calculate the length of the model car.

7 An apartment tower is 90 metres tall.
a Use an appropriate calculation to write this length in centimetres.
Using a scale of 1 : 500, the tower is reduced to make a scale model.
b Identify which number in this ratio represents the actual height of the tower.
c Use the scale to calculate the height of the model tower.

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APPLICATIONS

Example 28a

8 A developer plans to build a new high-rise apartment tower. She has a model tower built to a scale of 1 : 2000. The model is 18 cm tall with a square base of side length 4 cm.

- **a** Use the scale to calculate the actual side length of the base of the building.
- **b** Use division to convert this measurement to metres.
- **c** Use the scale to calculate the actual height of the building, in centimetres.
- **d** Use division to convert this measurement to metres.
- **e** If each floor level is 3 m tall, determine how many floors there are in this building.

9 The HMS *Endeavour*, commanded by Captain Cook, had a length of 30 m. A model of this ship is built to a ratio of 1 : 40.

- **a** Explain why you would divide by 40 to calculate the length of the model ship.
- **b** Calculate the model length, in metres.
- **c** Use multiplication to convert the model length to centimetres.
- **d** Cook's cabin on the HMS *Endeavour* was around 1 m wide by 2 m long. Convert these dimensions to centimetres, and then calculate the length, width and floor area for Cook's cabin on this model.

James Cook's apprenticeship studies included maths, navigation and astronomy.

10 **a** A scale model of a caravan is to be created. Explain why a scale of 1 : 50 would be more appropriate to use than a scale of 1 : 10 000.

- **b** A scale model of a city is to be created. Explain why a scale of 1 : 10 000 would be more appropriate to use than a scale of 1 : 500.

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Example 28b

11 The McLaren P1 racing car is 4.588 m long. A detailed scale model of this car has been built to a scale of 1 : 14.

a Calculate the length of the model in metres.

b Use multiplication to convert this length of the scale model to millimetres.

Hint The model length is *smaller* than the real length, so *divide* by the scale ratio.

12 Refer to this map of Australia and answer the questions.

a Use a ruler to measure the 0 to 500 km scale in millimetres.

b Use your result from part **a** to determine how many millimetres on the map represents an actual distance of 100 kilometres.

c Use a ruler to measure the direct distance between Brisbane and Cairns in millimetres.

d Use your answer from part **b** to estimate the actual direct distance from Brisbane to Cairns, correct to the nearest 100 kilometres.

e Use a ruler to measure the direct distance between Brisbane and Perth in millimetres.

f Use your answer from part **e** to estimate the actual direct distance from Brisbane to Perth, correct to the nearest 100 kilometres.

g Use the scale to estimate the actual direct distance from Broome to Sydney.

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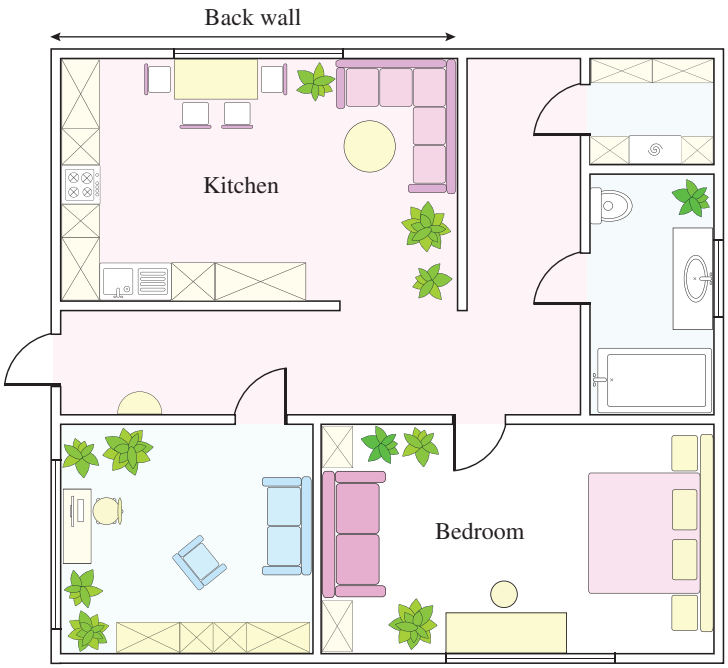
13 In the unit plan shown, the back wall of the kitchen is 4 m long.

a Determine an appropriate scale for this unit plan.

b Use this scale to find the dimensions of the bedroom.

c Calculate the area of the actual bedroom floor.

d Carpet costs \$66 per m^2 laid. Calculate how much it will cost to carpet the bedroom.



14 West Australian open-cut mines require huge dump trucks. The mining truck shown has a vertical height of 7.37 m (measured at the front left).

a Convert the vertical height of the mining truck to centimetres.

b Use a ruler to measure the height of the truck in the photo at the front left, in centimetres.

c Use your answers to parts **a** and **b** to calculate the scale for this photo.

d Use your ruler to measure the diameter of the front tyre of the truck in the photo.

e Use the scale to calculate the diameter of the front tyre of the actual truck, correct to the nearest 10 cm.

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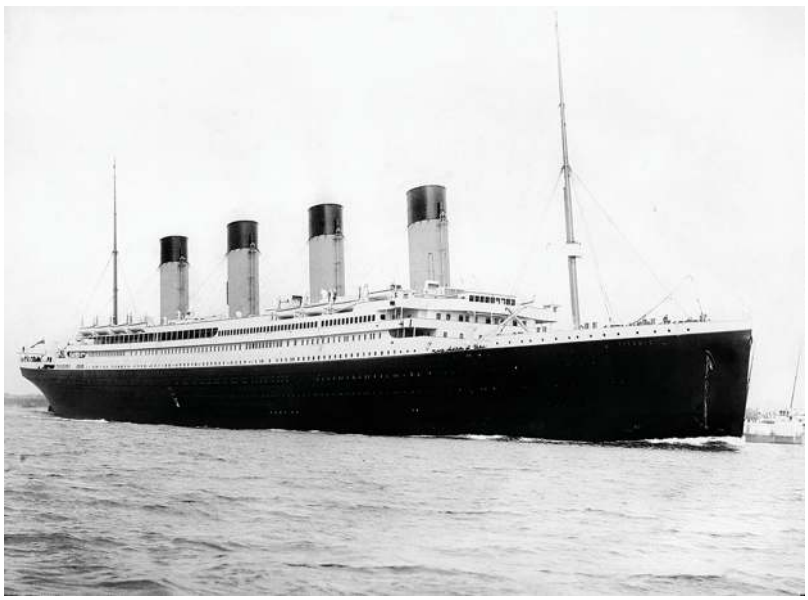
15 Sam has found a model of the RMS *Titanic* for sale, which is advertised as having a scale of 1 : 570. The actual RMS *Titanic* had a total length of 269.1 m, a maximum width of 28.2 m and a maximum height of 53.3 m. Calculate the length, width and height of the model.



16 Rachel has built a model of her family house. The model is 32 cm long, 24 cm wide and has a height of 9 cm. Rachel's actual house is 16 m long, 12 m wide and 4.5 m high. Determine the scale ratio for the model, in simplified form.

17 The person in the photograph is 1.9 metres tall in real life. Determine the scale for the photograph, and then use this scale to calculate the height that the dog (in sitting position) appears to be.

18 Julie owns a block of land that is 24 m by 36 m. She is planning to build a rectangular house that is 12 m by 18 m, but she is struggling to decide where to position the house on the land.

The council requires a 6-m setback from the street frontage (i.e. the house must be built at least 6 m back from the front of the block) and a minimum of 2 m space between all sides (left, right and back) and the edges of the block.
A scale of 1 : 120 will be used to draw a scale plan of the block and the outside of the rectangular house.

a Use all of the information to design a plan for where Julie's house should be situated on the block.

b Justify your placement of the house in its position on the plan.

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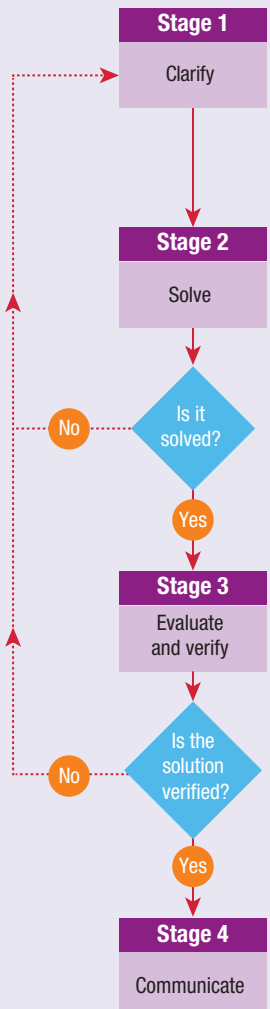
Mathematical thinking process task

CREATING A SCALE MODEL OF A CAR

Background: Scale models are used in the car industry to develop new designs.

Task: Your task is to construct a scale model of a car of your choice. You can do this in the form of a drawing or with the use of clay or plasticine. You will need measuring tape, rulers and either grid paper, modelling clay or plasticine.

To complete this task, follow the mathematical thinking workflow diagram below and use the steps listed as a guide.



Stage 1: Clarify
Interpret the task, gather the key information and identify the mathematics which could help to complete the task

1. Decide on the car you will use to make your scale model.
2. Decide on the maximum size of your model.
3. Decide what measurements on the car will be needed (e.g. various heights, lengths, widths, wheel diameter, etc.).

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

4. Create an appropriate scale to use (e.g. 5 cm = 100 cm).
5. Obtain measurements of the actual real car for which you are creating a model.
6. Use your scale to calculate measurements for the scale model.
7. Make your scale model, using pencil and grid paper, or with modelling clay/plasticine.

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources and verify the reasonableness of the solution

8. Compare your model with the actual car. Which parts of your model look correct? Which parts do not? Revisit steps in stage 1 (Clarify) and stage 2 (Solve) to make adjustments to your model.
9. Does your model now look in proportion to the actual car?

Stage 4: Communicate
Communicate findings in a systematic and concise manner

10. Communicate your findings in a short report, outlining the decisions involved in making your model, how you adjusted your model, and the calculations you used along the way.

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Puzzle

Mechanics are problem solvers. They use various tools, diagnostic tests and their own intuition and experience to put the clues together and identify the faults with the engine they are working on.

An interesting automechanical fact can be found by solving the puzzle on this page. Answer the questions and use the answers to fill in the letters in the grid below.

Mechanics working on a plane engine.

Calculate the missing values in the following equivalent ratios.

A $3 : 7 = \square : 56$ **B** $\square : 9 = 16 : 36$ **C** $4 : 7 : 11 = 28 : \square : 77$

Determine the highest common factor (HCF) of each of the following pairs of numbers.

D 12, 30 **E** 48, 80

Use division to simplify the following fractions.

F $\frac{30}{42}$ **G** $\frac{75}{135}$

Use what you have learned to simplify each of the following ratios.

H $\frac{3}{7} : \frac{5}{8}$ **I** $2\frac{4}{5} : 1\frac{3}{4}$ **J** $24 : 36$

K $12 : 18 : 21$ **L** $25\,\text{mm} : 50\,\text{mm}$ **M** $30\,\text{mL} : 4.5\,\text{L}$

For each of the following, calculate the size of each part when the quantity is divided in the given ratio.

N 175 in the ratio of $3 : 4$ **O** 175 in the ratio of $2 : 5$

P Determine the scale ratio when a length of 6 cm on a scale drawing represents an actual distance of 30 m.

$75 : 100$	$\frac{5}{9}$	16

$\frac{5}{7}$	$24 : 35$	$1 : 20$	$1 : 150$	$75 : 100$

$50 : 125$	$24 : 35$	$56 : 35$	6	$1 : 150$	49	$1 : 20$	16	16	$56 : 35$

$50 : 125$	$24 : 35$	$4 : 6 : 7$	16	$1 : 120$	$1 : 150$

$50 : 125$	16	$1 : 120$	16

$2 : 3$	$4 : 6 : 7$	16	$1 : 120$	24	$75 : 100$	16	6

4	$1 : 500$

$\frac{5}{9}$	24	$56 : 35$	6

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Chapter checklist

☐ I understand the fundamentals of ratios and their relationship with fractions.

1 Write $\frac{5}{9}$ as a ratio.

2 Write 21 : 17 as a fraction.

3 Express 8 apples to 11 oranges as a ratio.

☐ I can demonstrate my understanding of ratios.

4 Complete the equivalent ratio 20 : 36 = 30 : ☐.

5 If the oil-to-fuel ratio for an outboard motor is 1 : 25, calculate how much oil is needed for 5 litres of petrol.

☐ I can express a ratio in simplest form, using whole numbers.

6 Use division to simplify the ratio 28 : 35.

7 Use division to simplify the ratio 18 : 30 : 48.

8 Write the ratio $\frac{4}{7} : \frac{3}{5}$ so that the fractions have a common denominator, and use this to simplify the ratio.

☐ I can determine the ratio of two quantities in simplest form.

9 Convert the ratio 15 cm : 3 m to the same units, and then use division to simplify.

10 Convert the ratio 20 weeks : 2 years to the same units, and then use division to simplify.

11 A gardener uses 15 mL of pesticide for every 1 litre of water. Identify the ratio of pesticide to water, then write this as a simplified ratio.

☐ I can divide a quantity into a given ratio.

12 Divide 630 kg in the ratio 4 : 5.

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13 Uncle John leaves $200 000 000 to be split in the ratio 5 : 7 : 8 to his nephews, Logan, Lucas and Luther. Determine how much Luther receives.

☐ I can use a ratio to describe a simple scale.

14 Determine the scale ratio when a length of 30 mm on a scale drawing represents an actual distance of 4.5 m.

15 A developer wishes to create a model of a sample house. The house is 15 m wide, 21 m long and 3 m tall. Calculate the dimensions (width, length and height) of the model, using a scale of 1 : 50.

☐ I can determine the overall change that occurs when given multiple percentage changes.

16 Determine the overall change when $724 is increased by 38% and then increased by 24%.

17 Determine the overall change when $278 000 is decreased by 18% and then increased by 45%.

18 Milk costs the farmer 28 cents per litre to produce. The farmer sells it with an 80% mark-up to the retailer, who then marks it up by another 150%. Calculate the total cost increase of the milk per litre.

☐ I can calculate simple interest for different rates and time periods.

19 Calculate the value of n, in years, when the time period is 39 fortnights.

20 Calculate the value of i, the annual interest rate, if the interest rate per time period is 0.25% per week.

21 Calculate the simple interest earned on $2800, invested over a total of 18 months at 6.8% p.a.

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Chapter review

Section 10D

1 During one year, it rained for 73 days and it was dry on all the other days. Determine the ratio of rainy days to dry days for that year.

2 Mark is a bartender and mixes 30 mL of rum with 240 mL of ginger ale. Determine the ratio of rum to ginger ale, in simplest form.

Section 10E

3 A swim school has 7 teachers and 56 students. In simplest form, determine the ratio of teachers to students.

Section 10F

4 Robyn does twice as much cardiovascular exercise as weights in the gym. Interpret this information by writing it as a ratio. Determine the equivalent ratio if she does 30 minutes of cardio exercise.

5 A local dam is stocked with three types of fish – catfish, yellowbelly and barramundi – in the ratio 2 : 12 : 15. The dam is emptied and all the fish are relocated, of which 24 are catfish.

a Use the ratio to calculate how many yellowbelly you would expect to be relocated.

b Use the ratio to calculate how many barramundi you would expect to be relocated.

6 Lads Landscapes Supplier has 14 cubic metres of garden soil, consisting of 4 cubic metres of fertiliser to 10 cubic metres of ordinary soil.
If they wish to make more garden soil and they have 22 cubic metres of fertiliser, how much ordinary soil do they need to mix in with the fertiliser to maintain the same mixture?

7 Martha is a great tennis player and has won 42 games and lost 18 games so far in her career. If Martha wins 105 games in the next 3 years, calculate how many games you would expect her to have lost if she maintains her win : loss ratio.

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Section 10G

8 Moo Moo Milk Company produces 5600 bottles of flavoured milk per day, producing five chocolate for every three iced coffee and two strawberry milk.

a Calculate how many more bottles of chocolate milk than strawberry milk are made each day.

b Determine how many more bottles of chocolate milk than strawberry milk will be produced in a standard year.

9 Grace and Lilly both put money into their superannuation for their retirement. Grace saves $120 for every $90 that Lilly saves. After 10 years of saving, they have $109 200 in total. Calculate how much each has saved.

10 The Year 11 students have won 25.2 kg of lollies in a competition to make up an advertising slogan for the Honey Sweetie confectionary company. There are 45 boys and 60 girls. If each student receives an equal share, calculate how many kilograms of lollies the girls will receive.

Section 10M

11 Sam has a scale model of a container ship with a ratio of 1 : 200. His model is 1.6 m long. Calculate the length of the actual ship.

12 Igor wants to make a scale model of a skateboard ramp to show his parents what he would like to build in the backyard. The actual ramp is to be 8 m long and 3 m tall. Decide on an appropriate scale for his model if it is to have a maximum length of 50 cm and, hence, calculate the dimensions of Igor's model. Show all your working.

Section 10B

13 Farmer Joe harvests macadamia nuts at a cost of $1.70 per kilogram and sells them with a 150% mark-up to the markets. The market sells them to the retailer with a 75% mark-up. The retailer marks up the macadamia nuts another 200%. Calculate the overall change in the cost of the macadamia nuts, per kilogram.

Section 10C

14 Gavin invests $37 000 over a total of 3 years, earning 8.6% p.a. simple interest. Calculate the amount of interest he receives.

15 Susan invests $734 over a total of 4 years, earning 2.3% per month. Calculate the amount of interest she receives.

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Section 10B

16 Medusa purchases an old collectable car from a farmer for $12 000. She spends a further 60% of the purchase price to restore the car. Medusa advertises the car for sale after adding a mark-up of 80% to the total amount she has already spent on the car. Calculate the amount the car's value has increased by, when compared with the original purchase price.

Section 10C

17 Eliza invests $4200 at an interest rate of 4.8% p.a. over 20 months. Calculate how much money Eliza will have at the end of the 20 months.

18 Sam borrows $24 000 for a new car, which is to be repaid over 54 months at 13.8% p.a. simple interest. He must pay back the loan in 54 equal monthly payments. Calculate the total amount that Sam must pay back to the bank, including interest.

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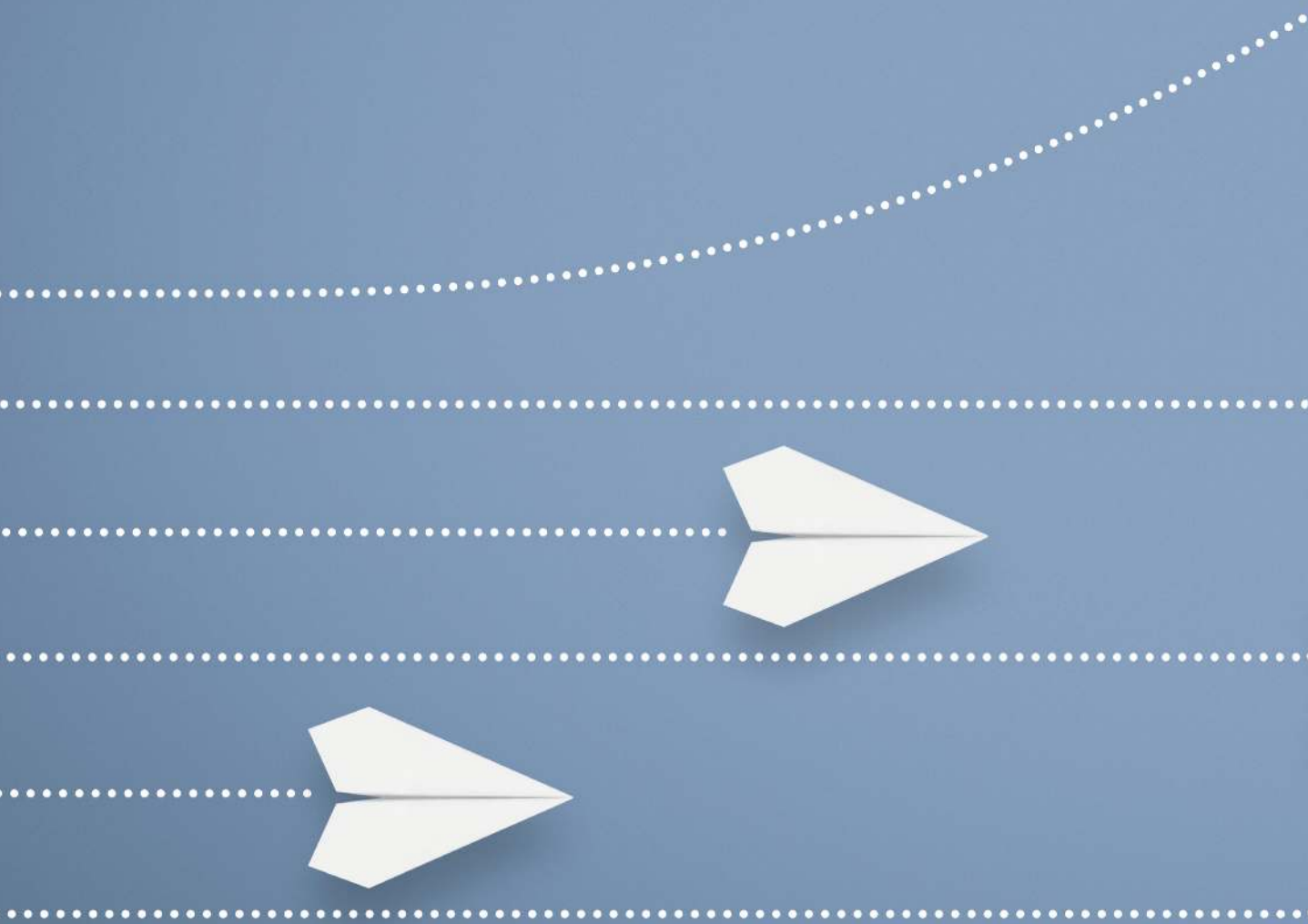
11 Rates



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In this chapter

Syllabus reference

Unit 2 Topic 2.3: Rates and ratios

Rates

- review identifying common usage of rates, such as km/h
- convert units for rates; for example, km/h to m/s, mL/min to L/h
- complete calculations with rates, including solving problems involving direct proportion in terms of rates
- use rates to make comparisons
- use rates to determine costs

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Pre-test

1 Solve the following questions.

a $360 \div 3$
b $120 \div 60$
c $3 \div 9$
d $9 \div 36$
e $56 \div 7$
f $4 \div 8$

2 Simplify the following fractions.

a $\frac{25}{50}$
b $\frac{39}{13}$
c $\frac{6}{14}$
d $\frac{45}{21}$
e $\frac{5}{9}$
f $\frac{18}{27}$

3 In each pair of fractions, which is the larger one?

a $\frac{15}{6}$ or $\frac{8}{3}$
b $\frac{7}{9}$ or $\frac{5}{7}$
c $\frac{11}{17}$ or $\frac{9}{13}$

4 What do each of the following units measure?

a centimetres (cm)
b dollars ($)
c seconds (s)
d litres (L)
e kilograms (kg)
f degrees Celsius (°C)

5 Convert the following measurements to the units shown in the brackets.

a 3600 ml (L)
b 132 km (m)
c 270 minutes (hours)
d 5.5 years (days)
e $65 (cents)
f 14 000 g (kg)

6 Give two units of measurement that can be used to calculate the following.

a speed
b fuel usage
c cost of oranges

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11A Review of rates

LEARNING GOAL

- Review rates and their uses

WHAT YOU NEED TO KNOW

- Rates show the relationship between different types of quantities with different units.
- For example: A car's speed is a rate, usually written in kilometres per hour (km/h).
- Rates are always written with two different units, unlike simplified ratios which are written without units.
  - For example: the rate $\$50/10\,\text{m} = \$5/\text{m}$
  - For example: the ratio $60\,\text{cents} : \$1 = 60 : 100 = 3 : 5$
  - The word 'per' is used instead of the words 'divided by' or instead of the dividing symbol '/' (also known as a solidus).
- When simplifying rates, we find the amount of the first quantity per 1 unit of the second quantity.
  - For example: 32 km walked over 8 hours would be simplified to how many kilometres walked in 1 hour. Dividing both parts by 8 gives $4\,\text{km/h}$.
- It is conventional to write rates in their simplest form.
- The average rate is the change in the first quantity divided by the change in the second quantity.
  - For example: Jason's report increased from 500 words to 2000 words from 8 p.m. to 8:40 p.m. Average typing rate is change in words/change in time

$$\text{Average typing rate} = 1500\,\text{words}/40\,\text{min}\ (\div 40) = 37.5\ \text{words/min}$$

Example 1 Writing a rate and simplifying it

Use division to write each of the following as simplified rates.

a $34.95 is the cost of a 5-kg piece of ham.

b Monica took 36 minutes to drive 8 laps in her go-cart.

WORKING		THINKING
a	$\$34.95/5\,\text{kg} = \$6.99/\text{kg}$	◂····· Divide both sides by 5.
b	$\frac{36\ \text{min}}{8\ \text{laps}} = 4.5\text{min/lap}$	◂····· Divide both sides by 8.

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Example 2 Applying rates in a real-world situation

Kirra cycles 42 km in 72 minutes. Skye finishes a 30-km cycling race in 58 minutes. Calculate the rate of change of distance (i.e. speed) in km/minute, correct to one decimal place, for both cyclists. Who has the fastest rate?

WORKING	THINKING
Kirra 42 km/72 min = 0.6 km/min	Write the rate from the information given. Calculate the rate per minute by dividing both sides by 72 and rounding to one decimal place.
Skye 30 km/58 min = 0.5 km/min	Write the rate from the information given. Calculate the rate per minute by dividing both parts by 58 and rounding to one decimal place.
Kirra has the fastest rate.	Identify that Kirra's cycling rate is a larger number and write your answer as a short sentence.

Exercise 11A

Example 1

1 Use division to write each of the following as simplified rates. Round your answer to one decimal place if needed.

- **a** 300 km travelled in 5 hours
- **b** $7 for 4 kg of apples
- **c** $18 000 for 600 m^2 of land
- **d** 12 tries in six games
- **e** 660 mL of rainfall in 30 days

Example 2

2 Kane cycles 50 km in 108 minutes. Archie finishes a 42-km cycling race in 90 minutes. Determine who has the fastest rate of movement.

3 Amanda purchases 4 kg of chips and 16 litres of soft drink for a party. She has invited 32 guests to her party, where the chips and drink will be shared equally.

- **a i** Calculate the number of grams in 4 kg.
  - **ii** Calculate how many grams of chips each guest receives. Express this as a rate of grams/person.
- **b i** Calculate the number of millilitres in 16 litres.
  - **ii** Calculate how many millilitres of soft drink each person receives. Express this as a rate with appropriate units.

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11B Converting units

LEARNING GOALS

- Convert between speed in km/h and speed in m/s
- Convert the units of various rates by applying unit conversion rules
- Apply conversion of units to rates in real-world situations

When is it essential to convert the units of a rate?

To compare rates, they each need to have identical units. For example, there are many units of speed, such as miles/h, km/h and m/s. So to compare speeds, the units must be the same.
Rate units also need changing when the given rate for the application of a product doesn't match the required amount for a particular task. People working in trades regularly convert the manufacturer's stated units of application to units that are appropriate for the job they are working on.

A tiler can convert a rate of 1 litre of water per 4 kg tile adhesive to a usable rate of 125 mL of water per 500 g tile adhesive.

WHAT YOU NEED TO KNOW

- Converting speed units
  - To convert from m/s to km/h, *multiply* by 3.6: $\text{m/s} \times 3.6 \rightarrow \text{km/h}$
  - To convert from km/h to m/s, *divide* by 3.6: $\text{km/h} \div 3.6 \rightarrow \text{m/s}$
- When converting from mph (miles per hour) to km/h, use the conversion 5 miles = 8 km.
  - 5 mph = 8 km/h
    Hence, 1 mph $= \frac{8}{5}$ km/h and 1 km/h $= \frac{5}{8}$ mph.

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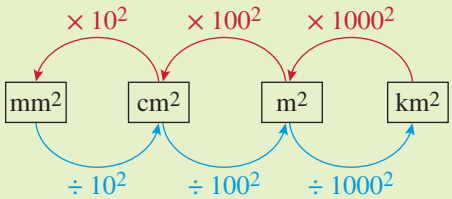
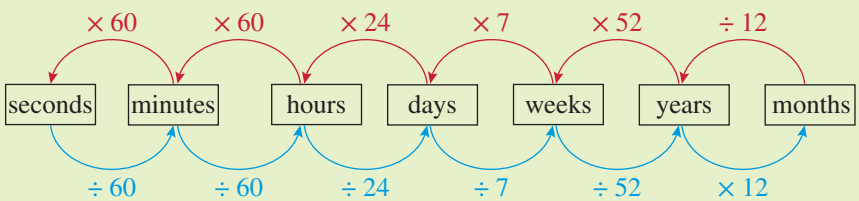
- To convert between area units, square the length unit conversion factors.



- Other area unit conversions
  - 1 hectare = 1 ha = 10 000 m^2
  - 1 ha = 2.47 acres
- Time conversion units



- Useful time unit conversions
  - $\frac{\text{number of weeks}}{52} \times 12 = \text{number of months}$
  - $\frac{\text{weekly wage} \times 52}{12} = \text{monthly wage}$
- When converting to another unit of measure, it is important to think whether your answer should be greater or smaller to decide whether to multiply or divide by the conversion number.

Example 3 Converting between metres per second and kilometres per hour

a Convert 24 m/s to km/h, given that 1 m/s = 3.6 km/h.
b Convert 100 km/h to m/s, given that km/h ÷ 3.6 gives m/s. Round your answer to 2 decimal places.

WORKING	THINKING
a 24 m/s × 3.6 = 86.4 km/h	1 m/s = 3.6 km/h, To convert from m/s to km/h, multiply by 3.6.
b 100 km/h ÷ 3.6 = 27.78 m/s	km/h ÷ 3.6 = m/s To convert from km/h to m/s, divide by 3.6, then round to 2 dp.

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Example 4 Converting units of rates by applying unit conversion rules

Convert the following rates to the units shown in the brackets.

a $\$12/\text{m}$ $(\$/\text{cm})$

b 21 700 cans/month (cans/year)

WORKING	THINKING
a $\$12/\text{m} \div 100$ $= \$0.12/\text{cm}$	1 cm is $\frac{1}{100}\times$ smaller than 1 m, so it will cost *less* per cm, so *divide* by 100.
b 21 700 cans / month $\times$ 12 = 260 400 cans / year	1 year is 12× larger than 1 month, so *more* cans will be produced per year than per month, so *multiply* by 12.

Example 5 Applying conversion of units to rates in real-world situations

In 2016, American athlete Kendra Harrison set a world record in the 100 m hurdles event, with a speed of approximately 8.2 m/s.

a Determine her speed in km/h.

b Kendra is faster than which of the following animals? A greyhound running at 60 km/h, a cat running at 48 km/h, a mouse moving at 14 km/h or a snake travelling at 24 km/h.

WORKING	THINKING
$8.2\,\text{m/s} \times 3.6$ $= 29.52\,\text{km/h}$	1 m/s = 3.6 km/h, so multiply 8.2 by 3.6.
Kendra is faster than both the mouse and the snake.	Compare with the animal speeds and write the answer in a sentence.

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Exercise 11B

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

a To convert from m/s to km/h, _____ by 3.6.

b _____ by 3.6 to convert from km/h to m/s.

c To convert between area units, _____ the length unit conversion factors.

d $\frac{\text{Number of } \square}{52} \times 12 = \text{Number of } \_\_\_\_\_$

e $\frac{\square \text{ wage} \times 52}{12} = \_\_\_\_\_ \text{ wage}$

Example 3

2 Convert the following speed units, rounding to one decimal place if needed.

a Convert metres/second to kilometres/hour, given that 1 m/s = 3.6 km/h.

i 20 m/s **ii** 12 m/s

iii 4.25 m/s **iv** 67.5 m/s

b Convert kilometres/hour to metres/second, given that km/h ÷ 3.6 produces m/s.

i 60 km/h **ii** 110 km/h

iii 80 km/h **iv** 6 km/h

At 100 km/h on a dry road, a car's stopping distance is 100 m.

ISBN 978-1-009-25785-5

Example 4

3 Convert the following rates to the units shown in the brackets.

a \$80/litre (\$/mL)
b \$250/kg (\$/gram)
c \$1 200 000/hectare (\$/m^2)
d 24 000 cans/day (cans/hour)
e 2500 mL/year (litres/year)
f 1.25 m/min (cm/min)
g 0.125 kg/year (grams/year)
h 7600 tonnes/month (kg/month)

Hint 1 hectare = 10 000 m^2
1 tonne = 1000 kg

4 Harry earns \$1400/week, which is based on a 36-hour, 5-day working week. Determine the following rates of Harry's income.

Hint Monthly wage = weekly wage × 52 ÷ 12

a \$/day
b \$/fortnight
c \$/hour
d \$/year
e \$/decade
f \$/month

5 Juanita earns \$2060/week, which is based on a 36-hour, 5-day working week. Find her income over the following different time periods.

a \$/day
b \$/fortnight
c \$/hour
d \$/year
e \$/decade
f \$/month

Hint \$/month is found by multiplying \$/week by 52 and then dividing by 12.

APPLICATIONS

Example 5

6 Alyssa worked for 74 hours to paint the interior of a new house. She was paid \$3515 for this job.

a Calculate her rate of pay in \$/h.
b Alyssa works a 40-hour week. Determine her rate of pay in \$/week.

Paint coverage rate is expressed in m^2 per litre.

7 A swimming pool has developed a slow leak and is leaking water at a rate of 10 mL per second.

a Use a suitable calculation to express this rate in litres per hour.
b The pool contains approximately 48 000 litres of water. If the leak goes unnoticed, determine how long it will take for the pool to empty.

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8 The Indianapolis 500 is a 500-mile car race held in Indiana, USA. Convert the following record speeds from mph to km/h, rounded to two decimal places.

Hint $5 \text{ mph} = 8 \text{ km/h}$
$1 \text{ mph} = \frac{8}{5} \text{ km/h}$

a The fastest lap speed ever is 239.26 mph, recorded during a practice session.
b The fastest lap speed recorded during a race is 236.103 mph

9 The scale on a map is 5 cm to 100 km. Determine the scale of the map in mm per km.

10 After heavy rain, 15 000 litres of water flows into a dam in 2.5 hours. Calculate the rate at which water flows into the dam in:
a litres/hour **b** litres/minute

11 Julia rides 8 km on her bike in 45 minutes. Calculate Julia's average speed in km/h.

12 It is estimated that your ears continue to grow at a rate of 0.22 mm per year as you age, as a result of gravity. If George has ears that are 7.2 cm long at the age of 50, how long will his ears be, in cm, when he is 80 years old?

13 The average sloth moves at a speed of 240 metres per hour. Convert this rate to km/h and then determine how long it takes, in hours, for the sloth to travel 0.6 km.

14 Chris decides to grow her hair long and wants it to be 60 cm longer to cover her entire back. Hair grows at about 0.15 m per year. Convert this rate to cm/year, and hence, determine how long it will take Chris to grow her hair to cover her back.

15 A cheetah runs at approximately 30.5 m/s. Convert this to km/h and determine the road speed limit that would be best suited for a car travelling at the same speed. The various speed limits are: 40 km/h, 50 km/h, 60 km/h, 80 km/h, 100 km/h and 110 km/h.

16 Some of the slowest-moving creatures on Earth are the sloth, snail, tortoise and worm. Use suitable calculations to determine which of the creatures is the slowest.
The average speeds are: snail 0.013 m/s, worm 0.2 cm/s, tortoise 0.48 km/h and sloth 240 m/h.

ISBN 978-1-009-25785-5

11C Solving problems using rates and direct proportion

LEARNING GOALS

- Find an unknown value, using rates and direct proportion
- Change rates, using direct proportion
- Solve problems involving direct proportion in real-world contexts

Who finds it essential to use direct proportion in their work?

Every person who uses a product for a smaller or larger application than the rate stated by the product's manufacturer finds it essential to use direct proportion. Countless people regularly use direct proportion, including chemists, farmers, cooks, nurses, doctors, mechanics, construction workers, plumbers, electricians, currency converters, engineers, hairdressers and DIY people. For example, a home gardener who needs to mix fertiliser for a 100-m^2 garden uses direct proportion to convert the manufacturer's rate appropriate for farms of 70 kg/hectare (i.e. 70 kg/10 000 m^2) to a home garden rate of 700 g/100 m^2.

Landscapers use direct proportion to adjust fertiliser application rates, to calculate quantities suitable for home gardens and lawns.

WHAT YOU NEED TO KNOW

- Two quantities in **direct proportion** increase together or decrease together at a constant rate.
  - For example: Ellie's wages are in direct proportion to the hours she works. Ellie's wages increase at a rate of \$20/h as her hours of work increase.
- All parts of a rate must be multiplied or divided by the same factor to maintain equivalence.
  - For example: If chlorine is added to a swimming pool at a rate of 40 g/10 000 L, then for a pool with five times the volume (i.e. 50 000 L) the amount of chlorine required is $5 \times 40\text{ g} = 200\text{ g}$.
- An unknown value is represented by a variable, such as x or y. These variables indicate a value that is to be calculated.

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Example 6 Finding an unknown value, using rates and direct proportion

Money, in $, is in direct proportion to time, in hours. Calculate the value of x if $24 in 1 hour is equivalent to $$x$ in 6 hours.

WORKING	THINKING
$\times 6$ ($24/1 hour = $144/6 hours) $\times 6$ $x = \$144$	As the number of hours is multiplied by 6, also multiply the dollars by 6.

Example 7 Changing rates, using direct proportion

The Hart family consume six bottles of juice in three days. Determine how many bottles they would consume in:

a one day

b eight days

WORKING	THINKING
a $\div 3$ (6 bottles/3 days = 2 bottles/1 day) $\div 3$ The Hart family would consume two bottles in one day.	Three days divided by 3 gives one day, so divide the six bottles by 3 to get two bottles.
b $\times 8$ (2 bottles/1 day = 16 bottles/8 days) $\times 8$ The Harts would consume 16 bottles in eight days.	One day multiplied by 8 gives eight days, so multiply the two bottles by 8 to get 16 bottles.

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Example 8 Solving a direct proportion problem in a real-world context

A Holden Commodore V8 police car has a fuel consumption rate of 12.6 litres per 100 km. If the car starts with a full 80-litre fuel tank, determine how far it could travel before its fuel tank is empty. Round your answer to the nearest km.

WORKING	THINKING
÷ 12.6 ↙ 12.6 litres/100 km ↘ ÷ 12.6 1 litre/7.94 km × 80 ↘ 80 litres/635 km ↙ × 80	Write the fuel consumption rate in litres/100 km. Fuel used is in direct proportion to the distance travelled. So divide both parts of the rate by 12.6 to find the distance per 1 litre of fuel.
The car should be able to travel 635 km.	Multiply both parts of the rate by 80 to find the distance travelled per 80 litres of fuel. Round your answer.

Exercise 11C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

a Two quantities in _____ proportion increase together or _____ together at a _____ rate.

b To calculate an _____ rate, both parts of the rate must be multiplied or _____ by the _____ factor.

Example 6

2 In each of the following, calculate the value of x if:

a 80 km travelled in 1 hour is equivalent to x km travelled in 4 hours

b \$12.75 earned in 1 hour is equivalent to \$$x$ earned in 6 hours

Hint First write the rate, then apply the same operation to both parts of the rate.

c 9 litres used in 1 minute is equivalent to x litres used in 4.5 minutes

d two questions completed in 1 minute is equivalent to x questions completed in 1 hour

e 210 km travelled in 3 hours is equivalent to x km travelled in 9 hours

f \$90 earned in 4 hours is equivalent to earning \$$x$ in 1 hour

g 120 litres leaked in 6 hours is equivalent to 40 litres leaked in x hours

h 30 hours in 21 days is equivalent to 10 hours in x days

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Example 7

3 Find the value of x and y in the following.

a 10 500 cans produced in 7 days
$= x$ cans produced in 1 day
$= y$ cans produced in 10 days

b It costs \$56 for 4 litres
$= \$x$ for 1 litre
$= \$y$ for 11 litres

c 200 metres travelled in 8 minutes
$= x$ m travelled in 1 minute
$= y$ m travelled in 30 minutes

d \$45 earned for working 3 hours
$= \$x$ earned for working 1 hour
$= \$y$ earned for working 5 hours

e 350 km travelled in 5 hours
$= x$ km travelled in 1 hours
$= y$ km travelled in 8 hours

f 108 texts sent in 6 hours
$= x$ texts sent in 1 hour
$= y$ texts sent in 13 hours

APPLICATIONS

4 At milking time, a dairy produces 22 litres/minute.

a Determine how many litres of milk are produced in 60 minutes.

b Determine how many litres are produced in 2 hours of milking.

Milking machines increase the rate of milk flow per hour.

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5 In a fruit and vegetable shop, 4 kg of tomatoes cost $4.80.
- **a** Determine the cost per kilogram.
- **b** Calculate the cost of purchasing 2.5 kg of tomatoes.

6 Nectarines are sold for $3 per kilogram when they are in season.
- **a** Determine the cost of:
  - **i** 2 kg of nectarines
  - **ii** 4.5 kg of nectarines
- **b** Calculate how many kilograms of nectarines can be purchased for $10.50.

7 A restaurant uses 750 litres of olive oil in a 30-day period.
- **a** Calculate the rate at which olive oil is used, in litres/day.
- **b** Determine how many litres of olive oil the restaurant would expect to use in one year.
- **c** The restaurant purchases good-quality olive oil for $10.40 per litre. Calculate how much it would spend on olive oil in one year.

Example 8

8 The new BMW turbo diesel police cars claim a fuel consumption of 5.1 L/100 km. If a car starts with a full 70-litre fuel tank, determine how far it could travel before its fuel tank is empty.

9 The Morello family's water bill states that they use an average of 550 litres per day of water. Calculate the number of litres of water they would use in one year.

10 If four ice-creams cost $18, calculate the cost of five ice-creams.

11 Calculate the cost of paving a driveway that is 5 m wide and 8 m long if paving costs $75 per square metre.

12 A team of bricklayers can lay 12 000 bricks, used to build one house, in four days. After 38 days' work at this rate, calculate how many houses this team has built.

13 A group of four friends goes camping and need to buy $210 worth of food for the trip. If five more people decide to join them, determine the cost of food for the entire group.

Bricklayers can be paid at a rate of $1.80/brick and work at a rate of 400 bricks/day.

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14 At a school camp, two teachers are legally required to supervise a maximum of 13 students, swimming in the river. If three teachers are available, determine how many students are able to go swimming at a time.

Hint Round down when the maximum value is a decimal answer.

15 The Nguyen family leave home for their family holiday. They leave home at 6 a.m. They plan to have morning tea at a town that is 275 km away from home. If they travel at an average speed of 85 km/h, calculate at what time they would expect to stop for morning tea.

16 When travelling on a highway, a hybrid car has a fuel consumption of 4.1 litres per 100 kilometres. If this car travels at an average speed of 95 km/h for 2 hours, how much fuel will it consume?

17 A modern Porsche drives 13.7 km on 1 litre of fuel. Calculate how many litres would be needed for this Porsche to drive from Exmouth to Broome, which is a distance of 1370 km.

18 The supervision rates of school students for swimming are three teachers per 21 primary school students and two teachers per 30 high school students.

If 174 students are swimming and 18 teachers are required, determine the ratio of primary students to secondary students.

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11D Comparisons using rates and determining best value

LEARNING GOALS

- Compare rates of price per equal weight or volume to find the best-value buy:
  - with a calculator, and
  - without a calculator.
- Solve best-value rate problems in real-world situations

When is it essential to compare rates?

Examples of when rates are compared include determining the best value for money, the most efficient production rates and the most economical rates of energy usage. Also, we can compare our pay rates to the Australian award pay rates, and we all need to compare our car's speed to the speed limit.

Milk is one of the many items sold in various-sized containers. Comparing costs of equal volumes or weights will identify the best buy for a product.

When a company makes decisions about large investments, it will consider and compare many types of rates. For example, a new building with a higher cost rate in $\$/m^2$ may be compared with alternative buildings that have lower cost rates. The company may consider whether the higher cost building will be longer lasting than one with a lower cost rate, which would require more frequent maintenance.

WHAT YOU NEED TO KNOW

- To find the **best buy**, convert each price option to the cost of equal amounts.
  - For example: The option on the left is the best buy of these two brands of chainsaw oil.

$$\div 3 \left(\begin{array}{l}\$76.50/6\text{ L} \\ = \$25.50/2\text{ L}\end{array}\right) \div 3 \qquad \div 4 \left(\begin{array}{l}\$106/8\text{ L} \\ = \$26.50/2\text{ L}\end{array}\right) \div 4$$

- The best value can be either the lowest rate or the highest rate, depending on the situation.
  - For example: The lowest rate is the best value for a car's fuel economy in litres/100 km.
  - The highest rate of data download, in Mbps (megabits per second), is the best value.

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Example 9 Comparing rates to find the best-value buy, without a calculator

Without a calculator, determine the cost of the following sugar brands per 100 grams, and hence, determine the best buy.
500 grams of Soft Sugar costs $2.25, and 300 grams of Sun Sugar costs $1.50.

WORKING	THINKING
Soft Sugar $2.25 ÷ 5 = $0.45/100 grams	Since 500 grams is 5 × 100 grams, divide $2.25 by 5.
Sun Sugar $1.50 ÷ 3 = $0.50/100 grams	Since 300 grams is 3 × 100 grams, divide $1.50 by 3.
It is the best value to buy the 500 g packet of Soft Sugar.	The lowest cost per 100 g is the better buy. Write the answer as a sentence.

Example 10 Comparing rates to determine the best-value buy, using a calculator

Determine the best value (i.e. the cheaper deal) between these products: 1.5 litres of Frank's Lemonade for $1.80 and 2.5 litres of Lucy's Lemonade for $2.80.

WORKING	THINKING
Frank's Lemonade ÷ 1.5 ($1.80/1.5 litres = $1.20/L) ÷ 1.5	Write the information as the rate $/litre. 1.5 L ÷ 1.5 = 1 L, so we also divide $1.80 by 1.5.
Lucy's Lemonade ÷ 2.5 ($2.80/2.5 litres = $1.12/L) ÷ 2.5	2.5 L ÷ 2.5 = 1 L, so we also divide $2.80 by 2.5.
2.5 L of Lucy's Lemonade is the best buy.	The lowest cost per litre is the best value.

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Example 11 Solving best-value rate problems in real-world situations

A carton of 20 cans of lemon soft drink costs $10.95 and a carton of 36 cans of lemon soft drink costs $20.95. Which is the cheapest way to buy the lemon soft drink?

WORKING		THINKING
$10.95/20 cans	$20.95/36 cans	Write as the rate $ per can.
$10.95 ÷ 20	$20/36	Divide $10.95 by 20 to find the cost per can.
= 0.55/can	= 0.58/can	Divide $20.95 by 36 to find the cost per can.
The best value is to buy the carton of 20 cans.		The lowest cost per can is the best value.

Exercise 11D

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** To find the _____ buy, convert each price option to the cost of _____ amounts.
- **b** The best value can be either the _____ rate or the _____ rate, depending on the situation.
- **c** The _____ rate is _____ value for a car's fuel economy, in litres/100 km.
- **d** The _____ rate of data download, in Mbps, is the _____ value.

Example 9

2 Without a calculator, determine the cost of the following items per 100 grams, and hence, determine the best buy.

Hint First write each as a rate per 100 g.

- **a** 500 grams of washing detergent for $2.25 and 1.2 kg of washing detergent for $5.04.
- **b** 300 grams of dim sims costs $1.95 and 1 kg of dim sims costs $6.10.
- **c** 750 grams of flour costs $1.65 and 500 grams of flour costs $1.05.
- **d** 250 grams of cereal costs $3.20 and 600 grams of cereal costs $7.74.

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Example 10

3 Determine the best value (i.e. the cheaper deal) between each of the following.

Hint First write each rate per equal quantity of the second amount.

- **a** 2.4 litres of cordial for $3 or 1.75 litres of cordial for $2.10?
- **b** 250 grams of coffee for $5.70 or 400 grams of coffee for $10.20?
- **c** 1.5 kg of oranges for $4.80 or 2.5 kg of oranges for $8.25?
- **d** Phone internet plan A is 3 GB for $22 or plan B is 5 GB for $36?

APPLICATIONS

4 At the supermarket, dishwasher soap tablets can be purchased in a box of 40 for $14 or a box of 320 for $79.

- **a** Calculate the cost per tablet for each box.
- **b** Determine how much cheaper it is per tablet for the best buy.

Calculating the best buy can result in big savings over a year.

5 A bottle of milk can be bought in several different sizes at different prices. At a supermarket, a 600-mL bottle costs $1.20 and a 2-litre bottle cost $3.60.

- **a** Determine the cheapest way to buy the milk.
- **b** Explain why the cheapest option may not be the most suitable in some circumstances.

Example 11

6 A carton of 24 cans of Super soft drink costs $12.99 and a carton of 30 cans of Cool soft drink costs $16.99. Compare these prices and determine the cheapest way to purchase the cans of soft drink.

Hint Write each price in $ per can.

7 A service station sells 600-mL bottles of soft drink for $3.20 and 1-litre bottles of soft drink for $5.50. Use a suitable calculation to determine the cheapest way to buy the soft drink.

8 Loose grapes can be purchased at a price of $4 per kilogram. The same type of grapes also can be purchased in a pre-packed container for $3 per 600 grams. Calculate how much more expensive it is per kilogram to buy the pre-packed grapes.

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9 Ibrahim and Yohann are fit bushwalkers who like to compare their long-distance walking speeds to see who is the faster. Ibrahim walks 60 km in 8 hours and Yohann walks 55 km in 7 hours. Which bushwalker has the faster walking rate?

10 A store is selling bulk potato chips at $5.49 for a 600-gram packet. If a 175-gram packet of the same type of chips costs $1.59, determine the cheapest way to buy the chips.

11 Hayden's car uses 40 litres of fuel to travel 520 km and Aria's car uses 50 litres of fuel to travel 645 km.

a Determine whose car is the most economical.

b If fuel costs $1.49/litre, calculate how much it will cost in fuel for each car owner to travel 904 km.

12 Bethany's car uses 65 litres of fuel to travel 780 km and Teri's car uses 55 litres of fuel to travel 638 km.

a Determine whose car is the most economical.

b If fuel costs $1.57/litre, calculate how much it will cost in fuel to travel 1726 km.

13 Sheldon is arranging a party for 18 people and will be catering for some treats and beverages per person. Sheldon determines that each person will drink 600 mL of soft drink, eat two party pies, four tiny sausages and 100 grams of corn chips.

Soft drink costs $1.80 for 1.25 litres or $2.80 for 2 litres.
Party pies cost $3.42 for 18 pies or $4.80 for 24 pies.
It costs $4.20 for 24 little sausages or $5.40 for 30 little sausages.
Corn chips cost $5.49 for 600 grams or $2.39 for 250 grams.

Using the prices given, calculate the following. Remember that the best value might not be the cheapest way to purchase items if the quantity doesn't match how much is required.

a the quantity of each item to buy

b the best buys for each item

c the cheapest cost for supplying these items for Sheldon's party

ISBN 978-1-009-25785-5

Mathematical thinking process task

PLANNING A ROAD TRIP

Background: Planning a trip requires the comparison of many different cost rates and rates based on time, such as speed.

Task: Your task is to plan a return trip, for two people, to a destination that is more than 8 hours' drive away from your home town. Then you are to compare the costs of driving with either flying or travelling by bus or train to your chosen destination.

You will use rates to determine the costs and the time required to make your round trip. For example, if driving from Geraldton to Carnarvon, you might average 80 km/h (i.e. when considering breaks).

To determine the cost of flying, research either airline sites or popular sites such as webjet.com.au.

You will then compare both costs and travel times, and draw a conclusion for the best-value mode of travelling, for two people, between your two chosen towns.

To complete this task, follow the mathematical thinking workflow diagram on the following page and use the steps listed as a guide.

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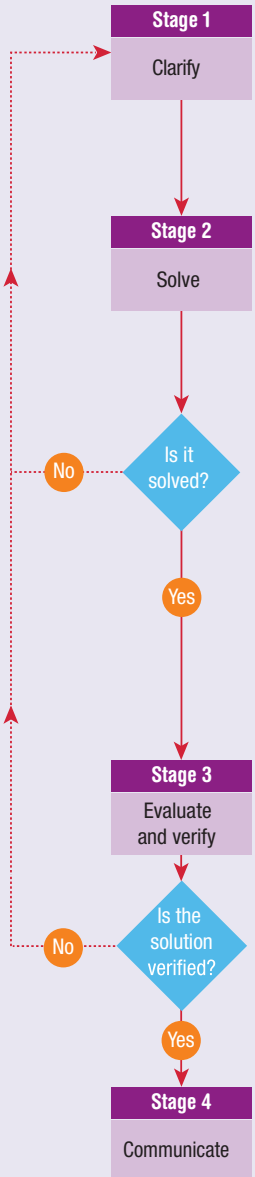

Stage 1: Clarify
Interpret the task, gather the key information and identify the mathematics which could help to complete the task

1. Decide on the car that you will drive.
2. Investigate possible journeys you might take.

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

3. Determine the car's fuel economy by referring to a website such as carsales.com.
4. Search Google to determine the distance between your two towns.
5. Determine the current cost of fuel, in $ per litre.
6. Calculate the fuel required, in litres.
7. Calculate the cost of fuel needed for the car trip.
8. Calculate the total time needed (including all breaks and stop-overs) to drive the return journey.
9. Use a suitable website to find the costs and travel times to fly or to travel by bus or train for a return journey between these two places.

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources and verify the reasonableness of the solution

10. Compare the total costs of the trip when driving with flying or travelling by bus or train.
11. Compare the travel times of driving with flying or travelling by bus or train between your town and chosen destination.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

12. Communicate your findings in a short report, describing whether you decided to drive or fly or travel by bus or train to complete your journey. In your report, refer to the different costs and time calculations that you have made.

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Puzzle

The first cash register was invented in 1879 in America. It was a purely mechanical adding machine, with its design based on a tool that counted the number of revolutions of a certain 'item'.

You will discover this item when you solve the following questions and use the answers to fill in the question letter above the answer in the grids below.

Use division to write each of the following as simplified rates (rounded to two decimal places if needed).

A \$9 per 4 kg

E 280 jumps in 15 minutes

Convert the following rates to the units shown in the brackets.

H 250 cm/hour (m/hour)

I three cans per minute (cans/hour)

Calculate the following speeds in km/h, given that $1\text{ m/s} = 3.6\text{ km/h}$.

L 35 m/s

M 7.25 m/s

Determine the value of x in each of the following.

O \$4.50 for 1 kg is equivalent to \$$x$ for 7 kg

P \$52 for 2 kg is equivalent to \$$x$ for 5 kg

R If Joey can wrap three presents in 20 minutes, determine how many presents he can wrap in 3 hours.

S Determine which is the best value: a 180-gram packet of chips that costs \$2.25 or a 150-gram packet of chips that costs \$1.75?

T For each of the following vehicles, calculate the fuel economy, in km/litre, and then determine which is the most economical: the Mercury, which travels 600 km with 48 L, or the Jupiter, which travels 520 km with 40 L.

2.25	150	12.5	18	2.25	26.1	150	2.5	180	130

130	27	31.5	130	18	126	126	18	27

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ISBN 978-1-009-25785-5

Chapter checklist

☐ I can understand the fundamentals of rates.

1 Identify which of the following are examples of rates:
16 m/s, 12 cm^2, 12 cans per second, \$6/kg, \$4 to 80 cents.
2 Recall the typical unit that would be used to describe the speed of an aeroplane.

☐ I can simplify rates and calculate average rates.

3 Write as a rate and use division to simplify: 21 goals in three games.
4 Calculate the average growth rate of a dog that has grown 12 cm between January 2019 and January 2022.

☐ I can convert the units of rates.

5 Convert 1750 m/h to km/h.
6 Convert 47 m/s to km/h.
7 Convert 90 km/h to m/s.

☐ I can determine the value of an unknown, using direct proportion.

8 125 litres used in 15 hours = 25 litres used in x hours. Find x.
9 \$14 for 2 kg = \$$x$ for 9 kg. Find x.
10 Linda can swim 200 m in 12 minutes. Calculate how long it will take her to swim 700 m at the same rate.

☐ I can find the best value by comparing rates.

11 Calculate the cost, per 100 grams, to determine the best value between 150 grams of tea for \$1.38 and 250 grams of tea for \$2.20.
12 Determine the best value if pine timber costs \$24 for 4.8 m and cypress timber costs \$15.60 for 3 m.
13 Fuelo station sells 60 litres of petrol for \$85.80 and Peteral station sells 70 litres of petrol for \$101.50. Determine which service station has the cheapest petrol.

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Chapter review

Section 11B

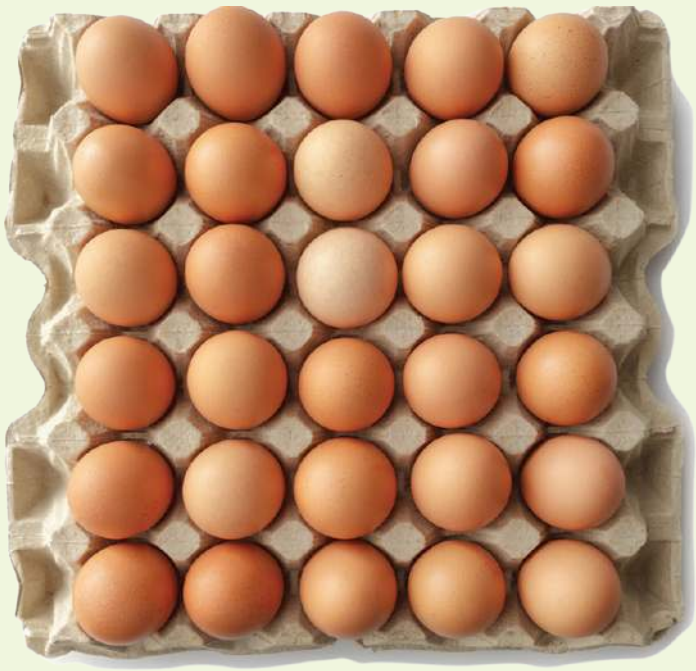
1 Leonard is a professional tiler and can lay $4\,m^2$ in 20 minutes.

a Write this as a rate of m^2/ min.

b Use a suitable calculation to convert the rate to the rate of m^2/hour.

c Determine how long it would take Leonard to lay $126\,m^2$ of tiles.

2 The local fruit shop sells eggs at \$4.20 for a dozen and \$9 for a tray of 30. Determine which is the cheapest way to buy eggs.



3 A 175-gram packet of biscuits costs \$2.50 and a 125-gram packet of the same brand costs \$1.80. Determine which packet is the best value. Explain your answer.

4 A family uses 9.8 units of electricity a day. Calculate how much they will use in a 90-day billing period.

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Section 11D

5 Lynn's car uses 38 litres of fuel to travel 480 km and Mike's car uses 52 litres of fuel to travel 745 km.

a Determine whose car is the most economical.

b If fuel costs \$1.49/litre, calculate how much it will cost, in fuel, for Lynn to travel 400 km.

6 At the supermarket, a brand of biscuit can be purchased in a box of 20 for \$3.60 or a box of 36 for \$5.75. Determine whether the box with 20 biscuits or the box with 36 biscuits is the best buy.

7 Adam paints 24 m^2 in 8 hours and Jarli paints 180 m^2 in 5 hours and 30 minutes. Which painter is faster and by how much?

Section 11C

8 You and two friends hire a boat for \$12 per hour and agree to share the cost. Just before you leave, two more friends join you and agree to share the cost. How much do you save when the two extra friends join you?

9 A patio measuring 7 m long and 4 m wide is to be paved. If paving costs \$62 per m^2, how much will it cost to pave the patio?

10 The birth rate in 2020 was 12.08 babies per 1000 people. If the population was 24 000 000, how many babies were born in 2020?

11 A people-mover car has a fuel economy of 8.1 litres per 100 km.

a Calculate how many litres of fuel is required to travel 1280 km.

b If fuel costs \$1.57 per litre, calculate how much it costs in fuel to travel that distance.

12 If a group of painters can paint 480 m^2 in 10 hours, determine how many hours it will take to paint 1380 m^2.

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12 Time

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In this chapter

Syllabus reference

Unit 2 Topic 2.4: Time and motion

Time

- use of units of time, conversions between units and fractional, digital and decimal representations
- represent time using 12-hour and 24-hour clocks
- calculate time intervals, for example, time between, time ahead and time behind
- interpret timetables, such as bus, train and ferry timetables
- use several timetables and electronic technologies to plan the most time-efficient routes
- interpret complex timetables, such as tide charts, sunrise charts and moon phases
- compare the time taken to travel a specific distance with various modes of transport

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Pre-test

1 Express the following statements as 12-hour times. You are not required to specify p.m. or a.m., so, for example, 'nine' would be '9:00 hours'.

a twelve
b three seventeen
c half past seven
d quarter to five
e quarter past nine
f five to eleven

2 Convert the following to seconds.

a 2 minutes
b half an hour
c 2 hours

3 Convert the following to hours.

a half a day
b three and a half days
c 2 weeks

4 Write the following times in words. (You are not required to specify p.m. or a.m.)

a 9:00 **b** 4:30 **c** 11:45
d 8:15 **e** 12:12 **f** 6:49

5 Calculate what the original time was half an hour ago if the time is currently:

a 9:45 a.m. **b** 6 p.m. **c** 12:15 a.m.

6 If it is 4:45 p.m. now, state what the time will be in:

a half an hour **b** 2 hours **c** 12 hours

7 Calculate what time it will be in 2 hours and 45 minutes if the time is currently:

a 5:15 a.m. **b** 6:45 p.m. **c** 10:55 a.m.

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12A Units of time and representing time

LEARNING GOALS

- Convert between the units of time
- Understand spoken language phrases that state the time
- Convert a time in digital format to time written with the:
  - minutes expressed as a fraction of an hour
  - minutes and seconds expressed as a decimal number
- Convert from time written as a decimal to the digital format of hours : minutes : seconds
- Convert between 12-hour clock time and 24-hour clock time
- Apply time conversions in real-life situations

Why is it essential to understand the various ways that time is written?

Universal formats for writing the time enable worldwide communication which is vital in complex global organisations and when travelling across different time zones.

Time coded into computer language enables humans to communicate with machines, computers and robots; for example, when managing driverless mining trucks and trains.

There are countless applications requiring time measurements and calculations, such as calculating wages and charges, communicating appointment times and starting and finishing times, understanding transport schedules and school and university timetables and computer calculations.

International travel requires time calculations using 24-hour, 12-hour and both digital and analogue formats.

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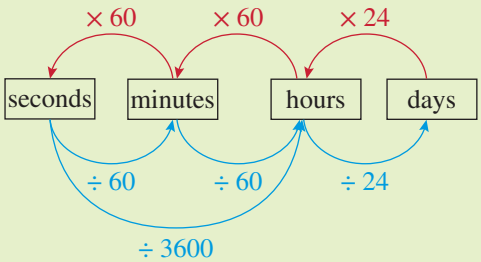
WHAT YOU NEED TO KNOW

- Conversion of **time units**



- When we speak about time, we use either fractions of an hour or minutes past or before the hour.
  - For example: 5:30 is spoken as 'half past five' or 'five thirty'.
    9:20 is spoken as 'twenty minutes past 9' or 'nine twenty'.
    11:45 is spoken as 'a quarter to twelve' or 'eleven forty-five'.
    We do not say 'eleven and three-quarters'.
    $2{:}15 = 2\frac{1}{4}$ is spoken as 'a quarter past two' or 'two fifteen'.
- Time in **digital format** is expressed as **hours : minutes : seconds**.
- When converting minutes to a fraction of an hour, divide the minutes by 60 and simplify the fraction.
  - For example: 3:15 = 3 hours and 15 minutes
    $= 3 \text{ hours and } \frac{15}{60} \text{ hours}$ ($\frac{\text{minutes}}{60} = \text{hours}$)
    $= 3\frac{1}{4} \text{ hours}$
- Converting minutes and seconds to decimal hours
  Either method 1 or 2 can be used.
  - **Method 1:** Convert seconds to minutes, then convert the total minutes to hours.

    For example: 2:23:10.5 = 2 hours, 23 minutes and 10.5 seconds
    $= 2 \text{ h} + \left(23 + \frac{10.5}{60}\right) \text{min}$ (seconds ÷ 60 = minutes)
    $= 2 \text{ h} + 23.175 \text{ min}$
    $= 2 \text{ h} + \frac{23.175}{60} \text{ h}$ (minutes ÷ 60 = hours)
    $= 2.38625 \text{ h}$
    $= 2.39 \text{ h}$
  - **Method 2:** Convert minutes to seconds (× 60) and add on the number of seconds. Then divide the total number of seconds by 3600 to give decimal hours.

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For example: 4:35:20 = 4 hours, 35 minutes and 20 seconds

$$= 4\text{ h} + (35 \times 60 + 20)\text{ s}$$
$$= 4\text{ h} + 2120\text{ s}$$
$$= 4\text{ h} + \frac{2120}{3600}\text{ h}$$
$$= 4.589\text{ h}$$

seconds ÷ 3600 = hours

- Converting time given in decimal form to time in digital format.
  First multiply the decimal part of the hours by 60 to give minutes in decimal form.
  Then multiply the decimal part of the minutes by 60 to give seconds in decimal form, and round to the nearest second.
  - For example: 5.378 = 5 hours + 0.378 hours

$$= 5\text{ h} + 0.378 \times 60\text{ min}$$
$$= 5\text{ h} + 22.68\text{ min}$$
$$= 5\text{ h} + 22\text{ min} + 0.68 \times 60\text{ s}$$
$$= 5\text{ h} + 22\text{ min} + 40.8\text{ s}$$
$$= 5{:}22{:}41$$

hours × 60 = minutes

minutes × 60 = seconds

- Clocks and timepieces can display the time in two different ways.
  - **24-hour clock** time is presented as how many hours and minutes since midnight.
  - **12-hour clock** time is presented by splitting the day into two 12-hour parts: a.m. and p.m.
  - **a.m.** is from midnight to midday and **p.m.** is from midday to midnight.
    For example:

12-hour clock	24-hour clock
12 a.m.(midnight)	00:00
9:45 a.m.	9:45
12 p.m. (midday)	12:00
2:30 p.m.	14:30

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Example 1 Convert time in digital format to time with the minutes expressed as a fraction of an hour

For each of the following times given in digital format:

i Write the minutes as a fraction of an hour and use division to simplify.

ii Write down how you would say each fractional time.

a 4:15 p.m. **b** 8:30 a.m. **c** 12:45 p.m.

WORKING	THINKING
a i $4{:}15\text{ p.m.} = 4\frac{15}{60}\text{ h}$	4:15 p.m. = 4 h and 15 min
$= 4\frac{1}{4}\text{ p.m.}$	Work out 15 min out of 60 min:
	HCF = 15, so $\frac{15}{60} = \frac{1}{4}$.
ii a quarter past four	
b i $8{:}30\text{ a.m.} = 8\frac{30}{60}\text{ h}$	8:30 a.m. = 8 h and 30 min
$= 8\frac{1}{2}\text{ a.m.}$	Work out 30 min out of 60 min:
	HCF = 30, so $\frac{30}{60} = \frac{1}{2}$.
ii half past eight	
c i $12{:}45\text{ p.m.} = 12\frac{45}{60}\text{ h}$	12:45 p.m. = 12 h and 45 min
$= 12\frac{3}{4}\text{ p.m.}$	Work out 45 min out of 60 min:
	HCF = 15, so $\frac{45}{60} = \frac{3}{4}$.
ii a quarter to one	As there is only $\frac{1}{4}$ of an hour before 1 p.m., this time is spoken as 'a quarter to one'.

Example 2 Convert time in digital format to time with the minutes and seconds expressed as a decimal

Express the following times, given in digital format, to times with the minutes and seconds calculated as a decimal, rounding to two decimal places.

a 7:35 **b** 2:43 **c** 8:15:48 **d** 5:33:27

WORKING	THINKING
a $7{:}35 = 7 + \frac{35}{60}$	7 h is the whole number.
$= 7.58\text{ h}$	$35 \div 60 = 0.583\,33$ h
	Round your answer to two decimal places.

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b $2{:}43 = 2 + \frac{43}{60}$
$= 2.72\text{ h}$

◄ 2 h is the whole number.
$43 \div 60 = 0.7167\text{ h}$
Round your answer to two decimal places.

c $8{:}15{:}48 = 8\text{ h} + \left(15 + \frac{48}{60}\right)\text{ min}$
$= 8\text{ h} + 15.8\text{ min}$
$= 8\text{ h} + \frac{15.8}{60}\text{ h}$
$= 8.26\text{ h}$

◄ 8 is the whole number of hours.
48 seconds ÷ 60 = 0.8 min
15.8 minutes ÷ 60 = 0.263 h
Round your answer to 0.26 h.
The time in decimal format is 8.26 h.

d $5{:}33{:}27 = 5\text{ h} + (33 \times 60 + 27)\text{ s}$
$= 5\text{ h} + 2007\text{ s}$
$= 5\text{ h} + \frac{2007}{3600}\text{ h}$
$= 5.56\text{ h}$

◄ This solution uses the alternative method of first converting minutes and seconds to total seconds.
5 is the whole number of hours.
$33 \times 60 = 1980$ s
1980 s + 27 s = 2007 s
2007 s ÷ 3600 = 0.5575 h and rounds to 0.56 h.

Example 3 Convert time written in decimal form to the digital format of hours : minutes : seconds

The following times are written in decimal form. Express these times in the digital format of hours : minutes : seconds, using multiplication to express the decimal part as minutes and seconds.

a 9.6 h **b** 11.3 h **c** 12.253 h

WORKING	THINKING
a $9.6\text{ h} = 9\text{ h} + (0.6 \times 60)\text{ min}$ $= 9{:}36\text{ a.m.}$	9 h stays the same. Multiply the decimal place by 60, so $0.6 \times 60 = 36$ min. Write the solution in the form of hours : minutes. The answer is a.m. because 9 h is less than 12 h in the 24-hour clock.

... *Continued*

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b $11.3\text{ h} = 11\text{ h} + (0.3 \times 60)\text{ min}$
$= 11{:}18\text{ a.m.}$

◄ 11 h stays the same. $0.3 \times 60 = 18$ min. Write the solution in the form of hours : minutes. The answer is a.m. because 11 h is less than 12 h in the 24-hour clock.

c $12.253\text{ h} = 12\text{ h} + (0.253 \times 60)\text{ min}$
$= 12\text{ h} + 15.18\text{ min}$
$= 12\text{ h} + 15\text{ min} + (0.18 \times 60)\text{ s}$
$= 12\text{ h} + 15\text{ min} + 11\text{ s}$
$= 12{:}15{:}11\text{ p.m.}$

◄ 12 h stays the same. $0.253 \times 60 = 15.18 = 15\text{ min} + 0.18\text{ min}$. $0.18 \times 60 = 10.8\text{ s} = 11\text{ s}$. Write the solution in the form of hours : minutes : seconds. The answer is p.m. because 12 h have passed in 24-hour clock time.

Example 4 Convert from a 12-hour clock to the 24-hour clock

Express these 12-hour digital times as 24-hour times.

a 8:30 a.m. **b** 12:07 a.m. **c** 2:35 p.m. **d** 10:17 p.m.

WORKING	THINKING
a 8:30 a.m. = 08:30	From 1 a.m. to 11:59 a.m., 12-hour and 24-hour time is the same, except single digit hours start with a zero.
b 12:07 a.m. = 00:07	From 12 a.m. to 12:59 a.m., the time is represented as 00:00 to 00:59 on the 24-hour clock.
c 2:35 p.m. $= (2 + 12)\text{ h} + 35\text{ min}$ $= 14{:}35$	Between 1 p.m. and 11:59 p.m., add 12 to the hour, but the minutes remain the same.
d 10:17 p.m. $= (10 + 12)\text{ h} + 17\text{ min}$ $= 22{:}17$	Between 1 p.m. and 11:59 p.m., add 12 to the hour, but the minutes remain the same.

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Example 5 Convert from the 24-hour clock to a 12-hour clock

Express the following 24-hour times as 12-hour digital times.

a 00:59 **b** 03:37 **c** 12:15 **d** 16:45 **e** 23:08

WORKING	THINKING
a 00:59 = 12:59 a.m.	00:59 means 59 min after midnight, as 24-hour time measures the time from midnight. It is a.m. time on the 12-hour clock.
b 03:37 = 3:37 a.m.	03:37 means 3 h and 37 min after midnight. It is a.m. time on the 12-hour clock.
c 12:15 = 12:15 p.m.	12:15 means 12h and 15 min after midnight, so change to p.m. time.
d 16:45 = (16 − 12) h + 45 min = 4:45 p.m.	16:45 means 16 h and 15 min after midnight. Subtract 12 from the number of hours for p.m. time.
e 23:08 = (23 − 12) h + 8 min = 11:08 p.m.	23:08 means 23 h and 8 min after midnight. Subtract 12 from the number of hours for p.m. time.

Example 6 Converting time representations in a real-world situation

Jack is an army recruit and he has received a text message from his Lieutenant General saying to meet for a training session at 17:30 hours. Jack wears a 12-hour digital watch. Determine what the time will be on Jack's watch when he must meet his Lieutenant General.

WORKING	THINKING
17:30 = 5:30 p.m. = (17 − 12):30 Jack must meet with his Lieutenant General at 5:30 p.m.	Between 13:00 and 23:59, subtract 12 from the number of hours for p.m. time. Minutes stay the same.

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Exercise 12A

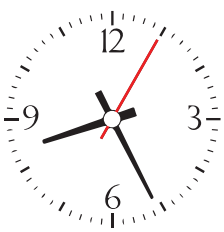
FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** Time in digital format is expressed as _____:_____:_____.
- **b** To convert minutes to a fraction of an hour, _____ the minutes by ___ and simplify the fraction.
- **c** To convert minutes and seconds to decimal hours:
  - **i** First _____ the number of minutes by ___ and add on the number of _____.
  - **ii** Then divide this total number of seconds by _____ to convert the seconds to decimal hours.
- **d** To convert time given in decimal form to time in digital format:
  - **i** First multiply the decimal part of the hours by ___ to give minutes.
  - **ii** Then multiply the decimal part of the minutes by ___ to give seconds.
- **e** 24-hour clock time is presented as hours and minutes since _____.

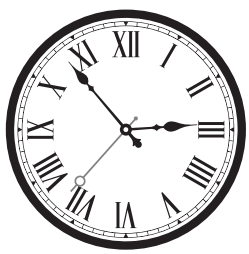
2 Identify the time shown on the following analogue clocks, and write this in the 12-hour digital time format of hours : minutes : seconds.

a



b

c



Example 1

3 For each of the following times given in digital format:

- **i** Write the minutes as a fraction of an hour and use division to simplify.
- **ii** Write how each fractional time is spoken.

Hint: Convert the minutes to a fraction out of 60 and simplify.

a 5:15 a.m. **b** 7:20 p.m. **c** 12:30 a.m.

d 10:45 p.m. **e** 6:30 p.m.

Example 2

4 Express the following times, in digital format, to times with the minutes and seconds calculated as a decimal, rounding to two decimal places.

a 9:35 a.m. **b** 5:43 p.m.

c 12:18 p.m. **d** 1:55 a.m.

e 3:16:30 a.m. **f** 8:33:27 p.m.

g 11:23:45 a.m. **h** 9:08:12 p.m.

Hint: Convert minutes and seconds to total seconds, then ÷ 3600 for decimal hours.

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Example 3

5 The following times are written in decimal form. Express these times in the digital format of hours: minutes: seconds, using multiplication to express the decimal part as minutes and seconds.

a 10.5 h **b** 9.7 h
c 5.3 h **d** 12.8 h
e 7.1 h **f** 1.29 h
g 3.55 h **h** 7.38 h
i 11.78 h **j** 2.94 h

Hint Decimal part of hour ×60 = minutes
Decimal part of minutes ×60 = seconds

Example 4

6 Express the following 12-hour digital times as 24-hour times.

a 8:37 a.m. **b** 4:48 p.m.
c 12:28 p.m. **d** 1:55 a.m.
e 3:16 a.m. **f** 8:33 p.m.
g 11:23 a.m. **h** 12:08 a.m.

Hint 24-hour time states time from midnight.

Example 5

7 Express the following 24-hour times as 12-hour digital times.

a 09:35 **b** 15:33
c 12:49 **d** 01:55
e 21:25 **f** 17:16
g 23:23 **h** 13:45
i 00:12 **j** 11:50

Hint 12 a.m. is midnight and 12 p.m. is midday or noon.

APPLICATIONS

8 This past week, Steven worked the following number of hours and minutes. Express the number of hours that he worked each day in decimal form.

a 7 h 30 min **b** 4 h 45 min
c 9 h 15 min **d** 5 h 20 min
e 6 h 40 min **f** 7 h 25 min

9 Using your answers from Question **8**, calculate the total number of hours that Steven worked that week.

10 The following times were collected from a school's cross-country event. They are written in the form of hours : minutes : seconds. Express these times in decimal form, rounding to two decimal places where necessary.

a 1:02:30 **b** 1:12:48 **c** 1:20:55
d 1:17:27 **e** 1:13:08 **f** 1:01:58

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11 **a** Consider the two runners from Question **10** who finished with the times 1:02:30 and 1:01:58. Explain which runner is the faster of the two.

b Using the decimal times you calculated in Question **10**, organise them in order from the fastest runner to the slowest runner.

12 Quinn has received a payslip from her boss, showing that she worked for 40.8 hours this past week. Determine how many hours and minutes this represents.

Example 6

13 Sarah works at a local bakery. Her roster is in 24-hour time, as shown in the table. Sarah wears a 12-hour digital watch. Determine the equivalent 24-hour times for each start and finish time.

Date	Start time	Finish time
Monday 3/3/22	08:30	13:30
Tuesday 4/3/22	08:30	16:30
Thursday 6/3/22	13:00	17:00
Friday 7/3/22	11:00	20:00

14 Matthew would like to watch the tennis tonight, which starts at 11:45 p.m. He is going to bed early so that he can have a sleep before the game and then still be up early for work the next day. Matthew's clock uses 24-hour time and he wants the alarm to go off at 11:40 p.m.
Express 11:40 p.m. in 24-hour time to determine the time that Matthew should enter into his clock so that he does not miss the tennis.

15 When Lachlan was mowing the lawn one afternoon, he noticed his watch had stopped at a quarter to eleven. He asked his friend for the time and he replied, "It's twenty to three."

a If the house clock uses 24-hour digital time, what time did his friend see on the clock?

b Calculate for how long Lachlan's watch had stopped, if it had stopped:

i that morning

ii the previous night

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12B Calculating time intervals

LEARNING GOALS

- Calculate the hours : minutes : seconds between two given times
- Calculate the clock time which is a certain number of hours ahead or behind a given clock time
- Apply clock time calculations in real-life situations

When is it essential to calculate time intervals?

When working with time schedules, it is important to accurately calculate the length of a time interval, in hours, minutes and seconds, between two clock times.

For a medical emergency, the patient's arrival time at the hospital must be accurately calculated.

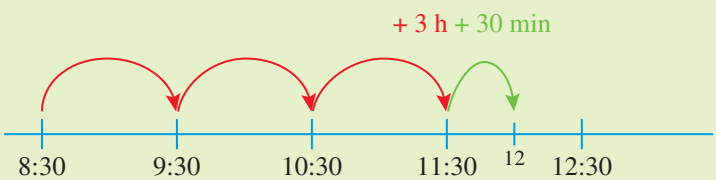
For example, many wage calculations require knowing the exact time worked; the cost of hiring expensive machines (e.g. a helicopter) is based on the exact time of usage; the crucial time for a political meeting or medical procedure will affect earlier events that day; and when travelling, we certainly need to know the time interval between connecting flights or train departures.

WHAT YOU NEED TO KNOW

- Basic time measurements

  24 hours in 1 day
  60 minutes in 1 hour
  60 seconds in 1 minute
  Midnight is 12 a.m., the middle of the night.
  Midday or noon is 12 p.m., the middle of the day.

- Calculate time intervals using a timeline
  - For example: The time difference between 8:30 a.m. and 12 p.m. is 3 h and 30 min.



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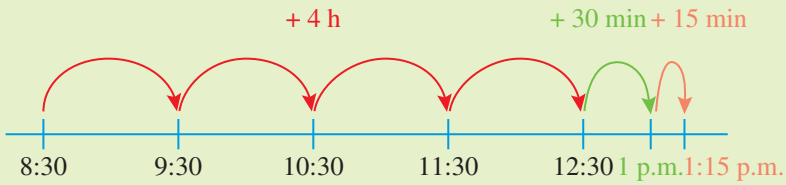
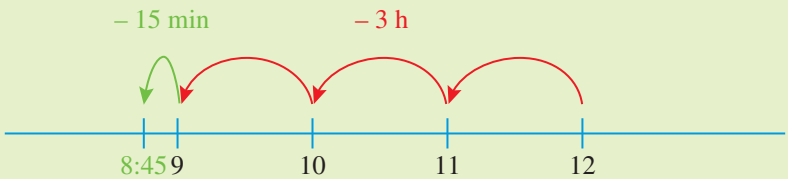
- For example: The time 4 h and 45 min after 8:30 a.m is 1:15 p.m.



- For example: The time 3 h and 15 min earlier than midday is 8:45 a.m.



Example 7 Calculate the hours : minutes : seconds between two given times

Calculate the length of time between each of these time periods.

a 1:30 p.m. to 7 p.m.

b 12:45 p.m. to 3:30 p.m.

c 8:47 a.m. to 12:08 p.m.

WORKING	THINKING
a 1:30 p.m. to 7 p.m. 1.30 p.m. to 6:30 p.m. = 5 h 6:30 p.m. to 7 p.m. = 30 min Time = 5 h 30 min	1:30 p.m. to 6:30 p.m. = 5 h and then 6:30 p.m. to 7 p.m. = 30 min. Adding both parts together gives 5 h and 30 min.
b 12:45 p.m. to 3:30 p.m. 12:45 p.m. to 2:45 p.m. = 2 h 2:45 p.m. to 3:00 p.m. = 15 min 3:00 p.m. to 3:30 p.m. = 30 min Time = 2 h 45 min	Break up the time into parts that are manageable. 15 min +30 min = 45 min and add the hours back in, giving 2 h and 45 min.
c 8:47 a.m. to 12:08 p.m. 8:47 a.m. to 11:47 a.m. = 3 h 11:47 a.m. to 12 p.m. = 13 min 12 p.m. to 12:08 p.m. = 8 min Time = 3 h 21 min	Break up the time into parts that are manageable. 13 min + 8 min = 21 min and add the hours back in, giving 3 h and 21 min.

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Example 8 Calculate the number of hours ahead of a given time

Calculate what time it will be in 2 h and 15 min when the current time is:

a 8:30 a.m. **b** 2:17 p.m. **c** 10:47 p.m.

WORKING	THINKING
a 8.30 a.m. + 2 h = 10:30 a.m. 10:30 a.m. + 15 min = 10:45 a.m. Time is 10:45 a.m.	Add the hours and minutes to the current time in parts. So 8:30 a.m. plus 2 h = 10:30 a.m. Plus 15 min = 10:45 a.m. (Check your calculations by adding together the hours and minutes that you added on, so 2 h + 15 min.)
b 2:17 p.m. + 2 h = 4:17 p.m. 4:17 p.m. + 10 min = 4:27 p.m. 4:27 p.m. + 5 min = 4:32 p.m. Time is 4:32 p.m.	Add the hours and minutes to the current time in parts. So 2:17 p.m. plus 2 h = 4:17 p.m. plus 10 min = 4:27 p.m. plus 5 min = 4:32 p.m. (Check your calculations by adding together the hours and minutes that you added on, so 2 h + 10 min + 5 min = 2 h 15 min.)
c 10:47 p.m. + 2 h = 12:47 a.m. 12:47 a.m. + 10 min = 12:57 a.m. 12:57 a.m. + 5 min = 1:02 a.m. Time is 1:02 a.m.	Add the hours and minutes to the current time in parts. So 10:47 p.m. plus 2 h = 12:47 a.m. (be mindful of the p.m. to a.m. change here) plus 10 min = 12:57 a.m. Plus 5 min = 1:02 a.m. (Check your calculations by adding together the hours and minutes.)

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ISBN 978-1-009-25785-5

Example 9 Calculate the number of hours behind a given time

Calculate what was the time 2 h and 41 min ago when the time is currently:

a 12 p.m. **b** 8:43 a.m. **c** 11:35 a.m.

WORKING	THINKING
a 12 p.m. − 2 h = 10 a.m. 10 a.m. − 30 min = 9:30 a.m. 9:30 a.m. − 10 min = 9:20 a.m. 9:20 a.m. − 1 min = 9:19 a.m. Time was 9:19 a.m.	Break up 41 min into 30 min +10 min +1 min. Check your calculations by adding together the hours and minutes that you subtracted, so 2 h + 30 min +10 min + 1 min = 2 h 41 min.
b 8:43 a.m. − 2 h = 6:43 a.m. 6:43 a.m. − 40 min = 6:03 a.m. 6:03 a.m. − 1 min = 6:02 a.m. Time was 6:02 a.m.	Break up 41 min into 40 min +1 min. Check your work by adding the hours and minutes.
c 11:35 a.m. − 2 h = 9:35 a.m. 9:35 a.m. − 30 min = 9:05 a.m. 9:05 a.m. − 10 min = 8:55 a.m. 8:55 a.m. − 1 min = 8:54 a.m. Time was 8:54 a.m.	Break up 41 min into 30 min + 10 min + 1 min. Check your work by adding the hours and minutes.

Example 10 Applying clock time calculations in a real-life situation

George went to his local shopping centre and parked in a 2.5 h carpark. He arrived at 12:17 p.m. and was back at his car by 2:57 p.m. Determine whether or not George could have a fine for parking too long.

WORKING	THINKING
12:17 p.m. + 2 h = 2:17 p.m. 2:17 p.m. + 30 min = 2:47 p.m. George needed to be back at his car by 2:47 p.m. and was 10 min late. George could have a parking fine.	Calculate the latest time that George would need to leave the carpark to avoid a fine. Add the 2 h and 30 min in parts to his arrival time. George went over the 2.5 h parking limit.

ISBN 978-1-009-25785-5

Exercise 12B

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** There are __ hours in 1 day.
- **b** There are __ minutes in 1 hour.
- **c** There are __ seconds in 1 minute.
- **d** Midnight is __ a.m., the middle of the _____.
- **e** Twenty minutes to midnight is written as __:__ _._. in 12-hour time.
- **f** Twenty minutes to 1 a.m. is written as __:__ _._. in 12-hour time.
- **g** Midday or noon is 12 _._., the middle of the _____.

Example 7 **2** Calculate the length of time, in hours : minutes : seconds, between each of these time periods.

- **a** 10:30 a.m. to 5:10 p.m.
- **b** 11:45 a.m. to 2:30 p.m.
- **c** 10:47 a.m. to 12:08 p.m.
- **d** 8:15 a.m. to 2:40 p.m.

Hint Break the time into manageable parts.

Example 8 **3** Calculate what time it will be in 3 h and 20 min when the current time is:

- **a** 9:30 a.m.
- **b** 1:15 a.m.
- **c** 11:17 p.m.
- **d** 3:47 p.m.
- **e** 8:39 p.m.

Hint Add the hours and minutes in manageable parts.

Example 9 **4** Calculate what was the time 2 h and 28 min ago when the time is currently:

- **a** 12:30 p.m
- **b** 9:50 a.m.
- **c** 12:35 a.m.
- **d** 7:40 p.m.
- **e** 2:25 a.m.

Hint Subtract the hours and minutes in manageable parts.

5 Calculate the missing times to complete the table.

Start time	End time	Time elapsed (in h and min)
8:40 a.m.	12:25 p.m.	**a**
b	4:50 p.m.	2 h 25 min
11:25 a.m.	**c**	5 h 22 min
1:08 p.m.	11:54 p.m.	**d**
2:34 a.m.	**e**	9 h 44 min
f	6:12 p.m.	7 h 15 min
10:36 a.m.	5:18 p.m.	**g**

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ISBN 978-1-009-25785-5

APPLICATIONS

6 The image displays the simultaneous time in various world cities. Calculate the time difference, in hours, between these cities.

- **a** Hong Kong (clock shows a.m. time) and Sydney (clock shows p.m. time on the same day).
- **b** Moscow (clock shows a.m. time) and Sydney (clock shows p.m. time on the same day).
- **c** Sydney (clock shows p.m. time) and New York (clock shows p.m. time on the previous day).
- **d** Sydney (clock shows p.m. time) and London (clock shows a.m. time on the same day).

Example 10

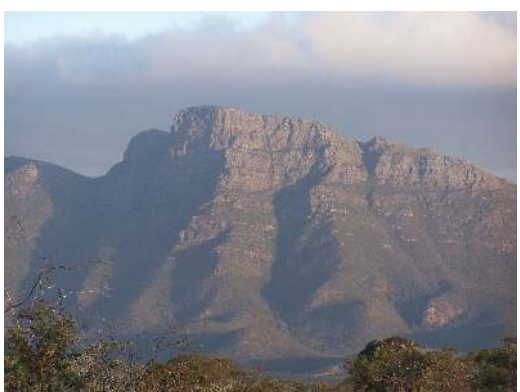
7 Jan went to her local shopping centre and parked in a carpark with a 2.5 h limit. She arrived at 10:57 a.m. and was back at her car by 1:42 p.m. Determine whether or not Jan could have a fine for parking too long.

8 Peter and his family are off on a camping holiday. It is currently 7:10 a.m. and the GPS is estimating that their destination arrival time is approximately 11:37 a.m. Calculate how long Peter will be driving.

9 Mahalia is rostered to work from 8:15 a.m. to 3:30 p.m. Calculate the length of her shift, in hours and minutes.

10 Ella and Madi went to see a movie that started at 4:05 p.m. They called their parents to pick them up as soon as they left the cinema at 6:18 p.m. Determine how long they were at the cinema, in hours and minutes.

11 Billy left his car at 6:17 a.m. and hiked the Bluff Knoll trail. If he arrived back to his car at 5:09 p.m., for how long did Billy hike?



Bluff Knoll is the highest peak of the Stirling Range. Hikers should be aware of clock times and walking speed to check their map location.

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12 Hayden's work roster is shown. Calculate the total number of hours he worked that week.

Date	Start time	Finish time
Monday 3/5/22	8:30 a.m.	1:30 p.m.
Tuesday 4/5/22	9:30 a.m.	2 p.m.
Thursday 6/5/22	1 p.m.	5:30 p.m.
Saturday 8/5/22	11 a.m.	6 p.m.

13 Charlie is off on a holiday to Geraldton, WA. She knows that the trip will take 4.5 hours and she plans to leave at 8:15 a.m. At what time would Charlie expect to arrive at Geraldton.

14 Jason is booked in for an MRI on his shoulder. He is told that the scan will take around 55 min. He looks at the time on his way into the room and it is 2:24 p.m. At what time would Jason expect to have completed his MRI?

15 Doug and Dena are planning to see a movie that starts at 9:15 a.m. and runs for 1 h and 50 min. They have an appointment at 11:10 a.m. next door to the cinemas. Determine whether they could see the movie and still make the appointment on time.

16 Alana climbed Bluff Knoll and when she reached the top, at 8:22 a.m., she checked her smartwatch and noticed that she had been climbing for 47 min. At what time did Alana begin her climb?

17 Bethany is at the hair salon, having some foils put in her hair. The hairdresser says the foils will take 45 min. The hairdresser begins to wash the foils out of Bethany's hair at 12:08 p.m. At what time did the hairdresser finish applying the foils?

18 Dean is a truck driver and his workplace insists that he stops for a break after approximately 3 h of driving. At 10:08 a.m., after 3 h and 15 min of continuous driving, Dean finds a nice spot to stop for his morning tea, his first break of the day. At what time did Dean begin driving that day?

ISBN 978-1-009-25785-5

12C Interpreting timetables and finding the best route

LEARNING GOALS

- Read and interpret train, bus and ferry timetables
- Select the best leaving times and routes for public transport
- Select the best route from multiple timetable options

When are timetables essential?

Timetables are essential for communicating the times of multiple future events. We can efficiently meet up in groups using timetables, such as for school classes, work rosters, sporting events, appointments and medical procedures.

Reading public transport timetables is a valuable skill, especially when travelling in India where, on average, over 22 million people travel by train every day.

Timetables or schedules are especially important for public transport, as they communicate, between the public and the drivers, the future arrivals and **departures** of cars, buses, ferries, trams, trains or planes. Such communication is essential for a smoothly run transportation service.

WHAT YOU NEED TO KNOW

- **Public transport timetables**, like the one on the following page, display information.
  Reading across, the *first row* is the list of location names of all the bus stops or train stations.
  Reading down, the *column* for a location shows all the departure times from that location.
  Reading across each *row*, listed next to each route number, are the departure times from each stop on the route. Each of these rows is a service.
  A timetable may use a single point instead of a colon between the hours and minutes, like the one on the following page (but the time is not decimal time).

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For example, 11.30 a.m. = 11:30 a.m. = half past eleven.

Timed Stops							
Stop No.		12212	11140	10729	10780	10786	20944
Route No.		Elizabeth Quay Bus Stn	Booragoon Bus Stn	Marmion Reserve	Fremantle Cemetery	Carrington St / Rockingham Rd	Asquith St / Beckett Cl
Monday to Friday							
am	114	7:00	7:18	7:21	7:29	7:35	7:50
	114	7:31	7:52	7:56	8:04	8:10	8:25
	114	8:01	8:22	8:26	8:36	8:42	8:59
	114	8:31	8:52	8:56	9:04	9:10	9:27
	114	9:01	9:22	9:26	9:34	9:40	9:57
	114	10:01	10:19	10:23	10:31	10:37	10:53
	114	11:00	11:19	11:23	11:31	11:37	11:54
pm	114	12:00	12:19	12:23	12:31	12:37	12:54
	114	1:01	1:19	1:23	1:31	1:38	1:55
	114	2:00	2:19	2:23	2:31	2:38	2:56
	114	2:30	2:49	2:53	3:03	3:10	3:29
	114 S	-	3:11	3:16	3:26	3:33	3:55
	114	3:00	3:21	3:26	3:36	3:43	4:03
	114	3:30	3:51	3:55	4:04	4:11	4:29
	114	3:50	4:11	4:15	4:24	4:31	4:49
	114	4:10	4:31	4:35	4:44	4:51	5:09
	114	4:30	4:51	4:56	5:05	5:12	5:30
	114	4:46	5:07	5:11	5:21	5:27	5:44
	114	5:01	5:22	5:26	5:34	5:40	5:57
	114	5:16	5:37	5:41	5:50	5:56	6:13
	114	5:30	5:51	5:54	6:03	6:09	6:26
	114	5:45	6:06	6:09	6:17	6:22	6:39
	114	6:00	6:21	6:24	6:32	6:37	6:52
	114	6:15	6:34	6:37	6:44	6:49	7:04
	114	6:35	6:54	6:57	7:04	7:09	7:24
	114	7:25	7:44	7:47	7:54	7:59	8:13
	114	8:25	8:41	8:44	8:51	8:56	9:10
	114	9:35	9:51	9:54	10:01	10:06	10:20
	114	10:35	10:51	10:54	11:00	11:05	11:18
	114	11:35	11:51	11:53	11:59	12:03	12:16

Reading the given table, down from Elizabeth Quay Bus Station, we see that, on weekdays, the bus for route 114 departs from Elizabeth Quay Bus Station at 7:00 a.m., 7:31 a.m., 8:01 a.m., 8:31 a.m., etc.
Reading across the second row, we see that the first bus for route number 114 departs from Elizabeth Quay Bus Station at 7:00 a.m. and is expected to arrive at Booragoon Bus Station at 7:18 a.m., Marmion Reserve at 7:21 a.m., Fremantle Cemetery at 7:29 a.m., etc.

- How to read timetables when choosing the start and finish locations:
  1. Select your departure location and go down to the time when you choose to depart.
  2. Run your finger across the row until you reach the desired finish location and read the **arrival time**.
- For example, bus route number 114 departs Elizabeth Quay Bus Station at 8:31 a.m. and arrives at Carrington St/Rockingham Rd at 9:10 a.m.

ISBN 978-1-009-25785-5

Example 11 Reading and interpreting a train timetable

This train timetable shows the schedule from the Train Terminal to the Botanical Gardens. Use the timetable information to answer the following questions.

Train route 178 → Terminal to the Botanical Gardens: Monday–Friday						
Station	**a.m.**	**a.m.**	**p.m.**	**p.m.**	**p.m.**	**p.m.**
Train Terminal	9:06	10:37	12:04	2:06	5:06	7:04
Sports Centre	9:19	10:50	12:17	2:19	5:19	7:17
Shopping Centre	9:25	10:56	12:23	2:25	5:25	–
University Campus	9:44	11:15	12:42	2:44	5:44	7:38
Botanical Gardens	9:48	11:19	12:46	2:48	5:48	7:42

a Determine how many times each day a train for route 178 departs the Train Terminal.
b Determine how many stops this train makes en route to the Botanical Gardens.
c Identify which departing train does not make all the stops. What might be a reasonable explanation for this?
d Determine how long it takes for the train to get from the Terminal to the Botanical Gardens.
e Paul catches the 10:37 a.m. train from the Terminal. At what time would he expect to arrive at the Botanical Gardens?
f Stacey catches the 5:19 p.m. train from the Sports Centre. At what time would she expect to arrive at the University Campus?

WORKING	THINKING
a There are six trains along this route each day, departing from the Terminal.	Count the number of departure times in the row across from the Terminal station.
b Five stops	Count the number of stations.
c The 7:04 p.m. train does not stop at the Shopping Centre, perhaps because the Centre is closed at this time.	Note the dash (–) on the 7:04 p.m. train at the Shopping Centre station.
d 9:06 to 9:48 (or 10:37 to 11:19, etc.) 9:06 to 9:46 = 40 min 9:46 to 9:48 = 2 min 40 + 2 = 42 min	Calculate the time interval, in parts, from the departure time to the arrival time.
e 11:19 a.m.	Read off the departure and arrival times on the timetable.
f 5:44 p.m.	Read off the departure and arrival times on the timetable.

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Example 12 Selecting the best leaving times and bus routes

Josh lives in Perth, near the Elizabeth Quay Bus Station, and works Mondays to Fridays in Fremantle, starting work at 9 a.m.
He catches a bus from Elizabeth Quay Bus Station to Fremantle Bus Station and then walks for 15 minutes to his workplace.
Use the Perth 160 bus timetable shown to answer the following questions.

Timed Stops								
Stop No.		22973	12213	10245	11139	10907	10790	10428
Route No.		Hale St / WACA	Elizabeth Quay Bus Stn	Canning Hwy / Ogilvie Rd	Booragoon Bus Stn	Archibald St / Jemerson St	Collick St / Chadwick St	Fremantle Stn
Monday to Friday								
am	160	-	-	-	5:58	6:05	6:14	6:33
	160	-	-	-	6:16	6:23	6:32	6:53
	160	-	-	-	6:36	6:43	6:52	7:13
	160	-	-	-	6:55	7:03	7:12	7:33
	160	6:37	6:52	7:01	7:09	7:17	7:26	7:53
	160	-	-	-	7:25	7:34	7:45	8:13
	160	7:07	7:22	7:31	7:41	7:50	8:01	8:30
	160 W	7:27	7:42	7:52	8:02	8:12	8:23	8:53
	160 W	7:47	8:04	8:14	8:24	8:34	8:45	9:10
	160 A	8:07	8:25	8:35	8:45	8:54	9:05	9:33

a Identify two times and their bus routes that Josh could catch to work in the mornings.
b Give a reason for which time and route is better.

WORKING

a Route 1: The 160 bus route at 6:52 a.m.
Route 2: The 160 bus route at 7:22 a.m.
b It would be best for Josh to aim for the 7:22 a.m. bus, as that would give him time to get to work and settle in for the day. If he caught the 6:52 a.m. bus he would arrive almost an hour early.

THINKING

Need to check the timetable and make sure that Josh arrives at Fremantle no later than 8:40 a.m., as this allows him the 15 min of walking time.

Example 13 Selecting the best route from multiple timetable options

Keeley has an interview with a clothing retailer at Kardinya Shopping Centre.
She lives in Perth and must catch a train from Perth to Fremantle then a bus to the Kardinya Shopping Centre for her 9:15 a.m. interview.
Use the information in the following timetables to select which train and then which bus Keeley should catch to arrive at her interview on time? Justify your selection of times.

... Continued

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Train Timetable

To Fremantle

Pattern		Perth 99007	City West 99202	West Leederville 99212	Subiaco 99222	Daglish 99232	Shenton Park 99242	Karrakatta 99252	Loch Street 99262	Claremont 99282	Swanbourne 99292	Grant Street 99302	Cottesloe 99312	Mosman Park 99322	Victoria Street 99332	North Fremantle 99342	Fremantle 99352
Monday to Friday																	
am		5:15	5:17	5:19	5:21	5:22	5:24	5:26	5:27	5:30	5:31	5:33	5:35	5:36	5:38	5:40	5:45
		5:30	5:32	5:34	5:36	5:37	5:39	5:41	5:42	5:45	5:46	5:48	5:50	5:51	5:53	5:55	6:00
		5:45	5:47	5:49	5:51	5:52	5:54	5:56	5:57	6:00	6:01	6:03	6:05	6:06	6:08	6:10	6:15
		6:00	6:02	6:04	6:06	6:07	6:09	6:11	6:12	6:15	6:16	6:18	6:20	6:21	6:23	6:25	6:30
		6:15	6:17	6:19	6:21	6:22	6:24	6:26	6:27	6:30	6:31	6:33	6:35	6:36	6:38	6:40	6:45
		6:30	6:32	6:34	6:36	6:37	6:39	6:41	6:42	6:45	6:46	6:48	6:50	6:51	6:53	6:55	7:00
		6:45	6:47	6:49	6:51	6:52	6:54	6:56	6:57	7:00	7:01	7:03	7:05	7:06	7:08	7:10	7:15
		7:00	7:02	7:04	7:06	7:07	7:09	7:11	7:12	7:15	7:16	7:18	7:20	7:21	7:23	7:25	7:30
		7:10	7:12	7:14	7:16	7:17	7:19	7:21	7:22	7:25	7:26	7:28	7:30	7:31	7:33	7:35	7:40
		7:20	7:22	7:24	7:26	7:27	7:29	7:31	7:32	7:35	7:36	7:38	7:40	7:41	7:43	7:45	7:50
		7:30	7:32	7:34	7:36	7:37	7:39	7:41	7:42	7:45	7:46	7:48	7:50	7:51	7:53	7:55	8:00
		7:40	7:42	7:44	7:46	7:47	7:49	7:51	7:52	7:55	7:56	7:58	8:00	8:01	8:03	8:05	8:10
		7:50	7:52	7:54	7:56	7:57	7:59	8:01	8:02	8:05	8:06	8:08	8:10	8:11	8:13	8:15	8:20
		8:00	8:02	8:04	8:06	8:07	8:09	8:11	8:12	8:15	8:16	8:18	8:20	8:21	8:23	8:25	8:30
		8:10	8:12	8:14	8:16	8:17	8:19	8:21	8:22	8:25	8:26	8:28	8:30	8:31	8:33	8:35	8:40
		8:20	8:22	8:24	8:26	8:27	8:29	8:31	8:32	8:35	8:36	8:38	8:40	8:41	8:43	8:45	8:50
	D	8:25	8:27	8:29	8:31	8:32	8:34	-	-	-	-	-	-	-	-	-	-
		8:30	8:32	8:34	8:36	8:37	8:39	8:41	8:42	8:45	8:46	8:48	8:50	8:51	8:53	8:55	9:00
		8:40	8:42	8:44	8:46	8:47	8:49	8:51	8:52	8:55	8:56	8:58	9:00	9:01	9:03	9:05	9:10
		8:50	8:52	8:54	8:56	8:57	8:59	9:01	9:02	9:05	9:06	9:08	9:10	9:11	9:13	9:15	9:20
		9:00	9:02	9:04	9:06	9:07	9:09	9:11	9:12	9:15	9:16	9:18	9:20	9:21	9:23	9:25	9:30
		9:15	9:17	9:19	9:21	9:22	9:24	9:26	9:27	9:30	9:31	9:33	9:35	9:36	9:38	9:40	9:45
		9:30	9:32	9:34	9:36	9:37	9:39	9:41	9:42	9:45	9:46	9:48	9:50	9:51	9:53	9:55	10:00
		9:45	9:47	9:49	9:51	9:52	9:54	9:56	9:57	10:00	10:01	10:03	10:05	10:06	10:08	10:10	10:15

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Bus Timetable

Route 511, 513 - To Murdoch

Timed Stops									
Stop No.	10435 / Stand 8	10754	19831	19802	26888	19689	10489	10495	23718 / Stand 12
Route No.	Fremantle Stn	Fremantle College	Ralston St / Dearle St	Winterfold Rd / Stock Rd	Coolbellup Shop Ctr	Somerville Bvd / Keall Pass	Kardinya Shop Ctr	Murdoch University	Murdoch Stn
Monday to Friday									
am 511	-	-	-	5:40	-	5:48	-	-	5:59
513	-	-	6:12	-	6:15	-	6:20	6:22	6:29
511	-	-	-	6:10	-	6:18	-	-	6:29
511	6:14	6:21	-	6:27	-	6:36	-	-	6:48
513	6:23	6:30	6:38	-	6:42	-	6:48	6:50	6:58
511	6:32	6:39	-	6:45	-	6:54	-	-	7:08
513	6:37	6:44	6:52	-	6:56	-	7:02	7:05	7:13
513	6:47	6:54	7:02	-	7:06	-	7:12	7:15	7:23
511	6:47	6:54	-	7:00	-	7:09	-	-	7:23
513	-	-	7:12	-	7:16	-	7:22	7:25	7:33
511	6:59	7:06	-	7:13	-	7:22	-	-	7:38
513	7:05	7:12	7:21	-	7:25	-	7:31	7:35	7:43
511	7:11	7:18	-	7:25	-	7:35	-	-	7:53
513	7:18	7:26	7:35	-	7:39	-	7:45	7:49	7:58
511	7:26	7:34	-	7:41	-	7:51	-	-	8:08
513	7:31	7:39	7:48	-	7:53	-	7:59	8:03	8:13
511	7:39	7:48	-	7:56	-	8:06	-	-	8:23
513	7:45	7:54	8:04	-	8:09	-	8:16	8:19	8:29
511	7:49	7:58	-	8:09	-	8:19	-	-	8:34
513	8:01	8:10	8:20	-	8:25	-	8:32	8:35	8:45
511	8:01	8:10	-	8:20	-	8:30	-	-	8:45
513	-	-	8:38	-	8:42	-	8:48	8:51	9:00
511	8:19	8:28	-	8:36	-	8:45	-	-	9:00
513	8:34	8:43	8:51	-	8:55	-	9:01	9:04	9:14
511	8:34	8:43	-	8:50	-	8:59	-	-	9:14
513	8:49	8:58	9:07	-	9:11	-	9:17	9:20	9:29
513	-	-	9:23	-	9:27	-	9:33	9:36	9:44
511	9:05	9:13	-	9:20	-	9:29	-	-	9:44
513	9:20	9:28	9:37	-	9:41	-	9:47	9:50	9:59
513	9:35	9:43	9:52	-	9:56	-	10:02	10:05	10:14
511	9:35	9:43	-	9:50	-	9:59	-	-	10:14
513	10:04	10:13	10:22	-	10:26	-	10:32	10:35	10:44

WORKING

Possible solution:
Train: depart Perth 8:00 a.m., arrive 8:30 a.m. Fremantle
Bus: Bus 513, depart 8:34 a.m. Fremantle, arrive 9:01 a.m. Kardinya Shopping Centre.
This is 1 possible route that Keeley can take that allows her to arrive at her interview location before 9:15 a.m., while also reducing the amount of time she spends waiting around.

THINKING

Multiple solutions are available. Ensure that the chosen route is reasonable and can be clearly justified.

ISBN 978-1-009-25785-5

Exercise 12C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words in the following statements.

- **a** Reading across, the *first row* is the list of _____ names of all the bus stops or train stations.
- **b** Reading down, the *column* for a location shows all the _____ times from that location.
- **c** Reading across each *row*, listed next to each route number, are the departure times from each _____. Each of these rows is a service.

Example 11

2 The train timetable from the Main Terminal to City Parklands is shown. Use this information to answer the following questions.

Train route 121 → Main Terminal to City Parklands: Monday–Friday						
Station	**a.m.**	**a.m.**	**p.m.**	**p.m.**	**p.m.**	**p.m.**
Main Terminal	07:07	09:38	12:05	02:07	05:07	07:05
Stadium	07:19	09:50	12:17	02:19	05:19	07:17
Entertainment Centre	07:26	09:57	12:24	02:26	05:26	–
University Campus	07:44	10:15	12:42	02:44	05:44	07:38
Factory Outlets	07:49	10:20	12:47	02:49	–	–
City Parklands	07:56	10:27	12:54	02:56	05:53	07:50

- **a** Determine how many times each day a train for route 121 departs the Main Terminal.
- **b** Determine how many stops this train makes en route to City Parklands.
- **c** Identify the departing trains that do not make all the stops.
- **d** Calculate how long it takes for the train to get from the Main Terminal to City Parklands.

> Hint Add or subtract times in manageable parts of a time interval.

- **e** Determine how long it takes the 7:44 a.m. train from University Campus to arrive at City Parklands.
- **f** Kelly catches the 9:38 a.m. train from the Main Terminal. At what time would she arrive at City Parklands?
- **g** Harrison catches the 5:19 p.m. train from the Stadium. At what time would he arrive at the University Campus?

ISBN 978-1-009-25785-5

Example 11

3 The bus timetable from Bunbury Bus Station to Bunbury Health Campus is shown. Use this timetable to answer the following questions.

Route 828, 829 - To Bunbury Health Campus

Timed Stops							
Stop No.		72030	*	71838	71879	72119	72048
Route No.		Bunbury Bus Stn	Bunbury Plaza	Yorla Rd / Mondak Pl	Frankel St / Woodrow St	South Regional TAFE	Bunbury Health Campus
Monday to Friday							
am	828	7:05	7:09	7:14	-	7:21	7:25
	829	7:20	7:25	-	7:31	7:39	7:43
	828	7:55	7:59	8:04	-	8:11	8:15
	828	8:20	8:24	8:30	-	8:38	8:43
	829	9:20	9:25	-	9:31	9:39	9:44
	828	9:50	9:54	10:00	-	10:07	10:11
	829	10:20	10:25	-	10:31	10:39	10:44
	828	10:50	10:54	11:00	-	11:07	11:11
	829	11:20	11:25	-	11:31	11:39	11:44
	828	11:50	11:54	12:00	-	12:07	12:11
pm	829	12:20	12:25	-	12:31	12:39	12:44
	828	12:50	12:54	1:00	-	1:07	1:11
	829	1:20	1:25	-	1:31	1:39	1:44
	828	1:50	1:54	2:00	-	2:07	2:11
	829	2:20	2:25	-	2:31	2:39	2:44
	828	2:50	2:54	3:00	-	3:07	3:11
	829 S	3:05	3:15	-	3:26	3:37	3:42
	829 H	3:15	3:20	-	3:25	3:33	3:36
	828	3:50	3:55	4:01	-	4:08	4:12
	829	4:20	4:25	-	4:30	4:38	4:41
	828	4:50	4:55	5:00	-	5:08	5:12
	829	5:15	5:20	-	5:25	5:33	5:36
	828	5:45	5:50	5:55	-	6:02	6:06
	829	6:15	6:20	-	6:25	6:33	6:36
	828 T	9:10	9:14	9:18	-	9:24	9:27

a Kadeem has just visited his wife who has had a baby at the Bunbury Health Campus. He must then go to the Bunbury Plaza to buy some baby supplies. If he arrives at the Bunbury Health Campus bus stop at 11:50 a.m., when will the next bus on route 828 or 829 arrive at his destination?

b Meghan has just finished her lecture at the South Regional TAFE, and it is 2:10 p.m. She is going to the Bunbury Health Campus. At what time will the next bus depart from the TAFE?

c Angelo has a job interview at the Bunbury Health Campus at 12:30 p.m. He finishes a shift at his job on Woodrow St at 11 a.m. Determine if he can catch a bus after 11 a.m. that will allow him to get to his interview on time or whether he needs to rearrange his shift.

ISBN 978-1-009-25785-5

4 The Perth–Mandurah train timetable is displayed. Use this timetable to answer the following questions.

To Mandurah

Pattern	Perth Underground 99602	Elizabeth Quay 99612	Canning Bridge 99632	Bull Creek 99642	Murdoch 99652	Cockburn Central 99662	Aubin Grove 99672	Kwinana 99682	Wellard 99692	Rockingham 99702	Warnbro 99712	Mandurah 99732
Monday to Friday (continued)												
W	8:56	8:58	9:04	9:08	9:11	9:14	-	-	-	-	-	-
	9:01	9:03	9:09	9:13	9:16	9:19	9:22	9:27	9:31	9:35	9:39	9:52
W	9:06	9:08	9:14	9:18	9:21	9:24	-	-	-	-	-	-
	9:11	9:13	9:19	9:23	9:26	9:29	9:32	9:37	9:41	9:45	9:49	10:02
W	9:16	9:18	9:24	9:28	9:31	9:34	-	-	-	-	-	-
	9:23	9:25	9:31	9:35	9:38	9:41	9:44	9:49	9:53	9:57	10:01	10:14
	9:38	9:40	9:46	9:50	9:53	9:56	9:59	10:04	10:08	10:12	10:16	10:29
	9:53	9:55	10:01	10:05	10:08	10:11	10:14	10:19	10:23	10:27	10:31	10:44
	10:08	10:10	10:16	10:20	10:23	10:26	10:29	10:34	10:38	10:42	10:46	10:59
	10:23	10:25	10:31	10:35	10:38	10:41	10:44	10:49	10:53	10:57	11:01	11:14
	10:38	10:40	10:46	10:50	10:53	10:56	10:59	11:04	11:08	11:12	11:16	11:29
	10:53	10:55	11:01	11:05	11:08	11:11	11:14	11:19	11:23	11:27	11:31	11:44
	11:08	11:10	11:16	11:20	11:23	11:26	11:29	11:34	11:38	11:42	11:46	11:59
	11:23	11:25	11:31	11:35	11:38	11:41	11:44	11:49	11:53	11:57	12:01	12:14
	11:38	11:40	11:46	11:50	11:53	11:56	11:59	12:04	12:08	12:12	12:16	12:29
	11:53	11:55	12:01	12:05	12:08	12:11	12:14	12:19	12:23	12:27	12:31	12:44
pm	12:08	12:10	12:16	12:20	12:23	12:26	12:29	12:34	12:38	12:42	12:46	12:59
	12:23	12:25	12:31	12:35	12:38	12:41	12:44	12:49	12:53	12:57	1:01	1:14
	12:38	12:40	12:46	12:50	12:53	12:56	12:59	1:04	1:08	1:12	1:16	1:29
	12:53	12:55	1:01	1:05	1:08	1:11	1:14	1:19	1:23	1:27	1:31	1:44
	1:08	1:10	1:16	1:20	1:23	1:26	1:29	1:34	1:38	1:42	1:46	1:59
	1:23	1:25	1:31	1:35	1:38	1:41	1:44	1:49	1:53	1:57	2:01	2:14
	1:38	1:40	1:46	1:50	1:53	1:56	1:59	2:04	2:08	2:12	2:16	2:29
	1:53	1:55	2:01	2:05	2:08	2:11	2:14	2:19	2:23	2:27	2:31	2:44
W	1:58	2:00	2:06	2:10	2:13	2:16	-	-	-	-	-	-
	2:03	2:05	2:11	2:15	2:18	2:21	2:24	2:29	2:33	2:37	2:41	2:54
W	2:08	2:10	2:16	2:20	2:23	2:26	-	-	-	-	-	-
	2:13	2:15	2:21	2:25	2:28	2:31	2:34	2:39	2:43	2:47	2:51	3:04
W	2:18	2:20	2:26	2:30	2:33	2:36	-	-	-	-	-	-
	2:23	2:25	2:31	2:35	2:38	2:41	2:44	2:49	2:53	2:57	3:01	3:14
W	2:28	2:30	2:36	2:40	2:43	2:46	-	-	-	-	-	-
	2:33	2:35	2:41	2:45	2:48	2:51	2:54	2:59	3:03	3:07	3:11	3:24
W	2:38	2:40	2:46	2:50	2:53	2:56	-	-	-	-	-	-
	2:43	2:45	2:51	2:55	2:58	3:01	3:04	3:09	3:13	3:17	3:21	3:34

a Rachel is planning to visit her friend who lives in Mandurah. She is leaving from Elizabeth Quay Station and is hoping to be in Mandurah just before 12 p.m. What is the latest time she could catch the train?

b Tracey is staying at a hotel in Perth and is hoping to catch a train to Cockburn Central to meet her mum for lunch at 12 noon. Advise her as to which train she could catch to be in Cockburn on time.

c Tom and Lizzy are meeting their friend in Warnbro. At what time would Tom need to catch the train from Bullcreek in order to meet Lizzy, who left Elizabeth Quay at 2.15 p.m., on the same train, so that they can continue the journey together to Warnbro?

d How long will it take Lizzy to travel from Elizabeth Quay to Warnbro?

5 Jerry has just returned from his holiday in Cambodia. His plane lands at 8:30 a.m. at Perth Airport (Terminal 4) and he expects it will take him 1 hour to go through passport control, collect his luggage, go through customs and make his way to the train station. Jerry then needs to catch a bus to Elizabeth Quay terminal, where he will catch a train to get to Mandurah.

ISBN 978-1-009-25785-5

Use the timetable given in Question **4**, together with the following timetable, to determine a plan for Jerry's trip from the airport to Mandurah. Organise the information in a table, such as the one shown. Justify your selection of times.

Location	Mode	Departure time	Destination	Arrival time
Terminal 4				

Route 40 - To Perth

	Timed Stops					
	Stop No.	15570	14487	14499	10070	12202
	Route No.	Perth Airport T3 and T4	Gt Eastern Hwy / Fauntleroy Av	Gt Eastern Hwy / Belmont Av	Victoria Park Transfer Stn	Elizabeth Quay Bus Stn
Monday to Friday						
am	40	5:34	5:39	5:45	5:51	6:04
	40	6:08	6:13	6:20	6:27	6:42
	40	6:38	6:43	6:50	6:57	7:13
	40	7:11	7:17	7:25	7:33	7:51
	40	7:40	7:46	7:56	8:05	8:24
	40	8:14	8:20	8:30	8:39	8:58
	40	8:47	8:53	9:00	9:08	9:27
	40	9:16	9:22	9:30	9:37	9:55
	40	9:46	9:52	10:00	10:07	10:25
	40	10:16	10:22	10:30	10:37	10:55
	40	10:46	10:52	11:00	11:07	11:25
	40	11:16	11:22	11:30	11:37	11:55
	40	11:46	11:52	12:00	12:07	12:25
pm	40	12:16	12:22	12:30	12:37	12:55
	40	12:46	12:52	1:00	1:07	1:25
	40	1:16	1:22	1:30	1:37	1:55
	40	1:46	1:52	2:00	2:07	2:25
	40	2:16	2:22	2:30	2:37	2:55
	40	2:46	2:52	3:00	3:08	3:27
	40	3:16	3:22	3:30	3:38	3:56
	40	3:50	3:56	4:05	4:13	4:33
	40	4:05	4:11	4:20	4:29	4:49
	40	4:30	4:36	4:45	4:53	5:15
	40	5:00	5:06	5:15	5:24	5:46
	40	5:31	5:37	5:45	5:53	6:11
	40	5:51	5:57	6:05	6:12	6:29
	40	6:22	6:28	6:35	6:42	6:58
	40	6:44	6:49	6:55	7:01	7:17
	40	7:14	7:19	7:25	7:31	7:46
	40	7:41	7:46	7:52	7:58	8:13
	40	8:11	8:16	8:22	8:28	8:43
	40	8:41	8:46	8:52	8:58	9:12
	40	9:11	9:16	9:22	9:28	9:42
	40	9:41	9:46	9:52	9:58	10:12
	40	10:12	10:17	10:22	10:27	10:41
	40	10:42	10:47	10:52	10:57	11:10
	40	11:13	11:17	11:22	11:27	11:39
	40	11:43	11:47	11:52	11:57	12:09
am	40	12:13	12:17	12:22	12:27	12:38

Route 40 - To Perth

	Timed Stops					
	Stop No.	15570	14487	14499	10070	12202
	Route No.	Perth Airport T3 and T4	Gt Eastern Hwy / Fauntleroy Av	Gt Eastern Hwy / Belmont Av	Victoria Park Transfer Stn	Elizabeth Quay Bus Stn
Saturday						
am	40	6:19	6:24	6:30	6:36	6:48
	40	6:49	6:54	7:00	7:06	7:18
	40	7:19	7:24	7:30	7:36	7:48
	40	7:49	7:54	8:00	8:06	8:19
	40	8:18	8:23	8:30	8:37	8:50
	40	8:48	8:53	9:00	9:07	9:22
	40	9:18	9:23	9:30	9:37	9:52
	40	9:48	9:53	10:00	10:07	10:24
	40	10:18	10:23	10:30	10:37	10:54
	40	10:47	10:52	11:00	11:08	11:24
	40	11:17	11:22	11:30	11:38	11:53
	40	11:47	11:52	12:00	12:07	12:24
pm	40	12:17	12:22	12:30	12:37	12:53
	40	12:48	12:53	1:00	1:07	1:23
	40	1:18	1:23	1:30	1:37	1:54
	40	1:48	1:53	2:00	2:07	2:23
	40	2:18	2:23	2:30	2:37	2:54
	40	2:48	2:53	3:00	3:07	3:23
	40	3:17	3:22	3:30	3:37	3:53
	40	3:47	3:52	4:00	4:07	4:22
	40	4:17	4:22	4:30	4:37	4:52
	40	4:48	4:53	5:00	5:07	5:23
	40	5:18	5:23	5:30	5:37	5:53
	40	5:48	5:53	6:00	6:07	6:22
	40	6:17	6:23	6:30	6:37	6:52
	40	6:48	6:54	7:00	7:07	7:22
	40	7:18	7:23	7:30	7:37	7:52
	40	7:48	7:53	8:00	8:07	8:22
	40	8:24	8:29	8:35	8:41	8:56
	40	9:24	9:29	9:35	9:41	9:56
	40	10:24	10:29	10:35	10:40	10:54
	40	11:25	11:30	11:35	11:40	11:53

ISBN 978-1-009-25785-5

6 Emma and Rob are on holiday on the Gold Coast, staying at the Northcliffe Hotel. They are planning to visit Movie World for the day and wish to arrive before the 9 o'clock opening time. They have a tram stop right outside their hotel, at Northcliffe, and they plan to catch the tram to Helensvale Station and then catch a bus from Helensvale to Movie World.
Determine a plan for their trip, using the following table, ensuring that they arrive on time before the Movie World gates open at 9 a.m.

Location	Mode	Departure time	Destination	Arrival time

Train timetable

Northcliffe station	7:33 am	7:41 am	7:48 am	7:56 am	8:03 am
Surfers Paradise station	7:35 am	7:42 am	7:50 am	7:57 am	8:05 am
Cavill Avenue station	7:37 am	7:45 am	7:52 am	8:00 am	8:07 am
Cypress Avenue station	7:40 am	7:47 am	7:55 am	8:02 am	8:10 am
Surfers Paradise North station	7:41 am	7:49 am	7:56 am	8:04 am	8:11 am
Main Beach station	7:44 am	7:51 am	7:59 am	8:06 am	8:14 am
Broadwater Parklands station	7:46 am	7:54 am	8:01 am	8:09 am	8:16 am
Southport South station	7:48 am	7:55 am	8:03 am	8:10 am	8:18 am
Southport station	7:51 am	7:58 am	8:06 am	8:13 am	8:21 am
Nerang Street station	7:52 am	8:00 am	8:07 am	8:15 am	8:22 am
Queen Street station (Southport)	7:56 am	8:03 am	8:11 am	8:18 am	8:26 am
Griffith University station (Southport)	7:59 am	8:06 am	8:14 am	8:21 am	8:29 am
GCUH	8:00 am	8:08 am	8:15 am	8:23 am	8:30 am
Parkwood East station	8:05 am	8:12 am	8:20 am	8:27 am	8:35 am
Parkwood station	8:08 am	8:15 am	8:23 am	8:30 am	8:38 am
Helensvale Station	8:11 am	8:19 am	8:26 am	8:34 am	8:41 am

Bus timetable

Route **TX7**

Helensvale to Coomera

servicing Helensvale, Wet 'n' Wild, Movie World, Oxenford, Dreamworld and Coomera

Monday to Friday

map ref	Route number	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7	TX7
		am	am	am						am	am	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
F	Helensvale station, stop C	8.44	8.59							11.34	11.44	12.14	12.44	1.14	1.44	2.14	2.44	3.14	3.44	4.14	4.44
E	Wet 'n' Wild	8.49	9.04							11.40	11.50	12.20	12.51	1.21	1.51	2.21	2.51	3.21	3.51	4.21	4.51
D	Movie World	8.53	9.08	Departs every 10 minutes						11.44	11.54	12.24	12.55	1.25	1.55	2.25	2.55	3.25	3.55	4.25	4.55
C	Siganto Dr near Trade Winds Dr	8.58	9.13	between 9am and 11.30am						11.49	11.59	12.29	1.00	1.30	2.00	2.30	3.00	3.30	4.00	4.30	5.00
B	Dreamworld	9.05	9.20							11.56	12.06	12.36	1.08	1.38	2.09	2.39	3.10	3.40	4.10	4.39	5.08
A	Coomera station	9.10	9.25							12.01	12.11	12.41	1.13	1.43	2.14	2.44	3.15	3.45	4.15	4.44	5.13

ISBN 978-1-009-25785-5

APPLICATIONS

Example 13

7 The Brisbane City Cat ferry timetable is given. Use the timetable to answer the following questions.

Departs Terminal:	am	am	am	am	am	am	am	pm	pm	pm	pm	pm	pm
Northshore Hamilton	10.19	10.34	10.49	11.04	11.19	11.34	11.49	12.04	12.19	12.34	12.49	1.04	1.19
Apollo Road	10.24	10.39	10.54	11.09	11.24	11.39	11.54	12.09	12.24	12.39	12.54	1.09	1.24
Bretts Wharf	10.27	10.42	10.57	11.12	11.27	11.42	11.57	12.12	12.27	12.42	12.57	1.12	1.27
Teneriffe	10.33	10.48	11.03	11.18	11.33	11.48	12.03	12.18	12.33	12.48	1.03	1.18	1.33
Bulimba	10.37	10.52	11.07	11.22	11.37	11.52	12.07	12.22	12.37	12.52	1.07	1.22	1.37
Hawthorne	10.42	10.57	11.12	11.27	11.42	11.57	12.12	12.27	12.42	12.57	1.12	1.27	1.42
New Farm Park	10.47	11.02	11.17	11.32	11.47	12.02	12.17	12.32	12.47	1.02	1.17	1.32	1.47
Mowbray Park	10.51	11.06	11.21	11.36	11.51	12.06	12.21	12.36	12.51	1.06	1.21	1.36	1.51
Sydney Street	10.54	11.09	11.24	11.39	11.54	12.09	12.24	12.39	12.54	1.09	1.24	1.39	1.54
Riverside	11.02	11.17	11.32	11.47	12.02	12.17	12.32	12.47	1.02	1.17	1.32	1.47	2.02
QUT Gardens Point	11.12	11.27	11.42	11.57	12.12	12.27	12.42	12.57	1.12	1.27	1.42	1.57	2.12
South Bank 2	11.16	11.31	11.46	12.01	12.16	12.31	12.46	1.01	1.16	1.31	1.46	2.01	2.16
North Quay	11.20	11.35	11.50	12.05	12.20	12.35	12.50	1.05	1.20	1.35	1.50	2.05	2.20
Milton	11.26	11.41	11.56	12.11	12.26	12.41	12.56	1.11	1.26	1.41	1.56	2.11	2.26
Regatta	11.30	11.45	12.00	12.15	12.30	12.45	1.00	1.15	1.30	1.45	2.00	2.15	2.30
Guyatt Park	11.34	11.49	12.04	12.19	12.34	12.49	1.04	1.19	1.34	1.49	2.04	2.19	2.34
West End	11.37	11.52	12.07	12.22	12.37	12.52	1.07	1.22	1.37	1.52	2.07	2.22	2.37
UQ St Lucia	11.41	11.56	12.11	12.26	12.41	12.56	1.11	1.26	1.41	1.56	2.11	2.26	2.41

a Saylesh is catching the ferry from Northshore Hamilton to Milton. He hops on the 10:49 a.m. ferry. At what time will he arrive at his destination?

b Graham also arrives at Milton at 11:56 a.m. from the same ferry, but he had hopped on at Hawthorne. Identify the time at which he would have caught the ferry.

c Josie has a lecture to attend at QUT every Thursday. The lecture starts at 12 p.m. and she catches the ferry from Bretts Wharf.

i Determine the latest time she could catch the ferry so that she is not late for the lecture.

ii At what time would she expect to arrive at QUT Gardens Point if she catches this ferry?

Hint For parts c to e, justify (i.e. give reasons for) your final choice of departure time.

d Ashleigh is taking his daughter Ella to the Queensland Museum at South Bank, and he has promised she can attend the 1 p.m. insect show. If they are catching the ferry from the Apollo Road terminal, at what time should they catch the ferry, considering that there is a 10-minute walk from the South Bank terminal to the Museum, and they will still need to purchase tickets for the show when they arrive?

e Tania and Mick have a lunch reservation for 1:30 p.m. at West End. They are catching the ferry from the New Farm Park terminal. What is the latest time they could catch the ferry and get to lunch on time?

A Brisbane ferry with the Storey Bridge in the background.

ISBN 978-1-009-25785-5

12D Comparing travel times with various modes of transport

LEARNING GOAL

- Compare the time taken to travel a specific distance with various modes of transport

When is it essential to compare travel times?

When travelling, there are often various modes of transport available that can take you to your destination. It is important to look at the most time-efficient manner when deciding on the mode of transport, as this can save you a lot of time, especially if it is a journey that you take on a regular basis, such as your daily trip to work.

WHAT YOU NEED TO KNOW

- **To calculate the time difference in travel times** (the time difference between departure and arrival in hours and minutes), first count the number of hours that have passed and then the number of minutes.
  - For example: Find the time difference between 2:43 p.m. and 4:08 p.m.
    2:43 p.m. to 3:43 p.m. = 1 h
    3:43 p.m. to 4 p.m. = 17 min
    4 p.m. to 4:08 p.m. = 8 min
    So, 1 h + 17 min + 8 min = 1 h 25 min.
- **Compare travel times** to decide which is the best travel option.
  Calculate the hours and minutes that it takes for each option, then check that the shortest time will still allow you to arrive before the required time.
- Use timetables to **determine departure and arrival times**.
  - For example: Using the following timetable, if you are travelling by train from Perth to Karrakatta and you depart at 6.45 a.m., you would arrive at 6.56 a.m.

To Fremantle

Pattern	Perth 99007	City West 99202	West Leederville 99212	Subiaco 99222	Daglish 99232	Shenton Park 99242	Karrakatta 99252
Monday to Friday							
am	5:15	5:17	5:19	5:21	5:22	5:24	5:26
	5:30	5:32	5:34	5:36	5:37	5:39	5:41
	5:45	5:47	5:49	5:51	5:52	5:54	5:56
	6:00	6:02	6:04	6:06	6:07	6:09	6:11
	6:15	6:17	6:19	6:21	6:22	6:24	6:26
	6:30	6:32	6:34	6:36	6:37	6:39	6:41
	6:45	6:47	6:49	6:51	6:52	6:54	6:56
	7:00	7:02	7:04	7:06	7:07	7:09	7:11
	7:10	7:12	7:14	7:16	7:17	7:19	7:21
	7:20	7:22	7:24	7:26	7:27	7:29	7:31
	7:30	7:32	7:34	7:36	7:37	7:39	7:41
	7:40	7:42	7:44	7:46	7:47	7:49	7:51
	7:50	7:52	7:54	7:56	7:57	7:59	8:01
	8:00	8:02	8:04	8:06	8:07	8:09	8:11

Cambridge Senior Maths for Western Australia
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ISBN 978-1-009-25785-5

Example 14 Comparing the travel times of various bus trips for a specific route

Chloe is heading to the Heritage Village from the centre of Perth, but she does not have a car. She has found that there are three bus options for her to choose from. Analyse each of the following options to determine the travel time, and hence, decide which option Chloe should choose if she wishes to minimise her travel time.

Option 1: Bus that departs at 9:16 a.m. and arrives at 10:29 a.m.
Option 2: Bus that departs at 9:20 a.m. and arrives at 10:18 a.m.
Option 3: Bus that departs at 9:31 a.m. and arrives at 10:15 a.m.

WORKING	THINKING
Option 1: 9:16 a.m. to 10:16 a.m. = 1 h 10:16 a.m. to 10:29 a.m. = 13 min Takes 1 hour and 13 minutes.	Calculate the amount of time that each option takes by breaking up the time into parts.
Option 2: 9:20 a.m. to 10:18 a.m. = 58 min Takes 58 minutes.	
Option 3: 9:31 a.m. to 10:01 a.m. = 30 min 10:01 a.m. to 10:15 a.m. = 14 min Takes 44 minutes.	
Chloe should take the bus that departs at 9:31 a.m., as it takes the least amount of time to get there.	Select the option that takes the least amount of time.

Example 15 Comparing bus and ferry timetables to find the earliest arrival time

It is currently 5 p.m. and Anthony is at South Bank, wanting to get to QUT Gardens Point. He has two options for public transport.

Option 1: Anthony can catch a ferry from South Bank 1 terminal and go straight to QUT Gardens Point, which then requires a 9-minute walk to access the campus.
Option 2: Anthony can get a bus at the South Bank bus station and go to Elizabeth Street, which then also requires a 9-minute walk to reach the QUT campus.

... Continued

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Use the following timetables to determine the best option for Anthony's travel if he wishes to travel for the least amount of time. Justify your choice.

Elizabeth Street plus a 9-minute walk to QUT

South Bank Busway Station	1:10 pm	2:10 pm	3:10 pm	4:10 pm	5:10 pm
Cultural Centre Station	1:13 pm	2:13 pm	3:13 pm	4:13 pm	5:13 pm
Elizabeth Street Stop 81 near George St, Brisbane City	1:20 pm	2:20 pm	3:20 pm	4:20 pm	5:20 pm

Ferry timetable

Departs Terminal:	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
UQ St Lucia	**4.22**	....	4.33	....	**4.40**	4.48	....	**5.00**	5.03	....	5.18	**5.20**	....	5.33	....
West End	**SE**	....	4.38	....	**SE**	4.53	....	**SE**	5.08	....	5.23	**SE**	....	5.38	....
Guyatt Park	**SE**	....	4.41	....	**SE**	4.56	....	**SE**	5.11	....	5.26	**SE**	....	5.41	....
Regatta	**SE**	....	4.45	....	**SE**	5.00	....	**SE**	5.15	....	5.30	**SE**	....	5.45	....
Milton	**SE**	....	4.49	....	**SE**	5.04	....	**SE**	5.19	....	5.34	**SE**	....	5.49	....
North Quay	**4.37**	....	4.55	....	**4.55**	5.10	....	**5.15**	5.25	....	5.40	**5.36**	....	5.55	....
South Bank 1	....	....	4.59	....	....	5.14	....	....	5.29	....	5.44	**SE**	....	5.59	....
QUT Gardens Point	....	....	5.03	....	....	5.18	....	....	5.33	....	5.48	**SE**	....	6.03	....
Riverside	....	5.09	5.13	5.24	....	5.28	5.39	....	5.43	5.54	5.58	**5.50**	6.09	6.13	6.24
Sydney Street	....	5.17	5.21	5.32	....	5.36	5.47	....	5.51	6.02	6.06	**SE**	6.17	6.21	6.32
Mowbray Park	....	5.20	5.24	5.35	....	5.39	5.50	....	5.54	6.05	6.09	**SE**	6.20	6.24	6.35
New Farm Park	....	5.24	5.28	5.39	....	5.43	5.54	....	5.58	6.09	6.13	**SE**	6.24	6.28	6.39
Hawthorne	....	5.29	5.33	5.44	....	5.48	5.59	....	6.03	6.14	6.18	**SE**	6.29	6.33	6.44
Bulimba	....	5.34	5.38	5.49	....	5.53	6.04	....	6.08	6.19	6.23	**SE**	6.34	6.38	6.49
Teneriffe	....	5.38	5.42	5.53	....	5.57	6.08	....	6.12	6.23	6.27	**6.07**	6.38	6.42	6.53
Bretts Wharf	....	....	5.48	....	....	6.03	....	....	6.18	....	6.33	**SE**	....	6.48	....
Apollo Road	....	....	5.51	....	....	6.06	....	....	6.21	....	6.36	**6.15**	....	6.51	....
Northshore Hamilton	....	....	5.56	....	....	6.11	....	....	6.26	....	6.41	6.20	....	6.56	....

WORKING	THINKING
Bus option is 5:10 p.m. to 5:20 p.m. = 10 min plus a 9-min walk = 19 min, so arrive at 5:29 p.m.	Check the bus departure and arrival times to calculate the length of time for the trip.
Ferry option is 5:14 p.m. to 5:18 p.m. = 4 min plus a 9-min walk, which is 13 min in total, so arrive at 5:27 p.m.	Check the ferry departure and arrival times to calculate the length of time for the trip.
The ferry option is the best, as it requires less travel time and Anthony will arrive at QUT earlier than the bus option.	Choose the shortest travel time that arrives the earliest.

ISBN 978-1-009-25785-5

Exercise 12D

FUNDAMENTALS

1 Write down the missing numbers in the following calculation of the time difference, in hours and minutes, between 2.43 p.m. and 4:08 p.m.

a 2:43 p.m. to _____ p.m. = 1 h

b 3:43 p.m. to 4 p.m. = ___ min

c 4 p.m. to ____ p.m. = ___ min

d 1 h + ___ min + ___ min = __ h __ min

APPLICATIONS

Example 14

2 Suzanna is travelling from Eaton to Preston Beach and she has found two options for transport. Analyse each of these options by determining the travel time and, hence, decide which option Suzanna should choose if she wishes to minimise her travel time.

Option 1: Departs Eaton at 10:51 a.m. and arrives in Preston Beach at 12:26 p.m.

Option 2: Departs Eaton at 9:21 a.m. and arrives in Preston Beach at 11:26 a.m.

3 Noah is travelling from Kelmscott Station to Cannington Station. He can either catch a train or a bus. Analyse each of the following options by determining the travel time, and hence, decide which option Noah should choose if he wishes to minimise his travel time.

Option 1: Train departing Kelmscott at 8:18 a.m. and arriving at Cannington at 8:46 a.m.

Option 2: Bus departing Kelmscott at 8:35 a.m. and arriving at Cannington at 9:01 a.m.

ISBN 978-1-009-25785-5

Example 15

4 Refer to the following bus and train timetables and answer the following questions.

Train timetables

To Perth

Pattern	Armadale 99191	Sherwood 99181	Challis 99771	Kelmscott 99161	Seaforth 99151	Gosnells 99141	Maddington 99131	Kenwick 99121	Beckenham 99111	Thornlie 99104	Cannington 99101	Queens Park 99091	Welshpool 99081	Oats Street 99071	Carlisle 99061	Victoria Park 99051	Burswood 99041	Claisebrook 99021	McIver 99011	Perth 99005
Monday to Friday																				
am T	-	-	-	-	-	-	-	-	-	5:17	5:22	5:24	5:26	5:28	5:29	5:31	5:33	5:37	5:38	5:40
C	5:10	5:13	5:14	5:16	5:19	5:21	5:25	5:27	5:29		-	5:31	-	-	5:35	-	-	-	5:42	5:43	5:45
T	-	-	-	-	-	-	-	-	-	5:33	5:38	5:40	5:42	5:44	5:45	5:47	5:49	5:53	5:54	5:56
C	5:25	5:28	5:29	5:31	5:34	5:36	5:40	5:42	5:44	-	5:46	-	-	5:50	-	-	-	5:57	5:58	6:00
T	-	-	-	-	-	-	-	-	-	5:48	5:53	5:55	5:57	5:59	6:00	6:02	6:04	6:08	6:09	6:11
C	5:39	5:42	5:43	5:45	5:48	5:50	5:54	5:56	5:58	-	6:00	-	-	6:04	-	-	-	6:11	6:12	6:14
T	-	-	-	-	-	-	-	-	-	6:03	6:08	6:10	6:12	6:14	6:15	6:17	6:19	6:23	6:24	6:26
C	5:55	5:58	5:59	6:01	6:04	6:06	6:10	6:12	6:14	-	6:16	-	-	6:20	-	-	-	6:27	6:28	6:30
T	-	-	-	-	-	-	-	-	-	6:18	6:23	6:25	6:27	6:29	6:30	6:32	6:34	6:38	6:39	6:41
C	6:10	6:13	6:14	6:16	6:19	6:21	6:25	6:27	6:29	-	6:31	-	-	6:35	-	-	-	6:42	6:43	6:45
T	-	-	-	-	-	-	-	-	-	6:33	6:38	6:40	6:42	6:44	6:45	6:47	6:49	6:53	6:54	6:56
C	6:23	6:26	6:27	6:29	6:32	6:34	6:38	6:40	6:43	-	6:45	-	-	6:49	-	-	-	6:57	6:58	7:00
T	-	-	-	-	-	-	-	-	-	6:47	6:52	6:54	6:56	6:58	7:00	7:02	7:04	7:08	7:09	7:11
C	6:38	6:41	6:42	6:44	6:47	6:49	6:53	6:55	6:58	-	7:00	-	-	7:04	-	-	-	7:12	7:13	7:15
T	-	-	-	-	-	-	-	-	-	7:03	7:08	7:10	7:12	7:14	7:16	7:18	7:20	7:24	7:25	7:27
C	6:53	6:56	6:57	6:59	7:02	7:04	7:08	7:10	7:13	-	7:15	-	-	7:19	-	-	-	7:27	7:28	7:30
T	-	-	-	-	-	-	-	-	-	7:18	7:23	7:25	7:27	7:29	7:31	7:33	7:35	7:39	7:40	7:42
C	7:08	7:11	7:12	7:14	7:17	7:19	7:23	7:25	7:28	-	7:30	-	-	7:34	-	-	-	7:42	7:43	7:45
C	7:13	7:16	7:17	7:19	7:22	7:24	7:28	7:30	7:33	-	7:35	-	-	7:39	-	-	-	7:47	7:48	7:50
T	-	-	-	-	-	-	-	-	-	7:33	7:38	7:40	7:42	7:44	7:46	7:48	7:50	7:54	7:55	7:57
C	7:23	7:26	7:27	7:29	7:32	7:34	7:38	7:40	7:43	-	7:45	-	-	7:49	-	-	-	7:57	7:58	8:00
C	7:28	7:31	7:32	7:34	7:37	7:39	7:43	7:45	7:48	-	7:50	-	-	7:54	-	-	-	8:02	8:03	8:05
T	-	-	-	-	-	-	-	-	-	7:48	7:53	7:55	7:57	7:59	8:01	8:03	8:05	8:09	8:10	8:12
C	7:38	7:41	7:42	7:44	7:47	7:49	7:53	7:55	7:58	-	8:00	-	-	8:04	-	-	-	8:12	8:13	8:15
C	7:43	7:46	7:47	7:49	7:52	7:54	7:58	8:00	8:03	-	8:05	-	-	8:09	-	-	-	8:17	8:18	8:20
T	-	-	-	-	-	-	-	-	-	8:03	8:08	8:10	8:12	8:14	8:16	8:18	8:20	8:24	8:25	8:27
C	7:53	7:56	7:57	7:59	8:02	8:04	8:08	8:10	8:13	-	8:15	-	-	8:19	-	-	-	8:27	8:28	8:30
C	8:00	8:03	8:04	8:06	8:09	8:11	8:15	8:17	8:20	-	8:22	-	-	8:26	-	-	-	8:34	8:35	8:37
T	-	-	-	-	-	-	-	-	-	8:20	8:25	8:27	8:29	8:31	8:33	8:35	8:37	8:41	8:42	8:44
C	8:12	8:15	8:16	8:18	8:21	8:23	8:27	8:29	8:32	-	8:34	-	-	8:38	-	-	-	8:46	8:47	8:49
T	-	-	-	-	-	-	-	-	-	8:35	8:40	8:42	8:44	8:46	8:48	8:50	8:52	8:56	8:57	8:59
C	8:28	8:31	8:32	8:34	8:37	8:39	8:43	8:45	8:47	-	8:49	-	-	8:53	-	-	-	9:00	9:01	9:03
T	-	-	-	-	-	-	-	-	-	8:50	8:55	8:57	8:59	9:01	9:02	9:04	9:06	9:10	9:11	9:13
C	8:45	8:48	8:49	8:51	8:54	8:56	9:00	9:02	9:04	-	9:06	-	-	9:10	-	-	-	9:17	9:18	9:20

To Mandurah

Pattern	Perth Underground 99602	Elizabeth Quay 99612	Canning Bridge 99632	Bull Creek 99642	Murdoch 99652	Cockburn Central 99662	Aubin Grove 99672	Kwinana 99682	Welland 99692	Rockingham 99702	Wambro 99712	Mandurah 99732
Monday to Friday (continued)												
W	8:56	8:58	9:04	9:08	9:11	9:14	-	-	-	-	-	-
	9:01	9:03	9:09	9:13	9:16	9:19	9:22	9:27	9:31	9:35	9:39	9:52
W	9:06	9:08	9:14	9:18	9:21	9:24	-	-	-	-	-	-
	9:11	9:13	9:19	9:23	9:26	9:29	9:32	9:37	9:41	9:45	9:49	10:02
W	9:16	9:18	9:24	9:28	9:31	9:34	-	-	-	-	-	-
	9:23	9:25	9:31	9:35	9:38	9:41	9:44	9:49	9:53	9:57	10:01	10:14
	9:38	9:40	9:46	9:50	9:53	9:56	9:59	10:04	10:08	10:12	10:16	10:29
	9:53	9:55	10:01	10:05	10:08	10:11	10:14	10:19	10:23	10:27	10:31	10:44
	10:08	10:10	10:16	10:20	10:23	10:26	10:29	10:34	10:38	10:42	10:46	10:59
	10:23	10:25	10:31	10:35	10:38	10:41	10:44	10:49	10:53	10:57	11:01	11:14
	10:38	10:40	10:46	10:50	10:53	10:56	10:59	11:04	11:08	11:12	11:16	11:29
	10:53	10:55	11:01	11:05	11:08	11:11	11:14	11:19	11:23	11:27	11:31	11:44
	11:08	11:10	11:16	11:20	11:23	11:26	11:29	11:34	11:38	11:42	11:46	11:59
	11:23	11:25	11:31	11:35	11:38	11:41	11:44	11:49	11:53	11:57	**12:01**	**12:14**
	11:38	11:40	11:46	11:50	11:53	11:56	11:59	**12:04**	**12:08**	**12:12**	**12:16**	**12:29**
	11:53	11:55	**12:01**	**12:05**	**12:08**	**12:11**	**12:14**	**12:19**	**12:23**	**12:27**	**12:31**	**12:44**

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ISBN 978-1-009-25785-5

Bus timetable

Route 204, 205 - To Murdoch

	Timed Stops						
	Stop No.		13464	20779	20804	10520	23711	26628
	Route No.		Maddington Stn	Bridge Rd / Gateway Bvd	Warton Rd / Holmes Rd	Livingston Marketplace	Murdoch Stn	Murdoch University
Monday to Friday								
am	205	A	5:12	5:17	-	5:24	5:36	-
	204	A	5:34	-	5:39	5:46	5:59	-
	205	M	5:47	5:57	-	6:06	6:19	-
	204	M	6:03	-	6:13	6:23	6:37	-
	205	M	6:19	6:29	-	6:38	6:53	-
	204	M	6:33	-	6:43	6:53	7:08	-
	205	M	6:49	7:00	-	7:10	7:28	-
	204		6:54	-	7:05	7:15	7:33	7:40
	205		7:08	7:20	-	7:30	7:48	7:58
	204		7:14	-	7:25	7:35	7:53	8:03
	205		7:23	7:35	-	7:45	8:03	8:13
	204		7:29	-	7:40	7:50	8:08	8:18
	205		7:38	7:50	-	8:00	8:18	8:28
	204		7:44	-	7:55	8:06	8:24	8:34
	205		7:49	8:02	-	8:13	8:30	8:40
	204		7:56	-	8:07	8:18	8:35	8:44
	205		8:09	8:22	-	8:33	8:50	8:59
	204		8:30	-	8:41	8:52	9:07	9:16
	205		8:45	8:57	-	9:07	9:22	9:31
	204		9:10	-	9:21	9:30	9:44	9:52
	205		9:40	9:52	-	10:00	10:14	10:22
	204		10:10	-	10:21	10:30	10:44	10:52
	205		10:40	10:52	-	11:00	11:14	11:22
	204		11:10	-	11:21	11:30	11:44	11:52
	205		11:40	11:52	-	12:00	12:14	12:22
pm	204		12:10	-	12:21	12:30	12:44	12:52
	205		12:40	12:52	-	1:00	1:14	1:22
	204		1:17	-	1:28	1:38	1:52	2:00
	205		1:48	2:00	-	2:08	2:22	2:30
	204		2:17	-	2:28	2:38	2:52	3:01
	205		2:28	2:40	-	2:48	3:02	3:11

Dennika has a lecture at Murdoch University at 9:30 a.m. She is departing from Maddington and must be on time for her lecture. She is meeting a friend at Murdoch Station and they will then take 6 minutes to drive to the University. By considering the timetables given:

a Identify the possible transport options for Dennika to travel from Maddington to Murdoch Station that will allow her to arrive for her lecture before 9:30 a.m.

b Analyse each option by determining the travel time.

c Determine the best mode of transport for Dennika if she wishes to travel for the least amount of time.

ISBN 978-1-009-25785-5

5 Lian and Phil are on holiday in Queensland, staying at a hotel on Breaker Street, Main Beach. They wish to go to a friend's house that is near Broadbeach South Station for breakfast at 9 a.m. They have two possible modes of transport to get to their destination.

Option 1: Catch the tram from Main Beach Station and hop off at Broadbeach South Station.

Option 2: Catch a bus from the Main Beach Tourist Park and hop off at Broadbeach South Station.

Using the following tram and bus timetables, advise Lian and Phil of the best option and give reasons for this selection.

Tram timetable

Main Beach station	8:33 am	8:40 am	8:48 am	8:55 am	9:03 am
Surfers Paradise North station	8:36 am	8:43 am	8:51 am	8:58 am	9:06 am
Cypress Avenue station	8:38 am	8:45 am	8:53 am	9:00 am	9:08 am
Cavill Avenue station	8:41 am	8:48 am	8:56 am	9:03 am	9:11 am
Surfers Paradise station	8:43 am	8:50 am	8:58 am	9:05 am	9:13 am
Northcliffe station	8:44 am	8:52 am	8:59 am	9:07 am	9:14 am
Florida Gardens station	8:46 am	8:54 am	9:01 am	9:09 am	9:16 am
Broadbeach North station	8:49 am	8:56 am	9:04 am	9:11 am	9:19 am
Broadbeach South station	8:51 am	8:58 am	9:06 am	9:13 am	9:21 am

Bus timetable

Route 705

Sea World/Main Beach to Broadbeach

servicing Sea World/Main Beach, Surfers Pardise, Pacific Fair and Broadbeach South station

Monday to Friday

map ref	Route number	705	705	705	705	705	705	705	705	705	705	705	705	705	705	705	705	705	705	705
		am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am
H	Sea World	4.07	4.37	5.07	5.37	6.07	6.22	6.37	6.52	7.07	7.22	7.37	7.52	8.07	8.22	8.37	8.52	9.07	9.22	9.37	9.52
G	Main Beach Tourist Park	4.09	4.39	5.09	5.39	6.09	6.24	6.39	6.54	7.09	7.24	7.39	7.54	8.09	8.24	8.39	8.54	9.09	9.24	9.40	9.55
F	Surfers Paradise, Beach Rd	4.16	4.46	5.16	5.47	6.18	6.33	6.48	7.03	7.18	7.34	7.48	8.04	8.19	8.34	8.49	9.04	9.19	9.34	9.49	10.04
E	Old Burleigh Rd near Wharf St	4.19	4.49	5.19	5.50	6.22	6.37	6.52	7.07	7.22	7.38	7.53	8.09	8.24	8.39	8.54	9.09	9.24	9.39	9.54	10.09
D	Broadbeach Mall - Surf Pde	4.23	4.53	5.23	5.54	6.28	6.43	6.58	7.13	7.28	7.44	7.59	8.14	8.30	8.45	9.00	9.15	9.30	9.45	10.00	10.15
C	The Star Casino	4.26	4.56	5.26	5.57	6.31	6.46	7.01	7.16	7.31	7.47	8.02	8.17	8.33	8.48	9.03	9.18	9.33	9.48	10.03	10.18
B	Pacific Fair	4.30	5.00	5.30	6.02	6.36	6.51	7.06	7.21	7.36	7.52	8.07	8.22	8.38	8.53	9.09	9.24	9.39	9.54	10.09	10.24
A	Broadbeach South station	4.33	5.03	5.33	6.05	6.39	6.54	7.09	7.24	7.39	7.55	8.10	8.25	8.41	8.56	9.12	9.27	9.42	9.57	10.12	10.27

ISBN 978-1-009-25785-5

6 Ryan starts work at 7 a.m. in Perth and lives in Warwick. He must catch either a bus or a train to work daily. If he catches the train, he must drive 7 minutes from home to Warwick Station first, but he lives only a 2-minute walking distance from the bus stop on Beach Rd.

a Use the following timetables to identify at least three travel options for Ryan.

b Analyse each of these options by calculating the total travel time.

c Discuss which travel option you believe to be the best for Ryan, giving reasons for your choice.

Bus timetable

Route 387, 388, 389 - To Perth

Timed Stops										
Stop No.		18307	12308	12553	18584	12331	12344	12354	12593	
Route No.		Belgrade Rd / Esther Loop	Wanneroo Shop Ctr	Warwick Stn	Erindale Rd / Balcatta Rd	Wanneroo Rd / Beach Rd	Wanneroo Rd / Harrison St	Dog Swamp Shop Ctr	Charles St / Scarborough Beach Rd	Perth Busport
Monday to Friday										
am	388	-	-	5:18	-	5:22	5:28	5:34	5:37	5:47
	389	5:31	5:35	-	-	5:48	5:54	6:00	6:03	6:13
	388	-	-	6:06	-	6:11	6:17	6:23	6:26	6:37
	389	6:09	6:13	-	-	6:27	6:35	6:41	6:44	6:55
	388	-	-	6:38	-	6:43	6:51	6:58	7:01	7:12
	389	6:35	6:40	-	-	6:55	7:03	7:10	7:13	7:26
	388	-	-	7:00	-	7:05	7:13	7:20	7:23	7:36
	389	6:55	7:00	-	-	7:15	7:23	7:30	7:34	7:53
	388	-	-	7:20	-	7:25	7:33	7:41	7:45	7:59
	389	7:12	7:17	-	-	7:34	7:43	7:52	7:57	8:14
	388	-	-	7:38	-	7:44	7:53	8:02	8:06	8:23
	389	7:32	7:37	-	-	7:54	8:03	8:12	8:17	8:33
	388	-	-	7:59	-	8:05	8:15	8:23	8:27	8:43
	388	-	-	8:14	-	8:19	8:28	8:36	8:40	8:56
	389	8:12	8:17	-	-	8:34	8:44	8:51	8:55	9:09
	388	-	-	8:47	-	8:52	9:01	9:08	9:11	9:24
	389	8:45	8:50	-	-	9:06	9:14	9:21	9:24	9:37
	388	-	-	9:15	-	9:21	9:29	9:36	9:39	9:51

Train timetable

To Perth

Pattern	Butler 99901	Clarkson 99891	Currambine 99881	Joondalup 99871	Edgewater 99861	Whitfords 99851	Greenwood 99841	Warwick 99831	Stirling 99821	Glendalough 99811	Leederville 99801	Perth Underground 99602	Elizabeth Quay 99012
Monday to Friday													
am	4:51	4:56	4:59	5:03	5:06	5:09	5:12	5:15	5:19	5:22	5:25	5:28	5:32
	5:03	5:08	5:11	5:15	5:18	5:21	5:24	5:27	5:31	5:34	5:37	5:40	5:44
	5:14	5:19	5:22	5:26	5:29	5:32	5:35	5:38	5:42	5:45	5:48	5:51	5:55
K	-	5:23	5:26	5:30	5:33	5:36	5:39	5:42	5:46	5:49	5:52	5:55	5:59
	5:29	5:34	5:37	5:41	5:44	5:47	5:50	5:53	5:57	6:00	6:03	6:06	6:10
	5:39	5:44	5:47	5:51	5:54	5:57	6:00	6:03	6:07	6:10	6:13	6:16	6:20
	5:49	5:54	5:57	6:01	6:04	6:07	6:10	6:13	6:17	6:20	6:23	6:26	6:30
K	-	5:59	6:02	6:06	6:09	6:12	6:15	6:18	6:22	6:25	6:28	6:31	6:35
	5:59	6:04	6:07	6:11	6:14	6:17	6:20	6:23	6:27	6:30	6:33	6:36	6:40
	6:09	6:14	6:17	6:21	6:24	6:27	6:30	6:33	6:37	6:40	6:43	6:46	6:50
K	-	6:19	6:22	6:26	6:29	6:32	6:35	6:38	6:42	6:45	6:48	6:51	6:56
	6:19	6:24	6:27	6:31	6:34	6:37	6:40	6:43	6:47	6:50	6:53	6:56	7:01
K	-	6:30	6:33	6:37	6:40	6:43	6:46	6:49	6:54	6:57	7:00	7:03	7:07
	6:31	6:36	6:39	6:43	6:46	6:49	6:52	6:55	7:00	7:03	7:06	7:09	7:13
K	-	6:41	6:44	6:48	6:51	6:54	6:57	7:00	7:05	7:08	7:11	7:14	7:18
	6:41	6:46	6:49	6:53	6:56	6:59	7:02	7:05	7:10	7:13	7:16	7:19	7:23
K	-	6:51	6:54	6:58	7:01	7:04	7:07	7:10	7:15	7:18	7:21	7:24	7:28
	6:51	6:56	6:59	7:03	7:06	7:09	7:12	7:15	7:20	7:23	7:26	7:29	7:33
K	-	7:01	7:04	7:08	7:11	7:14	7:17	7:20	7:25	7:28	7:31	7:34	7:38
	7:01	7:06	7:09	7:13	7:16	7:19	7:22	7:25	7:30	7:33	7:36	7:39	7:43
K	-	7:11	7:14	7:18	7:21	7:24	7:27	7:30	7:35	7:38	7:41	7:44	7:48
	7:11	7:16	7:19	7:23	7:26	7:29	7:32	7:35	7:40	7:43	7:46	7:49	7:53
K	-	7:21	7:24	7:28	7:31	7:34	7:37	7:40	7:45	7:48	7:51	7:54	7:58
	7:21	7:26	7:29	7:33	7:36	7:39	7:42	7:45	7:50	7:53	7:56	7:59	8:03

ISBN 978-1-009-25785-5

12E Interpreting complex timetables

LEARNING GOAL

- Read and interpret complex tables, such as tidal times, moon phases and sunrise/sunset charts

When is it essential to interpret complex timetables?

Timetables are not only used for information about public transport. They are also used to display other kinds of important information, including tidal charts, moon phases and sunrise/sunset charts.

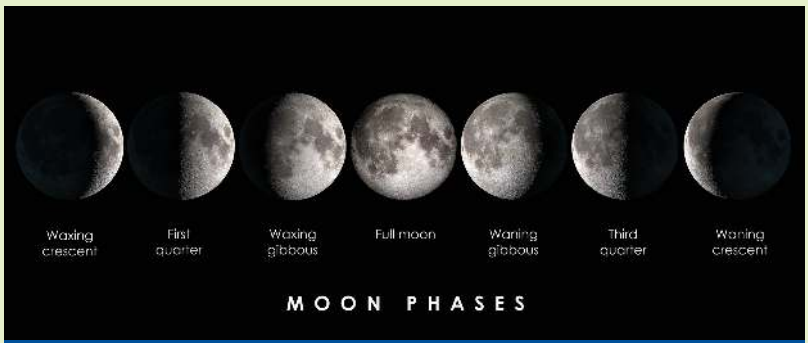
Tidal charts record the high and low tides of local coastal areas; moon phase charts record when the sunlit part of the moon has a variety of shapes, such as the full moon and new moon; and sunrise and sunset charts tell us the times for the first and last rays of sunshine for the day.

As the tide rises relentlessly along Rainbow Beach, Qld, every year it submerges some 4WDs! Local tide times and the period of safe travel is essential knowledge.

WHAT YOU NEED TO KNOW

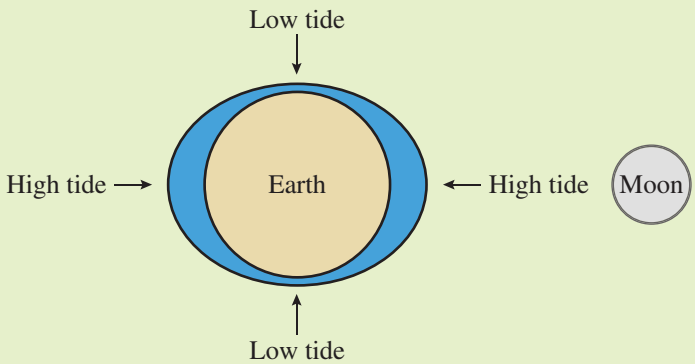
- **Sunrise/sunset charts** Sunrise and sunset are measured when the first ray of light (sunrise) or last ray of light (sunset) can be seen. The Sun will always rise in the east and set in the west because the Earth spins in an easterly direction. Sunrise and sunset times vary from place to place and throughout the year.
- **Moon phases** The lunar phase or phase of the Moon is the shape of the directly sunlit portion of the Moon as seen from the Earth. The lunar phases cyclically change, as the Moon orbits the Earth, according to the shifting positions of the Moon and Sun relative to the Earth.

ISBN 978-1-009-25785-5

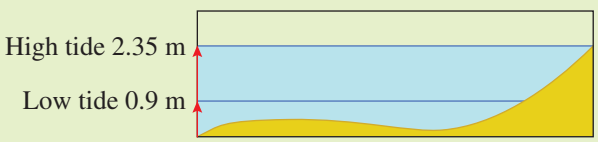


The Moon's phase cycle repeats, on average, every 29.53 days.

- **Tides** are the rise and fall of local sea levels that are caused by gravitational forces from the Moon and Sun, combined with the rotation of the Earth. The diagram shows a simplified Earth with exaggerated tide heights.



- **Tide tables** for all major coastal locations, with predicted tide times and heights, are published each year by the Bureau of Meteorology. Marine and Harbours WA produces WA tide tables, sunrise and sunset times, and moon rise and set times.
- There are two high tides and two low tides every 24 hours. Tide heights vary from place to place and from day to day. Each high tide and low tide are approximately six hours apart. The safest beach driving time is within two hours of low tide.
  - For example: The diagram illustrates a 0.9-metre low tide, increasing to a 2.35-metre high tide.



  - The Australian Maritime Safety Authority operates the tide gauge network around the coast.

ISBN 978-1-009-25785-5

Example 16 Reading and interpreting a tidal chart

Terry plans to go on a 4WD adventure, leaving Sunday morning, 26 November, and returning Tuesday morning, 28 November. His trip includes a one-hour drive up the beach, driving only within two hours of low tide.

a Identify the time and height of the first low tide on Sunday 26 November.

b Determine the times that Terry could start his beach driving on that day.

c Identify the time Terry should start his return on Tuesday.

SAT 25 Nov	**SUN** 26 Nov	**MON** 27 Nov	**TUE** 28 Nov	**WED** 29 Nov
▼ 5:15 a.m. 0:72 m	▲ 12:45 a.m. 1:24 m	▲ 2:07 a.m. 1:28 m	▲ 3:16 a.m. 1:39 m	▲ 4:11 a.m. 1:53 m
▲ 12:14 p.m. 1:67 m	▼ 6:14 a.m. 0:8 m	▼ 7:29 a.m. 0:83 m	▼ 8:51 a.m. 0:82 m	▼ 10:02 a.m. 0:76 m
▼ 7:05 p.m. 0:76 m	▲ 1:12 p.m. 1:63 m	▲ 2:17 p.m. 1:63 m	▲ 3:18 a.m. 1:66 m	▲ 4:12 p.m. 1:69 m
	▼ 8:12 p.m. 0:73 m	▼ 9:09 p.m. 0:67 m	▼ 10:00 p.m. 0:58 m	▼ 10:46 p.m. 0:47 m

WORKING	THINKING
a Low tide is at 6:14 a.m., with height 0.8 m.	Low-tide times are beside the 'down arrowhead' symbol and have lower heights.
b 26 November: Start beach driving between 4:14 a.m. and 7:14 a.m.	Start between 6:14 a.m. minus 2 hours and 6:14 a.m. plus 1 hour (as the beach trip takes 1 hour).
c 28 November: Start return from camp between 6:51 a.m. and 9:51 a.m.	Start between 8:51 a.m. minus 2 hours and 8:51 a.m. plus 1 hour (as the beach trip takes 1 hour).

Exercise 12E

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words.

a The _____ of the Moon is the shape of the directly _____ portion as seen from the Earth.

b Tides are the rise and fall of local ____ levels, caused by _____ forces from the Moon and Sun and the rotation of the Earth.

c There are __ high tides and __ low tides every 24 hours.

d The safest beach driving time is within __ hours either side of low tide.

e Tide tables are published each year for all major coastal _____, with predicted tide _____ and _____.

ISBN 978-1-009-25785-5

2 Use the information in the Moon phases timetable to answer the following.

A new moon.

- **a** Identify the date in November when there is a new moon.
- **b** Identify the date in April when there is a first quarter moon.
- **c** Identify the date in July when there is a full moon.
- **d** Identify the month in which there are two full moons.
- **e** Identify the month in which there are two first quarter moons.

Moon phases for Perth, 2022

Lunation	New Moon		First Quarter		Full Moon		Third Quarter		Duration
1200			3 Jan	2:45 pm	11 Jan	5:21 am	17 Jan	10:58 pm	29d 16h 29m
1201	25 Jan	7:42 am	2 Feb	11:41 am	9 Feb	5:33 pm	16 Feb	8:17 am	29d 17h 50m
1202	24 Feb	1:32 am	3 Mar	5:57 am	10 Mar	3:47 am	16 Mar	7:34 pm	29d 17h 56m
1203	24 Mar	7:28 pm	1 Apr	8:21 pm	8 Apr	12:35 pm	15 Apr	8:56 am	29d 16h 58m
1204	23 Apr	12:25 pm	1 May	6:38 am	7 May	8:45 pm	15 May	12:02 am	29d 15h 13m
1205	23 May	3:38 am	30 May	1:29 pm	6 Jun	5:12 am	13 Jun	4:23 pm	29d 13h 03m
1206	21 Jun	4:41 pm	28 Jun	6:15 pm	5 Jul	2:44 pm	13 Jul	9:28 am	29d 10h 51m
1207	21 Ju1	3:32 am	27 Jul	10:32 pm	4 Aug	1:58 am	12 Aug	2:44 am	29d 9h 09m
1208	19 Aug	12:41 pm	26 Aug	3:57 am	2 Sep	3:22 pm	10 Sep	7:25 pm	29d 8h 19m
1209	17 Sep	9:00 pm	24 Sep	11:54 am	2 Oct	7:05 am	10 Oct	10:39 am	29d 8h 31m
1210	17 Oct	5:31 am	23 Oct	11:22 pm	1 Nov	12:49 am	8 Nov	11:46 pm	29d 9h 36m
1211	15 Nov	3:07 pm	22 Nov	2:45 pm	30 Nov	7:29 pm	8 Dec	10:36 am	29d 11h 09m
1212	15 Dec	2:16 am	22 Dec	9:41 am	30 Dec	1:28 pm			29d 12h 44m

3 Use the information in the tidal chart to answer the following questions.

Ningaloo Reef, WA – January 2022

WED 3 JAN		THU 4 JAN		FRI 5 JAN		SAT 6 JAN		SUN 7 JAN		MON 8 JAN		TUE 9 JAN	
LOW	4:04 a.m. 0.19 m	LOW	4:51 a.m. 0.32 m	HIGH	12:02 a.m. 2.22 m	HIGH	12:59 a.m. 2.12 m	HIGH	2:11 a.m. 2.06 m	HIGH	3:39 a.m. 2.08 m	HIGH	5:07 a.m. 2.22 m
HIGH	11:00 a.m. 3.45 m	HIGH	11:46 a.m. 3.36 m	LOW	5:40 a.m. 0.51 m	LOW	6:35 a.m. 0.75 m	LOW	7:41 a.m. 1.00 m	LOW	9:06 a.m. 1.21 m	LOW	10:44 a.m. 1.29 m
LOW	5:10 p.m. 0.67 m	LOW	6:05 p.m. 0.73 m	HIGH	12:36 p.m. 3.19 m	HIGH	1:30 p.m. 2.97 m	HIGH	2:32 p.m. 2.74 m	HIGH	3:43 p.m. 2.54 m	HIGH	4:56 p.m. 2.40 m
HIGH	11:10 p.m. 2.31 m			LOW	7:08 p.m. 0.79 m	LOW	8:18 p.m. 0.84 m	LOW	9:28 p.m. 0.85 m	LOW	10:35 p.m. 0.82 m	LOW	11:32 p.m. 0.76 m

- **a** On the evening of Thursday 4 January, determine the time when the tide will be at its lowest.
- **b** On the morning of 9 January, determine the depth at the lowest tide.
- **c** On Saturday 6 January, determine whether the tide is at the lowest in the morning or the evening.
- **d** An average king tide for this region is around 3.8 metres. Identify the day on which the tide comes closest to this.
- **e** Identify the day and time when the tide reaches the lowest height.

ISBN 978-1-009-25785-5

4 This chart lists the sunrise and sunset times for Port Hedland, WA, in July 2020. Use the data to answer the questions.

2020	Sunrise/Sunset		Daylength	
Jul	Sunrise	Sunset	Length	Difference
1	6:50 am (66°)	5:46 pm (294°)	10:56:07	+0:12
2	6:50 am (66°)	5:47 pm (294°)	10:56:23	+0:15
3	6:50 am (66°)	5:47 pm (294°)	10:56:39	+0:16
4	6:50 am (66°)	5:47 pm (294°)	10:56:57	+0:17
5	6:50 am (66°)	5:48 pm (294°)	10:57:15	+0:18
27	6:46 am (70°)	5:56 pm (290°)	11:09:14	+0:43
28	6:46 am (70°)	5:56 pm (290°)	11:09:58	+0:44
29	6:46 am (70°)	5:56 pm (290°)	11:10:44	+0:45
30	6:45 am (71°)	5:57 pm (289°)	11:11:30	+0:46
31	6:45 am (71°)	5:57 pm (289°)	11:12:17	+0:47

a On 4 July, identify the times that sunrise and sunset take place.
b Of the days between 1 and 31 July, identify the day that has the most amount of daylight.
c Identify the date in July that has the earliest sunrise.
d Identify the date in July that has the latest sunset.
e What is the difference in length of the daylight hours from 1 to 31 July?

APPLICATIONS

5 In some areas, Aboriginal and Torres Strait Islander peoples held ceremonies during a full moon. Ceremonies involved large groups of people storytelling, singing and dancing and took place often in the past. Having a ceremony during a full moon holds the advantage of brighter light and a higher chance of calm weather.
Alinta is planning to organise a full moon ceremony in the middle of the year. Use the moon phases chart to identify possible dates for this ceremony.

Lunation	New Moon		First Quarter		Full Moon		Third Quarter		Duration
1200			3 Jan	2:45 pm	11 Jan	5:21 am	17 Jan	10:58 pm	29d 16h 29m
1201	25 Jan	7:42 am	2 Feb	11:41 am	9 Feb	5:33 pm	16 Feb	8:17 am	29d 17h 50m
1202	24 Feb	1:32 am	3 Mar	5:57 am	10 Mar	3:47 am	16 Mar	7:34 pm	29d 17h 56m
1203	24 Mar	7:28 pm	1 Apr	8:21 pm	8 Apr	12:35 pm	15 Apr	8:56 am	29d 16h 58m
1204	23 Apr	12:25 pm	1 May	6:38 am	7 May	8:45 pm	15 May	12:02 am	29d 15h 13m
1205	23 May	3:38 am	30 May	1:29 pm	6 Jun	5:12 am	13 Jun	4:23 pm	29d 13h 03m
1206	21 Jun	4:41 pm	28 Jun	6:15 pm	5 Jul	2:44 pm	13 Jul	9:28 am	29d 10h 51m
1207	21 Ju1	3:32 am	27 Jul	10:32 pm	4 Aug	1:58 am	12 Aug	2:44 am	29d 9h 09m
1208	19 Aug	12:41 pm	26 Aug	3:57 am	2 Sep	3:22 pm	10 Sep	7:25 pm	29d 8h 19m
1209	17 Sep	9:00 pm	24 Sep	11:54 am	2 Oct	7:05 am	10 Oct	10:39 am	29d 8h 31m
1210	17 Oct	5:31 am	23 Oct	11:22 pm	1 Nov	12:49 am	8 Nov	11:46 pm	29d 9h 36m
1211	15 Nov	3:07 pm	22 Nov	2:45 pm	30 Nov	7:29 pm	8 Dec	10:36 am	29d 11h 09m
1212	15 Dec	2:16 am	22 Dec	9:41 am	30 Dec	1:28 pm			29d 12h 44m

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6 Sarrita wants to know what the moon will be like on her birthday. She is planning an outdoor birthday party and she has heard that the weather is more likely to be calm on the day of a full moon. Sarrita's actual birthday is 10 September, and she would like to have her party on a Saturday night, so she is considering having it on 7 September.
Use the Moon phases table provided to advise whether this date is ideal or whether there is a date around the time of Sarrita's birthday that is more likely to have calm weather.

2022 Lunar Phases - Perth Time

● New Moon	◖ First Quarter	○ Full Moon	◗ Third Quarter	Lunation
Jan. 6, Su *11:29*	Jan. 14, Mo *16:46*	Jan. 21, Mo *15:17*	Jan. 28, Mo *07:12*	1188
Feb. 5, Tu *07:04*	Feb. 13, We *08:26*	Feb. 20, We *01:53*	Feb. 26, Tu *21:29*	1189
Mar. 7, Th *02:05*	Mar. 14, Th *20:26*	Mar. 21, Th *11:43*	Mar. 28, Th *14:11*	1190
Apr. 5, Fr *18:52*	Apr. 13, Sa *05:06*	Apr. 19, Fr *21:12*	Apr. 27, Sa *08:19*	1191
May 5, Su *08:47*	May 12, Su *11:13*	May 19, Su *07:11*	May 27, Mo *02:34*	1192
June 3, Mo *20:02*	June 10, Mo *16:00*	June 17, Mo *18:31*	June 25, Tu *19:48*	1193
July 3, We *05:17*	July 9, Tu *20:56*	July 17, We *07:39*	July 25, Th *11:20*	1194
Aug. 1, Th *13:12*	Aug. 8, Th *03:32*	Aug. 15, Th *22:31*	Aug. 24, Sa *00:58*	1195
Aug. 30, Fr *20:38*	Sept. 6, Fr *13:11*	Sept. 14, Sa *14:35*	Sept. 22, Su *12:43*	1196
Sept. 29, Su *04:27*	Oct. 6, Su *02:48*	Oct. 14, Mo *07:10*	Oct. 21, Mo *22:41*	1197
Oct. 28, Mo *13:40*	Nov. 4, Mo *20:23*	Nov. 12, Tu *23:37*	Nov. 20, We *07:13*	1198
Nov. 27, We *01:07*	Dec. 4, We *16:58*	Dec. 12, Th *15:14*	Dec. 19, Th *14:59*	1199
Dec. 26, Th *15:15*				1200

7 Answer the following with reference to the sunrise/sunset chart shown below.

a Kieran is going fishing on 17 December. His theory is that the fish are always biting right at the time of sunrise. Determine the time on this day that he will need to be out fishing.

b Casey has put up her Christmas lights on the front of her house. She has set up a timer for the lights to come on once the sun has gone down, as she is not always home in the evenings. At which time should Casey set her timer to ensure that her Christmas lights come on at around sunset?

2022	Sunrise/Sunset	
Dec	**Sunrise**	**Sunset**
▼ **14**	4:48 am →(117°)	6:36 pm ←(243°)
▼ **15**	4:49 am →(117°)	6:36 pm ←(243°)
▼ **16**	4:49 am →(117°)	6:37 pm ←(243°)
▼ **17**	4:49 am →(117°)	6:38 pm ←(243°)
▼ **18**	4:50 am →(117°)	6:38 pm ←(243°)
▼ **19**	4:50 am →(117°)	6:39 pm ←(243°)
▼ **20**	4:51am →(117°)	6:39 pm ←(243°)
▼ **21**	4:51 am →(117°)	6:40 pm ←(243°)
▼ **22**	4:52 am →(117°)	6:40 pm ←(243°)

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c Stefan is a pilot who flies a plane that requires VFR (visual flight referencing). This means that he flies, in daylight, between sunrise and sunset. Between 14 and 22 December, approximately how many daylight hours per day are available for Stefan to charter this plane?

Sunrise over the Glasshouse Mountains, Sunshine Coast, Queensland

8 Andrea is studying at university and works casually, running photography courses. The most popular courses are her sunrise and sunset photography courses.
To allow the best opportunity for sunrise and sunset photos, she meets her students on the roof of a tall building 20 minutes before the official sunrise and sunset times.

Bunbury, WA

March	Sunrise	Sunset
1	5:41 a.m. → (99°)	6:18 p.m. ← (261°)
2	5:41 a.m. → (99°)	6:17 p.m. ← (262°)
3	5:42 a.m. → (98°)	6:16 p.m. ← (262°)
4	5:42 a.m. → (98°)	6:15 p.m. ← (263°)
5	5:43 a.m. → (97°)	6:14 p.m. ← (263°)
6	5:43 a.m. → (97°)	6:13 p.m. ← (263°)

Use the information in the chart shown above to identify:

a the sunrise and sunset times for Andrea to meet her students on 4 March

b the degrees from true north (e.g. 99°) of the sunrise and sunset on 6 March

c the difference in sunrise times between 1 March and 6 March

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9 Answer the following, using the information given in the tidal chart shown.

TODAY 4 Jan	**FRI** 5 Jan	**SAT** 6 Jan	**SUN** 7 Jan	**MON** 8 Jan
▼ 3:57 a.m. 0.39 m	▼ 4:43 a.m. 0.54 m	▼ 5:30 a.m. 0.77 m	▲ 12:24 a.m. 3.05 m	▲ 1:22 a.m. 2.94 m
▲ 10:20 a.m. 4.17 m	▲ 11:08 a.m. 4 m	▲ 11:56 a.m. 3.78 m	▼ 6:21 a.m. 1.03 m	▼ 7:23 a.m. 1.29 m
▼ 5:01 p.m. 0.65 m	▼ 5:48 p.m. 0.74 m	▼ 6:34 p.m. 0.87 m	▲ 12:48 p.m. 3.53 m	▲ 1:45 p.m. 3.31 m
▲ 10:41 p.m. 3.28 m	▲ 11:31 p.m. 3.17 m		▼ 7:21 p.m. 1.01 m	▼ 8:12 p.m. 1.11 m

a Patrick is going crabbing during the low tides on Friday. If the safest times to do this are within two hours of low tide, determine the suitable times on this day for Patrick to be on the beach, catching crabs.

b On Sunday morning, Kelly and Henry are heading out fishing while on holiday in Cervantes. Kelly insists that the best time to fish is the hour before high tide, as it brings the fish with it. Identify a suitable time to recommend for them to head out fishing if they are to follow Kelly's theory.

c On Thursday 4 January, Carlene and Brian left their dinghy tied to the jetty and went to explore the town. When they returned to their dinghy at 5 p.m., they discovered that it was sitting on the sand, as the tide had gone out. If Carlene and Brian must wait a minimum of 3 hours after low tide before it is deep enough to use the dinghy, at what time will it be deep enough so that they can head back home?

Example 16

10 On Wednesday 24 January, Damien is taking his family on a 4WD adventure from Perth to Jurien Bay. The trip requires a one-hour drive on the beach. Upon arriving at Jurien Bay, Damien would like to spend as much time as possible there. However, he is mindful of the tides, as he has heard that many people have been caught out on this beach and their vehicles destroyed by the high tide. Damien is travelling only during daylight and he wishes to be on the beach only within two hours either side of the low tide. Remember that he must also allow for the one hour of travel time on the beach.

SUN 21 Jan	**MON** 22 Jan	**TUE** 23 Jan	**WED** 24 Jan	**THU** 25 Jan
▼ 3:58 a.m. 0.55 m	▼ 4:36 a.m. 0.63 m	▼ 5:21 a.m. 0.72 m	▲ 12:31 a.m. 1.49 m	▲ 1:45 a.m. 1.53 m
▲ 10:37 a.m. 1.9 m	▲ 11:11 a.m. 1.83 m	▲ 11:49 a.m. 1.75 m	▼ 6:19 a.m. 0.81 m	▼ 7:36 a.m. 0.88 m
▼ 5:05 p.m. 0.63 m	▼ 5:43 p.m. 0.65 m	▼ 6:26 p.m. 0.66 m	▲ 12:36 p.m. 1.67 m	▲ 1:38 p.m. 1.59 m
▲ 10:51 p.m. 1.5 m	▲ 11:36 p.m. 1.49 m		▼ 7:19 p.m. 0.65 m	▼ 8:22 p.m. 0.63 m

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For each of the following, use the information given and refer to the tidal chart shown on the previous page.

a Determine the times that Damien could start driving on the beach to head to Jurien Bay.

b Determine the times that he would need to leave Jurien Bay to travel safely on the beach.

c Calculate how long Damien and his family could stay at Jurien Bay.

11 A 'rule of thumb' is that, after low or high tide, the tide height changes by 25% in the first and second hours, 50% in the third and fourth hours, and 25% in the fifth and sixth hours.

Use this rule of thumb to answer the following questions about the changes in tide height at Hay Point (south of McKay) on 1 February, which are presented in the table.

1 FEBRUARY Hay Point	
Time	**Height (in m)**
05:19	0.38
11:26	7.09
18:02	0.58
23:50	5.81

Find the change in tide height, in metres, to one decimal place:

a from the first low tide to the first high tide

b over the first two hours after the first low tide

c in the third and fourth hour after the first low tide

d in the last two hours before the first high tide

e from the first high tide to the second low tide

f in the third and fourth hour after the first high tide

ISBN 978-1-009-25785-5

Mathematical thinking process task

DAY TRIP

Background: Timetables are essential for using public transport, and creating your own personalised timetable is an excellent way to plan a trip and keep to time.

Task: Your task is to plan a day trip around a city near you. You must use at least two forms of public transport during your trip. Make sure you keep a copy of the timetables you use, to show your teacher the departure and arrival times. With this information, create a timetable that details your plan for the day.

To complete this task, follow the mathematical thinking workflow diagram on the next page and use the steps listed as a guide

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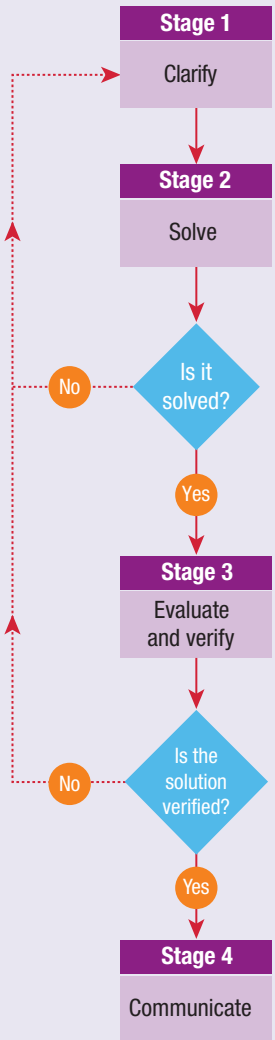

Stage 1: Clarify
Interpret the task, gather the key information and identify the mathematics which could help to complete the task

1 Decide on the destination city.
2 Research interesting things to do in this city.
3 Look up public modes of transport available.

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

4 Decide on at least two destinations or activities that will be completed during the day trip.
5 Look up relevant transportation timetables, and use these to identify suitable routes to help plan your trip around the city.
6 Plan your trip and decide on relevant time intervals. Consider where you are starting from. How long does the bus take? How much time would you need at the museum, etc.?

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources and verify the reasonableness of the solution

7 Check that you have allowed yourself enough time to complete the activity and then make it to your mode of transport.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

8 Communicate your findings in the form of a timetable, detailing the activities that took place and the transportation that was used, in an ordered manner. You might like to use this table as a guide.

Time	Activity
8 a.m.	
9 a.m.	
10 : 30 a.m.	
12 p.m.	
1 : 15 p.m.	
3 p.m.	

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ISBN 978-1-009-25785-5

Puzzle

WHAT IS TIME MANAGEMENT?

Good time management skills are essential in many careers. They enable you to work smarter, meaning you get more done in a shorter space of time – even when time is short and the pressure is mounting. Failing to manage your time well may cause stress and potentially decreases your rate of success. There are three key aspects to becoming skilled in time management. The first two are organisation and planning. To find the third, solve the puzzle below. Find the answer to each question and write the letter of the question above its answer.

1 Calculate the number of hours, in decimal time, for the following time periods.

A 8 a.m. to 1:30 p.m.

B 7 a.m. to 2:15 p.m.

C 12:18 p.m. to 7:32 p.m.

D 4:51 a.m. to 11:29 p.m.

2 Use a suitable calculation to convert the following decimal hours to the form hours : minutes : seconds.

I 3.42 hours

N 7.16 hours

O 2.97 hours

3 Express the following 12-hour times as 24-hour times.

P 3:55 p.m.

R 12:11 a.m.

4 Express the following 24-hour times as 12-hour times.

S 19:22

T 12:30

7.23	2:58:12

7:9:36	2:58:12	12:30 p.m.

15:55	0:11	2:58:12	7.25	0:11	5.5	7.22 p.m

12:30 p.m.	3:25:12	7:9:36	5.5	12:30 p.m.	6.63

ISBN 978-1-009-25785-5

Chapter checklist

☐ I can use units of time and convert between fractional, decimal and digital times.

1. Yesterday, Josef worked for 6 hours and 15 minutes. Write the minutes as a fraction of an hour and use division to simplify, then write how the fractional time is spoken.
2. Last week, Rania worked for 32 hours and 27 minutes. Calculate the number of hours that she worked, in decimal form.

☐ I can use 12-hour and 24-hour time.

3. Bronwyn has just received a text from her brother, in Canada, asking her to call him at Australian time 00:30. What time should she set her 12-hour alarm clock?
4. Oliver is a drill sergeant in the army and he has instructed his troupe to meet him in two and a half hours from now. If the instruction was given at 12:45 p.m., at what time must Oliver's troupe meet again, using 24-hour time?

☐ I can calculate time intervals, including the time between, the time ahead and the time behind.

5. Joel has just finished working from 8:15 a.m. to 2:50 p.m. Calculate the length of his shift.
6. Katrina is driving to Karijini National Park for a bushwalk. She expects the trip to take 7 h and 45 min. If Katrina leaves home at 5:20 a.m, determine the time that she would expect to arrive.

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ISBN 978-1-009-25785-5

I can interpret timetables for buses, trains and ferries.

7 Tammie must catch a bus from Dianella Plaza to Perth airport. She must be at the airport by 10:30 a.m. Use the following timetables on this page and the next to identify the latest time she could catch the bus from Dianella Plaza.

Route 960 - To Curtin University

Timed Stops										
Stop No.		26333 / Stand 3	12799	12808	12814	Zone B	10140 / Stand D	11714	27934 / Stand 7	11699 / Stand 1
Route No.		Mirrabooka Bus Stn	Dianella Plaza	Edith Cowan University Mount Lawley	North Perth Plaza	Perth Busport	Victoria Park Transfer Stn	Albany Hwy / Kent St	Curtin Central Bus Stn	Curtin University Bus Stn
Monday to Friday										
am	960	5:09	5:14	5:19	5:22	5:30	5:39	5:44	5:51	5:57
	960	5:28	5:34	5:39	5:42	5:50	5:59	6:04	6:12	6:18
	960	5:48	5:54	5:59	6:02	6:10	6:20	6:25	6:33	6:39
	960	6:05	6:11	6:16	6:19	6:27	6:38	6:43	6:51	6:58
	960	6:16	6:22	6:27	6:30	6:38	6:49	6:54	7:02	7:09
	960	6:24	6:30	6:35	6:39	6:48	6:59	7:04	7:12	7:19
	960	6:32	6:38	6:43	6:47	6:56	7:07	7:12	7:20	7:27
	960	6:39	6:46	6:51	6:55	7:04	7:16	7:21	7:30	7:37
	960	6:46	6:53	6:58	7:02	7:11	7:23	7:29	7:38	7:45
	960	6:52	6:59	7:04	7:08	7:17	7:30	7:36	7:45	7:52
	960	6:57	7:04	7:09	7:13	7:23	7:36	7:42	7:51	7:59
	960	7:02	7:09	7:14	7:19	7:29	7:42	7:48	7:57	8:05
	960	7:08	7:15	7:20	7:25	7:35	7:48	7:54	8:03	8:11
	960	7:12	7:19	7:25	7:30	7:41	7:54	8:00	8:10	8:18
	960	7:17	7:24	7:30	7:35	7:47	8:00	8:06	8:16	8:24
	960	7:22	7:30	7:36	7:41	7:53	8:07	8:13	8:23	8:31
	960	7:27	7:36	7:42	7:47	7:59	8:13	8:19	8:29	8:37
	960	7:32	7:41	7:48	7:53	8:05	8:19	8:25	8:35	8:43
	960	7:37	7:46	7:53	7:58	8:11	8:25	8:31	8:41	8:49
	960	7:42	7:51	7:59	8:04	8:17	8:31	8:37	8:47	8:55
	960	7:47	7:56	8:04	8:10	8:23	8:37	8:43	8:53	9:01
	960	7:53	8:02	8:10	8:16	8:29	8:43	8:49	8:59	9:07
	960	7:59	8:08	8:16	8:22	8:35	8:49	8:55	9:04	9:12
	960	8:04	8:13	8:22	8:28	8:41	8:55	9:01	9:10	9:18
	960	8:11	8:20	8:29	8:35	8:47	9:01	9:07	9:16	9:23
	960	8:20	8:28	8:37	8:42	8:54	9:08	9:14	9:23	9:30
	960	8:28	8:36	8:44	8:49	9:01	9:15	9:21	9:30	9:37
	960	8:38	8:45	8:53	8:58	9:09	9:23	9:29	9:38	9:45
	960	8:47	8:54	9:01	9:06	9:17	9:30	9:36	9:45	9:52
	960	8:57	9:04	9:10	9:14	9:25	9:38	9:44	9:53	10:00
	960	-	-	9:20	9:24	9:35	9:47	9:53	10:02	10:09
	960	9:17	9:24	9:30	9:34	9:45	9:57	10:03	10:12	10:19
	960	-	-	9:40	9:44	9:55	10:07	10:12	10:21	10:28
	960	9:37	9:44	9:50	9:54	10:05	10:17	10:22	10:31	10:38

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Route 380 - From Perth

Timed Stops					
Stop No.		12201	10142	11304	27377
Route No.		Elizabeth Quay Bus Stn	Victoria Park Transfer Stn	Belmont Forum / Wright St	Perth Airport T1 and T2
Monday to Friday					
am	380	5:30	5:41	5:53	6:04
	380	6:32	6:45	6:58	7:09
	380	7:29	7:45	7:59	8:12
	380	8:25	8:41	8:55	9:08
	380	9:25	9:41	9:54	10:07
	380	10:25	10:41	10:54	11:07
	380	11:25	11:41	11:54	12:07
pm	380	12:25	12:41	12:54	1:07
	380	1:25	1:41	1:54	2:07
	380	2:25	2:41	2:55	3:08
	380	3:25	3:41	3:55	4:08
	380	4:32	4:50	5:04	5:17
	380	5:32	5:50	6:04	6:17
	380	6:37	6:52	7:04	7:17
	380	7:37	7:50	8:02	8:15
	380	8:37	8:50	9:02	9:14
	380	9:37	9:49	10:01	10:13
	380	10:37	10:48	10:59	11:11
	380	11:48	11:59	12:09	12:20

☐ I can use timetables and/or electronic technologies to plan and compare the most time-efficient routes.

8 Shae lives in Maroochydore and studies full-time at the University of Western Australia (UWA). She catches a bus four mornings a week and must be on campus by 10:30 a.m. to attend lectures. Shae has two options.

Option 1: Travel with her housemate to Warwick Station by 9:00 a.m. every morning and then catch a bus from there. There is a 12-minute walk from the Perth Bus Station to the Elizbeth Quay Bus Station.
Option 2: Catch a train and bus directly from Joondalup Station to UWA.

Use the following two bus and one train timetables to determine the best route for Shae if she wishes to spend the least amount of time travelling to university.

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Bus timetables

Route 387, 388, 389 - To Perth

Timed Stops										
Stop No.		18307	12308	12553	18584	12331	12344	12354	12593	
Route No.		Belgrade Rd / Esther Loop	Wanneroo Shop Ctr	Warwick Stn	Erindale Rd / Balcatta Rd	Wanneroo Rd / Beach Rd	Wanneroo Rd / Harrison St	Dog Swamp Shop Ctr	Charles St / Scarborough Beach Rd	Perth Busport
Monday to Friday										
am	388	-	-	5:18	-	5:22	5:28	5:34	5:37	5:47
	389	5:31	5:35	-	-	5:48	5:54	6:00	6:03	6:13
	388	-	-	6:06	-	6:11	6:17	6:23	6:26	6:37
	389	6:09	6:13	-	-	6:27	6:35	6:41	6:44	6:55
	388	-	-	6:38	-	6:43	6:51	6:58	7:01	7:12
	389	6:35	6:40	-	-	6:55	7:03	7:10	7:13	7:26
	388	-	-	7:00	-	7:05	7:13	7:20	7:23	7:36
	389	6:55	7:00	-	-	7:15	7:23	7:30	7:34	7:53
	388	-	-	7:20	-	7:25	7:33	7:41	7:45	7:59
	389	7:12	7:17	-	-	7:34	7:43	7:52	7:57	8:14
	388	-	-	7:38	-	7:44	7:53	8:02	8:06	8:23
	389	7:32	7:37	-	-	7:54	8:03	8:12	8:17	8:33
	388	-	-	7:59	-	8:05	8:15	8:23	8:27	8:43
	388	-	-	8:14	-	8:19	8:28	8:36	8:40	8:56
	389	8:12	8:17	-	-	8:34	8:44	8:51	8:55	9:09
	388	-	-	8:47	-	8:52	9:01	9:08	9:11	9:24
	389	8:45	8:50	-	-	9:06	9:14	9:21	9:24	9:37
	388	-	-	9:15	-	9:21	9:29	9:36	9:39	9:51
	389	9:15	9:20	-	-	9:36	9:44	9:51	9:54	10:06
	388	-	-	9:45	-	9:50	9:58	10:05	10:08	10:20
	387	-	-	10:00	10:06	-	10:13	10:20	10:23	10:35
	388	-	-	10:16	-	10:21	10:29	10:36	10:39	10:51
	389	10:15	10:20	-	-	10:36	10:44	10:51	10:54	11:06
	388	-	-	10:46	-	10:51	10:59	11:06	11:09	11:21
	387	-	-	11:01	11:07	-	11:14	11:21	11:24	11:36
	388	-	-	11:16	-	11:21	11:29	11:36	11:39	11:51
	389	11:15	11:20	-	-	11:36	11:44	11:51	11:54	**12:06**
	388	-	-	11:46	-	11:51	11:59	**12:06**	**12:09**	**12:21**
pm	387	-	-	**12:01**	**12:07**	-	**12:14**	**12:21**	**12:24**	**12:36**

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Route 995 - To Claremont

Timed Stops			
Stop No.	12218 / Stand B4	10390	28528 / Stand 6
Route No.	Elizabeth Quay Bus Stn	University of WA	Claremont Stn
Monday to Friday			
am	995 D	6:05	6:12	6:21
	995 D	6:30	6:37	6:48
	995 D	6:50	6:57	7:08
	995 A	7:00	7:07	7:18
	995 A	7:15	7:23	7:35
	995 D	7:26	7:36	7:48
	995 A	7:36	7:46	8:00
	995 D	7:48	7:58	8:15
	995	7:58	8:08	8:23
	995 D	8:08	8:19	8:31
	995 A	8:20	8:31	8:42
	995	8:31	8:42	8:53
	995 D	8:42	8:53	9:04
	995 A	8:53	9:04	9:16
	995	9:03	9:14	9:26
	995 D	9:13	9:22	9:34
	995 A	9:28	9:37	9:49
	995 D	9:44	9:53	10:04
	995	9:59	10:08	10:19
	995 D	10:14	10:23	10:34
	995 A	10:29	10:38	10:49
	995 D	10:44	10:53	11:04
	995	10:59	11:08	11:19
	995 D	11:14	11:23	11:34
	995 A	11:28	11:37	11:49
	995 D	11:43	11:52	12:04
	995	11:58	12:07	12:19
pm	995 D	12:13	12:22	12:34
	995 A	12:28	12:37	12:49
	995 D	12:43	12:52	1:04

ISBN 978-1-009-25785-5

Train timetable

To Perth

Pattern	Butler 99901	Clarkson 99891	Currambine 99881	Joondalup 99871	Edgewater 99861	Whitfords 99851	Greenwood 99841	Warwick 99831	Stirling 99821	Glendalough 99811	Leederville 99801	Perth Underground 99602	Elizabeth Quay 99612
Monday to Friday (continued)													
W	-	-	-	-	-	8:34	8:37	8:40	8:45	8:48	8:51	8:54	8:58
	8:22	8:27	8:30	8:34	8:37	8:40	8:43	8:46	8:51	8:54	8:57	9:00	9:03
W	-	-	-	-	-	8:45	8:48	8:51	8:55	8:58	9:01	9:04	9:08
	8:32	8:37	8:40	8:44	8:47	8:50	8:53	8:56	9:00	9:03	9:06	9:09	9:13
W	-	-	-	-	-	8:55	8:58	9:01	9:05	9:08	9:11	9:14	9:18
	8:42	8:47	8:50	8:54	8:57	9:00	9:03	9:06	9:10	9:13	9:16	9:19	9:25
PW	-	-	-	-	-	9:05	9:08	9:11	9:15	9:18	9:21	P 9:24	-
	8:56	9:01	9:04	9:08	9:11	9:14	9:17	9:20	9:24	9:27	9:30	9:33	9:40
	9:11	9:16	9:19	9:23	9:26	9:29	9:32	9:35	9:39	9:42	9:45	9:48	9:55
	9:26	9:31	9:34	9:38	9:41	9:44	9:47	9:50	9:54	9:57	10:00	10:03	10:10
	9:41	9:46	9:49	9:53	9:56	9:59	10:02	10:05	10:09	10:12	10:15	10:18	10:25
	9:56	10:01	10:04	10:08	10:11	10:14	10:17	10:20	10:24	10:27	10:30	10:33	10:40
	10:11	10:16	10:19	10:23	10:26	10:29	10:32	10:35	10:39	10:42	10:45	10:48	10:55
	10:26	10:31	10:34	10:38	10:41	10:44	10:47	10:50	10:54	10:57	11:00	11:03	11:10
	10:41	10:46	10:49	10:53	10:56	10:59	11:02	11:05	11:09	11:12	11:15	11:18	11:25
	10:56	11:01	11:04	11:08	11:11	11:14	11:17	11:20	11:24	11:27	11:30	11:33	11:40
	11:11	11:16	11:19	11:23	11:26	11:29	11:32	11:35	11:39	11:42	11:45	11:48	11:55
	11:26	11:31	11:34	11:38	11:41	11:44	11:47	11:50	11:54	11:57	**12:00**	**12:03**	**12:10**
	11:41	11:46	11:49	11:53	11:56	11:59	**12:02**	**12:05**	**12:09**	**12:12**	**12:15**	**12:18**	**12:25**

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I can interpret complex timetables, such as tide charts, moon phases and sunrise/sunset charts.

9 Aaron wishes to find the earliest time to catch the 20-minute ferry trip to Fraser Island on Wednesday morning, 7 February. The ferry also transports vehicles to the island. Upon arrival, Aaron will need to drive along the beach to get to his destination. As beach travel is safe only within 2 hours of low tide, determine the earliest time he can catch the ferry.

TUE 6 Feb	**WED** 7 Feb	**THU** 8 Feb
▲ 12:53 a.m. 3.08 m	▲ 1:51 a.m. 2.95 m	▲ 3:06 a.m. 2.88 m
▼ 7:03 a.m. 1.3 m	▼ 8:11 a.m. 1.55 m	▼ 9:40 a.m. 1.67 m
▲ 1:10 p.m. 3.2 m	▲ 2:08 p.m. 2.95 m	▲ 3:16 p.m. 2.79 m
▼ 7:36 p.m. 1.15 m	▼ 0:31 p.m. 1.28 m	▼ 9:37 p.m. 1.35 m

ISBN 978-1-009-25785-5

Chapter review

Section 12A

1 Michael works full-time at a discount store. His roster is in 24-hour time and he is working tomorrow from 11:00 to 19:45. Express Michael's start and finishing times in 12-hour time.

2 Anita ran a 10-km marathon in 1:05:39. Express this time in decimal form.

Section 12B

3 Dimitri's payslip states that he worked for 28.8 h this past week. Dimitri had recorded that he has worked for 28 h and 45 min. Determine if Dimitri's payslip is correct. Justify your answer with mathematical working and discuss how Dimitri's boss could have concluded that this was correct.

4 Joan is booked in for an x-ray on her knee. She is told that the scan will take around 12 minutes. She looks at the time on her way into the room and it is 1:09 p.m.

a Determine the time when Joan would expect her x-ray to be completed.

b Express the expected finishing time in 24-hour time.

5 Justine went to her local public library at 9:15 a.m. and returned home at 12:02 p.m. Determine how long she was out, in hours and minutes.

6 Kyosti has just finished his English lesson. All lessons go for 80 min and the time is now 2:56 p.m. At what time did Kyosti's lesson begin?

Section 12C

7 Use the information in the Perth train timetable, shown on the next page, to answer the following questions.

a Mike is catching the 6.20 a.m. train from Claisebrook to Kelmscott. Calculate how long this train trip would take.

b i Paul is travelling from McGiver Station to Armadale Station. He looks at his watch and catches what he believes to be the 7:05 a.m. train. At what time would he expect to arrive at his destination?

ii Paul has just realised that his watch is 3 minutes ahead of time and he has actually caught the 7:08 a.m. train. Paul has caught the train to Thornlie Station, so he will need to change at Cannington Station to the Armadale line. Determine the time when he should arrive at his destination.

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c Jacquie lives in Victoria Park and is catching the train to a big furniture sale in Maddington. She would like to be there by 8:15 a.m. so that she can grab some good bargains. Decide which train you would recommend Jacquie should catch.

d David is meeting a friend at 8 a.m. at a cafe that is a 5-minute walk from the Thornlie train station. Considering that he is leaving from Oats Street Station, decide which train would be best for David to catch.

To Thornlie / Armadale

Pattern	Perth 99004/99005	McIver 99012	Claisebrook 99022	Burswood 99042	Victoria Park 99052	Carlisle 99062	Oats Street 99072	Westfield 99082	Queens Park 99092	Cannington 99102	Thornlie 99103	Beckenham 99112	Kenwick 99122	Maddington 99132	Cosmolls 99142	Seaforth 99152	Kelmscott 99162	Challis 99172	Sherwood 99182	Armadale 99192
Monday to Friday																				
am T	5:05	5:07	5:08	5:12	5:13	5:15	5:17	5:19	5:21	5:23	5:28	-	-	-	-	-	-	-	-	-
	5:11	5:13	5:14	5:18	5:19	5:21	5:23	5:25	5:27	5:29	-	5:30	5:32	5:34	5:38	5:40	5:43	5:45	5:47	5:50
T	5:20	5:22	5:23	5:27	5:28	5:30	5:32	5:34	5:36	5:38	5:43	-	-	-	-	-	-	-	-	-
	5:27	5:29	5:30	5:34	5:35	5:37	5:39	5:41	5:43	5:45	-	5:46	5:48	5:50	5:54	5:56	5:59	6:01	6:03	6:06
T	5:35	5:37	5:38	5:42	5:43	5:45	5:47	5:49	5:51	5:53	5:58	-	-	-	-	-	-	-	-	-
C	5:47	5:49	5:50	-	-	-	5:57	-	-	6:01	-	6:02	6:04	6:06	6:10	6:12	6:15	6:17	6:19	6:22
T	5:50	5:52	5:53	5:57	5:58	6:00	6:02	6:04	6:06	6:08	6:13	-	-	-	-	-	-	-	-	-
C	6:02	6:04	6:05	-	-	-	6:12	-	-	6:16	-	6:17	6:19	6:21	6:25	6:27	6:30	6:32	6:34	6:37
T	6:05	6:07	6:08	6:12	6:13	6:15	6:17	6:19	6:21	6:23	6:28	-	-	-	-	-	-	-	-	-
C	6:17	6:19	6:20	-	-	-	6:27	-	-	6:31	-	6:32	6:34	6:36	6:40	6:42	6:45	6:47	6:49	6:52
T	6:19	6:21	6:22	6:26	6:27	6:29	6:31	6:33	6:35	6:37	6:42	-	-	-	-	-	-	-	-	-
C	6:32	6:34	6:35	-	-	-	6:42	-	-	6:46	-	6:47	6:49	6:51	6:55	6:57	7:00	7:02	7:04	7:07
T	6:35	6:37	6:38	6:42	6:43	6:45	6:47	6:49	6:51	6:53	6:58	-	-	-	-	-	-	-	-	-
C#	6:43	6:44	6:46	-	-	-	6:53	-	-	6:57	-	6:58	7:00	7:02	7:06	7:08	7:11	7:13	7:15	7:18
C	6:47	6:49	6:50	-	-	-	6:57	-	-	7:01	-	7:02	7:04	7:06	7:10	7:12	7:15	7:17	7:19	7:22
T	6:50	6:52	6:53	6:57	6:58	7:00	7:02	7:04	7:06	7:08	7:13	-	-	-	-	-	-	-	-	-
C#	6:58	6:59	7:01	-	-	-	7:08	-	-	7:12	-	7:13	7:15	7:17	7:21	7:23	7:26	7:28	7:30	7:33
C	7:02	7:04	7:05	-	-	-	7:12	-	-	7:16	-	7:17	7:19	7:21	7:25	7:27	7:30	7:32	7:34	7:37
T	7:05	7:07	7:08	7:12	7:13	7:15	7:17	7:19	7:21	7:23	7:28	-	-	-	-	-	-	-	-	-
C#	7:13	7:14	7:16	-	-	-	7:23	-	-	7:27	-	7:28	7:30	7:32	7:36	7:38	7:41	7:43	7:45	7:48
C	7:17	7:19	7:20	-	-	-	7:27	-	-	7:31	-	7:32	7:34	7:36	7:40	7:42	7:45	7:47	7:49	7:52
T	7:20	7:22	7:23	7:27	7:28	7:30	7:32	7:34	7:36	7:38	7:43	-	-	-	-	-	-	-	-	-
C	7:32	7:34	7:35	-	-	-	7:42	-	-	7:46	-	7:47	7:49	7:51	7:55	7:57	8:00	8:02	8:04	8:07
T	7:35	7:37	7:38	7:42	7:43	7:45	7:47	7:49	7:51	7:53	7:58	-	-	-	-	-	-	-	-	-
C	7:47	7:49	7:50	-	-	-	7:57	-	-	8:01	-	8:02	8:04	8:06	8:10	8:12	8:15	8:17	8:19	8:22
T	7:52	7:54	7:55	7:59	8:00	8:02	8:04	8:06	8:08	8:10	8:15	-	-	-	-	-	-	-	-	-
C	8:02	8:04	8:05	-	-	-	8:12	-	-	8:16	-	8:17	8:19	8:21	8:25	8:27	8:30	8:32	8:34	8:37
T	8:07	8:09	8:10	8:14	8:15	8:17	8:19	8:21	8:23	8:25	8:30	-	-	-	-	-	-	-	-	-
C	8:17	8:19	8:20	-	-	-	8:27	-	-	8:31	-	8:32	8:34	8:36	8:40	8:42	8:45	8:47	8:49	8:52
T	8:22	8:24	8:25	8:29	8:30	8:32	8:34	8:36	8:38	8:40	8:45	-	-	-	-	-	-	-	-	-
C	8:32	8:34	8:35	-	-	-	8:42	-	-	8:46	-	8:47	8:49	8:51	8:55	8:57	9:00	9:02	9:04	9:07
T	8:35	8:37	8:38	8:42	8:43	8:45	8:47	8:49	8:51	8:53	8:58	-	-	-	-	-	-	-	-	-
C	8:45	8:47	8:48	-	-	-	8:55	-	-	8:59	-	9:00	9:02	9:04	9:08	9:10	9:13	9:15	9:17	9:20
T	8:52	8:54	8:55	8:59	9:00	9:02	9:04	9:06	9:08	9:10	9:15	-	-	-	-	-	-	-	-	-

ISBN 978-1-009-25785-5

Section 10D

8 On Friday 6 April, Liza and her family are heading to Stradbroke Island for a 4WD beach driving adventure, returning Monday morning.

a Use the table to identify the low-tide times on Friday and Monday.

b It is safe to drive on the beach only within two hours either side of low tide. Determine the safe times to drive on the beach on Friday and Monday.

FRI 6 Apr	**SAT** 7 Apr	**SUN** 8 Apr	**MON** 9 Apr	**TUE** 10 Apr
6:04 a.m. 0.83 m	12:21 a.m. 2.08 m	1:18 a.m. 2 m	2:33 a.m. 1.96 m	3:47 a.m. 2 m
11:42 a.m. 1.72 m	6:58 a.m. 0.91 m	8:13 a.m. 0.94 m	9:33 a.m. 0.9 m	10:37 a.m. 0.81 m
5:41 p.m. 0.72 m	12:34 p.m. 1.6 m	1:48 p.m. 1.54 m	3:17 p.m. 1.57 m	4:26 p.m. 1.69 m
	6:29 p.m. 0.83 m	7:35 p.m. 0.91 m	8:58 p.m. 0.92 m	10:15 p.m. 0.85 m

North Stradbroke Island is called Minjerribah by the Quandamooka Traditional Owners.

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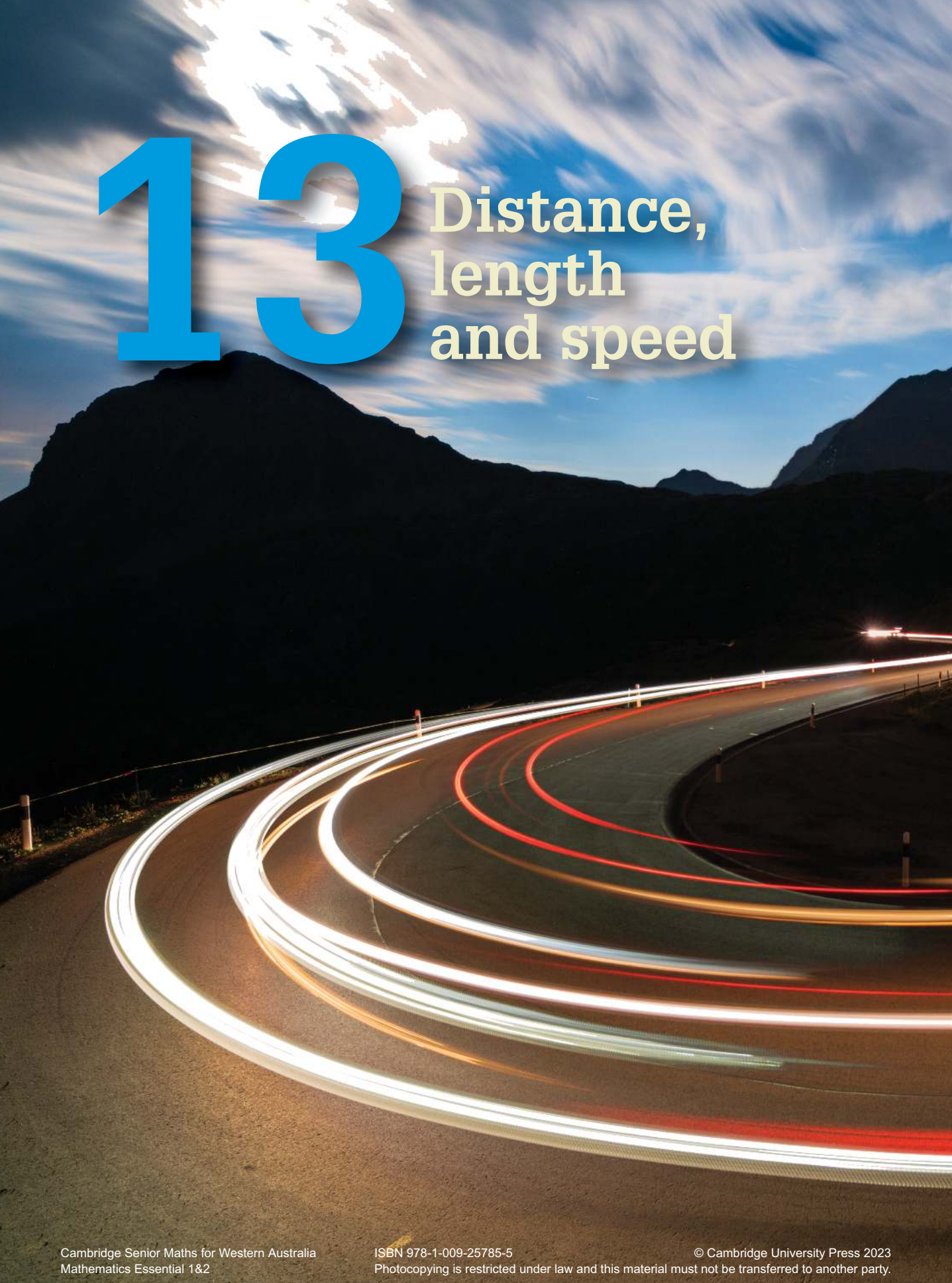
ISBN 978-1-009-25785-5



13 Distance, length and speed

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In this chapter

Syllabus reference

Unit 2 Topic 2.4: Time and motion

Distance and length

- use scales to calculate distances and lengths on plans, maps and charts
- plan routes for practical purposes, accounting for local conditions

Speed

- identify the appropriate units for different activities, such as walking, running, swimming and flying
- calculate speed, distance or time using the formula: speed = distance/time
- calculate the time or costs for a journey from distances, estimated from maps
- interpret distance versus time graphs
- calculate and interpret the average speed

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Pre-test

1 Convert the following distances to metres.
- **a** 1 km
- **b** 2.35 km
- **c** 75 mm

2 Convert the following to kilometres.
- **a** 3000 m
- **b** 575 000 cm
- **c** 250 m

3 Calculate the distance covered in 2 h and 30 min when travelling at the following speeds. (Write your answers in km.)
- **a** 60 km/h
- **b** 45 000 m/h
- **c** 300 m/min
- **d** 13 m/s
- **e** 792 m/h
- **f** 33 mm/s

4 Write each of the following as simplified rates, rounding to two decimal places if needed.
- **a** $10 for 3 kg of bananas
- **b** 452 L in 8 min
- **c** 14 m of yarn in 4.5 h
- **d** 867 MB in 259 s
- **e** 100 m in 9.58 s
- **f** 7400 km in 9.7 h

5 Find the average rate of change in the following situations, rounding to three decimal places if needed.
- **a** Talia jogged 19 km in 2 h.
- **b** Frederic played a total of 694 notes on the piano in 5 min.
- **c** Marina boiled water for 3 min from 21°C to 100°C.
- **d** The flight from Brisbane to Perth was 3624.98 km and took 5.75 h.
- **e** The tree was 15 years old and is currently 22 m tall.
- **f** The Earth rotates 61 times in 60.83 days.

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6 Convert the following rates to the units shown in the brackets.

- **a** \$4.50/kg (\$/g)
- **b** \$58 493/year (\$/week)
- **c** 8 MB/s (KB/s)
- **d** 450 calories/kg (calories/100 g)
- **e** 0.54 km/s (m/s)
- **f** \$3.50/ft (\$/inch, where 1 ft = 12 inches)

7 Use the following steps to convert the speed of a plane, 902 km/h, to m/s.

- **a** Convert 902 km/h to m/h.
- **b** Convert the result from part **a** to m/min.
- **c** Convert the result from part **b** to m/s.

8 For each comparison, determine which is faster.

- **a** 7000 km in 9.5 h or 5000 km in 7.5 h?
- **b** 15 cm in 3 min or 90 mm in 2 min?
- **c** 900 m in 200 min or 2.1 km in 6 h?

9 Lucas is deciding between three brands of soup: A, B and C. He finds out that a can of brand A costs \$2.50 for 485 mL, a can of brand B costs \$4.85 for 900 mL and a can of brand C costs \$3.50 for 615 mL. Which brand of soup is the best value to buy?

10 Fatima and Reiko are selling apples at a farmers' market. Buying 12 apples from Fatima costs \$15, and buying eight apples from Reiko costs \$9.

- **a** Who sells apples at a cheaper rate?
- **b** Jiyeon wishes to buy 20 apples. How much money would she save if she buys her apples from the cheaper stall rather than the more expensive stall?

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ISBN 978-1-009-25785-5

13A Calculating distances using scales from maps

LEARNING GOALS

- Revise how to convert a scale distance to an actual distance, using a scale ratio
- Convert a map distance to an actual distance, using the line segment scale given on a map

When is it essential to calculate distances from maps?

When travelling where the internet is unavailable, it is certainly essential to be able to interpret maps. There are many Australian regions without internet access where people such as pilots (e.g. Flying Doctors and farmers), truck drivers, tourists, army personnel, workers (e.g. road, railway and electricity) and geologists all find map-reading skills essential.

You can use a map scale to calculate actual distances from the map and work out how long it will take to travel between various places along a route.

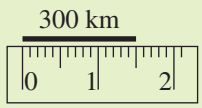
Adventurers must be able to calculate distances using national park maps, orienteering maps and topographical maps.

WHAT YOU NEED TO KNOW

- **Actual distance** is distance measured on the ground, usually in kilometres. **Map distance** is distance measured on the map with a ruler, usually in centimetres or millimetres.
- The **scale** ratio on a map shows the ratio of map distance to actual distance.

$$\text{Scale ratio} = \text{map distance} : \text{actual distance}$$

- A scale ratio of 1 : 20 000 means the actual distances are 20 000 times longer than the distances measured on the map.
  - For example: Finding the actual distance for a map distance of 4 cm.
    Actual distance $= 4\text{ cm} \times 20\,000$
    $= 80\,000\text{ cm}$
    $= 800\text{ m}$
- Many maps include a **line segment scale** instead of a scale ratio. The scale for such maps is determined by first measuring the length of the line segment with a ruler, then calculating the scale.
  - For example: A line segment labelled as 300 km is found to be 15 mm long.
    The line segment scale is $300\text{ km} \div 15\text{ mm} = 20\text{ km/mm}$.



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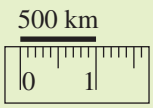
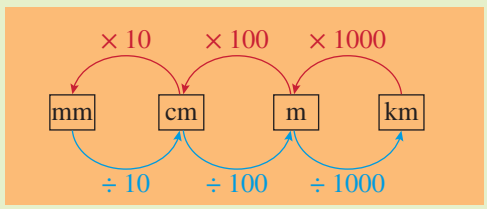
- To calculate actual distances using a map's labelled line segment scale:
  1. Measure the length of the line segment and calculate the line segment scale by the method described above.
  2. Measure the map distance with a ruler.
  3. Actual distance = map distance × scale
     - For example: Using the line segment scale below, find the actual distance for a 45 mm map distance.



$$\begin{aligned}\text{Line segment} &= 500\text{ km per } 10\text{ mm}\\ \text{Scale} &= 50\text{ km/mm}\\ \text{Map distance} &= 45\text{ mm}\\ \text{Actual distance} &= 45\text{ mm} \times 50\text{ km/mm}\\ &= 2250\text{ km}\end{aligned}$$

- Conversion of **length units**



Example 1 Convert a scale distance to an actual distance, using a scale ratio

A map has a scale of 1:25 000.
Find the actual distance for each of the following scaled distances.

a 130 mm **b** 24.8 cm

WORKING	THINKING
a Actual distance = 130 mm × 25 000 = 3 250 000 mm = 3250 m = 3.25 km	Multiply 130 mm by the scale factor of 25 000, giving an answer in mm. Now convert mm to m and then to km. [mm (÷1000) → m (÷1000) → km]
b Actual distance = 24.8 cm × 25 000 = 620 000 cm = 6200 m = 6.2 km	Multiply 24.8 cm by the scale factor of 25 000 to give an answer in cm. Now convert cm to m and then to km. [cm (÷100) → m (÷1000) → km]

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Example 2 Convert a map distance to an actual distance, using the line segment scale given on a map

Using the map of New Zealand shown:

a Measure the line segment scale in mm and calculate the scale in km/mm, to one decimal place.

b Measure directly on the map the distance, in mm, between Auckland and Invercargill, and then convert this map distance to the actual distance, to the nearest 10 km.

WORKING	THINKING
a Line segment = 200 km per 11 mm Scale = 18.2 km/mm	Measure the line segment in mm. Calculate km/mm. Round to 1 dp.
b Map distance = 65 mm Actual distance = 65 mm × 18.2 km/mm = 1180 km	Measure directly on the map the distance, in mm, between Auckland and Invercargill. Actual distance = map distance in mm × km/mm

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Exercise 13A

FUNDAMENTALS

1 Show your understanding of terms by writing down the missing words and units.

a The scale ratio on a map shows the ratio of _____ length : _____ length.

b A scale ratio of 1 : 5000 shows actual lengths are 5000 times _____ than the ____ lengths.

c Many maps include a line _____ scale instead of a scale ratio.

d To calculate actual distances using a map's line segment scale:

1 Measure the ____ segment scale in ___.

2 Calculate km/___ from the line segment scale.

3 The _____ distance in km = ____ distance in __ × scale in __/__.

Example 1

2 Find the actual distance for each of the scaled distances. Give your answer in appropriate units.

Hint Multiplying by the scale factor does not change the units.

a scale 1 : 1000

i 6 cm **ii** 120 mm

b scale 1 : 20 000

i 4 cm **ii** 300 mm

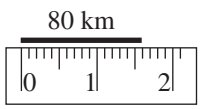
c scale 1 : 8000

i 13 mm **ii** 3.75 cm

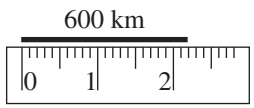
Example 2

3 Each of the following line segment scales has a ruler in cm and mm shown with it. Write the length of each line segment, to the nearest mm, and find each scale in km/mm. Give your answers to one decimal place.

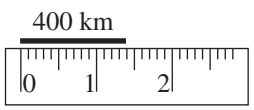
a



b



c



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4 Using the map of Western Australia shown:

a Measure the line segment scale, in mm, and calculate the scale, in km/mm, to one decimal place.

b Between each two places listed below, first measure, in mm, the distance directly on the map and then convert each map distance to the actual distance, to the nearest 10 km.

i Perth to Kalgoorlie

ii Esperance to Carnarvon

iii Exmouth to Broome

iv Albany to Mt Magnet

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5 Using this map of Perth and its given scale, find the approximate driving distances between the following places.

a Hotel Grand Chancellor (Hotel 29) to Saville Parks Suite (Hotel 7)

b Quest on James (Hotel 31) to Museum (Blue Circle 8)

c Chifley on St George (Hotel 26) to Goodearth Hotel (Hotel 2)

d Swan Bell (Blue Circle 4) to Railway Stations (Blue Circle 15)

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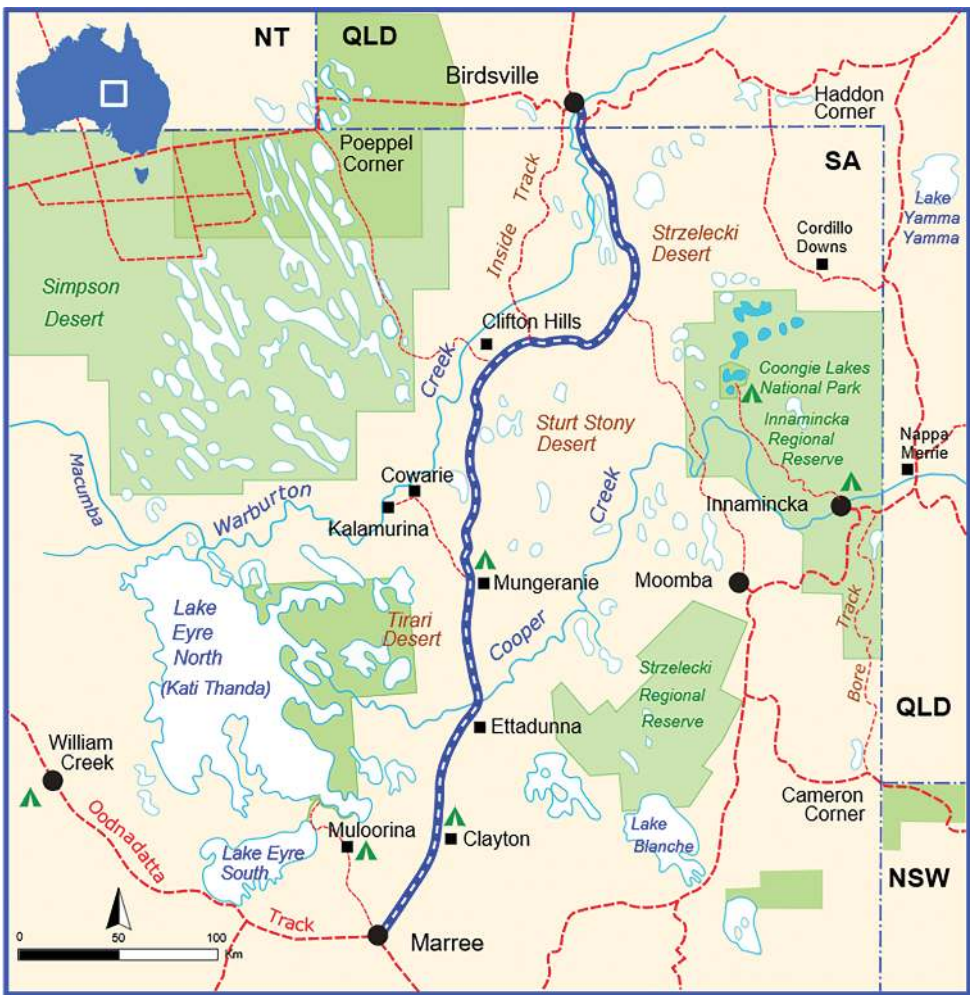
6 The map below shows the Birdsville Track, which was developed in the late 1800s as part of the stock route for walking cattle from North Queensland to a railway station at Marree, South Australia.



Use this map and its given scale to calculate the following direct distances.

a Birdsville to Innamincka

b Mungeranie to Ettadunna

c Marree to Cameron Corner (where the state borders of SA, QLD and NSW meet).

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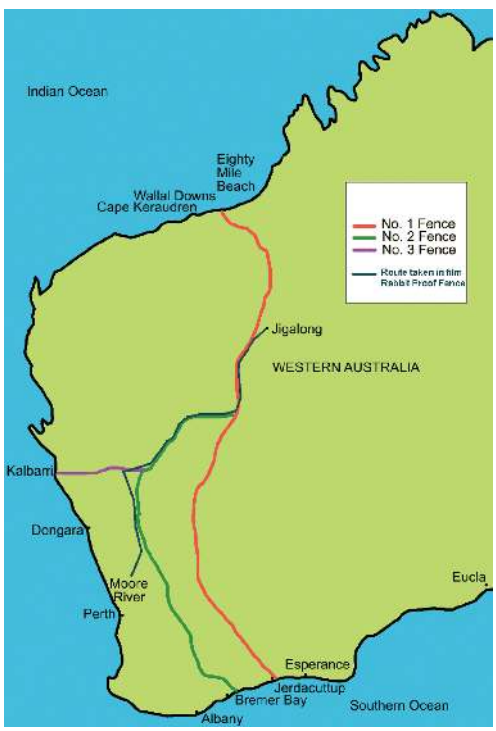
7 Archie is looking at this map of Australia. He believes that the largest distance between any two towns in Australia is either between Exmouth and Noosa Heads or Hobart and Darwin. Calculate both approximate distances and determine which is the larger distance and by how many km.

8 This map of Western Australia shows the three rabbit-proof fences that were constructed in the early 1900s.
The red line is the Number 1 fence that crosses WA from south to north.



a Given that the direct distance from Esperance to Perth is 600 km, calculate the map's scale in km/mm.

b Using a length of cotton or string to assist you, determine the length of Number 1 fence, to the nearest 10 km.
The Dingo fence measures 5614 km from Queensland to South Australia and is the longest unbroken fence in the world.

c By what percentage is the length of the Dingo fence greater than the length of the Number 1 rabbit-proof fence in WA?

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13B Investigating shortest distances through trial and error or systematic methods

LEARNING GOALS

- Use trial and error to determine the shortest route between two places
- Use a systematic method to determine the shortest route between several places

When is it essential to calculate the shortest route from maps?

The shortest distance between any network of locations will typically be the fastest and most economical route. Shortest-distance routes are calculated for road networks, rail networks, electricity networks, water supply networks, airline flight paths and communication networks.

Delivery companies calculate the shortest possible route for an economical and efficient service.

Even small local networks find the shortest distance for a route, such as for a parcel delivery route or for a community nurse to visit several of his patients in a country area or across various suburbs.

WHAT YOU NEED TO KNOW

- The shortest route is the route that has the smallest total distance.
- The shortest route can be calculated in two ways:
  - **Trial and error**: distances are calculated for each route and compared.
  - **Systematically**: the shortest distance is selected from each point on the route to the next.
- The scale on a map can be expressed in two ways:
  - As a **scale ratio**; for example, map distance : actual distance = 1 : 5000.
  - As a **line segment scale**; for example, 400 km.
- To calculate actual distances using a map's labelled line segment scale:
  1. Measure the length of the line segment and calculate the line segment scale.
  2. Measure the map distance with a ruler.
  3. Actual distance = map distance × scale

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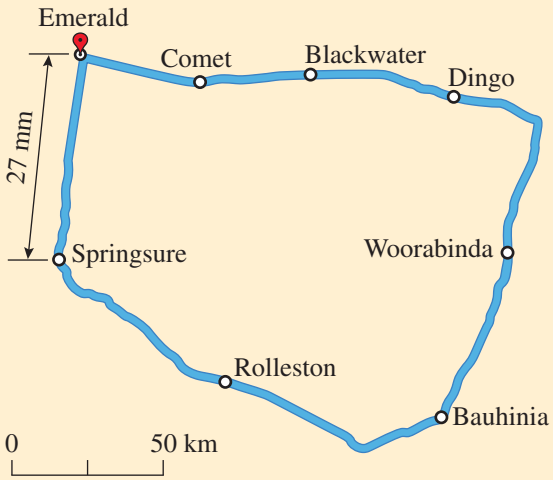
Example 3 Use trial and error to determine the shortest route between two towns

Determine the shortest road distance, in km, from Emerald to Woorabinda, via Rolleston or via Dingo.



WORKING	THINKING
Line segment = 50 km per 20 mm Scale = 2.5 km/mm	First calculate the scale ratio of the map. The line segment scale is 20 mm in length. For every mm on the map, the real-life distance is $\frac{50}{20}$ = 2.5 km/mm.
Emerald to Woorabinda via Rolleston: Map distance = 110 mm Actual distance = 110 mm × 2.5 km/mm = 275 km	The approximate map distance is 110 mm, so multiply by 2.5 km/mm.
Emerald to Woorabinda via Dingo: Map distance = 80 mm Actual distance = 80 mm × 2.5 km/mm = 200 km	The approximate map distance is 80 mm, so multiply by 2.5 km/mm.
The trip via Dingo is 75 km shorter.	Write your answer as a sentence.

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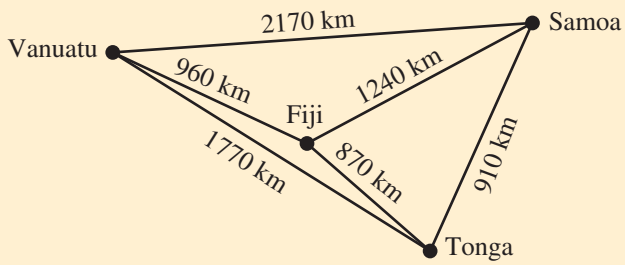
Example 4 Systematically calculate the shortest route between several places

Tevita Transport delivers cargo by plane throughout the Pacific Islands. Its home base is in Fiji.
Using the distances in the diagram, systematically calculate the shortest route, starting from Fiji and flying to Samoa, Vanuatu and Tonga, in any order, and returning to Fiji.



WORKING	THINKING
Shortest distances between islands: Fiji to Tonga (870 km) Tonga to Samoa (910 km) Samoa to Vanuatu (2170 km) Vanuatu to Fiji (960 km)	List the islands in pairs with each connecting distance, and calculate the overall shortest distance. The shortest flight from Fiji is 870 km to Tonga, and the shortest flight from Tonga is 910 km to Samoa. Vanuatu is the only island left to connect, being 2170 km from Samoa, then a 960-km return flight to Fiji.
Shortest distance: $= 870 + 910 + 2170 + 960$ $= 4910$ km	Add the distances to find the shortest distance overall.

Exercise 13B

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words and units.

a The scale on a map can be expressed in two ways:

i As a _____ _____, which is _____ length : _____ length.

ii As a line _____ scale.

b The steps to calculate actual distances using a map's line segment scale are:

1 Measure the line _____ in mm.

2 Calculate __/___ from the line segment scale.

3 Actual distance in km = ____ distance in units of __ × scale in __/__.

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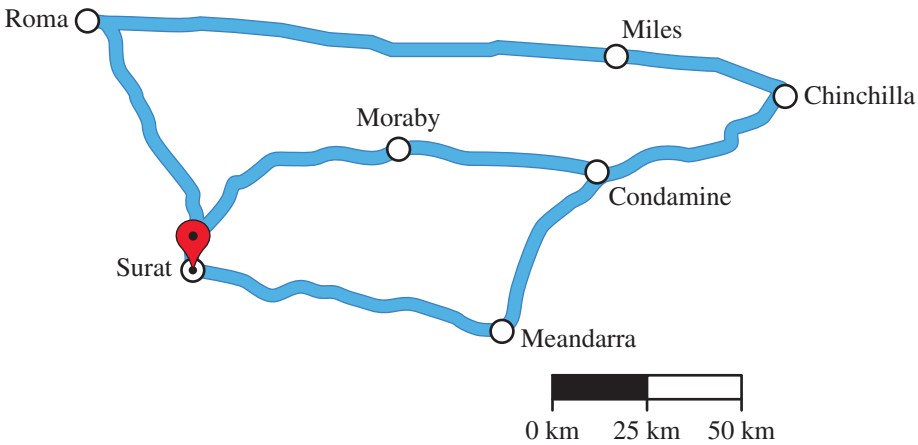
Example 3

2 Charlie is driving his cattle truck from Chinchilla to Surat and would like to know the shortest route to take. Use this map to answer the following questions.



a Apply your knowledge of scale to the map, to calculate and compare the approximate lengths of these three possible routes for Charlie's cattle truck. (Answer may vary depending on the accuracy of measurement.)
- **i** Chinchilla to Surat via Roma
- **ii** Chinchilla to Surat via Condamine and Moraby
- **iii** Chinchilla to Surat via Condamine and Meandarra

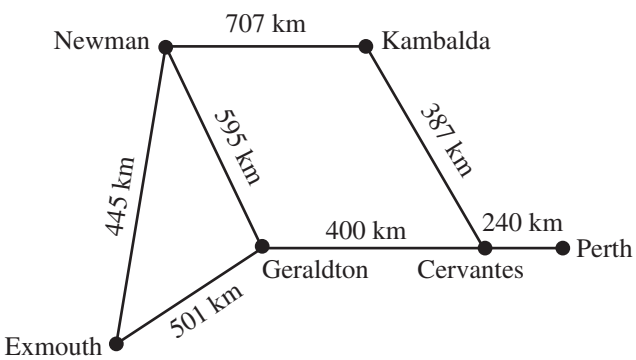
b Advise Charlie of the shortest route to take and its length.

APPLICATIONS

Example 4

3 Parcel Planes fly out from Perth and deliver lightweight freight to Western Australian towns. Today they must deliver supplies to Cervantes, Geraldton, Exmouth, Newman and Kambalda.
Given the sketch below, systematically calculate the shortest route for Parcel Planes to deliver supplies to all these towns, starting from and returning to Perth.

List the cities in pairs with each connecting distance and calculate the overall shortest distance.



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4 Daisy Delivery Service flies parcels from Sydney to three regional centres in New South Wales: Dubbo, Tamworth and Bourke.

a Use the map shown on the following page to find the total distance to travel from Sydney to Tamworth to Dubbo to Bourke and back to Sydney.

Hint First measure the line segment scale and calculate km/mm.

b Find the total distance to travel from Sydney to Tamworth to Bourke to Dubbo and back to Sydney.

c Compare the two routes and determine the shortest route for the Daisy Delivery Service.

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5 Angela and Deon are holidaying in Egypt. They decide to visit the following localities, starting and finishing their holiday in Cairo: Alexandria, Minya, Beni Suef, Dakhla Oasis, Sohag and Port Said.
Use a systematic method to determine the shortest route and its total distance for Angela and Deon to visit all the places on their list. Calculate direct distances between towns, to simplify the solution.

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6 Donna's Deliveries is a start-up air-freight delivery service in the planning stages of development. Its goal is to offer an economical and efficient delivery service to east coast Australian capital cities and Adelaide. The business must now analyse which capital city will be best suited as its home base.

a Set up an Excel spreadsheet to calculate the flight distances to each capital city that Donna's Deliveries will service, if the home base is based in:

i Melbourne **ii** Sydney

The Excel image on the next page shows the table for calculating one-way distances from Melbourne to each city. Enter the distances from the map in cells C2 to C6. Work out the scale in km/mm and enter it in cells D2 to D6 (the same number will go in each of these cells). Enter a formula into cell E2 that will calculate the actual distance, based on the values in cells C2 and D2, and fill down and enter the sum formula into cell E7.

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	A	B	C	D	E
1	**From**	**To**	**Map direct distance in mm**	**Scale in km/mm**	**Actual one-way direct distance in km**
2	Melbourne	Adelaide			
3	Melbourne	Hobart			
4	Melbourne	Sydney			
5	Melbourne	Canberra			
6	Melbourne	Brisbane			
7				**Total**	

b Research shows that 60% of all return flights to Sydney can be used for deliveries and 40% of all return flights to Melbourne can be used for deliveries, and hence, are profitable flight distances. A profitable flight distance means the company is making a profit by transporting goods when flying those kilometres. Calculate the total expected profitable flight distances, to the nearest 10 km, from a home base in Sydney and from a home base in Melbourne.

c Would you advise Donna's Deliveries to set up its home base in Sydney or Melbourne? Justify your conclusion and suggest other aspects that should be considered as well as profitable flight distances.

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13C Expressing speed as a rate and finding average speed

LEARNING GOALS

- Write speed as a rate and simplify to suitable units
- Calculate average speed from the distance travelled in a given time

Where is an average speed calculation essential?

To catch speeding drivers, each vehicle's average speed between two speed cameras is calculated.

To reduce the trauma and cost of road accident deaths and injuries, governments are installing more point-to-point speed cameras, which can monitor every vehicle's average speed over long distances. A surveyor uses maps to determine the shortest distance between the two cameras, so speeds are not overestimated. The average speed cameras use automatic number plate recognition technology and record the exact time each number plate is identified. The known distance between two cameras is divided by the travel time to give each car's average speed. If the average speed is above the speed limit, then a fine is automatically issued.

WHAT YOU NEED TO KNOW

- Speed is an example of a **rate** in units of distance/time. To express speed in the simplest way, we find the distance travelled per 1 unit of time. This is done by dividing both parts of the rate by the time taken.
  - For example:

$$\div 5 \left(\begin{array}{c} 400 \text{ km}/5 \text{ h} \\ = 80 \text{ km/h} \end{array}\right) \div 5 \qquad \div 40 \left(\begin{array}{c} 100 \text{ m}/40 \text{ s} \\ = 2.5 \text{ m/s} \end{array}\right) \div 40$$

- The most common units of speed are **metres per second** (m/s) and **kilometres per hour** (km/h).
- When travel is between two known locations, we can determine the distance covered. One way that locations can be identified is by their distance from a nearby town.
  - For example, for travel from 50 km south of Ayr to 400 km south of Ayr, the distance travelled $= 400 - 50 = 350$ km.

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- In speed calculations, time taken has to be in one time unit. We cannot use, for example, 12:31 or 3 h 5 min.
- During a journey, speed varies, even for a vehicle with cruise control. The **average speed** is a measure of overall speed, expressed as the distance travelled per 1 unit of the time taken.
- To calculate a journey's **average speed**, determine the total distance travelled and the total time taken for the whole journey. Then use the same method for calculating speed given at the top of this box on the previous page. (The speeds given there are actually average speeds). Note that you cannot calculate average speed by adding speeds at certain times and calculating their average.
- To calculate speed in km/h when the time is not a whole number of hours, write the time in hours as a decimal number.
  First convert seconds to minutes, then convert the total minutes to hours.
  - For example: $2:23:10.5 = 2\text{ h }23\text{ min }10.5\text{ s}$

$$= 2\text{ h} + \left(23 + \frac{10.5}{60}\right)\text{ min}$$

seconds ÷ 60 → minutes

$$= 2\text{ h} + 23.175\text{ min}$$

minutes ÷ 60 → hours

$$= 2\text{h} + \frac{23.175}{60}\text{ h}$$

$$= 2.38625\text{ h}$$

$$= 2.39\text{ h (rounded to 2 dp)}$$

- Converting between speed units: m/s × 3.6 = km/h, and km/h ÷ 3.6 = m/s.

Example 5 Writing average speed as a rate in suitable units

For these events, write the average speed as a rate of distance/time and simplify.

a Tara's best time for running 200 m is 40 s. Calculate her average speed, in m/s.

b Peter takes 8 h and 24 min to drive 588 km in a car rally on gravel roads through hilly country. Calculate his average speed, in km/h.

WORKING	THINKING
a Average speed	Write the average speed as a rate of distance/time.
$= 200\text{ m}/40\text{ s}$	Divide both parts by 40 to find the distance per 1 s.
$= 5\text{ m/s}$	
5 m/s	Write the average speed in units of m/s.

... *Continued*

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WORKING	THINKING
b Time $= 8 + \frac{24}{60} = 8.4$ h	First find the time in hours, then work out 24 min as a decimal; that is, $24 \div 60 = 0.4$ h.
Average speed $= 588\text{km}/8.4\text{ h}$ $= 70\text{ km/h}$	Write the average speed as a rate of distance/time. Divide both parts by 8.4 to find the distance per 1 h. Write the average speed in units of km/h.

Example 6 Calculating average speed from distance travelled in a given time

For the following journeys, determine the distance travelled and the average speed.

a Tao takes 3 h to drive from 50 km south of Townsville to 320 km south of Townsville.

b Hannah takes 2 h and 24 min to fly from 100 km west of Longreach to 350 km east of Longreach.

WORKING	THINKING
a Distance $= 320\text{ km} - 50\text{ km}$ $= 270\text{ km}$ Average speed $= 270\text{ km}/3\text{ h}$ $= 90\text{ km/h}$	Both locations are south of Townsville, so subtract to find the distance travelled. Write the average speed as distance/time. Find the distance per 1 h by dividing both parts by 3. Write the answer with the units km/h.
b Distance $= 100\text{ km} + 350\text{ km}$ $= 450\text{ km}$ Average speed $= 450\text{ km}/2.4\text{ h}$ $= 187.5\text{ km/h}$	Add the distances, as the locations are west and east of Longreach. Convert hours and minutes to decimal: 2 h 24 min $= 2 + 24/60\text{ h} = 2.4\text{ h}$ Divide both parts of the distance/time rate by 2.4 to give km/h.

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Example 7 Calculating average speed in a real-world context

At the 2011 Cycling Australia Track National Championships, Jack Bobridge set a world record for the 4000 m individual pursuit in a time of 4 min 10.534 s. Calculate Jack's average speed, in m/s and km/h. Round your answers to two decimal places.

WORKING	THINKING
Time $= 4 \times 60 + 10.534$	Find the time in seconds when calculating speed in units of m/s.
$= 250.53$ s	Keep two decimal places.
Average speed $= 4000$ m/250.53 s	Write the average speed as a rate of distance/time.
$= 15.97$ m/s	Divide both parts by 250.53 to give units of m/s.
$= 15.97 \times 3.6$ km/h	Multiply by 3.6 to convert from m/s to km/h.
$= 57.49$ km /h	Round the answer to two decimal places.

Exercise 13C

FUNDAMENTALS

1 Demonstrate your understanding of terms by writing down the missing words, values and units in the following statements.

- **a** Units of speed are metres per second (_/_) and kilometres per hour (__/_).
- **b** Speed is an example of a _____ in units of distance/_____.
- **c** When simplifying a speed rate, we find the distance moved per _____ of the time taken.
- **d** To calculate a journey's average speed:
  - **1** Determine the _____ travelled.
  - **2** Write the _____/____ rate and divide both parts of the rate by the _____ taken.

2 For the following movements, select the most suitable units of speed from:

m/year km/h m/s m/min m/h

- **a** walking down the street
- **b** driving a racing car
- **c** running in a 200-m race
- **d** a turtle walking
- **e** a moving glacier
- **f** flying between capital cities

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Example 5

3 Calculate the average speed for each of the following, displaying the appropriate units to one decimal place.

a The world record for the 100-m sprint is 9.58 s, held by Usain Bolt in 2009.

b A non-stop flight from Auckland, New Zealand, to Doha, Qatar, travels 14 534 km in 17 h.

c The starfish is one of the world's slowest creatures, travelling 32 m in an hour.

d Paula Radcliffe (Great Britain) ran the 42-km London Marathon in just over 2 h and 15 min.

e Eamon Sullivan (Australia) achieved a world record for completing the 100-m freestyle in 47 s.

f Jack Bobridge (Australia) completed the 4000 m cycling individual pursuit in 4.17 min.

Hint: seconds ÷ 60 → minutes
minutes ÷ 60 → hours

Example 6

4 Calculate the average speed, in km/h, for the following journeys. Round each answer to one decimal place.

a A car starts at a location 100 km north of Blackall and drives to a location 340 km north of Blackall in 2.5 h.

b A cyclist rides from 50 km west of Gympie to 178 km west of Gympie in 4 h.

c A plane takes off 150 km west of Mt Isa and flies to a location 325 km east of Mt Isa in 2.5 h.

d A group of backpackers on Fraser Island drive from the Lake Wabby car park, which is 6 km north of Eurong, to Hook Point, which is 32 km south of Eurong, taking 54 min.

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APPLICATIONS

5 Calculate the average speed, in km/h, for these journeys. Round each answer to one decimal place where needed.

Will Power's car, 2015 Indianapolis 500.

- **a** One of the longest non-stop passenger flights in the world is from Singapore to New York, USA. It is 15 322 km and the flight time is approximately 19 h.
- **b** The Tilt Train leaves Rockhampton at 7:10 a.m. daily and arrives at Roma Street Station, Brisbane, at 2:50 p.m. The distance travelled is 639 km.
- **c** In the 2022 Berlin Marathon, Eliud Kipchoge (of Kenya) set the world record for the fastest man by running the 42.195 km in 2 h 01 min 09 s.
- **d** In 2018, Will Power, from Toowoomba, became the first Australian to win the Indianapolis 500. His time was 2 h 59 min 42.6365 s for the 500-mile race.
- **e** A Ford Falcon drove the 67 laps (each lap is 2.423 km) of the Winton Super Sprint in a total time of 1 h 36 min 36 s.

6 Anna Beck won the elite women's section of the 2018 'Easter in the Alice' mountain bike competition. The race takes place in the rocky landscape of West MacDonnell Ranges, Central Australia, Northern Territory. Anna's stage distances and times were:

Stage	Distance (km)	Time (h:min:s)
1	83	04:07:34
2	24	01:28:16
3	34	01:44:14


West MacDonnell Ranges, NT

- **a** Calculate Anna's average speed for stage 1, in km/h.
- **b** Calculate the total distance for the race; that is, stages 1, 2 and 3.
- **c** Calculate the total time that Anna took to complete the three race stages.
- **d** Calculate Anna's average speed over the three stages, in km/h.

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Example 7

7 Hamish has purchased a pet hermit crab and notices that it moved 20 cm in 1 h. Determine the units of speed Hamish could use to describe the speed of his hermit crab to his friends.

8 Grace is staying in Green Town, which is 50 km east of Perth, and must drive to Tea Tree, which is 128 km east of Perth, in only 40 min. Calculate the average speed she must drive her car.

9 Kayleigh needs to travel 360 km from Billabong to the Geraldton Airport in four hours. Calculate the average speed that she will need to travel to arrive on time.

10 Bruce is 900 m from the bus stop and has only 5 min until the bus leaves. Calculate the speed, in m/s, he must run to catch the bus.

11 Perth trains travel 13 km from Perth Station to Cannington Station, and 27 km from Perth Station to Ellenbrook Station. If the train trip from Cannington Station to Ellenbrook Station takes a quarter of an hour, calculate the train's average speed, in km/h, for this section.

12 Angus lives 90 km west of Coolgardie, in outback Western Australia. He has an 11:30 a.m. specialist appointment in Merridin, which is 248 km west of Coolgardie. If Angus starts his journey at 10:06 a.m., does he risk a speeding fine if he drives fast enough to arrive at his appointment on time?

13 Ruby and Jade are both in a 400-m race. However, Ruby starts 80 m ahead of the start line because she is younger. Both girls finish together after exactly 60 s. How much faster did Jade run than Ruby, in m/s? Give your answer to one decimal place.

14 If Anton drives at 100 km/h for 2 h and at 80 km/h for half an hour, calculate his average speed, to one decimal place.

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13D Using the speed formula to calculate speed, distance and time

LEARNING GOALS

- Know the 'speed triangle' and use it to write the formulas for speed, distance and time
- Calculate speed, distance and time by substitution into the relevant formulas
- Apply the speed, distance and time formulas to solve real-life situations

When are calculations using speed, distance and time essential?

All journey scheduling is based on calculations using speed, distance and time. Public transport schedules are the basis for digital journey planners. Google Maps calculates route times using distance and the speed of driving or cycling or walking.

Airline companies use average speed and distance to calculate flight times. Arrival and departure schedules are essential knowledge for travellers to plan a journey with sufficient time between interconnecting flights, trains and buses.

Engineers schedule the transport of millions of tonnes of Australian coal and metal ores across vast distances from the mine to ships. Supply schedules are precisely determined with maths programming, using many variables, including speed, distance and time.

The average speed (85 km/h) of a road train and its capacity (120 tonnes) are used to schedule the delivery of mined ore to each ship when in port.

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WHAT YOU NEED TO KNOW

- The speed formula:

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \qquad s = \frac{d}{t}$$

  - For example:

$s = \frac{d}{t}$

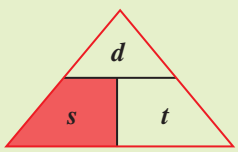
$s = \frac{300\,\text{km}}{3\,\text{h}}$

$s = 100\,\text{km/h}$

$\text{Speed} = \text{distance/time}$
$= 300\,\text{km}/3\,\text{h}$
$= 100\,\text{km/h}$

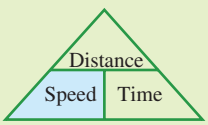
  The speed formula can be rearranged to calculate times and distances, using speeds.

- The speed triangle can be used to help remember the speed formula and its rearrangements that make distance or time the subject.



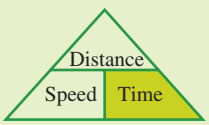
- The three formulas for speed, distance and time are:

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \qquad s = \frac{d}{t}$$



$$\text{Distance} = \text{speed} \times \text{time} \qquad d = s \times t$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}} \qquad t = \frac{d}{s}$$



Example 8 Calculating speed, distance and time by substituting into the relevant formulas

For the following journeys, first write the appropriate formula and then substitute the given values into the formula to calculate the unknown value. Round each answer to one decimal place and include appropriate units.

a Calculate the average speed when distance = 270 km and time = 3.75 h.

b Calculate the time taken when the average speed is 5.2 m/s and distance is 200 m.

c Calculate the distance travelled if average speed = 77 km/h and time = 2.5 h.

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WORKING	THINKING
a $s = \frac{d}{t}$	Write the speed formula: $s = \frac{d}{t}$.
$s = \frac{270\text{ km}}{3.75\text{ h}}$	Substitute the given values with their units.
$s = 72\text{ km/h}$	Calculate the result and round the answer to one decimal place. Include the units of km/h.
b $t = \frac{d}{s}$	Write the formula for time: $t = \frac{d}{s}$.
$t = \frac{200\text{ m}}{5.2\text{ m/s}}$	Substitute the given values with their units.
$t = 38.5\text{ s}$	Calculate the result and round the answer to one decimal place. Include the units of seconds.
c $d = s \times t$	Write the formula for distance: $d = s \times t$.
$d = 77\text{ km/h} \times 2.5\text{ h}$	Substitute the given values with their units.
$d = 192.5\text{ km}$	Calculate the result and round the answer to one decimal place. Include the units of km.

Example 9 Applying the distance formula to solve a real-life situation

Lee used cruise control and travelled at a constant speed of 110 km/h along a highway. If he started at 1:00 p.m., how far had he travelled by 1:55 p.m.?

WORKING	THINKING
Speed = 110 km/h	Identify and write down the values given in the question.
Time = 0.75 h	1:55 p.m. − 1:10 p.m. = 45 min, which is 45 ÷ 60 = 0.75 h when converted to decimal time.
Distance = speed × time	Identify what it is you must find and write the required formula.
= 110 km/h × 0.75 h = 82.5 km	Substitute the given values and calculate the result.
Lee travelled 82.5 km.	Write your answer in words.

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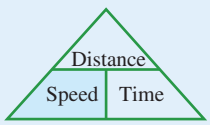
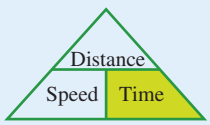
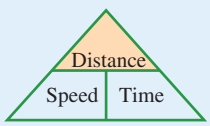
Exercise 13D

FUNDAMENTALS

1 Demonstrate your understanding of terms by filling in the missing words and letters in the following formulas.

Speed triangles	Formulas using words	Formulas using pronumerals
	Speed $=$ $\frac{\square}{\square}$	$s = \frac{\square}{\square}$
	Time $=$ $\frac{\square}{\square}$	$t = \frac{\square}{\square}$
	Distance $=$ ____ $\times$ ____	$d =$ _ $\times$ _

Example 8

2 For each of the following journeys, first write the appropriate formula and then substitute the given values to calculate the unknown value. Round each answer to one decimal place and include appropriate units.

a Find the average speed when:
- **i** distance $= 20$ m, time $= 5$ s
- **ii** distance $= 320$ km, time $= 4$ h
- **iii** distance $= 400$ m, time $= 55$ s
- **iv** distance $= 840$ km, time $= 9.5$ h

b Find the time when:
- **i** speed $= 28$ m/s, distance $= 1500$ m
- **ii** speed $= 100$ km/h, distance $= 1250$ km
- **iii** speed $= 88$ km/h, distance $= 620$ km
- **iv** speed $= 42$ m/s, distance $= 1240$ m

c Find the distance when:
- **i** speed $= 18$ m/s, time $= 4$ s
- **ii** speed $= 60$ km/h, time $= 3$ h
- **iii** speed $= 80$ km/h, time $= 6.5$ h
- **iv** speed $= 14$ m/s, time $= 12.5$ s

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3 Eva cycles at an average speed of 12 km/h and plans to ride along a 20-km mountain bike trail in Pemberton. Use the time formula to find the time, in hours and minutes, that Eva's cycle trip is expected to take.

4 Henry delivers parcels and has calculated that his average speed is 55 km/h when driving around the suburbs. Use the distance formula to calculate how far Henry has driven after 4.25 h of parcel deliveries.

APPLICATIONS

Example 9

5 Tom Speeder drove the 1000-km Bathurst race in 12 h. Calculate his overall race speed.

6 The World Record for the 4000-m individual pursuit is 250 s. Calculate the overall race speed.

7 The school record for the 800-m race is 4 min and 20 s. If Terry runs the race with a speed of 3.2 m/s, determine if he will break the record and calculate by how much.

8 Jasmine notices a snail in her front yard one morning. If the snail moves at 47 m/h, calculate how long it will take for the snail to reach Jasmine's vegetable garden, which is 30 m away.

9 Nick went for a drive around the city at a speed of 60 km/h. If he left home at 2 p.m., calculate how far he had travelled by 4:30 p.m.

10 Alana is driving from Esperance to Perth, and according to her internet search, it is 872.2 km and will take 9.62 h. Calculate the speed of the trip.

11 Craig flew 1020 km/h from the Gold Coast to Mackay at a speed of 340 km/h. If he left at 2:30 p.m., at what time did he arrive at Mackay?

12 Ezekiel flew from the Albany Airport to Perth at an average speed of 586 km/h. His plane left Albany Airport at 2:45 p.m. and landed at 5:15 p.m. at Perth Airport. Use the distance formula to calculate the length of this flight.

Hint Start each question by drawing the speed triangle.

13 Leigh drove 623 km from Mt Barker to Perth, averaging a speed of 89 km/h. If she left at 6 a.m., use the time formula to help calculate the time Leigh arrived in Perth.

Hint Write the relevant formula as the first line of each solution.

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14 Boyce ran the 200-m race in 28 s, and he ran the 800-m race in 118 s. Use the speed formula to determine in which race Boyce ran fastest, and by how much. Give your answer to one decimal place.

15 Melody went for a run at an average speed of 12 km/h for 45 min due west of her house. Use the distance formula to calculate how far Melody has run from her home.

16 A small plane departs from Perth Airport at 3:10 p.m. and arrives at Pearce Airport at 4:25 p.m. If the plane flies at an average speed of 388 km/h, determine the flight distance for this plane's journey.

17 In a handicap race in which Jacinta is given a 'head start', Jacinta and Wanita start at the same time. Jacinta runs at 8 m/s and Wanita runs at 10 m/s. If they finish exactly together after 20 s, calculate the length of Jacinta's 'head start'.

Hint A 'head start' is starting at a distance ahead of the start line.

18 Gillian jumps from a plane at 6000 m altitude and freefalls at an average speed of 190 km/h until she opens her parachute at 1000 m above ground. If Gillian falls at an average speed of 28 km/h with her parachute open, calculate the total time of her descent, in minutes, to one decimal place.

19 The highway between Northam and Perth has a 60-km section with a 110 km/h speed limit and a 45-km section with a 100 km/h speed limit.

a If a car is driven exactly at the speed limit for each relevant distance, calculate its average speed for the trip from Northam to Perth.

b Calculate how much longer the trip will take if night roadworks require a speed limit of 40 km/h on 12 km of the 100 km/h section and a speed limit of 60 km/h on 18 km of the 100 km/h section.

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13E Calculating journey time and costs using maps

LEARNING GOALS

- Calculate the time for a journey, using map distances and a given speed
- Calculate the cost for a journey, using map distances and a given speed
- Calculate journey time and costs, using map distances in a real-world situation

Who would find it essential to use a map for estimating the time and cost for a journey?

Civil engineers and surveyors use detailed maps for accurate distance calculations when estimating construction times and costs for new routes, such as new railway lines, roads, power lines, and water, oil and gas pipelines. Such infrastructure development is essential for Australia's economy.

These small planes are at Birdsville for the annual horse races. Flight costs are calculated using distance, speed and the operating cost per hour.

Both commercial and private plane operators, such as passenger and freight services, farmers, flying doctors and miners, all calculate flight distances, times and costs to ensure that their business stays profitable.

Furniture removal companies also make distance, time and cost estimations to ensure the prices charged cover their operating costs.

WHAT YOU NEED TO KNOW

- To calculate **actual distances** using a map's line segment scale:
  1. Measure the **line segment** scale, in mm.
  2. Calculate **the scale ratio** in km/mm (to one decimal place) from the line segment scale given.
  3. Measure the **map distance**, in mm, and convert this map distance to the actual distance by:

$$\text{Actual distance in km} = \text{map distance in mm} \times \text{scale in km/mm}$$

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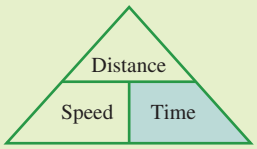
- Calculate time by dividing the distance by the speed. The distance units must be the same.
  - For example: When $d = 240$ km and $s = 60$ km/h, then $t = 240 \div 60 = 4$ h.

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$t = \frac{d}{s}$$



- The cost of a journey is the cost per hour × the number of hours.
  - For example: A plane's operating cost is \$450/h and a flight takes 2.4 h.
    Cost of the flight = \$450 × 2.4 = \$1080
- A complex problem is a series of simple problems.

Refer to this map of Western Australia for **Examples 10**, **11** and **12** and Questions **2**, **5–9**, **11** and **12**.

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Example 10 Calculating the time for a journey, using map distances and a given speed

A small passenger plane flies at 320 km/h directly from Albany to Perth.
Use the Western Australia map given to complete the following.

a Measure the 250-km line segment scale, in mm, and calculate the scale in km/mm, to one decimal place.

b Measure the map direct distance, in mm, from Albany to Perth and then convert this map distance to the actual distance.

c Estimate the time that a small plane will take to fly from Albany to Perth.

WORKING	THINKING
a Line segment = 250 km per 20 mm Scale = 12.5 km/mm	Measure the line segment, in mm. Calculate the scale in km/mm, rounding the answer to one decimal place.
b Map distance = 35 mm Actual distance = 35 mm × 12.5 km/mm = 437.5 km	Measure the direct map distance between the two towns, in mm. Actual distance = map distance in mm × scale in km/mm
c $t = \frac{d}{s}$ $= \frac{437.5\text{ km}}{320\text{ km/h}}$ = 1.4 h	Time = $\frac{\text{distance}}{\text{speed}}$ Write the formula for time and substitute the known values. Write the time estimated in hours.

Example 11 Calculating the cost of a journey, using map distances and a given speed

A chartered plane costs approximately $2500 per hour of flight time to cover its operating costs. For a plane that flies at 375 km/h, use the Western Australia map given previously to estimate the cost for a chartered flight between Kalgoorlie and Perth.

... Continued

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WORKING	THINKING
Scale = 12.5 km/mm	Write the scale, which we already calculated in **Example 10.**
Map distance = 50 mm	Measure the direct map distance between the two towns.
Actual distance = 50 mm × 12.5 km/mm = 625 km	Actual distance = map distance in mm × scale in km/mm
$t = \frac{d}{s}$ $= \frac{625\text{ km}}{375\text{ km/h}}$ $= 1.67\text{ h}$	Time = $\frac{\text{distance}}{\text{speed}}$ Write the formula for time and substitute the known values to solve.
Cost = 1.7 × $2500 = $4250	Cost = time × cost per hour

Example 12 Calculating journey time and costs, using map distances in a real-world context

Dell owns a private plane in Esperance and plans to fly to Geraldton for a family wedding that starts at 4 p.m. The plane costs $280/h to fly and has a speed of 300 km/h. Determine at what time Dell should take off from Esperance to arrive in Geraldton with an hour to spare before the wedding starts. Estimate the cost of the total trip (there and back).

WORKING	THINKING
Scale = 12.5 km/mm	Write the scale, which we already calculated in **Example 10**.
Map distance = 82 mm	Measure the direct map distance between the two places.
Actual distance = 82 mm × 12.5 km/mm = 1025 km	Actual distance = map distance in mm × scale in km/mm
$t = \frac{d}{s}$ $= \frac{1025\text{ km}}{300\text{ km/h}}$ $= 3.4\text{ h} = 3\text{ h }24\text{ min}$	Time = $\frac{\text{distance}}{\text{speed}}$ Write the formula for time and substitute the known values to solve.
Dell must leave by 11:36 a.m.	3 p.m. is an hour earlier than the wedding start time of 4 p.m. So Dell must take off 3 h and 24 min before 3 p.m.
Cost = $280 × 6.8 h = $1904	The total trip will take 6.8 hours of flying time. Cost = time × cost per hour

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Exercise 13E

FUNDAMENTALS

1 Demonstrate your understanding of terms by filling in the missing words and units in the following statements.

a To calculate actual distances using a map's line segment scale:

i Measure the ____ segment scale, in ___.

ii Calculate the scale in km/___ from the line segment scale.

iii The _____ distance in km = ____ distance in __ × scale in __/__.

b Time = $\frac{\square}{\square}$; $t = \frac{\square}{\square}$

c The cost of a journey is the cost per _____ × the number of _____.

Example 10

2 Rob and Mary fly their small plane from Albany to Geraldton for the annual races. Their plane flies at 320 km/h . Use the Western Australia map given previously to complete the following.

a Measure the 250-km line segment scale, in mm, and calculate the scale, in km/mm, to one decimal place.

b Measure the map direct distance, in mm, from Albany to Geraldton and then convert this map distance to the actual distance.

c Estimate the time it will take Rob and Mary to fly from Albany to Geraldton.

3 Using the following map of Australia, determine the direct distance and travel time between each of the two towns listed when given the speed of travel between each place.

a Brisbane – Townsville, 280 km/h

b Alice Springs – Uluru, 85 km/h

c Noosa Heads – Brisbane, 100 km/h

d Hobart – Sydney, 650 km/h

e Perth – Cairns, 620 km/h

f Melbourne – Darwin, 560 km/h

Hint First calculate the map scale in km/mm.

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Example 11

4 A private jet costs approximately $4000 per hour of flight time to hire and flies at an average speed of 480 km/h. Use the map of Australia above to determine the approximate costs for chartering this jet for each of the following journeys.

- **a** Airlie Beach to Canberra
- **b** Broome to Adelaide
- **c** Bundaberg to Snowy Mountains
- **d** Albany to Newcastle
- **e** Melbourne to Sydney
- **f** Brisbane to Sydney

APPLICATIONS

5 Hindley Cruises runs an all-inclusive boat cruise from Albany to Esperance. Complete the following for a one-way trip on this cruise.

- **a** Using the map of Western Australia on page 618, determine the direct distance, in km, from Albany to Esperance.
- **b** If the boat's operating costs are $584/km, determine the total cost to run this cruise.
- **c** Assuming there are 500 passengers, calculate the minimum price per passenger to cover operating costs.

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Example 12

6 Jay promptly leaves work at Kalgoorlie at 11 a.m. to visit his mother, who is in the Perth Hospital. Visiting hours are from 5 p.m. until 8 p.m. Using the map of Western Australia on page 618, determine approximately how long Jay will have visiting his mother. Assume he drives at an average speed of 90 km/h.

7 The Adams family are planning a weekend away at Margaret River. They will leave their home in Geraldton by Friday 8 a.m. and will travel to Margaret River via Perth. Due to Friday afternoon traffic, their average speed is estimated at 80 km/h. Using the map of Western Australia on page 618, calculate when the Adams family can expect to arrive at Margaret River.

8 Use the map of Western Australia on page 618 to complete the following.

Delia is a florist in Perth and needs to make a last-minute delivery to Bunbury by 2 p.m. for a wedding. If she leaves at 11:30 a.m. and travels at an approximate speed of 100 km/h, will she make the delivery in time? If yes, how much time will Delia have to spare?

Hint First, find the approximate distance between Toowoomba and Dalby.

9 Aiden's car has broken down in Albany. Two towing companies arrive: one that will tow the car to Perth and one that will tow the car to Esperance. Perth Towing costs $9.50 per km and Esperance Towing costs $10 per km. Use the map of Western Australia on page 618 to determine the approximate distances from Albany to each of the towns, and hence, determine which company has the cheaper offer and by how much.

10 Lukas owns a vineyard in the Barossa Valley, South Australia, and is hiring a small plane to fly to Toowoomba, to attend a winemaking conference. The plane's average speed is 280 km/h and it costs $340 per hour of flight time. Using the map of Australia given in Question **3**, calculate how much it will cost for Lukas to fly from the Barossa Valley to Toowoomba and back again.

11 Nigel and Naomi are cotton farmers in Geraldton. They have booked tickets to a jazz concert in Bunbury for 7 p.m. Saturday. Nigel is a pilot and the couple have their own plane, which flies at 270 km/h.

Nigel and Naomi plan to fly from Geraldton Airport to Perth Airport and then drive a hire car, driving at an average speed of 70 km/h.

Using the map of Western Australia on page 618, calculate the latest time that Nigel and Naomi will need to take-off from Geraldton Airport in order to arrive at the jazz concert half an hour before it starts.

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12 Trevor is living in Adelaide and has a meeting in Margaret River, WA. He has chartered a plane from Adelaide that flies at an average speed of 600 km/h, leaving at 8 a.m. in the morning. He has also hired a car at Perth airport to drive to Margaret River and will average 70 km/h for this part of his journey.

Using the maps of Australia given in Questions **3** and Western Australia on page 618, calculate the time Trevor should schedule his meeting, given that Adelaide is in a time zone that is 1 hour 30 minutes ahead of Perth time.

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13F Interpreting distance versus time graphs

LEARNING GOALS

- Identify distances from distance–time graphs when given the time
- Identify speeds from distance–time graphs
- Calculate speeds from distance–time graphs
- Describe journeys from distance–time graphs

Why do people find it essential to use distance–time graphs?

Distance–time graphs are a pictorial representation of speed. By recording a person's distance over time we are also recording their speed, though not directly, it has to be calculated. On the distance–time graph, speed is represented by the slope of the line.

Distance–time graphs are also used to show the motion of vehicles and objects. They are used in science to analyse motion.

In this section, we deal with linear graphs that have line segments which are straight. This means the speeds are constant or average speeds for the different segments of the journey or route. Speed changing over time—acceleration (speeding up) and deceleration (slowing down)—are not are featured in this section. They would result in curved or irregular graph lines.

The distance for a distance–time graph is measured from the starting point, along the route (not the straight line distance to the start). The distance could be measured at regular time intervals or only when the speed changes. Distance–time graphs do not provide any information about the shape of the route—they are not maps. The only direction information provided is whether the motion is away from or towards the starting point.

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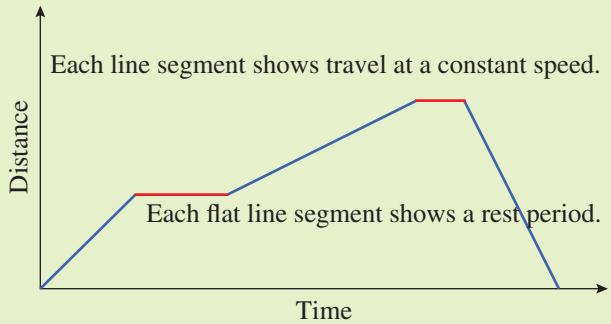
WHAT YOU NEED TO KNOW

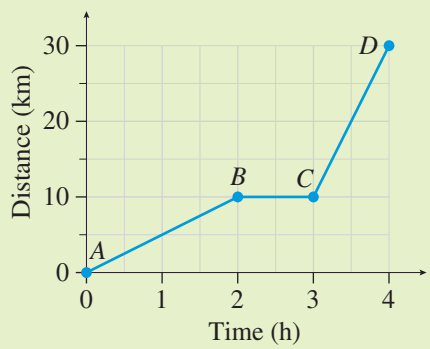
- A **distance–time graph** has distance on the vertical axis and time on the horizontal axis.
- Each plotted point has **coordinates** (time, distance).
- Each **straight-line segment** shows a section of a journey travelled at a **constant speed**.



- A horizontal line segment has no distance change; hence, the speed is zero (i.e. the object or person is at rest).
- A line sloping up to the right is the journey away from the starting point (outbound). A line sloping down to the right is the journey back to the starting point (return).
- **Steeper lines** show faster speeds than flatter lines.
- For each line segment:

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

  - For example: A bicycle journey is shown in the following distance–time graph.



Line segment AB:

$$\text{Speed} = \frac{10\,\text{km}}{2\,\text{h}} = 5\,\text{km/h}$$

Line segment BC:

$$\text{Speed} = 0$$

Line segment CD:

$$\text{Speed} = \frac{20\,\text{km}}{1\,\text{h}} = 20\,\text{km/h}$$

- When drawing a distance–time graph:
  1. Identify the total distance and create the vertical axis scale.
  2. Identify the total time and create the horizontal axis scale.
  3. Plot the given points and join them with a ruler to create line segments.

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Example 13 Interpreting a distance–time graph

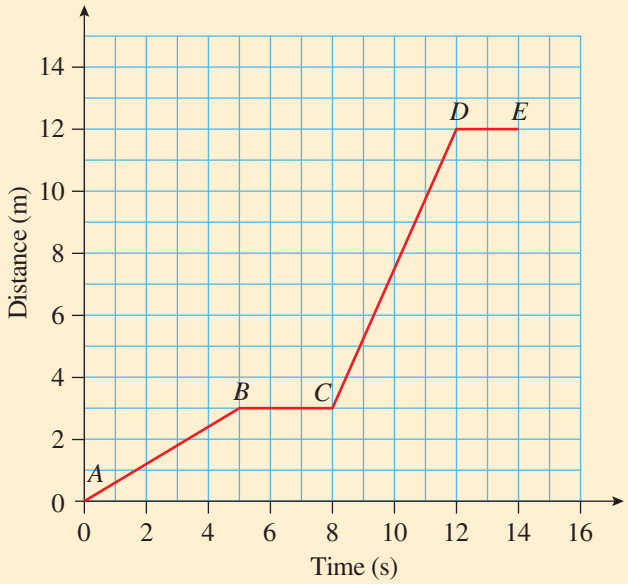
Refer to the graph shown at right and complete the following.

a What is the distance travelled in the first 5 s?

b What is the speed of the journey from A to B?

c Which part of the journey has the longest rest period?

d Which part of the journey has the fastest speed?



WORKING	THINKING
a Distance = 3 m	Draw a vertical line from the axis 'Time' at 5 s up to the graph, and then a horizontal line across to the distance axis.
b Speed = distance ÷ time = 3 m ÷ 5 s = 0.6 m/s	Use the formula to find the speed from A to B.
c From B to C	Identify the longest horizontal line.
d From C to D	Identify the steepest line segment.

Example 14 Interpret the distance–time graph in a real-world situation

The first 14 seconds of Alice's movements during a netball match are shown in the graph in **Example 13**. Describe, in words, Alice's movements for the first 14 seconds of her game.

WORKING	THINKING
Alice ran her slowest for the first 5 s and then rested for 3 s before running her fastest for the next 4 s and then rested for the last 2 s.	Using the slope of the line, identify when Alice was running fast, slow and not running at all. Use that information to describe her run in words.

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Exercise 13F

FUNDAMENTALS

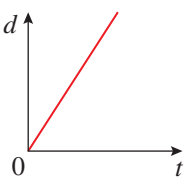
1 Demonstrate your understanding of terms by writing down the missing words and units in the following statements.

- **a** A distance–time graph has distance on the _____ axis and time on the _____ axis.
- **b** Each plotted point has coordinates (_____, _____).
- **c** Each _____ line segment shows a journey section travelled at a _____ speed.
- **d** For each line segment, speed = _____ ÷ _____.
- **e** A horizontal line segment shows the speed is _____.
- **f** Steeper lines show _____ speeds than flatter lines.
- **g** When drawing a distance–time graph:
  - **1** Identify the total _____ and create the vertical axis scale.
  - **2** Identify the total _____ and create the horizontal axis scale.
  - **3** _____ the given points and join them with a ruler to create line _____.

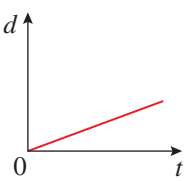
Example 13

2 Shown are distance–time graphs, where the distance is d and the time is t. Match the descriptions **a** to **c** with the graphs **A** to **C**.

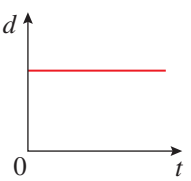
A



B



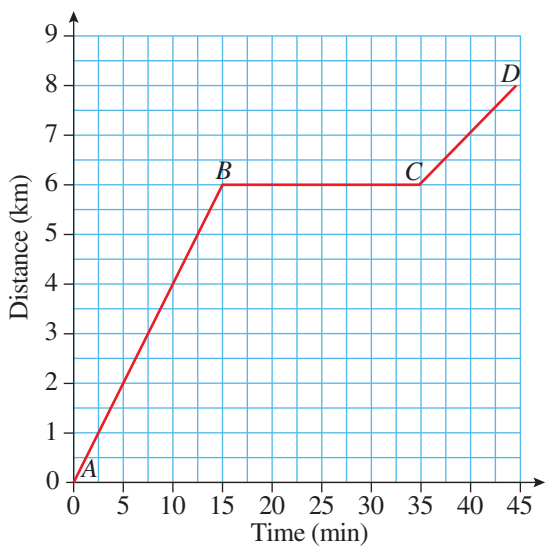
C



a slowest speed but not at rest **b** stopped **c** fastest speed

3 Refer to the graph shown and complete the following.

- **a** What is the distance travelled in the first 15 min?
- **b** Calculate the speed of the journey from A to B.
- **c** What is the distance travelled from B to C?
- **d** Calculate the speed of the journey from B to C.
- **e** What is the distance travelled from C to D?
- **f** Calculate the speed of the journey from C to D.



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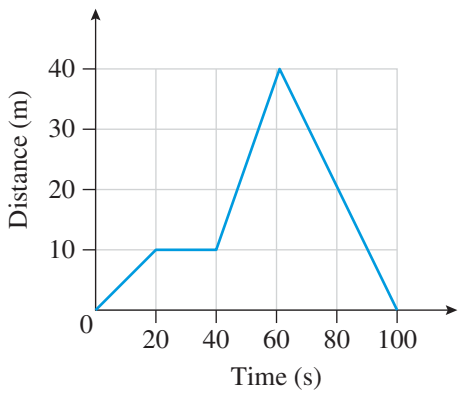
APPLICATIONS

Example 14

4 Kemar and Ramon are Commonwealth Games 100 m athletes. If Kemar takes 10.5 s and Ramon takes 11 s to complete the race, construct a graph to illustrate the race.

5 Jenni watches her hen in the backyard and keeps track when she opens the chicken coop. She records its movements and displays them on a graph, shown below. Describe, in words, the hen's movements.

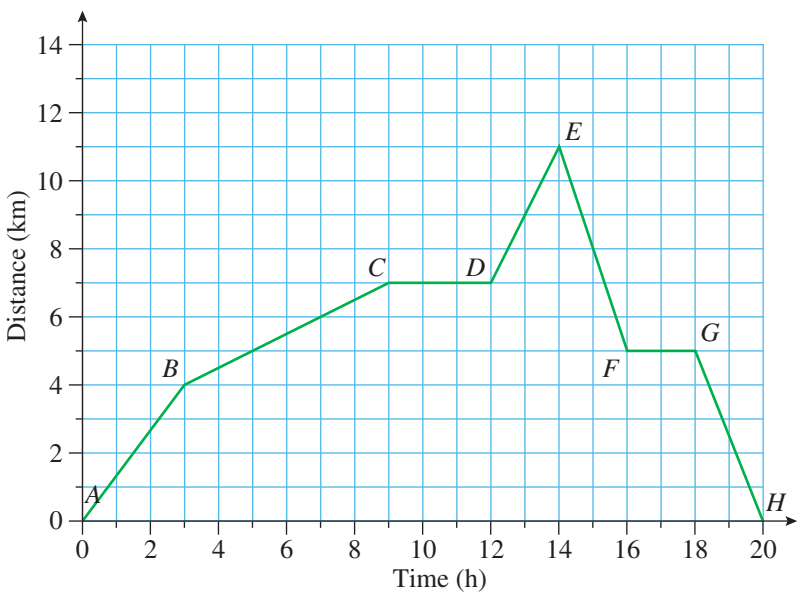


6 The graph shown describes Kim's 12-km walk around Mt Babinga. Identify the part of Kim's journey where she:

a was at rest the longest

b walked at her slowest speed but was not at rest

c walked at her fastest speed



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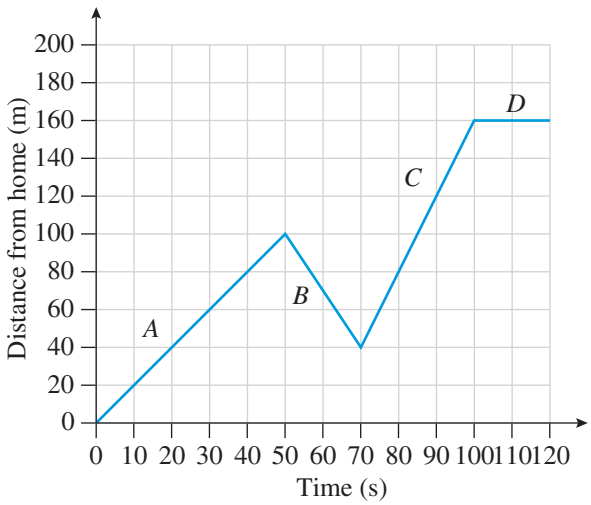
7 Ken is riding his bike around his neighbourhood. The graph below illustrates his ride for the first 2 min. Identify the part of Ken's journey where he:

a was at rest

b cycled at his slowest speed

c cycled at his fastest speed



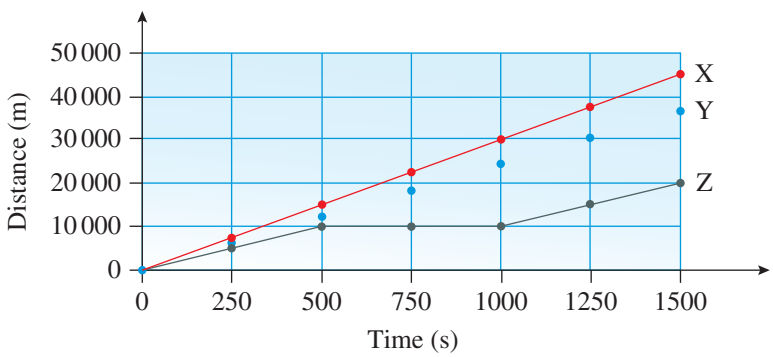
8 Draw a distance–time graph to show the following information about Lana's run.

- Starting from her home, Lana runs 3 km in 12 min.
- She then has a 5-min rest.
- She then runs 4 km farther from home, taking 15 min.
- After a 10-min rest, Lana runs home in 21 min.

9 Illustrate Aamir's journey with a distance–time graph.

Aamir drives 120 km in 2 h. He stops for lunch for 30 min, then drives 240 km in the next 3 h. After another 30-min break, Aamir drives a further 400 km in 4 h.

10 The graph shows three prototype vehicles, labelled X, Y and Z. The company decides to produce and market the fastest vehicle. From the graph, determine the fastest vehicle, giving reasons for your decision.



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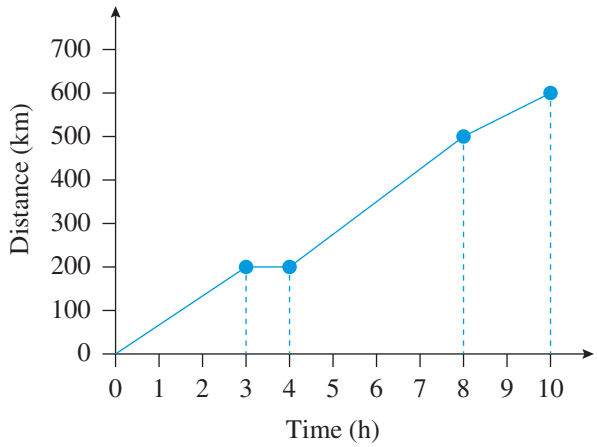
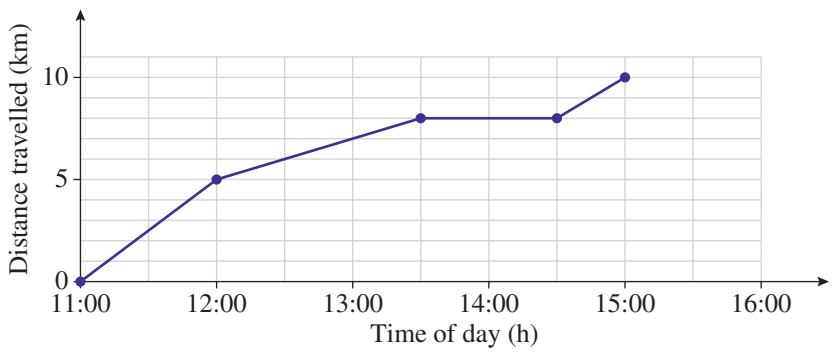
11 Chen is driving and his journey is illustrated by the graph shown. Using your imagination and the information from the graph, write a story describing Chen's car trip.



12 Using your imagination, write a short story that could be illustrated by the following graph.



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Mathematical thinking process task

HOLIDAY

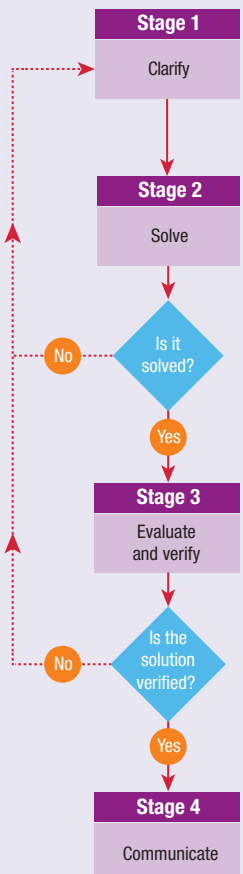
Background: Travel websites, particularly for flights, provide accessible time and cost data, while online mapping applications provide distance data, enabling speed to be calculated.

Task: Your task is to plan a return trip by plane, in Australia, to a minimum of two places and then returning home. You are to use a travel website to plan your journey. For each flight you will need to identify or calculate the flight distance, the flight (duration) time, the speed of the plane and the cost of the flight.

Also, for each flight you will need to use the cost and journey time to calculate the cost per hour you are paying.

To conclude, you will identify the fastest and cheapest leg of your trip.

To complete this task, follow the mathematical thinking workflow diagram below and use the steps listed as a guide.



Stage 1: Clarify
Interpret the task, gather the key information and identify the mathematics which could help to complete the task

1. Decide which travel site you will use.
2. Decide to which destinations you will be flying.

Stage 2: Solve
Apply existing mathematical knowledge and strategies to obtain a solution

3. Research the distance between your destinations using Google Maps.
4. Use the travel website to identify the travel time between each of your destinations and the cost of the flight.
5. Calculate the plane's speed for each flight.
6. Calculate the cost per hour of each leg of your journey.

Stage 3: Evaluate and verify
Analyse information and data from a variety of sources and verify the reasonableness of the solution

7. Compare the speed of each flight and its flying costs per hour.
8. Identify which flight is the cheapest and quickest.

Stage 4: Communicate
Communicate findings in a systematic and concise manner

9. Communicate your findings in a short report, referring to your calculations of cost and speed.

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Puzzle

The Bugatti Veyron used to be the world's fastest car, up until 6 November 2017, when another car hit a top speed of 284.5 miles/h or 458 km/h. Solve the puzzle below to find out the name of the World's Fastest Car, on that date, and the country in which it was made.

Solve the following questions and use each answer to fill in the question letter in the grids below.

K Calculate the average speed of a 42-km marathon run in 3.5 h.

O Calculate the average speed of a 100-m race run in 10 seconds.

E Calculate the distance travelled by a car travelling at a speed of 132.3 km/h for 3 hours.

W Julia travelled 450 km in 5 h. Using the appropriate formula, calculate the speed at which she travelled.

R Calculate the distance travelled if Toby drove at 72 km/h for 6 h.

N Calculate, to one decimal place, the amount of time Alyssa spent travelling if she flew 1650 km at 420 km/h.

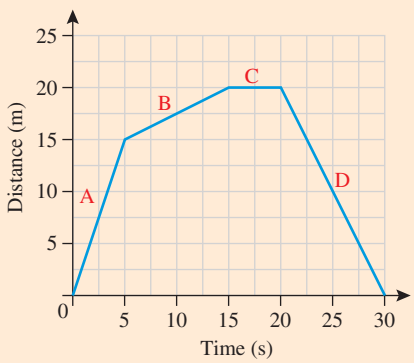
A Imogen ran 200 m in 53 s. Calculate how fast she ran, to one decimal point.

I Michael rode his bicycle for 12.5 h at 36 km/h. Calculate how far Michael cycled.

G Use the information in the graph on the right to calculate the speed of section B.

D Use the information in the graph on the right to determine how long the person was at rest.

S Use the information in the graph on the right to calculate the speed of section A.



12	10	396.9	3.9	450	0.5	3	396.9	0.5	0.5

3.8	0.5	396.9	432	3.8		432	3

3	90	396.9	5	396.9	3.9

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Chapter checklist

The questions in this checklist refer to the map of New Zealand shown.

☐ I can calculate scale distance from maps.

1 Find the actual distance, with an appropriate unit, for a measured map length of 35 mm, using a scale of 1 : 250.

2 Calculate the distance between the following towns.

a Rotorua and Gisborne

b Christchurch and Auckland

☐ I can determine the shortest route from maps.

3 Calculate the shortest path between Queenstown and Christchurch by calculating the distance when travelling via Harihari and the distance when travelling via Timaru. Determine the shortest route.

4 Kiwi Flight delivers parcels once a day from Auckland to the towns of Gisborne, Hamilton, Napier and Rotorua. Calculate the shortest path for the delivery run, using direct distances.

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☐ I can calculate average speed and determine the appropriate units of speed.

5 Describe the speed of an athlete competing in the 400-m event, choosing the appropriate unit of speed from: km/h, m/s, m/h or km/year.

6 Calculate the average speed of a sloth that travels 32 m in 60 s.

☐ I can calculate speed, distance or time from the formula: speed = distance/time.

7 Using the formula, calculate the speed when travelling a distance of 720 km in 10 h.

8 Using the formula, calculate the distance when travelling at a speed of 26 m/s in 37 s.

9 Using the formula, calculate the time taken to travel a distance of 378 km at 89 km/h.

10 Nazeem drove 230 km at 92 km/h. If she left home at 3 p.m., at what time did she arrive at her destination?

☐ I can calculate time and costs for a journey from distances estimated from maps when given a travelling speed.

11 Use the map of Australia given in **Exercise 13E**, question 3, to solve the following questions.

a Estimate the time taken to travel from Geraldton to Alice Springs when travelling at 420 km/h.

b Connie chartered a private jet, costing $2160 per hour of flight time. If the jet travels at 440 km/h, use the map to calculate the approximate cost of flying from Hobart to the Barossa Valley.

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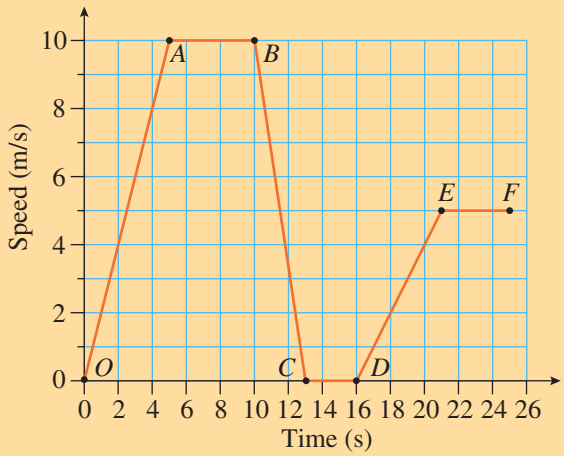
I can interpret distance–time graphs.

12 Use the information in the graph shown to complete the following.

- **a** Name the sections in which the object is at rest.
- **b** Name the section in which the object has the fastest speed.
- **c** Calculate the object's speed from D to E



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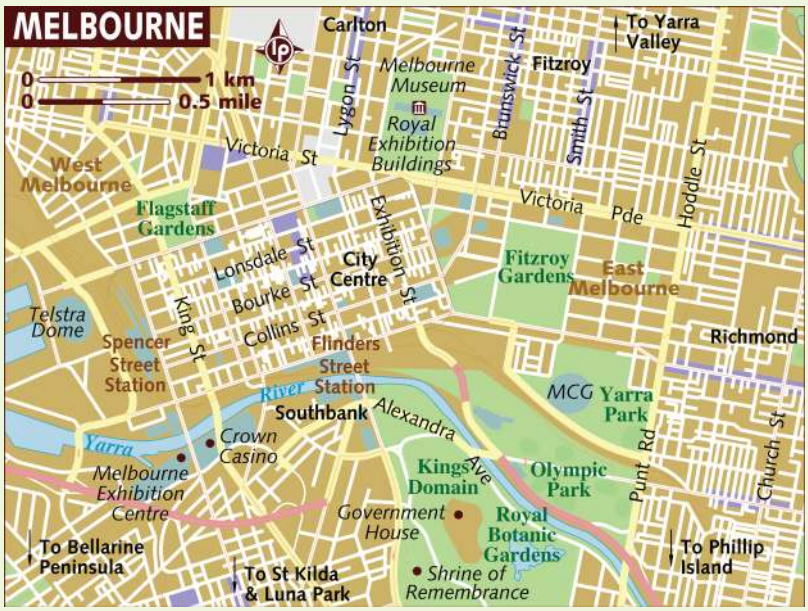
Chapter review

Use the map of Melbourne below question 1c given to complete questions 1–3.



Section 13A

1 Using the city map of Melbourne and its scale, find the approximate direct helicopter distances between:

- **a** Melbourne Exhibition Centre and Crown Casino
- **b** Telstra Dome (the old name for Docklands Stadium) and the Shrine of Remembrance
- **c** Melbourne Museum and Government House

2 Crown Casino is considering buying a gondola to give patrons from the Casino a romantic ride to the Melbourne Exhibition Centre and back. If the Casino charges $2 per 100 m, how much would a return gondola ride cost? Use the map above to help answer this question.

3 Val and John are visiting Melbourne from Brisbane and plan to walk directly from the Shrine of Remembrance to Government House. If they take 4 minutes to walk 100 m, approximately how long will it take them? Use the map above to help answer this question.

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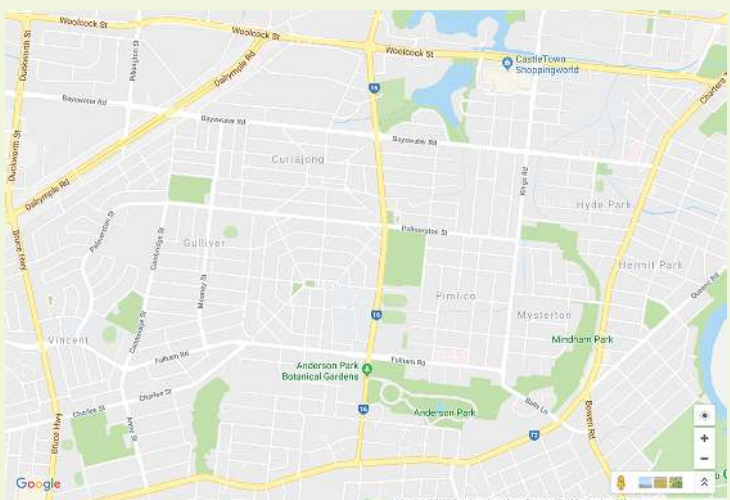
Section 13B **4** Use this map of NSW to answer the following questions.

a Find the total distance to fly from Sydney to Canberra to Mildura to Broken Hill.

b Find the total distance to fly from Sydney to Orange to Bourke to Broken Hill.

c Compare the two routes and determine the shortest route.

Section 13B **5** Zara lives on the corner of Charles Street and Anne Street, as shown on this street map of Townsville. Determine the shortest route from her home to CastleTown Shoppingworld.



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Section 13C

6 Vlad must drive from Dalby to Brisbane to catch a flight departing at 10:30 a.m. The distance to the airport is 210 km and he drives at an average speed of 84 km/h. Calculate the latest time Vlad must leave Dalby if he must arrive 45 min before the departure time to allow for check-in.

7 Willow lives 750 m from her local train station. If she has only 5 min to catch the train, calculate the speed, in m/s, she must run.

Section 13D

8 Laiza participated in a charity car rally, driving 500 km in 5 h 36 min. Calculate her overall average speed, to one decimal place.

9 Ibrahim ran west from his home for 45 min at a speed of 9.2 km/h. Calculate the distance he must travel to get back home.

10 Natalia must get to her grandmother's house by 5 p.m. If Natalia rides her bike at 15 km/h and her grandmother lives 20 km away, calculate when she must leave home to arrive at her grandmother's house on time.

11 Adrian drove 476 km at a speed of 94 km/h. If he left home at 7 a.m., calculate when he arrived at his destination.

12 Jessica runs the 800-m event at 4.2 m/s and Nikki runs the same event at 4.1 m/s. Calculate the time difference between the two girls at the finish line.

13 Osman leaves home at 11 a.m. and drives at an average speed of 80 km/h until 2:45 p.m. Calculate how far he has travelled in that time.

Section 13F

14 Hayley walked 8 km in 2 h. She stopped for lunch for 30 min and then walked a further 4 km in the next 15 h. After stopping for 30 min for a coffee, she walked a further 10 km in 4 h. Illustrate Hayley's journey with a distance–time graph.

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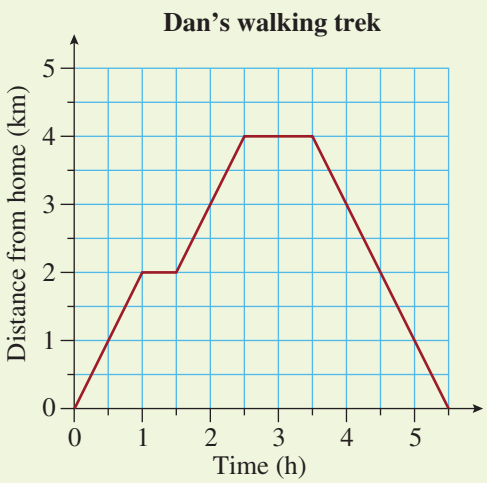
Section 13E Use the map of Canada given to complete questions **15, 16** and **18**.

15 Kyle hires a private plane to fly from Montreal to Calgary, which costs $1720 per hour. If the plane flies at 480 km/h, calculate the approximate cost of Kyle's flight.

16 Kelly takes a taxi from Regina to Winnipeg. The taxi costs $64 per hour and travels at an average speed of 88 km/h. From the map, find an approximate direct distance, and hence, calculate the cost of Kelly's journey.

17 Dan went on a walking trek, as shown in the graph. Using your imagination and the information from the graph, write a story describing his trek.



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18 Use the map of Canada on the previous page to complete this question. Merlin has a deer farm and lives near Edmonton. He takes a limousine from Edmonton to Calgary and then takes a chartered flight to Toronto, to attend an important business dinner at 7 p.m. The limousine, which costs $82 per hour, travels at an average speed of 80 km/h directly from Edmonton to Calgary, and the plane, which costs $1600 per hour, flies at 420 km/h. From the map, find the approximate distances and then calculate:

a when Merlin should leave to make it to the business dinner on time

b how much the journey will cost.

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Glossary

A

Accuracy [p. 10] How close a measurement is to the real value

Actual distance [p. 588] The actual distance between two points on map, usually in units of kilometres

Area [p. 90] The measurement of space enclosed by the boundary of a 2D shape

Arrival time [p. 543] Occurring once every year

Association [p. 182] A relationship where one variable can be used to predict the other

Average rate [p. 57] The total change in one quantity divided by the total change in the second quantity

Average speed [p. 57] The average rate at which distance changes with time

Axes [p. 188] The vertical and horizontal lines that define the number plane quadrants

Axes scale [p. 188] A system of ordered marks at fixed intervals referred to as a standard in measurement

Axes/sector labels [p. 188] Words of numbers used to name the axes of a graph and/or sectors of a pie graph

B

Back-to-back stem-and-leaf plot [p. 399] A method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem' in a stem-and-leaf plot

Bar graph [p. 160] A diagram in which horizontal rectangles or columns are used to represent data

Best buy [p. 511] A purchase where the largest amount of a particular item is obtained for each dollar spent

Bias [p. 362] The external influence of responses, which could potentially cause the results to be different from the true results

BIDMAS [p. 15] Gives the order that mathematical operations must be evaluated in from left to right. Brackets first, then indices, followed by either division or multiplication and then finally either addition or subtraction

Bimodal [p. 411] A dataset is bimodal if it has two modes; this means that there is not a single data value that occurs with the highest frequency, but two data values have the same highest frequency

Box plot [p. 391] A visual representation of the five-number summary of a dataset

Broken scale [p. 188] An axis with a break or gap in its values

C

Calories [p. 139] Unit of energy that is used for food

Capacity [p. 127] The quantity of fluid a container can hold

Categorical data [p. 222] Data that can be put into separate and distinct categories, like red or blue

Centimetres [p. 74] Unit of measurement for length (100 cm = 1 m)

Cluster [p. 356] To gather around or gather together

Colon [p. 451] The symbol:, which is used in ratios and operates similarly to the horizontal line of a fraction

Column graph [p. 160] A graphical representation of a single set of categorical or discrete data, where columns are used to show the frequency of scores

Composite shape [p. 85] A 2D shape made up of two or more basic 2D shapes

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Continuous data [p. 222] Data that can take any numerical value. This is data that is measured

Conversion [p. 74, 91] Changing (converting) a value to the equivalent value expressed in a different form

Conversion graph [p. 173] Specific line graphs used to convert a measurement from one unit to another unit

Cost price [p. 50] The cost to the owner when goods are purchased by a store

Cumulative frequency [p. 342] The frequency of the score plus the frequency of all the scores less than that score. It is the progressive total of the frequencies

D

Data [p. 222] Information gathered by observation, survey or measurement; e.g. in a probability experiment. Singular of data is 'datum'.

Data series [p. 235] A list of data on one row or column that is in the same category of data

Dataset [p. 222] Information from different trials or sources

Deciles [p. 341] Any of the nine values that divide a ranked dataset into ten equal parts

Decimals [p. 7] Numbers written after the decimal point in a number

Denominator [p. 5] The number in a fraction below the horizontal line. It is the number of equal parts into which the whole is divided

Departure time [p. 542] Time when a bus or train leaves the stop

Digital format [p. 526] A method of displaying time, where the time of day is displayed as digits

Direct proportion [p. 505] The relationship between two quantities that increase or decrease at the same rate (also called direct variation)

Discount [p. 51] An amount subtracted from a price, usually to sell goods faster

Discount price [p. 51] The final price you pay once the discount has been subtracted

Discrete data [p. 222] Data that can take only particular numerical values. This is data this is counted

Distance [p. 81] The measure of how far apart two objects are. See length.

Distance–time graph [p. 626] A line graph showing the distance travelled on the vertical axis and the time on the horizontal axis

Dot plot [p. 273] A graph in which each dot represents one data value

E

Equivalent fraction/ratio [p. 465] Fractions and ratios that represent the same amount. They can be reduced to the same basic fraction or ratio

Estimating [p. 75] An informal guess. To estimate means to approximate an quickly without calculating exactly

Excluded value [p. 160] A value not part of a line segment in a step graph. Represented by an open (hollow) circle

F

Factor [p. 457] A whole number that will divide into another number without a remainder

Five-number summary [p. 379] A method of summarizing a set of data using the minimum value, the lower or first quartile (Q_1), the median (Q_2), the upper or third quartile (Q_3) and the maximum value; forms the basis for a box plot

Formula:

simple interest = principal × rate × time

Formulas [p. 71] A special type of equation that shows the relationship between different variables

Fortnight [p. 445] A 2 week period; 14 days in a row

Fraction [p. 5] Numbers expressed as the division of one number by another

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Glossary

Frequency (distribution) table [p. 254] A table summarising data by showing all possible scores, from lowest to highest, in one column, and the frequency of each score in another column

Frequency [p. 254] The number of times a data value occurs

Fully simplified fraction [p. 457] A fraction or ratio in which both of their numbers are divided by their highest common factor (HCF)

G

Gap [p. 356] A section of a graph that contains no data

Goods and Services Tax (GST) [p. 51] A government levy or tax included in the purchase price of merchandise. In Australia, this rate is 10%.

Graph [p. 222] The representation of data in a diagram

H

Hectare (ha) [p. 91] A unit of measurement for area equal to 10 000 m^2

Highest common factor (HCF) [p. 457] The largest number that is a factor of all the given factors

Histogram [p. 263] A special type of column graph with no gaps between the columns; it can represent class intervals

I

Included value [p. 160] A value part of a line segment in a step graph. Represented by a closed circle

Interest [p. 444] The amount of money paid (earned) for borrowing (lending) money over a period of time

Interquartile range [p. 329] A measure of the spread within a numerical dataset; it is equal to the upper quartile (Q_3) minus the lower quartile (Q_1), that is, $IQR = Q_3 - Q_1$

K

Kilocalorie (kcal) [p. 139] Unit of energy that is used for food, where 1 kcal = 1 calorie

Kilojoules (kJ) [p. 139] Unit of energy that is used in food, where 1 kcal = 4.2 kJ

Kilometre (km) [p. 75] A unit of metric measurement for length equal to 1000 metres (1 km = 1000 m)

Kilowatt hours (kWh) [p. 140] Unit of energy used to measure electricity usage

L

Leading digit approximation [p. 17] A way to quickly get an estimate for an expression by rounding the first digit in each number of the expression to simplify calculations

Length [p. 74] The measurement of the distance from end to end along a line; the longest dimension of a 2D shape or 3D object

Length units [p. 75] Units of measurement for length. Examples include millimetre, centimetre, metre and kilometre

Line graph [p. 173] A graph that shows the data as points joined with line segments

Line segment scale [p. 588] A way of displaying a map's scale, by displaying a length of line to show the relationship between the scale distance of the map and the actual distance

Lowest common denominator (LCD) [p. 457] The LCD of two fractions is the LCM of the denominators

Lowest common multiple (LCM) [p. 457] The LCM of two numbers is the lowest whole number that is exactly divisible both numbers

M

Map distance [p. 588] The distance measured on a map using a ruler; it is related to the actual distance by the scale

Marked price [p. 50] The selling price, equal to the sum of the cost price and mark-up

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Mark-up [p. 50] The amount added to the cost price to find the selling price (usually expressed as a percentage of the cost price)

Mass [p. 122] The amount of matter (material) in a body

Mass unit [p. 122] Metric units for measuring mass are the gram (g) and kilogram (kg)

Mean [p. 316] the mean of a list of numbers is the sum of the data values divided by the number of values in the list; in everyday language, the mean is commonly called the average

Measure of central tendency [p. 315] A measure of location of a distribution. Measures of centre include the median and mean

Median [p. 316] The value in a set of ordered data values that divides the data into two parts of equal size; when there are an odd number of data values, the median is the middle value; when there are an even number of data values, the median is the average of the two central values

Mental mathematics strategies [p. 24] Breaking a difficult calculation into several easy calculations

Metre [p. 74] Standard unit of metric measurement for length equal to 100 centimetres

Millimetre [p. 74] Unit of metric measurement for length equal to one thousandths of one metre (1000 mm = 1 m)

Misleading [p. 188] Giving the wrong idea or impression. Can be used to describe graphs if they make the data look better or worse than it actually is.

Mode [p. 315] The most frequently occurring value in the dataset

Moon phases [p. 562] The shape of the visible (sunlit) Moon

N

Negatively skewed [p. 410] A data distribution with a long tail to the left

Numerator [p. 5] The number in a fraction above the horizontal line

Numerical data [p. 222] Data that is measured using numbers

O

Order of operations [p. 15] The order that mathematical operations must be evaluated in an expression, it is given by BIMDAS

Outlier [p. 279] A value that is much larger or much smaller than the rest of the data

P

Parallel box plot [p. 398] Box plots of different data sets placed on the same axes to allow a comparison to be made

Per [p. 20] Per means 'for each'. It is used when the number of units of one quantity is given for each unit of another quantity. This is a rate. Per may be abbreviated to 'p' but, in units, it is usually shown as a forward slash '/' (also known as a solidus). This is also a symbol for division because to calculate a rate you have to divide one quantity by another.

Per annum (p.a.) [p. 444] Each year

Per cent (%) [p. 20] A fraction with one hundred equal parts where only the numerator is shown followed by the percentage sign %. The sign indicates that the denominator of the fraction is 100

Percentiles [p. 341] The values that divide a ranked set of data into 100 equal parts

Perimeter [p. 81] The total distance (length) around the outside of a figure

Perpendicular [p. 134] The height of a 2D shape or 3D object measured at right angles to the base

Picture graph [p. 167] A graph where the size of each sector of the circle represents a particular amount (also called a pie chart)

Pie graph [p. 167] A graph where the size of each sector of the circle represents a particular amount (also called a pie chart)

Positively skewed [p. 410] A data distribution that has a long tail to the right

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Glossary

Principal [p. 445] An amount of money invested in a financial institution or loaned to a person or business

Prism [p. 134] A solid where each cross-section in a particular direction is exactly the same

Profit [p. 50] The amount of money made by selling something for more than its cost

Q

Quantiles [p. 341] A set of values that divide an ordered dataset into equal parts

Quartiles [p. 341] The quartiles of a ranked set of data values are the three points that divide the dataset into for equal parts

R

Range [p. 329] The difference between the highest and lowest numbers in a set

Rate [p. 56] The number of units of one quantity for each single unit of another quantity

Ratio [p. 451] A method used to compare two or more quantities measured in the same units

Respectively [p. 451] In the same order at that listed

Right angle [p. 134] An angle equal to 90°

S

Scale [p. 477] A ratio that compares a drawing or a model to the real object

Scale drawings [p. 477] A drawing that represents the actual object

Scale factor [p. 477] The number by which you multiply each side length to enlarge or reduce the size of a shape

Simple interest [p. 444] Percentage rate per year paid on a loan or investment amount. Simple interest is generally charged on money borrowed or invested for a short period of time.

Solid [p. 127] Having three dimensions (length, breadth and thickness) as a geometrical body or figure

Speed [p. 56] A measure of how fast an object is moving

Spread [p. 329] A measure of the degree to which data values are clustered around some central point in the distribution. Measures of spread include the standard deviation (s), the interquartile range (IQR) and the range (R)

Spreadsheet [p. 229] A table comprised of rows and columns for entering data and headings. Computer spreadsheet programs allow you to enter formulas that will manipulate data electronically and update results immediately when data is changed.

Standard deviation [p. 330] A summary statistic that measures the spread of data values around the mean

Stem-and-leaf plot [p. 273] A graph that lists numbers in order, grouped in rows

Step graph [p. 160] A special type of line graph that jumps in steps

Sunrise/sunset charts [p. 562] A table showing sunrise and sunset times

Symmetric [p. 410] A data distribution in which the data values are evenly distributed around the mean. In a symmetric distribution, the mean and the median are equal

Systematic (route calculations) [p. 596] Selection method where the shortest distance is selected from each next location

T

Table [p. 285] An arrangement of data using rows and columns of boxes

Tally marks [p. 254] Line strokes used to record data, usually in groups of 5

Tide tables [p. 563] A table used to show a specific location's high and low tide heights and times

Tides [p. 563] The rise and fall of local sea levels caused by the gravitational pull of the Moon and Sun

Time [p. 526] Period of existence that is measured in seconds, minutes, hours, days, weeks, months, years and so on; this process considered as a whole

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Time difference [p. 554] The length of a period of time between two times

Travel time [p. 554] Period of time needed to reach a destination from the starting point

Trends [p. 235] A general shape or direction created by data

Trial and error [p. 596] A method of solving problems where multiple varied attempts are done until the problem is solved

Two-dimensional (2D) [p. 91] Having two dimensions, like a surface, but no depth; requiring two coordinates to specify a point

Two-way table [p. 182] A table that displays data from two categories that are often connected to each other

U

Uniform [p. 411] Having all the parts or elements the same; sharing characteristics

Units of area [p. 90] Units of measurement for area. Examples include square millimetres. Square centimetres and square metres

Units of length [p. 74] Units of measurement for length. Examples include millimetre, centimetre, metre and kilometre

Units of volume [p. 127] Units of measurement for volume. Examples include cubic millimetres, cubic centimetres and cubic metres

V

Variability [p. 330] The amount by which data points in a dataset differ from the mean and from each other, which can also be measured by the spread or range

Variable [p. 71] A symbol, or the quantity it signifies, that may represent any one of a given set of number and other objects

Volume [p. 127] The size, measure or amount of anything in three dimensions; the space occupied by a body or substance in cubic units; the SI unit of volume is the cubic metre (m^3)

Where principal is the amount borrowed or invested, rate is the percentage charged per time period (e.g. daily, monthly or yearly) for interest, and time is the total number of time periods of the loan or investment. Note that when using the formula, the units of time for the interest rate and total number of time periods must match. If the time is stated in years and the interest rate is given in a percentage per month, first convert the time in years to months.

X

***x*-axis [p. 235]** The horizontal axis on the number plane

***x*-coordinate [p. 626]** The horizontal distance on the x-axis

Y

***y*-axis [p. 235]** The vertical axis on the number plane

***y*-coordinate [p. 626]** The vertical distance on the y-axis

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Answers

Chapter 1

Pre-test

1 **a** 252 **b** 195

2 **a** $30 + 40 = 70$, $40 + 50 = 90$. 83 is between 70 and 90 so reasonable.

b $150 - 30 = 120$, $160 - 20 = 140$. 127 is between 120 and 140 so reasonable.

c $20 \times 30 = 600$, $30 \times 40 = 1200$. 514 is not between 600 and 1200 so not reasonable.

3 **a** $\frac{5}{9}$ **b** $\frac{1}{3}$ **c** 75% **d** 4.73

4 **a** 8640 **b** 101 000

c 153.463 **d** 87

5 18 balls

6 **a** 36 **b** 10 **c** 12 **d** 18

7 **a** $\frac{1}{2}$ **b** $\frac{89}{100}$

8 **a** 71.4% **b** 125.0%

9 **a** $\frac{3}{5}$ **b** $\frac{17}{20}$

10 **a** 90.0% **b** 32.1%

Exercise 1A

1 **a** larger, smaller, smaller, larger

b larger, larger, smaller, smaller

c smaller **d** 20, 5, 5

2 **a** $\frac{4}{11}$ **b** $\frac{1}{2}$ **c** $\frac{7}{9}$

d 76% **e** 5.54 **f** 256.23

g 56.759%

3 **a** $\frac{2}{9}$ **b** $\frac{1}{8}$ **c** $\frac{3}{7}$

d 43% **e** 3.34 **f** 583.034

g 43.582%

Exercise 1B

1 **a** simplifying, mental

b decimal places, accuracy, same, add one

2 **a** 1860 **b** 95 000 **c** 300 **d** 349

3 **a** 1863.6

b 947.64 to 2dp.

c 254.983

d 53

Exercise 1C

1 **a** most basic, combines

b opposite, difference

c product, repetitive addition

d multiplication, splitting

2 27

3 **a** 131 **b** 192

4 **a** 72 **b** 170

5 \$153.50

6 \$9.00

Exercise 1D

1 multiple operations, BIMDAS

i brackets

ii indices

iii left to right, divide

iv add, first

2 **a** 24 **b** 21 **c** 64 **d** 28

3 **a** 5 **b** 18 **c** 10 **d** 13

Exercise 1E

1 estimation, approximation, reasonable, Leading Digit

2 **a** 120 **b** 80 **c** 1100 **d** 4000

3 **a** 1000 **b** 4000

c 210 000 **d** 300 000

4 **a** Reasonable **b** Reasonable

c Not reasonable **d** Not reasonable

5 **a** Reasonable **b** Reasonable

c Not reasonable **d** Reasonable

6 John wrote down the incorrect number, the number of bolts is between 170 and 180.

7 Yes, the number of coat hangers is reasonable as the number of dresses is between 4000 and 9000.

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Exercise 1F

1 a 100, %
b numerator, divided by
c decimal, divide by 100
d i $\frac{1}{20}$ **ii** 0.125 **iii** $66\frac{2}{3}$

2 a $\frac{3}{5}$ **b** $\frac{39}{100}$ **c** $\frac{77}{50}$ **d** $\frac{239}{1000}$

3 a 42.9% **b** 76.5% **c** 51% **d** 209.3%

4 a 0.57 **b** 0.68 **c** 0.68 **d** 1.67

5 a $\frac{1}{5}$ **b** $\frac{18}{25}$ **c** $1\frac{1}{4}$ **d** $\frac{31}{40}$

6 a 0.55 **b** 0.63 **c** 1.45 **d** 0.06

7 a 80% **b** 41%
c 102.5% **d** 45.1%

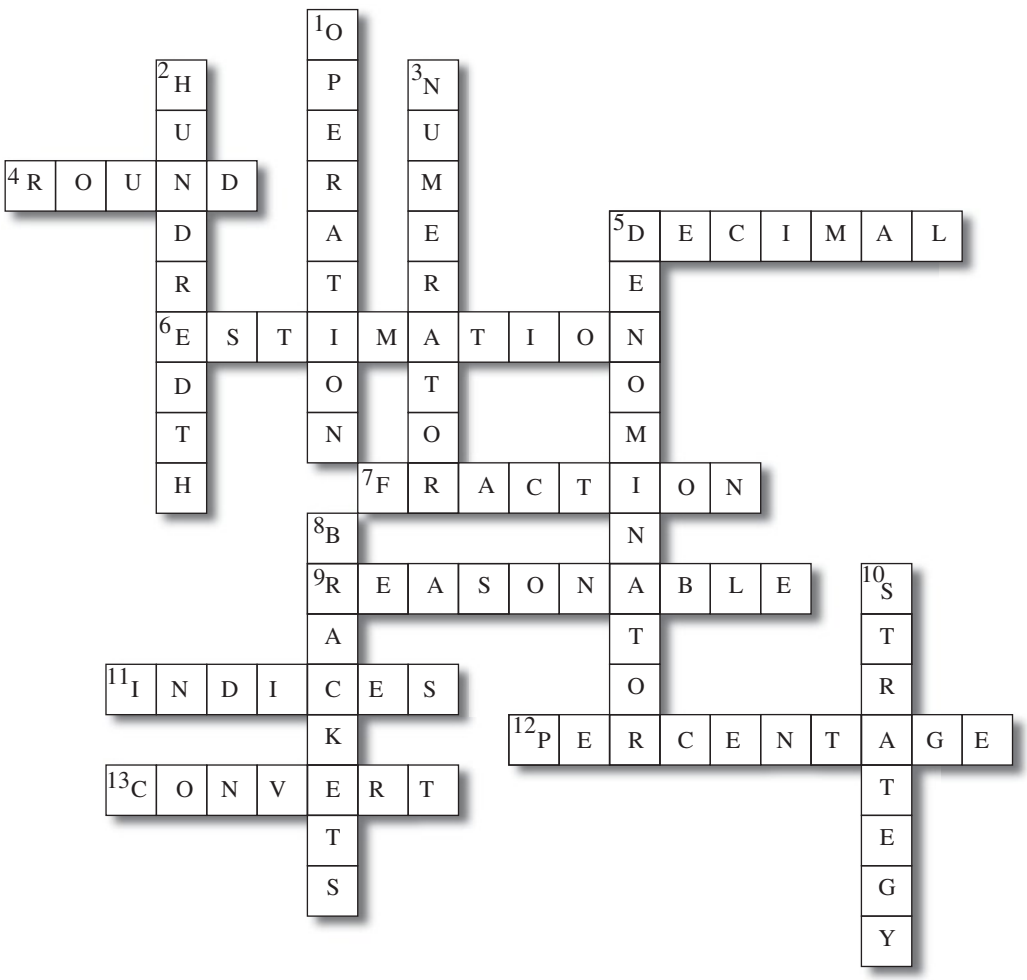
Exercise 1G

1 a 204 **b** 253 **c** 675 **d** 144
e 777 **f** 342 **g** 1248 **h** 2080

2 a 8.4 **b** 8.25 **c** 6 **d** 11.4
e 1.2 **f** 6.5 **g** 34.4 **h** 5.12

3 a 131.3 **b** 135.3 **c** 265.5
d 551.4 **e** 198.4 **f** 855

Puzzle



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Chapter checklist

1 **a** $\frac{5}{13}$ **b** $\frac{8}{10}$
2 29%
3 756.23
4 600
5 725.370
6 26 objects
7 35 flowers
8 **a** 33 **b** 10
9 $20 + 180 = 200$, $30 + 190 = 220$, 208 is between 200 and 220 so is reasonable.
10 $150 \times 25 = 3750$, $160 \times 30 = 4800$, 300 is not between 3750 and 4800 so is not enough.
11 0.2, 20%
12 0.47, $\frac{47}{100}$
13 $\frac{1}{4}$, 25%
14 **a** 918 **b** 6.3

Chapter review

1 **a** 348 **b** 322 **c** 24 **d** 19
2 **a** 140 **b** 80
c 240 000 **d** 500 000
3 **a** $60 + 30 = 90$, $70 + 40 = 110$. 95 is between 90 and 110 so reasonable.
b $200 - 20 = 180$, $300 - 30 = 270$. 184 is between 180 and 270 so reasonable.
c $100 \times 300 = 30\,000$, $200 \times 400 = 80\,000$. 48 546 is between 30 000 and 80 000 so reasonable.
d $500 \times 300 = 150\,000$, $600 \times 400 = 240\,000$. 138 131 is not between 150 000 and 240 000 so not reasonable.
4 Smallest estimation is $10 \times 20 = 200$ so Lisa will have at least enough.
5 $100 + 200 = 300$, $200 + 300 = 500$. 363 is between 300 and 500 so reasonable.
6 **a** $\frac{4}{13}$ **b** $\frac{1}{6}$ **c** $\frac{7}{9}$
d 69% **e** 3.41 **f** 554.93
g 56.454%
7 **a** 1590 **b** 63 000
c 725.476 **d** 48
8 No, there are 6 extra bales.
9 **a** 28 **b** 20 **c** 46 **d** 28
10 **a** $\frac{1}{5}$ **b** $\frac{31}{20}$
11 **a** 33.3% **b** 152.0%
12 **a** 0.38 **b** 1.44
13 **a** $\frac{7}{10}$ **b** $2\frac{1}{4}$
14 **a** 0.28 **b** 0.349
15 **a** 70% **b** 109.9%

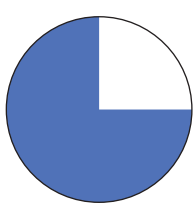
Chapter 2

Pre-test

1 **a** $\frac{1}{2}$ **b** $\frac{2}{5}$ **c** $\frac{1}{20}$
d $\frac{9}{10}$ **e** $\frac{1}{8}$
2 **a** $\frac{3}{10}$ **b** $\frac{3}{4}$ **c** $\frac{99}{100}$
d $1\frac{1}{4}$ **e** $\frac{19}{25}$
3 **a** Length **b** Money
c Time **d** Mass
e Capacity **f** Volume
g Length **h** Mass
4 **a** distance and time
b volume and distance
c dollars and kg
d amount of energy and time
e number of heart beats and time

Exercise 2A

1 **a** 100, 100
b per cent, %, 100, fraction
c simplifying, fraction
d division, multiplication
2 **a** 50% **b** 10% **c** 25%
3 **a** 40% **b** 10% **c** 70%
4 ~80%
5



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6 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $\frac{1}{4}$
d $\frac{2}{5}$ **e** $\frac{1}{2}$ **f** $\frac{3}{4}$
g 1 **h** $\frac{3}{20}$ **i** $\frac{3}{5}$
j $\frac{6}{25}$ **k** $\frac{19}{20}$ **l** $\frac{22}{25}$

7 **a** 100 **b** 10 **c** 2
d 4 **e** 1 **f** 3
g 5 **h** 5, 4 **i** 3, 2
j 4, 3 **k** 2, 3

8 **a** \$20 **b** \$200 **c** \$20
d \$421 **e** \$70 **f** \$600
g \$6000 **h** \$660

9 **a** 14 kg **b** \$47.52
c 74.4 km **d** 47.4 L
e 512 mL **f** \$18 750
g 420 kg **h** 338.7 tonnes

10 **a** 920.4 **b** 920 students

11 **a** \$5880 **b** \$89 880

12 **a** \$117 **b** \$663

13 **a** 150 g **b** 0.67% **c** 1 g
d 13.3% **e** 20 g **f** 9.3%
g 14 g

14 30 marks

15 296 km

16 8400 students

17 \$2720

18 **a** \$960 **b** \$8960

19 IMGood (employs 4195 female employees)

Exercise 2B

1 **a** **i** same **ii** fraction
iii 100, A/B, 100%
b %

2 **a** 50% **b** 25% **c** 30%
d 40% **e** 35% **f** 2.7%
g 17.5%

3 **a** 12.5% **b** 10% **c** 30%
d 12.5% **e** 15.4% **f** 16.5%
g 260% **h** 125%

4 **a** **i** 4000 g **ii** 12.5%
b **i** 3000 mL **ii** 25%
c **i** 8000 m **ii** 15%
d **i** 120 min **ii** 25%
e **i** 1000 c **ii** 4.2%
f **i** 21 days **ii** 23.8%
g **i** 5000 cm **ii** 1.8%
h **i** 3200 mm **ii** 27.5%
i **i** 1800 s **ii** 26.7%
j **i** 168 h **ii** 11.9%

5 **a** 36% **b** 32% for both

6 **a** $\frac{240}{699}$ **b** 34.3% discount

7 75%

8 55.6%

9 34.3%

10 **a** accommodation 45.8%
b airfares 24.2%
c food 30%

11 **a** 2000 m **b** 40%

12 30%

13 22.9%

14 2.5%

15 7.5%

16 3.1%

17 640 g, 32%

Exercise 2C

1 **a** cost
b profit, marking, cost price
c mark-up, percentage
d marked, retail
e decrease, marked
f discounted, subtracted
g GST, 10

2 **a** **i** \$120 **ii** \$1320
b **i** \$60 **ii** \$460
c **i** \$20 **ii** \$180
d **i** \$7 **ii** \$42
e **i** \$700 **ii** \$3200
f **i** \$87 **ii** \$145
g **i** \$12 915 **ii** \$362 915
h **i** \$511.88 **ii** \$5761.88
i **i** \$164.37 **ii** \$363.37

3 **a** **i** \$25 **ii** \$100
b **i** \$69.50 **ii** \$625.50
c **i** \$900 **ii** \$6600
d **i** \$75 **ii** \$425

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e **i** \$60 **ii** \$180
f **i** \$3.51 **ii** \$74.79
g **i** \$51.58 **ii** \$247.42
h **i** \$2563.20 **ii** \$26 236.80

4 **a** \$5 **b** \$28.5 **c** \$300
d \$5.6 **e** \$17.4 **f** \$6.99
g \$1699.9 **h** \$42 650

5 **a** \$10.49 **b** \$59.46

6 **a** \$88.50 **b** \$206.50

7 **a** \$4.50 **b** \$49.50

8 **a** \$113.85
b Australia \$115.50; NZ cheaper by \$1.65

9 \$737.50

10 \$300

11 \$308

12 **a** Triangle Shop **b** \$1.44

13 **a** Time-off Travel
b \$13.45

14 \$8.32 profit

Exercise 2D

1 **a** different, different
b with, without
c one
d change, change

2 **a** rate **b** rate
c not a rate **d** rate
e not a rate **f** rate
g not a rate **h** rate

3 **a** m/s **b** \$/kg
c \$/L **d** words/min
e kj/100 g **f** km/h
g $\$/m^2$ **h** runs/over

4 **a** 60 km/h **b** \$1.75/kg
c $\$300/m^2$ **d** 2 tries/game
e 22 mL/day **f** 167 rev/s
g 32 customers/h **h** 2857.1 bottles/h
i 55.6 km/h **j** 1.3 jumps/s

5 **a** 400 km/day **b** 8 kg/year
c 10 km/h **d** 2.4 lollies/min
e 0.4 cm/h **f** 1.33 points/min
g 1.5°C/h **h** 7 cm/year

6 **a** 0.46 km/min
b 0.47 km/min
c Archie

7 **a** **i** 8 km/L
ii 0.125 L/km
b **i** 2.08
ii 12.5 L/100 km

8 **a** **i** 4000 g
ii 125 g/person
b **i** 16 000 mL
ii 500 mL/person

9 \$0.40/sausage

10 179 cm

11 3000 members/year

12 10 h

13 **a**

Time (in hours)	Cost of hire from Brooke's Boat	Cost of hire from Jessica's Jetty
1	\$32	\$16
2	\$44	\$32
3	\$56	\$48

b 5 h
c **i** Brooke **ii** \$12

Puzzle

Water in a jellyfish.

Chapter checklist

1 $\frac{7}{20}$
2 \$540
3 36
4 40%
5 20%
6 25%
7 \$405
8 \$337.50
9 \$59 800
10 20 km/h, 24 pings per millisecond, \$1000/tonne.

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11 km/h
12 40 apples/box
13 5 cm/month

Chapter review

1 93 students
2 $1450
3 90%
4 30%
5 12.5%
6 $252 000
7 61.5%
8 $187.20
9 $2.50/mango
10 7.62 L/100 km
11 **a** 53.6 ml/min **b** 0.3 kg/day
c 1.3 m^3/min

Chapter 3

Pre-test

1 **a** 4 sides **b** 4 corners
2 **a** Degrees **b** 3
3 **a** Angle B **b** 90 degrees
4 Lines A and C
5 **a** **i** Opposite sides are parallel
ii Square
b **i** Opposite sides equal
ii Rectangle
c **i** All sides are equal
ii Equilateral Triangle

Exercise 3A

1 **a** $1400 **b** $1500 **c** $1425
2 380 km
3 **a** $10.50 **b** $14.40 **c** $30
4 **a** P = 14 **b** P = 46 **c** P = 23
5 **a** 10°C **b** −17.8°C
c 100°C **d** 33.3°C
6 **a** **i** 1 h 50 min
ii 2 h 56 min
iii 3 h 6 min
iv 2 h 38 min
b 5:15 p.m.

Exercise 3B

1 **a** 500 cm **b** 70 mm
c 2 cm **d** 2000 m
e 370 cm **f** 1700 m
g 4.9 m **h** 7000 mm
i 1 200 000 mm **j** 0.03 km
2 **a** 3 cm **b** 5 m **c** 25 mm
d 2.5 km **e** 500 cm **f** 1700 m
3 Any sensible response will be acceptable.
4 **a** m **b** mm **c** mm
d cm **e** km **f** mm
g cm **h** m **i** km
j km
5 5.86 m
6 4500 m
7 4200 mm
8 1.8 m
9 15 m
10 Diamond – 6 mm
Mobile – 12 cm
Gum tree – 32 m
Skyscraper – 0.1 km
Perth to Port Hedland – 1300 km
11 **a** 5 cm **b** 1.75 cm **c** 10 cm
12 4 m

Exercise 3C

1 **a** 40 cm **b** 66 cm **c** 200 mm
d 6 m **e** 17.1 m
2 **a** 56 m **b** 320 mm **c** 4010 m
3 80 cm
4 300 m
5 4 m
6 **a** 85.6 m **b** 256.8 m

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Exercise 3D

1 **a** 40 cm **b** 66 mm **c** 6 m
2 **a** 48 cm **b** 480 mm
c 12.6 km **d** 200 m
3 56 m
4 57.4 m
5 1.96 m
6 66.6 m

Exercise 3E

1 **a** multiplication, smaller
b smaller, larger
c 10 000
d 0.0001
2 **a** 50 000 cm^2 **b** 700 mm^2
c 2 cm^2 **d** 2 000 000 m^2
e 500 ha **f** 37 000 cm^2
g 1 700 000 m^2 **h** 0.049 m^2
i 0.0527 ha **j** 0.05 cm^2
k 0.0005 km^2 **l** 375 ha
3 **a** 7 000 000 mm^2
b 4 000 000 000 cm^2
c 0.0028 m^2
d 1 200 000 000 000 mm^2
e 0.0000003 km^2
4 0.025 km^2
5 16 418 000 000 m^2
6 1900 mm^2
7 0.16 m^2
8 3.236 ha
9 0.92 m^2
10 7.14 m^2
11 18 000 mm^2

Exercise 3F

1 **a** 20.25 cm^2 **b** 7 m^2
c 272 cm^2 **d** 51 710.76 mm^2
e 209 cm^2 **f** 24.96 m^2
g 38.53 m^2
2 **a** 9 cm^2 **b** 40 cm^2
c 6716 mm^2 **d** 769 mm^2
3 5.25 m^2
4 7.84 m^2
5 13.12 m^2
6 17.1 m^2
7 **a** 2.2 ha **b** 22 sheep

Exercise 3G

1 **a** 30 cm^2 **b** 53.76 m^2
2 **a** 95 cm^2 **b** 5600 mm^2
c 8.6 km^2 **d** 1323 m^2
3 **a** 16.25 m^2 **b** 4500 mm^2
c 2100 mm^2
4 63 m^2
5 1.195 m^2
6 131 m^2

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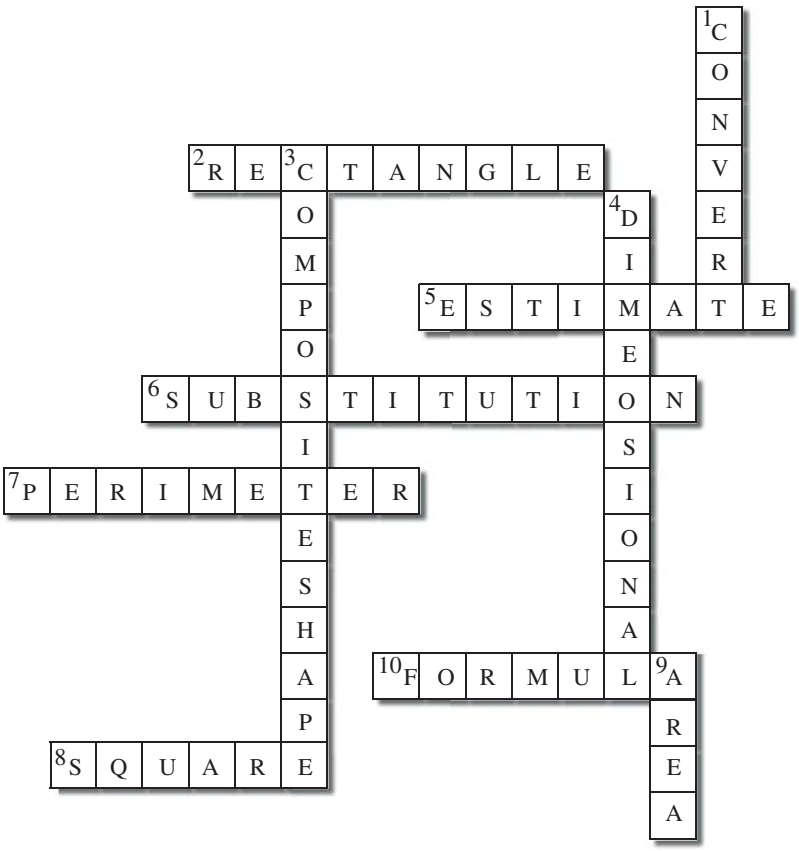
Puzzle



Chapter checklist

1 **a** cm **b** m **c** mm **d** km

2 Using rulers, estimation

3 **a** iii **b** ii **c** i **d** iii

4 **a** 40 mm **b** 3 cm
c 4500 m **d** 730 cm

5 **a** 11.7 cm **b** 7 m **c** 160 mm

6 **a** 11 m **b** 400 cm

7 **a** mm^2 **b** km^2

8 **a** square centimetres
b square metres
c hectares

9 **a** cm^2 **b** m^2 **c** m^2 **d** ha

10 **a** 60 000 cm^2 **b** 900 mm^2
c 3.5 cm^2 **d** 2 700 000 m^2
e 600 ha **f** 7 000 000 mm^2

11 **a** 33.64 cm^2 **b** 231 mm^2
c 52 m^2

12 **a** 8 m^2 **b** 28 cm^2

13 **a** 162 m^2 **b** 60 000 mm^2

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Chapter review

1 **a** 3800 cm **b** 150 mm
c 4.4 cm **d** 179 cm
2 1.8 m
3 28 m
4 21 m
5 192 000 cm^2
6 2.719 ha
7 1.81 m^2
8 3840 cm^2
9 35 m^2

Chapter 4

Pre-test

1 **a** Milli represents one thousandth of a metre
b Kilo represents one thousand metres
2 **a** 1 mm^3 **b** 1 cm^3 **c** 1 m^3
3 1000 mL
4 B
5 grams, kilograms and tonnes
6 joules, kilowatts, calories

Exercise 4A

1 **a** 850 g **b** 973.4 g
c 2.3 kg **d** 7500 kg
e 32 000 mg **f** 3.57 t
g 4 700 000 mg **h** 85 000 g
i 0.078 kg **j** 3.56 t
2 3450 g
3 450 000 mg
4 52 310 000 kg
5 1.26 g
6 **a** 2900 kg **b** 3020 kg
7 4.05 kg
8 3700 g
9 13.64 t

Exercise 4B

1 **a** milligram, extremely, (any sensible response)
b Grams
c water, kilogram
d tonnes
2 **a** mg **b** g
c t **d** g
3 Any sensible answers will be accepted
4 **a** Tonnes
b Kilograms
c Grams
d Kilograms or tonnes
e Grams
f Grams
g Milligrams
h Tonnes
i Grams
j Kilograms
k Tonnes
l Milligrams
5 **a** **ii** **b** **iii** **c** **i**
d **iii** **e** **ii** **f** **i**
g **i** **h** **iii** **i** **ii**
j **i**
6 Any sensible response will be accepted
7 Weight ranges from 7.5 kg to 15 kg total, hence hire ute.
8 Answers can vary from 200 kg to 350 kg or even more, hence only the ute is needed.
9 Any sensible answer above 1 t and 3 t will be accepted, hence hire the 3-tonne truck.
10 Any sensible answer above 5 tonnes will be accepted, hire the 10-tonne truck.
11 Difficult to tell without more information.

Exercise 4C

1 **a** larger, smaller
b division, cm^3 or mm^3
c hold, solid, liquid
d millilitre

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e 1000
f kilolitre
g kilolitres

2 a 860 000 000 m^3
b 18 400 mm^3
c 2 700 000 cm^3
d 0.276 cm^3
e 1 380 000 000 m^3
f 68 000 mm^3
g 0.000129 m^3
h 2 700 000 000 mm^3

3 a 265 mL **b** 412 m^3
c 6000 cm^3 **d** 2.128 L
e 27 cm^3 **f** 0.001247 kL
g 2700 kL **h** 27.5 kL
i 0.267 ML **j** 800 mL
k 0.628 L **l** 3 200 000 L

4 0.8 m^3
5 1 089 000 000 cm^3
6 12 m^3
7 5000 mL
8 2 500 000 L
9 10 000 L
10 562 000 ML

Exercise 4D

1 a 15 625 cm^3 **b** 6 m^3
c 229 405 mm^3 **d** 3.375 m^3
e 0.08 km^3 **f** 437 920 cm^3
g 783.43 cm^3

2 a 27 cm^3 = 27 mL
b 4.284 m^3 = 4.284 kL
c 102240 cm^3 = 102.24 L
d 21952 m^3 = 21.952 ML
e 48720 mm^3 = 48.72 mL

3 a 2295.61 cm^3 = 2.30 L
b 9.03 m^3 = 9.03 kL

4 0.76 m^3
5 91 125 cm^3
6 413.1 mL
7 4.9 L
8 134.4 kL

Exercise 4E

1 a calories, kilojoules, (kJ)
b kilocalories, (kcal)
c ÷, 4.2, ×, 4.2
d kilojoules, 100 g, 100 ml
e more, consumed (or eaten)

2 a i 286 Cal **ii** 108 Cal
iii 18 Cal **iv** 235 Cal
b i 2293 kJ **ii** 244 kJ
iii 1319 kJ **iv** 764 kJ

3 a 6360 kJ, 1514 Cal
b 1914 kJ, 456 Cal
c 2670 kJ, 636 Cal
d 3050 kJ, 726 Cal
e 1128 Cal, 4738 kJ
f 1827 Cal, 7673 kJ
g 70 Cal, 294 kJ
h 4350 Cal, 18270 kJ

4 a 336 Cal **b** 132 Cal
c 490 Cal **d** 1658 Cal
e 155 Cal **f** 127 Cal
g 64 Cal **h** 903 Cal
i 211 Cal **j** 2729 Cal

5 a 1848 Cal **b** 73.92%
c 8.034 Cal/min **d** 230 min

6 a 1196 Cal **b** 59.8%
c 8.84 Cal/min **d** 135 min

7 a $83.23 **b** $5.26 **c** $161.00

8 a 120 kWh **b** $41.48

9 1530 Cal
10 248 Cal
11 Chocolate bar is 210 Cal; hiking burns all the energy in 30 min but cycling does not.
12 55 min
13 Answers will vary; total calories burned must exceed 1700 Cal.

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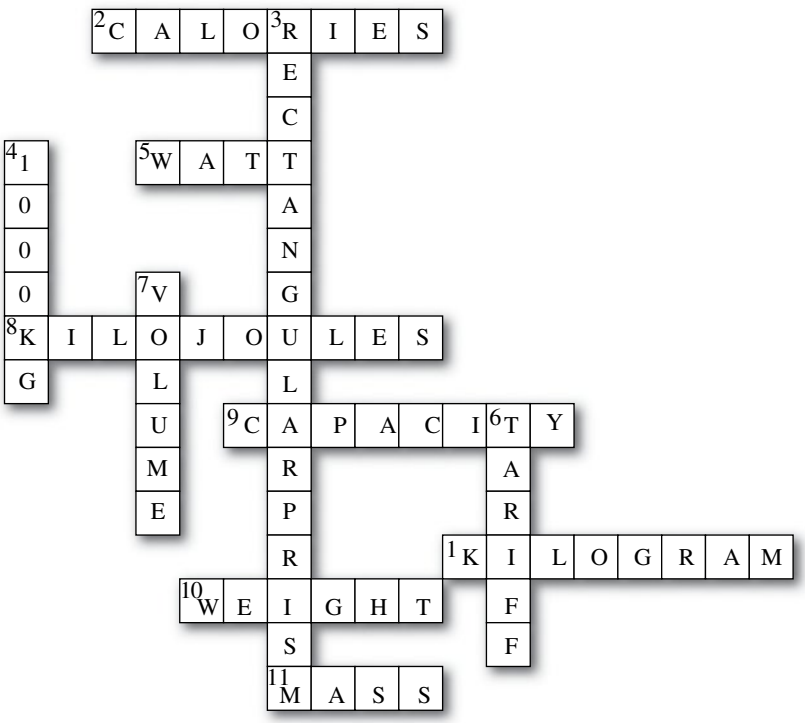
Puzzle



Chapter checklist

1 **a** mm^3 **b** cm^3 **c** m^3

2 **a** $1740\ mm^3$

b $2\,670\,000\ cm^3$

c $2.34\ cm^3$

d $100\,000\,000\,000\ cm^3$

3 **a** 75 mL **b** $2500\ cm^3$

c 6.172 L **d** 3200 kL

4 **a** $636.1\ mm^3$ **b** $8.4\ m^3$

c $2250\ cm^3$

5 **a** 91.13 mL **b** 2.73 mL **c** 95.42 mL

6 **a** $22\,99.97\ cm^3 = 2.30$ L

b $16.8\ m^3 = 16.8$ kL

c $91\,751.81\ mm^3 = 91.75$ mL

7 **a** mg **b** g **c** kg **d** t

8 **a** 27 g **b** 973.4 g

c 2.3 kg **d** 7500 kg

e 3.57 t **f** 0.094 kg

9 **a** kilograms **b** tonnes

c grams **d** milligrams

10 Answers can vary, as long as they are reasonable

a 700 kg **b** 5400 t

c 20 g **d** 4 mg

11 5865 kJ

12 555 Cal

Chapter review

1 $0.002987\ m^3$

2 $1\,400\,000\ cm^3$

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3 125 L
4 1.14 kL
5 **a** 117 649 cm^3 **b** 1056 cm^3
c 28 652 mm^3
6 9.6 kg
7 23.36 kg
8 **a** Milligrams **b** Grams
c Tonnes **d** Kilograms
9 **a** B **b** C **c** A **d** B
10 1538.46 g
11 32.22 min

Chapter 5

Pre-test

1 **a** 4 **b** 3 **c** walk
2 **a** 17 **b** Toyota **c** 9
3 **a** 6 **b** 21
c 4 **d** 9
4

	Class A	Class B	Total
Boys	12	16	28
Girls	13	10	23
Total	25	26	51

Exercise 5A

1 **a** horizontal **b** line
c included **d** excluded
e categories **f** bar
2 **a** **i** $20 **ii** $40 **iii** $30
b If there is no initial charge, then people could hire the trimmer for less than 1 hour and presumably receive free hire of the service.
3 **a** 8 **b** 5
c dog **d** guinea pig
4 **a** $6 **b** 200 g
5 **a** April 2017 to July 2017
b 2100 kWh
c Fran and Cindy have air conditioning installed, resulting in more power being used in the warmer months.
6 **a** $15 **b** $30 **c** $35
d 60 GB (included) to 75 GB (excluded)
e 90 GB (included) to 120 GB (excluded)
f $25
7 **a** Number of commencements
b January
c This is the beginning of the year during which school/university holidays have just ended; Students researching degrees or study courses.
d 2000 students
8 **a** 20%
b 7.5%
c South Australia
d Northern Territory
e Yes; South Australia has the highest proportion of households that use solar electricity.
9 **a** 17.4% **b** 24.2%
c Informal care decreased from 28.4% in 2011 to 24.2% in 2014 to 22.2% in 2017.

Exercise 5B

1 **a** sectors
b percentage
c symbols, icons or pictures
d key/legend
e information visually
2 **a** fire **b** 44% **c** $750 000
3 **a** Mari **b** 25 **c** 26%
d drawing two-fifths of a car
e Bar graph would probably be better. The current graph measures sales only over 5 cars sold per person; this measurement is too inaccurate.
4 **a** 90 students **b** 27.59%

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5 a Advantages of a pie graph:
- displays relative proportions of multiple classes of data
- size of the circle can be made proportional to the total quantity it represents
- summarises a large dataset in visual form
- can be visually simpler than other types of graphs
- permits a visual check of the reasonableness or accuracy of calculations
- requires minimal additional explanation
- can be easily understood due to widespread use in business and the media.

b Disadvantages of a pie graph:
- does not easily reveal exact values
- many pie charts may be needed to show changes over time
- fails to reveal key assumptions, causes, effects or patterns
- can be easily manipulated to yield false impressions.

c Values of the exact percentages have not been shown.

6 a 15 **b** 6
c none **d** customer service
e tutor **f** 65
g 33 **h** 97

7 a 9 **b** 7
c disagree **d** 8
e 26 **f** 44
g 6

8 a i Eastern Asia, South-Eastern Asia, Sub-Saharan Africa, Southern Asia
ii Eastern Asia decreased 134 365 000 people; South-Eastern Asia decreased 74 445 000 people; Sub-Saharan increased 38 535 000 people; Southern Asia decreased 16 205 000
b The world hunger problem is improving since the number of people who are affected by world hunger has decreased.

Exercise 5C

1 a variables **b** straight
c horizontal **d** convert

2 a Year
b Unemployment rate (as a%)
c

Year	Unemployment rate (%)
1	3.2
2	6
3	6.8
4	7
5	5.2
6	5.4
7	5.6
8	6.8
9	7.2
10	7.6

d Unemployment rate has increased overall.

3 a $15 **b** $60
c $40 **d** $100

4 a $120 **b** 38 m^2 **c** $170
d Various answers possible, such as conversion rate between currency, converting between values of trade etc.

5 a 76 **b** 140
c 146 **d** 30
e Sandra started to sprint or had to run uphill
f 17
g Sandra stopped for a rest or slowed down to a walk

6 a i 6 miles **ii** 37 miles
iii 26 miles **iv** 61 miles
b i 16 km **ii** 48 km
iii 55 km **iv** 68 km

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7 a 6000 kWh
b 5700 kWh
c 6250 kWh
d February
e This would be possibly the hottest month, so the family would possibly be using air conditioning more.
f 300 kWh

8 a 212°F
b 32°F
c 38°C

Exercise 5D

1 a

	A	B	Total
C	8	5	13
D	2	7	9
Total	10	12	22

b

	A	B	Total
C	13	48	61
D	32	25	57
Total	45	73	118

2 a

	A	B	Total
C	4	7	11
D	9	6	15
Total	13	13	26

b

	A	B	Total
C	17	24	41
D	20	5	25
Total	37	29	66

3 a 16 **b** 21
c 9 **d** 13
e 6 **f** 2

4 a 82 **b** 67
c 24 **d** 6.1%

5 a 200 **b** female
c sports car **d** 20
e 9.4%

6 a 7.29% **b** 87.5%
c 6.25% **d** 11.46%

7 a 15.43% **b** 13.53%
c 23.35% **d** 68.03%

Exercise 5E

1 a misleading **b** biased
c broken **d** size

2 a Graph 1 is misleading as the scale is broken.
Graph 2 is not misleading as the scale is not broken.
b Graph 1 is not misleading as the scale is not broken.
Graph 2 is misleading as the scale is broken.

3 a no
b That company sales have maintained the same level over the years 2014 to 2017.
c The angle at which the graph has been presented makes it difficult to determine how the sales have changed. It gives the impression that the number of sales has remained constant, whereas they could have been decreasing.

4 a That the unemployment problem has been solved.
b The title is biased, telling the reader what to think, and the scale is broken, making a small change look much bigger. Also, there is no data before Feb 2017; earlier data could show a different trend from the more recent data presented here.

5 a No, the trends appear to be different.
b Graph A, as there is a greater range of data.
c Its misleading because the scale of the graph is too small, making it difficult to see any changes.

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d It will definitely be used as it will mislead the parents into believing that the enrolment numbers have been quite consistent throughout the years.

6 a That soccer is more popular than rugby league.

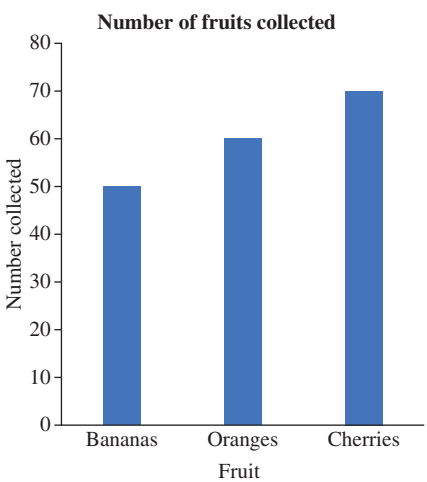
b The columns are not the same width. This makes the column for soccer look much bigger than that for rugby league when, in fact, its height is less.

c John is biased towards soccer, trying to convince his friend that it is more popular than rugby league.

7 a 50 bananas, 60 oranges and 70 cherries were collected.

b The graph is misleading as it makes it appear there are more bananas because the bananas occupy the most area and are furthest to the right, whereas there are more cherries.

c



8 a Walking is the most popular method of getting to school.

b The icons used to represent one student are not the same height. So the height of the walk column is much taller than the others when, in fact, bus is the most popular response.

9 a The larger the business, then the more likely that the business has a social media presence.

b Larger businesses have social media. Therefore, growing and successful businesses succeed by utilising social media.

c By categorising the business types without revealing the number of businesses that fall into each category, could deem the data to be inaccurate.

Exercise 5F

1 a 38.5% **b** 78.13%

c 57.89% **d** 50%

e The table shows there are differences in each position of the police force with regard to males and females.
Males have a greater percentage in the constable (60.1%) and sergeant (78.1%) roles. In the inspector role, females dominate (57.9%). However, in the role of chief inspector there are an equal number of males and females.
With males having a greater percentage in two out of the four roles, as well as being equal in one role, it would be acceptable to state that males dominate in this particular study.

2 a health

b China 20%; Australia 30%

c The pie charts show the proportion of money spent on various household expenses in China and Australia in 2022. We can see that, in Australia, the greatest proportion of expenditure (30%) is on food, whereas in China food accounts for just 20% of the total. In contrast, the greatest single expense in China is recreation (31%) compared with 25% in Australia. Food and transport

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come in equal second place in China (20%), whereas in Australia housing and recreation come in equal second place (25%). In China, another major expense is transport (20%), but this is much lower in Australia (10%). In both countries the smallest percentage of expenditure is on health care.
Overall, the data indicates that, in both countries, food, housing and recreation are the main expenses, and that in China transport and recreation take up a higher proportion of total expenditure than in Australia.

3 a 5 min

b i 15 min **ii** 30 min **iii** 19 min

c pH 5

d Answers may vary. All of the sweet substances have the possibility of causing tooth decay. However, sugar derived from honey and fruit are likely to cause less decay than cane sugar.

4 a The graph shows the ups and downs of the Australian share market since 1900. The graph's overall trend is it's distinctly heading in one direction – up.

b ending of World War I, The Great Depression, decimal currency introduced, OPEC oil embargo begins

c Check with your teacher.

d Check with your teacher.

5 a satisfied 72.4%; dissatisfied (overweight) 22.4%; dissatisfied (underweight) 5.3%

b satisfied 65.2%; dissatisfied (overweight) 17.4%; dissatisfied (underweight) 17.4%

c The data shows that 65.2% of male survey participants are satisfied with their body weight, and 72.4% of female participants are satisfied. Of those females who are not happy with their body image, there is a higher percentage who consider themselves overweight. Of those males who are not happy with their body image, the percentage is equal for over and underweight. This information shows that there is a higher percentage of participants who are satisfied than dissatisfied for both males and females, therefore the data suggests that these body image categories are not related to gender.

6 a Highest to lowest internet penetration: North America (77.4%), Oceania/Australia (61.3%), Europe (58.4%), Latin America/Caribbean (34.5%), Middle East (29.8%), Asia (21.5%), Africa (10.9%).

b Highest to lowest growth percentage of internet users: Africa (2357.3%), Middle East (1825.3%), Latin America/Caribbean (1032.8%), Asia (621.8%), Europe (352%), Oceania/Australia (179%), North America (146.3%).

c Founded on the given statistics, it can be noted that the regions with the lowest existing percentage of internet penetration are also the regions in which internet penetration is growing at the quickest rates. The internet penetration in Asia is not growing nearly as fast as it is in Africa, the Middle East and Latin America.

7 a 40%

b 25%

c Answers will vary. Australia ranks lowest among OECD countries for the relative income of people with disabilities, at 27th. Overall employment rates for people with disabilities remain low, with workforce participation at around 54% compared to 83% for people without disabilities. 1.2 million people with disabilities report difficulties using public transport.

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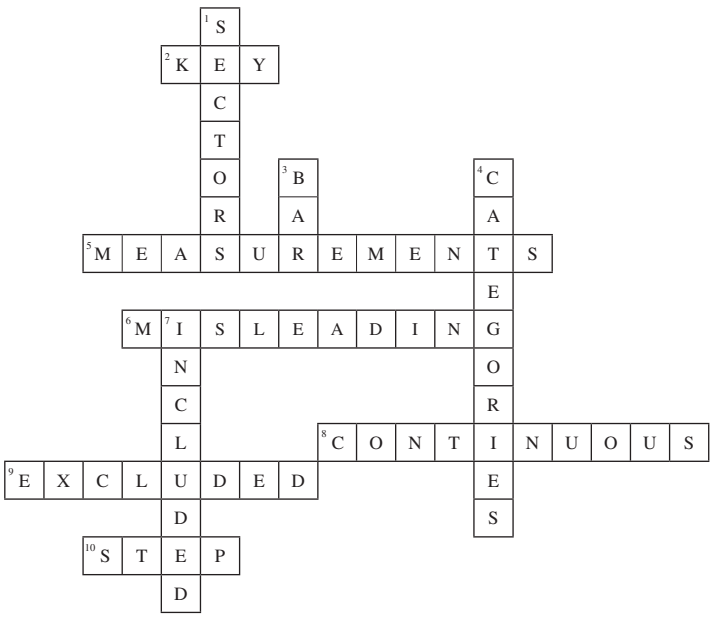
Puzzle



Chapter checklist

1. $4
2. 25 min
3. 12%
4. six magpies
5. 22°C
6. 1190 Cal
7. 20
8. This graph has been altered, as the right-hand column should be narrower. Also, the scale has been manipulated and the baselines omitted to create a larger gap in the data than is really there. The graph does not start at zero and the columns are different widths.
9. female

Chapter review

1 a 5 **b** 8
c Tuesday, Thursday and Friday of the first week, and Wednesday of the second week
d Janet worked the same number or more hours than Dan.

2 a 58 mm
b 34 mm
c February
d November to April, as these are the six months with the highest levels of rainfall.

3 a 8 a.m. **b** 1 p.m.
c 14 (inclusive) to 16 (exclusive)

4 a 65.6% **b** 14.5%
c 19.9% **d** 86.0%

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5 a 6%
b 15%
c 75–84 years
d Males more likely to have diabetes for almost all the age groups, as graph shows males have a higher proportion of having diabetes.

6 a 9 **b** 6
c 80% **d** 47.5%

7 a In Week 8 the politician had a big drop in popularity rating.
b The scale is broken, making a small drop seem much bigger.
c The producer of this graph possibly would like readers to think that the politician has done something bad to receive a big drop in popularity.

8 a i Petrol prices have increased steadily over half a year.
ii The horizontal axis is extremely skewed in time; that is, the leap from last year to last week is too big in actual time to be represented by the same length as from last week to today. Shortening the time axis in this way eliminates many of the petrol price fluctuations that have occurred, which could change the message of the graph completely.
iii The horizontal axis needs to be more consistent in scale, such as months. The price per litre on the horizontal axis should be labelled and also increased in its range; e.g. $1.00 to $2.50.
b i The population of hawks has made a strong comeback since 1975.
ii The number of birds is misleading, whereby do the values along the vertical axis represent the overall percentage of hawks compared to the number of hawks in the local area or does it represent a multiple of birds?
iii Change the measurement of the vertical axis to better units; e.g. hundreds or thousands.

9 a The graph is comparing the number of short-term visitors to Australia in 2015 and 2016.
b December 2016
c May 2015

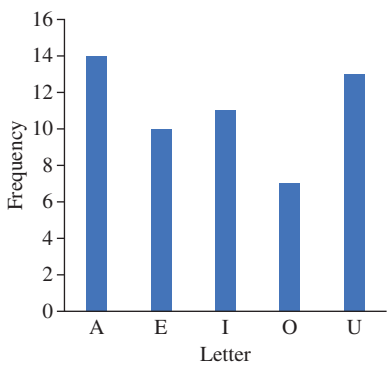
10 a The growth rate of plants were measured according to the amount of sunlight they received.
b No because between the fourth and sixth day the fastest growth rate recorded was for the plant that received 9 h of sunlight.
c 9 ha day
d Adding a title for the graph and having more plants being recorded would be suggested improvements but, otherwise, the graph is pretty good.

Chapter 6

Pre-test

1 a line graph
b pie graph
c histogram
d stem-and-leaf diagram

2 a 14 **b** 7
c



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3

State	Frequency
NSW	5
Qld	17
Vic.	6
SA	5
WA	5
NT	2
ACT	1
Tas.	1

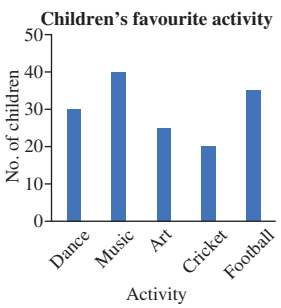
4 a 170 kg **b** 230 kg **c** 280 kg

Exercise 6A

1 a column graph

b Olivia should use the column graph as it can display relative numbers in multiple categories.

c



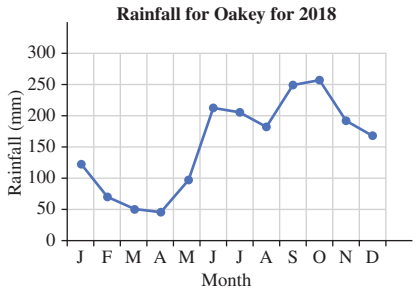
2 a line graph

b histogram

3 a line graph

b To better compare the change in rainfall throughout the year

c



4 a Stem-and-leaf plot

b Stem-and-leaf plot will suit this type of data, showing all counts neatly. Its presentation works well in this case because the sample size is low and allows Jeff to easily compare the number of snakes in each zoo.

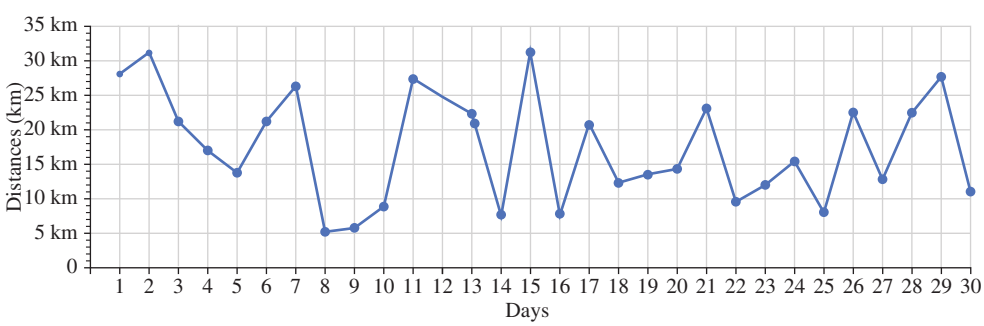
c

Stem	leaf
1	
2	0 6 8 8
3	0 2 6 6 7 8 8
4	1 2 6 6

5 a Line graph

b This allows Brandon to better measure average travel time.

c



6 a bar graph

b Bar graphs help summarise the large data in a visual form, while displaying the relative number in multiple categories.

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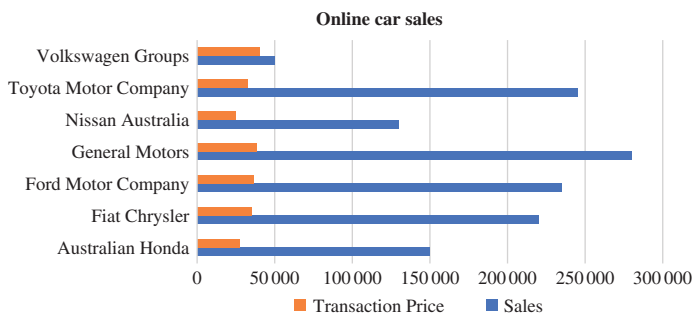
c



7 a Answers may vary. Many graphs suitable for presenting small samples; e.g. dot plots, bar graphs.

b Displaying small samples and counting a relative number of observations can be done easily with a bar graph or a dot plot.

c Check with your teacher.

8 a two-way table

b Two-way table will allow the respective number of people in their respective categories to be displayed.

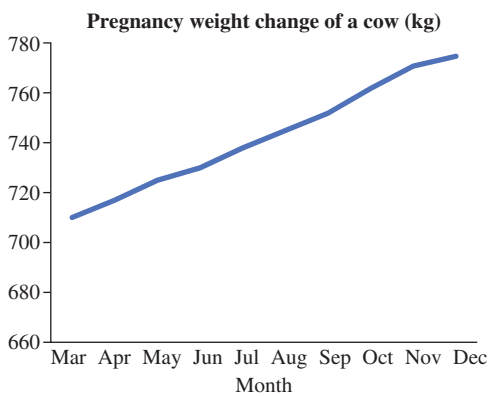
c

	Aiming for a HSC	Aiming for TAFE	Apprenticeship	Total
Male	16	46	3	65
Female	12	51	3	66
Total	28	97	6	131

9 a line graph

b A line graph allows better analysis of change over a period of time.

c



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Exercise 6B

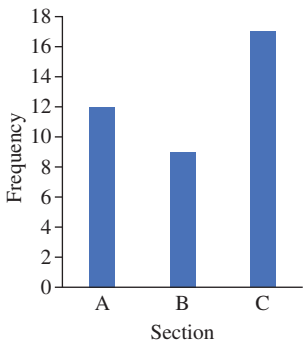
1 a

Colour	Frequency
red	5
green	7
blue	3

b

Response	Frequency
strongly agree	3
agree	6
neutral	2
disagree	3
strongly disagree	2

2



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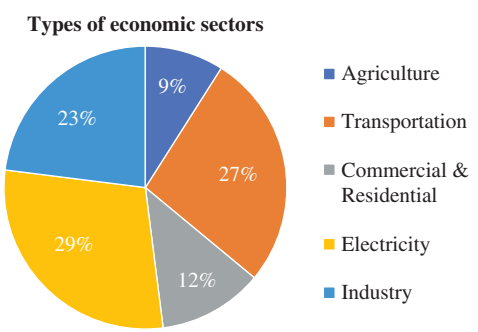


4 a table

b To allocate relative number of meals with multiple categories

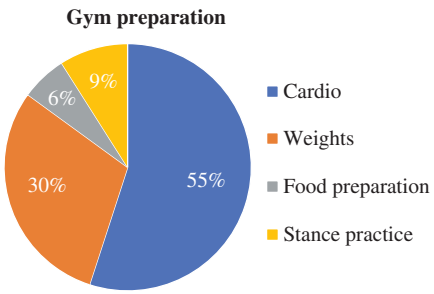
c

Meal	Frequency
chicken	7
beef	4
vegetarian	5

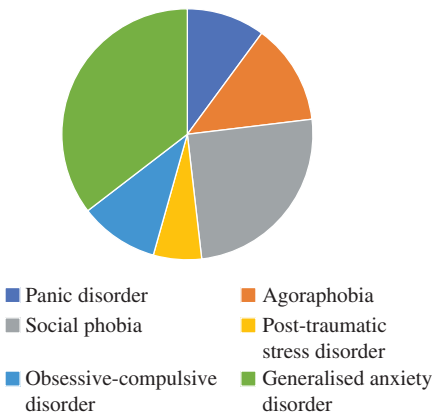
5 a pie chart

b The pie chart can easily show the cycle of an individual's daily life, where the proportion of activities is shown visually.

c

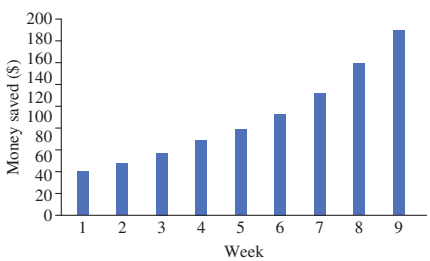


6 a



b Generalised anxiety disorder is the most common.

7 a



b Yes, Samantha seems to be saving around $30 a week and she has about $190 in Week 9.

8 a column graph

b Columns graphs can show the frequency of multiple categories.

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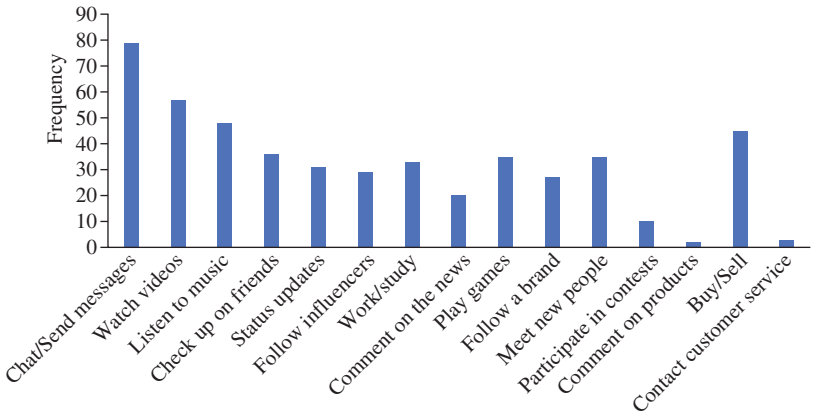
c

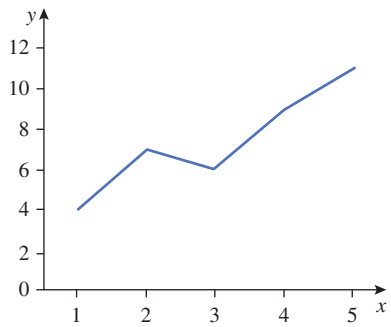


Exercise 6C

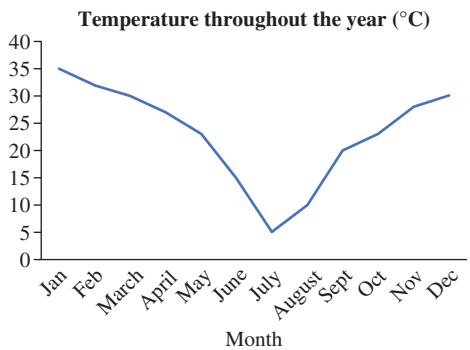
1 a 7

b 11

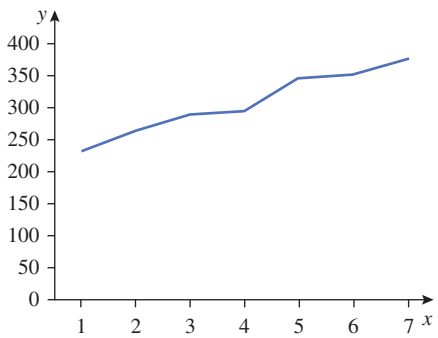
c



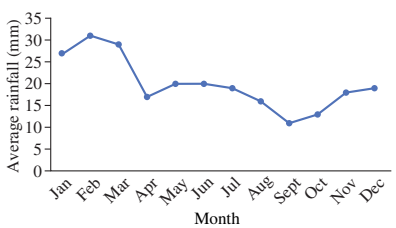
2 a



b



3 a



b According to the graph, the wettest months are February and March, and the driest months are September and October.

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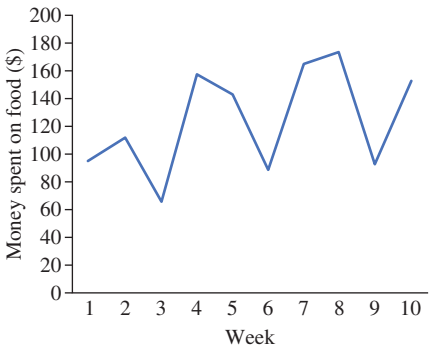
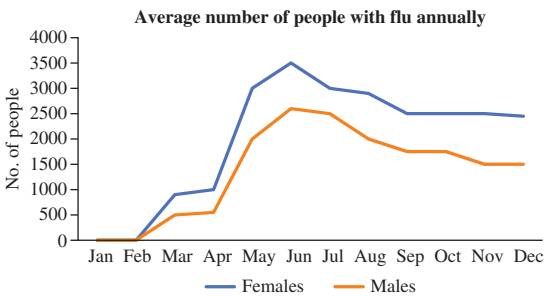
4 a



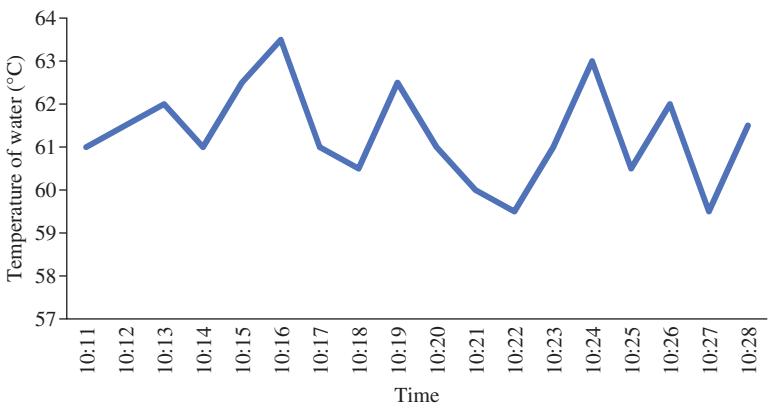
b According to the graph, the amount of money that Cameron is spending on food is gradually increasing.

5 a



b From the graph, we can observe that during the summer months of January and February fewer people catch the flu. Females seem to catch the flu more frequently than men.

6 a



b This hot water boiling system is not working efficiently, as the temperature drops below 61°C on numerous occasions.

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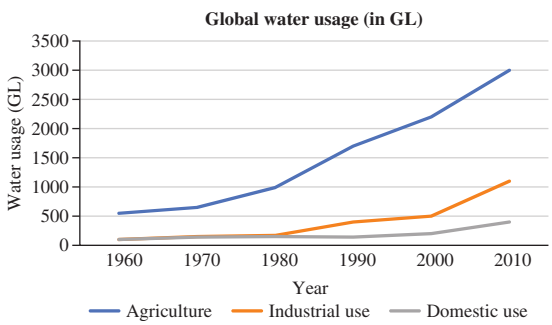
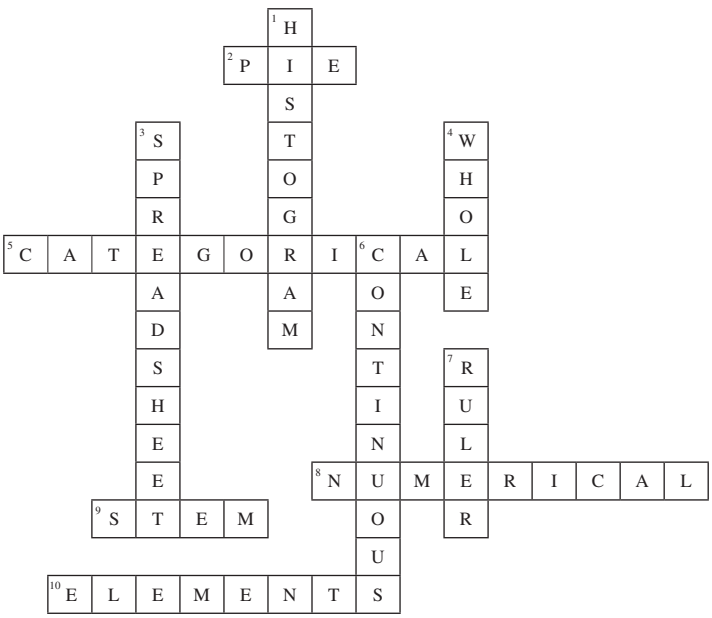
7 a



b All types of water usage is increasing as time passes, although agricultural water usage is growing the most rapidly compared to other usage types.

Puzzle



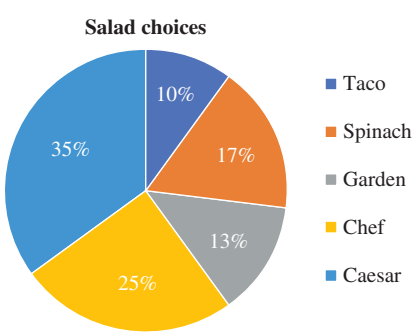
Chapter checklist

1 bar graph

2

Response	Frequency
Strongly agree	2
Agree	4
Neutral	2
Disagree	5
Strongly disagree	7

3



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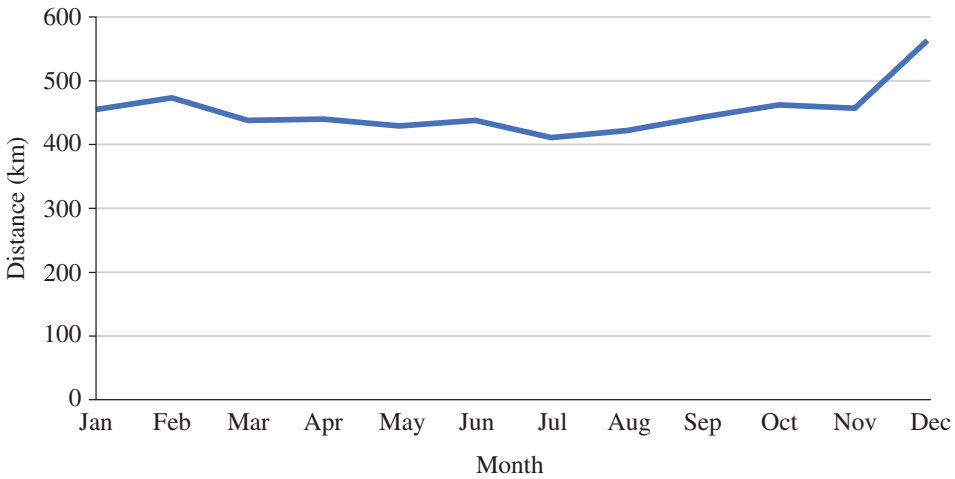
4



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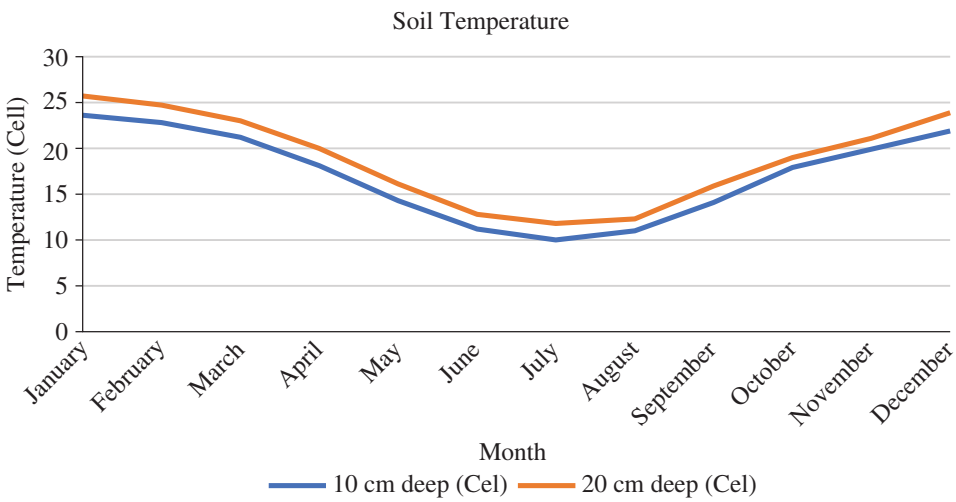
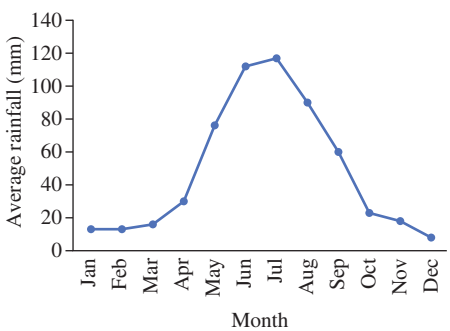


Chapter review

1 pie graph

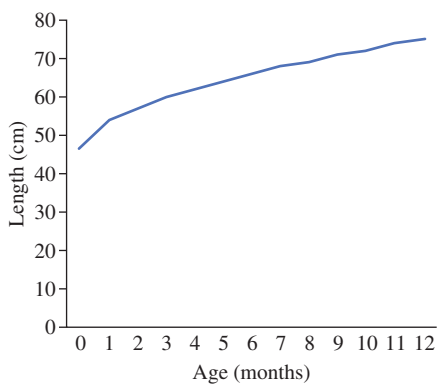
2 histogram

3 a



b We can conclude from the graph that the wettest month is July and the driest month is December.

4



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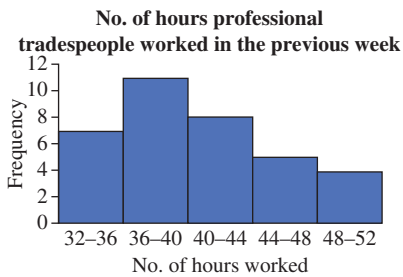
5

Response	Frequency
very positive	5
positive	8
neutral	3
negative	2
very negative	2

6



Chapter 7

Pre-test

1 **a** 4 **b** 37 **c** 13
d yes **e** 3
2 **a** yes (37) **b** 12 **c** 7
d 35 **e** 6

Exercise 7A

1 **a** words, numbers
b categorical
c numerical
2 **a** categorical **b** numerical
c categorical **d** categorical
e categorical **f** numerical
3 **a** categorical (words; e.g. bus, walk, car)
b numerical (numbers; e.g. 0, 1, 2 …)
c categorical (words; e.g. hip-hop, Country, popular)
d categorical (words; e.g. pizza, burgers, Thai)
4 **a** categorical (words; e.g. speed, fatigue, condition)
b numerical (numbers; e.g. 0, 1, 2)
c numerical (numbers; e.g. 63 km, 108 km)
d categorical (words; e.g. cannabis, meth, amphetamines)
e numerical (numbers; e.g. 0.00, 0.02)
f categorical (words; e.g. positive, negative)
5 **a** categorical (words; e.g. yes or no)
b numerical (numbers; e.g. 0, 1, 2)
c categorical (words; e.g. dog, cat, bird)
d numerical (numbers; e.g. 3.6 kg)
e numerical (numbers; e.g. 5 years)
f categorical (words; e.g. outside, laundry, my bed)
g numerical (numbers; e.g. $12, $30)
h numerical (numbers; e.g. 1 time, 2 times, 3 times)
i numerical (numbers; e.g. 2, 4)
j categorical (words; e.g. black, brown)
6 **a** categorical (words; e.g. yes, no)
b numerical (numbers; e.g. 14 days, 200 days)
c categorical (words; e.g. New Zealand, India)
d categorical (words; e.g. convention, business)
e categorical (words; e.g. New Zealand, India)
7 **a** Answers will vary; e.g. How much are you willing to spend on formal tickets? What time should the formal start?
b Answers will vary; e.g. Where should we hold the formal? Do you want your parents to attend the formal?
8 **a** Answers will vary; e.g. How many hours do you train a week? How much do you spend on sport registration? How many team sports do you participate in?
b Answers will vary; e.g. What sport do you play? What is the highest level you have played at? Have you represented your sport at State level?

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Exercise 7B

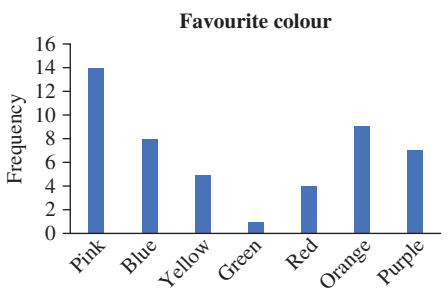
1 **a** frequency **b** count
c grouped **d** frequency
e checking **f** categorical
g 5 and 10

2 **a**

Snack	Frequency
Packet of chips	8
Muesli bar	16
Hot dog	10
Potato wedges	4
Pizza slice	22

b **i** 4 **ii** 16
c 22
d **i** 22 steps **ii** 11 steps
iii 5 steps
e Count up in 5s; best to have between 5 and 10 steps.

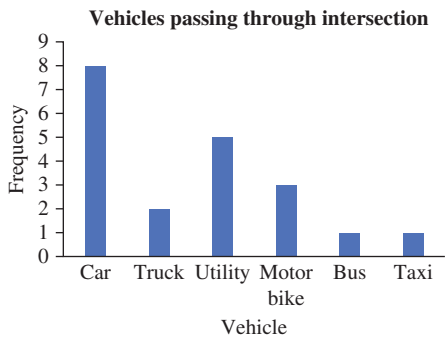
3 **a** 9 **b** 27
c 11 **d** pink
e **i** 14 steps **ii** 7 steps
iii 3 steps
f Count up in 2s
g



4 **a**

Category	Tally	Frequency
Car	𝍸 |	8
Truck	||	2
Utility	𝍸	5
Motorbike	||	3
Bus	|	1
Taxi	|	1
Total	**20**	**20**

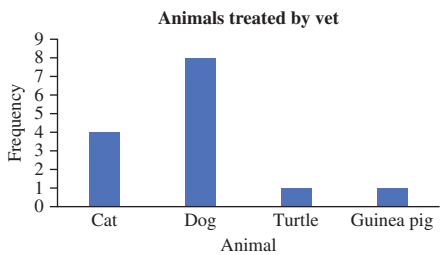
b car
c



5 **a**

Animal	Tally	Frequency
Cat	||||	4
Dog	𝍸 |||	8
Turtle	|	1
Guinea pig	|	1
Total	**14**	**14**

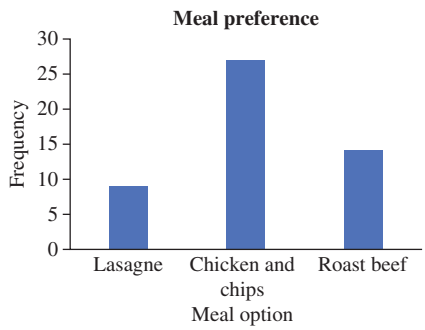
b dog
c



6 **a**

Meal	Tally	Frequency
Lasagne	𝍸 ||||	9
Chicken and chip	𝍸 𝍸 𝍸 𝍸 𝍸 ||	27
Roast beef	𝍸 𝍸 ||||	14
Total	**50**	**50**

b chicken schnitzel and chips
c



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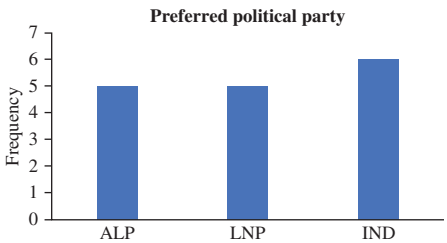
7 a

Party	Tally	Frequency
ALP	𝍸	5
LNP	𝍸	5
IND	𝍸 |	6

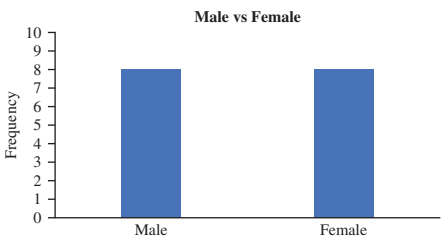
b



c

Gender	Tally	Frequency
Male	𝍸 |||	8
Female	𝍸 |||	8

d



8 a

Region	
Oceania (New Zealand, PNG…)	1 500 000
North-West Europe (UK, France…)	1 400 000
South-East Europe (Italy, Spain…)	200 000
Americas (Canada, USA, Mexico…)	1 100 000
North Africa and the Middle East (Israel, UAE…)	100 000
South-East Asia (Thailand, Indonesia…)	1 400 000
North-East Asia (China, Japan, Korea…)	2 500 000
Southern and Central Asia (India, Pakistan…)	400 000
Sub-Saharan Africa (South Africa…)	100 000
Total	8 700 000

b

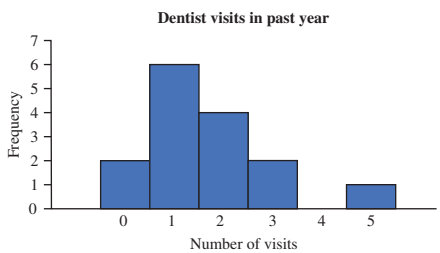
Exercise 7C

1 a column, no
b horizontal
c frequency
d touch
e gap

2 a

Visits	Tally	Frequency
0	||	2
1	𝍸 ||	6
2	||||	4
3		2
2		0
5	||	2
Total	**15**	**15**

b one visit per year
c count up in 1s
d



3 a 40 students
b 20–24 min
c 2s

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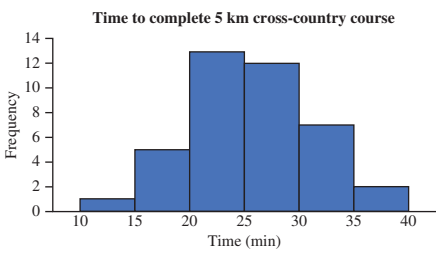
d



4 a 22 h

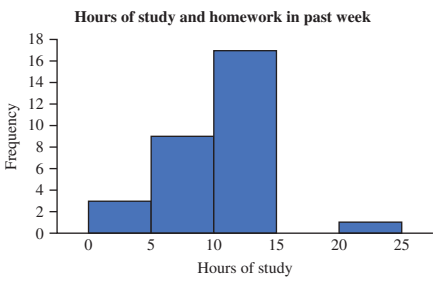
b 1s > 22 steps, 2s > 11 steps, 5s > 5 steps; therefore, 5s is best.

c Groups of 5 and need to start at 0, so 0–4, 5–9, 10–14 etc.

d

Hours	Tally	Frequency
0–4	|||	3
5–9	𝍸 ||||	9
10–14	𝍸 𝍸 𝍸 ||	17
15–19		0
20–24	|	1
Total	**30**	**30**

e

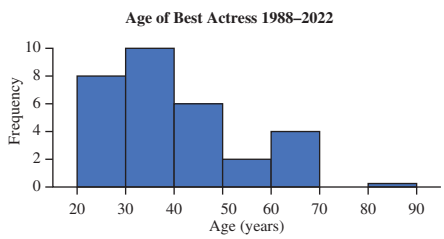


5 a youngest = 22, oldest = 64, group in 10s

b

Age Best Actress	Tally	Frequency
20–29	𝍸 |||	8
30–39	𝍸 𝍸	10
40–49	𝍸 |	6
50–59	||	2
60–69	||||	4
70–79		0
80–89		0
Total	**30**	**30**

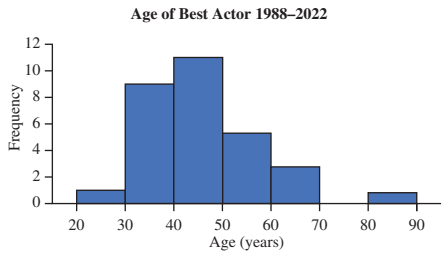
c



d

Age Best Actor	Tally	Frequency
20–29	|	1
30–39	𝍸 ||||	9
40–49	𝍸 𝍸 |	11
50–59	𝍸	5
60–69	|||	3
70–79		0
80–89	|	1
Total	**30**	**30**

e



f The range is similar, with most ages between 20 and 70 years.

g Differences include highest frequency for Best Actress is between 20 and 40 years. Highest frequency for Best Actor is 40–50 years. Best Actor has a frequency of only 1 in 20–30 years.

6 a No gap; would expect a close range of ages from 15 to 17 years.

b Yes. Although the students' ages would not produce a gap, the teachers would have a greater range and likely some gaps between the ages of the older teachers.

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c Yes, you would expect a wide range of ages amongst a random group of people.

d Yes, you would expect a wide range of ages amongst a random group of people.

7 a false **b** true

c false **d** true

e true

Exercise 7D

1 a plots, distribution

b data, range, dot

c leaf, order

d stem, leaf

2 **A** and **iii**, **B** and **iv**, **C** and **ii**, **D** and **i**

3 a 10 students

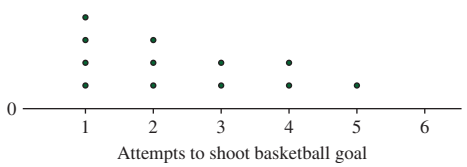
b 5, 7.5, 9, 15, 15, 20, 20, 20, 25, 45

c 20 min

d 45 min

e 5 min

4



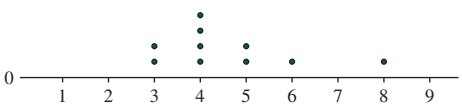
Attempts to shoot basketball goal

5 a 18 students

b 13 students

c 4 cm

6 a



Puppies born per litter

b Most common litter size is four puppies.

c Eight puppies in one litter is unusual.

7 a lowest = 61, highest = 151

b Use 6 to 15 for the stem.

c **Student Heart Rates after doing 1 minute of jumping jacks**

Stem	Leaf
6	1
7	
8	4 6 8
9	2 9 9 9
10	
11	3 4
12	1 2 4 4 7 9
13	8
14	4 7 8
15	1

Key: 6|1 = 61

8 a

Stem	Leaf
1	6 7 8
2	0 2 5 5 8 8
3	1 1 3

b 33°C

c 16°C

9 a

Stem	Leaf
2	2 5 6 6 8 8 9 9
3	0 2 3 3 3 4 5 5 6 9
4	4 5 5 5 5 9
5	1 4
6	1 1 2 4
7	
8	

b

Stem	Leaf
2	9
3	2 3 6 7 7 8 8 8 9
4	0 1 1 3 4 5 5 6 6 7 8
5	0 0 2 3 5
6	0 0 0
7	
8	4

c The Best Actress ages are younger than those for the Best Actors. For Best Actress, the most common ages are 20–30 years and 30–40 years. The most common age group for Best Actor is 40–50 years, with 11 actors in their 40s receiving a Best Actor award over the period studied.

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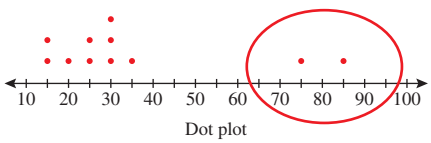
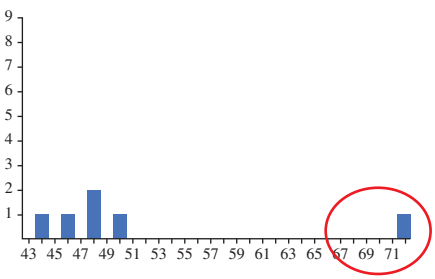
Exercise 7E

1 outlier, long, above, below

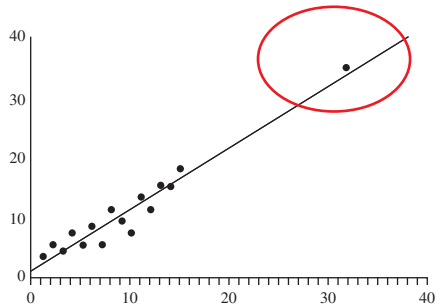
2 a



b

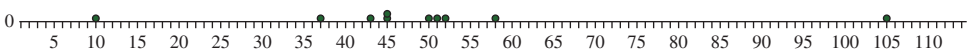


c



3 a 10, 37, 43, 45, 45, 50, 51, 52, 58, 105

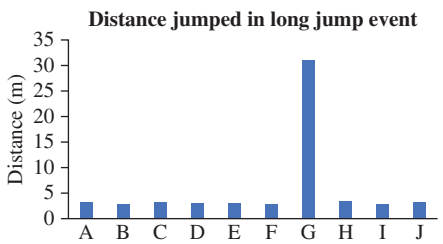
b



c 10 and 105 are outliers.

d 10; perhaps the last day is on a weekend and the business has reduced working hours.
105; perhaps the business ran a promotion and lots of people signed up.

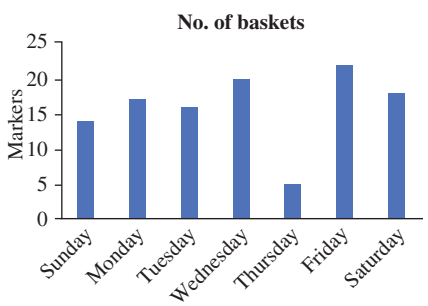
4 a



b student G (31 m is an outlier.)

c 31 m is likely to be a mistake and should be 3.1 m.

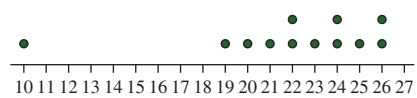
5 a



b Thursday, with only five baskets picked, is an outlier.

c The worker may not have worked a full day for whatever reason (e.g. rain, heat, illness etc.)

6 a



b 10 is an outlier.

7 a United States result of 121 medals is an outlier.

b Outlier(s) for Gold: USA, Silver: USA, Bronze: Canada and United States.

c The size and wealth of the United States increases its performance at the Olympic Games.

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8 a Outliers: 416 mm and 413 mm; 2013 (416 mm) and 2011 (413 mm) are a long way above the rest of the data and are outliers. The 2004 reading of 198.7 mm could also be considered unusual enough to be an outlier

b No outliers; Although there are some high readings in the 200s, there are enough of them to not be considered different enough to be outliers.

c Outlier: 313.6 mm; The 2017 reading (313.6 mm) is a lot different from the rest of the data.

d No outliers; All data is low and close together; no outliers.

e Outlier: 134.2 mm; The 2015 reading (134.2 mm) is a lot different from the rest of the data.

f Outliers: 105 mm, 77.8 mm, 72.6 mm; July looks to be a traditionally dry month with the mean being only 28 mm. The 1998 reading (105 mm) is very different. You could also consider the 1999 (77.8 mm) and 2008 (72.6 mm) readings to be outliers.

g No outliers; All data is low and close together; no outliers.

h Outliers: 74.2 mm, 119 mm, 92.6 mm, 104 mm; September looks to be a traditionally dry month with lots of data below 25 mm. The mean is 36.5 mm. The rainfall in 1997, 1998, 2010 and 2016 are all above 70 mm, so would be considered unusual and outliers.

i No outliers; the data is quite spread out.

j Outlier: 165.2 mm; The 2001 reading of 165.2 mm is a lot different from the rest of the data.

Exercise 7F

1 a information, equal

b trends, between

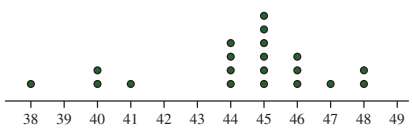
c simple, message

d careful, lose, features

2 a

Age of mother	Tally	Frequency
38	|	1
39		0
40	||	2
41	|	1
42		0
43		0
44	||||	4
45	$\cancel{\text{IIII}}$ |	6
46	|||	3
47	|	1
48	||	2
Total	**20**	**20**

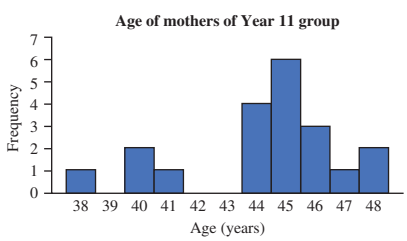
b i



ii

Stem	Leaf
3	8
4	0 0 1 4 4 4 4 5 5 5 5 5 5 6 6 6 7 8 8

iii



c The dot plot and histogram give a quick overview of the data distribution and are visually appealing. The stem-and-leaf plot is not suitable as the range is too small.

3 a The data is presented in the form of a table but without showing the table outline.

b Yes, there are more mobile subscriptions than people, so 128% indicates this. Many business people would have a separate phone for personal and business calls.

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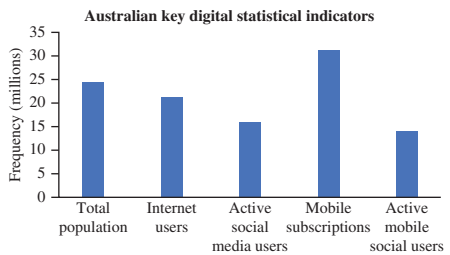
c

Category	Frequency (millions)
Total population	24.48
Internet users	21.18
Active social media users	16
Mobile subscriptions	31.35
Active mobile social Users	14

d



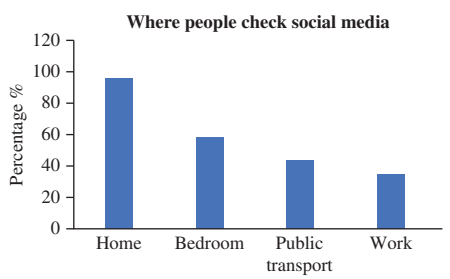
e The column graph gives a good visual comparison of the adoption of digital technology, especially with respect to the population presented in the first column.

4 a The data is presented in a table form with graphics added.

b

Category	Frequency (%)
Home	96
Bedroom	59
Public transport	43
Work	35

c



d The information does not work well as a dot plot because it is categorical data.

e A column graph is the most suitable method to present the information, as you can visually compare the values to each other. Graphics could be added for visual appeal.

5 a The data has been presented in a column graph.

b

Age	Frequency (estimated)
15–19	40
20–24	40
25–29	100
30–34	300
35–39	550
40–44	1000
45–49	1800
50–54	2100
55–59	2050
60–64	2350
65–69	2500
70–74	1700
75–79	1300
80–84	850
85+	980

c No, grouped data does not work well as a dot plot.

d Since the data shows the frequency of grouped numerical data, a histogram is the most appropriate method and should be drawn without spaces between the columns.

6 a The data is presented as a bar graph. (Note — a horizontal column graph is called a bar graph)

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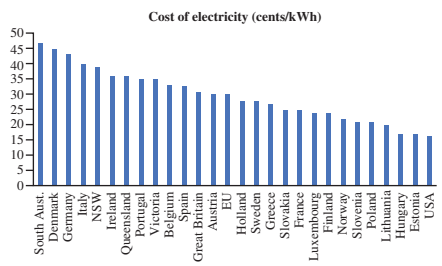
b

Region	Cost of electricity (cents/kWh)
South Aust.	47
Denmark	45
Germany	43
Italy	40
NSW	39
Ireland	36
Queensland	36
Portugal	35
Victoria	35
Belgium	33
Spain 33	33
Great Britain	31
Austria	30
EU	30
Holland	28
Sweden	28
Greece	27
Slovakia	25
France	25
Luxembourg	24
Finland	24
Norway	22
Slovenia	21
Poland	21
Lithuania	20
Hungary	17
Estonia	17
USA	16

c The data could not be drawn as a dot plot as it is categorical data.

d



e The newspaper graph does not show a scale and has exaggerated the difference between expensive and cheaper regions.

f The newspaper method is suitable but is misleading in the difference between what regions pay for electricity.

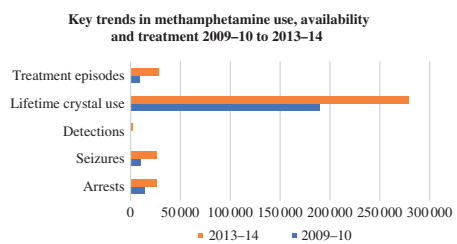
g A column graph is a suitable method but it should be drawn to a scale so as to not be misleading.

h The newspaper appears to be saying the electricity providers are making a profit because their share prices have been increasing.

7 a The information has been presented in a table, with multiple columns indicating the frequency for two different time periods.

b Using a table means that the numbers are easily read rather than have to be estimated on a graph.

c



d Since the data is categorical, either a table or column graph is the most suitable method to present the data.

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e The table is more suitable because the data has a large range of values. Including the data on lifetime crystal use makes the other categories seem to have little significance on the column graph, when they absolutely do.

8 a column graph

b A comparison can easily be made between usage of desktop and mobile phone for accessing the internet.

c

Country	Desktop	Mobile
USA	2100	5500
Canada	2300	3900
France	1800	4000
Germany	1700	3300
Italy	1750	2800
Spain	1800	4600
UK	1800	4600
Argentina	1800	6500
Brazil	1800	4500
Mexico	1500	3900
India	1500	2900
Indonesia	1500	4000
Malaysia	1500	3800

d No, it could not be shown as a dot plot as it compares two different pieces of information.

e The column graph is the best way to display the information, as it is easy to see differences in the data.

9 a column graph

b The differences are easy to see. The use of colour makes it easy to identify different countries.

c

Country	Smartphone	Tablet	Laptop	Desktop
UK	75	52	66	39
France	77	45	71	50
Germany	78	43	72	51
Italy	85	51	68	51
USA	69	45	61	47
Japan	58	25	66	38
Australia	77	46	67	49
Spain	87	55	73	56
Sweden	78	44	69	41

d The information could also be presented in a table.

e The column graph is a suitable way to display the data. It is attractive with the use of colour and trends can be seen easily. The data for each country does not add to 100% so a pie chart would not be suitable.

10 a bar chart

b The main search reasons are shown at the top with lower values shown below it. The percentages on the bar emphasise the importance of Food & Beverage searches on mobile devices above other reasons like banking.

c A picture graph would add interest and explain the categories. A table also would be useful to display the information.

d The bar chart is a suitable method and shows the information well.

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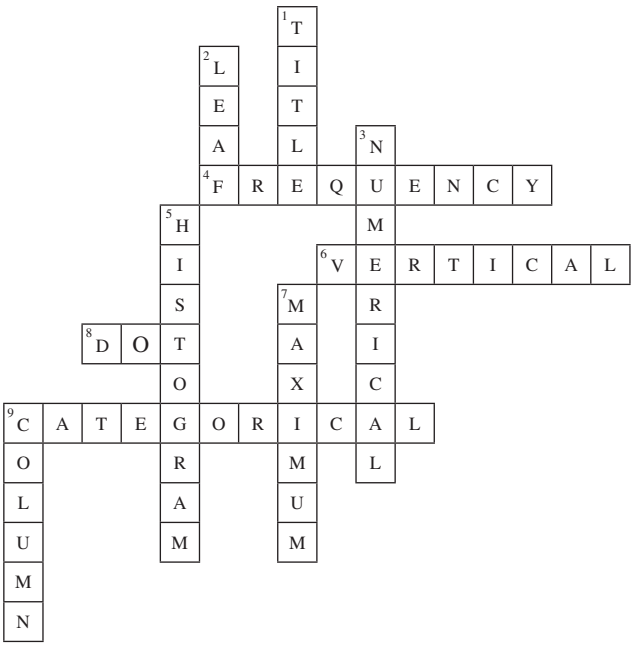
Puzzle



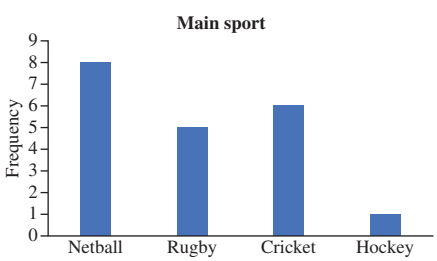
Chapter checklist

1 a numerical **b** categorical

2 a

Sport	Tally	Frequency
Netball	𝍸 | | |	8
Rugby league	𝍸	5
Cricket	𝍸 |	6
Hockey	|	1
Total	**20**	**20**

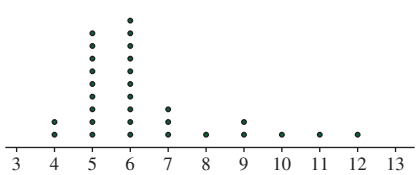
b



3 a

Hours of study	Tally	Frequency
4	| |	2
5	𝍸 | | | |	9
6	𝍸 𝍸 |	10
7	| | |	3
8	|	1
9	| |	2
10	|	1
11	|	1
12	|	1
Total	**30**	**30**

b



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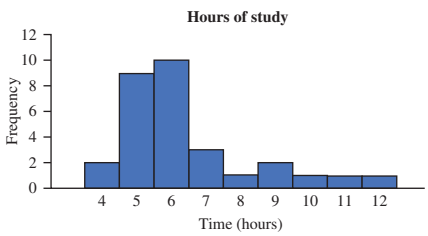
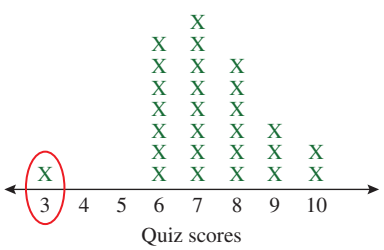
c

Stem	Leaf
0	4455555555566666
	66666777899
1	012

d



4



5 a i dot plot: yes
ii stem plot: yes
iii column graph: yes
iv histogram: yes

b i dot plot: Yes but it would be difficult to make sense of large frequencies.
ii stem plot: Yes but all data would be on the same line.
iii column graph: Yes, this would work well but numbers would be better presented as a histogram with no gaps between columns.
iv histogram: Yes, this would work very well.

c histogram

Chapter review

1 a numerical (0, 1, 2 etc.)
b categorical (dog, cat)
c categorical (AMP, BHP, NAB)
d numerical (1500, 3000)
e numerical (132.6, 156.8)
f categorical (unleaded, diesel, gas, electricity)

2 a Answers will vary. Examples: How often do you have lunch from the canteen? How much do you spend on lunch? On a scale of 1 to 5 how would you rate the canteen food?

b Answers will vary. Examples: What days do you have lunch at the canteen? Do you think the food is good at the canteen?

3 a

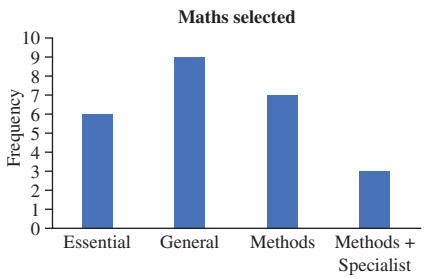
Maths	Frequency
Essential	6
Applications	9
Methods	7
Methods + Specialist	3

b General

c i nine steps
ii five steps
iii two steps

d either 2s or 1s

e



4 a

Age (years)	Frequency
18	7
19	8
20	6
21	11
22	4
23	5
24	5
25	4
Total	**50**

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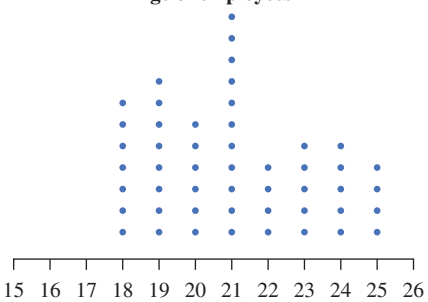
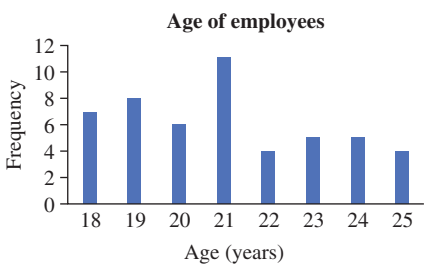
b



c

Stem	Leaf
1	888888899999999
2	000000111111111122
	223333334444455555

d



e A histogram is the easiest graph to interpret.

5 a 4; perhaps an entry error.

b 3, 115, 126; unknown reason

c 14; some of the raisins have been eaten from that box.

d −85; perhaps an entry error.

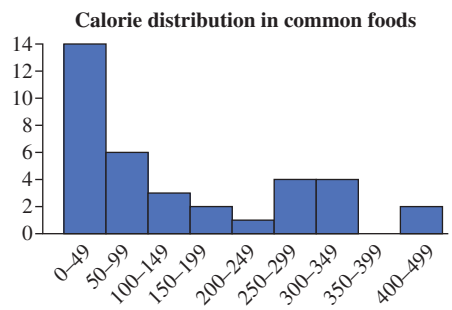
6 a numerical data

b lowest = 8 (cucumber), highest = 420 (pork)

c Counting up in 50s takes nine steps.

Calories	Frequency
0–49	14
50–99	6
100–149	3
150–199	2
200–249	1
250–299	4
300–349	4
350–399	0
400–449	2
Total	**36**

d

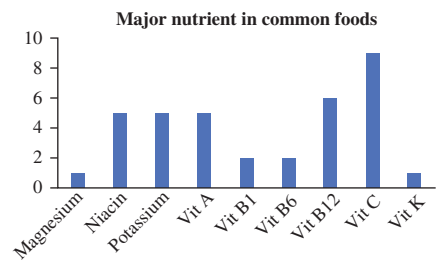


7 a categorical

b

Major nutrient	Frequency
Magnesium	1
Niacin	5
Potassium	5
Vitamin A	5
Vitamin Bl	2
Vitamin B6	2
Vitamin B12	6
Vitamin C	9
Vitamin K	1
Total	**36**

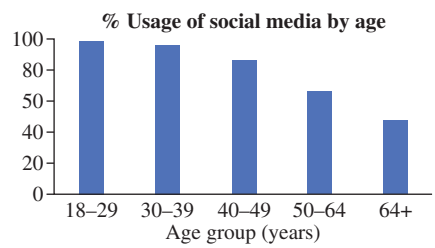
c



8 a info graphic

b Visually appealing and retains information

c



d No, dot plots are for numerical data that is not grouped.

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e A column graph is most appropriate. Although we have numerical data, the groups are not the same size, so a histogram is not appropriate.

9 A and iii, B and i, C and v, D and ii, E and iv

10 A and ii, B and iii, C and i

Chapter 8

Pre-test

1 **a** 20.71 **b** 0.5 **c** 0.33

2 1, 4, 5, 6, 7, 9, 10, 11, 13, 13, 17, 19, 21
Median = 10

3 **a** 26.5 **b** 44 **c** 63.5

4 **a** 25% **b** 43.75%

5 **a** $x = 1$ **b** $x = 9$

6 **a** 4 **b** 49 **c** 0.25

7 24 minutes

8 smallest: 41
largest: 95
most common (mode): 78

Exercise 8A

1 **a** centre

b mode, mean, median

c Mode

d two, bimodal

e average, sum, divided

f $\bar{x} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$

g middle, ascending

h even, two

2 **a** **i** 5.2 **ii** 5 **iii** 5 and 6

b **i** 77.2 **ii** 85 **iii** 85

c **i** 8.0 **ii** 7.95 **iii** no mode

d **i** 20.1 **ii** 19 **iii** 18

e **i** 2.1 **ii** 2 **iii** 3

3 **a** Mode = 1 **b** Mode = 12

c Mode = 75%

4 **a** Mean = 121 **b** Median = 115

c Mode = 115

5 **a** Mean = \$886 **b** Median = \$850

c Mode = Bimodal (\$700 and \$900)

6 **a** Mean = 1.9 **b** Median = 2

c Mode = Bimodal (1 and 2)

7 **a** **i** Mean = 16.5 **ii** Mode = 12

iii Median = 14

b **i** Mean = 222.5 **ii** Mode = 225

iii Median = 225

c **i** Mean = 57.4 **ii** Mode = 52

iii Median = 55

d **i** Mean = 115.9 **ii** Mode = 100

iii Median = 115

8 **a** **i** Mean = 2.1 **ii** Mode = 2

iii Median = 2

b **i** Mean = 5.2 **ii** Mode = 6

iii Median = 5.5

c **i** Mean = 1.7 **ii** Mode = 1

iii Median = 1.5

d **i** Mean = 1.8

ii Mode = no mode

iii Median = 2

Exercise 8B

1

Measure of central tendency	Advantages	Disadvantages
Mean	All data is taken into account. Easy to understand and calculate.	Extreme values distort data. Percentages and ratios could be challenging.
Median	Simple to understand and calculate. Not affected by extreme values.	May not be able to sort. All items may not be taken into account. An even number of data points requires an extra calculation step.
Mode	Simple to find. Not affected by extreme values.	Not based on all the data Can have more than one mode or no modes at all.

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2 **a** Mean = 8
b Median = 8
c Mode = 7
d Mean and median shoe sizes do not help determine which sizes the shop should stock. But the size which is the mode (7) should be stocked the most, and sizes which occur the least should have the least stock. So mode is the better measure.

3 **a** Mean = 16
b Median = 15
c Mode = no mode
d The better measure would be the mean, as it takes all results into account and gives Noah the best idea of his overall performance. There is no mode and the median does not take all results into account.

4 **a** Jamie:
i Mean = 82.4 **ii** Mode = 85
iii Median = 83.5
Scarlett:
i Mean = 86.7 **ii** Mode = 95
iii Median = 89
b Scarlett's higher scores were shown in the mean and the median, as she had several high results. The mode scores were irrelevant to their actual results.

5 Median = 1

6 **a** Mean = $257
b Median = $75
c Mode = $50
d The son would suggest the mean, as it takes into account the extreme $2350, which brings the mean up to $257. The father is trying to be fair and puts his son in the middle of his friends' range. The younger sister may just be choosing the mode as it happens to be at the lower end of the range and is closer to what she gets.
In this situation the median would be the best measure of tendency, as it is not affected by the outlier. Since it is the middle of the ordered values, there are as many friends who get less as there are friends who get more.

7 Student A Mean = 37.3; Student B Mean = 38.3; Student C Mean = 37.3
Student A Median = 45; Student B Median = 43; Student C Median = 42
Student A Mode = 45; Student B Mode = Bimodal (30 and 50); Student C Mode = 48
The fairest measure of central tendency for this scenario would be the mean, as it takes into account all results of the students, whether high or low. The median only takes into account the middle result and the mode is only found by results being the same rather than the overall score of the student. The math award should go to Student B.

Exercise 8C

1 **a** range
b difference, highest, lowest
c interquartile range
d measure, average
e outside
f mean, median

2 Range: 15, IQR 9, SD 5.5

3 **a** $Q_1 = 14$, $Q_3 = 20$, IQR = 6
b 5. One value (0) is lower than this.
c 29. No values are greater than this.
d There is an outlier in the data, the value 0.

4 **a** 3 and 25
b 25 – 3 = 22, range is 22.

5 **a** 20, 37, 40, 75, 80, 85, 100
b $Q_2 = 75$
$Q_1 = 37$
$Q_3 = 85$
c IQR = 48

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6 **a** 5 **b** 2.4

7 **a** Mean 89

b The standard deviation for Carrie's friends losing weight is 16.66 from the mean (89).

c 72, 75, 85, 105, 120

d Range = 48

e IQR = 30

8 **a** The mean = 135.85

b The standard deviation = 18.19.

c 102, 120, 140, 148.5, 163

d Range = 61

e IQR = 28.5

9 Mean 29.38, standard deviation 2.29

10 Mean 3.28%, standard deviation 0.62%

11 **a** 14.9

b 2 is an outlier.

c Mean without outlier = 17. The outlier 2 does affect the mean, lowering it from 17 to 14.9.

d The outlier is either a valid score because Cooper had an off day, in which case it should be kept, or it is an error in writing down the score, in which case it could be removed. Cooper would know if he had an off day, if he didn't then he must have made an error recording it.

12 **a** The median is $40 000.

b $120 000 is an outlier.

c The median without the outlier is $37 000. The outlier does not affect the median much (with the outlier it is $40 000).

d The outlier can be removed as it does not have much effect on Faith's research into the prices of cars.

Exercise 8D

1 **a** quantiles **b** running total

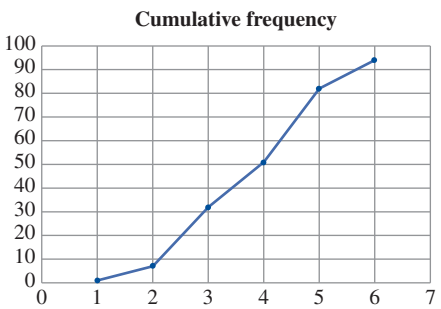
c three, four **d** value

e nine, ten **f** 99, 100

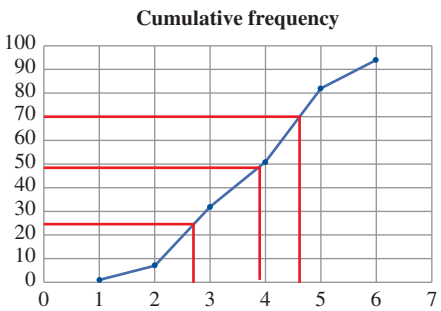
2

Score	Frequency	Cumulative frequency	Cumulative frequency %
1	1	1	1.1%
2	6	7	7.4%
3	25	32	34.0%
4	19	51	54.3%
5	31	82	87.2%
6	12	94	100.0%
Total	**94**		

3



4



Q_1: 2.7

Q_2: 3.9

Q_3: 4.7

5 **a** 7

b 5

c 9

d A quarter of the matches won lie below Q_1 and three-quarters lie above it.
Half the matches won lie below Q_2 and half lie above it.
Three-quarters of the matches won lie below Q_3 and a quarter lie above it.

6 **a** Age 16.5 years **b** Age 15 years

c Age 17.7 years

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7 a Andrew's score lies between the 7th and 8th deciles.
Beni's score lies between the 9th and 10th deciles.
b 7 tenths of the students did worse than Andrew and two tenths (one fifth) did better.
9 tenths of the students did worse than Beni and less than one tenth did better.

8 a Brolga A's mass is between the 5th and 6th percentiles.
Brolga B's mass is between the 81st and 82nd percentiles.
b Brolga A is heavier than 5% of the brolgas and lighter than 94%.
Brolga B is heavier than 81% of the brolgas and lighter than 18%.

9 a Virat is 17.7 years old.
b Zara is 15 years old.
c 50% or the visitors to the park are between the ages of Zara and Virat.

10 It would be better to receive a high percentile, as the higher percentile means a higher grade.

11 The 85th percentile would mean that Troy has been waiting longer than others. It means that 85% of the other people waited 32 minutes or less and 15% waited 32 minutes or longer, which is a bad outcome.

12 Katrina and Elliot can only afford 25% of the houses in their area, as the other 75% cost $350 000 or more.

Exercise 8E

1 a displayed
b difference(s)
c histogram, column
d mean
e cluster
f gap

2 a Not accurate. The distribution does not have an outlier.
b Not accurate. The distribution does not have a gap.
c Accurate: The data is tightly packed around the mean.

3 The data is spread out and loosely packed around the mean.
The dispersion is widely scattered from 8 to 19.
There is one cluster between 8 and 19.
The data has one gap between 2 and 7.
0–1 could be considered as an outlier.

4 January's data shows a spread that is tightly packed around the mean with a wide value. The data has a gap between 149 and 179, with what could be considered an outlier at 179.
February's data is spread out around the mean with a wide value. There are two main clusters, with a small gap between 143 and 161. February does not contain an outlier.
February's data is loosely packed and more spread out than January. Only January's data contains an outlier. Both contain wide values. February has 3 less observations making it slightly less dense.

5 Graph A's data is spread out around the mean. The data has a gap between 2.25 and 6.75. Graph A has what could be considered an outlier at 6.75.
Graph B's data is tightly packed around the mean with a small value. This graph does not contain an outlier.
Graph A's data is loosely packed and more spread out than Graph B. Only Graph A's data contains an outlier. Both contain small values and only one cluster.

6 Semester 1's data is spread out around the mean. The data has a gap between 30 and 51. There are two clusters, one from 0–30, and the other from 51–90.

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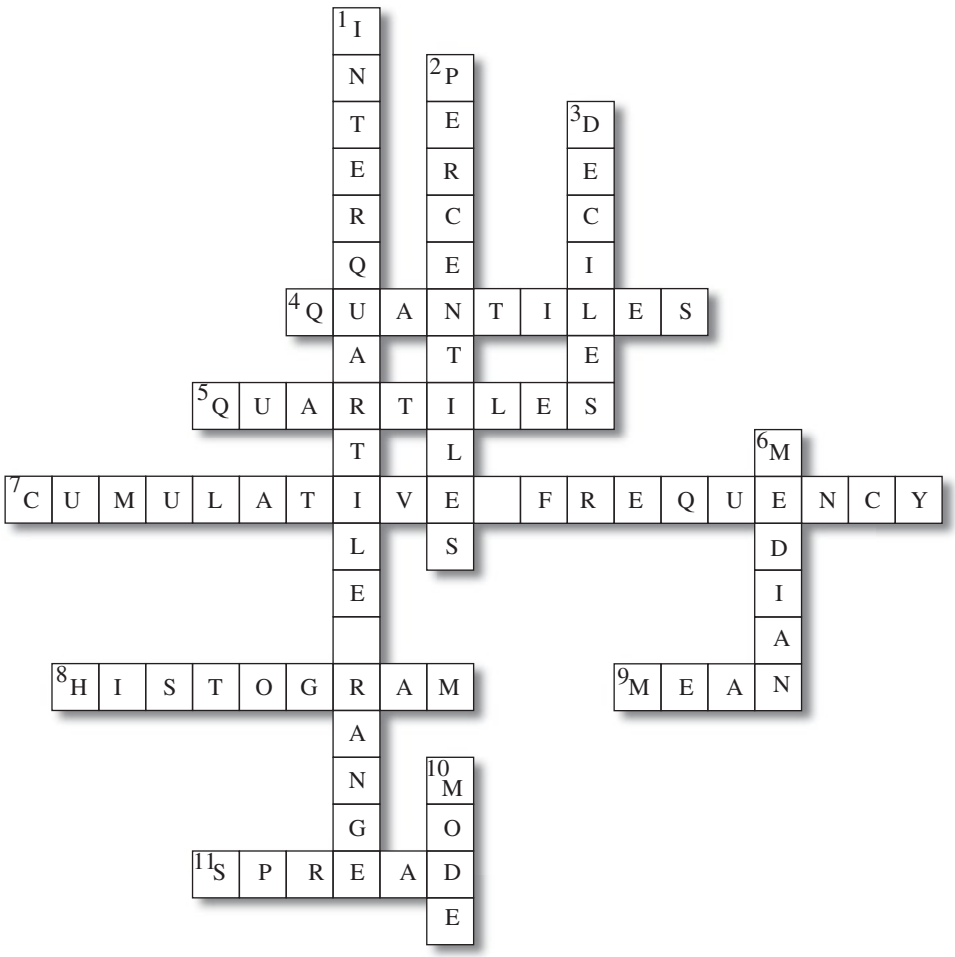
Semester 2's data is spread out around the mean. This graph could contain an outlier at 0–10. Semester 2 has two gaps, one at 10–21, and the other at 50–81. Semester 2 also has two clusters, the first at 21–50, the second at 81–100.

Both graphs have the same size spread, and both have two distinct clusters. Semester 2 is the only graph that could contain an outlier. Semester 1 has one gap, whereas Semester 2 has two gaps.

Puzzle



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Chapter checklist

1 Mode = 7

2 Mean = 6.5
Median = 7

3 Median is the better measure of central tendency, as it is not so affected by the outlier ($1 700 000), which would affect the mean. There is no mode.

4 a Mode and median

b The mean, because when you calculate it you add up all the values in the dataset before dividing by the number of values.

5 If the dataset has outliers and is not very big, the mean is misleading as a measure of central tendency.
If the dataset is skewed towards one particular section, the median is misleading as a measure of central tendency.
If the dataset does not have a value that occurs many times, or is bimodal, the mode is misleading as a measure of central tendency.

6 Q_1 Lower Quartile = 5
Q_2 Median = 7
Q_3 Upper Quartile = 8

7 5

8 3

9 98th percentile, 2% of the class did better.

10 The data is spread out with its dispersion being widely scattered from 0 to 49 500. There is one main cluster between 0 and approximately 11 250. The data has five gaps; one between 11 250 and 13 500; one between 13 500 and 22 500; one between about 23 000 and 27 000; one between about 27 500 and 31 500; and one between 31 500 and 49 500. 49 500 could be considered as an outlier.

11 Range = 45, IQR = 24,
Standard deviation = 14.19

12 The range is quite large, so the data is spread out. The interquartile range is a lot less and may be more representative. The standard deviation is quite high also indicating that the data is quite scattered.

13 107 is an outlier. The mean is 38.9 with the outlier and 32.7 without it. The median is 38.5 with the outlier and 37 without it.

14 Features of a dataset that could make the range misleading:
Datasets with outliers
Features of dataset that could make the standard deviation misleading:
Datasets with outliers
Widely scattered with dense and less dense regions

Chapter review

1 a Mode = 65 b Mean = 62.15
c Median = 65

2 a The median or mode would be better than the mean because calculation of the mean adds up the values of all data points and extreme values will give a mean that is quite different from most of the data points. The median and the mode are not affected much by the inclusion of extreme values.

b The mean is best as its calculation adds up the values of all data points.

c The median is best because it is the middle value when the data is ordered so it always has an equal number of data points above it as below it.

d The mode is the best as it is the only measure based on the value that occurs most often in the dataset.

3 The data is spread out with its dispersion being widely scattered from 0 to 21. There is one main cluster between 0 and 13. The data has three gaps; one around 13; one around 16; one between 18 and 20. 21 could be considered as an outlier.

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4 **a** Mean = 7.25
b Median = 6.5
c Mode = There is no mode
d Mean is the better measure of central tendency, as it is the average score of all grades. The median score does not take into account all grades recorded and there is no mode in this dataset.

5 **a** 40 **b** 26
c 60 **d** 26

6 **a** 64 g **b** 83%

7 Range = 45 − 0 = 45
IQR = 7 − 1 = 6
Standard deviation = 13.36

8 Range = 27 − 16 = 11
IQR = 25 − 19 = 6
Mean = 22.33
Five-number summary = 16, 19, 22.5, 25, 27
Standard deviation = 3.5

9 Range = 4
Mean = 6.17
Standard deviation = 1.40

10 **a** **i** Median = 35.5
ii Upper quartile = 46
iii Lower quartile = 15
iv IQR = 31
b 107 is an outlier.
c There may have been a lot of fish around that day.
d The outlier is more than double any of the other observations, therefore it may be an error. It should only be removed if there is evidence that it is an error (i.e. a mistake in adding up or recording the number that day).

11 Mitchell needs to make at least $80 on Saturday to meet his budget.

12 Jackie earns more than 78% of her colleagues' wages and 22% of them have a higher wage. Jackie needs to consider whether they do more hours or more skilled work than her. If she believes she contributes considerably more to the success of the workplace than 78% of her colleagues she is probably justified in asking for a raise.

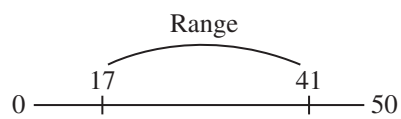
13 **a** The standard deviation of each group is:
A = 2.28
B = 0
C = 8.28
b Dataset C has the largest standard deviation.
c Yes, since by inspection a mean can be estimated for each dataset, and dataset C has data values that are further away from the mean compared to the other two datasets.

Chapter 9

Pre-test

1 12, 23 and 46

2 Range = 24



3

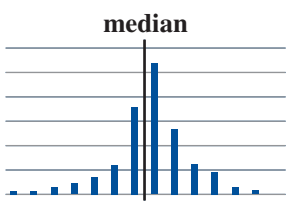
Stem and Leaf Plot

4	1
5	2 7 8
6	5 6
7	0 5 8 8 8
8	0 0
9	5

4 Answer is C

5 Most students: class E
Least students: class A

6



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Exercise 9A

1 a smallest, largest
b minimum
c Q_1, Q_3
d median, middle
e maximum

2 a 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 5, 7, 9
b 7.7, 7.9, 8.5, 8.6, 8.7, 8.7, 8.9, 9, 9.2, 9.6
c 41, 42, 42, 43, 45, 45, 46, 46, 47, 48, 49, 65
d \$34.70, \$35.80, \$36.00, \$36.30, \$36.50, \$45.90

3 a Min = 3; Max = 9
b Min = 8.0; Max = 8.9
c Min = 15; Max = 28
d Min = 0.5; Max = 0.88

4 a 5 **b** 4 **c** 3.5
d 15.5 **e** 14 **f** 60.5

5 4, 6, 7, 7, 12, 13, 13, 16, 16, 17, 27, 29, 45, 46, 48, 50, 54, 67, 72, 83, 84, 86

6 a 0, 0, 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 7
b Min. = 0 and Max. = 7
c Median = 2
d $Q_1 = 0$
e $Q_3 = 3$
f Min. = 0, $Q_1 = 0$, Median = 2, $Q_3 = 3$, Max. = 7

7 a 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 7, 9
b Min. = 2 and Max. = 9
c Median = 5
d $Q_1 = 4$
e $Q_3 = 6$
f Min = 2, $Q_1 = 4$, Median = 5, $Q_3 = 6$, Max. = 9

8 a 8, 9, 10, 10, 11, 12, 12, 12, 12, 12, 15, 15, 16, 17, 18, 19, 21, 28, 29, 34
b Min. = 8 and Max. = 34
c Median = 13.5
d $Q_1 = 11.5$
e $Q_3 = 18.5$
f Min. = 8, $Q_1 = 11.5$, Median = 13.5, $Q_3 = 18.5$, Max. = 34

9 a 98, 101, 103, 105, 107, 109, 112, 114, 116, 118, 119, 127, 127, 127, 128, 134, 135, 135, 137, 138, 139, 145, 145, 146, 147, 148, 149, 150, 150, 150, 155, 155, 155, 157
b Min. = 98 and Max. = 157
c Median = 135
d $Q_1 = 116$
e $Q_3 = 148$
f Min. = 98, $Q_1 = 116$, Median = 135, $Q_3 = 148$, Max. = 157

10 a Min. = 24, $Q_1 = 32$, Median = 53.5, $Q_3 = 67$, Max. = 87
b Min. = 27, $Q_1 = 43$, Median = 65, $Q_3 = 88$, Max. = 105
c Min. = 12, $Q_1 = 35$, Median = 73, $Q_3 = 97.5$, Max. = 117
d Min. = 1.05, $Q_1 = 2.84$, Median = 3.975, $Q_3 = 6.5$, Max. = 9.56

11 Min. = 106, $Q_1 = 107.5$, Median = 115, $Q_3 = 121$, Max. = 130

12 Min. = 200, $Q_1 = 400$, Median = 775, $Q_3 = 900$, Max. = 1 200

13 Min. = 0, $Q_1 = 1$, Median = 2, $Q_3 = 3$, Max. = 5

14 a Interest rates

3	2 5 6 7 7
4	0 5 6
5	3 4 4 6 8
6	0 0 4 4 5
7	0 0 0 0 3
8	4

key 3|2 = 3.2

b Min. = 3.2, $Q_1 = 4.25$, Median = 5.7, $Q_3 = 6.75$, Max. = 8.4

15 D

16 B

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17 a Number of cars per 20 min

1	9
2	0 0 5 6 7 8 8
3	0 4 6
4	0 4 5 5 5 5 6

b Min. = 19, Q_1 = 26, Median = 32, Q_3 = 45, Max. = 46

Exercise 9B

1 a visual, dataset

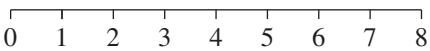
b scale

c whisker

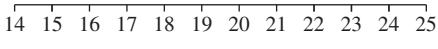
d Q_3

e inconsistent, surprising

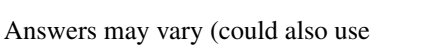
2 a



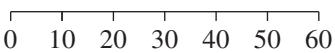
b



c

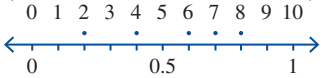


d Answers may vary (could also use increments of 5)

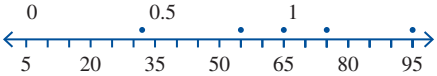


3 a

b



c



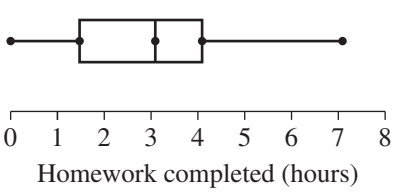
4 a 3, 9, 12, 18, 28

b 5, 30, 35, 45, 80

c 20, 120, 180, 320, 360

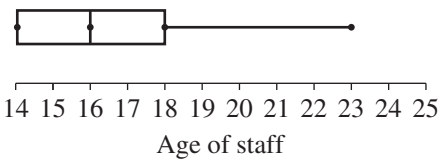
5 a Min. = 0, Q_1 = 1.5, Median = 3, Q_3 = 4, Max. = 7

b



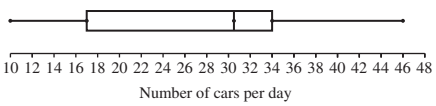
6 a Min. = 14, Q_1 = 14, Median = 16, Q_3 = 18, Max. = 23

b



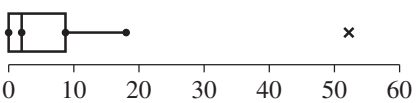
7 a Min. = 10, Q_1 = 16.5, Median = 30.5, Q_3 = 34, Max. = 46

b



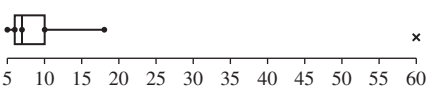
8 a

b



9 a 5 5 5 6 6 6 6 6 7 7 7 8 8 8 9 10 10 11 14 14 15 18 60

b



10 Min. = 30, Q_1 = 20, Median = 51, Q_3 = 70, Max. = 78

11 Min. = 9, Q_1 = 20, Median = 51, Q_3 = 68, Max. = 76

12 a At least 75% of the children are over 8 years of age.

b No, as there are many datasets that could fit this information.

c 25% of the children

d 5 years of age

e 75% of the children

13 a 75% of the trees are older than 32 years of age.

b 72 years

c 25% of the trees

d 87 years old

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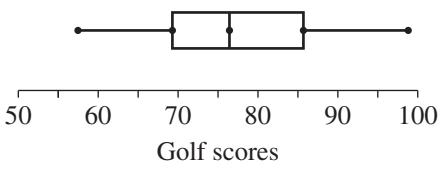
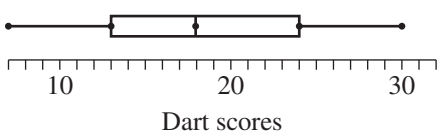
14 a Min. = 58, Q_1 = 69, Median = 76, Q_3 = 86, Max. = 99

b



15 a Min. = 7, Q_1 = 13, Median = 18, Q_3 = 24, Max. = 30

b



16 C

17 C

Exercise 9C

1 a two
b same
c measure, sensitive, outliers
d outlier
e single, two
f separated, compared

2 a minimum
b lower quartile whisker
c Q_1
d median, Q_2
e IQR (Interquartile Range)
f Q_3
g upper quartile whisker
h maximum
i range

3 a Title
b Stem column
c Girls' leaf column
d Boys' leaf column
e Girls' key
f Boys' key

4 a The median for males is 65, which is lower than the females' median of 75 beats per minute.
b The range of the male's pulse rates is 40, compared to the range of the females being larger at 55.
c The IQR for males is 15, being less than the females' IQR of 25.
d Modelled response: In general the resting pulse rate for females is higher, and so is the variability. Both genders had the same minimum pulse rates.

5 a The median for both datasets is 15 spelling errors.
b The range of the Year 11 errors is 45, compared to the range of the Year 12 being smaller at 35.
c The IQR for Year 11 is 10, being less than the Year 12 IQR of 15.
d Modelled response: Judged by the median number of errors being the same, the general number of errors is similar in Year 11 and 12. The variability is also similar: while the range is greater in Year 11, the IQR is greater in Year 12, so it evens out. The maximum number of errors was highest for Year 11 at 50, but this could be an outlier. The minimum number was lower in Year 12, but again this could be an outlier.
NOTE: *this is an example where it would have been very helpful to know the means of the datasets, in order to decide if there really is a difference between them.*

6 a The median for Year 7 is 9 hours, which is lower than the Year 12 median of 14 hours per week.
b The range of the Year 7 students homework time is 12 hours, compared to the range of the Year 12 being 4 hours longer at 16.
c The IQR for Year 7 is 9, being less than the Year 12 IQR of 11 hours of homework per week.

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d Modelled response: The homework time for Year 12 students is longer, as shown by the median, but also has greater variability.

7 a The median for performing burpees before a game of netball is 32, which is lower than the end of season median of 37 burpees.

b The range of burpees before the season is 27, compared to the range after the season being slightly larger at 30.

c The IQR for before is 15, being less than the end of season IQR of 22.

d Modelled response: The ability to do burpees at the end of the season of netball is higher, though the range of ability is greater, meaning that a few players did not improve very much.

8 a The mean values of each distribution cannot be calculated since we don't have the data values; however, the median of the 2016 Olympics distribution is represented as being higher than the median of the Australian qualifiers distribution. These datasets show that the Australian qualifiers' long jumps were generally shorter than the Olympic long jumps, though there is one outlier in the Australian qualifiers of 8.15 m which is the same as the longest jump at the 2016 Olympics.

b Ignoring the outlier in the Australian qualifiers' distribution, the distribution of the Olympic long jumps is more spread out than the Australian qualifiers' distribution. The datasets show that the IQR and range of the Olympic jumps is larger. Therefore, the Olympic long jumps vary more (are more spread out) than the Australian qualifiers' jumps.

c The shortest jump at the Olympics (the minimum value, 7.7 m) was the same as the third quartile of the Australian qualifiers' jumps. This means that all the Olympic jumps were longer than 75% of the Australian qualifiers' jumps, therefore it can be concluded that most of the Olympic jumps were longer.

9 Answers may vary. Modelled response.
Range: Popular 8–30 min, new 10–30 min.
Mean: Popular 17.5 min, new 17.3 min.
Median: Popular 16 min, new 16 min.
Mode: popular 13 and 23 min, new 10 and 16 min. The datasets are very similar, and they do not justify the new café in claiming to have the fastest service. Although the popular café had the two shortest delivery times, overall there was very little difference between them.

10 The median for the highest score for the girls was 130, while the median for the highest score for the boys was 143. The mode for the highest score for the girls was both 125 and 150, while the mode for the highest score for the boys was 162. The range for the highest score for the girls was 100, while the range for the highest score for the boys was 90.
The highest score for the girls was 203, while the highest score for the boys was 196. We are told that the mean highest score was slightly higher for the boys than the girls, and the median for the boys was higher. The mode does not tell us anything useful. While the highest score of 203 was by a girl, overall the girls' highest scores were slightly lower than the boys' highest scores.

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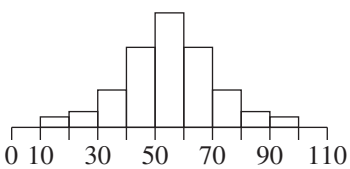
11 Answers may vary. Modelled response. The delay distributions from the 20th of December to the 25th of December are different from the delay distributions for the month before and after those dates. Both the median delay and the variability (spread) of the delays are greater from December 20th to 25th. By the 26th the delays are almost the same as before the 20th, but still slightly higher.

Exercise 9D

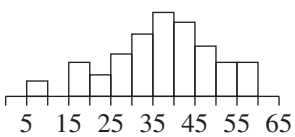
1 a mirror **b** approximately
c left, right **d** Negatively, left
e Bimodal, two

2 a Approximately symmetric
b Negatively skewed
c Bimodal
d Symmetric

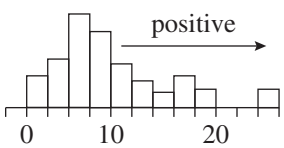
3 a



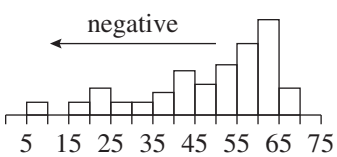
b



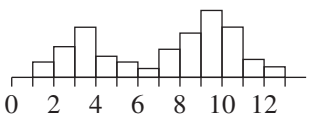
c



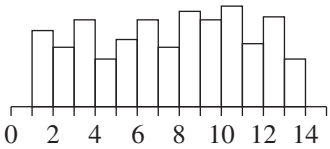
d



e



f



4 a The First 11 throws histogram is very nearly a symmetric shape, whereas the Second 11 throws histogram is only very roughly symmetric.
b Both histograms have a centre of 3.5. The spread for both histograms is exactly the same, being five.

5 a The Monday histogram is positively skewed, as the distribution's peak is higher on the left, with the tail stretching to the right.
The Tuesday histogram is negatively skewed, as the distribution's peak is higher on the right, with the tail stretching to the left.
b The Monday histogram has a centre of 10.5, with the Tuesday histogram also having a centre of 10.5. The spread of both histograms is exactly the same at 19 m/s.

6 a The Term one histogram's shape is bimodal as it is double-peaked at 30% and 90%, whereas the Term two histogram is approximately symmetric.
b The Term one histogram has a centre of 55, whereas the Term two histogram has a centre of 60. Therefore, the difference for the centres is 5. The spread for Term one is 90 and the spread for Term two is 80, which is also a difference of 10.

7 a Course A is positively skewed, as the results are higher on the left. Course B is approximately symmetric as the results are approximately the same on both sides of the peak.

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b Course A's results lean towards scores that are failing the course. More students on Course B achieve high marks and the centre of spread is higher too. So the student may feel that she has a higher chance of passing and getting a high score on Course B.

8 The Before program histogram is positively skewed, as the peak is higher on the left of the distribution. The After program histogram displays an approximately symmetric shape, as the distributions are approximately the same on both sides of the peak.
The locations are different for the histograms, with the Before program's location covering 40–55% and the After program's location covering 20–40%.
The spread for the After program (25%) is larger than the spread for the Before program (20%).

9 a The Zambia histograms are both positively skewed, with the shorter bars being in the positive section of the distribution.
The Zambia histograms have the same location and same range of 80. Females outnumber males slightly for ages up to about 24, and above that the sexes have approximately the same numbers in each age group.

b The Swedish histograms are both bimodal, with peaks at 50–54 and 35–39. There are slightly more males than females in each age group up to about 60, and above that, there are more females and males. While there are a few thousand females older than 100, the number of males is too small to appear on the graph.
The Swedish histograms have similar locations and ranges.

c In Zambia, the youngest in the population are the most numerous, indicating a rapidly-growing population. Each older age group is smaller than the preceding one, indicating high birth or death rates or a combination of the two. In Sweden a greater proportion of the population is old compared to Zambia, and people live longer. In Sweden the total population is probably not growing and, as death rates for children are unlikely to have increased, we can say the birth rate has declined over the past 60 years. In Zambia there are more females than males under 24 and then the sexes have about equal numbers in each age group, but in Sweden males exceed females up to about 60, and above that there are more females than males.

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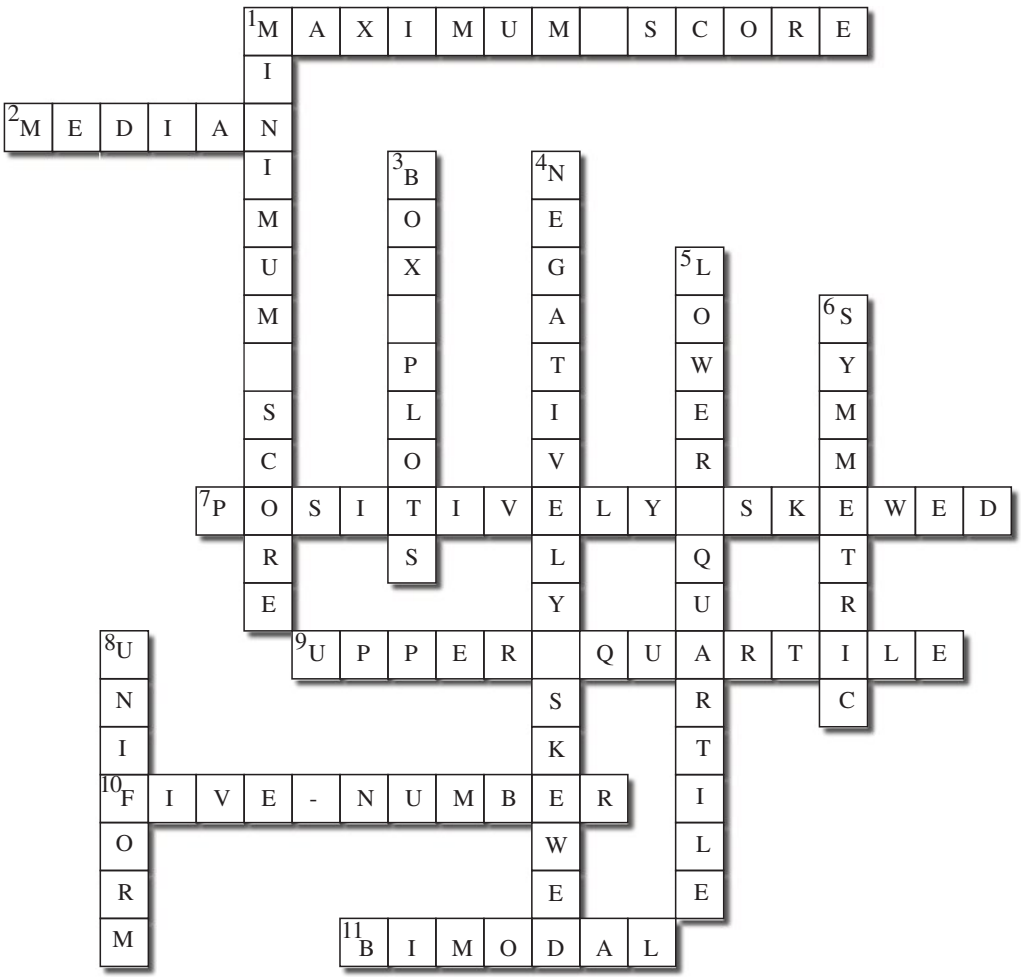
Puzzle



Chapter checklist

1 minimum, lower quartile, median, upper quartile, maximum

2 Minimum – the smallest number in the dataset
Lower quartile – the median of the lower half of the dataset
Median – the number that falls exactly in the middle of the dataset
Upper quartile – the median of the upper half of the dataset
Maximum – the largest number in the dataset

3 Min. = 4, Q_1 = 5.5, Median = 8, Q_3 = 9, Max. = 13

4 Min. = 4, Q_1 = 5.5, Median = 8, Q_3 = 9, Max. = 13

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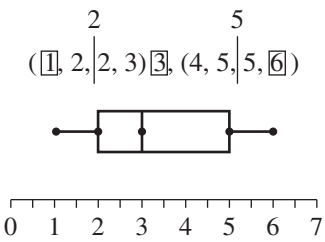
5



6 The barramundi in location B are longer, the median for location A is 80 cm, whereas for location B it is 40 cm longer at 120 cm. 25% of the barramundi at B are longer than 170 cm, while the longest barramundi at A was only just over 170 cm. The variability (spread) of length at the two locations is similar, location B has a slightly smaller IQR (70 cm compared to 75 cm) but a larger range (140 cm compared to 130 cm).

7 Batch B tomatoes took a little bit longer to ripen than batch A. The median was 11 days for A and 13 days for batch B. The variability was similar, with B being slightly less (A had a range of 10 days and IQR of 8 days, B had a range of 9 days and an IQR of 6 days).

8 a Cow A's histogram is bimodal, as it has a double peak. Cow B's histogram is uniform or multimodal, having a fairly even distribution.

b Both Ty and James's histograms are approximately symmetric, as the shape of both graphs is approximately the same on both sides.

c Team A's histogram is negatively skewed, whereas Team B's histogram is positively skewed.

Chapter review

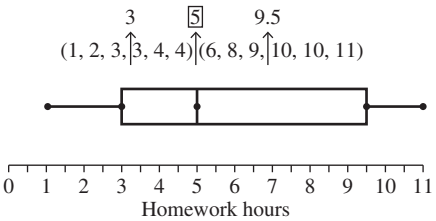
1 Min. = 3, Q_1 = 4, Median = 5, Q_3 = 7.5, Max. = 14

2 Min. = 0, Q_1 = 1, Median = 2, Q_3 = 2.5, Max. = 4

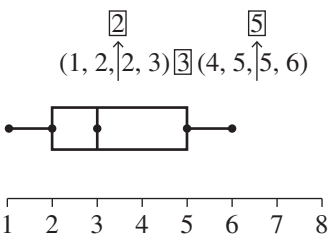
3 Min. = 4, Q_1 = 5.5, Median = 8, Q_3 = 9, Max. = 13

4 a Min. = 1, Q_1 = 3, Median = 5, Q_3 = 9.5, Max. = 11

b



5



6 Answers may vary. Modelled response. Varieties A and B are shorter than C and D. The median heights are similar for A (55 cm) and B (50 cm). The median height for C is 65 cm and is greatest for D (85 mm). The variability indicated by the range is similar for all four varieties, but the IQR indicates variety B has the lowest variability.

7 a The median life expectancy of males in 1988 (69) was 6 years less than in 2022 (75). The range is 36 for 1988 and 35 for 2022. The IQR in 1998 was 17 and was less than in 2022 (IQR=25).

b In conclusion, the median life expectancy for these particular countries has increased over the last 34 years, and the variability in life expectancy between countries has increased as shown by the increase in the IQR.

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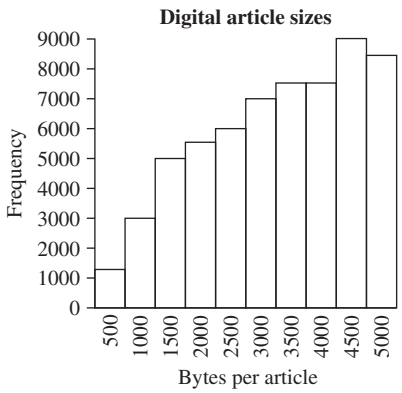
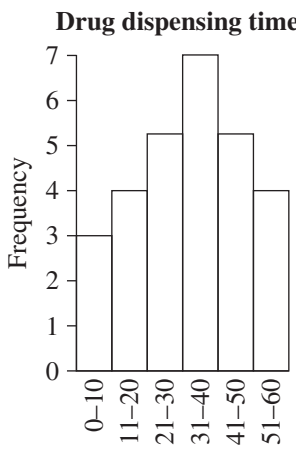
8 The 2021 histogram's shape is bimodal as it is double-peaked at April and December. The 2022 histogram's shape is uniform, as it has a fairly constant distribution. Both histograms have their centre between June and July. The spread for both histograms is 12.

9 a i



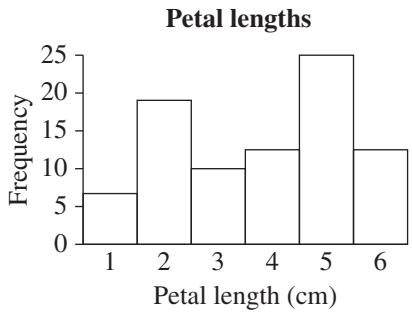
ii The Digital article sizes histogram is negatively skewed, and the peak of the distribution is higher on the right.

b i



ii The drug dispensing histogram is approximately symmetric with a peak almost in the centre.

c i



ii The Petal length histogram is bimodal, as the frequency numbers have two peaks, which would produce a bimodal histogram.

10 Answers may vary. Modelled response.
The median heights for GA/GD and GK/GS (both about 166 cm) are different from the height distributions of WA/WD and C (about 158 and 156 cm). The variability is similar for GA/GD and GK/GS, and is less than for WA/WD and C. GK/GS has the lowest variability and C has the highest. Positions involving goal have higher median heights and lower variability than those that do not.

11 Both male and female histograms are bimodal as they display double peaks. The male histogram has peaks at ages 25–29 and ages 40–44, whereas the female histogram has peaks at ages 25–29 and ages 50–54. The location for both histograms is similar, as well as the spread.

Chapter 10

Pre-test

1 a 40 **b** 18 **c** 27
d 6 **e** 4 **f** 8

2 a $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{3}{4}$
d $\frac{1}{4}$ **e** $\frac{5}{7}$ **f** $\frac{5}{8}$

3 a 9 **b** 27 **c** 30

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4 **a.** 0.25 **b** 36 **c** 2.5
d 750 **e** 2300 **f** 3600
5 **a** $\frac{1}{4}$ **b** $\frac{2}{13}$ **c** $\frac{1}{2}$
d $\frac{2}{5}$ **e** $\frac{5}{12}$ **f** $\frac{1}{20}$
6 **a** \$15 **b** \$75 **c** \$40
7 **a** 6 **b** 20 **c** 3
8 **a** 12 **b** 60 **c** 30
9 **a** 2 : 1 **b** 3 : 1 **c** 5 : 3

Exercise 10A

1 **a** \$20 **b** \$200 **c** \$20
d \$421 **e** \$70 **f** \$600
g \$6000 **h** \$660
2 **a** 14 kg **b** \$47.52
c 74.4 km **d** 47.4 L
e 512 mL **f** \$18 750
g 420 kg **h** 338.7 tonnes
3 **a** 12.5% **b** 10% **c** 30% **d** 12.5%
e 4.2% **f** 23.8% **g** 1.8% **h** 27.5%
4 **a** \$1320 **b** \$460
c \$180 **d** \$42
e \$180 **f** \$74.79
g \$247.42 **h** \$26 236.80
5 296 km
6 **a** $\frac{240}{699}$ **b** 34.3%
7 \$59.46
8 \$737.50

Exercise 10B

1 **a** percentage, change
b don't
c increase, decrease, step, step
d original, percentage, previous
e final amount, original amount
2 **a** \$80 **b** \$163.97
c \$727.82 **d** \$24 691.50
e \$252 **f** \$326 795.25
g \$534.08 **h** \$220
3 **a** \$120 **b** \$121
c The second percentage is based on the already increased amount.
4 **a** \$0.39 **b** \$1.06
5 **a** 92 marks
b improvement of 40 marks
6 **a** \$495 000 **b** \$598 950
c \$583 950
7 **a** \$630 **b** \$425.25
8 \$16.40
9 \$15.28
10 9720 L lost
11 ~35.37 tonnes
12 5.69% decrease

Exercise 10C

1 **i** **a** interest, \$
b principal, initial
c rate in %
d years
e per annum
ii 4, 12, 26, 52, 365
2 **a** $n = 1.5$ **b** $n = 0.75$
c $n = 1.5$ **d** $n = 2.25$
e $n = 2$ **f** $n = 2.5$
3 **a** $i = 18\%$ **b** $i = 9\%$
c $i = 3.25\%$ **d** $i = 22.75\%$
e $i = 10.4\%$ **f** $i = 36.4\%$
4 **a** \$680 **b** \$1008
c \$2925 **d** \$11 970
e \$1760 **f** \$4290
5 **a** \$6804 **b** \$456
c \$1664
6 I = \$819 000
7 **a** \$860.63 **b** \$405
c \$900
8 \$112.80
9 \$36 812.50
10 \$467.40
11 \$480 000
12 \$1842
13 \$8642
14 **a** \$2856 **b** \$7.82
15 option 1
16 **a** G3 = F3∗6.1%/12, F4 = G3 + F3

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b See spreadsheet

	A	B	C	D	E	F	G
1	**Penny's Term Deposit**						
2		Amount	Interest 6%			Amount	Interest 6.1%
3		20000.00	46.15			20000.00	101.67
4	fortnight 1	20046.15	46.26		Month 1	20101.67	102.18
5	fortnight 2	20092.41	46.37		Month 2	20203.85	102.70
6	fortnight 3	20138.78	46.47		Month 3	20306.55	103.22
7	fortnight 4	20185.26	46.58		Month 4	20409.78	103.75
8	fortnight 5	20231.84	46.69		Month 5	20513.53	104.28
9	fortnight 6	20278.53	46.80		Month 6	20617.80	104.81
10	fortnight 7	20325.32	46.90		Term Deposit Total	20617.80	104.81
11	fortnight 8	20372.23	47.01				
12	fortnight 9	20419.24	47.12				
13	fortnight 10	20466.36	47.23				
14	fortnight 11	20513.59	47.34				
15	fortnight 12	20560.93	47.45				
16	fortnight 13	20608.38	47.56				
17	Term Deposit Total	20608.38	47.56				

c The monthly term deposit is better by $9.42.

Exercise 10D

1 **a** order **b** same **c** units **d** same

2 **a** **i** 7 : 9 **ii** $\frac{7}{16}$ **iii** 9 : 16

b **i** 6 : 9 **ii** $\frac{6}{15}$ **iii** 9 : 15

c **i** 8 : 12 **ii** $\frac{8}{20}$ **iii** 12 : 20

3 **a** 2 : 3 **b** 8 : 13 **c** 22 : 37

d 45 : 78 **e** 15 : 2 **f** 9 : 4

g 14 : 19 **h** 7 : 12

4 **a** $\frac{3}{5}$ **b** $\frac{6}{11}$ **c** $\frac{31}{211}$

d $\frac{42}{77}$ **e** $\frac{7}{15}$ **f** $\frac{8}{17}$

g $\frac{9}{5}$ **h** $\frac{57}{36}$

5 **a** Student : teacher; i.e. 12students for every 1 teacher.

b For every 3 units of flour there will be 2 units of sugar.

6 **a** 5 : 8 **b** 3 : 5

c 11 : 14 **d** 7 : 12

e 8 : 11 **f** 4 : 5

7 **a** 8 : 9 **b** 9 : 14

c 14 : 17 **d** 8 : 9 : 14

8 **a** 26 : 26 **b** 13 : 26

c 13 : 39 **d** 4 : 48

9 40 : 3

10 **a** Ratio is 6 : 1, therefore there would be six children and one coach.

b If Konrad excludes himself, there will be five other children during his training session.

11 Answers will vary but approximately 1 : 8.

12 Answers will vary.

13 Answers will vary depending on the length of hands and feet; e.g. 10 cm : 15 cm.

Exercise 10E

1 **a** HCF, exactly **b** dividing, HCF

c dividing, HCF **d** LCD, exactly

e LCD

2 **a** 8 : 8 = 1 : 1

b 12 : 8 = 3 : 2

c 12 : 12 = 1 : 1

3 **a** **ii** 4 : 22 = 2 : 11

b **iii** 13 : 39 = 1 : 3

c **iv** 35 : 25 = 7 : 5

4 **a** 5 **b** 7 **c** 3 **d** 12

e 7 **f** 12 **g** 9 **h** 6

i 12 **j** 22 **k** 35 **l** 25

5 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

e $\frac{2}{3}$ **f** $\frac{13}{25}$ (already in simplest form)

g $\frac{2}{3}$ **h** $\frac{5}{6}$ **i** $\frac{5}{8}$

6 **a** 1 : 3 **b** 1 : 2 **c** 1 : 3

d 1 : 4 **e** 3 : 5 **f** 2 : 3

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g 2 : 5 **h** 1 : 3 **i** 2 : 9
j 4 : 1 **k** 27 : 8 **l** 3 : 5

7 a 1 : 3 : 6 **b** 1 : 4 : 5
c 1 : 3 : 5 **d** 4 : 6 : 9
e 3 : 4 : 10 **f** 4 : 9 : 16
g 2 : 5 : 7 **h** 2 : 3 : 4
i 2 : 5 : 7

8 a 6 **b** 35 **c** 24
d 42 **e** 20 **f** 50
g 18 **h** 48 **i** 40

9 a 3 : 2 **b** 4 : 3 **c** 8 : 15
d 9 : 14 **e** 3 : 4 **f** 65 : 44
g 20 : 27 **h** 168 : 235
i 2 : 5

10 a 300c : 40c = 15 : 2
b 50c : 750c = 1 : 15
c 275c : 25c = 11 : 1
d 75c : 425c = 3 : 17
e 180c : 300c = 3 : 5
f 120c : 400c = 3 : 10

11 a 6 h : 24 h = 1 : 4
b 26 weeks : 104 weeks = 1 : 4
c 4 weeks : 52 weeks = 1 : 13
d 26 weeks : 156 weeks = 1 : 6
e 20 h : 72 h = 5 : 18
f 0.5 years : 4 years = 1 : 8
g 60 h : 168 h = 5 : 14
h 450 min : 2880 min = 5 : 32

12 a 250 mm : 12 mm = 125 : 6
b 2 km : 12 km = 1 : 6
c 200 m : 4000 m = 1 : 20
d 5 mm : 15 mm = 1 : 3
e 2000 m : 400 m = 5 : 1
f 500 cm : 30 cm = 50 : 3
g 2 km : 12 km = 1 : 6
h 2000 m : 12 000 m = 1 : 6

13 a 5 mL : 2 L
b 5 mL : 2000 mL
c 1 : 400

14 a 1 : 8
b The station donates $1 for every $8 raised by the listeners

15 a $\frac{7}{4} : \frac{7}{6}$ **b** 12 **c** 3 : 2

16 7 : 5 : 9

17 30 cod : 45 flake : 12 coral trout = 10 : 15 : 4

18 1000 : 1 : 75

19 364 weeks : 7 weeks = 52 : 1

20 7 : 50

21 2 : 1

22 3 : 2

23 a 1 : 3 : 2
b Four people.

Exercise 10F

1 a dividing, denominator, same
b multiplied, same, equivalent

2 a $\frac{10}{20}$ **b** $\frac{14}{21}$ **c** $\frac{35}{40}$
d $\frac{20}{35}$ **e** $\frac{64}{88}$ **f** $\frac{15}{36}$
g $\frac{22}{24} = \frac{55}{60}$ **h** $\frac{26}{50} = \frac{130}{250}$
i $\frac{6}{21} = \frac{14}{49}$ **j** $\frac{25}{30} = \frac{100}{120}$
k $\frac{35}{60} = \frac{49}{84}$ **l** $\frac{\frac{40}{320}}{3} = \frac{30}{80}$

3 a Examples include:
4 : 6, 10 : 15, 20 : 30 etc.
b Examples include:
6 : 10, 15 : 25, 30 : 50 etc.
c Examples include:
22 : 18, 55 : 45, 110 : 90 etc.
d Examples include:
8 : 12 : 16, 20 : 30 : 40, 40 : 60 : 80 etc.

4 a 20 **b** 18 **c** 28
d 1 **e** 11 **f** 4
g 25 **h** 14 **i** 40
j 12, 15 **k** 8, 15 **l** 4, 36
m 6, 21 **n** 20, 5 **o** 7, $\frac{35}{3}$

5 a When equating the ratios into fractions we see that $\frac{2}{5}$ does not equal $\frac{8}{11}$.
b Answers may vary; e.g.
4 : 10, 6 : 15, 8 : 20 etc.

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6 4
7 15 green lights.
8 8
9 5.652 m
10 27
11 12 goals
12 7.5 kg
13 160 mL
14 1920 mL or 1.920 L

Exercise 10G

1 a sum, ratio
b one, sum
c total, total
2 a 6 **b** 8 **c** 20 **d** 15
3 a $\frac{3}{10}$ **b** $\frac{7}{10}$
4 a $\frac{6}{11}$ **b** $\frac{5}{11}$
5 a $\frac{2}{7}$ **b** $\frac{5}{7}$
6 a 2 : 5 **b** 1 : 10 **c** 7 : 10
7 a \$10 : \$90 **b** \$25 : \$75
c \$30 : \$70 **d** \$28.58 : \$71.43
8 a \$400 : \$800 **b** \$200 : \$1000
c \$500 : \$700 **d** \$840 : \$360
9 a \$10 : \$30 **b** \$30 : \$60
c \$60 : \$90 **d** \$80 : \$140
e 12 kg : 20 kg **f** 350 kg : 100 kg
g 20 L : 28 L **h** 96 L : 36 L
i 160 g : 40 g
j 35 lollies : 14 lollies
10 a \$50 : \$100 : \$150
b 54 kg : 81 kg : 135 kg
c 8000 L : 24 000 L : 32 000 L
d \$600 000 : \$400 000 : \$250 000
e 90 m : 90 m : 120 m
f 10 chocolates : 40 chocolates : 30 chocolates
g 100 plants : 50 plants : 30 plants
h \$1000 : \$350 : \$150
11 a 39
b $\frac{20}{29}$ ewes; $\frac{19}{39}$ lambs
c 2700 ewes and 2565 lambs.
12 a 5 : 4 **b** 9
c $\frac{4}{9}$ **d** 40
13 a 10
b $\frac{1}{10}$ accident fatal, $\frac{9}{10}$ non-fatal
c 217 fatalities compared to 1953 non-fatalities
14 a Ariana **b** 11 **c** $\frac{5}{11}$
d $\frac{2}{11}$ **e** \$25 000 **f** \$10 000
g \$15 000
15 a Jarred paid for three tickets and Ryan paid for his own ticket.
b 3 : 1
c Jarred paid \$42 for three tickets
16 Megan receives \$63 740 and Luke receives \$50 992.
17 Brock receives 166.67 g, Jasper receives 333.33 g and Ruth receives 500 g
18 Gretel receives \$120 000, Helga receives \$120 000 and Marie receives \$360 000
19 a 5 L of cordial and 45 L of water
b \$7.65
20 a 300 mL
b \$12
21 Cost of cement = \$225
Cost of sand = \$180
Cost of gravel = \$360
Total costs = \$765
22 1500 g

Exercise 10H

1 a drawing, actual **b** 1, 100 cm or 1 m
c second **d** smaller, multiply
e larger, divide
2 a i 1 km
ii 3 km
iii 4.4 km
b i 1.5 km
ii 10 km
iii 7.5 km
c i 200 m
ii 1 km
iii 64 m

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d i 450 m
ii 68.75 m
iii 562.5 m

3 a i 5 m
ii 30 mm
iii 20 m
b i 60 mm
ii 20 m
iii 70 mm
c i 20 m
ii 10 mm
iii 70 mm
d i 6 mm
ii 0.5 mm
iii 75 cm

4 a 1 : 2000 **b** 1 : 200 000
c 1 : 30 000 000 **d** 1 : 1200
e 15 000 : 1 **f** 1400 : 1

5 40 000 cm

6 a 500 cm
b 1 represents the length of the model and 50 represents the length of the actual car. This is because in a scale ratio, the larger object is the larger number.
c 10 mm

7 a 9000 cm
b 500
c 18 cm

8 a 8000 cm
b 80 m
c 36 000 cm
d 360 m
e 360 m ÷ 3 m = 120, so there are 120 floors in this building.

9 a Divide by 40 since the model of the ship represents the ratio of 1 : 40
b 0.75 m
c 75 cm
d Area is 5 cm × 2.5 cm = 12.5 cm^2

10 a A caravan is approximately 4 m long at most; and at 1 : 10 000, the model will be 0.4 mm, which is too small. 1 : 50 is more appropriate.
b A city can be a couple of kilometres long and wide. For example, if a city is 5 km long and wide, the model will be 80 m long and wide, which is impractically large. In this case, 1 : 10 000 is more appropriate.

11 a 0.328 m
b 328 mm

12 a 13 mm
b 2.6 mm
c 33 mm
d 1300 km
e 85 mm
f 3300 km
g 3000 km

13 a 1 : 67
b 2.4 m wide by 4 m long
c 9.6 m^2
d Carpet cost = $633.60

14 a 737 cm
b 3.5 cm
c 1 : 211
d 1.5 cm
e 320 cm

15 Scale = 1 : 570
Scale length = 472.1 mm
Scale width = 49.5 mm
Scale height = 7.9 mm

16 1 : 50

17 The person is 39 mm in the picture, giving a scale of $\frac{1900}{39}$ = 48.7, therefore approximately 1 : 49. Height of dog in sitting position in picture is 14 mm, therefore actual height of the seated dog is 0.686 m.

18 a Answers will vary.
b Answers will vary. Potential factors to consider include but are not limited to fencing, backyard size and neighbours.

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Puzzle

The first windscreen wipers were operated by hand.

Chapter checklist

1 5 : 9
2 $\frac{21}{17}$
3 8 : 11
4 54
5 200 mL
6 4 : 5
7 3 : 5 : 8
8 20 : 21
9 1 : 20
10 5 : 26
11 3 : 200
12 280 kg : 350 kg
13 $80 000 000
14 1 : 150
15 30 cm wide 42 cm long, 6 cm tall
16 $514.91
17 $52 542
18 98c
19 $n = 1.5$
20 13%
21 $285.60

Chapter review

1 1 : 3
2 1 : 8
3 1 : 8
4 2 : 1 is the ratio of cardio to weights, so 30 min : 15 min
5 **a** 144 **b** 180
6 55 m^3
7 7 : 3 = 105 : 45, so we expect that Martha will lose 45 games in the next 3 years.
8 **a** 1680 **b** 613 200
9 Grace will have saved $62 400 and Lilly will have saved $46 800.
10 14.4 kg
11 320 m
12 50 cm : 8 m = 50 cm : 800 cm = 1 : 16. The model will be 50 cm long and 18.75 cm tall.
13 $20.61/kg
14 $9546
15 $810.34
16 $22 560
17 $4536
18 $38 904

Chapter 11

Pre-test

1 **a** 120 **b** 2
c $\frac{1}{3}$ or 0.333333 …
d $\frac{1}{4}$ or 0.25 **e** 8 **f** $\frac{1}{2}$ or 0.5
2 **a** $\frac{1}{2}$ **b** $\frac{3}{1}$ or 3
c $\frac{3}{7}$ **d** $\frac{15}{7}$
e $\frac{5}{9}$ (can't be simplified)
f $\frac{2}{3}$
3 **a** $\frac{8}{3}$ **b** $\frac{7}{9}$ **c** $\frac{9}{13}$
4 **a** length or distance
b money
c time
d capacity or amount of liquid
e weight or mass
f temperature
5 **a** 3.6 L **b** 132 000 m
c 4.5 h **d** 2007.5 days
e 6500 c **f** 14 kg
6 **a** distance and time (e.g. km and hour)
b capacity and distance (e.g. L and km)
c money and weight (e.g. $ and kg)

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Exercise 11A

1 **a** 60 km/h **b** $1.75/kg
c $300/m^2 **d** 2 tries/game
e 22 mL/day
2 Archie
3 **a** **i** 4000 g
ii 125 g/person
b **i** 16 000 mL
ii 500 mL/person

Exercise 11B

1 **a** multiply
b divide
c square
d weeks, months
e weekly, monthly
2 **a** **i** 72 km/h **ii** 43.2 km/h
iii 15.3 km/h **iv** 243 km/h
b **i** 16.7 m/s **ii** 30.6 m/s
iii 22.2 m/s **iv** 1.7 m/s
3 **a** $0.08/mL **b** $0.25/g
c $120/m^2 **d** 1000 cans/h
e 2.5 L/year **f** 125 cm/ min
g 125 g/year **h** 7 600 000/month
4 **a** $280/day **b** $2800/fortnight
c $38.89/h **d** $72 800/year
e $728 000/decade
f $6066.67
5 **a** $412/day
b $4120/fortnight
c $57.22/hour
d $107 120/year
e $1 071 200/decade
f $8926.67/month
6 **a** $47.5/h
b $1900/week
7 **a** 36 L/h
b 1333.33 h, 55.56 days or 7.94 weeks
8 **a** 382.82 km/h
b 377.76 km/h
9 0.5 mm per km
10 **a** 6000 L/h
b 100 L/min
11 10.67 km/h
12 13.8 cm
13 0.24 km/h, therefore it takes 2.5 hours to move 0.6 km
14 15 cm/year, therefore it will take 4 years
15 109.8 km/h, therefore the 110 km/h speed limit
16 worm

Exercise 11C

a direct, decrease, constant
b equivalent, divided, same
2 **a** $x = 320$ **b** $x = 76.5$
c $x = 4.5$ **d** $x = 120$
e $x = 630$ **f** $x = 22.5$
g $x = 2$ **h** $x = 7$
3 **a** $x = 1500$, $y = 15\,000$
b $x = 14$, $y = 154$
c $x = 25$, $y = 750$
d $x = 15$, $y = 75$
e $x = 70$, $y = 560$
f $x = 18$, $y = 234$
4 **a** 1320 L **b** 2640 L
5 **a** $1.2/kg **b** $3
6 **a** **i** $6
ii $13.5
b 3.5 kg
7 **a** 25 L/day
b 9125 L/year
c $94 900/year
8 1372.55 km
9 200 750 L
10 $22.50
11 $3000
12 9 houses
13 $472.50
14 19 students
15 9:14 a.m.
16 7.79 L
17 100 L
18 84 primary : 90 secondary

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ISBN 978-1-009-25785-5

Exercise 11D

1 **a** best, equal
b lowest, highest
c lowest, best
d highest, best

2 **a** \$0.45/100 g or \$0.42/100 g; 1.2 kg is the best value.
b \$0.65/100 g or \$0.61/100 g; 1 kg is the best value.
c \$0.22/100 g or \$0.21/100 g; 500 g is the best value.
d \$1.28/100 g or \$1.29/100 g; 250 g is the best value.

3 **a** \$1.25/L or \$1.2/L; 1.75 L is the best value.
b \$2.28/100 g or \$2.55/100 g; 250 g is the best value.
c \$3.20/kg or \$3.30/kg; 1.5 kg is the best value.
d \$7.33/GB or \$7.20/GB; plan B is the best value.

4 **a** \$0.35/box (box of 40, \$0.25/box (box of 320)
b \$0.10 cheaper

5 **a** The 2 L bottle is the best value.
b Especially for food, it has an expiry date and might go bad before you can consume it all, which will result in wastage.

6 The carton of 24 cans is the best value.
7 The 600 mL bottle is the best value.
8 More expensive by \$1/kg.
9 Yohann has the fastest walking rate.
10 175-g packet is the best value.

11 **a** Hayden's car is 7.69 L/100 km; Aria's car is 7.75 L/100 km. So Hayden's car is the most economical.
b \$103.61 for Hayden, \$104.42 for Aria

12 **a** Bethany's car is 8.33 L/100 km; Teri's car is 8.62 L/100 km. So Bethany's car is the most economical.
b Bethany \$225.82, Teri \$233.61

13 **a,b** 5×2 L and 1×1.25L soft drink
2×18 pack of party pies
3×24 pack of little sausages
3×600 g packets of corn chips
c Total = \$51.71

Puzzle

A steamship propeller.

Chapter checklist

1 16 m/s, 12 cans/s and \$6/kg
2 km/h
3 7 goals/game
4 4 cm/year
5 1.75 km/h
6 169.2 km/h
7 25 m/s
8 $x = 3$
9 $x = 63$
10 42 min
11 250 g packet is best value.
12 4.8 m of pine timber is best value.
13 Fuelo has the best value petrol.

Chapter review

1 **a** 0.2 m^2/min
b 12 m^2/h
c 10.5 h

2 The carton of 30 eggs is best value.
3 The 175 g packet is best value.
4 882
5 **a** Mike's car is more economical.
b Lynn's cost is \$47.18.

6 A box of 20 is \$0.18 per biscuit, a box of 36 is \$0.16 per biscuit; therefore a box of 36 is better value.
7 Adam is 3m^2/hour and Jarli is 32.73m^2/hour, therefore Jarli is quicker by 29.73m^2/hour.
8 Originally \$6, now \$3, therefore save \$3

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9 Area is 28 m^2, therefore cost is $28 \times 62 =$ \$1736

10 $24\,000\,000/1000 \times 12.08 = 289\,920$ babies born this year.

11 **a** $8.1 \times 12.8 = 103.68$ L

b $1.57 \times 103.68 =$ \$162.78

12 48 m^2/hour therefore $1380/48 =$ 28.75 hours

Chapter 12

Pre-test

1 **a** 12:00 hours **b** 3:17 hours
c 7:30 hours **d** 4:45 hours
e 9:15 hours **f** 10:55 hours

2 **a** 120 s **b** 1800 s **c** 7200 s

3 **a** 12 h **b** 84 h **c** 336 h

4 **a** nine o'clock **b** half past four
c quarter to twelve
d quarter past eight
e twelve minutes past twelve
f eleven minutes to seven

5 **a** 9:15 a.m. **b** 5:30 p.m.
c 11:45 p.m.

6 **a** 5:15 p.m. **b** 6:45 p.m.
c 4:45 a.m.

7 **a** 8:00 a.m. **b** 9:30 p.m.
c 1:40 p.m.

Exercise 12A

1 **a** hours, minutes, seconds
b divide, 60
c **i** multiply, 60, seconds
ii 3600
d **i** 60
ii 60
e midnight

2 **a** 8:26:05 **b** 10:08:58
c 2:53:37

3 **a** **i** $5\frac{1}{4}$
ii quarter past five
b **i** $15\frac{1}{3}$
ii twenty minutes past seven
c **i** $12\frac{1}{2}$
ii half past twelve
d **i** $10\frac{3}{4}$
ii quarter to eleven
e **i** $18\frac{1}{2}$
ii half past six

4 **a** 9.58 **b** 17.72 **c** 12.3
d 1.92 **e** 3.275 **f** 20.56
g 11.40 **h** 21.14

5 **a** 10:30 **b** 9:42
c 5:18 **d** 12:48
e 7:06 **f** 1:17:24
g 3:33 **h** 7:22:48
i 11:46:48 **j** 2:56:24

6 **a** 08:37 **b** 16:48 **c** 12:28
d 01:55 **e** 03:16 **f** 20:33
g 11:23 **h** 00:08

7 **a** 9:35 a.m. **b** 3:33 p.m.
c 12:49 p.m. **d** 1:55 a.m.
e 9:25 a.m. **f** 5:16 p.m.
g 11:23 p.m. **h** 1:45 p.m.
i 12:12 a.m. **j** 11:50 a.m.

8 **a** 7.5 h **b** 4.75 h **c** 9.25 h
d 5.33 h **e** 6.67 h **f** 7.42 h

9 40.92 h

10 **a** 1.04 **b** 1.21 **c** 1.35
d 1.29 **e** 1.22 **f** 1.03

11 **a** 1:01:58 is faster, which is 1.03 in decimal form. Whereas the decimal form of 1:02:30 is 1.04, which is larger. Therefore, the runner who finished fastest is the student with the time of 1:01:58.
b 1.03, 1.04, 1.21, 1.22, 1.29, 1.35

12 40 h 48 min

13

Date	Start time	Finish time
Monday 3/3/19	8:30 a.m.	1:30 p.m.
Tuesday 4/3/19	8:30 a.m.	4:30 p.m.
Thursday 6/3/19	1 p.m.	5 p.m.
Friday 7/3/19	11 a.m.	8 p.m.

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14 23 : 40 hours

15 **a** 14:40 hours

b **i** 3 h 55 min

ii 15 h 55 min

Exercise 12B

1 **a** 24 **b** 60

c 60 **d** 12:00, night

e 11:40 p.m. **f** 12:40 a.m.

g p.m., day

2 **a** 6 h 40 min **b** 2 h 45 min

c 1 h 21 min **d** 6 h 25 min

3 **a** 12:50 p.m. **b** 4:35 a.m.

c 2:37 a.m. **d** 7:07 p.m.

e 11:59 p.m.

4 **a** 10:02 a.m. **b** 7:22 a.m.

c 10:07 a.m. **d** 5:12 p.m.

e 11:57 p.m.

5 **a** 3 h 45 min **b** 2:25 p.m.

c 4:47 p.m. **d** 10 h 46 min

e 12:18 p.m. **f** 10:57 a.m..

g 6 h 42 min

6 **a** 3 h **b** 7 h

c 16 h **d** 11 h

7 2 h 45 min, so Jan could have a fine.

8 4 h 27 min

9 7 h 15 min

10 2 h 13 min

11 10 h 52 min

12 5 + 4.5 + 4.5 + 7 = 21 h

13 12:45 p.m.

14 3:19 p.m.

15 Movie would finish at 11:05 a.m., so they could still make the appointment on time.

16 7:35 a.m.

17 11:23 a.m.

18 6:53 a.m.

19 **a** 11:35 a.m. **b** 2 h15 min

c 1 h 30 min **d** 1:35 p.m.

Exercise 12C

1 **a** location

b departure

c stops on the route

2 **a** six times per day

b six stops, including City Parklands

c 5:07 p.m. and 7:05 p.m. Entertainment centre and factory outlets are probably closed at these times so there's no need to stop.

d 49 min **e** 12 min

f 10:27 a.m. **g** 5:44 p.m.

3 **a** 12:11 p.m. **b** 2:39 p.m.

c He can catch the 11:31 a.m. to arrive at 11:44 a.m.

4 **a** 11:10 a.m.

b 11:38 a.m. from Perth underground.

c 2:25 p.m.

d 49 minutes

5

Location	Mode	Departure time	Destination	Arrival time
Terminal 4	Bus Route 40	9:46 a.m.	Elizabeth Quay	10:25 a.m.
Elizabeth Quay	Train Mandurah line	10:40 a.m.	Mandurah	11:29 a.m.

6

Location	Mode	Departure time	Destination	Arrival time
Northcliffe	tram	7:48 a.m.	Helensvale	8:26 a.m.
Helensvale	bus	8:44 a.m.	Movie World	8:53 a.m.

7 **a** 11:56 a.m. **b** 11:12 a.m.

c **i** depart 11:12 a.m.

ii arrive 11:57 a.m.

d 11:54 a.m. **e** 12:32 a.m.

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ISBN 978-1-009-25785-5

Exercise 12D

1 **a** 3:43 **b** 17 **c** 4:08, 8
d 17, 8, 1 h and 25 min

2 Option 1: 1 h 35 min; option 2: 2 h 5 min. So option 1 is best.

3 Option 1: 28 min;
option 2: 26 min. So option 2 is best.

4 **a** 8:45 a.m. bus or 8:43 a.m. train
b By travel time: bus 37 min, train 38 min
c According to least amount of travel time, bus is the best mode of transport.

5 Option 1: tram 18 min;
Option 2: bus 32 min.
So tram travel is the better option, with choice of arriving at either 8:51 a.m. or 8:58 a.m.

6 **a** Bus route: Catch 6:27 a.m. bus to arrive Perth 6:55 a.m. is 30 min total travel time.
Train route 6:38 a.m. Warwick station arrive Perth 6:56 a.m.
Total travel time is 25 min (including driving time).
b bus route takes 30 min, train route takes 25 min
c So the driving to train is faster by may cost more.

Exercise 12E

1 **a** phase, sunlit **b** sea, gravitational
c two, two **d** two
e locations, times, heights

2 **a** 15 November **b** 1 April
c 5 July **d** November
e May

3 **a** 6:05 p.m.
b Tide lowest at 1.29 m at 10:44 a.m.
c morning
d Wednesday 3rd January 11:00 a.m.
e Wednesday 3rd January at 4:04 a.m. with lowest tide depth of 0.19 m.

4 **a** 6:50 a.m. and 5:47 p.m.
b July 31st
c July 30th and 31st
d July 30th and 31st
e 16 min 10 s

5 6 June or 5 July

6 Sarrita should hold her party on the 14th September as it will be a full moon.

7 **a** 4:49 a.m.
b approx. 6:30 p.m.
c From approx. 5 a.m. to 6:30 p.m.; i.e. 13.5 h available.

8 **a** 5:22 a.m. and 5:55 p.m.
b 97° and 263°
c 2 min

9 **a** Any suggested time between 4:30 a.m. and 7:30 a.m. is correct (allowing for daylight, tides, travel time and maximum time on beach)
b ~ 11:48 a.m. **c** ~ 9:41 p.m.

10 **a** Between 5 a.m. (assuming sunrise) and 7.19 a.m; 5:19 p.m and 7:19 p.m., assuming sunset is somewhere around/soon after 8:19 p.m.
b 8:19 a.m. or approx. 7–7:30 p.m. (taking into account sunset).
c Assuming earliest start time (5 a.m.) and latest leave time (7:30 p.m.), this would equal approx. 12 h 30 m.

11 **a** 6.71 m increase **b** 1.68 m increase
c 3.36 m increase **d** 1.68 m increase
e 6.51 m decrease **f** 3.26 m decrease

Puzzle

Do not procrastinate.

Chapter checklist

1 $6\frac{1}{4}$ h, six and a quarter hours
2 32.45 h
3 12:30 a.m.
4 15:15
5 6 h 35 min
6 1:05 p.m.
7 9:04 a.m.

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8 Option 1: 9:00 a.m. to 10:23 a.m. = 1 hr 23 min
Option 2: 9:38 a.m. to 10:23a.m. = 55 min.
Option 2 has less travel time.
9 5:51 a.m.

Chapter review

1 11:00 a.m. to 7:45 p.m.
2 1.094 h
3 Dimitri's records show 28.75; his boss must have rounded to one decimal place.
4 **a** 1:21 p.m.
b 13 : 21
5 2h 47 min
6 1 : 36 p.m.
7 **a** 32 min
b **i** 7:37 a.m. **ii** 7:48 a.m.
c 7:43 a.m. Thornlie line and change at Cannington
d 7:42 a.m.
8 **a** 6:04 a.m., 5:41 p.m. on Friday and 9:33 a.m. and 8:58 p.m. on Monday.
b Friday 4:04 a.m. to 8:04 a.m. Friday
3:41 p.m. to 7:41 p.m. Monday
12:33 a.m. To 4:33 a.m. Monday
6:58 p.m. To 10:58 p.m.

Chapter 13

Pre-test

1 **a** 1000 m **b** 2350 m **c** 0.075 m
2 **a** 3 km **b** 5.75 km **c** 0.25 km
3 **a** 150 km **b** 112.5 km **c** 45 km
d 117 km **e** 1.98 km **f** 0.297 km
4 **a** \$3.33/kg **b** 56.5 L/min
c 3.11 m/h **d** 3.35 MB/s
e 10.44 m/s **f** 762.89 km/h
5 **a** 9.5 km/h
b 138.8 notes per min
c 26.33°C/ min
d 630.43 km/h
e 1.47 m/year
f 1.00 rotation/day
6 **a** \$0.0045/g or 0.45 cents/g
b \$1124.87/week
c 8000 KB/s
d 45 calories/100 g
e 540 m/s
f \$0.29/inch
7 **a** 902 000 m/h **b** 15 033m/min
c 250.56m/s
8 **a** 736.84km/h vs 666.67km/h → 7000 km in 9.5 h is quicker.
b 50 mm/min vs 45 mm/min → 15 cm in 3 min is quicker.
c 4.5m/min vs 5.83m/min → 2.1km in 6 h is quicker.
9 A: \$0.52/100mL
B: \$0.54/100mL
C: \$0.57/100mL; therefore brand A is the cheapest.
10 **a** Fatima \$1.25/apple, Reiko \$1.125/apple; therefore, Reiko sells the cheapest apples.
b \$2.50

Exercise 13A

1 **a** map, actual
b larger, map
c segment
d Step 1: line, mm Step 2: mm
Step 3: actual, map, mm, km/mm
2 **a** **i** 60 m **ii** 120 m
b **i** 800 m **ii** 6 km
c **i** 104 m **ii** 300 m
3 **a** 16 mm, 5 km/mm
b 22 mm, 27.27 km/mm
c 14 mm, 28.57 km/mm
4 **a** 28.57 km/mm
b **i** Measured: 17 mm, Actual: ~490 km
ii Measured: 41 mm, Actual: ~1170 km
iii Measured: 29 mm, Actual: ~830 km
iv Measured: 25 mm, Actual: ~710 km

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5 a ~4500 m **b** ~1300 m
c ~3000 m **d** ~1600 m
6 a ~190 km **b** ~70 km **c** ~200 km
7 Exmouth to Noosa is the larger distance by approximately 220 km.
8 a Measured: 32 mm, Scale: 18.75 km/mm
b 1800 km
c 212%

Exercise 13B

1 a i scale ratio, map, actual
ii segment
b 1: segment **2:** km/mm
3: map, mm, km/mm
2 a i ~258 km **ii** ~182 km
iii ~192 km
b Route 2: Chinchilla–Condamine–Moraby–Surat is the shortest, with a distance of approximately 182 km.

3 Perth–Cervantes 240 km
Cervantes–Kambalda 387 km
Kambalda–Newman 707 km
Newman–Exmouth 445 km
Exmouth–Geraldton 501 km
Geraldton–Cervantes 400 km
Cervantes–Perth 240 km
Total distance travelled = 2920 km
4 a ~1500 km **b** ~1350 km
c The shortest route is Sydney to Tamworth to Bourke to Dubbo to Sydney.
5 Cairo–Port Said ~150 km
Port Said–Alexandria ~175 km
Alexandria–Dakhla Oasis ~400 km
Dakhla Oasis–Sohag ~160 km
Sohag–Minya ~150 km
Minya–Beni-Suef ~80 km
Beni-Suef–Cairo ~75 km
Total distance travelled = ~1190 km

6 a

	A	B	C	D	E
1	From	To	Map direct distance in mm	Scale in km/mm	Actual one-way direct distance in km
2	Melbourne	Adelaide	16	35.71428571	444
3	Melbourne	Hobart	15	35.71428571	417
4	Melbourne	Sydney	18	35.71428571	500
5	Melbourne	Canberra	12	35.71428571	333
6	Melbourne	Brisbane	35	35.71428571	972
7				Total	2667
8	From	To	Map direct distance in mm	Scale in km/mm	Actual one-way direct distance in km
9	Sydney	Adelaide	30	35.71428571	833
10	Sydney	Hobart	27	35.71428571	750
11	Sydney	Melbourne	18	35.71428571	500
12	Sydney	Canberra	6	35.71428571	167
13	Sydney	Brisbane	19	35.71428571	528
14				Total	2778

b

15	PFD Melbourne	PFD Sydney
16	1370 km	2140 km

c Sydney, as the distance between cities is not that much greater than Melbourne, but there is a much higher profitable flight distance for Sydney. One consideration is where the most popular places are for deliveries and prioritise making the distances to those cities minimal.

Exercise 13C

1 a m/s, km/h **b** rate, time
c 1 unit
d i distance
ii distance, time, time

2 a km/h **b** km/h
c m/s **d** m/h
e km/year **f** km/year
3 a 10.4 m/s **b** 854.9 km/h
c 32 m/h **d** 18.7 km/h
e 2.1 m/s **f** 57.6 km/h

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ISBN 978-1-009-25785-5

4 **a** 96 km/h **b** 32 km/h
c 190 km/h **d** 42.2 km/h
5 **a** 806.4 km/h **b** 83.3 km/h
c 20.9 km/h **d** 267.1 km/h
e 100.8 km/h
6 **a** 20.12 km/h
b 141 km
c 7:20:04
d 19.22 km/h
7 cm/hour
8 102 km/h
9 90 km/h
10 3 m/s
11 56km/h
12 Angus travelling at 112.9 km/h, therefore he risks a speeding fine, as the limit is 110 km/h.
13 Jade was approximately 1.33 m/s faster.
14 96 km/h

Exercise 13D

1 distance/time, s = d/t
distance/speed, t = d/s
speed × time, d = s × t
2 **a** **i** 4 m/s **ii** 80 km/s
iii 7.3 m/s **iv** 88.4 km/h
b **i** 53.6 s **ii** 12.5 h
iii 7.0 h **iv** 29.5 s
c **i** 72 m **ii** 180 km
iii 520 km **iv** 175 m
3 1 h 40 min
4 233.75 km
5 83 km/h
6 16 m/s
7 Yes, he will, and by 10 seconds
8 38.3 minutes
9 150 km
10 90.7 km/h
11 5:30 p.m.
12 1465 km
13 1 p.m.
14 200 m race in 7.1 m/s, 800 m race in 6.8 m/s, so Boyce ran faster in the 200 m race.
15 9 km
16 485 km
17 40 m
18 3.7 min
19 **a** 105.5 km/h **b** 18 min 38 s

Exercise 13E

1 **a** **i** Step 1: line, mm
ii Step 2: mm
iii Step 3: actual, map, mm, km/mm
b distance/speed, t = d/s
c hour, hours
2 **a** 12.5 km/mm
b 70 mm, giving an actual distance of 875 km.
c 2.7 h = 2 h 42 min
3 **a** ~3.4 h **b** ~3.6 h **c** ~1.5 h
d ~1.5 h **e** ~4.8 h **f** ~4.9 h
4 **a** ~$12 400 **b** ~$18 000
c ~$9600 **d** ~$22 800
e ~$5400 **f** ~$5400
5 **a** ~450 km **b** ~$262 800
c ~$525.60 each
6 ~2 h
7 ~5:00 p.m
8 Yes, Delia will make the delivery by 30 min.
9 ~$101.50 cheaper towing to Esperance
10 ~$3180
11 1:30 p.m.
12 Any time after 1:30 p.m.

Exercise 13F

1 **a** vertical, horizontal
b (time, distance)
c straight, constant
d distance travelled, time taken
e zero
f faster
g **1:** distance **2:** time **3:** plot, segments

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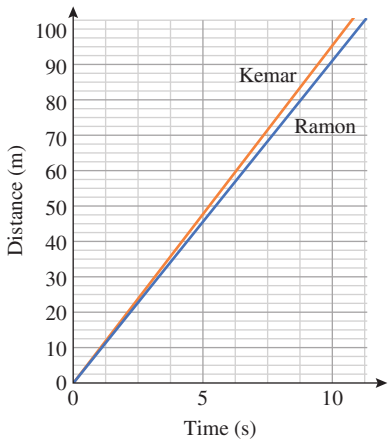
2 **a** B **b** C **c** A

3 **a** 6 km **b** 24 km/h
c 0 km **d** 0 km/h
e 2 km **f** 12 km/h

4

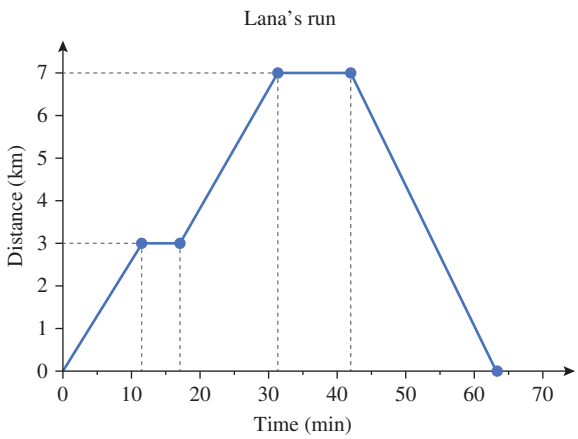


5 The hen walked 10 m in 20 s in the direction away from the coop. It stayed still for 20 s, then ran 30 m further away in 20 s, and then returned 40 m in 40 s to the coop.

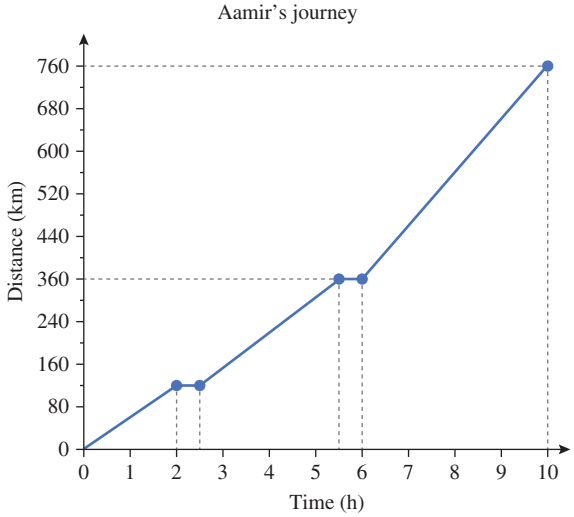
6 **a** *C* to *D* **b** *B* to *C* **c** *D* to *E*

7 **a** D **b** A **c** C

8



9



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10 prototype X
11 Chen drove 200 km in 3 h and then stopped for an hour. He drove 300 km in the next 4 h and 100 km in the last 2 h.
12 Check with your teacher.

Puzzle

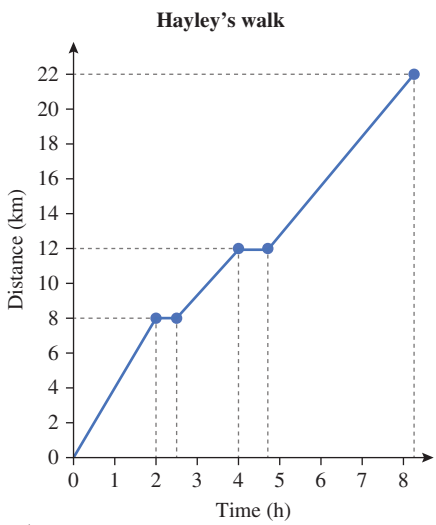
Koenigsegg Agera RS Sweden

Chapter checklist

1 8.75 m
2 **a** ~139 km **b** ~739 km
3 ~340 km Queenstown via Timaru to Christchurch
4 ~830 km Auckland to Hamilton to Napier to Gisborne to Rotorua to Auckland
5 m/s
6 0.53 m/s
7 72 km/h
8 962 m
9 4.25 h
10 5:30 p.m.
11 **a** ~4.2 h or 4 h 10 min
b ~$5184
12 **a** *AB* and *EF* **b** *BC*
c 1 m/s

Chapter review

1 **a** ~200 m **b** ~2.65 km
c ~2.5 km
2 $8
3 18 min
4 **a** ~1190 km
b ~1170 km
c The first route found in part **a** is short by about 20 km.
5 Approximately 4.7 km
6 7:15 a.m.
7 2.5 m/s
8 89.29 km/h
9 6.9 km
10 3:40 p.m.
11 12:04 p.m.
12 4.6 s
13 330 km
14



15 ~$9890
16 ~$370.91
17 Dan walks 2 km slowly for 1 h and has a 30-min rest. He then walks another 2 km at 2 km/h for 1 h and rests for an hour before returning home at the same pace.
18 **a** 9:11 a.m.
b ~$10 019.50

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